ENHANCING FINITE MATHEMATICS WITH GLOBAL AWARENESS

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Abstract

This paper describes the project of modifying the course, *Finite Mathematics*, to meet a Global Awareness course requirement. The components of the revised course are described, including topics, in-class activities, and student evaluation methods, along with a summary of the results of a general education assessment of the course.

Introduction

Like other liberal arts colleges, Mary Washington College (MWC) includes a mathematics requirement in its general education program. To meet this requirement, courses must develop "an understanding of mathematical thought and the ability to conceptualize and apply mathematical logic to problem-solving." Students must take two courses from within this general education goal, one of which must be offered by the Department of Mathematics. *Finite Mathematics* is one course that meets this requirement. This course is typically taken by students in non-science disciplines and assumes that students have completed high school algebra. The goals of this 100-level course are to demonstrate how mathematics is part of our everyday lives, to help students gain a mathematical perspective, and to accomplish these tasks while easing the math anxiety many students feel. While the topics included in *Finite Mathematics* can provide the means for accomplishing these tasks, teaching the course with a global awareness emphasis can help provide an incentive for learning those topics. It can also enhance students' interest in and appreciation for the mathematics they are learning.

The general education program at MWC also includes a requirement that students take two courses that carry a Global Awareness (GA), Across-the-Curriculum (ATC) designation. A course may receive this designation after a review by the Global Awareness Committee to assess the extent to which it meets the GA requirements: (1) awareness and knowledge of global issues, (2) respect for other languages and cultures, and (3) appreciation of the United States in its international context. Each of these goals has a number of objectives that provide suggestions for how that goal may be met. Although a course need not meet all of the objectives under each

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of the goals to receive GA designation, it must meet all three goals. A GA course may be within any discipline and may be at any level.

As first taught at MWC, *Finite Mathematics* included sets, counting, and probability and logic, taking advantage of the underlying similarity in the structure of these topics. While mathematicians can recognize and appreciate this similarity, most students who take this course do not. As promoted by the Virginia Collaborative for Excellence in the Preparation of Teachers (VCEPT), instructors began to experiment with adding and/or replacing topics and with adding an ATC designation. The alternate topics depend on the interests and expertise of individual faculty and include graph theory, consumer mathematics, number systems, voting schemes, statistics, patterns, and tessellations. In addition to a GA ATC designation added by one faculty member, two other faculty members have added a Speaking Intensive ATC designation to their sections of this course.

As well as being one of a group of courses that meets the mathematics general education requirement, *Finite Mathematics* is also a required course for Elementary Education students at MWC. In a recent offering of this course, approximately 13% of those enrolled were in the Education program, which is roughly twice the percentage of Education students campus-wide. The general nature of these topics provides a good foundation for developing early math skills. When taught as a GA course, the topics further provide vast enrichment opportunities through the mathematical exploration of other cultures and countries.

As a GA course, *Finite Mathematics* is divided into two parts. Topics were chosen that offer a way either (1) to compare how cultures have developed and use number systems or (2) to compare cultures or countries using mathematics. In the first part of the course, the topics are further divided into two main sections: (1) development of counting, numbers, and number systems; and, (2) the mathematics of symbols and patterns. The second part also has two sections: (1) sets and counting, and (2) probability and statistics. By first showing students how numbers and number systems were developed, they gain an appreciation for the numbers and mathematics that are a part of their daily lives. Using sets, probability, and statistics together to compare countries provides students with a mathematical perspective of the world around them. Finally, the GA emphasis provides an engaging environment and a unified purpose that can help students to overcome their math anxiety. The topics and activities for each of these four sections are described in detail below, along with methods of instruction. Student evaluation and an assessment of the course as a general education course are also discussed. See Appendix A for a list of web resources.

The Development of Counting and Number Systems

By looking at and working with number systems from other cultures, students gain an understanding of how numbers were developed and an appreciation for the different types of number systems.

