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How Heterogeneous is Productivity? A Comparison of Gross Output and Value Added

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Abstract

In this paper, we study the theoretical and empirical relationship between gross output and value-added models of production. Using plant-level data from Colombia and Chile, we find that estimates of a gross output production function imply fundamentally different patterns of productivity heterogeneity compared to a value-added specification. Our estimates suggest that the specification of the technology may be more important than controlling for the endogeneity of inputs. Insights derived under value added, compared to gross output, could lead to significantly different policy conclusions.

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1 Introduction

There is a large literature studying heterogeneity of productivity at the firm level. Dhrymes (1991), Bartelsman and Doms (2000), Syverson (2004), Collard-Wexler (2010), and Fox and Smeets (2011), among others, document that there is a large amount of productivity dispersion within narrowly defined industries. This finding is related to the growing research agenda on the misallocation of inputs (Hsieh and Klenow, 2009, Asker, Collard-Wexler, and De Loecker, 2014; Bils, Klenow, and Ruane, 2017). The literature has also shown that there is a close relationship between productivity and many other dimensions of firm-level heterogeneity, such as importing of intermediate inputs (Amiti and Konings, 2007; Kasahara and Rodrigue, 2008; Halpern, Koren, and Szeidl, 2015; De Loecker et al., 2016), exporting of output (Bernard and Jensen, 1995; Bernard and Jensen, 1999; Bernard et al., 2003), wages (Baily, Hulten, and Campbell, 1992), research and development (Doraszelski and Jaumandreu, 2013, 2015), and demand-side heterogeneity (Foster, Haltiwanger, and Syverson, 2008; Pozzi and Schivardi, 2016; Blum et al., 2017). Some of these relationships have been obtained under gross output models of production, while others have been obtained in the context of models with value-added production functions that subtract out intermediate inputs.

Besides restrictions due to data availability, it is not clear why some researchers choose to work with gross output specifications and others value added for studying productivity. On the one hand, the problem of the firm is typically written in terms of gross output. In addition, production processes require intermediate inputs. On the other hand, value-added models require less data, involve smaller dimensional problems, and map directly into macroeconomic aggregates that avoid the double-counting of intermediate inputs.

In this paper, we study whether conclusions based on estimates of productivity obtained from models of gross output production functions differ substantively from those based on value added. We begin by analyzing the relationship between a gross output model, as described in Section 2 below, and a value-added model that relates measures of output that subtract out intermediate inputs (value added) to capital, labor, and productivity. We first examine the so-called "restricted profit" formulation of value added, in which intermediate inputs are replaced in the profit function by their conditional demand (as a function of other inputs and productivity), as in Bruno (1978) and Diewert (1978). We show that their duality results, which lead to the well-known result of using input revenue shares to rescale value-added objects up to their gross output counterparts, do not apply to the model of production we study.

We then analyze an alternative derivation of value added from gross output based on specific parametric assumptions, which allow one to separate the contribution to output of capital, labor, and productivity (the value-added production function) from that of intermediate inputs. We refer to this approach as "structural value added". We show that, unless the production function is a very specific version of Leontief in value added and intermediate inputs, value added cannot be used to identify features of interest (including productivity) from the underlying gross output production function.

Having established that moving between gross output and value added is not straightforward and requires additional assumptions, we then examine whether the differences between these two approaches matter empirically. In order to do so, we apply the nonparametric identification strategies developed by Gandhi, Navarro, and Rivers (2017) and Ackerberg, Caves, and Frazer (2015) (henceforth GNR and ACF) to plant-level data from Colombia and Chile. We study the underlying patterns of productivity under gross output (using GNR) compared to value-added (using ACF) specifications.

We find that productivity differences become substantially smaller and sometimes even change sign when we analyze the data via gross output rather than value added. For example, the standard 90/10 productivity ratio taken among all manufacturing firms in Chile is roughly 9 under value added (meaning that the 90th percentile firm is 9 times more productive than the 10th percentile firm), whereas under the gross output estimates this ratio falls to 2. Moreover, these dispersion ratios exhibit a remarkable degree of stability across industries and across the two countries when measured via gross output, but exhibit much larger cross-industry and cross-country variance when measured via value added. We further show that, as compared to gross output, value added estimates generate economically significant differences in the productivity premium of firms that export, firms that import, firms that advertise, and higher wage firms.

In contrast to the view expressed in Syverson (2011)—that empirical findings related to productivity are quite robust to measurement choices—our results illustrate the empirical importance of the distinction between gross output and value-added estimates of productivity. Our findings highlight the empirical relevance of the assumptions being invoked that allow a researcher to work with either value-added or gross output models of production. They suggest that the distinction between gross output and value added is at least as important, if not more so, than the endogeneity of inputs that has been the main focus of the production function estimation literature to date.

