MATHEMATICS IN HANDS-ON SCIENCE FOR LIBERAL ARTS STUDENTS

M.I. SOBEL
Brooklyn College of the City University of New York
msobel@brooklyn.cuny.edu

We describe a number of experiments from the courses called, *General Science 9*, part of the science program for elementary education majors at Brooklyn College. These courses provide hands-on learning experiences for students who are insecure and weak in science and mathematics. Quantitative thinking is a central element in most of the students' work. Mathematics is taught in a concrete and intuitive way, as a direct outgrowth of their needs; first, in analysis of data, and second, in discovering underlying theory. The science program has been developed through cooperation among faculty from the School of Education and the science departments.

This paper examines the synergy between teaching science and teaching mathematics, with special reference to the education of non-science majors. Our discussion grows out of experiences developing and teaching inquiry-based courses for an audience of elementary education majors.

In a traditional introductory science course, particularly in the physical sciences but to some extent also in the biological sciences, mathematical treatment of the subject is taken for granted. When it comes to science for the liberal arts major, however, it is often argued that mathematical treatment is inappropriate, or at least not feasible because students' skills are weak. Thus, texts emphasize conceptual learning [1, 2]. It is nevertheless true that the mathematics that students are supposed to have learned, in high school for example, or in remedial work in college, should prepare them for a reasonable level of quantitative thinking in connection with science or, indeed, with other subjects as well [3]. Moreover, the argument that science is intrinsically mathematical, while out of fashion in some circles, nevertheless retains merit – particularly if one accepts the view that science is fundamentally theory-based, and that theories are expressed mathematically. Thus the science class, even one that is dominantly hands-on, should

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proceed from experiment and analysis to the construction and testing of theory, the latter expressed in mathematical terms.

Pursuing this objective and struggling with student weaknesses, we find that new ways of approaching math education present themselves. Manipulations with numbers that are the result of measurement (and so have limited accuracy) obey different rules than manipulations with exact numbers. Scientific notation, powers of ten, and metric prefixes all become useful tools, not annoying obstacles. Geometrical relationships and theorems follow from hands-on measurements, reviving the root meaning of the word "geometry." The utility of much of algebra turns out to be exaggerated, but some simple algebraic operations are constantly in demand, and become second nature. The concept of the "function," stripped of abstraction, is more important than anything else.

Below, we give two examples of combined math/science units which are treated in the course *General Science 9*, part of the science curriculum for education students at Brooklyn College. The New York Collaborative for Excellence in Teacher Preparation (NYCETP) supported this curriculum as it grew out of extensive collaboration among faculty in the School of Education and the science departments.

Students majoring in early childhood and elementary education take a series of science courses, beginning with the 8-credit science component of the Core Curriculum. The Core is required of all undergraduates, and the science component consists of four survey courses: two credits each in Chemistry, Physics, Biology, and Geology. In addition to the Core, education students take Gen. Sci. 9 (four credits), Gen. Sci. 10 (three credits), and Gen. Sci. 20 (two credits); the latter two paired with courses on pedagogy in science, math, and technology. Gen. Sci. 9 represents a series of courses from which the students choose one. At present, the following courses are available:

- General Science 9.1: Geophysics
- General Science 9.2: Light and Visual Perception
- General Science 9.3: Biology and Chemistry of Everyday Life

A fourth course, General Science 9.4: Studies in Paleobiology, is in preparation.

These courses meet for five hours per week in a room containing laboratory tables and seats for class discussion. The group is small, up to 24 students. Learning is based primarily on hands-on activities that engage students with observation of the natural world. During the class period, students go back and forth from lab work and collection of data, to working individually or in small groups on data analysis, to writing reports, and to class-wide discussion led by the instructor. Thus, the class models, at a higher level, the kind of science classroom that is desired at the elementary school level.

A good deal of the student's time is spent writing a lab report, and this is largely done in the classroom, not at home. Thus, he/she has the opportunity to go back to the lab table, or to the data, to re-evaluate or repeat procedures. The final report, which sometimes represents 6 or 8 hours of work, can be an impressive portfolio of achievement. Grades on lab reports make up almost half the student's final grade.

