STUDENT DISCOVERY AND LEARNING THROUGH PRECALCULUS CBL PROJECTS

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In the precalculus course which is being piloted at Virginia Commonwealth University we have interwoven four student CBL (Calculator-Based Laboratory) projects throughout the semester to foster student participation and discovery through supervised group work. These activities are scheduled on specific class laboratory days throughout the semester and are coordinated with topics as they are developed in the text. These topics range from an introduction to functions to specific applications of functions including the quadratic exponential, and trigonometric functions. For each experiment, the apparatus, including appropriately programmed CBL's, is set up in the classroom by the instructor and assistants at two to four stations. Students are then organized into groups of five or six and conduct their experiments at the established stations. A written description of the experiment (including goals and expectations, step-by step process instructions, and hints for making the desired mathematical connections) has been given to each student at the prior class meeting. Following a brief overview of the current experiment by the instructor, the student groups perform the experiment and transfer the data collected by the CBL to their own TI-83 or TI-82 calculators for interactive study and analysis. The class instructor and a student assistant (undergraduate or graduate) assist groups throughout the process as needed. The general group task in each of these experiments is to analyze the functional data points collected by the CBL and stored and graphed under the STAT PLOT procedure on their TI-83 or TI-82 calculator; to use algebraic techniques to find an appropriate mathematical function (quadratic, exponential, trigonometric etc.) which will closely model the situation depicted by the data points; to graph the function (entered as Y_1 in their TI-83 calculator) simultaneously with the STAT PLOT of the data points; and to refine their function rule until their graph closely fits the graphed data points.

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In addition to leading the students to actively discover the specific characteristics and controlling features of these mathematical functions, the knowledge of use of which are critical to the impending study of calculus, we believe that this student work provides invaluable insight into how scientists use mathematics to determine functions which model physical situations. For example, when the student encounters a textbook exercise like "If a certain bacteria grows according to the functional rule $f(x) = 1000\ e^{.3062x}$ where the time x is measured in minutes, when will the population reach one million?", some insight will already have been instilled as to how such a function was determined. The instructor can remind the class that they, in fact, have similarly determined function rules from collected data. In contrast to receiving on "faith" a general answer like "Those functions are statistically determined by scientists after many experimental trials" which still gives little insight into the mechanisms for determining the function rule, the students can now relate to their own experience in modeling data with function rules. The students should also have a better feel for what the variables x and $f(x)$ represent.

The CBL laboratory experiment also provides a natural opportunity for students to describe their mathematical discoveries and observations in a written report form. Guided by the written expectations and grading criteria provided, each student is required to synthesize the group findings and analysis to an extent that he/she can accurately describe the goals, processes, and results in a written form which their peers can expect to comprehend.

Our preliminary evaluation of these CBL laboratory experiments in pilot sections of the precalculus course show a very positive effect in generating student interest and in their ability to make mathematical connections to real world physical situations. Instructors agree that the students' written reports are generally very good, although wide variations exist in the effort and creativity which student writers show in striving to generate reader interest in their reports. Preliminary analysis of performance on test questions directly keyed to the analysis of laboratory projects indicate student achievement comparable to that on other questions. In the first pilot sections, written student reports were required on all of the laboratory projects. Constraints due to the magnitude of grading have led us to explore a balance between written report grades and specific test question analysis to provide evaluation of student learning from the CBL laboratories. Additional evaluation of the impact of CBL activities is planned during the next course offerings.

We will give a sample student instructional handout along with a possible set of reportable analyses for one of our four CBL laboratory experiments. Similar descriptions of the other three laboratories will be available commercially.

Before we present the "Pendulum" experiment which motivates the modeling of periodic phenomena with sine and cosine functions, we will give some tips for the experimental set-up which, based on our experiences, should be helpful. First, since the CBL motion detector measures distance from an origin on a horizontal line, we found it necessary to swing a large object in order for the detector to retain horizontal visual contact as the object swung in its arc. Old basketballs proved to work very well when hung from a ceiling on a six to eight foot length of monofilament fishing line. Since the basketballs do not need to maintain firm inflation, they can usually be obtained at minimal costs from flea markets or thrift stores. Also, a small nail inserted into the valve stem of the ball provides a convenient means to tie the ball to the monofilament line. Thus, except for the cost of the CBLs and motion detectors, several experimental stations can be set up with very little cost. Two stations proved ample to serve six groups of five or six students during a fifty minute laboratory session, even when some repetitions were needed by several groups. We found that nice graphical results could be obtained with three or four pendulum swings with minimal observed damping results. Other self explanatory tips are interspersed in the description which follows.