The rest of the paper is organized as follows. In Section 2 we describe the basic setup of the model. Section 3 provides an overview of the assumptions needed to interpret a value-added production function as being derived from an underlying gross output production function. In Section 4 we describe the Colombian and Chilean data and show the results comparing gross output to value added for productivity measurement. In particular, we show evidence of large differences in unobserved productivity heterogeneity suggested by value added relative to gross output. Section 5 concludes with an example of the policy relevance of our results.

2 Model

In this section we first describe a standard economic model of the firm based on the model underlying the "proxy variable" approach of Olley and Pakes (1996), Levinsohn and Petrin (2003) (henceforth LP), and ACF, which has become a widely-used approach to estimating production functions and productivity in applied work. We focus only on the main assumptions of the model, and point the interested reader to ACF and GNR for the more detailed assumptions needed for identification. We first describe the structure of the data and the nature of input decisions. We then write down the model of production and the process for productivity. Next we derive the expected profit maximization problem of the firm.

2.1 Data and Definitions

Our data consists of firms $j = 1, \ldots, J$ over periods $t = 1, \ldots, T$.[1] Firm j's output, capital, labor, and intermediate inputs are given by $(Y_{jt}, K_{jt}, L_{jt}, M_{jt})$ respectively, and their log values will be denoted in lowercase by $(y_{jt}, k_{jt}, l_{jt}, m_{jt})$. Since we will assume that firms operate in a competitive environment in both the output and intermediate input markets, we let P_t denote the output price, and ρ_t the price of intermediate inputs faced by the firm.

Let $\mathcal{I}_{jt}$ denote the "information set" of the firm in period t, which we model as a set of random variables. It consists of all information the firm can use to solve its period t decision problem. If the choice of a generic input is a function of $\mathcal{I}_{jt-1}$, then we say it is a *predetermined* input in period t, as it was effectively chosen at (or before) $t-1$. If an input's optimal period t choices are affected by lagged values of that same input, then we say the input is *dynamic.* If an input is predetermined, dynamic, or both, we say it is *non-flexible.* If an input is chosen in this period and its choice does not depend on lagged values, so it is neither predetermined nor dynamic, then we say it is *flexible*.

2.2 The Gross Output Production Function and Productivity

We assume that there exists a gross output production function F that summarizes how the firm transforms inputs into output, up to a factor neutral (Hicks neutral) productivity shock ν_{jt}.

Assumption 1. *The relationship between output and the inputs takes the form*

$$\begin{aligned} Y_{jt} &= F\left(k_{jt}, l_{jt}, m_{jt}\right) e^{\nu_{jt}} \iff \\ y_{jt} &= f\left(k_{jt}, l_{jt}, m_{jt}\right) + \nu_{jt}. \end{aligned} \tag{1}$$

The production function f is differentiable at all $(k, l, m) \in \mathbb{R}^3_{++}$, and strictly concave in m.

We decompose the productivity shock as $\nu_{jt} = \omega_{jt} + \varepsilon_{jt}$. ω_{jt} is the part of productivity that is known to the firm before making its period t decisions, whereas ε_{jt} is an ex-post productivity

[1] Throughout this section we assume a balanced panel for notational simplicity.

shock realized only after the period decisions are made. The stochastic properties of both of these components is explained next.

Assumption 2. *$\omega_{jt} \in \mathcal{I}_{jt}$ is known to the firm at the time of making its period t decisions, whereas $\varepsilon_{jt} \notin \mathcal{I}_{jt}$ is not. Furthermore ω_{jt} is Markovian so that its distribution can be written as $P_\omega(\omega_{jt} \mid \mathcal{I}_{jt-1}) = P_\omega(\omega_{jt} \mid \omega_{jt-1})$. The function $h(\omega_{jt-1}) = E[\omega_{jt} \mid \omega_{jt-1}]$ is continuous. The shock ε_{jt} on the other hand is independent of the within-period variation in information sets, $P_\varepsilon(\varepsilon_{jt} \mid \mathcal{I}_{jt}) = P_\varepsilon(\varepsilon_{jt})$.*

