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Mathematics Tasks as Experiential Therapy for Elementary Preservice Teachers

In what unique ways can mathematics tasks contribute to pre-service teachers' understanding of subject matter and pedagogy? And what school mathematics tasks can usefully be included in a pre-service program? To contribute to answering these questions, we report on the selection and choice criteria for mathematics tasks that we use in an elementary pre-service program. We see these tasks as experiential therapy. We believe that for teachers to see mathematics, and consequently mathematics teaching and learning, in new ways then they need to personally experience mathematics in new ways. We discuss at length one of the tasks, the Consecutive Terms task. The evidence that we have gathered shows that teachers' engagement with such tasks may help them become better positioned to teach mathematics in what we refer to as "warm" ways, in that re-service teachers begin to revisit their mathematical experiences and start seeing mathematics as different from the stereotypical view of a cold, rigid, individual endeavour.

Key Words: Pre-service teacher education, content and pedagogy, mathematics teacher preparation, doing mathematics, mathematical thinking, mathematics tasks.

Background and Theoretical Framework

In what unique ways can mathematics tasks contribute to pre-service teachers' understanding of subject matter and pedagogy? And what school mathematics tasks can usefully be included in a pre-service program?

In our research we focus on doing school mathematics within a one-year after degree elementary pre-service program. Over the past three years we have added a school "mathematics tasks" component to the methods course. The goal is to immerse pre-service teachers in doing mathematics in which they are prompted to think mathematically, they deeply attend to school mathematics content and experience the pleasure of mathematical insight (Author 2 & Author 1, 2005).

Certain researchers claim that doing school mathematics tasks with pre-service teachers is a way of exploring the mathematics that teachers need to know (Ball, 2003) and encouraging them to be curious about mathematics (Simmt, Gordon, Davis & Towers, 2003). Emphasis on doing mathematics is central to our framework. For us doing mathematics moves beyond rules and procedures to include activities such as exploring patterns, seeking generalizations, and selecting appropriate tools, activities that encourage mathematical thinking. We view teacher participation in mathematics tasks as an all-at-once source of interruption of unhelpful school mathematics experiences, as motivation to learn mathematics and as modelling for teaching mathematics.

Developing the pedagogical understanding teachers need in teaching mathematics goes hand in hand with developing their own mathematical thinking (Wilson, Cooney & Stinson, 2005). Many elementary pre-service teachers' classroom experiences have been negative or the experiences, even when positive, have left the pre-service teachers with a narrow understanding of what doing mathematics involves (Cooney, 1999). There is a body of research on how teacher practices (Koehler & Grouws, 1992), knowledge (Fennema & Franke, 1992; Ball 1990), conceptions (Thompson, 1992), and belief systems (Cooney 1994) influence teaching. These researchers conceive that re-organization, interruption of the flow of experience is one of the major roles of teacher education courses. They present varied tasks as forms of intervention. We use mathematics tasks to offer pre-service teachers mathematics experiences that help them to become better positioned to learn and teach mathematics in what we refer to as warm ways, in that pre-service teachers begin see mathematics as different from the stereotypical view of a cold, rigid, individual endeavour.

Fennema and Franke (1992) observe that teachers are reflective and thoughtful individuals. We add that they are experiential and affective individuals. Their experiences of and feelings about doing, learning and teaching mathematics are key in shaping future experiences. We draw from the scant research on students' affect and meta-affect (see *Educational Studies in Mathematics*, Special Issue Affect in Mathematics, 63(2), 2006; McLeod, 1992). None of this research has looked at doing mathematics as experiential intervention or therapy. By experiential therapy[a] we mean that for teachers to see

[a] We derived the use of the word therapy from the way students came to commonly refer to the component as. In mathematics education literature this comes close to intervention of mathematics anxiety, in

mathematics, and consequently mathematics teaching and learning, in new ways then they need to personally experience mathematics in these new ways, and that this experience of doing mathematics is a form of therapy. In this paper we primarily focus on the selection and choice criteria for tasks.

Mathematics Tasks: Design and Use

Mathematics tasks trigger reflection and are a source of (re-) experiencing mathematics. In selecting and designing tasks we consider that a large number of our pre-service students confess not to be fond of mathematics and to be anxious about teaching mathematics. They have experienced mathematics as a cold activity. The challenge lies in selecting tasks that would be *helpful* in offering mathematical experiences that are flexible, imaginative, connected and insightful.

