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A COMPARISON STUDY OF USING ORIGAMI AS A TEACHING TOOL IN MIDDLE-SCHOOL MATHEMATICS CLASS IN NORTH AMERICA AND CHINA

by

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ABSTRACT

This Major Paper compares origami-based mathematics school activities in North America and China. It introduces the current situation of the use of origami in mathematics classes to identify the similarities and differences of using origami as a teaching approach in these two regions. The Paper also attempts the analysis from various perspectives, including mathematics reform, mathematics learning system and environment, as well as the benefits of using origami in mathematics class, and how it relates to students' mathematics achievement. Both North American and Chinese mathematics educators' focus on integrating origami into middle schools' mathematics classes can be found in the study. Some same or similar origami activities are used in the two regions; however, origami in Chinese classes is more often used as an auxiliary teaching activity, while teachers in North America also teach origami itself such as modular origami.

Keywords: educational comparison, origami, mathematics education, China, North America

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Chapter 1: Introduction

Background

Mathematics is everywhere in our lives. As the base of science, mathematics is one of the essential subjects in every level of science education in both North America and China. This is especially true for the intermediate level of education; it is the key level because it expands on what students learned at the base level and also affects further mathematics learning. Furthermore, mathematics plays an important role every day in school life, and it is also taken seriously by educators, policy makers, students, and their parents. Mathematics educators are always seeking more effective ways to teach mathematics, and many governments have reformed mathematics education curricula in recent years. With the wave of reforms, more and more new mathematical education concepts have been proposed, and some have reconsidered how mathematics teachers teach and express mathematical knowledge. This has caused extensive discussions among teachers, who are the key to the success of any mathematics education reform movement, whether in America (Battista, 1994) or in China (Hu, 2004).

There is common trend in North American and Chinese mathematics education reforms, which is to transform mathematics classrooms from teacher-centred to student-centred, and emphasize students' learning of mathematics through exploring by themselves. Adding novel mathematical activities can help achieve this goal. It can effectively stimulate students' interest in learning and improve students' abilities to observe, question, analyze problems, solve problems, collect information and actively acquire knowledge. Therefore, the use of diverse mathematical activities in the mathematics classrooms is more and more common. Mathematics teachers from all around the world try their best to make mathematics lessons become more interesting. Elementary mathematics teachers in North America often use various mathematics activities in teaching mathematics. Thinking back to my elementary school experience as a student in China, I do not remember having many mathematics activities in my class. Most of the time, we used the instructional paradigm in which teachers are at the centre of the class and students just listen and do what they are asked. However, situation is changing because of the mathematics education reform in China. Nowadays, mathematics teachers put interesting activities in the mathematics classrooms and origami is one of them.

In China, origami has a long history. It has been used mainly in the development of children’s intelligence and their interests. Origami as a kind of manual activity as well as artistic activity is deeply loved by people because of its many possibilities. Origami is an activity that is full of imagination and creativity. People exercise both their hands and brains when they do origami. As such, origami affects one’s dexterity (i.e., hands), focus, patience, attention, and problem solving (i.e., brain). With the rich theory of foreign origami geometry (Geretschläger, 1995), origami has been widely used in many fields such as aerospace design, automotive design, architectural design and precision instrument manufacturing. In the field of mathematics education, origami is an important way to learn and explore mathematics. In North America, mathematics teachers also integrate origami into their teaching.

This paper is about a comparison of educational use of origami in middle school mathematics classes in the two regions. The educational research encourages people to conduct comparisons because of increasing globalization; however, the comparative education field is still developing. There are many definitions for comparative education, and one of the existing definitions states that,

> comparative education is a field that attempts to establish the world's education problems and, by considering the similarities and differences between these problems, examine them in the context of a country's values and conditions so as to develop proposed solutions that are unique to the country. (Dede & Baskan, 2011, p. 2)

It is known that the character and content of a country's educational system may be determined and shaped by a variety of internal and external factors (Dede & Baskan, 2011). For example, ethnic and religious structure, economic situation, and political and cultural parameters may all have impacts on shaping and directing an educational system. This situation has made the foundation for many debates in this field for more than half a century and has helped to develop new approaches to education management (Dede & Baskan, 2011). Therefore, generally speaking, comparative education is an area developed by examining the education systems of other countries (Kelly, Altbach & Arnove, as cited in Dede & Baskan, 2011). To investigate the differences and what causes these differences can be one of the simple definitions of comparative studies (Artigue & Winsløw, 2010).

The internationalization of mathematics education promotes collaboration and research. It involves perspectives and practices from other teaching contexts, which may be geographically distant. Based on that context, internationalization has led to reflections on the differences between international scholars' perspectives, practices, and teaching contexts through large-scale surveys. Given that mathematics is one of the three basic and common subjects of the school curriculum, it has figured prominently in these large-scale surveys (Phillips & Schweisfurth, 2014).

Educators can learn a lot from comparative education because different countries have different experiences, and different experiences can help share good ideas. Over the years, different countries have designed different solutions for common problems. Learning from comparing experiences is more effective than trying to determine the label of the policy maker's theoretical approach (Psacharopoulos, 1990). It can contribute to policymakers' and decision makers' better understanding of education systems. The findings from such research are valuable resources for the administration of education systems.

Over the past few decades, people have expressed interest in large mathematical performance-based international surveys using neutral tests, which may or may not take into account the changing focus of the participating education systems (Artigue & Winsløw, 2010). System comparisons are common in many academic research areas (Phillips & Schweisfurth, 2014). In Phillips and Schweisfurth's study, with the reforms of mathematics curriculum, both North American and Chinese educators made efforts to meet the new requirements of teaching. Traditional teaching ways were replaced by new teaching approaches. Educators in the two regions continue to try their best to change their classrooms and seek many ways to achieve the new teaching goals. Therefore, comparing the educational systems, mathematics teaching goals and mathematics learning environments in the two regions is necessary. This is the basis for comparing the use of origami in mathematics classes in North America and China.

Research Question

The aim of this Major Paper is to identify the similarities and differences of using origami as a teaching tool in mathematics class in North America and China. This question will be investigated from the following three perspectives:

1. The reasons for integrating origami into mathematics classes.
2. The current use of origami in mathematics classes in North America and China.
3. The applications of origami in the middle mathematics classes in North America and China.

Significance of the study

As two very different major educational regions, North America and China and their educational philosophies are important to compare. With the reforms of mathematics education in these two geographic areas, there are some shared beliefs about mathematics education, which is that students should be at the centre in the classroom and that they need to learn mathematics through hands-on learning. Origami is used by mathematics teachers in the two regions although origami originated in Asia.

There are various articles that describe use of origami in middle-school mathematics classes in both North America and China. Scholars have already addressed many aspects of origami teaching (in both English and Chinese literature), such as the relationship between origami and mathematics, and the application of origami in middle-school mathematics classrooms. Also research studies described trial use of origami in middle schools. However, the North American and Chinese literature on the topic are distinct. There appear to be no existing comparative studies in this field. Therefore, this study extends the existing research by comparing the similarities and differences of using origami in two areas. By doing so, mathematics teachers can learn from each other and also give further suggestions on origami-based mathematics classrooms.

