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Essays on Microeconomic Problems in Multilateral Settings

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Abstract

This thesis consists of three chapters on microeconomic problems in multilateral settings. In the first chapter, I use a model with two local bidders and a global bidder in a simultaneous ascending auction. I find that the simultaneous ascending auction does not allocate two heterogeneous objects efficiently. This result holds with and without resale and is independent of the resale mechanism. Then I use a fixed resale mechanism with take-it-or-leave-it offers after the auction to study the effects of resale on allocation efficiency. In cases where the two local bidders win the objects inefficiently, no resale occurs. Therefore, whenever the simultaneous ascending auction overdiffuses the objects to the local bidders, resale cannot correct this inefficiency. However, whenever the simultaneous ascending auction overconcentrates the objects to the global bidder, the global bidder can resell the objects to the local bidders.

The second chapter studies the effects of resale on the efficiency of simultaneous ascending auctions with more general resale mechanisms. In this paper, speculation by the local bidders is allowed in the simultaneous ascending auction. I look for efficient mechanisms in multilateral settings, if they exist given the beliefs, and use these mechanisms as the resale mechanisms after the auction. The simultaneous ascending auction can grossly misallocate the objects by allocating the objects to the speculators but there exist efficient resale mechanisms that can restore full efficiency.

In the final chapter, I use a model with a committee and two project sponsors. The committee members decide which one of the two projects to approve. Each project sponsor can choose to disclose information about his project to select committee members. If a committee member receives information from a sponsor, he can choose to investigate the project at a cost to learn his own payoff from the project. After that, the committee members decide which project to implement. As competition between the projects gets stronger, there is more information disclosure from the sponsors.

Keywords: Simultaneous ascending auction, resale, group persuasion

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Chapter 1

Introduction

Musical chairs is a game in which the number of players is always more than the number of chairs. The players are successively eliminated and one player eventually wins the game. In elections, there are typically multiple candidates running for a given position. For most job postings, there are more applicants than there are openings. Other examples where there are more people competing for fewer items or positions abound. My thesis consists of three essays in which I study two such environments using microeconomic models to study optimal solutions in multilateral settings. In both environments that I study, there is a priori uncertainty over what a desirable outcome would be. In addition, the rules of the game and the details of the environment influence the extent to which this uncertainty can be resolved and the desirable outcomes can be achieved. The first two essays deal with the effects of resale on allocation efficiency after simultaneous ascending auctions. The third essay studies the dynamics of group persuasion.

The thesis is organized into three chapters. The first chapter deals with the challenges of allocating multiple heterogeneous objects efficiently. Simultaneous ascending auctions have been used to allocate electromagnetic spectrum licences in the United States and other countries around the world. A global bidder whose value for a specific bundle of related objects is higher than the sum of the standalone values for the objects faces the exposure problem. On the other hand, local bidder, who values only one object, does not face the same problem. Zheng (2012) proposes jump bidding in simultaneous ascending auctions to help to mitigate the exposure problem. When jump bidding is allowed, simultaneous ascending auctions overly concentrate the objects to a global bidder and never overly diffuse the objects to local bidders. I use a fixed resale mechanism to study the effects of resale after a simultaneous ascending auction without jump bidding. I find an equilibrium in which resale improves the allocation efficiency relative to the benchmark equilibrium without resale. I also find that the chosen resale mechanism can only partially mitigate the exposure problem for the global bidder. When the global bidder loses the objects inefficiently, resale never takes place. However, whenever the global bidder wins the objects inefficiently, he can resell the objects to the local bidders.

The second chapter investigates the effects of resale in the presence of speculators who are active bidders in the simultaneous ascending auctions. Since bans on post-auction trade are difficult to enforce, there may be speculators who do not value the objects but wish to make a profit through post-auction trade with other bidders who actually value the objects. Williams (1999) extends the Myerson and Satterthwaite (1983) bargaining problem to multilateral settings. I use this extension to look for the existence of efficient resale mechanisms that can be used after the simultaneous ascending auction to restore efficiency. Even when the simultaneous ascending auction grossly misallocates the objects by letting the speculators win the auction, there exist efficient resale mechanisms that can rectify the situation and result in efficient outcomes.

The final chapter investigates the dynamics of group persuasion. In cases where a committee is making a decision, strategies for persuading the individuals in the committee can be different from the strategies for persuading a single decision maker. Selective communication, where the sponsor of an idea or project talks to select committee members only, and persuasion cascades, where the sponsor targets key committee members and obtains their approval before using their support for his project to convince the other committee members to approve his project, may be used for group persuasion. I study the problem where a committee decides which of two projects to approve. A sponsor of a project can choose to disclose information about his project to select committee members. Each committee member can investigate a project at a cost, if he receives information from the project's sponsor, and learn his own payoff from implementing the project. Then, the committee members vote which project to approve. I find that stronger competition between the sponsors leads to more information disclosure to the committee members.

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Chapter 2

Simultaneous ascending auction with resale

2.1 Introduction

This paper investigates the effects of resale on a simultaneous ascending auction. A simultaneous ascending auction is an auction format that allows a seller of multiple heterogeneous objects to sell these objects simultaneously, yet separately. Each object is auctioned via an English auction; these English auctions are held simultaneously. Although there is only one seller, there is no coordination across the separate English auctions. This auction format was first adopted by the US Federal Communications Commission (FCC) in 1994 to sell electromagnetic spectrum licences. Since then, the simultaneous ascending auction has become one of the standard methods to conduct spectrum auctions in the US and around the world. Furthermore, a version of the simultaneous ascending auction has also been used to sell divisible goods in electricity and gas markets.

In such an auction with multiple objects, there may be local bidders and global bidders. A local bidder values only one particular object, whereas a global bidder's payoff from winning his desired bundle of related objects is higher than the sum of the standalone values for the objects if those objects are complementary. This synergy may arise due to various reasons. In the case of spectrum licences, for example, a nationwide telecommunications company may be able to reduce its costs if it can win licences to operate in adjacent geographic regions. Although complementarities between objects can give a global bidder a boost in payoff, they also create a dilemma for him. In a simultaneous ascending auction, when the price of an object is above its standalone value and the prices of the other objects in a global bidder's desired bundle are still uncertain, should he continue bidding or drop out? If he is an aggressive competitor and continues to bid in the auction, the complementary objects may turn out to be so expensive that he ends up getting a negative payoff from acquiring his desired bundle. However, if he chooses to be cautious and drops out now, he then loses the opportunity to possibly acquire his desired bundle at a total price that gives him a positive payoff. The upshot of this is that the auction's efficiency may be adversely affected. Meng and Gunay (2017) use simulation methods and show that, in some cases, the probability of inefficient allocations in the simultaneous ascending auction can be up to 9 percent. The problem that each global bidder faces is known as the exposure problem. Goeree and Lien (2014) document some disadvantages which result from the exposure problem that were previously unknown for the simultaneous ascending auction: this auction format can lead to outcomes in which local bidders win the objects at very low prices, and more bidders competing in the auction may actually decrease the seller's revenue.

Resale has been proposed as a solution to the problem of inefficiency of auctions in general because the possibility of resale may mitigate inefficiency. Most of the work in the economics literature on auctions with resale focus on single-object auctions. Hafalir and Krishna (2008) study how resale affects the revenue and efficiency outcomes of first-price auctions. In their model, resale happens through monopoly pricing where the winner of the first-price auction makes a take-it-or-leave-it offer to the loser. Interestingly and perhaps surprisingly, Hafalir and Krishna show that resale may actually decrease efficiency. Garratt and Tröger (2006) investigate how a speculator affects standard auctions with resale. A speculator is a bidder who is commonly known to have zero value for the object being auctioned. In first-price and Dutch auctions with resale, they conclude that speculators do not profit and that the opportunity to resell the good after the auction can be detrimental to efficiency. However, in second-price and English auctions, there are multiple equilibria: the efficient equilibrium in which bidders bid their values exists, along with inefficient equilibria in which the speculator wins the auction and makes a positive profit from resale. With collusion in English auctions, Garratt, Tröger, and Zheng (2009) find that even a high-value bidder prefers collusion to value-bidding in the auction when the spoils from collusion are divided among members of the bidding ring through an arbitrary resale mechanism that the reseller is free to choose. However, the equilibria constructed are less efficient than the value-bidding equilibrium, so the possibility of resale creates inefficiency.

The literature on multiple-object auctions, however, has largely ignored resale even though resale of spectrum licences has indeed occurred[1] and bans on post-auction trade are difficult, if not impossible, to enforce.[2]

Xu, Levin, and Ye (2012) study auctions with synergy and resale. In their model, there are two objects and two global bidders. The two objects are sold sequentially via second-price sealed bid auctions; each bidder only learns his value for the second object being auctioned after the first auction. After both auctions, resale occurs either as a monopoly or as a monopsony take-it-or-leave-it offer. They find that the resale mechanism has an impact on bidding strategies in the auctions: whereas no equilibrium in which the bidders reveal their types with positive probability exists under a *monopoly* take-it-or-leave-it offer in the resale stage, the bidders are willing to use increasing bidding strategies in equilibrium under a *monopsony* take-it-or-leave-it offer in the resale stage. However, the effect of resale on the probability of exposure is ambiguous in the latter case. In addition, we note that there is sufficient information revealed through their model of the sequential auctions that it is clear in some cases immediately after the auctions whether resale can generate a higher surplus.

[1] See, for example, Cramton (2004).
[2] See Hafalir and Krishna (2009).

Filiz-Ozbay, Lopez-Vargas, and Ozbay (2015) use experiments to study multiple-object auctions with resale with multiple local bidders and a global bidder with private information. They consider two cases: a generalized Vickrey auction that allows package bidding followed by resale and simultaneous second-price auctions followed by resale. In both cases, resale takes place in the form of take-it-or-leave it offers made by the winners. In the case of a generalized Vickrey auction followed by resale, an equilibrium that allocates the objects efficiently in the auction stage exists. However, when the objects are auctioned via simultaneous second-price sealed bid auctions, the final allocation is inefficient even after post-auction trade.

Various resale mechanisms have been studied in the literature. As noted above, Garratt, Tröger, and Zheng (2009) allow the winner of the auction to choose an arbitrary resale mechanism. This is rare in the literature: the other main paper in the literature that allows the reseller of the good to choose his resale mechanism is Zheng (2002). In that paper, there are multiple stages where resale can occur; the current owner of the good at each stage is allowed to choose his resale mechanism. In most papers, however, a resale mechanism is specified and bidders have no say in that choice of mechanism. In this paper, we take the approach of using a fixed resale mechanism as well.

In this paper, we study the problem theoretically with local bidders and a global bidder who participate in a simultaneous ascending auction. The global bidder suffers from the exposure problem in our model and we investigate if resale mitigates the exposure problem for this global bidder. We fix the mechanism by which post-auction trade can take place and analyze whether the opportunity of resale improves the allocation efficiency. Resale through the fixed resale mechanism does not completely mitigate the exposure problem for the global bidder because resale never takes places whenever the global bidder loses both objects inefficiently (see Propositions 2.4.1 and 2.4.2). However, the global bidder can resell the objects to the other bidders whenever he wins both objects inefficiently. We prove that there exists an equilibrium in which resale improves the allocation efficiency relative to the benchmark equilibrium without resale (see Proposition 2.7.1).

2.2 Benchmark: Simultaneous ascending auction

2.2.1 Model

The simultaneous ascending auction is modelled as in Zheng (2012). There are two objects that are denoted by A and B. There are two categories of bidders: local bidders who value only one object and global bidders for whom the two objects are complements. There are three bidders: 1, 2 and 3. Bidder 1 values only object A and bidder 2 values only object B. Their values for the objects, denoted by v_{1A} and v_{2B} respectively, are identically and independently distributed on $[0, \overline{v}]$ according to a continuous distribution F_L. For simplicity, bidder 1 is not allowed to bid for object B and bidder 2 is not allowed to bid for object A. Bidder 3 is a global bidder. His standalone value for each object is 0. His value for the package of both objects, $\gamma \in [0, 2\overline{v}]$, is drawn from a continuous distribution F_G. The values v_{1A}, v_{2B} and γ are private information; everything else is commonly known.

Each bidder's payoff is equal to his value of the object(s) he eventually owns minus his total payment.

The two objects are auctioned via separate English auctions which start simultaneously. Each English auction is modelled by the "clock model" as in Albano, Germano, and Lovo (2001). In the separate English auctions, the prices p_A and p_B start at 0 and increase at the same exogenous speed. When a bidder drops out from an auction for a given object, the clock stops temporarily in both auctions, and all bidders have a chance to exit at the same price. Ties are broken in favour of the global bidder. When there is only one bidder left in the auction for object j, where $j \in \{A, B\}$, the object is then sold to the remaining bidder at the current price. The bidders' actions during the separate English auctions are commonly observed. Consequently, at the end of the simultaneous ascending auction, each bidder knows the identities of the winners and the prices paid by these winners to the auctioneer.

We use perfect Bayesian equilibrium as the solution concept in this paper. We consider pure strategies only. In the auction stage, each bidder decides the price(s) at which he should drop out of the English auction(s).

2.2.2 Equilibrium

First, we show that the simultaneous ascending auction cannot allocate the objects efficiently.

Proposition 2.2.1 ***(Corollary of Proposition 2 of Zheng (2012))*** *There is no equilibrium in which the simultaneous ascending auction allocates the objects efficiently in the auction stage of the game.*

Proof Suppose that there exists an equilibrium in which the objects are allocated efficiently in the auction stage. In this equilibrium, let $b_{ij}(v)$ be the price at which bidder i with value v drops out of the auction for object j when every other bidder is still participating in the simultaneous ascending auction.

Consider a realization of values (v_{1A}, v_{2B}, γ) where $v_{1A} + v_{2B} < \gamma$. Then efficiency requires that bidder 3 wins both objects in the simultaneous ascending auction. This implies that at least one of the following two inequalities has to hold:

$$b_{1A}(v_{1A}) < b_{3A}(\gamma) \tag{2.1}$$

$$b_{2B}(v_{2B}) < b_{3B}(\gamma) \tag{2.2}$$

Assume that Inequality (2.1) is true. This assumption is without loss of generality because Inequality (2.2) must be true otherwise and that case is symmetric to the case we are considering.

Next, consider a realization of values $(v_{1A}, v'_{2B}, \gamma)$, where v_{1A} and γ are as above and v'_{2B} is such that $v_{1A} + v'_{2B} > \gamma$. In this case, efficiency requires that the local bidders win the objects in the simultaneous ascending auction. Therefore, both the following inequalities should hold:

$$b_{1A}(v_{1A}) > b_{3A}(\gamma) \tag{2.3}$$

$$b_{2B}(v'_{2B}) > b_{3B}(\gamma)$$

Note that Inequality (2.3) contradicts Inequality (2.1) from the previous case. Therefore, an equilibrium that allocates the objects efficiently in the simultaneous ascending auction cannot exist. $\blacksquare$

The result of the above proposition is quite general. In fact, it holds with and without resale in the game. Moreover, the proof does not depend on the details of the post-auction trade mechanism.

Since the simultaneous ascending auction cannot allocate the objects efficiently, for the remainder of the paper, we consider resale and investigate whether resale can improve the allocation efficiency.

Proposition 1 of Zheng (2012) describes a perfect Bayesian equilibrium. This is used as the benchmark for the simultaneous ascending auction without resale in this paper.

2.3 Model: Simultaneous ascending auction with resale

We study the allocation efficiency with the following fixed resale mechanism after the auction stage.

After the simultaneous ascending auction, the bidders are allowed to trade sequentially in a fixed order. In the resale stage, the winners of the objects in the auction make take-it-or-leave-it offers to the potential buyers. While all the bidders observe if an object has been sold or not in the resale stage, a bidder not involved in the post-auction trade does not observe the sale price. The order of trade in the resale stage depends on the allocation of the objects after the auction.

- If the local bidders have won in the auction, bidder 1 can sell object A to bidder 3 before bidder 2 can sell object B to bidder 3.

- If the global bidder has won one of the objects, the local bidder who has won the other object can sell that object to bidder 3 before bidder 3 can sell to the other local bidder.

- If the global bidder has won both objects, he can sell the objects to the local bidders. First, the global bidder makes take-it-or-leave-it offers to the local bidders simultaneously. Then the local bidders simultaneously accept or reject the offers. Finally, the global bidder decides whether to trade or not.

In the resale stage, a local bidder who has won an object in the auction chooses the price at which he wants to sell the object to the global bidder; a local bidder who has not won an object in the auction decides whether to accept the global bidder's take-it-or-leave-it offer if the global bidder makes an offer. If the global bidder has not won anything in the auction, he decides whether to accept the take-it-or-leave-it offers made by the local bidders. If he has won an object in the auction, first he has to decide whether to accept the take-it-or-leave-it offer made by the local bidder who has won the other object in the auction, then he chooses the price at which he wants to offer the object(s) that he currently owns to the other local bidder. If the global bidder has won both objects in the auction, he chooses the take-it-or-leave-it offers to the local bidders and decides whether to trade with the local bidders after they have chosen whether to accept or reject his offers.

2.4 Improbability of resale

Since bidder 1 is not allowed to bid for object B and bidder 2 is not allowed to bid for object A, there are four possible allocations after the simultaneous ascending auction.

1. Bidder 1 has object A; bidder 2 has object B.

2. Bidder 1 has object A; bidder 3 has object B.

3. Bidder 3 has object A; bidder 2 has object B.

4. Bidder 3 has both objects A and B.

Consider the first case where the local bidders have won the objects in the auction. This is a case of sequential common agency, where the global bidder is the common agent. Since each bidder learns some information about the other bidders' private values during the auction, the beliefs about the other bidders' values may be updated after the auction. As the following propositions show, there is zero probability of resale when the objects are owned by the local bidders.

First, I show the result for a special case where bidder 3's value is common knowledge. The result for this special case is used later in Lemma 2.4.3, and its proof is much more transparent than for the case where bidder 3's value is uncertain.

Proposition 2.4.1 *Let $v_{1A} > 0$ and $v_{2B} > 0$ with probability 1. Suppose bidder 1 has object A and bidder 2 has object B. Suppose also that bidder 3's value γ is commonly known. Then there is zero probability of resale in equilibrium.*

Proof If bidder 3 has bought object A from bidder 1, it is optimal for bidder 2 to sell object B to bidder 3 at the price of γ if $\gamma > v_{2B}$ and to keep object B otherwise since γ is revealed when bidder 3 drops out of the auction. If bidder 1 charges $\widehat{p}_A$ and bidder 2 charges $\widehat{p}_B$, bidder 3's payoff from buying object A from bidder 1 and object B from bidder 2 is

$$\gamma - \widehat{p}_A - \widehat{p}_B = \gamma - \widehat{p}_A - \gamma = -\widehat{p}_A.$$

This is non-negative only if $\widehat{p}_A = 0$. However, with probability 1, bidder 1's value for object A is more than 0 so it is not optimal for him to charge 0 for object A. Therefore, resale happens with zero probability in equilibrium. ∎

It is not essential in the above proposition that bidder 3's value is commonly known. In fact, the same result holds when bidder 3's value is private information, as the following proposition shows. Although it may appear that there is redundancy in having two propositions with the same result, the above proposition, where it is assumed that bidder 3's value is commonly known, is used in the proof of Lemma 2.4.3.

Proposition 2.4.2 *Let $v_{1A} > 0$ and $v_{2B} > 0$ with probability 1. Suppose bidder 1 has object A and bidder 2 has object B. Suppose also that bidder 3's value $\gamma \in [\underline{\gamma}, \overline{\gamma}]$ is private information. Then there is zero probability of resale in equilibrium.*

Proof Suppose to the contrary that there exists an equilibrium in which resale occurs with strictly positive probability. We can then characterize this equilibrium in the following way.

Suppose it is optimal for bidder 1 with value v_{1A} to sell object A in such a way that the probability that bidder 3 buys object A from him is

$$p_{3A}(v_{1A}, \gamma) = \begin{cases} 1 & \text{if } \gamma \geq \widehat{\gamma}_1(v_{1A}) \\ 0 & \text{if } \gamma < \widehat{\gamma}_1(v_{1A}) \end{cases}$$

where $\widehat{\gamma}_1(v_{1A}) \in \mathbb{R}_+$. If object A is not sold to bidder 3, it is not profitable for bidder 3 to buy object B from bidder 2 unless bidder 2 offers to sell the object at the price of 0. However, since $v_{2B} > 0$ with probability 1, it is with probability 1 that bidder 2 is unwilling to sell object B at the price of 0. Therefore, there is zero probability of resale if bidder 3 does not buy object A from bidder 1. On the other hand, conditional on object A being sold to bidder 3 earlier in the resale stage, suppose it is optimal for bidder 2 with value v_{2B} to sell object B so that the probability that bidder 3 buys object B from him is

$$p_{3B}(v_{2B}, \gamma) = \begin{cases} 1 & \text{if } \gamma \geq \widehat{\gamma}_2(v_{2B}) \\ 0 & \text{if } \gamma < \widehat{\gamma}_2(v_{2B}) \end{cases}$$

where $\widehat{\gamma}_2(v_{2B}) \in \mathbb{R}_+$. The take-it-or-leave-it offers from bidders 1 and 2 to bidder 3 result in the outcomes described above. Since we are considering pure strategies only, restricting attention to take-it-or-leave-it offers is without loss of generality. In particular, note that if it is optimal for bidder 3 with value γ to accept the offer at a certain price, it must be optimal for bidder 3 with value γ', where $\gamma' > \gamma$, to accept the offer as well since he has a higher value and he pays the same price for the object.

First, we establish some properties of $\widehat{\gamma}_2(v_{2B})$. For all v_{2B}, $\widehat{\gamma}_2(v_{2B}) \geq \underline{\gamma}$. Otherwise, there exists v_{2B} such that bidder 2 posts a price $\widehat{\gamma}_2(v_{2B}) < \underline{\gamma}$. In this case, $p_{3B}(v_{2B}, \gamma) = 1$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$. However, if bidder 2 sets the price at $\underline{\gamma}$ instead, it remains true that $p_{3B}(v_{2B}, \gamma) = 1$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$. This is a profitable deviation. If $\widehat{\gamma}_2(v_{2B}) > \overline{\gamma}$ for all v_{2B}, $p_{3B}(v_{2B}, \gamma) = 0$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$. Since bidder 3 anticipates that there is no resale of object B, he is unwilling to buy object A from bidder 1 unless the price is 0 which occurs with probability 0 since $v_{1A} > 0$ with probability 1. The upshot is that there is no resale and we are done. Therefore, for the rest of the proof, we focus on the case where $\underline{\gamma} \leq \widehat{\gamma}_2(v_{2B}) \leq \overline{\gamma}$.

The profit of bidder 2 with v_{2B} who offers to sell object B at the price $\widehat{\gamma}_2(v_{2B})$ is

$$\pi_2(\widehat{\gamma}_2(v_{2B}), v_{2B}) = [1 - F_3(\widehat{\gamma}_2(v_{2B}))][\widehat{\gamma}_2(v_{2B}) - v_{2B}] + v_{2B}$$

where F_3 describes the distribution of γ conditional on bidder 3 having bought object A. For bidder 2's profit to be positive, $\widehat{\gamma}_2(v_{2B}) \geq v_{2B}$ in case of resale. Moreover, $\widehat{\gamma}_2(v_{2B})$ is non-decreasing in v_{2B}. To see why this is the case, consider v_{2B} and v'_{2B}, where $v'_{2B} \neq v_{2B}$. Then, the inequalities

$$\pi_2(\widehat{\gamma}_2(v_{2B}), v_{2B}) \geq \pi_2(\widehat{\gamma}_2(v'_{2B}), v_{2B})$$

and

$$\pi_2(\widehat{\gamma}_2(v'_{2B}), v'_{2B}) \geq \pi_2(\widehat{\gamma}_2(v_{2B}), v'_{2B})$$

have to hold. After adding the two inequalities and some simplification, we obtain

$$[F_3(\widehat{\gamma}_2(v'_{2B})) - F_3(\widehat{\gamma}_2(v_{2B}))](v'_{2B} - v_{2B}) \geq 0.$$

If $v'_{2B} > v_{2B}$, then $F_3(\widehat{\gamma}_2(v'_{2B})) \geq F_3(\widehat{\gamma}_2(v_{2B}))$. If $F_3(\widehat{\gamma}_2(v'_{2B})) > F_3(\widehat{\gamma}_2(v_{2B}))$, then $\widehat{\gamma}_2(v'_{2B}) > \widehat{\gamma}_2(v_{2B})$ since F_3 is non-decreasing. On the other hand, if $F_3(\widehat{\gamma}_2(v'_{2B})) = F_3(\widehat{\gamma}_2(v_{2B}))$, then $\widehat{\gamma}_2(v'_{2B}) = \widehat{\gamma}_2(v_{2B})$. To see why this is the case, suppose $\widehat{\gamma}_2(v'_{2B}) < \widehat{\gamma}_2(v_{2B})$. Then bidder 2 with value v'_{2B} is selling to the same types of bidder 3 as bidder 2 with value v_{2B} but at a lower price.

Charging $\widehat{\gamma}_2(v_{2B})$ is a profitable deviation. Therefore, $\widehat{\gamma}_2(v'_{2B}) \geq \widehat{\gamma}_2(v_{2B})$.

Furthermore, it is never optimal for bidder 2 to sell the object at the price of 0. Since $\widehat{\gamma}_2(v_{2B}) \geq \widehat{\gamma}_2(0)$ for all v_{2B}, it suffices to show that $\widehat{\gamma}_2(0) > 0$. We know that $\widehat{\gamma}_2(0) \geq \underline{\gamma} \geq 0$ because we are considering the case where $\widehat{\gamma}_2(v_{2B}) \geq \underline{\gamma}$ for all v_{2B}. In particular, if $\underline{\gamma} > 0$, we are done. Next, consider the case where $\underline{\gamma} = 0$. If bidder 2 with $v_{2B} = 0$ sells object B at the price of 0, his profit is 0. If $F_3(\epsilon) = 1$ for all $\epsilon > 0$, then it means that the resale of object A happens with zero probability. Otherwise, there exists $\epsilon > 0$ such that $F_3(\epsilon) < 1$. Therefore, if bidder 2 raises the resale price of object B to ϵ, his profit is

$$\pi_2(\epsilon, 0) = [1 - F_3(\epsilon)]\epsilon > 0,$$

so this is a profitable deviation. Therefore, $\widehat{\gamma}_2(v_{2B}) > 0$ for all v_{2B}.

Denote bidder 3's continuation payoff conditional on γ from having object A as $U_3(\gamma)$. More specifically,

$$U_3(\gamma) = \mathbb{E}_{v_{2B}}[p_{3B}(v_{2B}, \gamma)]\gamma - \mathbb{E}_{v_{2B}}[T(v_{2B}, \gamma)]$$

where $T(v_{2B}, \gamma)$ is the transfer from bidder 3 to bidder 2 conditional on (v_{2B}, γ). By a standard argument, we can show that $\mathbb{E}_{v_{2B}}[p_{3B}(v_{2B}, \gamma)]$ is weakly increasing in γ and

$$U_3(\gamma) = U_3(\underline{\gamma}) + \int_{\underline{\gamma}}^{\gamma} \mathbb{E}_{v_{2B}}[p_{3B}(v_{2B}, t)]\, \mathrm{d}t$$

so $U_3(\gamma)$ is weakly increasing in γ.

Next, we establish some properties of $\widehat{\gamma}_1(v_{1A})$. For all v_{1A}, $\widehat{\gamma}_1(v_{1A}) \geq \underline{\gamma}$. Otherwise, there exists v_{1A} such that bidder 1 with v_{1A} offers object A at the price $s_1(v_{1A}) < U_3(\underline{\gamma})$. In this case, $p_{3A}(v_{1A}, \gamma) = 1$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$. However, if $s_1(v_{1A}) = U_3(\underline{\gamma})$ instead, it remains true that $p_{3A}(v_{1A}, \gamma) = 1$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$. This is a profitable deviation. If $\widehat{\gamma}_1(v_{1A}) \geq \overline{\gamma}$ for all v_{1A}, $p_{3A}(v_{1A}, \gamma) = 0$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma})$. Then there is no resale with probability 1 and we are done. Therefore, for the rest of the proof, we focus on the case where $\underline{\gamma} \leq \widehat{\gamma}_1(v_{1A}) < \overline{\gamma}$.

In addition, it is never optimal for bidder 1 to sell the object at the price of 0. Using a revealed preference argument as above, we can show that $s_1(v_{1A})$ is non-decreasing. Therefore, it suffices to show that $s_1(0) > 0$. If bidder 1 with $v_{1A} = 0$ sells object A at the price of 0, his profit is 0. Since resale is assumed to occur with strictly positive probability in this equilibrium, there exists a set $\mathcal{S}$ of positive probability measure such that $\mathbb{E}_{v_{2B}}[p_{3B}(v_{2B},\gamma)] > 0$ for every $\gamma \in \mathcal{S}$. Therefore, there exists $\alpha \in (\underline{\gamma}, \overline{\gamma})$ such that $\Pr\{\mathcal{S} \cap (\underline{\gamma}, \alpha)\} \neq 0$ and

$$U_3(\alpha) = U_3(\underline{\gamma}) + \int_{\underline{\gamma}}^{\alpha} \mathbb{E}_{v_{2B}}[p_{3B}(v_{2B}, t)]\,\mathrm{d}t > U_3(\underline{\gamma}) \geq 0.$$

However, if bidder 1 raises the price to $U_3(\alpha)$, his profit is

$$\pi_1(U_3(\alpha), 0) = \Pr\{\gamma \geq \alpha\} U_3(\alpha) > 0,$$

so this is a profitable deviation. Hence, $s_1(v_{1A}) > 0$ for all v_{1A}.