General Science 9 provides a realistic model of a science research lab, where student-scientists first informally investigate (play with) a new phenomenon, then set up systematic and quantitative measurements, then analyze (play with) the data, and finally, look for explanations of their results in terms of simple theoretical principles.

Students in this program are quite weak in science and mathematics. They may have had some high school algebra, but remember very little of it. They heartily dislike mathematics, are certain that they are very poor math students, and cannot see any reason for studying it at all. Some students come into the class hating science, in some cases as a consequence of their experience in the Core course. Some feel that science can be interesting, but all feel that science is not for them: it is too hard and too mathematical. On the whole, these students represent the norm among those Brooklyn College students who do not major in math or science.

Observations over a period of about six years suggest that these students' major weakness is in abstract thinking, that they have done very little of it, find it stressful, and tend to surrender to their stress. Consequently, we attempt in these courses to approach subjects, both in science and math, as concretely as possible, and to develop abstract ideas gradually as an outgrowth of concrete studies. Algebraic work is largely limited to

equations like "ab = c," or sometimes "ab = cd," but students must work with these equations frequently, exploring them in a wide variety of forms and contexts.

The Shadow Experiment

The shadow experiment [4,5] in Gen. Sci. 9.2 is shown schematically in Fig. 1. The light source is a lamp with a non-frosted bulb. The filament is oriented vertically (the experimental layout is a horizontal plane), so that the source is almost a point. The "object" is a card that the students make (about 8 cm wide), and it casts a shadow on the screen. Students are to investigate how the width of the shadow, S, varies as things move. The lamp has a reflector, so that initially there is a complicated umbra/penumbra. After some observations and discussion, the reflector is covered with black paper to simplify experimental conditions (to create the point source).
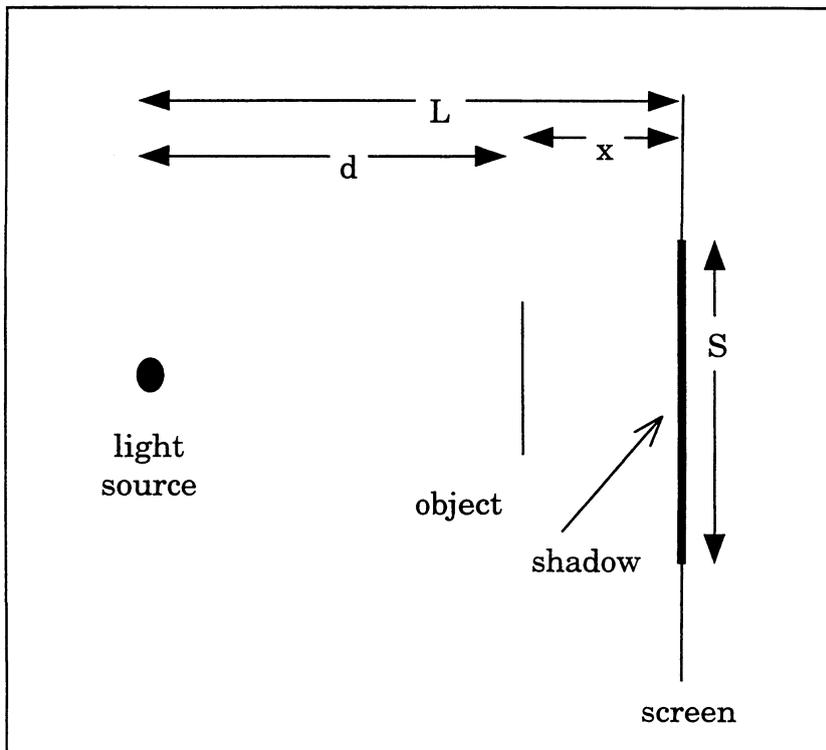

Fig. 1 - The shadow experiment

Students first spend some time playing with the pieces to see how the shadow size changes. We then get together to discuss how to control variables systematically. It is useful to disabuse the students of the notion that controlling the variables is a straightforward or obvious process — as is typically implied in discussions of "the scientific method." In this experiment, there are three variables (d, x, and L). Although it is clear that only two of the three need be studied experimentally since they are not independent, it is not at all easy to convince students of this point. Furthermore, once we agree to study two of these variables, one choice of two variables will make interpretation of the experiment much clearer than other choices. While students are not led through a full mathematical discussion of these points, they do begin to see, after working on the experiment for some time, something of the subtlety involved in planning research.