Our research considers a mathematical task to include problems that may be offered to students for mathematical exploration. Since learning is influenced by the tasks given (Mason, Watson & Zaslavsky, this issue), we specify that the tasks should be *variable entry* to encourage participation (Simmt, 2000); *non-routine* (Polya, 1945/1973), *non-standard* (Schoenfeld, 1985) or *rich learning tasks* (Flewelling & Higginson, 2001) so as to go beyond mere practice of procedures, to invoke mathematical insight and to focus on mathematical *structure* (Lampert, 1991). Author 1 (2005) refers to such tasks as *dynamically attracting* and *structuring* since they encourage students to engage in mathematical thinking. One example of the tasks we use, one that we made up, is the

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psychology to intervention by first acknowledging that it is the unique experiences of cold mathematics that

Making Ten question: If the answer is ten, what was the question? What (data, graph) patterns emerge when we solve questions like $\_ + \_ = 10$ and consider similar equations like $\_ + \_ = 6$ *or* $\_ x \_ = 12$? A second example is the Consecutive Terms task, adopted from Mason, Burton & Stacey (1982/1985). Elsewhere, in Author 1 and Author 2 (forthcoming), we have described other tasks that we use, including the well-known Handshake task.

We include the Consecutive Terms task among our tasks mainly because it vividly highlights patterning in mathematics. To make a complete package of tasks we include other tasks that vividly highlight different aspects of mathematics thinking. The Making Ten problem has many answers; it begins with the answer and asks for the question. It also connects numbers to data to graphing. The Handshake problem has a multitude of solution forms and strategies.

The tasks we use with pre-service teachers serve two inter-related purposes. First, they offer opportunities for experiencing mathematics in ways that encourage participation and evoke mathematical insight. DeBellis & Goldin (2006) say that "intimate mathematical experiences include emotional feelings of warmth, excitement … affection … They build a bond" and can meta-affectively be harnessed for successful mathematics learning (p.137). A metaphor for understanding further what we mean by "warm Mathematics" is to see mathematics activity as the personified behaviour of "cold" or "warm" molecules. Warm molecules, unlike cold molecules, are not set in their ways. They move about and into new environments. They are quick to interact, to react to

triggered the unhelpful positioning.

change and to exchange energy. Second, as pre-service teachers pay attention to their experiences and consider classroom implications, mathematics tasks serve as pedagogical models for informing pre-service teachers' classroom practice.

Over the past three years we have strengthened the mathematics tasks component of our program. The component currently has three dimensions: a) mathematical exploration—solving a mathematical task and extending the mathematics of the task, b) the articulation of what is personally learned or felt—paying attention to personal learning experiences, and c) pedagogical reflection on the task—considering implications for the classroom. We have gathered empirical evidence concerning potential influences of the mathematical tasks on pre-service teacher experiences.

Data and Analysis

Because mathematics tasks are a new component of our mathematics teacher education program we are still negotiating for time, space and resources with our institution. We have implemented and researched its implementation in a variety of settings: in an elective course of 30 students in 2005; as a compulsory component of our program done in a large auditorium setting with 440 students in 2005; as an entirely online component using interactive content and a discussion board that allowed for multimodal communication for groups of 30 students in 2006; and as in-class mathematics workshops in 2007. Thus, our own expertise in the design of mathematics tasks for teachers has developed through different settings.

We conducted interpretive analyses of data from over 400 pre-service students (ages 21-50) who consented to participate in the 2004 and 2005 study. The data included end-of- task reflections, online discussions, end-of-year culminating mathematics writing and a transcript from a follow-up interview. Thompson (1992) cautions the use of verbal data to study teacher beliefs suggesting that "data about their mathematical behavior as they encounter tasks … would be useful (p.135)." Students' written responses, written from (rather than about) experience (Mason, 1994) with the tasks is our main source of data. Using content analysis we collected what Mason (1994) dubs brief-but-vivid quotes from the data. These quotes described changes in views, insightful learning experiences, or some form of disequilibrium. We then interpretively studied these quotes to understand the extent of the changes or newness, the re-organization in experiences. We searched for commonalities among (a) how their ways of doing and learning mathematics had changed—we called this *mathematical attentiveness* (b) the ways in which students' emotions about mathematics had changed—we called this *mathematics therapy*, and (c) how they had began to analyze more useful learning and teaching possibilities—we called this *pedagogical responsiveness*.

