3-15-2013

A multi-objective facility location model for closed-loop supply chain network under uncertain demand and return

Saman Hassanzadeh Amin
University of Windsor

Guoqing Zhang
University of Windsor

Recommended Citation

Amin, Saman Hassanzadeh and Zhang, Guoqing. (2013). A multi-objective facility location model for closed-loop supply chain network under uncertain demand and return. *Applied Mathematical Modelling*, 37 (6), 4165-4176.
https://scholar.uwindsor.ca/mechanicalengpub/12

A multi-objective facility location model for closed-loop supply chain network under uncertain demand and return

Saman Hassanzadeh Amin, Guoqing Zhang *

Department of Industrial and Manufacturing Systems Engineering,
University of Windsor, Windsor, ON, Canada N9B 3P4

* Corresponding author, Tel: 519-253-3000 Ext. 2637, Fax: 519-973-7062
Email addresses: gzhang@uwindsor.ca (G. Zhang); hassanzs@uwindsor.ca (S. H. Amin)

A multi-objective facility location model for closed-loop supply chain network under uncertain demand and return

A closed-loop supply chain (CLSC) network consists of both forward and reverse supply chains. In this paper, a CLSC network is investigated which includes multiple plants, collection centres, demand markets, and products. To this aim, a mixed-integer linear programming model is proposed that minimizes the total cost. Besides, two test problems are examined. The model is extended to consider environmental factors by weighed sums and ε-constraint methods. In addition, we investigate the impact of demand and return uncertainties on the network configuration by stochastic programming (scenario-based). Computational results show that the model can handle demand and return uncertainties, simultaneously.

Keywords: Reverse logistics (RL); Closed-loop supply chain (CLSC); Mixed-integer linear programming (MILP); Multi-objective programming; Stochastic programming

1. Introduction

Supply chain management (SCM) has received a lot of attentions. There are two types of supply chains: forward and reverse supply chains. The forward supply chain (FSC) contains of series of activities which result in the conversion of raw materials to finished products. Managers try to improve forward supply chain performances in areas such as demand management, procurement, and order fulfilment [1, 2]. Reverse supply chain (RSC) is defined as the activities of the collection and recovery of product returns in SCM. Economic features, government directions, and customer pressure are three aspects of reverse logistics [3]. The integration of a forward supply chain and a reverse supply chain results in a closed-loop supply chain (CLSC) [4]. In other words, there are both forward and reverse channels in CLSC networks.

Several investigations have been done about forward facility location models. Facility location models try to answer the following questions: How many facilities should be open? Where each facility should be located? What is the allocation? Which set of collection centres should be opened and operated? What products should be processed in these open facilities? Some authors have examined facility location models for closed-loop supply chain networks (such as [5]). The objective of these models is to determine decision variables of both forward and reverse channels. Minimization of total cost is considered as main objective function. A minority of authors not only considered the total cost, but also they took into account other factors by multi-objective models (such as [6]). On the other hand, some researchers investigated uncertainty in CLSC configuration (for instance [7]). Uncertainties in supply and demand are two major sources of vagueness in SCM. Uncertainty in supply is appeared because of the mistakes or delays in the supplier's deliveries. Demand uncertainty is defined as inexact forecasting demands or as volatility demands [8, 9, 10]. Uncertain return is another important source of ambiguity in reverse logistics. To our knowledge, most of authors have not taken into account multi-objective closed-loop supply chain models under uncertainty. Thus, it is valuable to examine integrated models including multi-objective models with uncertain parameters.

In this paper, a facility location model is proposed for a general closed-loop supply chain network. The model is designed for multiple plants (manufacturing and remanufacturing), demand markets, collection centres, and products. The goal is to know how many and which plants and collection centres should be open, and which products and in which quantities should be stock in them. The objective function minimizes the total cost. In this paper, two test problems are examined. In addition, the model is developed to multi-objective by considering environmental factors including environmental friendly materials and clean technology. Then, the model is solved by two methods including weighted sums and ε-constraint methods. Furthermore, trade-off surfaces of test problems are examined. The multi-objective model also is extended by stochastic programming (scenario-based) to examine the effects of uncertain demand and return on the network configuration. Finally, computational results are discussed and analysed. This research is among the first investigations that consider multi-objective mathematical models under uncertainty in CLSC network configuration.

The organization of this paper is as follows. Literature review is discussed in Section 2. In Section 3, a general network is described. In Section 4, the mathematical model is provided. Then, two test problems are presented in Section 5. An extension to multi-objective programming is provided in Section 6. In addition, the model is developed by stochastic programming in Section 7. Finally, conclusions are discussed in Section 8.

2. Literature review

Jayaraman et al. [11] presented a mixed-integer linear programming model to determine optimal quantities of remanufactured products and used parts in a reverse supply chain network. Fleischmann et al. [5] extended a forward logistics model to a reverse logistics system and discussed the differences. They utilized mixed-integer linear programming model. Kannan et al. [12] proposed a model using genetic algorithm and particle swarm techniques. They applied the model by considering two cases including a tyre manufacturer and a plastic goods manufacturer. Kannan et al. [13] developed a mathematical model for a case of battery recycling. However, they did not consider uncertainty of parameters. Amin and Zhang [14] designed a network based on product life cycle. They utilized mixed-integer linear programming to configure the network. Fleischmann et al. [15], Rubio et al. [16], Guide and Van Wassenhove [4], and Akcali and Cetinkaya [17] provided literature review and survey for the papers of RL and CLSC.

Multi-objective and goal programming models have been developed by some authors for CLSC networks. Some of the papers have been categorized in Table 1. Krikke et al. [18] considered minimization of the supply chain costs, energy use, and residual waste of a closed-loop supply chain network. Pati et al. [19] formulated a mixed-integer goal programming model to determine the facility location, route and flow of different varieties of recyclable wastepaper CLSC network. They examined minimization of the reverse logistics cost, maximization of the product quality improvement, and environmental benefits. Du and Evans [20] developed a bi-objective model for a reverse logistics network by considering minimization of the overall costs, and the total tardiness of cycle time. Gupta and Evans [21] proposed a non-preemptive goal programming approach to model a closed-loop supply chain network. Pishvaee et al. [22] considered minimization of the total costs, and maximization of the responsiveness of a logistics network.

