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Elementary Teachers' Definitions and Usage of Inquiry-Based Mathematics Instruction

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Elementary Teachers’ Definitions and Usage of Inquiry-based Mathematics Instruction

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy at Virginia Commonwealth University.

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Dedication and Acknowledgement

This work is dedicated to my family. First, to my husband, Keith and son, Landon. Thank you for your continued support, encouragement and patience through this whole journey. To my parents, brother and extended family, your encouragement through this process helped me through and I hope I have made you all proud. To my grandparents in heaven, I could feel your support and thought of you often during this journey.

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Abstract

Current educational leaders call for students to build his or her own mathematical understanding from experiences, coupled with feedback from peers, teachers, and themselves and gain a conceptual understanding of mathematics. Researchers agree that inquiry in the elementary mathematics classroom can help increase conceptual understanding.

This case study focused on how elementary teachers define inquiry-based mathematics and implement it in their classrooms. Interviews, observations and lesson analysis were used to investigate what identities, relationships and activities look like in an elementary classroom that uses inquiry.

All of the participants felt "problem solving" and "working collaboratively" were essential for inquiry but each teacher defined them differently. Questioning was also an important feature of inquiry according to the teachers. Professional development seemed to have a strong impact on why these teachers use inquiry in their classrooms. As far as the relationships necessary to teach using inquiry, teachers did not indicate that administrators' nor peers' support were necessary to continue using this type of pedagogy in their classrooms. The participants believed that including inquiry in mathematics was a best practice and continued to incorporate inquiry because they felt it allowed their students to gain a deeper understanding of mathematics.

The local field of each teacher influenced the planning they did before the lesson and the activities they included in their inquiry instruction. The written plans of each

participant differed greatly. The requirements of the district had an effect on how much detail the participants included in their planning documents. Also, whether they were planning for their entire grade level or just themselves influenced how much detail was included. Another aspect of the mathematics classroom that was influenced by the local field was including a software program, which is expected to be a part of students' daily mathematics instruction. The various ways inquiry is carried out and how the local field influences this is important for educators at all levels to understand.

This study has implications for teachers, administrators and teacher educators. Inquiry means a variety of things to elementary teachers within this study. If mathematics educator leaders, teachers and administrators want to infuse more inquiry into the classroom, the many ways it is carried out needs to be understood.

Chapter 1

Introduction

Background of the Study

Both the Common Core State Standards (CCSS) and the National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics call for mathematics instruction that will improve the country's "mile wide and inch deep" (Schmidt, Houang, & Cogan, 2002) mathematics curriculum. These documents propose that students should have a conceptual understanding concerning mathematics (CCSS, 2017) while actively building new knowledge from a combination of experience and prior knowledge (NCTM, 2000). Conceptual understanding refers to "an integrated and functional grasp of mathematical ideas" (National Research Council, 2003, p. 141). While not cited by name, the descriptions of what mathematics instruction should share similarities with various descriptions of inquiry-based instruction (IBI) (i.e., conceptual understanding and problem solving).

Descriptions of IBI vary among different researchers. With the multitude of descriptions of inquiry-based mathematics, some of which are quite vague, it is not surprising that the mathematics community does not agree on one definition of IBI. Some definitions, such as Dorier & Garcia's (2013), are imprecise in nature; IBI "refers to a student-centered paradigm of teaching mathematics, in which students are invited to work in ways similar to how mathematicians work" (p. 837). Artigue and Blomhoj

(2013), on the other hand, describe IBI more specifically as “diverse forms of activities combined in inquiry processes: [that involve] elaborating questions; problem solving; modeling and mathematizing; searching for resources and ideas; exploring; analyzing documents and data; experimenting; conjecturing; testing, explaining, reasoning, arguing and proving; defining and structuring; connecting, representing and communicating” (p. 808). Although this definition illustrates how complex IBI can be, it also gives numerous examples of what students and teachers could be doing in an IBI classroom. Lewis (2013) similarly describes the identity of students in an IBI classroom as “tak(ing) a more active, central role in their learning” and “develop(ing) methods to solve mathematical and real-life application problems” (p. 400). Several of the activities Lewis (2013) describes include learning to “formulate conjectures, present partial and sometimes incorrect solutions for peer feedback, and work to revise their and others’ work to meet grade-appropriate standards of rigor” (p. 400). With a variety of descriptions of IBI in the literature, it is not surprising that practitioners would also have various interpretations of what IBI looks like in the mathematics classroom. The context in which they are teaching (Dorier & Garcia, 2013) and their beliefs about mathematics (Stipek et al., 2001; Philipp, 2007) also influence teachers’ definitions of IBI. If the types of activities described by Artique & Blomhoj (2013) are happening in a mathematics classroom, students have an opportunity to gain a conceptual understanding of mathematics and learn how to use mathematics within other domains. Teachers need to allow for problem solving and reasoning to occur to enhance students’ understanding of mathematics, but, as research shows, many do not employ these techniques in their classrooms (Brodie, 2011; Kisa & Stein, 2015; Sullivan, Clarke & Clarke, 2016).

The CCSS and NCTM documents include descriptions of teacher competencies such as knowing how students learn and assessing them appropriately. There are various reasons given in the existing literature as to why many elementary school teachers do not teach mathematics using the methods suggested in the mathematics education literature (Goldsmith & Schifter, 1997; Grant, 1998; Shepard, 2006). Some researchers believe that this is due to the short span of time preservice teachers spend learning about mathematics methods in college, as well as to the differences in how they learned math when they themselves were elementary school students (Nolan, 2012; Raymond, 1997; Steele, 2001). Another potential influence that could explain the difficulty in navigating mathematics pedagogy is the context in which individual teachers teach (Lewis, 2013). For example, if a teacher does not have a strong, cohesive support system that endorses the pedagogical approach they want to implement, they are less likely to take the time necessary for developing such lesson plans. Still others feel that even if a school provides a supportive environment and a teacher believes in a specific pedagogy for the mathematics classroom, the teacher might still hold more traditional beliefs about what mathematics is which leads to less problem solving and group discussions (Kuntze, 2011; Lui & Bonner, 2016). Overwhelmingly, the literature supports the idea that teachers' beliefs have a substantial influence on what they decide to do in their classroom (Ball, 2000; Makar, 2007; Stipek, Givvin, Salmon, & MacGyvers, 2001).

To teach mathematics using a conceptually focused approach, as CCSS and NCTM suggest, teachers need to understand what that entails, believe doing so is worthwhile, and know how to plan for and employ the mathematics content. Even those teachers that comprehend how to teach mathematics conceptually will face barriers to this style of teaching (Marshall, Horton & Smart, 2009; Towers, 2010). The perceived roles of the teacher and the students, the relationships between stakeholders in the school, and the culture of the school can all influence how a teacher teaches mathematics. This study investigated how elementary school teachers define IBI and what the related identities, relationships, and activities look like in the elementary mathematics classroom.

Overview of the Literature

What is IBI?

As mentioned, there are various descriptions of inquiry-based instruction (IBI). However, many claim the same roots, specifically, in Dewey's philosophy of education (1938). Dewey's philosophy posits experience and reflective thinking as cornerstones of successful learning. He defines inquiry as "the controlled or directed transformation of an indeterminate situation into one that is as determinate in its constituent distinctions and relations as to convert the elements of the original situation into a unified whole" (1938, p. 108). The interplay between using known information and figuring out a problem (unknown information) is what inquiry looks like to Dewey. Pedagogy involving the type of inquiry Dewey describes rose in popularity during the constructivist movement of the 1950s and then again in the 80s. According to Dewey, the student does not serve as a passive receptacle for knowledge, but rather an active agent engaged in meaningful and relevant tasks; inquiry in education, then, requires students to apply learned principles through reflection and experimentation (Artique & Blomhoj, 2013). There have been many interpretations of Dewey's philosophy and how his teachings should be applied in modern mathematics classrooms. Researchers agree that for students to have a conceptual understanding of mathematics they need to be solving mathematics problems, formulating questions, and building new understandings using rich tasks (Henningsen & Stein, 1997; Artique & Blomhoj, 2013). This classroom environment, the creation of which is the teacher's responsibility, allows students an opportunity to understand mathematics conceptually. Across the numerous descriptions of IBI, there are similar activities that take place. In general, problem solving of rich tasks, discourse between students, and allowing multiple paths to a solution are three activities that continuously appear in explanations of the use of IBI in the mathematics classroom.

A second mathematics pedagogy, direct instruction, does not contain the elements described above. Direct instruction is based upon a banking model of education. The teacher, as the holder of the knowledge, gives the information to the student, the receptacle for the knowledge (Lui & Bonner, 2016). This style of teaching often focuses on learning procedures without any connection to meaning, understanding, or the applications that require these procedures (NCTM, 2014). Students are not encouraged to work together to solve problems, and they often learn only one way to perform an operation. Munter, Stein, and Smith (2015) describe direct instruction as "pedagogy consist(ing) of describing an objective, articulating motivating reasons for achieving the objective and connections to previous topics; presenting requisite concepts (if they have not been presented previously); demonstrating how to complete the target problem type; and providing scaffolded phases of guided and independent practice, accompanied by corrective feedback" (p. 7). Comparing the underlying epistemologies of the various definitions of both IBI and direct instruction is the clearest way to see the differences. The main difference between the two pedagogies can be summarized as what it means to know and execute mathematics. The choice of what activities teachers use in their classrooms is one way to determine which type of pedagogy they value. Lessons that contain problem solving tasks, thoughtful questioning stems, and group work require the teacher to have certain competencies in order for the teaching to be effective and for students to learn successfully.

Teaching Using IBI

Teachers who understand the value of inquiry-based mathematics and apply its teaching techniques can enhance students' critical thinking and reasoning skills--skills necessary for success in the technological age in which we live. In order for students to experience inquiry in mathematics, teachers must "create the conditions that will allow students to take their own effective mathematical actions" (Smith, 1996, p. 393). This is an example of the identity of a teacher. Based on Shulman's (1986) concept of pedagogical content knowledge, Mathematical Knowledge for Teaching (MKT) (Ball, Thames & Phelps, 2008) demonstrates the immense amount of knowledge that teachers need to have in order to be effective in the mathematics classroom. According to Ball et al. (2008), "teachers need to know mathematics in ways useful for, among other things, making mathematical sense of students' work and choosing powerful ways of representing the subject so that it is understandable to students" (p. 404). There are six domains of MKT covering both subject matter knowledge and pedagogical content knowledge. These domains involve familiarity with the mathematics concepts at the levels below and above the grade a teacher teaches, knowing how the mathematics are used outside the classroom, and an understanding of the best strategies to teach each mathematical topic for which they are responsible. All of these are further examples of the identity of the teacher in classroom.

Using a teaching method based on any one of the inquiry-based approaches described in the literature brings with it more specific teacher knowledge, practices, and dispositions. Though there is not an agreed-upon set of such capacities, Towers (2009) has compiled a list of seven practices gathered from experts in the field of mathematics education (p. 247). These practices include understanding the provisional nature of knowledge and the complexity of the teaching/learning relationship, knowing how to "teach for understanding" including fluency in teaching with manipulatives, guiding small-group work, capitalizing on students' multiple solutions, and having the ability to understand and draw out the deep structure of the discipline so learners develop their abilities to reason and connect ideas. They also include some personal beliefs and social skills such as being comfortable with ambiguity and uncertainty, being committed to exploring student thinking, building a community of inquiry in the classroom, and being responsive to students. These social skills describe the relationships that teachers foster in a classroom that uses any of the inquiry-based approaches.

Overall, teachers that effectively use techniques associated with the various descriptions of IBI understand that the subject of mathematics is about more than just computing quickly. As Artigue and Blomhoj (2013) summarize, "it is essential for the teacher to select appropriate experiences, to guide students' reflections on these experiences so that their educational potential actually emerges, and to organize inquiry activities so that knowledge, in particular subject matter knowledge, progressively accumulates" (p. 799). This type of mathematics classroom does not just happen. Rather, fostering such an environment takes a considerable amount of planning.

Planning for Instruction

Researchers agree that planning for mathematics lessons is an important aspect of the teaching process (Shulman, 1986; Schoenfeld, 1998; Brown, 2009; Roche, Clarke, Clarke & Sullivan, 2014), yet they have not placed an emphasis on studying this topic. Roche et al. (2014) describes teachers' planning as "an artifact of their work as teachers" that "represents a picture of their intentions" (p. 856). Planning has also been described as a personal process (Roche et al., 2014) and as being intensely contextual, which would explain the dramatic differences in three stages of lesson planning Leiken & Kawass (2005) studied. The researchers had teachers write a lesson plan after giving them a task. The teachers then solved the problem and their lesson plans were modified based on their positioning of the task in the curriculum. Finally, the teachers were shown a video of students working on the task. The adjustments to the lesson plan after viewing the video included new "targets that may be achieved by means of this problem, the management of learning and the mathematical challenge they planned for their pupils" (Leiken & Kawass, 2005, p. 253). This shows that teachers considered their students while engaged in the planning process and took into account whether their plans were appropriate given the presumed level of their students. The relationship between teacher and student is, therefore, an important element of the planning stage.

There is currently little research on planning for instruction in the elementary school mathematics classroom. Older studies and studies from other countries suggest that teachers' personal experiences with the mathematics they are going to teach and their awareness of the processes involved during learning are both crucial for effective planning (Leikin & Kawass, 2005). Prior research has found that teacher beliefs do have an effect on the type of pedagogy they use but that this does not mean that what they believe mathematics pedagogy should look like is what is ultimately represented in their classrooms. Studies on teacher beliefs of mathematics and mathematics teaching have used a range of research designs (case studies, belief inventories) as well as participants (pre-service, in-service, and most commonly, secondary school teachers).

Teacher Beliefs about Mathematics

Research on teacher beliefs has shown inconsistencies between what teachers believes about mathematics, what they believe about mathematics pedagogy, and how they choose to teach mathematics to their students (Raymond, 1997; Skott, 2001). What is consistent in the literature is that teacher beliefs do affect teaching practices (Stipek et al., 2001; Peterson et al., 1989; Philipp, 2007). Their experiences in a mathematics classroom as students influence mathematics educators' beliefs (Skott, 2001), the professional development they take part in (Peterson et al., 1989; Callopy, 2003), and their understanding of the nature of teaching and learning (Raymond, 1997; Hoyles, 1992). All of these influences affect how each teacher chooses to teach mathematics and what teaching mathematics could look like in their classroom.

Social Field Theory

In order to understand teachers' definitions and usage of IBI in the elementary mathematics classroom, it is important to keep in mind the social field of mathematics education as defined by social field theory. Social field theory allows us to focus on how the teachers define and act in the field of elementary school mathematics because it is considered a socially created field. We can focus on the perceptions of how the identities, relationships, and activities in the field influence the instruction in the classroom. The theory of field and disposition allows us to ask questions of teachers regarding the elements of the field they are teaching in, and it also allows for a deeper understanding of their pedagogical decisions. For instance, how does a teacher activate their definition of the field at a particular school or district (what I refer to as their local field) in their lesson plans and their instruction? My study looks at how teachers navigated their local (or micro) field and how that influences the manifestation of IBI in their classrooms.

Rationale for the Study

Focusing on how elementary teachers define and use IBI will facilitate the understanding of how teachers activate their definitions of a theory of learning. It can also help other teachers and districts as a whole move to improve the effectiveness of their mathematics instruction. There is a substantial body of research on the relationship between teachers' mathematical knowledge, their beliefs, and their instructional planning (Lui & Bonner, 2016), how teachers' beliefs affect mathematics teaching practices (Cady, Meier, & Lubinski, 2006; Fennema et al. 1996) and how teachers' beliefs shifted as a result of using a specific curriculum (Collopy, 2003). Currently, however, there is no research examining what teachers perceive to be the elements of effective inquiry-based mathematics instruction or how they enact their definition of this type of instruction. Researching how teachers' perceptions of IBI influence their lesson plans and pedagogical decisions could explain a key difference between teachers that use inquiry-based instruction and those that do not. In between teachers' perceptions and their instructional practices lies the lesson planning stage. Therefore, studying the teachers who currently subscribe to some type of IBI approach, listening to their thoughts and opinions, and observing their practices could potentially lead to the changes necessary to improve the teaching and learning of elementary school mathematics. It may also shed light on the slow progression of the reform movement in the mathematics education community toward a more inquiry-based instructional model.

Understanding how the interaction between teachers' perceptions of IBI and their understanding of their local fields influences the pedagogical choices they make may help inform and improve decision-making for both current and future teachers. Thus, rather than deliberately teaching specific pedagogies, teacher educators can discern what in-service teachers perceive is possible in the classroom and allow pre-service teachers an opportunity to reflect on current pedagogical techniques. By exploring teachers' perceptions of how to plan for and engage students in any type of IBI, teacher educators can better align their curricula to meet students' needs in the areas of both planning and execution, regardless of teacher experience level in mathematics. Investigating how teachers' local fields (their schools and/or districts) influences what they include in their planning documents and instruction can help explain why some districts or schools have more teachers that use one of the various descriptions of inquiry. Lesson planning is an expected practice in most school districts, as well as in pre-service teacher courses. Reflecting on how teachers' understandings of their local fields influences the choices they make when planning and implementing any type of IBI in the mathematics classroom may help both current and future teachers recognize the importance of effective planning.

Research Questions

The purpose of this study was to investigate how elementary school teachers who identify as using inquiry-based practices in the mathematics classroom define and act in the field of elementary school mathematics by interviewing them and observing their lesson planning processes and delivery. The following research questions guided this inquiry:

1. What do elementary teachers believe IBI looks like in an ideal classroom? Specifically,
    a. What are the identities in an IBI classroom?
    b. What are the relationships in an IBI classroom?
    c. What are the activities in an IBI classroom?
2. How does teachers' understanding of their local fields influence their perceptions of what IBI can look like in their mathematics classrooms? Specifically,
    a. What can the identities look like in their IBI classroom?
    b. What can the relationships look like in their IBI classroom?
    c. What can the activities look like in their IBI classroom?
3. In what ways do teachers believe the interaction between (1) their perceptions of IBI and (2) their understanding of their local fields influences the choices they make when planning and instructing?

Overview of the Methodology

This study was qualitative in nature. Utilizing qualitative research strategies allowed me to study teachers' experiences in the setting those experiences take place in every day- the teachers' own classrooms. Data collection consisted of interviews, observations, and document analysis. My intent was to collect each individual teacher's opinion on the definition and usage of inquiry-based techniques. These opinions were different for each individual, and I wanted to avoid the impact of a group mentality on what teachers revealed. Therefore, I individually interviewed six teachers from two different school districts. I chose multiple districts because of the potential that differences in the local field could be due to different aspects of the districts themselves, such as the size, demographics, or socio-economic status of each district. I personally transcribed all of the interviews and used Atlas.ti software to code the data. I collected one lesson plan and observed one lesson from each teacher, the selection criterium for which was that the teacher felt the lesson illustrated well how she uses inquiry-based techniques.

Chapter 2

Review of Literature

Social Field Theory

Since elementary mathematics is socially constructed, it can look very different in different settings. Various aspects of the field in which mathematics is taught influences the delivery of the mathematics. The field also influences how teachers think about the subject. Bourdieu (1992) defines a *field* as "…a network, or a configuration, of objective relations between positions" (p. 97). He suggests that there are many possible fields, all "historically constituted areas of activity with their specific institutions and their own law of functioning" (Bourdieu, 1990, p. 87). These fields and the positions within them are continuously shifting and changing as the occupants of those positions strive for goods and resources (capital) specific to the field of mathematics education (Ferrare & Apple, 2015). Outside forces can also cause shifting within a given field. Bernstein (1996) uses the term *pedagogic device* to describe a similar theory of actors selecting and adapting to a given field. The use of language and the situations that occur are always changing and shifting based on what actors are using the language and acting in the field. As Gee (1999) explains, "we continually and actively build and rebuild our worlds not just through language, but through language used in tandem with actions, interactions, non-linguistic symbol systems, objects, tools, technologies, and distinctive ways of thinking, valuing, feeling, and believing" (p. 11). He goes on to explain that sometimes what is built is similar to the past and sometimes it is not, but "language-in-action" is there throughout the process. In a school building, this social process sets the rules regarding how its actors should communicate and with whom (Collin, 2014).

While Bourdieu was describing fields at a macro level, a recent overview of his work examines field theory on a more micro (or local) level. As Ferrare and Apple (2015) explain, “Bourdieu’s primary emphasis on the macro view of cultural fields obscures an understanding of how educational actors directly experience and make sense of the pedagogic qualities inherent in local field positions, practices and meanings” (p. 45). They argue that positions in the field occupied by actors (teachers, administrators, and students) operate in these fields on a daily basis and thus affect the pedagogies used therein. These fields are made up of elements (one of which is the actors) that define how the field looks and how those in the field experience it. According to Gee (2014), just a few of these elements in this socially constructed field are identities (actors), relationships, activities, and forms of knowledge.

In mathematics education, the identities are the stakeholders who do specific jobs and perform different actions in the school building, e.g., as a classroom teacher, a math coach, or principal. An individual’s understanding of the field and their personal experiences in and with that position influence how they perform the job; this is what Bourdieu calls habitus. For Bourdieu, habitus is a central concept in social field theory that closely aligns with identity. He explains (1979), “The habitus is a system of durable, transposable dispositions” that influence the actions and behaviors of a given person. A teacher’s identity is malleable and can look different based on the field they are in and the experiences they have had in that specific field. Thus, the identity of an elementary mathematics teacher can range from a facilitator who traverses the classroom posing questions while students discover the mathematics for themselves to the omniscient one who must dispense all of the knowledge. These different identities would indeed change the activities and the structure of the classroom environment, as well as the interactions between the teacher and students.

The relationships between the identities also shape the field. For example, the relationship between a student and a teacher or a teacher and a principal can look different depending on the elements in the local field. Ferrare and Apple (2015) explain that the relationships at the local level shape how the field looks to the actors working in that field: "The structure of the space of positions is determined by the state of relations between the positions at a given point in time. This means that any change in the state of relations between the positions results in a shift in the entire structure of the field" (p. 46). Thus, the relationship between teacher and principal could be as partners with a common goal of student understanding, or it could be a dictatorship wherein the principal mandates a specific curriculum that teachers must follow. Similarly, the relationship between parents and teachers in a school can influence the teachers' identities and the accepted style of teaching in the classroom. As Gee (1999) explains, in order to have particular identities and make visible and recognizable to others what a school is doing requires that actors "act, value, interact, and use language in sync with or in coordination with other people and with various objects in appropriate locations and at appropriate times" (p. 14). In sum, the relationships between stakeholders as well as the activities that occur in the classroom influence the local field and vice-versa.

As mentioned, the field influences the activities that occur, for instance, proctoring a high stakes test, delivering a lesson on fractions, or questioning students about a problem they are solving. In turn, the activities also influence how the field looks. Ferrare and Apple (2015) describe the field as an "arena of struggle" in which actors compete over resources that will ensure success. Teachers want their students to be successful (as students' success reflects the success of teaching), and if administrators view that success as passing a standardized test, then the teacher will take the necessary steps to ensure success on that test. However, if test scores are not the be-all and end-all when it comes to mathematical understanding, then the teacher might feel at liberty to use more open-ended tasks that allow students to problem solve. "It is this feel for the game that enables some actors the freedom to know when to take risks – to engage in subversion strategies – and when to 'dig in' and fight to conserve the present rules of engagement" (Ferrare & Apple, 2015, p. 48). As mentioned above, relationships and identities influence the activities, but the activities also affect the relationships and identities. Thurs, there is an overlap between the identities, relationships, and activities and all three are tightly connected.

Finally, the forms of knowledge in the field of mathematics education include traditional mathematics pedagogy and inquiry-based pedagogy, to name just a few of the strategies used to deliver content. Gee (1999) uses the term Discourses to describe different ways of being and doing: "If you put language, action, interaction, values, beliefs, symbols, objects, tools, and places together in such a way that others recognize you as a particular type of who (identity) engaged in a particular type of what (activity) here and now, than you have pulled off a Discourse (and thereby continued it through history, if only for a while longer)" (Gee, 1999, p. 18). In a classroom that uses traditional mathematics, the teacher often teaches the topic for the day and then has the students practice the concept independently, using similar examples. For those teachers with a Discourse of inquiry, mathematical discussions may occur between students working together to solve a real-life problem. Each type of pedagogy will have different activities, teacher-student relationships, and teacher identities. Since Discourses are "embedded in a medley of social institutions" (Gee, 1999), they are constantly changing based on the elements in the field.

Those who operate in such fields and institutions (on a local level) are continuously struggling with each other to define and redefine the elements that constitute the fields and institutions. To reiterate, the elements of a field are not static; external forces and actors in the field influence the identities, relationships, and activities. In the field of mathematics education, people disagree over how to teach the content, how to differentiate for gifted and special education students, who should make decisions about the curriculum (students, teachers, administrators) and the amount of time spent on specific topics and at what grade level.

