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Mathematics-for-Teachers (and Students)

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Abstract

What mathematics do elementary teachers need and how might such mathematics be provided in a teacher education program? In this paper, we discuss the development of a mathematics-for-teachers component for our elementary (K-8) preservice education program. Our mathematics-for-teachers program has evolved from an elective course for 20 preservice teachers, to 440 preservice teachers working in small groups in an auditorium setting, to a fully online component. The mathematics-for-teachers component immerses preservice teachers in mathematics experiences that many of them have never had, namely, experiences where they attend deeply to mathematical relationships and have opportunities to sense the pleasure of mathematical insight. As such, our primary goal is experiential therapy (Gadanidis & Namukasa, 2005), rather than content knowledge.

Recently, at an orientation assembly, we asked our in-coming group of

440 elementary preservice teachers how they felt about mathematics.

When asked if they loved mathematics, 15-20 hands went up. When asked

if they hated mathematics, a sea of hands filled the auditorium. As one

elementary preservice teacher commented, "I hated math. I absolutely

despised it. I still remember sitting at my desk in grade one. I was sobbing

quietly, because I was struggling a bit, and I didn't finish my math on

time. Thus I had to stay in at recess. Awful isn't it!" Another preservice

teacher said, "Math is like an iguana. As long as it blends into its

environment I don't mind it. But once I have to hold it I'm not so fond of

it." Given that most elementary (K-8) teachers have to teach mathematics

(in the province of Ontario), we have a responsibility to try to help them

change their outlook towards the subject. We're assuming that to do this,

we need to engage preservice teachers with doing mathematics, and not

just learning about pedagogy. We're also assuming that engaging them

with more school-like mathematics—the type of mathematics that turned

them off the subject in the first place—would not be the most effective

approach to take.

   In this paper, (1) we discuss the evolution of our mathematics-for-

teachers program, from an elective course for 20 preservice teachers, to

440 preservice teachers working in small groups in an auditorium setting,

to a fully online component; (2) we define what we mean by mathematics-

for-teachers and distinguish it from pedagogical content knowledge

(Shulman, 1987) and specialized content knowledge (Ball, Bass, Sleep, &

Thames, 2005); and (3) we elaborate on our approach for offering

mathematics-for-teachers in an online environment.

Evolution of Our Mathematics-for-Teachers Program

Most Ontario preservice teacher education programs are composed of 5

months of in-class instruction and 3 months of practicum experience. The

program for elementary (K-8) teachers must prepare them to teach all

subject areas and, consequently, this leaves little time for mathematics

education. Accordingly, Ontario elementary preservice teachers typically

receive 25-30 hours of mathematics education instruction.

   Prior to 2001, our Faculty of Education mathematics education

program for elementary pre-service teachers consisted of 8 two-hour

workshops (with approximately 28 pre-service teachers per class) and 9

hours of lectures (approximately 440 pre-service teachers in a large

auditorium). The lectures gave preservice teachers the mathematics

education theory that they would then experience and discuss in a more

hands-on approach in the workshops. In reviewing our program, we

realized that both students and instructors valued the workshops but

viewed the large lectures as not very effective. Our preference was to

replace the large lectures with more small-group workshops. However, it

has been difficult to do this, given monetary and logistic constraints in our

Faculty of Education. Replacing the 9 lecture hours with 9 additional

hours of workshops would necessitate more instructional time and space.

Our Faculty of Education, whose ideal capacity is 650 preservice teachers,

currently has about 850 preservice teachers (440 in K-8 and 410 in 9-12).

In 2001, as a compromise solution, we replaced the large lectures with 9

online modules accompanied by a structured online discussion, where

preservice teachers were organized in small discussion groups. The online

content came from the lecture notes. Unlike the lectures, the online

discussion offered preservice teachers the opportunity to discuss the

online content in small group settings. It also gave preservice teachers a

first-hand experience with online teacher education which is used with

increasing frequency in the school districts where they will be employed.

The online discussion was assessed which ensured participation by

preservice teachers. In the first year of implementation, instructors noticed

that most preservice teachers came to the workshops much better prepared

in terms of having read and thought about the course readings.

