HISTORY OF A PROBLEM SOLVING COURSE FOR PRE-SERVICE K-8 TEACHERS

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Abstract

James Madison University's mathematics course in problem solving for prospective K-8 teachers has gone through many changes since we introduced it as a course for prospective middle school teachers four years ago. This paper will discuss the original design of the course, the changes made and their reasons, and the current state of the course.

Introduction

In Spring 2000, the National Science Foundation (NSF) awarded a grant to the Department of Mathematics at James Madison University (JMU). Department head David Carothers, joined by local school systems and JMU faculty, had proposed that our department, in conjunction with local middle school and high school teachers, develop upper level mathematics courses for students who plan to teach middle school.

The plan was to develop four courses: a problem solving course; an algebra course; a geometry course; a course in the ideas of calculus. Two-person teams, composed of a mathematics department faculty member and a middle school mathematics teacher, worked together during the summer to develop the courses. The problem solving course began as *MATH 303 – Mathematical Problem Solving*, and ran for the first time in Fall 2000 as an experimental course.

Problem Solving—Pilot Course

Ginger Carico, a seventh grade teacher at a local middle school, worked with me to develop the problem solving course. Besides developing the course, Ms. Carico would come to JMU to teach six times during the Fall 2000 semester; I would teach the other sessions. We decided that the students would spend most of the time working on classic puzzle problems. This worked well because Ms. Carico used many of those same problems in her seventh grade classroom. At that time, JMU did not have a large collection of mathematics manipulatives available, so Ms. Carico planned to bring many manipulatives and other materials with her when

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she taught the class, composed of twelve students. The course began with the painted cube problem

> Suppose a large cube is painted red. The large cube is then cut into n^3 smaller cubes by making vertical and horizontal cuts. How many cuts will be necessary to produce the n^3 smaller cubes? How many of the smaller cubes will have zero, one, two, and three red painted faces?

Although a couple of the students solved this easily, it was a real challenge for most of the students. They were hindered by not wanting to use manipulatives, even though I provided several different types. They seemed to think that college students shouldn't need to use such things, but should know immediately how to solve any problem. By the end of the first week, when the problem write-up was due, most of the students had solved the problem by making a table for various n's, but they were unable to go beyond the table and write expressions in n for the number of cubes with one or two painted faces.

This situation would probably not occur today. Our expectations for prospective teachers in mathematics courses are now much higher, and the students would have been exposed to problem solving of this type throughout their earlier mathematics courses. However, the students in that first class found problem solving difficult. They complained bitterly that the problem was too hard. The very next class period, Ms. Carico presented the same problem as one that she gave to her seventh grade pupils. This was a real eyeopener for my students, and their attitudes changed slightly.

One reason that middle school and high school students are often able to solve problems that prospective teachers cannot is simply that students who really enjoy mathematics in middle and high school often choose to major in science or mathematics when they go to college, rather than to prepare to teach in grades K-8. Thus, the students in the mathematics courses for prospective teachers are sometimes less interested in mathematics than many middle school and high school students.

The course progressed with standard problems like the locker problem and several problems from Stevenson's *Exploratory Problems in Mathematics* [1]. These latter problems were really too difficult for the students and would probably still be so today. Toward the end of

the course, the students worked in two teams to determine the volume and surface area of the mathematics/biology building on campus. They enjoyed this project and, I think, learned a great deal from it.

At the end of the semester, I felt that the next time I taught the course, I wanted less of an emphasis on puzzle problems and a greater emphasis on what I think of as mathematics, which includes building from basic concepts into a beautiful structure with surprising results.

Problem Solving—Revision 1

The second version of the course, *MATH 207 – Mathematical Problem Solving*, was developed in Spring 2002 when it was decided that the problem solving course would be taken by all prospective teachers, not just those who were concentrating in mathematics and science.

When that happened, other faculty members began to teach the course. Judy Kidd, one of the other faculty members, and I decided that we would use a "Harvard-style" pre-calculus book by Hughes-Hallett, et al. [2]
The reason for this decision was that we wanted to challenge the students and also to provide a strong algebra review, because we had noticed very weak algebra skills among our students.

This version of the course was not a success. The students detested the book, which is strongly oriented to problem solving (as are all the books by Hughes-Hallett). Although the book gives examples of solved problems, those problems were, in the students' eyes, never "like" the problems that they were expected to solve for homework. In addition, there was no answer key in the student textbook. The uncertainty generated by these aspects of the text made the course unpleasant for the students. Ms. Kidd and I also felt that the book was too hard for our students.

Problem Solving—Revision 2

The third version of the course, *MATH 207 – Mathematical Problem Solving*, started in Fall 2003. Ms. Kidd decided to use a high school textbook on problem solving, *Crossing the River with Dogs* [3]. I decided to use handouts rather than have a text.

I think my course was more successful than either of my previous versions had been. We began by studying logic, which was at the time not part of the two preliminary courses. We *then* saw how logical equivalence could be used to design efficient circuits, and we actually built some circuits. We also connected formal logic to Boolean searches on the Internet. Next, we moved on to the study of triangular numbers and their relationship to counting and probability. This part of the course was particularly difficult for the students. Finally, we studied simple strategy games and their relationship to axiomatics and proofs.

One problem that is always present with a problem solving course is the issue of grading. Grading student write-ups of problems is extremely time consuming and difficult. With about sixty students in problem solving classes, along with about ninety in other courses, this simply became overwhelming. Future versions of the course must address this difficulty.

Problem Solving—Revision 3

The next version of the problem solving course is scheduled for next fall. At that time, this course will become simply a third semester of general mathematics for prospective teachers. We will complete the material in our current textbook for the first two semesters of mathematics for elementary school teachers. Our current book, by Betsy Darken, has problem solving interwoven in all topics, and we need the additional semester to provide adequate time and coverage for probability, statistics, and functions [4].

Conclusion

While in some ways a separate course in problem solving is a nice luxury, it is not really possible in mathematics to separate content from problem solving. The new approach should be better for the students and more comparable to the way mathematicians approach their subject. ■

References

[1] F.W. Stevenson, *Exploratory Problems in Mathematics*, National Council of Teachers of Mathematics, Reston, VA, 1991.

[2] E. Connally, D. Hughes-Hallett, and A. Gleason, et al., *Functions Modeling Change: A Preparation for Calculus*, John Wiley and Sons, New York, NY, 2000.

[3] K. Johnson, T. Herr, and J. Kysh, *Crossing the River with Dogs: Problem Solving for College Students*, Key College Publications, Emeryville, CA, 2004.

[4] B. Darken, *Fundamental Mathematics for Elementary and Middle School Teachers*, Kendall/Hunt Publishing Co., Dubuque, IA, 2003.