Some authors have examined uncertainty in CLSC network configuration. Table 1 shows the summary of the articles. Salema et al. [7] extended the reverse logistics model of Fleischmann et al. [5] and took into account uncertainty in demand and return by defining scenario-dependent cases. They utilized mixed-integer programming and Branch & Bound technique and solved the problem by CPLEX. Francas and Minner [23] proposed a two-stage stochastic model to design a closed-loop network under uncertain demand and return. Pishvaee et al. [24] proposed a deterministic optimization model for a reverse logistics network. Then, they developed a stochastic model. However, environmental factors have not been considered in the model. Lee and Dong [25] proposed a two-stage stochastic programming model for a closed-loop supply chain network. They also developed a solution approach by Simulated Annealing. Pishvaee and Torabi [26] developed a possibilistic mixed integer programming model to deal with uncertainty in closed-loop supply chain configuration. Shi et al. [27] proposed a mathematical model to maximize the profit of a remanufacturing system by developing a solution approach based on Lagrangian relaxation method. Wang and Hsu [28] proposed an interval programming model where the uncertainty has been expressed by fuzzy numbers. Shi et al. [29] studied a production planning problem for a multi-product closed-loop system. The authors considered uncertain demand and return by stochastic programming. Pishvaee et al. [30] proposed a robust optimization model for a closed-loop supply chain network to consider uncertainty. Amin and Zhang [31] developed an optimization model under uncertain demand and decision environment for a CLSC. Vahdani et al. [32] applied fuzzy multi-objective robust optimization to configure a CLSC network.

The research papers of Table 1 have not considered multi-objective and uncertainty issues in CLSC configuration, simultaneously. In this paper, we develop a multi-objective model under uncertainty for a CLSC network.

Table 1
CLSC configuration models

	Multiple parts	Multiple products	Multiple suppliers	Multiple scenarios	Multiple customers	Multiple recyclers	Multiple collection facilities	Multiple distributors	Multiple plants	Multiple warehouses	Multiple disposal centers	Multiple disassembly centers
Multi-objective models for CLSC												
[18]	x	x	x	x	x	x						
[19]		x	x		x			x				
[20]		x					x		x	x		
[21]	x	x	x		x				x			
[22]					x		x	x			x	
CLSC under uncertainty												
[7]		x		x	x				x	x		x
[23]		x							x			
[24]				x	x			x			x	
[25]		x			x				x			
[26]					x	x	x	x	x			
[27]		x										
[28]			x		x			x	x			x
[30]					x		x	x			x	
[31]	x	x	x		x							
[32]		x	x		x		x	x	x			

3. Network description

In this section, a general closed-loop supply chain network is described. Fig. 1 shows the network which includes plants, collection centres, and demand markets. The plants can manufacture new products and remanufacture returned products. The products are sent to demand markets by plants. Then, the returned products are sent to collection centres. Collection centres have the following responsibilities: collecting of used products from demand markets, determining the condition of the returns by inspection and/or separation to find out whether they are recoverable or not, sending recoverable returns to the plants, sending the unrecoverable returns (because of economic and/or technological reasons) to the disposal centre. The objective is to know how many and which plants and collection centres should be open, and which products and in which quantities should be stock in them.

The following assumptions are made in the network configuration:

- The model is designed for a single period.

- All of the returned products from demand markets are collected in collection centres.
- Locations of demand markets are fixed.
- Locations and capacities of plants and collection centres are known in advance.

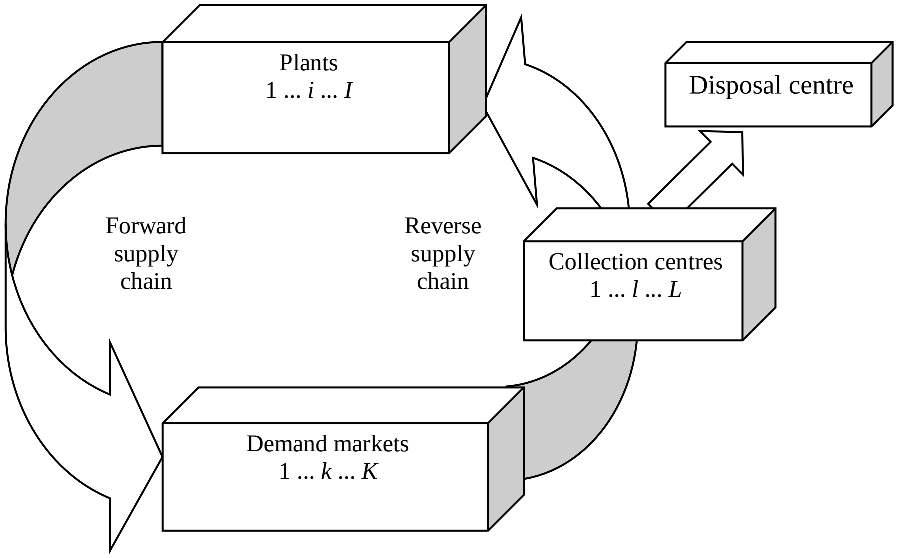


Fig. 1. The closed-loop supply chain network

4. Mathematical model

The network can be formulated as a mixed-integer linear programming model. Sets, parameters, and decision variables are defined as follows:

Sets

I = set of potential manufacturing and remanufacturing plants locations ($1 \ldots i \ldots I$)

J = set of products ($1 \ldots j \ldots J$)

K = set of demand markets locations ($1 \ldots k \ldots K$)

L = set of potential collection centres locations ($1 \ldots l \ldots L$)

Parameters

A_j = production cost of product j

B_j = transportation cost of product j per km between plants and demand markets

C_j = transportation cost of product j per km between demand markets and collection centres

D_j = transportation cost of product j per km between collection centres and plants

O_j = transportation cost of product j per km between collection centres and disposal centre

E_i = fixed cost for opening plant i

F_l = fixed cost for opening collection centre l

G_j = cost saving of product j (because of product recovery)

