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By

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AN AQUEOUS ENVIRONMENTAL SIMULATION MODEL FOR MID-SOUTH LAKES AND RESERVOIRS *

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ABSTRACT

Quantitative relationships and associated computer program has been developed to simulate some of the major physical, chemical and biological processes occuring within the aqueous phase of lakes and reservoirs. The model was developed, in part, to study the eutrophic development of these water bodies. Emphasis is upon lakes in the Mid-South U.S.A. The physical model reflects the general environment in this region and includes a single stratified period. The chemical subsystem includes nitrogen, phosphorus, oxygen and carbon. The biological subsystem includes phytoplankton, zooplankton, omnivorous fish, carnivorous fish and aerobic bacteria.

The model differential equations are solved numerically with the IBM Continuous System Modeling Program (CSMP). The output results (graphical or numerical) of critical eutrophic parameters can be obtained as a function of time (Julian Day), depth and distance down-lake. The model has been adjusted to field data from Beaver Reservoir in Northwest Arkansas. A comparison of the adjusted simulation and the field data is presented along with examples of use of the model for predictive purposes. The final completion report includes an appendix that contains the program listing, documentation and case studies.

KEY WORDS: MATHEMATICAL MODEL*, LAKE SIMULATION*, EUTROPHIC PREDICTION, COMPUTER SIMULATION, AQUEOUS ENVIRONMENT

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INTRODUCTION

Inland lakes and man-made reservoirs constitute a sizable freshwater resource in the mid-south region of the U.S.A. Maintaining this water in a state of high quality for multiple uses will be a never-ending challenge as population increases and associated cultural developments crowd the lake shores and tributaries. A means of assessing the impact of proposed or present cultural developments upon the lake ecosystem is desirable. The regulation of chemical and energy inputs, which enter the lake through a combination of the tributaries, runoff or point sources, is the major means of controlling and manipulating water quality within a lake or reservoir.

Comprehensive computer simulation models provide a means for assessing the impact of proposed or present cultural developments. These models are also able to predict proposed lake restoration programs and in this sense serve as tools of water quality control.

The primary objective of this project was to develop a mathematical model of the biological, chemical, and physical phenomena occurring within Arkansas lakes and lakes in neighboring states. Some of these lakes and reservoirs are receiving and/or will be receiving aqueous heat loads, industrial wastes, agricultural wastes and urban wastes. Emphasis was on forecasting the effects of these loads on critical eutrophic parameters. Lakes and reservoirs are often the sinks of untreated or partially treated waste discharges, resulting in some beneficial and not so beneficial effects upon the water environment.

ACHIEVEMENT OF PROJECT OBJECTIVES

A. Objectives

The primary objective of this project was to develop a mathematical model of the biological, chemical, and physical phenomena occuring within Arkansas lakes and lakes in neighboring states. The procedure proposed to achieve this objective consisted of five phases:

Phase I: Collect, review and evaluate the available observational data over the past years by scientists and engineers on Mid-South lakes and reservoirs. This data includes the basic water chemistry information, nutrient concentrations, biota present (and concentrations), temperature, heavy metals, benthic deposits, geologic character of reservoir bottom and sides, etc. Where observational data is missing, literature data from lakes of similar limnology will be used.

Phase II: Employ the above data and determine the fundamental phenomena model constants that will mimic the biological, chemical and physical processes characteristic of Mid-South lake environments.

Phase III: Develop a general purpose computerized simulation model. The simulation will be performed on a digital computer employing the IBM continuous systems modeling program (CSMP).

Phase IV: Check the simulation model by comparing its output with the observational data available on Lake Fayetteville and Beaver Reservoirs located in Northwest Arkansas. Make the necessary adjustments as needed to assure a reasonable simulation.

Phase V: Document and make the model available to local state agencies, planning commissions, private industries and municipalities. The model will be polished and finished in this phase and placed in a state of readiness for use by individuals or by personnel at the State Office of Water Resources, University of Arkansas, Fayetteville, Arkansas.

B. Extent of Achievement

The primary objective has been achieved. Each phase outlined above has been completed and this report constitutes the achievement of the last phase of the project. A deterministic lake ecosystem simulation model has been developed which is capable of simulating the gross features of the biological, chemical and physical processes occuring within Mid-South lakes and reservoirs. The generality of the model allows the forecasting of critical water quality parameters within the aqueous environment with emphasis placed upon the effects and results of cultural eutrophication.

Although the primary objective was achieved, several important model facets, originally proposed, were not included. The benthic organisms portion of the biological system was not included. This exclusion was partly due to a general lack of quantitative descriptions of this important subsystem. Heavy metals and associated toxic effects plus pH was not included in the chemical subsystem. In general, toxic material (including metals) information and concentration levels are available but time did not allow the inclusion of this facet. A realistic pH model would have necessitated the inclusion of several other chemical species outside the scope and resources of the project. The physical sybsystem does not contain a metalimnion. An accepted quantitative description of the metalimnion does not exist. The thermocline plane was devised to create two chambers: epilimnion and hypolimnion.

Several important model facets, not originally proposed, were developed and have been included. The developments are innovations of this research project and are presented as such in a later section of this report.

SIMULATION MODEL STRUCTURE

A. Outline of Model

A lake is a complex ecosystem involving interactions between biological, chemical and physical subsystems. The simulation model developed here represents the major actors within this complex ecosystem. If it were possible to include all the lake environmental variables and parameters known to exist astronomical cost would be required to develop a model. This may not even guarantee success. Ultimately, a model has to be weighed among generality, realism, precision and cost to fit the need of such a development. Our inability to formulate the universal model leads to the approach of constructing models for specific lakes. This model is then adjusted to simulate the field data of this specific lake. It should be understood that the application of the model to other lakes is with a certain amount of risk if specific and critical field data are not available for tuning purposes.

The physical structure components of the model consists of hydraulic flows, elements of water volume, geometric shapes, etc. All are interrelated by simplistic though realistic concepts. The lake volume as a whole is subdivided into N subvolumes. The subvolumes are arbitrarily drawn to partition the lake into small segments. Segments or subvolumes are arbitrarily drawn to partition the lake into small segments. Segments or subvolumes are interconnected by water flow in the downlake direction only. Natural shoreline morphology, tributaries and bays may provide natural boundaries for subdivision.

Mid-South lakes typically stratify (i.e. normally warm water above cold) thermally only during spring, summer and early fall. Reverse stratification (i.e. cold water above warm) if evident, is weak and short-lived. In general, these lakes are unstratified (or completely mixed) from Julian Day 1 (J=1) to the vernal equinox (March 21, J=80), stratified (generally unmixed in the vertical direction) throughout the summer until the autumnal equinox (September 23, J=266) or beyond and unstratified to the end of the Julian Year (J=365). This seasonal variation was employed to implement the vertical structure in lake subvolumes.

During the unstratified periods the water contained in lake subvolumes was assumed to be completely mixed. No thermal or concentration gradients were assumed to exist during this period. During the stratified period the water in the subvolumes was divided into two chambers each completely mixed. The upper chamber (i.e., epilimnion) consisted of warm water. This upper chamber was separated from the lower chamber (i.e., hypolimnion) consisting of cold water, by the thermocline. The thermocline is a plane horizontal to the lake surface at which the thermal gradient ($\partial T/\partial z$) is a maximum. The depth of the thermocline increases from the vernal equinox (t=0) to the autumnal equinox as the square root of time t. Water flow was assumed to occur through the entire volume element during unstratified periods, but only through the epilimnion during the stratified period. Surface area-elevation characteristic curves were employed to quantify the variation of water volume with elevation in subvolumes and this was an exogenous variable in the model.

Water flow, surface temperature and sunlight were employed as exogenous variables of the physical portion of the model. These variables were inputted as a function of Julian Day. All subvolumes were assumed to have the same surface temperature and sunlight intensity relationship with time. The bottom lake temperature was assumed constant throughout the year.

Chemical species accounted for in each subvolume by component material balances included: nitrogen, phosphorus, oxygen and carbon. Nitrogen and phosphorus are basic nutrients for phytoplankton (microscopic floating aquatic plants such as blue-green algae) which constituted the primary production biological unit for the prey-predator ecosystem structure. Oxygen critical to a healthy aerobic lake environment, and carbon (organic and oxygen consuming) are closely related to the biological system and are a basic necessity in realistic modeling.

Diffusional transfer of all chemical species in the vertical direction was taken into account. Diffusion can occur in either direction from the hypolimnion to the epilimnion based on concentration difference. Nutrients and carbon compounds can upsurge from the bottom muds (water-mud interface) into the hypolimnetic waters. Interphase transfer of oxygen occurs at the air-water interface. Fickian diffusion is quantified by turbulent (eddy) diffusion coefficients and not tied to the thermal profile. All chemical species are transferred from subvolume to subvolume by interconnecting water flows.

Biological species accounted for in each subvolume by component material balances included: Phytoplankton, zooplankton (microscopic animals that feed on phytoplankton such as daphnia), omnivorous fish (fish that feed on both phytoplankton and zooplankton), carnivorous fish (fish that feed on omnivorous fish) and aerobic bacteria (microscopic organisms that feed on organic matter). Phytoplankton is produced in the nutrient rich, sunlit zone of the epilimnion. "Rainout" (i.e., settling) occurs and they become distributed throughout the water column. The prey-predator structure commences with zooplankton grazing upon phytoplankton, followed by grazing of omnivorous fish and carnivorous fish upon zooplankton and phytoplankton.

Phytoplankton settle downward and are carried from subvolume to subvolume by the water flow. Zooplankton and bacteria are assumed to be uniformly distributed throughout each chamber and carried by flow from subvolume to subvolume. Fish is also assumed to be uniformly distributed throughout each chamber, but mobile enough to resist flow between subvolumes. Only phytoplankton displays any vertical distribution.

Michaelis-Menton and Putter kinetic rate expressions were used in formulating both chemical and biological prey-predator relationships. For each subvolume and chamber of the lake, fourteen non-linear, first order, deterministic ordinary differential equations were formulated based on component (or species) material (mass) balances.

An objective of this work was to produce a user oriented simplified lake ecosystem model which contains a great deal of generality with a certain degree of realism, precision and economy. The remainder of the introduction contains critical information on the use of the simulation model.

The language used in the model simulation was IBM 360/155 CSMP (Continuous Simulation Modeling Program) which accepts most Fortran IV statements. It also provides an application-oriented input language that accepts problems in the form of a system of ordinary differential equations. All exogenous variables such as lake geometry, flow rates, chemical and biological concentrations associated with inflows, sunlight intensities, and temperatures can be inputted as functions of depth or Julian Day as actual field data permit, otherwise, thay can be inputted as constants. Initial calculations of all the fixed parameters such as the volumes and surface areas of subvolumes, seepage rates and runoff rates are performed in the initial section. All the initial conditions such as the initial concentrations of the chemical and biological species which are used in solving the differential equations are also included in the initial section. All the terms which associate with the physical, chemical and biological systems and which remain constant throughout the simulation are listed in the constant statements. All calculations of variables and integrations of the ordinary differential equations which associate with the physical, chemical and biological systems are listed in sequence with the proper subvolume notations inside the dynamic section. Proper sequence and notations are extermely important at this point as to eliminate the possibility of making errors in the simulation. Limitations and selections of the final and intermediate input or output variables used in the simulation to ensure realistic values are performed in the procedure section. Detail descriptions of the input and output formats are provided in the main computer program.

The step size for integration, simulation output intervals and program duration are listed inside the timer section. Integration technique in solving the differential equations is listed inside the method section. The selection of the proper step size and integration method are critical and interrelated. Too large a step size will induce instability or greater error even though a more sophisticated form of the integration method is used. Too small a step size will also induce instability or greater error if a simpler integration method is used. However, if the step size used is small and a sophisticated form of integration method is also used, the simulation cost may be very high. Therefore, the optimum step size and integration method have to be searched by using different combinations of step sizes and integration methods to obtain a stable, realistic and economical simulation.

The variable names used in the model theory are similar to those used in the model simulation. The equations formulated in the model can be applied to any subvolume, therefore, subvolume names are not included. All the input variables used in the model simulation are defined in the theory section with the proper dimensions. In order to obtain a proper simulation, the exact dimensions have to be used. The model thus simulated will give all output biomass concentrations of the chemical and biological systems in mg/l.

A detail list of eutrophic parameters was compiled from different sources and presented in this paper. These ranges can then be applied and adjusted to tune the simulation model to the available field data. Typical example can be found in this paper when the model was applied to a portion of Beaver Reservoir and tuned to the available field data.

As a predictive tool in water quality control, the "tuned" model can then be used to simulate the effects of different sources of impoundments to the lake ecosystem. These predictions can easily be performed by eliminating those parameters which associate with the sources such as inflows, runoff and seepage in the simulation model. A detail illustration of such a predictive application is presented in an appendix to this report.

Furthermore, the model theory is not limited to the simulation of the physical, chemical and biological systems listed in this paper. Similar forms of material balances and reaction kinetics can be included to formulate other species not accounted for in this model such as anaerobic bacteria, benthic animals, calcium, iron, etc., provided that eutrophic parameters such as reaction rate constants, half saturation constants, etc. can be obtained.

B. Innovations in this Model

Seasonal-thermal structure: The physical life-cycle of the lake has been incorporated into the simulation. The annual life-cycle for water bodies can be structured and connected to the seasonal thermal structure. In general, the period of time between the vernal equinox and the autumnal equinox is the dominant productive period for lakes in the temperate zones. This period is roughly from March 21 until September 23, Julian Day 80 to Julian Day 266 respectively for a total of 186 days. This is also the heating period for the lake. During this period the net heat transfer rate into the lake is positive, and lake stratification occurs. Increased solar radiation upon the lake surface and increased air temperature coupled with a favorable water density-temperature relationship causes a stable water column consisting of warm water above and cold water below.

The time of the vernal equinox is important to model structure. Prior to this time the lake is at nearly uniform conditions (isothermal) with respect to temperature. Ice may cover the surface and commences to melt rapidly. A temperature throughout of 4°C to 8°C is not uncommon. This is also the calendar time at which the heat flux rate changes from negative to positive, The lake is stratified during the period between the vernal and autumnal equinoxes.

The time of the autumnal equinox is also important with respect to model structure. It marks the approximate end of the stratified structure of the lake and the beginning of a more mixed state with respect to temperature and nutrients. At this time the net heat flux rate changes from positive to negative. Plunging cold water triggered by heat loss is the primary reason for the rapidly occuring mixed state and the subsequent fall "turnover". A less elaborate physical model is possible for the time period between autumnal and vertical equinoxes, unless the lake becomes reverse stratified in the winter. In the case of lakes which experience reverse stratification, there are two periods of stratification coupled with two periods of mixing.

The calendar year was chosen as the time period for the simulation model; Julian Day 1 through Day 365. The onset of stratification (J~80) and the end of stratification (J~266) are two important model times which divide the calendar year into three time subdivisions: unstratified, stratified and unstratified. Three subdivisions are sufficient for most Mid-South lakes.

The above describe calendar year time-structure is necessary for the switching on and off of other critical physical processes occuring in lakes. These physical processes include chemical species movement (i.e. eddy diffusion) and thermal energy movement (i.e. temperature) in the vertical direction.

Two chamber stratification model: A simplistic thermal model was invoked in order to obtain analytical expression for the thermal profile and the thermocline depth during the stratified period. A semi-infinite slab model was used and is considered good for deep lakes and reasonable for shallow lakes of depth greater than 40 meters (5). The temperature profile expression is

$$\theta(x,t) = \{\theta(0,t) - \theta_o\}\exp(-x^2/4\alpha^{(t)}t) + \theta_o \qquad (1)$$

where: t = time lapse commencing with stratification, t,
θ_o = isothermal lake temperature at t=0,degrees,
$\theta(0,t)$ = lake surface temperature, degrees,
x = water depth measured from air-water interface, l,
$\alpha^{(t)}$ = turbulent thermal diffusivity, l^2/t.

The thermocline depth expression is

$$X_{tc} = \sqrt{2\alpha^{(t)}t} \qquad (2)$$

These equations give an adequate representation of the lake temperature profile and downward migrating thermocline position during the stratified period for purposes of the ecosystem simulation. Equation 2 was used to create an epilimnion and a hypolimnion and mark the separation point between each. A modification of Equation 1 yielded average temperatures for the mixed epilimnion and the mixed hypolimnion. The surface temperature (i.e., $\theta(O,t)$) is applied as an exogenous variable to the model. The inlake temperature was assumed equal to the surface temperature during the unstratified period.

Vertical diffusion in the water column: Pronounced vertical gradients of chemical species can exist in the lake water column. The turbulent diffusion movement of material in the vertical direction is drastically reduced during the stratified period and this can effect the primary production of the ecosystem. A simplistic diffusion model was invoked in order to obtain analytical expressions for the rate of movement of chemicals up and down the water column. A fickian diffusion model along with a semi-infinite slab structure was used to develop explicit expressions for concentration profiles and flux rates of nutrients leaving the water-sediment interface (7). The concentration profile expression for component A is

$$C_A(X',t) = (C_{AO} - C_{Ai}) \{1 - \mathrm{erf}\, X'/\sqrt{4D^{(t)}t}\,\} + C_{Ai} \qquad (3)$$

where: C_{Ai} = initial water column concentration at start of the stratified period, m/l^3,

C_{AO} = concentration in water at mud-water interface, m/l^3,

X' = distance upward from mud-water interface, l,

$D^{(t)}$ = turbulent diffusivity, l^2/t.

The mass flux rate expression for material entering the hypolimnion from the bottom sediments is,

$$N_{AO} = \sqrt{\frac{D^{(t)}}{\pi t}}\,(C_{AO} - C_{Ai}) \qquad (4)$$

where N_{AO} is the mass flux per unit of sediment interfacial area and had dimensions of $m/l^2 \cdot t$. Equation 3 was employed to estimate $D^{(t)}$ and it was found to be closely related to $\alpha^{(t)}$. Equation 4 was used to account for the movement of nutrients nitrogen and phosphorus from the sediments up the water column into the hypolimnion. During the period t increased continuously, however during the unstratified period an average flux rate based on a t of two weeks was employed. A modification of Equation (4) was employed as an expression for the flux rate of all diffusible between the hypolimnion and epilimnion.

Omnivorous Predation: The biological system quantitative description was based in large part upon the analysis presented by O'Connor, et.al. (10, 16) and Yang and Johnson (3). In general, a combination of Michaelis-Menton and Putter kinetic rate expressions were employed.

At first a prey-predator hierarchy consisting of phytoplankton, zooplanktoon and a single fish category was envisioned. The simulation model was incapable of mimicing the field data and the fish were partitioned into a omnivore and carnivore subgroups. The net growth rate equation for a living biomass in an aquatic ecosystem is typically written

$$r_g = R_{gc} \, C_{of}^{m1} \qquad (5)$$

where: R_{gc} = unit growth rate of species
C_{of} = concentration of the species (i.e., omnivore fish)
$m1$ = constant coefficient

The unit growth rate coefficient R_{gc} for species C may be limited by the food concentration (i.e., zooplankton and phytoplankton). The relationship is conveniently expressed in terms of Monod's equation for two nutrient food-limited growth:

$$R_{gc} = R_{gco}\left(\frac{C_A}{K_{FA}+C_A}\right)\left(\frac{C_z}{K_{Fz}+C_z}\right) \qquad (6)$$

where: R_{gco} = maximum growth rate of species
C_A, C_z = concentrations of phytoplankton and zooplankton
K_{FA}, K_{Fz} = half saturation constants for phytoplankton and zooplankton.

Repeated simulations, adjusting growth constants, etc. within accepted (i.e., published) ranges, with Equations 5 and 6 for omnivore fish resulted in an inadequate qualitative representation of the phytoplankton and zooplankton field data.

Omnivore fish are not dependent upon both food sources being available at all times so that Equation 6 is unrealistic. This equation was modified to reflect a unit growth rate dependent on only one/and or both food sources being available:

$$R_{gc} = R_{gco} \left\{ \frac{C_A}{K_{FA} + C_A} + \frac{C_z}{K_{Fz} + C_z} \right\} \qquad (7)$$

The predation rates within the phytoplankton (phyto) and zooplankton (zoo) material balances were also modified to reflect this omnivorous predation. For phytoplankton:

$$r_a = \frac{R_{gc}}{e} \; f_{OA} \; C_{Of}{}^{m1} \left(\frac{A_a}{A_a + A_z} \right) \tag{8}$$

and for zooplankton:

$$r_z = \frac{R_{gc}}{e} \; f_{oz} \; C_{of}{}^{m1} \left(\frac{A_z}{A_a + A_z} \right) \tag{9}$$

where: r_a and r_z = omnivore fish predation rate upon phyto and zoo,
f_{oa} and f_{oz} = preference factors of omnivore upon phyto and zoo,
e = omnivore assimilation efficiency,
A_A = $C_A/(K_{FA} + C_A)$, phyto availability and,
A_z = $C_z/(K_{FZ} + C_Z)$, zoo availability

Use of these modified rate equations resulted in a much improved simulation of the field data both in a qualitative and quantitative sense.

C. General Model Assumptions

- Lake was divided into convenient segments starting from tributaries and proceeding downstream.
- During unstratified period(s), each segment was assumed completely mixed.
- During stratified period(s), each segment was assumed to consist of epilimnion and hypolimnion and each being completely mixed.
- Flows were assumed to proceed from segment to segment in the downstream direction.
- Tributary flows entered the first segment in each reach.
- Distributed flows (as seepage and runoff) were lumped as a single input to each segment that applied.
- Each segment was assumed to have similar uniform geometry.
- All segments received the same amount of photoactive radiant energy input per unit of exposed surface and had the same temperature distribution.
- Radiant energy, temperature and stream flows were forcing functions.
- Chemical system included soluble nitrogen (total) and phosphorus (total), oxygen, and organic matter.
- Phytoplankton (a single lumped species) was the primary contributor for a prey-predator biota system.
- Other biological species were zooplankton, omnivorous fish, carnivorous fish and bacteria. (All were assumed as lumped species, respectively).
- Nutrient recycling was accounted for at all major biota levels.
- Eddy diffusion transfer of all chemical (dissolved) species between epilimnion and hypolimnion during stratification.
- Gas (oxygen) exchange through air-water interface.
- Nutrients upsurge into overlying lake waters from rich bottom sediment during stratified and unstratified periods.
- Oxygen absorbed into bottom sediment.
- A combination of Michaelis-Menton and Putter kinetic expressions quantify the chemical and biological reaction rates.
- Model consisted of interrelated non-linear deterministic, dynamic system of ordinary differential equations.

D. Model Structure - Beaver Reservoir as example

Sections of Beaver Reservoir (Figure 1) were used as an example to the application of the modeling concepts. Three lake segments (Figure 2) were used. The first segment began from the intersection of White River and Highway 45 bridge to eighteen river miles downstream at WTR (White River Reach). The second segment began from War Eagle Creek to eleven river miles downstream at HPT (Hoffman's Point). The third segment began at the confluence of the first and second segments to eight river miles downstream at WWK (Water Works). Each segment was treated as a separate, completely mixed reactor with first and second segments in parallel and both were feeding into the third segment (Figure 2). It was also assumed that there was no back flow upstream between segments.

The flow of the White River (1) before joining Fayetteville Treatment Plant, the flow of the Treatment Plant (2) and the flow of War Eagle Creek (2) were inputted to the model using function generators. Flows into each segment were assumed well mixed before entering (Figures 3, 4 and 5).

The summer stratification period of Beaver Lake was assumed to start on Julian day 80 and end on Julian day 270. During stratification, both epilimnion and hypolimnion were treated as separate, completely mixed reactors with turbulent diffusion of chemical components as the main interaction. It was assumed that flow occurred only through the epilimnion. During unstratified periods, the entire segment was considered as a single, completely mixed reactor. It was assumed that flows occurred in the entire segment.

Three types of systems were considered in Beaver Reservoir as physical, chemical and biological. The physical system consisted of surface water temperature fluctuations, flow conditions, lake geometry, thermocline locations and sunlight intensities. The chemical system consisted of nutrients (nitrogen and phosphorus), oxygen and organic matter. Inorganic carbon input from air was assumed to be in excess for growth requirement (3). Therefore, it was not considered in this model. Daily pH fluctuations were ignored in this model. The biological system consisted of phytoplankton, zooplankton, two types of fish (omnivores and carnivores), and bacteria. Concentrations of nutrients, oxygen, organic matter, phytoplankton, zooplankton and bacteria were limited to their minimum detectable concentrations in