Since ω_{jt} is Markovian and known at time t, we will refer to ω_{jt} as persistent productivity, ε_{jt} as ex-post productivity, and $\nu_{jt} = \omega_{jt} + \varepsilon_{jt}$ as total productivity. If we express $\omega_{jt} = h(\omega_{jt-1}) + \eta_{jt}$, by construction η_{jt} satisfies $E[\eta_{jt} \mid \mathcal{I}_{jt-1}] = 0$. η_{jt} can be interpreted as the, unanticipated at period $t-1$, "innovation" to the firm's persistent productivity ω_{jt} in period t. We normalize $E[\varepsilon_{jt} \mid \mathcal{I}_{jt}] = E[\varepsilon_{jt}] = 0$, without loss of generality. Given this normalization, the expectation of the level of the ex-post shock becomes a free parameter which we denote as $\mathcal{E} \equiv E[e^{\varepsilon_{jt}} \mid \mathcal{I}_{jt}] = E[e^{\varepsilon_{jt}}]$.[2] Furthermore, since input demand is by construction a function of $\mathcal{I}_{jt}$, it follows that $E[\varepsilon_{jt} \mid k_{jt}, l_{jt}, m_{jt}] = 0$. The following assumption formalizes the timing of the firm's input choices. Our interest is in the case in which the production function contains both flexible and non-flexible inputs. We focus on the case of a single flexible input in the model and assume that the rest are predetermined for simplicity.

Assumption 3. *Intermediate inputs m_{jt} is a flexible input, i.e., it is chosen at time t independently of the amount of m the firm employed in the previous period. We treat capital k_{jt} and labor l_{jt} as predetermined, i.e., as chosen in the previous period (hence $k_{jt}, l_{jt} \in \mathcal{I}_{jt}$).*

2.3 The Firm's Problem

In what follows, we write down the problem of a profit maximizing firm under perfect competition. From this, we derive the explicit intermediate input demand equation that is key for both the LP/ACF

[2] See Goldberger (1968) for an early discussion of the implicit reinterpretation of results that arises from ignoring $\mathcal{E}$ (i.e., setting $\mathcal{E} \equiv E[e^{\varepsilon_{jt}}] = 1$ while simultaneously setting $E[\varepsilon_{jt}] = 0$) in the context of Cobb-Douglas production functions.

proxy variable approach, as well as for the GNR strategy. The following assumption formalizes the environment in which firms operate.

Assumption 4. *Firms are price takers in the output and intermediate input market, with ρ_t denoting the common intermediate input price and P_t denoting the common output price facing all firms in period t. Firms maximize expected discounted profits.*

Under Assumptions 1, 2, and 4, the firm's profit maximization problem with respect to intermediate inputs is

$$\max_{M_{jt}} P_t E\left[F\left(k_{jt}, l_{jt}, m_{jt}\right) e^{\omega_{jt}+\varepsilon_{jt}} \mid \mathcal{I}_{jt}\right] - \rho_t M_{jt}, \tag{2}$$

The first-order necessary condition for a maximum is given by

$$P_t \frac{\partial}{\partial M_{jt}} F\left(k_{jt}, l_{jt}, m_{jt}\right) e^{\omega_{jt}} \mathcal{E} = \rho_t, \tag{3}$$

which implies

$$m_{jt} = \mathbb{M}_t\left(k_{jt}, l_{jt}, \omega_{jt}\right). \tag{4}$$

Intermediate input demand equation 4 has two key properties: a) it depends on a single unobservable (to the econometrician) ω_{jt}, and b) it can be inverted to solve for productivity as a function of the observables. This result is commonly employed to justify the use of the (inverted) demand for intermediate inputs to proxy for the unobserved productivity ω_{jt} in LP/ACF. These are the scalar unobservablility and strict monotonicity assumptions of LP/ACF. Even though we generated them from the profit maximization problem of the firm, we call them assumptions as they are commonly invoked on their own without explicit reference to the profit maximization problem.

The key to the identification result in GNR is precisely to notice that equation 4 contains more information than just scalar unobservability and monotonicity. This becomes clear when one notices that condition 3, from which equation 4 is derived, is an explicit function (a partial differential equation) of the production function f. It is this additional information that allows the GNR strategy to recover the gross output production function.

3 Value Added

In the literature on productivity, a common alternative empirical approach to gross output is is to employ a value-added production function by relating a measure of the output of a firm to a function of capital and labor only. Typically output is measured empirically as the "value added" by the firm (i.e., the value of gross output minus expenditures on intermediate inputs). One potential advantage of this approach is that, by excluding intermediate inputs from the production function, it avoids the identification problem associated with flexibly chosen intermediate inputs discussed in GNR.

The use of value added is typically motivated in one of two ways. First, a researcher may feel that a value-added function is a better model of the production process, as a primitive assumption. For example, suppose there is a lot of heterogeneity in the degree of vertical integration within an industry, with firms outsourcing varying degrees of the production process. In this case, a researcher may feel that focusing on just the contributions of capital and labor (to the value added by the firm) is preferred to a gross output specification including intermediate inputs.

The second motivation is based on the idea that a value-added function can be constructed from an underlying gross output production function. This value-added function can then be used to recover objects of interest from the underlying gross output production function, such as firm-level productivity $e^{\omega_{jt}+\varepsilon_{jt}}$ and certain features of the production technology (e.g., output elasticities of inputs) with respect to the "primary inputs", capital and labor. This approach is typically justified either via the restricted profit function or by using structural production functions. As we discuss in more detail below, under the model described in Section 2, neither justification generally allows for a value-added production function to be isolated from the gross output production function.