Teachers often struggle with negotiating the field of mathematics education from various viewpoints and attempt to discern which one they should navigate in their current positions and fields. The teacher's disposition (what Bourdieu calls *habitus*) for the field of mathematics education develops over time and influences how they teach. Since teachers have experienced the field of mathematics education for many years as students, they have become accustomed to a certain standard, and their experiences inform their view of how instruction should be conducted in a classroom. Since NCTM's Principles and Standards (2000) was either not yet published or just newly published when today's teachers were elementary students, and there was not a huge push nationwide to teach mathematics using inquiry, many of the current elementary teachers were probably taught mathematics in a direct instruction format. The teachers are therefore accustomed to direct instruction and recognize certain kinds of identities, relationships, activities, and forms of knowledge as acceptable. In direct instruction, the teacher is the authority (identity) who uses the “banking” model of teaching (form of knowledge) to teach students who intake the information without asking questions (relationship). Those within the field socially create that field; therefore, the standard way of seeing the field of mathematics strengthens when others in the same social field use a non-inquiry based mathematics pedagogy, usually called direct instruction. This is typical of many US teachers of elementary mathematics. Ultimately, a teacher’s understanding of their local field coupled with an understanding of IBI interacts to produce the type of pedagogy the teacher decides to implement.

Inquiry Based Mathematics Instruction (IBI)

Some researchers believe that Dewey’s philosophy of education (1938), which posits experience and reflective thinking as cornerstones of true learning, inspired inquiry in education. Problem solving and “realistic mathematics education” (Freudenthal, 1973) increased in popularity during the constructivist movement of the 1950s and again in the 1980s. According to Dewey (1938), the student does not serve as a passive receptacle for knowledge, but is rather an active agent engaged in meaningful and relevant tasks; inquiry in mathematics, then, requires students to apply the principles being taught through reflection and experimentation (Artique & Blomhoj, 2013). The following is a brief history of the descriptions of mathematics pedagogy involving inquiry in the literature and a description of three elements that have been used historically to describe IBI.

History and Definitions of IBI

As mentioned previously, inquiry in education is not a new concept. Its philosophical basis began with Dewey (1933) who advocated for planning experiential activities for students around the content they are learning. Curriculum reforms in the 1960s, 1980s, and 1990s all focused on the processes of inquiry in the classroom, predominantly in science education. Throughout the decades, there have been different interpretations of what inquiry comprises and what it looks like in the classroom. Much of the research on inquiry in the classroom has been on science education, but the reform movements on inquiry teaching focus on all subjects. In the 1960s, the focus was on viewing knowledge as tentative rather than absolute with the result that everything had the potential to be under continuous review (Massialas, 1969). Inquiry was seen as a motivational device, with the teacher providing the problems and the opportunities for students to theorize. Moving into the 1970s, the view of inquiry was as a self-directed experience where the teacher established the ideal environment for exploration, and the students produced the questions for investigation (Beyer, 1979). Then, in the 1980s, inquiry continued to focus on students solving problems in the classroom, but the goal shifted from students finding a solution on their own, to a more holistic goal of creating citizens who wanted to and were able to inquire as adults (Hawkins & Pea, 1987). During the turn of the century, inquiry learning meant not only learning about a topic but also learning the process of inquiry (Schön, 1992). Schön (1992) describes inquiry learning as solving problems but not necessarily discovering a solution. A popular learning theory at the time, situated cognition, posits that you cannot separate the learner from the environment in which they are learning or the activities in which they are participating while learning.

Situated Cognition theory is a learning theory based upon the work of Dewey (1938), Vygotsky (1978, 1926/1997), Leont'ev (1978, 1981), and Luria (1976, 1979). Brown et al. (1989) described the theory's implications and goals for learning as involving more than merely an acquired set of self-contained skills and producing meaning through interaction and activity. IBI in math connects skills through the presentation of rich tasks and allows students to interact as they solve problems. Another goal of situated cognition is to create a community of learners in which each student becomes a member of a "culture of learning" (Brown et al., 1989, p. 39). Jenlink (2013) describes these communities as having "authentic experiences with opportunities to examine ideas, develop(ing) underlying concepts, and engag(ing) in activities to successfully complete a learning experience" (p. 186). The goals are very similar to the components in IBI in mathematics.

However, a number of critiques of this theory have been posited, which have challenged the assumptions of situated cognition. Studies have been conducted (Carraher, Carraher, & Schliemann, 1985; Lave, 1988), which found that students did not transfer the mathematical understandings they had in the classroom to the "real world." Critics argue, therefore, that the learning that occurs in the context of the classroom, even if "real world" problems are solved, is not transferable outside of the classroom and that school learning is necessarily bound by the context in which it occurs. Boaler (1993) has disagreed with the conclusions of these studies and argued that if students could not construct their own meaning in the classroom, then transferability would be difficult. The findings, however, should not preclude teachers from using “real world” contexts but should focus mathematics educators on considering the nature and variety of contexts they are using, as well as the nature of students’ learning.

In the early 2000s, mathematics education researchers focused their efforts on teacher training on pedagogy similar to IBI. They focused on describing specific competencies that teachers require in order to enact and support inquiry in the classrooms. For students to experience IBI in mathematics, teachers must “create the conditions that will allow students to take their own effective mathematical actions” (Smith, 1996, p. 393). Based on Shulman’s (1986) concept of pedagogical content knowledge, Mathematical Knowledge for Teaching (MKT) (Ball, Thames & Phelps, 2008) illustrates the immense amount of knowledge teachers require in order to be effective in the mathematics classroom. According to Ball et al. (2008), “teachers need to know mathematics in ways useful for, among other things, making mathematical sense of students’ work and choosing powerful ways of representing the subject so that it is understandable to students” (p. 404). There are six domains of MKT spread between subject matter knowledge and pedagogical content knowledge. This model clearly illustrates how much more knowledge a teacher requires than simply knowing the mathematics content found in the curriculum. It touches on three elements in the field of mathematics education: the identities of the teacher and students, relationships, and activities.

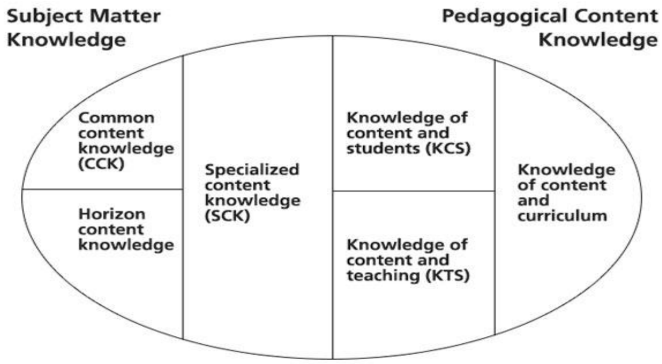

Figure 1. Domains of Content Knowledge for Teaching. Adapted from "Content Knowledge for Teaching: What Makes it Special?," by Ball, D., Thames, M., & Phelps, G., 2008, Journal of Teacher Education, 59, p. 403. Copyright 2008 by Sage Publications.

As shown in Figure 1, subject matter knowledge involves "common" content knowledge, which is the knowledge that is relevant to people who do not teach mathematics. Horizon content knowledge is the understanding of the necessary mathematics immediately prior to and following the concepts for which the teacher is responsible, as well as the connections among topics. Specialized content knowledge suggests teachers need to know how to explain math concepts in ways that make sense to students.

The pedagogical content knowledge required for effective teaching includes knowledge of mathematics content in conjunction with the students, the curriculum, and the teaching. This half of the domain "comprises blends of mathematical knowledge together with other kinds of knowledge, such as knowledge of students' thinking in a particular content domain, or knowledge of likely effective approaches to or materials for teaching specific content ideas" (Selling, Garcia & Ball, 2016, p. 37). The domains of

MKT that Ball et al. (2008) present illustrates how simply knowing the mathematics is not sufficient to make one an effective teacher.

The literature stresses additional competencies to support more teachers using inquiry in the mathematics classroom. Although there is not an agreed-upon set of such capacities and they change as the field changes, Towers (2009) has compiled a list of practices gathered from experts in the field of mathematics education (p. 247). These practices include understanding the provisional nature of knowledge and the complexity of the teaching/learning relationship; knowing how to "teach for understanding," including fluency in teaching with manipulatives, guiding small-group work and capitalizing on students' multiple solutions; and having the ability to understand and draw out the deep structure of the discipline so learners learn to reason and connect ideas. The practices also include some personal beliefs and social skills such as being comfortable with ambiguity and uncertainty, having a commitment to explore student thinking, building a community of inquiry in the classroom, and being responsive to students. This portrait of the identity of a teacher of inquiry-based mathematics and the activities in an inquiry-based classroom has changed over time as the field has changed. Most recently, the focus in the field of mathematics education in higher education has been on how students learn more when they make mistakes and how a teacher should handle such situations in the classroom.

Darling-Hammond (2016) lists six practices, similar to those espoused by Towers (2009), that are exhibited by "teachers who succeed at developing deep understanding of challenging subjects for an array of students, including those traditionally thought to be 'at risk'" (p. 86). Teaching using IBI means not always having the answers, and being comfortable switching between the roles of a teacher and learner. When working on rich tasks, atypical solutions and ideas may surface that the teacher did not consider ahead of time. Rather than panic or stop the activity, an inquiry-oriented teacher would allow students to share their strategies and as a group of learners (teacher included) reason out what they know to be true. Inevitably, mistakes will be made but as Boaler (2016) explains, “when mathematics is taught as an open and creative subject, all about connections, learning and growth, and mistakes are encouraged, incredible things happen” (p. 20). Through the struggle, the teacher constantly assesses her students to identify their strengths and learning approaches. As Darling-Hammond (2016) explains, “[the teachers] understand assessment as a measure of their teaching as well as a measure of student learning” (p. 86). To be able to conduct this type of formative assessment while students are working, the tasks that are selected should be accessible yet challenging and engaging, as well as include not only mathematics content appropriate for the students but also the Mathematics Process Standards (NCTM, 2001). The complex and ever-changing identities of the teacher and students, the relationship between them, and the activities that occur in a mathematics classroom are the elements of the field that have intrigued mathematics education researchers over time. Since these elements are not static, the people that operate in the field are always redefining what the field looks like for them and attempting to act accordingly.

Over the years, the roles (or identities) of the teacher and student have changed, as has the ultimate goal of using inquiry-based pedagogy in the classroom. The backbone of inquiry instruction, which is problem-solving and investigating, has remained consistent over time but what that looks like and how it is interpreted for teachers and students in the classroom has never been clear.

For the purposes of this study, enacting successful IBI means students are solving mathematics problems, formulating questions, and building new understandings using rich tasks. This classroom environment, which teachers create, affords students the opportunity to understand mathematics conceptually. In order for teachers to implement the elements that are described as part of IBI in the research, both mathematical and pedagogical content knowledge are necessary to create an exciting environment in which students can thrive (Ball, Hill & Bass, 2005; Baumert et al., 2010; Hill et al., 2008). These elements are part of the teacher's identity in the classroom. The following section reviews what the research says inquiry-based mathematics teaching could look like in today's classroom and what teachers need to do to ensure it is effective.

As history has evolved, there have been many different components to IBI in the mathematics classroom. I will be focusing on the problem solving of rich tasks, the discourse between students, and the multiple paths to a solution allowed when students are solving problems. I have selected these elements because they have remained consistent in the IBI literature throughout the years and teachers who subscribe to this description of IBI would most likely use these elements in their definitions of IBI and the types of activities used in an ideal IBI classroom.

Key Components of IBI

Using real-world tasks and problem solving in the mathematics classroom has been popular at different points in history. In the 1950's after World War II, "increased criticism of public education in general and mathematics education in particular, as well as the perception of the growing threat from Soviet technological prowess, ultimately gave rise to several projects to improve school mathematics" (Kilpatrick, 2014, p. 330). This New Math Era featured an abundance of federally funded programs aimed at increasing all students' understanding of mathematics in all students. The focus of the discipline was again on discovery learning and "the goal of mathematics education was understanding and not simply the manipulation of symbols" (Woodward, 2004, p. 17). The "why" became more important than the "what" and mathematicians worked together with psychologists to create curricula and study the results of this new way of teaching. Again, as the 21st century approached, the world of research in mathematics education was alive with studies that demonstrated just how weak U.S. students were in math compared to the rest of the world. Classroom instruction shifted from a focus on learning facts and procedures without knowing why to a more Piagetian teaching and learning style, i.e., a child-centered classroom focused on students' thinking and active involvement. The following is a brief explanation of what problem solving, communication, and representations have looked like in the mathematics classroom since the turn of the century.

Problem Solving

Inquiry-based mathematics instruction allows students to work in ways similar to a mathematician (Artique & Blomhoj, 2013). If someone were to enter a room that is engaging in this type of IBI, they might see groups of students working together to solve real-life problems in multiple ways with the goal of finding the most efficient strategy. Problem-solving tasks have been at the heart of many explanations of IBI throughout history because research demonstrates that students are better able to learn mathematics through tasks than through any other method (Anthony & Walshaw, 2009; Ruthven, Laborde, Leach, & Tiberghien, 2009; Sullivan, Clarke & Clarke, 2013). Henningsen and Stein (1997) define an effective mathematical task as “a classroom activity, the purpose of which is to focus students’ attention on a particular mathematical concept, idea or skill” (p. 528). The task also has “more than one solution strategy, [is able] to be represented in multiple ways, and demand[s] that students communicate and justify their procedures and understandings in written and/or oral form” (Stein, Grover & Henningsen, 1996, p.456).

When problem solving, the relationship between teacher and student becomes a partnership in mathematical discovery. The role of the teacher is to give the students a task and act as a facilitator, while students are expected to use their understanding of mathematics to solve the task. Once students are engaged in the problem, common practices that emerge are discussion with peers, modeling the problem in various ways, and often solving smaller problems along the way. Research by Stein et al. (1996, 2000) has demonstrated that tasks can be sorted based on the level and type of thinking that they have the potential to elicit. They created the Task Analysis Guide (2000), which identifies two categories of mathematical tasks with high-level cognitive demand and two with low-level cognitive demand. Solving problems that focus on memorization or procedures without connections requires a low-level of cognitive demand while tasks that focus on procedures with connections or “doing mathematics” have the potential to elicit high-level cognitive demand (See Table 1). Problems such as those on the right-hand side of Table 1 push students to use higher-order thinking skills, as well as, critical thinking skills, compared with the left-hand side, which limits the modes of access for students and reinforces mathematics as a subject of facts and procedures.

Table 1

Task Characteristics

Low-Level Cognitive Demands	High-Level Cognitive Demands
Memorization Tasks	*Procedures With Connections Tasks*
• Involve either producing previously learned facts, rules, formulae, or definitions or committing them to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous – such as tasks involving exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlie the content being learned or reproduced.	• Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections Tasks	*Doing Mathematics Tasks*
• Are algorithmic. • Require limited cognitive demand for successful completion. Little ambiguity about what needs to be done and how. • Have no connection to concepts or meaning that underlie the procedure being used.	• Require complex and non-algorithmic thinking. • Require students to explore and understand the nature of mathematical concepts, processes or relationships.

• Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanations or explanations that focus solely on describing the procedure that was used.	• Demand self-monitoring or self-regulation of one's own cognitive processes. • Require students to access relevant knowledge in working through the task. • Require students to analyze the task. • Require considerable cognitive effort and possibly anxiety due to the unpredictable nature of the solution process required.

Note. Adapted from The Task Analysis Guide (Stein et al., 2000).

During problem solving, students are interacting with mathematics differently than in the past. They are thinking at higher levels, making decisions on possible methods, and connecting prior knowledge with the new ideas in the problem; in sum, students are developing a conceptual understanding of mathematics. Understanding is not something a teacher can teach; rather, for this type of understanding to occur, the students have to be actively involved and make the connections themselves (Schoenfeld, 1992). Part of being actively involved is allowing students to converse with each other throughout the problem-solving process. Discussion not only allows students to learn from each other but also permits the teacher to assess the students informally on their mathematical understandings. This further supports the teacher's role as facilitator in the classroom.

Classroom Discourse

Allowing students to communicate by sharing ideas and clarifying their understanding while engaged in mathematics has also been an essential piece of effective mathematics pedagogy (NCTM, 2000; Boaler, 2016). In traditional mathematics classrooms, there is no need for students to communicate with each other; students are merely passive learners to the teachers' demonstrations (Smith, 1996). We now know, however, that through discourse, students hone their reasoning skills, build their understandings, and come to see mathematics as sensible and doable (Kilpatrick, Swafford & Findell, 2001; Smith, 1996). In 2000, when NCTM published the Principles and Standards for School Mathematics, the authors included a Communication Standard containing four guidelines that all mathematics programs should have in terms of what students should be able to do. These are:

- Organize and consolidate their mathematical thinking through communication;
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- Analyze and evaluate the mathematical thinking and strategies of others;
- Use the language of mathematics to express mathematical ideas precisely.

The various descriptions of IBI contain these guidelines throughout the problem-solving process and the sharing of solutions. Through small group and classroom discussions, students are learning both directly (mathematical strategies, procedures, and facts) and indirectly (social skills, respect for others, and listening skills). Establishing a supportive learning environment through discourse is essential for students' confidence and motivation (Walshaw & Anthony, 2008; Henningsen & Stein, 1997). A classroom that encourages discussion between students also suggests a view of mathematics that is about "having equal partners in thinking, conjecturing, exploring and sharing ideas" (Chapin, O'Connor, & Anderson, 2009) rather than one in which one person is the keeper of the answers. This relationship between students is a learned behavior for those who have not experienced it and for a teacher who has never led this type of classroom.

Lastly, communication in the mathematics classroom allows teachers to hear what students understand and how they use the language of mathematics. "Asking students to talk about mathematical concepts, procedures, and problem solving … makes clear to them what they do and do not understand and what other students think about these same issues" (Chapin et al., 2009). I would also argue that it allows the teacher to hear the misconceptions and be able to address them at an individual level. Mathematics is a language and, in mathematics, using correct vocabulary is an important piece in effectively communicating in mathematics (Ball, Hill, & Bass, 2005). For students to understand the vocabulary, they should be the ones using it repeatedly for ultimate internalization.

Multiple Representations/Entry Points

Allowing students to solve a problem in a way that makes sense to them has been a long-standing aspect of many descriptions of IBI, starting with Dewey (1897). When solving the types of problems that Dewey's inquiry encourages, students need to be able to demonstrate their understanding in a way that makes sense to them. Multiple entry points to a problem allow all learners, regardless of background, to tackle the problem at their level of understanding rather than having them remember a specific procedure. "Teachers are asked to focus on learners' thinking and ways to knowing, to respect their diversity and to build on what they know" (Brodie, 2011, p. 174). Along with having multiple ways to tackle a mathematics problem, students need to know that there are various ways to both reach and represent a solution.

Another characteristic of rich tasks is that they have multiple exit points. Students illustrate their thinking in a variety of ways and as Leinwand (2009) explains, "very rarely do more than half of the students [in a class] process the math being taught, see the math being taught, or feel the math being taught in the same way their teacher is seeing it" (p. 21). Ensuring that alternative solution strategies are shared is an important piece in many descriptions of IBI. Artigue and Blomhoj (2013) explain that once students have worked through a task, "emphasis is put on the collective sharing and discussion of different attempts and solutions allowing students to discover and discuss alternative approaches and solutions, and to clarify their own ideas" (p. 803). Once students see that there are multiple ways to solve mathematical tasks, they will realize that mathematics is not always a procedural discipline but can have many different solution strategies. This view of the field of mathematics education is new to teachers and students who have never experienced it. Since the elements of a field are never static, different people see the field in different ways based on what they have experienced.

Not only should students be able to represent their understanding in a way that makes sense to them but they should also learn to translate between the different representations to increase their mathematical understanding. Representational fluency should not be measured by the number of ways a student can solve a problem but rather by "whether and how representations become connected or linked to one another" (Stein et al., 1996). More representations do not equate with deeper learning. Choosing meaningful tasks, presenting them in a logical order, and helping students see the connections are all parts of the teacher's job.

Importance of Inquiry

Teachers who subscribe to IBI understand that it has many benefits including the potential to increase students' cognitive level (Marshall, Horton, & Smart, 2009), curiosity (von Renesse & Ecke, 2017), discovery and exploration in the classroom (Engel & Randall, 2009), and achievement (Carpenter, Fennema, Peterson, Chiang & Loef, 1989). Ball (2003) even describes components of IBI as making a positive difference in students' life chances and their future participation in society. The common theme running through the research is that using practices consistent with inquiry-based instruction allows students to truly "do mathematics" (Artigue & Blomhoj, 2013).

National organizations such as NCTM, the Mathematical Association of America (MAA), and the National Research Council (NRC) have all published documents describing pedagogies found in IBI and point to the importance of students developing a conceptual understanding of mathematics. Dewey (1938) saw learning through doing and the development of general habits of mind for learning as an essential function of education. Allowing students the time to develop a conceptual understanding of the mathematics they are learning is important for their problem-solving and reasoning skills, as well as their ability to connect the concepts together.

Carpenter et al. (1989) researched classrooms where the teachers had participated in a month-long professional development focused on children's development of problem-solving skills in addition and subtraction. Forty first-grade teachers (half in the treatment group, half in the control group) participated in the study. Those in the treatment group participated in a workshop with the initial goal of familiarizing them with strategies students use to solve addition and subtraction problems. Throughout the four weeks, researchers studied problem types, and the processes students use to solve them, and related these processes to the levels of the problems. Teachers also had time to brainstorm instructional strategies and questioning techniques they could use to facilitate their instruction, which was in line with inquiry-based techniques. Finally, teachers evaluated instructional materials based on the knowledge and instructional methods they had learned in the workshop. The researchers collected data on teacher beliefs, instructional practices, and student achievement in the form of classroom observations, surveys, and student achievement tests. Students in the classrooms of the teacher treatment group not only outperformed their peers in the teacher control group on numbers facts and complex addition and subtraction problems, but they were also more confident in their ability to solve math problems, felt they had a greater understanding of mathematics, and were more cognitively guided in their beliefs than their peers.

The various descriptions of IBI all have many benefits to both teachers and students. Goodchild, Fuglestad, and Jaworski (2013) describe those in an inquiry community as "approach(ing) practice with a questioning attitude, not to change everything overnight, but to start to explore what else is possible, to wonder, to ask questions, and to seek to understand by collaborating with others in the attempt to provide answers" (p. 396). The community the teacher builds in the classroom can only exist if the teacher believes this type of pedagogy is both appropriate and beneficial to student success.

Teacher Beliefs

Research has demonstrated that what teachers believe to be true about mathematics teaching and learning affects how they teach (Cady, Meier, & Lubinski, 2006; Philipp, 2007; Thompson, 1992). In general, if teachers believe that mathematics involves many calculations to arrive at a single answer, they teach concepts as isolated procedures. Alternatively, if teachers believe mathematics has tools to solve problems in other disciplines and there are multiple ways to solve these problems, they teach using real-world tasks and allow students to work together (Peterson et al., 1989; Stipek et al., 2001). These forms of knowledge are part of the ever-changing field of mathematics education. In Bourdieu's terms, a teacher's habitus develops over time and through their experiences develops a way to act in that field. A person who attends a typical US public school may develop a disposition whereby teachers are the authority in the classroom and students follow the teachers' directions. Most teachers who have experienced learning mathematics in this way will most likely teach using similar strategies. However, teachers' dispositions towards the teaching of mathematics can change through different experiences in and out of the classroom.

Peterson et al. (1989) conducted a quantitative study that examined the relationships between teachers' beliefs, their content knowledge and students' achievement in mathematics. Teachers answered questions about their beliefs on how to teach addition and subtraction through questionnaires and structured interviews. Four subscales were designed to measure interrelated but separate constructs: how children learn mathematics, the relationship between mathematical skills and understanding and problem solving, the basis for sequencing instruction, and how addition and subtraction should be taught. They found that teachers who believed in pedagogy related to IBI used more word problems and spent more time developing children's counting strategies than those who believed mathematics should be learned through direct instruction (i.e., the students receiving knowledge from their teacher. In the elementary grades, learning about the four operations is a large part of the curriculum. The fact that teachers' beliefs about operations influence their pedagogy can lead us to believe that their beliefs also affect other mathematics content.

Stipek et al. (2001) studied teacher beliefs and practices and included observations in their methodology. The researchers also investigated other constructs related to teacher beliefs, such as motivation, confidence, and enjoyment in teaching mathematics. The researchers found that four dimensions of beliefs were strongly associated with each other: 1. Mathematics is a set of operations to be learned; 2. Students' primary goal is to procure correct solutions; 3. The teacher needs to exercise complete control over mathematics activities; 4. Extrinsic rewards and grades are effective strategies for motivating students to engage in mathematics. Similar to Peterson et al. (1989), they also found that traditional beliefs are associated with traditional practices. This means that those who use traditional, direct instruction techniques emphasized performance and speed in the students rather than understanding and the use of various strategies. Stipek et al. (2001) explain that the attention of such teachers was more on "how much students knew in general, relative to other students, rather than on students' interpretations and understandings of particular math concepts" (p. 223). Researchers believe that because of the strong relationship between beliefs and practice more teachers would teach using inquiry techniques if their beliefs about mathematics changed. The teacher's local field affects how they implement their version of IBI, if at all. Based on the identities and relationships within a teacher's local field, the supports and barriers could look very different.

