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# Pricing Policies for a Dual-Channel Retailer with Cross-Channel Returns 

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#### Abstract

Many retailers are adopting a dual-channel retailing strategy (DCRS) in which products are offered through two channels: physical stores and online stores. Due to regulations or competitive measures, such a strategy allows customers who find a purchase unsatisfactory to obtain a full refund through a same-channel return or a cross-channel return. No papers have collectively studied the aforementioned types of customer returns in a dual-channel context. This paper studies optimal pricing policies for a centralized and decentralized dual-channel retailer (DCR) with same- and cross-channel returns. How dual-channel pricing behaviour is impacted by customer preference and rates of customer returns is discussed. It is found, through sensitivity analysis, that when a channel with significant customer preference faces a high rate of returns, decentralized channels generate a greater system profit for retailers than coordinated channels that have a unified pricing strategy. A DCR with a Stackelberge scheme has the proclivity to be more profitable when under the leadership of a channel with a high rate of returns and significant customer preference.


Keywords: Dual channel, resalable returns, reverse supply chains, pricing, Stackelberge game, Nash game

## 1. INTRODUCTION

This work analyzes the problem of customer returns under a dual-channel retailing system. It is motivated by the fact that online customers may return their unsatisfactory purchases to a retailer's physical store where they can be resold again (e.g. such the case in WalMart, Costco, and Target). Consequently, the two channels are greatly intermingled and a decision for one channel should not be taken in isolation. The main goal of this research is to develop proper methodologies of price management and inventory control management when the demand is deterministic.

Both the rapid development of the Internet and the growth of third-party logistics providers have inspired $80 \%$ of US retailers to adopt a dual-channel retail strategy (Zhang et al. 2010). Such a strategy offers retail businesses several advantages. For example, it allows retailers to reach wider segments of customers and increase revenue (Ryan, Sun and Zhao 2013). Also, it enables retailers to satisfy increasing customer demands for multiple channels through which to shop. Thus, using a DCRS increases customer loyalty and satisfaction (Zhang et al. 2010). According to Chiang and Monahan (2005), such a strategy adds flexibility to a retailer's supply chain. This flexibility allows customers to view a product's description online and purchase it at a physical store, order a product online and pick it up from a physical store, or purchase a product online and return it to a physical store. A DCR can use differential pricing strategy to direct customer traffic depending on a retailer's best interest (Zhang et al. 2010).

However, DCRS also comes with sever drawbacks. As Webb and Hogan (2002) have stated, "goal incompatibility" (between physical stores and online stores, for example) is an inevitable result if channels are not integrated. Channels may generate internal conflict due to scarce resources (for example, a tight budget or few customers) or tight objectives (for example, a targeted revenue and profit). Such a conflict will in turn spark competition and trigger a price war that may harm the parties involved. Consequently, channels may limit their cooperation and customers are inspired to change companies due to confusion and agitation (Steinfield 2004). This competition may be so intense that one channel may sabotage another. For example, Levi Strauss and Best Buy had terminated their online stores after a few years of their first operational trial due to internal competition (Yan 2010; Falk et al. 2007). Webb and Hogan's research supports this; they found that $66 \%$ of 50 interviewed retail businesses viewed channel conflict as the most troublesome issue that is faced when they run dual-retailing channels.

Intuitively, one may argue that all of those drawbacks could be eliminated by integration and price coordination. That is true. However, most dual-channel retailers (i.e., Target, Nike, Kmart, Barnes and Noble, Jo-Ann Fabric and Craft Stores, and Kohl's) still use decentralized teams to run their stores due to the high cost of coordination and the operational difficulties associated with it (Zhang et al. 2010; Yan et al. 2010; Neslin and Shankar 2009; Yan 2008; Berger et al. 2006; Webb and Hogan 2002; Schoenbachler and Gordon 2002). Also, a variety of managerial skills are needed for different channels; thus, some retailers may even outsource the management of unfamiliar or newly opened channels to a third party. An example is Toys"R"Us, which outsourced the management of its online channel to Amazon (Berger et al. 2006).

Since online customers do not experience products prior to purchasing them, the transfer from the traditional retailing system to the DCRS may cause a noticeable growth in the number
of customer purchase returns. According to Akcay, Boyaci and Zhang (2013), Mostard and Teunter (2006), and Vlachos and Dekker (2003) fashion products purchased in person through physical channels can have return rates as high as $35 \%$. In contradistinction, fashion products purchased online can have return rates as high as $75 \%$. Also, the implementation of the full refund policy increases the number of customer purchase returns. However, many retailers use this policy in order to increase customer loyalty, provide customer satisfaction, boost sales, and/or comply with country legislations. In the United States and Canada, yearly returns to merchants total between $\$ 100$ and $\$ 10$ billion dollars of products, respectively (Akcay, Boyaci and Zhang 2013; Chen and Bell 2009; Su 2009). While defective returns constitute only $5 \%$ of all customer returns, a significant amount of returned apparel is of good quality and can be resold several times without a recovery process (Akcay, Boyaci and Zhang 2013; Su 2009).

Many DCRs (for example, Wal-Mart or Toys"R"Us) allow same-channel returns, wherein an item purchased from their physical store is returned to their physical store, or an item purchased from their online store is returned to their online store. However, many also allow cross-channel returns, wherein a product purchased from their online store may be returned to their physical store. Allowing cross-channel returns is vital for online stores as such a policy increases sales and customer satisfaction and allow physical stores to create additional crossselling opportunities (Zhang et al. 2010; Cao and Li 2015). However, if cross-channel returned items are not offered at the physical store, then the items must be shipped to the online store (Zhang et al. 2010). Otherwise the ownership of such items is transferred to the physical store. This is done by conducting an inventory transfer that is subject to the retailer's internal rules. The retailer's policy and practice of having cross-channel returns can be acquired through partial integration. As Cao and Li (2015) have stated, channels will only have full integration when
prices align to meet the retailer's goals and objectives. That is to say, cross-channel returns do not contradict the fact that channels of a DCR may still undergo price competition.

Several studies have considered competition and possible coordination strategies between dual channels, which are owned by either the same retailer or different enterprises. The customer returns topic has also been thoroughly studied in single retailer or two retailers systems. However, few papers have studied customer returns under a DCRS. To the best of our knowledge, there is no work related to a DCRS that has collectively considered all forms of same-channel returns and cross-returning an original online store's item to the physical store. Also, there is no published paper that has studied the impact of cross-channel returns on both stores of a DCR, especially when those returns are resalable. For example, the effect of crosschannel returns on channels' pricing policies and inventory management has not being studied yet. Therefore, this paper investigates optimal pricing policies for a centralized and decentralized DCR with same-channel returns and cross-channel returns. Both centralization with differential and unified pricing schemes, and competition in regards to theoretical game frameworks are addressed.

This paper is organized in a logical manner. Related works are reviewed in Section 2 while the paper's statement of the problem is in Section 3. Then centralized and decentralized management styles are both discussed in Sections 4 and 5, respectively. Section 6 discusses the paper's sensitivity analysis and Section 7 provides managerial insights. Finally, conclusions and suggested future research are presented in Section 8.

## 2. RELATED WORKS

This section sheds the light upon two streams of literature. The first stream addresses dual-channel systems under two settings: manufacturer-retailer setting and multi-channel
retailing setting. Considerable research works have analyzed systems that contain a manufacturer that sells a single product to customers through both a manufacturer or supplier-owned online store and an independent retail store(s). Several different types of competition take place between the two channels, including competition in price (David, and Adida 2015; Balakrishnan, Sundaresan and Zhang 2014; Ryan, Sun and Zhao 2013), competition in services (Dan, Xu and Liu 2012), and competition in product availability (Takahashi et al. 2011; Chiang and Monahan 2005). Existing literature shows that decentralized systems are a representation of a situation wherein each channel seeks to maximize its own profit in the presence of cannibalization (David, and Adida 2015; Ryan, Sun and Zhao 2013; Dan, Xu and Liu 2012; Huang, Yang and Zhang 2012; Chen, Zhang and Sun 2012; Hua, Wang and Cheng 2010). In a coordinated or centralized duopolistic system, each player maximizes its own profit. However, it is done within the boundaries of a contract (David, and Adida 2015; Ryan, Sun and Zhao 2013; Chen, Zhang and Sun 2012). A fully coordinated or centralized monopolistic system uses a sole decision maker to maximize the system's total profit (Huang, Yang and Zhang 2012; Hua, Wang and Cheng 2010).

Other papers in literature have examined the situation wherein a multi-channel retailer offers the same product in both self-owned online and physical stores. Yan et al. (2010), Yan (2010), and Yan (2008) all studied Bertrand-Nash, online-Stackelberge and retailer-Stackelberge games to model the price competition that stems from operating a multi-channel retailing system. Each of the studies stated that the Stackelberge games always outperform the Bertrand-Nash game. Huang and Swaminathan (2009) studied four pricing strategies commonly used by monopolistic DCRs. They also considered a duopoly case in which an established DCR faces competition from a new offline entrant. Additionally, Berger et al. (2006) examined the profit enhancement induced by a multi-channel retailer that integrates the advertisement efforts of both
online and physical stores. Each study presented above examined the possible coordination strategies or policies between different competing channels in a dual-channel system. None of the papers have considered customer returns and the impact that the returns have had on optimal pricing strategies.