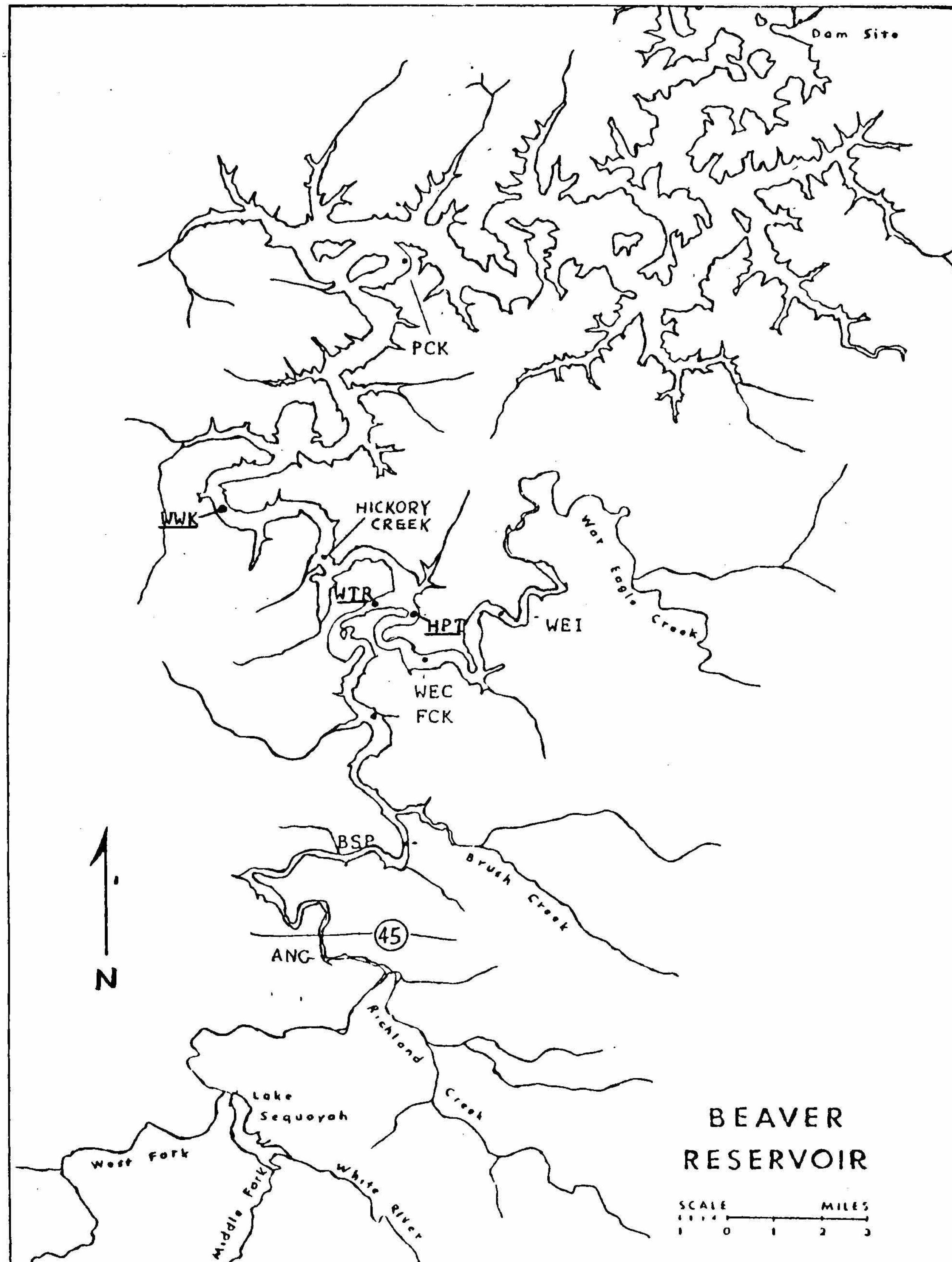

Figure 1. Beaver Reservoir Map

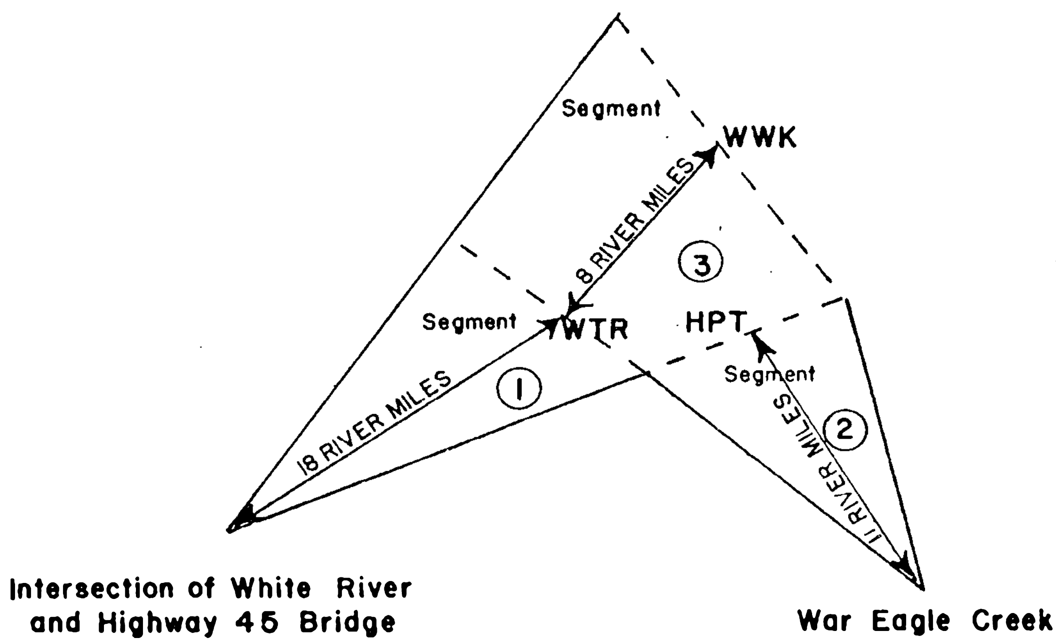

Figure 2. Beaver Reservoir Segmentations

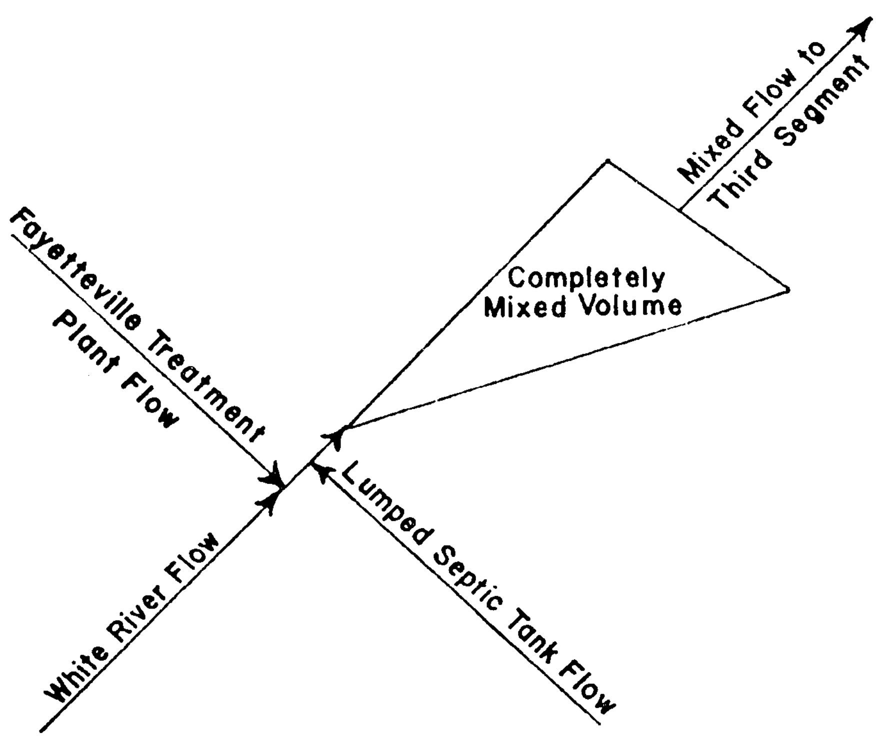

Figure 3. First Segment Flow Pattern

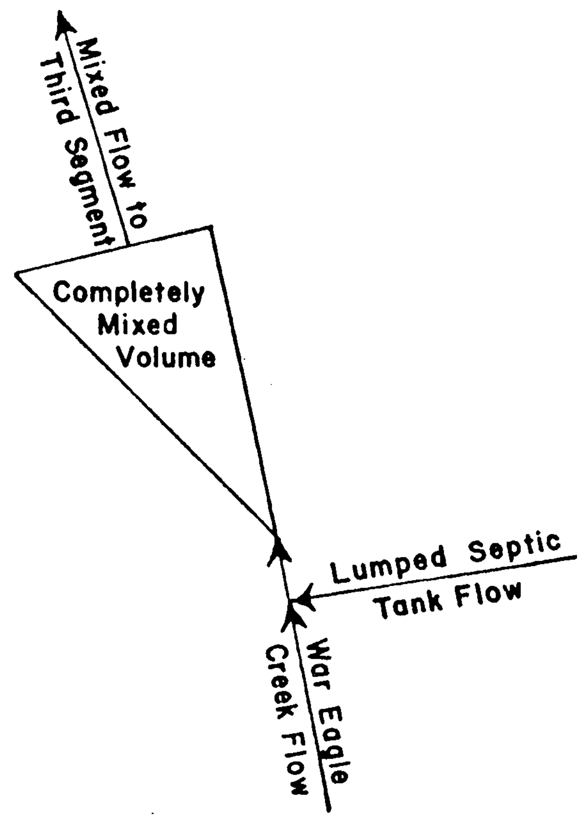

Figure 4. Second Segment Flow Pattern

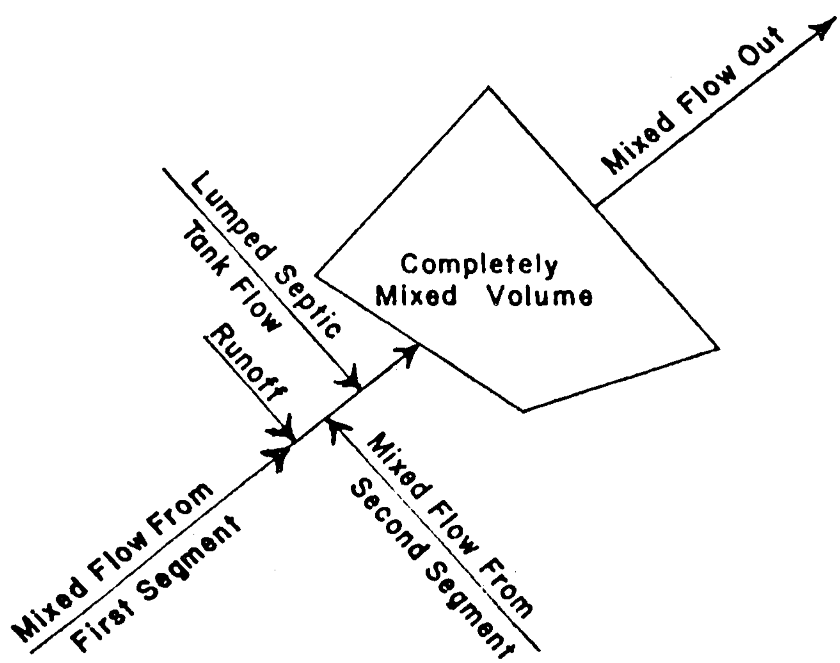

Figure 5. Third Segment Flow Pattern

the model simulation for realistic purpose. Overall ecosystem interrelationships and pictoral descriptions of system dynamics in both unstratified and stratified periods of the lake ecosystem are shown in Figures 6, 7 and 8 respectively.

For chemical and biological components, the general mass balance was used as the basis for developing differential equations for the specific species under consideration. It was as follows (6):

$$\frac{dC}{dt} = \left(\frac{Q_o}{V}\right)(C_I - C) + \Sigma \text{ Sources} - \Sigma \text{ Sinks}$$

where: C = outlet and volume segment of a species concentration, (mg/l)
Q_o = volumetric flow rate to each segment, (ft^3/day)
V = Volume of the segment where flow occured, (ft^3)
C_I = inlet concentration of that species, (mg/l)
Σ sources = those phenomena associated with biological growth, diffusion in, upsurge, etc., (mg/l-day)
Σ sinks = those phenomena associated with biological respiration, mortality, predation, diffusion out, etc., (mg/l-day)

The IBM System/360 CSMP (Continuous System Modeling Program) language was used in the entire model simulation with Fourth-order Runge Kutta fixed step as the integration method. The step size was 0.2 day and simulation period was one year. The simulation time was less than six minutes.

E. Physical System Components

1. Lake Geometry Estimation

Volumes and surface areas of lakes were inputted as functions of elevation to the model by the use of the function generators. Each lake could be sectioned into different segments as geometry permitted. The bottom of each segment was assumed to have a uniform slope or elevation drop per river mile (Figure 9). From the data on Beaver (4), the elevation drops in feet per river mile for the first and third segments were 5.97 and the second segment was 2.98. Then, the maximum depth of each segment was obtained from the following:

$$MAXDPF = (S)(DI)$$

where: MAXDPF = Maximum depth of each segment, (feet)
S = Bottom slope of each segment, (feet/river miles)
DI = Distance of each segment, (river miles)

It was further assumed that the lake structure was similar. In other words, when each segment was transposed to the bottom of the dam (where the area

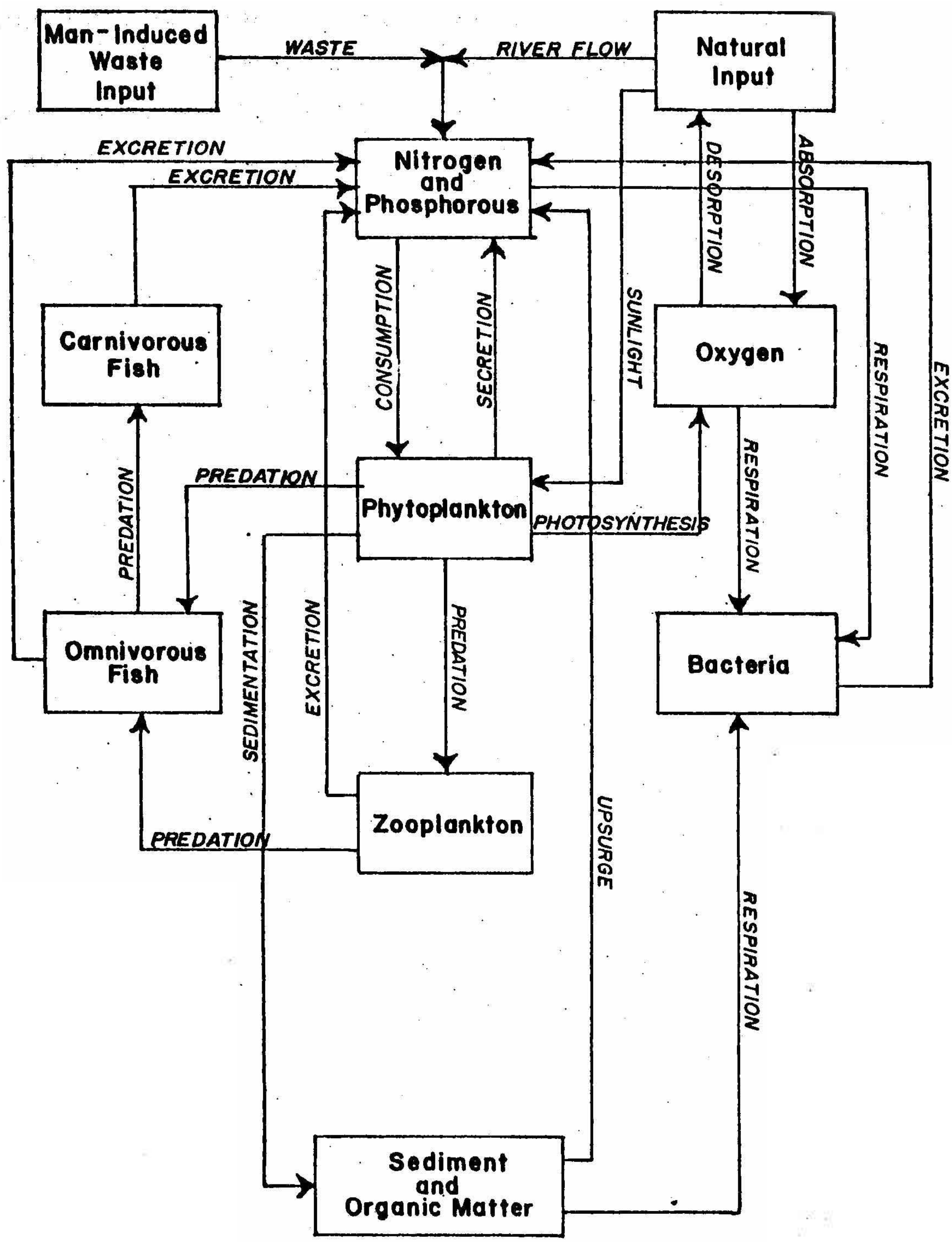

Figure 6. Lake Ecosystem Interrelationships

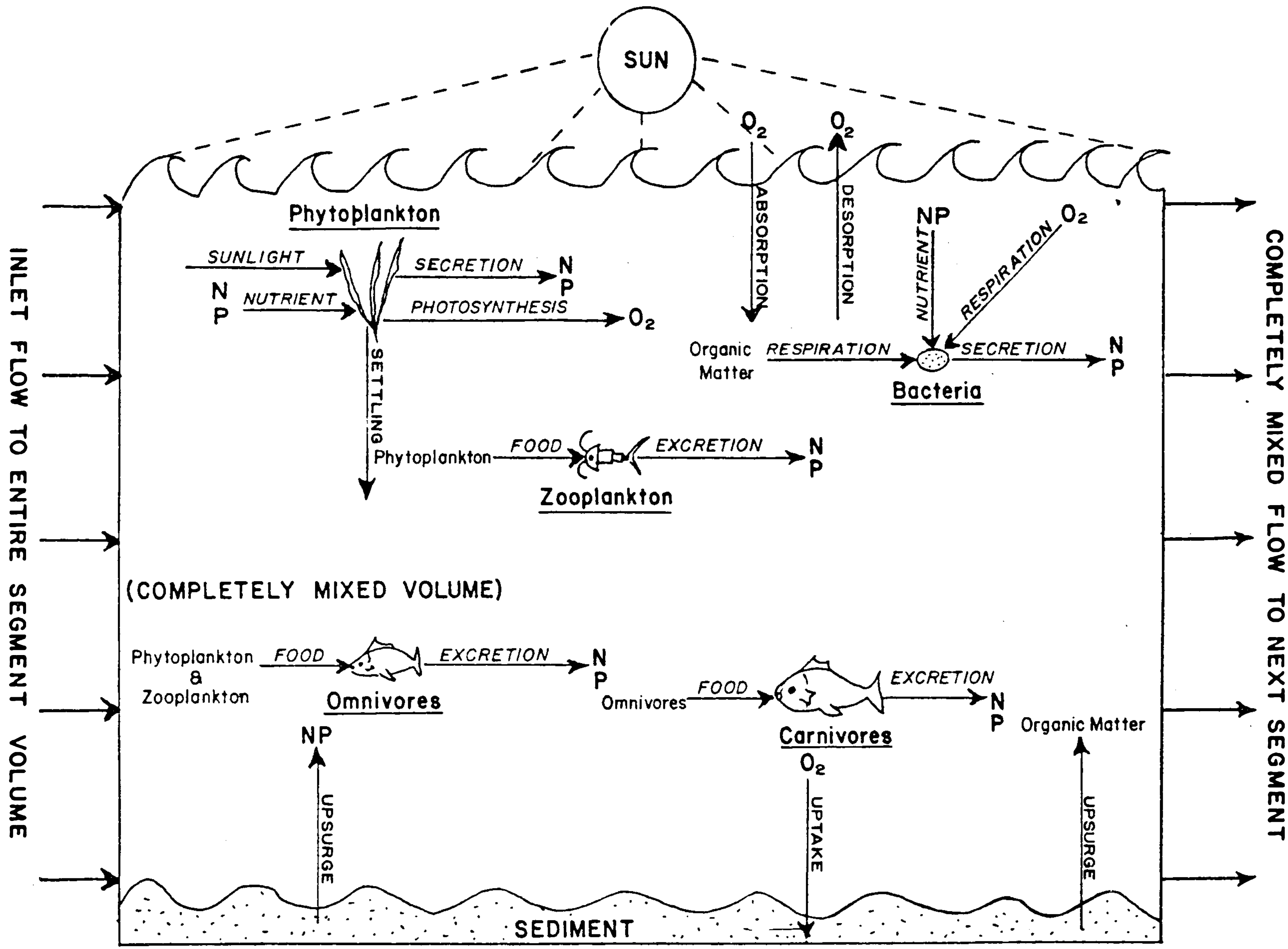

Figure 7. Dynamic Ecosystem in Unstratified Completely Mixed Volume

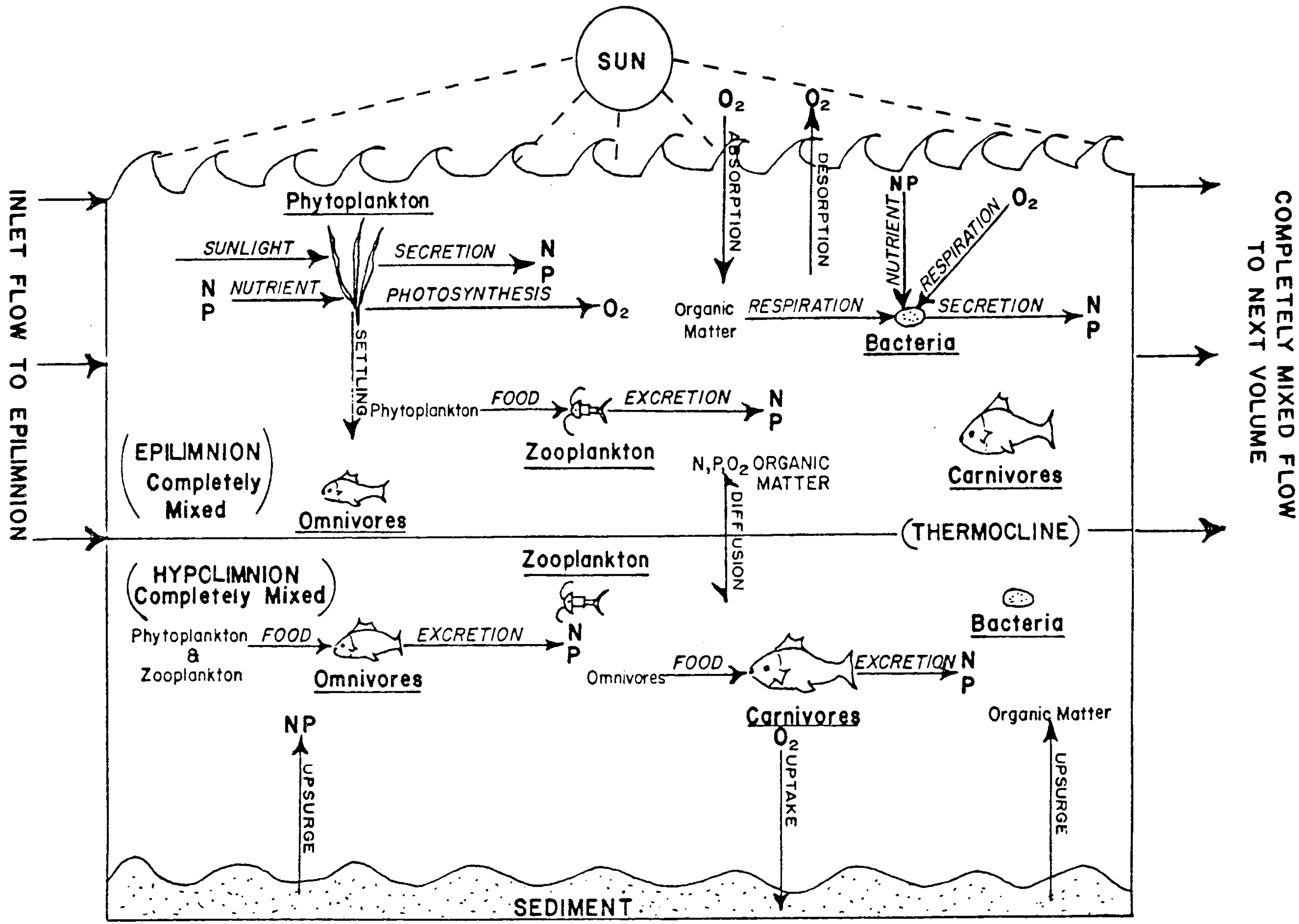

Figure 8. Dynamic Ecosystem in Stratified Completely Mixed Epilimnion and Hypolimnion Volumes

and volume were zero) (Figure 10), its surface area and volume were assumed to remain unchanged. With the addition of the maximum depth of each segment to the elevation of the bottom of the dam, the actual transposed elevation was obtained. With this transposed elevation, actual segment surface area and volume were obtained directly from the actual hydrological inputs. As summer stratification commenced, thermocline began to form acting as a membrane, separating the epilimnion and hypolimnion. In order to obtain the epilimnion and hypolimnion volumes and thermocline area, thermocline position had to be located as a function of time. It was formulated as follows (5), (Figure 10):

$$X_{TC} = \sqrt{(2.0)(TDIFS)(T_L)}$$

where: X_{TC} = thermocline depth from lake surface, (m)
TDIFS = diffusion constant at stratification, (square meter/day)
T_L = model time or lapsed time since the lake was isothermal,(days)

After the thermocline of the segment was obtained, the difference in height between the maximum depth and thermocline depth was transposed, using the same approach as before, to the bottom of the dam. Hypolimnion volume and thermocline area could be obtained. The difference between the total segment volume and hypolimnion volume gave the epilimnion volume.

2. Flows

Inlet flow rates to any segment were considered as functions of time or constants, depending on available data. All flows were combined before entering into each segment. Also, it was assumed that there was no back flow between segments. The seepage rate from septic tanks along the shore of the lake was formulated as follows:

$$Q_{SEEP} = \frac{(WOPCD)(POP)(DI)}{(TRBL)(7.48)}$$

where: Q_{SEEP} = septic tanks' seepage rate, (ft^3/day)
WOPCD = waste output per capita per day, (gal./capita-day)
POP = total population along the shore of the lake, (capita)
TRBL = total river miles of the lake, (river miles)

Runoff was also taken into consideration as another source of input of flow and nutrients. The fact was that leaching of soluble compounds of nitrogen and phosphorus from poultry farming areas into lake system might stimulate eutrophication process. The runoff rate was formulated as follows:

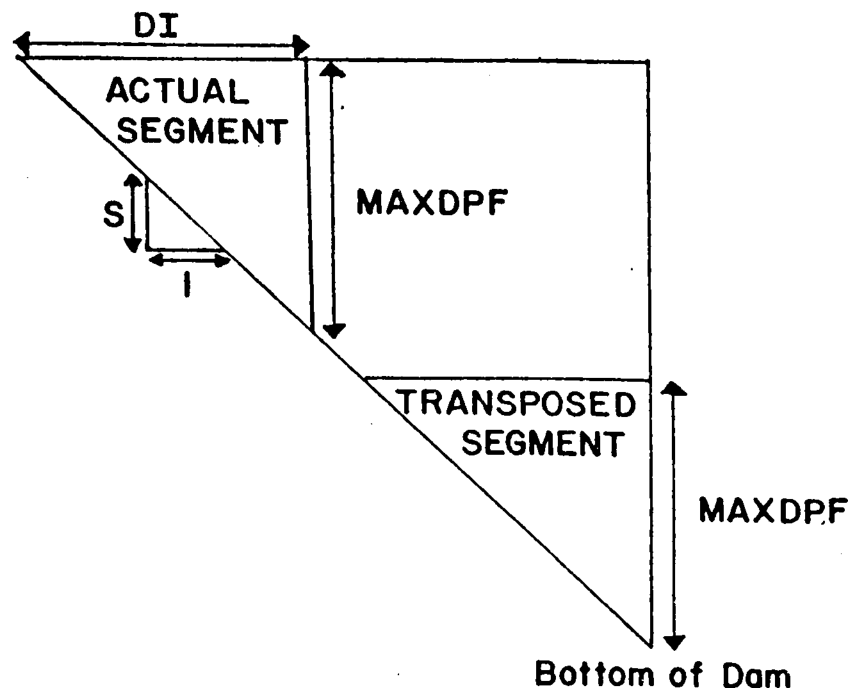

Figure 9. Transposing Segment to Bottom of Dam

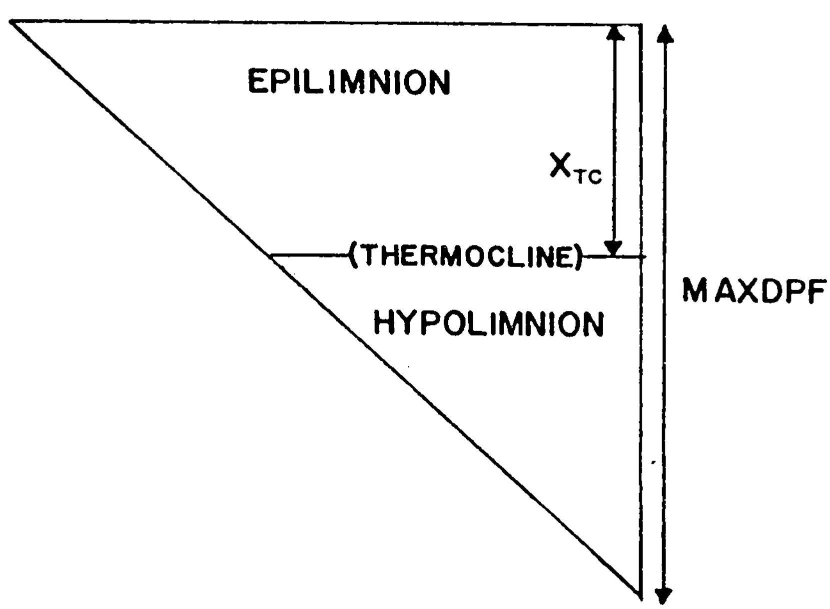

Figure 10. Thermocline Location in Stratified Period

$$RUNOFF = (RNCFSM)(SM)(3600.0)(24.0)$$

where:
- RUNOFF = runoff rate into any segment, (ft^3/day)
- RNCFSM = runoff rate per area per unit time, (ft^3/sec.-mile^2)
- SM = total estimated area for leaching, (mile^2)

Flows were assumed to enter the entire volume segment in unstratified period(s) and in epilimnion in stratified period(s). This assumption was verified by the fact that warmer water tends to stay in the upper layer. Also, flows from septic tanks were assumed as a single lumped flow and combined with the other inlet flows before entering each segment. This concept was further illustrated in the case of Beaver Reservoir (Figures 7, 8 and 9).