Student Laboratory Description – Pendulum

I. Background

When a pendulum swings back and forth its horizontal motion can be described by a periodic function. In this lab we will use the CBL to measure the horizontal distance of a swinging basketball from the "center". We will call the spot where the pendulum hangs at rest the "center". The CBL will take measurements every tenth of a second. The data curve will actually be damped (its amplitude will decrease as the pendulum arc decreases) because of air resistance, etc., but we will find a good approximation of the motion using the basic trigonometric functions cosine and sine.

II. Lab

Step 1. The classroom calculator will have the program SWING installed. Clear out any functions that may have been stored in the calculator, especially under Y_1.

Step 2. The pendulum will be set up with motion detector and CBL unit ready to go. Make sure the class calculator, with the program SWING installed, is connected to the CBL unit using the link provided.

Step 3. Turn the CBL unit on by hitting the red ON button. Turn your calculator on and hit the PRGM button. Use the arrow keys to scroll down to the program called SWING and hit the ENTER button twice.

You will see the introductory screen. Hit ENTER and then hit ENTER again to collect the data.

Step 4. Let the ball hang at rest and hit ENTER, making sure the ball is still. This center point of the swing will serve as the point from which the horizontal distance of the ball will be measured.

Step 5. Hold the ball out (no closer than 14 in. from motion detector, keeping the string taut, but not stretched, and keeping the ball aligned with the motion detector). Hit ENTER.

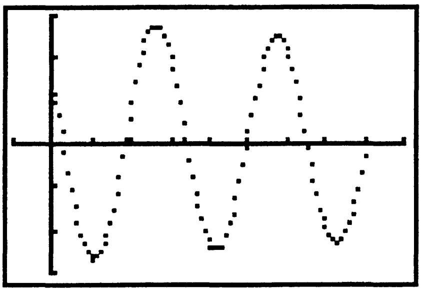
Step 6. Have one student count down and hit ENTER, starting the program at the same time that another student releases the ball. Be sure to let go, <u>without</u> giving the ball a push.

Step 7. You should now have a plot (graph) of data on the calculator, the x-axis scale representing time and the y-axis scale the horizontal distance of the ball from the center. Does it look like a cosine curve? If not return to the program and repeat the experiment. Keep trying until you get a nice cosine curve.

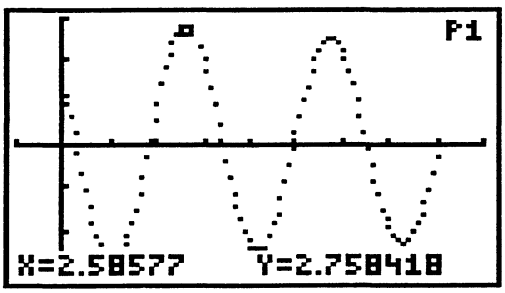
Step 8. Disconnect your calculator from the CBL unit. Transfer the data to your own calculators (TI-82 or TI-83).

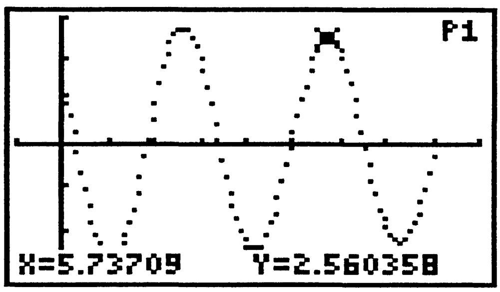
Sample Experiment Responses:

After completing step 8 of part "II. Lab" the student has a calculator with the following sample graph.



Trace to find the extrema of your data curve. What are the coordinates of two consecutive local maximum points?