Regardless of the motivation for value added, the objects from a value-added specification, particularly productivity, will be fundamentally different than those from gross output. Under the first motivation, this is because productivity from a primitively-specified value-added setup measures differences in value added holding capital and labor fixed, as opposed to differences in gross output holding all inputs fixed. The results in this section show that under the second motivation, the value-added objects cannot generally be mapped into their gross output counterparts if only the value-added objects are available. A key exception is the linear in intermediate inputs Leontief specification that we discuss below, a version of which is employed by ACF.

3.1 Restricted Profit Value Added

The first approach to relating gross output to value added is based on the duality results in Bruno (1978) and Diewert (1978). We first briefly discuss their original results, which were derived under the assumption that intermediate inputs are flexibly chosen, but excluding the ex-post shocks. In this case, they show that by replacing intermediate inputs with their optimized value in the profit function, the empirical measure of value added, $VA_{jt}^{E} \equiv Y_{jt} - M_{jt}$, can be expressed as:

$$VA_{jt}^{E} \quad = \quad F\left(k_{jt}, l_{jt}, \mathbb{M}_t\left(k_{jt}, l_{jt}, \omega_{jt}\right)\right) e^{\omega_{jt}} - \mathbb{M}_t\left(k_{jt}, l_{jt}, \omega_{jt}\right) \equiv \mathcal{V}\left(k_{jt}, l_{jt}, \omega_{jt}\right), \tag{5}$$

where we use $\mathcal{V}(\cdot)$ to denote the value-added function in this setup.[3] This formulation is sometimes referred to as the restricted profit function (see Lau, 1976; Bruno, 1978; McFadden, 1978).

In an index number framework, Bruno (1978) shows that elasticities of gross output with respect to capital, labor, and productivity can be locally approximated by multiplying estimates of the value-added counterparts by the firm-level ratio of value added to gross output, $\frac{VA_{jt}^{E}}{GO_{jt}} = (1 - S_{jt})$, where GO stands for gross output and S_{jt} is the intermediate input share of output.[4] For productivity, the result is as follows:

$$\left(elas_{e^{\omega_{jt}}}^{GO_{jt}}\right) = \left(elas_{e^{\omega_{jt}}}^{VA_{jt}^{E}}\right)\left(\frac{VA_{jt}^{E}}{GO_{jt}}\right) = \left(elas_{e^{\omega_{jt}}}^{VA_{jt}^{E}}\right)(1 - S_{jt}) \tag{6}$$

See the Appendix for the details of this derivation. Analogous results hold for the elasticities with respect to capital and labor by replacing $e^{\omega_{jt}}$ with K_{jt} or L_{jt}.

While this derivation suggests that estimates from the restricted-profit value-added function can

[3] Technically, $VA_{jt}^{E} \equiv \frac{P_t Y_{jt}}{\overline{P}_t} - \frac{\rho_t M_{jt}}{\overline{\rho}_t}$, where $\overline{P}_t$ and $\overline{\rho}_t$ are the price deflators for output and intermediate inputs, respectively. The ratio $\frac{P_t}{\overline{P}_t}$ is equal to the output price in the base year, P_{BASE}, and similarly for the price of intermediate inputs. Since P_{BASE} and ρ_{BASE} are constants, they are subsumed in the constants in the F and $\mathbb{M}_t$ functions. For ease of notation, we normalize these constants to 1.

[4] These results were originally derived under a general form of technical change. We have augmented the results here to correspond to the standard setup with Hicks-neutral technical change as discussed in Section 2.

be simply multiplied by $(1 - S_{jt})$ to recover estimates from the underlying gross output production function, there are several important problems with the relationship in equation (6). First, this approach is based on a local approximation. While this may work well for small changes in productivity, for example looking at productivity growth rates (the original context under which these results were derived), it may not work well for large differences in productivity, such as analyzing cross-sectional productivity differences.

Second, this approximation does not account for ex-post shocks to output. As we show in the Appendix, when ex-post shocks are accounted for, the relationship in equation (6) becomes:

$$\underbrace{\left(\frac{\partial GO_{jt}}{\partial e^{\omega_{jt}}}\frac{e^{\omega_{jt}}}{GO_{jt}}\right)}_{elas^{GO_{jt}}_{e^{\omega_{jt}}}} = \underbrace{\frac{\partial VA^E_{jt}}{\partial e^{\omega_{jt}}}\frac{e^{\omega_{jt}}}{VA^E_{jt}}}_{elas^{VA^E_{jt}}_{e^{\omega_{jt}}}} (1 - S_{jt}) + \left[\frac{\partial M_{jt}}{\partial e^{\omega_{jt}}}\frac{e^{\omega_{jt}}}{GO_{jt}}\left(\frac{e^{\varepsilon_{jt}}}{\mathcal{E}} - 1\right)\right] \tag{7}$$

The term in brackets is the bias introduced due to the ex-post shock. Ex-post shocks prevent one from being able to use the observable shares S_{jt} to convert value-added objects into their gross output counterparts. Analogous results for the output elasticities of capital and labor can be similarly derived.

