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COLLABORATIVE EXPLORATIONS

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Virginia Mathematics and Science Coalition

National Alliance of State Science and Mathematics Coalitions

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MATH, SCIENCE AND ADVENTURES IN SPACE

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When asked to speak to you at this 1997 Summer Colloquium, it was with a great deal of reluctance and a bit of arm-twisting that I agreed – not because I don't appreciate the goals and purpose for this institute, but because it is my personal policy not to talk at great length on a subject about which I know very little. For some people that is not at all inhibiting, but it makes me uncomfortable so I try to avoid doing it. So I won't talk to you about years of experience in classrooms, about research in teaching methods, or about innovative new ways to teach math and science. What I can tell you from first-hand experience is the profound effect you, as teachers, can have on your students.

Along with three other scientists, I spent about a year and a half training for my last Space Shuttle mission. We worked closely with the principal investigators to learn all the procedures we would use to conduct more than 30 experiments during our 16-day stay in the Spacelab module onboard the Space Shuttle Columbia. After one particularly long day of training, over Chinese food and a couple of bottles of wine if I remember correctly, we began to reminisce about how we got where we were; how we got interested in math and science and ultimately got a ride in space. Remarkably, for each of us it was the same: a teacher in middle or high school who made all the difference in us and on our careers.

For me it was my high school physics teacher who helped me realize that science was interesting and exciting and what I wanted to do with my life. I don't recall any particular encouragement for me as the only girl in my physics class. It was just a statement of fact that anyone who was willing to try could learn physics. For that I thank him.

I also have to thank another teacher in another country for the exciting experiences I have had. Wernher von Braun admitted to a strong dislike for math as a student, even failing in the subject, until someone introduced him to the field of rocketry. He quickly realized that he could not follow his passion for space flight unless he understood mathematics. So he learned

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math. He learned it so well that he became a tutor for fellow students in math and physics. As an engineering student and a member of an amateur rocket society who used an abandoned Army ammunition dump as a launch site for their experiments, von Braun's talent and energy were recognized by the military. Where von Braun and his colleagues saw a vehicle for space travel, the German Army saw an instrument of war. In 1932, when the Army offered von Braun the opportunity to pursue his education and rocket research with funding, he accepted and became their top civilian rocket specialist at the age of twenty. Only a few years later, von Braun became technical director of a secret rocket development center at Peenemunde, as Germany began gearing up for another war. Von Braun never gave up the dream of space flight and, in fact, was once arrested by the SS and imprisoned for spending too much time working on a rocket for space rather than a weapon for the war effort. In the final months of World War II, as the Russian Army was about to take over Peenemunde, von Braun and his team hid hundreds of secret documents about the dreaded V-2 rocket in an abandoned mine shaft, and traveled hundreds of kilometers south using forged orders to pass checkpoints in order to surrender to the American Army. He believed that his best chance to continue his dream of space flight would be in America. He and more than a hundred German rocket specialists came to the United States to become the nucleus of the American Space Program. His Redstone rocket put Alan Shepard into space. His Saturn V rocket took twelve men to the moon.

To Wernher von Braun's teacher: Thanks for igniting that spark in him which ultimately allowed me to travel in space and orbit the earth 650 times. And thanks to all the other teachers who lit those sparks in all the other scientists and engineers who were and still are sending men and women to Earth orbit and robots to explore Mars, Venus, Jupiter and beyond our solar system. I am personally indebted to them all.

In part to repay that debt and also to help keep the dream of space exploration alive for the next generation, I believe that those of us who are privileged to live that dream have an obligation to bring back as much as we can to those who have not, or not yet, had that grand adventure. As you may know, astronauts are selected for their technical abilities, not because they are particularly articulate. So we take a lot of pictures. I'm sure you have seen some of those remarkable photographs of the Earth from an astronaut's vantage point. If you have not seen the IMAX films shot in space, you should definitely not miss any opportunity to see them. What you will see in IMAX is very close to what I saw from space—the Earth passing by at five miles per second; the blue of the oceans, the distinctive colors of the continents. What you cannot get from films or photographs is what it feels like to launch, to float, or to watch the Earth "go by."

I'll do my best to describe to you an indescribable experience. T-0, the moment of launch, does not feel as violent as you might expect watching all that fire come out of the rear end of the rockets. It felt to me like a firm push on my back, not unlike the firm but unrelenting push it takes to get your kids out the door in the morning to catch their school bus. That firm push is followed by a bumpy ride during first stage, when the solid rocket boosters are burning. If you can imagine lying on your back in a hard chair, riding in the back of a pickup truck on railroad tracks with a bowling ball on your chest, that would be close to what I felt during first stage. That lasts for about two minutes until the solid rocket boosters have done their job. They separate from the rest of the stack with a boom and a lot of fire as separation rockets push them away. The boosters parachute into the Atlantic Ocean to be recovered and reused on another flight. You continue "uphill" with the three main engines on the Orbiter, which is a smooth, almost electric motor-feeling ride. Toward the end of the ascent, you experience 3 G's, or three times the acceleration of gravity. That can become a little uncomfortable, feeling like a weight is pressing down on your chest as you try to breathe. But it only lasts for a couple of minutes.

The single most magical moment of the whole flight is main engine cutoff, or MECO, when you go from feeling like you have a gorilla sitting on your chest to having your feet float up in front of you. The first time I experienced MECO, I had to check to see that those were my feet because I didn't put them there. The sensation of floating is one of complete freedom. You can move effortlessly in any direction. World-class gymnastics moves are easy even for those of us over the age of 18, provided the rules can be amended so that after a spectacular 3½ rotation maneuver it is permissible to land on the ceiling instead of the floor. It is also a sense of frustration because you can't stay where you want without using your hands for positioning. Simple tasks take twice or three times as long because nothing, including your own body, stays where you want it to. Imagine trying to work in an office or a lab with a strong wind blowing through. You can't simply put down a pencil or a *screwdriver or a piece* of paper because it will not be there when you want it. Imagine having to hold yourself in place with at least one hand while trying to work with the other. It takes a lot of thought, planning, velcro and bungees to set up a work site when you are floating.

There are three impressions of space flight that I hope to remember all of my life. The first is about our place in the Universe. I came to understand how ancient peoples believed that the Earth was the center of the Universe. I could float in the middle of a spaceship traveling at 17,500 miles per hour and yet feel none of the usual cues of motion: no vibrations, no wind noise, no centrifugal force pressing me against the wall as I traveled in a circle.[1] It appeared to me as if I were fixed in the Universe while the Earth turned in front of me just to give me a show, and the sun did a dance around both of us. I felt like the Universe was centered on me and my spaceship. Fortunately, someone more clever than I developed a heliocentric model of the Universe, which we know to be correct.

The second lasting impression I have is about the Earth. I expected to and tried to see the Earth as a fragile, blue marble in the blackness of space. That is an image from Apollo astronauts who went to the moon. Sadly, human space flight is now limited to low Earth orbit, so the view of Earth we get is dramatically different. If you were able to shrink the Earth to the size of a basketball (12 inch diameter), the Shuttle would fly only about ½ inch above the surface. The moon would be about 30 feet away.[2] Flying on the Space Shuttle, I

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[1]Most of us have experienced the sensation of flying in an airplane on a clear day. Looking out the window, we feel like we are moving very slowly relative to the earth below. Even though we do not hear wind noises or feel vibrations, all our experience tells us that we are moving and the earth is not. We might imagine that we are flying in a straight line over a map, every minute bring us closer to our destination.
Orbiting two or three hundred miles above the earth, we can see that the earth is indeed round. If we were traveling in a straight line, we would go whizzing past this planet and on to the next. We don't *feel* like we are traveling in a circle (no centrifugal force). We don't *feel* like we are moving at all (no wind or vibration). Therefore, in spite of all our experiences and common sense, we may *feel* like we are fixed in space and the earth is turning in front of us just to show off its splendor.

[2]For these calculations I used round figures and English units because those are the facts I remember from my pre-metric childhood. The radius of the earth is about 8,000 miles; the diameter of a basketball about 12 inches; the highest Space Shuttle orbit has been about 360 miles; and the distance to the moon about 250,000 miles. We can set up the following ratios:

$$\frac{12\ inches}{8000\ miles} = \frac{X\ inches}{360\ miles} = \frac{Y\ inches}{250{,}000\ miles}$$

Solving for our scaled-down shuttle orbit (X), we get 0.5 inches. The distance to the moon (Y) is 375 inches, or 31 feet. The distance from the earth to the sun in the real universe is about 93 million miles. In our scaled down solar system it is just a little over 2 miles.

never felt separated from the Earth but I did get a new perspective. The Earth I saw was a living, breathing, powerful thing. I could look out the windows during any night pass and within seconds see lightning somewhere. Frequently there would be sympathetic discharges where one lightning bolt would trigger several more nearby. There is a tremendous amount of energy evident in the electrical activity of our atmosphere. I would look for and photograph volcanoes for scientists to study and compare changes over the years. As powerful as they are to us, volcanoes like Soufriere Hills in Montserrat or Kilauea in Hawaii appear as only the tiniest pimples on the face of the Earth. Looking at the sun's reflection on the oceans, I could see eddy currents churning in the water all the time -- not major currents like the Gulf Stream, but random motions. If you have even been to the beach after a storm and been tumbled by the waves, you know how powerful the ocean is. Imagine the energy it takes to keep all the water of all the oceans in constant motion. If I had not been a believer in plate tectonics before I flew in space, I would certainly be a convert now. I could see cracks in the surface of the Earth, like the Great Rift Valley which extends 4000 miles from Jordan in southwestern Asia southward through eastern Africa to Mozambique. I could see the effect of India crashing into Asia at geologic speed and, in the process, forming the Himalayas, which are the highest mountains on Earth and still growing. Compared to all the energy involved in these natural processes in the Earth, we humans seem very small and powerless.

My feeling after seeing the Earth from an astronaut's vantage point is that the Earth has been here for several billions years before us and will be here a long time after we are gone. We will not hurt the Earth. What we should worry about is ourselves, our children, and our grandchildren. We are the ones who are fragile. We can live in a very limited chemical environment and a very narrow temperature range. We are capable of making ourselves extinct by our own actions, but we will not hurt the Earth.

The third impression I always want to remember is about neighbors. We think of neighbors as the people who live close to us. If we travel across the state and meet someone from our hometown, we immediately feel that we have a relationship with them; we are neighbors. If we travel to California and meet someone from Virginia, again we have a neighborly feeling even though their home may be hundreds of miles from ours. Still we are neighbors. If we travel to California and meet someone who even knows someone in Virginia, he is sure that we know them too because we are neighbors. It occurred to me that I could circle the whole Earth in about the same time it took to drive around the City of Houston, my home at the time. If I felt that if all Houstonians were, in a way, my neighbors, then perhaps all the people within the globe I had just circled were my neighbors, too. Maybe if we all had the opportunity to see our world from that point of view, we would learn to get along better as neighbors.

How did I get so lucky to experience the thrill of space flight? There are a number of factors including good health, having the educational credentials NASA was looking for, and a lot of luck. Certainly my life's choices had something to do with being prepared to answer NASA's call for volunteers, but I also credit my parents for good health genes, fate for the stroke of luck, and dedicated teachers who ignited that spark in me.

I don't believe you can light a spark in every one of your students. I know I can't. I try to keep them all awake, some of them interested and a few excited about the possibilities that lay before them. There is a group of engineering students at the University of Virginia who called themselves the Space Advancement Society, reminiscent of the amateur rocket societies who kept the science of rocketry alive during the 1930's. The UVa Space Advancement Society members have drafted me as their faculty advisor this year. My job, as it was explained it to me, was to sign papers when they need evidence of adult supervision and otherwise stay out of their way. Actually, they were very polite; they invited me to tag along on their rocket launching adventures but they are clearly a self-motivated and self-directed group of students. Sometimes I wonder if there is young Wernher von Braun, Robert Goddard, Alan Shepard, Neil Armstrong, Sally Ride, or Shannon Lucid among them, and if have I done my part to ignite that spark in them.

I was very fortunate to be a crew member on four missions on the Space Shuttle. I gave that up last summer to accept a new mission: to do what I can to improve math and science teaching and learning in Virginia. It wasn't so long ago, at least in some states, that citizens had to pass a literacy test in order to vote. It was believed that if a person could not read, he could not be informed on the issues and, therefore, could not cast an informed vote. Those literacy requirements were forbidden by an act of Congress in 1965 because they disenfranchised groups of people and because, by that time, a citizen could become informed without reading by listening to radio or television. Today mathematical and scientific knowledge is increasing in importance as our society becomes more technology based. The decisions we as citizens must make require us to be technologically literate. Our students must acquire thinking, reasoning and problem solving skills in order to become informed adults and participate in a knowledgeable way in the democratic process in this country. As director of the University of Virginia Center for Science, Mathematics, and Engineering Education, I join with you in your goals to support and promote math and science in K-12 classrooms across the Commonwealth. And as a parent of three children in the public schools, I deeply and personally appreciate what you and this Collaborative are doing for all our children and for the future of Virginia. ■

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ON THE RELATIONSHIPS BETWEEN MATHEMATICS AND SCIENCE IN SCHOOLS

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1. The current scene

In our elementary schools, mathematics and science play completely different roles. Mathematics is established as the second most important subject behind reading. It is taught in virtually every classroom almost every day. It is tested. In many areas of the country, mean mathematics achievement scores for each school are published, and the quality of a school's program is judged in some way on how well the students in that school perform on the test. In some states of the country, there are state-wide tests for graduation from 8th grade that include mathematics. Now there is the possibility of a national test at 8th grade.

None of this is true of science in elementary schools. Science is not taught everywhere, and even where it is taught, it is seldom taught daily. Although it appears on test batteries, few schools are judged by how well their students do in science, and seldom do mean science scores in science appear in newspapers. Though it is certainly possible that somewhere a state-wide science test is required for graduation from 8th grade, I know of no state in the country that has such a requirement. However, recently a number of states, including Virginia, have instituted tests in science and other disciplines at the eighth grade level that are to be used in making promotion decisions.

It is sometimes said that mathematics in our schools is in bad shape – I have been among those who have said this – but mathematics in our elementary schools is in great shape compared to science. Except for health, science in our elementary schools is a catastrophe. That on international tests compared to other countries we have performed better in science than in mathematics can be interpreted as evidence that science is in even worse shape in other countries.

Perhaps the best piece of evidence we have for the current condition of elementary

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school science is to look at what the high school science curriculum expects students to know upon leaving elementary school. Ninth grade science can assume almost nothing, and so it does assume nothing. Even in places where science is taught for 8 years, the senior high school tends to ignore what their entering students know. High school science teachers do not trust even middle school science teachers trained in science to teach anything; it all is retaught. In contrast, 9th grade mathematics is able to assume that most students know addition, subtraction, multiplication, and division with whole numbers, fractions, decimals, some percent, some basic measurement, some formulas, even a little bit of graphing and algebra and geometry in many school systems. High school mathematics teachers are often like their science counterparts in that they do not trust middle school mathematics teachers to teach some content – they think only they can teach algebra and geometry – but they do trust these teachers to teach arithmetic, and in fact they want all arithmetic taught before students get to high school. As far as I know, there is no counterpart in science.

The situation in science education is, fortunately, a little better in grades 9-12. Science is taught everywhere. Biology is taken by virtually all students. Earth science or general science or environmental science courses exist in almost all schools, and virtually all schools that do not teach chemistry or physics at least know that they should be teaching those subjects. A student who takes all the science that is offered in most high schools today can learn quite a bit of science. In some places in the country, that student will have biology taught by an individual with a bachelor's degree in biology, chemistry taught by someone with a bachelor's degree in chemistry, and physics taught by someone with a bachelor's degree in physics. No other subject area in the high school can boast that each year it has specialists teach its courses.

This specialization has caused problems that are known to us all. Biology classes have almost no chemistry in them, even though today no one can be up to date in biology without a reasonable knowledge of chemistry. Chemistry courses have almost no physics in them, but they could use some physics. The same situation used to exist in mathematics. There used to be very little algebra in geometry and very little geometry in algebra, but that has evolved over the century so that now the courses are not air-tight. Part of the reason that this could be done in mathematics is that every mathematics teacher is supposed to be able to teach both algebra and geometry. Unlike science, no mathematics teacher comes to a school with a B.A. degree in geometry, or a B.A. in algebra, or a B.A. in calculus.

Even though science at the secondary level is in better shape than it is at the elementary level, science at the senior high school level still does not have the status of mathematics. The requirements for graduation in science are about a year less than those for mathematics. Mathematics comprises one-half of the SAT exams; science plays only a cameo role in some of the verbal portion. High schools have far more mathematics teams than they have science teams. Though at least as many students enroll in biology as in algebra, one of the most fundamental concepts in all of biology - evolution - is ignored in order to avoid controversy, and I do not know how one can do earth science without tackling the age of the earth. But perhaps this is only analogous to doing mathematics without using calculators and computers, in the sense that there are people who wrongly believe that knowledge does not change and techniques do not change, and refuse to enter the 20th century.

There are some positive trends. In the National Assessment of Educational Progress *NAEP 1994 Trends in Academic Progress* [1], it is reported that over half of today's 17-year-olds report having taken chemistry, about the same percent as report having taken a second year of algebra. The percent of 17-year-olds taking physics increased from 11% in 1986 to 18% in 1994, paralleling an increase from 6% in 1978 to 13% in 1994 in the 17-year-olds taking precalculus or calculus. But there is a fundamental difference in the subjects that masks these similarities. A large percentage of the students who are not in these standard college-prep mathematics courses are still taking mathematics, some a couple of years behind their peers, others in consumer mathematics courses. In contrast, those that are not taking higher level science courses in high school are likely to be taking no science at all. There may be a few places that offer physics courses and chemistry courses that are meant for all students, but they are not in the majority. Again, the situation is bad in mathematics but it is worse in science.

I have gone to great pains to indicate that, at least in schools, mathematics and science are quite different animals. (Perhaps I should add that there used to be another difference. It used to be that science departments in middle and senior high schools had a budget, whereas mathematics departments did not. But computers have changed that, and at the expense of science. It is more glamorous to have a computer terminal than a microscope.)

Despite these differences, which I feel are major differences between the ways in which the subjects are treated, the two subject areas are often treated in the same breath as if they were alike and as if their problems were alike. In announcing Goals 2000 [2], President Bush asserted that the U.S. should be first in the world in science and mathematics by the year 2000 as if it would be as easy to be first in one as in the other. Eisenhower funds have been available in science and mathematics but not other subjects. There are programs for recruiting science and mathematics teachers in some states; seldom do these programs want one without the other. And, of course, there is the School Science and Mathematics Association, not the School Science and Social Science Association, or the School Mathematics and Other Languages Association. No other pair of subjects in our schools are so often grouped together.

It is interesting, then to ask: If these subjects have developed to be so different in our schools, why are they so aligned in people's minds? Are the schools correct to treat the subjects differently, or should they treat them alike? Should the subjects be taught together? Many national groups, including the School Science and Mathematics Association, have emphasized the integration of science and mathematics. That is why in this article I focus on the relationships between mathematics and science, or between science and mathematics, if you prefer that order, and spend some time also examining the question of integrated curricula.

2. Science in mathematics

In my undergraduate education as a mathematics major, I saw very few applications to science, and none after calculus. I was, after all, a mathematics major, and that meant – in the early 60s when I went to school – that I was a "pure" mathematics major, not an applied mathematics major. There were no majors in applied mathematics then.

Not until ten years later, in the early 70s, did I get converted to the belief that applications were important. I shall tell you how that happened. I came to the University of Chicago in 1969. My colleague, Max Bell, was one of the few people who during the height of the new math era felt that there needed to be applications in the curriculum, and he first convinced me of one thing – that the word problems then found in books were not applications of mathematics, and in fact that they hindered the learning of applications of mathematics because they implicitly taught students that mathematics had no applications. I mean, who really wants to know the age of Mary who is half as old as her father will be when she is as third as old as her mother was?

In 1973, Max asked me if I would teach a course in applications of mathematics because he did not have the time to teach it. I responded positively, but only if someone else could teach it with me, for that is how uncomfortable I felt with this material at the time. But when I went to devise a reading list for the course, I was astonished. There were the "mathematics for carpenters and nurses and biologists and technicians...books", the statistics and other applied mathematics books, the collections of applications, some standard science books, and a good number of nice articles. Without a great deal of trouble, the reading list grew to 86 items, and it was clear how narrow my education had been. It is sad that only through this exercise did I learn that applied mathematics was as large a field as pure mathematics.

It then became clear that if one was to enlarge the number of students who learned a significant amount of mathematics, one had to incorporate this huge domain into the curriculum in some way. It began for me with a vengeance in 1974 with a course entitled *Algebra Through Applications* [3], later distributed by the National Council of Teachers of Mathematics, and has continued until the present day with the work of the University of Chicago School Mathematics Project.