Students do two experiments, one keeping d fixed and varying L, the second keeping L fixed and varying d. The graph of S vs. L is their first example of a linear relationship. They plot the points, draw the best straight line close to the points and through the origin, and then find the slope using one point on the line. All this is somewhat new, since most of their prior graphing experience was with integers. Working with measured numbers, with, say, three significant figures, is unfamiliar. A line that fits points only approximately is also something new. We create an entirely new kind of arithmetic when we use measured numbers.

We stress that linearity is a fundamental relationship, the one to which others are usually compared. Important real-life problems depend on whether a functional relationship is linear or not (the biological effects of low level radioactivity, for example). Students may have spent hours analyzing straight-line graphs using slopes and y-intercepts, but they have not been told the significance of what they have done. A linear relationship can be expressed in many ways: as a graph (which is the first way that we see it here), as a proportion, or as an equation. We see all of these in the shadow experiment.

Data for fixed L and variable d are also plotted. The graph of S vs. d represents a decreasing function. A smooth curve fits the points rather well. Students also plot S vs. 1/d. This is quite a bizarre step to them, and they cannot appreciate it without considerable background work. To this end, students receive homework assignments in

which they start with equations like, $y = 3/x$, $y = 0.5/x$, make tables of x, y, and $1/x$, and plot y vs. x and $1/x$. By doing a number of simple examples, they see how the $1/x$ graph comes out as a straight line. (In fact, a few students need practice like this even for the linear relationship, and they do homework problems on $y = 2x$, $y = 0.4x$, etc.) Thus, we learn that a decreasing relationship may or may not be an inverse proportion.

Teaching this inverse relationship probably would not succeed if it were done only in this experiment, but there are two or sometimes three other places in the course where it comes up. One is in an experiment on parallax which I won't elaborate, where the parallax shift is inversely proportional to the distance of an object from the viewer. Another is in the ripple tank experiment, discussed later.

Data in the shadow experiment are also looked at as direct and inverse proportionalities:

$$S_1/L_1 = S_2/L_2 = \text{etc.,} \quad \text{and} \quad S_1d_1 = S_2d_2 = \text{etc.}$$

Results are usually convincing, but small discrepancies lead naturally to a discussion of measurement uncertainties, round-off, and significant figures.