Topics — This section of the course begins by addressing the question, is counting universal? Students consider the existence of a "number sense" and the "limit of four" that is prevalent in many number systems [1,2]. Class discussions center on reasons why early humans might need to know how many they had of something, leading to the use of mediating objects such as tally sticks and pebbles [1-4]. Through examples, students are led to discover the limitations of these methods. The five uses for numbers (number, size, form, order plus unique identification) are then discussed. Students are encouraged to find examples in their daily encounters with numbers. In the preface of Brian Butterworth's *What Counts*, a tour through just one page of a daily newspaper reveals 51 separate numbers; a convincing example of just how prevalent and important numbers are in our lives [5]. Early counting systems, such as those used by the two-counting and five-counting cultures described in *Pi in the Sky*, are discussed along with number words from different cultures [1]. Students are asked to consider the analytical, linguistic, and writing skills needed to develop a useful number system. The development of simple grouping number systems from basic tallies is explored, along with the similarity between these systems and the "five-barred gate" method of tallying still used today [2]. A comparison is made between the four types of number systems: simple grouping, ciphers, multiplicative grouping, and positional [1-3]. As an introduction to positional systems, bases other than 10 are explored. Bases less than 10 seem to be easier for students to grasp so more time is spent preparing students for the base 20 Mayan and base 60 Babylonian systems. The historical development and use of ten separate number systems is examined: Egyptian hieroglyphs, hieratic and demotic ciphers, Chinese brush form and counting rods, Babylonian, Mayan, Roman, and the Greek Ionian and Attic systems. By comparing how these different number systems work, students discover the tradeoffs that result between memorizing more symbols compared to having fewer symbols to write and the relative ease of performing computations in one system compared to another. The development and use of fractions, decimals, and negative numbers [2,3] in some number systems are also examined, along with a history of zero [6].

Activities — To record their daily encounter with numbers, students create a scrapbook of examples for each of the five uses for numbers along with number words and symbols from other number systems. To learn more about oral counting, students investigate the counting system used by Hawaiians [7] and discover its similarities to a ciphered number system. Students then compare the finger counting system used by the Masai [4] with the system described by the Venerable Bede [1,2,4,7]. To study more about mediating objects, students investigate the Incan quipu and other knot-and-string methods of counting [2,4,7]. To ease difficulties encountered in dealing with a system other than base 10 and with a different set of numerals, students can create a base four system using O, I, V, N, and M. For more practice with bases other than 10, students investigate the base 20 Kaktovik Inupiaq numbers created by school children to support their oral counting system. The Chinese counting rod number system can be found in the early use of "Pascal's" triangle by the Chinese and can be used to expand students' practice with this system [2,7]. Students can construct a simple Chinese abacus and learn to do basic calculations [2,8]. The use of an abacus in other cultures can also be explored. Figure 1 shows an abacus created in *Microsoft Word* that was created for a worksheet and could easily be incorporated in an online activity.

Number stories are common in many cultures and countries. Examples include Horus' eye in ancient Egypt [2,4], the stories about King Shirham of India and about the Tower of Brahma [9], and the collection of stories in *The Man Who Counted* [10]. Students can investigate the underlying mathematics behind these stories or create number stories of their own. Further explorations can include number mysticism and gematria [2,3], the uses of numbers in cryptology [2], the history of *pi* [11], and number games and puzzles [2]. *Multicultural Mathematics* [12] provides many ideas for activities to enrich lessons in the early and middle school grades, while *The Crest of the Peacock* [13] provides a history of non-European mathematics.

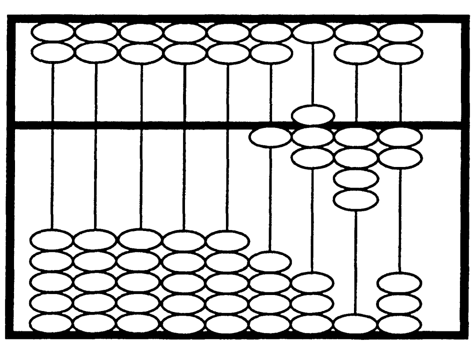

Figure 1 Chinese Abacus showing the number 1,742.