But there was a significant aspect to the incorporation of applications of mathematics into the mathematics curriculum that is of particular relevance here. The best applications for the mathematics students were *not* those to science, but to everyday events. Given a choice between half-life and compound interest as an example of exponential functions, there was no question: choose compound interest because students care more about it, and they have more knowledge of the context, and so they are more interested in it. If you want to apply trigonometry to the finding of inaccessible distances, then use the distance between two mountain peaks rather than the distance between planets, because if you use astronomy you will have to teach astronomy, but you do not have to teach students what a mountain is. If you wish to discuss trajectories, use a basketball, baseball, or football, not an arbitrary projectile or a rocket.

Of course, in our materials we do discuss astronomy, and half life, and rocket trajectories, but not as the first examples. I would imagine that those of you who teach science have learned the same thing. For instance, in chemistry an internal application of chemistry is not as appealing to students as the applications to things we eat, or to purification of water or air, or to plastics we use. It is natural that the best examples are those that shed new light on something familiar to us, because we are free to concentrate on the new light rather than on learning the context of the example.

Some of you may think that we became very constrained because we tended not to use science examples, but that was not even close to the reality. Pick up a newspaper and examine the number of numbers on a page. In various countries, I have found invariably the *median* number of numbers on a newspaper page is somewhere between 120 and 150. The *mean* number of numbers is far higher – the last time I calculated it for a Chicago newspaper, the mean number of numbers on a page was over 500, due to sports pages, want ads, the weather page, and business pages.

These numbers are used in many ways. For instance, on the day I completed this paper, there were 77 numbers on page 1 of the *Chicago Tribune*, with the following kinds of uses: *counts*, with a wide variety of counting units, and often large, such as: 1.3 million member teamster's union, 75 high schools, two taverns; *measures*, and there are more of them than counts: many ages, such as a 66-year-old woman; money, often in large amounts: including a $28,570 inheritance, $383 million deficit (a negative number), 50 minutes, 100 miles, two weeks ago; uses we call *locations on a scale*, including: many dates; section 2, page 9; temperatures, such as 76 degrees; ordinals such as first; uses we call *ratio comparisons*, such as: a 27 percent reduction over last year's school funding; uses we call *identifications or codes*, including: Interstate 80, bar code number, Local 743 of a union. And there are *estimates* of counts and measures: hundreds of little towns, thousands of farmers, the 1950s (an interval).

Inside the pages, there are more numbers. The second page of the Tribune has an index and a list of phone numbers and subscription rates, and a table of lottery numbers from four states, and even the competitor Chicago Sun-Times' second chance lottery! All articles in the business and sports sections are filled with numbers. There are many stock averages --

actually rather complex weighted averages, not to mention the stock prices themselves. There are the various statistics used in sports, ranging from simple scores to earned run averages and quarterback ratings that are most easily analyzed using algebra. There are all sorts of advertisements with discounts given as percents and there are annual percentage rates for investments. Some of the advertisements contain dimensions of the articles being offered, some contain computer specifications, powers of zoom lenses, power capacities of stereo systems, and other technical information. There are advertisements discussing mortgage payments, capacities of wine bottles in liters and milliliters (sometimes with the units missing!). There is the extensive weather page, with wind directions and precipitations and barometric pressure and all sorts of other statistics.

I have not attempted to be exhaustive in my listing of numbers in the newspaper. I do not think it is needed to make the point. To read a newspaper today requires that the reader be able to process mathematical information to an extent far beyond that required even one generation ago. It is often said that we are in an information age; it is the case that much of that information is numerical or pictoral, and thus is mathematical. *But the mathematical information is usually not related to science.*

It is often said that mathematics is the language of science. This is true but mathematics is now an important language of communication in many disciplines, including economics and sports and consumer products and services. I am not sure that mathematics ever was the handmaiden of science; if it was, it certainly is not today.

3. Mathematics in science

Now let me consider the other direction. How much mathematics is there in our science courses? Here the situation is spotty. In good science curricula at the elementary and middle school levels, there is usually a lot of mathematics related to data gathering and presentation, often including far more graphing than mathematics teachers realize. Where elementary school science is taught well, the mathematics in that science at a particular grade level is often at a level quite a bit beyond that in the corresponding mathematics lessons at that grade level. I have seen graphing in 2nd grade science that is at a level that some 8th grade mathematics teachers think is beyond their students. This practice demonstrates that material learned in context is usually far more accessible than material learned in the abstract.

Enter high school, however, and the situation is different. There is very little mathematics in some earth science and environmental science and general science courses. Nor is there much mathematics in school biology. Indeed, there seems to be a general policy of not using mathematics in the science courses taken by the vast majority of students. Surely there is almost no algebra - no formulas, no mathematical generalizations. And there is very little geometry. In chemistry, finally, students see algebra and a little bit of geometry. In physics, of course, the mathematical content runs throughout. The upshot of all this is that the student is told that you need mathematics for all of science, but the current science curriculum suggests you only really need mathematics beyond arithmetic in the *physical* sciences.

Is there science in newspapers? On page 1 of a recent issue of the Chicago Tribune there is mention of the first symptoms of Alzheimer's disease, and there is a weather forecast. These represent two of the oldest interests in all of science, medicine and weather. On page 2 there is an article about a 53-year-old woman who is pregnant with the sperm of her husband and the eggs of another woman. On page 3 is an announcement of the next shuttle flight from Cape Canaveral. There are two articles dealing with abortion on the first five pages.

In the second section, there is a lead article on angioplasty. On page 3 there are articles about the two beluga whales that had recently died, and about a dispute over dredging in the Chicago river that might have caused the flood in the Chicago loop. On the weather page are the phases of the moon, the rising times for the planets, all the weather statistics we have come to expect, and a couple I had never noticed before: an aviation forecast that includes turbulence (reported as moderate) and icing (freezing level near 12,000 feet).

In the business section, there is an article on supercomputer software and all sorts of data on the values of metals and livestock and grains and other commodities. In the sports section, there is no article that I would say could be identified as science, though there are articles and data on horse racing and greyhound racing. In Section 5, called Tempo, there is a review of *The Eye of the Elephant: Life and Death in an African Wilderness*, and an article entitled "2 mathematicians explain the function of sea foam". In Section 6, the want ads, there is occasional science required: what is meant by dogs being wormed or being allergenic, various kinds of woods in furniture, land descriptions, and so on. In the last section there is an article that claims that a new sweater "repels electro-magnetic energy, shielding your body from electric shocks, bad vibes and nasty moods". It costs only $425 to $1,275, you'll be happy to know. Another article speaks of the effects of wearing red lipstick. There are all sorts of articles on fashion, in which it would be helpful to be up on natural and synthetic fabrics.

Advertisements tout luggage as water resistant, promote furs and diamonds (which I view as part of our natural world), and speak of quartz tuning of TVs, bass boast, mega bass in a CD-player and "surround sound" in speakers, laser-quality output from a printer, energy saving dishwashers, humidity controlled crispers in refrigerators, and resolution of TV screens.

So there is quite a bit of science lurking in a daily newspaper, just as there is a huge amount of mathematics. Indeed, the amount of science surprised me about as much as the amount of mathematics surprised me when I first analyzed it. But the science as presented in newspapers does not involve mathematics beyond arithmetic, beyond the understanding of measurement and scales and all sorts of units.

The separation that occurs between the subjects in the newspaper mirrors what happens in teaching. I have gone into a geometry class on the day of an eclipse of the moon, and found it to be ignored. No one can condone such artificial separation.

So we have the following situation: If we view the newspaper as being representative of what is important to people, or what they need to know in their daily lives, most of the mathematics does not involve science and most of the science does not require mathematics beyond simple arithmetic. One of the reasons that there is so little mathematics beyond arithmetic in newspapers is that journalism is one of the few college majors that requires almost no mathematics. Journalists also tend to take very little science.

4. Connecting mathematics and science

Gauss wrote, "Mathematics is the queen of the sciences." What does this mean? One interpretation begins with the fact that mathematics is the language of *generalization* of patterns, both numerical and geometrical. The sciences study the patterns of things, both animate and inanimate, in our world. So you cannot get along without mathematics.

Science is one of the targets of the language of mathematics; science studies the contexts. This is the reason that there are so many more stories about science in the newspaper than there are stories about mathematics. Who wants to read about a language? To carry the analogy perhaps farther than it should be carried, it is far more interesting for most people to learn about the Russians than to learn Russian. But if you want to *really* learn about the Russians, you need Russian! Mathematics is, of course, more powerful than Russian because it is a worldwide language, and it seems to be a natural language. As Galileo stated (in his essay *Il Saggiatore*, 1623): "The universe...cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is written. It is written in the language of mathematics, and its characters are triangles, circles, and other geometrical figures, without which it is humanly impossible to understand a single word of it; without these, one is wandering about in a dark labyrinth." Or as Sir James Jeans wrote (in *The Mysterious Universe*, 1930), "All the pictures which science now draws of nature and which alone seem capable of according with observational fact are mathematical pictures...From the intrinsic evidence of his creation, the Great Architect of the Universe now begins to appear as a pure mathematician."

Because you do not need much mathematics to understand the contexts of science, there have been dozens – perhaps hundreds – of superb science programs produced and aired on television. Thousands of articles have appeared in *Scientific American* and other magazines that attempt to explain science, with very little attention to mathematics. With regard to reaching the general public, science is far ahead of mathematics, and it has not needed mathematics to do so.

There is a fundamental reason why science does not always need mathematics. Science studies things that we can see (perhaps with the help of special tools), or things whose effects we can see – physical things even if the science is not a physical science. Thus, a visual medium, such as motion pictures or television or computer screens, is a natural medium for the transmission of science ideas, for the seeing of patterns. Even people who do mathematics today can get along with a little less knowledge of the traditional written mathematics than they used to – functions can be seen and analyzed using a graphing calculator.

Another reason why science does not always need mathematics is that the concepts of science are not, in their roughest state, mathematical any more than they are English. Mathematics is a language we often use to describe science, but it is not itself science. Concepts such as entropy and many ideas in genetics can be described in mathematical terms, but they begin non-mathematically. We may define work in physics in terms of other quantities, but before we start we have a non-mathematical idea of what we want work to be. The energy of an atom bomb may be explained in part by the equation $E = mc^2$, but energy is a far broader concept than is suggested by the variable E in that formula.

5. Should mathematics and science be integrated?

By "integrated", I mean "Should they be taught together?" –at any level. Should children in primary school receive their science instruction along with mathematics? Should older students learn physics and calculus together? What about middle school, where there is a movement to integrate many things?

It is interesting that the journal *School Science and Mathematics* began in 1901 with the title *School Science*. In 1903, George Myers, a professor at the University of Chicago, was asked to be the departmental editor for Mathematics and Astronomy for the Mathematical Supplement to *School Science*. Ultimately, this supplement became integrated into the journal and the title was changed to what it is today.

Myers worked hard, however, to break down barriers between geometry and algebra. Many of the things algebra teachers take for granted today, such as the use of graphs, number lines, and area diagrams in algebra, were innovations in his work. He also used equations in geometry, which also was novel. It could be argued that his books *First Year Mathematics* and *Second Year Mathematics* were the first unified school mathematics series published in the United States. In both courses he used applications, again an innovation for his time.

But Myers did not believe that science and mathematics should be taught together. He encouraged the correlation of work in science and mathematics. So even the person who may have, more than anyone, been responsible for the name School Science and Mathematics, was not in favor of their integration.

Yet there are still some who think that they should be taught together. I assume that there are people who are members of the School Science and Mathematics Association for this very reason. Didn't Gauss say that mathematics was the queen of the sciences? In the writing that I have seen from American Association for the Advancement of Science Project 2061, it certainly seems that the authors are recommending that mathematics and science be taught together, particularly at the elementary school level.

But they shouldn't, and here are the reasons:

- Integration takes time from one for the other. In fact, a skeptic might argue that the reason Project 2061 recommends teaching math and science together is that the authors don't think as much arithmetic needs to be taught in the elementary school, so combine it with science, and at least get some science there.
- Integration requires that teachers know both mathematics and science, when we often have trouble finding teachers who know one of these areas.
- Integration makes it more difficult to have sustained treatment of the big ideas – proof, evolution, etc. – and always is frustrating to the teacher who cannot spend time on one for too long.
- Mathematics and science utilize fundamentally different modes of thinking, views of valid reasoning. For science, the scientific method is preeminent, with knowledge created through experiment and induction. For mathematics, the logical method is required, with knowledge created through deduction and proof.
- Integration has virtually never worked, despite the good intentions of many bright people.
- Integration is not good for science, because many of today's scientific issues are very much connected with social sciences and politics and ethics, and to have mathematics as a requirement for discussing those issues is a barrier.
- It is good to have the same ideas taught in different places. If spheres are taught in mathematics and sphere packing is discussed in chemistry, students appreciate the utility of mathematics in science. If trajectories are taught in both mathematics and physics, students get two chances to appreciate that science.
- Integration is not good for mathematics, because the preponderance of today's applications of mathematics are not to science, and the important concepts are far more general than their scientific applications would suggest.

6. Should there be more connections between mathematics and science in schools?

YES, fundamentally because it is dishonest to do mathematics without science or science without mathematics.

It hurts science when mathematics is purposely ignored, because it keeps from students a language that could enhance their understanding of the field – if they had more experience with it. This does not mean that mathematics should always be there, but that it should not be kept from being there.

Similarly, it hurts mathematics when science is ignored, because it keeps from students some of the most important reasons for having mathematics, and some of the great advances in history. Students of mathematics should learn of both of Galileo's new sciences – not just the dropping of objects down an inclined plane or from the Leaning Tower, but also the second new science, which showed the effects of changes of scale on objects; students of calculus should learn of Newton – not merely as mentioned in the margin but in specific terms of the problems that led him to develop calculus.

Should they be correlated, as George Myers recommended? This question does not have a simple answer. There are a number of ways to correlate. One way is to use the same language. I recall one time teaching the wrapping function approach to sines and cosines in an advanced algebra class. This is an approach where triangles are not used; a number line is wrapped around the unit circle and sines and cosines are defined in terms of arc lengths, not angles. It must have been three or four weeks before we showed the applications to right triangles. Two students came up to me after class and said – you mean these are the same sines and cosines we have been studying in physics all year! I never used the wrapping function approach again. The languages of science and mathematics are often different, and we should not make school an artificial place where they are the same, but we should be careful that our students see the connections.

A second way to correlate is to examine curricula of both mathematics and science together, and to make certain that ideas are taught in one area at about the same time as the same ideas in the other, and that the prerequisites are there when needed. No chemistry teacher wants to teach first-year algebra, but they often have to. Similarly, no mathematics teacher wants to teach students what is meant by velocity or acceleration. It is very inefficient to do this. But I must say that, except for broad ideas, it does not bother me that vectors are taught in physics and in later high school mathematics, that probability may be found in biology and in junior high school mathematics, that systems of linear equations are solved in chemistry and algebra, that data collection and analysis might be both curricula. These ideas are always approached differently in the two areas, which enriches the ideas and gives students a greater appreciation of their importance.

A third way to correlate is to have a unit which involves both areas. This can be very effective, particularly at the elementary school level when the same teacher may be teaching both, or in grades 6-8, when in some schools interdisciplinary activities are not rare. But it takes time - not only time for the science and for the mathematics, but time to connect them. Unless that connection time is available, the result is often frustration on the part of teachers that they cannot get through their own agendas.

I would much prefer a unit which involves more than just mathematics and science - one that involves all the subject areas - but this is very difficult to manage and takes even more time. Before high school, the time should be taken not from science or from mathematics, but from language arts or English, because one thing that needs greater attention in schools is the use of nonfiction in the teaching and learning of reading and writing. Another reason for my preference for a total experience over one that merely combines science and mathematics is my hope that we can bridge the gulf identified by C.P. Snow (*The Two Cultures and the Scientific Revolution,* 1959): "Literary intellectuals at one pole - at the other scientists...Between the two a gulf of mutual incomprehension."

7. The role of mathematics and science in social science

Finally, I would like to make a few comments on the relationships between mathematics and science and the social sciences.

When mathematics and science are discussed together, it makes it too easy for people to think that math/science types are different from other types of people, and that these areas are arcane, not of relevance to the average person. Yet we all know that both areas have much to offer the social sciences, and we need a concerted effort to inform our colleagues in those areas of the importance of our disciplines.

The place to start is not in their courses, but in ours. How can a knowledge of science and mathematics help voters and government officials make wise decisions regarding the ozone layer, or regarding pollution control or waste cleanup or population growth, to mention some of the major issues of our day? How can a knowledge of mathematics or science help a student understand his or her stereo, or computer, or kitchen appliance? How can a knowledge of mathematics or science help people to make wise consumer decisions? Why are mathematics and science required in so many college majors? We cannot expect others to answer these questions for us.

8. Summary

We who are in mathematics and science are linked very much in the public eye. This linkage is both natural and historical, natural because, as so many have said, the laws of the universe are awesomely mathematical, and historical because at one time these were virtually the only sciences and the only mathematics. There is also a practical link, namely that in order to be a scientist, one has to know quite a bit of mathematics. And in order to be an engineer, one has to know both. I do not question the importance of mathematics for scientists.

But there are downsides to that link. If a person is poor in mathematics – and that today still often means being poor in computational mathematics – that person is often discouraged from doing anything in science. Science is too important to be so constrained.

If a person is good at one of our subjects, it is a surprise for people to learn that they are may not be particularly interested in the other. But there are fine mathematicians who are not involved with science, because there is much of mathematics that is not scientific; and there are fine scientists who are not particularly good at mathematics, and who need very little of it, because there is much of science that is not mathematical.

Yet the link is so strong that the public has a notion that there are "math-science" types, and C.P. Snow's quote is evidence that even the educated public has this view. As long as that stereotyping exists, we will not have the public support we need for our disciplines. We need to educate our students of our connections with each other, but we also need very much to educate our students of the importance of our disciplines for their everyday life, for their roles as consumers, citizens, and wage earners.

One would think that the connections would be easiest to make at the lower elementary school level, where the same teacher has the responsibility for teaching science, social studies, mathematics, and language arts. But in point of fact, the least science is done at that level. Part of the reason for this situation has to be that the teachers who are empowered with the task of teaching science are not only unprepared, but often possess unfavorable attitudes towards science. The same is true for mathematics, but not to as great an extent.

I believe that we will not get significant improvement in the mathematics or science taught in our elementary schools until we have people teaching these subjects who care about them, who view themselves as having a responsibility to keep up with these subjects, and – most important – who have the time to come to meetings or read articles such as this one. This can only be done by most teachers if they specialize. You know the line – if we can have music teachers, PE teachers, art teachers, why not math teachers and science teachers? And I believe that, with specialist teachers at the elementary school level we would have a chance for the coordination that would enrich all of our courses, and also enrich language arts and social studies.

The problems at the elementary school level in science lead to fundamental problems at the high school level. Because virtually no science knowledge is assumed, there is too much to teach at the high school level. The courses are unwieldy.

But there is another problem that must be solved before science will improve at the high school level. There must be a greater number of connections made between the courses, and students should not have to wait until 11th grade to study significant amounts of chemistry or physics. Science is too important to our lives to have so few students encounter some of its most important concepts.

And so, if it is possible to summarize these points in just a few lines, what it is that we need to promulgate:

Science is needed by everyone, not just mathematicians.

Mathematics is needed for everything, not just science.

For these reasons, we need to stress the connections between science and the everyday world of our students; we need to stress connections between mathematics and the everyday world of our students.

But also for these reasons, the subjects should not be taught together as one integrated whole.

However, we need more connections within all of our subjects, and we need to break down the barriers that keep important concepts from our students because they are linked with courses that come later. ■

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MATHEMATICS FOR GENERAL EDUCATION: ANOTHER RULE OF THREE

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1. General Education Students

A commonly held view of mathematics faculty is that general education (GE) students, those compelled to study mathematics as a degree requirement, comprise, by far, the most difficult audience to please. Indeed, many departments of mathematics are happy if these students can go elsewhere, philosophy, computer science or business, for example, to fulfill such a requirement.

Our union stewards, if we had them, would grieve the resulting employment loss to the mathematical community, not a trivial issue these days. But the availability of alternatives to mathematics for GE requirements for this large population may, in many instances, protect the mathematics department. We know these students dislike mathematics, so if we can shift the burden, or blame, to other disciplines, the English majors and History majors and all those other majors who later acquire influence over science research budgets won't have such negative feelings about mathematics. In turn, many of these students are only too happy to join the conspiracy to export jobs to mathematically underdeveloped departments. Perhaps most disturbing is the possibility that a potential K-8 teacher would do 'mathematics' in this way.

There are students, however, frequently uninterested in mathematics as mathematics, who nevertheless have very positive feelings toward the subject. Engineers are quite aware that they cannot survive without the tools provided in calculus, differential equations, linear

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algebra, probability and so on. As students, these people are avid consumers of our mathematical wares. When an engineering student asks, "What's this stuff good for?" it is a gesture of friendship.