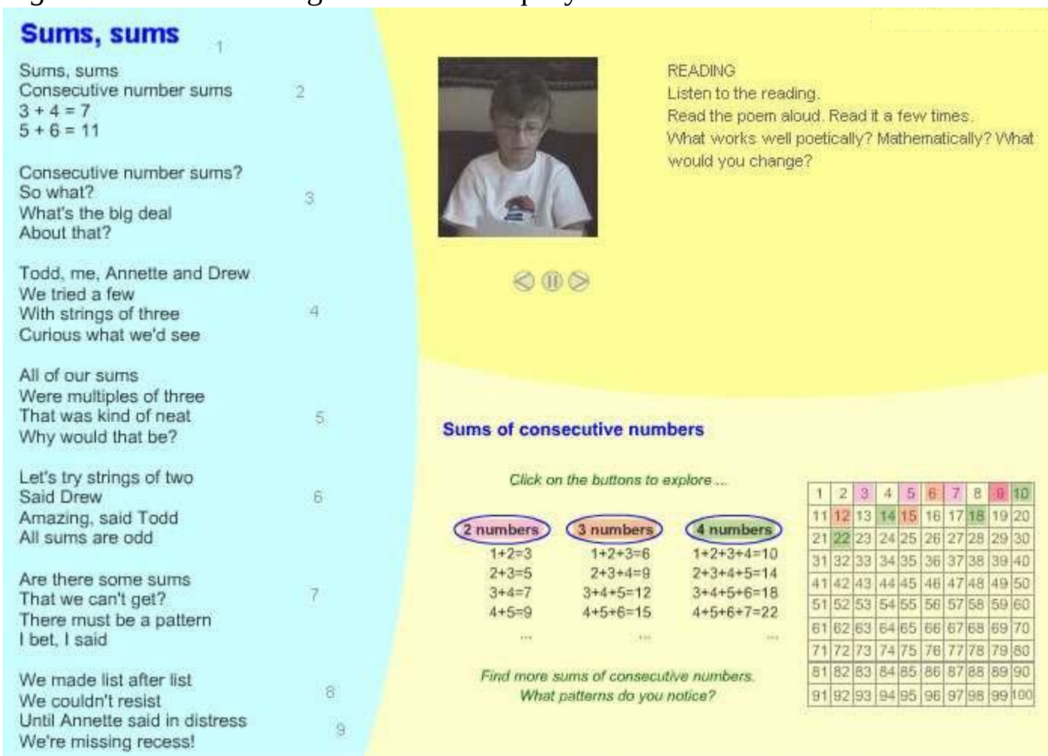
The Consecutive Terms Task

> Some numbers can be expressed as the sum of a string of consecutive positive integers. For example, 6 = 1 + 2 + 3; 11 = 5 + 6; 18 = 3 + 4 + 5 + 6; 15 = 7 + 8; What numbers have this property?

This task is about addition, in a patterning and problem-solving context. It is a task that Author 1 (2005) has used with school students in extra-curricular research. Paired school students of varied grades and mathematics interest usually begin the task by brainstorming about numbers that have the property. “What will 9 be?” “–9 will be 1 + 2 + 3 + 4 that does not work. 2 + 3 + 4, that works!” They soon turn to writing, at first to record and later to organize their thinking and to systematically search for patterns. The task gradually shifts to a task about strings of addends; Strings that begin with one (e.g. 1 + 2, 1 + 2 + 3), two, three and so on or strings made up of two (e.g. 1+ 2, 2 + 3, 3 + 4), of three and so on consecutive numbers. From these strings, students generate sequences and make observations. For instance, they observe that all odd numbers can be generated by summing two consecutive numbers. Strings that begin with one generate the sequence of triangular numbers. Students continue to notice patterns. They make conjectures and seek to verify them. The task offers opportunities for experiencing warm and insightful mathematics. It involves ideas about summing, sequences, and patterning interacting with each other.