H_j = disposal cost of product j

P_{ij} = capacity of plant i for product j

Q_{lj} = capacity of collection centre l for product j

t_{ik} = the distance between location i and k generated based on the Euclidean method (t_{kl} and t_{li} are defined in the same way). t_l is the distance between collection centre l and disposal centre

d_{kj} = demand of customer k for product j

r_{kj} = return of customer k for product j

α_j = minimum disposal fraction of product j

Variables

X_{ikj} = quantity of product j produced by plant i for demand market k

Y_{klj} = quantity of returned product j from demand market k to collection centre l

S_{lij} = quantity of returned product j from collection centre l to plant i

T_{lj} = quantity of returned product j from collection centre l to disposal centre

Z_i = 1, if a plant is located and set up at potential site i, 0, otherwise

W_l = 1, if a collection centre is located and set up at potential site l, 0, otherwise

$$Min\ z_1 = \sum_i E_i Z_i + \sum_l F_l W_l + \sum_i \sum_k \sum_j (A_j + B_j\, t_{ik}) X_{ikj} + \sum_k \sum_l \sum_j C_j\, t_{kl}\, Y_{klj}$$

$$+ \sum_l \sum_i \sum_j (-G_j + D_j\, t_{li}) S_{lij} + \sum_l \sum_j (H_j + O_j\, t_l) T_{lj}$$

s.t.

$$\sum_i X_{ikj} \geq d_{kj} \qquad \forall k, j, \tag{1}$$

$$\sum_l \sum_j S_{lij} + \sum_k \sum_j X_{ikj} \leq Z_i \sum_j P_{ij} \qquad \forall i, \tag{2}$$

$$\sum_{l} Y_{klj} \leq \sum_{i} X_{ikj} \qquad \forall k, j, \qquad (3)$$

$$\alpha_j \sum_{k} Y_{klj} \leq T_{lj} \qquad \forall l, j, \qquad (4)$$

$$\sum_{k} \sum_{j} Y_{klj} \leq W_l \sum_{j} Q_{lj} \qquad \forall l, \qquad (5)$$

$$\sum_{k} Y_{klj} = \sum_{i} S_{lij} + T_{lj} \qquad \forall l, j, \qquad (6)$$

$$\sum_{l} Y_{klj} = r_{kj} \qquad \forall k, j, \qquad (7)$$

$$Z_i, W_l \in \{0, 1\} \qquad \forall i, l, \qquad (8)$$

$$X_{ikj}, Y_{klj}, S_{lij}, T_{lj} \geq 0 \qquad \forall i, k, l, j, \qquad (9)$$

The objective function is minimization of the total cost. The first and second parts show the fixed costs of opening plants and collection centres, respectively. The third part represents the production and transportation costs of new products. The forth part is related to product recovery and transportation costs of returned products. Besides, the fifth part represents the total recovery and transportation costs of returned products from collection centres to plants. Besides, the sixth part calculates disposal and transportation costs.

The constraint (1) ensures that the total number of each manufactured product for each demand market is equal or greater than the demand. Constraint (2) is a capacity constraint of plants. Constraint (3) represents that forward flow is greater than reverse flow. Constraint (4) enforces a minimum disposal fraction for each product. Constraint (5) is capacity constraint of collection centres. Constraint (6) shows that the quantity of returned products from demand market is equal to the quantity of returned products to plants and quantity of products in disposal centre for each collection centre and each product. Constraint (7) shows the returned products. Constraint (8) ensures the binary nature of decision variables while Constraint (9) preserves the non-negativity restriction on the decision variables.

5. Application of the proposed model

Copier remanufacturing has been investigated in some papers such as [5]. Major manufacturers such as Canon are reselling and remanufacturing used copy machines collected from their customers. During an initial inspection at a collection site, quality standards of used machines are checked to make sure the returned products have certain quality standards. Remanufacturing is often carried out in the original manufacturing plants using the same equipment. Machines that cannot be reused as a whole may still provide a source for reusable spare parts. The remainder is typically sent to a disposal centre.

The goal of this section is to show the application of the mathematical model by numerical examples. To this aim, two test problems are examined. In the test problem 1, a deterministic example is considered. Data of costs and minimum disposal fraction are adopted from [5]. Table 2 shows the data in detail. The potential locations for manufacturers, demand markets, collection centres, and disposal centre were generated from uniform distribution between 0 and 100 units of distance on the x and y coordinates. Test problem 1 consists of deterministic parameters. However, it is hard to estimate the values of parameters in real world. In the test problem 2, it is supposed that parameters (except demand and return) follow uniform distribution. The reason is that each parameter under uniform distribution can be shown by two numbers (not exactly one). Table 2 shows the values. The objective is to consider a realistic model by using uniform distribution.

Table 2
Data for copier remanufacturing example

Test problem 1		
I = 4 (number of plants)	C_j = 0.005	H_j = 2.5
J = 3 (number of products)	D_j = 0.003	P_{ij} = 84,000
K = 5 (number of demand markets)	O_j = 0.00155	Q_{lj} = 34,000
L = 4 (number of collection centres)	E_i = 5,000,000	d_{kj} = 30,000
A_j = 15	F_l = 500,000	r_{kj} = 10,000
B_j = 0.01455	G_j = 7	α_j = 0.4
Test problem 2		
I = 4 (number of plants)	C_j = uniform (0.0045, 0.0055)	H_j = uniform (2.25, 2.75)
J = 3 (number of products)	D_j = uniform (0.0027, 0.0033)	P_{ij} = uniform (75,600, 92,400)
K = 5 (number of demand markets)	O_j = uniform (0.0014, 0.0017)	Q_{lj} = uniform (30,600, 37,400)
L = 4 (number of collection centres)	E_i = uniform (4,500,000, 5,500,000)	d_{kj} = 30,000
A_j = uniform (13.5, 16.5)	F_l = uniform (450,000, 550,000)	r_{kj} = 10,000
B_j = uniform (0.0131, 0.0160)	G_j = uniform (6.3, 7.7)	α_j = uniform (0.27, 0.33)

Test problems have been solved by CPLEX 9.1.0. CPLEX is an optimization software package which is suitable for solving mixed-integer linear programming problems. All computational work was performed on a personal computer (32-bit operating system, 2.33 GHz CPU, and 4.00 GB). The model statistics are 797 non-zero elements, 78 single equations, 189 single variables, and 8 discrete variables. The objective value (total cost), in the test problem 1 is 17,878,724 (solved in 0.031 seconds) and in the test problem 2 is 17,406,850 (solved in 0.124 seconds). Fig. 2 and Fig. 3 show the optimal networks for test problems 1 and 2, respectively (product 2). It can be seen that in the test problem 1, plants 1 and 3 are open. However, plants 2 and 3 work in the test problem 2. In addition, different collection centres are open in the test problems 1 and 2. As a result, considering uniform distribution not only changes the total cost of network configuration, but also it alters the open facilities.