We would all like to teach mathematics to students who are intrigued by the structural aspects of the subject. Indeed, some GE courses in mathematics are designed to introduce students to axiomatic methods; the instructors presumably aiming to show students how mathematics is done by those in the trade. Such an inward view of mathematics for GE imposes a very great burden on both the instructor and the student; the former to avoid frustration and the latter to avoid boredom. But engineering students *voluntarily* take mathematics courses. This being so, we need to ask: assuming there were actually an explicit choice in the matter, would we be better off if we taught non-science students as if they were engineers or as if they were mathematicians?

2. Engineering Students

Mathematicians know that mathematics, besides being a very entertaining way to while away the time, is an indispensable tool for analysis of the real world. It is the latter characteristic, not the former, that will convince lay people that mathematics and mathematicians deserve support. And how shall this claim of indispensability be verified? We can just make it repeatedly and hope that it sticks. This style of argument is used regularly in public venues. It requires the aid of lobbyists. A better way might be to get the students themselves to do some work and see that without mathematics, they cannot understand why particular pieces of real data are important or what those data have to say about design decisions.

It is fair to assume that this understanding is precisely what attracts engineering students to mathematics. But the desire for genuine external application often makes engineering students an imperfect fit for many mathematics classes. On one hand, these are talented, hard-working, quantitatively oriented students who are often among the top performers in our classes. On the other, engineering students get impatient if confronted with too much 'theory'. In fact, if they and their engineering faculty get really annoyed, one finds mathematics courses taught in engineering departments. *This* migration to mathematics courses outside the mathematics department is cause for rather more concern, as we know from frequent panel sessions on the subject at mathematics meetings. In addition to the too-much-theory problem, lack of deep familiarity with the engineering application on the part of mathematicians surely plays an important role in this phenomenon. From the point of view of abstract mathematics, this is, therefore, not a completely positive situation.

Perhaps a better way to grasp what engineers like in their mathematics courses is to examine the most highly mathematical texts that are used in engineering courses. These can already be found at the sophomore-junior level. Courses in linear circuit analysis, for example, cover a lot of ground that might otherwise be found in linear algebra, complex function theory and differential equations courses. A mathematician would complain that too much is left out of that coverage. However, the entire context of the treatment is connected with lumped circuit elements, a physical setting that is simultaneously crucial to the electrical engineer and forbidding to someone not introduced to Coulomb, Faraday and Ohm, the (hard) freshman physics parents of the hardware. The EE student who does not think of a transfer function in physical terms will suffer the consequences.

We all know that those EE students who take a standard course in linear algebra will have the benefit of mathematical preparation useful for many things besides circuit analysis. In fact, because there are different kinds of engineers, that standard course is precursor to courses in mechanical, civil and chemical engineering as well. The real point here is that until the day the EE student works with the actual circuits, that student's faith in much of linear algebra relies mainly on a promise. Similar situations occur in chemical thermodynamics, mechanics of materials, and numerous other engineering regimes.

3. The Rule of Three

In an attempt to abstract from these examples, we identify the characteristics of a pleasurable mathematical experience for an engineering student.[2]

1. Students should encounter real data, reliably gathered from a familiar setting, as

[2]No one will mistake this 'Rule of Three' with its predecessor, made famous by the Harvard Calculus Consortium [1].

the raw material for mathematical problems;

2. Students should perform some analysis on that data, typically some calculation; the methods of analysis should conform to the practices of people who work in the setting;
3. The results of the analysis should cast a genuinely informative light on the setting of the problem.

The typical engineer maintains an interest in mathematics for just so long as it continues to provide these services. When these things are not happening, the engineering students complain. And if those complaints are legitimate, what does this say about the way we treat GE students?

Take some examples that are imposed on GE students:

a) Problems in high-school algebra that begin: "I was three years old when my uncle's first wife ..." and end, "How much does my dog weigh?" fail on all three counts;
b) the techniques of symbolic logic fail on the second count as they are rarely employed by lawyers or others in the arguing business;
c) the fitting of simple functions to statistical data, with no supporting model, fails on the third count;
d) mathematical modelling with no verifying reference to external data fails on the first count.

Notwithstanding lapses relative to our three ground rules, many mathematicians have found these kinds of problems a convenient, and sometimes effective way of approaching general education students. Our intent here is to capture what engineering students find attractive about mathematics and to describe what we might do to induce a similar reaction in others.

It is more-or-less obvious why engineers want their mathematics to conform to this 'Rule of Three'. They live their professional lives in a sea of real data. Furthermore, the standard methods of analysis are standard because they have been found useful; engineers who use arcane mathematical techniques are bound to be poorly understood by their colleagues. Lastly, there is little reason for an engineer to do a computation if it doesn't give any information about the problem at hand.

For only slightly different reasons, this Rule of Three is also pertinent to non-science students. Let us assume that we want these students to escape the 'mathematics as a textbook exercise' view.

1. To omit Rule 1 is to confine the students' work to toy problems. Trying to find a good garbage pickup route in a town with six streets will convince few students that graph theory is a useful analytical tool.
2. To omit Rule 2 is to have students doing mathematics that is irrelevant to the practice in the external discipline. This kind of work isolates the students from practitioners of that discipline just as we are trying to convince those students that mathematics is useful. No one in penology has much interest in the rate at which the searchlight scans the prison wall, even though this is a favorite problem in the calculus text.
3. To omit Rule 3 is to use the setting as an excuse to do mathematics. Say we fit a quadratic function to the incidence of hepatitis B from 1984 to 1996. This may be a good exercise in polynomial algebra. But what do we now know about the etiology of hepatitis?

Now, general education students are not aspiring engineers. Besides being quantitatively less literate than their engineering sisters and brothers, they have no 'subject' that requires mathematics in any crucial fashion. In particular, prospective K-8 teachers want to teach, they do not want to engineer. This means that an effective approach to engineering-style mathematics for non-science students will require a simultaneous time-efficient introduction to some subject of common acquaintance, if not common interest. That subject will have to entail some identifiable mathematical considerations. This raises a disturbing prospect. An instructor who wants students to employ our Rule of Three will have to know something about the setting. To put things bluntly, the instructor will have to know something besides mathematics. This kind of familiarity with the setting of an applied problem is atypical in a mathematics course.

Take a simple example. In Chapter 1 of the Harvard calculus book [1], there is a discussion of the near exponential behavior of the population of Mexico during a certain time period. In treating this problem, most instructors will utterly ignore the fact that Mexico is under consideration. The tabular data and its management are the issue. There are at least three valid reasons why the instructor does this:

a) This course is labeled mathematics, not sociology;
b) The instructor doesn't know anything about the factors that influence Mexican population growth;
c) Time taken to talk about Mexico will be lost to important mathematics.

While these reasons are quite legitimate in a calculus class where students and instructor have a common mathematical agenda, they are not particularly germane to a GE course. In fact, the reverse is true: If an instructor wants to get students to employ mathematics as a tool in understanding some external setting, the ambience of that setting is going to be virtually as important as the mathematics itself. The fact that both mathematics and real context are desirable almost surely demands that we confine that context as narrowly as possible.

4. Implementing the Rule: An example

With very considerable advice from engineers and scientists at the NASA-Langley Research Center in Hampton, Virginia, material has been developed for a GE mathematics course which is designed to conform to the standards described above. The development of the course, "The Mathematics of Powered Flight", was largely prompted by the recent establishment of an 'everyone takes mathematics' requirement at William and Mary. The physical setting is airplane flight. Hence, we try to prod students to consider what aspects of flying can best be understood by doing elementary mathematics. 'Elementary' is very much an operative word here. A beginning course in aerodynamics is not the agenda. In fact, unless there is some elementary analysis available for a problem, we simply ignore it. The advertised prerequisites are high school algebra (algebra I is sufficient) and geometry. Some students have taken physics or chemistry in high school, others have not.

Those looking for ways to improve a GE mathematics course are often determined to make the course 'interesting' for students. This is obviously a desirable goal. It is for us, however, something of a fringe benefit. Rather than hoping that students be intrigued by the subject of airplane flight, we merely ask that they recognize its familiarity. Do we not all have ears that pop with changing altitude? Do we not all see air-resistance-driven violations of Galileo's law of falling bodies, $v = gt$? Do we not all see mysterious 'road' signs near the runway at an airport? Ultimately, the idea is not, "Do you find this interesting?" Instead, we ask, "Do you see that your computations help you to understand what is going on here?"

This is, we think, not a trivial issue for prospective K-8 teachers. It is one thing for a teacher to tell ten year olds that some *other* people use mathematics to help understand the world. It is quite another for that teacher to say to those children, "I used some algebra (or some geometry or some trigonometry) to work out certain engineering problems." No one need confess an interest or lack thereof in the problem itself.

As it happens, many students do get interested, precisely because of familiarity. One student in a class has made parachute jumps. Students are impressed when we calculate a terminal velocity (about 20 feet per second) that matches what jumpers are told in training. Moreover, a student may use a map of the main airport at his or her hometown to find sources of data. The high-level navigation maps that students use present a picture of the continental United States that has no political boundaries, but instead is full of geometric data, much of which can be verified by measurement or calculation.

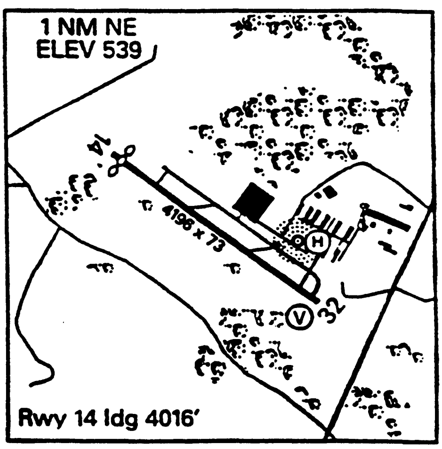
It is safe to say that *every* student is attentive when the Microsoft Flight Simulator is used to illustrate some piece of mathematics. Perhaps because it looks like (and is sometimes advertised as) a game, one may think of managing all the gauges on the panel to keep from crashing. In some sense this is true, for the Simulator as well as for a real airplane. For our purposes, the computational and geometric content of the gauges are the things that bear exploitation.

Students who do a group taco project can directly manipulate a three dimensional geometric figure to see how it can be analyzed with the tools of *plane* trigonometry. Some purchase blowup globes of the earth to help with work on latitude and longitude. Some cut oranges to study the definitions of those angles. Some even acquire a feel for geography (where is Topeka ?) that they missed in school.

A brief account of several topics will give the reader an idea of the course content.

We can begin with a map of a runway, for instance, the airport at Gaithersburg, a Maryland suburb of the District of Columbia. This map is taken from [2]. It identifies the single bi-directional runway at Gaithersburg as 14-32. The student must learn that Runway 14 specifies a runway that heads in a direction 140° east of north; that is, in a southeasterly direction. It is, therefore, a fact of geometry that the opposite heading must be 320°. For this purpose, runway direction are rounded to the nearest 10°. Among other things, this means that a student in an airplane who looks out the window during the ground taxi will be perpetually reminded of a fact of geometric/arithmetic invariance: The identifiers on the red runway signs must always differ by 18. For some students, it is fair to assume that they know a little something about trigonometry. If this be the case, the material can proceed somewhat faster. Otherwise, one must take an hour to show students some similiar right triangles and get them to look up sine and cosine function values on a hand calculator.

One may continue, then, with more real data by calling (301) 977-2971, a telephone number that will yield a current account of the flying weather near Gaithersburg. Included in that weather report is a vector, namely the wind direction and speed. Students must now calculate the component of that wind in the direction of the Gaithersburg runway. This is an exercise carried out by all working pilots. As a matter of life and limb, they are always concerned about the relationship between the wind and the direction along which their wheels are rolling on concrete. Incidentally, as with much of mathematics that is applied in the world, terminology is adapted to the setting rather than to ideas of textbook mathematics. This is the case with wind data, which is *not* provided in the standard Cartesian mode.



GAITHERSBURG AIRPORT

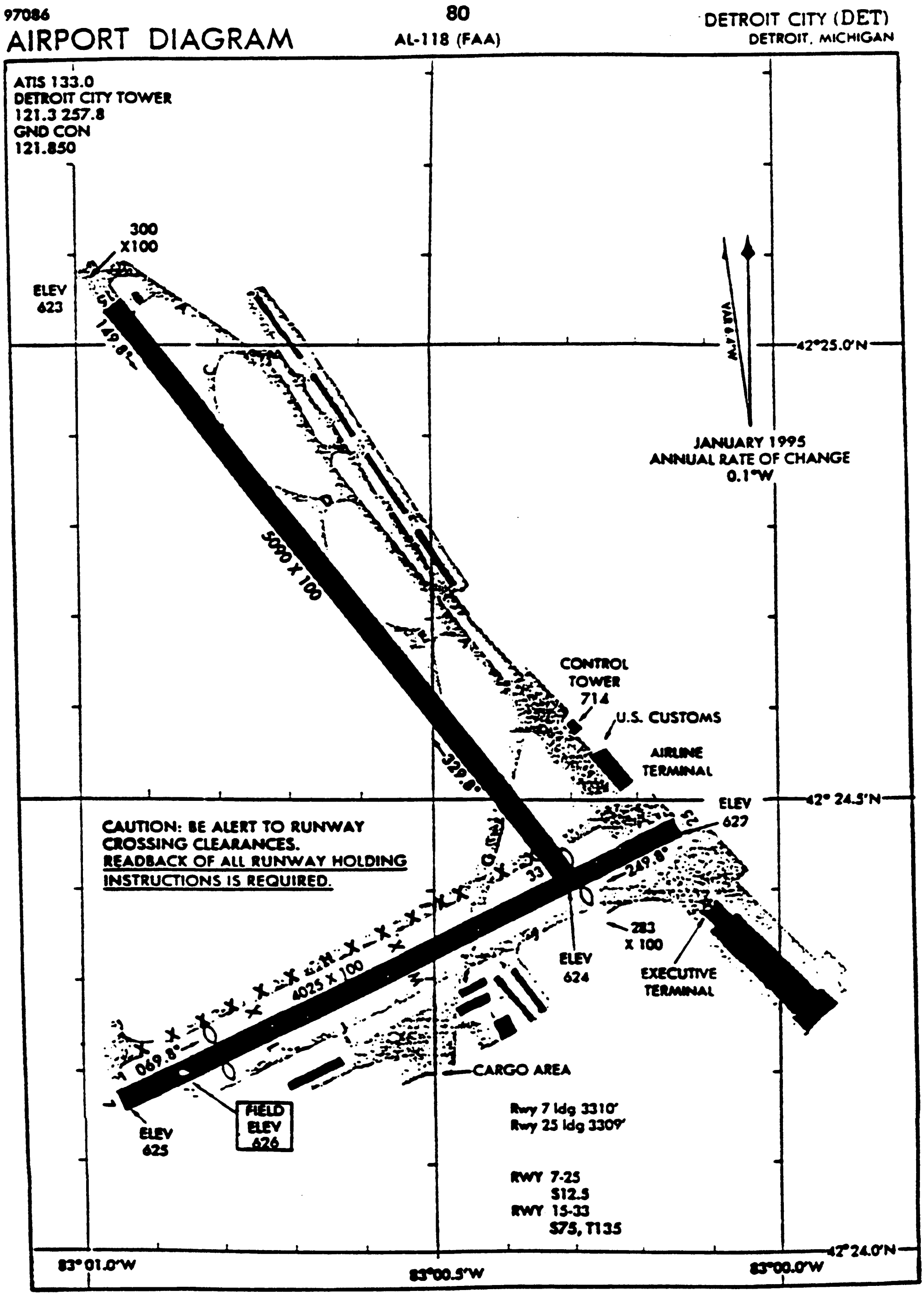
Next, a related issue. Why was the 14-32 direction chosen for Gaithersburg in the first place? It may well be that in particular cases, the runway layout must conform to the geometry of the land available for construction. More typically, the pattern of prevailing winds plays a critical role. It is no accident that runways 5-23 are found at Norfolk International, Oceana Naval Air Station and the Chesapeake Muni airport, all on the southside of the James River in Tidewater, Virginia. For our purposes, we can have students examine wind histories provided in [3]. These histories have been reformatted in [4], specifically for use in an FAA runway layout program [5]. Each individual student is given the airport diagram for a major airport in the continental United States. With the aid of the FAA program, the student may study the connection between the direction of the runway(s) and the wind history provided.

The diagrams for large airports are sources of other useful data. For example, at the Detroit City airport, one sees that runway 7 actually has compass direction 69.8°. Now it is easy for students to measure the geometric heading of that runway by using a protractor set against the 83°00.5' meridian. That geometric heading may be combined with the magnetic variation displayed at the top right of the airport diagram in the shape of a wedge. A harpoon arrow points to compass north, a spade head arrow points true (= geometric) north. The actual measured variation in 1995 was 6.4° west of north. All this information should fit together into a neat addition.

Notice as well that the Detroit City airport has an elevation of 626 feet above sea level. This may be verified with the aid of the Microsoft Flight Simulator. The simulator will permit one to place an aircraft at the Detroit airport and read the altimeter. Better yet, the user may change the weather, in particular, the ambient pressure, to see what *quantitative* effect this has on the altimeter reading. In order to do this, we have to work out a solution to the hydrostatic equation for a compressible fluid. Using this model, the pressure, P_h, at altitude h, is given by

$$P_h = P_0 \exp(-\kappa h),$$

where P_0 is sea-level pressure measured on a particular day (and announced in an airport weather service), h is in feet and $\kappa = 8.05 \times 10^{-2}/(14.7 \cdot 144)$. In fact, the notation used in the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ANNUAL RATE OF CHANGE
0.1°W
CONTROL
TOWER
714
U.S. CUSTOMS
AIRLINE
TERMINAL
329.8°
42° 24.5'N
ELEV
627
CAUTION: BE ALERT TO RUNWAY
CROSSING CLEARANCES.
READBACK OF ALL RUNWAY HOLDING
INSTRUCTIONS IS REQUIRED.
33
249.8°
283
X 100
4025 X 100
ELEV
624
EXECUTIVE
TERMINAL
069.8°
CARGO AREA
FIELD
ELEV
626
ELEV
625
Rwy 7 ldg 3310'
Rwy 25 ldg 3309'
RWY 7-25
S12.5
RWY 15-33
S75, T135
42°24.0'N
83°01.0'W
83°00.5'W
83°00.0'W

formula, eminently readable for a mathematician, is replaced for student consumption by

$$P_h = P_0 \times (.999962)^h, \tag{1}$$

a formula derived from a discrete version of the hydrostatic equation.

Students are required to examine the structure of this calculation: All the numbers have an important physical meaning: 8.05×10^{-2} is the weight density of air in pounds per cubic foot; 14.7 is standard air pressure in pounds per square inch; 144 is the number of square inches in a square foot. In observing the instrument panel, the student will see that the pilot must take account of weather changes in order to get correct altimeter readings. Dangerous altimeter errors resulting from failure to attend to the weather may be calculated using (1).

Equation (1) is also important when one studies the pressurization schedule of an aircraft. Maintaining ground level pressure in the passenger cabin of an aircraft at altitude is not feasible because repeated cycling through such large pressure differentials would quickly destroy the airframe. In order to simultaneously protect the aircraft and keep passengers comfortable, certain standard constraints are enforced. For fear of oxygen deficit, passengers should not 'feel' as if they are any higher than 8000 feet. Equation (1) determines the corresponding minimum pressure. But the maximum pressure differential for a certain aircraft might be, say, 8 pounds per square inch. Now it is time for students to compute: How high can this aircraft fly?

The solution to the hydrostatic equation is itself copied from that of an earlier problem: How rapidly does an unpowered missile lose horizontal velocity as it passes through a resisting medium? Arguing from conservation of momentum, we show that the velocity, as a function of *distance*, falls off exponentially. By further exploiting this result, we can establish a rule for the calculation of terminal velocities for bodies falling in air:

$$g = C_D \frac{A}{W} \frac{\rho V_{terminal}^2}{2}. \tag{2}$$

In this equation, g is the acceleration due to gravity, C_D the drag coefficient for the body, A its cross sectional area, W its weight, ρ the weight density of air, and $V_{terminal}$ the terminal velocity of the body.

Equation (2) appears to require a heavy dose of physics. But, in fact, the word 'acceleration' is never used in the development and (2) is itself obtained mainly from a proportionality argument together with the intuitively reasonable conservation of momentum. Indeed, the treatment of physical laws is sufficiently remote from beginning physics that the idea of mass is never used either. For that matter, we consistently employ British Engineering Units, first because they are familiar to students and second because much published aeronautical data is given in that system. Students may employ (2) to work out terminal velocities for baseballs and parachutes. Results are a good match with published data.

At least as important as the availability of the terminal velocity formula is its structure. In general, C_D depends on the Reynolds number of the corresponding flow, and therefore on the velocity itself. But, for speeds up through 300 knots, C_D is nearly independent of velocity. Otherwise, C_D depends almost entirely on the character of the surface of the body (rough or smooth), and on its geometry (streamlined or blunt). Hence, we may pose for the student:

> Two smooth balls are dropped at the same time from the Tower of Pisa. One is made of iron and is one inch in diameter. The other is made of aluminum and is three inches in diameter. Which hits the ground first?