In all settings (*auditorium, online or small group*) we provided selected materials to support mathematical engagement: manipulative materials, computing materials, graphical organizers or online interactive pages. For the Consecutive Terms (CT) task, in the auditorium setting we provided a worksheet with a pre-drawn table for organizing sums and calculators plus cube-a-links. In the online settings in addition to space and tools for discussion we incorporated a video clip of a Grade 4 student wondering about what happens when you add consecutive numbers (see Figure 1), an online applet, and a poem designed around the task. It is important to mention that whatever the setting, we used it as an opportunity to model presentations that could illuminate the mathematical potential of the task. Discussions about the possibilities and constraints presented by each presentation method are beyond the scope of this paper.

Figure 1. Interactive Page for Online Inquiry or Extensions

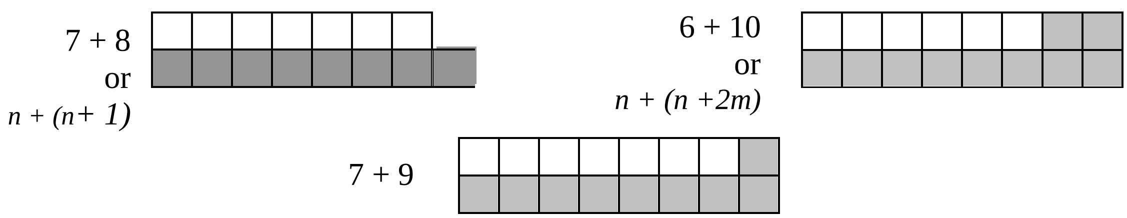


In this task pre-service teachers, as do school students, explore patterns involved in adding consecutive numbers. To explain the pattern “An odd and an odd always equal an even” some students use algebra (for instance, $n + (n + 1) = 2n + 1$ *or* $n - 1 + n = 2n - 1$, which is always odd when n is an integer.) Other students use the geometry of numbers (see Figure 2). Some seek to find if this pattern extends to the sums of other even numbers. By the time students notice that many numbers have the consecutive terms property except {1, 2, 4, 8 …}, they have generated, described and explained a multitude of patterns including:

- Adding three consecutive numbers gives you a multiple of three, i.e. 1 + 2 + 3 = 6 or in general terms $n + (n + 1) + (n + 2) = 3n + 3$. This pattern extends to other strings. Adding any odd string also gives you a multiple of the number of items in the string (5 + 6 + 7 + 8 + 9 + 10 + 11 = 56, 56 being a multiple of 7).
- Numbers that are sums of two consecutive terms, the odd numbers can be described by $2n + 1$, where n is greater or equal to 1. Numbers that are sums of three consecutive addends {6, 9, 12, 15 …} can be described by $3(n + 1)$, $3m$ or 6
    - $+ 3n$ or $3n + 3$, depending on how one defines m or n. The sum of four consecutive addends can be expressed as $10 + 4n$ or as $6 + 4n$. The pattern continues.
- All numbers except powers or exponents of two have the property.
- Summing non-zero consecutive numbers is equivalent to multiplying the median number with the number of items in the string. For instance, 5 + 6 + 7 + 8 + 9 + 10 = $7.5 \times 6 = 45$.

- Sums of strings of two numbers are odd, of four are even, of six are odd, of eight even and so forth.
- Many of these patterns extend to negative integers.

Figure 2. The geometry of sums of odd and even numbers



What is Learned and Felt

For all mathematics tasks, we emphasize end-of-session reflection. Pre-service teachers complete the "What I learned and felt" exit slips. We summarize the reflections and discuss them at the beginning of the next task. This aspect of the mathematics tasks assists pre-service teachers in becoming aware of themselves as mathematics learners. It also encourages reflective practice and serves as feedback to the instructors. Following meta-cognitive (Schoenfeld, 1992), meta-awareness (Mason, 1994), meta-affective (DeBellis & Goldin, 2006) and second-order observation research (Author 1, 2005), when pre-service teachers rise above their mathematical activity, which might include insight or frustrations, learning new things or facing mental blocks, they pay attention to their experiences as well as envision possibilities for their upcoming teaching. Below is a highlight of what students wrote down as positively learned and felt about the consecutive terms task:

> Found it engaging because there was a multitude of rich patterns to discover. This activity made me recognize the importance of patterns in math. I have never done anything like this in math before. I thought it was really interesting to learn something new about adding – a skill I thought I knew everything about.