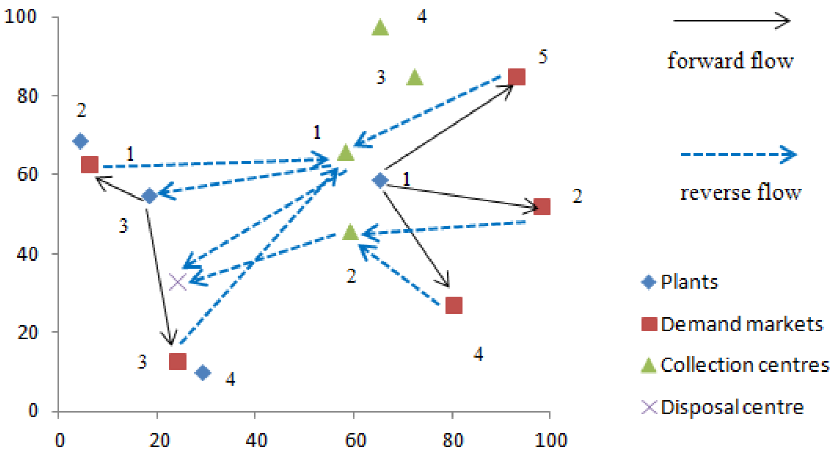


Fig. 2. Optimal closed-loop supply chain network (test problem 1, product 2)

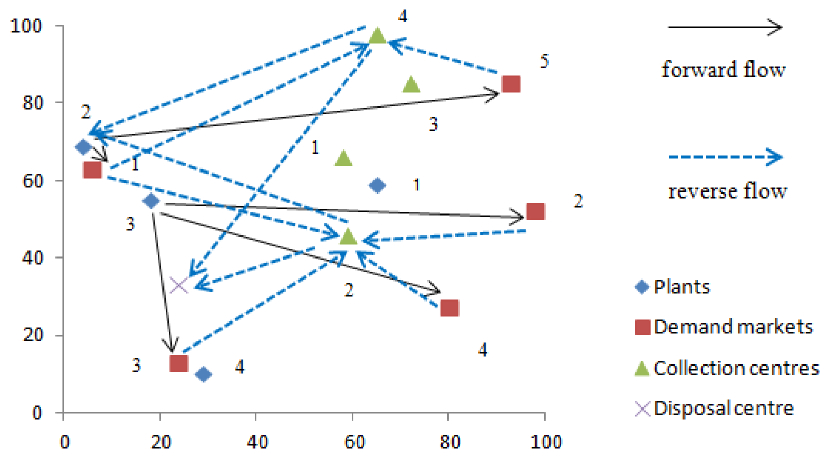


Fig. 3. Optimal closed-loop supply chain network (test problem 2, product 2)

6. An extension to multi-objectives

In the mentioned mathematical model, the total cost is minimized. However, environmental issues also should be considered. To this aim, new parameters are defined. M_{ij} is parameter of using environmental friendly materials by plant i to produce product j. Recyclable materials is an example of this parameter [33]. Another parameter is N_{lj} which is defined as parameter of using clean technology by collection centre l to process product j. Clean technology consists of renewable and recycling energy such as solar power [34]. Both of two parameters are qualitative and should be determined by decision makers. These two parameters are between 0 and 1. Some decision making techniques such as analytic hierarchy process (AHP) can be helpful to convert qualitative assessments to quantitative results. AHP method has different stages including developing hierarchy of problem, constructing pairwise comparison matrix, synthesization, and consistency test. The second objective function can be written as Eq. (10).

$$Max\ z_2 = \sum_i \sum_j M_{ij} \left(\sum_k X_{ikj} + \sum_l S_{lij}\right) + \sum_l \sum_j N_{lj} \left(\sum_k Y_{klj} + \sum_i S_{lij} + T_{lj}\right) \tag{10}$$

6.1. *Solution approach*

To solve the multi-objective problem, two methods are utilized including weighted sums method, and *ε*-constraint method. These methods can transform our problem to a mono-objective optimization problem. Weighted sums method is the most popular multi-objective method. However, determining the weights is a challenge. To compare the results, we also apply *ε*-constraint method. For more information you can refer to [35].

6.1.1. *Weighted sums method*

In this method, objective functions are combined by assigning appropriate weights. The weights (w_1 and w_2 in this case) are determined by decision makers. Some methods such as AHP also can be applied in determining the weights of objectives. It is noticeable that w_1, $w_2 \geq 0$ and $w_1 + w_2 = 1$. Eq. (11) shows the formula for our problem.

$$\begin{aligned} &Min\ z = w_1 z_1 - w_2 z_2 \\ &s.t. \\ &Eq. (1)-(9) \end{aligned} \tag{11}$$

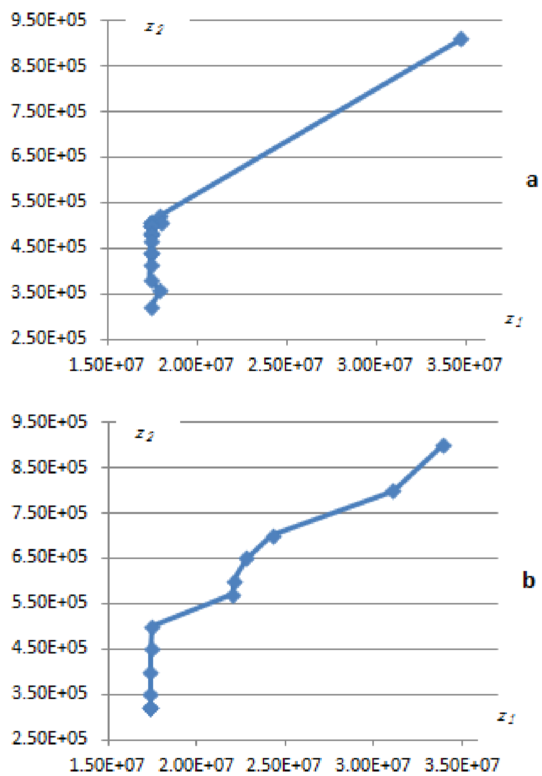
6.1.2. *ε-constraint method*

In this method, the multi-objective optimization problem is transformed to a mono-objective optimization problem with additional constraints. The objective function with a high priority is considered as objective function. Other objectives are written as constraints by using a constraint vector *ε*. The transformed problem is written in Eq. (12).