Organization of the Study

The paper consists of three chapters in total. The first chapter provides general ideas of the study and background knowledge about origami in mathematics education at the middle-school level in three countries—China, the United States and Canada—and the reasons why this topic is significant. Comparative education is also mentioned in this chapter. The second chapter introduces the mathematics education systems and learning environments in the two regions—China and North America. In addition, several articles about origami and mathematics and mathematics education reforms in the two regions are presented in this part. This chapter also present some origami-based class examples at the middle-school level. Conclusions and implications in the last chapter complete this Paper, and also provide implications for further study.

Chapter 2: Literature Review

In this study, all literature sources consist of published academic articles, most of which are written within the recent 10 years. Because this is a comparison study between North America and China, some of the articles are in Chinese and they could be searched on the Chinese academic websites. The paper also borrows information from official documents for policy of mathematics education, and images are from official origami websites or mathematics textbooks in both North America and China.

Another notable aspect is that there are three phases in Chinese schools, namely elementary school (usually 1-6 grades), middle school (usually 7-9 grades), and high school (usually 10-12 grades); however, in North America, middle school is usually composed of students in grades 6-8. The classes of Grade 9 in Chinese schools are basically review classes for preparing Zhongkao test which is an entrance test for high schools. Therefore, middle school refers to grade six to grade eight in this study, as this study is conducted in Canada.

What is Origami?

The word 'origami' comes from Japanese. Origami is explained in the Merriam-Webster dictionary as "the Japanese art of process of folding squares of paper into representational shapes" (Merriam-Webster dictionary, n.d.). The history of origami followed the invention of paper, and it was a result of paper's use in society. Origami comes from Asian cultures and China also has Chinese-style origami; however, there is no agreement on the origin of origami, and the origin of origami is not the main problem discussed in this study.

Origami meets Mathematics

In the modern society, people usually think that origami is a fun activity for children. Children can make various shapes or objects through origami. For example, children can fold paper into a bird, a flower, etc. Origami as an art form is not only a fun activity for children. For some people, the word "origami" points to extremely complex organic and geometric shapes, which find applications in different domains, from designing safer airbags in cars to telescopes. Modern origami is closely related to science and can be perfectly integrated with mathematics and science. There are three interdisciplinary subjects that exist, namely: origami mathematics, computer origami, and origami engineering technology. *Origami mathematics* studies the mathematical principles in origami. *Computer origami* uses computers to seek new solutions and reasoning for problems caused by origami. *Origami engineering technology* is to use origami approach to solve engineering problems encountered in practical use (Dai, 2013). There is even a term "Origametria" which combines the words "origami" and "geometry." The Israeli Origami Center (IOC) created this word to describe their innovative program, which uses origami to teach geometry curriculum (Golan & Jackson, 2009). Origami indeed has many relations to science and mathematics. Tom Hull, a mathematician at Merrimack College in Andover says, "There's a particular appeal of origami to mathematicians. I tend to view origami as being latent mathematics" (Cipra, 2001, para. 2). There are even specific conferences dedicated to studies of origami, for example, the International Meeting of Origami, Science, Math, and Education (OSME). This peer-reviewed academic conference is the "quadrennial meeting on the intersections between origami, science, mathematics, and education (with additional ties to history, art, and design, among other fields)" (OSME Steering Committee, n.d., para. 1).

Origami is becoming more and more popular in the mathematics classrooms whether in North America or China. There are many articles that show how to integrate origami into mathematics classrooms. Many studies show that paper-folding as one of the arts can be an effective mathematics teaching tool (Boakes, 2008). It has been proved that there are many benefits of origami to mathematics classes. Paper folding as a type of hands-on activity can help students better understand mathematical concepts and increase students' ability to communicate mathematically (Cipoletti & Wilson, 2004). Different intelligences are combined in origami activity. Students begin to understand and experience some of the complexities and interconnections of aspects of life and perseverance when they practice origami (Sze, 2005). As a hands-on learning, origami helps students to learn important mathematical skills. According to Olson, when they fold and crease paper into different shapes and structures, they build skills involving spatial reasoning, following precise directions in sequence, fractions, geometry, and more (as cited in Sze, 2005, p. 1).

In addition, it is generally difficult for people to understand abstract things. Students also often experience difficulties with abstract mathematical concepts regardless of whether they have difficulties learning. Paper folding can relieve this problem. Teachers need to develop origami lessons that address mathematical goals; otherwise, origami could be nothing more than a cute activity (Georgeson, 2011). In the middle school curriculum, origami supports the natural development of students' spatial visualization, especially relevant for studying the content of geometry. The National Council of Teachers of Mathematics (1989) proposes using everyday objects, like paper, to help students to explore geometric relationships and vocabulary (as cited in Cipoletti & Wilson, 2004, p. 1). Origami is useful for teachers teaching geometry concepts because there is a large number of geometric properties implied during the origami process evident in the creases left on the paper. In addition, it teaches the skill to visualize two- and three-dimensional objects. The National Mathematical Standards are dedicated to developing these skills through practical experience, and origami is a commonly accepted method (Boakes, 2009). Students can be eventually guided to transfer from a two-dimensional to a three-dimensional product by folding and unfolding a piece of paper during constructing and deconstructing concepts (Sze, 2005). For example, in the curriculum area of geometry and measurement, students often mistake area for volume or have difficulty understanding how changes in one dimension affect measurements in another. To solve this problem, students may use origami to make paper models of cubes and explore relationships in the process. Students can benefit from making models of other polyhedral shapes through origami, which will lead them to further algebraic investigations (Georgeson, 2011).

Moreover, in the curriculum area of "space and graphics," China's new mathematics curriculum standards attach great importance to cultivating students' hands-on abilities. The standards also encourage for students to experience the formation and development of mathematics knowledge through hands-on explorations. This can be observed both in mathematics teaching materials and mathematics classroom teaching in primary and middle schools. Chinese teachers also began to use origami to explore the discovery and proof of some mathematical propositions (Chinese Ministry of Education, 2012).

Paper is an everyday object and activities with it can incorporate geometric relationships. Origami tasks can help students move from concrete to abstract reasoning. Not only geometry, but algebra is also too abstract to many middle school students.

Origami can also be applied in learning algebra. To connect algebra to something concrete and real can interest and motivate students to develop the concepts and strengthen learning processes (Georgeson, 2011).

Moreover, origami is not only an effective teaching tool for students from general settings but also for students in special education. Origami can benefit students who have learning disabilities. In special education, art therapy is considered a related service modality (IDEA as cited in Sze, p. 1). Many such students need to be taught by using the special education approaches, and art therapy can play a critical role for them (Sze, Murphy, & Smith, 2004). Origami as one of the methods of art therapy is an ancient method for healing (Sze, 2005). For example, students who are deaf or hard-of-hearing benefit from origami activities. There are articles that show that many deaf and hard-of-hearing students have significant delays in learning mathematics (Chen, 2005), as they are more likely to be concrete learners. Origami can be very useful for them in learning mathematics because many of them need to learn it through seeing and feeling (Chen, 2005).

Math anxiety. Students having mathematical anxiety is a common phenomenon. Students at all levels of education experience mathematics anxiety in today's educational systems (Finlayson, 2014). Mathematics anxiety can be defined in many ways because different people have different reactions to learning and doing mathematics. Richardson and Suinn state, "Mathematics anxiety involves feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (Richardson & Suinn, 1972, p. 551).