There are two more follow-ups to the shadow experiment, aimed at "explaining" the data, as the next step beyond analysis. Here, one hopes to make the point that science, including experimental research, rests on theoretic underpinnings. One seeks simple principles to explain observed relationships. First, students are asked to explain qualitatively why S increases with L by simply drawing two diagrams of the apparatus, one with large L and one with small L, with d held constant (as in Fig. 2). Before they can do this, they have to come to the understanding that the pairs of P_1, P_2 and Q_1, Q_2 determine the width of the shadow, since light rays travel in straight lines. We generally come to this conclusion in a group discussion without too much prodding from the instructor.
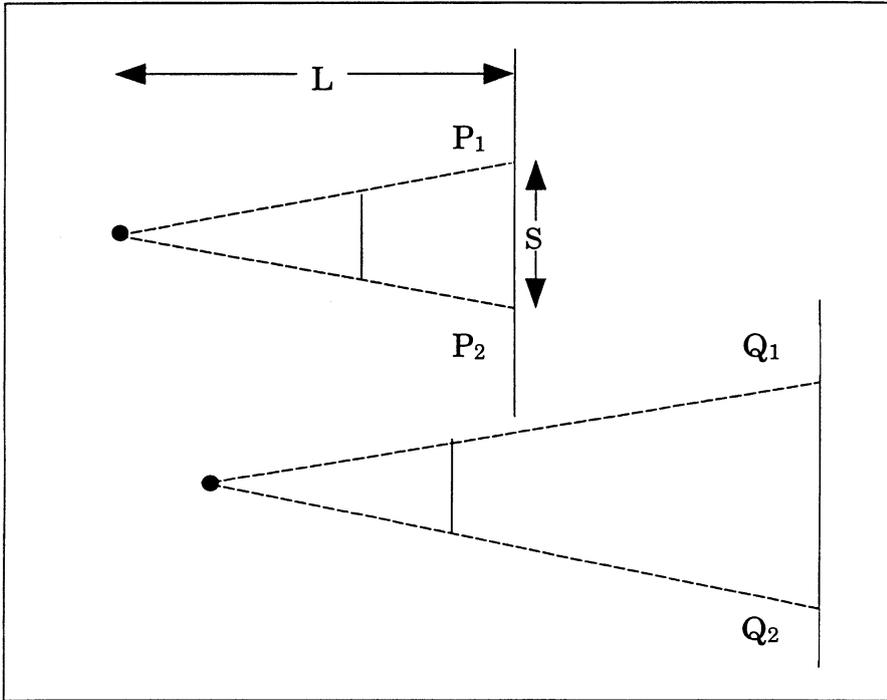
Fig. 2 – Student exercise to demonstrate qualitatively the dependence of S on L

But in the next step, in which students must draw figures to show the increase with L, they are surprisingly poor. They don't draw diagrams accurately, they don't draw lines straight, they fail to keep other lengths constant, and they don't know that they should make L _very_ different in the two cases in order to demonstrate clearly the increase in S. Significantly, it is not natural to students to draw a diagram in a schematic form, as is done in Figs. 1 and 2 here, for example. They prefer to draw realistic diagrams, with perspective, and showing various details of the apparatus that are irrelevant to the scientific or mathematical questions at issue. This illustrates their unfamiliarity with abstraction. We note that this kind of diagrammatic analysis is the kind of "problem-solving" skill that math reform tries to introduce in the early grades [6], and an exercise such as this, combined with real lab observations, would be a useful example. Whether students can successfully develop these skills remains to be seen.

Finally, we deduce the quantitative relation in this experiment by using the diagram in Fig. 1, and the proportionality of sides in similar triangles:

$$S/L = w/d,$$

where w is the width of the object. Deriving this equation calls for a detour into some geometry, discussed below. The equation for S is then used to show the proportionality of S to L and to $1/d$, to deduce the slopes of these two straight lines, and to compare them with slopes found from the students' graphs.

Students come to understand proportionalities by working with them repeatedly. Many other hands-on subjects in science or math suggest themselves for the study of proportions: mass/density/volume, time/speed/distance, scaling on a map, unit conversion, etc. They are not what might be considered "college math," but it is well-known that college students today are poor in these areas [7], and the science class should be used to bring home the issue.

Experiments with Angles

Both Gen. Sci. 9.1 and 9.2 require students to work with relations among angles. We study the laws of refraction and reflection of light in both courses. In 9.1, they serve as analogs for refraction and reflection of seismic waves. Other uses of angles are discussed below. Students have some feel for angles on a piece of paper, but not in real space, and they have little awareness that angles are the key to visual perception. A number of exercises are used in these courses to give students an understanding of angles on a concrete level, as opposed to a more abstract or set-theoretic treatment of the subject.   The close connections relating angles, similarity in geometric figures, and proportions leads to reinforcement in the study of these subjects.