$$\begin{aligned}&Min\ z = z_1 \\ &s.t. \\ &z_2 \geq \varepsilon \\ &Eq.(1)-(9)\end{aligned} \tag{12}$$

6.2. ***Trade-off surfaces***

The goal of multi-objective programming models is to find efficient solutions. An efficient solution has the property that it is impossible to improve any one objective values without sacrificing on at least one other objective. The small number of efficient solutions produces the trade-off surface or Pareto front [35, 36]. In this section, the test problem 2 is solved by two mentioned methods and trade-off surfaces are depicted in the Fig. 4. To this aim, different weights are assigned and the values of objective functions are calculated. In addition, the trade-off surface of the problem is obtained by changing the value of ε. As mentioned before, CPLEX 9.1.0 is utilized to solve the problem. In this example, it is supposed that M_{ij} and N_{li} have uniform distribution between 0 and 1.



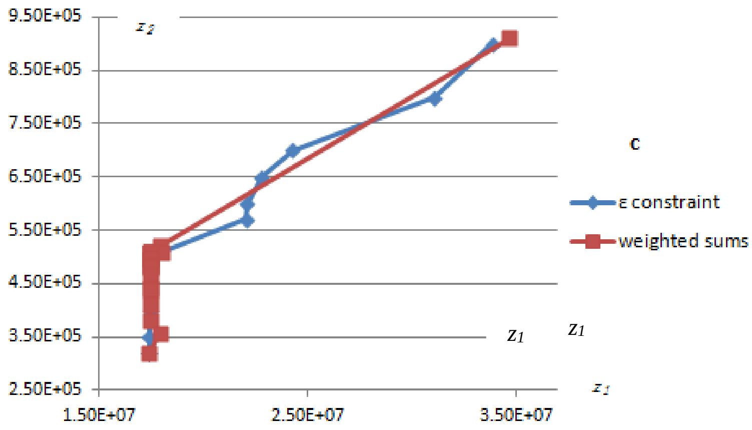


Fig. 4. Trade-off surfaces for the test problem 2: (a) weighted sums method, (b) ε-constraint method, (c) weighted sums and ε-constraint methods

It is easy to use weighted sums method, but it can be applied only to the convex sets. This is a weakness of this method that makes it difficult to identify the trade-off surface of the problem. The ε-constraint method can be applied for non convex problems. However, it is very sensitive to the selection of parameter ε. A good choice can provide a good spread of solutions on the trade-off surface. This issue can be considered as a weakness of this method.

It can be seen in the Fig. 4 that weighted sums method cannot identify some solutions between 17,891,000 and 34,684,000 values of the first objective function. However, ε-constraint method can obtain more solutions. As a result, for the test problem 2, ε-constraint method is more efficient rather than weighted sums method. The values of objective functions of ε-constraint method have been written in the Table 3. The numbers of open facilities (plants and collection centres) also have been written. We can see that results of some test problems in Table 3 are different from Fig. 3. For example, collection centres 2 and 4 are open in Fig. 3 (single objective). However, collection centres 2 and 3 are open in some cases in Table 3 (multi-objective). This issue shows the effect of second objective function on the results. In addition, we show the sensitivity analysis of ε according to the objective function in Fig. 5.

Table 3
Results of ε-constraint method

ε	Value of the first objective	Value of the second objective	Open plants	Open collection centres
50,000	17,407,000	319,120	2, 3	2, 4
100,000	17,407,000	319,120	2, 3	2, 4
200,000	17,407,000	319,120	2, 3	2, 4
300,000	17,407,000	319,120	2, 3	2, 4
350,000	17,407,000	350,000	2, 3	2, 4
400,000	17,413,000	400,000	2, 3	2, 4
450,000	17,440,000	450,000	2, 3	2, 3
500,000	17,473,000	500,000	2, 3	2, 3
600,000	22,094,000	600,000	2, 3, 4	2, 3
650,000	22,794,000	650,000	2, 3, 4	2, 3
700,000	24,298,000	700,000	2, 3, 4	1, 2, 3
800,000	31,091,000	800,000	1, 2, 3, 4	2, 3
900,000	33,870,000	900,000	1, 2, 3, 4	1, 2, 3

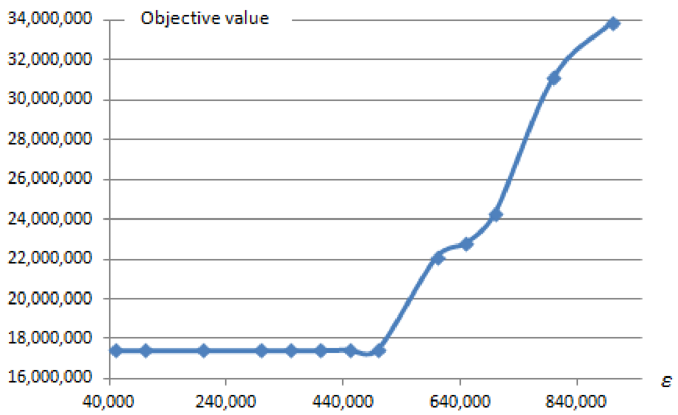


Fig. 5. Sensitivity analysis of ε

7. An extension to consider uncertainty

Several parameters have uncertain values in practice. Uncertainty in demand is major source of uncertainty in supply chain management. Uncertain return is another important source of vagueness in reverse logistics. It is useful to take into account this issue in the optimization model.