Mathematics anxiety can affect children's success greatly throughout their education and their adult life (Rossnan, 2006). Addressing and relieving mathematics anxiety is the primary problem that needs to be approached and solved. Several articles mention how to do it. There is a relationship between mathematics anxiety and teaching style of mathematics in the classroom (Finlayson, 2014). Many mathematics classrooms still follow traditional delivery methods with a focus on development of skills (Finlayson, 2014). Teachers need to create a positive learning climate for their mathematics classes and teachers should permit students to use manipulatives and encourage them to use a variety of games and activities (Chavez & Widmer, 1982). Rossnan (2006) mentions the suggestions from Cruikshank and Sheffield, who argue that teachers need to implement the following seven important pedagogical approaches to prevent their students' mathematics anxious behaviours. The teachers are advised to do the following:

- Show that they like mathematics
- Make mathematics enjoyable
- Show the use of mathematics in careers and everyday life
- Adapt instruction to students' interests
- Establish short-term and attainable goals
- Provide successful activities
- Use meaningful methods of teaching so that mathematics makes sense.

Rossnan also states that teachers should give more attention to classroom design in order to reduce mathematics anxiety in the classroom, and they should strive to create a space that encourages each individual's strengths and successes. Mathematics needs to be relevant to students' everyday lives. Also, students enjoy experimenting. All teachers are recommended to incorporate technology, encourage cooperative learning, and use mathematics manipulatives in their mathematics lessons for all students (Rossnan, 2006).

The common thing that mathematics education researchers mention is to use manipulatives and mathematical games or activities to make students feel that mathematics is not detached from their daily lives. Since origami-based activity needs only what is readily available—paper, as a hands-on activity, origami can be an effective method for mathematics teachers to reduce their students' mathematics anxiety.

Conclusion. In conclusion, although a form of art, origami is tightly connected with both science and mathematics. It is also a helpful teaching tool in mathematics classrooms. Origami started to be used in mathematics education in various ways and grade levels because of its benefits for mathematics instruction (Arslan & Işıksal-Bostan, 2016). Mathematics educators at all levels should know that origami provides a good context for understanding powerful mathematical concepts (Wares, 2013) and that students' conceptual understanding in mathematics could be enhanced within origami activity (Arslan & Işıksal-Bostan, 2016).

Mathematics Education

In both China and the North America, middle school belongs to the compulsory education level, which means that school-age children and adolescents must receive this level of education. The Chinese Compulsory Mathematics Education Curriculum Standards (Chinese Ministry of Education, 2012) are compiled in a book created by the Chinese Ministry of Education. This book points out the direction for Chinese mathematics education for grades 1-9, and every Chinese school and mathematics educator need to follow these standards. The 2012 edition highlights four mathematics curriculum goals:

1. **Knowledge and skills**. Students have the ability to obtain important mathematical knowledge (including mathematical facts, mathematical activity experience), basic

mathematical thinking methods and necessary application skills for adapting to future social life and further development.

2. **Mathematical thinking**. Students have the ability to observe and analyze the real society by using mathematical thinking, solving problems that exist in their daily lives and other subjects, and enhancing the awareness of applied mathematics;
3. **Problem solving**. Students have the ability to experience the close connection between mathematics, nature and human society, to understand the value of mathematics, to enhance the understanding of mathematics and build confidence in learning mathematics;
4. **Emotional attitudes**. Students have the ability to practice and have the spirit of innovation. In addition, their emotional attitudes and general abilities are fully developed in the learning process. (Chinese Ministry of Education, 2012)

To large extent, school mathematics derives its content from the needs that students may have in their daily lives and future schooling. Thus, in addition to teaching mathematic knowledge, the first goal, teachers should also help students connect mathematics to their daily lives so that students can apply mathematical knowledge whenever possible. This is consistent with the second goal: developing mathematics thinking skills. However, the reality is that many people cannot connect their mathematical knowledge to their daily lives. There is an established concern that many people are unable to apply the mathematics they learn in schools outside of schools (Boaler, 2002). The third goal, to enable problem solving, is the expectation to bridge the gap between mathematics and real-life applications. In addition, with the reform of the Chinese Mathematics Curriculum, much more attention has been focused on the improvement of students' problem-solving ability. Using mathematics activities is one of the most important core concepts in the reform of Chinese modern mathematics education. The concept of mathematical activities was proposed because of the wave of mathematics education reforms. The students' mathematics activities are carried out by the students under the guidance of the teacher, using physical objects, models, mathematical language, and mathematical thinking methods as the operational tools to accomplish certain mathematical tasks. Such a collection of activities engages students in observation, listening, speaking, doing, and thinking about mathematics (Zhong, 2009). Chinese Compulsory Mathematics Education Curriculum Standards (Chinese Ministry of Education, 2012) also point out that effective mathematics learning activities cannot only rely on imitating and remembering what teachers do. Students need to experience and explore by themselves by being engaged in mathematics learning activities. Collaborating and communicating are also important ways of learning mathematics. Students need to have sufficient time and space for participating in mathematics activities. They understand mathematics, solve mathematical problems, and learn the basic mathematical knowledge, as well as skills and methods, through hands-on experience and exploration (Chinese Ministry of Education, 2012).

Similarly, the National Council of Teachers of Mathematics (NCTM) in North America was founded in 1920, and it has grown to become the world's largest mathematics education organization, serving members of the United States and Canada. NCTM has published a series of mathematical standards that outline the school mathematics vision in the United States and have been highly influential in the direction of mathematics education in Canada as well. In 2000, NCTM released the updated *Principles and Standards for School Mathematics*, and it is widely considered to be a more balanced and less controversial vision of reform than its previous version. The reform in mathematics education advocates changes in content and pedagogy as understood from the NCTM standards. The reform in pedagogy views the learner as participating in the construction of knowledge actively, which leads to changes in teaching practice, such as the following:

- Active student involvement in discovering and constructing mathematical relationships, rather than merely memorizing procedures and following them by rote.
- The use of concrete materials, graphics calculators, graphs and tables, or other representations as a means to help students grasp abstract concepts.
- Group work, including students sharing and justifying their ideas.
- Student writing (including drawings, diagrams, charts) to encourage reflection on mathematical ideas, and oral presentation to promote communication of those ideas.
- The use of context, whether imaginary or real world, as a way to capture student interest in problems and as "a framework or structure upon which to secure concepts and study them" (Robinson & Robinson, as cited in Herrera & Owens, 2001, p. 89).
- Teacher as orchestrator of classroom discourse and facilitator of learning experiences. (Herrera & Owens, 2001).

Chinese Compulsory Mathematics Education Curriculum 2011 Version (Chinese Ministry of Education, 2012) proposes four goals and, like the reform in mathematics pedagogy from the NCTM standards, is the guidance for achieving the goals.