Supports and Barriers to using Inquiry-based practices

Researchers who subscribe to any type of IBI in mathematics and/or focus their studies on mathematics classrooms agree that mathematical content knowledge is essential for teaching using this type of pedagogy and could be a barrier to teaching using IBI (Hill, Rowan, & Loewenberg Ball, 2005; Hill, Sleep, Lewis, & Loewenberg Ball, 2007). However, there are reports of other possible barriers to teaching using inquiry, such as discomfort with student struggle (Stein, Grover, & Henningsen, 1996; Lewis, 2014), beliefs about the nature of mathematics teaching and learning (Stipek et al., 2001; Beswick, 2007; Olson & Barrett, 2004; Philipp, 2007), and administrative support (Towers, 2009). However, there is little research on supports for those who use a version of IBI in their mathematics classroom.

As explained previously, the biggest barriers to using any type of IBI are the teacher's beliefs and the content knowledge. Both of these factors influence a teacher's instructional strategies. Researchers have found that the pedagogy used when teachers were students highly influences their current beliefs (Towers, 2009) as well as their choice of pedagogy (Nolan, 2012). Since it is not an option to change the pedagogy used in the past, professional development on inquiry-based strategies is necessary in order for teachers to know and understand what it is, as well as possibly change their beliefs about mathematics pedagogy. Many researchers have studied teachers who have gone through professional development (Peterson et al., 1989; Carpenter et al., 1989) and have demonstrated that beliefs can change. In their study, for example, Carpenter, et al. (1989) found that after teachers attended a workshop focused on CGI (Cognitively Guided Instruction) strategies, they agreed more that children can construct their own mathematical knowledge: "CGI teachers agreed more with the belief that instruction should facilitate children's construction of knowledge" and that "skills should be used on understanding and problem solving" (p. 526). This demonstrates that part of a teacher's identity, specifically their beliefs, have an effect on their pedagogy.

Lack of support in the school building (local field) is also a major barrier for teachers, particularly novice teachers. Whether it is resistance from other teachers or administration (Towers, 2009), if teachers do not receive support in their buildings, they tend to move towards the type of instruction that is occurring around them rather than risk being the odd man out (Allen, 2009). In addition, if the administrator who is assessing them does not understand or support an inquiry-based pedagogy, it makes sense that the teachers will use the strategies understood by their evaluator. The limited amount of research that does examine the supports of IBI discusses how collegiality and sustained support from a mentor helps teachers endure and develop expertise in teaching using the various IBI techniques (Lewis, 2014; Makar, 2007).

Another barrier discussed in the literature is the heightened anxiety due to the shift from teacher-centered to student-centered instruction that many types of IBI require. This shift in classroom relationships can affect student behavior, the amount of time the teacher is instructing the students, the teacher's planning process, and the teacher's response to unexpected student ideas. Based on the descriptions of IBI in the research, which include rich tasks and discussion, teachers spend less time standing at the front of the classroom telling students how to solve problems. The stereotypical classroom, where desks are in rows, students learn an algorithm, and then practice said algorithm, is not conducive to inquiry pedagogy. Discussions about strategies and ideas need to occur between students, and this can lead to students engaging in off-task behavior and/or uncertainty on the part of the teacher. For this reason, Towers (2009) lists “a level of comfort with ambiguity and uncertainty” as well as “a commitment to building a community of inquiry in the classroom” (p. 247) to her list of practices and dispositions attributed to inquiry-oriented teachers. In order for this type of IBI to occur, supports need to be in place, and the teacher must have the right disposition and specific competencies to carry it out effectively.

Teacher competencies necessary to support IBI

There are a number of specific competencies that teachers require to have in order to enact and support any type of IBI in their classrooms. “In order to be able to plan for and support [IBI] for students, the teachers need to experience and exercise inquiry in mathematics themselves” (Artigue & Blomhoj, 2013, p. 807). As previously discussed, they need to understand the mathematical content at a deep level, be able to plan for discussion in the classroom, and be comfortable with uncertainty and unexpected situations occurring in the classroom.

Planning for IBI

Planning for any IBI can be a stressor for some teachers, particularly novice teachers (Lewis, 2014). Designing or finding rich tasks and orchestrating discussions are two important pieces of IBI for which a teacher needs to plan in advance.

Designing worthwhile tasks is one of the teacher’s roles when using IBI and is where activities and identities overlap in the classroom. The most effective task permits student choices in addition to different entry points, as it allows each student to build on previous knowledge and expand her understanding with more difficult skills. It is extremely important for the teacher to invest a great deal of energy into selecting effective tasks. Stein et al. (1996) explain that "the mathematical tasks with which students become engaged determine not only what substance they learn but also how they come to think about, develop, use, and make sense of mathematics" (p. 459). If the task includes the process standards set by NCTM, it will naturally allow students to build on their current knowledge. The five process standards are problem-solving, reasoning & proof, representations, connections, and communication (NCTM, 2000). Woven through each grade level, these standards are just as important as the five content strands in NCTM's Principles and Standards for School Mathematics (2000). The key to planning is to identify what the students should come to know and understand about mathematics as a result of the lesson. This includes the mathematics content but also the process goals that will occur throughout the lesson. Teachers must not only consider not only these standards when planning but also the level of their students and often, the pacing guide of their district.

Although little research has been conducted on planning for elementary mathematics in the United States. there has been research on task development (Stein et al. 1996) and pedagogical decisions during teaching that enhance student thinking (Ball, 1993; Henningsen & Stein, 1997; Lampert & Ball, 1998), all of which reveal complex thinking in the planning of lessons. As of yet, however, research in mathematics education has not honed in on the thoughts and considerations teachers have while planning. In one of the few studies to have been conducted in this area, however, Fernandez and Cannon (2005) compared the lesson planning processes of Japanese and U.S. teachers. With a sample size of 61 middle school teachers, the researchers interviewed participants on their thoughts and attitudes towards planning. Similarities were found between the two nationalities (approached lesson planning with a focus on mathematics, shaped lessons by changing questions and problems, believed that achieving a mathematical goal was a characteristic of a good lesson) but stark differences were found regarding what they focused on while planning. Japanese teachers focused more on the process of student learning and creating the dispositions for learning mathematics that allow them to discover new ideas themselves. Conversely, U.S. teachers focused on teaching a specific mathematical topic and this took precedence over the type of learning process in which the students engaged. The forms of knowledge and activities that are valued in the local fields or the teachers' role that is accepted may explain this difference. These findings highlight the importance of planning in the teaching process and if studied further could help create professional development that could assist teachers in using best practices for mathematics.

Planning for a mathematics lesson can be seen as a process that straddles teachers' beliefs (beliefs influence planning) and the teaching that occurs (planning impacts teaching). Based on the connections between beliefs, planning, and teaching, it is difficult to obtain a full understanding of the needs of teachers to implement inquiry in their mathematics classrooms if planning is not included in the research. Mathematics education needs more studies, both a large and small scale, to grasp how teachers plan mathematics lessons at all levels.

Teaching using IBI

Teachers that effectively use IBI understand that mathematics is more than merely a process of computing quickly. Artigue and Blomhoj (2013) summarize that “it is essential for the teacher to select appropriate experiences, to guide students’ reflections on these experiences so that their educational potential actually emerges, and to organize inquiry activities so that knowledge, in particular, subject matter knowledge, progressively accumulates” (p. 799). This type of effective mathematics instruction must be learned, practiced, and planned. During the actual lessons, teachers need to be able to orchestrate discussion and be flexible in allowing students to make mistakes. These teacher identities and activities are valued in many IBI classrooms.

One important component of implementing any type of IBI in the mathematics classroom is orchestrating discussions. The relationship between teacher and students needs to be well established and acceptable in the local field in which the teacher is operating. Smith and Stein (2011) identify five practices that were designed to “help teachers to use students’ responses to advance the mathematical understanding of the class as a whole by providing teachers with some control over what is likely to happen in the discussion as well as more time to make instructional decisions by shifting some of the decision making to the planning phase of the lesson” (p. 7). The five practices are anticipating, monitoring, selecting, sequencing, and connecting. Prior to engaging the five practices Smith and Stein (2011) argue that setting the goals for the lesson and choosing the appropriate task need to be conducted during the planning phase. This is also where the first practice of anticipating occurs. During the planning stage, the teacher will anticipate students’ responses and reactions to different strategies. Considering which strategies are most helpful in addressing the mathematical goals of the lesson should also occur during the planning stage. The rest of the practices occur during the lesson.

Once students are engaged in the task, the teacher monitors their work and listens to the conversations that are occurring. This is where the teacher needs to be flexible and patient. When students are engaged in productive struggle, it demonstrates a positive engagement with the mathematics. The teacher should not feel the need to interfere and correct. They should be comfortable with uncertainty (Lampert & Ball, 1998). Then, after giving students sufficient time to work on the task, the discussion begins. First, the teacher selects and sequences which students will share their strategies. The selection is far from random and takes into account mathematical thinking, not necessarily the level of the student. Smith and Stein (2011) describe selecting as “the act of purposefully determining what mathematics students will have access to – beyond what they were able to consider individually or in small groups – in building their mathematical understanding” (p. 44). The teacher’s role is now to control what the whole class will discuss which leads to how the sharing is sequenced. The mathematics should build coherently so that it is accessible to all students. Finally, the last practice is connecting. The teacher needs to think of questions to make the mathematics “visible and understandable” (Smith & Stein, 2011, p. 49). The questions posed should focus on linking mathematical ideas and representations so students can create a complete understanding of the mathematics they are studying.

It is no easy task to teach the use of any of the IBI pedagogies in the mathematics classroom. Teachers need to possess certain traits and feel comfortable with orchestrating classroom discussion. Moreover, the identity of the teacher, the teacher-student relationship, and the activities that occur need to be accepted in the local field as elements of good mathematics instruction.

Summary

This review of the literature has provided an overview and history of how IBI has looked in the mathematics classroom to date. It describes how the essential components, teacher competencies, and some supports and barriers to implementation of inquiry-based mathematics instruction have been described historically. The changing field of mathematics education makes for varying identities, relationships, and activities in the classroom. Teacher disposition and content knowledge also influence the mathematics pedagogy that occurs in the classroom. Previous research has found that professional development can influence teacher beliefs and support from administration, while peers (local field) can help teachers sustain a more inquiry-based pedagogy for mathematics. Therefore, if teachers have had professional development in mathematics that supports inquiry-based practices and they believe this to be best practice, their lesson plans and pedagogical decisions should match those of IBI. However, there are sometimes outside forces in their local field that influence instruction in the classroom. Moreover, no research exists on how teachers who use IBI define the pedagogy. What do the identities, relationships, and activities look like in the classroom? How does their understanding of their local field influence what inquiry-based pedagogy looks like in their classrooms? Finally, there appears to be no research on what planning looks like for inquiry-based mathematics and whether the planning matches the instruction that actually occurs. How do teachers who subscribe to IBI incorporate it into their lesson plans and instruction?

This study attempts to fill these gaps by asking teachers what inquiry means to them, looking at teacher's lesson plans, and observing inquiry lessons.

Chapter 3

Methods

This multiple site, qualitative case study framed within the interpretive paradigm sought to understand teachers' definitions and usage of teaching using inquiry-based pedagogical techniques. The following research questions guided the data collection and analysis:

1. What do elementary teachers believe IBI looks like in an ideal classroom? Specifically,
   a. What are the identities in an IBI classroom?
   b. What are the relationships in an IBI classroom?
   c. What are the activities in an IBI classroom?
2. How does teachers' understandings of their local fields influence their perceptions of what IBI can look like in their mathematics classrooms? Specifically,
   a. What can the identities look like in their IBI classroom?
   b. What can the relationships look like in their IBI classroom?
   c. What can the activities look like in their IBI classroom?
3. In what ways do teachers believe the interaction between (1) their perceptions of IBI and (2) their understanding of their local fields influences the choices they make when planning and instructing?

Research Design

Denzin and Lincoln (2011) describe qualitative research as “a set of interpretive, material practices that make the world visible” (p. 3). While examining the relationship between each teacher’s definition and usage of inquiry-based mathematics instruction, I focused on the identities, relationships, and activities within each unique setting. My goal was to study each teacher in their local field to make sense of how each individual understands and practices IBI in their classroom.

Case study research explores a real-life, contemporary bounded system over time through in-depth data collection using multiple pieces of information (Creswell, 2013). The researcher then reports the themes that emerge through case analysis. I conducted a multisite study, and thus have multiple cases on which I collected data. By treating each teacher as a separate case, I am acknowledging that everyone has a unique local field, professional and personal experiences, and pedagogical beliefs. Miles et al. (2014) emphasize that a multiple-case design is particularly useful if the cases are “chosen to be critical, extreme, or unique or revelatory” (p. 30.). This study focused on elementary teachers who were identified as using IBI in their mathematics classrooms. District mathematics supervisors identified teachers that use problem-solving, questioning, and rich tasks in their instruction, since these are the elements identified in the literature as using some form of IBI (Towers, 2009; Artique & Blomhoj, 2013). By studying teachers who subscribe to an inquiry-based pedagogical approach, this study aimed to add to the research on what influences the use of IBI in the elementary mathematics classroom. In the interpretive paradigm, the researcher tries to understand the subjective experiences of individuals and not necessarily generalize beyond the cases studied. Specifically, this study aimed to understand elementary teachers' definitions and usage of inquiry-based mathematics instruction.

District Selection

My research took place in two different school districts in a southeastern state of the United States. Both districts have provided teachers with professional development within the past five years on inquiry-based mathematics instruction. I included multiple districts to address the potential differences between the local fields. Within each district, I included schools of different sizes, demographics, and socio-economic status.

District A

The first district was suburban and in 2017 served approximately 18,000 students in 25 schools. There were 15 elementary schools, four middle schools, four high schools, one alternative school, and one technical school in District A. Every year, teachers in this district can choose their professional development activities. The professional development offered in 2016 through 2017 focused on incorporating the new Standards of Learning (SOL's) into the existing mathematics curriculum. Suggestions on how to incorporate the new SOL's (sample lesson plans and activities) and how they align with NCTM standards were shared during the professional development sessions. I taught in three different elementary schools within this district between 2002 and 2011 (four years as a classroom teacher and five years as a math specialist). In addition, I delivered professional development seminars to many of the elementary teachers that work in the district. I continue to converse with many teachers in this district and have a child attending an elementary school there.

District B

The second district is a larger school district with approximately 61,000 students attending 61 schools; 38 of these are elementary schools, 12 are middle schools, and 11 are high schools. In addition, there is one technical center. During the 2016-2017 and 2017-2018 school years, professional development in this district focused on implementing the new math SOL's, increasing discussion in the classroom with math talks, allowing students to use various strategies when solving problems, drawing with problem-solving, and building a growth mindset within students. I taught two courses for math specialists (summers of 2016 and 2017), in which two of the teachers in the class taught in this district. The lead mathematics supervisor and I enrolled in a master's degree program together between 2003 and 2007. I also have close relationships with two principals in the district.

Participant Selection

I used a purposeful sampling technique to recruit a pool of teachers for this study. As Maxwell (2013) explains, in purposeful sampling, participants are "selected deliberately to provide information that is particularly relevant to your questions and goals, and that can't be gotten as well from other choices" (p. 97). I asked the math coordinator in each of the districts to recommend teachers who already believed in IBI as an effective instructional strategy and practiced the IBI techniques described in the research to be part of IBI (Appendix A). I chose a total of six teachers because "the evidence from multiple cases is often considered more compelling and the overall study is therefore regarded as being more robust" (Yin, 2014, p. 57). I also believed that conducting six case studies would illustrate the various descriptions of IBI found in the literature. Once I had a list of ten teachers from each district, I started from the top of each list and choose the first three teachers, making sure they taught different grade levels. Through email (Appendix B), I asked if they were willing to be interviewed and observed as part of my study. I waited for each teacher to respond to me before emailing the next, enabling me to thoughtfully choose the next participant and get a variety of grade levels. If a teacher told me she was not interested, I emailed the next teacher on the list. If a teacher was interested in participating in my study, I then chose another teacher of a different grade level to contact. Of the ten teachers contacted, three teachers did not agree to participate. Once a teacher agreed to participate, I sent an email to the principal of that school to inform them that one of their teachers would be part of the study. I interviewed and observed three teachers from each district and obtained a varied sample of kindergarten through fifth-grade teachers.

As for the district in which I used to teach, I made sure to choose teachers I did not know from my previous years of employment. I used my experience as a teacher to establish rapport, trust, and use common language with the participants. When scheduling observation and interview times (through e-mail), I made sure to let the teachers know I understood that the beginning of the year was busy and allowed them to choose a day and time that was convenient for them.

Data Collection

To understand teachers' perceptions of IBI, as well as how they plan for and implement it in their classrooms, it was important to choose data collection procedures that would provide as much detailed information as possible: semi-structured interviews, classroom observations, and document analysis (of lessons plans and district materials). Since each teacher's perspective and situation are different, I treated each as an individual case. The interviews were conducted between late August and early October, while the observations took place in the first six weeks of the academic year. All participants were responsive to e-mails and scheduling was never an issue.

In each phase of data collection, I focused on identities, relationships and activities. I asked teachers about all three elements in the interviews, watched for aspects of all three when I observed lessons, and looked for how the planning documents revealed the identities, relationships, and activities in the classroom. I also wrote a memo after the pre-observation interview and the observation.

Interviews

I conducted semi-structured, individual interviews with each teacher. Semi-structured interviews allow the interviewer to ask each participant the same questions in the same order and lessen interviewer error. Each teacher had different perceptions regarding IBI. The semi-structured interviews allowed me to question the ways in which their local field influenced their instructional decisions. Taking Yin's (2014) description of an interview as "resembling guided conversations rather than structured queries," I often asked follow-up questions not on my protocol to gain more clarity, or if further probing was necessary. The pre-observation and post-observation interviews provided insight into teachers' beliefs about the identities, relationships, and activities of IBI, their perceptions of their local field and how these interact to influence their planning and instructional strategies. I allowed the participants to choose the location of the pre-observation interview because I wanted to conduct the interviews where they felt most comfortable answering my questions. All but one of the pre-observation interviews took place in the teachers' classrooms. One of the teachers wanted to meet at a restaurant for the interview.

The pre-observation interview (Appendix C) included approximately 15 open-ended questions and sought to collect data on each participant's professional background, perceptions about the identities, relationships, and activities in mathematics teaching and learning, and their local teaching context. Each interview took approximately 45 minutes and was recorded and transcribed for analysis. I conducted a post-observation interview to allow the teacher to explain her decision-making throughout the lesson and to allow me to ask follow-up questions about the components of IBI the teacher described as being in the lesson. During the post-observation interview, I also asked about the identities, relationships, and activities I observed in the classroom. Conducted over the telephone, each post-observation interview took approximately 15 minutes. The post-observation interviews took place on the same day as the classroom observations. I conducted these interviews soon after the observations so the teacher and I each had clear recollections of what occurred during instruction.

Reflective Memos

Memos are one of the most important techniques used in qualitative research (Maxwell, 2012). They help the researcher make sense of the topic being studied through reflection, analysis, and self-critique. I believe writing memos throughout the data collection process is a great way for me, as the researcher, to absorb the vast amount of information being collected and acknowledge any bias I might have pertaining to the information. I handwrote memos throughout my data collection process, after each interview and observation. After data collection, I used my memos as another source of data for coding.

Document Analysis

I asked all the teachers that I interviewed to send me the lesson plan for the day I observed. These documents became part of each hermeneutic unit. I looked specifically for how the identities, relationships, and activities of the math classroom were reflected in the document. I collected their lesson plans and other instructional materials (worksheets, tasks, and informal assessments) as available and used them to validate statements made by the participants and help to determine the teachers' instructional goals of the teachers. All documents were uploaded and coded using the qualitative data software Altas.ti.

Observations

To determine how the teachers incorporated inquiry-based mathematics instruction, I performed classroom-based observations. Observations "provide a direct and powerful way of learning about people's behavior and the context in which this occurs" (Maxwell, 2013, p. 103). Once the pre-observation interviews were complete, I asked each participant to look at their schedules and let me know which day between September 4th and October 30th they felt would include IBI. Allowing the teachers to choose the observation day allowed me to see how they integrated IBI into their mathematics classroom and helped me to understand how they perceived IBI pedagogy.

The observation protocol I used is an edited version of the Mathematics Scan (M-Scan) created by researchers at the University of Virginia (Walkowiak et al., 2013). The M-Scan was created to measure the quality of mathematics instruction and is based on the NCTM standards. Two publications served as guidance for the protocol: *Mathematics Teaching Today: Improving Practice, Improving Student Learning* (NCTM, 2007) and *Principles and Standards for School Mathematics* (NCTM, 2000). Both documents describe mathematics pedagogy that is very similar to IBI. The dimensions of the measure were also linked to the five Process Standards (NCTM, 2000) and the seven Content Standards for Teaching and Learning Mathematics (NCTM, 2007). The Process Standards are "the mathematical processes students draw on to acquire and use their [mathematical] content knowledge" (NCTM, 2000), while the Content Standards are the specific mathematical understandings, knowledge, and skills students are expected to acquire in each grade band. The M-Scan is a tool that helps teachers see how the Standards suggested by NCTM can be integrated into classroom instruction. The eight dimensions constituting these principles are lesson structure, multiple representations, mathematical tools, cognitive depth, mathematical discourse community, explanation and justification, problem-solving, and connections and applications. The M-Scan dimensions describe the same pedagogical techniques that research has described as inquiry-based mathematics instruction (Artique & Blomhoj, 2013; Smith, 1996; Towers, 2009). For the purposes of aligning this study with social field theory (the theoretical framework being used), I employed the framework questions that related to the identity of the teacher, identity of the student, relationship between the teacher and students, and activities that occurred in the classroom. I then put each question into the dimensions that matched my research questions focused on the identities, relationships, and activities in the classroom (Appendix E). Reducing the M-Scan helped me focus on those elements. Table 2 details the research questions and associated data sources.

Table 2

Research Questions and Corresponding Data Sources

Research Questions	Data Source
1.What do elementary teachers believe IBI looks like in an ideal classroom? Specifically: a. What are the identities in an IBI classroom? b. What are the relationships in an IBI classroom? c. What are the activities in an IBI classroom?	Pre-observation interviews
2.How does teachers' understanding of their local fields influence their perceptions of what IBI can look like in their mathematics classrooms? Specifically: a. What can the identities look like in their IBI classroom? b. What can the relationships look like in their IBI classroom? c. What can the activities look like in their IBI classroom?	Pre-observation interviews, post-observation interviews, lesson plans
3. In what ways do teachers believe the interaction between (1) their perceptions of IBI and (2) their understanding of their	

local field influences the choices they make when planning and instructing?	Pre-observation interviews, post-observation interviews, observations, lesson plans

Data Analysis

The purpose of this study was to understand how teachers define and use inquiry-based techniques in their elementary mathematics classrooms. I collected, transcribed, categorized into codes and themes, interpreted, and reported on the data. After each pre-observation and post-observation interview, I personally transcribed the responses and wrote a memo about the experience. Analyzing the planning documents and observing a lesson helped me see if the teacher used IBI in the way she described to me during the interview. I questioned the teachers after the observation to develop what Yin (2014) describes as “converging lines of inquiry” (p. 120). Based on navigational tools, the desired “converging lines of inquiry” occurs when a finding or conclusion is found in multiple sources and therefore, is more convincing and accurate. Yin (2014) states, “a major strength of case study data collection is the opportunity to use many different sources of evidence” (p. 119).

Interview transcripts, observation field notes (which included the observation protocol and written notes), memos, and lesson plans were imported into the data analysis software program Atlas.ti. I created a hermeneutic unit for each case and uploaded all relevant documents. Data analysis was continuous and followed a combination of deductive and inductive coding techniques to arrive at a rich description of the cases and the themes of the cases. First, I used deductive coding based on the literature on IBI, my theoretical framework (identities, relationships, and activities), and my own experience in the mathematics classroom. I created a codebook with these pre-determined codes (Appendix F). While reading and re-reading the transcripts, I coded each case based on the pre-determined codes. I then used open coding to develop categories within each theme. I used the network view in Atlas.ti to sort the data into categories that described similar attributes of the case. Once the data for each participant was sorted, I created codes that described each set of quotes. This inductive coding process consisted of reviewing and comparing pre-observation interview data, observation data, and post-observation responses and looking for patterns reflecting the identities, relationships, and activities of the classroom. The final codebook consists of all the codes sorted into each pre-determined code (Appendix F).