In our review of our mathematics education program, we felt that we

needed to add a component where pre-service teachers re-experienced

mathematics. That said, there was no time available to do this in the

existing structure. In 2003, one of the authors (Gadanidis, 2005) offered

an elective Mathematics Course to experiment with what a mathematics-

for-teachers component for elementary teachers might be like. The

Mathematics Course consisted of nine 2-hour classes, and it was offered to

20 pre-service teachers. In 2004, we were able to add 8 hours of large

group lectures without giving up the online content and discussion that

replaced the original lectures. That is, we in effect added 8 more hours of

large group contact time to our mathematics education program. We used

this time to offer a mathematics-for-teachers component, where 440

preservice teachers worked on doing and discussing mathematics in

(approximately 110) small groups in an auditorium setting. Each hour-

long session focused on one mathematics task. Most of the 8 mathematics

tasks employed came from the elective Mathematics Course taught in the

previous year. We purchased concrete materials for each session and

brought them into the auditorium in large containers, and preservice

teachers used paper plates to carry the materials they needed to their

groups.

The auditorium-based mathematics-for-teachers component had six

important characteristics. First, doing mathematics became the starting

point. Most elementary teachers have narrow views of what mathematics

is and what it means to do mathematics (Fosnot & Dolk 2001; McGowen

& Davis 2001a; McGowen & Davis 2001b). Fosnot and Dolk (2001, 159)

suggest that "teachers need to see themselves as mathematicians," and

towards this end we need to foster environments where they engage with

mathematics and construct mathematical meaning. Second, the

mathematics experiences for pre-service teachers were designed to be

interesting and challenging enough to capture their interest and

imagination and to offer the potential for mathematical insight and

surprise (Gadanidis, 2004). Third, a collaborative environment was fostered, where pre-service teachers worked in small groups. Fourth, reflection was fostered. In the last 5 minutes of each session, pre-service teachers took the time to write about what they *learned* and what they *felt* during the class. Their ideas were compiled into a single document under the headings of "learned" and "felt" (anonymously), and this was distributed and briefly discussed at the beginning of the next session. This helped preservice teachers see what others learned and how they felt when doing mathematics. The learned/felt activity also served the purpose of taking attendance. Fifth, between sessions, preservice teachers had access to online interactive explorations of the activities they worked on in the auditorium. They also had access to an online discussion where they could collaborate to better understand and extend the mathematics of each activity. Last, the culminating assessment activity of the math sessions was a Math Essay. In the last workshop of the course, each preservice teacher randomly received one of the math activities explored in the mathematics-for-teachers component and had 30 minutes to 'discuss' one or two of the following: different solution approaches, mathematical extensions, what they learned from the activity, or pedagogical implications.

   In 2005, we made a number of changes to the mathematics-for-teachers component. First, it was converted to a fully online component. The decision to do this was, in part, due to the challenge of running over

100 small groups in an auditorium setting and, in part, due to our desire to

explore what might be possible in a fully online environment. We

contemplated replacing the 8 auditorium mathematics sessions with in-

class mathematics sessions, but this would necessitate more instructional

time and space, making it unfeasible due to resource constraints. We also

contemplated splitting into 4 smaller auditorium groups, but still faced the

limitations of space and human resources. With a large online continuing

teacher education program (approximately 5,000 online students), we have

ample online learning resources, experience and interest in our Faculty of

Education. Second, we reduced the number of mathematics tasks from 8 to

4, to allow for a longer sustained focus on each task. Third, the online

mathematics activities were redesigned, becoming more comprehensive

and incorporating video as well as text, graphics and interactive content.

Third, we designed three different online Mathematics-for-Teachers

courses: Measurement and Geometry, Number, and Algebra. Organizing

the activities into mathematics curriculum strands allowed us to offer three

distinct courses through our Continuing Teacher Education Program to

inservice elementary teachers and also to parents of elementary school

children. The four activities of the Algebra course were the ones that were

incorporated into the mathematics-for-teachers component of our

preservice program, and offered to all preservice teachers during the

regular teacher education program. The Measurement and Geometry

course was offered prior to their teacher education program, in August, as

an optional course that they would pay for and would appear as a quarter-

credit on their transcript. The course was offered to inservice as well as

preservice teachers, and thirty-six teachers enrolled in this course. The

Number course will be offered following their teacher education program

as an optional credit. We should note that all of our activities do cut across

curriculum strands, thus an activity whose main focus is Algebra may also

integrate Measurement, Geometry and Number concepts. We are not

confident that the strand breakdown is the best organization or the

activities we use; however, the strands do correspond with the curriculum

strands used in sanctioned mathematics curriculum and assessment

documents in the province of Ontario and are also common to other

jurisdictions.