Let $\underline{\gamma}_1 = \inf_{v_{1A}} \widehat{\gamma}_1(v_{1A})$ and let $\underline{\gamma}_2 = \inf_{v_{2B}} \widehat{\gamma}_2(v_{2B})$. First, we show that $\underline{\gamma}_1 = \underline{\gamma}_2$. If not, $\underline{\gamma}_1 < \underline{\gamma}_2$ or $\underline{\gamma}_1 > \underline{\gamma}_2$. First consider the case where $\underline{\gamma}_1 < \underline{\gamma}_2$. Then, if $\gamma \in (\underline{\gamma}_1, \underline{\gamma}_2)$, there is a strictly positive probability that bidder 3 buys object A and expects to be unable to buy object B. Since his standalone value for object A is 0, he is only willing to buy object A if bidder 1 charges 0 for it. However, as shown above, this is never optimal for bidder 1. Hence, for all v_{1A}, $p_{3A}(v_{1A},\gamma) = 0$ if $\gamma \in (\underline{\gamma}_1, \underline{\gamma}_2)$, contradicting the fact that there exists v_{1A} such that $\widehat{\gamma}_1(v_{1A}) \in (\underline{\gamma}_1, \underline{\gamma}_2)$ so $p_{3A}(v_{1A},\gamma) = 1$ for $\gamma \in [\widehat{\gamma}_1(v_{1A}), \underline{\gamma}_2)$. Now consider the case where $\underline{\gamma}_1 > \underline{\gamma}_2$. Then, if $\gamma \in (\underline{\gamma}_2, \underline{\gamma}_1)$, bidder 3 buys object A with zero probability on the equilibrium path. Therefore, it is not optimal for bidder 2 with v_{2B} such that $\widehat{\gamma}_2(v_{2B}) \in (\underline{\gamma}_2, \underline{\gamma}_1)$ to set the price at $\widehat{\gamma}_2(v_{2B})$ since he can raise the price to $\underline{\gamma}_1$ and sell to the same types and thus get a higher profit. Note that $\underline{\gamma}_1 < \overline{\gamma}$ since we are focusing on the case where $\underline{\gamma} \leq \widehat{\gamma}_1(v_{1A}) < \overline{\gamma}$; therefore, this event is on the equilibrium path.

Let $\underline{\gamma}_1 = \underline{\gamma}_2 = \widetilde{\gamma}$. Since $\underline{\gamma}_2 = \inf_{v_{2B}} \widehat{\gamma}_2(v_{2B}) \geq \widehat{\gamma}_2(0) > 0, \widetilde{\gamma} > 0$. If $\widetilde{\gamma} = \overline{\gamma}$, then the probability of resale is 0. Therefore, consider the case where $\widetilde{\gamma} < \overline{\gamma}$. Bidder 3 with $\gamma = \widetilde{\gamma} + \beta$, where

$0 < \beta < \min\{\overline{\gamma} - \widetilde{\gamma}, \frac{1}{2}s_1(0)\}$, has a strictly positive probability of buying both objects. His payoff is

$$\gamma - s_1(v_{1A}) - \widehat{\gamma}_2(v_{2B}) \leq \widetilde{\gamma} + \beta - s_1(0) - \widetilde{\gamma} < -\frac{1}{2}s_1(0) < 0.$$

On the other hand, he gets 0 if he does not buy the objects, so he would rather not buy the objects. This contradicts the definition of $p_{3A}(v_{1A}, \gamma)$ and $p_{3B}(v_{2B}, \gamma)$. Therefore, there can be no resale with positive probability in equilibrium. ∎

The two propositions above show that resale does not take place when the objects are "overly diffused"[3] to the local bidders after the auction. This is because at least one of the sellers demands the entire share of the global bidder's value for the package of objects. This problem is known as the "holdout problem" in the literature.[4] Therefore, if the global bidder drops out of the simultaneous ascending auction before he is sure that it is not efficient for him to own the objects, there is no guarantee that the equilibrium allocation of the entire game is indeed efficient.

On the other hand, if bidder 3 wins at least one object in the simultaneous ascending auction, the next proposition determines the outcomes of resale depending on the post-auction ownership of the objects and shows that the allocation after resale is efficient as long as the losers reveal their values in the auction. One way of achieving this revelation of values is for all the bidders to use separating strategies in the auction. More specifically, bidders use bidding functions in the auction stage such that $b_{ij}(v) \neq b_{ij}(v')$ if $v \neq v'$ for all v, v' in the support of bidder i's value, where $b_{ij}(v)$ is the price at which bidder i with value v drops out of the auction for object j. This result will be used in the next section to show that no such separating equilibrium exists.

Lemma 2.4.3 *Suppose all bidders use separating strategies in the auction stage. Let p_A and p_B be the respective prices at which objects A and B are sold in the simultaneous ascending*

[3]Zheng (2012) coins the term "overdiffusion" to describe the following kind of inefficiency: two objects go to two separate bidders when, in our notation, $\gamma > v_{1A} + v_{2B}$.

[4]See, for example, Kominers and Weyl (2012).

auction. Then, the following hold.

1. *If bidder* 1 *has object A and bidder* 2 *has object B after the auction, the bidders' payoffs after the resale stage are* $(v_{1A} - p_A, v_{2B} - p_B, 0)$.

2. *If bidder* 1 *has object A and bidder* 3 *has object B after the auction, the bidders' payoffs after the resale stage are* $(\max\{\gamma - v_{2B}, v_{1A}\} - p_A, 0, v_{2B} - p_B)$.

3. *If bidder* 3 *has object A and bidder* 2 *has object B after the auction, the bidders' payoffs after the resale stage are* $(0, \max\{\gamma - v_{1A}, v_{2B}\} - p_B, v_{1A} - p_A)$.

4. *If bidder* 3 *has objects A and B after the auction, the bidders' payoffs after the resale stage are* $(0, 0, \max\{\gamma, v_{1A} + v_{2B}\} - p_A - p_B)$.

Moreover, in cases 2, 3 and 4, the allocation is efficient.

Proof Since all bidders use separating strategies in the auction stage, a bidder's value is revealed when he drops out of the auction.

In case *1*, bidder 3's value for the package of objects is known whereas the values of bidders 1 and 2 remain unknown. By Proposition 2.4.1, there is zero probability of resale in equilibrium. Therefore, the bidders' payoffs are $(v_{1A} - p_A, v_{2B} - p_B, 0)$ since bidders 1 and 2 paid p_A and p_B in the auction.

In case *2*, bidder 2's value, v_{2B}, and bidder 3's value, γ, are known. According to the fixed mechanism for post-auction trade, bidder 1 can sell object A to bidder 3 before bidder 3 can sell to bidder 2. We solve the game from the back. If bidder 3 has object B only, he can sell the object to bidder 2 at the price of v_{2B} since this is the highest price that bidder 2 is willing to pay for the object. Then, the bidders' payoffs are

$$(v_{1A} - p_A, 0, v_{2B} - p_B). \tag{2.4}$$

On the other hand, if bidder 3 has bought object A from bidder 1 at the price of $\widehat{p}_A$, his payoff is $\gamma - p_B - \widehat{p}_A$ if he keeps both objects and $v_{2B} - p_B - \widehat{p}_A$ if he sells object B to bidder 2. Therefore, bidder 3 sells object B if $\gamma < v_{2B}$. In either case, bidder 2's payoff is 0. Now consider the first part of the resale stage where bidder 1 can sell object A to bidder 3. First consider the case where $\gamma < v_{2B}$. If bidder 3 buys object A at the price of $\widehat{p_A}$, his payoff is $v_{2B} - p_B - \widehat{p_A}$; if he doesn't buy object A, his payoff is $v_{2B} - p_B$ as shown in (2.4). Therefore, it is optimal for bidder 3 to buy object A if and only if $\widehat{p_A} \leq 0$. There is no trade in this case if $v_{1A} > 0$. On the other hand, if $v_{1A} = 0$, bidder 1 may be willing to trade at $\widehat{p_A} = 0$, but such a trade does not change the payoffs. Now consider the case where $\gamma \geq v_{2B}$. If bidder 3 buys object A from bidder 1 at the price of $\widehat{p_A}$, his payoff is $\gamma - p_B - \widehat{p_A}$; if he doesn't buy object A, his payoff is $v_{2B} - p_B$ from (2.4). Therefore, bidder 3 buys object A from bidder 1 if $\widehat{p_A} \leq \gamma - v_{2B}$. If there is indeed a sale, $\widehat{p_A} = \gamma - v_{2B}$. If bidder 1 sells object A to bidder 3 at the price of $\widehat{p_A} = \gamma - v_{2B}$, his payoff is $\gamma - v_{2B} - p_A$; if he keeps object A, his payoff is $v_{1A} - p_A$. Therefore, it is optimal for bidder 1 to sell object A to bidder 3 if $\gamma > v_{1A} + v_{2B}$. Consequently, if $\gamma > v_{1A} + v_{2B}$, bidder 1 sells object A to bidder 3 at the price of $\gamma - v_{2B}$ and bidder 3 keeps both objects, so the bidders' payoffs are

$$(\gamma - v_{2B} - p_A, 0, v_{2B} - p_B).$$

On the other hand, if $\gamma < v_{1A} + v_{2B}$, bidder 1 keeps object A and bidder 3 sells object B to bidder 2 at the price of v_{2B}, so the bidders' payoffs are

$$(v_{1A} - p_A, 0, v_{2B} - p_B).$$

In the case where $\gamma = v_{1A} + v_{2B}$, the payoffs are the same under either of the above two scenarios. Therefore, the bidders' payoffs are $(\max\{\gamma - v_{2B}, v_{1A}\} - p_A, 0, v_{2B} - p_B)$ and the allocation is efficient.

Case *3* is symmetric to case *2*.

Finally, in case *4*, bidders 1 and 2 have revealed their values, v_{1A} and v_{2B} respectively, when they dropped out of the auction. If bidder 3 keeps the objects, his payoff is $\gamma - p_A - p_B$. Since bidder 1 is willing to pay up to v_{1A} for object A and bidder 2 is willing to pay up to v_{2B} for object B, bidder 3's payoff is $v_{1A} + v_{2B} - p_A - p_B$ if he sells both objects. Therefore, it is optimal for bidder 3 to sell the objects to the other two bidders if $\gamma > v_{1A} + v_{2B}$. Since bidders 1 and 2 have to pay v_{1A} and v_{2B} respectively if bidder 3 sells the objects to them, they get 0 regardless of whether they participate in post-auction trade or not. Therefore, the bidders' payoffs are $(0, 0, \max\{\gamma, v_{1A} + v_{2B}\} - p_A - p_B)$ and the allocation is efficient. ∎

In the lemma above, a key assumption is that the losers in the auction reveal their values. With such information, whenever bidder 3 owns at least one object at the beginning of the resale stage (i.e. when there is no common agency problem), resale happens whenever it is efficient for the winners to sell the objects. Therefore, if the bidders are willing to use strategies that reveal their values if they lose in the auction in equilibrium, resale can improve the allocation efficiency and partially mitigate the exposure problem for the global bidder. Every loser in the auction gets a payoff of 0 after resale since the winner can take advantage of this revelation by selling the object to the loser at the highest price the loser is willing to pay for the object. As such, bidders are unwilling to reveal their values in the auction in equilibrium.

2.5 Efficiency with strictly increasing strategies

In this section and the next section, we assume that the bidders' values are continuously distributed.

Proposition 2.5.1 *Let v_{1A} and v_{2B} be distributed continuously on* $[0, 1]$ *according to F_L and γ be distributed continuously on* $[0, 2]$ *according to F_G. Suppose $b_{1A}(x) = b_{2B}(x)$ for all x and $b_{3A}(\gamma) = b_{3B}(\gamma)$ for all γ, where $b_{ij}(v)$ is the price at which bidder i with value v drops out of the auction for object j when every other bidder is still participating in the simultaneous ascending auction. Then there exists no equilibrium in which the bidders use strictly increasing strategies in the auction stage.*

Proof Suppose there is an equilibrium in which the bidders use strictly increasing strategies in the auction stage. To simplify notation, let $b_{1A}(x) = b_{2B}(x) = b_L(x)$ for all x and $b_{3A}(\gamma) =$

$b_{3B}(\gamma) = b_G(\gamma)$ for all γ. Define the following functions and bounds:

$$\begin{aligned} b_L^{-1}(y) &= \sup\{v : b_L(v) \leq y\}, \\ \underline{B}_L &= \max\{y : F_L(b_L^{-1}(y)) = 0\}, \\ b_G^{-1}(y) &= \sup\{\gamma : b_G(\gamma) \leq y\}; \\ \underline{B}_G &= \max\{y : F_G(b_G^{-1}(y)) = 0\}. \end{aligned}$$

where $y \in \mathbb{R}_+$. The inverse functions defined above are continuous, thus the lower bounds of the bids are well-defined. Then there are three cases to consider: $\underline{B}_L > \underline{B}_G$, $\underline{B}_L < \underline{B}_G$ and $\underline{B}_L = \underline{B}_G$.

1. $\underline{B}_L > \underline{B}_G$.

   Define $\widehat{\gamma}$ as

   $$\widehat{\gamma} = \sup\{\gamma : b_G(\gamma) < \underline{B}_L\}.$$

   Consider bidder 1 with v_{1A} such that $0 < v_{1A} < \frac{1}{2}\mathbb{E}[b_G(\gamma)|\gamma \in [0, \widehat{\gamma})]$. Then, $b_L(v_{1A}) > \underline{B}_L$; this implies that, in equilibrium, each of the three bidders may drop out of the auction first. Consider the following three events[5] which are conditional on v_{1A}:

   (a) Event E_1: Bidder 1 drops out before bidders 2 and 3.

   Bidder 3 wins object A and bidder 1's value is revealed because all the bidders use separating strategies. By Lemma 2.4.3, bidder 1's payoff is 0 regardless of whether bidder 2 or bidder 3 wins object B in the auction.

   (b) Event E_2: Bidder 2 drops out before bidders 1 and 3.

   Bidder 2 reveals his value and bidder 3 wins object B. Since the local bidders are

[5] Since all the bidders are using separating strategies in the auction, cases where at least two bidders drop out simultaneously occur with zero probability.

using symmetric separating strategies, it must be the case that $v_{2B} < v_{1A}$. Therefore,

$$\Pr\{E_2\} \leq \Pr\{v_{2B} < v_{1A} | v_{1A}\} = F_L(v_{1A}).$$

By Lemma 2.4.3, bidder 1 gets $\max\{\gamma - v_{2B}, v_{1A}\} - p_A$, where p_A is the price at which object A is sold in the auction, if he wins object A in the auction and 0 if he loses object A in the auction. Therefore, the maximum he can get is 2.

(c) Event E_3: Bidder 3 drops out before bidders 1 and 2.

Since bidder 3 drops out first with probability 1 if $\gamma < \widehat{\gamma}$,

$$\Pr\{E_3\} \geq \Pr\{\gamma < \widehat{\gamma}\}.$$

By Proposition 2.4.1, there is zero probability of resale. So bidder 1's expected payoff is at most $v_{1A} - \mathbb{E}[b_G(\gamma) | \gamma \in [0, \widehat{\gamma})]$.

Let bidder 1's payoff conditional on v_{1A} be denoted by $U_1(v_{1A})$. Then,

$$\begin{aligned}
U_1(v_{1A}) &\leq \Pr\{E_1\}(0) + \Pr\{E_2\}(2) + \Pr\{E_3\}(v_{1A} - p_A) \\
&\leq 2F_L(v_{1A}) + \Pr\{\gamma < \widehat{\gamma}\}\left(\frac{1}{2}\mathbb{E}[b_G(\gamma)|\gamma \in [0, \widehat{\gamma})] - \mathbb{E}[b_G(\gamma)|\gamma \in [0, \widehat{\gamma})]\right) \\
&\leq 2F_L(v_{1A}) - \frac{1}{2}\Pr\{\gamma < \widehat{\gamma}\}\mathbb{E}[b_G(\gamma)|\gamma \in [0, \widehat{\gamma})].
\end{aligned}$$

First, note that $\frac{1}{2}\Pr\{\gamma < \widehat{\gamma}\}\mathbb{E}[b_G(\gamma)|\gamma \in [0, \widehat{\gamma})] > 0$. In addition, F_L is continuous, $2F_L(0) = 0$ and $2F_L(1) = 2$. By the intermediate value theorem, there exists $\beta \in (0, 1)$ such that $2F_L(v_{1A}) - \frac{1}{2}\Pr\{\gamma < \widehat{\gamma}\}\mathbb{E}[b_G(\gamma)|\gamma \in [0, \widehat{\gamma})] < 0$ for all $v_{1A} \in (0, \beta)$. Therefore, for sufficiently small v_{1A}, $U_1(v_{1A}) < 0$. Since dropping out of the auction when the price is 0 is a profitable deviation, it cannot be the case that $\underline{B}_L > \underline{B}_G$ in equilibrium.

2. $\underline{B}_L < \underline{B}_G$.

Consider bidder 1 with sufficiently small $v_{1A} > 0$ such that $\Pr\{\gamma < v_{2B}\} > F_L(v_{1A})$ and $\underline{B}_L < b_L(v_{1A}) < \underline{B}_G$. We know by the intermediate value theorem that such v_{1A} exist.[6] In this case, events E_1, E_2 and E_3 are defined similarly as above. First, bidder 1's payoff is 0 in event E_1. Secondly, $\Pr\{E_2\} \le F_L(v_{1A})$ as above. Moreover, by cases (*2*) and (*4*) of Lemma 2.4.3, the post-resale allocation is efficient. Therefore, bidder 1 has object A after the resale stage with probability $\Pr\{v_{1A} + v_{2B} > \gamma | v_{1A}, E_2\}$. Let bidder 1's expected transfer in this event be denoted by $T_1(v_{1A}, E_2)$. Thirdly, since $b_L(v_{1A}) < \underline{B}_G$, $\Pr\{E_3\} = 0$. Therefore, bidder 1's payoff is

$$\begin{aligned} U_1(v_{1A}) &= \Pr\{E_1\}(0) + \Pr\{E_2\}[\Pr\{v_{1A} + v_{2B} > \gamma | v_{1A}, E_2\} v_{1A} + T_1(v_{1A}, E_2)] \\ &\le \Pr\{E_2\}[v_{1A} + T_1(v_{1A}, E_2)]. \end{aligned}$$

If bidder 1 with $v_{1A} = 0$ deviates and drops out of the auction at the price at which bidder 1 with $v_{1A} > 0$ drops out and rejects all resale offers in event E_1, his payoff is

$$U_1^d(0) = \Pr\{E_2\} T_1(v_{1A}, E_2).$$

On the other hand, if $v_{1A} = 0$, bidder 1's payoff is

$$U_1(0) = T_1(0) \ge 0$$

where $T_1(0)$ is the expected transfer conditional on $v_{1A} = 0$. If bidder 1 with $v_{1A} > 0$ deviates to the strategy of bidder 1 with $v_{1A} = 0$, his payoff is

$$U_1^d(v_{1A}) = \Pr\{\gamma < v_{2B}\} v_{1A} + T_1(0).$$

[6]Since $v_{2B} \in [0, 1]$ and $\gamma \in [0, 2]$, $0 < \Pr\{\gamma < v_{2B}\} < 1$. In addition, F_L is continuous, $F_L(0) = 0$ and $F_L(1) = 1$. Therefore, by the intermediate value theorem, there exists $\epsilon \in (0, 1)$ such that $\Pr\{\gamma < v_{2B}\} > F_L(v_{1A})$ for all $v_{1A} \in (0, \epsilon)$.

Then bidder 1 has a profitable deviation because

$$\begin{aligned} U_1(v_{1A}) &\leq \Pr\{E_2\}v_{1A} + T_1(0) \\ &\leq F_L(v_{1A})v_{1A} + T_1(0) \\ &< \Pr\{\gamma < v_{2B}\}v_{1A} + T_1(0) \\ &= U_1^d(v_{1A}). \end{aligned}$$

where the first inequality holds because $U_1(0) \geq U_1^d(0)$ by incentive compatibility.

3. $\underline{B}_L = \underline{B}_G$.

We show that bidder 1 also has a profitable deviation in this case, thus this cannot be an equilibrium. The argument we use here is similar to the argument above for the case where $\underline{B}_L < \underline{B}_G$. The difference is that, in this case, it is possible for bidder 3 to drop out before bidders 1 and 2. If bidder 3 is the first to drop out, $b_G(\gamma) < b_L(v_{1A})$ and $b_G(\gamma) < b_L(v_{2B})$ have to hold simultaneously.

$$\begin{aligned} \Pr\{E_3\} &= \Pr\{b_G(\gamma) < b_L(v_{1A}), b_G(\gamma) < b_L(v_{2B}) | v_{1A}\} \\ &< \Pr\{b_G(\gamma) < b_L(v_{1A}) | v_{1A}\} \\ &= \Pr\{\gamma < b_G^{-1}(b_L(v_{1A}))\} \\ &= F_G(b_G^{-1}(b_L(v_{1A}))). \end{aligned}$$

Moreover, by Proposition 2.4.1, there is zero probability of resale after the auction. Therefore, bidder 1 consumes object A and we denote his expected transfer in this case as $T_1(v_{1A}, E_3)$.

For bidder 1 with $v_{1A} > 0$,

$$\begin{aligned} U_1(v_{1A}) &= \Pr\{E_1\}(0) + \Pr\{E_2\}[\Pr\{v_{1A} + v_{2B} > \gamma | v_{1A}, E_2\} v_{1A} + T_1(v_{1A}, E_2)] \\ &\quad + \Pr\{E_3\}[v_{1A} + T_1(v_{1A}, E_3)]. \end{aligned}$$

If bidder 1 with $v_{1A} = 0$ deviates and drops out of the auction at the price at which bidder 1 with $v_{1A} > 0$ drops out and rejects all resale offers in event E_1, his payoff is therefore

$$U_1^d(0) = \Pr\{E_2\} T_1(v_{1A}, E_2) + \Pr\{E_3\} T_1(v_{1A}, E_3)$$

which implies that

$$T_1(0) \geq \Pr\{E_2\} T_1(v_{1A}, E_2) + \Pr\{E_3\} T_1(v_{1A}, E_3) \tag{2.5}$$

by incentive compatibility.

Since $v_{2B} \in [0, 1]$ and $\gamma \in [0, 2]$, $0 < \Pr\{\gamma < v_{2B}\} < 1$. Consider a monotonically decreasing sequence of v_{1A} that converges to 0. Denote this sequence as $\{v_n\}$. Let $y_n = F_L(v_n)$ and $z_n = F_G(b_G^{-1}(b_L(v_n)))$. Since F_L is a continuous function and $F_L(0) = 0$, the sequence $\{y_n\}$ converges to 0. $\{b_L(v_n)\}$ is a monotonically decreasing sequence because b_L is a strictly increasing function. Moreover, $\{b_L(v_n)\}$ is bounded, so it converges to $\underline{B}_L$. Then, $\{b_G^{-1}(b_L(v_n))\}$ converges to 0 since $\underline{B}_L = \underline{B}_G$, so $\{z_n\}$ converges to 0. Therefore, $\{y_n + z_n\}$ converges to 0. Since $\Pr\{\gamma < v_{2B}\} > 0$, there exists an integer N such that, for all $n \geq N$, $y_n + z_n < \Pr\{\gamma < v_{2B}\}$. Thus, there exists $\delta \in (0, 1)$ such that

$$\Pr\{\gamma < v_{2B}\} > F_L(v_{1A}) + F_G(b_G^{-1}(b_L(v_{1A})))$$

holds for all $v_{1A} \in (0, \delta)$. Therefore, for sufficiently small $v_{1A} > 0$, deviating to the strategy of bidder 1 with $v_{1A} = 0$ is profitable because

$$\begin{aligned}
U_1(v_{1A}) &\leq \Pr\{E_2\}[v_{1A} + T_1(v_{1A}, E_2)] + \Pr\{E_3\}[v_{1A} + T_1(v_{1A}, E_3)] \\
&\leq [\Pr\{E_2\} + \Pr\{E_3\}]v_{1A} + T_1(0) \\
&\leq [F_L(v_{1A}) + F_G(b_G^{-1}(b_L(v_{1A})))]v_{1A} + T_1(0) \\
&< \Pr\{\gamma < v_{2B}\}v_{1A} + T_1(0) \\
&= U_1^d(v_{1A}).
\end{aligned}$$

where the second inequality holds by Inequality (2.5).

Therefore, there does not exist an equilibrium in which the bidders use strictly increasing strategies in the auction stage. ∎

In the proposition above, it is not essential that the bidders bid symmetrically according to strictly increasing functions. The key part of the proof is that a bidder reveals his value by dropping out of the auction. In the resale stage, the seller can then sell the object at the highest price the potential buyer is willing to pay and the buyer gets 0 after resale. For some types of bidders, it is profitable to deviate and bid in a way such that their private information remains private so they are offered resale prices that are lower than what they are willing to pay.

In the previous section, we show that resale may improve allocation efficiency in certain situations when the losers have no private information. However, the above proposition shows that it is difficult to achieve efficiency once the entire game is taken into consideration.

2.6 Efficiency with weakly increasing strategies

In the previous section, we established that there exists no equilibrium in which the bidders are using strictly increasing strategies. In this section, we study the problem with weakly increasing strategies.

Proposition 2.6.1 *Let v_{1A} and v_{2B} be distributed continuously on $[0, 1]$ according to F_L and γ be distributed continuously on $[0, 2]$ according to F_G. Suppose all bidders use weakly increasing strategies. Suppose $b_{1A}(x) = b_{2B}(x)$ for all x and $b_{3A}(\gamma) = b_{3B}(\gamma)$ for all γ, where $b_{ij}(v)$ is the price at which bidder i with value v drops out of the auction for object j when every other bidder is still participating in the simultaneous ascending auction. Then there exists no equilibrium in which the objects are allocated efficiently after the resale stage.*

Proof Suppose there exists an equilibrium in which the post-resale allocation is efficient.

First, we note that efficiency requires that the local bidders use separating strategies when all three bidders are participating in the auction. To see why this is the case, suppose the contrary. Then, there exist $\underline{v}$ and $\overline{v}$, where $0 \leq \underline{v} < \overline{v}$, such that $b_{1A}(v) = b_{1A}(v')$ for all $v, v' \in [\underline{v}, \overline{v}]$. Consider the event where $v_{1A} \in [\underline{v}, \overline{v}]$, $v_{2B} \in [\underline{v}, \overline{v}]$ and $\gamma \in (2\underline{v}, 2\overline{v}]$. This is an event that occurs with strictly positive probability because $\Pr\{v_{1A} \in [\underline{v}, \overline{v}], v_{2B} \in [\underline{v}, \overline{v}], \gamma \in (2\underline{v}, 2\overline{v}]\} = [F_L(\overline{v}) - F_L(\underline{v})]^2[F_G(2\overline{v}) - F_G(2\underline{v})] > 0$.

In any efficient equilibrium of the game, bidder 3 cannot drop out of the auction before he is sure that $\gamma \leq v_{1A} + v_{2B}$. Otherwise, bidders 1 and 2 win the objects in the auction when the probability that $\gamma > v_{1A} + v_{2B}$ is strictly positive. By Proposition 2.4.2, there is zero probability of resale in this case, so this equilibrium is inefficient with strictly positive probability. Therefore, bidder 3 with $\gamma \in (2\underline{v}, 2\overline{v}]$ has to win both objects in the simultaneous ascending auction against bidders 1 and 2 with $v_{1A}, v_{2B} \in [\underline{v}, \overline{v}]$.

Note that, with strictly positive probability, resale is necessary to achieve efficiency because $v_{1A} + v_{2B} > \gamma$ with strictly positive probability in the event we are considering. Let p_1 and p_2 denote the offers bidder 3 makes at the resale stage after winning both objects to bidders 1 and 2 respectively. Then, $p_1 + p_2 \geq \gamma$ because bidder 3 would be better off keeping the objects otherwise. Bidder 1 is willing to accept bidder 3's offer p_1 if and only if $p_1 \leq v_{1A}$. Similarly, bidder 2 is willing to accept bidder 3's offer p_2 if and only if $p_2 \leq v_{2B}$. If bidder 3 sells only one object, say object A, and keeps the other object, object B, then $v_{1A} \geq p_1 \geq \gamma$. This is inefficient because $v_{2B} > 0$ with strictly positive probability; thus, $v_{1A} + v_{2B} > \gamma$ with strictly positive probability but object B is not assigned to bidder 2. On the other hand, if bidder 3 sells both objects to the local bidders, then $p_1 + p_2 \geq \gamma$ implies that $p_1 \geq \frac{\gamma}{2}$ or $p_2 \geq \frac{\gamma}{2}$. Without loss of generality, assume $p_1 \geq \frac{\gamma}{2}$. If $p_1 \geq \overline{v}$, the resale of object A to bidder 1 occurs with zero probability. However, $v_{1A} + v_{2B} > \gamma$ with strictly positive probability, so this is inefficient. If $p_1 < \overline{v}$, first consider the point $(v_{1A}, v_{2B}) = \left(\frac{\gamma}{2} - \epsilon, \overline{v}\right)$, where $\epsilon \in \left(0, \min\left(\overline{v} - \frac{\gamma}{2}, \frac{\gamma}{2}\right)\right)$.[7] Then,

$$v_{1A} + v_{2B} = \frac{\gamma}{2} - \epsilon + \overline{v} > \frac{\gamma}{2} - \left(\min\left(\overline{v} - \frac{\gamma}{2}, \frac{\gamma}{2}\right)\right) + \overline{v} = \max(\gamma, \overline{v}) \geq \gamma.$$

Therefore, it is efficient for the local bidders to own the objects. However, there is no resale in this case since bidder 1 is unwilling to buy object A because $v_{1A} < \frac{\gamma}{2} \leq p_1$. Similarly, we can find a neighbourhood of the point $(v_{1A}, v_{2B}) = \left(\frac{\gamma}{2} - \epsilon, \overline{v}\right)$ such that there is no resale even when it is efficient for the local bidders to own the objects. In particular, the event $\{v_{1A} \in (\underline{v}, p_1)\} \cap \{v_{1A} + v_{2B} > \gamma\}$ occurs with strictly positive probability. The upshot is that the allocation after the resale stage is inefficient with strictly positive probability, contradicting the assumption of efficiency. Therefore, in an efficient equilibrium, the local bidders have to use separating strategies while all three bidders are participating in the auction.