3. Water Temperature

Surface water temperatures were put in as function of time using a function generator. Bottom water temperature was assumed constant. During unstratified periods, water temperatures were assumed uniform in the entire volume segment approaching surface water temperatures. However, during stratified periods, epilimnion and hypolimnion temperatures were different. Therefore, thermocline temperatures were obtained and the average of thermocline and bottom temperatures gave the average hypolimnion temperatures as a function of time. They were obtained as follows (5):

$$\theta(X_{TC}, t) = T_B + (T_o(t) - T_B)(\exp - ^{(1/2)})$$

where:
- $\theta(X_{TC}, t)$ = thermocline temperature at time t, (°F)
- t = model time, (days)
- T_B = bottom water temperature, (°F)
- $T_o(t)$ = surface water temperature as function of time, (°F)

Therefore, average hypolimnion temperature as function of time was:

$$TH = [T_B + \theta(X_{TC}, t)]/2.0$$
$$= [(2)(T_B) + (T_o(t) - T_B)\exp^{(-1/2)}]/2.0$$
$$= 0.697(T_B) + 0.303(T_o(t))$$

where: TH = average hypolimnion temperature, (°F)

4. Sunlight Intensity

Since intensity of sunlight was an important factor for phytoplankton growth, maximum and minimum sunlight radiation data from the Weather Bureau (6) were fitted into equations as functions of time for modeling use. For Beaver Reservoir, it was as follows:

$$SMAX = \left\{188.3 + 97.1 \quad \sin\left[\frac{(2.0)(\pi)}{365.0} \; (t - 80.0)\right]\right\}$$

$$SMIN = \left\{105.9 + 63.3 \quad \sin\left[\frac{(2.0)(\pi)}{365.0} \; (t - 70.0)\right]\right\}$$

$$S_c = [1.0 + (0.01)(E)] \; [SMIN + R \; (SMAX - SMIN)]$$

where: SMAX = maximum sunlight radiation as function of time, (cal/cm^2 - day)

SMIN = minimum sunlight radiation as function of time, (cal/cm^2 - day)

S_c = actual intensity of sunlight at time t, (cal/cm^2 - day)

E = lake elevation in thousand feet

R = cloud cover factor.

F. Chemical System Components

1. Nutrient (Nitrogen and Phosphorus)

The nutrients nitrogen (total) and phosphorus (total) were considered in soluble form and were assumed to be released by all major trophic levels as a result of respiration, excretion and decay. Upsurge of nutrients from sediment was also considered in the model. For shallow lakes, sediment upsurge might stimulate phytoplankton growth during low nutrient inputs from tributary flows. Nutrients associated with flows and runoff were either considered as constants or functions of Julian days (if enough data could be obtained) in the simulation. Separate mass balance equations were employed for both epilimnion and hypolimnion during stratification. Slight modifications of the derived epilimnion equation was used to formulate the mass balance equation for unstratified period(s). They were as follows:

a. Epilimnion (completely mixed)

(1) Nutrient Mass Balance

$$\frac{dC_{N1}}{dt} = \left(\frac{Q_o}{V_T}\right) (C_{NI} - C_{N1}) + \sum_{i=1}^{n} (FR_i)\,(RESC_i)\,(C_i)^m + N2DF1$$

$$(SR)\,(GRCA_1)\,(C_{A1})^m$$

where: C_{N1} = epilimnion outlet and segment nutrient concentration, (mg/l)

C_{N1} = epilimnion inlet nutrient concentration, (mg/l)

V_1 = epilimnion volume

n = total number of major biological species

FR_i = fraction of nutrient in excretion of major species i (mg nutrient/mg species i excreta)

$RESC_i$ = respiration rate of major biological species i, (day^{-1})
C_i = concentration of major biological species i, (mg/l)
m = constant coefficient
N2DF1 = nutrient diffusion between epilimnion and hypolimnion during stratification, (mg/l-day)

SR = stochiometry ratio of nutrients uptake by phytoplankton for growth, (weight of nutrient uptake/weight of phytoplankton formed)
$GRCA_1$ = positive growth rate of phytoplankton in epilimnion (day^{-1})
C_{A1} = epilimnion phytoplankton concentration, (mg/l)

(2) Nutrient Diffusion (16)

$$N2DF1 = \frac{(AT)\,(ATC)\,(C_{N2} - C_{N1})}{(DZTMP)\,(V_2)\,(0.3048)}$$

where: AT = average vertical dispersion constant at stratification (5),(16),(m^2/day)

$$= (2.0)\,(DZTMP)\left(\sqrt{\frac{TDIFS}{(\pi)\,(1.0)}}\right)$$

DZTMP = average depth of entire lake, (m)
ATC = area of thermocline where diffusion occurred, (ft^2)
C_{N2} = hypolimnion nutrient concentration, (mg/l)
V_2 = hypolimnion volume, (ft^3)

b. Unstratified Whole Segment Volume (completely mixed)

During completely mixed or unstratified periods, thermocline ceased to exist. Therefore, diffusion of nutrients at the thermocline was absent. However, nutrient upsurge from sediment was still persisting with a higher diffusion rate due to turbulence. Inlet flows were previously assumed to flow through the entire volume segment. Epilimnion equations were modified as follows:

(1) Nutrient Mass Balance

$$\frac{dC_N}{dt} = \left(\frac{Q_o}{V_T}\right)(C_{NI} - C_N) + \sum_{i=1}^{n} (FR_i)\,(RESC_i)\,(C_i)^m + NUPS - (SR)\,(GRCA)\,(C_A)^m$$

where: C_N = outlet and entire segment nutrient concentration, (mg/l)
V_T = total segment volume, (ft^3)
C_A = phytoplankton concentration in entire segment, (mg/l)
GRCA = positive growth rate of phytoplankton in entire segment volume, (day^{-1})

(2) Nutrient Upsurge (7)

The upsurge rate was evaluated at the average depth of entire segment volume:

$$NUPS = (2.0)\left(\sqrt{\frac{TDIFU}{(\pi)(TMIX)}}\right) \quad (C_{NINF} - C_{NINI})\left(\frac{SAREA}{(V_T)(0.3048)}\right)$$

where: NUPS = nutrient upsurge rate to entire segment, (mg/l-day)
TDIFU = diffusion constant at unstratification, (m^2/day)
TMIX = mixing time lapsed since unstratification commenced, (day)
C_{NINF} = sediment-water interfacial nutrient concentration, (mg/l)
C_{NINI} = initial nutrient concentration in segment, (mg/l)
SAREA = surface area of segment, (ft^2)

c. Hypolimnion (completely mixed)

It was assumed that inlet flow did not flow through hypolimnion. Therefore, terms associated with flow were neglected. It was further assumed that phytoplankton grew only in epilimnion, then, nutrients released by phytoplankton were ignored. Furthermore, nutrient diffusion was formulated in the same manner as in epilimnion, except in opposite direction. Nutrient mass balance and upsurge were modified as follows:

(1) Nutrient Mass Balance

$$\frac{dC_{N2}}{dt} = \sum_{i=1}^{n-1} (FR_i)(RESC_i)(C_i)^m - N2DF1 + NUPS_2$$

where: $NUPS_2$ = nutrient upsurge from sediment to hypolimnion (mg/l-day)

(2) Nutrient Upsurge (7)

Upsurge rate was evaluated at average depth of hypolimnion:

$$NUPS_2 = (NUPK_2)(C_{NINF} - C_{NINI})\left(\frac{ATC}{(V_2)(0.3048)}\right)$$

where: $NUPK_2$ = nutrient upsurge constant to hypolimnion, (m/day)

$$= \sqrt{\frac{TDIFS}{(\pi)(TS)}} \left\{\exp\left[-\frac{(MAXDP - X_{TC})^2}{(16.0)(TDIFS)(TS)}\right]\right\}$$

TS = time lapsed since stratification commenced, (day)
MAXDP = maximum depth of segment (MAXDPF) in meters, (m)

2. Oxygen

Oxygen was transferred through the air-water interface by absorption or desorption. It was assumed to be released only by phytoplankton photosynthesis It was further assumed to be used up by bacteria in aerobic digestion of organic matter and by oxidation of sediment.

Oxygen associated with flows were also included. Development of oxygen mass balances in both epilimnion and hypolimnion in stratified period(s) and entire segment volume in unstratified period(s) were closely related to those of nutrient mass balances. The formulation of the diffusion of oxygen through the thermocline followed essentially the same mechanism as that of nutrients. However, there was no upsurge of oxygen from sediment except uptake by sediment which was assumed to occur in hypolimnion at stratification and entire volume segment at unstratification. Overall oxygen balances were formulated as follows:

a. Epilimnion (completely mixed)

$$\frac{dC_{021}}{dt} = \left(\frac{Q_o}{V_1}\right) (C_{021} - C_{021}) + O2SUF_1 + O2ALG_1 - O2DIF - O2OM_1$$

where: C_{021} = epilimnion outlet and segment oxygen concentration,(mg/l)

C_{021} = epilimnion inlet oxygen concentration, (mg/l)

$O2SUF_1$ = rate of oxygen transferred through air-water interface to epilimnion, (mg/l-day)

$$= \frac{(KL)\,(COS - C_{021})}{(H_{1F})}$$

H_{1F} = average depth of epilimnion in feet, (ft)

$$= \left(\frac{V_1}{SAREA}\right)$$

KL = liquid phase mass transfer coefficient for absorption or desorption of oxygen from or to atmosphere, (ft/day)

COS = oxygen saturation concentration in water at a given temperature (8), (mg/l)

$$= 14.5532 - 0.38217\,(T) + 0.0054258\,(T)^2$$

T = surface water temperature ($T_o(t)$) in degree centigrade, (^oC)

$O2ALG_1$ = rate of oxygen released by phytoplankton during photosynthesis in epilimnion (mg/l-day)

= $(GMMA)(GRCA_1)(C_{A1})^m$

GMMA = stochiometry ratio of oxygen produced per phytoplankton formed, $\left(\frac{mgO_2}{\text{mg phytoplankton}}\right)$

O2DIF = oxygen diffusion between epilimnion and hypolimnion during stratification, (mg/l-day)

$$= \frac{(AT)(ATC)(C_{021} - C_{022})}{(DZTMP)(V_1)(0.3048)}$$

C_{022} = hypolimnion oxygen concentration, (mg/l)

$O2OM_1$ = rate of epilimnion oxygen consumed by bacteria acting on organic matter, (mg/l-day)

= $(O2COM)(OMCN_1)$

O2COM = stochiometry ratio of oxygen consumed per organic matter consumed by bacteria, $\left(\frac{\text{mg oxygen consumed}}{\text{mg organic matter consumed}}\right)$

$OMCN_1$ = rate of epilimnion organic matter consumed by bacteria, (mg/l-day)

b. Unstratified Whole Segment Volume (completely mixed)

Similarly, oxygen mass balance in unstratified period(s) was derived from epilimnion equation except there was no diffusion across the thermocline, but there was oxygen used up by sediment for oxidation. It was as follows:

$$\frac{dC_{02}}{dt} = \left(\frac{Q_o}{V_t}\right) (C_{021} - C_{02}) + O2SUF + O2ALG - O2OM - O2SED.$$

where: C_{02} = outlet and entire segment oxygen concentration, (mg/l)

O2SUF = rate of oxygen transferred through air-water interface to entire segment volume, (mg/l-day)

$$= \frac{(KL)(COS - C_{02})}{(H_F)}$$

H_F = average depth of entire segment volume, (ft)

$$= \left(\frac{V_T}{SAREA}\right)$$

O2ALG = rate of oxygen released by phytoplankton during photosynthesis in entire segment volume, (mg/l-day)

= (GMMA)(GRCA)$(CA)^m$

GRCA = positive growth rate of phytoplankton in entire segment volume, (day^{-1})

C_A = phytoplankton concentration in unstratified period, (mg/l)

O2OM = rate of oxygen consumed by bacteria acting on organic matter in unstratified period, (mg/l)

= (O2COM)(OMCN)

OMCN = rate of organic matter consumed by bacteria in unstratified period, (mg/l-day)

O2SED = rate of oxygen consumed by sediment in unstratified period (9), (mg/l-day)

$$= \frac{(KOSED)\,(THSED)^T}{(H)}$$

KOSED = rate constant for oxygen consumed by sediment, (gm oxygen/m^2-day)

THSED = temperature coefficient for oxidation of sediment,

H = average depth of entire segment, (H_F) in meters, (m)

c. Hypolimnion (completely mixed)

Those terms associated with flows, phytoplankton and absorption (desorption) through the air-water interface were not considered in hypolimnion. Diffusion through the thermocline was taken in the opposite direction as that of epilimnion. Sediment uptake of oxygen was included in this model development. Overall equation was as follows:

$$\frac{dC_{022}}{dt} = O2DIF - O2OM_2 - O2SED_2$$

where: $O2OM_2$ = rate of hypolimnion oxygen consumed by bacteria acting on organic matter, (mg/l-day)

= (O2COM) ($OMCN_2$)

$OMCN_2$ = rate of epilimnion organ matter consumed by bacteria, (mg/l-day)

$O2SED_2$ = rate of oxygen consumed by sediment in hypolimnion, (mg/l-day)

$$= \frac{(KOSED)(THSED)^{THC}}{(H_2)}$$

THC = average hypolimnion temperature (TH) in degree centigrade, (°C)

H_2 = average depth of hypolimnion in meters, (m)

$$= \left(\frac{V_2}{ATC}\right)(03.048)$$

3. Organic Matter

A fraction of settling phytoplankton was assumed to have partially dissolved and returned as soluble organic matter into water. It was also assumed that organic matter released by other trophic levels was very low and treated as a constant input to the model for completeness. Organic matter was further assumed to have been released by sediment in soluble form. Upsurge into the water column was treated just as nutrient upsurge. Bacteria was assumed to be the sole predator of the organic matter. BOD (Biochemical Oxygen Demand -Total) was used as the basis in the material balance. Overall organic matter balances in both stratified and unstratified periods were formulated in a similar manner as oxygen balances and were as follows:

a. Epilimnion (completely mixed)

$$\frac{dC_{OM1}}{dt} = \left(\frac{Q_o}{V_1}\right)(C_{OMI} - C_{OM1}) + OMALG_1 + ROMO - OMCN_1$$

where: C_{OM1} = outlet and epilimnion organic matter concentration, (mg/l)

C_{OMI} = inlet epilimnion organic matter concentration, (mg/l)

$OMALG_1$ = rate of soluble organic matter released by settling phytoplankton in epilimnion, (mg/l-day)

= $(FSAD)(SETLA_1)(FOMA)$

FSAD = fraction of settling phytoplankton that can be utilized by bacteria

$SETLA_1$ = settling rate of phytoplankton in epilimnion, (mg/l-day)

FOMA = fraction of settling phytoplankton that was organic matter

ROMO = rate of organic matter input to water from other trophic levels, (mg/l-day)

$OMCN_1$ = rate of epilimnion organic matter consumed by bacteria, (mg/l-day)

$$= \frac{(KOM)(C_{OM1})(C_{B1})(THETE)^T}{(KSOM + C_{OM1})}$$

KOM = decay rate constant of organic matter decomposed by bacteria, $\left(\frac{\text{mg organic matter consumed}}{\text{mg bacteria-day}}\right)$

KSOM = half saturation constant for bacteria acting on organic matter, (mg/l)

C_{B1} = epilimnion bacteria concentration, (mg/l)

THETE = temperature coefficient for bacteria growth or respiration,

b. Unstratified Whole Segment Volume (completely mixed)

Similarly, material balance was constructed from epilimnion equation with minor changes as follows:

$$\frac{dC_{OM}}{dt} = \left(\frac{Q_o}{V_T}\right)(C_{OMI} - C_{OM}) + OMALG + ROMO - OMCN + ROMS$$

where: C_{OM} = outlet and entire volume organic matter concentration, (mg/l)

OMALG = rate of soluble organic matter released by settling phytoplankton in entire segment volume, (mg/l-day)

= (FSAD)(SETLA)(FOMA)

SETLA = settling rate of phytoplankton in entire segment volume, (mg/l-day)

OMCN = rate of entire segment volume organic matter consumed by bacteria, (mg/l-day)

$$= \frac{(KOM)(C_{OM})(C_B)(THETE)^T}{(KSOM+C_{OM})}$$

C_B = bacteria concentration in entire segment volume, (mg/l)

ROMS = rate of organic matter released by sediment to entire segment volume, (mg/l-day)

$$= \frac{(NUPK)(C_{OMINF} - C_{OMINI})}{H}$$

C_{OMINF} = water-sediment interfacial concentration of soluble organic matter, (mg/l)

C_{OMINI} = initial soluble organic matter concentration in segment volume, (mg/l)

c. Hypolimnion (completely mixed)

Similarly, flows to hypolimnion were not considered in the formulation of the model. Also, rate of soluble organic matter released by settling phytoplankton for epilimnion balance was assumed the same in hypolimnion with the fact that phytoplankton was settling continuously. The mass balance was as follows:

$$\frac{dC_{OM2}}{dt} = OMALG_1 + ROMO - OMCN_2 + ROMS_2$$

where: C_{OM2} = hypolimnion organic matter concentration, (mg/l)

$OMCN_2$ = rate of hypolimnion organic matter consumed by bacteria, (mg/l-day)

$$= \frac{(KOM)(C_{OM2})(C_{B2})(THETE)^{THC}}{(KSOM + C_{OM2})}$$

C_{B2} = hypolimnion bacteria concentration, (mg/l)

$ROMS_2$ = rate of organic matter released by sediment to hypolimnion, (mg/l-day)

$$= \frac{(NUPK_2)(C_{OMINF} - C_{OMINI})}{H_2}$$

G. Biological System Components

1. Phytoplankton

Phytoplankton growth was assumed to be affected by flows, nutrients (nitrogen and phosphorus) levels, sunlight, temperature, sunlight extinction coefficient, natural mortality, settling and predation by higher trophic levels of zooplankton and omnivorous fish. It was further assumed that phytoplankton growth occurred only in the epilimnion during stratified period(s), but in the entire segment volume during unstratified period(s). Material balances in stratified and unstratified periods were as follows (10):

a. Epilimnion (completely mixed)

$$\frac{dC_{A1}}{dt} = \left(\frac{Q_o}{V_1}\right)(C_{AI} - C_{A1}) + (GRCA_1 - RESC_A)(C_{A1}) - SETLA_1 - ROMFA - RAGZ$$

where: C_{A1} = outlet and epilimnion phytoplankton concentration, (mg/l)

$$GRCA_1 = (R1_1)(K1)\left(\frac{C_{N21}}{KMN + C_{N21}}\right)\left(\frac{C_{P1}}{KMP + C_{P1}}\right)$$

$R1_1$ = reduction in growth rate of phytoplankton due to nonoptimum sunlight conditions and total extinction coefficient in epilimnion averaged over depth and time

$$= \left(\frac{e}{K_{E1}(H_1)}\right)\left[\exp^{(-A1)} - \exp^{(-A_{o1})}\right]$$

e = exponential factor of 2.718

H_1 = average depth of epilimnion (H_{1F}) in meters, (m)

K_{E1} = total extinction factor due to epilimnion phytoplankton self-shading and other causes, (m^{-1})

$$= K_{EO} + 0.0088\,(P_1) + 0.054\,(P_1)^{2/3}$$

P_1 = chlorophyll a concentration in epilimnion, (μg/l)

$$= \left(\frac{C_{A1}}{60.0}\right)$$

K_{EO} = extinction coefficient due to other causes (other than phytoplankton), (m^{-1})

K_{EO} was obtained by using Beer's Law (6) with the assumption of 99% sunlight attenuation up to 4 meters deep:

$$I_z = (I_o)\left(\exp^{(-K_{EO})(Z)}\right)$$

with $\frac{I_z}{I_o} = 0.01$, $z = 4.0$ meters

I_z = intensity of sunlight at depth z, (cal/cm^2-day)

I_o = intensity of sunlight at water surface, (cal/cm^2-day)

$$A_1 = \left(\frac{S_c}{I_s}\right)\left(\exp^{-(K_{E1})(H_1)}\right)$$

$$A_{o1} = \frac{S_c}{I_s}$$

I_s = optimum radiation energy for phytoplankton growth, (cal/cm^2-day)

K1 = maximum specific growth rate of phytoplankton as function of temperature, (day^{-1})

$= (KGA)(THETA)^T$

= maximum specific growth rate constant of phytoplankton, (day^{-1})

THETA = temperature coefficient for phytoplankton, zooplankton and fish growth or respiration

$RESC_A$ = respiration rate of phytoplankton as function of temperature, (day^{-1})

$= KRA\,(THETA)^T$

KRA = respiration rate constant of phytoplankton, (day^{-1})

$$SETLA_1 = \frac{SVELA\,(C_{A1})}{(H_F)(0.3048)}$$

SVELA = settling rate constant of phytoplankton, (m/day)

KMN = half saturation constant of phytoplankton on nitrogen, (mg/l)

KMP = half saturation constant of phytoplankton on phosphorus, (mg/l)

C_{N21} = epilimnion nitrogen concentration, (mg/l)

C_{P1} = epilimnion phosphorus concentration, (mg/l)

Rate of predation of species i was related to the growth rate of predator by the following relationship (3):

$$\text{Predation rate} = \left(\frac{\text{growth rate of predator}}{\text{assimilation efficiency of predator}}\right)(\text{concentrate of predator})^m$$

The above equation was modified to formulate the predation rates of omnivorous fish and zooplankton on phytoplankton. They were as follows:

ROMFA = predation rate of omnivorous fish on plytoplankton, (mg/l-day)

$$= \left(\frac{GRCOM}{FE}\right) (PFOMA)\ (C_{OMF})^m \left(\frac{AAVIL}{AAVIL + ZAVIL}\right)$$

GRCOM = growth rate of omnivorous fish, (day^{-1})

FE = omnivorous or carnivorous fish assimilation efficiency,

$$\left(\frac{\text{weight of phytoplankton or zooplankton consumed}}{\text{weight of fish formed}}\right)$$

PFOMA = preference factor of omnivorous fish on phytoplankton

C_{OMF} = omnivorous fish concentration, (mg/l)

AAVIL = availability of phytoplankton for omnivorous fish growth

$$= \left(\frac{C_{AM}}{KFA + C_{AM}}\right)$$

C_{AM} = volumetric average concentration of phytoplankton of the entire segment volume in stratified period (s) or phytoplankton concentration in unstratified period (s), (mg/l)

$$= (C_{A1}) \left(\frac{V_1}{V_T}\right) \text{or } C_A$$

KFA = half saturation constant of omnivorous fish on phytoplankton, (mg/l)

ZAVIL = availability of zooplankton for omnivorous fish growth

$$= \frac{C_Z}{(KFZ + C_Z)}$$

C_z = zooplankton concentration in entire segment volume, (mg/l)

KFZ = half saturation constant of omnivorous fish on phytoplankton, (mg/l)

RAGZ = predation rate of zooplankton on phytoplankton, (mg/l-day)

$$= \left(\frac{GRZ}{ZE}\right) (C_Z)^m$$

GRZ = growth rate of zooplankton, (day^{-1})

ZE = zooplankton assimilation efficiency

The term $\left(\frac{AAVIL}{AAVIL + ZAVIL}\right)$ was created apart from the preference factors. Even if a species is preferred by a higher species, it will be predated upon only to the relative degree to which it is available.

b. Unstratified Whole Segment Volume (completely mixed)

Similar material balance equation was used, but based on entire segment volume, as follows:

$$\frac{dC_A}{dt} = \left(\frac{Q_o}{V_T}\right)(C_{AI} - C_A) + (GRCA - RESC_A)(C_A) - SETLA - ROMFA - RAGZ$$

where: $GRCA = (R1)(K1)\left(\frac{C_{N2}}{KMN + C_{N2}}\right)\left(\frac{C_p}{KMP + C_p}\right)$

R1 = reduction in growth rate of phytoplankton due to nonoptimum sunlight conditions and total extinction coefficient in entire segment volume averaged over depth and time

$$= \left(\frac{e}{K_E\,(H)}\right)\left(exp^{(-A_{11})} - exp^{(A_{01})}\right)$$

K_E = total extinction factor due to entire segment volume phytoplankton self-shading and other causes, (m^{-1})

$$= K_{EO} + 0.0088\,(P) + 0.054\,(P)^{2/3}$$

P = chlorophyll a concentration in entire segment volume, ($\mu g/l$)

$$= \frac{C_A}{60.0}$$

$$A_{11} = \left(\frac{S_c}{I_s}\right)\left(exp^{(-K_E)(H)}\right)$$

C_{N2} = nitrogen concentration of entire segment volume, (mg/l)

C_p = phosphorus concentration of entire segment volume, (mg/l)

$$SETLA = \frac{(SVELA)(C_A)}{(H_F)(0.3048)}$$

2. Zooplankton

All zooplankton were assumed to prey only on phytoplankton and be affected by flows and mortality rate. Also, since zooplankton were mobile, it was assumed that they existed in the entire volume of the segment all the year round. Therefore, only one material balance was needed for this model as follows:

$$\frac{dC_Z}{dt} = \left(\frac{Q_o}{V_T}\right)(C_{ZI} - C_Z) + (GRZ - RESC_Z - ZMORT)(C_Z) - ROMFZ$$

where: C_{ZI} = inlet zooplankton concentration, (mg/l)

GRZ = growth rate of zooplankton, (day^{-1})

$$=(MGRZ)(THETA)^T \left(\frac{C_{AM}}{KMA + C_{AM}}\right)$$

MGRZ= maximum growth rate constant of zooplankton, (day^{-1})

KMA= half saturation constant of zooplankton on phytoplankton, (mg/l)

$RESC_Z$ = respiration rate of zooplankton, (day^{-1})

$$=(K3)(THETA)^T$$

K3 = maximum respiration rate constant of zooplankton, (day^{-1})

ZMORT= zooplankton mortality rate constant, (day^{-1})

ROMFZ = predation rate of omnivorous fish on zooplankton, (mg/l-day)

$$= \left(\frac{GRCOM}{FE}\right) (PFOMZ)(C_{OMF})^m \left(\frac{ZAVIL}{AAVIL + ZAVIL}\right)$$

PFOMZ = preference factor of omnivorous fish on zooplankton

3. Fish

Two types of fish, omnivores and carnivores, were considered in this model. Omnivores were assumed to prey on both phytoplankton and zooplankton, depending on preference factors and the availability of each species as prey. Carnivores were assumed to prey on omnivores. Since fish were extremely mobile, both types of fish were unaffected by flows. Models based on weight of fish were formulated as follows:

a. Omnivores

$$\frac{dW_{OMF}}{dt} = (GRCOM - RESC_{OMF} - OMMORT)(W_{OMF}) - RCVOM$$

where: W_{OMF} = weight of omnivorous fish in entire segment volume, (gm)

$GRCOM = (MGROM)(THETA)^T \quad (AAVIL + ZAVIL)$

MGROM = maximum growth rate constant of omnivorous fish, (day^{-1})

$RESC_{OMF}$ = respiration rate of omnivorous fish, (day^{-1})

$= (REROM)(THETA)^T$

REROM = maximum respiration rate constant of omnivorous fish, (day^{-1})

OMMORT = mortality rate constant of omnivorous fish, (day^{-1})

RCVOM = predation rate of carnivorous fish on omnivorous fish, (mg/l-day)

$$= \left(\frac{GRCV}{FE}\right)(W_{CVF})^m$$

GRCV = growth rate of carnivorous fish, (day^{-1})

W_{CVF} = weight of carnivorous fish in entire segment volume, (gm)

b. Carnivores

$$\frac{dW_{CVF}}{dt} = (GRCV - RESC_{CVF} - CVMORT)(W_{CVF})$$

where: $GRCV = (MGRCV)(THETA)^T \left(\frac{C_{OMF}}{KCVOM + C_{OMF}}\right)$

MGRCV = maximum growth rate of carnivorous fish, (day^{-1})

KCVOM = half saturation constant of carnivorous fish on omnivorous fish, (mg/l)

C_{OMF} = omnivorous fish concentration in entire segment volume, (mg/l)

$$= \left(\frac{W_{OMF}}{V_T}\right)\left(\frac{1000.0}{28.32}\right)$$

$RESC_{CVF}$ = respiration rate of carnivorous fish, (day^{-1})

$= (RERCV)(THETA)^T$

RERCV = maximum respiration rate constant of carnivorous fish, (day^{-1})

CVMORT = mortality rate constant of carnivorous fish, (day^{-1})

4. Bacteria

Bacteria were assumed to depend on nutrients of nitrogen and phosphorus, organic matter and oxygen. In this model, only the aerobic type of bacteria was considered. Similar equations were employed to both stratified and unstratified periods, as follows:

a. Epilimnion (completely mixed)

$$\frac{dC_{B1}}{dt} = \left(\frac{Q_o}{V_1}\right)(C_{BI} - C_{B1}) + GRB_1 - (RESC_{B1})(C_{B1})$$

where: C_{BI} = inlet bacteria concentration, (mg/l)

GRB_1 = epilimnion growth rate of bacteria, (mg/l-day)

$$= Y\,(OMCN_1)\left(\frac{C_{N21}}{KSNB + C_{N21}}\right)\left(\frac{C_{p1}}{KSPB + C_{p1}}\right)\left(\frac{C_{021}}{KBO + C_{021}}\right)$$

Y = yield coefficient of bacteria formed per weight of organic matter consumed

KSNB = half saturation constant of bacteria on nitrogen, (mg/l)

KSPB = half saturation constant of bacteria on phosphorus, (mg/l)

KBO = half saturation constant of bacteria on oxygen, (mg/l)

$RESC_{B1}$ = respiration rate of bacteria in epilimnion, (day^{-1})

$= (KDB)(THETE)^T$

KDB = maximum respiration rate constant of bacteria, (day^{-1})

b. Unstratified Whole Segment Volume (completely mixed)

Similarly, during unstratified period(s), material balance was derived from epilimnion balance with some modification, as follows:

$$\frac{dC_B}{dt} = \left(\frac{Q_o}{V_T}\right)(C_{BI} - C_B) + GRB - (RESC_{B1})(C_B)$$

where: GRB = growth rate of bacteria in entire segment volume, (mg/l-day)

$$= (Y)(OMCN)\left(\frac{C_{N2}}{KSNB + C_{N2}}\right)\left(\frac{C_p}{KSPB + C_p}\right)\left(\frac{C_{02}}{KBO + C_{02}}\right)$$

$RESC_B$ = respiration rate of bacteria in entire segment volume, (day^{-1})

$= (KDB)(THETE)^T$

c. Hypolimnion (completely mixed)

Similarly, flows were not considered in the material balance. Material balance was as follows:

$$\frac{dC_{B2}}{dt} = GRB_2 - (RESC_{B2})(C_{B2})$$

where: GRB_2 = hypolimnion growth rate of bacteria, (mg/l-day)

$$= Y\,(OMCN_2)\left(\frac{C_{N22}}{KSNB + C_{N22}}\right)\left(\frac{C_{p2}}{KSPB + C_{p2}}\right)\left(\frac{C_{022}}{KBO + C_{022}}\right)$$

$RESC_{B2}$ = respiration rate of bacteria in hypolimnion, (day^{-1})

$$=(KDB)(THETE)^{THC}$$

H. Model Ecosystem Constants

Range or single value of ecosystem constants used in this model were compiled from different sources and presented to facilitate the use of these values of constants in adjusting the model for other lake uses. Also equivalent symbols used in the model simulation program were listed in Table I.