The Mathematics of Pattern and Design

By studying the basic symmetry operations, students explore the similarities in designs and patterns created and used by different cultures.

Topics — Beginning with the operations of rotation, reflection, translation and glide translation, students investigate the point, line, and plane groups [14,15]. Early and current methods of symbolism are studied for their symmetric properties, including symbols created by mimicking patterns from nature [16] and the development of sign structures [17]. Students are asked to consider the cultural use of patterns for aesthetic properties and for communication. Tessellations [14] and the work of Escher [18] are also explored.

Activities — Students are asked to create and identify patterns from each of the different point, line, and plane groups. The software package *Kaleidomania!* is especially useful in helping students explore the operations used to create different patterns. In addition to allowing students to create different patterns, *Kaleidomania!* also has a feature that shows how the pattern is generated. Figure 2 shows examples of point and line group patterns created using this software.

(a) Group 2mm pattern (two reflections)

(b) Group mt pattern (reflection and translation)

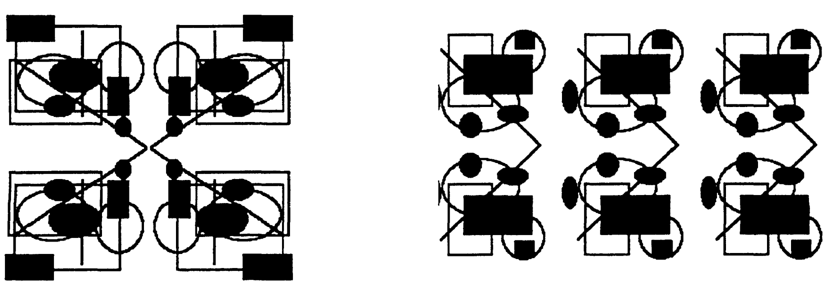

Figure 2 *Kaleidomania!* examples.

Students can explore the mathematical structure of logos in use today, many of which have been used by other cultures. For example, the logo for Mitsubishi is a Nordic rune used to induce madness [15] and is also called the triceps, meaning divine power [17]. Many cultural uses of patterns provide students with wonderful opportunities to explore their own and others' backgrounds. A few examples include: Russian Easter eggs; Mexican, Japanese, German paper cutting; American Indian basket weaving; Oriental carpets; American quilt patterns; and, Celtic knots. To record their explorations into patterns and symbols, students create a scrapbook of patterns and designs from different sources.

Sets and Counting

Organizing objects into collections with similar properties is an activity common to many human endeavors. By studying the properties of sets, students investigate how sets can be used to examine the similarities of different countries or cultures.

Topics — The topics in this section follow very closely the material provided in many finite mathematics or mathematics for the liberal arts texts. The text used for this course is found in Bello and Britton's *Topics in Contemporary Mathematics* [19]. Topics for sets include the basic definitions of set, universal set, empty set, and subset, along with the set operations and their properties and Venn diagrams. Special attention is paid to using Venn diagrams to summarize the results of surveys. Topics for counting include tree diagrams and the sequential counting principle, along with combinations and permutations.

Activities — To explore the use of sets as a problem solving tool, students choose a continent or region and create subsets of countries or states with similar properties that can be identified using an atlas (e.g., countries that have a coastline on the Atlantic, share a border with a given country or state, or include a specific mountain range). On a larger scale, students conduct a "survey" of the world's nations concerning the status of two international agreements, then create a table and Venn diagram to summarize the results, along with colored-in world maps to identify each of the four subgroups of countries created. The software *Amiglobe* or the CIA website can be used to conduct the survey and to locate each of the countries for the maps. An example survey and map are shown in Figures 3 and 4. A website for the map and the *Amiglobe* and CIA websites are given at the end of this paper.