Denote bidder 1's equilibrium payoff conditional on v_{1A} as $U_1(v_{1A})$. Then, for bidder 1 with $v_{1A} = 0$,

$$U_1(0) = T_1(0) \geq 0$$

where $T_1(0)$ is the expected transfer conditional on $v_{1A} = 0$. In addition, bidder 1 with $v_{1A} = 0$ consumes object A with probability $\Pr\{\gamma < v_{2B}\}$ since the allocation in this equilibrium is efficient. Therefore, if bidder 1 with $v_{1A} > 0$ deviates to the strategy of bidder 1 with $v_{1A} = 0$, his payoff is

$$U_1^d(v_{1A}) = \Pr\{\gamma < v_{2B}\} v_{1A} + T_1(0).$$

Now consider $v_{1A} > 0$. In equilibrium, each of the three bidders may drop out of the auction

[7] Since $p_1 < \overline{v}$ and $p_1 \geq \frac{\gamma}{2}$, $\overline{v} - \frac{\gamma}{2} > 0$. Thus, ϵ is indeed well-defined. Then, $v_{1A} \in \left(\max(0, \gamma - \overline{v}), \frac{\gamma}{2}\right)$.

first. Consider the following three events[8] which are conditional on v_{1A}:

1. Event E_1: Bidder 1 drops out before bidders 2 and 3.

   Bidder 3 wins object A and bidder 1's value is revealed because the local bidders use separating strategies. If bidder 3 does not offer object A for resale, bidder 1's payoff is 0. On the other hand, if bidder 3 offers object A to bidder 1, the optimal price for bidder 3 is v_{1A} because that is the maximum price that bidder 1 is willing to pay. Therefore, bidder 1's payoff is 0 regardless of whether or not bidder 3 resells object A.

2. Event E_2: Bidder 2 drops out before bidders 1 and 3.

   Bidder 2 reveals his value and bidder 3 wins object B. Since the local bidders are using symmetric separating strategies, it must be the case that $v_{2B} < v_{1A}$. Therefore,

   $$\Pr\{E_2\} \leq \Pr\{v_{2B} < v_{1A} | v_{1A}\} = F_L(v_{1A}). \tag{2.6}$$

   Moreover, bidder 1 has object A after the resale stage with probability $\Pr\{v_{1A} + v_{2B} > \gamma | v_{1A}, E_2\}$ since the allocation is efficient in this equilibrium. Denote bidder 1's expected transfer in this case as $T_1(v_{1A}, E_2)$.

3. Event E_3: Bidder 3 drops out before bidders 1 and 2.

   Suppose all three bidders are still participating in the simultaneous ascending auction. Let the current price in the auction for each object be p. Since the local bidders use fully separating strategies, bidder 3 knows that both the following inequalities must hold.

   $$\begin{aligned} b_{1A}(v_{1A}) &\geq p, \\ b_{2B}(v_{2B}) &\geq p. \end{aligned}$$

[8]We ignore cases where at least two bidders drop out simultaneously because the local bidders are using separating strategies, so ties happen with zero probability.

As explained earlier, in any efficient equilibrium of the game, bidder 3 cannot drop out of the auction before he is sure that $\gamma \leq v_{1A} + v_{2B}$. Consequently, at each price p, bidder 3 can drop out of both auctions only if $\gamma \leq 2\hat{v}$, where $\hat{v} = \inf\{v : b_L(v) \geq p\}$. Then,

$$\Pr\{E_3\} \leq \Pr\{\gamma \leq 2v_{1A}|v_{1A}\} = F_G(2v_{1A}). \tag{2.7}$$

Moreover, if bidder 3 drops out of the simultaneous auction before bidders 1 and 2, there is zero probability of resale by Proposition 2.4.2. So bidder 1 keeps object A and we denote his expected transfer in this case as $T_1(v_{1A}, E_3)$.

Therefore, for bidder 1 with $v_{1A} > 0$,

$$\begin{aligned} U_1(v_{1A}) &= \Pr\{E_1\}(0) + \Pr\{E_2\}[\Pr\{v_{1A} + v_{2B} > \gamma|v_{1A}, E_2\}v_{1A} + T_1(v_{1A}, E_2)] \\ &\quad + \Pr\{E_3\}[v_{1A} + T_1(v_{1A}, E_3)] \\ &\leq F_L(v_{1A})[v_{1A} + T_1(v_{1A}, E_2)] + F_G(2v_{1A})[v_{1A} + T_1(v_{1A}, E_3)]. \end{aligned}$$

If bidder 1 with $v_{1A} = 0$ deviates and drops out of the auction at the price at which bidder 1 with $v_{1A} > 0$ drops out but, in event E_1, rejects all offers at the resale stage, his payoff is

$$U_1^d(0) = \Pr\{E_2\}T_1(v_{1A}, E_2) + \Pr\{E_3\}T_1(v_{1A}, E_3).$$

Since

$$U_1(0) \geq U_1^d(0)$$

by incentive compatibility,

$$T_1(0) \geq \Pr\{E_2\}T_1(v_{1A}, E_2) + \Pr\{E_3\}T_1(v_{1A}, E_3). \tag{2.8}$$

Finally, since $v_{2B} \in [0, 1]$ and $\gamma \in [0, 2]$, $\Pr\{\gamma < v_{2B}\} > 0$. In addition, F_L and F_G are continuous, $F_L(0) + F_G(0) = 0$ and $F_L(1) + F_G(2) = 2$. Therefore, by the intermediate value theorem, there exists $\alpha \in (0, 1)$ such that

$$F_L(v_{1A}) + F_G(2v_{1A}) < \Pr\{\gamma < v_{2B}\}$$

holds for all $v_{1A} \in (0, \alpha)$. This implies that bidder 1 with sufficiently small v_{1A} has a profitable deviation since

$$\begin{aligned} U_1(v_{1A}) &\leq [\Pr\{E_2\} + \Pr\{E_3\}]v_{1A} + T_1(0) \\ &\leq [F_L(v_{1A}) + F_G(2v_{1A})]v_{1A} + T_1(0) \\ &< \Pr\{\gamma < v_{2B}\}v_{1A} + T_1(0) \\ &= U_1^d(v_{1A}) \end{aligned}$$

where the first inequality follows from Inequality (2.8) and the second inequality follows from Inequalities (2.6) and (2.7). This cannot be an equilibrium. ∎

In the proposition above, it is not essential that the bidders bid symmetrically according to weakly increasing functions. The key part of the proof is that, given the fixed resale mechanism, bidders 1 and 2 are required to use separating strategies to achieve efficiency, but a bidder who reveals his value by dropping out of the auction gets 0 after resale. For some types of bidders, it is profitable to deviate and bid in such a way that their private information remains private so they are offered resale prices that are lower than what they are willing to pay.

2.7 Improvement of efficiency through resale

The previous two sections provide some indication of how difficult it is to achieve efficiency when the entire game is considered. In this section, we describe an equilibrium where resale improves efficiency relative to the benchmark equilibrium without resale. Let v_{1A} and v_{2B} be independently and identically distributed uniformly on $[0, 1]$.

According to Proposition 1 in Zheng (2012), in the benchmark equilibrium without resale, bidders 1 and 2 bid up to their values while bidder 3 bids according to the following:

- When the price is 0, he participates in the auctions for both objects.
- If both auctions are still going on, he bids up to $p_j = \dfrac{\gamma + 1 - \sqrt{1 + 2\gamma - 2\gamma^2}}{3}$ if $\gamma < 1$ and $p_j = \dfrac{2\gamma - 1}{3}$ if $\gamma \geq 1$ for each object j.
- If he has won an object, he continues in the auction for the other object up to the price of γ.

Consider the following. Suppose the bidders bid according to the following in the simultaneous ascending auction.

- If the auctions for both objects are going on, $\forall i \in \{1, 2\}$, bidder i drops out at 0 if $v_{ij} \in [0, x)$ and bids up to $b_L > 0$ if $v_{ij} \in [x, 1]$. Bidder 3 drops out of both auctions at b_G, where $0 < b_G < b_L$; whereas he drops out from both auctions at M, where $M > b_L$, if he stayed in both auctions at price b_G.
- If only one of the auctions is going on and the global bidder has won the other object, the remaining local bidder drops out at $c > 0$ and the global bidder drops out at M, where $M > c$.
- If only one of the auctions is going on and a local bidder has won the other object, the remaining local bidder drops out at $\dfrac{1}{4(1-x)}$ and the global bidder drops out at M, where $M > \dfrac{1}{4(1-x)}$.

If a local bidder drops out of the auction at price 0, the global bidder believes that the local bidder's value for the object v_{ij} is given by updated prior distribution conditional on $v_{ij} \in [0, x)$. Otherwise, the global bidder believes that the local bidder's value for the object v_{ij} is given by updated prior distribution conditional on $v_{ij} \in [x, 1]$.

The simultaneous ascending auction is followed by the fixed resale mechanism defined above that is optimal given the beliefs. Then, we claim that an equilibrium defined by $\{x, b_L, b_G, c, M\}$ exists. Furthermore, the following proposition proves that there exists such an equilibrium with resale that improves efficiency relative to the benchmark equilibrium without resale.

Proposition 2.7.1 *Let v_{1A} and v_{2B} be distributed uniformly on $[0, 1]$. Let γ be commonly known. If $\gamma \in \left[1, \frac{5}{4}\right)$, then there exists an equilibrium where the bidders bid according to the strategies prescribed above in the simultaneous ascending auction followed by the fixed resale mechanism. This equilibrium weakly improves efficiency for every realization of values relative to the benchmark equilibrium without resale. The probability of inefficiency is reduced by $2\left(\frac{2-\gamma}{9}\right)^2$.*

Proof An equilibrium defined by $\{x, b_L, b_G, c, M\}$ such that

$$\begin{aligned} x &= \frac{2\gamma - 1}{3}, \\ b_G &= \frac{37}{54}\gamma - \frac{10}{27}, \\ \frac{2 + 11\gamma}{24} = b_L &< c < \frac{-22\gamma^2 + 40\gamma - 1}{24(2-\gamma)} \quad \text{and} \\ M &> \frac{5}{4} \end{aligned}$$

is an equilibrium with resale that improves efficiency relative to the benchmark equilibrium without resale.

First, it is necessary to establish the resale prices in equilibrium in order to show that $\{x, b_L, b_G, c, M\}$ indeed defines an equilibrium. We need to consider the following cases.

1. Bidder 3 wins both objects in the simultaneous ascending auction. Let F_1 be the updated cumulative distribution function for v_{1A} and F_2 be the updated cumulative distribution function for v_{2B}. Let p_1 be the resale price at which bidder 3 offers object A to bidder 1 and p_2 be the resale price at which bidder 3 offers object B to bidder 2.

(a) Bidder 3 believes that $v_{1A} \in [0, x)$ and $v_{2B} \in [0, x)$.

Since $x = \dfrac{2\gamma - 1}{3}$ and $\gamma \in \left[1, \dfrac{5}{4}\right)$, $x < \dfrac{1}{2}$. Therefore, in this case, there is no resale and bidder 3 consumes both objects.

(b) Bidder 3 believes that $v_{1A} \in [0, x)$ and $v_{2B} \in [x, 1]$.

In this case,

$$F_1(v) = \begin{cases} 0 & \text{if } v \leq 0 \\ \dfrac{v}{x} & \text{if } 0 < v < x \\ 1 & \text{if } v \geq x \end{cases}$$

and

$$F_2(v) = \begin{cases} 0 & \text{if } v \leq x \\ \dfrac{v - x}{1 - x} & \text{if } x < v < 1 \\ 1 & \text{if } v \geq 1 \end{cases} \cdot$$

Since $v_{2B} = 1$ with probability 0 and $\gamma \geq 1$, there is zero probability that bidder 3 is willing to resell to bidder 2 alone. Then, bidder 3 solves

$$\max_{p_1, p_2} [1 - F_1(p_1)][1 - F_2(p_2)](p_1 + p_2 - \gamma) + \gamma.$$

Consider the first order conditions:

$$\frac{x - 2p_1 - p_2 + \gamma}{x} = 0$$

and

$$\frac{1 - p_1 - 2p_2 + \gamma}{1 - x} = 0.$$

The optimal prices are

$$p_1 = \frac{\gamma + 2x - 1}{3} \tag{2.9}$$

and

$$p_2 = \frac{2 + \gamma - x}{3}. \tag{2.10}$$

Since $p_1 < x$ and $p_2 < 1$, resale takes place only if both bidders 1 and 2 accept bidder 3's offers. The expected profit is

$$\frac{325\gamma^2 - 166\gamma + 4}{162(2\gamma - 1)}$$

which is higher than the expected profits given by the boundary solutions. Therefore, the maximized expected profit is

$$\pi_{0,1}(\gamma) = \frac{325\gamma^2 - 166\gamma + 4}{162(2\gamma - 1)}. \tag{2.11}$$

(c) Bidder 3 believes that $v_{1A} \in [x, 1]$ and $v_{2B} \in [0, x)$.

This case is symmetric to case (b). Therefore, it is optimal for bidder 3 to offer objects A and B at the resale prices

$$p_1 = \frac{2 + \gamma - x}{3} \tag{2.12}$$

and

$$p_2 = \frac{\gamma + 2x - 1}{3} \tag{2.13}$$

to bidders 1 and 2 respectively. Resale occurs only if both bidders 1 and 2 accept bidder 3's offers. Therefore, the maximized expected profit is

$$\pi_{1,0}(\gamma) = \frac{325\gamma^2 - 166\gamma + 4}{162(2\gamma - 1)} \tag{2.14}$$

(d) Bidder 3 believes that $v_{1A} \in [x, 1]$ and $v_{2B} \in [x, 1]$.

In this case,

$$F_i(v) = \begin{cases} 0 & \text{if } v \leq x \\ \dfrac{v-x}{1-x} & \text{if } x < v < 1 \\ 1 & \text{if } v \geq 1 \end{cases}$$

for $i \in \{1,2\}$. Since $v_{1A} = 1$ with probability 0, $v_{2B} = 1$ with probability 0 and $\gamma \geq 1$, there is zero probability that bidder 3 is willing to resell to either bidder 1 or bidder 2 alone. Then, bidder 3 solves

$$\max_{p_1,p_2}[1 - F_1(p_1)][1 - F_2(p_2)](p_1 + p_2 - \gamma) + \gamma.$$

The first order conditions are

$$\frac{(1-p_2)(1+\gamma-2p_1-p_2)}{(1-x)^2} = 0$$

and

$$\frac{(1-p_1)(1+\gamma-2p_2-p_1)}{(1-x)^2} = 0$$

while the optimal prices are

$$p_i = \frac{1+\gamma}{3} \tag{2.15}$$

for $i \in \{1,2\}$. Since $p_i < 1$, resale occurs only if both bidders 1 and 2 accept bidder 3's offers. The expected profit is $\frac{11}{12}\gamma + \frac{1}{6}$ which is higher than the expected profits given by the boundary solutions. Therefore, the maximized expected profit is

$$\pi_{1,1}(\gamma) = \frac{11}{12}\gamma + \frac{1}{6}.$$

2. Bidder 3 wins object A and bidder 2 wins object B in the simultaneous ascending auction. Let F_1 be the updated cumulative distribution function for v_{1A}. Let p_1 be the resale price

at which bidder 3 offers object A to bidder 1 and p_3 be the resale price at which bidder 2 offers object B to bidder 3.

(a) Bidders 2 and 3 believe that $v_{1A} \in [0, x)$.

Then,

$$F_1(v) = \begin{cases} 0 & \text{if } v \leq 0 \\ \dfrac{v}{x} & \text{if } 0 < v < x \\ 1 & \text{if } v \geq x \end{cases} \cdot$$

If bidder 3 buys object B from bidder 2, his value for the package of objects A and B is γ. Since $\gamma \geq 1$ and it is believed that $v_{1A} \in [0, x)$, bidder 3 does not offer the objects for resale. On the other hand, if bidder 3 does not buy object B from bidder 2, his value for object A alone is 0. Therefore, bidder 3 solves

$$\max_{p_1} [1 - F_1(p_1)] p_1.$$

The first order condition is

$$1 - \frac{2p_1}{x} = 0$$

while the optimal price is

$$p_1 = \frac{1}{2} x. \tag{2.16}$$

Therefore, bidder 3's expected payoff is $\frac{1}{4}x$ if he does not buy object B from bidder 2. Hence, the highest price bidder 3 is willing to pay for object B is $\gamma - \frac{1}{4}x$. Consequently, if $v_{2B} < \gamma - \frac{1}{4}x$, bidder 2 offers object B to bidder 3 at the price

$$p_3 = \gamma - \frac{1}{4} x; \tag{2.17}$$

otherwise, he consumes object B.

(b) Bidders 2 and 3 believe that $v_{1A} \in [x, 1]$.

Then,

$$F_1(v) = \begin{cases} 0 & \text{if } v \leq x \\ \dfrac{v-x}{1-x} & \text{if } x < v < 1 \\ 1 & \text{if } v \geq 1 \end{cases} \cdot$$

If bidder 3 buys object B from bidder 2, his value for the package of objects A and B is γ. Since $\gamma \geq 1$ and it is believed that $v_{1A} \in [x, 1]$, bidder 3 does not offer the objects for resale. On the other hand, if bidder 3 does not buy object B from bidder 2, his value for object A alone is 0. Therefore, bidder 3 solves

$$\max_{p_1} [1 - F_1(p_1)] p_1.$$

The first order condition is

$$\frac{1 - 2p_1}{1 - x} = 0$$

and, using the fact that $x < \frac{1}{2}$, the optimal price is

$$p_1 = \frac{1}{2}.$$

Therefore, bidder 3's expected payoff is $\frac{1}{4(1-x)}$ if he does not buy object B from bidder 2. Hence the highest price bidder 3 is willing to pay for object B is $\gamma - \frac{1}{4(1-x)}$. Consequently, bidder 2 offers object B to bidder 3 at the price

$$p_3 = \gamma - \frac{1}{4(1-x)}$$

if $v_{2B} < \gamma - \frac{1}{4(1-x)}$ and consumes object B otherwise.

3. Bidder 1 wins object A and bidder 3 wins object B in the simultaneous ascending auction.

Let F_2 be the updated cumulative distribution function for v_{2B}. Let p_2 be the resale price at which bidder 3 offers object B to bidder 2 and p_3 be the resale price at which bidder 1 offers object A to bidder 3.

(a) Bidders 1 and 3 believe that $v_{2B} \in [0, x)$.

This case is symmetric to case 2(a). Therefore, if bidder 3 buys object A from bidder 1, there is no resale of object B to bidder 2. Otherwise, bidder 3 offers object B to bidder 2 at the price

$$p_2 = \frac{1}{2}x.$$

Bidder 1 offers object A to bidder 3 at the price

$$p_3 = \gamma - \frac{1}{4}x$$

if $v_{1A} < \gamma - \frac{1}{4}x$ and consumes object A otherwise.

(b) Bidders 1 and 3 believe that $v_{2B} \in [x, 1]$.

This case is symmetric to case 2(b). Therefore, if bidder 3 buys object A from bidder 1, bidder 3 does not offer the objects for resale. Otherwise, bidder 3 offers object B to bidder 2 at the price

$$p_2 = \frac{1}{2}.$$

Bidder 1 offers object A to bidder 3 at the price

$$p_3 = \gamma - \frac{1}{4(1-x)}$$

if $v_{1A} < \gamma - \frac{1}{4(1-x)}$ and consumes object A otherwise.

4. Bidder 1 wins object A and bidder 2 wins object B in the simultaneous ascending auction. Since γ is commonly known, there is zero probability of resale in equilibrium by

Proposition 2.4.1.

Now we consider the entire game and show that the proposed strategies constitute a perfect Bayesian equilibrium.

(I) Consider a history such that bidder 2 has dropped out at price $p_B = 0$, and thus the global bidder has won object B, while the auction for object A is still going on and the current price is $p_A \geq 0$. The case where the global bidder has won object A and the auction for object B is still going on is symmetric.

Suppose $p_A \in [0, c)$. The strategies prescribe that bidder 1 drop out when the price reaches c and bidder 3 drop out at M.

If both bidders follow the prescribed strategies, then bidder 3 wins both auctions and believes that $v_{1A} \in [x, 1]$, $v_{2B} \in [0, x)$. By case 1(c), bidder 3 offers the objects for resale at prices p_1 for object A and p_2 for object B, where p_1 and p_2 are defined by Equations (2.12) and (2.13) respectively. Resale occurs if both bidders 1 and 2 agree to buy. If $p_1 \leq v_{1A}$ and $p_2 \leq v_{2B}$, both bidders 1 and 2 are willing to buy the objects from bidder 3. Thus the payoff of bidder 1 is

$$\max\left\{0, \left(v_{1A} - \frac{2+\gamma-x}{3}\right)\Pr\left\{v_{2B} \geq \frac{\gamma+2x-1}{3} \mid v_{2B} \in [0, x)\right\}\right\}$$

and the payoff of bidder 3 is

$$\pi_{1,0}(\gamma) - c.$$

If bidder 1 drops out at price $p'_A \neq c$ such that $p'_A \in [p_A, M]$ instead, then bidder 3 still wins both objects; the beliefs and thus resale prices are unaffected, so bidder 1's payoff remains the same. If bidder 1 stays after the price goes above M instead, then bidder 1 wins object A. By case 3(a), the highest price at which bidder 3 is willing to pay for object A at the resale stage is $p_3 = \gamma - \dfrac{1}{4}x$. Thus the payoff of bidder 1 is

$$\max\left\{\gamma - \frac{1}{4}x, v_{1A}\right\} - M$$

which is negative. Hence bidder 1 does not have profitable deviations.

If bidder 3 drops out at price $p'_A \neq M$ such that $p'_A \geq c$ instead, then bidder 3 still wins object A and gets the same payoff as from following the prescribed strategy. If bidder 3 drops out at price $p'_A \in [p_A, c)$ instead, then bidder 1 wins object A. By case 3(a), the highest price at which bidder 3 is willing to pay for object A at the resale stage is $p_3 = \gamma - \dfrac{1}{4}x$. Thus the payoff of bidder 3 is $\dfrac{1}{4}x$. Since $\pi_{1,0}(\gamma) - c > \dfrac{1}{4}x$, bidder 3 does not have profitable deviations.[9]

At histories where $p_A \in [c, M]$, the strategies prescribe that bidder 1 drop out immediately and bidder 3 continue bidding until M. Verification that bidder 1 has no incentive to deviate in this case is nearly identical to that above. Ties are broken in favour of bidder 3; thus bidder 3 is indifferent between dropping out and continuing until M.

(II) Consider a history such that bidder 2 has dropped out at price $p_B > 0$, and thus the global bidder has won object B, while the auction for object A is still going on and the current price is $p_A \geq p_B$. The case where the global bidder has won object A and the auction for object B is still going on is symmetric.

Suppose $p_A \in [p_B, c)$. The strategies prescribe that bidder 1 drop out at price c and bidder 3 continue bidding and drop out when the price reaches M.

If both bidders follow the prescribed strategies, then bidder 3 wins both auctions and believes that v_{1A}, $v_{2B} \in [x, 1]$. By case 1(d), bidder 3 offers each object at p_i, where p_i is defined by Equation (2.15). Resale occurs if both bidders 1 and 2 agree to buy. If $p_i \leq v_{1A}$ and $p_i \leq v_{2B}$, both bidders 1 and 2 are willing to buy the objects from bidder 3.

[9] $\pi_{1,0}(\gamma) - c > \frac{325\gamma^2-166\gamma+4}{162(2\gamma-1)} - \frac{-22\gamma^2+40\gamma-1}{24(2-\gamma)} = \frac{112\gamma^3-510\gamma^2+210\gamma-5}{648(\gamma-2)(2\gamma-1)} > \frac{1}{4}x.$

Thus the payoff of bidder 1 is

$$\max\left\{0, \left(v_{1A} - \frac{1+\gamma}{3}\right)\Pr\left\{v_{2B} \geq \frac{1+\gamma}{3} \mid v_{2B} \in [x, 1]\right\}\right\}$$

and the payoff of bidder 3 is

$$\pi_{1,1}(\gamma) - c - p_B.$$

If bidder 1 drops out at price $p'_A \neq c$ such that $p'_A \in [p_A, M]$ instead, then bidder 3 still wins both objects; the beliefs and thus resale prices are unaffected, so bidder 1's payoff remains the same. If bidder 1 stays after the price goes above M instead, then bidder 1 wins object A. By case 3(b), the highest price at which bidder 3 is willing to pay for object A at the resale stage is $p_3 = \gamma - \dfrac{1}{4(1-x)}$. Thus the payoff of bidder 1 is

$$\max\left\{\gamma - \frac{1}{4(1-x)}, v_{1A}\right\} - M$$

which is negative. Hence bidder 1 does not have profitable deviations.

If bidder 3 drops out at price $p'_A \neq M$ such that $p'_A \geq c$ instead, then bidder 3 still wins object A and gets the same payoff as from following the prescribed strategy. If bidder 3 drops out at price $p'_A \in [p_A, c)$ instead, then bidder 1 wins object A. By case 3(b), the highest price at which bidder 3 is willing to pay for object A at the resale stage is $p_3 = \gamma - \dfrac{1}{4(1-x)}$. Thus the payoff of bidder 3 is $\dfrac{1}{4(1-x)} - p_B$. Since $\pi_{1,1}(\gamma) - c > \dfrac{1}{4(1-x)}$, bidder 3 does not have profitable deviations.[10]

At histories where $p_A \in [c, M]$, the strategies prescribe that bidder 1 drop out immediately and bidder 3 continue bidding until M. Verification that bidder 1 has no incentive to deviate in this case is nearly identical to that above. Ties are broken in favour of bidder 3; thus bidder 3 is indifferent between dropping out and continuing until M.

[10] $\pi_{1,1}(\gamma) - c > \frac{11}{12}\gamma + \frac{1}{6} - \frac{-22\gamma^2+40\gamma-1}{24(2-\gamma)} = \frac{3}{8(2-\gamma)} = \frac{1}{4(1-x)}$.

(III) Consider a history such that the global bidder has dropped out at price $p_B > 0$, and bidder 2 has won object B, while the auction for object A is still going on and the current price is $p_A \geq p_B$. The case where bidder 1 has won object A and the auction for object B is still going on is symmetric.

Suppose $p_A \in \left[p_B, \frac{1}{4(1-x)}\right)$. The strategies prescribe that bidder 1 drop out at price $\frac{1}{4(1-x)}$ and bidder 3 continue bidding and drop out when the price reaches M.

If both bidders follow the prescribed strategies, then bidder 3 wins object A at price $\frac{1}{4(1-x)}$; bidder 3 believes that $v_{1A}, v_{2B} \in [x, 1]$. By case 2(b), bidder 3 offers object A at the price $p_1 = \frac{1}{2}$ if he does not buy object B from bidder 2; bidder 2 offers object B to bidder 3 at the price $p_3 = \gamma - \frac{1}{4(1-x)}$ if $v_{2B} < \gamma - \frac{1}{4(1-x)}$ and consumes object B otherwise. The payoff of bidder 1 is

$$\max\left\{0, \left(v_{1A} - \frac{1}{2}\right)\Pr\left\{\gamma - \frac{1}{4(1-x)} \leq v_{2B} \leq 1 \mid v_{2B} \in [x, 1]\right\}\right\}$$

and the expected payoff of bidder 3 is zero.

If bidder 1 drops out at price $p'_A \neq c$ such that $p'_A \in [p_A, M]$ instead, then bidder 3 still wins both objects; the beliefs and thus resale prices are unaffected, so bidder 1's payoff remains the same. If bidder 1 stays after the price goes above M instead, then bidder 1 wins object A. By case 4, there is zero probability of resale. The payoff of bidder 1 is

$$v_{1A} - M$$

which is negative. Hence bidder 1 does not have profitable deviations.

If bidder 3 drops out at price $p'_A \neq M$ such that $p'_A \geq \frac{1}{4(1-x)}$ instead, then bidder 3 still wins object A and gets the same payoff as from following the prescribed strategy. If bidder 3 drops out at price $p'_A \in \left[p_A, \frac{1}{4(1-x)}\right)$ instead, then bidder 1 wins object A.

By case 4, there is zero probability of resale, and the payoff of bidder 3 is zero. Hence bidder 3 does not have profitable deviations.

Suppose $p_A \in \left[\frac{1}{4(1-x)}, M\right]$. The strategies prescribe that bidder 1 drop out immediately and bidder 3 continue bidding until M. The expected payoff of bidder 1 is the same as above and the expected payoff of bidder 3 is $\frac{1}{4(1-x)} - p_A \leq 0$. Verification that bidder 1 has no incentive to deviate in this case is nearly identical to that above. Ties are broken in favour of bidder 3; thus bidder 3 is indifferent between dropping out and continuing until M.

(IV) Consider a history such that both auctions are still going on and the current prices are $p = 0$. The strategies prescribe that, for $i \in \{1, 2\}$, bidder i drop out immediately if $v_{ij} \in [0, x)$ and continue until b_L if $v_{ij} \in [x, 1]$, and bidder 3 continue bidding and drop out when the price reaches b_G.

Suppose all bidders follow the prescribed strategies. If v_{1A}, $v_{2B} \in [0, x)$, then bidder 3 wins both objects at price 0 and, by case 1(a), there is no resale. If $v_{1A} \in [x, 1]$ and $v_{2B} \in [0, x)$, then bidder 3 wins object B at price 0, and the continuation game between bidders 1 and 3 was previously analyzed in I. The case where $v_{1A} \in [0, x)$ and $v_{2B} \in [x, 1]$ is symmetric. If v_{1A}, $v_{2B} \in [x, 1]$, then bidders 1 and 2 win objects A and B respectively at price b_G and, by case 4, there is no resale.