TABLE I

MODEL ECOSYSTEM CONSTANTS

THEORY CONSTANT	SIMULATION CONSTANT	RANGE OR SINGLE VALUE	UNIT	SOURCE
THETA	THETA	1.020	----------	(11)
		1.047	----------	(8)
THETE	THETE	1.047	----------	(8)
THSED	THSED	1.065	----------	(9)
C_{pINF}	CPINF, CPIFWE, CPIFWW	0.1 - 10.5	mg P/l	(12)
C_{NINF}	CNINF, CNIFWE, CNIFWW	1.8 - 80.0	mg N/l	(12)
C_{OMINF}	COMINF	4.0	mg C/l	*
SR (phosphorus)	AP	0.012	mg P/mg phytoplankton	(8)
		0.0378	mg P/mg phytoplankton	(13)
SR (nitrogen)	AN	0.085	mg N/mg phytoplankton	(8)
		0.432	mg N/mg phytoplankton	(13)
FR_i (phytoplankton-phosphorus)	FRPA	0.013	mg P/mg phytoplankton secretion	(8)
FR_i (phytoplankton-nitrogen)	FRNA	0.070	mg N/mg phytoplankton secretion	(8)
FR_i (fish - phosphorus)	FRPF	0.013	mg P/mg fish excreta	(8)

(Table I continued)

THEORY CONSTANT	SIMULATION CONSTANT	RANGE OR SINGLE VALUE	UNIT	SOURCE
FR_i (fish - nitrogen)	FRNF	0.07	mg N/mg fish excreta	(8)
m	M1AL, M1ZOO, M1OMF, M1CVF	0.67	----------	(3)
TDIFS	TDIFS	0.01 - 20.0	m^2/day	(6)
TDIFU	TDIFU	0.01 - 20.0	m^2/day	(6)
SVELA	SVELA	0.05 - 0.2	m/day	(11)
		0.5	m/day	(11)
KGA	KGA	1.5 - 2.0	day^{-1}	(8)
		1.0 - 2.0	day^{-1}	(11)
		1.5 - 2.5	day^{-1}	(11)
KRA	KRA	0.01	day^{-1}	(8)
		0.05	day^{-1}	(11)
K_{EO}	KEP1	1.15	m^{-1}	estimated ** from (6)
I_s	IS	43.2 - 86.4	cal/cm^2-day	(14)
KMN	KMN	0.014	mg N/l	(13)
		0.04 - 0.3	mg N/l	(11)
		0.2 - 0.3	mg N/l	(8)
		0.032	mg N/l	(10)
KMP	KMP	0.001	mg P/l	(13)
		0.03 - 0.05	mg P/l	(8), (11)
		0.006	mg P/l	(10)
PFOMA (normal)	PFOAI	0.75	----------	(15)

(Table I continued)

THEORY CONSTANT	SIMULATION CONSTANT	RANGE OR SINGLE VALUE	UNIT	SOURCE
PFOMZ (normal)	PFOZI	0.25	---------	(15)
MGRZ	MGRZ	0.20	day^{-1}	(8)
		0.15	day^{-1}	(11)
		0.25	day^{-1}	(11)
		0.08 - 0.18	day^{-1}	(10)
K3	K3	0.012	day^{-1}	(8)
		0.01	day^{-1}	(11)
KMA	KMA	10.0	mg/l	(8)
		0.5	mg/l	(11)
ZMORT	ZMORT	0.005	day^{-1}	(8)
		0.020	day^{-1}	(11)
ZE	ZE	0.7	wt. of zooplankton formed / wt. of phytoplankton consumed	(8)
MGROM	MGROM	0.1	day^{-1}	(8)
		0.02 - 0.03	day^{-1}	(11)
REROM	REROM	0.02	day^{-1}	(8)
		0.001	day^{-1}	(11)
OMMORT	OMMORT	0.005	day^{-1}	(8)
		0.010	day^{-1}	(11)
KFZ	KFZ	1.0	mg/l	(8)
		0.05 - 0.10	mg/l	(11)
KFA	KFA	0.05 - 1.0	mg/l	estimated from (8),
		0.5 - 10.0	mg/l	(11)

(Table I continued)

THEORY CONSTANT	SIMULATION CONSTANT	RANGE OR SINGLE VALUE	UNIT	SOURCE
FE	FE	0.60	$\frac{\text{wt. of fish formed}}{\text{wt. of prey consumed}}$	(8)
MGRCV	MGRCV	0.1 0.02 - 0.03	day^{-1} day^{-1}	(8) (11)
RERCV	RERCV	0.02 0.001	day^{-1} day^{-1}	(8) (11)
KCVOM	KCVOM	0.05 - 10.0	mg/l	estimated from (8), (11)
CVMORT	CVMORT	0.005 0.01	day^{-1} day^{-1}	(8) (11)
GMMA	GMMA	0.042 - 0.27	mg O_2 produced/mg phytoplankton	(9)
KL	KL	1.0 - 5.0	ft/day	(16)
KBO	KBO	1.0	mg/l	*
KOSED	KOSED	0.2 - 1.0	gm O_2/m^2-day	(9)
O2COM	O2COM	1.5	mg O_2 consumed/mg organic matter consumed	(8)
KOM	KOM	3.3 - 14.5	$\frac{\text{mg organic matter consumed}}{\text{mg bacteria-day}}$	(17)
KSOM	KSOM	65.0 - 355.0	mg/l	(17)
FOMA	FOMA	0.55	mg c/mg phytoplankton	(8)

(Table I continued)

THEORY CONSTANT	SIMULATION CONSTANT	RANGE OR SINGLE VALUE	UNIT	SOURCE
FSAD	FSAD	0.00001	-----------	*
Y	Y	0.40 - 0.67	$\frac{\text{wt. bacteria}}{\text{wt. organic matter}}$	(17)
KSNB	KSNB	0.014 - 0.3	mg N/l	estimated from (8), (10), (11), (13)
KSPB	KSPB	0.001 - 0.05	mg P/l	estimated from (8), (10), (11), (13)
KDB	KDB	0.045 - 0.180	day^{-1}	(17)
ROMO	ROMO	0.00001	mg organic matter/lit-day	*

* estimated and assumed by author
** estimated by assuming 99% attenuation of sunlight up to 4 meters deep and using Beers' Law

SPECIFIC APPLICATION RESULTS

A. Model Application to Beaver Reservoir

Hydrological data (surface area and volume) of Beaver Reservoir, annual White River and War Eagle flow rates, annual water surface temperatures of Beaver Reservoir, and average annual nutrient concentrations (nitrogen and phosphorous) associated with War Eagle flow (18) were listed in the main computer program as functional inputs. Flow from Fayetteville Treatment Plant (QFTP) was assumed constant at 360,000 cubic feet per day (2), and the bottom temperature of Beaver Reservoir was assumed constant at 43^{o} F.

Initial concentrations of oxygen, nutrients, organic matter, phytoplankton, zooplankton, omnivores, carnivores and bacteria in each of the three selected segments of Beaver Reservoir (WTR, HPT and WWK) were listed in Table II.

TABLE II

INITIAL CONCENTRATIONS USED IN BEAVER RESERVOIR SIMULATION

SIMULATION CONSTANT	DESCRIPTION	VALUE	UNIT	SOURCE
COI	initial oxygen concentration in WTR	9.0	mg/l	(19), (20)
COIWE	initial oxygen concentration in HPT	9.2	mg/l	(19), (20)
COIWW	initial oxygen concentration in WWK	9.8	mg/l	(19), (20)
COMINI OMI	initial organic matter concentration in WTR	3.0	mg/l	(21)
COMINW OMIWE	initial organic matter concentration in HPT	3.0	mg/l	(21)
COMINK OMIWW	initial organic matter concentration in WWK	3.0	mg/l	(21)
CNINI CNI	initial nitrogen concentration in WTR	1.118	mg/l	(19), (20)
CNINIW CNIWE	initial nitrogen concentration in HPT	1.0	mg/l	(19), (20)

(Table II continued)

SIMULATION CONSTANT	DESCRIPTION	VALUE	UNIT	SOURCE
CNINIK CNIWW	initial nitrogen concentration in WWK	0.931	mg/l	(19), (20)
CPINI CPI	initial phosphorus concentration in WTR	0.001	mg/l	(19), (20)
CPINIW CPIWE	initial phosphorus concentration in HPT	0.001	mg/l	(19), (20)
CPINIK CPIWW	initial phosphorus concentration in WWK	0.001	mg/l	(19), (20)
CAI	initial phytoplankton concentration in WTR	12.69	mg/l	(19), (20)
CAIWE	initial phytoplankton concentration in HPT	100.0	mg/l	(19), (20)
CAIWW	initial phytoplankton concentration in WWK	54.5	mg/l	(19), (20)
CZI	initial zooplankton concentration in WTR	0.0038	mg/l	(22)
CZIWE	initial zooplankton concentration in HPT	0.0038	mg/l	(22)
CZIWW	initial zooplankton concentration in WWK	0.0038	mg/l	(22)
CFI	initial concentration of both omnivores and carnivores	1.06	mg/l	(14)
FOMFI	initial fraction of omnivores	0.9	------------	(15)
BI	initial bacteria concentration in WTR	0.0003	mg/l	*
BIWE	initial bacteria concentration in HPT	0.0003	mg/l	*
BIWW	initial bacteria concentration in WWK	0.0003	mg/l	*

*assumed and estimated by author.

Ecosystem constants used in the Beaver Reservoir simulation were listed in Table III.

TABLE III

ECOSYSTEM CONSTANTS USED IN BEAVER RESERVOIR SIMULATION

ECOSYSTEM CONSTANT	VALUE	ECOSYSTEM CONSTANT	VALUE
THETA	1.02	THETE	1.047
THSED	1.065	CPINF	0.35
CPIFWE	0.5	CPIFWW	0.15
CNINF	6.5	CNIFWE	6.0
CNIFWW	4.50	COMINF	4.0
AP	0.10	AN	0.0135
FRPA	0.013	FRNA	0.07
FRPF	0.013	FRNF	0.07
M1AL	0.67	M1ZOO	0.67
M1OMF	0.67	M1CVF	0.67
TDIFS	0.30	TDIFU	1.50
SVELA	0.05	KGA	1.74
KRA	0.05	IS	86.40
KMN	0.014	KMP	0.001
PFOAI	0.75	PFOZI	0.25
MGRZ	0.138	K3	0.0115
KMA	10.0	ZMORT	0.005
ZE	0.70	MGROM	0.0267
REROM	0.02	OMMORT	0.005

(Table III continued)

ECOSYSTEM CONSTANT	VALUE	ECOSYSTEM CONSTANT	VALUE
KFZ	0.10	KFA	10.0
FE	0.60	MGRCV	0.028
RERCV	0.02	KCVOM	0.1
CVMORT	0.005	GMMA	0.042
KL	5.0	KBO	1.0
KOSED	1.0	O2COM	1.5
KOM	14.5	KSOM	65.0
FOMA	0.55	FSAD	0.00001
ROMO	0.00001	Y	0.40
KSNB	0.014	KSPB	0.001
KDB	0.18	KEP1	1.15

Those constants which were used in the formulations of lake geometry, segmentation, sunlight intensities, diffusion, upsurge, septic tanks and runoff rates for Beaver Reservoir simulation were listed in Table IV.

TABLE IV

PHYSICAL CHARACTERISTIC PARAMETERS USED IN BEAVER RESERVOIR SIMULATION

THEORY CONSTANT	SIMULATION CONSTANT	DESCRIPTION	VALUE	UNIT	SOURCE
S	DRPML	slope of river bottom of WTR and WWK	2.98	ft./river mile	(4)
S	DRPMWE	slope of river bottom of HPT	5.97	ft./river mile	(4)
------------	BDEL	bottom elevation of Beaver Dam	914	ft.	(4)
DI	D1	distance of WTR	18.0	river miles	(4)
DI	D2	distance of HPT	11.0	river miles	(4)
DI	D3	total distance of WTR and WWK	26.0	river miles	(4)
E	E	elevation of Beaver Reservoir above mean sea level	1.0	thousand ft.	(4)
R	R	cloud factor for Beaver Reservoir	0.634	------------	(23)
DZTMP	DZTMP	average depth of Beaver Reservoir	17.7	m	(5)
TMIX	TMIX	mixup time since lake was unstratified	15.0	day	(5)
------------	TZRO	time at the start of stratification	80.0	day	(5)
------------	TOUR	overturn time when stratification ceased	270.0	day	(5)
WOPCD	WOPCD	waste output per capita per day	60.0	gal./capita-day	(24)
POP	POP	population along the shore of Beaver Reservoir	8213	capita	(24)
TRBL	TRBL	total distance of Beaver Reservoir	73.0	river miles	(4)

(Table IV continued)

THEORY CONSTANT	SIMULATION CONSTANT	DESCRIPTION	VALUE	UNIT	SOURCE
RNCFSM	RNCFSM	runoff rate to WWK	2.18	ft.^{3}/mile^2-sec.	(25)
SM	SM	area affected by runoff	10.0	mile^2	*

*assumed and estimated by author.

All other nutrients, organic matter, oxygen, phytoplankton, zooplankton, and bacteria which were associated with tributary flows, septic tanks and runoff were listed in Table V.

TABLE V

CHEMICAL AND BIOLOGICAL INPUTS USED IN BEAVER RESERVOIR SIMULATION

THEORY CONSTANT	SIMULATION CONSTANT	DESCRIPTION	VALUE	UNIT	SOURCE
C_{PI}	CPW	phosphorus concentration in White River	0.11	mg/l	(26)
C_{N2I}	CNW	nitrogen concentration in White River	0.31	mg/l	(26)
C_{PI}	CPRUN	phosphorus concentration in runoff	6.0	mg/l	estimated from (27)
C_{N2I}	CNRUN	nitrogen concentration in runoff	14.0	mg/l	estimated from (27)
C_{PI}	CPFTP	phosphorus concentration in Fayetteville Treatment Plant flow	6.47	mg/l	(2)
C_{N2I}	CNFTP	nitrogen concentration in Fayetteville Treatment Plant flow	14.4	mg/l	(2)
C_{PI}	SCP	phosphorus concentration in septic tanks flow	0.066	mg/l	(28)
C_{N2I}	SCN	nitrogen concentration in septic tanks flow	0.635	mg/l	(28)
C_{OMI}	OMIN	organic matter concentration in mixed flow to WTR	3.0	mg/l	(26)
C_{OMI}	OMINWZ	organic matter concentration in mixed flow to HPT	3.0	mg/l	*
C_{OMI}	OMRUN	organic matter concentration in runoff	3.0	mg/l	*
C_{O2I}	COI1	oxygen concentration in mixed flow to WTR	9.8	mg/l	(26)
C_{O2I}	COI1WE	oxygen concentration in mixed flow to HPT	9.8	mg/l	estimated from (19)

(Table V continued)

THEORY CONSTANT	SIMULATION CONSTANT	DESCRIPTION	VALUE	UNIT	SOURCE
C_{O2I}	O2RUN	oxygen concentration in runoff	10.0	mg/l	*
C_{AI}	CAO	phytoplankton concentration in mixed flow to WTR	25.8	mg/l	*
C_{AI}	CAOWE	phytoplankton concentration in mixed flow to HPT	25.8	mg/l	*
C_{ZI}	CZO	zooplankton concentration in mixed flow to WTR	0.003	mg/l	*
C_{ZI}	CZOWE	zooplankton concentration in mixed flow to HPT	0.003	mg/l	*
C_{BI}	BIN	bacteria concentration in mixed flow to WTR	0.0003	mg/l	*
C_{BI}	BINWE	bacteria concentration in mixed flow to HPT	0.0003	mg/l	*

*assumed and estimated by author

In the entire simulation of Beaver Reservoir, each individual simulated result of any species (except both omnivores and carnivores) and its associated factor was subjected to limitation of either observed or realistic value. These limitations were used to ensure realistic output results. They were listed in Table VI.

TABLE VI

SELECTED BOUNDS (MAXIMUM AND MINIMUM) USED IN BEAVER RESERVOIR SIMULATION

SIMULATION CONSTANT	DESCRIPTION	VALUE	UNIT	SOURCE
PFMAX	maximum preference factor of omnivores on one prey if the other prey reached minimum value	1.0	–	*
PFMIN	minimum preference factor of omnivores on that prey which reached minimum value	0.0	–	*
XTCMIN	minimum depth of thermocline during stratification	3.0	M	(5)
CAMIN	minimum detectable phytoplankton concentration	1.20	mg/l	estimated from (19)
CZMIN	minimum zooplankton concentration	0.0005	mg/l	estimated from (22)
COMIN	minimum oxygen concentration in water	0.0	mg/l	(19)
OMMIN	minimum organic matter concentration in water	0.0	mg/l	*
B2MIN	minimum bacteria concentration in water	$3.0X10^{-9}$	mg/l	*

*assumed and estimated by author

B. Presentation of Results

The simulation model was applied to three selected segments of Beaver Reservoir. Selected model constants were chosen to "tune" the simulation model to the available field data. Both the simulated results and the available field data were superimposed (where available) and presented in Figures 11 through 26. Limited field data was available to compare with the simulation results.

The field data versus simulation data of phytoplankton concentrations in the three segments are presented in Figures 11, 12 and 13, respectively. Adjusted parameters are: maximum phytoplankton specific growth rate constant (KGA) is 1.74 day^{-1}; maximum phytoplankton respiration rate constant (KRA) is 0.05 day^{-1}; half saturation constant of phytoplankton on nitrogen (KMN) is 0.014 mg N/l; half saturation constant of phytoplankton on phosphorus (KMP) is 0.001 mg P/l; settling rate (SVELA) is 0.05 m/day; optimum sunlight intensity (IS) is 86.4 cal/cm^2-day; and reduction faction due to turbidity (KEP1) is 1.15 m^{-1}.

The field data versus simulation data of overall zooplankton concentrations in the three segments are presented in Figure 14. Adjusted parameters are: maximum zooplankton growth rate constant (MGRZ) is 0.138 day^{-1}; maximum zooplankton respiration rate constant (K3) is 0.0115 day^{-1}; and half saturation constant of zooplankton on phytoplankton (KMA) is 10.0 mg phytoplankton/l.

The simulated data of the combined omnivorous and carnivorous fish are presented in Figure 15. Rough field data is available only on the total fish population. Adjusted parameters are: maximum omnivores growth rate constant (MGROM) is 0.0267 day^{-1}; maximum omnivores respiration rate constant (REROM) is 0.02 day^{-1}; half saturation constant of omnivores on zooplankton (KFZ) is 0.10 mg zooplankton/l; half saturation constant of omnivores on phytoplankton (KFA) is 10.0 mg phytoplankton/l; maximum carnivores growth rate constant (MGRCV) is 0.028 day^{-1}; maximum carnivores respiration rate constant (RERCV) is 0.02 day^{-1}; and half saturation constant of carnivores on omnivores (KCVOM) is 0.10 mg omnivores/l.

The simulated data of bacteria concentrations in the three segments are presented in Figure 16. No field data was available. Adjusted parameters are: bacteria respiration rate constant (KDB) is 0.18 day^{-1}; half saturation constant of bacteria on nitrogen (KSNB) is 0.014 mg N/l; half saturation constant of bacteria on phosphorus

(KSPB) is 0.001 mg P/l; and yield coefficient of weight of bacteria formed per unit weight of organic matter consumed (Y) is 0.4 mg bacteria/mg organic matter.

The field data versus simulation data of nitrogen concentrations in the three segments are presented in Figure 17, 18 and 19, respectively. Adjusted parameters are: stochiometry ratio of nitrogen in phytoplankton (AN) is 0.10 mg N/mg phytoplankton, and water-sediment interfacial nitrogen concentrations in the three segments (CNINF), (CNIFWE) and (CNIFWW) are 6.5 mg N/l, 6.0 mg N/l and 4.5 mg N/l, respectively.

The field data versus simulation data of phosphorous concentrations in the three segments are presented in Figures 20, 21 and 22, respectively. Adjusted parameters are: stochiometry ratio of phosphorus in phytoplankton (AP) is 0.0135 mg P/l; and water-sediment interfacial phosphorus concentrations in the three segments(CPINF), (CPIFWE) and (CPIFWW) are 0.35 mg P/l, 0.50 mg P/l and 0.15 mg P/l, respectively.

The field data versus simulation data of oxygen concentrations in the three segments are presented in Figures 23, 24 and 25, respectively. Adjusted parameters are: liquid phase mass transfer coefficient (KL) is 5.0 ft/day; stochiometry ratio of oxygen produced per phytoplankton formed (GMMA) is 0.042 mg O_2/mg phytoplankton formed; and rate constant for oxygen consumed by sediment (KOSED) is 1.0 gm O_2/m^2-day.

Finally, the simulation data of organic matter concentrations in the three segments are presented in Figure 26. No field data was available. Adjusted parameters are: decay rate constant of organic matter decomposed by bacteria (KOM) is 14.5 mg organic matter consumed/mg bacteria -day; half saturation constant for bacteria acting on organic matter (KSOM) is 65.0 mg organic matter/l; fraction of settling phytoplankton that can be utilized by bacteria (FSAD) is 0.00001; rate of organic matter input to water from other trophic levels (ROMO) is 0.00001 mg/l-day; and water-sediment interfacial organic matter concentrations in the three segments (COMINF) is 4.0 mg/l.

This presentation of results specifically reflects the Beaver Reservoir ecosystem. Additional parameters for this ecosystem are shown in Tables III, IV and V.

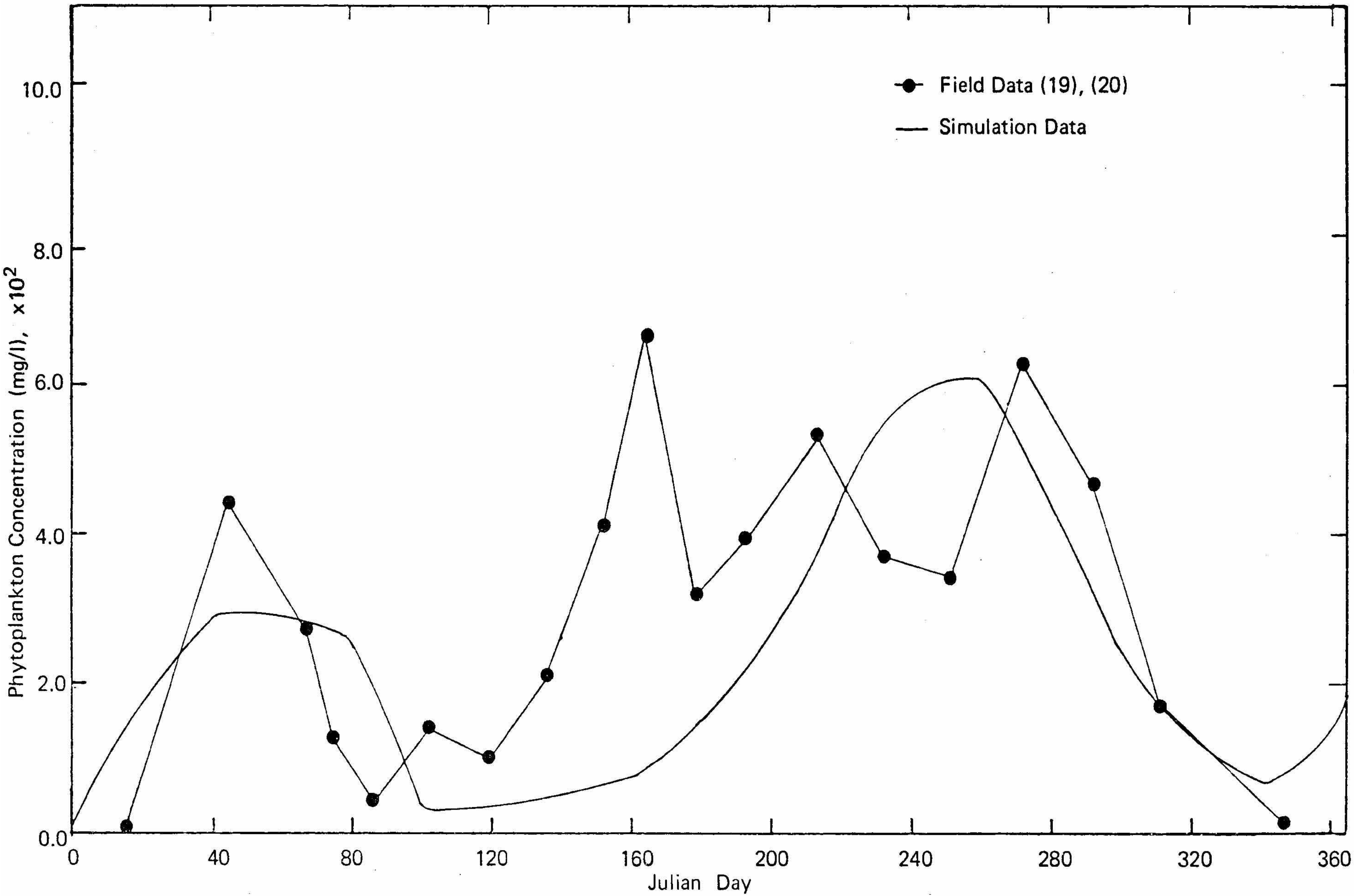

Figure 11. Field Data Versus Simulation Data of Phytoplankton Concentration in First Segment (WTR) in Beaver Reservoir.