Similarity is demonstrated by having students measure the sides of similar geometrical figures and verify the proportionality of corresponding sides. Angular functions are defined by having students measure lengths on paper, and determine appropriate ratios. They construct diagrams like those in Fig. 3 to study the radian measure of an angle, the "blip" (b/h in Fig. 3 (ii)), and the sine. We graph measurements gathered from the whole class, and they discover that (a) the radian measure is

proportional to the degree measure, and (b) the three functions are approximately equal for small angles.
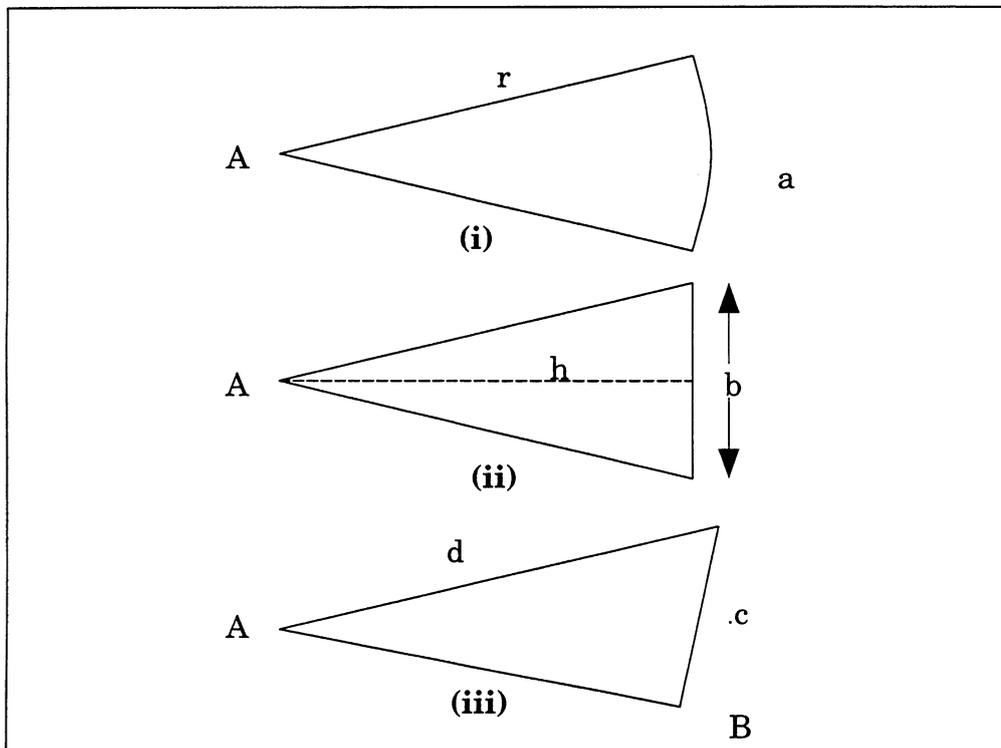

Figure 3 – Definitions of angular functions: (i) radian, (ii) blip, (iii) sine

The blip is used in an experiment on apparent size. Students hold up cards of different sizes (s) and place them at different distances (d) from the eye, so that they appear to be the same size. Students then verify that the ratios s/d are the same — either by calculating the ratios or by plotting s vs. d. The ratio s/d is the blip, size of the object divided by the distance from it to the eye. The idea that the eye measures angular size is quite new to students. (Indeed, there are usually one or two students in a class who simply cannot get it: they cannot abstract from the actual size of the object to the apparent size, and insist that the larger card appears larger than the smaller one, even when its apparent size is the same, or much smaller.)

We follow this experiment with the idea of using your finger at arm's length as a rough measure of the angular size of distant objects. Students can estimate, for example,

the angular size of a car at a distance of a block, and then check their result by pacing off the length of the block and noting the size of the car. Similarly, we can observe the near equality of the apparent size of the sun and the moon, and then calculate the blip from given astronomical numbers.