For bidder 1 with $v_{1A} \in [0, x)$, the expected payoff is

$$\max\left\{0, \left(v_{1A} - \frac{\gamma + 2x - 1}{3}\right) \Pr\left\{v_{2B} \geq \frac{2 + \gamma - x}{3}\right\}\right\} \tag{2.18}$$

and, for bidder 1 with $v_{1A} \in [x, 1]$, the expected payoff is

$$\Pr\left\{\frac{\gamma + 2x - 1}{3} \leq v_{2B} < x\right\} \max\left\{0, v_{1A} - \frac{2 + \gamma - x}{3}\right\} + \Pr\{x \leq v_{2B} \leq 1\}(v_{1A} - b_G). \tag{2.19}$$

For bidder 3, the expected payoff is

$$\begin{aligned}&\Pr\{v_{1A} \in [0, x), v_{2B} \in [0, x)\}\gamma + \Pr\{v_{1A} \in [0, x), v_{2B} \in [x, 1]\}[\pi_{0,1}(\gamma) - c] \\ &\quad + \Pr\{v_{1A} \in [x, 1], v_{2B} \in [0, x)\}[\pi_{1,0}(\gamma) - c] + \Pr\{v_{1A} \in [x, 1], v_{2B} \in [x, 1]\}(0). \qquad (2.20)\end{aligned}$$

Now let us consider various deviations for the bidders. We have previously shown that the prescribed strategies are sequentially rational for the bidders in cases when only one of the auctions is still going on. Thus for each bidder we will consider all alternative drop out prices provided that both auctions are still going on; if only one of the auctions is still going on, we will take it that the bidders are following the prescribed strategies.

First, let us consider "on path" deviations, i.e. when a given type of the local bidder mimics the behaviour of another type. Note that x and b_G are defined to ensure that bidder 1 of type $v_{1A} = x$ is indifferent between following the strategy prescribed for types in $[0, x)$ and for types in $[x, 1]$:

$$\left(x - \frac{\gamma + 2x - 1}{3}\right) \Pr\left\{v_{2B} \geq \frac{2 + \gamma - x}{3}\right\} = \Pr\{x \leq v_{2B} \leq 1\}(x - b_G).$$

Also, note that the expected payoff of bidder 1 is non-decreasing in v_{1A}. Thus no type of the local bidder has an incentive to mimic the behaviour of another type.

Next, suppose bidder 1 drops out at $p' \in (0, b_G)$. If $v_{2B} \in [0, x)$, then bidder 3 wins object B at price 0, and the continuation game was previously analyzed in I. If $v_{2B} \in [x, 1]$, then bidder 3 wins object A at price p', and the continuation game was previously analyzed in II, with the roles of bidders 1 and 2 being switched. The expected payoff of bidder 1 of type v_{1A} is then

$$\Pr\left\{\frac{\gamma+2x-1}{3} \le v_{2B} < x\right\} \max\left\{0, v_{1A} - \frac{2+\gamma-x}{3}\right\} + \Pr\left\{v_{2B} \ge \frac{1+\gamma}{3}\right\} \max\left\{0, v_{1A} - \frac{1+\gamma}{3}\right\}. \quad (2.21)$$

Since Expression (2.21) is equal to Expression (2.18) for $0 \le v_{1A} \le p^*$ where p^* is defined by Equation (2.9), Expression (2.21) is strictly less than Expression (2.18) for $p^* < v_{1A} \le x$, and Expression (2.21) is strictly less than Expression (2.19) for $x < v_{1A} \le 1$, this is not a profitable deviation.

Next, suppose bidder 1 drops out at $p' = b_G$ instead. If $v_{2B} \in [0, x)$, then bidder 3 wins object B at price 0, and the continuation game was previously analyzed in I. If $v_{2B} \in [x, 1]$, then bidder 3 wins object A at price b_G and bidder 2 wins object B at price b_G. By case 2(b), bidder 3 offers object A at the price $p_1 = \frac{1}{2}$ if he does not buy object B from bidder 2; bidder 2 offers object B to bidder 3 at price $p_3 = \gamma - \frac{1}{4(1-x)}$ if $v_{2B} < \gamma - \frac{1}{4(1-x)}$ and consumes object B otherwise. The payoff of bidder 1 of type v_{1A} is then

$$\Pr\left\{\frac{\gamma+2x-1}{3} \le v_{2B} < x\right\} \max\left\{0, v_{1A} - \frac{2+\gamma-x}{3}\right\} + \Pr\left\{\gamma - \frac{1}{4(1-x)} \le v_{2B} \le 1\right\} \max\left\{0, v_{1A} - \frac{1}{2}\right\}. \quad (2.22)$$

Since Expression (2.22) is equal to Expression (2.18) for $0 \le v_{1A} \le p^*$ where p^* is defined by Equation (2.9), Expression (2.22) is strictly less than Expression (2.18) for $p^* < v_{1A} \le x$, and Expression (2.22) is strictly less than Expression (2.19) for $x < v_{1A} \le 1$, this is not a profitable deviation.

Finally, suppose bidder 1 drops out at $p' \neq b_L$ such that $p' > b_G$ instead. The expected payoff of bidder 1 remains the same as if he dropped out at b_L, as prescribed for types in

$[x, 1]$. Hence this is not a profitable deviation.

Suppose bidder 3 simultaneously drops out from both auctions at $p' = 0$. If v_{1A}, $v_{2B} \in [0, x)$, then bidder 3 wins both objects at price 0 and, by case 1(a), there is no resale. If $v_{1A} \in [x, 1]$ and $v_{2B} \in [0, x)$, then bidder 3 loses object A and wins object B at price 0. By case 3(a), bidder 3's expected payoff is $\frac{1}{4}x$. The case where $v_{1A} \in [0, x)$ and $v_{2B} \in [x, 1]$ is symmetric. If v_{1A}, $v_{2B} \in [x, 1]$, then bidders 1 and 2 win objects A and B at price 0; by case 4, there is no resale. The expected payoff of bidder 3 is

$$\begin{aligned} &\Pr\{v_{1A} \in [0, x), v_{2B} \in [0, x)\}\gamma + \Pr\{v_{1A} \in [0, x), v_{2B} \in [x, 1]\}\frac{1}{4}x \\ &\qquad + \Pr\{v_{1A} \in [x, 1], v_{2B} \in [0, x)\}\frac{1}{4}x + \Pr\{v_{1A} \in [x, 1], v_{2B} \in [x, 1]\}(0). \end{aligned} \tag{2.23}$$

Since $\pi_{0,1}(\gamma) - c > \frac{1}{4}x$ and $\pi_{0,1}(\gamma) - c > \frac{1}{4}x$, Expression (2.23) is strictly less than Expression (2.20), so this is not a profitable deviation.

Next, suppose bidder 3 drops out from both auctions at $p' \in (0, b_L)$. This results in the same outcome and payoffs as when all bidders follow their equilibrium strategies. Hence this is not a profitable deviation.

Next, suppose bidder 3 plans to stay in both auctions until the price reaches $p' \in [0, b_L)$, and then drop out from the auction for object B only. If v_{1A}, $v_{2B} \in [0, x)$, then bidder 3 wins both objects at price 0 and, by case 1(a), there is no resale. If $v_{1A} \in [x, 1]$ and $v_{2B} \in [0, x)$, then bidder 3 wins object B at price 0; the continuation game between bidders 1 and 3 was previously analyzed in I. The case where $v_{1A} \in [0, x)$ and $v_{2B} \in [x, 1]$ is symmetric. If v_{1A}, $v_{2B} \in [x, 1]$, then bidder 2 wins object B at price p'; the continuation game between bidders 1 and 3 was previously analyzed in III, and bidder 3's expected payoff in it is zero. Therefore, bidder 3's expected payoff from this deviation is identical to the equilibrium expected payoff. Hence this is not a profitable deviation.

Finally, suppose bidder 3 drops out from both auctions at prices that are at least b_L. If v_{1A},

$v_{2B} \in [0, x)$, then bidder 3 wins both objects at price 0 and, by case 1(a), there is no resale. If $v_{1A} \in [x, 1]$ and $v_{2B} \in [0, x)$, then bidder 3 wins object B at price 0; the continuation game was previously analyzed in I. The case where $v_{1A} \in [0, x)$ and $v_{2B} \in [x, 1]$ is symmetric. Finally, if $v_{1A}, v_{2B} \in [x, 1]$, bidders 1 and 2 drop out simultaneously when the price is b_L. By case 1(d), bidder 3 offers objects A and B to bidders 1 and 2 at the same price $p_i = \dfrac{1+\gamma}{3}$. Resale occurs only if both bidders 1 and 2 accept bidder 3's offers; bidder 3 expects to get $\pi_{1,1}(\gamma) - 2b_L = 0$. Therefore, bidder 3's expected payoff from this deviation is identical to the equilibrium expected payoff. Hence this is not a profitable deviation.

Next, consider a history such that both auctions are still going on and the current prices are $p \in (0, b_G]$. Bidder 3 believes that v_{1A}, $v_{2B} \in [x, 1]$. The strategies prescribe that bidders 1 and 2 continue until b_L and bidder 3 continue bidding and drop out when the price reaches b_G.

Suppose all bidders follow the prescribed strategies. Then bidders 1 and 2 win objects A and B at price b_G and, by case 4, there is no resale. For bidder 1 with $v_{1A} \in [x, 1]$, the expected payoff is $v_{1A} - b_G$; for bidder 3, the expected payoff is zero.

Suppose bidder 1 drops out at $p' \in (p, b_G)$. Bidder 3 wins object A at price p'; the continuation game was previously analyzed in II, with the roles of bidders 1 and 2 being switched. The expected payoff of bidder 1 of type v_{1A} is then

$$\max\left\{0, \left(v_{1A} - \frac{1+\gamma}{3}\right) \Pr\left\{v_{2B} \geq \frac{1+\gamma}{3}\right\}\right\}$$

which is less than $v_{1A} - b_G$ since $b_G < \dfrac{1+\gamma}{3}$. Hence this is not a profitable deviation.

Next, suppose bidder 1 drops out at $p' = b_G$ instead. Bidder 3 wins object A at price b_G and bidder 2 wins object B at price b_G. By case 2(b), bidder 3 offers object A at the price $p_1 = \dfrac{1}{2}$ if he does not buy object B from bidder 2; bidder 2 offers object B to bidder 3 at the price $p_3 = \gamma - \dfrac{1}{4(1-\gamma)}$ if $v_{2B} < \gamma - \dfrac{1}{4(1-x)}$ and consumes object B otherwise. The payoff of bidder 1 of type v_{1A} is then

$$\max\left\{0, \left(v_{1A} - \frac{1}{2}\right)\Pr\left\{\gamma - \frac{1}{4(1-x)} \leq v_{2B} \leq 1\right\}\right\}$$

which is less than $v_{1A} - b_G$ since $b_G < \dfrac{1}{2}$. Hence this is not a profitable deviation.

Finally, suppose bidder 1 drops out at $p' \neq b_L$ such that $p' > b_G$ instead. The expected payoff of bidder 1 remains the same as if he drops out at b_L, as prescribed for types in $[x, 1]$. Hence this is not a profitable deviation.

Suppose bidder 3 drops out from both auctions at $p' \in [p, b_L)$. This results in the same outcome and payoffs as when all bidders follow their equilibrium strategies. Hence this is not a profitable deviation.

Next, suppose bidder 3 plans to stay in both auctions until the price reaches $p' \in [p, b_L)$ and then drop out from the auction for object B only. Bidder 2 wins object B at price p'. The continuation game between bidders 1 and 3 was previously analyzed in III, and bidder 3's expected payoff in it is zero. Hence this is not a profitable deviation.

Finally, suppose bidder 3 drops out from both auctions at prices that are at least b_L. Bidders 1 and 2 drop out simultaneously when the price is b_L. By case 1(d), bidder 3 offers objects A and B to bidders 1 and 2 at the same price $p_i = \dfrac{1+\gamma}{3}$. Resale occurs only if both bidders 1 and 2 accept bidder 3's offers; bidder 3 expects to get $\pi_{1,1}(\gamma) - 2b_L = 0$. Hence this is not a profitable deviation.

Next, consider a history such that both auctions are still going on and the current prices are $p \in (b_G, b_L]$. Bidder 3 believes that v_{1A}, $v_{2B} \in [x, 1]$. The strategies prescribe that bidders 1 and 2 drop out at b_L while bidder 3 is supposed to continue bidding and drop out when the price reaches M.

Suppose all bidders follow the prescribed strategies. Bidder 3 wins both objects and offers objects A and B to bidders 1 and 2 at the same price $p_i = \frac{1+\gamma}{3}$; resale occurs only if both bidders 1 and 2 accept bidder 3's offers. For bidder 1, the expected payoff is

$$\max\left\{0, \left(v_{1A} - \frac{1+\gamma}{3}\right)\Pr\left\{v_{2B} \geq \frac{1+\gamma}{3}\right\}\right\}$$

and bidder 3 expects to get

$$\pi_{1,1}(\gamma) - 2p_i$$

which is non-negative since $p_i \leq b_L = \frac{1}{2}\pi_{1,1}(\gamma)$.

Suppose bidder 1 drops out at $p' \in [p, b_L)$. Bidder 3 wins object A at price p', and the continuation game was previously analyzed in II, with the roles of bidders 1 and 2 being switched. As a result, bidder 3 wins both objects and offers objects A and B to bidders 1 and 2 at the same price $p_i = \frac{1+\gamma}{3}$. This results in the same outcome and payoff for bidder 1 as when all bidders follow their equilibrium strategies. Hence this is not a profitable deviation.

Finally, suppose bidder 1 plans to drop out at $p' > b_L$ instead. Bidder 3 wins object B at price b_L, and the continuation game was previously analyzed in II. As a result, bidder 3 wins both objects and offers objects A and B to bidders 1 and 2 at the same price $p_i = \frac{1+\gamma}{3}$. This results in the same outcome and payoff for bidder 1 as when all bidders follow their equilibrium strategies. Hence this is not a profitable deviation.

Suppose bidder 3 drops out from both auctions at $p' \in [p, b_L)$. Bidders 1 and 2 win objects A and B at price p' and, by case 4, there is no resale. The expected payoff of bidder 3 is zero. Hence this is not a profitable deviation.

Next, suppose bidder 3 plans to stay in both auctions until the price reaches $p' \in [p, b_L)$, and then drop out from the auction for object B only. Bidder 2 wins object B at price p'. The continuation game between bidders 1 and 3 was previously analyzed in III; bidder 3's expected payoff in it is zero. Hence this is not a profitable deviation.

Finally, suppose bidder 3 drops out from both auctions at prices that are at least b_L. Bidders 1 and 2 drop out simultaneously when the price is b_L. By case 1(d), bidder 3 offers objects A and B to bidders 1 and 2 at the same price $p_i = \frac{1+\gamma}{3}$. Resale occurs only if both bidders 1 and 2 accept bidder 3's offers; bidder 3 expects to get $\pi_{1,1}(\gamma) - 2b_L = 0$. Hence this is not a profitable deviation.

Finally, consider a history such that both auctions are still going on and the current prices are $p > b_L$. Bidder 3 believes that $v_{1A}, v_{2B} \in [x, 1]$. The strategies prescribe that bidders 1 and 2 drop out immediately; bidder 3 is supposed to continue bidding and drop out when the price reaches M.

Suppose all bidders follow the prescribed strategies. Bidder 3 wins both objects and offers objects A and B to bidders 1 and 2 at the same price $p_i = \frac{1+\gamma}{3}$; resale occurs only if both bidders 1 and 2 accept bidder 3's offers. For bidder 1, the expected payoff is

$$\max\left\{0, \left(v_{1A} - \frac{1+\gamma}{3}\right) \Pr\left\{v_{2B} \geq \frac{1+\gamma}{3}\right\}\right\}$$

and bidder 3 expects to get

$$\pi_{1,1}(\gamma) - 2p_i.$$

Suppose bidder 1 plans to drop out at $p' > p$ instead. Bidder 3 wins object B at price p, and the continuation game was previously analyzed in II. As a result, bidder 3 wins both objects and offers objects A and B to bidders 1 and 2 at the same price $p_i = \frac{1+\gamma}{3}$. This results in the same outcome and payoff for bidder 1 as when all bidders follow their equilibrium strategies. Hence this is not a profitable deviation.

Ties are broken in favour of bidder 3; thus bidder 3 is indifferent between dropping out immediately and continuing until M.

It remains to show that the allocation efficiency in this equilibrium with resale is higher than the allocation efficiency in the equilibrium without resale described in Proposition 1 of

Zheng (2012). In this equilibrium with resale, the allocation of the objects immediately after the simultaneous ascending auction ends and before the resale stage begins is exactly the same as the allocation of the objects in the equilibrium without resale. In particular, the objects are allocated in the following way:

- If $v_{1A} > x$ and $v_{2B} > x$, bidder 1 has object A and bidder 2 has object B.
- Otherwise, bidder 3 has both objects.

In the case where $v_{1A} < x$ and $v_{2B} > x$, by case 1(b), it is optimal for bidder 3 to offer the objects to bidders 1 and 2 at the prices p_1 and p_2, where p_1 and p_2 are defined by Equations (2.9) and (2.10) respectively, and sell both objects only if both bidders 1 and 2 accept the resale offers. Since $p_1 < x$ and $p_2 < 1$, resale occurs with probability

$$\Pr\{p_1 \leq v_{1A} \leq x\} \Pr\{p_2 \leq v_{2B} \leq 1\} = \left(\frac{1+x-\gamma}{3}\right)^2 = \left(\frac{2-\gamma}{9}\right)^2$$

which is strictly positive since $\gamma \in \left[1, \frac{5}{4}\right)$. The efficiency of the allocation is improved because, when resale occurs,

$$\gamma < p_1 + p_2 \leq v_{1A} + v_{2B}. \tag{2.24}$$

The first inequality in Expression (2.24) holds because

$$\gamma < \frac{5}{4} \implies \gamma < 2 \implies \gamma < \frac{8\gamma+2}{9} = \frac{\gamma+2x-1}{3} + \frac{2+\gamma-x}{3} = p_1 + p_2.$$

The second inequality in Expression (2.24) holds because each of bidders 1 and 2 is willing to buy an object from bidder 3 in the resale stage only if the resale price does not exceed his value, and bidder 3 is willing to sell the objects only if both bidders 1 and 2 are willing to buy the objects.

Similarly, in the case where $v_{1A} > x$ and $v_{2B} < x$, the efficiency of the allocation is improved through resale. Therefore, the allocation efficiency of the equilibrium with resale is higher than the allocation efficiency of the equilibrium without resale. ∎

The proposition above shows that, under some conditions, resale can indeed improve allocation efficiency. This result is driven by the fact that the equilibrium value of x coincides precisely with the global bidder's strategy in the game without resale. Now, we provide an illustrative example for the case where $\gamma = 1$. Then, as shown in Figure 2.1, it is efficient for bidders 1 and 2 to own the objects whenever $v_{1A} + v_{2B} > 1$ and for bidder 3 to own the objects otherwise. When the objects are allocated efficiently, the surplus is $\frac{7}{6}$.

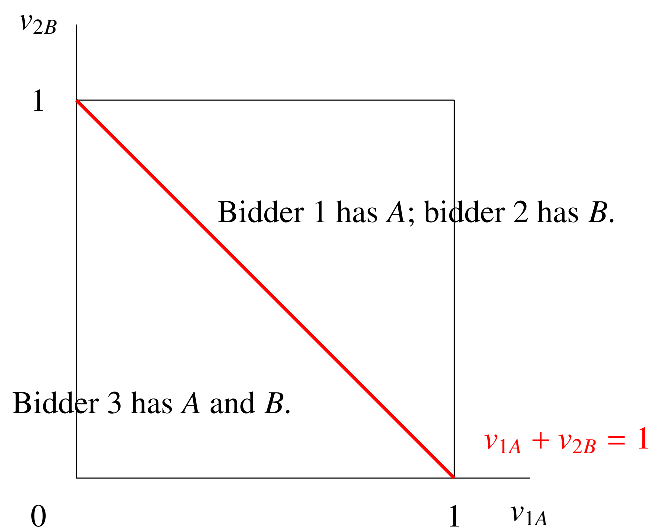

Figure 2.1: Efficient allocation of the objects for $\gamma = 1$.

In the benchmark equilibrium without resale, bidders 1 and 2 bid up to their values. If both auctions are going on, bidder 3 bids up to $p_j = \frac{1}{3}$ for each object j; if he has already won an object, he bids up to 1 for the other object. The object allocation after the auction is shown in Figure 2.2. The surplus is $\frac{31}{27}$.

In the equilibrium defined in the proposition above, the allocation of the objects after the simultaneous ascending auction is the same as in the benchmark equilibrium without resale. As shown in Figure 2.3, at the resale stage, some resale takes place when bidder 3 wins both objects. In particular, when bidder 3 believes that $v_{1A} \in [0, x]$ and $v_{2B} \in [x, 1]$, he offers object A to bidder 1 at the price $p_1 = \frac{2}{9}$ and offers object B to bidder 2 at the price $p_2 = \frac{8}{9}$. Similarly,

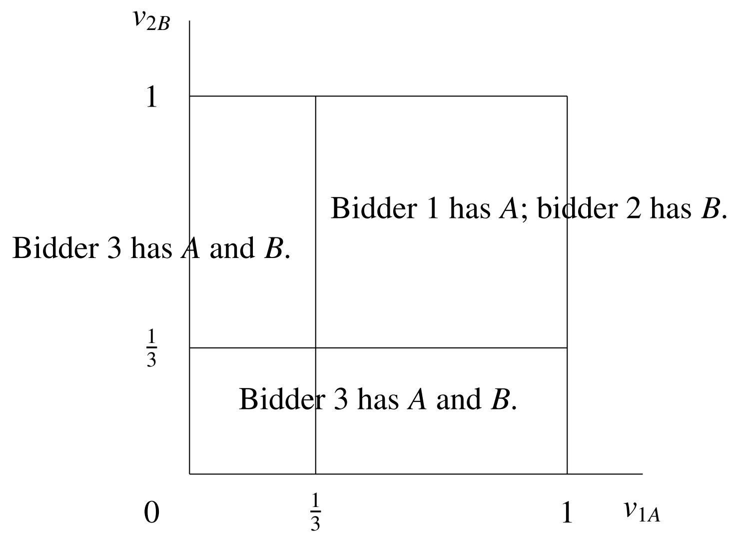

Figure 2.2: Allocation of the objects in the benchmark equilibrium without resale.

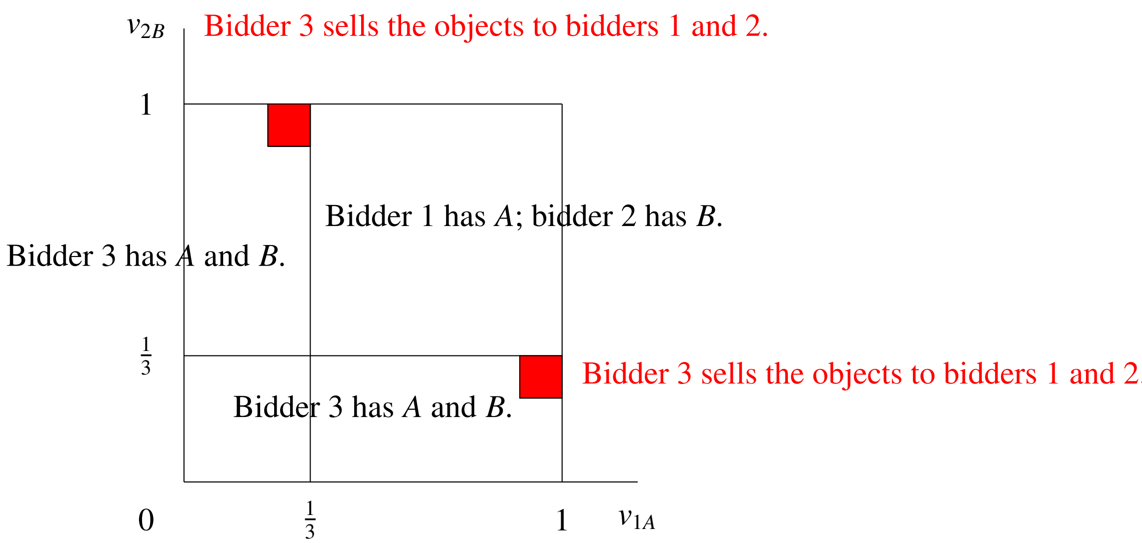

Figure 2.3: Allocation of the objects in the equilibrium with resale. As indicated in red, there is resale in some cases.

when bidder 3 believes that $v_{1A} \in [x, 1]$ and $v_{2B} \in [0, x]$, he offers object A to bidder 1 at the price $p_1 = \dfrac{8}{9}$ and offers object B to bidder 2 at the price $p_2 = \dfrac{2}{9}$. Resale occurs if both bidders 1 and 2 accept the offers. The extra surplus from resale is $\dfrac{4}{729}$, so resale through the fixed mechanism closes approximately 30% of the efficiency gap.

In particular, the example above shows that resale takes place when bidder 3 wins both objects inefficiently but not when bidders 1 and 2 win the objects inefficiently. Therefore, the fixed resale mechanism corrects overconcentration but not overdiffusion. This feature may be due to the fact that, when the objects are "overly diffused" to the local bidders, the local bidders make separate take-it-or-leave-it offers to the global bidder and at least one of the sellers demands the entire share of the global bidder's value for the package of objects. Therefore, a resale mechanism that does not allow at least one of the sellers to demand the entire share of the global bidder's value for the package of objects may be able to correct overdiffusion. One example of such a resale mechanism is one that allows the buyers to make take-it-or-leave-it offers to the sellers instead. Alternatively, resale mechanisms that give all the bargaining power to the global bidder at the resale stage, even if the global bidder does not win both objects in the auction, might improve efficiency as well.

More generally, the example above shows that, although the fixed resale mechanism can improve allocation efficiency under some conditions, it cannot restore full efficiency. This may be because the bidders have private information about their own values after the auction; resale does not always occur whenever resale is necessary to restore efficiency. This is not unique to the model studied in this paper. Therefore, the impossibility of efficiency may extend to other models with resale.

2.8 Conclusion

The simultaneous ascending auction is a standard method to sell multiple heterogeneous objects. Although secondary markets cannot be banned, the economics literature has little to say about simultaneous ascending auctions with resale. This paper is an attempt to investigate the effects of resale on the allocation efficiency of the simultaneous ascending auction.

By studying a model with two objects and three bidders, we find that there is no equilibrium in which the simultaneous ascending auction allocates the objects efficiently. This result holds with or without resale and this result is independent of the resale mechanism.

One of the bidders in the model is a global bidder for whom the exposure problem is a concern. Given our fixed resale mechanism, resale can only partially mitigate the exposure problem for the global bidder because resale does not take place when the objects are overly diffused to the local bidders after the auction but it is possible for the global bidder to resell the objects when the objects are overly concentrated in his hands.

Resale affects how the bidders behave in the auction. When the possibility of resale exists, the bidders anticipate resale and bid accordingly during the auction. Given our fixed resale mechanism, under some conditions, equilibria in which the bidders use fully separating strategies in the auction do not exist. This result could be driven by the fact that a loser in the auction gets a payoff of 0 after the resale stage because his value has been revealed in the auction. While information revelation can improve allocation efficiency, it is detrimental to the losers who may otherwise get positive payoffs.

Consequently, the improvement of efficiency that occurs in the resale stage after the losers reveal their information during the auction may be more difficult to achieve because it may still be uncertain after the auction whether resale should take place and, if resale should take place, what prices should be offered in order to ensure an efficient outcome.

Are there resale mechanisms that would result in fully efficient allocation? We study this problem using a mechanism design approach in a related paper.

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Chapter 3

Speculators in simultaneous ascending auctions

3.1 Introduction

This paper investigates the effects of resale on a simultaneous ascending auction. A simultaneous ascending auction is an auction format that allows a seller of multiple heterogeneous objects to sell these objects simultaneously, yet separately. Each object is auctioned via an English auction; these English auctions are held simultaneously. Although there is only one seller, there is no coordination across the separate English auctions. This auction format was first adopted by the US Federal Communications Commission (FCC) in 1994 to sell electromagnetic spectrum licences. Since then, the simultaneous ascending auction has become one of the standard methods to conduct spectrum auctions in the US and around the world.

Since resale of spectrum licences has occurred[1] and bans on post-auction trade are difficult to enforce,[2] there may be speculators in the auctions who do not value the licences but hope to make a profit by reselling the licences won in the auctions to other bidders. Garratt and Tröger (2006) investigate how a speculator affects standard auctions with resale. In first-

[1]See Cramton (2004).
[2]See Hafalir and Krishna (2009).

price and Dutch auctions with resale, they conclude that speculators do not profit and that the opportunity to resell the good after the auction can be detrimental to efficiency. However, in second-price and English auctions, there are multiple equilibria: the efficient equilibrium in which bidders bid their values exists, along with inefficient equilibria in which the speculator wins the auction and makes a positive profit from resale. In uniform-price auctions with complete information and resale, Pagnozzi (2010) finds that bidders with high values may strictly prefer to let speculators win in the auction so that they can acquire some units at lower prices at the auctions. Garratt, Tröger, and Zheng (2009) construct a family of equilibria in which one designated bidder wins the good without any competition; the winner then chooses an arbitrary resale mechanism to divide the spoils from collusion among members of the bidding ring. Even a high-value bidder prefers collusion to value-bidding in the auction. Rather than mitigating inefficiency, the possibility of resale creates inefficiency in these equilibria because the efficiency is lower than the efficiency of the value-bidding equilibrium.