Mathematics-for-Teachers

Two decades ago, Shulman (1987) suggested that teacher education (and

research) had "a blind spot with respect to content" and the emphasis was

solely "on how teachers manage classrooms, organize activities, allocate

time and turns, structure assignments, ascribe praise and blame, formulate

the levels of their questions, plan lessons, and judge general student

understanding" (p. 8). Since then, there is growing interest among

mathematics educators in what mathematical pedagogical content

knowledge (MPCK) could encompass. While we think that there needs to

be a dialectical relationship between content and pedagogy, the examples

of MPCK that we have seen seem counterproductive as they typically

define what mathematics teachers need to learn by trivializing what students need to learn. For example, MPCK tends to be defined by saying that students need to know a mathematics concept like *prime* or *multiplication* in two ways perhaps, but a teacher needs to know it in more ways. Likewise, Ball (2003) suggests that "teachers need to know the same things that we would want any educated member of our society to know, but *much more* (more understanding of the insides of ideas, their roots and connections, their reasons and ways of being represented)" (emphasis in original).

    Many elementary school students and educated members of society, for instance, think of multiplication only in terms of repeated addition or worse still as *times* devoid of any deeper meaning. This is, unfortunately, the case even for people who have been successful at mathematics (Ball & Bass, 2003). Educators and researchers acknowledge that this is a result of inadequate mathematics teaching. Students who have experienced rich mathematics also do think about multiplication in terms of areas or rows by columns; they can multiply fractions by fractions and decimals by decimals meaningfully. To base conceptualization of MPCK on the deficit of what students have not been taught is not very helpful. Teachers and students can both have rich mathematical understandings and attend to mathematics in deep and connected ways. The distinction made by the proponents of MPCK is not as dramatic as they suggest, and consequently does not warrant a "special" mathematics for beginning teachers as a

starting point in mathematics teacher education. We believe that the

starting point of the mathematics education for *both* students and teachers

should be a sophisticated and deep exploration of mathematics of which

we will give an example in the section on online mathematics.

We also do not agree with the conception of Mathematical Knowledge

for Teaching (MKT) being developed by Ball et al. (2005). For example,

Ball et al. give the example of the mathematical task, shown in Figure 1,

and suggest that "To teach, being able to

Perform this calculation is necessary.

This is *common content knowledge*.

$$\begin{array}{r} 307 \\ \underline{-168} \end{array}$$

Figure 1. A subtraction task

But being able to carry out the procedure is not sufficient for teaching it."

They identify four distinct domains of mathematical knowledge for

teaching:

1. *common content knowledge* (calculating the answer to 307-168)

2. *specialized content knowledge* (analyzing calculation errors)

3. *knowledge of students and content* (identifying student

   thinking that might have produced such errors)

4. *knowledge of teaching and content* (recognizing which

   manipulatives would best highlight place-value features

   of the algorithm)

The implication seems to be that the last three domains distinguish

what teachers need to know from what students need to know. But let us

imagine a classroom situation where a student is solving the problem in

Figure 1 on the blackboard and makes a mistake. We understand that the

teacher is "analyzing the calculation error," "is identifying student

thinking that might have produced the error," and is "thinking about which

manipulatives (or other modeling tools) would best highlight place-value

features of the algorithm" so that the student might realize the error made

and be able to make sense of the formal procedure. But what are the

students doing? Are they thinking? What are they thinking about? What

should they be thinking about? We suggest that they should also be invited

to attend to the calculation error, making conjectures about the thinking

that might have produced the error, and they should be thinking about how

they might model all of this so as to communicate their thinking to their

peers. In fact, these types of thinking are expectations for students in many

reform curricula including the Ontario mathematics curriculum for K-8

(Ontario Ministry of Education, 2005). It is not uncommon, for instance,

to find a task in a textbook that asks students to analyze an error that

another student has made. Namukasa (2005) argues that students should

also be invited to attend to their own and to each other's mathematical

thinking processes. This is also what the metacognitive, error, and

interpretive analysis research is about.

　　Another problem with the example that Ball et al. use to illustrate their

conception of MKT is the nature of their focus on a traditional algorithm.