Owing to the similarity in surface texture and geometry, one may assume the two drag coefficients are the same. Thus, this iron-aluminum problem requires students to calculate the weight to cross-sectional area ratio of the two bodies. The students themselves must look up the weight density of the two metals.

Working with drag coefficients leads us to a study of the conditions for smooth level flight of an aircraft. Students may demonstrate for themselves the significance of the weight to (wing) area ratio of the aircraft.

Returning to the magnetic variation indicator on the Detroit City map, we may take up a large family of navigation problems. First of all, a vast array of radio navigation aids is scattered across the country. Among other things, those navigation aids (they are commonly called VOR's, the letters individually articulated) broadcast a nominal magnetic north that is established by the FAA. The fact that magnetic variation slowly changes over time causes some curious 'disagreements' among nearby VOR's. These disagreements can be resolved with some elementary mathematics.

As far as navigation itself, we may begin with short distance approximations that make the earth flat near a particular site. Calculations of distances and flight headings may be verified with the Flight Simulator. For large distances, a somewhat deeper foray into trigonometry is required. But we do just enough to work out long distances on the curved earth and corresponding headings for great circle flights. We erect none of the standard structure of spherical trigonometry.

Central to this geometric analysis is a canonical form for a great circle. Let H be the most northerly point on the great circle, where H has latitude and longitude (λ_H τ_H). It is intuitively clear that the choice of H will fix the great circle. Then, a generic point P, with latitude and longitude (λ, τ) falls on that great circle if and only if

$$\tan\lambda_H = \frac{\tan\lambda}{\cos(\tau - \tau_H)}. \tag{3}$$

This result may be derived trigonometrically. But it may also be demonstrated with the aid of a cardboard and glue construction that is appropriate as a team project. Among other things, the student is asked:

A pilot flies a great circle path whose most northerly point is Reykjavik, Iceland. Does the airplane fly north or south of Cleveland?

The further usefulness of (3) may be seen from a simple formula for calculating distances and headings. Let an aircraft lie at $P = (\lambda, \tau)$ en-route to the 'high-point' $H = (\lambda_H, \tau_H)$ of a certain great circle. The angular separation, β, along that great circle between P and H satisfies

$$\sin\beta = \cos\lambda \sin(\tau - \tau_H). \tag{4}$$

The value of β, obtained from (4), determines the surface distance between P and H according to

$$distance = 2\pi \cdot 3440 \cdot \frac{\beta}{360},$$

where β is measured in degrees, the way pilots do. 3440 is the radius of the earth in nautical miles; we treat the earth as a sphere in these calculations. Furthermore, the (geometric) heading, γ must satisfy the equation

$$\cos \gamma = \tan \lambda \tan \beta, \qquad (5)$$

an equation whose similarity to (3) is shown to be no accident. Calculations in every specific case may be verified against navigation maps published by the National Ocean Service.

It might be argued that an entire semester devoted to airplane flight demands of students an interest they may not possess. But, as we have indicated above, 'interest', as such, is not really on our agenda. We do insist that the students' work be genuinely edifying in a context. It is essentially a matter of conserving time that, in our view, precludes one from jumping from airplanes to horticulture to bowling and elsewhere, hoping to hit a topic that will seize the imagination of almost every student. Finding even one topic in which mathematics actually contributes to GE students' understanding, as prescribed by our Rule of Three, is not so easy.

5. Conclusion

The course described above has found substantial popularity at William and Mary. A single instructor of record has complete responsibility for an individual section of the course. We are able to manage a class size of 55, meeting three hours per week in lecture-dialogue sessions. Approximately 15% of the typical graduating class will take this course. The FAA program [5], and the wind data base [4] are available on the student accessible server.

The wide ranging use of highly focused external data described in §4 is, for many mathematicians, a daunting matter. It might be suggested that team-teaching is an appropriate way to manage the foreign territory of flight applications. One great benefit is that the mathematics instructor would not be burdened by extraneous information. Moreover, a pilot, say, would surely be in a better position to explain much of the nuance of the physical application, presumably to the edification of the students.

But there are also some good reasons to avoid team teaching. First, it is expensive to pay two people for one job. Second, although there is indeed a real sense in which this is an 'interdisciplinary' course, the honest objective is to get students doing meaningful mathematics. In the hands of a pilot, for example, there is danger of turning this into a flying course; interesting perhaps, but not mathematics.

In this regard, another consideration is important. It should be clear from the description in §4 that there is little time spent on the development of mathematics for its own sake. The absence of any such development is driven, not by a disdain for mathematics, but rather by a desire to have GE students use their limited time to make a genuine connection between mathematical practice and the external world. However, it is one thing to avoid making mathematical developments and quite another to be unaware of them.

We are certain that serious mathematical training of the instructor is a valuable asset in the use of these course materials. In particular, preparation of those materials had to be carried out as a persistent search for mathematical problems whose solutions are elementary. That judgement could only be made on mathematical grounds. In particular, it is easy for a mathematician to omit an investigation that might be irresistible to a practitioner of some particular aspect of flight.

There is a mathematical quid pro quo. Some colleagues have suggested that flight scheduling problems might be an interesting way to get students to do some graph theory. There is, however, no obvious way in which GE students could construct, say, the flight timetable of Delta Airlines. Only if we could find a way for them to do such a real problem could we permit graph theory to enter the conversation without violating our *Rule of Three*.

All of these are issues that will confront any institution that tries to establish some real connections between intellectual disciplines. Real connections, if they are to be real, have to amount to something more than a narrative describing what other people can do in such a regime. The more prominent successful efforts along legitimate interdisciplinary lines are found at the advanced undergraduate or graduate level. We hope we have demonstrated that such things are also possible for general education students. ■

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ONE HOUR OF CHEMICAL DEMONSTRATIONS

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This article describes a diverse set of chemistry demonstrations especially selected to encourage student interaction and to be easily transported. The demonstrations may be presented at a level that can be tailored to any audience-- from very young children to high school students planning careers in science. An ideal environment is a small classroom with 20-30 students where everyone can take part in the discussion. Once the chemicals are prepared, the collection of demonstrations takes about ten minutes to set-up, and one hour (or less) to perform. Very little is needed at the visiting site, no more than a table and a pitcher of water. A single electrical outlet is useful, but not essential. In Table 2 the demonstrations are listed in the order suggested for their presentation, along with all chemicals and equipment needed. Emphasized below are original procedures developed in this laboratory as well as sources of materials, background chemistry and ideas for discussion.

The Color Chase

In the "color chase" adapted from an experiment in a magician's handbook [1], a pale yellow liquid turns wine-red, deep blue, sparkling yellow, and brilliant pink as it is poured from the first flask into four successive flasks each of which contains a tiny amount of chemical reagent. The size of the flasks or beakers depends upon the room in which the performance is to take place. A 1-L vessel with 500 mL liquid should be visible in a lecture hall. For a smaller room the 500 mL size would suffice. The amounts below are intended for 500 mL flasks, and may be adjusted as needed. To the first flask, which should be very dry, is added 0.06 to 0.07 g of the pale violet ferric ammonium sulfate, $FeNH_4(SO_4)_2{\cdot}12H_2O$ or FAS. This amount, which fills the tip of a spoon spatula, is the only one that must be measured accurately. The FAS can be crushed before weighing, however, prepare only what is needed, since the pulverized FAS begins to degrade within a day. The next two flasks contain spatulatips of sodium salicylate, 2-(HO)$C_6H_4CO_2Na$, and of sodium (or potassium) ferrocyanide, Na4Fe$(CN)_6 \cdot H_2O$. About 4-5 mL sodium silicate solution or "water glass", 37-40% Na_2SiO_3 (aq), are placed in the fourth flask. A few drops of phenolphthalein indicator solution are added to the fifth.

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The demonstration is started by adding distilled water to the solid FAS producing a light yellowish solution as the pale purple hexaaquo ion undergoes hydrolysis to form the yellow hydroxo species [2]:

$$[Fe(H_2O)_6]^{3+} = [Fe(H_2O)_5(OH)]^{2+} + H^+ \quad K = 10^{-3.0}$$
$$[Fe(H_2O)_5(OH)]^{2+} = [Fe(H_2O)_4(OH)_2]^{+} + H^+ \quad K = 10^{-6.3}$$

When the original concentration of FAS is 0.1% or less (0.02 M), the pH is greater than 3 and bridged species form. Within a minute or two a cloudy colloidal gel appears, and eventually ferric oxide, $Fe_2O_3 \cdot nH_2O$, precipitates. At higher concentrations ranging from about 0.2% to 0.4% (0.04 to 0.08 M) the pH falls to 2.6, and it takes hours for the solutions to become cloudy. At concentrations of 0.5% or more (greater than 0.1 M), the pH dips below 2.5 and little or no cloudiness develops. For this demonstration, the tiny quantity of FAS mixed with water (0.06 - 0.07 g in 250 mL) produces the very dilute 0.03% solution, which has a pH much greater than 3. Therefore, the water must not be added to FAS ahead of time or the solution will turn cloudy almost immediately and the iron(III) needed for the ensuing reactions will be depleted.

The fresh FAS solution is then poured into the second flask containing sodium salicylate giving a wine red color. The shade of red is *extremely* sensitive to the amount of iron(III) present, varying from pale orange to nearly opaque dark red within a small range of iron concentrations. If the color in flask 2 is the desired wine-red, the next color, produced by pouring the contents of flask 2 into flask 3, should be just the right shade of blue. The blue color is due to formation of Prussian Blue, ferric ferrocyanide, $Fe_4[Fe(CN)_6]_3$, a water-insoluble dark blue pigment used in printing inks. Too little iron(III) will make this color a pale green--too much-- a color so dark it is difficult to recognize as blue and may hardly be distinguishable from the red that preceded it. As the blue mixture is poured into the aqueous sodium silicate in flask 4, it is transformed into a clear yellow solution as the deep blue ferric ferrocyanide breaks down into an iron complex and ferrocyanide ion. Pouring the alkaline solution into the final container gives the typical phenolphthalein pink.

The discussion accompanying this demonstration can be adjusted to the level of the audience. For very young children, it can simply be said that colors are changing due to chemical reactions. For older students a description of the simplest reaction, the acid-base behavior of phenolphthalein, can be included, and later used in an inexpensive laboratory activity. Phenolphthalein turns pink in solutions of sodium carbonate (washing soda) or ammonia (cleaning products). A more detailed description of the chemistry may be appropriate for some high school students. The color chase can even be used as a demonstration for experienced chemists, by offering a contest in which the audience must identify the five reagents by noticing the colors that form, a non trivial task.

Invisible Writing

Revealing a message written in invisible inks, based upon the chemistry of iron [3], is a good way to welcome the audience. The message appears when sprayed with 10% (0.2M) ferric ammonium sulfate (FAS), the same source of iron(III) used above in the color chase. In this case the FAS solution remains clear since the concentration is high and the pH is well below 2.5. Best results are obtained when the iron solution is applied as a very fine mist using a spraying tool available in hardware stores.

Caution: *The fine mist can cause coughing, so be sure not to spray near anyone.*

The "painting" inks are dilute aqueous solutions that combine with iron(III) to produce deep blue, reds, and shades of black. Cotton swabs work better than paintbrushes.

Blue Ink

For blue, a 4% solution of sodium or potassium ferrocyanide is used. The color that results from spraying with Fe(III) is the deep blue Prussian Blue, the same compound formed in flask 3 of the color chase described above.

Red Inks

There are two different "inks" that combine with Fe^{3+} to give reds. The reaction of sodium salicylate (10%) with iron(III) gives the same purplish wine red iron salicylate complex produced in the second flask of the "color chase." A more orange hue is derived from using 1% thiocyanate, NH_4SCN or KSCN, to give intense red thiocyanate complexes of iron. However, the color of this complex begins to fade moments after it forms. The demonstrator can take advantage of this instability by creating a message that changes in some interesting way when the thiocyanate red is gone.

Black/Gray Ink

For blacks and grays the ink solution is aqueous tannic acid, which occurs in the bark and fruits of many plants, acorns, for example. The formula of commercial tannic acid, a yellowish water-soluble powder, is given as $C_{76}H_{52}O_{46}$. Depending upon the concentration of the tannic acid, the color of iron tannate can range from gray to black (0.2 - 2%).

Flash Paper

Flash paper is nitrated paper that burns dramatically with an instant bright flash and leaves little or no residue. A commercial version is available in magic and novelty shops.[2] A very fast method for making flash paper similar in size and burning properties to the commercial type has been described [4]. The nitrating mixture is 5 parts of very fresh concentrated HNO_3 mixed with 4 parts concentrated H_2SO_4 in a clean and dry 1 L beaker (the paper may not nitrate properly if the nitrating mixture is in a wide container such as a crystallizing dish, presumably because of the large surface area).

***Caution**: The concentrated acids should be handled in a hood with great care. Dispose of acids by first neutralizing with sodium bicarbonate, then pouring down the sink with running water.*

Once the acid mixture is cooled to about 40°C, a piece of very thin paper, such as the cheapest single-ply toilet tissue, is pushed gently beneath the liquid surface so that it is covered with acid, then soaked in the nitrating solution for 12 to 15 minutes. Very long pieces can be made by carefully folding the paper over on itself making sure that all surfaces are in contact with the nitrating solution. In this way a large supply of flash paper can be made in less than an hour. The nitrated papers are rinsed thoroughly with water, then stored in an ethanol bath until needed. To produce colored flames, the nitrated papers are dried, then coated with powdered salts to make colored-flame flash paper (green $BaSO_4$, orange $CaSO_4$, red-orange $SrCO_3$, blue CuBr, yellow NaCl, and violet KCl or KNO_3). For lighting the paper a candle in holder is more convenient than matches. Smaller pieces of flash paper can simply be lit and tossed in the air. A dramatic effect is achieved by wrapping a fresh (but not wet) flower in a larger piece of flash paper, then while holding on to the stem, burning the flash paper away. The flower seems to appear from nowhere out of the flame.

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[2]Distributed by D. Robbins & Company, Inc. Brooklyn, NY 11201.

Giant Bubbles

Large diameter soap bubbles have been prepared by using solutions containing 5% glycerol mixed with 10-12% dish detergent [5] or 2% dioctyl ester of sodium sulfosuccinate [6].The glycerol appears to function primarily as a non-drying agent. It has been found that the use of more concentrated detergent solutions (15-18%) appreciably increases bubble lifetime (See Table 1), making them not only longer-lasting, but much easier to prepare. No "aging" of the bubble solution nor any special coordination is required to create bubbles with diameters as large as 50-60 cm. The brands that were most effective included Ultra Joy and Dawn.[3] The bubble mixture is mixed in a container large enough to accommodate a wand with a diameter of about 20 cm, made by twisting a plastic-coated coat hanger. Studying bubble solutions can be adapted as a laboratory activity in which students compare the lifetimes of bubbles upon variation of detergent brand, concentration of detergent, and concentration of additives such as glycerol. Lifetime measurements should be made in a draft-free room by creating a bubble, catching it, then holding it in place on the wand until it deflates.

Table 1 Lifetime of Bubbles

Detergent Brand	Volume Detergent	Volume Water	Volume Glycerol	Conc % Detergent	Life of Bubble 30cm diameter (min)
Joy	100	700	40	12	0.75 - 1
Joy	150	650	40	18	2 - 3
Joy	200	600	40	23	2 - 3
Joy	150	690	0	18	2 - 3

The Elements

The display of a collection of elements generates immediate interest in chemistry and invites audience participation. Each element can be placed in its own transparent 20-mL screw-cap glass scintillation vial. The original packaging tray with compartments for 100 vials serves as an ideal portable storage case with unused spaces available to house small tools

[3] All concentrations refer to the Ultra type detergents which "require 1/3 less".

and reagents such as a magnet or phenolphthalein solution. A guide available for the assembly of an extensive set of elements includes prices, sources, and handling tips for each element (excluding gases or radioactive elements), as well as specific suggestions about how to use the elements in classroom situations [7-9]. Since the assembly of a complete collection is expensive and time-consuming, the element vials can be labeled and arranged, then filled as the occupying elements become available. The demonstrator can show students the explosive reaction that occurs when a pea-sized piece of potassium or sodium is dropped into water.

***Caution**: Be sure to use only small amounts of sodium or potassium.*

Students can help to assemble an initial starting set of elements from household materials such as carbon from pencil lead, aluminum foil, copper wire, iron nails, and tungsten in light bulbs.

Other Demonstrations

Sources of material and references for the rest of the demonstrations are included below.

Radioactivity

This flows naturally from the element discussion. For students who have studied a little chemistry, the idea can be introduced that an element with atomic number greater than 83 decays spontaneously. An inexpensive source of radioactive substances consisting of uranium and thorium ores, may be purchased from Central Scientific Company.[4] Placing a piece of paper, a student's notebook,and finally a piece of lead between the radiative source and the window of a portable Geiger counter demonstrates penetrative power. Students can be reminded that the lead apron they wear at the dentist protects them from similar radiation.

Memory Metal

A wire made of Ni-Ti alloy, twisted out of shape at room temperature, is returned to its original shape upon heating with a blow dryer or candle flame [10]. An inexpensive source for 3-inch samples of memory wire with a critical temperature of 50°C is Educational Innovations, Inc.[5]

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[4]Radioactive Mineral Collection 52630K; CENCO Central Scientific Company, Franklin Park, IL 60130.

[5]Educational Innovations, Inc., 151 River Road, Cos Cob, CT 06807 USA; Telephone (203)629-6049; ***http://www.teachersource.com***

Rubber Ball

The latex[6] needed to make a rubber ball is first mixed with an equal volume of water, a few drops of food color, then an equal volume of vinegar. The shapeless rubbery mass is removed with a gloved hand, then submerged in a container of water in order to squeeze out the excess water.

Caution: *Gloves must be used to handle the acidic mass.*

The finished lumpy ball bounces as soon as it is removed from the water, higher as it dries.

Nylon 6,10

The nylon rope forms most effectively with 6% hexamethylene diamine, $NH_2(CH_2)_6NH_2$, in 0.5 M NaOH and 4% sebacoyl chloride, $ClOC(CH_2)_8COCl$, in hexane or cyclohexane. Ready made solutions can also be purchased from Flinn Scientific.[7] The more dense aqueous hexamethylene diamine is poured first into a small beaker, then the other layer. Tweezers remove the nylon rope from the interface between the two layers. The solutions have a long shelf life, but small amounts should be tested before each use to be sure the rope forms.

Polyurethane Foam

Polyurethane foam is made by the combination of viscous solutions that contain a diisocyanate and a polyol. In this case it is much more convenient to purchase the components.[8] The Polyurethane Foam System consists of two parts. Part A contains the polyol and Part B the diisocyanate. The lids can become sealed shut. Be sure to check that they can be removed, particularly for Part B which has a much shorter shelf life than Part A.

Liquid Nitrogen

The properties of liquid nitrogen are particularly intriguing to students. Demonstrations can be as simple as the freezing of flowers or bananas. A more advanced activity suitable even for young children is the Meissner effect. A strong rare earth magnet is elevated above a superconducting pellet cooled to liquid nitrogen temperature. A superconductivity kit

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[6]Flinn Scientific Inc. P.O. Box 219. 131 Flinn St. Batavia, IL 60510.

[7]Flinn Scientific Inc. P.O. Box 219. 131 Flinn St. Batavia, IL 60510.

[8]Flinn Scientific Inc. P.O. Box 219. 131 Flinn St. Batavia, IL 60510.

containing a rare earth cobalt magnet and a ceramic superconducting disc composed of yttrium, barium, and copper oxides is available commercially.[9] Be sure to dry the pellet after each use.

Caution: *When transporting the liquid nitrogen in a car be sure the Dewar flask cannot topple over to avoid asphyxiation.*

During this demonstration the temperature of liquid nitrogen can be expressed using Kelvin, Celsius and Fahrenheit scales and then compared to familiar substances such as ice.

Conclusion

Early exposure of many students to chemistry is from the media where chemicals are often portrayed in a negative light, their toxicity emphasized rather than their usefulness. Hopefully, while engaging and entertaining students, the set of demonstrations described here will also help to counteract the negative impressions students may have absorbed about chemistry and chemicals.

Acknowledgement

We would like to thank Frederick R. Longo whose broad chemical insight provided us invaluable guidance. ■

[9]Edmund Scientific Co. Barrington, NJ 08007-1380 Telephone 609-573-6250.