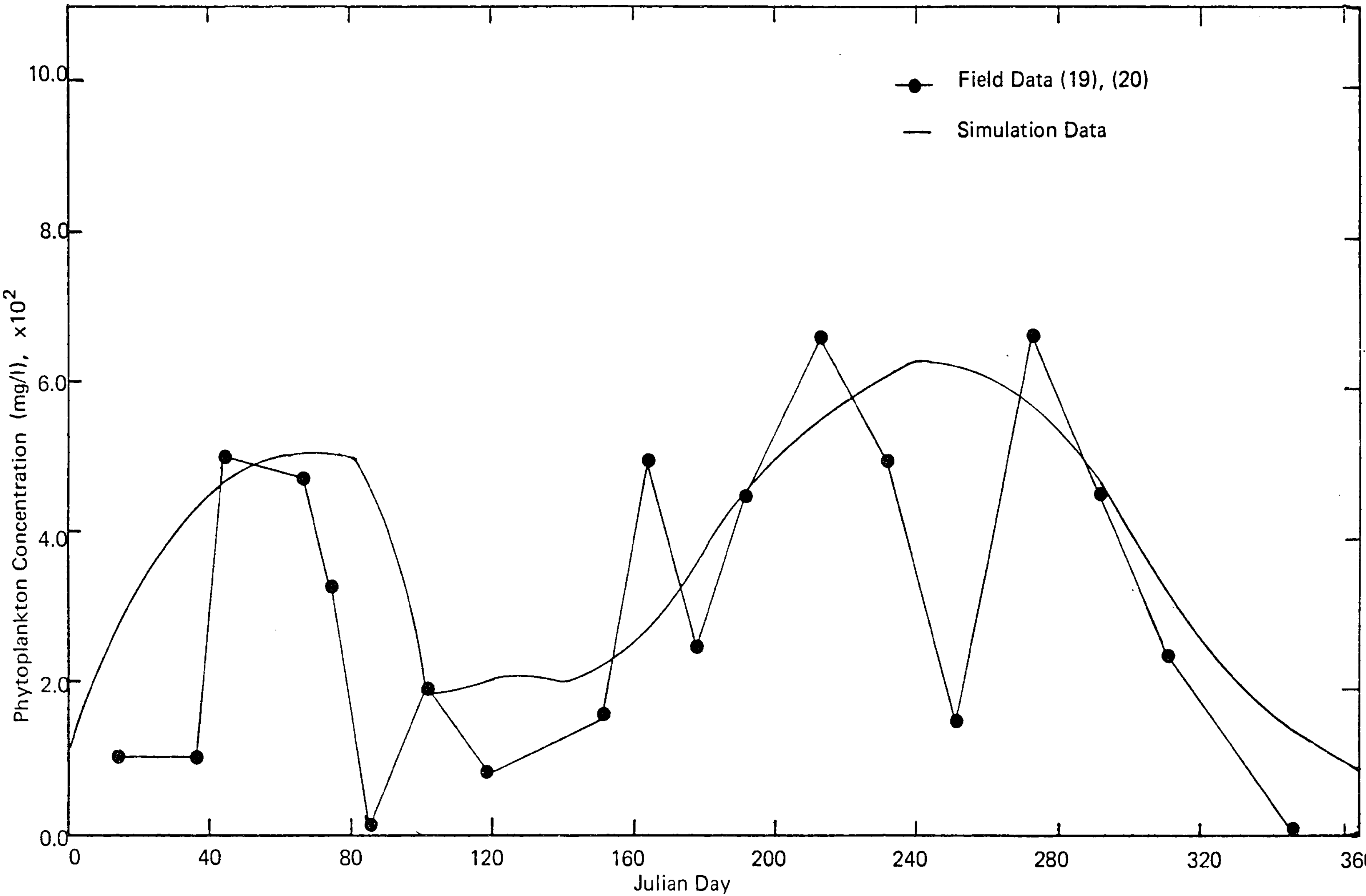

Figure 12. Field Data Versus Simulation Data of Phytoplankton Concentration in Second Segment (HPT) in Beaver Reservoir.

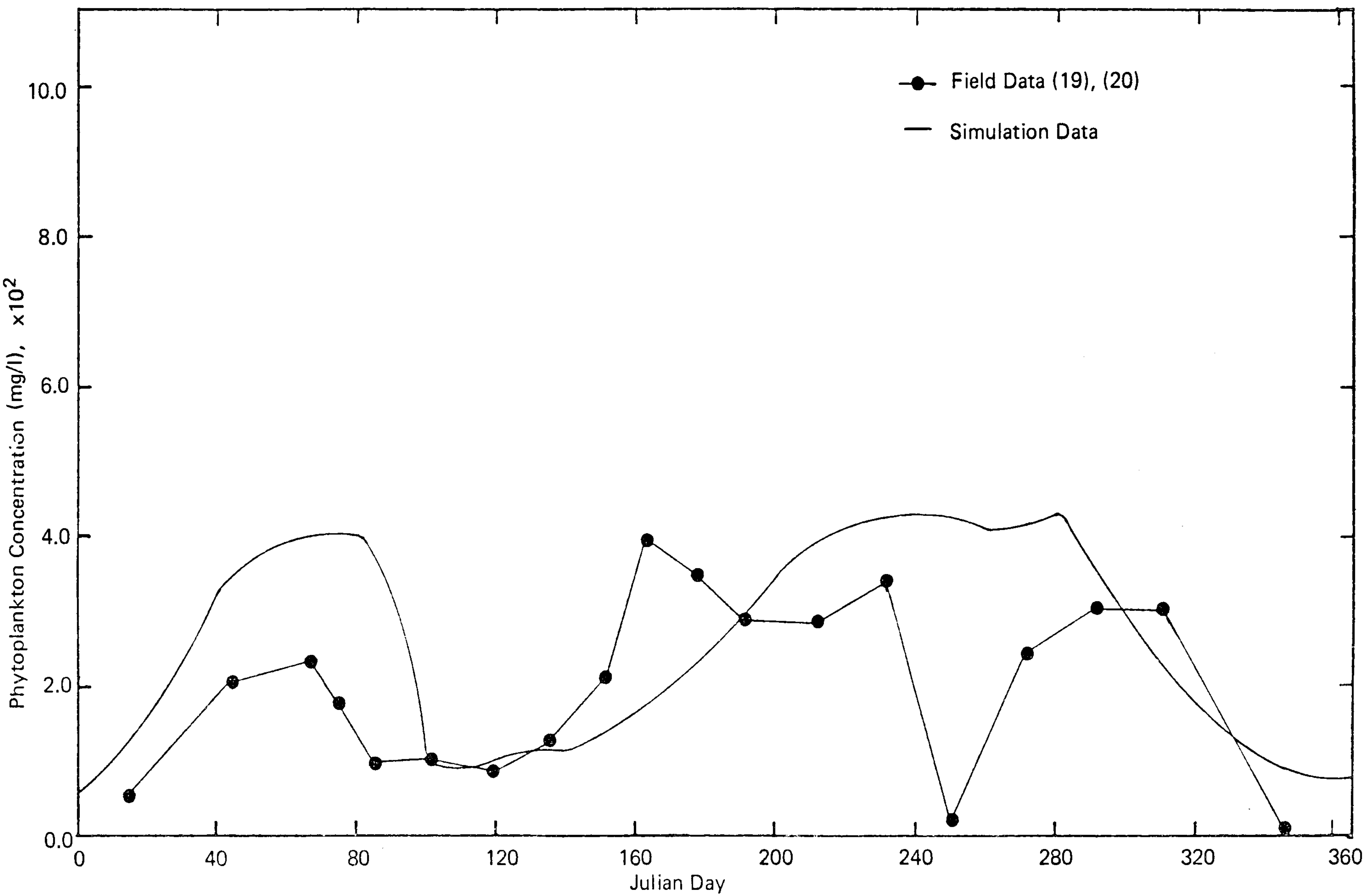

Figure 13. Field Data Versus Simulation Data of Phytoplankton Concentration in Third Segment (WWK) in Beaver Reservoir.

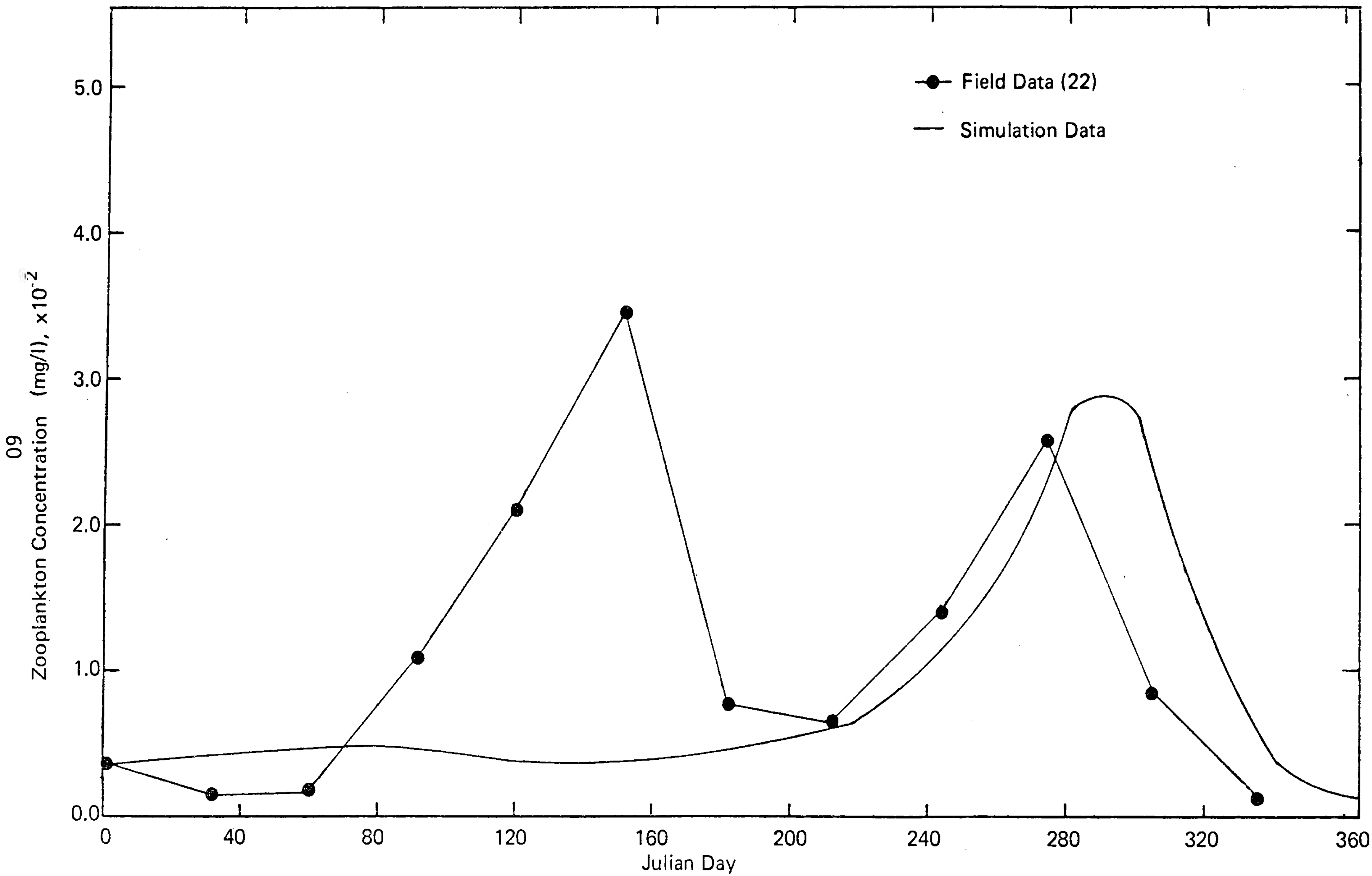

Figure 14. Field Data at Hickory Creek Versus Volumetric Average Simulation Data in Three Segments of Zooplankton Concentration in Beaver Reservoir.

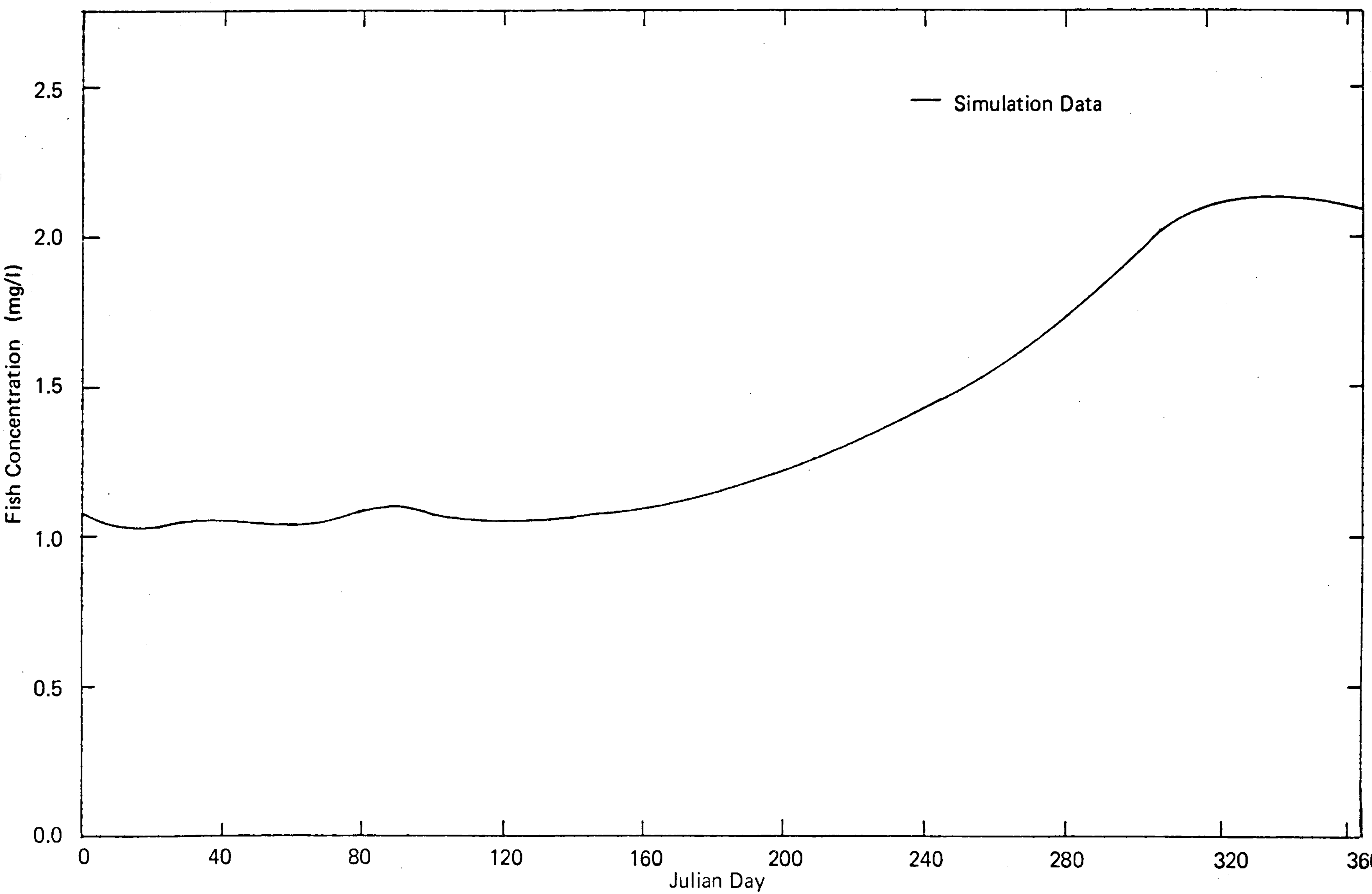

Figure 15. Volumetric Average Simulation Data of Both Omnivorous and Carnivorous Fish Concentration in Three Segments in Beaver Reservoir.

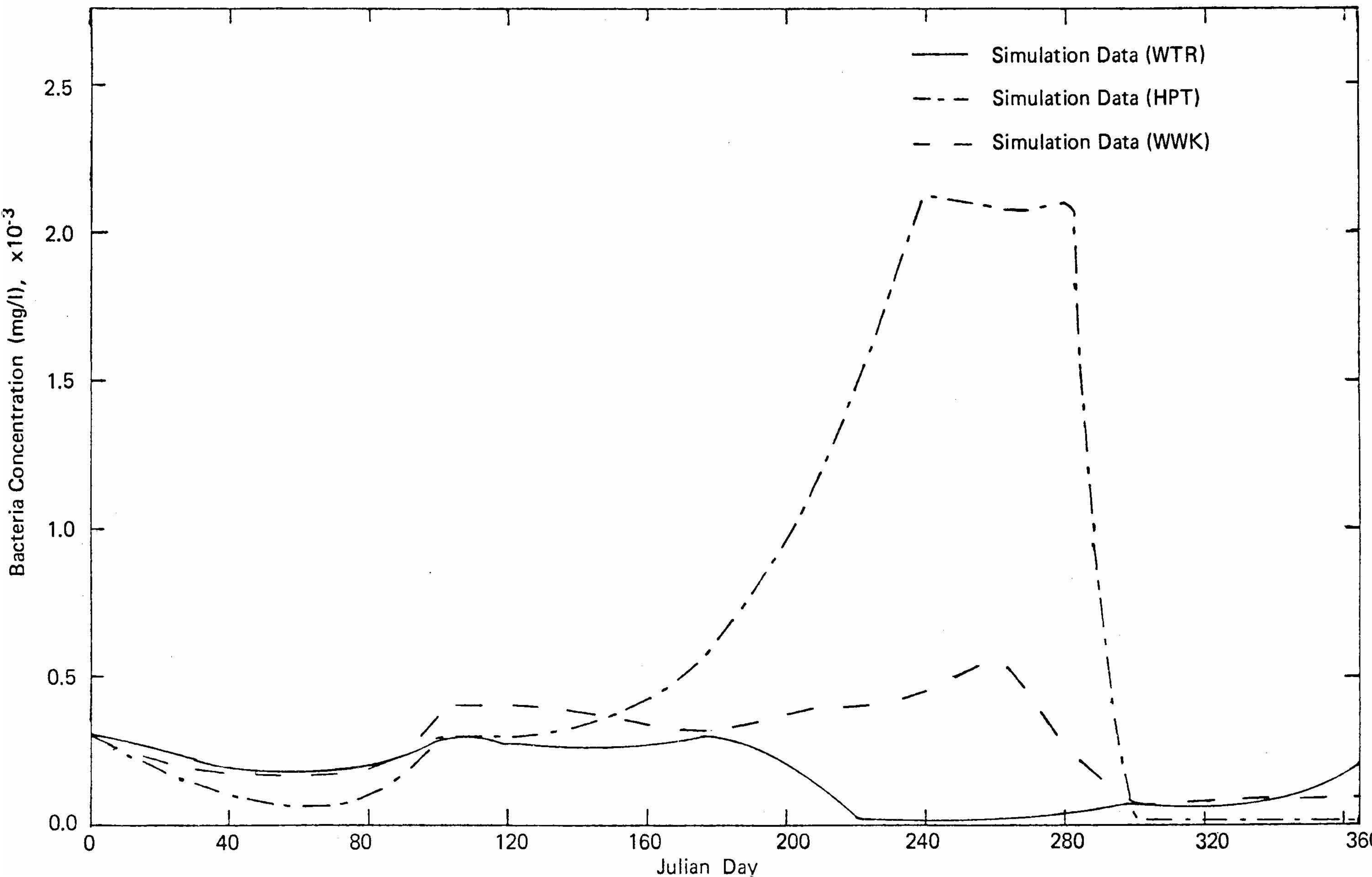

Figure 16. Simulation Data of Bacteria Concentration in Three Segments (WTR), (HPT) and (WWK) Respectively in Beaver Reservoir.

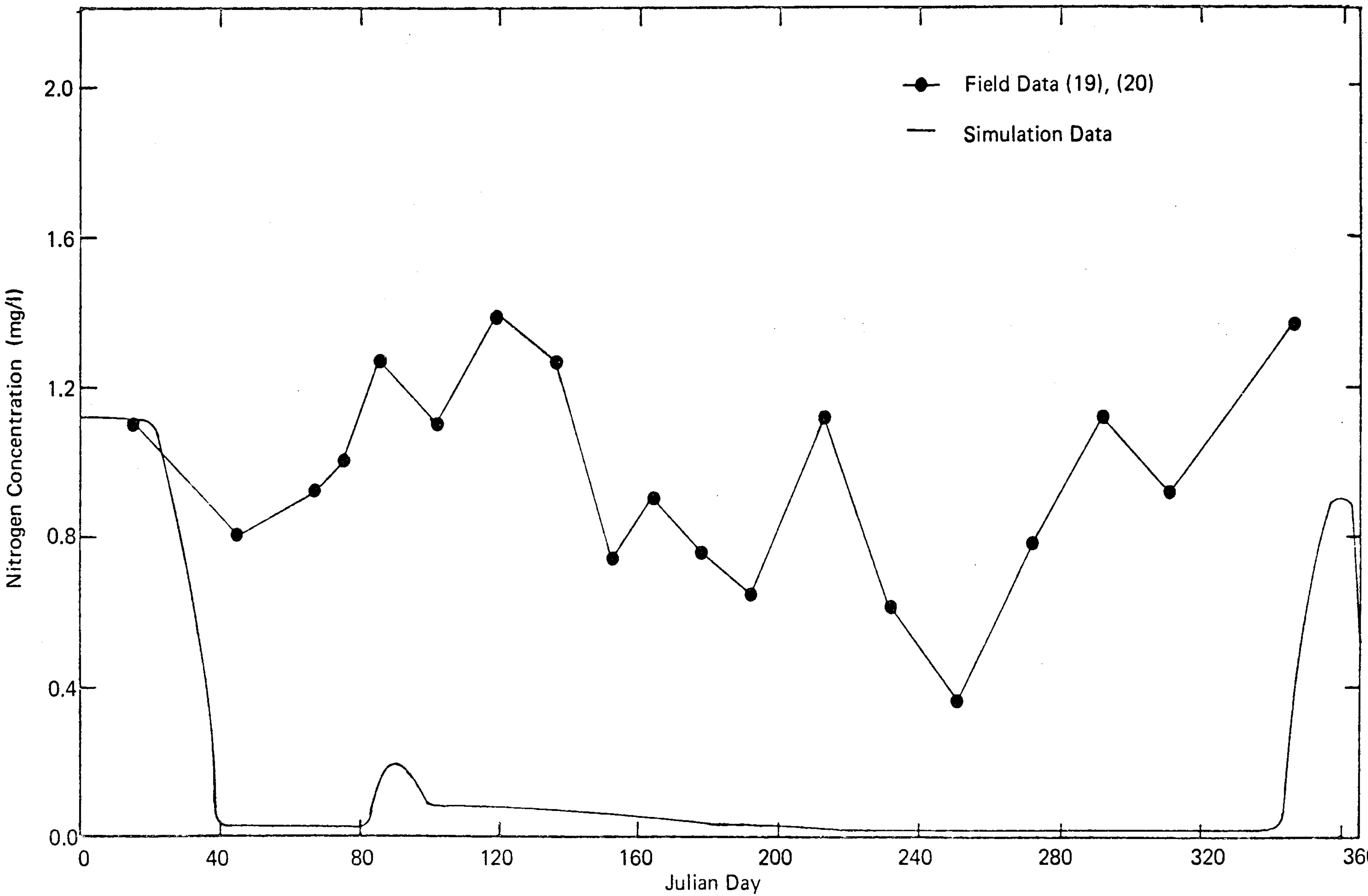

Figure 17. Field Data Versus Simulation Data of Nitrogen Concentration in First Segment (WTR) in Beaver Reservoir.

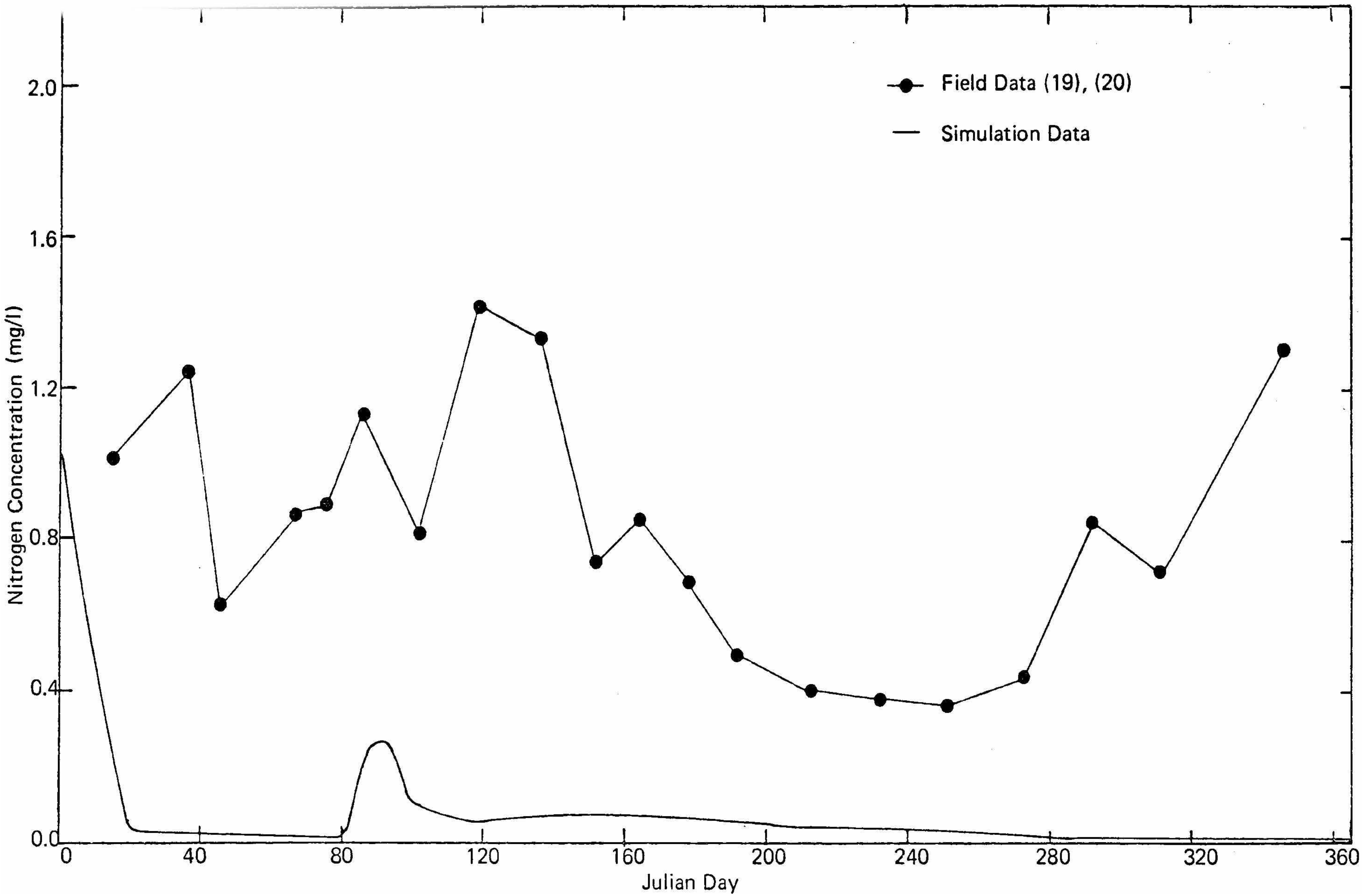

Figure 18. Field Data Versus Simulation Data of Nitrogen Concentration in Second Segment (HPT) in Beaver Reservoir.