In such an auction with multiple objects, there may be bidders who value the objects as well. These bidders may be local bidders or global bidders. A local bidder values only one particular object, whereas a global bidder's payoff from winning his desired bundle of related objects is higher than the sum of the standalone values for the objects if those objects are complementary. This synergy may arise due to various reasons. Although complementarities between objects can give a global bidder a boost in payoff, they also create a dilemma for him. In a simultaneous ascending auction, when the price of an object is above its standalone value and the prices of the other objects in a global bidder's desired bundle are still uncertain, should he continue bidding or drop out? If he is an aggressive competitor and continues to bid in the auction, the complementary objects may turn out to be so expensive that he ends up getting a negative payoff from acquiring his desired bundle. However, if he chooses to be cautious and drops out now, he loses the opportunity to possibly acquire his desired bundle at a total price that gives him a positive payoff. The upshot of this is that the auction's efficiency may be adversely affected. This problem that each global bidder faces is known as the exposure problem.

Xu, Levin, and Ye (2012) study auctions with synergy and resale. In their model, there are two objects and two global bidders. The two objects are sold sequentially via second-price sealed bid auctions; each bidder only learns his value for the second object being auctioned after the first auction. After the two auctions, they consider two fixed resale mechanisms (monopoly and monopsony take-it-or-leave-it offers). They find that the resale mechanism has an impact on bidding strategies in the auctions. When the resale mechanism is a monopsony take-it-or-leave-it offer, resale always improves efficiency.

Filiz-Ozbay, Lopez-Vargas, and Ozbay (2015) use experiments to study multiple-object auctions with resale with multiple local bidders and a global bidder with private information. They consider two cases: a generalized Vickrey auction that allows package bidding followed by resale and simultaneous second-price auctions followed by resale. In both cases, they fix the resale mechanism (take-it-or-leave-it offers made by the winners). A generalized Vickrey auction followed by resale can allocate the objects efficiently at the auction stage. However, when the objects are auctioned via simultaneous second-price sealed bid auctions, the final allocation is inefficient even after post-auction trade.

In the previous chapter, we study the problem with local bidders and a global bidder who participate in a simultaneous ascending auction. We do not allow speculators in the auction and fix the resale mechanism. We find that resale does not completely mitigate the exposure problem for the global bidder because resale never takes place whenever the global bidder loses both objects inefficiently. However, the global bidder can resell the objects to the other bidders whenever he wins both objects inefficiently.

In this paper, we study the problem with the same bidders who participate in a simultaneous ascending auction. In contrast with the previous chapter, we allow speculators in the auction and look for efficient resale mechanisms rather than using a fixed resale mechanism. Williams (1999) extends the Myerson and Satterthwaite (1983) bargaining problem to a multilateral setting; we use this extension to find the appropriate resale mechanism. In some cases, even though the simultaneous ascending auction may grossly misallocate the objects by having the speculators win the objects, there exist efficient resale mechanisms that can rectify the situation and result in efficient outcomes.

3.2 Model

There are two objects that are denoted by A and B. There are two categories of bidders: local bidders who value only one object and global bidders for whom the two objects are complements. There are three bidders: 1, 2 and 3. Bidder 1 values only object A and bidder 2 values only object B. Their values for the objects, denoted by v_{1A} and v_{2B} respectively, are independently and identically distributed uniformly on $[0, 1]$. Bidders 1 and 2 are allowed to bid for the objects that they don't value as well, so they are speculators in the auctions for those objects. These speculators are players in the game, just like bidder 3 who is a global bidder. Bidder 3's standalone value for each object is 0. His value for the package of both objects, γ, is distributed uniformly on $[0, 2]$. The values v_{1A}, v_{2B} and γ are private information; everything else is commonly known. In addition, there is another player called the "social planner" whose role is to offer resale mechanisms to the bidders to maximize efficiency. Since the social planner is a player, he knows the prior beliefs.

The simultaneous ascending auction is modelled as in Zheng (2012). The two objects are auctioned via separate English auctions which start simultaneously. Each English auction is modelled by the "clock model" as in Albano, Germano, and Lovo (2001). In the separate English auctions, the prices p_A and p_B start at 0 and increase at the same exogenous speed. When a bidder drops out from an auction for a given object, the clock temporarily stops in both auctions, and all bidders have a chance to exit at the same price. Ties are broken in favour of a speculator (specifically, bidder 2 in the auction for object A and bidder 1 in the auction for object B). If the speculator is not involved, then ties are broken in favour of bidder 3 (the global bidder). The tie-breaking rule is crucial to the equilibrium construction because the feature that the speculators own the objects at the resale stage helps to relax the participation constraints of the bidders at the resale process as much as possible since the speculators have no use value for the objects they have won in the auction. When there is only one bidder left in the auction for object j, where $j \in \{A, B\}$, the object is then sold to the remaining bidder at the current price. The bidders' actions during the separate English auctions are commonly observed. Consequently, at the end of the simultaneous ascending auction, each bidder knows the identities of the winners and the prices paid by these winners to the auctioneer. In addition, in equilibrium, the social planner knows the bidders' strategies in the auction and observes the outcome of the auction.

We use a refinement of perfect Bayesian equilibrium as the solution concept in this paper. We consider pure strategies only. In the auction stage, each bidder decides the prices at which he should drop out of the English auctions. In each resale subgame, there may be multiple equilibria; the chosen equilibrium is the one that maximizes the expected social surplus.

After the simultaneous ascending auction, the social planner updates his beliefs and offers a resale mechanism that maximizes efficiency for the given beliefs, subject to interim individual rationality and incentive compatible constraints. In addition, the resale mechanism is assumed to be budget balanced. More details on the resale mechanism will be provided below.

Each bidder's payoff is equal to his value of the object(s) he eventually owns minus his total payment plus what he gets from participating in the resale mechanism.

3.3 Resale mechanisms

Since each bidder is allowed to bid for both objects, there are nine possible allocations after the simultaneous ascending auction.

1. Bidder 1 has both objects A and B.

2. Bidder 1 has object A; bidder 2 has object B.

3. Bidder 1 has object A; bidder 3 has object B.

4. Bidder 2 has object A; bidder 1 has object B.

5. Bidder 2 has both objects A and B.

6. Bidder 2 has object A; bidder 3 has object B.

7. Bidder 3 has object A; bidder 1 has object B.

8. Bidder 3 has object A; bidder 2 has object B.

9. Bidder 3 has both objects A and B.

Consequently, since all the three bidders participate in the resale mechanism, there may be multiple sellers or multiple buyers, depending on the object allocation after the simultaneous ascending auction.

Williams (1999) characterizes mechanisms that are efficient, incentive compatible and budget balanced that can be applied to such multilateral settings. Any such mechanism is interim payoff-equivalent for every bidder to a Groves mechanism: the expected payoff of a bidder with a given valuation is the same as his payoff in a Groves mechanism.

After the simultaneous ascending auction, let bidder i's value be uniformly distributed on $[a_i, b_i]$. Let $p_{ij}(\theta)$ be the probability that bidder i owns object j, where the types of the bidders are defined by θ. Then, the allocation after the resale stage is efficient if

$$(p^*_{1A}(\theta), p^*_{2B}(\theta), p^*_{3A}(\theta), p^*_{3B}(\theta)) = (1, 1, 0, 0)$$

whenever $v_{1A} + v_{2B} > \gamma$ and

$$(p^*_{1A}(\theta), p^*_{2B}(\theta), p^*_{3A}(\theta), p^*_{3B}(\theta)) = (0, 0, 1, 1)$$

whenever $v_{1A} + v_{2B} < \gamma$. In the Groves mechanism, transfers x_1, x_2 and x_3 will be defined in Lemmas 3.3.1-3.3.5 below. In the *basic Groves mechanism*, constants k_1, k_2 and k_3 in the formulae for the transfers are all equal to 0.

By applying Theorem 3 in Williams (1999), we can determine if efficient resale mechanisms exist.

Lemma 3.3.1 *Suppose one of the following cases holds in the simultaneous ascending auction.*

- *Bidder* 1 *wins both objects A and B.*
- *Bidder* 1 *wins object A and bidder* 3 *wins object B.*

Then, an efficient mechanism at the resale stage exists if and only if the following condition is satisfied.

$$\begin{aligned} & 2\mathbb{E}_\theta[\max\{v_{2B}, \gamma - v_{1A}\}] \\ \leq \ & \mathbb{E}_{\theta_{-1}}[\max\{v_{2B}, \gamma - b_1\}] + \mathbb{E}_{\theta_{-2}}[\max\{a_2, \gamma - v_{1A}\}] + \mathbb{E}_{\theta_{-3}}[\max\{v_{2B}, a_3 - v_{1A}\}] \end{aligned}$$

Proof Suppose one of the cases holds in the simultaneous ascending auction. Then, the non-monetary payoffs are $p_{1A}v_{1A} - v_{1A}$ for bidder 1, $p_{2B}v_{2B}$ for bidder 2 and $p_{3A}p_{3B}\gamma$ for bidder 3. The Groves transfers x_1, x_2 and x_3 that are taken from the bidders are therefore defined in the following way.[3]

$$\begin{aligned} x_1(\theta) &= -p^*_{2B}(\theta)v_{2B} - p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma + k_1 \\ x_2(\theta) &= -p^*_{1A}(\theta)v_{1A} + v_{1A} - p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma + k_2 \\ x_3(\theta) &= -p^*_{1A}(\theta)v_{1A} + v_{1A} - p^*_{2B}(\theta)v_{2B} + k_3 \end{aligned}$$

[3]There are also Groves mechanisms where the constant k_i is replaced by a function that depends on valuations of the opponents of bidder i. Considering such mechanisms is not going to change the result because, from the interim perspective, they are equivalent to mechanisms with constants k_1, k_2 and k_3.

Bidder 1's interim utility in basic Groves mechanism is

$$\begin{aligned} U_1(v_{1A}) &= \mathbb{E}_{\theta_{-1}}[p^*_{1A}(\theta)v_{1A} - v_{1A} + p^*_{2B}(\theta)v_{2B} + p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma] \\ &= \mathbb{E}_{\theta_{-1}}[\max\{v_{2B}, \gamma - v_{1A}\}] \end{aligned}$$

which is non-increasing in v_{1A}, so it is minimized at $v_{1A} = b_1$. Define $\underline{U}_1 = \mathbb{E}_{\theta_{-1}}[\max\{v_{2B}, \gamma - b_1\}]$.

Bidder 2's interim utility in basic Groves mechanism is

$$\begin{aligned} U_2(v_{2B}) &= \mathbb{E}_{\theta_{-2}}[p^*_{2B}(\theta)v_{2B} + p^*_{1A}(\theta)v_{1A} - v_{1A} + p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma] \\ &= \mathbb{E}_{\theta_{-2}}[\max\{v_{2B}, \gamma - v_{1A}\}] \end{aligned}$$

which is non-decreasing in v_{2B}, so it is minimized at $v_{2B} = a_2$. Define $\underline{U}_2 = \mathbb{E}_{\theta_{-2}}[\max\{a_2, \gamma - v_{1A}\}]$.

Bidder 3's interim utility in basic Groves mechanism is

$$\begin{aligned} U_3(\gamma) &= \mathbb{E}_{\theta_{-3}}[p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma + p^*_{1A}(\theta)v_{1A} - v_{1A} + p^*_{2B}(\theta)v_{2B}] \\ &= \mathbb{E}_{\theta_{-3}}[\max\{v_{2B}, \gamma - v_{1A}\}] \end{aligned}$$

which is non-decreasing in γ, so it is minimized at $\gamma = a_3$. Define $\underline{U}_3 = \mathbb{E}_{\theta_{-3}}[\max\{v_{2B}, a_3 - v_{1A}\}]$.

Therefore, applying Theorem 3 in Williams (1999), an efficient mechanism exists if and only if the following holds:

$$\begin{aligned} & 2\mathbb{E}_{\theta}[\max\{v_{2B}, \gamma - v_{1A}\}] \\ \leq\ & \underline{U}_1 + \underline{U}_2 + \underline{U}_3 \\ =\ & \mathbb{E}_{\theta_{-1}}[\max\{v_{2B}, \gamma - b_1\}] + \mathbb{E}_{\theta_{-2}}[\max\{a_2, \gamma - v_{1A}\}] + \mathbb{E}_{\theta_{-3}}[\max\{v_{2B}, a_3 - v_{1A}\}]. \end{aligned}$$

∎

The main idea behind the result in Williams (1999) as applied to our settings is as follows. The left-hand side of the inequality in the statement of Lemma 3.3.1 is the sum of the expected transfers that have to be given to the bidders in the basic Groves mechanism. The right-hand side of that inequality is the sum of the interim expected payoffs of the most disadvantaged types for each bidder in the basic Groves mechanism. If the inequality as stated in Lemma 3.3.1 is satisfied, the bidders can be taxed by an amount sufficient to finance the Groves mechanism without violating any bidder's participation constraints.

Lemma 3.3.2 *Suppose bidder* 1 *wins object A and bidder* 2 *wins object B in the simultaneous ascending auction. Then, an efficient mechanism at the resale stage exists if and only if the following condition is satisfied.*

$$\begin{aligned}
& 2\mathbb{E}_{\theta}[\max\{0, \gamma - v_{1A} - v_{2B}\}] \\
\leq \; & \mathbb{E}_{\theta_{-1}}[\max\{0, \gamma - b_1 - v_{2B}\}] + \mathbb{E}_{\theta_{-2}}[\max\{0, \gamma - v_{1A} - b_2\}] + \mathbb{E}_{\theta_{-3}}[\max\{0, a_3 - v_{1A} - v_{2B}\}]
\end{aligned}$$

Proof Bidder 1 wins object A and bidder 2 wins object B in the auction, so the non-monetary payoffs are $p_{1A}v_{1A} - v_{1A}$ for bidder 1, $p_{2B}v_{2B} - v_{2B}$ for bidder 2 and $p_{3A}p_{3B}\gamma$ for bidder 3. The Groves transfers x_i are therefore defined as follow.

$$\begin{aligned}
x_1(\theta) &= -p^*_{2B}(\theta)v_{2B} + v_{2B} - p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma + k_1 \\
x_2(\theta) &= -p^*_{1A}(\theta)v_{1A} + v_{1A} - p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma + k_2 \\
x_3(\theta) &= -p^*_{1A}(\theta)v_{1A} + v_{1A} - p^*_{2B}(\theta)v_{2B} + v_{2B} + k_3
\end{aligned}$$

Bidder 1's interim utility in basic Groves mechanism is

$$\begin{aligned}
U_1(v_{1A}) &= \mathbb{E}_{\theta_{-1}}[p^*_{1A}(\theta)v_{1A} - v_{1A} + p^*_{2B}(\theta)v_{2B} - v_{2B} + p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma] \\
&= \mathbb{E}_{\theta_{-1}}[\max\{0, \gamma - v_{1A} - v_{2B}\}]
\end{aligned}$$

which is non-increasing in v_{1A}, so it is minimized at $v_{1A} = b_1$. Define $\underline{U}_1 = \mathbb{E}_{\theta_{-1}}[\max\{0, \gamma - b_1 - v_{2B}\}]$.

Bidder 2's interim utility in basic Groves mechanism is

$$\begin{aligned} U_2(v_{2B}) &= \mathbb{E}_{\theta_{-2}}[p^*_{2B}(\theta)v_{2B} - v_{2B} + p^*_{1A}(\theta)v_{1A} - v_{1A} + p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma] \\ &= \mathbb{E}_{\theta_{-2}}[\max\{0, \gamma - v_{1A} - v_{2B}\}] \end{aligned}$$

which is non-increasing in v_{2B}, so it is minimized at $v_{2B} = b_2$. Define $\underline{U}_2 = \mathbb{E}_{\theta_{-2}}[\max\{0, \gamma - v_{1A} - b_2\}]$.

Bidder 3's interim utility in basic Groves mechanism is

$$\begin{aligned} U_3(\gamma) &= \mathbb{E}_{\theta_{-3}}[p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma + p^*_{1A}(\theta)v_{1A} - v_{1A} + p^*_{2B}(\theta)v_{2B} - v_{2B}] \\ &= \mathbb{E}_{\theta_{-3}}[\max\{0, \gamma - v_{1A} - v_{2B}\}] \end{aligned}$$

which is non-decreasing in γ, so it is minimized at $\gamma = a_3$. Define $\underline{U}_3 = \mathbb{E}_{\theta_{-3}}[\max\{0, a_3 - v_{1A} - v_{2B}\}]$.

Therefore, applying Theorem 3 in Williams (1999), an efficient mechanism exists if and only if the following holds.

$$\begin{aligned} & 2\mathbb{E}_{\theta}[\max\{0, \gamma - v_{1A} - v_{2B}\}] \\ \leq\ & \mathbb{E}_{\theta_{-1}}[\max\{0, \gamma - b_1 - v_{2B}\}] + \mathbb{E}_{\theta_{-2}}[\max\{0, \gamma - v_{1A} - b_2\}] + \mathbb{E}_{\theta_{-3}}[\max\{0, a_3 - v_{1A} - v_{2B}\}] \end{aligned}$$

∎

Lemma 3.3.3 *Suppose one of the following cases holds in the simultaneous ascending auction.*

- *Bidder* 2 *wins object A and bidder* 1 *wins object B.*
- *Bidder* 2 *wins object A and bidder* 3 *wins object B.*
- *Bidder* 3 *wins object A and bidder* 1 *wins object B.*

Then, an efficient mechanism at the resale stage exists if and only if the following condition is satisfied.

$$\begin{aligned}
& 2\mathbb{E}_{\theta}[\max\{v_{1A} + v_{2B}, \gamma\}] \\
\leq \quad & \mathbb{E}_{\theta_{-1}}[\max\{a_1 + v_{2B}, \gamma\}] + \mathbb{E}_{\theta_{-2}}[\max\{v_{1A} + a_2, \gamma\}] + \mathbb{E}_{\theta_{-3}}[\max\{v_{1A} + v_{2B}, a_3\}]
\end{aligned}$$

Proof Suppose one of the cases holds in the simultaneous ascending auction. Then, the non-monetary payoffs are $p_{1A}v_{1A}$ for bidder 1, $p_{2B}v_{2B}$ for bidder 2 and $p_{3A}p_{3B}\gamma$ for bidder 3. The Groves transfers x_i are therefore defined as follow.

$$\begin{aligned}
x_1(\theta) &= -p^*_{2B}(\theta)v_{2B} - p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma + k_1 \\
x_2(\theta) &= -p^*_{1A}(\theta)v_{1A} - p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma + k_2 \\
x_3(\theta) &= -p^*_{1A}(\theta)v_{1A} - p^*_{2B}(\theta)v_{2B} + k_3
\end{aligned}$$

Bidder 1's interim utility in basic Groves mechanism is

$$\begin{aligned}
U_1(v_{1A}) &= \mathbb{E}_{\theta_{-1}}[p^*_{1A}(\theta)v_{1A} + p^*_{2B}(\theta)v_{2B} + p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma] \\
&= \mathbb{E}_{\theta_{-1}}[\max\{v_{1A} + v_{2B}, \gamma\}]
\end{aligned}$$

which is non-decreasing in v_{1A}, so it is minimized at $v_{1A} = a_1$. Define $\underline{U}_1 = \mathbb{E}_{\theta_{-1}}[\max\{a_1 + v_{2B}, \gamma\}]$.

Bidder 2's interim utility in basic Groves mechanism is

$$\begin{aligned}
U_2(v_{2B}) &= \mathbb{E}_{\theta_{-2}}[p^*_{2B}(\theta)v_{2B} + p^*_{1A}(\theta)v_{1A} + p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma] \\
&= \mathbb{E}_{\theta_{-2}}[\max\{v_{1A} + v_{2B}, \gamma\}]
\end{aligned}$$

which is non-decreasing in v_{2B}, so it is minimized at $v_{2B} = a_2$. Define $\underline{U}_2 = \mathbb{E}_{\theta_{-2}}[\max\{v_{1A} + a_2, \gamma\}]$.

Bidder 3's interim utility in basic Groves mechanism is

$$\begin{aligned} U_3(\gamma) &= \mathbb{E}_{\theta_{-3}}[p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma + p^*_{1A}(\theta)v_{1A} + p^*_{2B}(\theta)v_{2B}] \\ &= \mathbb{E}_{\theta_{-3}}[\max\{v_{1A} + v_{2B}, \gamma\}] \end{aligned}$$

which is non-decreasing in γ, so it is minimized at $\gamma = a_3$. Define $\underline{U}_3 = \mathbb{E}_{\theta_{-3}}[\max\{v_{1A}+v_{2B}, a_3\}]$.

Therefore, applying Theorem 3 in Williams (1999), an efficient mechanism exists if and only if the following holds.

$$\begin{aligned} & 2\mathbb{E}_{\theta}[\max\{v_{1A} + v_{2B}, \gamma\}] \\ \leq \quad & \mathbb{E}_{\theta_{-1}}[\max\{a_1 + v_{2B}, \gamma\}] + \mathbb{E}_{\theta_{-2}}[\max\{v_{1A} + a_2, \gamma\}] + \mathbb{E}_{\theta_{-3}}[\max\{v_{1A} + v_{2B}, a_3\}] \end{aligned}$$

■

Lemma 3.3.4 *Suppose one of the following cases holds in the simultaneous ascending auction.*

- *Bidder* 2 *wins both objects A and B.*
- *Bidder* 3 *wins object A and bidder* 2 *wins object B.*

Then, an efficient mechanism at the resale stage exists if and only if the following condition is satisfied.

$$\begin{aligned} & 2\mathbb{E}_{\theta}[\max\{v_{1A}, \gamma - v_{2B}\}] \\ \leq \quad & \mathbb{E}_{\theta_{-1}}[\max\{a_1, \gamma - v_{2B}\}] + \mathbb{E}_{\theta_{-2}}[\max\{v_{1A}, \gamma - b_2\}] + \mathbb{E}_{\theta_{-3}}[\max\{v_{1A}, a_3 - v_{2B}\}] \end{aligned}$$

Proof This is symmetric to Lemma 3.3.1. ■

Lemma 3.3.5 *Suppose bidder* 3 *wins both objects A and B in the simultaneous ascending auction. Then, an efficient mechanism at the resale stage exists if and only if the following* *condition is satisfied.*

$$
\begin{aligned}
& 2\mathbb{E}_{\theta}[\max\{0, v_{1A} + v_{2B} - \gamma\}] \\
\leq\ & \mathbb{E}_{\theta_{-1}}[\max\{0, a_1 + v_{2B} - \gamma\}] + \mathbb{E}_{\theta_{-2}}[\max\{0, v_{1A} + a_2 - \gamma\}] + \mathbb{E}_{\theta_{-3}}[\max\{0, v_{1A} + v_{2B} - b_3\}]
\end{aligned}
$$

Proof Bidder 3 wins both objects A and B in the auction, so the non-monetary payoffs are $p_{1A}v_{1A}$ for bidder 1, $p_{2B}v_{2B}$ for bidder 2 and $p_{1A}p_{2B}\gamma - \gamma$ for bidder 3. The Groves transfers x_i are therefore defined as follow.

$$
\begin{aligned}
x_1(\theta) &= -p^*_{2B}(\theta)v_{2B} - p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma + \gamma + k_1 \\
x_2(\theta) &= -p^*_{1A}(\theta)v_{1A} - p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma + \gamma + k_2 \\
x_3(\theta) &= -p^*_{1A}(\theta)v_{1A} - p^*_{2B}(\theta)v_{2B} + k_3
\end{aligned}
$$

Bidder 1's interim utility in basic Groves mechanism is

$$
\begin{aligned}
U_1(v_{1A}) &= \mathbb{E}_{\theta_{-1}}[p^*_{1A}(\theta)v_{1A} + p^*_{2B}(\theta)v_{2B} + p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma - \gamma] \\
&= \mathbb{E}_{\theta_{-1}}[\max\{0, v_{1A} + v_{2B} - \gamma\}]
\end{aligned}
$$

which is non-decreasing in v_{1A}, so it is minimized at $v_{1A} = a_1$. Define $\underline{U}_1 = \mathbb{E}_{\theta_{-1}}[\max\{0, a_1 + v_{2B} - \gamma\}]$.

Bidder 2's interim utility in basic Groves mechanism is

$$
\begin{aligned}
U_2(v_{2B}) &= \mathbb{E}_{\theta_{-2}}[p^*_{2B}(\theta)v_{2B} + p^*_{1A}(\theta)v_{1A} + p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma - \gamma] \\
&= \mathbb{E}_{\theta_{-2}}[\max\{0, v_{1A} + v_{2B} - \gamma\}]
\end{aligned}
$$

which is non-decreasing in v_{2B}, so it is minimized at $v_{2B} = a_2$. Define $\underline{U}_2 = \mathbb{E}_{\theta_{-2}}[\max\{0, v_{1A} + a_2 - \gamma\}]$.

Bidder 3's interim utility in basic Groves mechanism is

$$\begin{aligned} U_3(\gamma) &= \mathbb{E}_{\theta_{-3}}[p^*_{3A}(\theta)p^*_{3B}(\theta)\gamma - \gamma + p^*_{1A}(\theta)v_{1A} + p^*_{2B}(\theta)v_{2B}] \\ &= \mathbb{E}_{\theta_{-3}}[\max\{0, v_{1A} + v_{2B} - \gamma\}] \end{aligned}$$

which is non-increasing in γ, so it is minimized at $\gamma = b_3$. Define $\underline{U}_3 = \mathbb{E}_{\theta_{-3}}[\max\{0, v_{1A} + v_{2B} - b_3\}]$.

Therefore, applying Theorem 3 in Williams (1999), an efficient mechanism exists if and only if the following holds.

$$\begin{aligned} & 2\mathbb{E}_{\theta}[\max\{0, v_{1A} + v_{2B} - \gamma\}] \\ \leq\; & \mathbb{E}_{\theta_{-1}}[\max\{0, a_1 + v_{2B} - \gamma\}] + \mathbb{E}_{\theta_{-2}}[\max\{0, v_{1A} + a_2 - \gamma\}] + \mathbb{E}_{\theta_{-3}}[\max\{0, v_{1A} + v_{2B} - b_3\}] \end{aligned}$$

∎

Lemmas 3.3.1-3.3.5 cover all nine possible allocations after the simultaneous ascending auctions.

3.4 An efficient equilibrium

For collusion in English auctions, Garratt, Tröger, and Zheng (2009) construct a family of equilibria in which one designated bidder wins the auction without any competition. The spoils of collusion are then divided among members of the bidding ring through a resale mechanism that the reseller chooses. Without resale, such bidding strategies are weakly dominated; with resale, Garratt, Tröger, and Zheng (2009) show that such strategies are not weakly dominated.

I construct a similar equilibrium, where the speculators win the auction without any competition. Let $\beta > 0$. Suppose the bidders bid according to the following in the simultaneous ascending auction.

- Bidder 1 drops out of the auction for object A at 0 and drops out of the auction for object B at β.

- Bidder 2 drops out of the auction for object A at β and drops out of the auction for object B at 0.

- Bidder 3 drops out of both auctions at 0.

If a local bidder does not follow the prescribed strategies, the other local bidder and the global bidder believe that the local bidder's value v_{ij} is distributed uniformly on $[0, 1]$. If the global bidder does not follow the prescribed strategies, the local bidders believe that the global bidder's value γ is distributed uniformly on $[0, 2]$.

The resale mechanism is assumed to be budget balanced, so the following equation holds.

$$\mathbb{E}_\theta[x_1(\theta) + x_2(\theta) + x_3(\theta)] = 0 \tag{3.1}$$

If there exist multiple solutions, then an equitable solution is chosen as described below.

Let the net interim expected utility of bidder i with value v_i from participation in the resale mechanism be denoted by $\widetilde{U}_i(v_i)$. Individual rationality requires that $\widetilde{U}_i(v_i) \geq 0$ for every i and v_i. If there are multiple solutions, then we choose the solution that divides the extra surplus equally between the most disadvantaged types of the players (specifically, such that $\min_{v_{1A}} \widetilde{U_1}(v_{1A}) = \min_{v_{2B}} \widetilde{U_2}(v_{2B}) = \min_\gamma \widetilde{U_3}(\gamma)$).

If a fully efficient mechanism is feasible, then $\widetilde{U}_i(v_i) = U_i(v_i) - k_i$, where $U_i(v_i)$ is the net interim expected utility of player i with value v_i from participation in the basic Groves mechanism, and k_i is a constant transfer taken away from bidder i for balancing the budget. Note that $\min_{v_i} \widetilde{U}_i(v_i) = \underline{U}_i - k_i$, where $\underline{U}_i$ was defined in Lemmas 3.3.1-3.3.5 above. Thus individual rationality in this case can be written as

$$\underline{U}_i \geq k_i. \tag{3.2}$$

The constants k_1, k_2 and k_3 are chosen such that the extra surplus is divided equally among the three bidders so the following equation is satisfied.

$$\underline{U}_1 - k_1 = \underline{U}_2 - k_2 = \underline{U}_3 - k_3 \tag{3.3}$$

Then, we claim that an equilibrium where the bidders bid according to the prescribed strategies exists. Moreover, in this equilibrium, even though the simultaneous ascending auction allocates the objects to the speculators, efficiency is fully restored at the resale stage.

Proposition 3.4.1 *Let v_{1A} and v_{2B} be independently and identically distributed uniformly on $[0,1]$; let γ be distributed uniformly on $[0,2]$. Let $\beta \geq 2$. Then it is an equilibrium for the bidders to use the above prescribed strategies in the simultaneous ascending auction. The outcome of the overall game in this case is efficient.*

Proof First, it is necessary to establish the bidders' interim utilities in the resale stage. We need to consider the following cases in which k_{zi} denote constants where z indicates case z and i indicates bidder i.

1. Suppose bidder 1 wins both objects A and B. Suppose the beliefs are that v_{1A} and v_{2B} are independently and uniformly distributed on $[0,1]$ and γ is distributed uniformly on $[0,2]$.