Another concept that can be explained by the principles proposed by the two regions and common among them, is the idea of visible learning in mathematics. The core of visible learning is about making learning visible to everyone, so that teachers and students can recognize that learning is actually occurring. Learning has three phases, namely surface, deep, and transfer. Surface learning is to explore new concepts through experiences, then to connect them to procedures, vocabulary, and symbolic representations. Deep learning is to make mathematical connections among ideas and to form generalizations. Transfer learning is to apply mathematical understanding to new contexts and situations and thinking metacognitively (Corwin, 2016). Mathematics teaching should address all the three phases of this learning process. While teaching, teachers are challenged to choose tasks that can build students' thinking skills, their procedural understanding of mathematics, and fluency of ideas, and their applications. Teachers have to think about the tasks that students need to work on to develop those kinds of understandings and skills. At the classroom level, students realize how it is that they can acquire knowledge rather than just wait passively to be taught by someone else (Corwin, 2016). Although learning can occur through both active and passive involvement, much of students' learning comes from activities and projects in which the students are active participants, rather than passive recipients of knowledge given by the teacher. Mathematics activities can be organized into language activities and operational activities (Huang, Li, & Lin, 2012). These two types of mathematics activities all have a close relationship with mathematical thinking activities. In mathematics classrooms, they complement and transform each other. They play an important role in understanding mathematical knowledge, mastering basic skills and improving mathematical thinking ability for students.

Classrooms settings and constructivism learning theory. *Chinese Compulsory Mathematics Education Curriculum 2011 Version, Interpretation,* states that students need to have sufficient time and space for mathematics activities (Chinese Ministry of Education, 2012). Classroom settings provide a basis for mathematics activities. Moreover, the mathematics classroom atmosphere is related to mathematics achievement. Creating a positive atmosphere is often a potential goal of school reform initiatives (Wang & Eccles, 2016). For example, the impact of classroom ability grouping on the classroom atmosphere varies with the size of the class and the differences of the mathematics ability of the students (Wang & Eccles, 2016).

There are not many articles that talk about mathematics classroom settings. However, based on my own experience, there are differences in classroom settings between North America and China. In North America, every teacher has their own classroom, which is also their office. Students go to different classrooms to have classes. Thereby teachers in North America can make a decision on how to set up their classrooms based on their teaching plans and their students' needs. Flexible classroom settings are a good opportunity for organizing mathematics activities in the classrooms. Teachers can establish different settings based on the different types of mathematics activities. In China, the general organization of each classroom is fixed and teachers move between classrooms. Students stay in one classroom for most of the subjects excluding arts, music, and physical education. Things are different in some Chinese schools, but most schools keep the traditional classroom setting in which the desks and chairs are arranged line by line. Based on my own observation, below are two examples of one of the types of North American classroom settings (Figure 1) and the typical Chinese classroom setting (Figure 2).

Figure 1: North American classroom setting

Figure 2: Typical Chinese classroom setting

Clark states that inclusion applies to all students, and their success depends on the teacher's teaching practices and differentiated teaching based on individual student learning needs (as cited in DeFrancesco, 2015). Flexible classroom settings can give students more opportunities for doing mathematics activities and exploring mathematics. Because of the different nature and function of the classrooms, generally speaking, North America has more flexible classroom settings than China which can provide teachers and students more opportunities to have mathematics activities.

Flexible mathematics classroom settings are one of the results of acceptance of constructivism learning theory (Jonassen & Rohrer-Murphy, 1999). It is employed as the supportive theory in the study. Phillips explains that constructivism can date back many decades as a learning theory (as cited in Richardson, 2003, p. 1); however, it only has been accepted in the 1990's as theory or practice (Richardson, 2003). Constructivism as a paradigm states that learning is an active, constructive process. The learner is an information constructor. The cognitive hypothesis of constructive learning is different from the cognitive hypothesis of traditional teaching. Therefore, classical requirements and task analysis methods are not suitable for designing a constructivist learning environment (Jonassen & Rohrer-Murphy, 1999).

Regarding mathematics education reform, there is a tendency all over the world toward constructivist classrooms. As cited in Draper in 2002, Grant and Noddings state that because of the newest wave of reforms, there is a demand for mathematics teachers to change from teaching by telling, common in the traditional mathematics classes, toward the constructivist teaching paradigm.

John Dewey also points out in his book *Experience and Education* (2007) that there are two different sides of education: traditional education and progressive education. It has been considered a big shift from traditional education to progressive education. Traditional education is dogmatic and static, because learners only learn from textbooks and elders. Such education emphasizes teacher-centred learning where the teachers are the centre of the classrooms, and they pass the knowledge to the students directly. In contrast, progressive education is more free education because it is a purposeful learning process. Leaners learn from experience and experiment in progressive education. Students are the focus of the whole learning process in student-centred learning (Dewey, 2007).

As a learning theory, constructivism provides a theoretical basis for helping mathematics teachers to reform classrooms. This transition is to a model in which the learner and the teacher participate together, solving problems, engaging in inquiry, and constructing knowledge (Draper, 2002). Teachers who use constructivist learning theory will provide students with the opportunity to construct their own knowledge in the environment, and students have the opportunity to learn within a constructivist framework through good teaching practice (Sze, 2005).

Culture and Mathematics

The world is diverse. Although there are various cultures in the world, and every culture has its own special characteristics, our world is still increasingly becoming interconnected. Because mathematics is a scientific subject, some people believe that mathematics can be independent of culture. Therefore, it should be taught globally using the same curriculum and pedagogy (Brandt & Chernoff, 2015). Many think mathematics itself will not be influenced by cultural it factors and that it provides definite unchanging answers. It is assumed to be unlike arts or history subjects, which are influenced by cultural backgrounds. Conversely, other people view mathematics as a human activity that is tightly related to culture. As such, it can be greatly enriched by intellectual diversity in curriculum and pedagogy (Brandt & Chernoff, 2015). In fact, mathematics, like all other subject areas, is embedded in cultural contexts. In the past decade, more and more literature has dealt with the relationship between culture and mathematics, and described mathematics in culture (Barton, 1996). Cultural background frames the way we look at the world. It pervades the whole of who we are and strongly influences our perspectives. Each culture develops ways of doing things, styles and techniques, as well as responses to seeking explanations and seeking understanding and learning, or basically the ways and reasons for how and why we do things (D'Ambrosio, 2016).

The word "ethnomathematics" has been proposed in recent years. The definition of ethnomathematics in Oxford dictionary is "mathematical concepts and activities as existing in various cultural groups (especially non-literate ones); the study of this" ("Ethnomathematics," n.d.). Ethnomathematics which is culturally-based mathematics, should be (further) integrated into the mathematics classroom (Brandt & Chernoff, 2015). Indeed, in mathematics education, mathematics educators can combine their special cultural backgrounds in their mathematics classrooms. Teachers recognize that students are most motivated when they actively participate in their own learning and deal with the issues that they and their community are most concerned about (Zaslavsky, 1996). Western culture believes that Europe is the origin of modern mathematics, but many mathematical ideas actually began in Africa and Asia (Wiest, 2002). Origami is one of these examples. Origami comes from Asia. Teachers introduce cultural applications of origami into mathematics classrooms as a simple and interesting activity that Chinese students have been exposed to since kindergarten. Integrating origami into mathematics classrooms can motivate Chinese students because they deal with the mathematical issues in the way their community is concerned about. Origami needs students to do it themselves, and students can feel the fun of this activity in person. Although the origin is in Asia, students in North America participate in their mathematics learning using hands-on practice. Mikhail Gromov said “I would not only focus on mathematics but on science and art and whatever can promote creative activity in young people” (as cited in Raussen & Skau, 2010, p. 403). Students can explore mathematics through the process of doing origami, which can cultivate their creativity. There are writings that attempt to show that mathematics education can be more effective if examples are obtained from culturally-specific contexts, in particular, the relationship between cultural group thinking processes and mathematics education (Barton, 1996).