Table 2 Materials Needed for Demonstrations

Demonstration	Chemicals	Equipment/Supplies
Invisible Writing	ferric ammonium sulfate sodium salicylate potassium (or sodium) ferrocyanide tannic acid potassium or ammonium thiocyanate	spray-tool large sheets of paper Q-tips
Collecting the Elements	chemical elements	20-mL scintillation vials packing container
Radioactivity	uranium or thorium ore lead brick or sheet	Geiger counter
Memory Metal	Ni-Ti wire	hair dryer
Color Chase	ferric ammonium sulfate sodium salicylate potassium (or sodium) ferrocyanide sodium silicate (aq) phenolphthalein (aq)	5 flasks (500 or 1000 mL)
Polymers		
Nylon 6,10	sebacoyl chloride 1,6-diaminohexane sodium hydroxide hexane	beaker stirring rod tweezers
Polyurethane	Polyurethane Foam Kit	foam cup stirrer
Rubber	latex vinegar	250-mL beaker stirrer gloves large container (for water)
Flash Paper	sulfuric acid and nitric acid salts ethanol or commercial flash paper	matches flower hair dryer
Giant Bubbles	detergent (Joy or Dawn Ultra) glycerol	container wand
Liquid Nitrogen	liquid nitrogen	Dewar flask superconductor kit flowers bananas

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USING GRAPHING CALCULATORS TO INTEGRATE MATHEMATICS AND SCIENCE

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The computational, graphing, statistical and programming capabilities of today's graphing calculators make it possible for teachers and students to explore aspects of functions and investigate real-world situations in ways that were previously inaccessible because of computational constraints. Many of the features of graphing calculators can be used to integrate topics from mathematics and science. Here we provide a few illustrations of activities that use the graphing, parametric graphing, regression, and recursion features of graphing calculators to study mathematics in science contexts.

Increasingly, graphing calculators are being used in secondary mathematics teaching. Textbooks, teachers' guides, and even high stakes examinations are being written with expectations that teachers and students in high school mathematics courses (and in some cases middle school courses) are using graphing calculators. Graphing calculator manufacturers are sponsoring workshops and producing supplementary materials to help teachers learn how to use their latest products and incorporate them into mathematics curricula. Furthermore, state departments of education are now revising their curriculum guidelines to include graphing calculators. Indeed, the *Virginia Standards of Learning* (SOL's) for introductory algebra [1] stipulate: "...graphing utilities (graphing calculators or computer graphing simulators) should be used as tools to assist problem solving. Graphing utilities enhance the understanding of functions..."(p. 18). The Virginia SOL's also specify that graphing utilities be used in algebra, trigonometry, and mathematical analysis courses because they "enhance the understanding of realistic applications through mathematical modeling and aid in the investigations of functions and their inverses" (p. 25).

The NCTM *Curriculum and Evaluation Standards* [2], written before the current widespread use of graphing calculators, also advocates the use of graphing utilities in courses in algebra, trigonometry, and functions. Moreover, the Standards for grades 9-12 assume that "Scientific calculators with graphing capabilities will be available to all students at all times." (p. 124).

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The benefits of graphing calculators are numerous. The computational, graphing, statistical and programming capabilities of today's graphing calculators make it possible for teachers and students to explore aspects of functions and investigate real-world situations in ways that were previously not feasible because of computational constraints. Features of graphing calculators also make it possible to study traditional topics in new ways and in more depth and to study new topics that were previously impractical at the secondary level using only paper and pencil methods. Mathematical modeling, simulation, connecting multiple representations, and data analysis are examples of mathematical topics that can be studied more efficiently and effectively using graphing calculators.

Many of the features of graphing calculators can be used to integrate topics from mathematics and science. Here we provide a few illustrations of activities that use the graphing, parametric graphing, regression, and recursion features of the Casio 9850 Plus graphing calculator to study mathematics in science contexts. These activities can be carried out with other graphing calculators as well.

Sample Integrated Calculator Activities

Parametric Graphing

The parametric graphing features of graphing calculators allow students to dynamically simulate the actual paths of projectiles by generating equations for motion in both the x and y directions using *time* as a parameter. These paths can assist students in understanding the components of projectile motion and their associated equations by providing appropriate visual support. Without using parametric equations, students could only graphically represent the trajectory of a projectile with *time* as the x-variable and *height* as the y-variable, and of course graphs of such relationships do not simulate the actual paths of projectiles. Research documents, however, that many students interpret such height-time graphs as true paths of projectiles because they interpret the change in *time* as motion in the x direction [3-5]. This type of misinterpretation of a graph is sometimes referred to as *iconic interpretation* of a graph [6]. Such graphical misconceptions can be avoided, and even analyzed, when students use parametric graphing.

Rocket Simulation. This activity asks students to use parametric equations to simulate the actual path of a rocket launched straight up with an initial velocity of 98 meters/second.

Students are first asked to simulate both the constant upward motion of a rocket unaffected by gravity, and the downward freefall accelerating motion of a rocket due to the force of gravity. The parametric equations that generate dynamic graphs of these two motions are: $x_1=1$, $y_1=98t$, and $x_2=2$, $y_2=-(.5)9.8t^2$. Students are then asked to consider these two components of motion together.

The first screen shot shown in Figure 1 simulates the paths of one rocket fired straight up with a constant velocity of 98 m/s and another rocket freefalling for 10 seconds. The second screen shot shows the paths of the same two rockets after 20 seconds, but also includes the path of a third rocket launched with an upward velocity of 98 m/s *and* under the influence of gravity. The equation generating the path of the third rocket combines those of the first two, namely $x_3=3$, $y_3=98t-(.5)9.8t^2$. The third screenshot zooms in on the graph representing the actual path of the rocket (note: t/320 has been added to the x component to move the downward portion of the graph over 1 pixel from the upward portion).

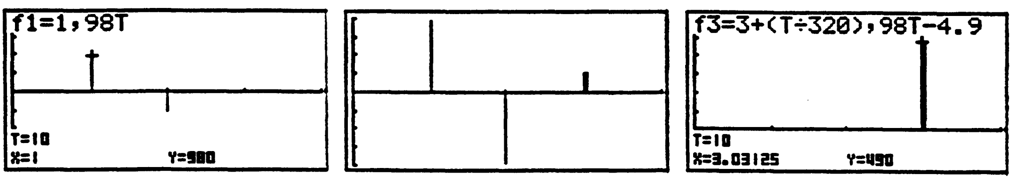

Figure 1: Rocket Simulation Screenshots.

Students are able see that after 10 seconds the distance traveled by the rocket moving upward at a constant velocity is greater than the distance traveled by the freefalling rocket, and can surmise that a rocket moving up and under the influence of gravity will still be above the ground. Also, students will see that at 20 seconds the distances traveled by both rockets are equal, and can predict that a rocket launched up at 98 m/s affected by gravity will hit the ground at 20 seconds. This prediction can then be verified graphically. Observing the three graphs over several time intervals, tracing the simulated paths, and comparing the distances traveled at various times allow students to observe graphically and numerically how constant velocity and acceleration are related.

Activities like this one, besides helping students to better understand velocity and acceleration, also help students connect the different terms of quadratic equations to formulas from Newtonian mechanics describing the different aspects of projectile motion.

Freefall. This activity, adapted from [7], gives students the gravity data shown in Table 1[1] and asks them to simulate the paths of objects freefalling from 500 feet above the surface of each of the planets.

Planet	Period	Distance from Sun	Gravity
Mercury	88	57.900	3.70
Venus	225	108.200	8.87
Earth	365	149.600	9.78
Mars	687	227.900	3.69
Jupiter	4332	778.300	23.12
Saturn	10760	1427.000	8.96
Uranus	30685	2869.328	8.69
Neptune	60189	4496.672	11.00
Pluto	90456	5913.500	0.66

Table 1: Planet Data.

The screen shots in Figure 2 show the parametric equations used to generate three of the graphs and the graphs of the paths of the objects freefalling from 500 feet for 9 seconds (with the names of planets abbreviated above each). This activity allows students to compare the effect of gravity dynamically by seeing the relative motions over time. Tracing various paths and relating the changes in distance over time to the equations helps students get a better feel for the effect of different gravitational constants on freefall motion.

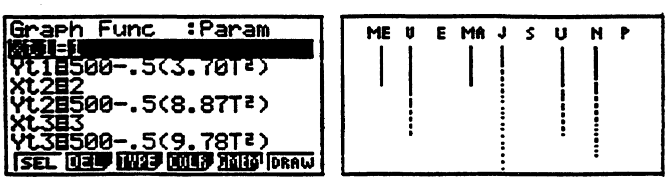

Figure 2: Freefall Screenshots.

[1]This data was found at *http://nssdc.gsfc.nasa.gov/planetary/planetfact.html*

Projectile Motion. This activity investigates non-vertical projectile motion and incorporates trigonometry (and calculus if desired). Students are asked to simulate the paths of three projectiles launched with an initial velocity of 64 feet/second at 30, 45, and 60 degrees, respectively. The screenshots in Figure 3 show the (truncated) parametric equations used to generate the graphs, the simulated paths, and the simulated paths with the derivative tracing feature activated (for use with calculus students).

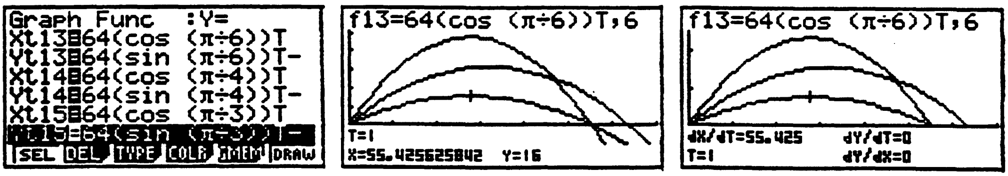

Figure 3: Projectile Motion Screenshots.

Activities such as this, with appropriate questioning and follow-up tasks, help students connect mathematical equations to formulas describing motion, connect coefficients in equations to features of graphs, and understand how derivatives represent slope at a point.

Curve Fitting

Graphing calculators make it possible for students to plot data, visually explore relationships between variables, and determine the equations of best-fitting curves in two ways: by using the graphing features to successively approximate best-fit curves or by using the regression capabilities to calculate least-squares regression equations.

Deriving Kepler's Third Law. This activity, adapted from [8], gives students the data in Table 1 and asks them to plot the approximate average distance from the sun (length of the semi-major axes) versus the period for each planet. From the plots students can easily conjecture that a relationship exists between these variables, and using the least-squares regression capabilities of the calculator, they can calculate the coefficients of the curves of best fit. The screen shots in Figure 4 show the plotted data points, the coefficients of the power function of best fit, and the drawn regression curve.

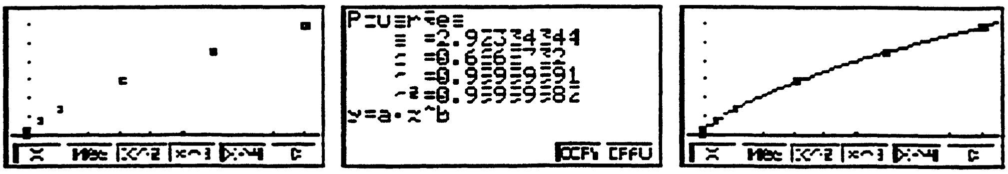

Figure 4: Kepler's Third Law Screenshots.

Students can see that $y = 2.9x^{.66}$, or distance = 2.9(period)$^{2/3}$, fits the data almost perfectly, and can relate the square of the period to the cube of the approximate distance from the sun by using basic algebra. Hence, they are deriving Kepler's Third Law. Activities like this one help students better understand that formulas in science are derived from data and that mathematics plays an integral role in such derivations.

Monthly Temperature. This activity gives students the temperature data in Table 2 and asks them to plot the temperature data and examine the variation over a year.[2]

	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Jan	Feb	Mar	Apr
Wash. DC	54	64	73	77	75	68	57	46	37	34	36	45	54
Verk. Russia	5	32	54	57	48	36	5	-35	-53	-57	-48	-25	5
Buenos Aries	63	55	48	50	52	55	59	66	72	73	73	70	63

Table 2: Average Monthly Temperature Data.

One version of this activity is to ask students to fit a sine curve to the data without using the regression feature of the calculator. (Not all graphing calculators have a sine regression feature). The screen shots in Figure 5 show a plot of the data, a sequence of sine curves approximating the data with one coefficient being adjusted at a time, and the coefficients of a regression equation calculated using the regression feature of the graphing calculator. Here the first sine curve uses an amplitude coefficient derived from examining the maximum and minimum values of the temperature [$y = 21.5 \sin(x)$], the next curve adds a vertical shift coefficient also derived from the maximum and minimum values [$y = 21.5 \sin(x) + 55.5$], the

[2] A fuller version of this activity, by R. Ward, can be found at *http://pen1.pen.k12.va.us:80/Anthology/Pav/MathSci/calc/TEMP*

next adjusts the period ($2\pi/12$) for 12 months [$y = 21.5 \sin(.52x) + 55.5$], and the fourth curve adjusts the horizontal shift [$y= 21.5 \sin(.52x - .59) + 55.5$]. The last coefficient can be approximated either visually, through use of the trace feature, or algebraically using one data point and the previously found coefficients.

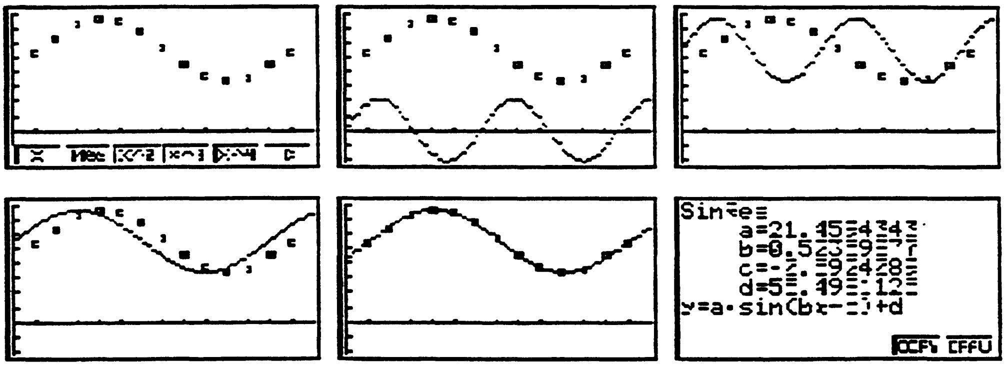

Figure 5: Washington, DC Temperature Screenshots.

Students are then asked to compare the derived equations and discuss their similarities and differences.

Figure 6 shows the plots of the data for all three cities.

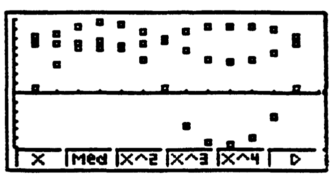

Figure 6: Temperature Plots for Three Cities.

Students can be asked to relate relevant aspects of the geography of the cities to amplitude and phase differences between the graphs and the coefficients of the equations describing them. Activities like this help students develop an understanding of how different coefficients affect the graphs of trigonometric functions.

Recursion

Graphing calculators facilitate the study of many topics from discrete mathematics. Examples of such topics include matrices, combinatorics, and recursion.

Solution Mixture Problem. This activity asks students to first solve the following problem:

> Consider two containers, A and B, containing 100 cc of *solution a* and *solution b*, respectively. Ten cc of *solution a* is taken from container A and placed in container B. The solutions in container B are then mixed up and 10 cc of this blend is placed in container A. Determine if there is then more of *solution a* in container B or more of *solution b* in container A.

Students usually try to solve this problem using a combination of intuition and algebra, and a substantial number of them do not solve it correctly. After this simple case is resolved, we ask students to predict what would happen if this process is continued many times. Students are then asked to calculate the amount of *solution a* in each container after each iteration. Using algebraic equations to answer this question can be complicated and inefficient. It is easier to solve this problem using recursion. At each iteration, the number of cc of *solution a* in container A and container B, respectively, can be represented as:

$$a_{n+1}=(10/11)a_n+(1/11)b_n$$
$$b_{n+1}=(1/11)a_n+(10/11)b_n$$

The graphing calculator screenshots in Figure 7 show these recursive equations (truncated), a table listing the number of cc of *solution a* in containers A and B, and a plot of the amount of *solution a* in each container at each step. Notice the amounts converge to 50 cc from above and below.

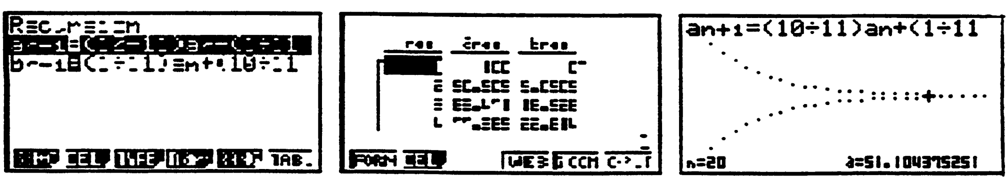

Figure 7: Solution Mixture Problem Screenshots.

Discussion

The graphing calculator activities described above use mathematics to model and analyze problems arising in scientific situations. Such activities provide contexts for school mathematics topics. It is widely acknowledged that such contexts can be helpful to learners, and that traditional mathematics instruction has been woefully inadequate in this regard [2, 9, 10].

Each of the above activities makes use of numerical, algebraic, and graphical representations of mathematical functions. Research has shown that many students have difficulty connecting multiple representations of functions. Activities such as those presented above can facilitate the making of such connections [2, 11-13].

These applications not only help students better understand the mathematics involved, but also help students develop better understanding of aspects of the science involved, namely, scientific concepts, how scientific laws are derived, and how the doing of science is facilitated by mathematics. Teachers and students could further explore science and its connection to mathematics by designing and conducting their own experiments using data collection devices and probes developed for use with graphing calculators. ■

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DEVELOPING MATHEMATICS ENRICHMENT WORKSHOPS FOR MIDDLE SCHOOL STUDENTS: PHILOSOPHY AND SAMPLE WORKSHOPS

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This paper describes our approach to organizing enrichment activities using advanced mathematics topics for diverse audiences of middle school students. We discuss our philosophy and approaches for the structure of these workshops, and then provide sample schedules and resource materials. The workshops cover activities on the following topics:

Graphing Calculators
The Chaos Game
Statistical Sampling
CT Scans--the reconstruction problem
The Platonic and Archimedean solids
The Shape of Space
Symmetry
The Binary Number System and the game of NIM
Graph Theory: Proof by Counterexample

Overview

The University of Minnesota Talented Youth Mathematics Program (UMTYMP) has created programs to capture and maintain the interest of middle school students who enjoy mathematics. The activities discussed in this paper have been developed and implemented for two of our intervention programs: Project YES (Young Emerging Scholars) and Project PRIME (Professions and Recreations: Intermediate Mathematics Enrichment). The overall objectives for both programs are:

1) to provide a comfortable and interesting setting for students to learn and enjoy mathematics and become aware of its applications
2) to provide opportunities for students to do mathematics in a stimulating, small-group environment
3) to allow students to learn mathematics using innovative visualization and graphics
4) to help students to become aware of communication problems in mathematics and improve their communication skills

[1] The intervention programs and this study were supported in part by grants from the Bush Foundation (St. Paul, Minnesota), the National Science Foundation, and the Tensor Foundation (through the M.A.A.)

5) to enable students to better understand how advanced mathematics and science courses enhance career opportunities in mathematics and related areas
6) to familiarize students with a variety of careers and disciplines that rely upon mathematics.

The enrichment activities may be used as stand-alone activities that do not rely on prior student knowledge, group involvement, or connections to the larger community of mathematicians and scientists. One way, however, to maximize the effectiveness of these enrichment activities is to develop them as a series of events that focus on a central theme and that take place on a regular basis or are part of a larger ongoing program.

The primary goal of Project YES is to provide a rich and diverse program of enrichment activities to support the further study of mathematics and to enhance interest in career opportunities in mathematics and the sciences for 7th and 8th grade students. Project YES especially encourages female students, students of color, and economically disadvantaged students to participate. Since the program began in 1990, 100 to 125 students have participated each academic year. Project YES consists of a comprehensive academic year program of monthly activities and a four-week summer enrichment institute. It is targeted at students identified as motivated to study mathematics by a variety of school and community sources, parents, and other non-traditional resources, as well as those who nearly obtained admission into UMTYMP. Families are invited to attend several of the activities, and parents are involved in helping their children participate in the program. The program helps students learn how to appreciate mathematical ideas gleaned from advanced topics. It does not pursue acceleration in one subject, but instead develops a series of challenging mathematical and enrichment activities covering a broad range of topics. An important aspect of the activities is the student friendships that develop. Students thrive when they are given the opportunity to spend time with students like themselves who are interested in math and like to be challenged. Learning and teaching emphasize hands-on activities. Students build models and actively use graphing calculators and computer software. In addition to mathematics workshops, the program includes presentations by professionals who use math and science in their careers.

Based on the success of Project YES, Project PRIME, which focuses on a younger population with an interest in mathematics enrichment, was started in 1996. Project PRIME was initially developed to reach out to females in the fifth and sixth grades from a diverse group of inner-city schools. An enrollment of fifty students was anticipated. Based on an overwhelming response from students, parents, and teachers, however, the program was opened to a larger audience. Two hundred young females registered for the 1996-97 academic year program; the project was opened to male students in 1997. A week-long institute was developed for the summer of 1997.