The second stream addresses customer returns under three settings: single retailer setting, two retailers setting, and multi-channel retailing setting. Before we review the papers under the different settings, it is adequate to examine the researchers findings in regard to the different refund policies. A large body of work on customer returns has examined a refund policy that is exogenously determined as a full refund or a Money Back Guarantee (MBG) (Chang and Yeh 2013; Chen and Bell 2013; Choi et al. 2013; Akcay, Boyaci and Zhang 2013; Wang, Tung and Lee 2010; You, Ikuta and Hsieh 2010; Vlachos and Dekker 2003). Other papers have compared a system's performance with no refund policy to a system's performance with a full refund policy (Chen and Grewal 2013; Choi et al. 2013; Chen and Bell 2012; Chen and Zhang 2011). Several papers that conducted such a comparison also examined a partial refund policy ( $\mathrm{Li}, \mathrm{Xu}$ and Li 2013; Su 2009; Yalabik, Petruzzi and Chhajed 2005). Hsiao and Chen (2012) found that the optimal refund policy may exceed the full price of the item. Su (2009), Chen and Bell (2009), and Yalabik, Petruzzi and Chhajed (2005) all argued that a full refund policy is not optimal as it overwhelms retailing systems. In contrast, Chen and Zhang (2011) argued that a full refund policy may be optimal in the presence of competition. However, Hu, Li and Govindan (2014) and Su (2009) claimed that the optimal refund policy depends upon the refunded product's salvage value. According to $\mathrm{Li}, \mathrm{Xu}$ and Li (2013), retailers should offer either a lenient return policy with a low quality and a low price or a strict return policy with a high quality and a high price. Moreover, Yu and Goh (2012) stated that retailers should enforce a return policy that takes
the nature of products and their condition upon return into consideration. Akcay, Boyaci and Zhang (2013) encouraged retailers to reduce the number of returns they receive by controlling selling prices and enforcing a refund policy with restocking fees. However, Hu and Li (2012) argued that offering a manufacturer buyback price equivalent to the retailer's refund price is the optimal coordinating mechanism.

Additionally, many papers have considered a retailer faces returns from unsatisfied customers. Akcay, Boyaci and Zhang (2013) studied a system wherein customers could differentiate between a new sell and a resell but their product valuation was uncertain. Yu and Goh (2012) examined a retailer facing eight different scenarios. The eight scenarios had several combinations that consisted of whether or not returns occurred within a grace period, whether or not returns were accompanied by a penalty, and whether or not returns were recoverable. Additionally, according to You, Ikuta and Hsieh (2010), a single selling period can be divided into N countable sub-periods. Each sub-period is associated with a probability of return. Chen and Bell (2009) did not allow "as good as new" returns to be sold in the same period in which they were sold, but did allow them to be salvaged in a single-period setting or resold in the following period in a multi-period setting. Vlachos and Dekker (2003) studied six different systems according to whether or not returns could be resold in the primary market, whether or not resalable returns needed a recovery process, and whether or not the needed recovery process was associated with fixed or variable costs. Wang, Tung and Lee (2010) investigated a system wherein customer returns could be resold several times. Selling periods were divided into three sub-periods: a period in which sales consumed both new and returned stocks, a period in which sales only consumed returned stocks, and a period in which there were only returns, not sales. Li , Xu and Li (2013), Hsiao and Chen (2012), and Mukhopadhyay and Setaputra (2007) discussed
the interrelationship between price, refund policy, and quality. Choi et al. (2013) and Mukhopadhyay and Setoputro (2005) examined a system in which demand is linearly dependant on price, return, and modularity level, while return is linearly dependant on return policy.

Several other papers have considered customer returns when two retailers compete in the same market. Chen and Bell (2012) examined a system with two customer behaviours: returnsensitive customers willing to pay more and enjoy the privilege of returning a product if it is a mismatch, and price-sensitive customers willing to pay less and keep the product if it is a mismatch. Both a returnable channel and a non-returnable channel are thus considered in Chen and Bell's study. Furthermore, Chen and Grewal (2013) studied Stackelberge and Bertrand-Nash competitions in situations wherein a new channel competes with a well-established retailer that offers a full refund policy. Additionally, Chen and Zhang (2011) studied Stackelberge and Bertrand-Nash competitions between two retailers that both offered a full refund policy. Balakrishnan, Sundaresan and Zhang (2014) studied the browse and switch behavior exerted by consumers on the brick and mortar stores. The effect of such a behavior on system's profits and prices are examined when returns are allowed for online purchases only. Ofek, Katona and Sarvary (2009) studied competion in pricing and assistance level between two retailers that may operate a single channel ("bricks") or dual-channels ("bricks and clicks").

Widodo et al. (2010) and Widodo et al. (2009) studied both Nash and Stackelberge competitions between a retailer's physical and online channels. Contrary to practice, they studied returns that were only allowed for online purchases. Online customers were allowed to return items to the online store (a same-channel return) or the physical store (a cross-channel return). The study assumed that returns could be exchanged but not refunded.

One should note that no author has collectively considered all forms of same-channel returns and cross returning an original online store's item to the physical store. Also, the impact of cross-channel returns on both stores has not being studied yet. There is thus a research gap in this area. Accordingly, this paper studies both same-channel and cross-channel returns encountered by a DCR. It analyzes the effect that such returns have on optimal prices. Centralization with differential and unified pricing strategies is addressed. Additionally, this paper studies theoretical game competition between stores using the following frameworks: the Online-Leader Stackelberge game, the Physical-Leader Stackelberge game, and the Nash game.

## 3. PROBLEM STATEMENT

This paper considers merchants that run both a physical store and an online store. It examines two coordination schemes: one in which channels are managed collectively in a centralized setting and one in which channels are managed competitively in a decentralized setting. Customers may receive a full refund for purchases returned within a merchant-specified time period. The probability that a product purchased from a physical store is returned to a physical store is $0 \leq r \leq 1$. The probability that a product purchased from an online store is returned to an online store is $0 \leq w \leq 1$. The probability that a product purchased from an online store is cross-returned to a physical store is $0 \leq v \leq 1$ (Figure 1). The assumption of ratios for returns has been implemented in literature before, such as in works by Chen and Grewal (2013), Mostard and Teunter (2006), Mostard, Koster and Teunter (2005), Vlachos and Dekker (2003), and many more.


Figure 1: A dual-channel retailer with same- and cross-channel returns
Akcay, Boyaci and Zhang (2013) have stated that a sold item is often returned "as good as new"; thus, it can be resold at least one more time during a selling season. Therefore, for a returned product to be resalable it must be returned in its original packaging and condition. We assume that a returned product has a resalability rate of $k_{r}$ if the item was purchased from and returned to a physical store, $k_{o}$ if the item was purchased from and returned to an online store, and $k_{o r}$ if the item was purchased from an online store but cross-returned to a physical store. Due to the seasonal length constraint, all same-channel resalable returns can be resold once more from their original channels. Regardless of the number of times an item is sold in the online store, all cross-channel resalable returns can be resold once from the physical store. Readers are reminded here that those assumptions are most likely valid when a retailer faces a single selling period such as in the apparel industry.

Each returned item is associated with a return collection cost of the value $d$. If an item is returned as not resalable or as resalable after the end of the selling season, then its salvage value, $s$, is acquired by selling the item in a secondary market. The unit's salvage value must be less than or equal to the unit's purchasing cost $s \leq c$; otherwise the profit function would be unbounded above. Due to salvaging, the retailer will have no inventory after the end of the
selling season. Thus, the holding cost is not considered in this paper. Items that are purchased from or returned to the online store will cost the store a per-unit shipping expense of $t$.
$D_{r}$ and $D_{o}$ denote total customer sales within the physical store and the online store, respectively. It follows from sales certainty that a store will experience no further sales once this totality is met. Therefore, a channel will have no back orders. $Q_{r}$ and $Q_{o}$ are the order quantities placed at the physical store and the online store at the beginning of the selling season, respectively. Since the ordering process is not repetitive and is not related to the order quantity, then its related cost can be safely ignored. The parameter $\alpha$ represents the base level of sales, or the sales level when items are offered to customers free of charge (Chen, Zhang and Sun 2012; Huang, Yang and Zhang 2012). If $0 \leq \theta \leq 1$ is the degree of customer preference for the physical store, then $\alpha_{r}=\alpha \theta$ is the physical store's base level of sales. Similarly, if $1-\theta$ is the degree of customer preference for the online store, then $\alpha_{o}=\alpha(1-\theta)$ is the online store's base level of sales. According to Hua, Wang and Cheng (2010), different products lead to different degrees of customer preference for the physical store. For example, products that are customized, require a high level of examination prior to being purchased (such as used cars, clothes, shoes, or eyeglasses), or require after-sale services (such as electronics) better fit physical stores. In contradistinction, products that do not require a high level of examination in regards to their quality level prior to being purchased, standardized, or mature (such as books and CDs) better fit online stores. Physical and online store sales functions are given as:
$D_{r}=\alpha_{r}-\beta p_{r}+\gamma p_{o}$ and
$D_{o}=\alpha_{o}-\beta p_{o}+\gamma p_{r}$, respectively.
$\beta$ is an ownership-price sensitivity that measures the rate at which sales are affected by a channel's own price. $\gamma$ is the cross-price sensitivity that reflects the degree of cannibalization
between two channels. A channel's cross-price sensitivity has a lesser effect on sales than a channel's ownership-price sensitivity, which is $\gamma<\beta$. Linear sales functions in a dual-channel system were utilized in Ryan, Sun and Zhao (2013), Huang, Yang and Zhang (2012), Chen, Zhang and Sun (2012), Bin, Rong and Meidan (2010), and more.