We use the sine of the angle in an experiment on refraction. This is a typical geometrical optics experiment in which one traces the path of a light ray through a rectangular block of lucite. I omit details of the setup; students obtain the angle of incidence and the angle of refraction for a ray emerging from lucite into air (Fig. 4). They then plot A_{air} vs. A_{luc} and obtain points that are fit by a straight line fairly well, up to about $A_{air} = 45$ degrees. Above that angle, the points veer systematically away from the line. They then find the sines of the angles, using our measured graph, plot $\sin A_{air}$ vs. $\sin A_{luc}$, and get a straight line close to all the points, verifying Snell's Law of Refraction.
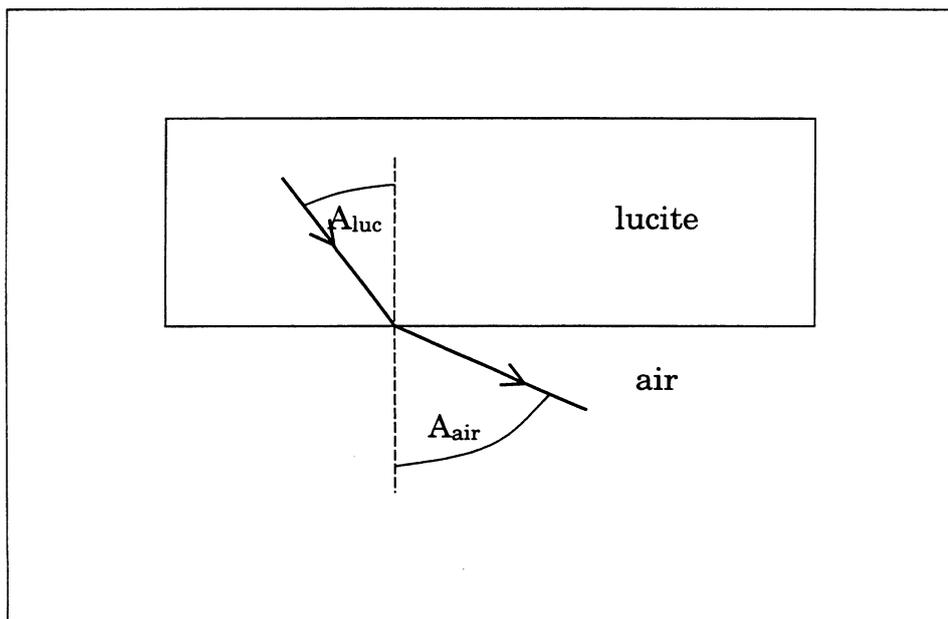

Figure 4 – The refraction experiment

Toward the end of the term in Gen. Sci. 9.2, we do two experiments that introduce the different colors of the spectrum and the wave character of light. The first is a dispersion experiment, in which we use helium-neon lasers with three different wavelengths, 544, 594, and 612 nm (green, yellow, and orange) [8]. The lasers are placed in a fixed stand and the light is refracted through a prism, striking a screen about 4 m away. We measure the shift in the refraction angle from one beam to the others. These shifts are small angles (up to .02 radians), and students see that they could not be measured with a protractor. We can only use the blip, which is easily determined from the shift of the laser point on the screen; we then recognize that the blip and the radian are equal for small angles, and then convert radians to degrees using an experimentally determined constant.

The last part of the course occupies at least 6 hours of lab time. It involves the two-slit experiment with light, in parallel with the analogous ripple tank experiment which looks at the interference pattern with two wave sources in water. Here, the fundamental equation, applying to both experiments, also involves the sine (Fig. 5):
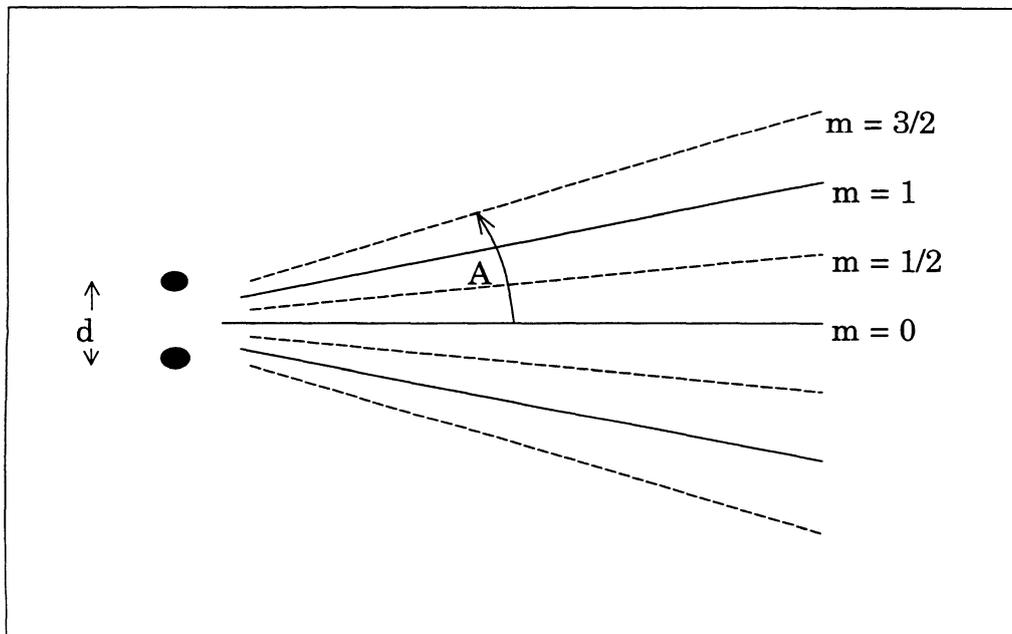
$$\sin A = m\lambda/d,$$