Teachers need to selectively choose classroom resources and supplement materials with additional resources to make the classroom full of multicultural mathematics, and they can also review the resources to improve their ability to teach mathematics from a multicultural perspective and broaden their ability to incorporate instructional ideas and activities (Wiest, 2002). Instructional materials such as textbooks, worksheets, and posters should reflect the rich diversity of people who populate the world. Pictures, names, and content included in mathematics materials should portray a wide range of people and lifestyles (Wiest, 2002). Origami activity can be shown in both North American schools’ mathematics textbooks and Chinese schools’ mathematics textbooks. Here are two of the examples: origami activity in North American mathematics textbook (Figure 3) and origami activity in Chinese mathematics textbook (Figure 4).

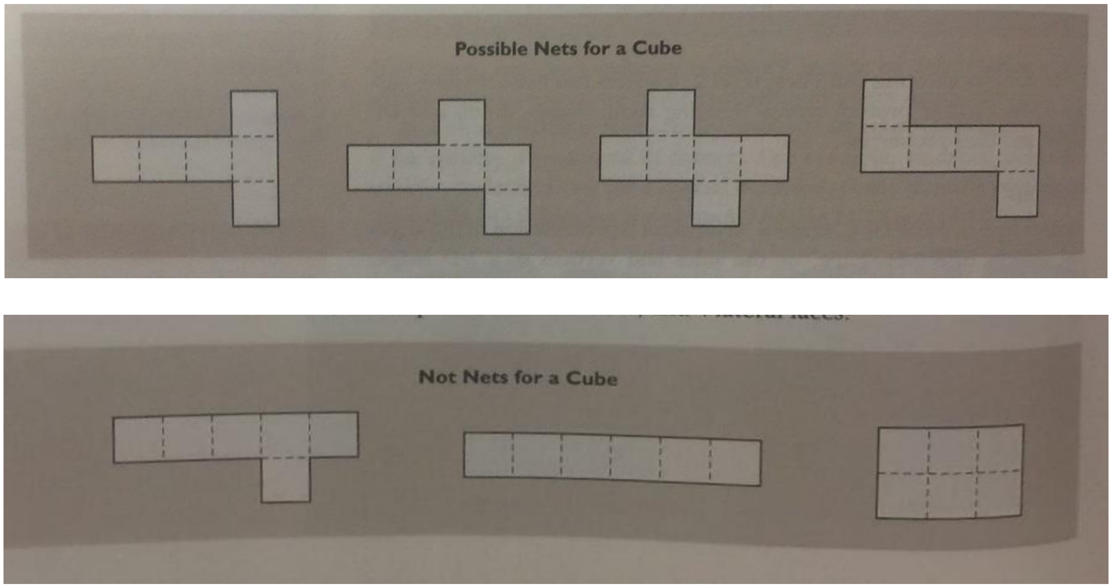
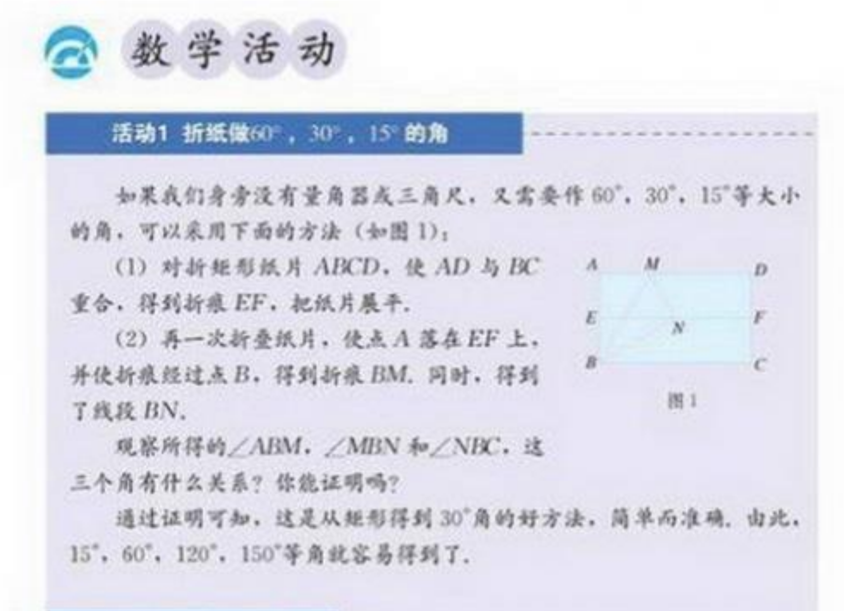


Figure 3: One of the origami activities in North America schools' mathematics textbook

Images from *Making Math Meaningful To Canadian Students, K-8* (Small, 2013)



数学活动

活动1 折纸做60°，30°，15°的角

如果我们身旁没有量角器或三角尺，又需要作60°，30°，15°等大小的角，可以采用下面的方法（如图1）：

（1）对折矩形纸片ABCD，使AD与BC重合，得到折痕EF，把纸片展平.

（2）再一次折叠纸片，使点A落在EF上，并使折痕经过点B，得到折痕BM. 同时，得到了线段BN.

观察所得的∠ABM，∠MBN和∠NBC，这三个角有什么关系？你能证明吗？

通过证明可知，这是从矩形得到30°角的好方法，简单而准确. 由此，15°，60°，120°，150°等角就容易得到了.

Figure 4: One of the origami activities in Chinese schools' mathematics textbook

Images from *Chinese mathematics textbook for the eighth grade (Renjiao Version)*

(Chinese Ministry of Education, 2013)

Chinese in the Figure 4 is translated into English:

> If we want to get the angles of 60 degrees, 30 degrees, 15 degrees, etc. without protractors or triangle rulers, we can use the following method (pictured):
>
> (1) Folding the rectangular paper ABCD in half so that AD and BC coincide. EF is the crease. Flattening the paper.
>
> (2) Folding the paper again so that the point A falls on the EF, and the crease passes through the point B. The crease BM appears. At the same time, the line segment BN is obtained.
>
> What is the relationships between the angles <ABM, <MBN and <NBC by observing? Can you prove it?
>
> Through proving we can know that it is an easy and accurate method to get the angle of 30 degrees from rectangles. Therefore, we can easily get the angles of 15 degrees, 60 degrees, 120 degrees and 150 degrees, etc.

Mathematics classrooms should reflect and embrace the cultural diversity in the teaching and learning of mathematics. As Wiest states:

> Exposing students to the contributions of members of their own and other cultures can help them gain confidence, self-esteem, and a sense of belonging, as well as respect for the mathematical thinking of all cultures. Teaching mathematics from a multicultural perspective can also help shake students' fundamental beliefs about mathematics itself. Students may come to see mathematics as a human endeavor done by real people to serve real needs and interests. (Wiest, 2002, p. 53)

If mathematics is related to culture, school mathematics classes should be more inclusive, so that students develop a greater interest in mathematics through culturally-based activities. If students grow their interests in mathematics, they have opportunity to see the mathematics extended beyond the classrooms, and that it has real importance in the ‘real’ world (Brandt & Chernoff, 2015). Although origami comes from Asia, teachers using it in North America can increase students’ interests in learning. In addition, students will know other countries’ cultures and engage with ideas from other unique cultures of the world.

