INFORMAL GEOMETRY IN THE PREPARATION OF TEACHERS: A NEW MATHEMATICS COURSE AT THE UNIVERSITY OF VIRGINIA

L. D. PITT
University of Virginia, Charlottesville, VA 22903
ldp@virginia.edu

Students require a rich variety of hands-on geometric experiences before they progress to more formal traditional geometric instruction. This fact has often been ignored in the mathematics preparation of today's teachers. At the University of Virginia a new general education geometry course, *The Shape of Space*, is being developed that focuses on obtaining deep understandings of elementary geometry through physical and visual activities.

Introduction

Deficiencies in the geometry education of American students have long been observed and documented. Geometry occupies a central place in the elementary and middle level mathematics curriculum, and geometric concepts, representations, and patterns contribute to students learning measurement and number concepts. Notwithstanding this central role, this material has been neglected to such an extent that in 1993 Geddes and Frotunato [1] stated that ". . . many middle grades students could be described as geometry deprived" (p. 212) and argued further that this deprivation is a likely impediment to the student's mathematical progress. This deprivation was prominently documented in the Third International Mathematics and Science Study (TIMSS) [2] which identified geometry and measurement as the only areas of mathematics where American 8th graders fall notably below the average of the 41 participating nations.

Our geometric deprivation in Virginia is currently partially hidden by the fact that the SOL Geometry test scores were better than the Algebra test scores, but the belief that this issue can temporarily be put off is based on a misinterpretation of the data. In fact, the geometry tests were taken by the much smaller and more select group of students that had taken geometry. When, in the future, the geometry test is administered to (almost) all students, we will likely discover that the geometry scores will cause intense alarm.

The Journal of Mathematics and Science: Collaborative Explorations Volume 2 No 2 (1999) 117-120

Two further observations are critical to my thesis. The first is that geometry is learned developmentally, and the developmental process can not effectively be bypassed. It is only a slight exaggeration to assert that when children do not have an appropriately rich developmental sequence of informal geometry experiences in which they are learning through their eyes and fingers, they do not learn geometry. And, without these experiences their overall mathematics development, especially that which rests on non-routine problem solving and critical thinking skills will suffer.

The second observation is that most children today are living in a geometry void. This is an obvious point, but one that is often ignored at the expense of our children. The active life of building, storing, measuring, sewing, and cooking that once was common in America is gone and has not been replaced with one of equal educational power. Nothing in their daily life, outside of cars, video games, and athletics, requires measurement, geometric awareness and analysis, or visualization. If students are to find the essential informal geometry experiences that once were part of daily life, a preponderance of those experiences must come from within the schools.

By and large, this geometry is absent in today's schools and few of our teachers are prepared to teach the hands-on geometry that is needed. In Virginia the seriousness of these dimensions is magnified by the Teacher Licensure Requirements that we have discussed at this conference. Our teacher preparation programs must include a new geometry dimension or our children will suffer.

A Response

As part of the NSF collaborative VCEPT, the University of Virginia is responding to these concerns by developing a new general education geometry course that is primarily focused on the needs of future K-8 teachers. The course is a 100 level course in informal geometry titled *The Shape of Space*. Currently it is being piloted for the second time and will be submitted for College approval this spring. The course title attempts to capture the flavor and philosophy of the course. To the extent the course is successful, students in *The Shape of Space* divide their time between geometry activities involved with drawing, building, coloring, measuring, and analyzing. Communicating what they learn is also stressed. This is our goal but candor requires an admission that the goal is not always reached.

When approved *The Shape of Space* will be a three credit course with two hours of lecture and two hours of laboratory. At present the course is divided into three approximately equal sections. The first part consists of measurement and estimation activities of a highly hands-on nature. This includes working with rulers, protractors, compasses, strings, and calipers. The conceptual development of geometry is built upon a foundation of activities where students measure, calculate and estimate lengths, areas, surface areas, and volumes.

A separate module is spent on analysis of geometric shapes, properties, and concepts. Students explore basic topics such as angle sums for triangles and properties of parallel lines. Paper folding is a large part of this section of the course. A few investigations of how these properties might change in a curved space together with geometric explorations on the surface of a balloon are included.

The final part of the course explores symmetry and other geometric properties from a transformational viewpoint. In this piece elementary symmetry groups, symmetry patterns, and tessellations are discussed. Similarity and proportional reasoning are also included here.

The universal problem of overly zealous teachers with too much material and too little time occurs in this course. Perhaps, because the goal is to overcome a lifetime of sensory deprivation in one semester, the problem is worse here than in many other subjects. However, this kind of geometry is fun and it is my firm belief that the exact topics covered are not of primary importance. What is most important is the active engagement of future teachers in geometric explorations and analysis. Teachers need to experience geometry and know that it is fun. We have failed whenever a teacher enters their profession without ever having smiled while holding something in their hands and saying, "Oh! I see!" If this is not happening, our efforts are not on target. Students need experiences in building, visualizing, and figuring things out. They need to do this at developmentally appropriate levels, and in my view this means that they need to experience success in finding explanations of phenomena that interest them.

Supporting Theory

There are three axioms which underlie this course, its philosophy, and its format. First is the body of research centering on the so called van Hiele Model of how children (and adults) learn

geometry [3], [4]. Children do not succeed in geometry if their development of understanding is at a different level than the instruction they are receiving. A second axiom is a personal article of faith that elementary geometry is an empirical science. Its roots are all based in experience and observation. This is a non-technical personal expression of truths similar to those behind the van Hiele model. Teaching and learning geometry will succeed when students come to geometry classes with a sufficiently rich treasure of experiences that combine physical manipulation, visualization, exploration, and analysis. Finally, informal geometry is the foundation on which children build much of their mathematical and scientific world. I believe, as Hilton, Holton, and Peterson [5] stated, that algebra gives us tools for solving problems, but geometry gives problems that we wish to solve. ■

References

[1] D. Geddes and I. Fortunato, "Geometry: Research and Classroom Activities, Research Ideas for the Classroom: Middle School Mathematics," in D. T. Owens, (ed.), *NCTM Research Interpretation Project*, Macmillan, 1993.

[2] *Mathematics Achievement in the Middle School Years, a report from the TIMSS*, TIMSS International Study Center, Boston College, Boston, 1997.

[3] D. Fuys, D. Geddes, and R. Tischler, "The van Hiele Model of Thinking in Geometry among Adolescents", Research Monograph No. 4, *Journal for Research in Mathematics Education*, 1988.

[4] P. van Hiele and D. van Hiele-Geldof, *Selected Writings of Dian van Hiele-Geldof and P. M. van Hiele*, Brooklyn College, 1984.

[5] P. Hilton, D. Holton, and J. Pedersen, *Mathematical Reflections: In a Room With Many Mirrors*, Springer Verlag, 1996.