The focus of the analysis was on how the identities, relationships, and activities influenced the way each teacher enacted IBI during mathematics instruction. Data analysis continued until all interviews, document analyses, and observations were complete. The goal was to obtain well-supported conclusions grounded in the continuous analysis of the study.

Positionality

As the researcher is the primary instrument in qualitative work, it was critical for me to assume a role that was best suited to help answer the research questions. I maintained an observer role while observing in the classrooms for multiple reasons. Having a presence in the classroom was already a slight distraction for the students; therefore, strictly observing and not taking part in any part of instruction was important for me to see how the teacher implemented the mathematics instruction. Second, I have a passion for mathematics education and my experiences could have potentially influenced the teachers' instructional decisions. Last, as mentioned previously, I have connections from my previous work in the districts used in this study, and knew many of the principals in the schools. I have been a mathematics educator for sixteen years and have worked with many students and teachers that do not have a conceptual understanding of mathematics. I do not feel mathematics should be as difficult as it is for so many students. If students had more opportunities to learn mathematics using real-life problems and discussing their thinking process when solving problems, I believe that more students would not only better understand mathematics but enjoy it. Inquiry-based mathematics has tools that can help both teachers and students gain a better conceptual understanding and I would like to see more teachers using inquiry-based techniques in their classrooms. Even in today's classroom with state standardized tests, I feel it is possible for all students to have a conceptual understanding of the mathematics they are learning. For these reasons, I wrote memos after the observations I conducted to make sure my own values, knowledge, and opinions did not influence my data analysis. This increased the credibility of my findings.

Validity

Validity is a point of contention among many qualitative researchers. I agree with Maxwell (2013) that validity refers to the "correctness or credibility of a description, conclusion, explanation, interpretation, or other sort of account" (p. 122). Two threats to validity are researcher bias and reactivity (Maxwell, 2013); for example, my analysis of the data could differ from that of another researcher with the same data. I thus took precautions during data collection and analysis to minimize these threats. To ensure an adequate amount of data, I interviewed six teachers and collected lesson plans from each one. I gained the trust of the participants and made them feel comfortable by first introducing myself as a teacher. The participants were informed about the goals of my study, and ensured that this was not an evaluation of how they teach mathematics. I also told each participant that the information I gathered would not be shared with their administration. I wrote memos throughout the data analysis process to acknowledge my own values and expectations. Triangulation of all data sources including interviews, unit plans, and observations helped make valid assertions that were grounded in the data.

With multiple data sources, another threat to validity was overlooking discrepant data. I rigorously examined the supporting data as well as the discrepant data "to assess whether it [was] more plausible to retain or modify the conclusion" (Maxwell, 2013, p. 127). If I found something that did not fit with my initial conclusions, I revisited these rare instances for further analysis. I also used peer debriefing to support the credibility of my findings (Lincoln & Guba, 1985). A fellow mathematics teacher educator, Kristina Anthony, who uses inquiry-based techniques and has also worked with in-service teachers, reviewed my initial analysis. I shared the initial themes found within the cases and the conclusions I made based on them. I have worked with her for six years and she knows my thoughts about IBI and how I feel it should be used in the classroom. I asked her beforehand to keep her knowledge of my values and perspective in mind while she questioned me about my findings. This helped me further identify my biases and minimized them during the analysis phase. She reviewed the dimensions I choose to use for each teacher based on the interview responses concerning what inquiry looks like in their classroom. I shared my memos and the conclusions I drew with her to find possible areas of bias that I might have missed. I also met with her after I finished transcribing and began finding themes to stimulate discussions of alternative interpretations (Lincoln & Guba, 1985). Ultimately, I wanted the peer debriefing to increase the "credibility of the project" (Denzin & Lincoln, 1994, p. 513) and authenticate my interpretations (Green, 1994).

Chapter 4

Results

This chapter provides a presentation of the findings with details from four data sources: pre-observation interview, observation notes, post-observation interview, and lesson plan documents. First, it will offer a detailed case description for each of the six teachers participating in this study in the order in which I interviewed them. Each case includes an extensive description of the observed lesson and how the teachers define the identities, relationships, and activities within their inquiry classrooms. Next, it will combine all six cases and discuss results relating to each research question.

I discuss the first two research questions through the thematic categories derived from the data after cross-case analysis. By looking at each participant's network on identities, relationships, and activities, I listed the common themes between all six cases. As described in Chapter 2, these three elements of the field are the basis of my theoretical framework and part of my initial codebook. The codes came from a combination of deductive and inductive codes generated by the participant's words when my initial codes did not suffice. My final codebook can be found in Appendix F.

Narratives

Ms. Miller

Ms. Miller was inspired to become a teacher by her 5^{th}-grade teacher. She explained, "He did cool things and did not let me hide. I loved that." She started her early childhood degree and loved it so much that she decided to enroll in a four-year elementary education program. After teaching in New England at a Higher Order

Thinking (HOT) school for 27 years, Ms. Miller and her family moved south. She has been teaching in Smith County for five years.

I interviewed Ms. Miller in August, 2018. She requested this time, as it was the most convenient for her schedule. I observed that her classroom was bright and colorful with the desks in groups of four or five. In the 2018-2019 school year she had 23 students in her class. The interview lasted approximately 45 minutes.

I visited Ms. Miller's classroom on a Thursday in the middle of October, 2018. She wanted to wait for me to observe a lesson until she felt the students had a good grasp of the routines of the classroom and her expectations during the lesson. The class that I observed lasted for 1 hour. When I arrived for the observation, the class was engaged in an activity that had students moving around and acting out specific animals. Mathematics began a little later than usual, and students missed their snacks. The students moved their desks back to the correct setup, got their snacks, and understood that they had to change their focus to the mathematics lesson.

The lesson began with a spiral review game of Simon Says Geometry. The last topic covered was lines, rays, and points, so to review these vocabulary words Ms. Miller had the students stand up and play a game. The well-known game of Simon Says had a twist in which the students had to create the mathematics term with their body. An example she used was "Simon says make a ray." Students lifted their arms and had one hand pointing while the other was in a fist. This simulated a ray because it has an endpoint and goes forever in only one direction. The students seemed to enjoy getting out of their seats to do this activity. They were smiling and laughing while illustrating geometrical terms with their arms.

After the quick review game, Ms. Miller explained that they were going to do an activity called “What’s My Rule.” According to Ms. Miller’s lesson plan, the goal for the students during this mathematics lesson according to Ms. Miller’s lesson plan was to “define a polygon and classify figures as polygons or not polygons.” Each student received a sticky note with a figure on it. There was a large piece of paper on the wall, split into two columns, “Polygons” and “Not Polygons.” Students had to decide where their figure should go. Ms. Miller stood next to the chart and validated whether their choice was correct or not. Once all the students had placed their figures in one of the two columns, she asked everyone what they noticed about the chart. Before the whole class discussion, the students wrote down up to three things they noticed about the chart.

During the discussion, Ms. Miller wrote down what students were saying about the figures that classified as polygons and those that were not polygons. Once the students shared ideas, she wrapped up the discussion and sent them back to their desks where, in partners, they sorted different figures into the two groups. As part of this activity they had to justify the placement of each figure they placed by saying, “This is a polygon because...” or “This is not a polygon because...” After the students sorted the figures, they had to discuss the question “Why isn’t a circle a polygon?” Once they finished the discussion, they raised their hands to have their choices checked by Ms. Miller. If everything looked correct, the students were given an individual sorting assignment consisting of cutting and gluing figures at their desks.

At the end of class, most students were involved in the individual sorting assignment. Ms. Miller stopped them to check how everyone felt about their understanding of polygons. She told them to hold up the number of fingers that expressed how they felt about describing and classifying polygons (four fingers represented "I can describe and classify," three fingers represented "I can kind of describe and classify," two fingers represented "I can't really describe or classify," one finger represented "I do not understand polygons"). Students held up their fingers in front of their chest because they knew that this was an individual and private response between each of them and Ms. Miller. This is how, in her words, she "takes a pulse of the classroom."

Beliefs about IBI and the influence of the local field in the classroom

Identities. For Ms. Miller, the identity of a teacher is to be "a facilitator of learning" in the classroom. In her words, she is "not the sage on the stage but the guide from the side." She describes the teacher as the one posing questions, assessing the learning, allowing students to discover and creating a culture in the classroom where these can all occur.

In my analysis, a recurrent theme in Ms. Miller's descriptions of her role in the classroom is that of a question poser. She said, "I like to pose questions like 'I wonder' or 'what would happen if..." She said that she asks students about what they are wondering about concerning the current topic and then she talks aloud saying what she wonders about to encourage students to think deeper about the mathematics. During my observation, I witnessed Ms. Miller ask the class "What do you notice?" once all the sticky notes were up on the "Polygon or Not a Polygon" poster. She first allowed them to write down what they noticed and then asked volunteers to share their insights. During the partner activity, while the students sorted shapes, Ms. Miller walked around and asked questions such as "why did you put that shape there?" and "how do you know that one is a polygon?" As I observed the lesson, it never happened that a student asked a question to Ms. Miller about the mathematical content and she gave the direct answer. Instead, Ms. Miller posed a question back to the students for them to figure out the answers by themselves. During the post-observation interview, I asked her when she was thought she was facilitating learning during the lesson and she replied that she was doing it throughout the lesson. She said, "In the beginning, I was not telling them I was kind of facilitating and trying to get them to think and talk quietly with their partner about it." These examples support both her pedagogic style of "giving kids the tools so they can solve the problems" and her description of the teacher's identity as a facilitator.

While posing questions throughout the lesson, Ms. Miller is constantly assessing the students' level of understanding. She describes walking around while the groups discuss a question or task and asks questions based on their current thinking processes. Ms. Miller uses many Kagan Structures, and one of her favorites is "Find Someone Who." Kagan Structures "carefully engineer student interaction to maximize cooperation, communication, and active engagement by all" (Kagan, 2009, p.1). The "Find Someone Who" structure consists of two students partnering up and asking each other a question they both completed on a worksheet. Ms. Miller explains, "I like 'Find Someone Who' because the one [problem] that no one is doing is the one we do together at the end." She explained that she realized that when students avoid a problem, it is usually a tougher one that everyone needs to review. This is an example of how Ms. Miller informally assesses her students every day. She repeatedly used the phrase, "taking a pulse" to describe how she checks her students' understanding. For example, the strategies that she used during my observation consisted in walking around during partner work and the independent assignment to observe and listen to the students, asking them to put up their fingers to indicate how they felt about sorting and describing polygons and asking her Twitter question at the end of class. Every day Ms. Miller tries to ask a Twitter question to assess her students' understanding and takes the responses into account to form her groups for the following day. She asks one or two questions that the students answer independently on a sticky note. They put the sticky notes on the "Twitter Question" poster at the front of the room, and Ms. Miller collects them. Ms. Miller explains, "The Twitter question really tells me if they know it or if they don't."

Another common theme I discovered during my analysis was that Ms. Miller wants "students to think and discover" and "figure it out themselves." I witnessed this during my observation when she allowed the students to determine whether their shape was a polygon or not rather than giving them the answers. The teacher has to set the expectations for the class in order for this to happen. During the pre-observation interview, Ms. Miller explained, "It takes a while for them to know my expectations and to hold them accountable for them." This statement supports her request for having me come in for the observation over a month after the beginning of the school year. She wanted her students to know the procedures and expectations before I came to observe a typical mathematics lesson. Part of the identity of a teacher for Ms. Miller is setting the expectations and tone for the classroom.

As for what the student's role is in her classroom, Ms. Miller described students as actively engaging in what they are learning. She wants students to work together, figure out what they need to solve problems and know that it is all right to try things and occasionally get frustrated. As mentioned, the students in Ms. Miller's classroom sit in groups of four or five. This intentional grouping supports the expectation that discussion and problem solving will occur between peers. As third graders, she expects them to check their homework with each other, work on problems together and "take charge of their learning." This leads me to conclude that the identity of a student in Ms. Miller's classroom is that of one who is independent and not afraid to admit confusion.

During Ms. Miller's interview, she said that one of the reasons why she likes to teach mathematics is because "there is a lot of cooperation going on. They are learning from each other." I witnessed these relationships during my observation when partners were explaining why a specific shape was a polygon or not a polygon. There were instances in which one partner was explaining their reasoning, and the other students listened intently to the reasoning rather than arguing. It appeared that the listening student initially did not agree with the classification his peer was giving, but after hearing the explanation understood the decision to put the shape under the polygon column. This is an example of how Ms. Miller's students are learning from and with each other. The identity of a student in her classroom includes working with peers to solve problems.

During the interview, Ms. Miller described her classroom as an environment where "it is okay to make mistakes. You learn from it. If it doesn't work, try something else." During the observation, Ms. Miller was very positive in her responses to students. Rather than saying, "No that is not right" she asked the students questions that would allow them to rethink their hypotheses. She described that a favorite activity in her mathematics class is number talks. Number talks are usually single problems that can be solved in more than one way, and then students share with the class their solutions. Ms. Miller said, "Number talks are fabulous because even if they are wrong, we have talked about it and as a group have...the light bulb has gone off, and it is okay to make mistakes."

Relationships. "The power of yet." This statement describes the relationship between teacher and student in Ms. Miller's classroom. She has created a supportive environment of mutual help for the students while making them understand that they might not have all the tools to answer the questions that arise. The lesson I observed included mostly group work and discussion in which the students got along and helped each other if there was any confusion. Whether it is the relationship between students or between Ms. Miller and the students, a belief that they are all learning and exploring together is evident. I observed that students were comfortable asking each other questions about the content as well as asking Ms. Miller. There were also instances I witnessed in which students figured something out about a particular shape (concavity) and seemed excited to share their discovery with Ms. Miller.

Ms. Miller does not act like the only person in the classroom that has the answers. She often tells them she is wondering about something and asks them to explore to find the answer. While I was in the classroom, she said, "I am wondering what characteristics a shape has to have in order to be named a polygon." During my observation, I also witnessed the students wondering aloud about the characteristics of a polygon. I assume they feel comfortable doing this so that their peers can piggyback on the thought process of the group. Some of the students' statements were wrong, and their peers respectfully countered with a different explanation or discovery. The relationships in Ms. Miller's classroom are respectful and honest.

Mutual respect between Ms. Miller and her students allows them to express what each individual needs to foster their understanding. Ms. Miller told me that her students are comfortable telling her that they need to be in her group during mathematics centers. “A few kids might say, ‘Ms. Miller can I be in your group because I am really struggling with this today.’ They need to own it.” She does small group teaching during learning centers to those students still stuck on a concept. Allowing her students to tell her they need help and having an open dialogue about the students’ mathematical understanding is how I would describe the relationships in her classroom. During the pre-observation interview, she described what she does when a student is confused:

> I ask them "What do you think you could do? Maybe get some advice or help from somebody at your table or partner." If everybody in your group has that question, you need to show me what you have done first. You have to prove to me that you have tried different things. And then I might say, "Well what about..." And sometimes I stop the whole class and say, "We are stuck over here and I'm not sure ...anyone have an idea."

This quote illustrates how Ms. Miller models problem-solving strategies, and what learning looks like in her classroom. She checks-in often with students to see what they understand and if there are any misconceptions. I observed a student saying, “I don’t know about this one” talking about one of the shapes they were sorting. This leads me to believe that her students feel comfortable saying that they are confused. She wants all of her students to “own their own learning” and be responsible learners. She modeled this throughout the math block by allowing the students to explore and investigate polygons.

Activities. Ms. Miller’s mathematics class has a balance between collaborative problem solving and independent practice. Allowing the students to discuss the content using Kagan Structures such as “Rally Coach” and “Find Someone Who” (collaborative) while asking a Twitter question (independent) at the end of each mathematics class. Ms. Miller assesses her students daily. She told me that she uses various manipulatives like tangrams and base ten blocks to allow the students to use concrete models and discover on their own. Once she feels that the students have had enough collaborative practice and discussion about the topic of the day, she makes sure to include an independent practice of some type. Her Twitter question is specific, and each student answers on his or her own sticky note. This feedback is how she creates groups for the following day. Overall, the most used activities in Ms. Miller's classroom during mathematics are questions, problem-solving, and exploration. To maximize engagement, activities are usually collaborative between peers.

Ms. Miller uses questions to assess her students understanding and to get them thinking in different ways about the mathematics. During the pre-observation interview, she told me, "I pose questions all the time." I witnessed Ms. Miller constantly asking questions and encouraging her students to ask each other questions throughout the polygon lesson. During the observation, I watched one pair of students ask each other why they placed a shape in a specific column. Ms. Miller has created a classroom environment in which the students feel comfortable questioning each other and defending their thought processes.

She tries to infuse problem-solving into every mathematics lesson. While I was observing, the students were engaged in problem-solving to figure out which shapes were polygons. Ms. Miller instructed them to defend their placement of each shape. When she heard most of the students using the same explanation for every shape ("It is a polygon because it has straight sides"), she stopped the class and edited the instructions asking them to say two reasons why each shape was or was not a polygon. This change on the spot illustrated the caring relationship she has with her students. It shows she was listening to their interactions and wanted to stretch their mathematical abilities. This led to more problem solving for the students because they had to figure out which qualities were accurate to explain each shape.

Ms. Miller often tells her students “I wonder” with a specific idea she wants them to investigate. She also encourages her students to wonder aloud and investigate the questions they have. “I really work with them. ‘I am wondering if I could...’ and maybe you can and maybe you can’t. Let’s explore it.” She feels that elementary students are naturally curious and likes to harness that to enrich their learning experience. Many of the activities Ms. Miller does are in reaction to a student’s idea and interest in a topic. When I asked her in the post-observation interview what part of the lesson consisted of exploration, she said that is was the sorting shapes with a partner. She explained that they had to figure out where each shape belonged and explore new shapes they had not seen before.

The interaction between perceptions of IBI and understandings of local field

During the pre-observation interview, Ms. Miller said she appreciates the support at her school and at the district level. She described her administration as “spectacular” and says that if she wants to do an activity for which she does not have what she needs, they try to get it for her. There is a mathematics coach accessible to all teachers in the building and a curriculum specialist for elementary mathematics at the district level. She explained that any new information from the State is always shared with the teachers and she feels well-informed of any changes. Speaking of the curriculum specialist for mathematics and the mathematics coach, Ms. Miller said, “They are available. That is huge. They are always available.”

Ms. Miller said that there is not a mathematics program in her district, but there are online resources that the district provides. She feels open to trying new and different activities in the classroom:

> We are encouraged to take risks. We are encouraged to try something new, and if it didn't work, it's just as important to know what you will never do again or know that you will regroup and go back to this tomorrow. It is valued.

Ms. Miller’s planning documents are very detailed and illustrate the thought she puts into each lesson. The document lists the standard along with a link to the essential knowledge and skills identified by the State Department of Education. She lists the vocabulary and anticipated misconceptions at the top of the document. After that, she lays out the activities for the lesson in detail. By looking at her planning documents, you can see the time and thought she puts into each day. She said that her administration does not look at the teacher’s lesson plans consistently. This shows a trusting relationship between administration and teachers in the building.

From the observation and interviews, I can conclude that Ms. Miller feels supported and trusted by her administration to use what she feels is the best mathematics pedagogy in her classroom. She tries new activities in her classroom because the local field she is a part of allows her to experiment. She is confident in the strategies she uses and feels her students can easily adapt to them if they have not been in a similar classroom before. Ms. Miller uses the first month of the school year to set standards and expectations for her classroom. This allows her students to become comfortable with how mathematics will look during the rest of the school year.

Ms. Washington

Ms. Washington is the only career switcher in my study. She worked in an energy clearinghouse before realizing she wanted to teach. After going back to school to get her teaching degree, Ms. Washington taught in a few different states before landing at her current school, which she has been at for five years. As a fifth grade teacher, Ms. Washington enjoys watching the many different ways students get to the meaning of the mathematics.

I interviewed Ms. Washington in early September and observed two weeks after the pre-observation interview. She only teaches Mathematics and Science for fifth graders. She has two groups of students, each for half of the school day. Her morning class is an advanced fifth-grade mathematics class that learns sixth grade content. I observed this class while they worked on integer operations. The class did not have individual desks but instead tables with four or five chairs at each. Ms. Washington explained in the pre-observation interview that having tables allows the students to discuss and share with their peers. "If they can learn to interact with each other a whole new world opens up."

Ms. Washington began the class asking the students what they know about integers. Students offered examples (4, -2), non-examples (1/2, 1.5) and a definition ("whole numbers including negatives"). Students seemed confident and comfortable with what an integer was, and it was obvious they have talked about them in class previously.

Ms. Washington explained what the activity of the day was going to be while also reminding the students of the available resources.

Each table had a bucket of manipulatives that included base ten blocks, dice, chips, and two-color counters. She also pointed out the bookshelf of other manipulatives if they wanted to use something that was not in the provided bucket. Ms. Washington asked the students, “What can you do if you don’t know where to start?” Many students raised their hands, and Ms. Washington called on a few. The students said, “ask a tablemate” and “draw a picture” and “repeat what you already did.” I wasn’t sure what the students meant by the last suggestion but Ms. Washington then said, “You are still learning when you try the same strategy.” She handed out a worksheet with four word problems on it and asked them to read the first problem to themselves.

This was the first word problem:

> A scuba diver sits at the surface of the water. He dives underwater 20 feet to watch some fish. A shark comes his way, so he swims back up 20 feet. The shark continues to swim toward him, so he climbs 10 feet higher into the boat. What is the height of the scuba diver at the end of his dive?

The other three problems involved an airplane, a football play, and a roller coaster. Ms. Washington asked the students to circle words or information in the problem that might be important. Once she saw that many of them circled the numbers in the problem and the question at the end she let them get started. The students worked by themselves, but many asked questions to their tablemates or shared manipulatives with them. Some questions I heard between the students were “How did you come up with that?”, “What was your strategy?”, and “What came to mind when you read the question?”

Ms. Washington walked around while the students worked. She looked at their work, praised some (“way to think of two different ways”), redirected others (“math talk please") and asked questions ("how did the chips help you solve this?"). After approximately twenty minutes, Ms. Washington had circled the entire classroom and felt that everyone had at least one strategy and an answer. She asked students to share their strategies on the Smartboard. Two students went up and drew their solutions. Each one had a symbol for the scuba diver, and both showed arrows going down twenty feet and then up twenty feet. One of the students' drawings stopped there while the second one drew a square to represent the boat with an arrow to the top of the square, representing the ten feet at the end of the problem. The students explained their pictures and Ms. Washington thanked each of them for sharing.

At this point, Ms. Washington had not revealed yet what was the final answer is to the problem. She drew a vertical line on the whiteboard and had the students walk through the problem with her. She explained, "Each time the scuba diver makes a change there needs to be a mathematics symbol to show the change." She reads the problem aloud while mimicking the movements on her vertical line and writing -20 + 20 + 10 on the board. She asked the students what the final answer was, and they all said, "Positive ten."

During the rest of the mathematics class, the students worked through the other three problems, and Ms. Washington walked around asking them questions and observing their work. She encouraged them to use a strategy that made sense to them. The plan for the following day was to use these four problems to come up with rules on integer operations.

Beliefs about IBI and the influence of the local field in the classroom

Identities. Ms. Washington describes the teacher in an inquiry classroom as "a facilitator." The teacher is the one who plans the lesson, allows the students to think and keeps the overall goal in mind. For Ms. Washington, the overall goal is to "figure out how to problem solve" and to "think critically."

The planning that goes into Ms. Washington's inquiry lessons is more extensive than a regular math lesson. She explained during the pre-observation interview "the planning has to be very, very purposeful." This purposeful planning was evident in her planning document. Ms. Washington includes students' misconceptions, the main lesson objectives and an "I Do, We Do, You Do" instructional plan for each mathematics lesson. "You have to think of all the ways where they could possibly go astray and then how to bring them back in." Ms. Washington's inquiry lessons allow the students to explore the concept, which means there are opportunities for students to connect to something they already know or discover new ideas that they want to explore. Both of these activities have side effects that can lead students to get off track and the teacher needs to get them focused on the goal of the lesson. According to Ms. Washington, this is the hardest part of inquiry pedagogy.

According to Ms. Washington, inquiry means allowing the students to think for themselves and truly explore. The teacher needs to reserve time for this exploration but also be aware that the aim that the teacher has in the onset of the lesson might not be where you end up.

> You could have this awesome lesson planned and all it takes is one kid saying, 'Well what about..' And then the whole class goes in that direction and then the time is up and you say 'Let's come back tomorrow.' It's hard to overcome.

Stepping back and seeing where the students' discussion goes is part of the teacher's identity in Ms. Washington's inquiry classroom. She believes "they can figure it out, make their own connections." As a fifth grade teacher, it is often hard to get the students to understand that she wants them to explore. "The kids come to me in 5th grade, and it is already ingrained in them to ask 'What about the [standardized] test?'" During the observation, Ms. Washington walked around and complimented students on their work, but did not answer any questions about integers.