Figure 5 – The interference experiment using light waves or water waves

where m represents the order of the interference fringe (m = 0 for the central maximum, m = ½ for the first destructive interference point, m = 1 for the first constructive interference point, etc.), d the spacing between the two sources, and λ the wavelength. The two-slit experiment which established beyond doubt the classical wave character of light is one of the most important experiments in the history of physics. Because of the subtlety of the phenomenon of interference, indeed because of the subtlety of the concept of waves altogether, it seemed essential to couple this experiment with the more concrete and tangible experiment with water waves. It is immensely instructive for students to try to see the parallel between the two experiments, even as the surface features of the two experiments differ so markedly; including the fact that, in the ripple tank we mark the line of destructive interference whereas with light fringes, we measure to the points of constructive interference.

At present, we are not able reliably to measure the wavelength of the water waves in the ripple tank. However, we can measure the angles up to m = 1/2, 3/2, and 5/2, choose three different values of d, and verify the dependence of the angle on m and d (see Fig. 5). The angle is drawn to the center point between the sources and is measured with a protractor. The sine of the angle is determined from our previous studies.

In the case of the light experiment, we use lasers with four wavelengths (the three used in the dispersion experiment, plus one at 633 nm). The slit-spacing d is fixed. The interference pattern is on a screen about 2 m from the slit, and the distance from the m = 4 bright spot on one side, to the m = 4 bright spot on the other side is measured. Then, the angle here is determined from the blip, and we again replace the blip by the sine. We usually get an impressive straight-line graph of angle vs. wavelength.

Conclusion

The purpose of these experiments is to teach subjects in science -- visual perception, refraction, interference, etc. Along the way, mathematics is introduced and reinforced from many directions. The mathematics is not an abstract entity created by a book or a teacher, but a real thing in front of the student, and it is _the_ tool the student needs to make sense of his/her findings. Furthermore, the same mathematical concepts turn up, perhaps unexpectedly, but repeatedly through the term, to the point where the student begins to see that certain important things are happening. It is not a matter of

drill vs. conceptual learning [9], but of the student's active and continuous involvement with the concept.

Similarly, one can envisage concrete, hands-on approaches that are continually tied to applications in science other subjects, and to many fields of mathematics, including statistics, and plane and solid geometry, etc. Where such an approach cannot be envisaged, perhaps an *ab initio* evaluation of the appropriateness of that field in the curriculum is called for.

Finally, we observe that the approaches being discussed here should be effective at earlier stages in the educational process. It is then possible that the more hands-on and concrete development of mathematics in the early years, especially with constant reinforcement in the science and social science classes, may allow a more secure growth of abstract thinking skills in the teen years. Thus prepared, a more sophisticated college student, adept at abstract analysis, may be appreciate and be given the deeper and more satisfying experience in science and mathematics that he/she deserves.   ■

Bio

Michael Sobel is Professor of Physics at Brooklyn College of CUNY. His interests are in theoretical nuclear physics, nuclear arms control, inquiry-based science for liberal arts students, and mathematics reform.

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