	Signed International Agreement?	
Environmental Issue?	Yes	No
Yes	55	5
No	100	33

Figure 3. Results of a survey of countries that have signed the International Agreement on Desertification and those that currently have this environmental problem.

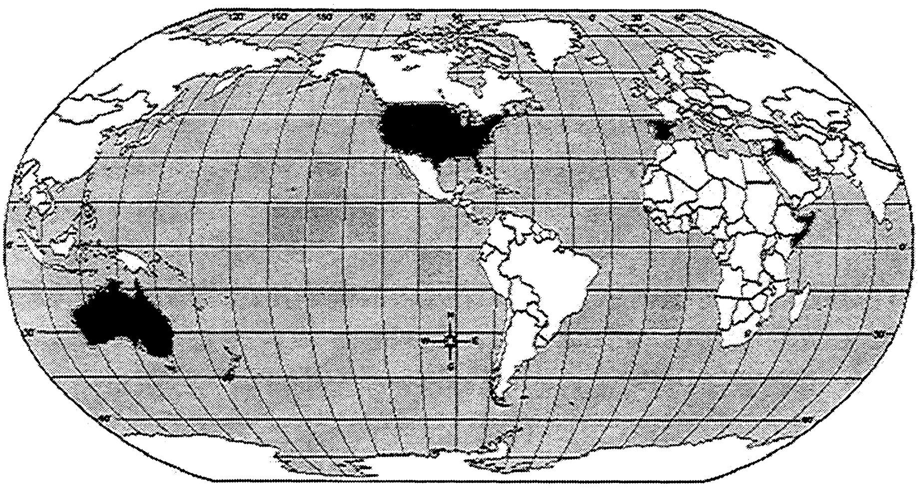

Figure 4. Countries that have not signed the International Agreement on Desertification, but have this environmental problem.

Students can also conduct a survey of other students on their opinions or knowledge of world issues and summarize the results as in the preceding example. Activities for counting topics are harder to "match" to a GA theme, but include skill development, particularly for preparing students for learning probability.

Probability and Statistics

The skills of reading and interpreting graphs, as well as selecting and making an appropriate graph for a given data set, are becoming increasingly important as more and more quantitative information is presented using graphs. In this section, students learn to use basic graphical measures and descriptive statistics to compare cultures or countries.

Topics — As with sets and counting, the topics for this section follow very closely the material available in many finite mathematics and liberal arts mathematics texts. Topics include the definitions of probability and independent events, methods of assigning probability, probability rules, measures of center, and basic graphical techniques. There are also other useful texts for exploring the uses of different graphs [20-25].

Activities — To investigate the uses of different graphs, students are given a data set and asked to decide on a "best" way to display the data along with an explanation for their choice. To explore the characteristics of different countries, students can compare mortality tables or pyramid charts that show the age distribution by sex. The global math activities in the workbook [26] provide many other graphical methods for comparing countries. To record their daily encounters with different graphs, students collect example charts, graphs, and comparative statistics for different countries. Students can also use software such as *Excel* or SPSS© to create graphs for data collected for a selected country to compare to the United States. Data for these projects can be collected using *Amiglobe* or the CIA website. Figure 5 shows a sample graph produced using *Excel.*

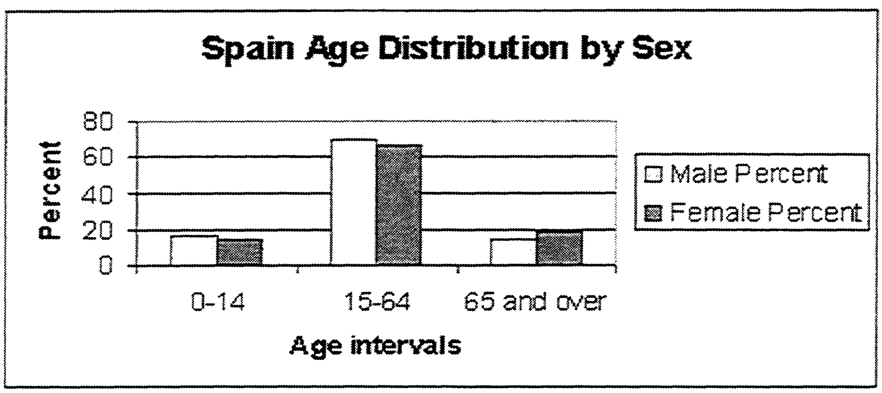

Figure 5. Age Distribution by Sex for Spain.