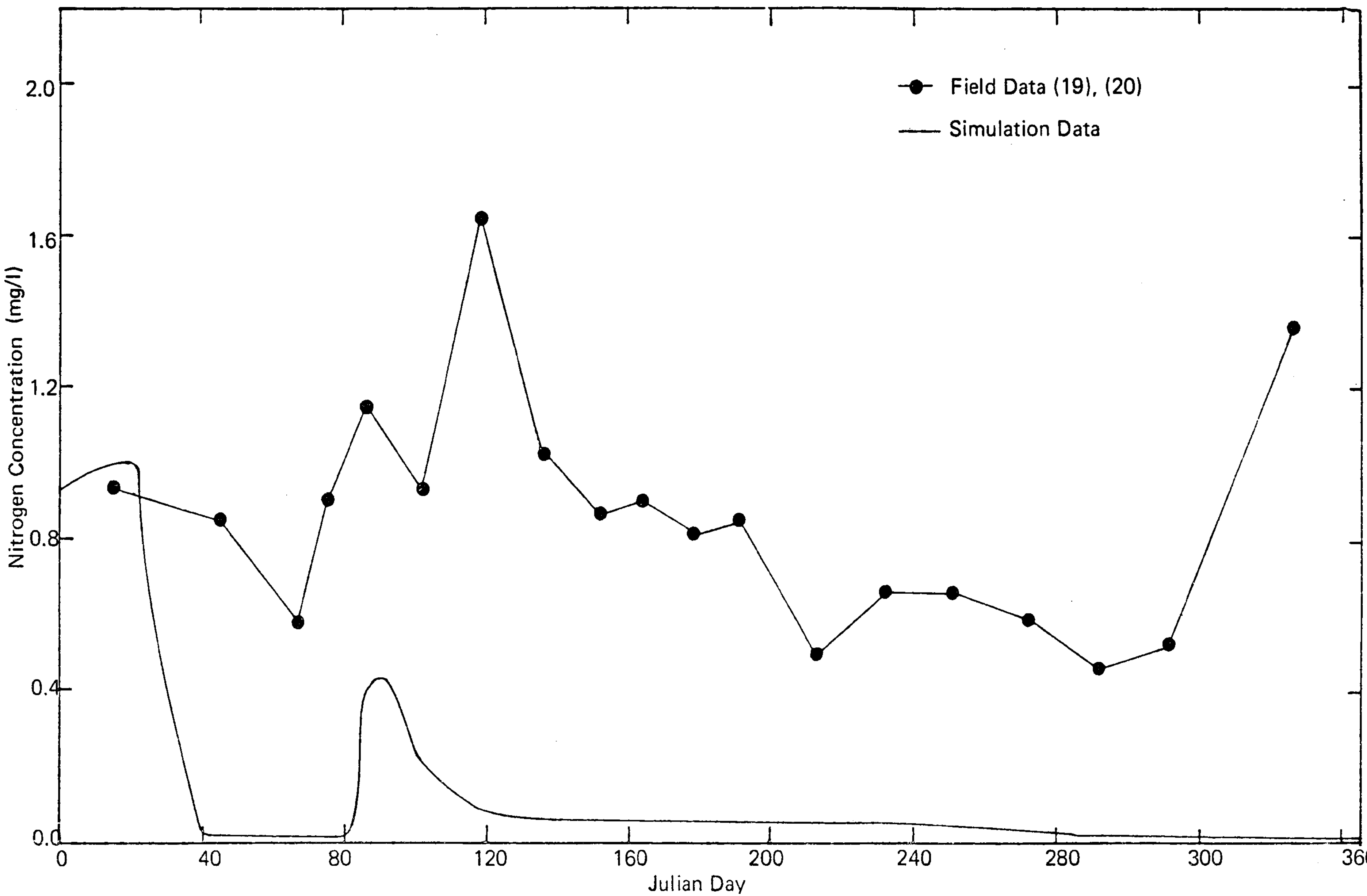

Figure 19. Field Data Versus Simulation Data of Nitrogen Concentration in Third Segment (WWK) in Beaver Reservoir.

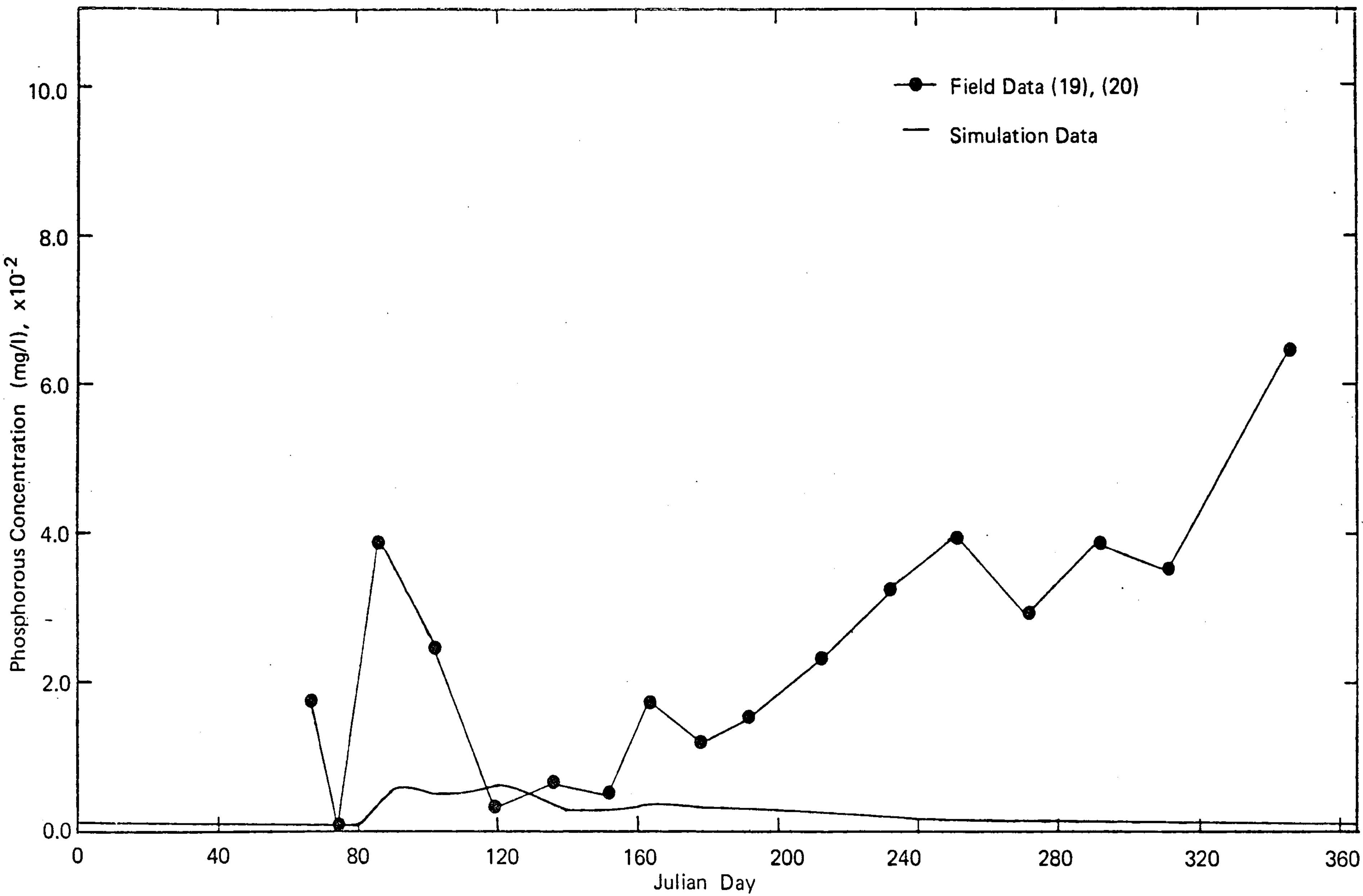

Figure 20. Field Data Versus Simulation Data of Phosphorus Concentration in First Segment (WTR) in Beaver Reservoir.

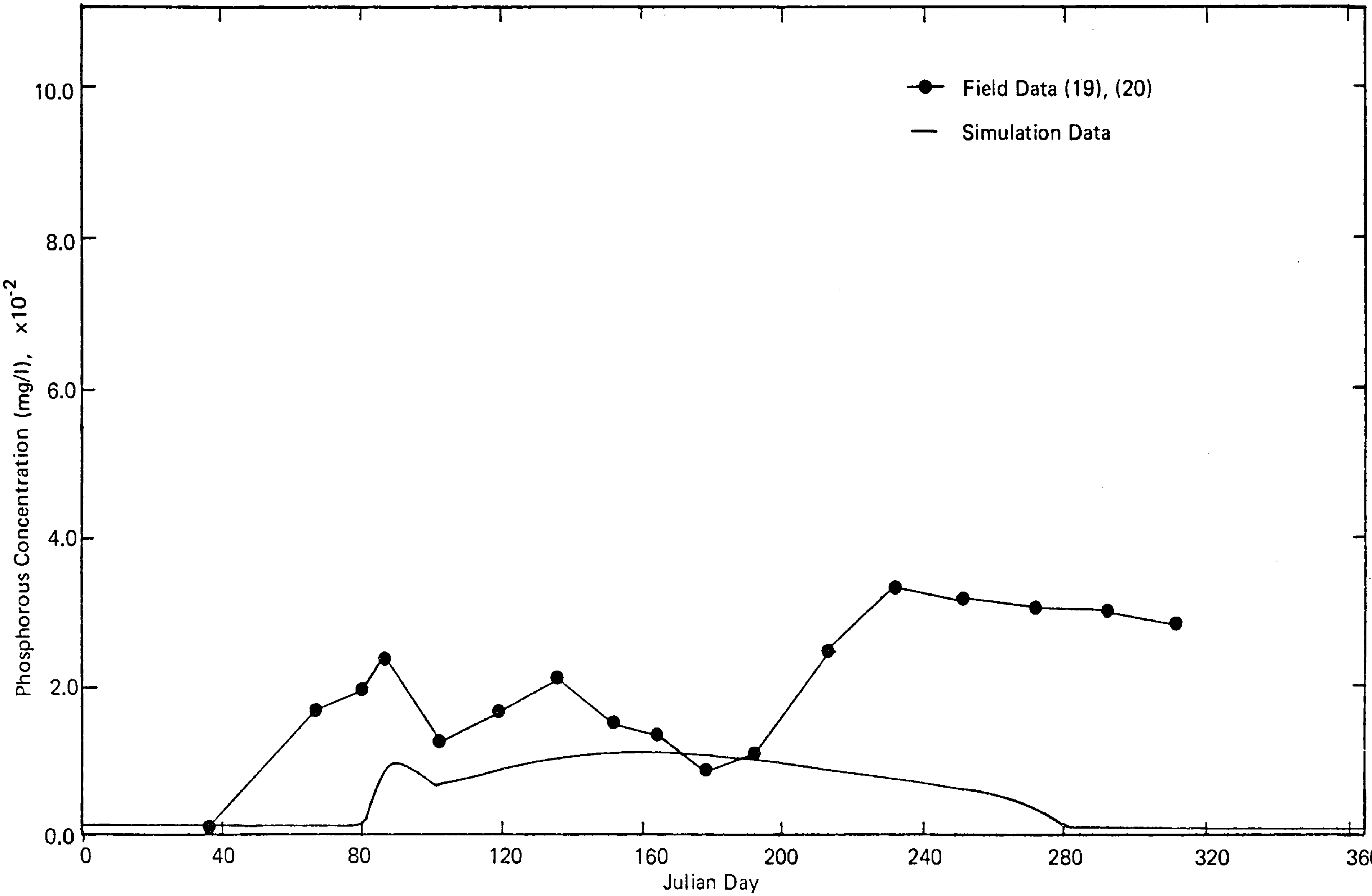

Figure 21. Field Data Versus Simulation Data of Phosphorus Concentration in Second Segment (HPT) in Beaver Reservoir.

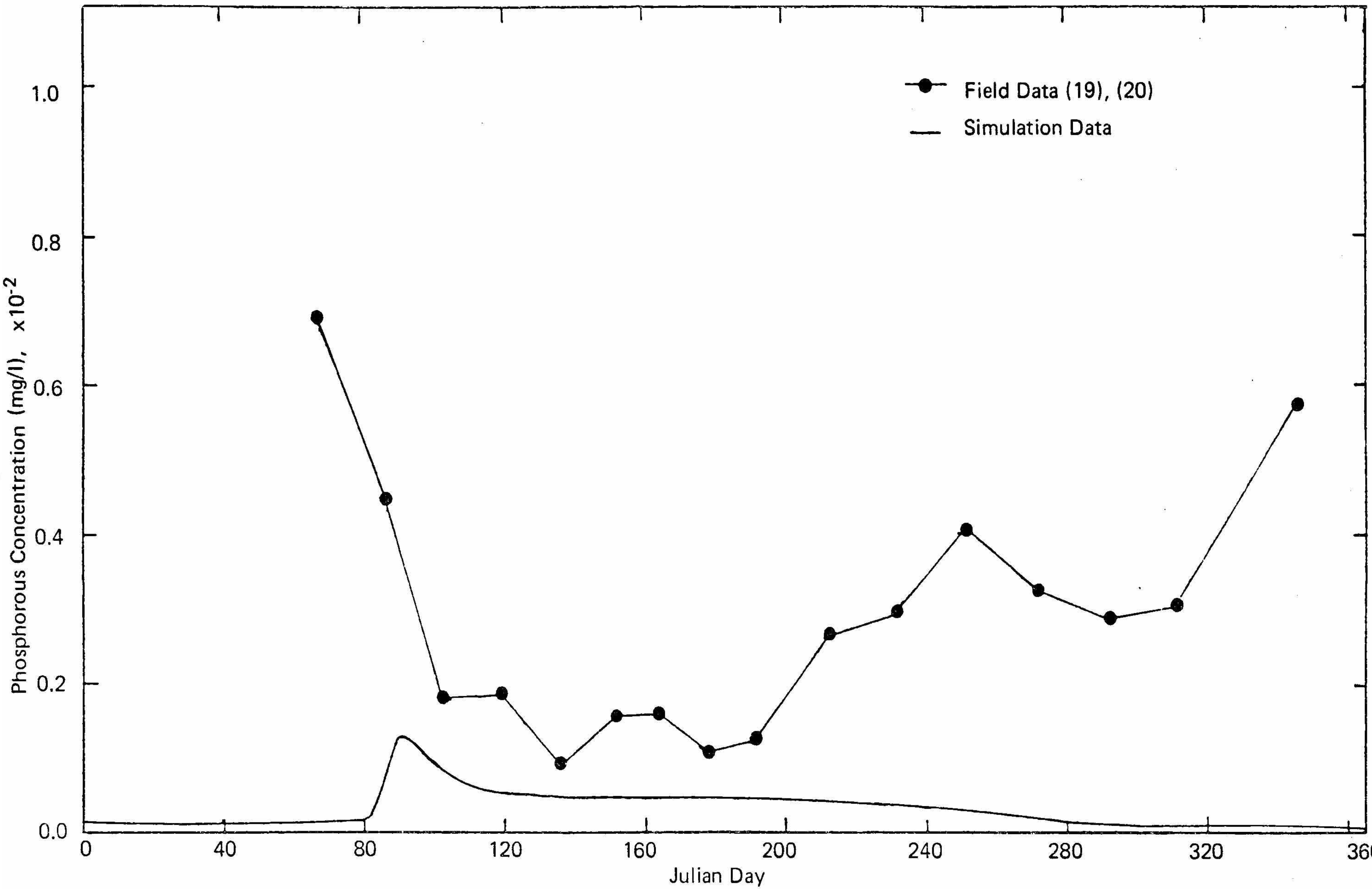

Figure 22. Field Data Versus Simulation Data of Phosphorus Concentration in Third Segment (WWK) in Beaver Reservoir.

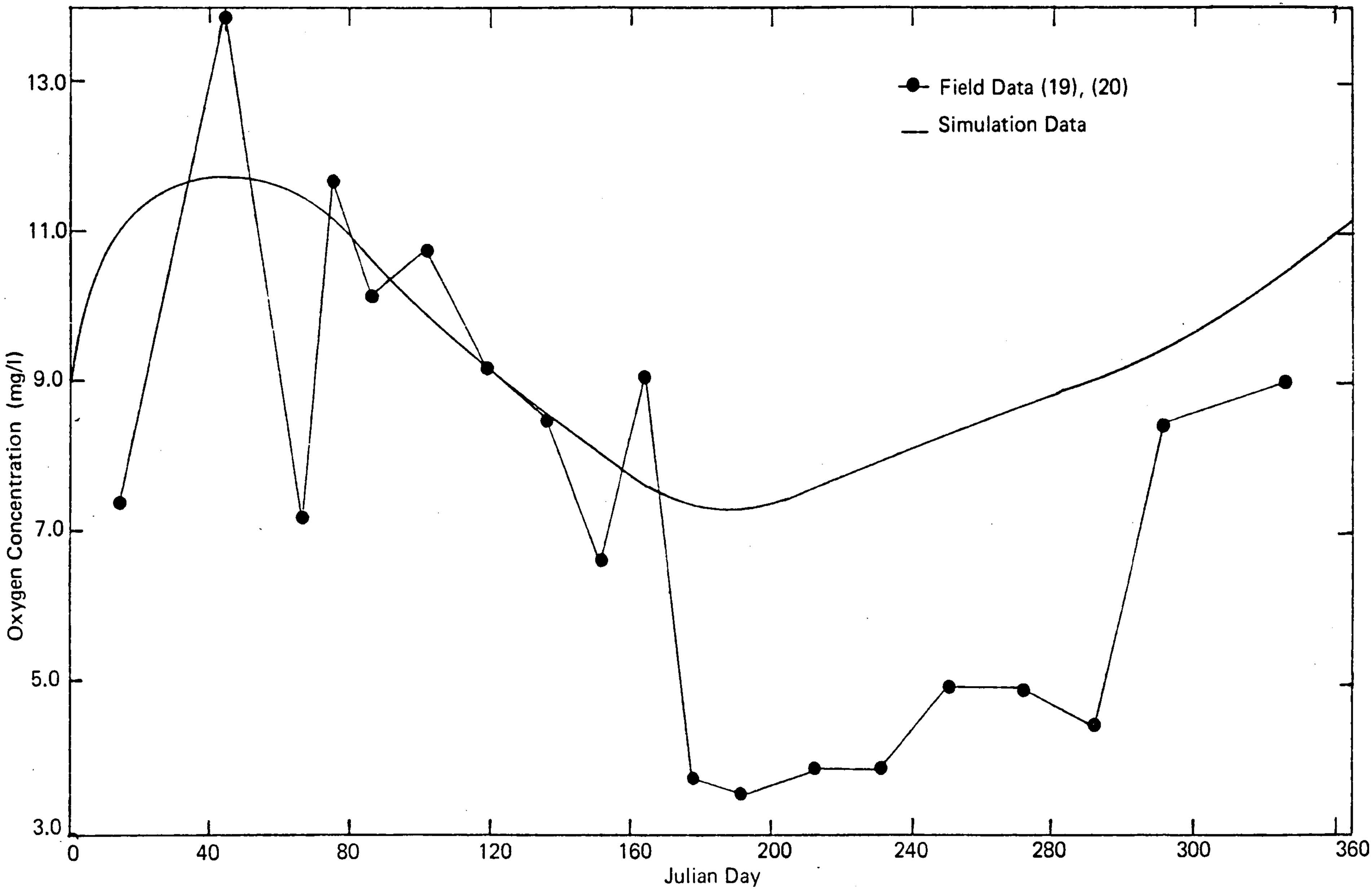

Figure 23. Field Data Versus Simulation Data of Oxygen Concentration in First Segment (WTR) in Beaver Reservoir.

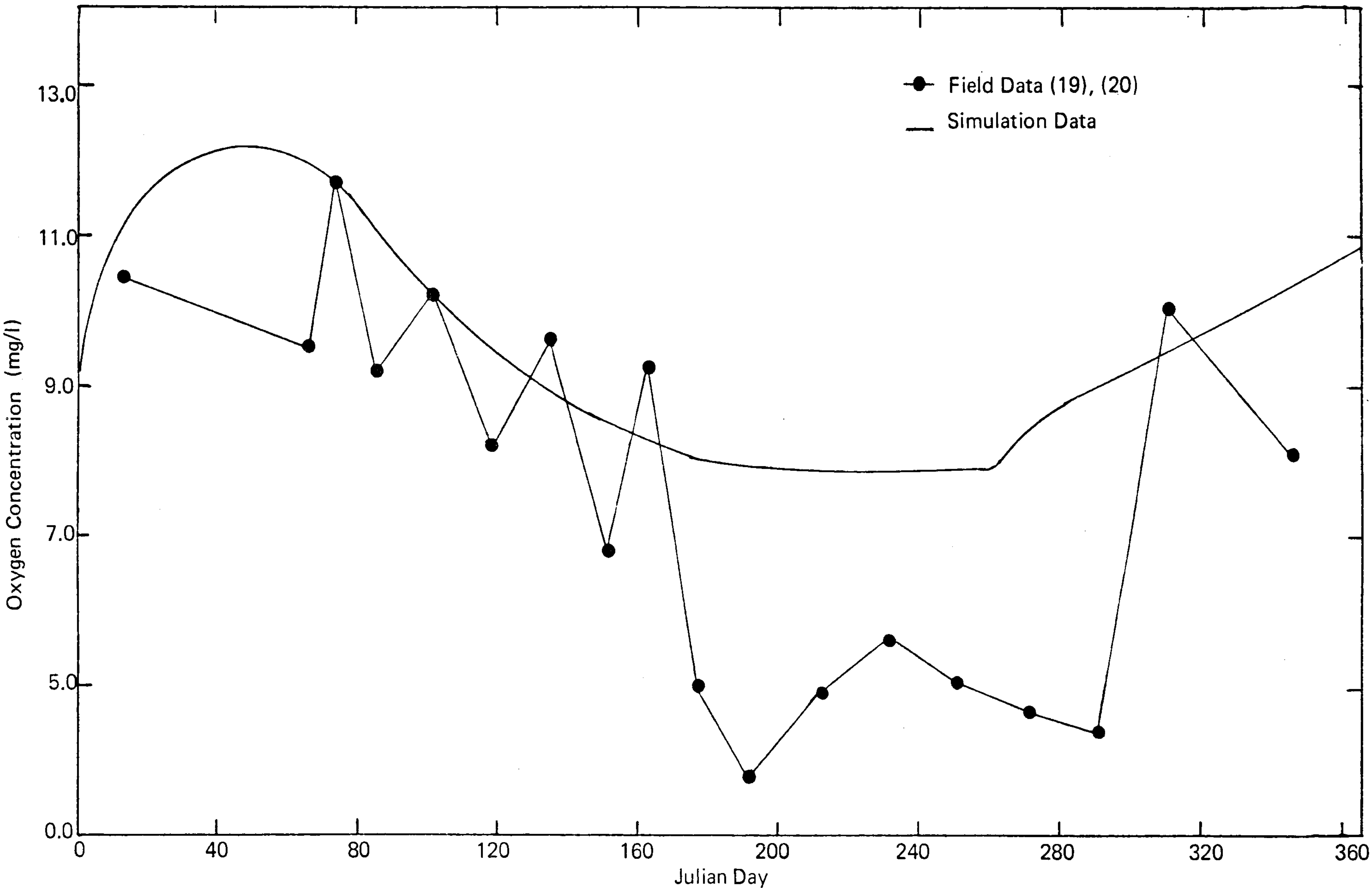

Figure 24. Field Data Versus Simulation Data of Oxygen Concentration in Second Segment (HPT) in Beaver Reservoir.

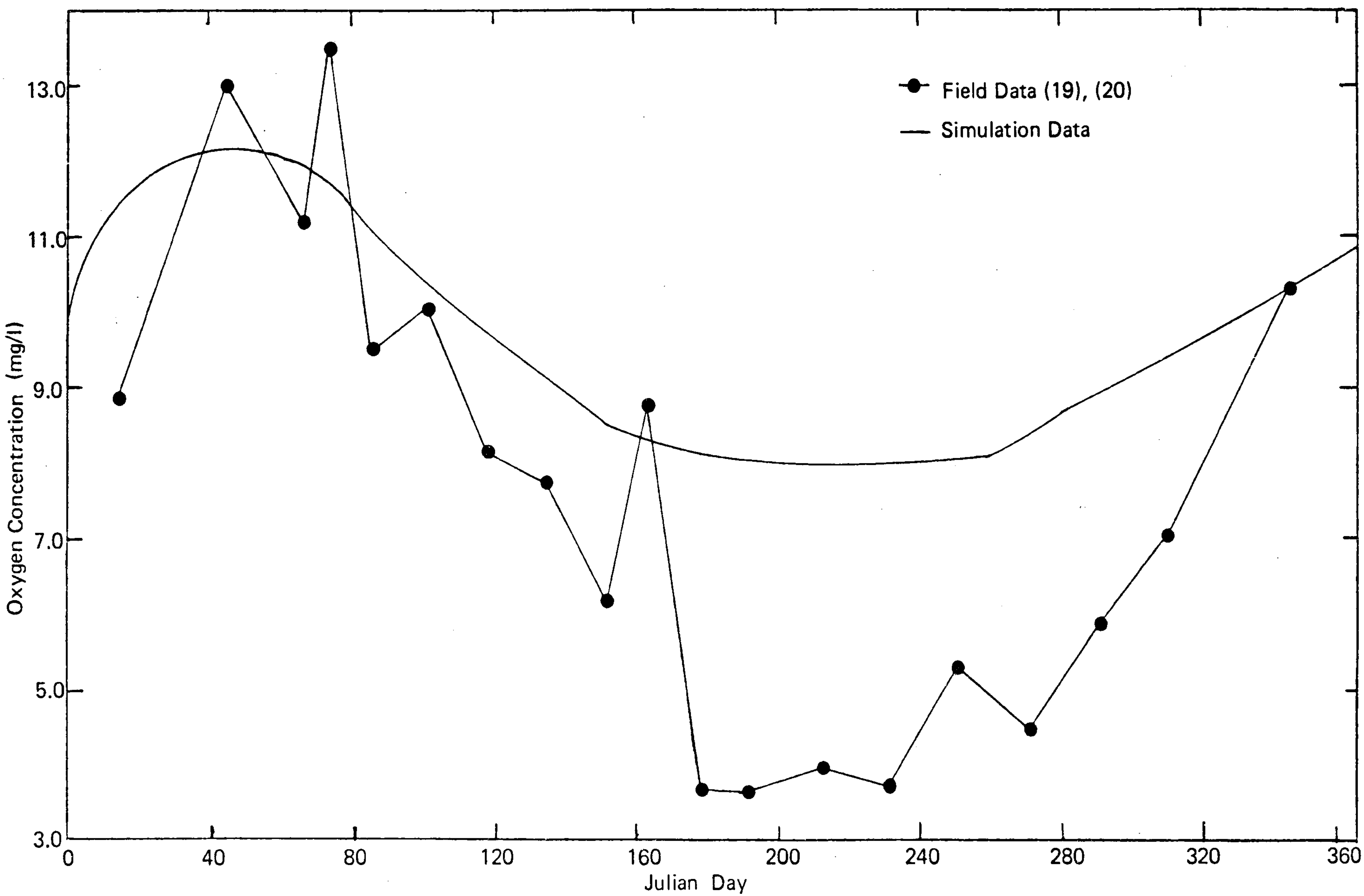

Figure 25. Field Data Versus Simulation Data of Oxygen Concentration in Third Segment (WWK) in Beaver Reservoir.