The 1996-97 PRIME program included a variety of events throughout the academic year: fall orientation, two hands-on mathematical workshops focusing on geometry through the use of visualization software applications, the annual math fair with exhibits and activities from a wide range of institutions, and a day of science and engineering lab visits and career speakers. Students also received subscriptions to the bi-monthly PRIME Times newsletter, which offered them the chance to write in and share their solutions to a variety of math puzzles.

In the fall of 1997, 355 male and female students enrolled in both Project PRIME (225 5^{th} and 6^{th} graders) and Project YES (130 7^{th} and 8^{th} graders). Based on the current waiting lists for these programs, it is anticipated that 50 to 60 more students will be invited to participate in the spring events. The students will attend three-hour workshops scheduled throughout the year. Project PRIME includes five activities, featuring hands-on workshops centered on the mathematics of knots. Project YES has eight activities, including interactive workshops and computer labs on topics ranging from graphing calculator functions to combinatorics *(http://www.math.umn.edu/itcep/)*. Both of these programs strive to give students a sense of participation in the culture of mathematics and a sense of how their interest in this culture can help them throughout their lives.

All of the workshops provide excellent teacher preparation opportunities for pre-service students. Each workshop is developed and taught by post-doctoral fellows. Secondary school teachers and mathematics and mathematics education graduate students who are enrolled in teacher certification programs also teach the workshops. Each instructor is assisted by an undergraduate student with an interest in mathematics education. The student/instructional team ratio is 25:2.

While many of the students who participated in Project YES and Project PRIME were identified as potentially talented in mathematics, the evaluations showed all of the participants maintained a high level of interest and involvement in the programs. For example, the survey results of the October 1997 Project PRIME workshop indicated that 94% thought it was fun or great and for the October 1997 Project YES workshop, 97% thought it was fun or great. Thus, we feel that these workshop activities are useful and stimulating for a diverse audience of middle school students. All that is required might be some modification in the length and possibly the depth of some of our activities. We strongly encourage any teacher to use these activities with students who are interested in new approaches to mathematics, and to use them to help students better understand the variety of mathematical applications.

Developing Workshops

Our workshops for middle school students always include multiple activities built around one main topic. As the workshop progresses, its segments build on knowledge or discoveries from earlier segments. As experience within a topic grows, more student questions develop and are addressed.

By the end of the session, we try to achieve some level of mathematical closure. This does not require that every mathematical question be resolved by the time that the students leave. In fact, we often give students open problems or additional materials to take home. At the end of each activity, students should feel that some interesting conclusions were reached as a result of their investigations, and some will wish to explore the topic further. (They should find enough gold to convince them that there is more gold to be found.)

While part of each workshop (typically the introductory material) is led in a lecture/discussion style, a significant portion of the workshop consists of hands-on activities that students accomplish in pairs or groups of 3-4 students. By pushing students to probe for answers in their own minds and from their own experiences, the hands-on activities strongly reinforce the learning process. Within a cooperative group, students share materials, make conjectures, and draw conclusions together through discussion.

In addition to being enjoyable and helping to build social relationships between students, we want the students to appreciate some of the educational benefits of cooperative learning. To this end, activities promoting a pooling of intellectual resources are created, allowing students to progress through new material more quickly than they might on their own. Opportunities for processing their thinking aloud through discussion seems to make the material more tractable for students. Verbal discussion forces students to make their reasoning precise enough that others can understand it. The small group discussions encourage active participation from students who would not feel as comfortable sharing their thoughts with the entire classroom of students.

Generally, we begin each workshop with all participants in a common session before breaking into separate classrooms and then further into groups. In addition to introducing the main topic, we often initiate a brief physical or verbal exercise (e.g., a resounding cheer for prime numbers) to get everyone in motion. There is a palpable energy and excitement when a large number of students gets together to discover the motivation for the mathematical journey upon which they are about to embark. This format also means that only one teacher needs to prepare remarks for the opening session! At the end of the workshop, we reconvene as a large group for a wrap-up discussion. During the closing time we have participants answer a short (4 to 5 question) evaluation of the event. The feedback from these evaluations is used when planning future events.

The components that we weave together to form a workshop are often activities that were designed by other teachers and curriculum developers in a format that is nearly ready for our purposes. If we are lucky, these pieces may already have been tested with prior groups of students, and are both realistic and effective. Some student activity sheets are used in their original form. Otherwise, we might cut and paste prepared activity sheets to make them fit our workshop vision. Sometimes we find articles from teachers that sketch an activity that can be done, and we fill in our own details. There is simply no need to start to build a program from sand and water when it can be built from bricks and sound scaffolding! This consideration is especially relevant during the school year when time is scarce.

We have also found that readily available software and manipulatives often enhance our examination of a topic. Many of the software applications we have used were obtained for free via the internet, most often from the NSF Geometry Center *(http://www.geom.umn.edu)* and from Rick Parris' Peanut Software *(http://www.exeter.edu/~rparris)*. Other excellent web sources are the Mathematics Forum, Swarthmore College *(http://www.forum.swarthmore.edu)*, and MEGAMATH *(http://www.c3.lanl.gov/mega-math/welcome.html)*. We use Polydron manipulatives (800-452-9978) for a variety of geometry topics. A one-time purchase of a classroom set of Polydrons has been a worthwhile investment. We frequently utilize lending programs of graphing calculator companies, which may include delivery of measurement devices for collecting "real world" data and activity booklets, if needed.

In our opinion, at some point in a teacher's development, the experience of combining basic mathematical concepts with a goal of creating some enrichment curricula is important. This is most realistically done during a summer period when teachers can focus on curriculum development as a project. At the NSF Geometry Center's 1994, 1995, and 1996 summer workshops for secondary teachers, for example, teachers from several states nationwide were involved in the development of curricula surrounding the "Shape of Space" video and the "KaleidoTile" software. The deeper understanding that the teachers gained while developing the curricula enabled them to use the materials more effectively in their classrooms. Many of them have gone on to develop their own enrichment programs.

In the remainder of this paper we describe in detail some of the workshops that we have developed for Projects YES and PRIME. For each example, we provide an event schedule and a description of activities and/or resource list. In cases where we created original material, copies of the activity sheets are listed on our web site, making these active documents available to teachers planning similar activities *(http://www.math. umn.edu/itcep/)*.

Graphing Calculator Workshop

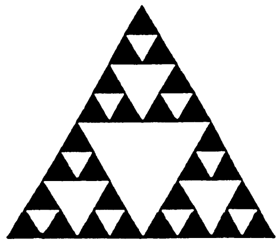
One of our most successful activities has been the Graphing Calculator Workshop, which was developed for a four-hour format. The data that is used for graphing is collected from sounds, motions, and temperatures. These investigations can be explored using the human voice or musical instruments, bodily movements or any source of motion such as bouncing balls, and any substance that responds to changes in temperature such as water. This enables the teacher to vary the lesson each time it is presented by using a variety of sources for the experiments.

Graphing Calculator Curriculum	
1.	Interactive demonstrations to small groups (15 to 10 students) on the basic use and functions of the graphing calculator, led by an instructor and teaching assistant.
2.	Rotation of teams through three stations using the graphing calculators with Calculator-Based Laboratory systems (CBLs) to graph data from experiments that explore motion, sound, and temperature. (CBLs are portable hand-held data collection devices with sensor probes for collecting 'real world' data that can be retrieved by graphing calculators for analysis.) A refreshment break is provided between two of the rotations.
3.	Reconvening of the groups to discuss their experiences and the implications of their findings.

The introductory demonstrations include instructions on using the calculator to perform basic arithmetic calculations, using a coordinate system, definition of a function, using the calculator to construct a function table and graph, and tracing and zooming on a graph. The ideas for the CBL activities are from Texas Instruments' CBL System Experiment Workbook [1]. More information can be obtained from 1 (800) TICARES. Student comments are very supportive of the positive effects of this workshop—the relevance of using graphs when solving problems in the classroom and the real world.

The Chaos Game

We spent several days during the summer YES program using The Chaos Game to explore dynamical systems. The Chaos Game, based on an idea of Kevin Lee, College of St. Catherine, St. Paul, MN, is an interactive game that develops a fractal on its game board. It can be played with pencil, paper, and a die. The game starts with only the vertices of an equilateral triangle each assigned 2 numbers between 1 and 6, and a random point inside the triangle. On successive moves the die rolls determine where to place the new dot by locating the new point halfway between the previous point and the triangle vertex chosen by that die roll. The compellingly beautiful results of applying the game's algorithm can best be viewed on a computer. The process forms the Sierpinski Triangle fractal as its end result or basis of attraction:



This topic and its mathematical justification can be spread over several days, with other fractal topics interspersed, as listed below:

The Chaos Game Curriculum	
Day 1	
1.	Explanation of the game and use of half-rulers.
2.	Students play the game on transparencies, with a partner, and then the class witnesses and analyzes the results.
3.	Collaboration to create a giant paper Sierpinski triangle for the classroom wall; Computation of the number of triangles at each stage.
4.	Collaboration to create a giant Sierpinski tetrahedron.
5.	Discussion of self-similarity.
6.	Worksheets for thinking about why The Chaos Game works.
7.	Coloring Pascal's triangle to make the Sierpinski Triangle.
8.	Introduction of contractions and rotations into The Chaos Game, and the use of fractals in storing and generating pictures.
Day 2	
1.	Play Fractalina (computerized version of The Chaos Game) on the computer.
2.	Play The Chaos Game (target version) as a class, on the giant Sierpinski Triangle.
Day 3	
1.	Play The Chaos Game (target version) in pairs on the computer.

Sources for material are the NCTM's Fractals for the Classroom publications [2] and Bob Devaney's home page at Boston University *(http://math.bu.edu/DYSYS/dysys.html)*, which has Java computer applet versions of the games.

Statistical Sampling

Using materials from the estimation section of the Challenge of the Unknown video and activity book (Norton & Company), this workshop introduces a variety of strategies for estimation. The meatiest of these is statistical sampling, which is introduced via a shark population example. Students emulate the approach indicated in the video using a pail of goldfish crackers. We structured this curriculum for a two-hour workshop.

Statistical Sampling Curriculum	
in auditorium	
1.	Introduction using envelope/winner experiment where the group tries to estimate how many people in the room received "winner" cards in their envelopes; discussion of uses of estimation in students' lives and in the larger world.
in individual classrooms	
2.	Numerical calculation warm-up (from book).
3.	Grocery bill totaling example.
4.	Discussion of how to estimate the number of M&Ms in a jar, and submission of estimates.
5.	Using a grid to estimate the crowd size at the Million Man March.
	(snack break--estimate the cost of the food for the event)
6.	Tag and Recapture statistical sampling worksheets and goldfish activity (from book).
7.	Gathering data for eye color sampling experiment; use the classroom proportions to estimate the proportions among all participants.
in auditorium	
8.	Collect totals for eye color experiment and compare classroom predictions to the actual value.

CT Scans

This topic was spread over several days in our summer YES program, and concluded with a visit to the radiology department of a hospital to see the CT scanners in use. It combines materials from the New Tools for New Technologies video and activity book set [3] published by COMAP (800 77C-OMAP), a May 1996 article in the Mathematics Teacher on "Medical Applications of Systems of Linear Equations"[4], and a Newton's Apple episode on 3D MRI brain scans.

CT Scans Curriculum	
1.	New Tools... video introduces the problem of reconstruction of information.
2.	Graph theory problem: Students try to reconstruct a graph from subgraphs (from New Tools... activity book).
3.	Discussion: interpretation of solutions to a set of linear equations; using linear equations to model the CT scan reconstruction problem (from article).
4.	Students work on activity sheets from article.
5.	Visit local hospital for lecture by a physicist and tour of the radiology department.
6.	View video on 3D MRI brain scans.

Platonic and Archimedean Solids

This is one of our longest-running workshops, based on The Platonic Solids video and The Platonic Solids Activity Book [5] from Key Curriculum Press (800 995 MATH). The KaleidoTile software may be downloaded from the NSF Geometry Center's web page at *http://www.geom.umn.edu/software/download/*. Materials for use with the software were prepared by teachers at the Center, Summer 1995.

Platonic and Archimedean Solids Curriculum	
in auditorium	
1.	As students arrive, they begin to cut out two-dimensional nets that they will use later to build a cube and an octahedron that fit together, demonstrating duality.
2.	Using activities P1 and P3 in the activity book, students deduce a method for computing interior angle measure for regular polygons.
in individual classrooms	
3.	Using Polydron manipulative pieces or heavy paper, students work in groups to build regular polyhedra, i.e., polyhedra that satisfy the properties that (a) all faces are regular polygons, (b) all faces are the same, and (c) each vertex has the same number of faces coming together; each group might be assigned a particular number and type of face for its vertices, e.g., restricting its solid to have exactly 3 pentagons at each vertex.
4.	Based on the angle sizes determined in the introductory session, the class discusses why there are exactly five Platonic solids.
5.	Discover duality by filling in a chart of the number of faces and vertices for the Platonic solids.
6.	Optional, if time allows: Build other symmetric polyhedra, as described in activity 3 of the activity book.
7.	Watch The Platonic Solids video.
8.	With KaleidoTile software, examine relationships and transformations between some of the Platonic and the Archimedean solids. Sample activities are described in the KaleidoTile materials.

Shape of Space

In this workshop, we begin to construct a view of four-dimensionality and of spaces that are not Euclidean. Of course, we are restricted to looking at these from a three-dimensional perspective, so we do this by analogy with the way that a three-dimensional world would appear to a two-dimensional creature. We use soap bubbles on plastic straw and paper clip models of tetrahedra and cubes to try to visualize four-dimensional extensions of these objects. Other activities are based around Jeff Weeks' video, "The Shape of Space." Review copies of this video are available on loan by contacting the NSF Geometry Center at the University of Minnesota, (612-626-0888) or admin@geom.umn.edu. The activities, such as playing the game of Dots on a torus, are collected in a set of materials written by teachers in Summer 1995 at the Center. The Shape of Space software created by Jeff Weeks is available on the internet via *http://www.geom.umn.edu/docs/forum/weeks_software*.

Symmetry

One special workshop that combines a significant amount of existing materials with a substantial component of our own ideas is our four-hour workshop on symmetry. The new ideas, offering additional approaches to the deep content of the existing materials, have developed naturally from using the existing materials and from doing additional reading.

Symmetry Curriculum	
1.	Introductory lecture/slide show on the basic types of mathematical symmetry and the 7 possible band patterns.
2.	Using Kali software (freeware) to see how students' initials look in the 7 band patterns.
3.	Making a chart of symmetries for regular polygons.
4.	Constructing paper snowflakes that exhibit 4-, 5-, and 6-fold rotational symmetry: What folding schemes are required?
Snack Break	
5.	Choreographing movements to show transformations between symmetry types.
6.	Reflecting and translating in xy-coordinate systems.
7.	Cutting pictures out of magazines that exhibit symmetry and identifying the types of symmetry.
8.	Discussing and summarizing *Why is this mathematics?*
9.	Viewing video[1] about reflection in 2 and 3 dimensions.

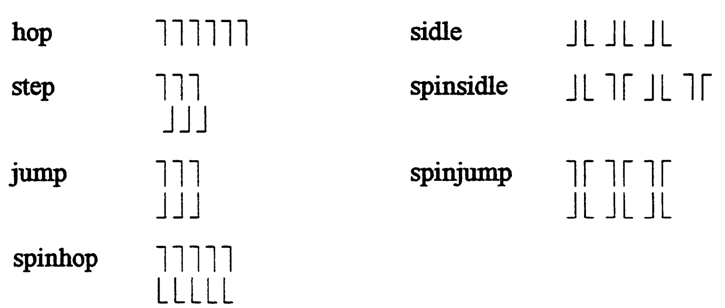
[1] The video shown is one produced by Anna Gardberg at the NSF Geometry Center, called "Mad About Mirrors." Unfortunately, it is not currently available to the public.

These activities were assembled for Project PRIME, and thus include some movement exercises that may be more popular with certain age groups than others.

Students who attended the symmetry workshop brought home their snowflakes and all of the worksheets that were distributed. Most of the motivation for these activities came from two sources: (1) a book called *Symmetry: A Unifying Concept* (Hargittai and Hargittai, Shelter Publications) [6], and (2) curricular materials developed by the NSF Geometry Center to go with its Kali software *(http://www.geom.umn.edu/locate/lori/kali/)*. The software is downloadable for free from the Center's web site *(http://www.geom.umn.edu/software/download/)*.

The movement choreography addressed two main themes. The first was experimentation with reflection and rotation symmetry, in pairs and circles of students. Following oral instructions, students formed reflective poses that assumed symmetry planes in locations specified by the leader. They made designs with k-fold rotational symmetry and stepped their designs around from equivalent position to equivalent position, until returning (after k changes) to the original position.

The second theme was that of the seven symmetry band patterns possible for a horizontal pattern of a repeated figure accomplished only by a combination of translation, relection, glide reflection, and half-turn transformations. These are translation ("hop"), translation and horizontal reflection ("step"), horizontal reflection and translation ("jump"), translation and two-fold rotation ("spinhop"), translation and vertical reflection ("sidle"), two-fold reflection and vertical reflection ("spinsidle"), and vertical and horizontal reflection ("spinjump"). See the figure below.



Each student in a group of five made a colored arrow on an index card. The group then chose a starting and ending symmetry band pattern for its arrows and planned a sequence of movements to change from one to the other using the transformations that were discussed. The groups performed these for one another, holding their arrow cards up for all to see.

The Binary Number System and the game of NIM

This material was spread over several days of our YES summer program, concluding with a presentation by a University of Minnesota professor about strategies for the game of NIM.

The Binary Number Curriculum	
in individual classrooms	
1.	To introduce the binary representation for the numbers 0-16, Doug Shaw conceived the idea of using student volunteers to form a human demonstration of a ripple counter. (Details of this activity are provided at the end of this section.)
2.	For each number 1-13, student groups receive a strip of cardstock; the students use the strip to make four equal-sized cards for that number, two labeled with the decimal representation, and two with the binary representation; decimal representations should be written in one color, binary ones in another, and the numbers should be underlined to indicate the bottom.
3.	Students play the familiar card game of "concentration," in which they try to locate pairs of cards bearing the same number; a number will be considered the same whether its decimal representation or its binary representation is shown.
4.	(Optional: binary arithmetic activities)
5.	Introduction to the game of NIM via playing the computerized version on the University of Toronto Mathematics Network's web page at *http://www.math.toronto.edu/mathnet/games*
in auditorium	
6.	Presentation of strategies for winning NIM, using binary number representations.

Human Binary Number Ripple Counter: A binary counting exercise developed by Assistant Professor Doug Shaw, School of Mathematics, University of Minnesota.

Human Binary Number Ripple Counter Exercise	
1.	Choose four student volunteers who are good at following directions, and invite them to the front of the class. They should sit in a row of chairs, facing the other students.
2.	There are two possible states for each student: sitting or standing. Define a "change of state" to be the action of moving from sitting to standing or from standing to sitting, depending on a person's starting state. (For example, if she/he is sitting, a change of state will be accomplished by standing up.) All students should start out in the sitting state.
3.	Students will need to change states in a prescribed way when the class claps hands: • The student on the right (from the class' point of view) will change state on every clap. • The one next to her/him will change state whenever the first one <u>sits down.</u> • The third will change state whenever the second <u>sits down</u>. • The fourth will change state whenever the third <u>sits down</u>. • Who will be changing most often? Who least often?
4.	Have the class clap together at a very slow rate (especially at first!), and have the first two people practice changing state. Start over and try it with the third. Start over, including the fourth, and continue until all four of them are seated again. This will take 16 claps. Try it again, a little faster, to make sure they have it. On the next time, return to a slow speed, but instead of clapping have the class count aloud. At "one" the first stands. At "two" the first sits and the second stands. At "three" the first stands and the second remains standing. At "four" the first sits, the second sits, the third stands. (If the students catch on quickly to the movements, another four can switch in for this counting section. Middle school students may find this difficult to coordinate, though.)

5.	Suggest that the name of the starting state, when they were all seated, will be 0. • Ask when the first person stood for the first time (1). • Ask when the second stood for the first time (2). • Ask when the third stood for the first time (4). • Ask when the fourth stood for the first time (8). • Ask when they all sat. (It was on the count of 16, but this was really the 0 state again.)
6.	After the discussion, have each one hold a card with a number on it. (The numbers, left to right, will be 8, 4, 2, and 1.) Now start counting with state changes again, but on "six" say "STOP". Note that the 4 and the 2 are standing, and 4+2=6. Let them start again, and stop them on 13. Note that 8+4+1=13, and 8, 4, and 1 are standing. • What is the biggest number that can be represented this way? (8+4+2+1=15). • What would be a natural way to represent 16? (An extra person on the left side!)

Graph Theory: Proof by Counterexample

This workshop was inspired by problems on the University of Victoria's Mathmania web page (*http://www.csc.uvic.ca/~mmania*). The progression of activities allows students to learn some vocabulary of graph theory representations, while exploring the notions of conjectures and counterexamples. With the workshop as preparation, they can then move to working independently to create new counterexamples that can expand the understanding of the as yet unsolved degree/diameter problem described on the web site.

