MATHEMATICS WITHOUT LECTURES: SMALL-GROUP LEARNING AT NEW YORK UNIVERSITY

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This article describes an effort to introduce small-group learning into the mathematics curriculum for the non-specialist at New York University. Starting in spring 1999, students were offered the choice of fulfilling their mathematics requirement in a small-group environment that included no formal lectures. The goal of these groups is to make the transition from relatively inactive, even passive, lectures to an experience that actively engages students in the process of doing mathematics. Contact time was restricted to two weekly classes run by a graduate student and was limited to enrollments of 15-16 students. The course is a small-group version of one that has been offered regularly since 1995, with a format that includes two traditional large lectures and one 100-minute workshop each week. Students in the College of Arts and Science and in the School of Education took the course, and the latter group included future K-12 teachers. Instructors for the small-group sections come from the graduate level Mathematics Education Group in the School of Education and the Mathematics Department in the Graduate School of Arts and Science.

Introduction

Motivating students who take mathematics as a liberal arts requirement for their baccalaureate degree is one of the most difficult problems facing mathematics educators. What should be the content? What pedagogical approach should be used? The trend in mathematics and science education is toward inquiry-based learning in which students are told as little as possible of what is now accepted scientific and mathematical fact. Instead, they are given the opportunity to discover scientific truths by observation, collect data, and draw conclusions by themselves [1,2,3] as a prelude to an introduction to the theory. One can add a laboratory experience to existing lecture-based courses, though this still can leave students with a passive experience in the lecture hall. Some educators now employ techniques for introducing small-group activity into the lecture experience, even if confronted with a large class of students [4].

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Another approach is to eliminate the lecture, which frequently has a large enrollment, and to increase the laboratory experience. Mathematics courses without lectures began to be offered at Dickinson College in 1991, through an option, which they call *Workshop Mathematics* [5]. This format of teaching students mathematics in small groups was introduced into the NYU math/science core program (the Morse Academic Plan) in the Spring 1999 semester, repeated in fall 1999 and is being done again in spring 2000. Educational research supporting this method of pedagogy has been appearing in the literature for a number of years [6]. Further motivation was provided in part by attendance in Quantitative Reasoning (the name given to the math part of the NYU core) lectures that are typically 75% of the enrolled students, but sometimes as low as 60%.

Poor lecture attendance and poor course evaluations by students prompted a discussion among the faculty on the Steering Committee that overlooks the math/science core. Members of this committee not only include faculty from the Mathematics Department, but also Kenneth Goldberg of the School of Education and Neville Kallenbach of the Department of Chemistry, both of whom have important roles in the New York Collaborative for Excellence in Teacher Preparation (NYCETP). Their work in the Collaborative informed their ideas about effective instruction, and a fresh perspective on mathematics education for the non-major was introduced into the discussions about Quantitative Reasoning.

Educational research points to something those engaged in college writing programs have known for some time: you learn writing not by listening to a lecture on writing, but by writing. This is how writing is taught at NYU. The small-group effort is based on the philosophy that one learns mathematics by doing mathematics and not by listening to lectures about it. This article documents an effort to introduce this mode of learning into the mathematics core curriculum at NYU.

Mathematics Education for the Non-Major at NYU

The math component of the NYU core curriculum in mathematics for the non-mathematics and non-science major consists of completion of a single course. The component, called Quantitative Reasoning (QR) is not a single course, but is currently a group of 3 courses, any one of which can be taken to fulfill the mathematics requirement. In the standard "lecture format," each course consists of two 75-minute lectures, given by a faculty member, and one 100-minute workshop, conducted by a graduate student, per week; total weekly contact time is 250 minutes. Lectures typically have enrollments of 126 students and the workshops have 21 students per section. One course, called *Mathematical Patterns in Nature*, has a textbook and workshop project book written by Frederick P. Greenleaf, who created the course [7, 8]. It was decided to translate this course from the lecture format, to a small-group format with classes run by graduate students and enrollments limited to 15 or 16 students.

Course Format: No Lectures

The approach that we took was to take the existing course, *Mathematical Patterns in Nature* and run it in two formats during the same semester: the lecture format described above and a small-group format without formal lectures. Adopting a new format for a course raises the question of contact time. Should the new format be designed to have the same amount of contact time as the lecture format?

Discussions led us to small-group classes that consisted of two 100-minute sessions. As a consequence, students enrolled in the small-group sections received 50 minutes less contact time each week than students in the lecture sections. In a 14-week semester, this amounts to 11.7 hours of contact time that students have free for other activities. This is a strong reason students find this new format appealing.

The small-group sections were able to cover the same material in a shorter amount of time, primarily due to the active nature of the classroom sessions. In a typical lecture, students watch the professor describe a topic in considerable detail and do a number of examples, but they usually do not work on or discuss problems themselves. Students will not attempt any close interaction with the material until they are out of the lecture and doing homework. This is not true for the small-group classes, which the Faculty Steering Committee felt removes a lot of the redundancy built into the traditional lecture format; whereby, students are not required to engage in class work even if they are present in the lecture hall, and spend time in the single weekly workshop reviewing material they watched the professor do in the lecture. Indeed, the attendance is much better for the small-group sessions since, among other reasons, it is possible to take attendance in a class of 15 students. Students also come to class knowing that they are going to work with other students. This compels the student to come prepared and ready to engage their fellow students and be engaged themselves.

Kenneth Goldberg introduced us to masters and doctoral candidates in mathematics education who had a particular interest in small-group learning. They were some of the instructors who taught the small-group format, while the remaining sections were taught by graduate students from the Mathematics Department. In addition to the teaching experience that the graduate students from the School of Education gained, the graduate students from the Mathematics Department had the opportunity to engage in discussions about teaching with students pursuing studies in education.