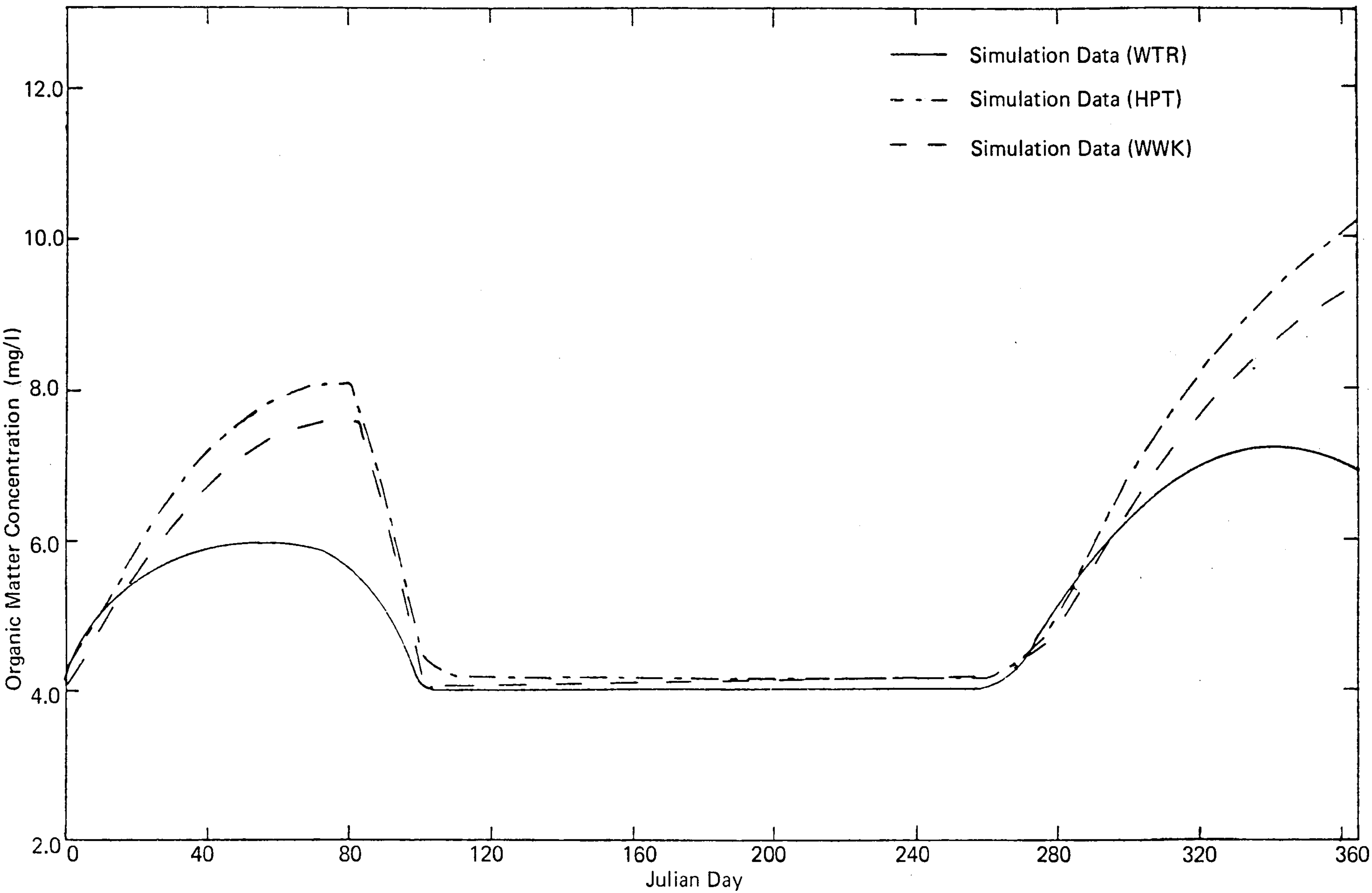

Figure 26. Simulation Data of Organic Matter Concentration in Three Segments (WTR), (HPT), and (WWK) Respectively in Beaver Reservoir.

C. Discussion of Results

The following sections contained a discussion of the Beaver Reservoir simulation results in comparison to the available field data of the various biological and chemical components. Where specific quantitative data is not available, discussion is limited to comparison with known and accepted qualitative behavior patterns.

1. Discussion of the Biological Component Phytoplankton

In all the three segments, the phytoplankton model equations predicted reasonably well the spring and the fall blooms as indicated in the actual field data (Figures 11, 12 and 13). The phytoplankton is the bottom element in the food chain. A high degree of simulation accuracy is desirable at this level.

2. Discussion of the Biological Component Zooplankton

The field data available for comparison was at Hickory Creek sampling point which is 1.0 river miles downstream from the WTR sampling station (Figure 2). The zooplankton model equations predicted only the fall bloom in the combined volumetric average of the three segments (Figure 14). The unpredicted spring bloom could be explained by the fact that, in the model assumption, all the different species of zooplankton were considered as one combined species and predated only on phytoplankton. However, some particular species might feed on some other forms of food besides phytoplankton. Furthermore, the model assumed that during stratified period, flow occurred only in the epilimnion and thus affected the zooplankton concentrations. However, some zooplankton species might migrate to hypolimnion. These limnological phenomena were not included in the zooplankton model equations.

3. Discussion of the Biological Component Fish

There were no available fish field data for comparison in the simulated results of the three segments. The volumetric average of both omnivores and carnivores in the three segments showed no significant changes in concentrations in the first half of the year. Then, there was a gradual increase in the fish concentrations in the last part of the year (Figure 15). This gradual increase could be explained by the fact that, in all three segments, both phytoplankton and zooplankton bloomed in the fall. The highest fish concentration simulated was 2.15 mg/l, which was close to that suggested by the Bureau of Sport Fisheries and Wildlife of 2.5 mg/l (15). This fish concentration is: weight of fish/volume water.

4. Discussion of the Biological Component Bacteria

There were no bacteria biomass field data available for comparison. The model equations predicted an increase in concentrations during the summer stratified period in all the three segments (Figure 16). The simulated results agreed somewhat with the sampling results of Drury (31). Also, the model considered only the aerobic types of bacteria. Therefore, the bacteria model was rather incomplete.

5. Discussion of the Chemical Components Nitrogen and Phosphorus

In all three segments, the nutrient model equations predicted comparatively lower concentrations of nitrogen and phosphorus than the field data (Figures 17, 18, 19, 20, 21 and 22). The failure of the nutrient model in predicting nutrients could be accounted for as a result of several factors. First, the model assumed that phytoplankton grew only in the epilimnion in stratified period(s), and thus depleted the nutrients in that zone. However, some portion of the phytoplankton might have grown in hypolimnion and this deviation was not included in the assumption of the model. Second, the actual mechanism of uptake of nutrients by different species of phytoplankton might vary. This was reflected in the selection of the stochiometry ratios (AN and AP) of nutrients used by phytoplankton for growth. In the Beaver Reservoir simulation, the lower limits of both AN and AP were used. However, the model still showed a very high uptake of nutrients by phytoplankton in the active zone. Third, the water-sediment interfacial concentrations of nutrients were unknown at this time and were assumed by the author. In some shallow lakes, upsurge from the rich sediment might increase the nutrient levels. Fourth, the model did not consider the contributions of nutrients by zooplankton and bacteria. This incompleteness might lower the simulated nutrient levels to some extent. Fifth, the concentrations of nutrients in the runoff that leached directly into the water bodies were not available and had to be assumed. This unknown contribution of nutrients from leaching might affect the simulated nutrient levels.

6. Discussion of the Chemical Component Oxygen

In all three segments, the oxygen model predicted fairly well as compared with the available field data, except in the period between the Julian days 180 and 280 (Figures 23, 24 and 25). In that period, the oxygen model predicted higher concentrations than the observed data. This could be explained by the fact that the amount of bacteria present in the water bodies was unknown in that period when the activity of bacteria was high. Therefore, the amount of oxygen used by bacteria in that period was unknown. Also, the oxygen model did not consider the oxygen used by fish and zooplankton for respiration.

7. Discussion of the Chemical Components Organic Matter

There were not enough BOD field data available for justifying the organic matter simulated results. However, the magnitudes of the simulated organic matter concentrations were close to that reported by Bennett (21). The simulated results showed increases in organic matter concentrations in the unstratified periods in all the three segments. (Figure 26). This could be explained by comparing with the simulated bacteria concentrations. During the unstratified periods, the activity of bacteria was low and organic matter concentrations began to build up. In the stratified period, the conditions became favorable for bacteria growth and this reduced the organic matter concentrations. This was a familiar phenomena of the prey-predator relationship.

8. Discussion of Computer System

The IBM System/360 CSMP (Continuous System Modeling Program) language is capable of solving 500 differential equations simultaneously with limited number of program statements. It also contains many special features such as forcing functional inputs, an initial calculations section, constant sections, various integration methods, variable step sizes, and print-plotting outputs. Therefore, it was employed to solve a total of fourteen differential equations in each of the three segments. All inputs were either in the form of functional inputs, initial conditions or constants. This provides easy adjustments to all parameters. Fourth-order Runge-Kutta with fixed step size of 0.20 day was used as the integration method. The entire simulation time was less than six minutes CPU time (Central Processing Unit), using IBM 370/155. Therefore, the cost of simulation was very economical for the three-element model of this study.

CONCLUSIONS

A deterministic aqueous ecosystem simulation model has been developed. This model, as with all ecosystem models, is incomplete and for this reason it is inadequate as a totally realistic simulator. The model has been developed upon the law of conservation of mass. Quantitative interactions between and among the biomass, chemical and physical subsystems are based upon the Michaelis-Menton and Putter biological kinetic relationships. Field data on a single lake indicate that the model is capable of simulating the major features, both quantitative and qualitative, present in a complex aqueous ecosystem.

The available knowledge concerning the qualitative and quantitative interactions of the biomass, chemical and physical subsystems plus the sophistication of present computer software packages to solve the differential equations, makes it possible to develop adequate aqueous ecosystem simulators from a "lake engineering" point-of-view. The essence of the dominant physical characteristics of the lake environment are important to a realistic simulation. The degree of sophistication employed for the physical subsystem in this model is gross compared to models which divide the vertical and lateral dimensions of the lake water volume into many subsections. A high degree of subsectioning is unwarranted in a general purpose simulator for several reasons. One main reason being the unavailability of sufficient supportative field data and another reason is that the end use of the model prediction rarely requires point-to-point information.

At this point, the general purpose simulation model is somewhat simple and incomplete. Simulation models are always incomplete. The model can be used effectively at present as a predictive tool, in an engineering sense, for existing lakes or as a planning tool for proposed lakes.

RECOMMENDATIONS

All simulation models can be improved continuously. The foremost recommendation however, is that the Water Resources Center provide a means for user application of, improving and updating this model over the next five years. The program is useful in its present form but its readiness to a broad spectrum of users, which include environmental engineers, consulting engineers, power company engineers, state health and pollution control agencies, planning commissions, etc. for specific applications has fallen short of the initial goal of the principal investigator. In essence quick turn-around on applications will be done with some difficulty without the assistance of someone intimately familiar with characteristics and limitations of the model and program.

Specific recommendations for improvement in the approximate order of significance are:

1. Resolve the nutrient (i.e., N and P) simulation difficulty as portrayed in Figures 17 through 22. Attention should be focused upon the stoichiometry of nutrient uptake by phytoplankton and/or a re-analysis of the nutrient recycling mechanism, from both a qualitative and quantitative point-of-view.
2. Calibrate the model based upon a more complete set of field data that includes fish (omnivore and carnivore), bacteria and organic matter.
3. Extend model to include the subsystems: benthic animals, organic particulates, anaerobic bacteria, sediment (degradation reactions, adsorption, etc.), metals and silica.
4. Continue to improve the CSMP simulation program to increase user efficiency. Improvement can be implemented readily by WRC personnel applying model to existing and farseeable water resources problems.

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A FICKIAN ANALYSIS OF LAKE SEDIMENT UPSURGE[1]

L. Thibodeaux and C. K. Cheng[2]

ABSTRACT: The fundamental hypothesis of this paper is that the stratified-mixed states which occur annually in lakes play a dominant role in the movement of nutrient-rich material from the lake bottom upward into the water column. The stratified-mixed history has been used previously to develop a Fourier type, semi-infinite slab energy model to represent the lake surface temperature, thermal profile, and thermocline position with Julian day (Thibodeaux, 1975). The present paper extends the semi-infinite slab idea to include a Fickian type model for the upsurge of nutrients from the sediment-water interface upward. Relations will be developed for the rate of nutrient upwelling into the epilimnion and the photoactive zone of a lake. A lake time constant emerges from the model equations. This constant is thought to be important in predicting the likelihood of a fall algal bloom. Results of model verification attempts on three lakes are presented.

INTRODUCTION

The period of time between the vernal equinox and the autumnal equinox is the dominant productive period for lakes in the temperate zones. This period is roughly from March 21 until September 23, Julian Day 80 and Julian Day 266 respectively, for a total of 186 days. This is also the heating period for the lake. During this period the net heat transfer rate into the lake is positive, and lake stratification occurs. Increased solar radiation upon the lake surface and increased air temperature coupled with a favorable water density-temperature relationship causes a stable water column consisting of warm water above and cold water below.

The time of the vernal equinox is important with respect to the present model. Prior to this time the lake is at nearly uniform conditions with respect to temperature (isothermal) and chemical species concentration. Ice may cover the surface and commences to melt rapidly. A temperature throughout of 4°C to 8°C is not uncommon. This time is model time zero (i.e., $t = 0$). This is also the calendar time at which the heat flux rate changes from negative to positive.

The time of the autumnal equinox is also important with respect to model structure. It marks the approximate end of the stratified structure of the lake and the beginning of a

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TABLE 1. General Lake Information.

Characteristic	Lake Shagawa	Lake Fayetteville	DeGray Reservoir
Location, state	N. Minnesota	N.W. Arkansas	S. Arkansas
Area, sq km	9.3	4.2	54.3
Mean depth, m	5.7	4.3	8.9
Maximum depth, m	14.	10.5	58.
Volume, cu m	53×10^6	3×10^6	794×10^6
Drainage basin, sq km	110		1173
Mean flow, cu m/year	83.4×10^6		
Eutrophic state	eutrophic	moderately eutrophic	mesotrophic

more mixed state with respect to temperature and nutrients. At this time the net heat flux rate changes from positive to negative. Plunging cold water triggered by heat loss is the primary reason for the rapid occurring mixed state and the subsequent fall "turnover." A less elaborate physical model is possible for the time period between autumnal and vernal equinoxes, unless the lake becomes reverse stratified in the winter. In the case of lakes which experience reverse stratification, there are two periods of stratification coupled with two periods of mixing.

ANALYSIS

The mathematics of diffusion in a semi-infinite slab (Bird, *et al.*, 1962) is employed as a quantitative tool to describe the nutrient upsurge phenomena. Sediments on the lake bottom occupy a volume enclosed by an interfacial (water-sediments) area A. Nutrient A is present in the sediment in concentration C_{As} (gr. A/gr.sed.). Interstitial water of nutrient concentration C_{Ao} is in equilibrium with the nutrient content within the sediments, C_{As}. The lake water is visualized as a solid body occupying the space from $X' = 0$ (the water-sediment interface) to $X' = \infty$ (far removed from the bottom) with concentration of C_{Ai}. Nutrient transport up the water column is Fickian and quantified by a constant turbulent mass diffusivity, $D_A^{(t)}$. At time $t = 0$, the bottom surface at $X' = 0$ is suddenly raised to a concentration C_{Ao} and maintained at this value.

The binary continuity equation for A in water can be put in the form

$$\frac{\partial C_A}{\partial t} = D_A^{(t)} \frac{\partial^2 C_A}{\partial X'^2} \tag{1}$$

The initial and boundary conditions are

I.C.: at $t \leqslant 0$, $C_A = C_{Ai}$ for all X' (1a)

B.C.1: at $X' = 0$, $C_A = C_{Ao}$ for all $t > 0$ (1b)

B.C.2: at $X' = \infty$, $C_A = C_{Ai}$ for all $t > 0$ (1c)

The solution to equation (1) as presented by Bird is

$$\frac{C_A - C_{Ai}}{C_{Ao} - C_{Ai}} = 1 - \text{erf}\, X'/\sqrt{4D_A^{(t)}\, t} \tag{2}$$

This equation represents the concentration of nutrient A in the water column as a function of distance from the sediment-water interface and time lapse since the column was mixed. With this equation the analysis of nutrient upsurge can be extended to include mass rates of transfer upward.

The sediment-water interface is stationary and the flux rate of nutrient A at this point is

$$N_{Ao} = -D_A^{(t)} \left.\frac{\partial C_A}{\partial X'}\right|_{X'=0} \tag{3}$$

The concentration gradient at the bottom of the lake is obtained from equation (2):

$$\partial C_A/\partial X' = -\frac{C_{Ao} - C_{Ai}}{\sqrt{\pi D_A^{(t)}\, t}} \exp\left(-X'^2/4D_A^{(t)}\, t\right) \tag{4}$$

at $X' = 0$. Substituting into equation (3) yields the instantaneous flux rate of nutrient A:

$$N_{Ao} = \sqrt{\frac{D_A^{(t)}}{\pi t}}\, (C_{Ao} - C_{Ai}) \tag{5}$$

A more useful quantity is the average flux rate for time period t, obtained by integrating equation (5) to obtain

$$\tilde{N}_{Ao} = 2\sqrt{D_A^{(t)}/\pi t}\, (C_{Ao} - C_{Ai}) \tag{6}$$

Equation (6) is an expression for the gross rate of material entering the water. The rate of nutrient arrival in the epilimnion and the photoactive zone of the lake is of interest also.

CASE I

A Lake with Well Developed Temperature and Nutrient Profiles

Nutrients entering the epilimnion from the sediment bed are capable of stimulating primary productivity. Nutrients enter the relatively well mixed epilimnion region due to the gradual deepening of the region and turbulent diffusion upward through the metalimnion. The flux rate of a dilute nutrient A through a horizontal plane located at a distance X below the surface is

$$N_{AI} = (C_A - C_{Ai})\,v - D_A^{(t)} \left.\frac{\partial C_A}{\partial X'}\right|_X \tag{7}$$

where v is the mass velocity of the thermocline downward. The lake depth (d), distance from the air-water interface (X), and distance from the water-sediment interface (X′) are related by

$$d = X + X' \tag{8}$$

The thermocline depth with time (Thibodeaux, 1975) can be expressed by

$$X_{Tc} = \sqrt{2a^{(t)}t} \tag{9}$$

where X_{Tc} is the thermocline distance from the air-water interface in meters
$a^{(t)}$ is the turbulent thermal diffusivity in sq m/day
t is the time lapse since the lake was in a mixed state in days

From equation (9),

$$v \equiv dX_{Tc}/dt = \sqrt{a^{(t)}/2t} \tag{10}$$

Substituting equations (2), (4), (9), and (10) into equation (7) yields

$$N_{A_I} = \sqrt{\frac{a^{(t)}}{2t}}\left[\left(1 - \operatorname{erf}\frac{d - \sqrt{2a^{(t)}t}}{\sqrt{4D^{(t)}t}}\right) + \sqrt{\frac{D^{(t)}}{\pi t}}\exp-\left(\frac{d - \sqrt{2a^{(t)}t}}{\sqrt{4D^{(t)}t}}\right)^2\right]\left(C_{Ao} - C_{Ai}\right) \tag{11}$$

This relation gives the instantaneous flux rate of nutrient A into the epilimnion due to the upsurge of components released from the sediment bed. The rate entering the epilimnion (equation 11) is less than the rate leaving the sediments (equation 5).

The fraction of nutrient A leaving the sediments which enter the epilimnion is significant because only this amount will stimulate algal growth. This fraction for Case I is defined as

$$\phi_I \equiv \tilde{N}_{AI}/\tilde{N}_{Ao} \tag{12}$$

Assuming $D^{(t)} = a^{(t)}$ and integrating equation (11) over time and dividing by equation (6) yields

$$\phi_I = \frac{1}{2\sqrt{\theta}}\int_0^{\theta}\left\{\sqrt{\frac{\pi}{2}}\;1 - \operatorname{erf}\frac{1-\sqrt{\theta}}{\sqrt{2\theta}} + \exp-\left(\frac{1-\sqrt{\theta}}{\sqrt{2\theta}}\right)^2\right\}\frac{d\theta}{\sqrt{\theta}} \tag{13}$$

where $\theta \equiv t/t_c$ and $t_c \equiv d^2/2D^{(t)}$, a lake time constant. From equation (9) it can be seen that $\theta \leqslant 1$. Figure 1 is a plot of ϕ_I versus θ. At $\theta = 1$, $\phi_I = 1$, and this implies that the thermocline has reached the lake bottom and all the nutrients are in the epilimnion.

CASE II

A Lake with a Poorly Developed or Non-Existent Thermal Profile and a Well Developed Nutrient Profile

Sunlight penetrates and effectively stimulates primary production down to a depth X_e. This depth depends upon the extinction coefficient of the water, self shading by algae, etc. The flux of nutrients arising from the sediments into this lighted region of the lake when no thermocline is present is

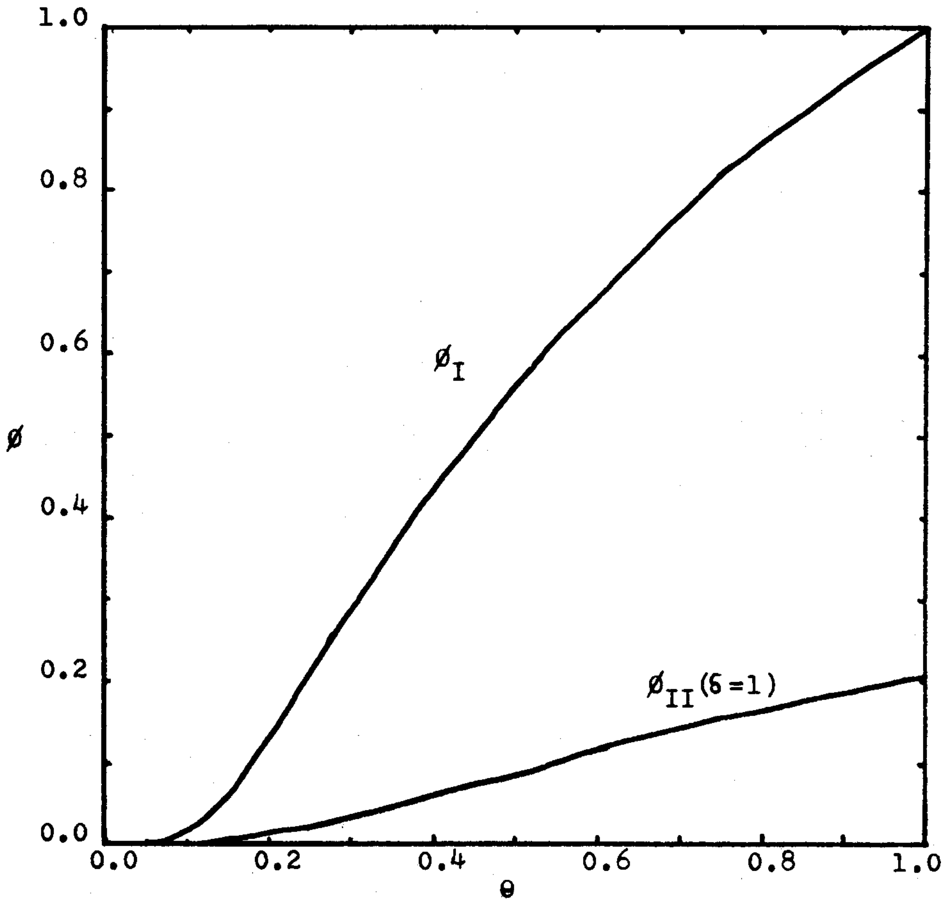

Figure 1. Fractions of Upsurging Nutrients in Epilimnion (ϕ_I) and At Surface (ϕ_{II}).

$$N_{A_{II}} = -D_A^{(t)} \left.\frac{\partial C_A}{\partial X'}\right|_{X_e} \tag{14}$$

Employing equation (4) yields

$$N_{A_{II}} = \sqrt{\frac{D^{(t)}}{\pi t}}\left[\exp - \frac{(d - X_e)^2}{4D^{(t)}t}\right]\left(C_{Ao} - C_{Ai}\right) \tag{15}$$

This is the instantaneous rate at which upsurging nutrient A enters a photoactive layer of depth X_e.

The fraction of nutrient A leaving the sediments which enters this photoactive layer can be defined as

$$\phi_{II} \equiv \tilde{N}_{A_{II}}/\tilde{N}_{A_O} \tag{16}$$

Integrating equation (15) over time and dividing by equation (6) yields

$$\phi_{II} = \frac{1}{2\sqrt{\theta}}\int_0^{\theta} \exp\left[-\frac{\delta^2}{2\theta}\right]\frac{d\theta}{\sqrt{\theta}} \tag{17}$$

where $\delta \equiv (d - X_e)/d$. θ appears here also, but in contrast to its use in equation (13), it can be greater than unity. Figure 2 shows the fraction entering the photoactive zone as a function of θ and the parameter δ. δ incorporates the distance of the photoactive zone from the lake bottom.

Uses and Implications of the above Fickian Analysis of Nutrient Upsurge from Bottom Sediments: From an ecosystem modeling point of view the above development provides equations for estimating the rate at which bottom residing nutrients enter lake water. Equation (6) gives the total rate; however, depending upon the lake thermal structure ϕ's from equations (13) and (17) will give the rate entering the epilimnion or photoactive zone:

$$\tilde{N}_A = 2\phi\sqrt{D^{(t)}/\pi t}\left(C_{Ao} - C_{Ai}\right) \tag{18}$$

The adequacy of describing upsurge by a single parameter, $D^{(t)}$, will be confronted later.

The time structure is also an important modeling parameter. Commencing at the vernal equinox the lake is stratified for several months, and t is on the order of 92 days. An unstratified period of approximately three months (91 days) ensues, and during this period t is of the order of one to two weeks. Another stratified (reverse) period of four months may occur, and here t is of the order of 121 days. A second unstratified period of two months (61 days) in which t is one to two weeks in length concludes the annual cycle. Lakes which stratify only once will have a less elaborate structure.

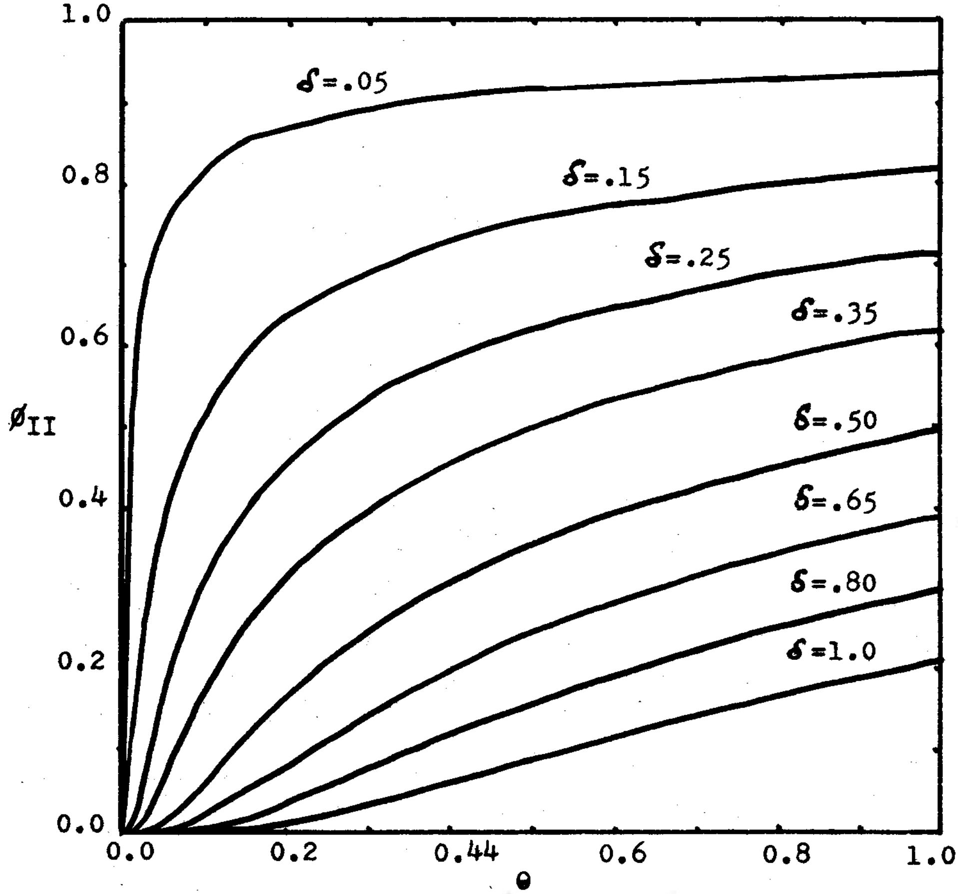

Figure 2. Fractions of Upsurging Nutrients in Lake Water Column (ϕ_{II}).