While walking around the classroom and observing her students' work, Ms. Washington said she always keeps the overall goal in mind. Rather than focusing on correct or incorrect answers, Ms. Washington is figuring out how to take their ideas and focus future lessons on the standards they need to learn. More than reaching the standard though, Ms. Washington wants her students to become problem solvers who can think critically about a given topic.

As for the student role in Ms. Washington's classroom, she just wants them to do the thinking and try to come up with various strategies for solving problems. She describes the students as "the guiding heartbeat" of the classroom and feels that if they are engaged in the activity, they will learn something. She expects them to make connections, learn from their peers and ask questions. Ms. Washington summed it up by saying, "Inquiry is about them [students] thinking for themselves" and "it is a lot more fun that way."

Relationships. During the pre-observation interview, Ms. Washington talked about students helping other students in the classroom. She uses Kagan strategies to increase discussion between students. She wants students to practice listening to their peers and “feed off each other’s ideas.” This discussion can get loud, but Ms. Washington explained, “I know they are definitely engaged. Engagement is not a problem in here.” The relationship between her students is an important part of Ms. Washington’s approach. To support the discussion and relationship-building in her classroom, Ms. Washington has traded in her desks for tables.

Each table has four or five students sharing the space. During my observation, the students shared the space on the table without any issues. They figured out where to put their binders and books, so they were not in their peers’ way and spoke respectfully to each other if they needed to move an item. Ms. Washington has created an environment in her classroom that fosters courteous relationships between students. “If they can learn to interact with each other a whole new world opens up.”

The relationship between the teacher and students in Ms. Washington’s inquiry classroom is also characterized by respect and understanding. It was apparent that the students in her class understand that this type of learning is a little different from what they have experienced in previous mathematics classrooms. Ms. Washington explained in the pre-observation interview that when she meets the students in September “they are all, ‘What is the answer, she wants me to say, and I'll say that.’” It is a transition from a direct instruction model to Ms. Washington’s inquiry model. Both the students and the teacher understand this and Ms. Washington jokes with them about “the struggle.” When I was observing, one way Ms. Washington got the students attention was to say, “The struggle is?” and the students responded with “real” to which she responded, “and good.” This mutual understanding between the teacher and students in her classroom is necessary for Ms. Washington to be able to introduce and use inquiry during her mathematics block.

Activities. The activities in Ms. Washington's inquiry classroom are used during units in which Ms. Washington feels that inquiry is possible. She admitted that she does not use as much inquiry during mathematics as she would like but that this also depends on the topic. When she is using inquiry, there are many discussions, hands-on learning, problem-solving and using various strategies to solve problems.

Ms. Washington wants her students to explore during mathematics but also discuss. The discussion needs to be on what they see and what they don't see. Ms. Washington said her mathematics classroom is very loud. "Everyone knows something about everything. You don't know all of it, but you know something. Starting there and having that conversation is awesome. They are talkers." When I observed, every student was exploring the manipulatives and talking with their tablemates. The discussion was focused and only occasionally off-task. Ms. Washington knows that sometimes students can get off task. She does not get too worried but instead asks guiding questions to get them back on track. "We do some Socratic seminar too, so we are always coming with those guiding questions about how to bring it back, not to center necessarily, but to bring it back to the goal, the overall objective."

I observed a lot of hands-on learning while I was in Ms. Washington's classroom. The bucket of manipulatives on every table for the students to use was a way for them to use concrete objects to show their thinking. Some students used base ten blocks, and others used two-color counters to illustrate integers. There were no restrictions on which manipulative to use or how to use them to show their thinking.

The word problems Ms. Washington used included topics not familiar to some students. She described scuba diving, football, and roller coasters so that all students, including her English as Second Language (ESL) students, would have an understanding of the context. If a student solved a problem using one strategy, Ms. Washington listened to the explanation, asked some questions to be sure the student understood and then challenged them to find another way to solve the same problem. The students with whom this exchange happened seemed excited and not at all upset that they needed to find a second strategy. This inquiry activity in Ms. Washington's classroom went smoothly, in part, because of the relationships that have been formed between everyone in the classroom.

The interaction between perceptions of IBI and understandings of local field

Ms. Washington has two mathematics classes each day. In one of her mathematics classes, she covers fifth-grade content and in the other, she covers sixth-grade content. During the pre-observation interview, she described how hard it is to deal with students that have different levels of understanding. "I have some students who based on their growth assessment are on a 1st or 2nd-grade math level. So it's really hard when I am trying to get them to really dig in to something and they are missing that basic knowledge, the basic building blocks." Ms. Washington feels that she uses activities that are naturally differentiated so that the students can all begin solving the problem. She also arranges the students so that each table is heterogeneous. With her focus on discussion and working together, she feels that students at different levels can all contribute to the conversation.

Ms. Washington does not feel that there is any support in her school for using inquiry in the math classroom. While she does not feel she is the only teacher using inquiry she does not plan with other teachers that use similar methods. She also feels that the district does not encourage this type of pedagogy through professional development. “It's sad that there is no professional development on ‘this is how you could do it.’” She explained that the professional development activities in which she was involved recently was on trauma-informed care. Ms. Washington understands that it is important to focus on “Maslow's needs before you are focus on that higher order thinking.”

The Ms. Washington’s district provides plenty of resources for mathematics, but she explained, “But it is more surface level, so it is up to the teacher to take it further.” It was apparent to me that Ms. Washington had many manipulatives for her students to use. She also described conversations on number sense and tasks that the district provides on their intranet. Although they are real-world problems, Ms. Washington feels, “it is pretty narrow focused - it is not true inquiry.” She feels that the activities involve higher order thinking and critical thinking tasks but not the inquiry style. Her district has an “I Can, We Can, You Can” model for their lessons and Ms. Washington feels that inquiry is “the opposite of what they tell you to do in [District B].” For this reason, she does not feel that the way she does inquiry, letting the students explore, is supported.

Ms. Thomas

Ms. Thomas was National Board Certified in 2010 and renewed this elite title in 2018. She enjoys the ‘A-ha’ moments in her mathematics class and the situations in which a student is applying what they learned to a different context or problem. Ms. Thomas has taught for fifteen years in the same school in fourth or fifth grades.

I interviewed Ms. Thomas at her home in mid-September and observed her class the same week. I observed Ms. Thomas while the fifth-grade students were working on double-digit multiplication (fourth-grade standard). Ms. Thomas finds that students still struggle with the computation in fifth grade.

The mathematics class began with a word problem that each student had to glue in their mathematics journal. Once everyone was ready, Ms. Thomas read the word problem aloud and went over the details to make sure the students understood the problem.

> Shawn has 440 baseball cards so far. By the end of the month he wants to have 500. The cards come in packs of 5. The first week of the month he bought 4 packs. The second week he bought 3 packs. The third week he bought 2 packs and the fourth week he bought 2 packs. Did he reach his goal of 500 cards?

"What is something we know? Talk to the people at your table." Ms. Thomas wanted to make sure that the students talked to their peers about all of the numbers in the problem before they started. She then told them to solve it on their own in their journal. Ms. Thomas walked around looking at the students' strategies and asked questions when there was something written down that she did not understand. She told me later that she likes to walk around so that she becomes aware of the students' strategies. Based on her goal for the lesson, Ms. Thomas can refer to her notes on what she saw from the students and decide on the order the strategies will be presented.

Ms. Thomas drew a large square on the Smartboard divided into four smaller squares. She calls this the UPS Check method and wants the students to use it when they are solving word problems. In the upper left-hand corner is the 'Understand' square. For this section, she asked the students to tell her the important information from the problem. The students responded with the overall question and the given weekly amounts. Ms. Thomas wrote, "Did he reach 500 cards?" and "1st week – 4 packs, 2nd

week – 3 packs, 3rd – 2 packs, 4th week – 2 packs" in the 'Understand' square. In the upper right-hand square is the Plan section. In this square, the students are supposed to draw a picture of the situation or write the operation(s) they will use before they actually solve the problem. For example, one student draw rectangles that represented each pack of cards with the amount written inside each one.

For the 'Solve' square in the lower left-hand section, she called on a student who described solving the problem by multiplying each week's total packs by five, getting the total by adding those products and then comparing that number to 500. A second student contributed by saying that it was necessary to add all of the packs and then multiplying by five. There were also two students who tried to use a bar model that they had been using for other word problems. A bar model is often used for addition and subtraction problems. The students who tried to use this method could not get it to work and asked Ms. Thomas to show them how to use the method for this specific problem. She showed them how to apply the method to the baseball card problem and explained that the bar model might not be the most efficient strategy.

The last square in the UPS Check method is the 'Check' square. In this section, Ms. Thomas has the students check their answer. She asked volunteers to share how they checked their work. One student counted by five's starting from 440. The student who shared the first strategy of multiplying each week's total by five checked his work by multiplying eleven by five and then adding that to 440. Another student who did not talk earlier asked Ms. Thomas to come over and see his work. She looked at his journal and complimented him on his strategy. He asked if he could share his answer with the other students using the document camera and she agreed. When the student put his journal under the document camera, he had numbers and arrows all over his paper. He explained his work to his peers, and it was apparent he understood what was on his paper and got the correct answer. Ms. Thomas concluded this part of the mathematics lesson by telling the students she was impressed with the many strategies that were used to solve the word problem.

The next part of the class was working on double-digit multiplication. Ms. Thomas wrote a multiplication problem (38 x 12) on the Smartboard. She showed the procedure for solving the problem using partial products. Students used a box like the one shown below when learning about partial products.

	30	8
10		
2		

Ms. Thomas solved each partial product (30 x 10, 30 x 2, 8 x 10, 8 x 2) and wrote the answers in the corresponding boxes. She then added all four partial products together to get the final product. Next, she solved the same problem but used the traditional U.S. algorithm (see below).

$$
\begin{array}{ccc}
 & 3 & 8 \\
\text{X} & 1 & 2 \\
\hline
 & 7 & 6 \\
3 & 8 & 0 \\
\hline
4 & 5 & 6
\end{array}
$$

Once she showed both strategies, she told the students, “you decide which one you like best.” Before she handed out a worksheet with eight two-digit multiplication problems on it for the students to practice, she connected the two strategies. Ms. Thomas put the two strategies next to each other and showed where the 76 and the 380 came from in the traditional U.S. algorithm and the partial products method. The students copied both problems in their mathematics journals so they could reference them if needed. The rest of the class time was used for practice on two-digit multiplication problems.

Ms. Thomas divided the students into small groups to work on the problems together. She had five students with her at the table and a special education teacher who comes in during the second half of the mathematics class took five students to a back table. The rest of the students worked in groups of two, three or four around the classroom. The students that were with one of the teachers had guided practice in that the teachers would ask them what their steps were and if they made a mistake, the teachers addressed it immediately. The students that were working in groups checked their answers with each other, and if they matched, the students assumed they were correct.

Beliefs about IBI and the influence of the local field in the classroom

Identities. For Ms. Thomas, being a teacher is all about encouraging the students to work hard and praising them when they do it. During the pre-observation interview, she said, “the teacher's job to make sure everyone is actively engaged and praise, praise, praise. Lots of support because they will shut down if they feel like they are going in the wrong direction or they will get frustrated easily.” The identity of a teacher for Ms. Thomas includes being a cheerleader for her students, supporting them in various ways, being the content specialist, questioning the students and reflecting on the lessons she prepares.

While I was observing Ms. Thomas' mathematics class at the beginning of the school year, I could not help but think that the students must feel very confident in her classroom. She encouraged the students to share their thinking with their peers and her. Her reaction to every student's work, whether correct or not, was encouraging. Some examples of what she said to various students were, "I love how you showed your work," "Thank you for checking your answer," "I like your strategy for keeping track of all those numbers" and "Beautiful way to show your thinking." She always began with a positive reinforcement and then if something was off with their mathematics she asked a question to make them think about their error. This strategy kept a positive tone in her classroom for the entire mathematics period.

The positive reinforcement was just one way Ms. Thomas supported the students in her classroom. Other strategies included asking the students to work independently while supporting them with readily available manipulatives, reminding them of past activities or A-ha moments in the classroom and encouraging questions. She used word problems that involved real life so every student, no matter their level of competence, could at least get started on the problem by drawing a picture or acting out the problem in their heads. Ms. Thomas sees herself as a coach in the mathematics classroom, which justifies her recurrent use of encouragement and support for the students. When I observed the class, she gave a mini-lesson on multiplying two-digit numbers and then had the students practice this skill in whatever way made sense to them. She showed them two different strategies (traditional U.S. algorithm and partial products) and then allowed them to choose the one they wanted to use. This mini-lesson/practice approach did feel coach-like because she did not give them too many things to practice at once but rather focused on one concept.

Ms. Thomas' view of inquiry mathematics sees the teacher as the content specialist who teaches the strategies that the students should then practice. This did not feel like direct instruction for a few reasons. The first reason why it felt more student-driven was that students choose the strategy that worked best for them. Secondly, she illustrated the connection between the two strategies rather than just teaching two separate procedures for the same problem. Even though her lesson plan states, "I am teaching two methods" and in the pre-observation interview she said, "On Thursday I will talk about partial products and multiplication instead of going straight to the U. S. algorithm," I felt she focused more on why the two methods worked rather than the procedures for each.

While explaining the two methods, she often questioned the students on where the numbers were coming from and how they knew they had the correct answer. This questioning is part of her identity as a teacher. While most of the students were doing the worksheet, Ms. Thomas worked with a small group at the back table. She guided and coached the small group of students just as she had done with the whole class. Her identity did not change just because she had a smaller group of students that needed more time with the teacher.

"It just takes a lot more time to teach inquiry." This quote describes the time commitment of Ms. Thomas so that she can teach inquiry mathematics. She explained that the teacher comes up with real-life scenarios rather than using "naked number" problems (problems with no context) which takes longer. In addition, planning how she will support her struggling students takes time. She does not just repeat what she said in the large group to the small group of struggling students but instead guides them asking questions to get them to think.

In Ms. Thomas' mathematics class, the students are "active learners" and "asking lots of questions." During the pre-observation interview, she explained,

> I find that the kids that are asking questions are the ones who are learning. The ones who have a question but don't ever ask it, they are not learning. They are still lost in confusion. I guess they are passively learning, because they might sometimes make connections based on what someone else is saying. But not that they thought out that answer.

Based on this idea, Ms. Thomas counts participation as part of the grade for each student. She encourages students to talk to each other, talk to her and reason on problems rather than just sit there passively. For Ms. Thomas, active learning equates to talking about mathematics and working together.

Ms. Thomas has a Smartboard and a document camera in her fifth-grade classroom. Both of these are tools she uses for students to share their work (document camera) and save previous strategies used by the students (Smartboard) so that the class can go back to them. When I observed, it was evident that the students were used to working together during mathematics. They were respectful, friendly and willing to answer each other's questions. One group had four students working together with half of them using partial products and the other half using the traditional U.S. algorithm. They looked at each other's' worksheets to see if they got the same answer and to find out whether the strategies "matched," i.e., whether the results were the same.

Relationships. In both interviews, Ms. Thomas spoke about the relationship between the teacher and the student. Whether it is "holding their hand" while they work through a problem or “watching them grow” as they discover a concept for themselves or “wanting them to know it is all right to make a mistake,” Ms. Thomas cares about her students and works to foster a trusting relationship.

During the post-observation interview, Ms. Thomas explained that she wished “she could clone herself because I realized that a lot of kids were struggling with their multiplication facts.” One student that was at the back table with Ms. Thomas was a new student to the school. Ms. Thomas asked her how she learned her multiplication facts in her old school so that she could support a strategy that worked for the student. This example shows what type of relationship she tries to create with each student. Rather than teaching the student a completely new way to multiply, she decided to support a familiar strategy for the student.

The relationship between Ms. Thomas and her students was evident during the observation when she allowed one of her students to share his strategy with the class. It was obvious that Ms. Thomas was trying to wrap up the discussion on the baseball example when the student asked her to see what he did in his journal. She looked at his journal and complimented him on his hard work. As she was walking away, he asked if he could share his work with the class. “If they ask me I try to let them share rather than saying, sorry we don’t have any more time.” By allowing him to share, she showed this student that he was important to her and that his strategy was just as effective as those previously shared.

Activities. The activities that took place in Ms. Thomas’ mathematics class mirror the identities of the teacher and student in her classroom. During the pre-observation interview, she described problem solving, examples, and questioning multiple times as the main activities in her mathematics classroom. These are also contained in the descriptions she gave during the pre-observation interview of the teacher and student roles' in an inquiry classroom.

Ms. Thomas described the teacher as the one coming up with word problems and scenarios that mimic real life and the student as the one doing the thinking in the classroom and solving the problems. Therefore, it is not a surprise that when I asked about how often she uses inquiry in her classroom, her response was "I'm asking questions and using stories every single day." For Ms. Thomas, using word problems daily is part of inquiry pedagogy.

Examples in her inquiry classroom include both students' examples of how they solved the problem and the different types of examples Ms. Thomas comes up with for her students. She wants students to "see multiple right answers" for the same problem. Most of the time students come up with various ways to solve a problem and share them with their peers. Sometimes Ms. Thomas led them to another strategy. While I was observing, she asked the class, "did anyone estimate for this problem?" The word problem examples need to be accessible to all students. She explained, "The point is to make it very low entry so that even a child who is not strong in math can get something." In the post-observation interview, she reflected, "I think if I did some more examples they would feel more comfortable with partial products. I think I am going to scrap the lesson I had planned for Monday and do more examples showing the connections between the two ways." Ms. Thomas believes that examples are a crucial piece of inquiry instruction.

Asking questions allows Ms. Thomas to assess her students' level of understanding. When asked about how she knows that learning is happening in her inquiry classroom she responded: "I think talking to one another, asking questions, it is not always that [student's understanding] comes out on paper." She also feels that students' questions, to her or their peers, are useful to assess their understanding. "Sometimes when they are questioning one another I can tell they are learning." An essential activity in Ms. Thomas' inquiry classroom is questioning.

The interaction between perceptions of IBI and understandings of local field

Ms. Thomas reported that she has all the necessary materials to enact inquiry in her mathematics classroom. She described having many manipulatives in a closet that all the teachers can use. Having the materials readily available is helpful in Ms. Thomas' inquiry mathematics class. Ms. Thomas could not think of any other supports at the building level. At the district level, she felt that the professional development for mathematics includes "little inquiry." This year teachers are watching videos that align with the topic they will teach. She described it as, "Mainly pedagogical with what would be an effective strategy for teaching this rather than give them these worksheets and show them this video and you're done."

Ms. Thomas claimed that she is one of only a few teachers in her building that use inquiry in the mathematics classroom. For this reason, there are not many other teachers to plan with or discuss possible activities. The mission of the school has to do with excelling and succeeding. Ms. Thomas feels that the mission is very general. She wants all her students to excel and succeed and said, "I have wholly expectations they are going to gain a lot of ground, they are going to grow a lot from when I started with them."

Ms. Smith

Ms. Smith has been teaching for twenty-nine years. She likes teaching mathematics because “the kids are the ones learning instead of me deciding what the kids need to do.” Teaching mathematics using inquiry is important to her because she feels that all kids can feel successful and “see themselves as mathematicians.”

When I contacted Ms. Smith to ask her to participate in my study, she was happy to help. We met at a local eatery per her suggestion. When I began asking questions about inquiry mathematics, she admitted that the term was not familiar. Ms. Smith told me about how she taught mathematics to her second graders and asked me to wait until mid-October to observe. She explained that she was still introducing the rotations in mathematics and by October, the students would have better understood the routines.

When I observed Ms. Smith’s mathematics class, she sent the students straight to centers. The three centers were:

1. Computer Center – Dream box (adaptive mathematics software)
2. Games – Shake and Spill, Three in a Row or Even to Win
3. Table with the teacher – Graphing activity with Ms. Smith

The students were at each center for twenty minutes. During the games, center students choose which game they wanted to play and with whom. Ms. Smith had given each student a magic number. The magic number was the number they used as the sum during Shake and Spill. Students put their magic number of two-color counters in a cup. They shook the cup and spilled out the counters. In their math journal, they wrote the addition equation that matched the shown chips. For example, if the student spilled three red counters and four yellow counters the equation was 3 + 4 = 7.

Three in a Row was a game made to practice subtraction facts. The digits from zero to nine were at the bottom of the game board with two paperclips, each one on a number. The main portion of the game board was Bingo-like with twenty-five squares. Each square had a number on it. Each player took turns moving one paperclip to a number and finding the difference. They marked the difference on the board with the goal of getting three differences in a row. Two people at a time played the game.

For the Even to Win game, the students used a deck of cards with the face cards removed. They flipped over two cards and found the sum. In their math journals, students would write the equation and identify whether the sum was even or odd. Either groups or single students could play Even to Win.

Ms. Smith worked on graphing with the students at her table. The previous day student had collected data on their favorite colors and made a pictograph. The day I observed, they created a bar graph using the same data. Ms. Smith was telling them step by step how to complete the bar graph. The y-axis had to count by two's in order for each bar to fit on the graph. Some of the data they had did not fall directly on a line and students struggled with how to represent odd totals. For instance, the number of students who like red was five, but there was no line for five. Instead, students needed to understand that halfway between the line for four and the line for six would represent five. Most of Ms. Smith's time was used explaining this to each student as they got to that part.

Beliefs about IBI and the influence of the local field in the classroom

Identities. In Ms. Smith's mathematics classroom, the teacher is a facilitator who is "orchestrating what will take place." Not only is the teacher setting up the classroom for inquiry mathematics to take place, but she is also giving the students' freedom to explore, and she is the content specialist. For Ms. Smith, this means that there will be some direct instruction during mathematics.

When Ms. Smith describes the identity of the teacher as a facilitator, part of this consists of having a supporting role for the students. "The teacher is the one that lets the kids know that they can explore and learn." She also described wanting her students to know that they are mathematicians and can solve the problems she gives them if they try hard. Another part of being a facilitator for Ms. Smith is getting the environment ready for the students to learn. When I asked Ms. Smith in the post-observation interview about how she facilitated learning the day I observed she said,

> I think I set up the room and the environment so that the kids were really in charge of their learning. I think I could step away from the group and they would be doing the same thing as if I were sitting there. I think I could step out of the room and probably 90% of the kids would not know I walked out.

While I was in Ms. Smith's mathematics classroom, I saw the teacher as the content specialist. The students graphed data they had collected on their favorite colors. Ms. Smith told them the type of graph they were going to create, explained how to set up the graph, and then the students made each bar on the graph. The direct instruction I witnessed supports what Ms. Smith explained during the pre-observation interview about some topics in mathematics. When I asked her how often she uses inquiry in her mathematics classroom her response was, "It depends on the topic; some units might be different because they are very content focused." Graphing, for Ms. Smith, is one of those topics that does not lend itself to inquiry pedagogy. As the content specialist, Ms. Smith also described teaching a game or activity and keeping the kids that "don't get it" with her and “working with them until they get it.” The identity of the teacher is the content specialist and the student is the independent learner, once they understand the activity.

The identity of the student that Ms. Smith described is one of an independent, confident learner who engages with the material, talks about mathematics and follows directions. Ms. Smith described a students’ role as “just to be on target, their role is to know that they can do it, that they know they can make mistakes, that they should be talking and they should be talking about math.”

The previous quote about Ms. Smith stepping out of the classroom and most students not noticing it is the type of independent learning that she expects during mathematics. Once she introduces a center activity, be it a game, computer program, or independent activity, she expects all of her students to be able to do the center by themselves. If they can do the center without asking her questions, she feels they are “on target,” engaged and understand the mathematics underlying the activity. On the other hand, at the table with her she wants students to be talking about mathematics and engaging with the material.

During my observation, the students were creating bar graphs with the y-axis counting by two’s. This was a new concept for the students, and they asked many questions about the activity. In the post-observation interview, when I asked Ms. Smith how she thought the lesson went her response was, “I felt like they were engaged. I felt like they had their hands on tasks and they were talking about math. I felt like they were focused.” This aligns with her explanation of the identity of a student in an inquiry classroom.

Relationships. During the pre and post-observation interviews, Ms. Smith talked mostly about the relationship between the teacher and the students. Although she also talked briefly about students working together, the interactions between students that I witnessed focused mainly on the rules of the game or activity rather than mathematics.

The identity of the teacher as the content specialist and the identity of the student as a confident, independent learner led to a small group direct instruction lesson and independent student centers. During the interviews, Ms. Smith spoke about "giving the student's freedom to explore" and "allowing them a choice in what they are doing." This understanding between the teacher and the students gave the students the impression that she trusted them to choose wisely and stay focused on the task. She reinforced this impression with her behavior management system. "In order for them to get the reward, they need to play the game the right way." The teacher as content specialist leads to the mathematics instruction coming from the teacher and not the independent centers.