7.1. *Stochastic programming*

The uncertainty in parameters can be modelled by stochastic programming. The goal of stochastic programming is to discover a solution that will perform well under any possible realization of the random parameters. The random parameters can be stated as continuous values or discrete scenarios [9]. In this paper, a scenario-based analysis is utilized to consider uncertainty. For more information, you can refer to [37, 38]. Suppose that vector *y* includes all binary variables. Besides, vector *x* has all non-negative variables. Moreover, *q* and *C* are vectors related to fix and variable costs, respectively. It is also assumed that *a*, *b*, *e*, and *f* are matrices. Minimization problem can be written as follow:

$$
\begin{aligned}
& Min\ z = q\,y + C\ x \qquad (13)\\
& s.t.\\
& a\ x\ \le b\\
& e\ x\ \le f\ y\\
& y \in \{0,1\} \quad x \ge 0
\end{aligned}
$$

Assume that there are *U* scenarios and scenario *u* can happen with probability p_u. The expected value of the objective function can be calculated by (14).

$$
\begin{aligned}
& Min\ z = q\,y + \sum_u p_u c_u x_u \qquad (14)\\
& s.t.\\
& a_u x_u \le b_u \qquad \forall u,\\
& e_u x_u \le f\ y \qquad \forall u,\\
& y \in \{0,1\} \quad x_u \ge 0 \quad \forall u
\end{aligned}
$$

To formulate the closed-loop supply chain network under uncertainty, new sets, parameters, and variables should be added to the previous definitions.

Sets

U = set of scenarios (1 ... *u* ... *U*)

Parameters

d_{kju} = demand of customer *k* for product *j* for scenario *u*

r_{kju} = return of customer *k* for product *j* for scenario *u*

p_u = probability of scenario *u*

Variables

X_{ikju} = quantity of product *j* produced by plant *i* for demand market *k* in scenario *u*

Y_{klju} = quantity of returned product j from demand market k to collection centre l in scenario u

S_{liju} = quantity of returned product j from collection centre l to plant i in scenario u

T_{lju} = quantity of returned product j from collection centre l to disposal centre in scenario u

The multi-objective stochastic model (scenario-based) can be written as:

$$Min\ z \;=\; \sum_i E_i Z_i + \sum_l F_l W_l + \sum_u \sum_i \sum_k \sum_j p_u (A_j + B_j\, t_{ik}) X_{ikju} + \sum_u \sum_k \sum_l \sum_j p_u C_j\, t_{kl}\, Y_{klju}$$

$$+ \sum_u \sum_l \sum_i \sum_j p_u (-G_j + D_j\, t_{li}) S_{liju} + \sum_u \sum_l \sum_j p_u (H_j + O_j\, t_l) T_{lju}$$

s.t.

$$\sum_i \sum_j M_{ij} (\sum_k X_{ikju} + \sum_l S_{liju}) + \sum_l \sum_j N_{lj} (\sum_k Y_{klju} + \sum_i S_{liju} + T_{lju}) \geq \varepsilon \qquad \forall u, \tag{15}$$

$$\sum_i X_{ikju} \geq d_{kju} \qquad \forall k, j, u, \tag{16}$$

$$\sum_l \sum_j S_{liju} + \sum_k \sum_j X_{ikju} \leq Z_i \sum_j P_{ij} \qquad \forall i, u, \tag{17}$$

$$\sum_l Y_{klju} \leq \sum_i X_{ikju} \qquad \forall k, j, u, \tag{18}$$

$$\alpha_j \sum_k Y_{klju} \leq T_{lju} \qquad \forall l, j, u, \tag{19}$$

$$\sum_k \sum_j Y_{klju} \leq W_l \sum_j Q_{lj} \qquad \forall l, u, \tag{20}$$

$$\sum_k Y_{klju} = \sum_i S_{liju} + T_{lju} \qquad \forall l, j, u, \tag{21}$$

$$\sum_l Y_{klju} = r_{kju} \qquad \forall k, j, u, \tag{22}$$

$$Z_i, W_l \in \{0, 1\} \qquad \forall i, l, \tag{23}$$

$$X_{ikju}, Y_{klju}, S_{liju}, T_{lju} \geq 0 \qquad \forall i, k, l, j, u, \tag{24}$$

7.2. *Computational results*

To consider the effects of uncertainty, scenario analysis is performed. The selected scenarios for analysis and discussion are listed in Table 4. Parameters of scenario 5 (base-case) are similar to the test problem 2. Each of the scenarios (1-9) represents different scenario reflecting variations in demand and return. Actually, different combinations of 10% increase and decrease in demand and return have been considered. In addition, the scenarios are compared in terms of changes in the value of objective function with respect to the base-case (scenario 5), as illustrated in Table 4 (e.g. (18,531,389-17,412,507)/17,412,507=6.43%). Besides, stochastic model has been solved and change in the value of objective function has been written in Table 4. Fig. 6 shows the value of objective functions in deterministic and stochastic models.

Sensitivity analysis of results shows that the optimum closed-loop supply chain network is very sensitive to changes in demand and return. As shown in Table 4, planning for a 10% increase in demand (scenario 6) would result to a network that has about 6.67% more cost than the base-case, while assuming 10% decrease in demand (scenario 7) reduces the cost about 6.49%. Deviations in cost also can be observed for return (scenarios 3 and 4). However, it can be seen that the effect of uncertainty in demand is higher than return because the demand has more significant contribution than return in the objective function. Such deviations in cost reveal that planning under uncertain situation (demand and return) is risky, and forecasts of vague parameters can be helpful. Results of the stochastic scenario (scenario 10) show that the stochastic programming model can obtain flexible optimum closed-loop supply chain configuration with the objective function near to the base-case (0.05% change). This observation shows that the proposed stochastic programming model takes into account the risks related to different sources of uncertainty including demand and return.

Minimum disposal fraction of product j (α_j) is an important parameter which is related to reverse supply chain. To show the effect of this parameter on the objective function, sensitivity analysis is performed. Fig. 7 shows the results for both of deterministic (base-case) and stochastic models. It can be seen that by increasing the parameters, the values of objective functions are increased.