Students reported negative feelings as well. Especially at the beginning of the course, some students found the mathematical experiences challenging and some were frustrated by the mathematical explorations. “Sessions seem to lack order as they don‘t connect to one another.” “I easily observed the patterns but did not understand the mathematical significance or explanations for the patterns and their algebraic expressions.” “I would appreciate having objectives and relevant curriculum expectations listed at the beginning of class."

Our focus on what pre-service teachers feel as they do mathematics should not be seen as a goal to eliminate negative emotions. We share these comments to illustrate students' experience of the task. In fact we do not rely only on selective positive evidence when we discuss students' reflections after each task. DeBellis and Goldin (2006), and Hannula (2006) observe that negative emotions such as fear, anger and frustration can both facilitate and discourage problem solving, depending on how these emotions are meta-affectively dealt with. Negative emotions emerged from gaps in mathematical understanding for some students, whereas for a majority these appeared to arise from tensions in beliefs and impermeability in belief structures (Cooney, 1994). In Cooney's (1999) study pre-service teachers whose belief systems were isolationist and thus less permeable found it hard to consider alternative views.

Pedagogical Reflections

With each of the tasks—Making Ten, Consecutive Task, Handshakes—different pedagogical issues arise. Mathematical tasks for teacher education, or the activity evoked by the tasks, for instance, bring to mind thoughts and discussions about the usefulness of the materials and teaching strategies used plus appropriateness of the task.

The initial feeling for many pre-service teachers was that a task as the Consecutive Terms task has no place in the elementary curriculum, since it did not directly fit under any Ontario grade expectation.[b] Students who strongly felt that the mathematics involved was insightful labored to show how the task indirectly fit with the curriculum or to explain how it generated many formulae. For instance, a student noted that it helps to mentally compute the sum of any consecutive addends using the formula—the median number times the length of the string. Students began to interrogate their understanding of relevant and tasks that fit with a curriculum. The Consecutive Terms task does indeed complexly fit in the grades 3-6 curriculum for Ontario, at once addressing several key curriculum expectation for the strands of Number Sense and Numeration, and Patterning and Algebra.

Pre-service students' comments illustrate that they gained much insight into patterning and its role in learning mathematics. One student commented that "Patterning is an important teaching tool. It is an important pre-algebra experience for students." And yet another "Finding patterns makes it easier to make predictions. Learning is easier if you

[b] This study was carried out in Ontario

see the pattern (you see a larger picture).” That pre-service teachers became aware of the centrality of patterning in mathematics learning can be a sign that they will adapt such tasks so as to enable their students to get the same mathematical experiences.

In terms of consequential gains, doing mathematics appeared to offer student teachers opportunities to consider and articulate useful teaching and learning experiences, and these were from a learner’s experience or point of view. We explicitly modelled teaching strategies and occasionally made observations about grade-level appropriateness, modification and assessment issues. Two themes illuminate how teachers considered ways of enhancing students’ learning experiences during classroom practice— pedagogical responsiveness: appropriate teaching strategies and reflections about learning.

Some student teachers initially resisted the idea of doing school mathematics problems. They said they felt “belittled” by doing problems suitable for primary grades—a common belief that in teacher education programs you do university not school content. We devoted part of one session to demystifying such unhelpful views. As student began to see the possible benefits of doing school mathematics, for a majority of students, it became a delight to step into the shoes of students to re-experience the insight, hard work and success of doing school mathematics. Students teachers began to pay attention to appropriate teaching strategies from a learner‘s point of view. Among the strategies were group work, rich problems, peer discussions, experimenting, integrating other subjects, and making connections. Students noted the importance of these teaching approaches. One student said, “I found this … to be an ‘eye-opener’ … how essential it is to share ideas." "The other strategy that was very useful is having students tackle the problem on their own before sharing and comparing with a partner, and eventually with the whole group. This helps broaden students' awareness." "The Predict-Observe-Explain method is useful because it teaches students that math is about *thinking not* just practicing learned formulas." (emphasis in original). These anecdotes show raised awareness of benefits of these approaches was raised. This was eye-opening, as one student put it. One student dubbed this kind of mathematical teaching and learning *liberating* pedagogy. "It made me feel that this type of 'liberating' teaching can also be used cross-curricular… I feel that the subject is no longer a mythological beast, but an area of interest." Whereas mythological beast may indicates an intense fear, liberating on the other hand suggests a feeling of relief, an empowerment.