During the interviews, Ms. Smith talked about students "getting a lot of direct intervention" and her "just really staying on top of it so that if they were making a mistake, I could catch it and then they can continue on and do the rest of it correctly." The idea of the teacher as the content specialist led to a form of direct instruction in Ms. Smith's mathematics class. The students looked at her for answers, and she explicitly told them what to do. The students were comfortable asking her questions about how to create the graph but did not make any inquiry working. She did not ask the students many questions while they worked but instead told them the next step if they were stuck. I observed Ms. Smith's identity as a content specialist but did not see the inquiry she described in the pre-observation interview. This could have been due to the topic she was teaching when I came to observe.

The day I observed the topic was graphing. Ms. Smith said in the pre-observation interview that there are some topics in which it is hard to incorporate inquiry. Number sense and operations are topics that Ms. Smith considered easier to teach using inquiry.

Activities. The activities that occur in Ms. Smith's inquiry mathematics class vary from day to day. She usually begins by posing a word problem for the students to solve and then moves into center rotations for the other mathematics block. The activities or games at the centers can change weekly depending on what Ms. Smith wants the students to practice. When I observed she did not present a word problem but instead sent the students directly to the centers.

Ms. Smith explained that while in the centers, students are exploring and talking about mathematics. Students' exploring is important to Ms. Smith as an inquiry mathematics teacher. When I asked what she thought inquiry mathematics was, she said, "I think that the kids are exploring, they are having fun, they are discovering things, and they do not even realize that they are discovering things." Ms. Smith had students working independently at centers for most of the mathematics block. While I observed students were playing games and recording in their math journals, but I did not see any exploring.

During the pre-observation interview, Ms. Smith described some activities as being part of her normal routine. The variation in her mathematics block did not allow me to witness a problem of the day, a word problem or a "teacher task" because I only observed one lesson. I observed Ms. Smith teaching graphing and connecting it to the data collection the students did the day before. There was a discussion between her and the students about the mathematics they were learning.

The interaction between perceptions of IBI and understandings of local field

Ms. Smith believes her district supports the type of inquiry teaching she uses in mathematics. The district provides many activities and manipulatives for the teachers, and she feels that they know she is dedicated and using best practices. She feels that as long as her benchmark tests show that her students are learning, the administration leaves her alone.

The training and professional development that Ms. Smith has been able to take have been precious. Her district often sends out information about online training for teachers. She was very grateful to have been a part of an online course by Jo Boaler, a well-known mathematics educator from Stanford. Along with the online training that the district provides, a math coach meets with Ms. Smith's grade level during their PLC (Professional Learning Community) meetings. She gets many ideas and feedback on the activities she does in her classroom.

Ms. Smith feels very fortunate to teach at a grade level that does not have a state mandated standardized test. She does not believe in standardized tests and does not want to teach a grade that requires one. The summative assessments are the same for each grade level, and teachers are forced to look at data surrounding the scores. Ms. Smith likes teaching and not analyzing data. She feels that administration primarily looks at scores on these summative tests and she would instead focus on pedagogy in the classroom.

Ms. Summer

Ms. Summer was so excited to share the new modules her district has created for the teachers this year; she started telling me all about them before our pre-observation interview began. I met with her in her first-grade classroom in late September to interview her and set up a date for the observation. Ms. Summer felt she uses inquiry every day in mathematics and so I was back less than a week later to observe a lesson.

Ms. Summer began the mathematics lesson using the Smartboard. There were four shapes on the board (heart, star, arrow, and decagon) all in different colors. The question asked to the students was ‘Which one does not belong?’ Half of the class immediately raised their hands to answer. Answers included: star (no red outline), heart (no straight sides), arrow (consists of two shapes put together), decagon (has many corners). Ms. Summer complimented each student on his or her observation and answered the question.

The next part of the class involved Ms. Summer writing the following word problem on the whiteboard:

> Isaiah had 3 Dogman books. He got 4 more in the last Scholastic order. How many Dogman books does he have now?

Before solving the problem, Ms. Summer asked the students to describe their problem-solving strategies. Students reported using tally marks, tens frame, counting on and using pictures in their head. Ms. Summer recorded each strategy on the board. Then a student explained how she used the counting on strategy to reach the solution, seven books. Ms. Summer asked the rest of the class if they also got seven and all agreed on the answer.

For the rest of the lesson period students rotated between two centers among those available. On the wall, Ms. Summer had created a chart in which each student’s picture is partnered with another student's picture. The partners are listed under a specific center each day. There were eight centers in total:

1. Ordinal Numbers – students use small toy animals and place them in a line. Then they fill out a worksheet that asks questions about ordinal places. For example, Which animal is second?
2. Tally Marks – students use popsicle sticks to mimic tally marks. Numbers from one to twenty are created.
3. Number Order – students have a set of six cards with two digit numbers on them. The cards are to be put in numerical order. For example, 89, 90, 91, 92, 93
4. Shape Monsters – students use shapes on a worksheet to create a picture of a monster
5. Number Recognition – students sort cards with numbers on them. Each number is represented in digit form, on a tens frame and with apples. Students are supposed to group all the cards that represented the same number together.
6. Ordinal Cards – students put cards with ordinal numbers in order.
7. Hundred Chart – the numbers one to one hundred are on the cards and students have to put them in order
8. Dreambox – computer program

While the students were working at a center, Ms. Summer walked around and observed. She occasionally asked a question to the students. For instance, a student working on the ordinal number toy center had put the animals in a line and started answering the questions on his worksheet. His animals were facing to the right but the student was counting starting on the left. Ms. Summer asked him where the front of the line was, and he correctly pointed to the animal furthest to the right. She asked him which animal was first and he thought about it for a few seconds before pointing to the animal at the front of the line.

Another example of how Ms. Summer asked questions to the students for them to discover an error or think more in-depth about mathematics was with the hundred chart group. The students were struggling with a large number of cards and did not have any organization. They were not talking with each other and had two separate hundred charts started. Ms. Summer walked over and asked if they had a plan for putting a hundred chart together. The two students looked at each other and shook their heads. Ms. Summer suggested they begin by putting the "column of tens" down first. She explained to the students that they know how to count by tens so starting by putting cards with ten, twenty, thirty, etc. in order could help them organize the rest of the cards. The students acknowledged what Ms. Summer said and began sorting the tens cards.

Once the two rotations of the centers were completed she had all the students come to the carpet for a quick mini-lesson on ordinal numbers. She asked four students to come up to the board and stand in a line. They all faced toward the left in a line, and Ms. Summer asked the rest of the class who was first, who was second, who was third and who was fourth. The students had no problem answering these questions. Ms. Summer then asked the four volunteers to turn around and face towards the right. She asked who was first in line now. After a few seconds, a few students raised their hand to answer. She called on a student, and she successfully answered the question. Ms. Summer asked, "Does the direction they are standing matter when you are figuring out their ordinal position?" This was the idea she wanted to share with the students. She drew a picture on the board of people facing towards the right and explained again that the way the people are facing impacts who is first and who is last in line.

Presumably, this mini-lesson was in reaction to the student who had trouble at the ordinal number center. Ms. Summer took this opportunity to catch any misconceptions of the other students before they worked at that center and clear up any confusion for the student who made a mistake. Her active strategy of pointing out detail about ordinal numbers but not telling them the answer matches her description of the identities, relationships, and activities in an inquiry classroom.

Beliefs about IBI and the influence of the local field in the classroom

Identities. Ms. Summer thinks mathematics is a "fun subject to teach." She has many interactive games and activities created for her first graders. She uses centers as the basis for her mathematics instruction. She occasionally teaches some mini-lessons on the topic before sending the students to centers. When asked about the role of the teacher she said, "You are not teaching, you are not the teacher." There is not much whole group direct instruction in her classroom, aside from her mini-lessons. Ms. Summer feels she is a guide rather than a teacher.

The identity of a teacher, according to Ms. Summer, includes preparing the lesson so that it runs smoothly and the students can work efficiently. Explaining the centers, writing a word problem and coming up with a mini-lesson are all examples of the tasks that the teacher performs. Once the students are working at their centers, Ms. Summer walks around, observes the students' work and listens for the mathematics talk that she expects from them. During the observation, she redirected students, complimented them when they were doing the activity correctly and asked organizational questions. For instance, she asked the group that was working on the hundred chart to explain how they were going to organize their cards. She asked a student working on ordinal numbers to clarify in which way he began counting the animals. Ms. Summer monitored students throughout the mathematics class to make sure they were on task and doing the activity correctly.

During the pre-observation interview, she also focused on the teachers believing in the type of instruction they want to enact in the mathematics classroom. Ms. Summer attended a Math Their Way workshop in her first year of teaching and it "opened her eyes" to number sense and algebra in a way she had not experienced as a student. She believes that a teacher needs to buy into the type of instruction she does. She said, "Some teachers do not like it" talking about the way she teaches mathematics using inquiry. Ms. Summer explained that the classroom seems chaotic at times when the students are busy working, and the teacher is not in complete control of the learning, which is why some teachers do not subscribe to inquiry teaching.

The chaos described by Ms. Summer is part of the student's identity in an inquiry classroom. The students are thinking, working together, coming up with various solutions and participating in active learning. When twenty students are doing this at once, it will be noisy.

Ms. Summer believes that the teacher should not give the answers in a mathematics class. Her description of inquiry learning in a mathematics class was very child-focused. Inquiry is "for them to discover it on their own and they need to share with each other their thoughts and ideas so that they can help each other process what they are processing." Therefore, the classroom environment involves students talking to each other and sharing their thoughts on the mathematics they are tackling. During the observation, I witnessed students working in groups of two or three. I heard no discussion from the groups that were doing the tally marks, ordinal numbers, number order, shape monsters and those on the computer. These students were working on their own even though another student was working on the same task. The group working on the number recognition and matching the cards was talking, but they were mainly correcting each other or asking for help to find a specific card. There was no discussion of why they put some cards together, how they figured it out or found another solution.

During the interview, Ms. Summer claimed that the cooperation between students help them to process the content they are working on. I did not hear conversation between the students that would help them to process what they were doing. I heard the students discuss organizing the cards they were using ("spread them out like this"), responsibilities in the small group ("you are doing this card") or telling each other what the card was representing ("that is six, not seven"). The students were participating in active learning as Ms. Summer defined it in the pre-observation interview.

The students were always moving and thinking about mathematics while working at the centers. I observed students moving cards, setting up popsicle sticks to look like tally marks, counting animals they had put in a line, and creating a large hundred chart on the floor. It was apparent that they were thinking about the mathematics involved in the task because many of them talked aloud as they worked or counted on their fingers. Except the computer station, the rest of the centers required the students to be actively doing something. No students were sitting passively in the classroom.

Relationships. Ms. Summer puts students in pairs, in part, so that they can learn from each other. She explains,

> They count on each other as a partner to make sure that they are working together and actually talking about what they are doing when they are on carpet. I want everyone to be engaged because if they are not then not only are they not learning but they are not helping someone else to learn as well.

The partners worked well together during the observation. As stated previously, some partners did not speak to each other but worked individually on the task. Others were respectful to each other in that they got along and occasionally asked a peer to hand them a card or popsicle stick. The description Ms. Summer gave of the student-to-student relationships in the interview did not match what I saw during my observation.

Ms. Summer, as the guide in the classroom, walked around as students worked and occasionally asked a question to one of the students. In the pre-observation interview, she claimed that she allows the students to discover and do the problem-solving during mathematics. Indeed, Ms. Summer was not doing the mathematics on behalf of the students when I observed. She was allowing them to explore, engage with the materials and work with their peers. Her description of the role of a teacher in an inquiry classroom and the interactions that take place between the teacher and the students were precisely what I witnessed in her classroom.

Activities. While the bulk of Ms. Summer's mathematics class is the work that the students do at the centers interacting with various activities, many other activities generally take place in her inquiry mathematics classroom. Number talks, calendar time, count around the room and dot images are a few examples of how she begins the lesson. All of these activities are interactive, and the students lead the discussion. Ms. Summer asks questions, but the students do most of the talking during the introductory activity.

During my observation, the activity called “Which does not belong” was the introductory activity. There were four shapes on the smartboard and the students had to pick the one that did not belong with the other shapes. They were also instructed to justify their choice. This activity was open-ended and had multiple solutions. Ms. Summer encouraged them to use the mathematics vocabulary they have learned and complimented the students on their explanations. Once the smartboard activity was complete, Ms. Summer told the students to move on to their first center of the day.

Ms. Summer uses various centers at which the students have to work each day. Throughout the week, they rotate between six to eight centers. At least two of the centers are on past topics. During the pre-observation interview, she explained why she includes previously learned topics.

> If we are doing shapes right now, a few of the math workstations will be shapes and then a couple of weeks I might throw a shape one in there, so they do not forget everything they learned about shapes. So spiral through the year where we are doing different concepts but then they do it again later on.

One of the centers is always on the computer using the Dreambox software. The rest of the centers are a mixture of games, hands-on activities, and worksheets, which the students are completing independently. As Ms. Summer says, “It is a little different every day.” For some of the centers, Ms. Summer expects the students to fill out a recording sheet of the work they completed. Ms. Summer uses the recording sheets to assess current and past topics as well as to help her plan for future centers.

The interaction between perceptions of IBI and understandings of local field

Ms. Summer feels supported in the way she teaches mathematics. Her principal believes in the inquiry techniques and encouraged other teachers to attend the same workshops that she has attended in the past. For instance, Ms. Summer read Debbie

Diller’s book *Math Work Stations: Independent Learning You Can Count On, K – 2* and went to a workshop on the book. She met with her principal to discuss what she had learned, and the principal sent half of the staff to the next workshop. This support allows Ms. Summer to feel confident that she is using pedagogy based on research and does not need to defend her strategies.

Ms. Summer described other supports at the building level such as her team and the mathematics lead at her school. “Our math head for the school is also very willing to talk about things and like I said we are doing a book this year that is on guided math. So, the whole school is reading this guided math book. When we have PD, we are going to get to talk about it as a school.” The encouragement from her administration and her peers helps Ms. Summer expand her repertoire of inquiry activities. It does not seem as daunting to her when other teachers are trying out activities and talk about them in professional development meetings. She also believes that the mission of the school supports her mathematics instruction. “I really do believe that they want all students to be successful and by the way I teach, they are going to be successful.”

At the district level, Ms. Summer also has numerous examples of supports that are available for all teachers. She described videos the district provides for each topic that include example activities, assessment ideas, and the vertical alignment of the topic. The suggestions are grade-level appropriate, which allows the teachers to integrate them directly into their classroom instruction. Ms. Summer said that she implements many of these ideas in her classroom in small groups or centers. The district supports her belief that small group instruction is the preferred way to teach mathematics in elementary school. “Whole group math, that is not going to really help kids really understand mathematics at all. They are going to hate math, they are not going to understand it, and we really need to get kids understanding what they are doing."

Ms. Summer could not think of any barriers to the way she teaches mathematics to her first graders. Her building administrator and the district support the identities, relationships, and activities she described as part of her inquiry-based mathematics teaching. Having these supports in place allows Ms. Summer to plan effectively with her peers and feel confident in her instruction.

Ms. Woods

Ms. Woods has been teaching either kindergarten or 1st grade for 35 years in the same school. She loved teaching kindergarten and knew from a very young age that she was going to be a teacher. During the pre-observation interview, Ms. Woods told me that "kindergarten is my first love" and I could tell during my observation that she adores her students. She was respectful and spoke in a gentle tone to every single child in her classroom.

I interviewed Ms. Woods six weeks into the school year. She agreed to be in my study in early September but did not want me to come to observe until mid-October. In our email correspondence, she explained that she was "establishing routines with a new group of kindergarteners" and "setting the tone and modeling." I observed Ms. Woods' class on a Tuesday in October while the students were working on one-to-one correspondence and number identification.

Ms. Woods began the lesson on the carpet to discuss the calendar. She asked the students what day of the week it was. They responded correctly and added a straw to their collection of days they have been in school. One student volunteered to come up and count all of the straws. The straws were grouped into tens and ones. Ms. Woods asked him how many groups of ten there were and he lost count the first time. The student looked at Ms. Woods when he was unsure, and she responded to him by saying, “How can you figure it out?” He began counting the groups again, and she enthusiastically said, “That is how you solve your own problem, by counting them again.” Once the students read the date orally, she asked them to move over to another area in the classroom where she had a rolling whiteboard set up.

On the whiteboard, Ms. Woods had the numbers ‘6’, ‘7’, and ‘8’ posted. She gave out cards to each student. On each card, there was a representation of one of the numbers. Examples included a picture of two hands holding up six fingers, two rows of four stars to represent eight, and a tens frame with seven squares filled in. One at a time the students came up and put their cards under the number their card represented. Ms. Woods asked the class after each one if they agreed by putting a thumb up or thumb down in the air. One student put the card with eight stars under the seven. Four students disagreed with his decision, and Ms. Woods asked them to share their thoughts. She chose a little girl who respectfully said, “I feel like it is 8.” Ms. Woods asked her to share her strategy, and she explained how she had counted the stars one by one. The student with the card counted again and agreed with his peer’s assessment that the card represented eight rather than seven.

The last card that was placed on the whiteboard was a tens frame with the top row of five filled in and the bottom row only having two cells filled in. The student with the card explained that they know “two and five make seven, so I know it is seven.” Another student shared that he saw four dots and three dots, which also totals seven. Ms. Woods complimented the students on their great thinking and concluded the activity by asking the students to show six on their fingers. Once everyone had their hands raised with six fingers, she asked them to show her a different way to show six. Many of the students looked perplexed by this command. After about thirty seconds, a few of the students understood and looked at their hands to manipulate their fingers to show six a differently. Once two of the students raised their hands with six fingers, the rest of the class understood and began solving the problem. Ms. Woods complimented the students again on thinking hard about the problem.

The next whole class activity was counting in a circle. The students sat in a circle, and Ms. Woods picked a student to begin counting starting at one. None of the students had an issue with this task. Then, Ms. Woods made the counting begin at the number four and continue around the circle. The first few students took just a few seconds to figure out what number to say, but once again, each student correctly said their number. Ms. Woods then told the students they were going to do centers and called them by name to tell them at which center they were going to work on that day. There were seven students on the computers working on the mathematics program Dreambox, eight students, in groups of two, playing number games and six students with her at a table working on representing numbers on ten frames. The students could choose from different partner games, including putting beads on sticks, stacking manipulatives to make numbers, rolling dice to make numbers or putting magnetic numbers in order. For the first two games, the students had to identify the number and then count out the correct amount to either place on the stick or stack in a pile. The activity with dice had the students figure out the total on both dice, write the numeral and count out Unifix cubes to match the number rolled. All of the students worked well together and seemed to be enjoying themselves while working. These activities lasted for twenty minutes.

The final activity focused on putting numbers in order. Ms. Woods had all the students come to the carpet. She gave them cards with string attached on them and the students had to wear them around their necks. Each card had one number on it from zero to seven. Ms. Woods asked the class if the students were in "counting order." A resounding "No" came from every student. She asked the eight students with cards to get in counting order. The students looked at the numbers on the cards around their neck as well as their peers' numbers and successfully got in numerical order. She asked how many numbers the students saw. One student called out "seven" presumably because that was the largest numeral. Ms. Woods said, "Does everyone agree that there are seven?" Four students' hands went up, and she called on a student who said there were eight numbers. The student had counted each card to figure this out. Ms. Woods then directed the students with the cards to do different things. Examples included, "If you are the number after three step forward" and "If you are the number that means nothing step backward." The students enjoyed this wrap-up activity and the class was concluded.

Beliefs about IBI and the influence of the local field in the classroom

Identities. Ms. Woods feels that the identity of a teacher should be that of a facilitator while the students are "doing the thinking and doing the work." She explained that she "roams around" and pulls out their thinking but they are the ones doing the mathematical thinking. In the pre-observation interview, Ms. Woods described the teacher as the one who understands her students and who asks them questions while they engage with the mathematics. Ms. Woods also described many jobs that the teacher does in the classroom. Since this is a kindergarten class, many of the roles were classroom management centered.

Ms. Woods welcomes students from a variety of backgrounds every year. Some of them enter kindergarten without having any prior experience with pre-school or daycare. Others have been in a structured program for three years before walking into kindergarten. Therefore, Ms. Woods feels that part of her role as a teacher is to get them acclimated to a routine for the first few months and allow them to explore the manipulatives and activities that they will be doing throughout the year. Her experience has taught her that if she does not allow for this period of exploration, the first time that the students were given beads or counters they are just going to play with them rather than using them for learning.

Ms. Woods' used various differentiation strategies during my observation. While working with a small group, she gave them numbers to represent on a tens frame. Not only did each student get a different number based on her knowledge of the numbers they knew, but each of them had different levels of scaffolding. She knows that she can give a number to a student that can work independently, while she guides and encourages another student who needs constant repetitions to keep working.

Another example of how well Ms. Woods knows her students is how she forms groups for mathematics activities. She told me in the pre-observation interview that "this group needed learning buddies at the beginning of the year." She puts the students in pairs to work together on a task. During my observation, every pair of students worked well together, helped each other and were respectful to each other the whole time. This illustrates how Ms. Woods has created a culture of respect in her classroom.

I witnessed various questioning techniques while I was in Ms. Woods' kindergarten class. Every time a students shared some answers she asked them, "how did you see that?" or "what was your strategy?" Rather than telling the students "yes, you are correct" or "no, that is not correct" she leads them to elaborate on their responses so that she could become aware of their thinking. The students were not hesitant in their explanations, and it was apparent that they are often asked to explain their thinking. Another type of questions asked by Ms. Woods were those that "tried to get them to thinking another way." When the students were sitting on the carpet, she asked them to show her six on their fingers. Once every student successfully showed six (most of them held up one full hand of five fingers and one finger on the other hand), she asked them to show the number six differently. This is an example of how she prompts them to think about mathematics differently without settling for just one way to complete a task.

As mentioned, Ms. Woods described many roles of the teacher that were focused on classroom management. These include putting the students in small groups, assigning learning buddies and giving them a problem to solve. She feels that her students are not only learning new content in kindergarten but also learning how to be students in a classroom. Therefore, the teacher has the responsibility to familiarize the students with new tasks.

As for the students' role in the mathematics classroom, Ms. Woods feels that learning from each other is their most important role. She tries to have a balance of structured and non-structured time where students get to have control of their learning and learn what it means to be an engaged student. She claims that "child-centered learning" is how she runs her mathematics class and for this to happen the students need to be doing mathematical thinking.

When asked about the students' role in an inquiry classroom, Ms. Woods described the "goals" she has for her students. These "goals" were procedural and focused on what it means to be a well-behaved and receptive student.

> We have learning goals. Their role is to get started on math immediately, work on math the whole time, make sure you are having math talk with your partner and clean up quickly and quietly to the gathering spot.

Probing further into what she felt the students' role was as a learner, she described how she wants them to "learn from each other and learn different strategies for solving a problem." She also wants students to learn how to "interact and play." This last statement acknowledges that some of the students never experienced a school setting before and need to learn what "play" means in kindergarten. This relationship of understanding between Ms. Woods and her students was apparent in her descriptions of both the teacher and student identities and during my observation.

Relationships. Ms. Woods described different relationships in her classroom. Both student-to-student and student-to-teacher relationships are essential in her kindergarten classroom. The identities she described for both students and teachers in an inquiry classroom mirror the relationships that she wants to foster in her classroom.

For Ms. Woods, the relationship between a kindergarten teacher and her students is one of understanding. The teacher understands the personalities and level of her students while the students understand and trust that their teacher knows what is best for them. During my observation, Ms. Woods had some structured time sitting on the carpet during which she expects the students to explain their thinking. There was also some less structured time spent in centers for the students to explore and learn independently. The entire mathematics class was child-centered in that the students were doing the thinking. There was no direct instruction during my observation. In Ms. Woods classroom the learning comes from working cooperatively and asking questions to the students. This illustrates how important the relationships are and how much trust there is in her classroom.

As mentioned, Ms. Woods assigns learning buddies at the beginning of the year because she is assessing the levels and personalities of all the students. Later in the year, "they will get to choose their learning buddy but until, you know, [they] learn how things are running and how we do our day." The trusting relationship between the teacher and the student in this process becomes a relationship between students as they pick their learning buddy. Once they find a learning buddy with whom they work well, Ms. Woods hopes they learn from each other, talk, possibly disagree and share.

Fostering these relationships is important for Ms. Woods to see her students grow. Since her lessons center on cooperative learning, students need to be willing to interact with Ms. Woods and their fellow kindergarteners. While I was observing a student counting beads to put on a string, he looked up at me and said, "I love school." In Ms. Woods' classroom, the relationships are not the only thing being fostered. The enjoyment of learning is also apparent.

Activities. The activities in Ms. Woods' classroom are very interactive and collaborative. She described both teacher-led activities and student-led activities as part of her mathematics class. During our pre-observation interview, she described many manipulatives that are available during mathematics. Examples included ten frames, rikenraks, counting bears, beads, and pattern blocks. These manipulatives are concrete representations that can be used to help students see mathematics differently than if they were only shown visual representations. There are activities in which Ms. Woods tells the students they have to use a manipulative but also an understanding that if students would like to use one during an activity, they are welcome to ask her.