Therefore, as an effective teaching tool in mathematics class, students have the opportunity to discover individual differences and commonalities in the learning styles in Western and Eastern cultures through origami construction (National Council of Teachers of Mathematics, 1975). Students can explore different perspectives, examine stereotypes, develop global awareness and celebrate the diversity in their own classrooms through a multiple intelligences approach (Reigeluth, as cited in Susan, 2005, p. 1).

Surveys on origami-based mathematics classes and examples

Many academic articles can be found on integrating origami into middle schools’ mathematics classes both in North America and China, and the focus on origami’s integration does not only stay on the theoretical level. Mathematics educators are encouraged to use origami in their actual mathematics teaching. After reading several teaching/lesson plans, two types of origami mathematics activities can be found in the two regions. One is to integrate origami into other mathematical knowledge: in another word, the process of folding paper is helpful for better understanding other mathematical concepts. This can be found in both the North American and Chinese middle schools’ mathematics teaching plans. For example, there is a teaching article in Chinese that introduces putting paper folding into lessons about “axisymmetric shapes” (i.e., those that have axes of symmetry). In the practical teaching of axisymmetric shapes, teachers can divide students into activity groups and guide each group to make axisymmetric graphic papers of different shapes, such as isosceles triangles, squares, rectangles, circles, isosceles trapezoids, etc., so that the students can feel the relationship between the two sides of the shape in this hands-on operation. Through the activity, students can realize that if a figure can be folded along a straight line and the parts on both sides of the line coincide with each other, then this shape is called an axisymmetric shape. Furthermore, students can make two of the same shapes that also meet the definition of axisymmetric. Students experiment by themselves to find if a graph is symmetrical about an axis with another graph.

This activity will enable students to have a clearer understanding of the definition. In this lesson, students are prone to have problems such as unclear definition of axisymmetric shape and being unable to accurately distinguish which figures are axisymmetric. Those problems will also have a certain impact on the later learning of isosceles triangle knowledge. In the past ways of teaching, teachers may have used pictures of the symmetrical shapes as examples, and students seldom had opportunities to practice by themselves. According to the interesting principle of culture-based teaching of mathematics, and the implementation of mathematics games into the classroom teaching strategy, the introduction of origami games into the teaching can enable students to better grasp the difference between the concept, nature and symmetry with respect to lines of axisymmetric figures (Li & Li, 2011).

Another example can be seen in a North American mathematics teaching plan. The teaching plan shows use of origami in teaching exponential functions. This activity is usually used at the beginning of the class for students folding papers so that they can get know to exponential functions' growth trends by analyzing the data. Below are the instructions:

1. Fold a sheet of white paper in half.
2. Unfold the sheet of paper and count the number of sections.
3. Refold and add a new fold.
4. Continue to fold and count the sections after you make each new fold.
5. Record the data in the table. (Ontario Association for Mathematics Education, 2008)

By introducing the class through this paper folding activity, students can get the function y=2^x which is one of the types of exponential functions. When they fold once, they can get two sections; When they fold twice, they can get four sections; When they fold the third times, they can get eight sections; And so on. In this example, the number of folds is x and the number of sections is y. Therefore, students can conclude the function y=2^x through the data. This activity is useful for deriving the definition of an exponential function. Origami is a helpful and practical way for students to understand other mathematical concepts. It is relevant to students' lives, and it can motivate students to learn mathematics and improve their innovative ability. The above two examples show the way of using origami as an auxiliary teaching activity. The other type of origami mathematics activities use origami as the whole mathematics class. Modular origami is one of them.

Compared to Chinese teachers use of traditional origami ways, which is to use a single sheet of paper to fold a figure, teachers in North America also construct modular origami. Modular origami is assembled by many sheets of paper to create a more complex geometric form (Zimmerman, 2016). Figure 5 shows some examples of modular origami.

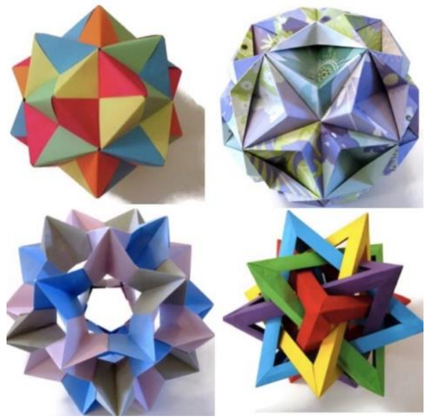


Figure 5. Examples of modular origami (Artful Math, 2016).

Different from integrating paper folding into other mathematics content, modular origami is more like learning mathematics through origami itself. Boakes (2008) mentions it in the steps to create origami-mathematics lessons that are designed around the model to be folded. Teachers should consider what the purpose of the model will be through searching and seeking out an origami model that is appropriate for their students' age and grade. Boakes suggests the steps:

1. Seek out an Origami publication that fits you and your students' needs. (i.e., level of difficulty, illustration quality)
2. Consider what mathematics concepts and/or vocabulary you wish to highlight as you choose your Origami model.
3. Go through the folding process of your Origami model. While doing so list vocabulary that matches with your goal set.
4. Fold the model again. At each step stop and write your teacher questions being mindful of your vocabulary list.
5. Answer your own teacher questions and record the kind of answers you expect to hear. (Boakes, 2008, p. 6)

Baicker also offers some instructions for teachers on teaching with origami:

- Try the activity on your own ahead of time to anticipate any areas of difficulty that students may encounter. Your completed activity provides a model for them.
- Think through the math concepts you want to highlight.
- Support students who need more help with following directions or with manipulating spatial relationships by marking landmarks on the paper with a pencil as you go around the classroom. You can make a dot at the point where two corners should meet, for example.
- Arrange the class in clusters and let students who have completed one fold assist other students. This will foster cooperative learning and help you address all students´ questions.
- Encourage students to make a soft fold and check that the edges line up properly to avoid overlapping. After they make adjustments, they can make a sharper crease using their fingernails.
- Have students unfold their origami projects to look at the interesting patterns and geometric figures they have created through their series of creases. Challenge them to create their own variations-and make their own diagrams showing how they did it. (Baicker, n.d.).

These methods of teaching origami-based mathematics classes demonstrate the advantages students derived from learning origami.

Teaching origami in mathematics classes even exists in the Chinese Zhongkao test, which is an entrance examination for senior high schools. It is an important test because it can decide which senior high school students can attend. Below are two examples of origami questions in the Zhongkao mathematics test. The first question is, as shown in the Figure 6 below: “in the rectangle ABCD, AB is equal to 3, BC is equal to 4, point E is a point on the side of BC, connect AE, and fold angle B along AE, so that point B falls at point B’. What is the length of BE when triangle CEB’ is a right triangle?”

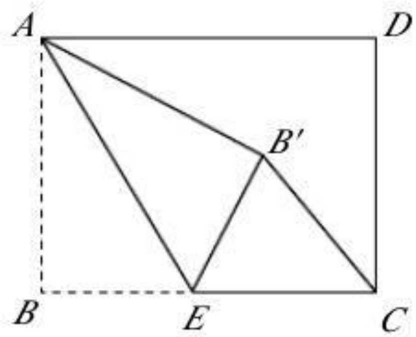


Figure 6. A folding rectangle task from 2013 Zhongkao Math test paper in Chinese Henan Province.