Some students might have experienced rich school curriculum, but as Cooney (1999) maintains they might have been too young or immature to notice. Experiencing warm and insightful mathematics evoked memories of the one teacher, the one class or the one school in which they had experienced the interactive and active kind of mathematics. It also offered fresh experiences from which to reflect the how to and why of such school mathematics. Modelling teaching approaches, exploring non-routine problem types and class routines and competences conducive to mathematics learning are important benefits of mathematics sessions. There was opportunity for teachers to try out some of these ideas during their practicum. Our research did not focus on evidence of use during practicum teaching.

Reflections about Learning

The pre-service teachers' comments about enhancing learning centered on using multiple solutions, a variety of representations, and patterning. They noted that deep and flexible mathematical thinking ensued from: (a) "Authentic problems" (b) sharing "knowledge and ideas in a group." (c) "Having students use more than one way to solve a problem." (e) Designing "an environment that fosters flexible thinking."(f) "Allowing students the opportunity to experiment and … to ask questions." Students noted other dos and don‘ts for successful mathematics learning.

The mathematical sessions were an opportunity for pre-service teachers to become more aware of potential impact of mathematical experiences on students' mathematics learning (Ingleton & O‘Regan, 2002). Some pre-service teachers might have experienced rich school curriculum, but as Cooney (1999) maintains, they might have been too young or immature to notice. Pre-service teachers do not have to wait to try out ideas in the classroom in order to get feedback. Their experiences and their awareness of themselves as learners (Zaslavsky, Watson & Mason, 2008) during the mathematics sessions provide feedback about different ways of learning mathematics. Barkatsas and Malone (2005) maintain that changes in teachers' beliefs about teaching and learning are influenced by feedback as evidence of valued learning. Scott (2005) says that in this way pre-service teachers' view the "benefits to learners as practical, achievable options" (p. 88).

Mathematical, Experiential and Pedagogical Re-organizations

At the end of the year we give a culminating assessment activity where pre-service teachers write a two-page mathematical essay exploring one of the problems. To encourage students to interpret the math essay task diversely, instructors ask students to imagine themselves sharing about the task on a professional development day: what would they say about the mathematics and pedagogy involved in the task? Analysis of the essays suggests that mathematics tasks lead teachers to the realization that mathematics may be learned and taught in ways that different from the procedure-dominated ways.

In the essays pre-service teachers talked about central characteristics of the tasks such as links to other mathematical concepts and potential for modifications for different grades. This is evidence of raised awareness of pre-service teachers' ability to identify as well as to value tasks with similar characteristics.

Generally, evidence of significant re-organization appears in cases where students confess to gaining a new understanding of mathematics as a whole or of an aspect of mathematics. One student commented that:

> I no longer think of math as simply being a 'cut and dry' right or wrong subject…instead, it has become an area … where I could be creative …. explore various methods … and where I could continually challenge myself.

Research on mathematics beliefs analyses metaphors of students to understand their views of mathematics. Student teachers enacted new metaphors for mathematics. "It is like being a detective trying to ‘crack’ the case; a different atmosphere; Working collaboratively and with visuals.”

For some students a different way of doing mathematics came with challenges.

> At first, I was extremely apprehensive. I did not want to break away from the traditional approach …. However, as I began to work with the problem and with my peers, I quickly saw the benefits of reversing the system.

Many pre-service teachers, albeit the reflective and adaptive according to Cooney (1994), came to a realization that mathematics can be flexible, connected and imaginative. Doing mathematics tasks contributed to interrupting unfavorable, or even to revisiting and learning from favorable, school mathematics experiences.

> This course has taught me that math can be a very engaging and teachable subject.... Students need to learn that thinking and experimenting is more advantageous than just practicing formulae.