Their example is overly concerned with how the algorithm works and how

students should learn it, rather than also focusing on other procedures for

subtracting the two numbers, which would build on the personal

knowledge and imagination of students and not simply on the rigid and

arbitrary rules of standard algorithms. In the section that follows we share

an example of the mathematics-for-teachers that we have been focusing

on, the aim of which is to immerse preservice teachers in mathematics

experiences that many of them have never had, namely, experiences where

they attend deeply to mathematical relationships and have opportunities to

sense the pleasure of mathematical insight.

Mathematics-for-Teachers Online

> I felt lost at first as I struggled to remember math concepts from
> childhood and adolescence. I felt confused. What did a poem have to
> do with math? I was perplexed. Was there not only one answer to a
> mathematical question? I felt apprehensive. How would I discuss a
> mathematical concept that I did not fully understand? Then as I got into
> the swing of things, I felt more confident with my opinions, my
> answers and most importantly myself. I felt cheerful that I was
> experiencing math as a student and that I would hopefully be able to
> empathize with my future students. I felt happy that math instruction
> could be made to be engaging. Finally, I was giddy that I was thinking
> about math, actually thinking about math and not doing everything else
> to avoid it.

The first problem explored in the Mathematics-for-Teachers Algebra

Course was *Making 10.* Preservice teachers are asked to find missing

numbers in the equation __ + _ = 10, and then plot them as ordered pairs on

a coordinate grid. Pre- service teachers expressed surprise that the ordered

pairs lined up. "I had the 'aha' feeling when I saw the diagonal line pattern

on the graph. That was my favourite part." Pre-service teachers also

noticed that the graph of_ +__ = 10 (that is, x + y = 10) could be used as a

visual proof of 12 + - 2 = 10 and 11 + - 1= 10, since (12,-2) and (11,-1)

line up with (10,0), (9,1), (8,2) and (7,3), thus satisfying the equation x + y

= 10. They also explored equations whose graphs were parallel and then

those that were not parallel to x + y =10. Such mathematical connections

appeared to be pleasing to pre-service teachers. "I loved the

adding/graphing we did and how you could take problems and branch out

… it really makes something in my mind click." This problem was first

used with two classes of fourth grade students when one of the authors

(Gadanidis, 2004) was invited to do a lesson on missing numbers (that is,

solving equations like _ + 3 = 7 and 5 + _ = 12). Using the above activity,

students did solve a lot of missing number problems. Typical classroom

activities on this topic would have students complete several unrelated

missing number problems. By adding the constraint that the sum of the

numbers is constant we generate a mathematical relationship among

solutions. Students also explored ways of changing the equation x + y = 10

so that the pattern of plotted ordered pairs might slope in a different

direction or might be curved. Preservice teachers also used the online

activity shown in Figure 2 (Gadanidis, 2005), which was based on the

fourth grade activity, and which incorporated (1) a mathematical poem, (2)

video annotations that pose extension problems and offer pedagogical

insights, and (3) an interactive exploration of functions and their graphs. A

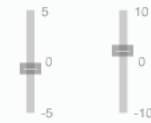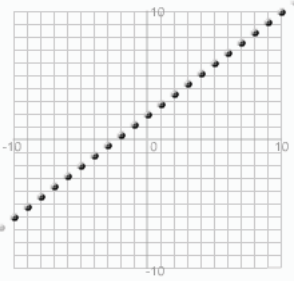mathematical poem is used as the centrepiece because, as the poet

Figure 2. The Pleasure of Making 10

Molly Peacock (1999) suggests, poetry is *screen-sized*. A poem is compact enough and cohesive enough to be held in one's mind as a whole. Poetry also makes use of image and metaphor, both of which help the reader sense deeper relationships to explore (Zwicky 2003).

Such mathematical experiences do offer preservice teachers opportunities to learn mathematical concepts. However, our primary goal is not to increase their mathematical content knowledge but to provide experiential therapy (Gadanidis & Namukasa, 2005). That is, our intention is to provide experiences that challenge and disrupt the mathematical discourse they have typically internalized through past school experiences, which is characterized by such views as: mathematics is a cold science— rather than an aesthetic, human experience (Gadanidis & Hoogland, 2003); mathematics is about learning procedures for getting correct

answers—rather than attending to and gaining insights about the

complexity of mathematical ideas (Gadanidis 2004); a good teacher makes

learning easy—rather than creating situations where students have to think

hard (Jonassen 2000); and, teaching should start with what a child already

knows and understands—rather than with what a child can imagine (Egan

1997).