Teaching Methods

The course is taught using modified lectures, where new material is interspersed with examples and activities. Class handouts were developed to supplement textbook [19] material on the development of numbers and number systems. The text used for this course does not include any material on patterns and symmetry, so class handouts were developed for these topics as well. The text does cover sets, counting, probability, and statistics so lectures on these topics are based on the text material. Most examples, activities, and assignments include a global theme. Whenever possible, current events are incorporated into class activities and discussions. A typical class might start with an introduction to new terminology and notation followed by examples and an in-class activity. Review summaries and practice problem worksheets are distributed before each exam. Because emphasis was placed on the underlying mathematics, and not on the memorization of symbols or labels, students are given handouts with each of the number systems and patterns studied to use for in-class activities, quizzes, and exams on these topics.

Student Assessment

In both the GA and non-GA versions of this course, students are assessed using in-class tests, take-home quizzes, and projects. Take-home quizzes consist of questions similar to those on the tests and the projects give students the opportunity to further explore topics from class. The biggest differences between the two versions of the course are in the tests and projects. In the non-GA course, four tests are given (Sets, Logic, Counting, and Probability) comprising 80% of the grade. Two individual class projects are assigned. For the first project, students conduct a survey of their classmates on a topic of their choice and organize the results in a Venn diagram. For the second, students design a scratch-off style lottery ticket, including the number and types of winning tickets, and calculate the probability of winning and the average winnings. In the GA course, three tests are given (Numbers and Numeration systems, Patterns and Sets, and Counting, Probability, and Statistics) accounting for 60% of the grade. Two group projects are also assigned. For the first project, students conduct a "survey" of countries on two international agreements and organize the results in a Venn diagram. They also investigate the terms of the agreement and produce maps to show the four subsets of countries identified by the Venn diagram. For the second project, students produce graphs depicting the population distribution by age and gender, birth and death rates, and the sex ratio at birth for a country of their choice to compare to graphs made for these variables for the United States. They also create a histogram for a variable of their choice to show the distribution for all countries and compare the United States and their country within that distribution.

Grade distributions for one section of each version of this course are provided in Table 1. The GA distribution is from Spring 2001 while the non-GA distribution is from Fall 1998. Grade distribution for all 100- and 200-level courses, including *Calculus I, II* and *III*, *Introduction to Statistics*, and *Finite Mathematics* (*MATH 110*), from Spring 2001 and Fall 1998, are also given. Although grade distributions vary by instructor and by course, students in the GA version of *Finite Mathematics* did as well or better than students in the non-GA version.

Table 1
Grade Distributions (percentages) by Course and for all Lower Level Mathematics Courses

	Spring 2001		Fall 1998	
Grade	*MATH 110* GA	All lower-level math courses	*MATH 110* Non-GA	All lower-level math courses
A	26	31	17	25
B	52	34	58	35
C	19	16	17	29

Course Assessment

As part of the evaluation of its general education program, MWC has conducted a survey of student course perceptions. The survey consists of ten pairs of statements to which students respond affirmatively if they agree with the statement. The first statement in each pair asks students to address the effectiveness of the particular course in meeting general education course goals while the second asks students to assess the impact of the course on their learning in that area. Ten of the twenty statements are relevant for evaluating the GA version of *Finite Mathematics*. The results for these questions (percentage of respondents who agree with the statement) from the Spring 2001 and Summer 2002 are provided in Table 2. A total of seven sections of *Finite Mathematics* were offered in the Spring 2001, one of which had the GA designation. Two sections of *Finite Mathematics* with a GA designation were also taught during the Summer 2002.