As a result of the points discussed above, estimates from the restricted profit value-added function cannot simply be "transformed" by re-scaling with the firm-specific share of intermediate inputs to obtain estimates of the underlying production function and productivity. How much of a difference this makes is an empirical question, which we address in Section 4. Previewing our results, we find that re-scaling using the shares, as suggested by equation (6), performs poorly.

3.2 "Structural" Value Added

The second approach to connecting gross output to value added is based on specific parametric assumptions on the production function, such that a value-added production function of only capital, labor, and productivity can be both isolated and measured (see Sims, 1969 and Arrow, 1972). We refer to this version of value added as the "structural value-added production function".

The empirical literature on value-added production functions often appeals to the extreme case of perfect complements (i.e., Leontief). A standard representation is:

$$Y_{jt} = \min\left[\mathcal{H}\left(K_{jt}, L_{jt}\right), \mathcal{C}\left(M_{jt}\right)\right] e^{\omega_{jt}+\varepsilon_{jt}}, \tag{8}$$

where $\mathcal{C}(\cdot)$ is a monotonically increasing and concave function. The main idea underlying the Leontief justification for value added is that, under the assumption that

$$\mathcal{H}\left(K_{jt}, L_{jt}\right) = \mathcal{C}\left(M_{jt}\right), \tag{9}$$

the right hand side of equation (8) can be written as $\mathcal{H}\left(K_{jt}, L_{jt}\right) e^{\omega_{jt}+\varepsilon_{jt}}$, a function that does not depend on intermediate inputs M_{jt}. The key problem with this approach is that, given the assumptions of the model, the relationship in equation (9) will not generally hold. Unless capital or labor is assumed to be flexible, firms either cannot adjust them in period t or can only do so with some positive adjustment cost. The consequence is that firms may optimally choose to *not* equate $\mathcal{H}(K_{jt}, L_{jt})$ and $\mathcal{C}\left(M_{jt}\right)$, i.e., it may be optimal for the firm to hold onto a larger stock of K_{jt} and L_{jt} than can be combined with M_{jt} if K_{jt} and L_{jt} are both costly (or impossible) to downwardly adjust.[5]

An exception to this, as discussed in ACF, is when $\mathcal{C}(\cdot)$ is linear (i.e., $\mathcal{C}\left(M_{jt}\right) = aM_{jt}$). In this case the relationship in equation (9) will hold, the right hand side of equation (8) can be written as a function of only capital, labor, and productivity, and we have that

$$Y_{jt} = \mathcal{H}\left(K_{jt}, L_{jt}\right) e^{\omega_{jt}+\varepsilon_{jt}}. \tag{10}$$

This does not imply though that VA^E can be used to measure the structural value-added production function, as $VA^E_{jt} \equiv Y_{jt} - M_{jt}$ will not be proportional to the value-added production function

[5]For example, suppose $\mathcal{C}\left(M_{jt}\right) = M_{jt}^{0.5}$. For simplicity, also suppose that capital and labor are fixed one period ahead, and therefore cannot be adjusted in the short run. When $M_{jt}^{0.5} \leq \mathcal{H}\left(K_{jt}, L_{jt}\right)$, marginal revenue with respect to intermediate inputs equals $\frac{\partial \mathcal{C}(M_{jt})}{\partial M_{jt}} aP_t$. When $M_{jt}^{0.5} > \mathcal{H}\left(K_{jt}, L_{jt}\right)$, increasing M_{jt} does not increase output due to the Leontief structure, so marginal revenue is zero. Marginal cost in both cases equals the price of intermediate inputs ρ_t. The firm's optimal choice of M_{jt} is therefore given by $M_{jt} = \frac{P_t}{\rho_t}0.5a^{\;2}$ if $\frac{P_t}{\rho_t}0.5a < \mathcal{H}\left(K_{jt}, L_{jt}\right)$. But when $\frac{P_t}{\rho_t}0.5a > \mathcal{H}\left(K_{jt}, L_{jt}\right)$, the firm no longer finds it optimal to set $\mathcal{H}\left(K_{jt}, L_{jt}\right) = \mathcal{C}\left(M_{jt}\right)$, and prefers to hold onto excess capital and labor.