The other example is as shown in the Figure 7 below; it is a net of a cube with a number on each face. Students are asked to find the number on the face opposite the number 2 in the folded cube.

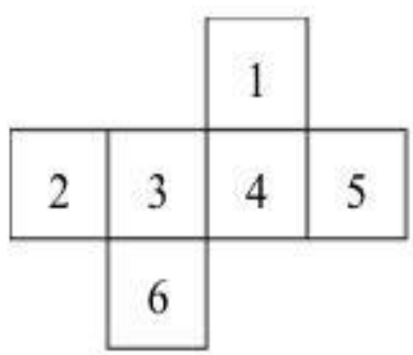


Figure 7. A cube net task from 2013 Zhongkao Math test paper in Chinese Henan Province.

This question is similar to the origami activity appearing in North American schools’ mathematics book *Making Math Meaningful to Canadian Students, K-8* (Small, 2013). The book gives some examples of possible nets for a cube that can or cannot be folded to create a top face, a bottom face, and four lateral faces.

There is a statement in the textbook *Making Math Meaningful to Canadian Students, K-8:*

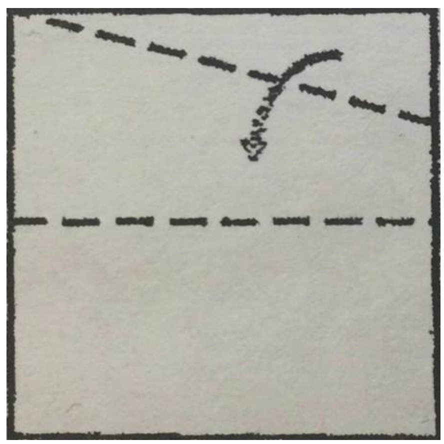
> A net is a 2-D representation of a 3-D shape that can be folded to re-create the shape. When students make nets, they focus particularly on the faces, and how the faces fit together to reform the shape. It is important for students to realize that there are often many different nets for a single shape. Even though the faces do not change, they can be connected in different ways. In creating the net for a cube, students work with the following properties of a cube: 6 congruent square faces, 3 pairs of opposite parallel faces, 3 faces joining at each vertex, and congruent edges that meet at right angles. However, students cannot assume that because a cube has 6 square faces, any grouping of 6 squares will create a net (Small, 2013, p. 360).

NCTM points out that various tools are recommended; the curriculum standards specifically support students' participation in the activities where they can switch from two-dimensional shapes to three-dimensional shapes, use two-dimensional shapes to make three-dimensional shapes, and deconstruct three-dimensional shapes into their two-dimensional equivalents (NCTM, 2000). The exercise of nets for a cube is one of the activities, and it can improve students' spatial ability. The activities in which students are building a three-dimensional model, drawing two-dimensional shapes, or working with manipulatives can enhance their spatial skills, which are important for students to develop when they study mathematics (Boakes, 2008).

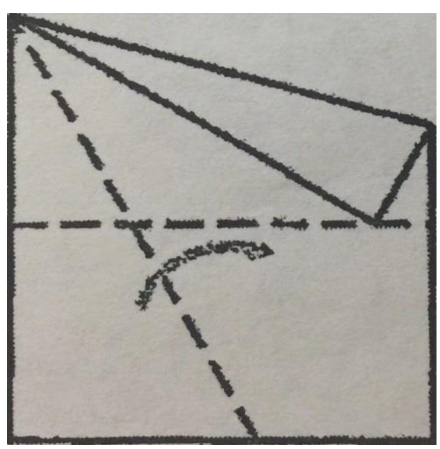
There is yet another origami example for determining angles that can be found in North American mathematics teachers' training program. The literature review has discussed one that appears in the Chinese mathematics textbook, which is for obtaining the angles of 60 degrees, 30 degrees, 15 degrees, etc. without protractors. This one is similar to that one but uses different paper shapes. Below are the steps:

Step 1: Start with a square piece of paper. Fold it in half horizontally.

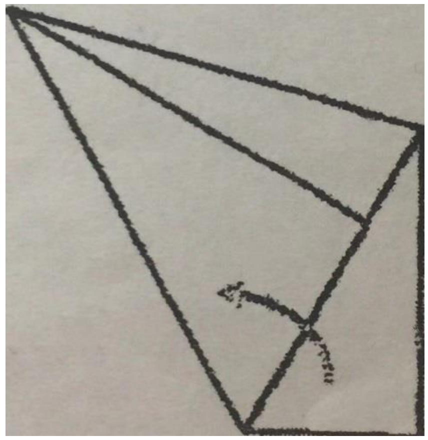
Step 2: Fold the top right vertex down to the horizontal crease line so that the new crease passes through the upper left vertex.



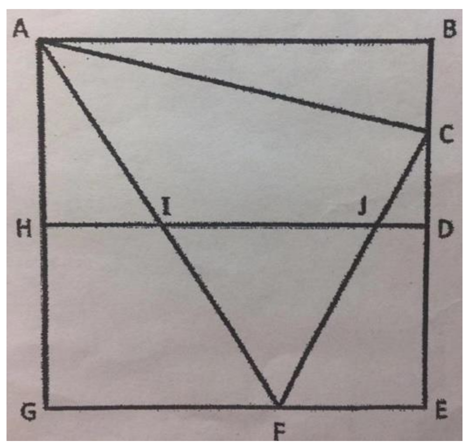
Step 3: Fold the bottom left vertex up to the horizontal crease line so that the new crease also passes through the upper left vertex.



Step 4: Fold along the edge created by the two folded triangles. The lower right vertex should come up and over the triangles.



Label each of the points to obtain the diagram as shown below:



Now, using logic and knowledge of angle relationships to determine all the unknown angles without using a protractor and explain the reasons. In this case, because of the repeated folding, the angle EFC, the angle CFA, and the angle AFG are all equal to 60 degrees. Then, according to this clue, the other angle can be obtained easily. Therefore, 60 degrees, 30 degrees, 15 degrees, etc. can be obtained through folding papers in the above way.

In addition to how to apply origami into mathematics classes, researchers in the two regions also focus on origami's influence on students. For investigating origami's impact on a group of students, there are research teams both in North America and China that set up origami trial classes to survey the relation between using origami and students' achievement. There are many articles about surveys like this, but here I only mention two of the research studies in North America and China just as examples.

Huang, Li, and Lin's (2009) research team set up a program called "Origami and Mathematics Thinking" in the middle school affiliated with Southwest University in China, in September 2009. They set up a specific activity room for the program. Each semester they selected 30 students to participate in this program through the voluntary registration method, and they carried out inquiry activities with the theme of "origami and mathematical thinking" two times per week at a fixed time. The activity was conducted in three phases each time. The first phase was called "Folding." It was mainly organized as a demonstration from a teacher, which students were expected to follow. The second phase was called "Thinking." The teacher asked questions according to the first phase of the origami operation, and the students answered the questions. In the final phase, on the basis of the basic grasp of the folding method and its principles, the teacher asked questions, and the students did the origami folding and answered questions. In the experiment, the researchers found that the students who were stronger in mathematical origami activities had better mathematical achievement (Huang et al., 2009).