Since the motion of the ball is periodic, trigonometric functions might help to describe its motion. We will first attempt to describe the horizontal position of the ball using a function of the form $y = A\cos(bx)$. Using the y-values of these points you can decide what your amplitude should be. What value do you want to use for your amplitude, A?

I chose A = 2.6 because it was between the two local maximum y-values.

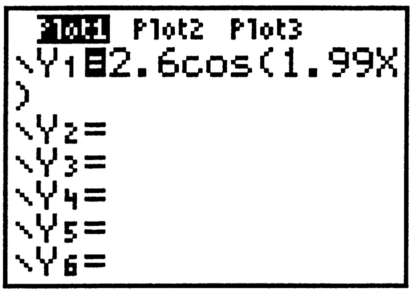
Using the x-values of these points you can decide what your period of your pendulum should be. What is the period?

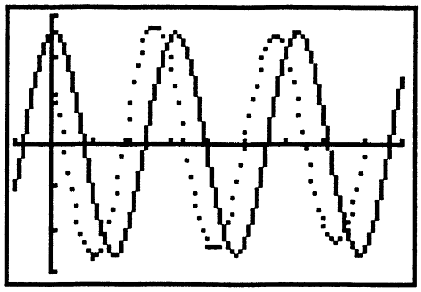
5.73709 - 2.58577 = 3.15132, so I wanted my period to be about 3.15132.
Knowing the period, what value do you want to use for your b?
$b = (2\pi)/\text{period} \sim (2\pi)/3.15132 \sim 1.99$

Putting in your values for A and b, what is your equation?

$y = 2.6\cos(1.99x)$

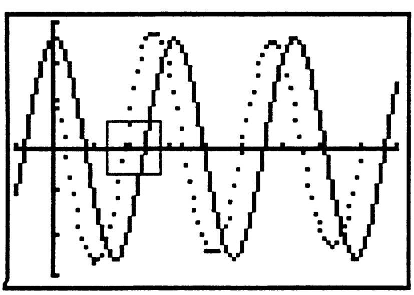
Graph this equation in Y1. Does it match the data well? Should you adjust your amplitude or period? Do so if necessary.





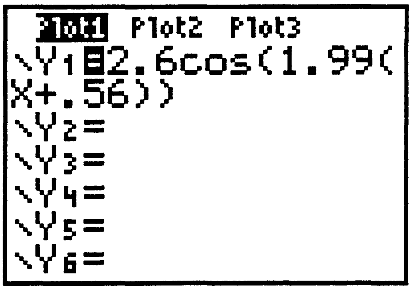
It matches pretty well. The period and amplitude look fine, but it seems offset on the x-axis a little bit.

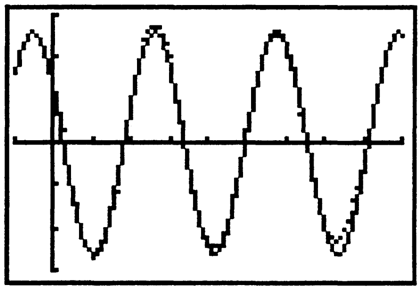
Shift your graph if necessary. Hint: Trace to find the difference in the x-values of an x-intercept on the data curve and the corresponding point on the curve obtained from your equation. Remember to add the distance c to your original x to obtain $y = A\cos[b(x+c)]$ for a left shift and to subtract the distance for a right shift. What is your equation now?



Distance between data curve and curve from the equation is .56.

Since c = .56, my new equation was $y = 2.6\cos(1.99x(x + .56))$.





Now rewrite the equation in the form $Y = A\sin[b(x + c)]$. What do you have to do to the equation above to get it in this form? Recall that the "general sine function" $y = A\sin[b(x + c)]$ must be shifted to the left by one fourth of a period to coincide with the "general cosine function" $y = A\cos[b(x + c)]$.

I needed to shift to the right (3.15132)/4 = .78783

This gave me the equation $y = 2.6\sin\{1.99[(x - .78783) + .56]\}$

So, my final equation was $y = 2.6\sin[1.99(x - .22783)]$

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