$\mathcal{H}(K_{jt}, L_{jt})\, e^{\omega_{jt}+\varepsilon_{jt}}$.[6]

4 Data and Application

In the previous section we showed that value-added production functions capture different objects compared to gross output. A natural question is whether these differences are relevant empirically. A recent survey paper by Syverson (2011) states that many results in the productivity literature are quite robust to alternative measurement approaches. He attributes this to the idea that the underlying variation at the firm level is so large that it dominates any differences due to measurement. This suggests that whether a researcher uses a value-added or gross output specification should not change any substantive conclusions related to productivity. In this section we show that, not only do the two approaches of gross output and value added produce fundamentally different patterns of productivity empirically, in many cases the differences are quite large and lead to very different conclusions regarding the relationship between productivity and other dimensions of firm heterogeneity.

We quantify the effect of using a value-added rather than gross output specification using two commonly employed plant-level manufacturing datasets. The first dataset comes from the Colombian manufacturing census covering all manufacturing plants with more than 10 employees from 1981-1991. This dataset has been used in several studies, including Roberts and Tybout (1997), Clerides, Lach, and Tybout (1998), and Das, Roberts, and Tybout (2007). The second dataset comes from the census of Chilean manufacturing plants conducted by Chile's Instituto Nacional de Estadística (INE). It covers all firms from 1979-1996 with more than 10 employees. This dataset has also been used extensively in previous studies, both in the production function estimation literature (LP) and in the international trade literature (Pavcnik, 2002 and Alvarez and López, 2005).[7]

[6] In the Appendix we also show that moving ω_{jt} and/or ε_{jt} inside of the min function presents a similar set of issues.

[7] We construct the variables adopting the convention used by Greenstreet (2007) with the Chilean dataset, and employ the same approach with the Colombian dataset. In particular, real gross output is measured as deflated revenues. Intermediate inputs are formed as the sum of expenditures on raw materials, energy (fuels plus electricity), and services. Real value added is the difference between real gross output and real intermediate inputs, i.e., double deflated value added. Labor input is measured as a weighted sum of blue collar and white collar workers, where blue collar workers are weighted by the ratio of the average blue collar wage to the average white collar wage. Capital is constructed using the perpetual inventory method where investment in new capital is combined with deflated capital from period $t-1$ to form capital in period t. Deflators for Colombia are obtained from Pombo (1999) and deflators for Chile are obtained from Bergoeing, Hernando, and Repetto (2003).

We estimate separate production functions for the five largest 3-digit manufacturing industries in both Colombia and Chile, which are Food Products (311), Textiles (321), Apparel (322), Wood Products (331), and Fabricated Metal Products (381). We also estimate an aggregate specification grouping all manufacturing together. We estimate the production function in two ways.[8] First, using the procedure in GNR we estimate a gross output production function using a complete polynomial series of degree 2 for both the elasticity and the integration constant in the production function. That is, we use

$$\begin{aligned} D_2^{\mathcal{E}}\left(k_{jt}, l_{jt}, m_{jt}\right) &= \gamma_0' + \gamma_k' k_{jt} + \gamma_l' l_{jt} + \gamma_m' m_{jt} + \gamma_{kk}' k_{jt}^2 + \gamma_{ll}' l_{jt}^2 \\ &\quad + \gamma_{mm}' m_{jt}^2 + \gamma_{kl}' k_{jt} l_{jt} + \gamma_{km}' k_{jt} m_{jt} + \gamma_{lm}' l_{jt} m_{jt} \end{aligned}$$

to estimate the intermediate input elasticity and

$$\mathscr{C}_2\left(k_{jt}, l_{jt}\right) = \alpha_k k_{jt} + \alpha_l l_{jt} + \alpha_{kk} k_{jt}^2 + \alpha_{ll} l_{jt}^2 + \alpha_{kl} k_{jt} l_{jt}$$

for the constant of integration. Putting all the elements together, the gross output production function we estimate is given by:

$$\begin{aligned} y_{jt} &= \left(\begin{array}{c} \gamma_0 + \gamma_k k_{jt} + \gamma_l l_{jt} + \frac{\gamma_m}{2} m_{jt} + \gamma_{kk} k_{jt}^2 + \gamma_{ll} l_{jt}^2 \\ + \frac{\gamma_{mm}}{3} m_{jt}^2 + \gamma_{kl} k_{jt} l_{jt} + \frac{\gamma_{km}}{2} k_{jt} m_{jt} + \frac{\gamma_{lm}}{2} l_{jt} m_{jt} \end{array} \right) m_{jt} \\ &\quad - \alpha_k k_{jt} - \alpha_l l_{jt} - \alpha_{kk} k_{jt}^2 - \alpha_{ll} l_{jt}^2 - \alpha_{kl} k_{jt} l_{jt} + \omega_{jt} + \varepsilon_{jt}, \end{aligned} \tag{11}$$

since $y_{jt} = \int \frac{D^{\mathcal{E}}(l_{jt}, k_{jt}, m_{jt})}{\mathcal{E}} dm_{jt} - \mathscr{C}\left(k_{jt}, l_{jt}\right) + \omega_{jt} + \varepsilon_{jt}$.