The logic to be digested in these problems is in the statement of the conjecture. Once students understand what the conjecture says, the method employed here is simply to find an example of a graph that contradicts the proposed conjecture, thereby disproving it directly.

Graph Theory Curriculum	
1.	Introduction to graph theory vocabulary; after introducing edges, vertices, and the definition of a *k*-regular graph, this sheet poses the first opportunity for students to try to disprove a conjecture by finding a counterexample.
2.	Introduction to cliques and independent sets for Ramsey Theory; this provides two more opportunities for students to devise a counterexample to a conjecture.
3.	Disproving a conjecture in matching theory.
4.	Definition of degree and diameter, and statement of the degree-diameter problem; in class or on their own, students can work on finding a planar graph with diameter 3 and degree 3 larger than those previously discovered.

Visit our web site (*http://www.math.umn.edu/itcep/*) for additional examples and activities from our programs.

Evaluation and Assessment Process

In order to assess the impact of the Project PRIME and Project YES enrichment activities, to analyze if we are meeting our primary objectives, and to gain useful data for refinements and future directions, student and parent surveys are used regularly to collect both qualitative and quantitative data.

Student attitudes about the concepts introduced at each workshop and their interest in the topic/s are measured by two to three open-ended questions on the survey. The questions are written by the curriculum developers and cite specific items from the lessons. For example, the Statistical Sampling Workshop survey asks the following two questions: "What part of the estimation problem did you find most interesting?" and "What did you learn from the Tag and Recapture Methods activity?"

On each survey, students are asked to rate their overall reaction to the workshop on a scale of 1 to 5: 1 no enjoyment, 2 little enjoyment, 3 neutral, 4 fun, and 5 great event. Demographic questions asking the students to list their gender, grade level, and reason for attending the event, (i.e., I wanted to attend, my parents wanted me to attend, and other) are also included on each survey.

Parent surveys are given at the end of each family activity and at the final event of the academic year. The questions probe whether or not the parents' expectations of the activities were met, to what extent their child shared his/her experiences with the family following the workshops, and what they felt their son/daughter gained from the experience.

The identical survey questions (except those that relate to a specific lesson) have been used for the past three years in order to enable us to do a longitudinal study. Those that are related to a specific lesson use the same format and are placed in similar context. After the results are collected and entered into our database, statistical reports are generated and descriptive analyses are written. These reports help us to assess the impact of the workshops and programs and to plan for the future.

Conclusion

The University of Minnesota Talented Youth Mathematics Program, which offers a nationally recognized rich and challenging curriculum, continues to serve 175 to 200 of Minnesota's most mathematically talented middle school students each academic year. However, a wider range of middle school students and their parents, teachers, and school coordinators has consistently expressed a strong interest in similar mathematics enrichment opportunities. To serve this audience, and to capture and maintain the interest of middle school students who enjoy mathematics at a variety of levels, we have developed and implemented several successful intervention programs. Over 350 middle school students participated in either Project YES or Project PRIME activities during the summer of 1996 and the 1996-97 academic year. Over 400 of these students, their family members, and teachers participated in our 1997 annual math fair, a one-day event of mathematical enrichment activities.

The activities that we developed for these programs sought 1) to broaden middle school students' understanding of mathematics by introducing topics that go beyond the standard curriculum, 2) to encourage meaningful insights, and 3) to provide opportunities for the students to become a part of the mathematics community. Several student evaluations have confirmed that our goals are being reached. For example, in response to the survey statement: *List the most interesting thing you learned today*, student comments ranged from "I learned the difference between Archimedean and Platonic solids" to "I learned how estimation is a very useful tool and works much faster than counting everything" to "I learned about how mathematics is not just numbers, and a lot about its daily uses."

In summary, we encourage all teachers to consider using our curricula as a model for developing engaging activities to build on middle school students' natural interest in the exploration of mathematics and its applications. The results can be very rewarding. To quote a parent's response to a Project YES survey, "Activities like these offer the opportunity to brainstorm, to experiment, and to create. The program was a fertile environment where ideas were exchanged and younger students got to experience a *creative* environment without stress or pressure. It was an ideal opportunity for youngsters to follow their mathematical ideas and see that doing so was worthwhile." ■

References

[1] *CBL™ System Experiment Workbook*, Texas Instruments, 1994.

[2] H. Peitgen et al., *Fractals for the Classroom*, Springer-Verlag, New York, 1992. Published in cooperation with the National Council of Teachers of Mathematics.

[3] J. Malkevitch, *Geometry: New Tools for New Technologies*, COMAP, 1993.

[4] D. Jabon et al., "Medical Applications of Systems of Linear Equations", *Mathematics Teacher*, **89** (5), 398-402 (1996).

[5] A. Fetter et al., *The Platonic Solids*, Key Curriculum Press, 1990.

[6] I. Hargittai and M. Hargittai, *Symmetry: A Unifying Concept*, Shelter Press, 1994.

STUDENT WORK SECTION

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Editor's Note: *The Journal will regularly feature samples of student work of a nature different from that traditionally expected in college courses. Students frequently become more engaged in the study of mathematics and science when they are given the opportunity and encouragement to place the mathematics or science that they are studying into a broader context, and to bring to bear their individual outside interests and abilities. Such is certainly the case in Contemporary Mathematics, a course designed for non-science majors - particularly prospective K-8 teachers - and currently being developed/refined by the Virginia Collaborative for Excellence in Teacher Preparation. In addition to completing traditional hour exams and quizzes, all 850 students currently enrolled in sections of this course are expected to complete two large projects and complete a substantial number of writing assignments, including two major papers in which they are required to describe the mathematics that they have studied to a non-technical reader.*

The following student paper is an example of such a paper; it accounted for 5% of the grade in the course. The author was a freshman Humanities and Science major who was, in our view, able to combine her interest in fictional writing with a clear exposition of the mathematics that she has learned.

SOLVING PROBLEMS INVOLVING HAMILTON CIRCUITS

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With a disturbing look of confusion, he sits at his desk, contemplating over the trip he must soon take. "Where do I start?" he questions himself. Sipping on a stale cup of black coffee, he looks down his nose through his glasses at the map resting upon the hard oak. Gray smoke rolls from his mouth, dissipating into the late night air after each drag of his cigarette.

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He thinks to himself, "I have four cities to visit in order to deliver my vacuums to my customers, yet I must stay in range of my budget when thinking of air fare. I don't want to pick a wrong route and end up paying far too much than I have to. Plus, I do need to return home so I can show up at the sales room on Monday. Oh, where should I begin? How do I go about solving this problem?"

"Well, my friend," a deep, radio-commercial voice rings from behind the confused salesman's head.

"Who's that?" the salesman shrieks, quickly turning his head to discover a tall, heavenly figure before him. The crisp, yet cheap white suit and peculiar fake gold ring floating above his head gives the salesman the notion that the unexpected guest seems a bit frugal.

"I overheard your complaints," the pale dressed man speaks. "Little did you probably know, I am your guardian salesman. Yes, guys in the sales business have them, too. Anyway, I thought this would be a perfect time for me to introduce myself personally and give you some helpful advice concerning your conflict."

Removing his glasses and rubbing his weary, disbelieving eyes, the salesman takes a closer examination at the character claiming to be his guardian angel. "Of course you can understand that I really don't believe a word you say, but if you have any suggestions, I'm up for them," the salesman shares.

"Alrighty then, make yourself comfortable and pay attention, I'm only gonna explain this once." With one stern look at the dingy wall, a blackboard appears. The tall angel struts to the board and begins his lecture.

"First, we need to address that the route you wish to take is called a Hamilton circuit. These circuits begin and end at the same vertex or point while visiting each exactly once. In order to reduce the amount of strain in finding the best route, you need an organized plan. I plan to show you the different procedures in which to go about solving your problem. Remember, each of these algorithms or in other words, easy processes, will solve your dilemma, yet they differ in efficiency and accuracy, plus you must follow the procedure precisely. It remains your decision to choose the suitable algorithm after you learn them." [1]

"I shall first explain the Brute Force algorithm, which allows you to view each Hamilton circuit and choose the most satisfactory. This trial and error process consists of the following rules:

1) List all of the Hamilton circuits, beginning and ending with the same vertex.
2) Add the variables or weights of each edge in each circuit.
3) Review all of the circuits and choose the optimal circuit, the one with the minimal combined weight. The optimal will result as the best answer.

Now let's apply this to your particular problem as I write it on the board. We first know that you have a total of 24 possible answers, since the first and last points are already designated as A or home. The first place to visit after leaving your hometown has four possibilities-- B, C, D, or E. The second stop only has three choices of cities due to the elimination of the first city (you don't want to backtrack). The third visit only has two selections, while the last city must take the leftover city. By multiplying four, three, two, then one, we end up with 24 possibilities. We must take in consideration duplicates, so we should finally divide by two and end up with 12 non-repeating sequences.

Now that you know the number of circuits you must review, let's list a few of them with their corresponding weights:

ABCDEA	500+305+320+302+205 = 1632 dollars	(dollars represent
ACBEDA	200+305+340+302+185 = 1332 dollars	air fare costs)
ADBCEA	185+360+305+165+205 = 1220 dollars	

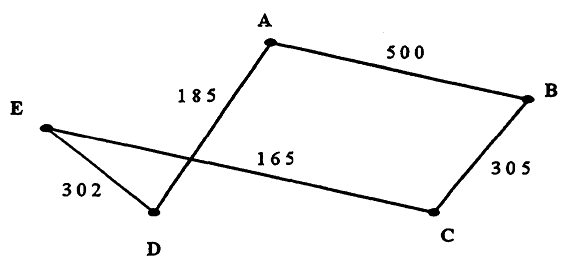
After repeating this process for every Hamilton circuit, find the one with the least numerical value. From the above circuits, and consequently all of the possibilities, the route ADBCEA proves to be the optimal choice.

While the Brute Force algorithm may produce the optimal circuit, it remains very time consuming when having to list every possible route. Not only does time pose a problem, it leaves room for error in naming each Hamilton circuit and in the arithmetic; but don't despair, I still have a few more tricks.

My second algorithm I will introduce is the Nearest Neighbor algorithm. Don't worry, Mr. Rogers didn't make this one up, it belongs to some qualified mathematician. This procedure enables you to find a possible Hamilton circuit in less amount of time. Now take a minute to study the procedure.

1) Start the circuit at the home vertex.
2) Pick the next point with the smallest weighted edge connected.
3) If a tie exists between edges, pick one at your discretion. Follow this rule until you've crossed each vertex exactly once.
4) Return to the home vertex.
5) Lastly, add the weights of the edges chosen.

After this process, you conclude with a route and its corresponding weight. This algorithm may seem perfect at first, but it contains a flaw in which to consider. Yes, it shortens the time required to find a route, but it does not guarantee an optimal solution. For example, the solution found through the graph of your routes shown doesn't agree with the optimal we found with the Brute Force algorithm.

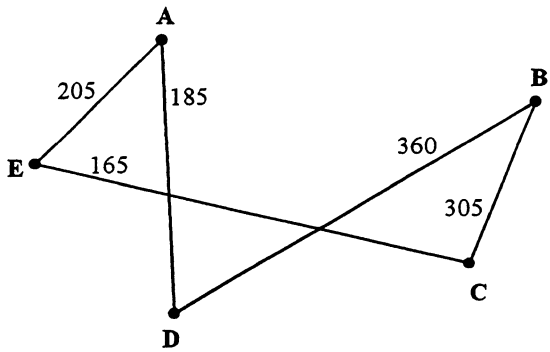


A-D-E-C-B-A 185+302+165+305+500 = 1457 dollars in air fare

Again, I want to describe yet another algorithm which you might find a bit more enticing to your needs-- the Repetitive Nearest Neighbor algorithm. Much like the Nearest Neighbor algorithm, this process allows you to reduce the time required to find a route and area for mistakes. It takes more time than the Nearest Neighbor algorithm, but still remains significantly more efficient than the Brute Force algorithm. Instead, it offers more choices of routes to choose from than the Nearest Neighbor algorithm, which in turn proves more accurate. Now listen close to the guidelines while applying them to your dilemma.

1) Pick a vertex and carry through with the Nearest Neighbor algorithm.
2) Continue this process with each vertex as the start and end.

3) After adding the weights of each circuit, choose the one with the smallest sum.
4) Rewrite the circuit so that the starting point is the necessary vertex.



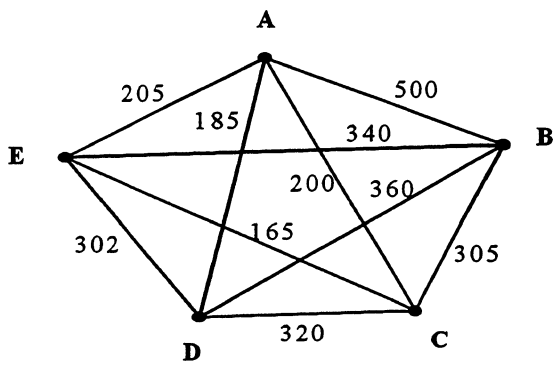
A-D-E-C-B-A = $1457
B-C-E-A-D-B = $1220
C-E-A-D-B-C = $1220
D-A-C-E-B-D = $1250
E-C-A-D-B-E = $1250

We find that the best solutions are the circuits starting with B and C, which consequently matches the value of the optimal found with the Brute Force algorithm. Look closely at the two solutions above, they have the same sequence, except for the beginning and ending cities, therefore they prove as the same circuit. Now we must rewrite the sequence so that it starts and finishes with A or your hometown-- A-D-B-C-E-A. Even if the route chosen does not start with the appropriate vertex, you can always pick up the circuit and follow it through, until you reach home again.

Lastly, I give you another algorithm, the Cheapest Link algorithm. Like the Nearest Neighbor and Repetitive Nearest Neighbor algorithms, it may not always result in an optimal circuit. It saves time in discovering a solution, and lessens the risk of human error. Unlike the Nearest Neighbor and Repetitive Nearest Neighbor algorithms, the Cheapest Link algorithm allows you to pick the edges with the minimal weights, without continuing a circuit from one point chosen to the next. For instance, the first edge picked may rest at one end of the graph while the second lies at the opposite end with no connection between the two.

Before explaining the procedure, I first must state that the Cheapest Link algorithm contains two very crucial rules in which to follow while performing the procedure. You cannot close a circuit while choosing edges until you cross each vertex once, or have three or more edges coming from one vertex. If you do not follow these rules, your resulting route cannot possibly satisfy the definition of a Hamilton circuit, therefore the circuit contains irrelevant edges and weights. If the concluding graph contains a smaller closed graph within itself, it either doesn't cross every vertex or must have three or more edges coming from one point. A Hamilton circuit can only cross each vertex once, if it has edges with the degrees of three or above meeting a single point, then it breaks the rule an a Hamilton circuit. Now, remember these rules while we review the procedure.

1) Choose the edge with the smallest weight, if a tie, decide at random.
2) Find the next smallest weighted edge and continue until you complete a Hamilton circuit.
3) Calculate the weights chosen to find the total value of the circuit.

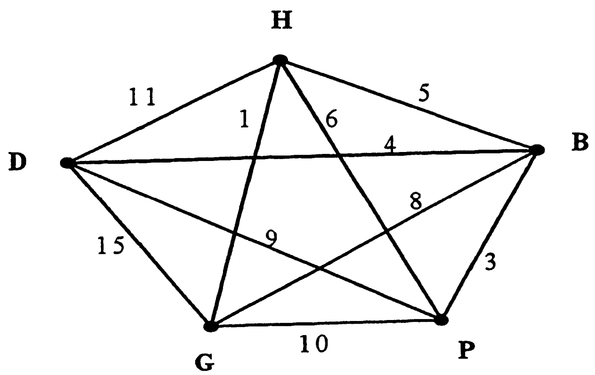


We can write the solution two ways: A-C-E-B-D-A or A-D-B-E-C-A. No matter what the order, the sum of the weights remain the same: 1250. The result of the Cheapest Link algorithm upon this problem varied from the optimal circuit. This proves that this procedure does not consistently offer the optimal solution, yet its efficiency in time and simplicity makes this algorithm a definite consideration when choosing a plan to find a Hamilton Circuit.

Not only do salesmen need the assistance of algorithms, other people such as families, students, and the bus driver may benefit from these procedures in their everyday lives. For example, a mother needs to make several errands in one day while traveling the least possible distance in order to save gas and time. After leaving home, she needs to visit the bank, post office, grocery store, K-mart, and drug store in no particular order before returning home. Let's explore the four algorithms to find solutions to this problem and determine which procedure proves most suitable.

Brute Force Algorithm:	H-B-P-G-D-H	5+3+10+15+11 = 44
	H-B-P-D-G-H	5+3+9+15+1 = 33
	H-B-G-P-D-H	5+8+10+9+11 = 43
	H-B-G-D-P-H	5+8+15+9+6 = 43
	H-B-D-P-G-H	5+4+9+10+1 = 29
	H-B-D-G-P-H	5+4+15+10+6 = 40
	H-P-B-G-D-H	6+3+8+15+11 = 43
	H-P-B-D-G-H	6+3+4+15+1 = 29
	H-P-G-B-D-H	6+10+8+4+11 = 39
	H-P-D-B-G-H	6+9+4+8+1 = 28
	H-G-B-P-D-H	1+8+3+9+11 = 32
	H-G-P-B-D-H	1+10+3+4+11 = 29

Optimal solution: H-P-D-B-G-H



Nearest Neighbor Algorithm: H-G-B-P-D-H 1+8+3+9+11 = 32

Repetitive Nearest Neighbor Algorithm:

H-G-B-P-D-H	1+8+3+9+11 = 32
B-P-H-G-D-H	3+6+1+15+11 = 36
P-B-D-H-G-P	3+4+11+1+10 = 29
G-H-B-P-D-G	1+5+3+9+15 = 33
D-B-P-H-G-D	4+3+6+1+15 = 29

Final Circuit: H-G-P-B-D-H

Cheapest Link Algorithm:	H-P-B-D-G-H	6+3+4+15+1 = 29
	H-G-D-B-P-H	1+15+4+3+6 = 29

With the Brute Force algorithm, we find the optimal circuit, yet it does not appear as the result of the other three algorithms. This alone shows that the Nearest Neighbor, Repetitive Nearest Neighbor, and Cheapest Link algorithms do not guarantee the optimal solution. Each procedure constitutes a Hamilton circuit, yet they vary in sequence and weight. The Brute Force's inefficient, bothersome process offers the optimal solution, yet some people prefer a faster procedure and sacrifice the difference in the weight of a non-optimal solution to the optimal. If the problem can produce a realistic number of possibilities, the Brute Force Algorithm could be used, but not every dilemma contains few enough vertices to make the application of this algorithm possible. When more involved graphs are in play, the Nearest Neighbor, Repetitive Nearest Neighbor, and Cheapest Link algorithms are needed."

"Now that I've thoroughly explained the algorithms available to you in order to solve your travel problem, which will you choose?" the angel questions as he places his chalk in the tray and glances at the salesman. With a deep sigh, the heavenly figure regains his breath from the tiresome lecture and walks over to the salesman. A slumped figure sits in the wooden chair with his forehead resting on his arms, asleep on his desk. With a caring hand and thoughtful mind, the angel takes a piece of paper and rewrites the algorithms on the paper for the salesman to refer upon. On another sheet of paper, he writes the optimal solution to the "Traveling Salesman Problem", then disappears, leaving a small cloud of sparkling dust suspended in the air and helpful tips for the salesman. ■

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[1] P. Tannenbaum and R. Arnold, *Excursions in Modern Mathematics*, 2nd Ed, Prentice Hall, Englewood Cliffs, NJ, 1995.

EDUCATIONAL RESEARCH ABSTRACTS

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Editor's Note: *The purpose of this Educational Research Abstract section is to present current published research on issues relevant to math and science teaching at both the K-12 and college levels. Because educational research articles are published in so many different academic journals, it is a rare public school teacher or college professor who reads all the recent published reports on a particular instructional technique or curricular advancement. Indeed, the uniqueness of various pedagogical strategies has been tacitly acknowledged by the creation of individual journals dedicated to teaching in a specific discipline. Yet many of the insights gained in teaching certain physics concepts, biological principles, or computer science algorithms can have generalizability and value for those teaching in other fields or with different types of students.*

While the teaching topic or instructional strategy will always guide the selection of journal articles presented, there are certain academic journals that typically publish research-based articles or practitioner experiences dealing with math and science teaching from elementary school through university level. If you are unfamiliar with any of the following journals, you may find perusing a recent issue to be a surprising -- and rewarding -- source of teaching and learning ideas.