Table 4
Scenario analysis

Deterministic models	797 non-zero elements, 78 single equations, 189 single variables, and 8 discrete variables.			
Scenario	Demand	Return	Probability	Change %
1	33,000	9,000	0.075	6.43
2	27,000	11,000	0.075	-3.53
3	30,000	11,000	0.1	0.23
4	30,000	9,000	0.1	-0.22
5 (base-case)	30,000	10,000	0.3	0.00
6	33,000	10,000	0.1	6.67
7	27,000	10,000	0.1	-6.49
8	33,000	11,000	0.075	6.91
9	27,000	9,000	0.075	-6.75
10 Stochastic model	Combination of nine scenarios 8,723 non-zero elements, 704 single equations, 1,630 single variables, and 8 discrete variables.			0.05

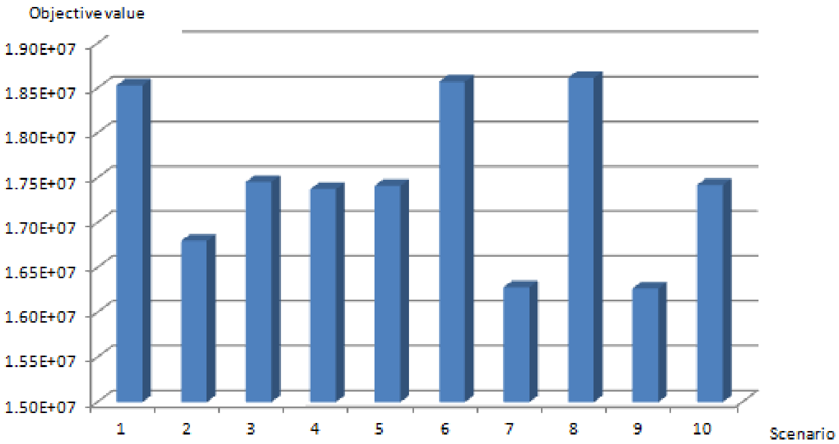


Fig. 6. Objective values of deterministic scenarios (1-9) and stochastic case (scenario 10)

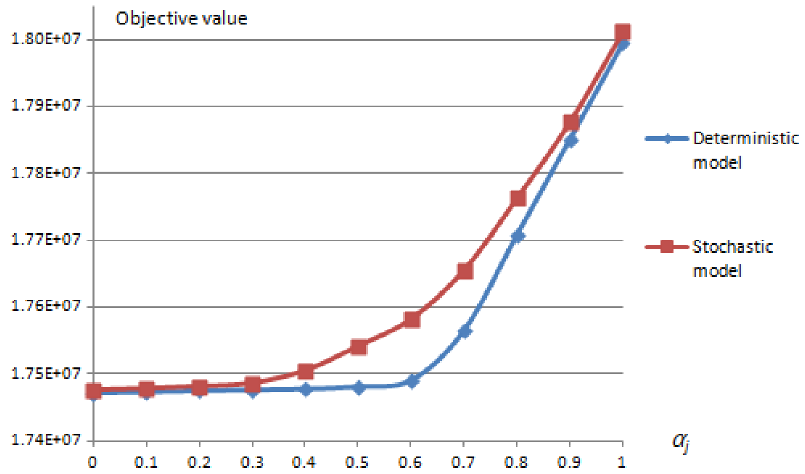


Fig. 7. Sensitivity analysis of α_j in deterministic (base-case) and stochastic scenarios

8. Conclusions

In this research, a facility location model is proposed for a closed-loop supply chain network. The model is designed for multiple plants, demand markets, collection centres, and products. To show the application of the mathematical model, two test problems are examined for a copier remanufacturing example. Besides, the model is extended to consider environmental objective. Two methods are utilized to solve the multi-objective programming model including weighted sums and ε-constraint methods. The results of test problem 2 show that ε-constraint method can obtain more efficient solutions than weighted sums method. Therefore, ε-constraint method is selected for this example. The model also is developed by stochastic programming (scenario-based) to examine the effects of uncertain demand and return on the network configuration. The computational results demonstrate that the stochastic programming model can gain flexible optimal closed-loop supply chain configuration with the objective function near to the base-case. This paper is among the first investigations that consider multi-objective mathematical models under uncertain environment in CLSC network configuration.

There are some potential future works. One of the weaknesses of scenario-based analysis is the small number of scenarios because of computational reasons. It is useful to examine the effects of uncertainty on the model by other methods such as robust optimization and compare the results. In this research, two qualitative factors (environmental friendly materials and using clean technology) have been considered. It is helpful to propose a new method based on some environmental standards such as

Eco-indicator 99. Another future research is to develop heuristic approaches such as Genetic Algorithm and Scatter Search because it is hard to solve large problems in a reasonable time. Meanwhile, the proposed model has been designed for a single period. The model can be developed to consider multiple periods. In this condition, the inventory level should be taken into account. Finally, it is valuable to apply the models in real cases and analyse the results.

Acknowledgments

The work of authors is supported by NSERC Discovery grant (298482). The first author thanks the Government of Ontario for an OGS. The authors would like to thank the editor and referees for their helpful comments.

References

[1] M.C. Cooper, D.M. Lambert, J.D. Pagh, Supply chain management: more than a new name for logistics, Int. J. Logist. Manage. 8 (1) (1997) 1-9.

[2] T. Abdallah, A. Farhat, A. Diabat, S. Kennedy, Green supply chains with carbon trading and environmental sourcing: Formulation and life cycle assessment, Appl. Math. Model. (In Press).

[3] M.T. Melo, S. Nickel, F. Saldanha-da-Gama, Facility location and supply chain management - A review, Eur. J. Oper. Res. 196 (2) (2009) 401-412.

[4] Jr.V.D.R. Guide, L.N. Van Wassenhove, The Evolution of Closed-Loop Supply Chain Research, Oper. Res. 57 (1) (2009) 10-18.

[5] M. Fleischmann, P. Beullens, J.M. Bloemhof-Ruwaard, L.N. Van Wassenhove, The impact of product recovery on logistics network design, Prod. Oper. Manage. 10 (2) (2001) 156-173.

[6] S.H. Amin, G. Zhang, An integrated model for closed loop supply chain configuration and supplier selection: Multi-objective approach, Expert Syst. Appl. 39 (8) (2012) 6782-6791.