Ms. Woods likes to introduce the "big idea" for the week on Monday. Therefore, she explained that "Monday's I do my big introduction, introduce the essential vocabulary, that is the one thing you cannot skip over. They always will have terminology. So, I like to stay whole group." During the rest of the week, she plans activities at the centers and the students rotate between them. These centers include games, the computer program "Dreambox" and activities at the table with Ms. Woods. The focus for the games is both on the new content that was introduced on Monday and previously learned content. The games that I witnessed were all hands-on and had the students interacting with each other. Students were independently working on one-to-one correspondence, writing numbers, counting orally, and representing numbers.

While the students on the computer and in the game centers worked independently, Ms. Woods focused on a small group of students at a table. The students had a blank tens frame for each of them. She would give each one of them a number to represent on the tens frame with chips. She differentiated the instructions by giving the students different numbers and asking questions that were appropriate for the student's level. The students in the group were all at different levels of understanding one-to-one correspondence to the number twenty. Therefore, Ms. Woods gave them a number that she felt they needed to practice. She assisted the students in different ways while working with them. She asked one student to point to each chip as he counted while also asking another student to show her the number twelve in a different way. The tasks that occur while the students are with Ms. Woods are supportive of the relationships described previously.

The interaction between perceptions of IBI and understandings of local field

Ms. Woods feels supported in the way she uses inquiry in her mathematics classroom. She listed the mathematics coach, the manipulatives and the district level mathematics supervisor as the most supportive aspects of her mathematics instruction. The mathematics coach is not at her school full time, but Ms. Woods feels that she can call on her any time if she needs suggestions on differentiation, a specific manipulative or just someone with whom to compare ideas.

The mathematics coach splits her time between two schools but plans with the kindergarten team at Ms. Woods' school weekly. She really feels this is an excellent way for her to use a resource the district has provided.

> She comes in and plans with us. 'What do you need?' She does not tell us what to do; she asks what do we need. I do not know how to use number paths, whatever it is, what do I do with this to extend them a little further? She is great to tap into.

The list of manipulatives that the school has compiled over the years is vast. Ms. Woods thinks that many concrete materials can be shared by the teachers. On one occasion, Ms. Woods wanted to try out a new manipulative with her students and the administration found the funds to purchase it. Also, the mathematics coach has borrowed items from other schools so that Ms. Woods can use them.

The mathematics supervisor for the district has collected videos and activities for each grade level. Before each unit, the teachers watch a short video that describes the big ideas for the topic and suggests hands-on activities. Ms. Woods has enjoyed each module and feels they are "very helpful" when she is planning for instruction.

When asked about support in the building and whether the mission of the school supported her way of teaching mathematics, Ms. Woods responded positively. "I know our administrators, I know our whole county is like this they want to walk in and see our kids involved, see the kids sharing and talking with one another and sharing their learning." Ms. Woods could not think of one barrier in her school or district that impedes the inquiry she uses in her mathematics instruction.

Themes across Cases

Research Question One: Inquiry in an Ideal Classroom

The Identity of the Teacher

The participants in my study described the identity of the teacher in an ideal inquiry classroom as a facilitator to learning. The most common ways of showing this identity, according to the participants, are questioning, organizing the classroom environment and allowing for exploration. The participants also described the teacher as the content specialist, but each participant saw the manifestation of this trait differently.

Four out of the six participants mentioned that asking questions is an essential role of the teacher in an inquiry classroom. Ms. Miller explained that a teacher should be "posing questions like 'I wonder' or 'what would happen if...'" In kindergarten, Ms. Woods felt that a teacher should be "stepping back and facilitating and asking them questions that maybe get them thinking another way." The word "facilitator" was the most used by the participants to describe a teacher in an inquiry classroom. When I spoke with Ms. Thomas, she agreed that a teacher should "facilitate by asking a lot of questions instead of telling." For this to occur, all the participants agreed that organizing the classroom was an essential part of the teacher's role.

Organizing the classroom for inquiry includes setting up areas in the classroom where the students have to work, collecting the necessary materials and planning the activities that will take place. Rotating between centers is how three out of the six participants organize their mathematics block. Ms. Summer talked a lot about how she organizes her centers and changes them out frequently. The centers in Ms. Summer's classroom are set up so that "the kids are working in pairs and they have a chart that they look at to see where they are going to go." Each center has different activities and the teacher is the one setting it all up. "When you are doing ten stations, and you are changing them out every couple weeks, this one stays then two days later you pick out another one. It can get a little insane." Gathering the materials that support student's learning is a part of the teacher's role in an inquiry classroom. Ms. Summer explains, "[the students] can actually see what they are doing and understand it on a whole other level" when they are using hands-on manipulatives. The organization takes much planning on behalf of the teacher. Ms. Washington summed up nicely what it takes to doing an inquiry lesson, "I think in an inquiry lesson, the planning takes a lot more than in a regular just math instruction. The planning has to be very purposeful."

Four out of the six participants mentioned using open-ended tasks and allowing for exploration as part of the teacher's identity. When Ms. Miller spoke about the students' identity, she mentioned that they often wonder about possible strategies and solutions. Her response to them is usually, "maybe you can and maybe you cannot. Let us explore it. Exploration and discovery are so great." According to Ms. Summer, a teacher needs to embrace exploration during mathematics for the inquiry to occur. Ms. Smith's favorite part of inquiry teaching is "the kids are exploring, they are having fun, they are discovering things, and they do not even realize that they are discovering things."

Three of the participants, when asked about the teacher's identity, mentioned their role as a content specialist. For Ms. Miller, as the content specialist she can ask guided questions and scaffold the content so that her students can think deeper. She described walking around and listening so that she can ask specific questions to each group based on their specific thinking process. Ms. Washington feels that as the content specialist she needs to teach in small groups. She presents a focus lesson and "the kids that do not get it stay with me, I will bring them to the blue kidney table and I will work with them." This type of direct instruction is a different description of the teacher as a content specialist than Ms. Miller's description. Ms. Thomas has yet another opinion of the teacher as the content specialist. Knowing the content well leads to the teacher's job of "help(ing) the kids make connections between their different ways of thinking and that is how you are going to help them grow." Lastly Ms. Washington explained why teachers need to be content specialists in the classroom. "You have to think of all the ways where [the students] could possibly go astray and then how to bring them back in."

The Identity of the Student

According to the participants in my study, the identity of a student in an ideal inquiry classroom is that of one who works with their peers, uses various problem-solving strategies, and engages with mathematics content. Four of my participants identified each of these descriptions when talking about students in an inquiry classroom.

Five participants described students working together as part of what students should be doing in an inquiry mathematics classroom. Ms. Summer described students' "shar[ing] with each other their thoughts and ideas." Ms. Miller said that there is "a lot of cooperation going on" within an inquiry mathematics classroom. In kindergarten, Ms. Woods explained that the students are "learning from each other, learn[ing] from different strategies that they might have in solving a problem. Just learning how to interact and play." In an inquiry classroom, at all levels, students are working together.

Three participants stated that during problem-solving students should use various strategies to solve the problems. Ms. Miller described giving the students the tools and "having them figure out what tool they might need to solve the problem." Ms. Thomas explained, "You hope they connect the strategies" that are shared. According to the participants, the students are doing the thinking in an inquiry classroom. Ms. Woods noted, "The kids take charge and the things they come up with are just amazing."

Five participants also mentioned active learning as an essential component of inquiry mathematics. When asked about the students' role, Mrs. Summer said, "They all need to be active participants, they all need to participate." Ms. Miller explained, "Inquiry is getting them involved in their work, and it becomes more meaningful." When I asked Ms. Smith how she knows that learning is occurring in an inquiry classroom, she said, "they are engaged in what they are working on." This active learning environment creates noise in the classroom as mentioned by two participants. Ms. Woods admitted, "I do not mind noise if it is good noise." The understanding between the teacher and the student that learning can be noisy is part of the relationship that is cultivated in an inquiry classroom.

Relationships

The only relationships discussed by the participants were those between students and between the teacher and the students. In an ideal inquiry classroom, there is an understanding that students will work together and learn from each other. The relationship between the teacher and the students is one built on an understanding of the type of mathematical thinking that occurs in an inquiry classroom.

All of the participants mentioned that the students work together during the mathematics class. Students solve problems together, discuss with each other and help each other to understand the content. When students get frustrated, Ms. Miller wants them to "get some advice or help from somebody at your table or partner." Ms. Thomas hopes that students will, "look at one another and understand what the other person is saying." Cultivating the relationship between students is important in an inquiry classroom because it encourages collaboration in problem-solving. As Ms. Miller said, "I will come around at the end, and ask 'how did problem-solving with your partner help you become a better mathematician?' It is not just about the math." While teachers are nurturing the relationships between the students in the classroom, they are also cognizant of the relationship between themselves and the students.

The relationship between the teacher and the students is based on an understanding of the described identities. The teacher is the facilitator in a child-centered classroom while the student is the one doing the thinking. Ms. Woods explained that a teacher should, "Roam around and guide and pull out their thinking. I like for [mathematics] to be more child-centered where they are doing the thinking, they are doing the work, and let me just facilitate that." Ms. Smith also agreed that students "are deciding what needs to take place and what they need to do and what materials they need to have instead of me saying ‘you need blocks to solve it.’” All of the teachers agreed that if both the teacher and the students respect the identities they described, then inquiry can occur in the mathematics classroom.

Activities

In an ideal inquiry classroom, the most common activities mentioned were the use of manipulatives, problem-solving, and the use of questioning. All of the participants mentioned these activities at least once during the pre-observation interview.

Three participants mentioned manipulatives as an essential component of inquiry pedagogy. Ms. Washington described an inquiry classroom with “a lot of hands-on activities.” Ms. Summer agreed that students should “use hands-on things, so it is not just numbers in their head.” Some of the manipulatives described by the teachers were base-ten blocks, tangram pieces, counters and ten frames.

Problem-solving was mentioned by five of the six participants as part of an inquiry classroom. Ms. Summer claimed that the teacher should “let [the students] do the problem-solving.” The problems students are solving are in different forms for different teachers. Word problems, ‘Which One Does Not Belong?’, a sorting task and making connections between strategies are all types of problem-solving situations that the teachers mentioned in the pre-observation interview. Therefore, the term problem-solving, just like that of content specialist, can be interpreted differently and include many different activities.

The use of questioning by the teacher to help the students to make a connection between ideas, dig deeper into the content and sometimes steer them into a different direction is part of an inquiry classroom. Ms. Washington explained that if a group is off task, a teacher can "ask guiding questions like, 'Have you tried this?' or 'What if I did this?'" Ms. Thomas reported that it is important to ask students about the various strategies that they are presenting so that they can truly understand them. The questions were, 'Out of all these strategies which one is the most comfortable to you and makes sense to you?' 'Which would be the most efficient?' Ms. Woods felt that even kindergarteners could ask questions to each other during mathematics class. She explained that they could learn from each other by asking 'how did you see that?' or 'how did you look at that?' Questioning throughout the mathematics block by both the teacher and students is essential in an inquiry classroom.

Research Question Two: How the Local Field Influences the Perception of IBI

The Identity of the Teacher

The most common attributes of the identity of the teacher that I witnessed during the observations were questioning, assessing both formally and informally, and facilitating the learning. These were also evident in the lesson plans of three teachers.

Ms. Washington's lesson plan had links to her formative and summative assessment and possible misconceptions on the topic of integers. She explained that this helps her to facilitate the learning and use guiding questions to address the misconceptions. In the post-observation interview, I asked Ms. Smith how she felt she facilitated the lesson I observed. She said, "I think I set up the room and the environment so that the kids were really in charge of their learning." When I spoke with Ms. Miller after the observed lesson, she reflected on her questioning during the lesson. "I should have had a few more guiding questions that I did not think I needed.” She said that she assumed that they knew more about polygons so when she asked about their attributes she expected more detailed responses from the students. Finally, all the teachers were assessing the students throughout the class period but did not mention that as an aspect related to the identity of a teacher. Four teachers collected a worksheet from the students at the end of the mathematics block. All of them informally assessed the students by asking questions and observing the students while they worked.

The Identity of the Student

Students in all of the classrooms that I observed worked together during the mathematics block. The students were doing the thinking, following the rules of the classroom, and actively engaging in mathematics.

Whether working in small groups or with a partner, the students were encouraged to talk about the mathematics and help each other if needed in every classroom I visited. Other than the small group in Ms. Smith’s room, there was no type of direct instruction during mathematics. Students were rotating through centers in three classrooms and working on word problems or tasks in the other three. In my notes on Ms. Washington’s classroom, I wrote ‘students were explaining their strategies to each other and asking questions to each other when they do not understand.’ In Ms. Woods’ class, there was a student who decided to work by herself, and I asked Ms. Woods about this in the post-observation interview. She explained, “if they have something that they really want to work on it is ok to get that game and work by yourself.” This is an example of the students actively engaging in their learning.

Relationships

The relationships that I observed during my visits were the same mentioned by the teachers in the pre-observation interviews: teacher to student and student to student. Two of the teachers asked me explicitly to visit the class at least four weeks from the beginning of the school year. Setting the routines of the classroom is vital if the teacher wants to use inquiry pedagogy during mathematics.

The teachers claimed that the students usually learn the routines of the classroom during the first couple of weeks. Ms. Washington spoke about the necessity to make the students realize that an inquiry classroom does not focus on the standardized test questions. She described, “The kids come to me in 5th grade, and it is already ingrained in them ‘What about the [standardized] test?’ ‘What about the [standardized test] question?’” Ms. Miller described the same situation in her third-grade classroom. “Some say, "I am not good at math, or my mom says I am not good at math just like her.’” Ms. Woods also explained that she allows the students to “play” with the manipulatives the first couple of weeks and “get it out of their system” because they need to use them for learning purposes later on. These are three examples of how the relationship between the teacher and the student in an inquiry classroom is based on an understanding of the roles. The students need to adjust to an inquiry mathematics class if they have never experienced that before.

Part of students’ understanding of their role in an inquiry classroom is learning how to work together. The teacher is not going to give the students all of the answers. Therefore, students need to learn how to work cooperatively with their peers. I saw students of all ages working together, complimenting each other and being respectful of each other's learning environment. Ms. Thomas described the different types of students in the classroom. The teacher is "helping kids work together because sometimes you will have ones that will be bossy and hog the whole collaboration. And you will also have the kids that are the bystander who let it all happen." For learning to occur, the student to student relationship is an essential aspect of the inquiry mathematics classroom.

Activities

The most common activities I saw in the participants' classrooms were students working in centers or solving problems given by the teacher, students working on a mathematics computer program, and students using manipulatives. These activities were common at all grade levels.

As mentioned, half of the classrooms I observed used centers for most of the mathematics block. These classrooms were the kindergarten, first-grade and second-grade. The centers allowed the teachers to work with a small group of students. Activities that were included in the centers were mathematics games, worksheets, working with the teacher and using Dreambox. The upper grades that did not have centers had students working in small groups on the same activity. The rare occasion in which I observed whole group instruction were during calendar time in kindergarten and the first five minutes of the mathematics block in first-grade and fifth-grade. The students worked individually on the adaptive mathematics computer program, Dreambox.

One activity that I observed in three of the classrooms was students use of computers to work individually on Dreambox. No one mentioned the program in the pre-observation interview, but it was a large segment of many students' mathematics blocks.

I asked Ms. Woods about the program, and she told me that her district requires kindergarteners to have sixty minutes on the program each week. She struggles with getting each student on the computer for that length of time.

Four of classrooms used some form of manipulatives in their lessons. The type of manipulative depended on the grade level and content. Ms. Washington's students used counters and base-ten blocks. Ms. Wood's students played with dice, counters and ten-frames. Ms. Thomas, whose students used pattern blocks, number cards and jungle animals, explained why she uses manipulatives in her classroom. "It allows them to connect their brains because a lot of times they do not picture anything in their brain. So I give them objects." I watched students play with jungle animals and identify their ordinal position in her first-grade classroom.

Other than Dreambox, the teachers described all the activities that I observed in each classroom as part of an ideal inquiry classroom. As a district requirement, Dreambox seems to be part of the teacher's local field that they have to incorporate into their inquiry mathematics lesson. The local field influences teacher's decisions about pedagogy in other ways as well. The following is a description of how the interaction between the teachers' perception of IBI and their understanding of their local field influences their choices in planning and teaching.

Research Question Three: How the Interaction between Perceptions of IBI and Understanding of Local Field Influences Teachers' Choices in Planning and Teaching

The teacher's local field influenced their planning and teaching for an inquiry mathematics lesson in three ways. First the district or building requirements for planning;

second, teachers' feeling of being supported in using inquiry in their mathematics classroom; third, the pressure of a standardized test at the end of the school year. These factors are beyond the teachers' control but influence the pedagogy they use in their classroom.

The planning documents I received from the participants varied in the amount of detail included. District A does not have a template or required lesson plan format for mathematics. The lesson plans I received from teachers in that district included the standard that was covered and a sentence or two about what will happen during the lesson. District B has a specific template, which every teacher is required to use, and the administration can access it. The lesson plans I received from teachers in District B included the standard, essential knowledge, common misconceptions, daily objective, available resources, a detailed instructional plan and formative assessment strategy. The stark difference in planning was not reflected in their lessons, but the requirements of their local field influenced the amount of planning for the lesson.

All but one teacher felt that they had the materials necessary to carry out inquiry pedagogy. Whether it was a sufficient number of manipulatives or online resources offered by the district, five teachers reported that they could use inquiry techniques with the materials provided. Ms. Woods enthusiastically described new modules created by her district to support mathematics instruction.

> We are actually taking modules before we teach every skill. We look at this module; this is the way it is presented; this is the way we want the kids practicing. The modules are very helpful, and then you take a little quiz at the end that is like 4 or 5 questions. There are also general resources for every topic. We are doing numbers to ten, so I can go and click, and they have games, vocabulary cards, lessons that you can choose from. It is really nice.

Some schools have access to a mathematics coach. They can request that the coach plan with them, brainstorm ideas for a specific topic, or observes a lesson. All of the teachers that have a mathematics coach agreed that this is a valuable resource.

Three participants teach a grade level that has a standardized test in mathematics at the end of the school year. All of them mentioned this test as a stressor that they are always thinking of when they are teaching mathematics. As previously mentioned, Ms. Washington found it sad that her students come in at the beginning of the year focused on the test they will take. “Being in a Title I [school] and always having to focus on that accreditation piece. It is hard and I do not really agree with the test.” Even Ms. Smith, who teaches in a grade level that does not require a standardized test at the end of the year, brought the test up during our pre-observation interview. “[The administration] want numbers, and one of the reasons I do not want to teach 3rd grade is because of the [standardized test]. That is the main reason why I want to stay away from it because I do not believe in them. I do not believe in them.”

Whether a teacher believes in the test or not, the current public education system requires the for specific grades. The local field can influence how a teacher feels about the tests and whether they influence the way they teach mathematics. According to the participants in my study, the tests are always in the back of their mind when planning and teaching.

Chapter 5

Discussion

This chapter presents a discussion on the findings concerning how elementary teachers understand IBI and how their local field influences that perception. Specifically, it will examine how the teachers described and implemented the identities, relationships, and activities in their mathematics classroom. Moreover, the interaction between the perception of IBI and the understanding of the local field and how that influences the pedagogical choices teachers make will be examined. In this section, I conclude the present study with a discussion of the results based on the three research questions and the theoretical framework. The rest of the chapter includes the study limitations, recommendations for future research, and implications for educational practice.

Discussion of Results

Just as the literature describes inquiry in various ways, each teacher had a slightly different interpretation of inquiry in the mathematics classroom. Similarly, their interpretation of the identities, relationships, and activities that were described each differed as well. For instance, five of the teachers described the students working together during mathematics. This description of the students' role looked very different in each of the five classrooms. Although the local field includes many stakeholders that influence teachers' perceptions, the relationships the teachers described did not include parents, administrators, or other teachers. This indicates the need for further discussion on the relationships that are necessary for teachers to implement IBI in their classrooms. Finally, some of the activities described in the literature that often appear in many types of IBI were not described or used by the teachers in the present study.

Different Interpretations Lead to Different Pedagogies

Just as the literature describes in various ways what IBI in the mathematics classroom looks like, each teacher in the present study interpreted inquiry differently. Discovering, posing word problems, and using hands-on activities were the most common terms used when the teachers were asked to define IBI in mathematics. Other descriptions of inquiry included the phrases child-centered, asking questions, using real life situations, and solving problems in different ways. This finding is consistent with the research concerning how inquiry is described, that is, it tends to differ depending on who is describing it and when, and according to their local field and teaching values. This is important because the field of education often uses buzzwords to describe best practices in the classroom. Thus, when an administrator or teacher uses the word “inquiry,” it is wise to have them describe what they mean by it. Not only do educators’ definitions of inquiry differ, but so do their interpretations of the activities that occur in an inquiry-based classroom.

Five of the six teachers mentioned problem-solving as a key activity in an IBI mathematics classroom. While often mentioned in the literature as part of inquiry and one of the NCTM’s five process standards, problem-solving has a different meaning for each teacher. The teacher’s disposition, local field, and the understanding of the mathematics content all influence a teacher’s understanding and implementation of problem-solving in their mathematics classroom. Ms. Miller’s professional background is at a higher-order thinking school that trains teachers how to deepen and expand best practices that focus on child-centered and experiential teaching and learning. Her understanding of problem-solving involves students talking through problems together and discovering new mathematics concepts independently. When I observed her lesson, she did not give the students the answers, but instead had them think and discuss with their peers about the attributes of a polygon. The students were discovering through Ms. Miller's questioning and the sorting activity she had planned. Ms. Miller's previous professional experiences, her awareness of her students, and the support she feels she receives from her local field combine to form her understanding and use of problem-solving in the classroom.

Ms. Summer's description of problem-solving was more about the teacher posing an open-ended problem and solving it in a way that made sense to the students. She explained how her district has provided resources for the first five minutes of the mathematics block that allow students to problem-solve. During my observation, I watched her pose multiple problems and allow the students to share their strategies for solving each problem. There was no questioning, like in Ms. Miller's mathematics class, but instead more of an informal assessment by Ms. Summer on students' prior knowledge about the content related to the problem. Ms. Summer has been trained using Math Their Way. This program uses a hands-on, activity-centered approach to teach basic mathematics content. Again, we see how a teacher's disposition influences how they interpret problem-solving and the pedagogy they use in their mathematics classroom.

All of the teachers in the study mentioned cooperative learning. This concept encompasses the identity of the student, the activities that occur in an inquiry-based classroom, and the relationships that are important. This concept of collaborative learning not only looked very different in each classroom, but each teacher had different reasons as to why they felt it was an important part of student learning. During the observations, I witnessed students sitting next to each other working independently on the same task, students talking with each other to solve a problem, and students working on a problem but without discussion. All of these manifestations of what can be described as "students working together" pose the same problem that the various descriptions of "problem solving" and "inquiry" pose. There are many interpretations and definitions of each of these popular phrases.

Education, like many other disciples, uses buzz words that, on the surface, sound great. If an administrator saw "problem solving" and "collaborative learning" in a lesson plan, many would be satisfied with the teacher's choice of pedagogical approach. However, because of the many interpretations of these terms, both administrators and educators alike need to clarify what they mean when they use them. If a teacher boasts that their students are "problem solving" in "collaborative groups," this could mean multiple things. The students could be working at the same table with no interaction but solving number sentences independently, or the students could be comparing and contrasting the various strategies they used to solve a complex task. It could also be a mixture of those two ideas. With all of these different interpretations, the idea of problem-solving that the literature espouses (Henningsen & Stein, 1997; Anthony & Walshaw, 2009) loses its validity. This is important because such a lack of clarity could lead to possible misunderstandings among teachers and administrators.

Relationships in the Field

During the pre-observation interview, I did not ask about relationships directly but all of the teachers discussed relationships when describing what inquiry means to them and what it looks like in their classroom. The relationships the teachers focused on during the interviews were those that exist in the classroom: the teacher and student relationship and the relationships among students. When discussing inquiry in the mathematics classroom, teachers did not mention the relationships between the teachers and the administrators, nor the relationships among teachers. Some of the teachers identified the administrator as a source of support for the use of inquiry-based strategies, but primarily as a source of financial support. Ms. Woods' explained that if teachers needed manipulatives, for example, the administrator would purchase them on their behalf. In addition, Ms. Summer described the administrator's support in sending teachers to a professional development opportunity outside the district.

There was some discussion about planning with other teachers, but many teachers explained that they were the only ones in their school who "taught this way." It was surprising to me that none of the teachers elaborated on their relationship with their administrator, nor on their relationships with other teachers when discussing the teaching and learning process. "Collaborative practice is increasingly seen as an important element of developing a school or classroom culture that supports student learning and teacher change" (Towers, 2009, p. 257). Whether it is taking part in effective professional development together (Carpenter et al., 1989) or collaborative lesson-planning (Smith et al., 2008; Kimmel, 2013) research shows that teacher collaboration helps to make connections within the mathematics content and stimulates reflection on professional practices. Some of the teachers in my study reported that they were the only ones in their grade level that use inquiry-based techniques and therefore do not feel comfortable collaboratively planning with their peers.