Boakes (2009) did a study that focused on origami's impact as a teaching tool in the American middle school mathematics classroom. His team conducted the study in a suburban middle school in southern New Jersey. They investigated the spatial visualization abilities and mathematical achievement of seventh-grade students taught by two different methods of instruction: a traditional teaching-based and an origami-based. There were 31 students as a control group who received traditional instruction over the course of a month-long geometry unit. Meanwhile, 25 students as a treatment group received traditional instruction with a collection of 12 origami-based mathematics lessons. The two groups were taught by the same classroom teacher. About 20 minutes was taken from the normal 80-minute class held daily for sessions of origami-mathematics lessons. The study implied that the origami-based instruction method is as beneficial as traditional instruction in the mathematics classroom, and it stands as an acceptable tool for improving children's spatial skills and geometry knowledge (Boakes, 2009).

The results show that there is positive influence on students in the two regions through origami mathematics classes. The above survey and other surveys, which are not mentioned in this paper, also reveal the positive influence. Students in Huang et al.'s (2009) research project had better mathematical achievement. Boakes' (2008) students who had done origami-mathematics on a regular basis stated that they enjoyed the experience, and it not only helped them understand the geometry they learned but also the other areas of mathematics. Students used the words: "helpful, exciting, interesting, easy, joyful, good job, enjoyable, awesome, fantastic, fine, and fun-filled," to describe their experience (Boakes, 2008, p.7). Students also commented:

> "I thought making the angles and discussing the lines of symmetry and such was a great and fun way of learning. Bringing all math topics into paper folding should be an actual activity in everyday math."
>
> "I thought this was so cool to do Origami. It was fun and educational" (Boakes, 2008, p. 7).

Generally speaking, Boakes students' responses were extremely positive.

Summary

Based on calls from mathematics reforms, teachers need to change the way they teach. Mathematics educators are constantly finding more effective ways to teach mathematics. Adding mathematics activities is one of them. Origami, as a hands-on learning mathematics activity, is popular in mathematics classrooms. Integrating origami into the mathematics cannot only relieve the concerns regarding the separation of mathematics and life, but also can cultivate students' mathematical skills. The creases generated during the origami process can demonstrate a large number of geometric properties. The process of doing origami is full of imagination, creativity and uses of both the hands and brain. Mathematical origami activities can develop students' mathematical thinking ability. Origami activities also create opportunities for communication and produce aesthetically pleasing objects to share with others while providing an opportunity to acquire knowledge that bridges diverse cultures.

Chapter 3: Conclusions and Implications

According to the origami examples which are shown in the literature review chapter, there are two usual ways mathematics teachers using origami in their middle mathematics class. One is the way of using origami as an auxiliary teaching activity, and the other type of origami mathematics activities use origami as the whole mathematics class such as modular origami.

Teachers in North America use modular origami more than Chinese teachers. Also, it is more common in North American origami mathematics classes to use models than in Chinese origami mathematics classes. One of the potential reasons behind the difference between North America and China may be due to mathematics class duration. Mathematics class in Chinese middle schools are usually 45-minutes in duration, and students typically have five or six of these classes per week. North America students typically spend as much time in mathematics class each week; however, they have fewer classes and longer duration. For example, the Ontario Ministry of Education (2016) prescribes math classes that last 60 minutes long based on research that suggests “extended blocks of time for mathematics learning” (p. 1). Although the specific situation varies in each school, this may be one of the potential reasons that there is more time to do detailed origami in North American middle school mathematics classes.

Potential limitations exist in this study. The study focuses on typical pedagogical approaches in North America and China. Although the examples come from the textbooks, the conclusions cannot be generalized for the whole regions or all teachers/schools. There are regional differences that exist even in the same country, and different schools have different school cultures. Rules on teaching vary. Taking classroom settings for example, some Chinese schools are changing in recent years. Some Chinese schools, especially international schools in China, have various classrooms settings so that they can adapt to different types of classes. In North America, because teachers can decide how to set up their classrooms, some North American teachers set up the classrooms desks in rows, which is like the traditional Chinese way. Therefore, the paper is based on the typical situation.

Mathematics is a scientific subject, and should be connected and adapted to society. Mathematics education reforms are proposed to solve the gap between mathematics and real lives. The innovation of mathematics education should go with the changes of the society, and it also provides opportunities for mathematics educators to change the way they teach.

Because origami is both fun and has applications to science and mathematics, it can be used to enhance mathematics learning, and there is a long tradition of using such approaches (Wares, 2013). There are various benefits to putting origami and mathematics together that are promoted by the mathematics reform movement. For example, it provides an engaging context in which teachers can introduce conceptual understanding of mathematical ideas and gives students hands-on learning experiences through which they can visualize abstract mathematical ideas in a concrete way (Wares, 2019). In addition, proper origami models not only help students explore crucial mathematical ideas but also introduce them to various cultural activities (Wares, 2013).

The purpose of this Major Paper is to compare use of origami in middle schools' mathematics classrooms in North America and China. Both mathematics educators in North America and China implement origami into mathematics classes, and many academic articles on this topic can be found in English and Chinese. They all do sample origami-based mathematics lessons and trial classes to study the impact on students. In addition, they all apply origami-based education theories into their practical mathematics classes in middle schools. Some same or similar origami activities are used by the two regions; however, Chinese classes more often use origami to put forward studying of other mathematical knowledge while teachers in North America also study origami itself in their classes. Different educational systems may lead to this difference. Another common thing is that the two regions include origami in their middle school mathematics textbooks, and there are even origami questions in Chinese provincial tests. That content is also mentioned in North American textbooks. It can be seen that both regions attach great importance to the application of origami in mathematics teaching, and the background is the same: The mathematics reform movement is consistent with the general educational theory called constructivism (Meyer & Meyer, 1999), as discussed in the literature review part.

Doing origami does not need complex equipment, it only needs ordinary paper. As a simple hands-on activity, origami is welcomed by mathematics teachers all over the world. Although Asia is the origin of origami, its cultural influences have spread into the North America. Origami offers a potential bridge between primary and secondary mathematics as an accessible practical activity (Pope, 2002), and there are already many articles to show it.

There are many things that can be studied in the comparison on how to use origami in middle school's mathematics classes. This study focuses on comparing the ways of teaching origami-based mathematics classes, gives specific examples on integrating origami into mathematical knowledge and examples of academic research in the two regions. Origami as a teaching tool in mathematics classes still has much potential. How to improve middle schools' origami-based mathematics classes, and how origami positively contributes to mathematics achievement can be further studied.

In addition to middle schools, origami is also widely applied in primary schools, secondary schools, and post-secondary schools. It is easy to create an origami-mathematics lesson that teachers of any grade level can do (Boakes, 2008). How other levels of education use origami in mathematics classrooms can be another interesting topic in the field. Moreover, though origami was originally from Asia, it is practiced all over the world in countries such as the United States, England, France, Germany, Belgium, Argentina, Singapore, Australia, and Italy (Lang, 2011), which demonstrates the utility of origami as a mathematics teaching tool in a global context. Although this study only focuses on North America and China, origami is used in mathematics classrooms around the world and its varied applications warrant further study. Thus, comparative studies can promote educational cooperation between countries by examining their unique applications of origami in mathematics classrooms and determining the benefits of origami as a teaching tool in these contexts.

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