Since the studied retailing system allows customer returns and a portion of those returns can be resold in the same selling season, then it is intuitive to see that a channel's order quantity is lower than its total sales. To further clarify this concept, an online store will sell its order quantity $\left(Q_{o}\right)$ and all of its same-channel resalable returns $\left(w k_{o} Q_{o}\right)$. Thus $D_{o}=Q_{o}\left(1+w k_{o}\right)$. The order quantity is given as the following:
$Q_{o}=\frac{D_{o}}{1+w k_{o}}$.
Due to the ratio $v$, a quantity of $v D_{o}$ is cross-returned from the online store to the physical store. A portion, $k_{o r}$ of this quantity, is resalable and can be resold once to satisfy part of the physical store's total sales $D_{r}$. Thus, the physical store will sell its order quantity $\left(Q_{r}\right)$, all of its same-channel resalable returns $\left(r k_{r} Q_{r}\right)$, and all of the cross-channel resalable returns $\left(v k_{o r} D_{o}\right)$. Consequently, $D_{r}=Q_{r}\left(1+r k_{r}\right)+D_{o} v k_{o r}$ and the order quantity is given as the following:

$$
\begin{equation*}
Q_{r}=\frac{D_{r}-D_{o} v k_{o r}}{1+r k_{r}} . \tag{2}
\end{equation*}
$$

Notice that the $D_{o} v k_{o r}$ is conditioned to be less than or equal to $D_{r}$ (i.e. $Q_{r} \geq 0$ ); otherwise the physical store would be overwhelmed by cross-channel returns that would allow the store to start its selling season without any quantity ordered from the supplier. Such a case is unrealistic; thus, its analytical complications are omitted from the calculations.

The following two sections examine the integration of a DCR under a centralized management using two pricing strategies: differential pricing mode and uniform pricing mode.

They also examine online and physical stores' equilibriums when the stores use three different competitive pricing schemes: Online-Leader Stackelberge game, Physical-Leader Stackelberge game and Nash game. In the Online-Leader Stackelberge game, a retailer's online store leads. It announces its selling price first and is followed by its physical store. In the Physical-Leader Stackelberge game, a retailer's physical store leads. The store announces its selling price first and is followed by its online store. However, in the Nash game both channels are equally powerful in price determination. Thus, they set their price strategies simultaneously. Table 1 presents a summary of the notations used in this paper.

Table 1: Notations

| Notation | Description |
| :--- | :--- |
| $\boldsymbol{r}$ | Probability an item purchased from a physical store is returned to the physical |
|  | store |
| $\boldsymbol{w}$ | Probability an item purchased from an online store is returned to the online store |
| $\boldsymbol{v}$ | Probability an item purchased from an online store is cross-returned to a physical |
| $\boldsymbol{k}_{\boldsymbol{r}}$ | Ptore |
| $\boldsymbol{k}_{\boldsymbol{o}}$ | Probability an item purchased from and returned to a physical store is resalable an item purchased from and returned to an online store is resalable |
| $\boldsymbol{k}_{\boldsymbol{o r}}$ | Probability an item purchased from an online store and cross-returned to a |
| $\boldsymbol{c}$ and $\boldsymbol{s}$ | Unit purchasing cost and salvage value, respectively |
| $\boldsymbol{d}$ and $\boldsymbol{t}$ | Return collection and shipping costs, respectively |
| $\hat{\boldsymbol{c}}$ | Amount a physical store pays to an online store for every cross-channel return in |

the decentralization scheme
$\boldsymbol{D}_{\boldsymbol{r}}$ and $\boldsymbol{D}_{\boldsymbol{o}}$ Retail and online stores' total sales including returns, respectively
$\boldsymbol{Q}_{\boldsymbol{r}}$ and $\boldsymbol{Q}_{\boldsymbol{o}} \quad$ Quantities ordered by retail and online stores, respectively
$\boldsymbol{\alpha}, \boldsymbol{\alpha}_{\boldsymbol{r}}$ and $\quad$ Enterprise, physical store and online store base levels of sale, respectively
$\alpha_{o}$
$\boldsymbol{\theta} \quad$ Customer preference for the physical store
$\boldsymbol{\beta}$ and $\boldsymbol{\gamma} \quad$ Ownership price and cross-price sensitivities of a channel, respectively
$\boldsymbol{p}_{\boldsymbol{r}}$ and $\boldsymbol{p}_{\boldsymbol{o}} \quad$ Retail and online store's prices, respectively
$\boldsymbol{\pi}, \boldsymbol{\pi}_{\boldsymbol{r}}$ and Enterprise, physical store and online store profits, respectively
$\pi_{o}$
4. CENTRALIZED DUAL-CHANNEL RETAILING SYSTEM

This section studies pricing policies in a centralized system wherein a retailer's physical and online stores are vertically integrated. One may assume the existence of a central decision maker who pursues the maximum total supply chain profit $(\pi)$. The central decision maker simultaneously determines the physical store's price, $p_{r}$, and the online store's price, $p_{o}$, to meet the retailer's goals and objectives.

The online store's profit function is modeled as the following:
$\pi_{o}^{c}=D_{o}\left[(1-w-v) p_{o}-t-w(d+t)+w\left(1-k_{o}\right) s+s \frac{\left(w k_{o}\right)^{2}}{1+w k_{o}}\right]-Q_{o} c$.
A portion from $D_{o},(1-w-v)$, is a final sale and contributes positively. Every sold item contributes negatively due to the shipped cost $t$ paid by the store. A $w$ portion from $D_{o}$ is returned to the online store and contributes negatively due to collection and shipping costs. A portion of $w\left(1-k_{o}\right)$ from $D_{o}$ is salvaged and contributes positively as it is returned as non-
resalable. The term $s \frac{\left(w k_{o}\right)^{2}}{1+w k_{o}}$ assures the salvaging of an item that end up being returned as resalable after the end of the selling season. The second term is the ordering cost for the quantity assigned to the online store.

The physical store's profit function is modeled as:
$\pi_{r}^{c}=D_{r}\left[(1-r) p_{r}-r d+r\left(1-k_{r}\right) s+s \frac{\left(r k_{r}\right)^{2}}{1+r k_{r}}\right]+v D_{o}\left[-d+\left(1-k_{o r}\right) s+s \frac{k_{o r} r k_{r}}{1+r k_{r}}\right]-Q_{r} c$.

In the first term, a portion of $(1-r)$ from $D_{r}$ is a final sale and contributes positively, a portion of $r$ from $D_{r}$ is returned to the physical store and contributes negatively due to its collection cost, and a portion of $r\left(1-k_{r}\right)$ from $D_{r}$ is salvaged and contributes positively as it is returned as a non-resalable item. The term $s \frac{\left(r k_{r}\right)^{2}}{1+r k_{r}}$ assures the salvaging of items that end up being resalable returns after the end of the selling season if the system experience no crosschannel returns. In the second term, a portion of $v$ from $D_{o}$ is cross-returned to the physical store and contributes negatively due to its collection cost. A portion of $v\left(1-k_{o r}\right)$ from $D_{o}$ is salvaged and contributes positively as it is cross-returned as a non-resalable item. The term $s \frac{k_{o r} r k_{r}}{1+r k_{r}}$ considers the increment in salvaged resalable returns at the physical store when the system experience cross-channel returns. The third term is the ordering cost for the items assigned to the physical store.