The Fickian analysis results can be used to determine whether a fall bloom of algae is likely for a given lake. The spring bloom can deplete the lake of a limiting nutrient, and an additional supply of this nutrient is needed if a fall bloom is to materialize. The depth of the lake (d) and the turbulent mass diffusivity ($D^{(t)}$) are important lake characteristics which regulate the time of travel of nutrients from the bottom sediments into the epilimnion. These two characteristics are contained in the time constant t_c (= $d^2/2D^{(t)}$). If one assumes that ten percent (ϕ_I = .1) of the limiting nutrient leaving the sediments is sufficient to stimulate increased algal growth in the epilimnion, from equation (13) (or figure 1), $\theta_I = 0.18$. This implies that the time (in days) required for a significant quantity (i.e., 10 percent) of the limiting nutrient to arrive at the epilimnion is

$$t_I = 0.18\,(d^2/2D^{(t)}). \quad (19)$$

In a similar fashion, if 10 percent ($\phi_{II} = .1$) arrives at the surface (i.e., $\delta = 1$), from equation (17) (or figure 1), $\theta_{II} = 0.54$, and

$$t_{II} = 0.54\,(d^2/2D^{(t)}). \tag{20}$$

Comparison of the nutrient travel times, t_I and t_{II}, indicates that in general, less time is required for lakes with a well developed thermal profile and a downward migrating thermocline.

In general, equations (19) and (20) give a range of times, in days, after the vernal equinox in which the photoactive region of a lake commences to receive detectable quantities of nutrients from the sediment bed that may stimulate a fall bloom. If the time computed is less than or equal to 186 days, a fall bloom is likely. If the time is greater than 186 days, the declining length of the photoperiod and low water temperatures may make a fall bloom unlikely.

INTERPRETATION OF FIELD DATA

In lakes where the bottom sediments contain large quantities of nutrient-bearing sludge it has been observed by the authors and others that the profiles develop from the bottom upward, exist for a certain period of time, and are then destroyed or rendered uniform. Lake Shagawa, Ely, Minnesota, has a bottom of which more than 70 percent is muck resulting from that city's wastewater discharge directly into the lake from its secondary treatment plant (Larson, *et al.*, 1974). The lake sediments contain large quantities of nitrogen and phosphorus. Figures 3 and 4 show the upsurge of phosphorus in Shagawa during the summer of 1973 and the upsurge of ammonia during the winter of 1972-73 (Larson, 1974).

Nutrient concentration profiles in Shagawa develop and move upward in the summer months of June, July, and August. Uniform concentration profiles are present in September, October, and November. Profiles redevelop and move upward in December, January, February, and March. Uniform concentration profiles appear again in April and May. The upsurge cycle commences again in June. The temperature structure has a similar behavior: stratified in the summer, mixed in the fall, reverse stratified in the winter, and mixed in the spring. The relatively stable water column during the stratified periods provides the necessary water stillness to observe upward moving nutrient profiles in these periods. "Turnover" in the fall and spring destroys the profiles.

The type of thermal structure and nutrient profile development was observed on Shagawa during 1972 and 1973, on Lake Fayetteville during 1969-1970 (Meyer, 1971), and on DeGray Reservoir during 1970 (Nix, 1974). The latter two water bodies have a very weak reverse stratified period or none, and therefore only one annual occurrence of upward moving nutrient concentration profiles was observed, and this was in the summer and early fall.

Equation (1) was used to determine average $D^{(t)}$ values for the three lakes. In all, 1463 observations of concentrations of ammonia, total phosphorus, and orthophosphate in Shagawa; 134 observations of ammonia in Lake Fayetteville; and 52 observations of nitrate and calcium in DeGray Reservoir were processed to obtain $D^{(t)}$ values.

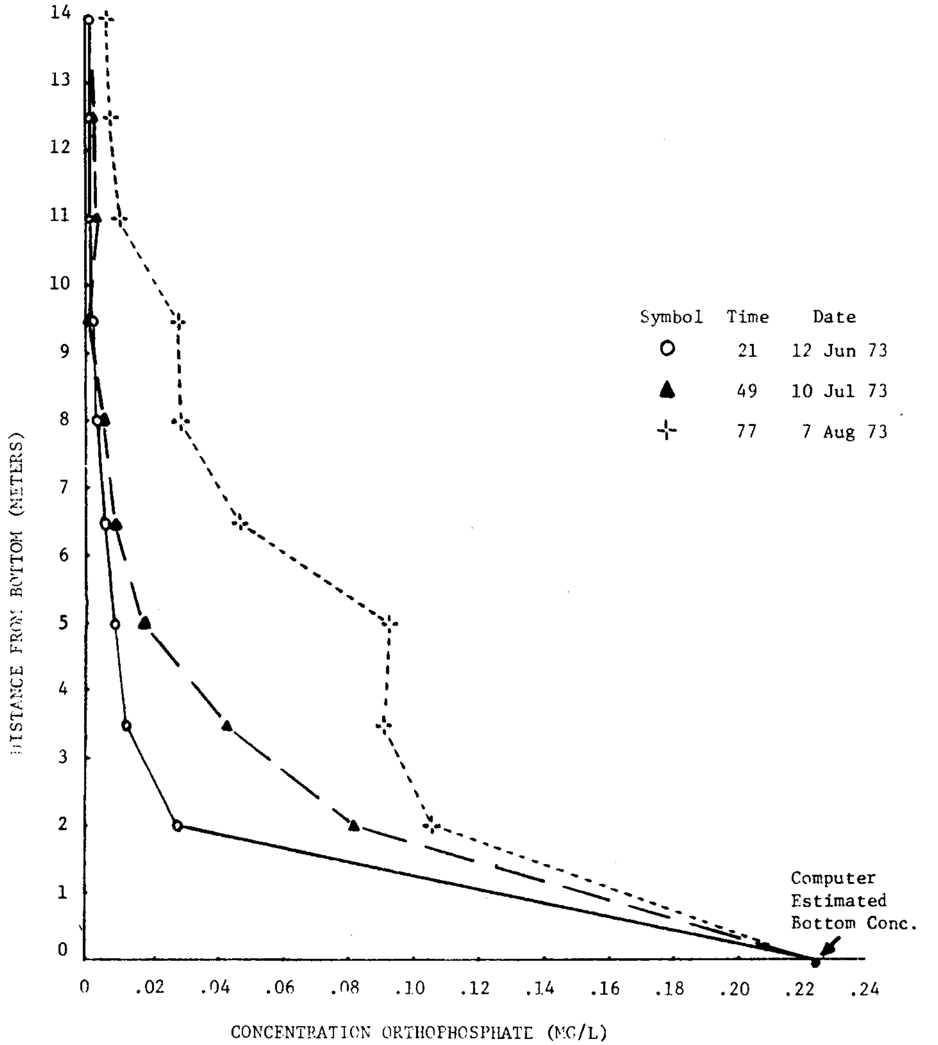

Figure 3. Shagawa Lake Orthophosphate Concentration Station LBS Summer 1973 Field Data.

Concentrations (C_A) and distances from the bottom (X') were obtained directly from the field data. Temperature and concentration profiles were used to establish the Julian day from which t commences and C_{Ai} C_{Ao} is unknown, since a sample at the water-sediment interface is difficult to obtain from the surface. A computer algorithm was devised to choose C_{Ao} and calculate $D^{(t)}$ so as to minimize the deviation of the calculated and actual profiles. Data from the top five meters of the lakes were excluded. Tables 2, 3, and 4 contain calculated $D_A^{(t)}$ and C_{Ao} values along with standard deviations and other constants for the three lakes of study. A limited number of turbulent model diffusivities, $a^{(t)}$, are computed from the thermal, semi-infinite slab model (Thibodeaux, 1975)

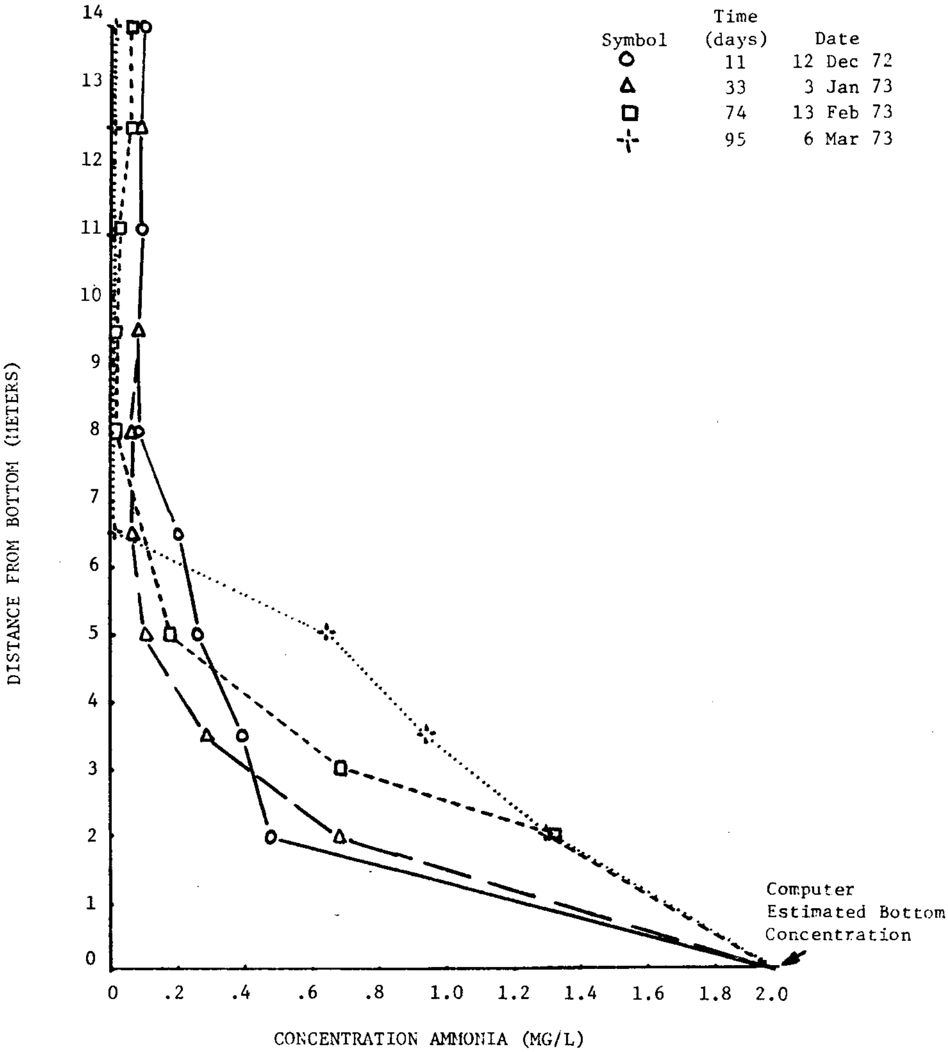

Figure 4. Shagawa Lake Ammonia Concentration Station LBS Winter 1972-1973 Field Data.

and are given for Lake Fayetteville and DeGray Reservoir. Shagawa is too shallow for this thermal model to apply, and therefore no $a^{(t)}$ values could be obtained.

DISCUSSION

The basic hypothesis of this semi-infinite slab model is that the mechanism is Fickian type with constant $D^{(t)}$ influenced mainly by water turbulence. This turbulence is induced mainly by surface winds, bulk hydraulic flow, and thermal gradients. A stratified

TABLE 2. Lake Shagawa Semi-Infinite Slab Model Results.

Year/Season/Species	No. observations	Time zero (Julian day)	C_{Ai}	Range of C_{Ao} search	Average $D^{(t)}$, sq m/day	σ of $D^{(t)}$, sq m/day	C_{Ao}
Sample Station LON:							
1972/Winter/TP[1]	28	318	.036	0.1 - 2.0	.036	.015	.271
1972/Winter/OP	31	318	.017	0.1 - 2.0	.044	.007	.194
1972/Winter/NH_3	14	318	.060	0.1 - 2.0	.105	.017	.467
1973/Summer/TP	30	142	.030	0.1 - 1.0	.051	.006	.375
1973/Winter/TP	39	335	.029	0.1 - 1.0	.031	.007	.394
1973/Summer/OP	31	142	.003	0.1 - 1.0	.041	.003	.208
1973/Winter/OP	44	335	.008	0.1 - 1.0	.045	.006	.248
1973/Summer/NH_3	26	142	.10	0.1 - 1.0	.053	.017	.584
1973/Winter/NH_3	24	335	.030	0.3 - 2.0	.050	.004 - .142[2]	.625
Sample Station LED:							
1972/Winter/TP	43	318	.047	0.1 - 3.0	.069	.006	.253
1972/Winter/OP	61	318	.021	0.1 - 2.0	.094	.002	.150
1972/Winter/NH_3	19	318	.077	0.5 - 3.0	.120	.019	.502
1973/Summer/TP	53	142	.033	0.7 - 3.0	.059	.014	.999
1973/Winter/TP	55	335	.036	0.1 - 1.0	.091	.007	.315
1973/Summer/OP	56	142	.001	0.2 - 2.0	.061	.017	.687
1973/Winter/OP	60	335	.013	0.1 - 1.0	.152	.004	.157
1973/Summer/NH_3	49	142	.01	0.7 - 3.0	.058	.027	1.95
1973/Winter/NH_3	40	335	.05	0.1 - 1.0	.095	.014	.764
Sample Station LBS:							
1972/Winter/TP	87	318	.048	0.7 - 8.0	.022	.0003 - .084	.918
1972/Summer/TP	65	157	.050	0.7 - 8.0	.171	.018	.869
1972/Winter/OP	101	318	.024	0.7 - 8.0	.029	.00035 - .101	.701
1972/Summer/OP	54	157	.018	0.7 - 8.0	.106	.020	.831
1972/Winter/NH_3	47	318	.070	0.7 - 8.0	.089	.034	.789
1972/Summer/NH_3	55	157	.056	0.7 - 8.0	.136	.037	1.60
1973/Winter/TP	70	335	.034	0.2 - 2.0	.136	.004	.282
1973/Summer/TP	55	142	.035	0.2 - 2.0	.120	.006	.424
1973/Winter/OP	70	335	.012	0.2 - 2.0	.149	.003	.201
1973/Summer/OP	60	142	.004	0.2 - 2.0	.135	.004	.224
1973/Winter/NH_3	45	335	.040	0.7 - 8.0	.100	.031	1.98
1973/Summer/NH_3	51	142	.01	0.7 - 8.0	.104	.014	.952

[1] TP ≡ total phosphorus, OP ≡ orthophosphate, NH_3 ≡ ammonia. Concentrations in mg/l.
[2] Range denotes 95 percent CI of $D^{(t)}$.

water column tends to stabilize and counteract the effects of wind, the bulk flow and thermal gradients, and therefore this analysis applies only during these periods.

The most persuasive evidence that a Fickian-type approach is reasonable is the time, distance concentration upsurge development. This development can be observed from published field data. The field data must be from lakes with fairly stable water columns and with no chemical or biochemical reactions so that the upsurge development can be observed over a period of several months. The field data of Larsen, Meyers, and Nix show these upsurge profiles particularly well. Figures 5 and 6 show the calculated profiles corresponding to figures 3 and 4 from Larsen's data.

The interpretation of a proposed model employing field data from a designed experimentation program is, at best, a difficult and somewhat ungratifying task. Existing field data were used for model verification purposes. Equation (1) was employed to obtain $D^{(t)}$ values for three lakes. Data from the top 13 to 15 ft were excluded as being influenced by the algae nutrient (i.e., N and P) reactions. Four quantities in this equation

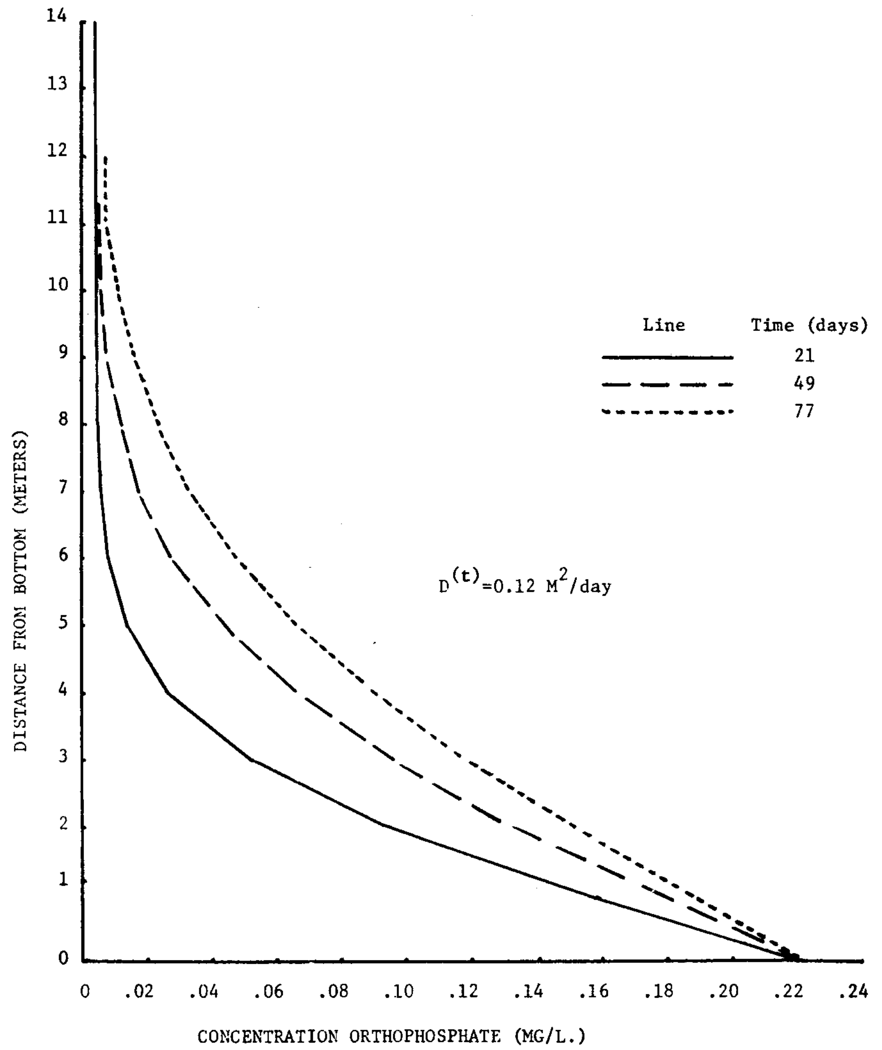

Figure 5. Shagawa Lake Orthophosphate Concentration Station LBS Summer 1973 Calculated Profiles.

(i.e., C_A, C_{Ai}, X', and t) can be fairly easily determined; however, two unknowns remain ($D^{(t)}$ and C_{Ao}). The additional constraint of minimizing profile concentration deviations with a search on C_{Ao} in effect provided an additional equation, so that the remaining two quantities can be determined. The Shagawa data indicate $D^{(t)}$ is independent of C_{Ao}; however, both $D^{(t)}$ and C_{Ao} were dependent on station in the same direction: LBS > LED > LON! Some functional relationship between these variables may logically be anticipated, considering how they were evaluated.

Thirty average $D^{(t)}$ values for Shagawa are shown in table 2. The overall average was .085 sq m/day. The range of the averages was from 0.022 to 0.17 sq m/day. Considering

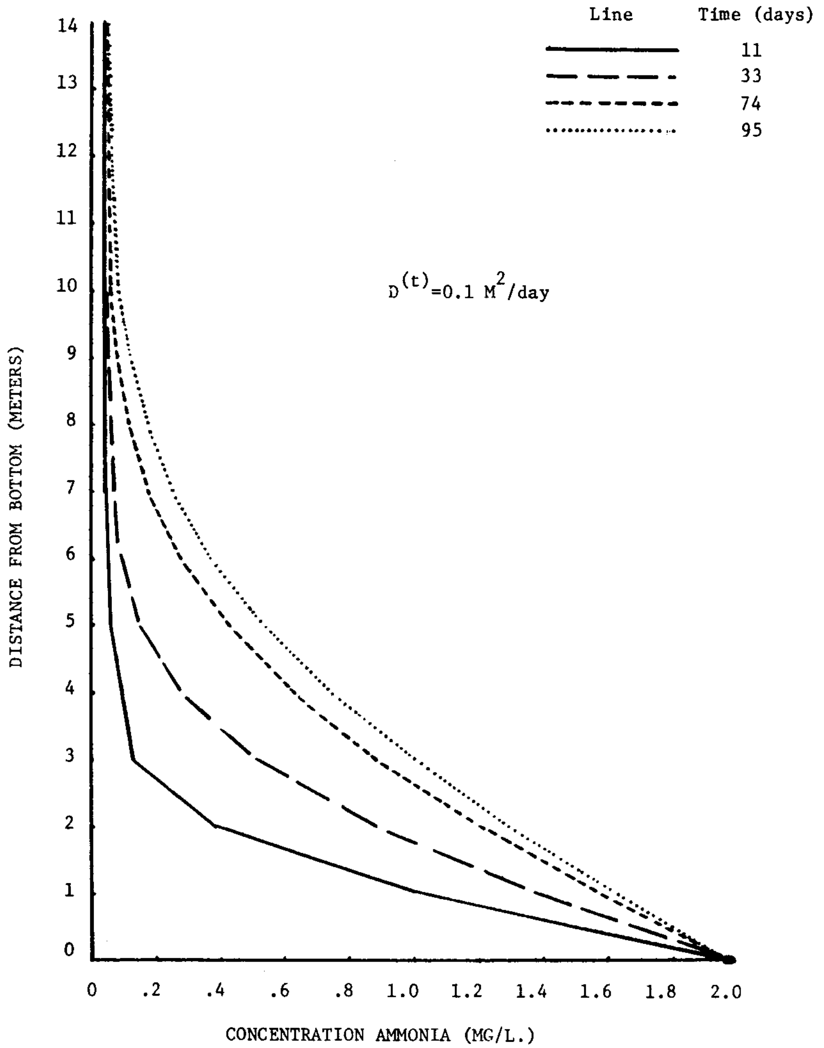

Figure 6. Shagawa Lake Ammonia Concentration Station LBS Winter 1972-1973 Calculated Profiles.

this range, $D^{(t)}$ showed no significant species variation. Overall averages of .079, .086, and .091 are indicated for total phosphate, orthophosphate, and ammonia respectively. Rich (1973) reports 0.026 to 17 sq m/day (0.003 to 2 sq cm/sec) as the range of turbulent diffusion coefficients in the deeper regions of lakes. $D^{(t)}$ also showed no dependence upon time or season of the year.

If the magnitude of $D^{(t)}$ is influenced primarily by water turbulence, it should be closely related to $\alpha^{(t)}$, the turbulent thermal diffusivity. $D^{(t)}$ and $\alpha^{(t)}$ values for Lake Fayetteville are 0.1 and 0.2 sq m/day respectively. $D^{(t)}$ and $\alpha^{(t)}$ values for DeGray Reservoir are approximately 1.3 and 0.74 sq m/day respectively. These values are roughly the same magnitude and possibly represent the same dispersive phenomena. Calculated $D^{(t)}$ values are particularly sensitive to errors in X', the distance from the sediment-water interface, as is shown by equation (1). There can be much error here, since distances are typically measured by "wire line" from the lake surface and X' must be computed using equation (8) knowing the exact depth d.

The concentrations of upsurging species at the sediment-water interface reported in tables 2, 3, and 4 are calculated values. This bottom concentration is dependent upon the species. Overall average values of 0.52, 0.36, and 1.0 mg/l are representative of total phosphorus, orthophosphate, and ammonia from Shagawa respectively. C_{Ao} ranges for Shagawa were as follows: total phosphorus, 0.25 to 1.0 mgP/l; orthophosphate, 0.16 to 0.83 mgP/l; and ammonia, 0.47 to 2.0 mgN/l. Gahler (1969) reported laboratory determined soluble nutrient concentrations in interstitial water from Shagawa sediments of 0.36, 0.15, and 3.5 mg/l for total phosphorus, orthophosphate, and ammonia respectively.

Woods, *et al.* (1975), reported phosphorus concentrations in water overlying anaerobic sediments containing glucose of ~0.25 to 8.0 mg/l. Tests with lactose and cornstarch impregnated sediments resulted in maximum phosphorus concentrations of 2.95 and 2.73 mg/l respectively. Woods and coworkers postulate and investigate (laboratory) fermentation gas-lift transport as the mechanism for moving water of high phosphorus content from the sediments to the interface. If this mechanism is present, one would anticipate that C_{Ao} would vary with lake and specific bottom conditions. Anaerobic conditions existed in the three lakes of this study. The Shagawa data indicate that C_{Ao} is not dependent upon season. A continuous supply of nutrients to the sediment-water interface via anaerobic fermentation gas-lift transport would tend to support boundary condition 1 (equation 1b) as reasonable. However, this assumption cannot be valid year after year, because eventually the sediment phosphorus content will be exhausted unless nutrient recycling occurs.

Larsen, *et al.* (1974), report 40 kg/day of phosphorus and 80 kg/day of nitrogen as reasonable inputs from the sediments of Shagawa between day 170 and day 230. Using equation (18) with $D^{(t)}$ = .085 sq m/day, t = 60 days, d = 5.7 m, area = 9.3 sq km with 70 percent coverage, C_{Po} = .52 mgP/l and C_{No} = 1.0 mgN/l results in a $\phi_I = 0.31$ and $\tilde{N}_P$ = 44.6 kgP/day and $\tilde{N}_N$ = 85.8 kgN/day. Equation (19) yields a t_I for Shagawa of 34 days, and Shagawa does experience fall blooms. t_I for Lake Fayetteville is 17 days, and it also has fall blooms. DeGray is a new reservoir with no significant quantity of nutrient-bearing sediments.

To and Randall (1975) report a maximum release rate of 5.8 mgP/sq cm/week from Occoquan Reservoir sediments incubated in pyrex glass culture tubes. Equation (18) with

TABLE 3. Lake Fayetteville Semi-Infinite Slab Model Results.

Year/Species	No. observations	Time zero (Julian day)	C_{Ai}	Range of C_{Ao} search	Average $D^{(t)}$, sq m/day	Range $D^{(t)}$, sq m/day	C_{Ao}
1969/NH_3	95	91	.70	6.0 - 20.	0.103	.014 - .453	10.3
1970/NH_3	39	91	.13	3.0 - 20.	0.091	.023 - .406	6.63

Year/Species	No. observations	Time zero (Julian day)	T_o, °C	Average $\alpha^{(t)}$, sq m/day	Range $\alpha^{(t)}$, sq m/day
1969/Temperature	66	91	8.5	0.21	.052 - .64
1970/Temperature	20	91	8.0	0.21	.058 - .58

TABLE 4. DeGray Reservoir Semi-Infinite Slab Model Results.

Year/Species	No. observations	Time zero (Julian day)	C_{Ai}	Range of C_{Ao} search	Average $D^{(t)}$, sq m/day	Range $D^{(t)}$, sq m/day	C_{Ao}
1970/NO_3	26	80	1.5	8.0 - 20.	1.06	.216[1]	12.1
1970/Ca	26	80	7.0	8.0 - 20.	1.54	.216[1]	14.6

Year/Species	No. observations	Time zero (Julian day)	T_o, °C	Average $\alpha^{(t)}$, sq m/day	Range $\alpha^{(t)}$, sq m/day
1970/Temperature	107	80	6.0	0.743	0.18 - 1.8

[1] Single value denotes α.

$D^{(t)}$ = .085 sq m/day, t = 7 days, and C_{P_O} = .52 mgP/l resulted in 45.3 mgP/sq cm/week. If a molecular mass diffusivity of 9 x 10^{-5} sq m/day is used, the calculated release rate is 1.5 mgP/sq cm/week. The magnitude of the dispersive phenomena in a culture tube is likely to be somewhere between that of a lake environment and molecular diffusion.

CONCLUSIONS

1. The upsurge of nutrients and other soluble material originating in lake bottom sediment area can be reasonably described with a Fickian-type model of constant $D^{(t)}$ (turbulent mass diffusivity) once the lake seasonal time cycle has been partitioned into mixed and stratified periods.
2. Quantitative relationships resulting from analysis of the model include a rate equation for the mass flux of a nutrient entering the photoactive zone of a lake (equation 18).
3. The likelihood of a fall algal bloom is dependent upon bottom residing nutrients upsurging and entering the nutrient-poor, photoactive lake zone before winter commences.
4. The time for a significant quantity of upsurging nutrient (10 percent) to enter the photoactive zone is directly related to the square of d, the mean depth of the lake, and inversely related to $D^{(t)}$. Equations (19) and (20) give the upsurge times.

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