   We have also developed an online discussion forum (Gadanidis, 2007)

whose features include (Wiki-style) editable postings, rich text postings,

and a draw tool with drawings embedded within postings. Such online

tools help enhance and enrich online mathematical communication. The

drawing in Figure 3 was created by a teacher in the online discussion of

the Geometry Course offered in August 2005, to illustrate how her legs

are positioned to form a triangle. In the text that accompanied the drawing,

the teacher discussed the three-dimensional figure she imagined being
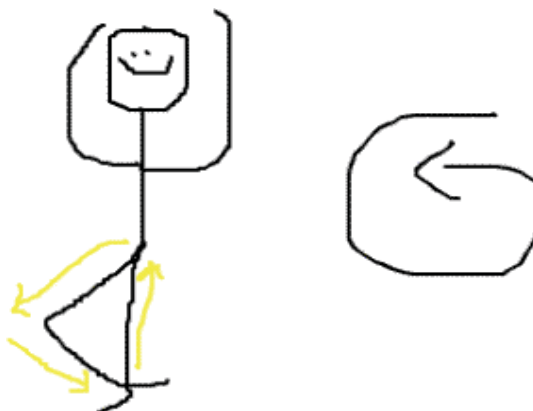
'cut' out of space as she twirls.

Figure 3. An online drawing

Looking Ahead

Pre-service teachers in the online Mathematics-for-Teachers: Measurement/Geometry Course (offered in the summer of 2005) overwhelmingly expressed that experiencing the Course helped change their view of mathematics and what it means to teach and learn mathematics. Similar views have been expressed by the preservice teachers in our other offerings of mathematics-for-teachers. In fact, the concept of doing mathematics as a therapeutic experience came from the original mathematics-for-teacher elective described above where preservice referred to the problem solving sessions as "math therapy." Statements such as the one below were common in preservice teachers' reflections on mathematics-for-teachers and its effect on their views and beliefs.

> Oh how the times have changed! In the few short months that have had the pleasure of "exploring big ideas in elementary school mathematics", my mathematical mindset has been overhauled. Now I feel *empowered by math*. I think mathematical experiences can change you. I was initially frustrated when I found that my classmates and myself were constantly being deprived of the solutions to mathematical situations that we were instructed to work through. The purpose of this type of an exercise soon became clear. It was the *process* of problem solving rather than the *accuracy* of the response that was being focused on. Soon I felt at ease in this pseudo-mathematical atmosphere. I started to discuss my ideas more openly with my classmates than I had initially. I had been programmed to withhold my ideas unless I was convinced that I knew the right answer, but when we knew that we were not going to be provided with the *correct* answer anyway, we were more open to discussing our individual strategies.

Despite such testimonials, however, a single course experience cannot create comprehensive or permanent change in teachers' perceptions of mathematics and mathematics teaching. Neither can we assume that such an experience will significantly affect teachers' classroom practice. Teaching is also greatly affected by accepted teaching practices in the wider school community (Buzeika, 1999; Ensor, 1998) and by conflicting priorities (Skott, 1999). However, such experiential therapy (Gadanidis & Namukasa, 2005) is an important starting point for change in teachers' perceptions and classroom practice (Gadanidis, Hoogland & Hill, 2002a, 2002b).

As we look back on the short history of developing a mathematics course for preservice teachers, we see three patterns emerging. First, our starting point has always been to involve preservice teachers in doing mathematics—mathematics where they have to attend deeply, mathematics that offers the potential of experiencing the pleasure of mathematical insight, mathematics that engages their imagination. Second, we have never viewed the mathematics we engage our preservice teachers with as "mathematics *only* for teachers"—we have viewed it as good mathematics, which is also good for students, and even parents. The three Mathematics-for-Teachers courses we have developed are also now offered through our Continuing Teacher Education Program to parents, as well as inservice teachers. And, in our classroom-based research projects, they are used with students. Last, we have been willing to experiment with

doing mathematics online. These patterns set the direction for our future

development of mathematics-for-teachers (and students!).

Acknowledgement

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