The total supply chain profit function can be modeled as:

$$
\begin{align*}
& \pi=\pi_{o}^{c}+\pi_{r}^{c}=D_{o}\left[(1-w-v) p_{o}-t-w(d+t)+w\left(1-k_{o}\right) s+s \frac{\left(w k_{o}\right)^{2}}{1+w k_{o}}-v d+\right. \\
& \left.v\left(1-k_{o r}\right) s+v s \frac{k_{o r} r k_{r}}{1+r k_{r}}\right]+D_{r}\left[(1-r) p_{r}-r d+r\left(1-k_{r}\right) s+s \frac{\left(r k_{r}\right)^{2}}{1+r k_{r}}\right]-Q_{o} c-Q_{r} c . \tag{5}
\end{align*}
$$

By replacing the quantity $Q_{o}$ with its function (1) and the quantity $Q_{r}$ with its function (2), the total supply chain profit can be transformed into the following:
$\pi=D_{o}\left[(1-w-v) p_{o}-t-w(d+t)+w\left(1-k_{o}\right) s+\frac{s\left(w k_{o}\right)^{2}-c}{1+w k_{o}}+v\left(\left(1-k_{o r}\right) s-d+\right.\right.$
$\left.\left.\frac{k_{o r}\left(c+s\left(r k_{r}\right)^{2}\right)}{1+r k_{r}}\right)\right]+D_{r}\left[(1-r) p_{r}-r d+r\left(1-k_{r}\right) s+\frac{s\left(r k_{r}\right)^{2}-c}{1+r k_{r}}\right]$.
One may reformulate the profit function (6) as the following:
$\pi=D_{o}\left[J p_{o}-B\right]+D_{r}\left[I p_{r}-A\right]=D_{o}\left[R E V_{o}^{c}\right]+D_{r}\left[R E V_{r}^{c}\right]$
Where;
$I=1-r>0, J=1-w-v>0, A=r d-r\left(1-k_{r}\right) s-\frac{s\left(r k_{r}\right)^{2}-c}{1+r k_{r}}$, and
$B=t+w(d+t)-w\left(1-k_{o}\right) s-\frac{s\left(w k_{o}\right)^{2}-c}{1+w k_{o}}-v\left(\left(1-k_{o r}\right) s-d+\frac{k_{o r}\left(c+s\left(r k_{r}\right)^{2}\right)}{1+r k_{r}}\right)$.
Notice that $R E V_{o}^{c}$ and $R E V_{r}^{c}$ is the revenue generated by satisfying a single sale from the online store and physical store, respectively. Thus, the optimal solution is subjected to the following constraints:
$D_{o} \geq 0, D_{r} \geq 0,\left[R E V_{o}^{c}\right] \geq 0,\left[R E V_{r}^{c}\right] \geq 0$, and $Q_{r} \geq 0$.
Section 4.1 presents an analysis of a situation wherein a central decision maker adopts a differential pricing strategy or does not add any constraint to prices. Section 4.2 studies a situation wherein a central decision maker adopts a unified pricing strategy or constrains prices so that they are equal.

### 4.1 DCRS under the Differential Pricing Strategy

It has been argued that differential pricing is the optimal strategy when higher prices are assigned to the channel with the highest operational costs (Zhang et al. 2010 and Yan 2008).

Neslin et al. (2006) have also argued in favour of differential pricing, but with higher prices
assigned to the channel with the lowest price-sensitive customers. However, several other authors have argued that a unified pricing strategy is not optimal for a DCR and that a channel's pricing strategy should be proportional to its customer's preference and its provided services (Chen, Zhang and Sun 2012; Dan, Xu and Liu 2012; Hua, Wang and Cheng 2010). Thus, this section investigates the effect customer preference and rates of return have on pricing policies when a sole DCR's manager chooses to run its enterprise using the differential pricing strategy.

Proposition 1: Under the differential pricing strategy, the profit function $\pi$ is strictly and jointly concave in $p_{o}$ and $p_{r}$, given that $4 \beta^{2} I J>\gamma^{2}(I+J)^{2}$. The system will perform at its best with the physical store's optimal price of $p_{C_{r}}^{*}$ and the online store's optimal price of $p_{C_{o}}^{*}$ :
$p_{C_{r}}^{*}=\frac{\left(\gamma J \alpha_{o}-\gamma^{2} A\right)(I+J)+\gamma \beta B(I-J)+2 J\left(\beta I \alpha_{r}+\beta^{2} A\right)}{4 J I \beta^{2}-\gamma^{2}(I+J)^{2}}$, and
$p_{C_{o}}^{*}=\frac{\left(\gamma I \alpha_{r}-\gamma^{2} B\right)(I+J)-\gamma \beta A(I-J)+2 I\left(\beta J \alpha_{o}+\beta^{2} B\right)}{4 J I \beta^{2}-\gamma^{2}(I+J)^{2}}$.
Proof: By substituting sales functions into (7), one may construct the Hussein matrix for the profit function as the following:
$H_{C}=\left[\begin{array}{cc}\frac{\partial^{2} \pi}{\partial p_{r}^{2}} & \frac{\partial^{2} \pi}{\partial p_{r} p_{o}} \\ \frac{\partial^{2} \pi}{\partial p_{o} p_{r}} & \frac{\partial^{2} \pi}{\partial p_{o}^{2}}\end{array}\right]=\left[\begin{array}{cc}-2 \beta I & \gamma(I+J) \\ \gamma(I+J) & -2 \beta J\end{array}\right]$.
Since $\frac{\partial^{2} \pi}{\partial p_{r}^{2}}<0$, the profit function is strictly and jointly concave in $p_{o}$ and $p_{r}$ given that $\left|H_{C}\right|>0$ or $4 \beta^{2} I J>\gamma^{2}(I+J)^{2}$. Intuitively, $p_{C_{r}}^{*}$ and $p_{C_{o}}^{*}$ can be found by simultaneously solving the firstorder conditions $\frac{\partial \pi}{\partial p_{r}}=0$ and $\frac{\partial \pi}{\partial p_{o}}=0$.

From Proposition 1, one may guarantee the existence of a unique optimal set of prices if the stated condition is satisfied. However, this condition may not be satisfied if $\gamma$ is very close to $\beta$ and the total return rate of a channel is much higher than what it is for the other channel. Those
cases are less likely to occur since $\gamma$ is expected to be much less than $\beta$. Also, a channel with excessive total return rate will, most likely, be eliminated or its return policy will, at least, be changed.

If we differentiate the optimal prices with respect to $\theta$ we get $\frac{\partial p_{C_{r}}^{*}}{\partial \theta}=\frac{\alpha J(2 \beta I-\gamma(I+J))}{4 J I \beta^{2}-\gamma^{2}(I+J)^{2}}$, $\frac{\partial p_{C_{o}}^{*}}{\partial \theta}=-\frac{\alpha I(2 \beta J-\gamma(I+J))}{4 J I \beta^{2}-\gamma^{2}(I+J)^{2}}$, and $\left|\frac{\partial p_{C_{r}}^{*}}{\partial \theta}\right|-\left|\frac{\partial p_{C_{o}}^{*}}{\partial \theta}\right|=\frac{\alpha \gamma(I+J)(I-J)}{4 J I \beta^{2}-\gamma^{2}(I+J)^{2}}$. It shows that the optimal price for a certain channel will not always increase as customers' preference for that channel increases with customer returns. It could instead increase or decrease depending on the signs $2 \beta I-$ $\gamma(I+J)$ and $2 \beta J-\gamma(I+J)$ for the physical store and the online store, respectively. That is, managers should not assume that higher customer preference for a certain channel drives prices in that channel up; they must first consider customer returns. The result for DCR with return is different from the previous observations on pricing strategies for DCR without returns: a higher base level of demand in a single sale channel leads to a higher selling price if customer returns are not considered (Dan, Xu and Liu 2012 and Hua, Wang and Cheng 2010). Also, as $\theta$ increases, the online store is found to have a higher corresponding rate of change in its optimal price than the physical store if $w+v<r$, an identical rate if $w+v=r$, and a lower rate if $w+$ $v>r$. If $\theta=\bar{\theta}$ and $0 \leq \bar{\theta} \leq 1$, then it is optimal for both channels to have a similar pricing strategy, i.e. $p_{C_{r}}^{*}=p_{C_{o}}^{*}$, where
$\bar{\theta}=\frac{\alpha J(2 \beta J-\gamma(I+J))-\gamma \beta(I-J)(A+B)+2 \beta^{2}(I B-J A)+\gamma^{2}(I+J)(A-B)}{\alpha\left(4 \beta I J-\gamma(I+J)^{2}\right)}$.
Note that $\bar{\theta}$ will mostly lie out of range if $\frac{\partial p_{C_{r}}^{*}}{\partial \theta}$ and $\frac{\partial p_{C_{o}}^{*}}{\partial \theta}$ are either positive or negative. One may observe that customer preference for a certain channel has a significant impact on the optimal prices of channels.

### 4.2 DCRS under the Unified Pricing Strategy

Webb and Lambe (2007) have stated that pricing strategy causes most of the conflicts that arise between channels. In addition, several authors have stated that one may avoid customer confusion and retain a business's image by using a unified price across all channels (Neslin and Shankar 2009; Webb and Lambe 2007; Berman and Thelen 2004). Consequently, $80 \%$ of all multichannel retailers choose to unify their pricing strategies across all channels (Ofek, Katona and Sarvary 2009). Thus, this section investigates the effect that customer preference and rates of return have on pricing policies when a sole DCR's manager choses to run the enterprise with a unified pricing strategy. Due to the added constraint (i.e., $p_{r}=p_{o}=p$ ), it is trivial that the profit generated by the unified pricing strategy is less than or equal to the profit generated by the differential pricing strategy.

Proposition 2: Under the unified pricing strategy, the profit function $\pi$ is strictly concave in $p$. Thus, there a unique optimal solution of $p_{U}^{*}$ exists that derives the optimal profit $\pi_{U}^{*}$ :
$p_{U}^{*}=\frac{1}{2}\left(\frac{(\beta-\gamma)(A+B)+\alpha_{o} J+\alpha_{r} I}{(\beta-\gamma)(J+I)}\right)$.
Proof: By substituting sales functions into (7) and constraining prices to be equivalent, i.e. $p_{r}=$ $p_{o}=p$, one may find that $\frac{\partial^{2} \pi}{\partial p^{2}}=-2(\beta-\gamma)(I+J)<0$. Thus, the profit function is strictly concave in $p$. The optimal unified price can be calculated by solving the first-order condition $\frac{\partial \pi}{\partial p}=0$.