- *Journal of College Science Teaching*
- *Journal of Research in Science Teaching*
- *Research In Collegiate Mathematics Education*
- *Teaching Children Mathematics*
- *Journal For Research In Mathematics Education*
- *The Mathematics Teacher*
- *The College Mathematics Journal*
- *The American Biology Teacher*
- *Computer Science Education*
- *Journal of Chemical Education*
- *Physics Teacher*
- *Physics Education*

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- *American Journal of Physics*
- *Journal of Geological Education*
- *Journal of Geoscience Education*
- *Journal of Science Education and Technology*
- *Teaching PreK-8*

In this first review the focus is on "active learning." Abstracts are presented according to a question examined in the published articles. Hopefully, such a format will trigger your reflections about exemplary math/science teaching as well as generate ideas about your own teaching situation. The abstracts presented here are not intended to be exhaustive, but rather a representative sampling of recent journal articles. Please feel free to identify other useful research articles on a particular theme or to suggest future teaching themes to be examined. Please send your comments and ideas via email to gmbass@facstaff.wm.edu *or by regular mail to The College of William and Mary, P.O. Box 8795, Williamsburg, VA 23185-8795.*

Learning Mathematics And Science Through Active Learning

In recent years national professional organizations have developed new curriculum and teaching standards for mathematics and science education. The National Council of Teachers of Mathematics in 1989 provided specific standards that emphasize active learning. "First, 'knowing' mathematics is 'doing' mathematics. A person gathers, discovers, or creates knowledge in the course of some activity having a purpose....instruction should persistently emphasize 'doing' rather than 'knowing that.'" With respect to student activities, the NCTM standards used two principles to guide their descriptions. "First, activities should grow out of problem situations; and second, learning occurs through active as well as passive involvement with mathematics." Finally, the standards provide specific examples of "active learning" in math. "This constructive, active view of the learning process must be reflected in the way much of mathematics is taught. Thus, instruction should vary and include opportunities for--

- appropriate project work;
- group and individual assignments;
- discussion between teacher and students and among students;
- practice on mathematical methods;
- exposition by the teacher."

The full set of NCTM standards can be found on the World Wide Web at the Eisenhower National Clearinghouse for Mathematics and Science Education (ENC) *http://www.enc.org/reform/journals/ENC2280/nf_280dtoc1.htm.*

The National Science Education Standards published in 1995 also continue this theme of active learning. "Student understanding is actively constructed through individual and social processes." From this key principle specific teaching standards were developed, among them Standard A: "Teachers of science plan an inquiry-based science program for their students." Consistent with the NCTM standards, the National Science Education Standards emphasize that "inquiry into authentic questions generated from student experiences is the central strategy for teaching science." The full text of the National Science Education Standards can be found on the World Wide Web at National Academy Press site *http://www.nap.edu/readingroom/books/nses/html/*

Does the use of active learning strategies enhance K-12 and college students' learning? As more teachers consider utilizing "active learning," it becomes critical to understand the issues involved in such an instructional strategy. What kinds of instructional objectives are best facilitated through active learning? When using active learning with various mathematical and science topics, what factors should a teacher consider? What have been the conclusions of empirical research studies examining active learning in the classroom? The following set of articles provides a representative sample of recent academic writings on the subject of active learning approaches.

- ***How well does active learning work in an introductory college biology course?***

A new two course sequence in introductory biology was developed to emphasize experience-based group learning rather than traditional lectures. This redesigned course required groups of students to work in project groups on topics such as the design of a closed life-support system for long-term space flight or the design of a unicellular organism to colonize a fictitious planet. An independent investigator examined two different groups of students (49 and 40 students respectively) who took these courses. He measured their attitudes and subsequent performance in advanced biology courses over a 15 month period. The authors report that many students would have preferred traditional lectures on traditional biology topics. However, they also acknowledged that this new project approach had increased students' interest in learning more biology and in investigating a wider array of biologically related problems. They also found differences in student preferences for working on projects and working in groups between the two classes of students studied. Follow-up interviews discovered that as biology majors preceded with their education, the value of project-oriented instruction became more evident to them. No differences were found on grades in three advanced biology courses when students who had taken the project biology course were compared to other students who had taken a more traditional introductory biology course. The authors conclude that this project-oriented approach requires a substantial time and effort commitment from both faculty and administrators, but that this investment is necessary if the process of creative scientific problem-solving is to be developed in students.

L. Goodwin, J. E. Miller & R. D. Cheetham, "Teaching freshman to think - does active learning work?", *BioScience*, **41 (10)**, 719-722, (1991).

- ***How can group activities encourage active learning in an introductory college chemistry course?***

John Frey describes an instructional approach for his introductory chemistry class in which he required teams of two or three students to complete several written assignments throughout the semester. These "homestudy" assignments were developed to encourage both cooperative group behavior and written presentations of the group's ideas. For example, sample questions in the first homestudy assignment included "What is the difference between a theory and a law?" and "What is meant by the term 'metalloid' and which elements are called metaloids?" Sample questions from the second assignment included "Compare kinetic energy and potential energy." and "What is the difference between an ion and a molecule? Give examples of each." Typically, these questions represented topics covered in the textbook, but eliminated from Frey's lectures so that he could cover more complex topics more slowly or allow more in-class discussion and group activities. Anonymous end-of-course student evaluations of these group assignments revealed that 94% of the students found the assignments helpful (either as motivation to read the text, guides for better understanding of broad concepts, or preparation for the course exams). However, 17% of the students felt that working in small groups was a bad idea while 67% expressed positive attitudes to the group work. Frey concludes that the use of homestudy group assignments can be an effective way to cover necessary course topics while at the same time freeing classtime for challenging activities that stimulate more active learning.

J. T. Frey, "Homestudy assignments: An experiment in promoting active learning in introductory chemistry", *Journal of College Science Teaching*, **26 (4)**, 281-282, (1997).

- ***To what degree does the use of student journals encourage more active learning in a high school physics course?***

In an attempt to stimulate more active rather than passive learning, a high school physics teacher implemented a mandatory journal in his college preparatory physics classes. K. David

Pinkerton calls his approach the Active Mental Processing Journal. It is a very structured, language-rich teaching technique that requires students to make three different journal entries per school day. These three daily entries include (1) class notes with personal embellishments that help them remember and understand key concepts; (2) a physics question related to the class topic, a newspaper article, or other daily event; and (3) an application of a physics concept mentioned in their journal, noting how it is an illustration of that concept. Pinkerton gives the students the last 2 to 4 minutes of each class to write the question and application in their journals. The journals are collected every three weeks and graded according to a specific set of criteria given to students. Pinkerton administered the 29-item Force Concept Inventory (designed by Hestenes, Wells and Swackhamer in 1992) as a pretest and posttest to measure the impact of this journal technique. The pretest-posttest change in the journal-oriented course was from a class mean of 8.3 to a mean of 19.1. Pinkerton's previous year's physics students, who took the course without the journal requirement, had scored a mean of 14.9 on the posttest administration of the Force Concept Inventory. He concludes that the use of this low-cost journal requirement is consistent with other active learning strategies that foster more "interactive engagement" on students' part. Pinkerton recommends teachers use such language-rich teaching techniques to foster active mental processing and help students internalize physics knowledge.

K. D. Pinkerton, "Low-tech solutions, high-tech results", *The Physics Teacher*, **34**, 30-34, (1996).

- ***What active learning techniques may promote independence and responsibility in organic chemistry?***

Because organic chemistry is often used as a "gatekeeper" course for further professional studies, a professor teaching that course is often faced with students who grudgingly take the course, but question the relevance of the material to their lives and career plans. Marlene Katz wanted to change this perception by designing a course that emphasized greater student ownership, active learning, accountability, and control. She created an organic chemistry course based on a Student-Directed Learning (SDL) philosophy that emphasizes the recognition of student affect and attitudes in learning, the need for support and incentives for struggling students, and the desirability of accessible and relevant course content. Katz reorganized course content around "Big Ideas" such as symbolic language, polarity, reactions, and mechanisms. These central themes were connected to specific skills that were taught in a hierarchy from simple to complex cognitive operations. She also rearranged classes from a traditional lecture to a "reverse Socratic" method where students initiate the teacher-student dialogue through their questions from assigned readings or lab skills. After this initial question/answer session, students choose from a menu of in-class activities aimed at developing the skills connected to the Big Ideas. Often these activities involve group work utilizing peer teaching with the professor being the "coach" who moves among the groups. Student assessment was done through a Mastery Learning strategy where alternative quizzes may be retaken until there are no significant errors in the student's work (thereby stressing the "process" of learning rather than the "performance" of learning.) During the four years she has used this student-directed technique, the average score on the ACS Organic Chemistry Exam has steadily increased to a class mean of 73 (compared to a class mean of 55 over the previous four years of the traditionally taught organic chemistry course.) More importantly, Katz reports the percentage of students scoring below the 50th percentile level has decreased from an average of 41% during the previous four years to less than 9% during the four years of the SDL classes. However, she acknowledges that some of the most valuable classroom innovations may not elicit warm responses from students. During her four years experience she admits to being dismayed at the unwillingness of many students to take over the direction of the class. She also was surprised at the negative responses of some students to their new role. Nevertheless, she concludes that students learning to take responsibility for their own learning can make all her frustrations and challenges worthwhile.

M. Katz, "Teaching organic chemistry via self-directed learning", *Journal of Chemical Education*, **73 (5)**, 440-445, (1996).

- ***How can partnerships between experienced teachers and disciplinary experts improve classroom instruction and student learning?***

Malcolm Wells was a high school physics and chemistry teacher who returned to Arizona State at the age of 49 to initiate doctoral research in physics. He had been a "hands-on" teacher for over two decades ever since his early experience with PSSC and Harvard Project Physics. He embraced a student-centered inquiry approach based on the *learning cycle* of Robert Karplus. Nevertheless, his students' performance on the Halloun and Hestenes Mechanics Diagnostic test was no better than students taught by traditional lecture-lab methods. Through his work at Arizona State, Wells was introduced to David Hestenes's theory of physics instruction with modeling. This modeling approach organizes physics content around a small number of basic models such as the "particle subject of a constant force." Students learn how models in physics are conceptual representations of physical objects and processes. They are introduced to representational tools such as force diagrams and motion maps that are essential for competent modeling and problem solving. The students also receive a detailed analysis of the procedural knowledge necessary for constructing scientific models. Wells adapted Hestenes's ideas into his own teaching by creating a modeling cycle which moves students through all aspects of model development, evaluation, and application in concrete situations. Instead of encouraging the "plug-and-chug" problem solving technique (plugging numbers in equations-and-chugging a little arithmetic), Wells and Hestenes take the position that every physics problem is solved by creating or adapting a known model to the specific problem. Evidence of the effectiveness of the modeling method was examined using the Force Concept Inventory and the Mechanics Baseline test as pretest and posttest measures. Students taught by teachers using traditional instruction were uniformly poor on those measures while students taught by teachers using the modeling method were dramatically better. Wells, Hestenes, and Swackhamer developed five-week summer workshops for introducing their modeling method to physics teachers. They concluded that teacher expertise is the critical factor in improving high school physics. They believe that teacher expertise can only be acquired through lifelong professional development during which the teacher draws on the resources of the physics community - specifically, partnerships between experienced teachers and physicists involved in educational research.

M. Wells, D. Hestenes and G. Swackhamer, "A modeling method for high school physics instruction", *American Journal of Physics*, **63** (7), 606-618, (1995).

- ***What role can physics projects play in helping high school students understand physics?***

In order to encourage her students to realize that "physics is everywhere," Joan Mackin decided to incorporate a more authentic way for her high school students to apply their knowledge. Her strategy was to require students to complete a project where they chose a physics-related topic (such as physics applications in other content areas, physics lessons for middle and elementary school, or student-developed experiments) and also chose the presentation method (such as the format of a poster display, oral talk, model, video tape, or lesson plan and also whether the project was an individual or group effort.) The projects were typically a semester long effort and equivalent to a regular test grade. The projects were judged according to the amount and accuracy of the research used and the quality of the presentation (pertinent information, interest level, and creativity.) Mackin believed these projects allowed students to explore areas of personal interest and also draw on their individual talents to produce the projects. Her evaluation of the success of this approach is that students will share their projects with others and talk about the projects with understanding. Yet she acknowledges that her efforts to teach physics in interesting and effective ways are more guided by intuition, student feedback, and classroom observation than by systematic collection of student learning evidence.

J. Mackin, "A creative approach to physics teaching", *Physics Education*, **31** **(4)**, 199-202, (1996).

- ***How could problem-based learning be used in a college biology course?***

Problem-based learning (PBL) is an instructional strategy requiring students to examine real-life problems, construct a knowledge base to address the problem, and justify the "best" solution with supporting evidence. Among the major features of PBL are (1) a focus on ill-structured, complex problems; (2) a commitment of several weeks to several months to work on a single problem; (3) the development of an entire curriculum around problems, not just an application of learned concepts; and (4) a change in teachers' roles as "cognitive coaches" guiding student inquiry rather than "knowledge dispensers." To implement PBL in an upper division biology course, Arámbula-Greenfield began by developing a list of problems that were ill-defined, complex, interesting, and required research data to validate a best course of action. For example, one of her problems identified the question of whether there were gender-related factors in the survival rates of patients undergoing treatment for heart attacks. She also determined that her students could access adequate research materials to analyze any of the problems they selected. She had her students work in teams of four or five students investigating one problem each six weeks of the semester. Most of the students' time is spent researching the topic outside of class. Her role was to support their efforts by providing feedback to each team on their research, analysis, and interpretation. In order to assess each student's understanding, Arámbula-Greenfield requires each individual to write a research paper reflecting his or her findings and each team to share their group findings with the rest of the class. Student evaluation of this PBL approach has been uniformly more positive on end of course evaluations than student evaluations in the previous traditional seminar course. Her experience with PBL lead her to conclude that it does not appeal to all students, especially those who prefer more competitive, abstract, and directed learning experiences. She also acknowledges that an instructor may view the new "cognitive coach" role as more challenging than the traditional role of "knowledge dispenser," She admits that students will acquire less breadth of knowledge with PBL which may make PBL inappropriate for some courses.

Because PBL is typically a new experience for students, they may become frustrated in trying to decide the amount of time and effort to be devoted to each problem. Nevertheless, Arámbula-Greenfield believes PBL can be an effective way for learning academic content, practicing independent learning, and applying critical thinking skills.

T. Arámbula-Greenfield, "Implementing Problem-Based Learning in a College Science Class", *Journal of College Science Teaching*, **26 (1)**, 26-30, (1996).

- ***What kinds of simple teaching devices can make active learning practical in the classroom?***

Believing that constructivistic, active learning approaches are necessary for effective student learning is not enough to make teachers change their traditional teaching techniques. These alternative, active learning techniques must also be usable in the everyday classroom. Richard Weisenberg, a college biology professor, proposes a simple yet effective way to allow students to engage in a variety of active learning activities. In his nonmajor biology classes he has been using "post-it notes" as low-tech instructional aids to create a variety of lessons. Teams of two or three students use the "post-it notes" to create concept maps of course concepts. Within the Learning Cycle method of A. E. Lawson (Exploration, Term introduction, and Concept application), Weisenberg has his students use the "post-it notes" for chemical modeling. He also uses them as models for chromosomes and genes and for any multi-step biological process such as glucose metabolism. Since one of the primary goals in the constructivist approach is to identify initial student understanding (or misunderstanding) in order to build new understanding, Weisenberg believes that familiar, concrete objects such as "post-it notes" can make a more visible link between students' existing concepts and new understandings. Using end-of-course evaluation questionnaires in the two courses where he used this technique, Weisenberg found students liked the group work especially the "post-it notes" activities. In a class of elementary education majors, over half commented they would use "post-it notes" in their own lessons when they became teachers.

R.C. Weisenberg, "Appropriate technology for the classroom - Using "post-it notes©" as an active learning tool", *Journal of College Science Teaching*, **26 (5)**, 339-344, (1997).

- ***What kind of inservice program can make hands-on science the predominant mode of instruction in elementary schools?***

Professional development programs for improving the quality of teaching science have generally focused on developing teachers' knowledge of science and creating more positive attitudes toward science. Observations in schools have often found that many teachers support the active learning goals of science education but have difficulty implementing those practices in their classroom. Therefore, an inservice program was developed to engage teachers in scientific inquiry rather than hear persuasive lectures on the importance of hands-on science. Over one academic year, twenty-eight elementary teachers were trained to be "mentor teachers" in the use of hands-on science investigations. During the subsequent summer these mentor teachers created 72 hands-on science lessons, each one identified for a specific K-5 grade level. Three Saturday workshops (a total of 20 hours) were held at each elementary school in the district led by these mentor teachers. At each of the workshops teachers worked in small groups on a variety of hands-on investigations appropriate for their own students. To evaluate the effectiveness of these mentor/colleague workshops, a teacher questionnaire was administered prior to the first workshop participation and as a posttest upon completion of the final workshop. Only the data from those teachers who had attended all twenty hours of the workshops were analyzed (Unfortunately only 48% of the 200 teachers who participated in the project met this criterion.) The analysis of the workshop evaluation questionnaire revealed a significant positive change in these teachers' attitudes toward teaching science and in their self-reported understanding of science. However, the teachers who attended the workshops did not significantly increase their classroom time for teaching science (teachers' average was about 145 minutes per week). They did indicate a change in the type of science instruction they used - more student hands-on investigations (from an average of 23 minutes per week to 54 minutes per week.) Because this district was only in the third year of implementing the hands-on workshops, no attempt was made to examine student science achievement as the ultimate outcome measure. Nevertheless, Shapley and Luttrell are encouraged that the mentor/colleague inservice approach reduced elementary teachers' anxiety toward science and increased their willingness to use active learning strategies with their students.

K.S. Shapely and H.D. Luttrell, "Effectiveness of mentor training of elementary colleague teachers", *Journal of Elementary Science Education*, **4 (2)**, 1-12, (1992).

- ***How can elementary and middle school teachers learn to teach chemistry with hands-on activities?***

Davis and George agree that teacher experience, teacher attitudes, and available materials can be a major barrier to implementing active learning in elementary and middle schools. They recognize that many K-8 teachers want to use hands-on activities with their students, but lack the academic training in specific science disciplines. They also believe that teachers overwhelmingly prefer to learn about new ideas and techniques from colleagues. Based on these premises, Davis and George designed a professional development program to prepare selected teachers to utilize hands-on activities to teach simple chemistry, to provide inservice workshops for their colleagues, and to serve as resource persons within their school districts. Teachers participated in an initial three-week summer workshop, a three-day session during the next February, and a three week workshop during the subsequent summer. During May of that subsequent academic year the teachers returned for a one day final evaluation of the project. Using the Shrigley and Johnson Science Attitude Scale as a pretest-posttest measure, these researchers found a strong positive improvement in the teachers' attitudes toward science. Using an 85-item chemistry content test they developed as a pretest-posttest measure, they found the teachers went from a mean of 52 to a mean of 62 during the two year project. An independent project evaluator added that "from the teachers' subjective perspectives, the methods and activities taught in this program, substantially improved young students' involvement in, knowledge of, and enthusiasm for science." However, no attempt was made to assess the students' knowledge of chemistry before and after changes in teachers' instructional activities. Davis and George conclude that programs such as theirs can indeed lead to a cadre of experienced teachers becoming inservice science specialists even though their initial college experiences lacked such a science emphasis.

S. A. Davis and A. George, "Chemistry for children: A program in precollege chemistry for elementary and middle school teachers", *Journal of Chemistry Education*, **74** **(1)**, 59-61, (1997).

- ***What can be the long term impact of a hands-on science workshop for middle school teachers?***

Consistent with a constructivist philosophy that hands-on science is the best foundation for developing middle school students' scientific interest and understanding, Hadfield and Lillibridge offered a one-week summer workshop emphasizing scientific process skills. Pretest-posttest data on the forty teachers who attended the workshop revealed significant improvements in knowledge of science concepts and in science teaching confidence. Visits to the teachers' classroom during the following academic year indicated they were using workshop materials and had disseminated these hands-on activities among their colleagues. A follow-up questionnaire was mailed to all forty teachers two years after the initial workshop to determine the lasting effects of that inservice experience. The return rate was 82.5% of the initial forty teachers. The vast majority of these respondents (88.5%) reported they spent either more time or equal time teaching science as they had the previous year. Nearly two-thirds of the responding teachers (64%) also indicated they thought their students enjoyed science more this school year than the previous year. Hadfield and Lillibridge conclude that the lasting effects of the hands-on science workshops, at least over the first two years, are a result of five key elements: (1) an introductory session on the importance of science education; (2) teacher construction of materials and guided practice; (3) teacher dissemination of activities; (4) administrative support; and (5) follow-up visits to their schools.

O. D. Hadfield and F. Lillibridge, "Can a hands-on, middle grades science workshop have staying power?", *The Clearing House*, **66 (4)**, 213-217, (1993). ■

AIMS & SCOPE

The Journal of Mathematics and Science: Collaborative Explorations is a forum for the exchange of ideas among college and university faculty from mathematics, science, and education as well as elementary and secondary school teachers. Articles are solicited that relate to all aspects of the preparation of prospective teachers of mathematics and science in grades K-8. The Journal is anonymously refereed, and appears twice a year.

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