[7] M.I.G. Salema, A.P. Barbosa-Povoa, A.Q. Novais, An optimization model for the design for a capacitated multi-product reverse logistics network with uncertainty, Eur. J. Oper. Res. 179 (3) (2007) 1063-1077.

[8] T. Davis, Effective supply chain management, Sloan Manage. Rev. 34 (4) (1993) 35-46.

[9] L.V. Snyder, Facility location under uncertainty: A review, IIE Trans. 38 (7) (2006) 537-554.

[10] G. Zhang, L. Ma, Optimal acquisition policy with quantity discounts and uncertain demands, Int. J. Prod. Res. 47 (9) (2009) 2409-2425.

[11] V. Jayaraman, Jr.V.D.R. Guide, R. Srivastava, A closed-loop logistics model for remanufacturing, J. Oper. Res. Soc. 50 (5) (1999) 497-508.

[12] G. Kannan, A. Noorul Haq, M. Devika, Analysis of closed loop supply chain using genetic algorithm and particle swarm optimization, Int. J. Prod. Res. 47 (5) (2009) 1175-1200.

[13] G. Kannan, P. Sasikumar, K. Devika, A genetic algorithm approach for solving a closed loop supply chain model: A case of battery recycling, Appl. Math. Model. 34 (3) (2010) 655-670.

[14] S.H. Amin, G. Zhang, A proposed mathematical model for closed-loop network configuration based on product life cycle, Int. J. Adv. Manuf. Technol. 58 (5) (2012) 791-801.

[15] M. Fleischmann, J.M. Bloemhof-Ruwaard, R. Dekker, E. Van Der Laan, J.A.E.E. Van Nunen, L.N. Van Wassenhove, Quantitative models for reverse logistics: a review, Eur. J. Oper. Res. 103 (1) (1997) 1-17.

[16] S. Rubio, A. Chamorro, F.J. Miranda, Characteristics of the research on reverse logistics (1995-2005), Int. J. Prod. Res. 46 (4) (2008) 1099-1120.

[17] E. Akcali, S. Cetinkaya, Quantitative models for inventory and production planning in closed-loop supply chains, Int. J. Prod. Res. 49 (8) (2011) 2373-2407.

[18] H. Krikke, J. Bloemhof-Ruwaard, L.N. Van Wassenhove, Concurrent product and closed-loop supply chain design with an application to refrigerators, Int. J. Prod. Res. 41 (16) (2003) 3689-3719.

[19] K.R. Pati, P. Vrat, P. Kumar, A goal programming model for paper recycling system, Omega. 36 (3) (2008) 405-417.

[20] F. Du, G.W. Evans, A bi-objective reverse logistics network analysis for post-sale service, Comput. Oper. Res. 35 (8) (2008) 2617-2634.

[21] A. Gupta, G.W. Evans, A goal programming model for the operation of closed-loop supply chains, Eng. Optim. 41 (8) (2009) 713-735.

[22] M.S. Pishvaee, R.Z. Farahani, W. Dullaert, A memetic algorithm for bi-objective integrated forward/reverse logistics network design, Comput. Oper. Res. 37 (6) (2010) 1100-1112.

[23] D. Francas, S. Minner, Manufacturing network configuration in supply chains with product recovery, Omega. 37 (4) (2009) 757-769.

[24] M.S. Pishvaee, F. Jolai, J. Razmi, A stochastic optimization model for integrated forward/reverse logistics network design, J. Manuf. Syst. 28 (4) (2009) 107-114.

[25] D. Lee, M. Dong, Dynamic network design for reverse logistics operations under uncertainty, Transport. Res. Part E. 45 (1) (2009) 61-71.

[26] M.S. Pishvaee, S.A. Torabi, A possibilistic programming approach for closed-loop supply chain network design under uncertainty, Fuzzy Sets Syst. 161 (20) (2010) 2668–2683.

[27] J. Shi, G. Zhang, J. Sha, S.H. Amin, Coordinating production and recycling decision with stochastic demand and return, J. Syst. Sci. Syst. Eng. 19 (4) (2010) 385-407.

[28] H. Wang, H. Hsu, Resolution of an uncertain closed-loop logistics model: An application to fuzzy linear programs with risk analysis, J. Environ. Manage. 91 (11) (2010) 2148-2162.

[29] J. Shi, G. Zhang, J. Sha, Optimal production planning for a multi-product closed loop system with uncertain demand and return, Comput. Oper. Res. 38 (3) (2011) 641-650.

[30] M.S. Pishvaee, M. Rabbani, S.A. Torabi, A robust optimization approach to closed-loop supply chain network design under uncertainty, Appl. Math. Model. 35 (2) (2011) 637-649.

[31] S.H. Amin, G. Zhang, A three-stage model for closed-loop supply chain configuration under uncertainty, Int. J. Prod. Res. (In press).

[32] Vahdani, B., Tavakkoli-Moghaddam, R., Modarres, M., Baboli, A. (2012). Reliable design of a forward/reverse logistics network under uncertainty: A robust-M/M/c queuing model, Transport. Res. Part E. (In press).

[33] J. Ruan, Z. Xu, Environmental friendly automated line for recovering the cabinet of waste refrigerator, Waste Manage. 31(11) (2011) 2319-2326.

[34] R. Kemp, M. Volpi, The diffusion of clean technologies: a review with suggestions for future diffusion analysis, J. Clean. Prod. 16 (1) (2008) S14-S21.

[35] Y. Collette, P. Siarry, Multi objective Optimization: Principles and Case Studies. Springer-Verlag, New York. (2003).

[36] V. Wadhwa, A.R. Ravindran, Vendor selection in outsourcing, Comput. Oper. Res. 34 (12) (2007) 3725-3737.

[37] J.R. Birge, F. Louveaux, Introduction to Stochastic Programming. Springer Series in Operations Research. Springer-Verlag, New York. (1997).

[38] W.B.E. Al-Othman, H.M.S. Lababidi, I.M. Alatiqi, K. Al-Shayji, Supply chain optimization of petroleum organization under uncertainty in market demands and prices, Eur. J. Oper. Res. 189 (3) (2008) 822-840.