From Proposition 2, one may differentiate the optimal unified price with respect to $\theta$ to get $\frac{\partial p_{U}^{*}}{\partial \theta}=\frac{\alpha}{2}\left(\frac{I-J}{(\beta-\gamma)(J+I)}\right)$. Thus, the optimal price will increase as $\theta$ increases under the condition $r<v+w$, and will decrease as $\theta$ increases under the condition $r>v+w$. Intuitively, the change in $\theta$ has no effect on the DCR's pricing strategy when $r=v+w$. One may notice that
the decision to increase or decrease the unified price solely depends on the values $r, v$, and $w$. This places an emphasis on customer returns when one selects pricing policies for dual-channel retailing systems.

## 5. DECENTRALIZED DUAL-CHANNEL RETAILING SYSTEM

According to Zhang et al. (2010), "most retail corporations manage their channels in a decentralized fashion and many of them maintain separate teams of inventory management." Falk et al. (2007) claim that integration may not be optimal if it is associated with a high implementation cost. As previously stated, a failure to centralize or integrate a DCR will trigger price and service competition that is normally initiated by cannibalization. Notice that a crosschannel return policy allows online stores to increase both sales and customer satisfaction and allows physical stores to create cross-selling opportunities. Assume that $\hat{c}$ is the amount a physical store pays to an online store for every cross-channel return. If $\hat{c}$ is constructed fairly, then it is of all channels' best interest to accept such a return policy. Thus, there is no contradiction between having a cross-channel return as an accepted practice and the fact that competition takes place between channels.

The performance of the competing channels is studied using two sequential games, namely the Online-Leader Stackelberge game, discussed in Section 5.1, and the Physical-Leader Stackelberge game, discussed in Section 5.2, and one simultaneous game, namely the Nash game, discussed in Section 5.3. Yan et al. (2010), Yan (2010), and Yan (2008) have stated that Target, Nike, and Kohl's are all good candidates for Stackelberge competition. They have also stated that a Stackelberge game always outperform a Nash game. Similarly, Lu and Liu (2013) have argued that a Stackelberge game influences the profitability of channels more effectively than a Nash game. In a competitive environment, each channel forms its own decision in
isolation to maximize its individual profit. One may assume that all sales function parameters, return rates, cost parameters, and decision rules are known to both competitors.

Due to decentralization, the profit functions below are constructed in a manner similar to formulas (3) to (7), with the exception that $\hat{c}$ is included in the formulation.
$\pi_{o}=D_{o}\left[(1-w-v) p_{o}-t-w(d+t)+w\left(1-k_{o}\right) s+v \hat{c}+\frac{s\left(w k_{o}\right)^{2}-c}{1+w k_{o}}\right]$
$\pi_{r}=D_{r}\left[(1-r) p_{r}-r d+r\left(1-k_{r}\right) s+\frac{s\left(r k_{r}\right)^{2}-c}{1+r k_{r}}\right]+v D_{o}\left[\frac{k_{o r}\left(c+s r k_{r}\right)}{1+r k_{r}}+\left(1-k_{o r}\right) s-\hat{c}-d\right]$

One may reformulate the profit functions (12) and (13) respectively as the following:
$\pi_{o}=D_{o}\left[J p_{o}-G\right]=D_{o}\left[R E V_{o}^{d}\right]$
$\pi_{r}=D_{r}\left[I p_{r}-A\right]+v D_{o} F=D_{r}\left[R E V_{r}^{d}\right]+v D_{o} F$
Where;
$G=t+w(d+t)-w\left(1-k_{o}\right) s-v \hat{c}-\frac{s\left(w k_{o}\right)^{2}-c}{1+w k_{o}}$, and $F=\frac{k_{o r}\left(c+s r k_{r}\right)}{1+r k_{r}}+\left(1-k_{o r}\right) s-\hat{c}-d$.
$F$ represents the savings or losses the physical store makes by accepting each cross-channel return. One may subject the optimal solution to the following constraints:
$D_{o} \geq 0, D_{r} \geq 0,\left[R E V_{o}^{d}\right] \geq 0,\left[R E V_{r}^{d}\right] \geq 0$, and $Q_{r} \geq 0$.
Since each channel aims to maximize it own profit in the competitive setting, the online store may over estimate the value of cross-channel returns $\hat{c}$. In return, the physical store may stop cooperating with the online store. Such a lack of cooperation may create havoc to the system and cause unnecessary practices such as returning all cross-channel returns back to the online store. Therefore, the following condition on the value of $\hat{c}$ should be satisfied:
$\hat{c} \leq \frac{k_{o r} c}{1+r k_{r}}+\frac{k_{o r} r r k_{r}}{1+r k_{r}}+\left(1-k_{o r}\right) s-d$

The right-hand side of the above relationship represents how a physical store should consider a cross-channel return. The first term denotes the physical store's valuation of a resalable crosschannel return. Since an item purchased by the physical store at the beginning of the selling season can satisfy $\left(1+r k_{r}\right)$ sales, it is worth a value of $c$. In contradistinction, since a resalable cross-channel return can only satisfy one sale it is worth a value of $\frac{c}{1+r k_{r}}$. The second term calculates the increase in salvaged resalable returns at the end of the selling season caused by each resalable cross-channel return. The third term denotes the physical store's gain, due to salvaging, from a non-resalable cross-channel return. The fourth term denotes the physical store's loss, due to the collection cost, from each cross-channel return.

### 5.1 DCRS under the Online-Leader Stackelberge Game (OLSG)

In contrast to the physical store, forming a customer base for the online store is not limited to the store's neighbourhood. Also, due to the advancement in cellular phones and IT, customers of a DCR may always check the prices of an online store before they conduct their purchases from a physical store. Additionally, online stores are normally considered to be the distribution centers of enterprises. Therefore, they can start the selling season before their competitors. For the aforementioned facts, the online store is considered to have more price influence on customers compared to the physical store. Thus, a retailer's online store will lead and its physical store will follow. In this game, the physical store optimizes its performance based on the online store's optimal price. The online store optimizes its performance based on the physical store's best response function.

Proposition 3: The physical store price of $p_{O_{r}}^{*}$ and the online store price of $p_{O_{o}}^{*}$ form the sole equilibrium solution for the Online-Leader Stackelberge Game:
$p_{O_{r}}^{*}\left(p_{o}\right)=\frac{1}{2}\left(\frac{\alpha_{r}}{\beta}+\frac{A}{I}+\frac{F v \gamma}{\beta I}+\frac{\gamma}{\beta} p_{o}\right)$
$p_{O_{o}}^{*}=\frac{G}{2 J}+\frac{\alpha_{o} \beta}{\left(2 \beta^{2}-\gamma^{2}\right)}+\frac{\alpha_{r} \gamma}{2\left(2 \beta^{2}-\gamma^{2}\right)}+\frac{A \beta \gamma}{2 I\left(2 \beta^{2}-\gamma^{2}\right)}+\frac{v \gamma^{2} F}{2 I\left(2 \beta^{2}-\gamma^{2}\right)}$.
Proof: Substitute sales functions into (14) and (15). Since $\frac{\partial^{2} \pi_{r}}{\partial p_{r}^{2}}=-2 \beta I<0$, then the physical store's profit function $\left(\pi_{r}\right)$ is strictly concave in $p_{r}$. Thus, given the online store's price $\left(p_{o}\right)$, the physical store's optimal price of $p_{O_{r}}^{*}\left(p_{o}\right)$ can be found by solving the first-order condition $\frac{\partial \pi_{r}}{\partial p_{r}}=$ 0. Substitute $p_{O_{r}^{*}}^{*}\left(p_{o}\right)$ into Eq. (14). Since $\beta \geq \gamma$, then $\frac{\partial^{2} \pi_{o}}{\partial p_{o}^{2}}=\frac{J\left(\gamma^{2}-2 \beta^{2}\right)}{\beta}$ is strictly negative. That is to say, given the physical store's best response function, the online store's profit function ( $\pi_{o}$ ) is strictly concave in $p_{o}$. This guarantees the existence of a unique equilibrium for the OLSG. To find the online store's optimal price of $p_{O_{o}^{*}}^{*}$ the first-order condition $\frac{\partial \pi_{o}}{\partial p_{o}}=0$ is solved. One may substitute $p_{O_{o}}{ }_{o}^{*}$ into Eq. (16) to get $p_{O}{ }_{r}^{*}$. The physical store and the online store equilibrium profits are denoted by $\pi_{O_{r}}^{*}$ and $\pi_{O_{o}}^{*}$, respectively.