   Then,

   $$2\mathbb{E}_\theta[\max\{v_{2B}, \gamma - v_{1A}\}] = \frac{19}{12}$$

   and

   $$\begin{aligned} \mathbb{E}_{\theta_{-1}}[\max\{v_{2B}, \gamma - 1\}] + \mathbb{E}_{\theta_{-2}}[\max\{0, \gamma - v_{1A}\}] + \mathbb{E}_{\theta_{-3}}[\max\{v_{2B}, -v_{1A}\}] &= \frac{7}{12} + \frac{7}{12} + \frac{1}{2} \\ &= \frac{20}{12}. \end{aligned}$$

   Therefore, by Lemma 3.3.1, an efficient mechanism at the resale stage exists.

Budget balance (Equation (3.1)) requires that $\frac{19}{12} = k_{11} + k_{12} + k_{13}$, whereas individual rationality (Inequality (3.2)) requires that $\frac{7}{12} \geq k_{11}$, $\frac{7}{12} \geq k_{12}$ and $\frac{1}{2} \geq k_{13}$. Finally, by the rule that the extra surplus is divided equally among the three bidders (Equation (3.3)), $\frac{7}{12} - k_{11} = \frac{7}{12} - k_{12} = \frac{1}{2} - k_{13}$. Therefore, $k_{11} = \frac{5}{9}$, $k_{12} = \frac{5}{9}$ and $k_{13} = \frac{17}{36}$.

Bidder 1's interim utility is

$$\begin{aligned} U_1(v_{1A}) &= \mathbb{E}_{\theta_{-1}}[\max\{v_{2B}, \gamma - v_{1A}\}] - k_{11} \\ &= \frac{1}{4}v_{1A}^2 - \frac{3}{4}v_{1A} + \frac{13}{12} - \frac{5}{9} \\ &= \frac{1}{4}v_{1A}^2 - \frac{3}{4}v_{1A} + \frac{19}{36}. \end{aligned}$$

Bidder 2's interim utility is

$$\begin{aligned} U_2(v_{2B}) &= \mathbb{E}_{\theta_{-2}}[\max\{v_{2B}, \gamma - v_{1A}\}] - k_{12} \\ &= \frac{1}{4}v_{2B}^2 + \frac{1}{4}v_{2B} + \frac{7}{12} - \frac{5}{9} \\ &= \frac{1}{4}v_{2B}^2 + \frac{1}{4}v_{2B} + \frac{1}{36}. \end{aligned}$$

Bidder 3's interim utility is

$$\begin{aligned} U_3(\gamma) &= \mathbb{E}_{\theta_{-3}}[\max\{v_{2B}, \gamma - v_{1A}\}] - k_{13} \\ &= \begin{cases} \frac{5}{6} - \gamma + \gamma^2 - \frac{1}{6}\gamma^3 - \frac{17}{36} & \text{if } \gamma > 1 \\ \frac{1}{2} + \frac{1}{6}\gamma^3 - \frac{17}{36} & \text{if } \gamma \leq 1 \end{cases} \\ &= \begin{cases} \frac{13}{36} - \gamma + \gamma^2 - \frac{1}{6}\gamma^3 & \text{if } \gamma > 1 \\ \frac{1}{36} + \frac{1}{6}\gamma^3 & \text{if } \gamma \leq 1 \end{cases}. \end{aligned}$$

2. Suppose bidder 1 wins object A and bidder 3 wins object B. Suppose the beliefs are that v_{1A} and v_{2B} are independently and uniformly distributed on $[0, 1]$ and γ is distributed

uniformly on $[0, 2]$.

Then, Lemma 3.3.1 applies and this case is the same as case 1 above.

3. Suppose bidder 2 wins object A and bidder 1 wins object B. Suppose the beliefs are that v_{1A} and v_{2B} are independently and uniformly distributed on $[0, 1]$; γ is distributed uniformly on $[0, 2]$.

Then,

$$2\mathbb{E}_{\theta}[\max\{v_{1A} + v_{2B}, \gamma\}] = \frac{31}{12}$$

and

$$\begin{aligned}\mathbb{E}_{\theta_{-1}}[\max\{v_{2B}, \gamma\}] + \mathbb{E}_{\theta_{-2}}[\max\{v_{1A}, \gamma\}] + \mathbb{E}_{\theta_{-3}}[\max\{v_{1A} + v_{2B}, 0\}] &= \frac{13}{12} + \frac{13}{12} + 1 \\ &= \frac{19}{6}.\end{aligned}$$

Therefore, by Lemma 3.3.3, an efficient mechanism at the resale stage exists.

Budget balance (Equation (3.1)) requires that $\frac{31}{12} = k_{3a1} + k_{3a2} + k_{3a3}$, whereas individual rationality (Inequality (3.2)) requires that $\frac{13}{12} \geq k_{3a1}$, $\frac{13}{12} \geq k_{3a2}$ and $1 \geq k_{3a3}$. Finally, by the rule that the extra surplus is divided equally among the three bidders (Equation (3.3)), $\frac{13}{12} - k_{3a1} = \frac{13}{12} - k_{3a2} = 1 - k_{3a3}$. Therefore, $k_{3a1} = \frac{8}{9}$, $k_{3a2} = \frac{8}{9}$ and $k_{3a3} = \frac{29}{36}$.

Bidder 1's interim utility is

$$\begin{aligned}U_1(v_{1A}) &= \mathbb{E}_{\theta_{-1}}[\max\{v_{1A} + v_{2B}, \gamma\}] - k_{3a1} \\ &= \frac{1}{4}v_{1A}^2 + \frac{1}{4}v_{1A} + \frac{13}{12} - \frac{8}{9} \\ &= \frac{1}{4}v_{1A}^2 + \frac{1}{4}v_{1A} + \frac{7}{36}.\end{aligned}$$

Bidder 2's interim utility is

$$\begin{aligned} U_2(v_{2B}) &= \mathbb{E}_{\theta_{-2}}[\max\{v_{1A} + v_{2B}, \gamma\}] - k_{3a2} \\ &= \frac{1}{4}v_{2B}^2 + \frac{1}{4}v_{2B} + \frac{13}{12} - \frac{8}{9} \\ &= \frac{1}{4}v_{2B}^2 + \frac{1}{4}v_{2B} + \frac{7}{36}. \end{aligned}$$

Bidder 3's interim utility is

$$\begin{aligned} U_3(\gamma) &= \mathbb{E}_{\theta_{-3}}[\max\{v_{1A} + v_{2B}, \gamma\}] - k_{3a3} \\ &= \begin{cases} \frac{4}{3} - \gamma + \gamma^2 - \frac{1}{6}\gamma^3 - \frac{29}{36} & \text{if } \gamma > 1 \\ 1 + \frac{1}{6}\gamma^3 - \frac{29}{36} & \text{if } \gamma \leq 1 \end{cases} \\ &= \begin{cases} \frac{19}{36} - \gamma + \gamma^2 - \frac{1}{6}\gamma^3 & \text{if } \gamma > 1 \\ \frac{7}{36} + \frac{1}{6}\gamma^3 & \text{if } \gamma \leq 1 \end{cases}. \end{aligned}$$

4. Suppose bidder 2 wins object A and bidder 3 wins object B. Suppose the beliefs are that v_{1A} and v_{2B} are independently and uniformly distributed on $[0, 1]$; γ is distributed uniformly on $[0, 2]$.

   Then, Lemma 3.3.3 applies and this case is the same as case 3 above.

5. Suppose bidder 3 wins object A and bidder 1 wins object B. Suppose the beliefs are that v_{1A} and v_{2B} are independently and uniformly distributed on $[0, 1]$; γ is distributed uniformly on $[0, 2]$.

   Then, Lemma 3.3.3 applies and this case is the same as 3 above.

6. Suppose bidder 2 wins both objects A and B. Suppose the beliefs are that v_{1A} and v_{2B} are independently and uniformly distributed on $[0, 1]$ and γ is distributed uniformly on $[0, 2]$.

Then, Lemma 3.3.4 applies and this is symmetric to case 1 above. Therefore, bidder 1's interim utility is

$$\begin{aligned} U_1(v_{1A}) &= \mathbb{E}_{\theta_{-1}}[\max\{v_{1A}, \gamma - v_{2B}\}] - k_{61} \\ &= \frac{1}{4}v_{1A}^2 + \frac{1}{4}v_{1A} + \frac{1}{36}. \end{aligned}$$

Bidder 2's interim utility is

$$\begin{aligned} U_2(v_{2B}) &= \mathbb{E}_{\theta_{-2}}[\max\{v_{1A}, \gamma - v_{2B}\}] - k_{62} \\ &= \frac{1}{4}v_{2B}^2 - \frac{3}{4}v_{2B} + \frac{19}{36}. \end{aligned}$$

Bidder 3's interim utility is

$$\begin{aligned} U_3(\gamma) &= \mathbb{E}_{\theta_{-3}}[\max\{v_{1A}, \gamma - v_{2B}\}] - k_{63} \\ &= \begin{cases} \frac{13}{36} - \gamma + \gamma^2 - \frac{1}{6}\gamma^3 & \text{if } \gamma > 1 \\ \frac{1}{36} + \frac{1}{6}\gamma^3 & \text{if } \gamma \leq 1 \end{cases}. \end{aligned}$$

7. Suppose bidder 3 wins object A and bidder 2 wins object B. Suppose the beliefs are that v_{1A} and v_{2B} are independently and uniformly distributed on $[0, 1]$ and γ is distributed uniformly on $[0, 2]$.

   Then, Lemma 3.3.4 applies and this is the same as case 6 above.

8. Suppose bidder 3 wins both objects A and B. Suppose the beliefs are that v_{1A} and v_{2B} are independently and uniformly distributed on $[0, 1]$ and that γ is uniformly distributed on $[0, 2]$.

   Then,

$$2\mathbb{E}_{\theta}[\max\{0, v_{1A} + v_{2B} - \gamma\}] = \frac{7}{12}$$

and

$$\begin{aligned}
&\mathbb{E}_{\theta_{-1}}[\max\{0, v_{2B} - \gamma\}] + \mathbb{E}_{\theta_{-2}}[\max\{0, v_{1A} - \gamma\}] + \mathbb{E}_{\theta_{-3}}[\max\{0, v_{1A} + v_{2B} - 2\}] \\
&= \frac{1}{12} + \frac{1}{12} + 0 \\
&= \frac{1}{6}.
\end{aligned}$$

Therefore, by Lemma 3.3.5, there does not exist an efficient mechanism at the resale stage. Bidder 3 of type $\gamma = 0$ will benefit the most from resale because he does not value the objects at all. Bidders 1 and 2 are not willing to pay more than their values v_{1A} and v_{2B} for objects A and B respectively. Thus the interim utility that bidder 3 can get from a mechanism at this stage is bounded above by 2.

Now we consider the entire game and show that the proposed strategies constitute a perfect Bayesian equilibrium.

(I) Consider a history where bidder 1 has won object B at price $p_B \in [0,\beta)$ while the auction for object A is still going on and the current price is $p_A \in [p_B,\beta]$. The case where bidder 2 has won object A at price $p_A \in [0,\beta)$ while the auction for object B is still going on is symmetric.

The strategies prescribe that bidders 1 and 3 drop out immediately and bidder 2 continue bidding until β.

If all three bidders follow the prescribed strategies, then bidder 2 wins object A and the beliefs are that v_{1A} and v_{2B} are independently and uniformly distributed on $[0, 1]$ and that γ is distributed uniformly on $[0, 2]$. By case 3, there exists an efficient mechanism at the resale stage. Then, in equilibrium, bidder 1's expected payoff is

$$U_1(v_{1A}) = \frac{1}{4}v_{1A}^2 + \frac{1}{4}v_{1A} + \frac{7}{36} - p_B, \tag{3.4}$$

bidder 2's expected payoff is

$$U_2(v_{2B}) = \frac{1}{4}v_{2B}^2 + \frac{1}{4}v_{2B} + \frac{7}{36} - p_A \tag{3.5}$$

and bidder 3's expected payoff is

$$U_3(\gamma) = \begin{cases} \frac{19}{36} - \gamma + \gamma^2 - \frac{1}{6}\gamma^3 & \text{if } \gamma > 1 \\ \frac{7}{36} + \frac{1}{6}\gamma^3 & \text{if } \gamma \leq 1 \end{cases}. \tag{3.6}$$

If bidder 1 drops out at price $p'_A \in [p_A, \beta]$ instead while bidders 2 and 3 follow the prescribed strategies, then bidder 2 still wins object A. The beliefs are unaffected, so bidder 1's expected payoff remains the same. If bidder 1 stays in the auction after the price is above β instead while bidders 2 and 3 follow the prescribed strategies, then bidder 1 wins object A. The beliefs remain the same. By case 1, there exists an efficient mechanism at the resale stage. Bidder 1's expected payoff is

$$(v_{1A} - \beta - p_B) + (\frac{1}{4}v_{1A}^2 - \frac{3}{4}v_{1A} + \frac{19}{36}) = \frac{1}{4}v_{1A}^2 + \frac{1}{4}v_{1A} + \frac{19}{36} - \beta - p_B$$

which is strictly less than Equation (3.4) since $\beta \geq 2$. Hence bidder 1 does not have profitable deviations.

If bidder 2 drops out immediately at p_A instead while bidders 1 and 3 drop out immediately as prescribed, bidder 2 wins object A since bidder 2 is a speculator in the auction for object A and ties involving speculators are broken in their favour. The beliefs are unaffected. By case 3, there exists an efficient mechanism at the resale stage. Bidder 2's expected payoff is the same as the expected payoff if he had followed the prescribed strategies. If bidder 2 stays in the auction after the price goes above p_A while bidders 1 and 3 drop out immediately as prescribed, then bidder 2 wins object A. The beliefs are unaffected and thus bidder 2's expected payoff is the same as if he had followed the prescribed strategies. Hence bidder 2 does not have profitable deviations.

If bidder 3 drops out at price $p'_A \in [p_A, \beta]$ instead while bidders 1 and 2 follow the prescribed strategies, bidder 2 wins object A. The beliefs are unaffected. By case 3, there exists an efficient mechanism at the resale stage. Bidder 3's expected payoff remains the same. If bidder 3 stays in the auction after the price is above β instead while bidders 1 and 2 follow the prescribed strategies, then bidder 3 wins object A. The beliefs are unaffected. By case 5, there exists an efficient mechanism at the resale stage. Bidder 3's expected payoff is

$$U_3(\gamma) = \begin{cases} -\beta + \frac{19}{36} - \gamma + \gamma^2 - \frac{1}{6}\gamma^3 & \text{if } \gamma > 1 \\ -\beta + \frac{7}{36} + \frac{1}{6}\gamma^3 & \text{if } \gamma \leq 1 \end{cases}$$

which is strictly less than Equation (3.6) since $\beta \geq 2$. Hence bidder 3 does not have profitable deviations.

(II) Consider a history such that both auctions are still going on and the current prices are $p_A = p_B = p \in [0, \beta]$. The strategies prescribe that bidder 1 drop out immediately of the auction for object A and continue until β in the auction for object B, bidder 2 continue until β in the auction for object A and drop out immediately of the auction for object B, and bidder 3 drop out immediately of both auctions.

Suppose all bidders follow the prescribed strategies. Then bidder 2 wins object A and bidder 1 wins object B. The beliefs are that v_{1A} and v_{2B} are independently and uniformly distributed on $[0, 1]$ and γ is uniformly distributed on $[0, 2]$. By case 3, there exists an efficient mechanism at the resale stage. Bidder 1's expected payoff is Equation (3.4) with p instead of p_B, bidder 2's expected payoff is Equation (3.5) with p instead of p_A, and bidder 3's expected payoff is Equation (3.6).

Now let us consider various deviations for the bidders. We have previously shown that the prescribed strategies are sequentially rational for the bidders in cases when only one of the auctions is still going on. We have only considered histories where either of the following is true:

(a) Bidder 1 has won object B and the auction for object A continues.

(b) Bidder 2 has won object A and the auction for object B continues.

It is straightforward to verify that these are the only relevant histories where only one auction continues that can be reached by unilateral deviations from the equilibrium strategy profile. Thus, for each bidder, we will consider all alternative drop out prices provided that both auctions are still going on; if only one of the auctions is still going on, we will take it that the bidders are following the prescribed strategies.

Suppose bidder 1 drops out of the auction for object A at $p'_A \in (p, \beta]$ and drops out of the auction for object B at $p'_B \geq p$ instead. Then bidder 2 wins object A and bidder 1 still wins object B. The beliefs are unaffected. By case 3, there exists an efficient mechanism at the resale stage. Bidder 1's expected payoff is the same as if he had followed the prescribed strategies. Therefore, this is not a profitable deviation.

Suppose bidder 1 drops out of the auction for object A at $p'_A > \beta$ and drops out of the auction for object B at $p'_B \geq p$ instead. Then bidder 1 wins both objects A and B. The beliefs are unaffected. By case 1, there exists an efficient mechanism at the resale stage. Bidder 1's expected payoff is

$$(v_{1A} - \beta - p) + (\frac{1}{4}v_{1A}^2 - \frac{3}{4}v_{1A} + \frac{19}{36}) = \frac{1}{4}v_{1A}^2 + \frac{1}{4}v_{1A} + \frac{19}{36} - \beta - p$$

which is strictly less than Equation (3.4) since $\beta \geq 2$. Therefore, bidder 1 does not have profitable deviations.

Suppose bidder 2 drops out of the auction for object A at $p'_A \geq p$ and drops out of the auction for object B at $p'_B \in (p, \beta]$ instead. Then bidder 2 still wins object A and bidder 1 wins object B. The beliefs are unaffected. By case 3, there exists an efficient mechanism at the resale stage. Bidder 2's expected payoff is the same as if he had followed the prescribed strategies. Therefore, this is not a profitable deviation.

Suppose bidder 2 drops out of the auction for object A at $p'_A \geq p$ and drops out of the auction for object B at $p'_B > \beta$ instead. Then bidder 2 wins both objects A and B. The beliefs are unaffected. By case 6, there exists an efficient mechanism at the resale stage. Bidder 2's expected payoff is

$$(v_{2B} - p - \beta) + (\frac{1}{4}v_{2B}^2 - \frac{3}{4}v_{2B} + \frac{19}{36}) = \frac{1}{4}v_{2B}^2 + \frac{1}{4}v_{2B} + \frac{19}{36} - p - \beta$$

which is strictly less than Equation (3.5) since $\beta \geq 2$. Therefore, bidder 2 does not have profitable deviations.

Suppose bidder 3 drops out of the auction for object A at $p'_A \in (p, \beta]$ and drops out of the auction for object B at $p'_B \in (p, \beta]$ instead. Then bidder 2 wins object A and bidder 1 wins object B. The beliefs are unaffected. By case 3, there exists an efficient mechanism at the resale stage. Bidder 3's expected payoff remains the same.

Suppose bidder 3 drops out of the auction for object A at $p'_A \in (p, \beta]$ and drops out of the auction for object B at $p'_B > \beta$ instead. Then bidder 2 wins object A and bidder 3 wins object B. The beliefs are unaffected. By case 4, there exists an efficient mechanism at the resale stage. Bidder 3's expected payoff is

$$U_3(\gamma) = \begin{cases} -\beta + \frac{19}{36} - \gamma + \gamma^2 - \frac{1}{6}\gamma^3 & \text{if } \gamma > 1 \\ -\beta + \frac{7}{36} + \frac{1}{6}\gamma^3 & \text{if } \gamma \leq 1 \end{cases}$$

which is strictly less than Equation (3.6) since $\beta \geq 2$.

Suppose bidder 3 drops out of the auction for object A at $p'_A > \beta$ and drops out of the auction for object B at $p'_B \in (p, \beta]$ instead. Then bidder 3 wins object A and bidder 1 wins object B. The beliefs are unaffected. By case 5, there exists an efficient mechanism at the resale stage. Bidder 3's expected payoff is

$$U_3(\gamma) = \begin{cases} -\beta + \frac{19}{36} - \gamma + \gamma^2 - \frac{1}{6}\gamma^3 & \text{if } \gamma > 1 \\ -\beta + \frac{7}{36} + \frac{1}{6}\gamma^3 & \text{if } \gamma \leq 1 \end{cases}$$

which is strictly less than Equation (3.6) since $\beta \geq 2$.

Suppose bidder 3 drops out of the auction for object A at $p'_A > \beta$ and drops out of the auction for object B at $p'_B > \beta$ instead. Then bidder 3 wins both objects A and B. The beliefs are unaffected. By case 8, there does not exist an efficient mechanism at the resale stage. An upper bound for bidder 3's expected payoff is

$$(\gamma - 2\beta) + 2$$

which is strictly less than Equation (3.6) since $\beta \geq 2$. Therefore, bidder 3 does not have profitable deviations.

Therefore, the prescribed strategies constitute an efficient perfect Bayesian equilibrium with resale. ∎

3.5 Conclusion

The simultaneous ascending auction is a standard method to sell multiple heterogeneous objects. This paper is an attempt to investigate the effects of resale on the allocation efficiency of the simultaneous ascending auction.

We study a model with two objects and three bidders. In the previous chapter, we find that there is no equilibrium in which the simultaneous ascending auction allocates the objects efficiently. This result holds with or without resale and this result is independent of the resale mechanism.

In this paper, we extend the model previously studied in the following ways.

- Rather than banning speculators, we allow speculators in the simultaneous ascending auction.
- Rather than using a fixed resale mechanism, there is a social planner whose role is to offer the bidders resale mechanisms that maximize efficiency.

Under some conditions, we find that it is possible for resale to restore efficiency. As shown in Chapter 2, it is impossible to achieve efficient allocation in all states of the world through the simultaneous ascending auction alone. Thus, post-auction misallocations have to be corrected through resale which takes place under residual asymmetric information that was not revealed in the course of the auction. The assumption about efficient resale mechanism ensures that the focus of resale is to achieve efficiency rather than to maximize the profit of the winners of the objects in the auction. The feature of the constructed equilibrium that the speculators own the objects at the start of the resale stage helps to relax participation constraints of the bidders in the resale process as much as possible because the speculators have no use value for the objects they have won in the auction.

Although we have some characteristics of the efficient resale mechanisms, we have not explored how some of these efficient resale mechanisms may be implemented. We leave this to future research.

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Chapter 4

Competing for a committee's approval

4.1 Introduction

The art of persuasion is practised early on in life, as when a child tries to convince his parents that he indeed deserves another cookie or ice cream. While research on the art of persuasion is well established in the economics literature, the focus of the literature has been on the persuasion of a single decision maker. Although there are many examples where the decision maker is indeed a single person, group decisions are becoming more prevalent. In Canada, committees in the House of Commons and the Senate hold considerable influence over legislative outcomes that potentially affect the entire country; academic appointments are made by committees or departments. Some persuasion strategies, such as "selective communication" (the sponsor of an idea or project talks to select members of the group only) and "persuasion cascades" (see next paragraph for details), can be relied upon only in group persuasion, so the dynamics of group persuasion can be much richer and more complex than what is already in the literature.

Caillaud and Tirole (2007) study persuasion cascades. Specifically, they show that, rather than trying to persuade every single member in a committee individually into adopting a project, the sponsor of the project stands to benefit by targeting key members in the committee and obtaining their approval before using their support for his project to convince the other members of the committee that his project is indeed worthwhile and beneficial for them. In addition, they show that the sponsor benefits more if the correlation between committee members' benefits is positive and higher. The size of the committee and how aligned the committee members' interests are to that of the sponsor also play a role in the sponsor's strategy and ability to get his project approved.

In many cases, however, there are multiple sponsors or projects competing for a committee's approval: for example, there are typically many applicants for a limited number of academic vacancies in universities. Hence, a very natural question that arises is, how would the presence of multiple sponsors change the game of group persuasion? How would the competition between sponsors play out?

We introduce a model with two senders and a two-member committee of receivers. Each sender is a sponsor of a project; the senders may disclose information about their projects to the committee. If committee members receive information from the project sponsors and wish to investigate further, they may do so at a cost. Finally, the committee members vote and approve one of the two competing projects.

Caillaud and Tirole (2007) assume that communication can take place only between the sender and committee members. The sender engineers the persuasion cascades by letting one committee member investigate and then revealing that investigator's support for the sender's project to the other committee member. Although Caillaud and Tirole find that the sponsor can obtain the same expected utility when communication channels between committee members exist, they acknowledge that their robustness result is fragile due to several reasons.

In contrast, we study cases where committee members can and cannot observe another committee member's investigation result separately. By studying a two-member committee with identical members and perfectly correlated benefits, we find that the ability to observe another committee member's investigation result has interesting implications. If committee members' benefits are perfectly correlated and each member is allowed to observe the result of an investigation carried out by another member, then this committee is equivalent to a dictatorial committee. On the other hand, if a committee member is not allowed to observe the result of an investigation carried out by another member, then this committee is equivalent to a committee with two identical members but with independent benefits. By studying the problem thus, we find different thresholds for the investigation cost in different circumstances.

Investigation of a project allows a committee member to learn his exact benefit from a project if that project is approved, so he knows not to vote for a project that will give him a negative payoff. However, investigation is not free, so a committee member chooses to investigate only if the cost of investigation is sufficiently low. Identical members of a two-member committee with independent benefits investigate less often than a dictator because their constraint on the investigation cost is more restrictive. The sponsor of the weaker project prefers a very low investigation cost, whereas his competitor benefits from a higher investigation cost and a committee that is equivalent to one with independent benefits.

4.2 Literature

This chapter is connected to several strands of literature. First, this chapter is related to a large literature on decision-making in committees. Persico (2004) studies a committee of identical agents who can each pay for a noisy signal of the actual state of the world; the agents cannot communicate prior to voting. Gerardi and Yariv (2003) allow for communication between committee members after they acquire the costly signals. Zhao (2018) studies a heterogeneous committee where each member can unilaterally acquire an imperfect signal about the state; all information acquired is publicly observable. In addition to the committee making the decision, this chapter builds a model with two senders competing for the committee's approval.

Secondly, there is some research on persuasion with multiple senders in the absence of investigation costs. For example, Gentzkow and Kamenica (2017) study games where multiple senders choose what information to communicate and find that the effect of competition between these senders on information revelation is ambiguous. In contrast, it is costly for committee members to investigate in this chapter.

Finally, this chapter is most closely related to Caillaud and Tirole (2007) and Perez-Richet (2012). Caillaud and Tirole (2007) build a one-sender/multi-receiver model of persuasion to study strategies that the sponsor of a proposal may employ to persuade a qualified majority of committee members into approving the proposal. Perez-Richet (2012) studies strategic information disclosure between multiple senders and a single receiver. Competing senders have information that is verifiable and equivocal; they decide whether to disclose this information to the receiver or not. Perez-Richet finds that, as competition increases, all candidates disclose information only if some of the candidates are unlikely to have favourable information. This chapter studies the case where there are two senders and two receivers, so there are essentially two sponsors competing for the committee's approval.

4.3 Model

The model in this chapter is largely based on the model in Caillaud and Tirole (2007). Here, however, there is an N-member committee that must decide which one of two proposed projects (A, B) to approve. These two projects are separately proposed by two sponsors, who are similarly denoted by A and B as well. Committee members simultaneously vote for project A or for project B; committee members are not allowed to abstain and the adopted project is determined by a voting rule to be defined later.

Sponsor A gets $s > 0$ if his project is adopted and 0 otherwise. Similarly, sponsor B gets $s > 0$ if his project is adopted and 0 otherwise. The sponsor's benefit s is common knowledge and each sponsor wants to maximize the expected probability that the project he has proposed is approved.

If project $j \in \{A, B\}$ is approved, committee member i gets r_{ij}, where $r_{ij} \in \{G, L\}$ and $L < 0 < G$. Committee member i's benefit from project j is a priori unknown to anyone; the realization of r_{ij} in case project j is implemented is not verifiable.

Committee member i can accept or reject project j on the basis of his prior $p_{ij} \equiv \Pr\{r_{ij} = G\}$. Otherwise, if sponsor j provides a detailed report to committee member i, the committee member may investigate the report and learn his exact benefit from project j. The sponsors do not have superior information and it is completely free for the sponsors to write and provide reports to the committee members. While the detailed reports provided by the sponsors do not contain information about r_{ij}, the reports contain sufficient details and data to let committee member i find out the consequences of the projects for his own benefit as long as he is willing to pay the investigation cost. If a committee member is given reports by both sponsors, he is restricted to investigating only one report, if he indeed chooses to investigate. Investigation is not verifiable and is therefore subject to moral hazard. The cost of investigation $c > 0$ is the same for all committee members. Another assumption about the investigation cost is that, if a committee member is indifferent between rubber-stamping (approving a project without having first investigated) and investigation, he would investigate. If committee member i does not investigate, he may use the correlation structure of benefits $\{r_{ij}\}_{i=1}^{N}$ to infer information about his own benefit based on the fact that another committee member has investigated and subsequently approved project j. All realized priors and the correlation structure of benefits $\{r_{ij}\}_{i=1}^{N}$ are assumed to be common knowledge.

For project $j \in \{A, B\}$, let $P_j = \Pr\{r_{1j} = r_{2j} = G\}$ denote the joint probability that both committee members benefit from project j. Different assumptions on P_j will be considered in this chapter: committee members' benefits could be independent, perfectly correlated, or positively correlated. Benefits are assumed to be independent across projects.

Caillaud and Tirole (2007) take a mechanism design approach in their analysis of the game with one sponsor and multiple committee members. However, it is not entirely obvious how a mechanism design approach would work for two sponsors and multiple committee members,[1] so a game is specified. Finally, the game proceeds as follow:

[1]Caillaud and Tirole (2007) note that building an equilibrium-mechanism-design methodology for competing sponsors is a very challenging endeavour. Yamashita (2010) studies a class of mechanism games with multiple principals and three or more agents; he notes that the result is ambiguous if there are only two agents.

1. Sponsors simultaneously decide to withhold information, to provide a detailed report to one of the committee members, or to send reports to both committee members.

2. Each committee member observes the decisions of the sponsors; if he receives at least one report, he decides whether to investigate or not.