Second, we estimate a value-added specification using the commonly-applied method developed by ACF, also using a complete polynomial series of degree 2:

$$va_{jt} = \beta_k k_{jt} + \beta_l l_{jt} + \beta_{kk} k_{jt}^2 + \beta_{ll} l_{jt}^2 + \beta_{kl} k_{jt} l_{jt} + \upsilon_{jt} + \epsilon_{jt}, \tag{12}$$

[8] For all of the estimates we present, we obtain standard errors by using the nonparametric block bootstrap with 200 replications.

where $\upsilon_{jt} + \ _{jt}$ represents productivity in the value-added model.

In Table 1 we report estimates of the average output elasticities for each input, as well as the sum, for both the value-added and gross output models. In every case but one, the value-added model generates a sum of elasticities that is larger relative to gross output, with an average difference of 2% in Colombia and 6% in Chile.

We also report the ratio of the mean capital and labor elasticities, which measures the capital intensity (relative to labor) of the production technology in each industry. In general, the value-added estimates of the capital intensity of the technology are larger relative to gross output, although the differences are small. According to both measures, the Food Products (311) and Textiles (321) industries are the most capital intensive in Colombia, and in Chile the most capital intensive is Food Products. In both countries, Apparel (322) and Wood Products (331) are the least capital intensive industries, even compared to the aggregate specification denoted “All” in the tables.

Value added also recovers dramatically different patterns of productivity as compared to gross output. Following Olley and Pakes (1996), we define productivity (in levels) as the sum of the persistent and unanticipated components: $e^{\omega_{jt}+\varepsilon_{jt}}$.[9] In Table 2 we report estimates of several frequently analyzed statistics of the resulting productivity distributions. In the first three rows of each panel we report ratios of percentiles of the productivity distribution, a commonly used measure of productivity dispersion. There are two important implications of these results. First, value added suggests a much larger amount of heterogeneity in productivity across plants *within* an industry, as the various percentile ratios are much smaller under gross output. For Colombia, the average 75/25, 90/10, and 95/5 ratios are 1.88, 3.69, and 6.41 under value added, and 1.33, 1.78, and 2.23 under gross output. For Chile, the average 75/25, 90/10, and 95/5 ratios are 2.76, 8.02, and 17.93 under value added, and 1.48, 2.20, and 2.95 under gross output. The value-added estimates imply that, with the same amount of inputs, the 95th percentile plant would produce more than 6 times more output in Colombia, and almost 18 times more output in Chile, than the 5th percentile plant. In stark contrast, we

[9]Since our interest is in analyzing productivity heterogeneity we conduct our analysis using productivity in levels. An alternative would be to measure productivity in logs. However, the log transformation is only a good approximation for measuring percentage differences in productivity across groups when these differences are small, which they are not in our data. We have also computed results based on log productivity. As expected, the magnitude of our results changes, however, our qualitative results comparing gross output and value added still hold. We have also computed results using just the persistent component of productivity, $e^{\omega_{jt}}$. The results are qualitatively similar.

find that under gross output, the 95th percentile plant would produce only 2 times more output in Colombia, and 3 times more output in Chile, than the 5th percentile plant with the same inputs.

Additionally, the ranking of industries according to the degree of productivity dispersion is not preserved moving from the value added to gross output estimates. For example, in Chile, the Fabricated Metals industry (381) has the smallest amount of productivity dispersion under value added, but the largest amount of dispersion under gross output, for all three dispersion measures.

The second important result is that value added also implies much more heterogeneity *across* industries, which is captured by the finding that the range of the percentile ratios across industries is much tighter using the gross output measure of productivity. For example, for the 95/5 ratio, the value-added estimates indicate a range from 4.36 to 11.01 in Colombia and from 12.52 to 25.08 in Chile, whereas the gross output estimates indicate a range from 2.02 to 2.38 and from 2.48 to 3.31. The surprising aspect of these results is that the dispersion in productivity appears far more stable both across industries and across countries when measured via gross output as opposed to value added. In the conclusion we sketch some important policy implications of this finding for empirical work on the misallocation of resources.

In addition to showing much larger overall productivity dispersion, results based on value added also suggest a substantially different relationship between productivity and other dimensions of plant-level heterogeneity. We examine several commonly-studied relationships between productivity and other plant characteristics. In the last four rows of each panel in Table 2 we report percentage differences in productivity based on whether plants export some of their output, import intermediate inputs, have positive advertising expenditures, and pay above the median (industry) level of wages.