If follows from Proposition 3 that by differentiating the equilibrium prices with respect to $\theta$ one may get: $\frac{\partial p_{O_{O}}^{*}}{\partial \theta}=-\frac{\alpha}{2}\left(\frac{2 \beta-\gamma}{2 \beta^{2}-\gamma^{2}}\right)<0, \frac{\partial p_{O_{r}}^{*}}{\partial \theta}=\frac{\alpha}{4 \beta}\left(\frac{4 \beta^{2}-2 \beta \gamma-\gamma^{2}}{2 \beta^{2}-\gamma^{2}}\right)>0$, and $\left|\frac{\partial p_{O_{o}}^{*}}{\partial \theta}\right|-\left|\frac{\partial p_{O_{r}}^{*}}{\partial \theta}\right|=$ $\frac{\alpha \gamma^{2}}{4 \beta\left(2 \beta^{2}-\gamma^{2}\right)}>0$. The aforementioned relationships indicate that a physical store's optimal price will increase as $\theta$ increases, while an online store's optimal price will decrease as $\theta$ increases. The follower's (physical store's) pricing strategy is always less affected by the change in $\theta$ than the leader's (online store's) pricing strategy. Also, $\frac{\partial p_{o_{r}^{*}}^{*}\left(p_{o}\right)}{\partial p_{o}}=\frac{\gamma}{2 \beta}>0$ or if the online store's best response increases by a single unit, then the physical store's best response will increase by half a unit at the most. Dan, Xu and Liu (2012) came to a similar conclusion for a dual-channel system without customer returns. This in fact shows how much control the leader has over the follower.

### 5.2 DCRS under the Physical-Leader Stackelberge Game (PLSG)

Similar to Yan et al. (2010) and Yan (2008), this section models a game wherein a physical store is the price leader due to its prevailing market power. Thus, the online store optimizes its performance based on the physical store's optimal price, while the physical store optimizes its performance based on the online store's best response function.

Proposition 4: The physical store price of $p_{R_{r}}^{*}$ and the online store price of $p_{R_{o}}^{*}$ form a unique equilibrium solution for the Physical-Leader Stackelberge Game:
$p_{R_{o}}^{*}\left(p_{r}\right)=\frac{1}{2}\left(\frac{\alpha_{o}}{\beta}+\frac{G}{J}+\frac{\gamma}{\beta} p_{r}\right)$
$p_{R_{r}}{ }^{*}=\frac{A}{2 I}+\frac{\alpha_{r} \beta}{\left(2 \beta^{2}-\gamma^{2}\right)}+\frac{\gamma \alpha_{o}}{2\left(2 \beta^{2}-\gamma^{2}\right)}+\frac{G \beta \gamma}{2 J\left(2 \beta^{2}-\gamma^{2}\right)}+\frac{\beta v \gamma F}{2 I\left(2 \beta^{2}-\gamma^{2}\right)}$.
Proof: Since $\frac{\partial^{2} \pi_{o}}{\partial p_{o}^{2}}=-2 \beta J<0$, then the online store's profit function $\left(\pi_{o}\right)$ is strictly concave in $p_{o}$. Thus, given the physical store's price $\left(p_{r}\right)$, the online store's optimal price of $p_{R_{o}}{ }^{*}\left(p_{r}\right)$ can be found by solving the first-order condition $\frac{\partial \pi_{o}}{\partial p_{o}}=0$. Substitute $p_{R_{o}}^{*}\left(p_{r}\right)$ into Eq. (15). Since $\beta \geq$ $\gamma$, then $\frac{\partial^{2} \pi_{r}}{\partial p_{r}^{2}}=\frac{I\left(\gamma^{2}-2 \beta^{2}\right)}{\beta}$ is strictly negative. Thus, given the online store's best response function, the physical store's profit function $\left(\pi_{r}\right)$ is strictly concave in $p_{r}$. This guarantees the existence of a unique equilibrium for the PLSG. To find the physical store's optimal price of $p_{R_{r}}{ }_{r}$ the firstorder condition $\frac{\partial \pi_{r}}{\partial p_{r}}=0$ is solved. One may substitute $p_{R_{r}}{ }_{r}$ into Eq. (18) to get $p_{R_{o}}{ }^{*}$. The physical store and the online store equilibrium profits are denoted by $\pi_{R_{r}}^{*}$ and $\pi_{R_{o}}^{*}$, respectively.

If follows from Proposition 4 that by differentiating the equilibrium prices with respect to $\theta$ one get: $\frac{\partial p_{R_{r}}^{*}}{\partial \theta}=\frac{\alpha}{2}\left(\frac{2 \beta-\gamma}{2 \beta^{2}-\gamma^{2}}\right)>0, \quad \frac{\partial p_{R_{o}}^{*}}{\partial \theta}=-\frac{\alpha}{4 \beta}\left(\frac{4 \beta^{2}-2 \beta \gamma-\gamma^{2}}{2 \beta^{2}-\gamma^{2}}\right)<0$, and $\left|\frac{\partial p_{R_{r}}^{*}}{\partial \theta}\right|-\left|\frac{\partial p_{R_{o}}^{*}}{\partial \theta}\right|=$ $\frac{\alpha \gamma^{2}}{4 \beta\left(2 \beta^{2}-\gamma^{2}\right)}>0$. The previous relationships indicate that a physical store's optimal price will
increase as $\theta$ increases, while an online store's optimal price will decrease as $\theta$ increases. Notice that the leaders in both Stackelberge games have the same rate of change. Similarly, the followers in both Stackelberge games have the same rate of change. Consequently, the follower's (online store's) pricing strategy is always less affected by the change in $\theta$ than the leader's (physical store's) pricing strategy. Also, $\frac{\partial p_{R_{o}}^{*}\left(p_{r}\right)}{\partial p_{r}}=\frac{\gamma}{2 \beta}>0$ or if the physical store's best response increases by a single unit, then the online store's best response will increase by half a unit at the most.

### 5.3 DCRS under the Nash Game

In a dual-channel Nash game, online and physical stores are equally powerful. The market has no price leader. Thus, prices are selected simultaneously in both channels. In this game, each store optimizes its performance given the rival's price.

Proposition 5: A unique Nash equilibrium exists under the physical store's price, $p_{N_{r}}^{*}$, and the online store's price, $p_{N_{o}}^{*}$ :
$p_{N_{r}}^{*}=\left(\frac{1}{4 \beta^{2}-\gamma^{2}}\right)\left(2 \beta \alpha_{r}+\gamma \alpha_{o}+\frac{4 \beta^{3} G}{J \gamma}+\frac{2 \beta^{2} A}{I}+\frac{2 v \beta \gamma F}{I}\right)-\frac{\beta G}{J \gamma}$
$p_{N_{o}}^{*}=\left(\frac{1}{4 \beta^{2}-\gamma^{2}}\right)\left(2 \beta \alpha_{o}+\gamma \alpha_{r}+\frac{2 \beta^{2} G}{J}+\frac{\beta \gamma A}{I}+\frac{v \gamma^{2} F}{I}\right)$.
Proof: The Game's Hussein matrix is formed as the following:
$H_{N}=\left[\begin{array}{cc}\frac{\partial^{2} \pi_{r}}{\partial p_{r}^{2}} & \frac{\partial^{2} \pi_{r}}{\partial p_{r} \partial p_{o}} \\ \frac{\partial^{2} \pi_{o}}{\partial p_{o} \partial p_{r}} & \frac{\partial^{2} \pi_{o}}{\partial p_{o}^{2}}\end{array}\right]=\left[\begin{array}{cc}-2 I \beta & I \gamma \\ J \gamma & -2 J \beta\end{array}\right]$.
Notice that each player's profit function is concave on the player's own decision variable, i.e. $\frac{\partial^{2} \pi_{r}}{\partial p_{r}^{2}}=-2 I \beta<0$ and $\frac{\partial^{2} \pi_{o}}{\partial p_{o}^{2}}=-2 J \beta<0$. Also, the determinant for the Hussein matrix is strictly positive, i.e. $\left|H_{N}\right|=I J\left(4 \beta^{2}-\gamma^{2}\right)>0$. Therefore, there exist a unique Nash equilibrium.

Consequently, given the competitor's price, a channel can find its own pricing strategy by solving the first-order condition as follows:
from $\frac{\partial \pi_{r}}{\partial p_{r}}=0$ we have $p_{r}\left(p_{o}\right)=\frac{\beta A+I \alpha_{r}+v \gamma F+I \gamma p_{o}}{2 \beta I}$, and
from $\frac{\partial \pi_{o}}{\partial p_{o}}=0$ we have $p_{o}\left(p_{r}\right)=\frac{\beta G+J \alpha_{o}+J \gamma p_{r}}{2 \beta J}$.
By solving the above system of equations simultaneously, one may find the physical store's and the online store's equilibrium prices. The physical store and the online store equilibrium profits are denoted by $\pi_{N_{r}}^{*}$ and $\pi_{N_{o}}^{*}$, respectively.