3. We consider two possible scenarios:

   (a) Each committee member observes the outcome of the investigation carried out by the other committee member. (See Section 4.6.1.)

   (b) Committee members do not observe the outcomes of other members' investigations. (See Section 4.6.2.)

4. If there is only one member in the committee, he approves either project A or project B. If there are two members in the committee, they simultaneously vote for either project A or project B; the decision rule from the vote is as follows.

		Member 2: A	Member 2: B
Member 1	A	A	$\frac{1}{2}A + \frac{1}{2}B$
	B	$\frac{1}{2}A + \frac{1}{2}B$	B

Table 4.1: Voting rule of the committee.

   That is, if both committee members vote for project j, then project j is approved. Otherwise, they toss a fair coin to determine which project to approve.

We use subgame perfect equilibrium with refinements as the solution concept in this chapter. We consider pure strategies only. If a committee member receives reports from both sponsors A and B and is indifferent between investigating project A and investigating project B, he tosses a fair coin to decide which project to investigate. Furthermore, in cases where both committee members are given reports but only one investigates, the committee members toss a fair coin to determine which committee member investigates. This is done such that there is efficiency at the investigation stage.

4.4 Dictator

First, consider the case where the decision-making committee consists of only one person. The dictator has priors $p_A = Pr\{G_A\}$ and $p_B = Pr\{G_B\}$, where G_A is defined as the dictator getting G from the project proposed by sponsor A and G_B is defined as the dictator getting G from the project proposed by sponsor B. (Since there is only one member, the subscript i denoting the committee member is dropped in this section.)

When rubber-stamping project j without investigation, the expected benefit to the decision maker is $p_jG + (1 - p_j)L$. The decision maker votes for whichever project gives him the higher expected benefit, so he votes according to his priors. If the decision maker is given a detailed report on project A and he chooses to investigate project A, his expected benefit is

$$p_AG + (1 - p_A)[p_BG + (1 - p_B)L] - c = (p_A + p_B - p_Ap_B)(G - L) + L - c.$$

That is, he realizes, as a result of his investigation, if he would get G or L from project A. Upon realizing that his benefit is G, he votes for project A; otherwise, he votes for project B even though he has no information about the project. His expected benefit from project B is $p_BG + (1 - p_B)L$. Since project A yields G with probability p_A and yields L with probability $(1 - p_A)$, his expected benefit is $p_AG + (1 - p_A)[p_BG + (1 - p_B)L] - c$ after taking into account the investigation cost. Similarly, if the decision maker is given a detailed report on project B and he chooses to investigate project B, his expected benefit is

$$p_BG + (1 - p_B)[p_AG + (1 - p_A)L] - c = (p_A + p_B - p_Ap_B)(G - L) + L - c.$$

Clearly, the decision maker is indifferent between investigating project A and investigating project B since his expected benefit is the same from investigating either project. Suppose that, if both sponsors provide detailed reports on the projects, the decision maker tosses a fair coin to decide which project to investigate.

Suppose $p_B > p_A$. (The case where $p_A > p_B$ is symmetric.) In this case, the decision maker investigates, if given the chance, if and only if his expected benefit from investigating a project is at least as large as his expected benefit from rubber-stamping project B:

$$[(p_A + p_B - p_A p_B)(G - L) + L - c] - [p_B(G - L) + L] \geq 0$$
$$\iff p_A(1 - p_B)(G - L) \geq c \quad (4.1)$$

If the investigation cost c is too high, the decision maker would rubber-stamp a project according to his priors rather than investigate any project, even if given the chance; thus, there is nothing the sponsors can do to affect the decision maker's choice of project. On the other hand, if the investigation cost c is sufficiently low, sponsors compete for the decision maker's approval. The sponsors' behaviour can then be captured by the following proposition.

Proposition 4.4.1 *Let $p_B > p_A$. Suppose the investigation cost c is sufficiently low, i.e. (4.1) is satisfied.*

1. *If $p_A + p_B > 1$, each sponsor provides the decision maker with a detailed report; the expected payoffs to the sponsors are $(\frac{1}{2}(1 + p_A - p_B)s, \frac{1}{2}(1 - p_A + p_B)s)$.*

2. *If $p_A + p_B < 1$, only sponsor A provides a detailed report to the decision maker; the expected payoffs to the sponsors are $(p_A s, (1 - p_A)s)$.*

Proof Since c is sufficiently low, the decision maker prefers to investigate when given the option. Sponsor j can then try to influence the decision maker's choice of project by providing a detailed report to the decision maker (denoted by Y_j) or not (denoted by N_j).

If both sponsors do not provide reports (N_A, N_B), then the decision maker cannot investigate even though he prefers to; hence, he votes according to his priors and approves project B since $p_B > p_A$ by assumption. Therefore, the sponsors' payoffs are $(0, s)$.

If sponsor A provides a report and sponsor B withholds information (Y_A, N_B), then the decision maker investigates project A since he prefers to investigate and has only one report to investigate. From the investigation, the decision maker learns if the project yields G or L. If the project yields G, he approves project A; if the project yields L, he approves project B. Therefore, he approves project A with probability p_A and approves project B with probability $(1 - p_A)$. The sponsors' payoffs are thus $(p_A s, (1 - p_A)s)$.

The other cases can be analyzed in a similar fashion.

		Sponsor B	
		N_B	Y_B
Sponsor A	N_A	$0,\ s$	$(1 - p_B)s,\ p_B s$
	Y_A	$p_A s,\ (1 - p_A)s$	$\frac{1}{2}(1 + p_A - p_B)s,\ \frac{1}{2}(1 - p_A + p_B)s$

Table 4.2: Payoffs of the sponsors in the benchmark case with a dictator making the decision.

If $p_A + p_B > 1$, Y_A is a dominant strategy for sponsor A. The Nash equilibrium of the game is (Y_A, Y_B), where both sponsors provide detailed reports to the decision maker.

On the other hand, if $p_A + p_B < 1$, N_B is a dominant strategy for sponsor B. The Nash equilibrium of the game is then (Y_A, N_B), where sponsor A provides a detailed report and sponsor B withholds information. $\blacksquare$

From Proposition 4.4.1, we see that the sponsor with the weaker project (in the eyes of the decision maker) gives information to the decision maker in every equilibrium; the sponsor with the stronger project either provides or withholds information. As such, if the sponsor with the stronger project withholds information, then the decision maker adopts the stronger project if and only if the weaker project fails to yield G, with probability $(1 - p_A)$. Since the objective of the sponsor is to maximize the expected probability that his project is approved, he can improve his chances of getting a positive benefit if he also provides information when p_B is high enough. On the other hand, if p_B is too low, then his chances of getting a positive benefit are improved by withholding information and simply relying on the weaker project yielding L.

This is different from the result found by Perez-Richet (2012), where the presence of weak candidates in the competition is required for full disclosure of information. The decision maker in the model studied by Perez-Richet (2012) prioritizes his investigations according to his priors, so strong candidates who do not disclose information incur a non-disclosure cost because the decision maker favours the strong candidates who provide information by investigating, and hence approving if the outcome is positive, those projects first.

Since the decision maker in this model is indifferent between investigating either project, the assumption is that he randomizes when deciding which project to investigate. According to Proposition 4.4.1, the stronger candidate chooses to withhold information when his opponent is very weak and disclose information when his opponent is relatively strong. Thus, it is rather the stronger opponents that lead to more information revelation here.

4.5 Two-member committee: identical members with independent benefits

Suppose there are two members in the committee ($N = 2$) with common priors ($p_{iA} = p_A, p_{iB} = p_B$ for all i). In this section only, assume that committee members' benefits $\{r_{ij}\}_{i=1}^2$ are independent for each $j \in \{A, B\}$. In particular, for $j \in \{A, B\}$, the joint probability that both committee members benefit from project j is $P_j = \Pr\{r_{1j} = r_{2j} = G\} = p_j^2$. Again, we consider the case with $p_B > p_A$ only, since the case where $p_A > p_B$ is symmetric.

If none of the committee members investigates, both members vote according to their priors. Since they have common priors, the project that has a higher probability of yielding G to the committee members is approved. With probability p_B, a committee member gets G; with probability $(1 - p_B)$, he gets L. Therefore, his expected benefit from project B is

$$p_B G + (1 - p_B)L = p_B(G - L) + L.$$

If committee member 1 investigates project A and committee member 2 does not investigate at all, then committee member 1 learns if he gets G or L from project A. Hence, he votes for A if he knows he will get G and votes for B otherwise. Therefore, he votes for A with probability p_A and votes for B with probability $(1 - p_A)$.

Committee member 2 has no information, so he votes according to his expected payoffs from projects, given the fact that committee member 1 has investigated. First, consider the case where he votes A. With probability p_A, both committee members vote for A; with probability $(1 - p_A)$, committee member 1 votes for B, so project A is approved with probability $\frac{1}{2}$ and project B is approved with probability $\frac{1}{2}$. His expected benefit is thus

$$p_A[p_A(G-L)+L] + (1-p_A)\left\{\frac{1}{2}[p_A(G-L)+L] + \frac{1}{2}[p_B(G-L)+L]\right\}$$
$$= \frac{1}{2}(p_A^2 + p_A + p_B - p_A p_B)(G-L) + L.$$

Now, consider the case where he votes B. With probability p_A, committee member 1 votes A, so project A is approved with probability $\frac{1}{2}$ and project B is approved with probability $\frac{1}{2}$; with probability $(1 - p_A)$, committee member 1 votes for B too, so B is approved. His expected benefit is thus

$$p_A\left\{\frac{1}{2}[p_A(G-L)+L] + \frac{1}{2}[p_B(G-L)+L]\right\} + (1-p_A)[p_B(G-L)+L]$$
$$= \left(\frac{1}{2}p_A^2 + p_B - \frac{1}{2}p_A p_B\right)(G-L) + L.$$

Since $p_B > p_A$, his expected benefit from voting B is higher. Therefore, he votes for project B. Since committee member 1 is investigating project A and voting for A if he gets G from the project and voting for B otherwise, after paying the investigation cost, his expected benefit is

$$p_A\left\{\frac{1}{2}G + \frac{1}{2}[p_B(G-L)+L]\right\} + (1-p_A)[p_B(G-L)+L] - c$$
$$= \left(\frac{1}{2}p_A + p_B - \frac{1}{2}p_A p_B\right)(G-L) + L - c.$$

If both committee members investigate project A, then

- with probability $P_A = p_A^2$, both get G from project A;

- with probability $p_A(1-p_A)$, committee member 1 gets G and committee member 2 gets L from project A;

- with probability $p_A(1-p_A)$, committee member 1 gets L and committee member 2 gets G from project A;

- with probability $(1-p_A)^2$, both get L from project A.

If a committee member gets G from project A, he votes for A; otherwise, he votes for B. According to the decision rule, a project is approved if both committee members vote for it, and the two projects are approved with equal probabilities if committee members disagree on which project to approve. Since they both investigate project A, they pay cost c each and have no information on project B. Hence, each committee member's expected benefit is

$$\begin{aligned} p_A^2 G + p_A(1-p_A)\left\{\frac{1}{2}G + \frac{1}{2}[p_B(G-L)+L]\right\} \\ + p_A(1-p_A)\left\{\frac{1}{2}L + \frac{1}{2}[p_B(G-L)+L]\right\} + (1-p_A)^2[p_B(G-L)+L] - c \\ = \left(\frac{1}{2}p_A^2 + \frac{1}{2}p_A + p_B - p_A p_B\right)(G-L) + L - c. \end{aligned}$$

The other cases can be analyzed similarly. Let $\Delta = G - L$. Denote no investigation by ϕ, investigation of project A by I_A, and investigation of project B by I_B. Then, the expected benefits can be summarized in the following table.

		Member 2		
		ϕ	I_A	I_B
Member 1	ϕ	$p_B\Delta + L$, $p_B\Delta + L$	$(\frac{1}{2}p_A^2 + p_B - \frac{1}{2}p_Ap_B)\Delta + L$, $(\frac{1}{2}p_A + p_B - \frac{1}{2}p_Ap_B)\Delta + L - c$	$\frac{1}{2}(p_A + p_B + p_B^2 - p_Ap_B)\Delta + L$, $(\frac{1}{2}p_A + p_B - \frac{1}{2}p_Ap_B)\Delta + L - c$
	I_A	$(\frac{1}{2}p_A + p_B - \frac{1}{2}p_Ap_B)\Delta + L - c$, $(\frac{1}{2}p_A^2 + p_B - \frac{1}{2}p_Ap_B)\Delta + L$	$(\frac{1}{2}p_A^2 + \frac{1}{2}p_A + p_B - p_Ap_B)\Delta + L - c$, $(\frac{1}{2}p_A^2 + \frac{1}{2}p_A + p_B - p_Ap_B)\Delta + L - c$	$(p_A + \frac{1}{2}p_B + \frac{1}{2}p_B^2 - p_Ap_B)\Delta + L - c$, $(\frac{1}{2}p_A^2 + \frac{1}{2}p_A + p_B - p_Ap_B)\Delta + L - c$
	I_B	$(\frac{1}{2}p_A + p_B - \frac{1}{2}p_Ap_B)\Delta + L - c$, $\frac{1}{2}(p_A + p_B + p_B^2 - p_Ap_B)\Delta + L$	$(\frac{1}{2}p_A^2 + \frac{1}{2}p_A + p_B - p_Ap_B)\Delta + L - c$, $(p_A + \frac{1}{2}p_B + \frac{1}{2}p_B^2 - p_Ap_B)\Delta + L - c$	$(p_A + \frac{1}{2}p_B + \frac{1}{2}p_B^2 - p_Ap_B)\Delta + L - c$, $(p_A + \frac{1}{2}p_B + \frac{1}{2}p_B^2 - p_Ap_B)\Delta + L - c$

Table 4.3: Payoffs of two identical committee members with independent benefits.

In this case, each committee member is indifferent between investigating project A and investigating project B, since investigating either would yield the same expected payoff. Each committee member investigates, if given the opportunity, if and only if his expected benefit from investigating a project is at least as large as his expected benefit from rubber-stamping project B; that is, investigating is a dominant strategy if and only if

$$\frac{1}{2}p_A(1-p_B)(G-L) \geq c. \tag{4.2}$$

Since the two committee members are identical and have common priors, both of them face the same constraint and would simultaneously agree to investigate (if given the opportunity) or not to investigate. Comparing inequality (4.2) with inequality (4.1) for the case of the dictator, we find that the constraint on c is more restrictive in the case for two committee members whose benefits are independent, so the two committee members do not investigate as often as the dictator.

If the investigation cost c does not satisfy the constraint above, both committee members prefer not to investigate, even if they receive detailed reports from the sponsors. Therefore, there is nothing a sponsor can do to increase his project's expected probability of approval by the committee. Since both committee members view project B more favourably, they approve project B.

Proposition 4.5.1 *Let $p_B > p_A$. Suppose the investigation cost c is sufficiently low, i.e. (4.2) is satisfied.*

1. *If $p_A + p_B > 1$, each sponsor gives detailed reports to both committee members; the expected payoffs to the sponsors are $(\frac{1}{2}(1+p_A-p_B)s, \frac{1}{2}(1-p_A+p_B)s)$.*
2. *If $p_A + p_B < 1$, only sponsor A gives committee members detailed reports; the expected payoffs to the sponsors are $(p_A s, (1-p_A)s)$.*

Proof Since c is sufficiently low, both committee members prefer to investigate. If a committee member is given detailed reports by both sponsors, he tosses a fair coin to decide which project to investigate since he is indifferent between investigating project A and investigating project B. Sponsor j has the following options: provide no information about his project (denote this strategy by ϕ_j), give a detailed report to committee member 1 (denote this by 1_j), give a detailed report to committee member 2 (denote this by 2_j), or give detailed reports to both committee members (denote this by $\{1,2\}_j$).

If both sponsors do not provide reports (ϕ_A, ϕ_B), committee members cannot investigate even though they prefer to. Thus, they vote according to their priors. Since they both view project B more favourably, project B is approved. Therefore, the sponsors' payoffs are $(0, s)$.

If sponsor A gives a detailed report to committee member 1 only and sponsor B withholds information $(1_A, \phi_B)$, committee member 1 investigates project A and committee member 2 is not allowed to investigate. Committee member 2 votes according to his priors and votes for project B. Upon investigation, committee member 1 learns his benefit from project A and votes accordingly: with probability p_A, he gets G and votes A, so committee members toss a fair coin to decide which project to approve; with probability $(1 - p_A)$, he gets L and votes B, so project B is approved. Therefore, project A is approved with probability $\frac{1}{2}p_A$, and the sponsors' payoffs are $(\frac{1}{2}p_A s, (1 - \frac{1}{2}p_A)s)$.

If sponsor A provides reports to both committee members and sponsor B withholds information $(\{1,2\}_A, \phi_B)$, then both committee members investigate project A since they prefer to investigate and have only one report to investigate. Each committee member learns if project A yields G or L for himself. If project A yields G, the committee member votes for project A; if project A yields L, the committee member votes for project B.

- With probability $P_A = p_A^2$, both get G from project A, so project A is approved.
- With probability $p_A(1 - p_A)$, committee member 1 gets G and committee member 2 gets L from project A, so project A is approved half the time.
- With probability $p_A(1 - p_A)$, committee member 1 gets L and committee member 2 gets G from project A, so project A is approved half the time.

- With probability $(1 - p_A)^2$, both get L from project A, so project B is approved.

The expected probability that project A is approved is thus

$$p_A^2 + \frac{1}{2} p_A (1 - p_A) + \frac{1}{2} p_A (1 - p_A) = p_A$$

and the expected probability that project B is approved is $(1 - p_A)$. The sponsors' payoffs are thus $(p_A s, (1 - p_A) s)$.

The other cases can be analyzed in a similar fashion.

		Sponsor B			
		ϕ	1_B	2_B	$\{1,2\}_B$
Sponsor A	ϕ	0, s	$\frac{1}{2}(1-p_B)s$, $\frac{1}{2}(1+p_B)s$	$\frac{1}{2}(1-p_B)s$, $\frac{1}{2}(1+p_B)s$	$(1-p_B)s$, $p_B s$
	1_A	$\frac{1}{2}p_A s$, $(1-\frac{1}{2}p_A)s$	$\frac{1}{4}(1+p_A-p_B)s$, $\frac{1}{4}(3-p_A+p_B)s$	$\frac{1}{2}(1+p_A-p_B)s$, $\frac{1}{2}(1-p_A+p_B)s$	$\frac{1}{4}(3+p_A-3p_B)s$, $\frac{1}{4}(1-p_A+3p_B)s$
	2_A	$\frac{1}{2}p_A s$, $(1-\frac{1}{2}p_A)s$	$\frac{1}{2}(1+p_A-p_B)s$, $\frac{1}{2}(1-p_A+p_B)s$	$\frac{1}{4}(1+p_A-p_B)s$, $\frac{1}{4}(3-p_A+p_B)s$	$\frac{1}{4}(3+p_A-3p_B)s$, $\frac{1}{4}(1-p_A+3p_B)s$
	$\{1,2\}_A$	$p_A s$, $(1-p_A)s$	$\frac{1}{4}(1+3p_A-p_B)s$, $\frac{1}{4}(3-3p_A+p_B)s$	$\frac{1}{4}(1+3p_A-p_B)s$, $\frac{1}{4}(3-3p_A+p_B)s$	$\frac{1}{2}(1+p_A-p_B)s$, $\frac{1}{2}(1-p_A+p_B)s$

Table 4.4: Payoffs for the two sponsors facing a committee with two identical members with independent benefits.

If $p_A + p_B > 1$, $\{1, 2\}_A$ is a dominant strategy for sponsor A. The Nash equilibrium of the game is $(\{1, 2\}_A, \{1, 2\}_B)$, where both sponsors provide detailed reports to both committee members.

However, if $p_A + p_B < 1$, ϕ_B is a dominant strategy for sponsor B. The Nash equilibrium of the game is $(\{1, 2\}_A, \phi_B)$, where sponsor A gives detailed reports to both committee members and sponsor B withholds information. $\blacksquare$

We noted earlier in the section that constraint (4.2) on the investigation cost c is more restrictive for a two-member committee with independent benefits than for a one-member committee (see constraint (4.1)). As a result, if c is very low, i.e. it satisfies (4.2), competing sponsors behave the same way and receive the same expected payoffs regardless of the composition and size of the committee. However, if c is in an intermediate range, i.e. it satisfies (4.1) but not (4.2), then identical committee members with independent benefits do not investigate, so they vote according to their priors and approve project B. Therefore, the sponsors' payoffs are $(0, s)$ when facing a committee with two identical members with independent benefits, rather than either $(\frac{1}{2}(1 + p_A - p_B)s, \frac{1}{2}(1 - p_A + p_B)s)$ or $(p_A s, (1 - p_A)s)$ which they would get when facing a one-member committee. When c is very high, committee members in both cases prefer not to investigate and they approve project B.

Committee size and composition are taken as given in this chapter. However, based on the analysis above, the sponsor with the stronger project would be most interested in influencing the investigation cost, and the composition and size of the committee. In particular, if c is in the intermediate range, he would prefer the committee to consist of two identical members with independent benefits rather than a dictatorial committee.

4.6 Two-member committee: identical members with perfectly correlated benefits

Suppose there are two members in the committee ($N = 2$) with common priors ($p_{iA} = p_A, p_{iB} = p_B$ for all i). Now, consider the case where the committee members' benefits are perfectly correlated. That is, for $j \in \{A, B\}$, the joint probability that both committee members benefit from project j is $P_j = \Pr\{r_{1j} = r_{2j} = G\} = p_j$. As before, suppose $p_B > p_A$, since the case where $p_A > p_B$ is symmetric.

If none of the committee members investigates, both members vote according to their priors. Since they have common priors, they approve project B since they view project B more favourably. With probability p_B, a committee member gets G; with probability $(1 - p_B)$, he gets L. Therefore, his expected benefit from project B is

$$p_B G + (1 - p_B)L = p_B(G - L) + L.$$

Since committee members' benefits are perfectly correlated, a committee member's ability to observe the outcome of the investigation carried out by the other committee member may affect his behaviour. Hence, we consider two scenarios:

1. Each committee member observes the outcome of the investigation carried out by the other committee member. (Section 4.6.1)

2. Committee members do not observe the outcomes of other members' investigations. (Section 4.6.2)

We note that it is unnecessary to consider these two scenarios separately when committee members' benefits are independently distributed.

4.6.1 Investigation results are observable

First, consider the case where each committee member observes the outcome of the investigation carried out by the other committee member. If a committee member investigates a project and gets G from the project, the other committee member knows that he gets G from the same project as well since benefits are perfectly correlated. Thus, they vote for the same project. On the other hand, if the committee member who investigates learns that his benefit from a project is L, the other committee member observes that he also gets L from that project, so they vote for the other project.

If committee member 1 investigates project A and committee member 2 does not investigate, then both committee members learn if they get G or L from project A. Since committee member 1 carries out the investigation, he has to pay c; on the other hand, committee member 2 observes the outcome of committee member 1's investigation, so he obtains the information without incurring a cost. With probability p_A, they get G from project A and approve project A; with probability $(1 - p_A)$, they get L from project A and approve project B. Since no one investigates project B, a committee member's expected benefit from project B is $p_B(G-L)+L$. Committee member 1's expected benefit is therefore

$$p_A G + (1 - p_A)[p_B(G - L) + L] - c = (p_A + p_B - p_A p_B)(G - L) + L - c.$$

Committee member 2's expected benefit is

$$p_A G + (1 - p_A)[p_B(G - L) + L] = (p_A + p_B - p_A p_B)(G - L) + L.$$

The other cases can be analyzed similarly. Let $\Delta = G - L$. Denoting no investigation by ϕ, investigation of project A by I_A, and investigation of project B by I_B, the expected benefits are summarized in the following table.

		Member 2		
		ϕ	I_A	I_B
Member 1	ϕ	$p_B\Delta + L$, $p_B\Delta + L$	$(p_A + p_B - p_A p_B)\Delta + L$, $(p_A + p_B - p_A p_B)\Delta + L - c$	$(p_A + p_B - p_A p_B)\Delta + L$, $(p_A + p_B - p_A p_B)\Delta + L - c$
	I_A	$(p_A + p_B - p_A p_B)\Delta + L - c$, $(p_A + p_B - p_A p_B)\Delta + L$	$(p_A + p_B - p_A p_B)\Delta + L - c$, $(p_A + p_B - p_A p_B)\Delta + L - c$	$(p_A + p_B - p_A p_B)\Delta + L - c$, $(p_A + p_B - p_A p_B)\Delta + L - c$
	I_B	$(p_A + p_B - p_A p_B)\Delta + L - c$, $(p_A + p_B - p_A p_B)\Delta + L$	$(p_A + p_B - p_A p_B)\Delta + L - c$, $(p_A + p_B - p_A p_B)\Delta + L - c$	$(p_A + p_B - p_A p_B)\Delta + L - c$, $(p_A + p_B - p_A p_B)\Delta + L - c$

Table 4.5: Payoffs for two identical committee members with perfectly correlated benefits when investigation results are observable.

Each committee member is indifferent between investigating project A and investigating project B. If

$$p_A(1 - p_B)(G - L) < c$$

the Nash equilibrium of the game is (No investigation, No investigation). Both committee members vote according to their priors and approve project B.

However, if

$$p_A(1 - p_B)(G - L) \geq c \tag{4.3}$$

the pure strategy Nash equilibria of the game are (No Investigation, Investigate A), (No Investigation, Investigate B), (Investigate A, No Investigation) and (Investigate B, No Investigation).

Proposition 4.6.1 *Let $p_B > p_A$. Suppose the investigation cost c is sufficiently low, i.e. (4.3) is satisfied.*

1. *If $p_A + p_B > 1$, each sponsor gives detailed reports to both committee members; the expected payoffs to the sponsors are $(\frac{1}{2}(1 + p_A - p_B)s, \frac{1}{2}(1 - p_A + p_B)s)$.*

2. *If $p_A + p_B < 1$, sponsor A is indifferent between giving information to committee member 1, committee member 2, or both committee members; sponsor B withholds information. The expected payoffs to the sponsors are $(p_A s, (1 - p_A)s)$.*

Proof Sponsor j has the following options: provide no information about his project (denote this strategy by ϕ_j), give a detailed report to committee member 1 (denote this by 1_j), give a detailed report to committee member 2 (denote this by 2_j), or give detailed reports to both committee members (denote this by $\{1, 2\}_j$).

If both sponsors do not provide information, committee members vote according to their priors. Since they both view project B more favourably, project B is approved. Therefore, the sponsors' payoffs are $(0, s)$.

If sponsor A gives a detailed report to committee member 1 only and sponsor B withholds information $(1_A, \phi_B)$, committee member 1 investigates project A and committee member 2 observes the investigation result. Since committee members' benefits are perfectly correlated, committee member 2 also votes for project A if project A yields G to committee member 1. Otherwise, both committee members vote for project B. Thus, project A is approved with probability p_A and project B is approved with probability $(1 - p_A)$. The sponsors' payoffs are $(p_A s, (1 - p_A)s)$.

If sponsor A gives information to both committee members and sponsor B withholds information $(\{1, 2\}_A, \phi_B)$, committee members play one of the pure strategy Nash equilibria where only one member investigates project A. Since committee members' benefits are perfectly correlated, if project A yields G to the investigating committee member, the other committee member also votes for project A. Otherwise, both committee members vote for project B. Thus, project A is approved with probability p_A and project B is approved with probability $(1 - p_A)$. The sponsors' payoffs are $(p_A s, (1 - p_A)s)$.

The other cases can be analyzed in a similar manner.

		Sponsor B			
		ϕ	1_B	2_B	$\{1,2\}_B$
	ϕ	0, s	$(1-p_B)s$, $p_B s$	$(1-p_B)s$, $p_B s$	$(1-p_B)s$, $p_B s$
Sponsor A	1_A	$p_A s$, $(1-p_A)s$	$\frac{1}{2}(1+p_A-p_B)s$, $\frac{1}{2}(1-p_A+p_B)s$	$\frac{1}{2}(1+p_A-p_B)s$, $\frac{1}{2}(1-p_A+p_B)s$	$\frac{1}{4}(3+p_A-3p_B)s$, $\frac{1}{4}(1-p_A+3p_B)s$
	2_A	$p_A s$, $(1-p_A)s$	$\frac{1}{2}(1+p_A-p_B)s$, $\frac{1}{2}(1-p_A+p_B)s$	$\frac{1}{2}(1+p_A-p_B)s$, $\frac{1}{2}(1-p_A+p_B)s$	$\frac{1}{4}(3+p_A-3p_B)s$, $\frac{1}{4}(1-p_A+3p_B)s$
	$\{1,2\}_A$	$p_A s$, $(1-p_A)s$	$\frac{1}{4}(1+3p_A-p_B)s$, $\frac{1}{4}(3-3p_A+p_B)s$	$\frac{1}{4}(1+3p_A-p_B)s$, $\frac{1}{4}(3-3p_A+p_B)s$	$\frac{1}{2}(1+p_A-p_B)s$, $\frac{1}{2}(1-p_A+p_B)s$

Table 4.6: Payoffs for two sponsors facing a committee with two identical members with perfectly correlated benefits and investigation results are observable.

If $p_A + p_B > 1$, $\{1,2\}_A$ is a dominant strategy for sponsor A. The Nash equilibrium of the game is $(\{1,2\}_A, \{1,2\}_B)$, where both sponsors provide detailed reports to both committee members.

However, if $p_A + p_B < 1$, ϕ_B is a dominant strategy for sponsor B. The Nash equilibria of the game are $(1_A, \phi_B)$, $(2_A, \phi_B)$ and $(\{1,2\}_A, \phi_B)$; sponsor A gives detailed reports to one of the committee members or both committee members, and sponsor B withholds information. ∎

Constraint (4.3) on the investigation cost c is the same as for a dictator in the committee (constraint (4.1)). Note that the results of Proposition 4.6.1 are almost identical to Proposition 4.4.1. Sponsor A is indifferent between providing information to committee member 1, committee member 2, or both committee members, because committee members are identical with perfectly correlated benefits and have the ability to observe results from investigations carried out by one another. Not surprisingly, under perfect correlation of benefits, a committee with two identical members who can observe each other's investigation result is equivalent to a committee with only one member.