Ms. Smith explained that she plans the mathematics lessons for the entire grade level, but she does not teach the same way as her colleagues. “I plan for other teachers. It is hard because I have to consider them. So sometimes I hold back because if I was just teaching math [the lesson plan] would have what I like to do.” Ms. Washington explained that she does not feel she is the only teacher using IBI in her school, but they do not do collaborative planning. She is excited about a new initiative in her school this year. “One thing we are starting this year, which is going to be really cool, is peer observations, where you can go and see other teachers. I think that would be amazing. You never get a chance to see anyone else [teach].” Both teachers admitted that they teach the way they feel is best for their students.

Whether collaborative planning is required or not, it seems that the teachers that subscribed to inquiry-based pedagogy use the strategies they feel work best in their classroom. The relationships that one might think are important when it comes to planning and teaching (i.e., relationships with colleagues and administrators) did not influence the pedagogical strategies used by the teachers in the present study. This is noteworthy because support from colleagues and administration might help other teachers include IBI techniques in their mathematics classrooms.

Activities in the IBI classroom

The literature on IBI in the mathematics classroom describes students using classroom discourse, problem-solving, and collaborative learning to share various solution strategies. During my observations, I saw different interpretations of each of these activities. I also witnessed the use of centers by teachers to teach small groups of students and differentiate content. One of those centers was the mathematics program Dreambox. Dreambox is an adaptive learning program that personalizes learning for each student. Districts purchase licenses for every student and recommend a certain amount of time for each student to be on the software every day.

While previous studies on inquiry-based learning have not delved into using technology in mathematics instruction, many have focused on the use of manipulatives (Larkin, 2015; Jones and Tiller, 2017). Dreambox uses manipulatives (e.g., ten frames, hundreds charts, RekenRek) in lessons, but they are obviously just movable pictures. Mathematics education research supports the use of concrete, pictorial (representational), abstract (CPA) alignment when teaching mathematical concepts (Purwadi, Sudiarta, & Suparta, 2019, Agrawal & Morin, 2016). If teachers do not use manipulatives in their classroom, then students will likely be introduced to these tools as pictures on screens. The teachers in this study used manipulatives during mathematics instruction, but I am not certain each of them used the same manipulatives that are used in the software. This could cause confusion for students if they have never experienced learning a concept using concrete objects prior to doing so on the computer. If the technologies jump to the pictorial before students have had access to the concrete form, how does that affect their understanding of the mathematical concept?

Ms. Woods, the kindergarten teacher, found it hard to get her students on the software for the recommended sixty minutes every week. She described students having a hard time using a mouse and typing in login information to access the software. The time she must use to help her students log in takes away from her whole-class instructional time. Other teachers in the study commented on how much time they were required to have their students use the online program, and that they did not feel they had a choice in the matter. The teachers thus planned their mathematics instructional time around the mandatory computer time required by the district. Whether the teacher's inquiry-based pedagogy matches that of the program is not clear. This is important because if the teachers' IBI conflicts with the technology they are using, this could cause student confusion.

Limitations

Although the findings of the present study shed light on IBI in the mathematics classroom, the research is not without limitations. First, the two districts handpicked a pool of possible participants who used inquiry-based techniques in their elementary mathematics classroom. From this pool of candidates, the participants volunteered to participate in the study. Their willingness to participate could be correlated with a strong understanding of IBI and/or knowledge of best practices. Other elementary teachers might not know how inquiry could be implemented in the elementary classroom. In addition, the teachers in the present study taught in districts in which all schools meet state accreditation standards. The districts provide professional development on best practices in education and allow teachers to use the pedagogical approaches that they are most comfortable with. The support and trust these district have for their teachers could increase teachers' confidence in discussing and demonstrating their chosen instructional practices.

The second limitation of this study relates to the difference in lesson plans collected from the participants. District A does not require teachers to create lesson plans or share them with their administration. District B provides a lesson plan template for teachers and requires them to share it with their administration. The lesson plans I received from the teachers in District A did not have enough information for me to analyze. The plans included the standard they were planning on teaching and a sentence about the activity or centers that would occur during the mathematics block. I could not use three of the lesson plans in my analysis, which made the amount of data for each participant unequal. It was also difficult to answer the planning part of my third research question, with only partial evidence of planning on the part of the teachers. For a true picture of how each teacher envisions IBI in their classroom, having a detailed lesson plan is essential.

Furthermore, for a researcher to obtain a comprehensive view of how teachers use IBI in the mathematics classroom, multiple observations are necessary. Due to time limitations, I was only able to observe each teacher once. While this provided me with an idea of what they thought IBI in the mathematics classroom looks like, one lesson does not give me the full picture of the pedagogy they use throughout the year. During the pre-observation interview, some of the participants mentioned activities or strategies they used in their inquiry-based classrooms, but I did not get to see these strategies in action during my observation.

Finally, the purpose of a case study is not to generalize but instead "illuminate a set of decisions: why they were taken, how they were implemented, and with what result" (Schramm, as cited in Yin, 2014, p. 15). The results of this study should not be generalized to elementary mathematics teachers or even teachers that subscribe to inquiry-based pedagogy. The views of the participants are based on their experiences in their local field during a two-month period and does not suggest that others, even in the same local field, would feel the same way.

Future Directions

The study presents exciting opportunities for future work in inquiry-based mathematics instruction and planning. Future studies could consider how teachers use centers during instruction in an inquiry-based classroom. This is an element of today's elementary classroom (especially primary) that was not present in the literature, but based on my findings, is a common practice among elementary teachers. Future studies could examine which types of inquiry-based activities are occurring within the centers when the teacher is not present. Furthermore, why do teachers feel centers are helpful when using inquiry-based techniques? While reading groups have been used for decades to differentiate instruction for students at different reading levels, would this be an effective solution to provide differentiated mathematics instruction?

This study included one lesson observation for each teacher. Future research could include more than one observation per teacher or even observation of an entire unit to better understand how often IBI is used throughout the unit. Longitudinal work could also help improve our understanding of the identities, relationships, and activities within IBI over time. How do ideas about IBI change based on changes in the local field? How do the relationships within the field change when there is a change in school leadership? How do these changes effect teachers who use inquiry-based techniques?

As mentioned previously, research on how teachers plan lessons is lacking. This crucial step in the teaching and learning cycle has been taken for granted in many schools and classrooms. To understand why and how teachers choose specific lessons and activities for their students, the planning stage needs to be further investigated. Previous studies have examined "lesson study" (a Japanese method of planning and reflecting) (Vrikki, Warwick, Vermunt, Mercer and Van Halem, 2017; Leavy and Hourigan, 2018), as well as how pre-service and novice teachers plan lessons (Lewis, 2014; Taylan, 2016). Although the focus of the present study is IBI in mathematics, planning in all subject areas should be studied and understood to obtain the full picture of IBI.

Implications

The results of the present study offer practical implications for stakeholders in inquiry-based pedagogy. The stories shared in this study can help districts, teacher educators, and teachers alike better understand how teachers understand inquiry-based pedagogy and how the local field affects their pedagogical decision-making. Although the findings of a small case study such as this one are not meant to be generalizable, these teachers can help mathematics educators of all types when discussing what IBI looks like in the elementary mathematics classroom.

Not all the teachers who participated in the present study had a professional development experience that cemented their choice of mathematics pedagogy, but they did agree that continued professional development in inquiry-based pedagogy helps teachers in multiple ways. A teacher feels supported when professional development opportunities offered by a district are in alignment with the type of pedagogy they use. The present study also found that professional development can alter teacher beliefs (Carney, Brendefur, Thiede, Hughes, and Sutton, 2016), which in turn affects the instructional strategies used in the classroom. In at least two of the cases in this study, changes in teachers' mathematics pedagogy occurred because of influential professional development experiences. Sustained professional development that includes looking at student work, reflecting on classroom experiences, and trying out inquiry techniques in their own classroom were reported to change the teachers' beliefs (Carpenter et al., 1989). If districts can provide teachers with such professional development opportunities, they will feel more supported and more teachers will be encouraged to use IBI in their classrooms. The importance of having a strong support system for inquiry-based pedagogy as reported by the teachers in this study is consistent with the findings of previous research (DuFour, Eaker, & Karhanek, 2004).

For many pre-service teachers, mathematics education courses in college are the first place they learn about inquiry-based pedagogy. Teacher educators should thus model inquiry-based practices while teaching the mathematics content that pre-service teachers need to know. In my own experiences, pre-service teachers re-learn much of the mathematics content they learned in elementary and middle school in a new way in teacher education programs. They often comment that they finally understand fractions or how long division works. Using student work and videos of students engaged in inquiry-based mathematics learning helps pre-service teachers understand what IBI could look like in the classroom. In addition, placing pre-service teachers in in-service teachers' classrooms who use inquiry-based techniques during student teaching will allow pre-service teachers to see the different styles of inquiry in the mathematics classroom and, more importantly, see that it is possible.

Finally, in-service elementary teachers that are interested in IBI but do not know where to begin should pair up with a teacher already using inquiry in their mathematics classroom. As mentioned previously, support for this type of pedagogy is important and talking with peers who are already using IBI effectively in their classrooms is a great way to introduce the pedagogical approach to an interested teacher. Teachers currently using inquiry-based techniques should advocate for professional development opportunities that supports inquiry-based mathematics instruction. One way to nudge a district to provide more opportunities for teachers is to get parents on board with this type of pedagogy. Having a parent night where students show the different activities they do during mathematics and how they understand the mathematics content they are learning helps parents to understand and support the use of inquiry-based pedagogy.

Conclusion

The teachers in this study should be applauded for their sustained efforts to use a type of pedagogy that might not be popular and/or supported by their local fields. Research has shown that many in-service teachers revert to the way they were taught (Cady et al., 2006) or abandon research-based practices in favor of more common traditional practices that their colleagues are using (Allen, 2009). The teachers in my study believe students should be active participants in their mathematics learning. They believe that simply giving students the answers will not help them understand nor remember the mathematical learning content. Other than one instance I would categorize as direct instruction that was in a small group setting, I would classify each of the classrooms I visited as student-centered, where the teacher acted as a facilitator of the students' thinking and learning.

Half of the classrooms I observed used centers in their instruction. The teachers felt that small groups of students working together facilitates the use of inquiry instruction. Students appeared comfortable discussing mathematics in partners or groups of three students while the teacher listened to the discussion and questioned the students accordingly. Teachers indicated that it was easier to informally assess while the students are in centers and thus one reason they chose to use centers during inquiry-based mathematics. The use of this practice in a classroom that uses inquiry supports student-centered pedagogy while also allowing the teacher to differentiate mathematics instruction.

The definitions of inquiry, problem solving, and cooperative learning varied among the participants in my study. The local field influences how each of these are defined but also influences the support that the school division will offer. To avoid confusion, teachers and administrators need to be clear in what constitutes as inquiry. The goal is not to have one definition but rather share the identities of a teacher and student and possible activities that could occur in an inquiry-based mathematics classroom. This open dialogue could also encourage more teachers to use inquiry techniques in their classroom. Teachers not familiar with inquiry or those who are unclear on how to include it in their instruction would feel supported if both teachers and administrators set aside time to discuss and collaborate.

Support for inquiry instruction is necessary. Some school districts provide support through professional development and some do not. When I heard that some of the teachers in my study did not feel that they received support from their school or had never received professional development training on inquiry-based pedagogy, I was both surprised and saddened. These teachers were identified as those who were using a type of IBI in their mathematics classrooms by their district supervisor; however, they did not feel supported. This illustrates how the local field can influence teachers' understandings and actions to a point. When a teacher believes that a type of pedagogy supports the goals they have for their students and they have witnessed student success through its use, they will find the strength to do what they feel is right for their students, with or without the support of their local field.

Schools want to see more research-based practices in the classroom. Practices that will increase the mathematics proficiency and understanding of students is important to administrators and teachers. Including inquiry in mathematics instruction can provide students opportunities to problem solve and discuss mathematics with their peers and teachers to gain a deeper understanding of mathematical concepts. Understanding what teachers who subscribe to an inquiry-based pedagogy in elementary mathematics believe, how they understand the identities, relationships, and activities within their inquiry-based classroom and how their local field affects their understanding is important.

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Appendix A

Email to Math Supervisors

Dear Math Supervisor,

I am writing to ask for your assistance with a research study I am conducting on elementary mathematics. I am looking for elementary teachers who use inquiry-based strategies in their instruction. Although there are many definitions of inquiry in the mathematics classroom, most of them include problem solving, rich classroom discussions, and multiple representations and entry points into tasks given to students. If you could provide me with a list of ten teachers (and their schools) that use these techniques, I will email them one at a time in the hope to get two participants in total.

Please let me know if you have any questions about the study or the type of teacher I need. Thank you in advance for your help.

Heather Nunnally
hmnunnally@vcu.edu

Appendix B

Email to Potential Participants

(date sent out), 2018
Elementary Teachers' Definitions and Usage of Inquiry-Based Mathematics Instruction

email to recruit participants

Dear ____________________,
I received your name from (district math supervisor)_____ as a teacher who uses inquiry-based mathematics strategies in your classroom. I hope you will consider participating in an important research project that attempts to better understand how teachers define and use inquiry pedagogy in the elementary classroom. This project is conducted through the School of Education at Virginia Commonwealth University. In an attempt to better understand how teachers define and use the pedagogy in their own classroom, the researchers will interview and observe teachers in their classroom.
Participating in this study is completely voluntary. If you choose to participate, your name and any other identifying information (school, descriptive information) will remain anonymous. If you choose to participate and change your mind at a later date, you may remove yourself from the study at any time with no retribution.
A consent form is attached. If you agree to participate, please sign the form and return to hmnunnally@vcu.edu by ____(1 week from receipt of email)_________.
If you have any questions, please contact:
Heather Nunnally
Virginia Commonwealth University
(804) 828-5231
hmnunnally@vcu.edu

Thank you for considering participation.

Sincerely,
Heather Nunnally

Appendix C

Semi-Structure Pre-Observation Interview Protocol

Pre-Observation Interview

1. Please tell me a little about your teaching history and experience.
    a. What made you decide to become a teacher?
    b. How long have you been teaching?
    c. Which grades have you taught?
2. What do you enjoy about teaching mathematics?
3. How would you define inquiry-based instruction (IBI) in mathematics?
4. What sorts of activities occur in an inquiry-based mathematics classroom?
5. What role does a teacher take in an IBI classroom?
6. What aspects of IBI do you use in your teaching of mathematics?
7. How often do you feel you use inquiry-based techniques in your classroom?
8. When you are teaching using IBI how would you describe your role as a teacher?
9. When you are teaching using IBI how would you describe the students' roles?
10. When you are teaching using IBI how do you know when learning is occurring in your classroom?
11. Do you feel there are any barriers that you face when trying to teach math using IBI? If so, what are those barriers?
12. What kind of support is available at the school level that helps you teach math using IBI?

13. What kind of support is available at the district level that helps you teach math using IBI?
14. What should I look for in your lesson that would relate to inquiry-based mathematics?
15. What should I look for in your planning documents that would relate to inquiry-based mathematics?
16. What should I look for in your planning documents that would support the roles you described for you and your students when you are using IBI?
17. Do you feel that the mission of the school supports IBI in mathematics?

Appendix D

Semi-Structured Post-Observation Interview Protocol

Post-Observation Interview

Share the observation protocol and my notes with the teacher.

1. How do you feel the lesson went?
2. Would you change anything about the lesson if you could do it again?
3. Question the teacher on things the teacher or students said or did that support or dispute the roles they described in the pre-interview. Ask why.
4. Question the teacher on things the teacher or students said or did that support or dispute the relationships they described in the pre-interview. Ask why.
5. Question the teacher on things the teacher or students said or did that support or dispute the activities they described in the pre-interview. Ask why.

Appendix E

Observation Protocol based on Mathematics Scan (M-Scan) Framework for Observing Teachers

Dimension	Questions
Task	• Do selected tasks connect to concepts, or do they mainly focus on memorization? • Are some of the tasks open-ended? • Are the activities mathematically related and coherent? • Are students engaged in problems that allow them to grapple with mathematical concepts, or are they doing exercises for which they are practicing an already learned procedure? • Does the lesson encourage multiple strategies to solve each problem?
Teacher Role	• Are feedback, modeling or examples included that promote complex thinking by students? • Are students encouraged to make conceptual connections during the lesson? • Are students' ideas, questions, and input frequently solicited? • Is the teacher monitoring students working throughout the class period?
Student Role	• Do students consistently participate throughout the math class and play a substantive role in directing the content of math discussion? • Do student explanations focus on conceptual understanding of the concept rather than procedural steps? • Are students often required to provide explanations and justify their reasoning?
Relationship between T and S	• Are students encouraged to make conceptual connections during the lesson? • Are students' ideas, questions, and input frequently solicited by the teacher?
Questioning	• Are questions asked by the teacher focused on mathematical thinking rather than on correct answers? • How often are "what, how, why" questions asked to solicit student explanations or justifications?
Discourse	• Do students often talk to each other and share mathematical thinking and language?

Source: Merritt, E., Rimm-Kaufman, S., Berry III, R., Walkowiak, T., & McCracken, E. (2010). A Reflection Framework for Teaching Mathematics. *Teaching Children Mathematics, 17*(4), 238-248.

Appendix F

Codebooks

I used deductive and inductive coding for this analysis. The initial codes were created based on the literature review for this study and the research questions. Using "open coding" allowed me to organize the categories I wanted to investigate. "Organizational categories function primarily as bins for sorting the data for further analysis" (Maxwell, 2013, p. 107).

Initial Codebook

Deductive Codes	**Description**	**Examples**	**Quotation**
Identity of Teacher	Descriptions of the teacher in the classroom	• roles • jobs	"I am going to teach them a vocabulary word."
Identity of Student	Descriptions of the student in the classroom.	• roles • jobs	"The kids talk through the problems."
Relationships	Interactions between stakeholders in the school.	• student to student • teacher to student • teacher to teacher • teacher to administration • teacher to parent	"Kids work together to figure out the next steps."
Activities	Any action that occurs in the classroom.	• lesson • game • procedure • manipulative • classroom routine	"There is lots of questioning."
Supports	Aids to teachers' selected pedagogy	• personal supports • school supports • district supports	"Our administration is spectacular here."
Barriers	Hindrances to teachers' selected pedagogy	• personal barriers • school barriers • district barriers	"Scheduling can be a barrier."
Inquiry	Descriptions of inquiry	• definitions • examples • opinions on	"Inquiry is getting them involved in their work."

Final Codebook

Inductive Codes	Category	Description	Quotation
Questioning	Identity of Teacher	Types of questions asked, when they are asking questions, why they are asking questions	“Asking them questions that get them thinking another way”
Structured	Identity of Teacher	Controlled part of instruction	“Some days I am more structured.”
Open-Ended	Identity of Teacher	Questions or activities that have multiple solutions	“Instead of me saying, ‘Let’s sort by..”, I will say ‘Let’s sort.’”
Knows Her Students	Identity of Teacher	Understanding the abilities and nuances of the students	“This group needed learning buddies.”
Teacher Roles	Identity of Teacher	Specific duty of the teacher	“I will teach them the essential vocabulary.”
Allow Discovery	Identity of Teacher	Permitting students to discover	“I really stick with allowing the kids to discover even with the pressure of the [standards].”
Classroom Culture	Identity of Teacher	Descriptions of culture in the classroom	“I was impressed by the culture she has created in her classroom.”
Facilitator	Identity of Teacher	Making the learning more accessible for students	“I am the facilitator of the learning.”
Reflective	Identity of Teacher	The teacher thinking about their teaching	“I needed better guiding questions.”
Assessing	Identity of Teacher	Evaluating student work	“I will be looking at their work as I walk around.”
Gives Kids Freedom	Identity of Teacher	Allows students flexibility in the classroom	“Giving the kids the freedom to have choices in what they are doing.”

Content Specialist	Identity of Teacher	Teacher knows what is to be taught	"I teach them the other strategies."
Logistical Manager	Identity of Teacher	Organizing the classroom for learning to occur	"I will get the materials they need."
Planner	Identity of Teacher	Planning of lessons	"Planning take a lot more time."
Keeps the Overall Goal in Mind	Identity of Teacher	Reflects on their goals for students	"Bringing it back to the ultimate goal."
Not Giving the Answer	Identity of Teacher	Teacher withholding final answer	"I answer procedural questions, but not the content questions."
Reminder	Identity of Teacher	Helping students remember	"Remember when you did..."
Supporter	Identity of Teacher	Being a support for students	"Making them work independently while holding their hand."
Cheerleader	Identity of Teacher	Verbally complimenting students	"I am walking around and encouraging."
Believer	Identity of Teacher	Believes students can do mathematics	"We all have to buy into it."
Guide	Identity of Teacher	Teacher advising students on mathematics	"I kind of guide their thinking."
Students Thinking	Identity of Student	Students doing the work	"Let them do the problem solving."
Variety of Ideas	Identity of Student	Multiple ways to think about mathematics	"All the different ways we came up with today."
Active Learning	Identity of Student	Students moving around or doing the mathematics	"They need to experience what they are doing, not just paper and pencil."
Working Together	Identity of Student	Students working with peers	"Kids are working in pairs."
Different Strategies	Identity of Student	Students using various ways to solve problems	"There are so many different ways for the kids to get to the meaning and understanding."
Questioning	Identity of Student	Students asking questions	"Kids that are asking questions are

			the ones that are learning."
Applying Knowledge	Identity of Student	Connecting what they know to other situations	"Try to extend it without me putting you in that situation."
Confident Learners	Identity of Student	Students being certain they can do mathematics	"They feel like they can do it."
Independent Learners	Identity of Student	Students making decisions on their own	"They are deciding what needs to take place."
Discussion	Identity of Student	Talking about mathematics	"Discuss what they are seeing and what they are not seeing."
Discovery	Identity of Student	Uncovering new information	"They don't even realize that they are discovering things."
Engaged	Identity of Student	Students interested in learning	"They were completely engaged in their centers."
Following Directions	Identity of Student	Students following rules of the classroom	"In order for them to get an award, they need to play the game the right way."
Learn from each other	Identity of Student	Learning from peers	"They will learn a little bit from their learning buddy."
Students take Control	Identity of Student	Students making decisions	"They are more in charge of their learning."
Responsibilities	Identity of Student	Students jobs in the classroom	"They need to clean up quietly and quickly to the gathering spot."
OK to be Wrong	Identity of Student	Knowing that being incorrect is alright	"It is ok to make mistakes."
Figure It Out	Identity of Student	Students working on mathematical problems	"Having them figure out what tool they need to solve the problem."
Student to Student	Relationships	Interaction between students	"It is ok to disagree with your partner."

Teacher to Student	Relationships	Interaction between the teacher and student	"I actually assign learning buddies."
Number Talks	Activities	Class discussions around mathematical topics	"I usually start with a number talk."
Problem Solving	Activities	Investigating problems given by the teacher	"I want them to focus on the story problem and how to solve it."
Games	Activities	Activity that is described as a game	"I will introduce the game that will be in the math workstation."
Organization	Activities	Mention of how the classroom is organized	"Mondays and Fridays, I like to stay whole group."
Word Problems	Activities	Mathematical problems with a context	"She used a word problem about baseball."
Examples	Activities	A model of what the students will do	"We will do more examples showing the connections between the two ways."
Questions	Activities	Specific questions the teacher asks	"Do you agree with your partner?"
Students Talking	Activities	Talk between students	"Great discussion between students of different levels."
Hands-On	Activities	Students using their hands during learning	"They are investigating the items at their table."
Exploring	Activities	Investigating concepts	"I wish I could give them more time to explore."
Centers	Activities	Small groups working on the same thing	"They rotate through stations."
Discussion	Activities	Talk between students or teacher and students	"Even if they are wrong, we have talked through it."
Teacher Teaching	Activities	Teacher delivering instruction	"I introduce the essential vocabulary."

Differentiation	Activities	Teaching the same concept in different ways	"Every group I am using the same manipulatives but doing something slightly different with them."
Using Manipulatives	Activities	Any mention of a manipulative	"They will use Five-frames and Ten-frames"
Independent Work	Activities	Students working by themselves	"Independent work at the end so I can see how they are doing."

Vita

Heather Marie Nunnally was born on January 4, 1980, in Cooperstown, New York, and is an American citizen. She graduated from Oneonta High School, Oneonta, NY in 1998. She received her Bachelor of Arts in Psychology from Muhlenberg College, Allentown, Pennsylvania in 2002. She taught in Hanover County Public Schools in Virginia for nine years. She received a Master of Education from The University of Virginia in 2007. She began teaching at Virginia Commonwealth University in August, 2011.