In response to Proposition 5, one may test the reaction of equilibrium prices when $\theta$ changes. Thus, the following differentiations are taken: $\frac{\partial p_{N_{r}}^{*}}{\partial \theta}=\frac{2 \beta \alpha-\gamma(\alpha-1)}{4 \beta^{2}-\gamma^{2}}$ and $\frac{\partial p_{N_{O}}^{*}}{\partial \theta}=$ $\frac{\gamma \alpha-2 \beta(\alpha-1)}{4 \beta^{2}-\gamma^{2}}$. Since $(\alpha-1) \approx \alpha$, then $\frac{\partial p_{N_{r}}^{*}}{\partial \theta} \approx \frac{\alpha(2 \beta-\gamma)}{4 \beta^{2}-\gamma^{2}}>0$ and $\frac{\partial p_{N_{o}}^{*}}{\partial \theta} \approx \frac{-\alpha(2 \beta-\gamma)}{4 \beta^{2}-\gamma^{2}}<0$. Thus, the physical store's optimal price will increase and the online store's optimal price will decrease as $\theta$ increases. Similar to the PLSG, $\left|\frac{\partial p_{N_{r}}^{*}}{\partial \theta}\right|-\left|\frac{\partial p_{N_{o}}^{*}}{\partial \theta}\right|=\frac{(2 \alpha-1)(2 \beta-\gamma)}{4 \beta^{2}-\gamma^{2}}>0$ or the online store's pricing strategy is less affected by the change in $\theta$ than the physical store's pricing strategy. Due to functions complexity, it is difficult to carry on a comparison between a channel's price and profitability under the different games. Thus, the comparison is done in the sensitivity analysis.

Lemma 1: In each competition scheme, there exists a threshold such that if $\theta$ is equivalent to that threshold, then the equilibrium prices are set equally.

## Proof:

If $\theta=\check{\theta}$ in the OLSG, then $p_{O_{o}}^{*}=p_{O_{r}}^{*}$; where
$\check{\theta}=\frac{2 \beta\left(2 \beta^{2}-\gamma^{2}\right)}{\alpha\left(8 \beta^{2}-\gamma^{2}-4 \beta \gamma\right)}\left(\frac{G}{J}-\frac{A}{I}-\frac{G \gamma}{2 J \beta}+\frac{4 I \beta \alpha+2 A \beta \gamma-2 I \alpha \gamma-A \gamma^{2}+2 v \gamma^{2} F}{2 I\left(2 \beta^{2}-\gamma^{2}\right)}-\frac{v \gamma^{3} F}{2 \beta I\left(2 \beta^{2}-\gamma^{2}\right)}-\frac{F v \gamma}{\beta I}\right)$.
If $\theta=\breve{\theta}$ in the PLSG, then $p_{R_{o}}^{*}=p_{R_{r}}^{*}$; where
$\breve{\theta}=\left(2+\frac{2 G \beta}{J \alpha}-\frac{2 A \beta}{I \alpha}+\frac{A \gamma}{I \alpha}\right) \frac{\left(2 \beta^{2}-\gamma^{2}\right)}{2\left(2 \beta^{2}-\gamma^{2}\right)+(2 \beta-\gamma)^{2}}-\left(\gamma+\frac{G \beta \gamma}{J \alpha}+\frac{\beta v \gamma F}{I \alpha}\right) \frac{(2 \beta-\gamma)}{2\left(2 \beta^{2}-\gamma^{2}\right)+(2 \beta-\gamma)^{2}}$.
If $\theta=\tilde{\theta}$ in the Nash game, then $p_{N_{r}}^{*}=p_{N_{o}}^{*}$; where
$\tilde{\theta}=\frac{1}{2 \alpha}\left(\alpha+\frac{\beta G}{J}-\frac{\beta A}{I}-\frac{v \gamma F}{I}\right)$.
A similar outcome can be found in the studies conducted by Dan, Xu and Liu (2012), Chen, Zhang and Sun (2012), and Hua, Wang and Cheng (2010). It follows from Lemma 1 that if customer preference for the physical store is lower than the threshold, then the selling price in the physical store is lower than the selling price in the online store. If customer preference for the physical store is higher than the threshold, then the selling price in the physical store is higher than the selling price in the online store. For example, when remanufactured or used items are offered for sale, customers are most likely eager to verify the quality of the offered items before completing a purchase. Thus higher prices should be offered in the physical store. While changing the $\theta$ value may impose a different outcome on the decentralized setting than on the centralized setting, in both cases it significantly impacts pricing decisions.

## 6. SENSITIVITY ANALYSIS

This numerical study aims to provide several key managerial insights by answering the following questions: Does a unified pricing strategy under centralized management have a higher total profit than competing dual channels? If not, under what conditions is this statement not correct? The latter's answer leads to the following question: Under what competition setting and conditions is the total performance best? How does a channel's pricing strategy compare to different cases? This study uses the following parameters:
$\hat{c}=\frac{k_{o r}\left(c+s r k_{r}\right)}{1+r k_{r}}+\left(1-k_{o r}\right) s-d, c=30, s=10, d=2, t=4, r=0.2, w=0.2, v=0.2$, $k_{r}=0.6, k_{o}=0.4, k_{o r}=0.4, \theta=\{0.45,0.65\}, \gamma=5, \beta=10$ and,$\alpha=15 k$.

### 6.1 Total System Performance under Unified-Pricing Strategy and Competition

If the centralization process eliminates conflict by including the unification of selling prices across all channels (Yan 2010), then an enterprise may be better off with uncoordinated channels. As presented in Figure 2a, when customer preference for the physical store, $\theta$, and the physical store's rate of return, $r$, are sufficiently high, competition between channels leads to a better total supply chain performance. Similarly, when customer preference for the online channel, $1-\theta$, and the online store's same-channel rate of return, $w$, are sufficiently high, an enterprise should encourage competition rather than coordination (Figure 2b). Indeed, embracing a sole price will reduce channel conflict but deprive the system of agility. That is, it is difficult for an enterprise to divert sales from a high return-rate channel to a low return-rate channel. It should be noted that centralization with a differential pricing strategy has not been considered in this section. Similar to Yan's 2008 and 2010 findings and Yan et al.'s 2010 findings, such a setting will lead to the best system performance for all applicable parameters, especially when coordination cost is not considered.


Figure 2: Total profit comparisons between centralization and decentralization with unified pricing scheme

### 6.2 System's Performance and Pricing Strategy under Competition Schemes

This section will compare the various Stackelberge games before addressing some features of the Nash game. In the sequential competitions, if customer preference for a physical store is high, then the total SC performs best under PLSG. If customer preference for an online store is high, then the total SC performs best in the OLSG (Figure 3a). Thus, a dual-channel system will have a higher chance of profitability when the channel with the highest customer preference leads. Cai, Zhang and Zhang (2009) came to a similar result and called it the "Stackelberge leadership dilemma." Notice that this result is true when the rates of return are not considered or low.


Figure 3: Comparison of total profit under different competition schemes
If $r$ is not low, then the total SC generates more profit in a PLSG than in an OLSG, even if customer preference for an online store is high (Figure 3b). Indeed, when $r$ increases, the necessity to switch sales from the retailer's physical store to its online store is higher. Since $p_{R_{r}}{ }_{r} \geq p_{O_{r}}^{*}$ and $p_{O_{o}}^{*} \geq p_{R_{o}}{ }_{o}^{*}$ are true at all times (Figure 4), then the physical store satisfies less sales and the online store satisfies more sales when in the PLSG than when in the OLSG. To elucidate, a physical store's manager sets prices depending on online store prices when the latter is the leader. Thus, the manager is distracted by the price competition and does not tackle the
issue of returns. However, one of his or her main concerns is to reduce the effect that returns have when he or she sets the price first in a PLSG.

Similarly, an online store will satisfy less sales and a physical store will satisfy more sales when in an OLSG than when in a PLSG. As $v$ or $w$ increase, there is a higher need to switch sales from the online store to the physical store. Therefore, if $v$ or $w$ is not low, then the dual channel generates more profit in an OLSG than in a PLSG -- even if customer preference for the physical store is high (figures omitted due to similar responses).

Similar to the findings of Yan et al. (2010), Yan (2010), and Yan (2008), the Stackelberge competition always has better channels' profits and thus system performance than the Nash competition (Figure 3). In general, the difference in a channel's profitability under a Stackelberge game verses a Nash game, i.e. $\pi_{O_{r}}^{*}-\pi_{N_{r}}^{*}, \pi_{R_{r}}^{*}-\pi_{N_{r}}^{*}, \pi_{O_{o}}^{*}-\pi_{N_{o}}^{*}$, and $\pi_{R_{o}}^{*}-\pi_{N_{o}}^{*}$, diminishes as any rate of return increases. However, under the OLSG and as $r$ increases, the online store's management does not get distracted by price competition and the channel is not increasingly troubled with returns. Consequently, the difference $\pi_{O_{o}}^{*}-\pi_{N_{o}}^{*}$ is enhanced. Similarly, under the PLSG and as $w$ or $v$ increases, the physical store's management does not get distracted by price competition and the channel is not increasingly troubled with returns. Consequently, the difference $\pi_{R_{r}}^{*}-\pi_{N_{r}}^{*}$ is enhanced.

Comparable to the outcomes of Yan et al. (2010), Yan (2010), and Yan (2008) where customer returns were not considered in their analysis, the Stackelberge competition induces higher equilibrium prices compared to Nash competition (Figure 4). This finding is intuitive, since Stackelberge game imposes a higher coordination level between channels. Additionally, both channels are equally powerful in the Nash game. This implies that there will be increased price competition. This provides an explanation for why the channels prices in a Nash game are
always lower than the channels prices in a Stackelberge game. Thus, enterprises should always employ the Stackelberge scheme to set channel prices in a competitive market.