Instead of a committee member being able to observe another committee member's investigation result, the same results can be obtained by allowing committee members to communicate before voting. Gerardi and Yariv (2003) find that, in large committees where members communicate prior to voting, each member has an incentive to save the investigation cost and benefit from the other members' investigations. In this chapter, even though there are only two committee members, there is also free riding of one committee member on the investigative effort of the other committee member. Since the benefits are perfectly correlated, free riding does not adversely affect the outcome because the free rider votes for the project that is in the interest of the committee member who investigated.

4.6.2 Investigation results are not observable

Now, suppose a committee member does not observe the outcome of the investigation carried out by the other committee member before voting.

If committee member 1 investigates project A and committee member 2 does not investigate, then committee member 1 learns if he gets G or L from project A. He votes for A if he knows he will get G and votes for B otherwise. Therefore, he votes for A with probability p_A and votes for B with probability $(1 - p_A)$.

Committee member 2 has no information, so he votes for the project that would give him a higher expected payoff. First, consider the case where he votes A. With probability p_A, both committee members vote for A; with probability $(1 - p_A)$, committee member 1 votes for B, so project A is approved with probability $\frac{1}{2}$ and project B is approved with probability $\frac{1}{2}$. His expected benefit is

$$p_A G + (1 - p_A)\left\{\frac{1}{2}L + \frac{1}{2}[p_B(G - L) + L]\right\} = (p_A + \frac{1}{2}p_B - \frac{1}{2}p_A p_B)(G - L) + L.$$

Now, consider the case where he votes B. With probability p_A, committee member 1 votes A, so project A is approved with probability $\frac{1}{2}$ and project B is approved with probability $\frac{1}{2}$; with probability $(1 - p_A)$, committee member 1 also votes for B, so B is approved. His expected benefit is thus

$$p_A\left\{\frac{1}{2}G + \frac{1}{2}[p_B(G - L) + L]\right\} + (1 - p_A)[p_B(G - L) + L]$$
$$= (\frac{1}{2}p_A + p_B - \frac{1}{2}p_A p_B)(G - L) + L.$$

Since $p_B > p_A$, his expected benefit from voting B is higher. Consequently, he votes for project B. Since committee member 1 is investigating project A and voting for A if he gets G from the project and voting for B otherwise, his expected benefit after paying the investigation cost is

$$p_A\left\{\frac{1}{2}G + \frac{1}{2}[p_B(G - L) + L]\right\} + (1 - p_A)[p_B(G - L) + L] - c$$
$$= (\frac{1}{2}p_A + p_B - \frac{1}{2}p_A p_B)(G - L) + L - c.$$

If both committee members investigate project A, then both get G with probability p_A and both get L with probability $(1 - p_A)$ from project A since benefits are perfectly correlated. Since they both investigate project A, they pay cost c each and have no information on project B. Hence, each committee member's expected benefit is

$$p_A G + (1 - p_A)[p_B(G - L) + L] - c = (p_A + p_B - p_A p_B)(G - L) + L - c.$$

The other cases can be analyzed similarly. Let $\Delta = G - L$. Denoting no investigation by ϕ, investigation of project A by I_A, and investigation of project B by I_B, the expected benefits are summarized in the following table.

		Member 2		
		ϕ	I_A	I_B
Member 1	ϕ	$p_B\Delta + L$, $p_B\Delta + L$	$(\frac{1}{2}p_A + p_B - \frac{1}{2}p_A p_B)\Delta + L$, $(\frac{1}{2}p_A + p_B - \frac{1}{2}p_A p_B)\Delta + L - c$	$(\frac{1}{2}p_A + p_B - \frac{1}{2}p_A p_B)\Delta + L$, $(\frac{1}{2}p_A + p_B - \frac{1}{2}p_A p_B)\Delta + L - c$
	I_A	$(\frac{1}{2}p_A + p_B - \frac{1}{2}p_A p_B)\Delta + L - c$, $(\frac{1}{2}p_A + p_B - \frac{1}{2}p_A p_B)\Delta + L$	$(p_A + p_B - p_A p_B)\Delta + L - c$, $(p_A + p_B - p_A p_B)\Delta + L - c$	$(p_A + p_B - p_A p_B)\Delta + L - c$, $(p_A + p_B - p_A p_B)\Delta + L - c$
	I_B	$(\frac{1}{2}p_A + p_B - \frac{1}{2}p_A p_B)\Delta + L - c$, $(\frac{1}{2}p_A + p_B - \frac{1}{2}p_A p_B)\Delta + L$	$(p_A + p_B - p_A p_B)\Delta + L - c$, $(p_A + p_B - p_A p_B)\Delta + L - c$	$(p_A + p_B - p_A p_B)\Delta + L - c$, $(p_A + p_B - p_A p_B)\Delta + L - c$

Table 4.7: Payoffs for two identical members with perfectly correlated benefits and investigation results are not observable.

Each committee member is indifferent between investigating project A and investigating project B, since investigating either would yield the same expected payoff. Investigating is a dominant strategy if and only if

$$\frac{1}{2}p_A(1 - p_B)(G - L) \geq c. \tag{4.4}$$

Comparing this constraint on c with inequality (4.1) from the case of the dictator, we find that the constraint on c is more restrictive in this case, so committee members do not investigate as often as the dictator; this constraint is the same as the constraint (4.2) for the case with two identical members with independent benefits.

If the investigation cost c does not satisfy the constraint above, then both committee members prefer not to investigate, even if they received detailed reports from the sponsors. Since both committee members view project B more favourably, they approve project B.

Proposition 4.6.2 *Let $p_B > p_A$. Suppose the investigation cost c is sufficiently low, i.e. (4.4) is satisfied.*

1. *If $p_A + p_B > 1$, each sponsor gives detailed reports to both committee members; the expected payoffs to the sponsors are $(\frac{1}{2}(1 + p_A - p_B)s, \frac{1}{2}(1 - p_A + p_B)s)$.*
2. *If $p_A + p_B < 1$, only sponsor A gives committee members detailed reports; the expected payoffs to the sponsors are $(p_A s, (1 - p_A)s)$.*

Proof Since c is sufficiently low, both committee members prefer to investigate. If a committee member is given detailed reports by both sponsors, he tosses a fair coin to decide which project to investigate since he is indifferent between investigating project A and investigating project B. Sponsor j has the following options: provide no information about his project (denote this strategy by ϕ_j), give a detailed report to committee member 1 (denote this by 1_j), give a detailed report to committee member 2 (denote this by 2_j), or give detailed reports to both committee members (denote this by $\{1, 2\}_j$).

If both sponsors do not provide reports (ϕ_A, ϕ_B), committee members cannot investigate even though they prefer to. Thus, they vote according to their priors. Since they both view project B more favourably, project B is approved. Therefore, the sponsors' payoffs are $(0, s)$.

Suppose sponsor A gives a detailed report to committee member 1 only and sponsor B withholds information $(1_A, \phi_B)$. Committee member 2 votes according to his priors and votes for project B. Upon investigation, committee member 1 learns his benefit from project A and votes accordingly: with probability p_A, he gets G and votes A, so committee members toss a fair coin to decide which project to approve; with probability $(1-p_A)$, he gets L and votes B, so project B is approved. Therefore, project A is approved with probability $\frac{1}{2}p_A$, and the sponsors' payoffs are $(\frac{1}{2}p_A s, (1-\frac{1}{2}p_A)s)$.

If sponsor A provides reports to both committee members and sponsor B withholds information $(\{1,2\}_A, \phi_B)$, then both committee members investigate project A since they prefer to investigate and have only one report to investigate. With probability p_A, both committee members learn that they get G from project A and approve project A; with probability $(1-p_A)$, both committee members learn that they get L from project A and approve project B. The sponsors' payoffs are $(p_A s, (1-p_A)s)$.

The other cases can be analyzed in a similar manner.

Sponsor A \ Sponsor B	ϕ	1_B	2_B	$\{1,2\}_B$
ϕ	0, s	$\frac{1}{2}(1-p_B)s$, $\frac{1}{2}(1+p_B)s$	$\frac{1}{2}(1-p_B)s$, $\frac{1}{2}(1+p_B)s$	$(1-p_B)s$, p_Bs
1_A	$\frac{1}{2}p_As$, $(1-\frac{1}{2}p_A)s$	$\frac{1}{4}(1+p_A-p_B)s$, $\frac{1}{4}(3-p_A+p_B)s$	$\frac{1}{2}(1+p_A-p_B)s$, $\frac{1}{2}(1-p_A+p_B)s$	$\frac{1}{4}(3+p_A-3p_B)s$, $\frac{1}{4}(1-p_A+3p_B)s$
2_A	$\frac{1}{2}p_As$, $(1-\frac{1}{2}p_A)s$	$\frac{1}{2}(1+p_A-p_B)s$, $\frac{1}{2}(1-p_A+p_B)s$	$\frac{1}{4}(1+p_A-p_B)s$, $\frac{1}{4}(3-p_A+p_B)s$	$\frac{1}{4}(3+p_A-3p_B)s$, $\frac{1}{4}(1-p_A+3p_B)s$
$\{1,2\}_A$	p_As, $(1-p_A)s$	$\frac{1}{4}(1+3p_A-p_B)s$, $\frac{1}{4}(3-3p_A+p_B)s$	$\frac{1}{4}(1+3p_A-p_B)s$, $\frac{1}{4}(3-3p_A+p_B)s$	$\frac{1}{2}(1+p_A-p_B)s$, $\frac{1}{2}(1-p_A+p_B)s$

Table 4.8: Payoffs for two sponsors facing a committee with two identical members with perfectly correlated benefits and investigation results are not observable.

If $p_A + p_B > 1$, $\{1,2\}_A$ is a dominant strategy for sponsor A. The Nash equilibrium of the game is $(\{1,2\}_A, \{1,2\}_B)$, where both sponsors provide detailed reports to both committee members.

However, if $p_A + p_B < 1$, ϕ_B is a dominant strategy for sponsor B. The Nash equilibrium of the game is $(\{1,2\}_A, \phi_B)$, where sponsor A gives detailed reports to both committee members and sponsor B withholds information. $\blacksquare$

Constraint (4.4) is the same as constraint (4.2) for a committee with two identical members with independent benefits. Note that the equilibria are also the same in both cases. To the sponsors, a committee with two identical members with independent benefits is equivalent to a committee with two identical members with perfectly correlated benefits if committee members cannot observe each other's investigation result. Intuitively, this makes sense because the inability to observe another committee member's investigation result leads to committee members acting independently, even though benefits are perfectly correlated. This is similar to committee members not being allowed to communicate before voting.

The sponsor with the weaker project prefers very low c so that committee members may be persuaded to investigate and approve his project. Thus, such a sponsor may be induced to dedicate resources to decrease the investigation cost for the committee members. In this chapter, we have assumed, as Caillaud and Tirole (2007) have, that sponsors are not allowed to bribe committee members. However, bribes to committee members, if allowed, may be a way to decrease the investigation cost that committee members incur.

On the other hand, the sponsor with the stronger project likes c to be at least in the intermediate range. Hence, he is unlikely to dedicate resources to bribe committee members even if he is allowed to. In addition, he prefers a committee with two identical members with either independent benefits or, if investigation results cannot be observed, perfectly correlated benefits.

In the cases considered thus far, the weaker sponsor always provides information whereas the strong sponsor withholds information if his opponent is rather weak and provides information if his opponent is strong. A more competitive environment induces more information revelation. This is in contrast with Perez-Richet (2012), where weaker candidates are required for full disclosure of information, and Gentzkow and Kamenica (2017), where the effect of competition on information revelation is ambiguous in general.

For the sponsor who is the a priori favourite, as the probability that his project yields G for the committee members increases, he switches from withholding information on his project to providing detailed reports to the committee members. Rather than relying on the opponent's project yielding L for the committee members, the a priori favourite sponsor allows the committee members to investigate his project if the probability of his project yielding G is sufficiently high. This is in contrast with Caillaud and Tirole (2007) where a sponsor relies on the committee to rubberstamp his project when the committee has sufficiently high priors about his project.

4.7 Two-member committee: general case

Let there be two members in the committee ($N = 2$) with diverse priors. Suppose that a committee member can observe the other committee member's investigation results. Committee member i has priors p_{iA} and p_{iB} about project A and project B respectively. Let committee members' benefits be positively correlated. In particular, for $j \in \{A, B\}$, if the joint probability that both committee members benefit from project j is $P_j = \Pr\{r_{1j} = r_{2j} = G\}$, assume that the Bayesian update of the prior on r_{ij} conditional on the other member benefiting from project j is larger than p_{ij}, i.e.

$$\widehat{p_{ij}} = \Pr\{r_{ij} = G | r_{kj} = G\} = \frac{P_j}{p_{kj}} > p_{ij}.$$

This assumption implies that the Bayesian update of the prior on r_{ij} conditional on the other member getting L from project j is $\widetilde{p_{ij}} = \Pr\{r_{ij} = G | r_{kj} = L\} = \dfrac{p_{ij} - P_j}{1 - p_{kj}} < p_{ij}$.

Suppose both committee members view project A more favourably: $p_{1A} > p_{1B}$ and $p_{2A} > p_{2B}$. By the assumptions on P_j, we know the following inequalities hold:

- $\widehat{p_{1A}} > p_{1A} > p_{1B} > \widetilde{p_{1B}}$
- $\widehat{p_{2A}} > p_{2A} > p_{2B} > \widetilde{p_{2B}}$

In addition, assume that each committee member has priors that are relatively close to each other such that the priors and updated beliefs are ordered in the following way:

- $p_{1B} > \widetilde{p_{1A}}$
- $\widehat{p_{1B}} > p_{1A}$
- $p_{2B} > \widetilde{p_{2A}}$
- $\widehat{p_{2B}} > p_{2A}$

If none of the committee members investigates, both members vote according to their priors. Since both committee members view project A more favourably, they approve project A. With probability p_{iA}, committee member i gets G; with probability $(1 - p_{iA})$, committee member i gets L. Therefore, committee member i's expected benefit from project A is

$$p_{iA}G + (1 - p_{iA})L = p_{iA}(G - L) + L.$$

If committee member 1 investigates project A and project member 2 does not investigate at all, committee member 1 learns if he gets G or L from project A and votes accordingly. Hence, he votes for project A with probability p_{1A} and votes for project B with probability $(1 - p_{1A})$.

Committee member 2's vote depends on which project gives him a higher expected payoff. With probability p_{1A}, committee member 1 votes for project A. If committee member 2 votes for project A too, his expected benefit is $\widehat{p_{2A}}(G - L) + L$. However, if committee member 2 votes for project B, project A is approved with probability $\dfrac{1}{2}$ and project B is approved with probability $\dfrac{1}{2}$. Therefore, his expected benefit is $\frac{1}{2}(\widehat{p_{2A}} + p_{2B})(G - L) + L$. Since $\widehat{p_{2A}} > p_{2B}$ by assumption, committee member 2 votes for project A. On the other hand, with probability $(1 - p_{1A})$, committee member 1 votes for project B. If committee member 2 votes for project

A, project A is approved with probability $\dfrac{1}{2}$ and project B is approved with probability $\dfrac{1}{2}$. Therefore, his expected benefit is $\frac{1}{2}(\widetilde{p_{2A}} + p_{2B})(G - L) + L$. However, if committee member 2 votes for project B too, project B is approved and his expected benefit is $p_{2B}(G - L) + L$. Since $p_{2B} > \widetilde{p_{2A}}$ by assumption, committee member 2 votes for project B.

Consequently, committee member 1's expected benefit is

$$p_{1A}G + (1 - p_{1A})[p_{1B}(G - L) + L] - c = (p_{1A} + p_{1B} - p_{1A}p_{1B})(G - L) + L - c$$

and committee member 2's expected benefit is

$$p_{1A}[\widehat{p_{2A}}(G - L) + L] + (1 - p_{1A})[p_{2B}(G - L) + L] = (P_A + p_{2B} - p_{1A}p_{2B})(G - L) + L.$$

Let $\Delta = G - L$. Denoting no investigation by ϕ_i, investigation of project A by I_{iA}, and investigation of project B by I_{iB}, the other cases are analyzed similarly and summarized in the following table.

		Member 2		
		ϕ	I_{2A}	I_{2B}
Member 1	ϕ	$p_{1A}\Delta + L$, $p_{2A}\Delta + L$	$(P_A + p_{1B} - p_{1B}p_{2A})\Delta + L$, $(p_{2A} + p_{2B} - p_{2A}p_{2B})\Delta + L - c$	$(P_B + p_{1A} - p_{1A}p_{2B})\Delta + L$, $(p_{2A} + p_{2B} - p_{2A}p_{2B})\Delta + L - c$
	I_{1A}	$(p_{1A} + p_{1B} - p_{1A}p_{1B})\Delta + L - c$, $(P_A + p_{2B} - p_{1A}p_{2B})\Delta + L$	$\frac{1}{2}(P_A + p_{1A} - p_{1A}p_{1B} - p_{1B}p_{2A} + 2p_{1B})\Delta$ $+L - c$, $\frac{1}{2}(P_A + p_{2A} - p_{1A}p_{2B} - p_{2A}p_{2B} + 2p_{2B})\Delta$ $+L - c$	$\frac{1}{2}(P_B + p_{1B} - p_{1A}p_{2B} - p_{1A}p_{1B} + 2p_{1A})\Delta$ $+L - c$, $\frac{1}{2}(P_A + p_{2A} - p_{1A}p_{2B} - p_{2A}p_{2B} + 2p_{2B})\Delta$ $+L - c$
	I_{1B}	$(p_{1A} + p_{1B} - p_{1A}p_{1B})\Delta + L - c$, $(P_B + p_{2A} - p_{1B}p_{2A})\Delta + L$	$\frac{1}{2}(P_A + p_{1A} - p_{1A}p_{1B} - p_{1B}p_{2A} + 2p_{1B})\Delta$ $+L - c$, $\frac{1}{2}(P_B + p_{2B} - p_{1B}p_{2A} - p_{2A}p_{2B} + 2p_{2A})\Delta$ $+L - c$	$\frac{1}{2}(P_B + p_{1B} - p_{1A}p_{2B} - p_{1A}p_{1B} + 2p_{1A})\Delta$ $+L - c$, $\frac{1}{2}(P_B + p_{2B} - p_{1B}p_{2A} - p_{2A}p_{2B} + 2p_{2A})\Delta$ $+L - c$

Table 4.9: Payoffs for two committee members with diverse priors and positively correlated benefits.

Each committee member is indifferent between investigating project A and investigating project B. Investigating is a dominant strategy for committee member 1 if and only if

$$p_{1B}(1 - p_{1A})(G - L) \geq c \tag{4.5}$$
$$\frac{1}{2}(p_{1A} - p_{1A}p_{1B} + p_{1B}p_{2A} - P_A)(G - L) \geq c \tag{4.6}$$
$$\frac{1}{2}(p_{1B} - p_{1A}p_{1B} + p_{1A}p_{2B} - P_B)(G - L) \geq c \tag{4.7}$$

Investigating is a dominant strategy for committee member 2 if and only if

$$p_{2B}(1 - p_{2A})(G - L) \geq c \tag{4.8}$$
$$\frac{1}{2}(p_{2A} - p_{2A}p_{2B} + p_{1A}p_{2B} - P_A)(G - L) \geq c \tag{4.9}$$
$$\frac{1}{2}(p_{2B} - p_{2A}p_{2B} + p_{1B}p_{2A} - P_B)(G - L) \geq c \tag{4.10}$$

Evidently, the constraints on the investigation cost c are more complicated for a multi-member committee with diverse priors. If c satisfies none of the above constraints, both committee members prefer not to investigate, even if they receive detailed reports from the sponsors. Since both committee members view project A more favourably, they approve project A.

Proposition 4.7.1 *Suppose the above assumptions on p_{1A}, p_{1B}, p_{2A}, p_{2B}, P_A and P_B hold. Suppose the investigation cost c is sufficiently low, i.e. (4.5)-(4.10) are satisfied. Also assume the following:*

- $p_{1A} + p_{1B} < 1$
- $p_{2A} + p_{2B} < 1$
- $p_{1A} + p_{2B} < 1$
- $p_{1B} + p_{2A} < 1$

Then, sponsor A withholds information and sponsor B provides a detailed report to the committee member who views project B more favourably. The sponsors' payoffs are $((1 - p_{iB})s, p_{iB}s)$*, where i is such that* $p_{iB} > p_{kB}$.

Proof Since c is sufficiently low, committee members prefer to investigate when given the choice. Sponsor j can then try to influence committee members into voting for project j by taking one of the following actions: withhold information (denoted by ϕ_j), give committee member 1 a detailed report (denoted by 1_j), give committee member 2 a detailed report (denoted by 2_j), or give both committee members detailed reports (denoted by $\{1, 2\}_j$).

If both sponsors withhold information (ϕ_A, ϕ_B), committee members can only vote based on their priors. Since both committee members view project A more favourably, project A is approved. Therefore, the sponsors' payoffs are $(s, 0)$.

Suppose sponsor A gives a detailed report to committee member 1 only and sponsor B withholds information $(1_A, \phi_B)$. Committee member 1 investigates project A and committee member 2 votes according to her updated beliefs. With probability p_{1A}, committee member 1 learns that project A yields G and votes for project A; committee member 2 votes for project A too since $\widehat{p_{2A}} > p_{2B}$. Therefore, project A is approved with probability p_{1A}. With probability $(1 - p_{1A})$, committee member 1 learns that project A yields L and votes for project B; committee member 2 votes for project B too since $p_{2B} > \widetilde{p_{2A}}$. Therefore, project B is approved with probability $(1 - p_{1A})$. The sponsors' payoffs are $(p_{1A}s, (1 - p_{1A})s)$.

The other cases are analyzed in a similar manner.

Sponsor B

Sponsor A	ϕ	1_B	2_B	$\{1,2\}_B$
ϕ	s, 0	$(1-p_{1B})s$, $p_{1B}s$	$(1-p_{2B})s$, $p_{2B}s$	$(1-\frac{1}{2}p_{1B}-\frac{1}{2}p_{2B})s$, $\frac{1}{2}(p_{1B}+p_{2B})s$
1_A	$p_{1A}s$, $(1-p_{1A})s$	$\frac{1}{2}(1+p_{1A}-p_{1B})s$, $\frac{1}{2}(1-p_{1A}+p_{1B})s$	$\frac{1}{2}(1+p_{1A}-p_{2B})s$, $\frac{1}{2}(1-p_{1A}+p_{2B})s$	$\frac{1}{4}(3+p_{1A}-p_{1B}-2p_{2B})s$, $\frac{1}{4}(1-p_{1A}+p_{1B}+2p_{2B})s$
2_A	$p_{2A}s$, $(1-p_{2A})s$	$\frac{1}{2}(1+p_{2A}-p_{1B})s$, $\frac{1}{2}(1-p_{2A}+p_{1B})s$	$\frac{1}{2}(1+p_{2A}-p_{2B})s$, $\frac{1}{2}(1-p_{2A}+p_{2B})s$	$\frac{1}{4}(3+p_{2A}-p_{2B}-2p_{1B})s$, $\frac{1}{4}(1-p_{2A}+p_{2B}+2p_{1B})s$
$\{1,2\}_A$	$\frac{1}{2}(p_{1A}+p_{2A})s$, $[1-\frac{1}{2}(p_{1A}+p_{2A})]s$	$\frac{1}{4}(1+p_{1A}+2p_{2A}-p_{1B})s$, $\frac{1}{4}(3-p_{1A}-2p_{2A}+p_{1B})s$	$\frac{1}{4}(1+2p_{1A}+p_{2A}-p_{2B})s$, $\frac{1}{4}(3-2p_{1A}-p_{2A}+p_{2B})s$	$\frac{1}{4}(2+p_{1A}+p_{2A}-p_{1B}-p_{2B})s$, $\frac{1}{4}(2-p_{1A}-p_{2A}+p_{1B}+p_{2B})s$

Table 4.10: Payoffs for two sponsors facing a two-member committee with diverse priors and positively correlated benefits.

If the four inequalities stated in the proposition hold, withholding information is a dominant strategy for sponsor A. The Nash equilibrium of the game is (ϕ_A, i_B), where i is such that committee member i's prior on project B is higher. That is, sponsor B gives a detailed report to committee member 1 if $p_{1B} > p_{2B}$, and vice versa. ∎

Proposition 4.7.1 examines a particular case of two competing sponsors persuading a two-member committee with diverse priors and positively correlated benefits. The sponsors' behaviours in equilibrium demonstrates that selective communication and persuasion cascades are indeed persuasion strategies that sponsors rely upon in group persuasion.

Clearly, there are many other cases to consider before we have a complete characterization of sponsors' behaviours when the committee consists of two members with diverse priors. We state the result of this case so that the reader has an indication of how the competition between two sponsors may play out, and leave the generalization of results to further research.

Although the results of Proposition 4.7.1 require many more assumptions on the priors, correlation structure and investigation cost, we note that the sponsor with the stronger project prefers to withhold information and to rely on committee members investigating and learning that the other project yields L when the priors on his own project are sufficiently low, as in earlier sections when we considered a dictatorial committee and a two-member committee with common priors.

This proposition also gives an indication of how non-trivial it is to completely analyze the group persuasion game with a general two-member committee and two competing sponsors.

4.8 Conclusion

Many decisions in large organizations are made by groups, but the economics literature has little to say about group persuasion. This chapter is an attempt to study some aspects of group persuasion with competing sponsors of projects.

By introducing a model with two senders and a N-member committee, where $N \in \{1, 2\}$, we find that a committee with two identical members with independent benefits is equivalent to a committee with two identical members with perfectly correlated benefits, when investigation results are not observed by others. On the other hand, a dictatorial committee is equivalent to a committee with two identical members who can observe each other's investigation result, if members' benefits are perfectly correlated. Identical members of a two-member committee with independent benefits face a more restrictive constraint on the investigation cost than a dictator, so they investigate projects less often. If the committee is equivalent to one with two identical members with independent benefits, rather than a dictatorial committee, the sponsor with the stronger project benefits. In addition, competing sponsors have different preferences on the investigation cost incurred by committee members: the sponsor with the weaker project prefers a very low investigation cost, whereas the sponsor with the stronger project prefers a high investigation cost.

There are still many open questions in this research field.

For a general two-member committee, there are too many different cases to analyze for this chapter. A complete characterization of sponsors' behaviours is still required.

In the model considered in this chapter, we specified a voting rule that is symmetric. Both committee members are pivotal in this voting model because each can increase the probability of approval for the project that he likes. Results may vary as the voting rule is changed. For example, if there is a status quo that yields 0 to every sponsor and every committee member, and this status quo is adopted whenever there is no unanimity in the committee, committee members may no longer be indifferent between project investigations. Since committee members' behaviours change, sponsors' behaviours may also change.

Caillaud and Tirole (2007) take a mechanism design approach to study group persuasion with one sponsor. In this chapter, we specified a game instead of taking a mechanism design approach, because it is not entirely obvious how a mechanism design approach would work with multiple sponsors and two committee members. Yamashita (2010) considers a class of mechanism games with multiple principals and three or more agents; he notes that the result is ambiguous if there are only two agents. Caillaud and Tirole also note that building an equilibrium-mechanism-design methodology for competing sponsors is a very challenging endeavour.

These and other open questions related to group persuasion are left to future research.

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Chapter 5

Conclusion

My thesis consists of three essays related to the problems of using microeconomic models to find optimal solutions in multilateral settings. The first two essays deal with the effects of resale after simultaneous ascending auctions. The third essay investigates the dynamics of group persuasion.

Chapter 2 deals with the challenges of allocating multiple heterogeneous objects efficiently where speculation by bidders is not allowed. Simultaneous ascending auctions have been used to allocate multiple heterogeneous objects, such as electromagnetic spectrum licences. As the objects are auctioned separately yet simultaneously, a global bidder faces the exposure problem. The upshot is that the allocation after a simultaneous ascending auction may be inefficient. I use a fixed resale mechanism to study the effects of resale and find an equilibrium in which resale improves the allocation efficiency relative to the benchmark equilibrium without resale. However, this fixed resale mechanism can only partially mitigate the exposure problem for the global bidder. This is because, with the fixed resale mechanism, resale never takes place when the objects are inefficiently allocated to the local bidders. However, when the objects are inefficiently allocated to the global bidder, he can resell the objects to the local bidders.

Chapter 3 shows that allowing speculation by local bidders in a simultaneous ascending auction is not necessarily detrimental to the allocation efficiency. Since bans on post-auction trade are difficult to enforce, some local bidders who are already participating in a simultaneous ascending auction may wish to make a profit through post-auction trade with other bidders. I look for the existence of efficient resale mechanisms that can be used at the resale stage after the simultaneous ascending auction. Even when the speculators win the objects at the auction, there exist efficient resale mechanisms that can restore efficiency. Therefore, if allocation efficiency is the desired outcome, restricting participation in the simultaneous ascending auction to bidders who have positive values for the objects only may not necessarily be optimal.

Chapter 4 investigates the dynamics of group persuasion which can be richer than the dynamics of persuading a single decision maker. While persuading a group, selective communication and persuasion cascades may be used. A committee decides to implement one of two projects. A sponsor of a project can provide information on his project to select committee members. If a committee member receives information from a sponsor, he can investigate that project at a cost and learn his own payoff from implementing that project. I consider cases where other committee members may or may not observe the results of his investigation. Then, the committee members vote which project to approve. I find that the a priori weaker sponsor always provides information to the committee, whereas the a prior favourite sponsor withholds information if the other project is weak enough and relies on the committee to rubberstamp his project. As the competition between sponsors gets stronger, there is more information disclosure from the sponsors to the committee members.

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