By exploring well-designed mathematical tasks with the students we increase the possibility that they will think about teaching mathematics in helpful and creative ways. Put in terms of teacher cognition and affect research our program re-organizes pre-service teachers’ potential to teach mathematics.

References

Ball, D. L. (1990). Breaking with experience in learning to teach mathematics: the role of a methods course. *For the Learning of Mathematics, 10* (2), 10-16.

Ball, D. L., & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. In E. Simmt & B. Davis (Ed.), *Proceedings of 2002 Annual meeting of Canadian Mathematics Education Study group,* pp3-14. Queens University.

Cooney, T. J. (1994). Research and teacher education: in search of common ground. *Journal for Research in Mathematics Education, 25*, 608-36.

Cooney, T. J. (1999). Conceptualizing teachers' ways of knowing. *Educational Studies in Mathematics, 38*(1-3), 163-87.

DeBellis, V., & Goldin, G. (2006). Affect and Meta-Affect in Mathematical Problem Solving: a Representational Perspective. *Educational Studies in Mathematics, 63*(2), 131-147.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.). *Handbook of Research on Mathematics Teaching and Learning* (pp. 147-164). New York: Macmillan.

Flewelling, G., & Higginson, H. (2001). *Realizing of vision of tomorrow's mathematics classroom: A handbook on rich learning tasks.* Kingston: MSTE.

Hannula, M. (2006). Motivation in Mathematics: Goals Reflected in Emotions. *Educational Studies in Mathematics,* 63(2), 165-178.

Koehler, M. S., & Grouws, D. A. (1992). Mathematics teaching practices and their effects. In D.A. Grouws (Ed.). *Handbook of Research on Mathematics Teaching* *and Learning* (pp. 115-126). New York: Macmillan.

Author 2. & Author 1. (2005). Math Therapy. *The Fifteenth ICMI Study: The Professional Education and Development of Teachers of Mathematics*, State University of Sao Paolo at Rio Claro, Brazil, 15-21 May 2005. [Retrieved 14 August 2006 from http://stwww.weizmann.ac.il/G-math/ICMI/log_in.html]

Mason, J., Watson, A., & Zaslavsky, O. (2008). Taken as Shared: a review of common assumptions about mathematical tasks in teacher education. *Journal of Mathematics Teacher Education*, 4-6, 205-215.

Mason, J., Burton, L., & Stacey, K. (1982/1985). *Thinking mathematically* (Rev. Ed.). First published in 1982. Wokingham: Addison-Wesley.

Mason, J. (1994). Researching from the inside in mathematics education: Locating an I-you relationship. *Proceeding of the 18th international conference for the psychology of mathematics education. Lisbon, Portugal, PMEXVIII*, vol. 1, (pp. 176-193).

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.). *Handbook of Research on Mathematics Teaching and Learning* (pp. 575-596). New York: Macmillan.

Author 1. (2005). *Attending in mathematics: A dynamic view about students' thinking.* Unpublished doctoral dissertation, University of Alberta.

Author 1 and Author 2 (forthcoming). How to feel about and teach/learn math: therapy, attentiveness and responsiveness.

Polya, G. (1945/1973). *How to solve it: A new aspect of mathematics (2nd ed.*) New Jersey: Princeton University Press.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. New York: Academic Press.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.). *Handbook of Research on Mathematics Teaching and Learning* (pp. 334-370). New York: Macmillan.

Skott, J. (1999). The multiple motives of teaching activity and the role of the teacher's school mathematical images. In O. Zaslavsky (Ed.) *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education*, (Vol. 4, pp. 209-216). Haifa, Israel.

Simmt, E., Davis, B., Gordon, L., & Towers, J. (2003). Teachers' mathematics: Cautious obligations. In N. A. Pateman, B. J. Dougherty, & J. T. Zilliox (Eds.), *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education held jointly with the 25th conference of PMENA*, vol. 1. (pp. 175-182). Honolulu: University of Hawai_i

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.). *Handbook of Research on Mathematics Teaching and Learning* (pp. 127-146). New York: Macmillan.

Wilson, P. S., Cooney, T. J., & Stinson, D. W. (2005). What Constitutes Good Mathematics Teaching and How it Develops: Nine High School Teachers' *Perspectives Journal of Mathematics Teacher Education*, 8 (2), 83.