According to Dan, Xu and Liu (2012) and Hua, Wang and Cheng (2010), the profit of the physical store (follower) grows in response to the increase in the online store's (leader's) pricing strategy. However, no practical justification is given for this price increment. Thus, the aforementioned finding can be further expanded under the context of customer returns. One may notice from Figure 4 that the increase in $v$ or $w$ forces the online store to raise its selling price in order to diminish the negative effect of returns and encourages the physical store to increase its selling price to capture more profit from the market. Consequently, the online store's profit will decrease and the physical store's profit will increase. This is true for any competitive scheme. A similar outcome is obtained when $r$ increases. Be informed that the impact a return rate has on stores' prices is not profound. A channel will be reluctant to dramatically increase its selling price and lose sales in favour of the competing channel. Also, it is worth noting that $\theta$ has a higher impact on physical store's profitability than online store's profitability under any competition scheme (Yan et al. 2010). However, the aforementioned statement is further boosted when $w$ or $v$ increases and reversed when $r$ increases.

(a) high $\theta$

(b) low $\theta$


Figure 4: Return rates' effect on physical and online store pricing strategies in competition schemes

### 6.3 Pricing Strategies under Centralized Management

A comparison of Figures 4 and 5 shows that channels have higher selling prices when coordinating rather than when decentralizing. Since the prices under the two settings are not equal, it can be stated that decisions in a decentralized setting deviate from the overall system's perspective. Indeed, coordination eliminates price competition, providing a chance for both channels to increase prices. Similarly, Yan et al. (2010) have indicated that differential prices set by a sole manager are higher than those set by competing channels. In contrast, Ryan, Sun and Zhao (2013) have indicated that coordination increases total supply chain profit, but at the same time decreases the prices of both channels.


Figure 5: Return rates' effect on physical and online pricing strategies under centralization schemes

Huang and Swaminathan (2009) have stated that a DCR with differential pricing strategy may price the online channel lower than the traditional channel under the condition that it has a higher market share. However, this study provides a different conclusion especially when customer returns are considered. To elucidate, as $w$ or $v$ increase within the differential case, the
online store's price increases in an attempt to decrease the negative effect of return. Consequently, the online store's sales will decline. The physical store should decrease the selling price to attract more customers and to shift part of the lost sales from the online store to the physical store (Figure 5 c to f ). Under extremely high $w$ and/or $v$, the online store can serve as an information channel and the physical store can serve as a transaction channel. When $r$ increases, channel prices are set such that sales shifts from the physical store to the online store (Figure 5 a and b ). If $r$ is extremely high, then the physical store can be used as a show room and most purchases can be directed to the online store (Neslin and Shankar 2009; and Steinfield 2004). Both channels operate in coordination to fulfill organizational-level goals rather than channellevel goals. Indeed, the compensation system in the centralized case should not depend upon the channel's profitability. It should instead depend upon the degree of coordination and the total supply chain's profitability. Comparing the above pricing strategy to that of both Stackelberge and Nash games wherein a store has no intention of losing customers in favour of the competing store, the findings in this section support Baal's (2014) hypothesis that "the higher the degree of harmonization, the greater the degree of cannibalization."

In the case of unification, it is difficult to mitigate the customer returns problem by shifting sales from one channel to another due to the pricing policy used. It has been found that the unified price should decrease if the rate of return for the channel with high customer preference increases. In contradistinction, if the rate of return for the channel with low customer preference increases, the unified price will increase (Figure 5).

## 7. MANAGERIAL INSIGHTS

This research has several implications in regards to the pricing strategies of a DCR wherein both same- and cross-channel returns have been considered. It has examined several
insights related to the centralization of a DCR under unified pricing or differential pricing schemes. It has also examined insights related to the decentralization of a DCR under OnlineLeader Stackelberge, Physical-Leader Stackelberge or Nash games.

It has been found that when customer preference for the physical channel is higher than a threshold value, then the retailer's set price should be higher in the physical channel than in the online channel. The threshold is defined as $\check{\theta}, \breve{\theta}$ and $\tilde{\theta}$ in the Online-Leader Stackelberge, Physical-Leader Stackelberge and Nash games, respectively. Such a situation may occur when products have been custom designed or when remanufactured or used items have been offered for sale. Consequently, customers are more likely to verify the design or quality of the offered items before completing their purchase. However, when customer' preference for the physical channel is lower than the threshold value, the retailer's set price should be lower in the physical channel than in the online channel. Such a situation may occur when the products that have been offered for sale do not require a high level of examination in regards to their design or quality level before being purchased. For example, products that are standardized or mature (such as books and CDs) better fit this category. In a centralized situation, the threshold value is defined as $\bar{\theta}$. Under valid, but unusual parameters, $\bar{\theta}$ could be higher than one, which means that the online store should always be priced higher than the physical store. Additionally, $\bar{\theta}$ could be less than zero, which means that the online store should always be priced lower than the physical store.

Centralization with a differential pricing scheme may cause a retailer to significantly shift sales from one channel to another, leaving the first channel with virtually no customers. Such a management style imposes hardship on the retailer when it comes to tailoring a compensation program that is fair for both channels and dependent on coordination level rather than on sales.

As a result, many retailers centralize decision-making under a unified pricing strategy. Thus, retailers should be aware of several important issues related to the price unification process. For example, when under a centralized DCR and a unified price strategy, a retailer's profit is not always higher than when under a decentralized DCR. Thus, the retailer should be careful in regards to encouraging or discouraging competition between channels. For example, if a channel experiences a sufficiently high customer preference and a same-channel rate of return, then it is better for the retailer to encourage competition rather than coordination. This could occur in the apparel industry, for example, wherein customers are increasingly inclined to use online stores despite having up to a $75 \%$ chance that they will return their purchases due to size or material mismatches.

Prices in the Nash game, when neither channel dominates the market, are lower than in the Stackelberge games, when a channel does dominate the market. This can be attributed to the higher competition or the lower level of coordination induced by the Nash game. Thus, both channels are worse off in the Nash game. Consequently, one may argue that Nash game leads to a lower total retailer's profitability. Also, one may think that a retailer will perform better if the channel with highest customer preference leads. Our study shows that this is not always the case. When one channel is dominant and all return rates are low, then a retailer will have a higher chance of profitability when the channel with a significantly high level of customer preference is the price leader. In contradistinction, when return rates are not low then a retailer will have a higher chance of profitability when the price leader is the channel with a significant rate of return, even if the competing channel has a high level of customer preference. This is due to the pricing behaviour occurs naturally in the Stackelberge games. It has been observed that a channel will always charge higher prices if it is leading rather than following. This may be due to the fact
that a price leader's main objective is to reduce the effect of return, while a price follower's main objective is to be competitive once it observes the leader's price. These results indicate that game schemes have a significant impact on retailer payoffs, and that the schemes have a substantial influence on sales and customer welfare.

## 8. CONCLUSIONS

This paper has studied the effect that same- and cross-channel customer returns have on a dual-channel retailer wherein an enterprise runs both a physical store and an online store. The results confirm that accounting for both types of returns is very important when calculating channels' optimal prices. Closed form formulas were assigned to the optimal unified price and the differential prices set by centralized management. The optimal prices for the competing channels were derived using the Online-Leading Stackelberge game, Physical-Leading Stackelberge game and Nash game.

It has been found that customer preference for a certain channel greatly affects the pricing strategy of that channel. For example, the optimal price for a channel facing competition will increase as customer preference for that channel also increases. Unlike common perception, the optimal price for a channel under centralized management and a differential pricing strategy could either increase or decrease when customer preference for that channel increases. The previous setting may not possess a customer preference value that causes both channels to optimally be priced equally. When the physical store's rate of return is less than all online store's rates of returns, then the optimal unified price will increase as customer preference for the physical store increases.

From the numerical example it has been observed that the prices set by centralized management are higher than those set by competing channels. When compared to Stackelberge
competitions, Nash competition imposes lower pricing strategies for both channels and lower total supply chain profitability. Under the Stackelberge scheme, the pricing strategy for a channel under leadership is at least as high as the pricing strategy for the same channel under fellowship. Consequently, total supply chain performance has a proclivity to be better when being led by the channel with high rate of returns and high customer preference. Such conditions promote competition over coordination with price unification.

In this work, we assumed the full rationality of all players. However, this is not always the case and players may have bounded rationality, may adopt a strategy that is not optimal, and may not know the rationality of other players. Thus, an evolutionary game theoretic approach can be used to examine price competition within a duopolistic dual-channel system. Besides, an evolutionary game theoretic approach may be used to study different return policies in dualchannel retailing where a store has a set of different policies to choose from namely: no refund, partial refund, and full refund. Unlike the classical game theory used in this paper, players in the aforementioned game dynamically compete or interact until an evolutionary stable strategy (ESS) is successfully achieved (Leboucher et al. 2016).

Another extension to this research is to consider multiple products and related constraints such as a total demand or budget limitation. The problem for the multi-product DCR with customer returns could be formulated as a complex nonlinear integer programming model and it may be difficult to find a closed form solution for the model. Similar to De et al. (2017), metaheuristic algorithms can be used to solve the problem and their corresponding system behaviour could be compared.

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