Size of the Experiment

In spring 1999, we directly compared the two formats by offering 126 seats in lecture version of *Mathematical Patterns in Nature* and 120 seats in a small-group version devoid of formal lecture. At final enrollment, 98 students chose the lecture version and 120 students chose the small-group version. The *only* sections that were filled to maximum capacity were the small-group sections.

Sample Activity

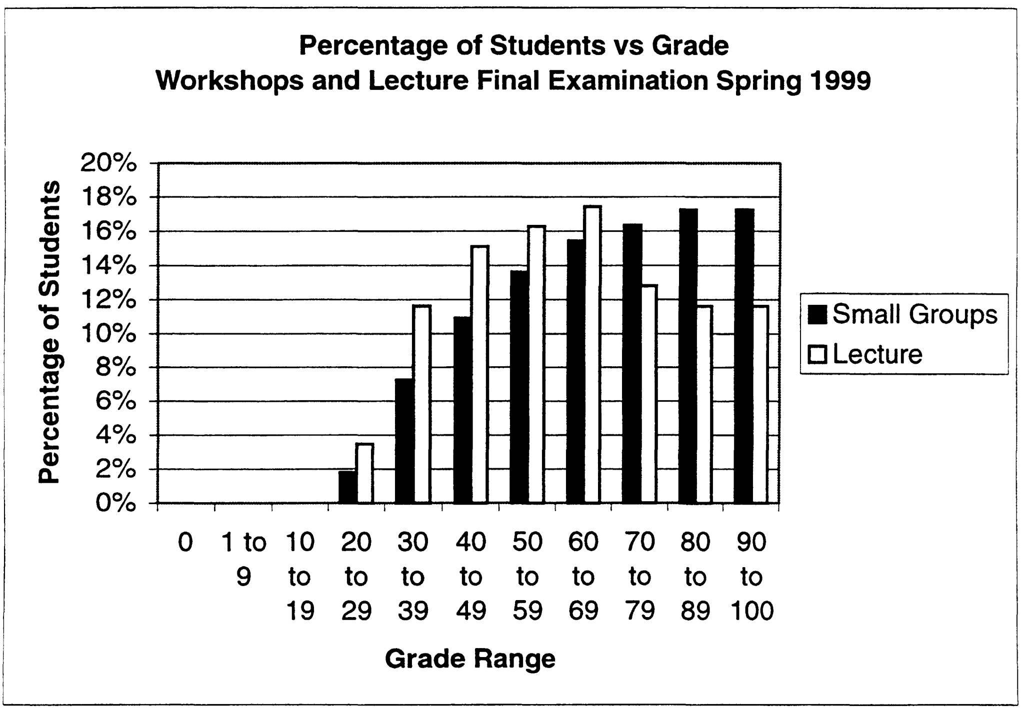
The curriculum for the course includes growth and decay problems, such as problems dealing with the growth of money under compounding. Students are asked to visit neighborhood banks and ask for current rates on 3-month CDs and savings accounts. The instructor for the course gives the students 5 stocks and 5 mutual funds, in addition to 3-month CDs and savings accounts, to choose from and make a hypothetical portfolio, assuming they had $10,000 to invest. Each week, the current stock price is recorded from the daily newspaper. Students calculate the present value of their portfolio each week.

Assessment

Early in our planning for the small-group format, it was decided that *all* students enrolled in *Mathematical Patterns in Nature* in the Spring 1999 semester should take the same final exam, regardless of format chosen. Exams were given at the same time and day for both groups. The syllabi for the lecture and small-group sections of *Mathematical Patterns in Nature* are identical, and they conducted essentially the same lab workshop projects. A comparison of the exam scores was then used to compare effectiveness of the two formats (see below). Further data to be considered includes: gender, class, school, and Math SAT scores. In addition, a pre-test was given at the beginning of the semester to get an idea of the knowledge that students brought into the class. These results will be published in the future.

Results of the Final Examination

Students in the lecture and workshop versions of the course took the same final examination. The exam was written by Frederick Greenleaf, who taught the lecture version of the course in spring 1999.



A graph of the percentage of students in the lecture receiving a grade within a 10 point-wide range was looked at and compared to the same data for all students in the small-group version of the course. This data is presented in the graph above. A greater percentage of the students in the workshop version of QR got scores of 70 or higher, while a greater percentage of students in the lecture version of QR got scores of less than 70. Specifically, 50% of students in the small-group based course achieved a grade of

70% or better on the final examination, as opposed to 37% of students in the lecture-based course.

Future Work

Results of the assessment activities outlined above will be published, based on the data that will be taken in the Fall 1999, Spring 2000, Fall 2000, and Spring 2001 semesters. Final exam results and student responses will be analyzed to see if students consistently prefer and do better in the small-group format.

Further, in the summer of 1999, a small-group version of another Quantitative Reasoning course, *Mathematics and the Computer*, was offered to 30 students in two sections. In spring 2000, the small-group version of *Mathematics and the Computer* will be offered with 60 seats distributed among 4 sections, thus maintaining a class size of 15. This will extend the small-group pedagogy to a second QR course. ■

Bio

Andre Adler is Coordinator of the Foundations of Scientific Inquiry program in the Morse Academic Plan. He holds a Ph.D. in Physics from New York University and has taught in the Physics Department, as well as courses in Mathematics and the Physical Sciences in NYU's Core Curriculum, the Morse Academic Plan.

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