The results suggest that students in the GA section respond as favorably as other *MATH 110* students to most of these statements and respond much more favorably to others. In particular, for the statements regarding the global nature of the course and relating subject matter to other disciplines, students in the GA sections report a substantially larger proportion of yes responses. Because the general education survey was designed to assess the complete array of general education course offerings, some questions may not be relevant for all courses. Consequently, some faculty instruct their students not to answer questions that are not pertinent to their course. While some of the *Finite Mathematics* instructors from the Spring 2001 may have omitted the questions on the global nature of the course and on how the course relates to other disciplines, the high percentage of yes responses for the GA sections indicates the GA designation is meeting these general education goals.

Table 2
Summary of General Education Course Evaluation for Finite Mathematics (percent yes)

Statement	Spring 2001 All sections n = 160	Spring 2001 GA section only n = 26	Summer 2002 Two sections both GA n = 16
Would you agree that the essence of this course is expressed in the description of a Goal 2 course?	91	88	94
I have gained in my understanding of mathematical thought.	80	77	81
In this course, an effort was made to present broad, even global, connections of this subject matter.	23	100	94
I have gained a better understanding of the broad, even global, connections of this subject matter.	20	88	94
In this course, an effort has been made to relate the subject matter to other fields of study.	56	83	75
I have gained a better understanding of how the field of study represented by this course is related to other fields.	57	75	81
This course has provided an introduction to the field it represents.	84	84	87
I have acquired an understanding of the basic concepts of this field of study.	86	88	93
This course has covered a range of topics that are clearly part of this field.	90	92	94
I have gained an appreciation for the breadth of areas covered by the field of study represented by this course.	79	81	94

Conclusion

The benefits for students taking the GA version of *Finite Mathematics* include a more focused learning experience and a greater emphasis on applications. The emphasis placed on other cultures promotes a greater appreciation for the development of numbers and mathematics. The topics covered also provide students with the means for quantitatively comparing different countries. For faculty, the benefits of teaching a GA (or other ATC designated course) include an opportunity to incorporate personal interests and to engage students more in their learning. A GA emphasis also provides a convenient method of incorporating more real-world activities and discussion into mathematics instruction.

In addition to the continued assessment of students' perceptions about the effectiveness of the GA version of *Finite Mathematics* in meeting general education course goals, additional work on this project will include collecting more examples of global awareness activities and topics to expand the course content, particularly for the sections on sets and counting, and for additional projects in statistics. This additional material will be combined with the examples and assignments described in this paper into a Global Awareness workbook for *Finite Mathematics*.■

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Appendix A

MWC General Education Requirements

http://www.mwc.edu/acsv/Level2Pages/GenEd_Sub_Pages/BABS98to01.htm

The Development of Counting and Numbers:

The History of Mathematics in the Americas
http://www.saxakali.com/COLOR_ASP/historymam.htm

Base Value Numbers http://www.psinvention.com/zoetic/basenumb.htm

Egyptian Number Translation http://www.psinvention.com/zoetic/tr_egypt.htm

The Maya Mathematical System http://www.mayacalendar.com/mayacalendar/f-mayamath.html

Kaktovik Inupiaq numerals http://www.col-ed.org/smcnws/potlatch/ak5.html

History of the abacus http://bethany.davis.students.noctrl.edu/5pagepaper.htm

A virtual abacus http://hometown.aol.com/edhobbs/applets/abacus/index.html

The Mathematics of Pattern and Design:

Kaleidomia! http://www.keypress.com/catalog/products/software/Prod_KaleidoMania.html

Sets and Counting:

Amiglobe http://amiglobe.com/

CIA website http://www.cia.gov/cia/publications/factbook/

Outline maps of the world http://www.eduplace.com/ss/ssmaps/wrldcount.html