Using the value-added estimates, for most industries exporters are found to be more productive than non-exporters, with exporters appearing to be 83% more productive in Colombia and 14% more productive in Chile across all industries. Using the gross output specification, these estimates of productivity differences fall to 9% in Colombia and 3% in Chile, and actually turn negative (although not statistically different from zero) in some cases.

A similar pattern exists when looking at importers of intermediate inputs. The average productivity difference is 14% in Colombia and 41% in Chile using value added. However, under gross output, these numbers fall to 8% and 13% respectively. The same story holds for differences in productivity based on advertising expenditures. Moving from value added to gross output, the estimated difference in productivity drops for most industries in Colombia, and for all industries in Chile. In several cases it becomes statistically indistinguishable from zero.

Another striking contrast arises when we compare productivity between plants that pay wages above versus below the industry median. Using the productivity estimates from a value-added specification, firms that pay wages above the median industry wage are found to be substantially more productive, with the estimated differences ranging from 34%-63% in Colombia and from 47%-123% in Chile. In every case the estimates are statistically significant. Using the gross output specification, these estimates fall to 9%-22% in Colombia and 19%-30% in Chile, representing a fall by a factor of 3, on average, in both countries.

Since intermediate input usage is likely to be positively correlated with productivity, we would expect that including (excluding) intermediate inputs in the production function will lead to smaller (larger) differences in productivity heterogeneity. Therefore, we would expect to see the largest discrepancies between the value-added and gross output productivity heterogeneity estimates in industries which are intensive in intermediate input usage. By looking at Tables 1 and 2 we can confirm that, for the most part, this is the case. When comparing the value-added and gross output productivity estimates, the largest differences tend to occur in the most intermediate input intensive industries, which are Food Products (311) in Colombia and Food Products (311) and Wood Products (331) in Chile. However, this is not always the case. For example, in Chile, the difference between the gross output and value-added estimates of the average productivity comparing advertisers and non-advertisers is actually the smallest in the Wood Products (331) industry.

In order to isolate the importance of the value-added/gross output distinction separately from the effect of having biased estimates from OLS, in Table 3 we repeat the above analysis without correcting for the endogeneity of inputs. We examine the raw effects in the data by estimating productivity using simple linear regression (OLS) to estimate both gross output and value-added specifications, using a complete polynomial of degree 2. As can be seen from Table 3, the general pattern of results, that value added leads to larger productivity differences across many dimensions, is similar to our previous results both qualitatively and quantitatively.

While the results in Table 3 may suggest that endogeneity is not empirically important, in Table 4 we provide evidence to the contrary. In particular, we report the average input elasticities based on estimates for the gross output model using OLS and using the GNR method to correct for the endogeneity of inputs. It is well known that endogeneity of inputs biases the coefficients on more flexible inputs upwards. Intuitively, the more flexible the input is, the more it responds to productivity shocks and the larger the correlation between the input and productivity. As our estimates in Table 4 demonstrate, OLS substantially overestimates the output elasticity of intermediate inputs in every industry, by an average of 34%. These results highlight the importance of controlling for the endogeneity generated by the correlation between input decisions and productivity.

An important implication of our results is that, while controlling for the endogeneity of inputs certainly has an effect, the use of value added versus gross output has a much larger effect on the productivity estimates. This suggests that the choice of gross output versus value added may be more important from a policy perspective than controlling for the endogeneity of inputs that has been the primary focus in the production function literature.

4.1 Adjusting the Value-Added Estimates

As discussed in Section 3.1, in the absence of ex-post shocks, the derivation provided in equation (6) suggests that the differences between gross output and value added can be eliminated by re-scaling the value-added estimates by a factor equal to the plant-level ratio of value added to gross output (i.e., one minus the share of intermediate inputs in total output). While this idea has been known in the literature for a while, this re-scaling is very rarely applied in practice.[10] As shown in Section 3.1, there are several reasons why this re-scaling may not work. In order to investigate how well the re-scaling of value-added estimates performs, we apply the transformation implied by equation (6) using the firm-specific ratio of value added to gross output $\frac{VA_{jt}^{E}}{GO_{jt}}$, which is readily available in the data. We find that this re-scaling performs quite poorly in recovering the underlying gross output estimates of the production function and productivity, leading to estimates that are in some cases

[10]See Petrin and Sivadasan (2013) for an example in which a version of this is implemented.

even further from the gross output estimates than the value-added estimates themselves.

In Tables 5 and 6 we report the re-scaled estimates as well as the value-added estimates using ACF and the gross output estimates using GNR. At first glance, the re-scaling appears to be working as many of the re-scaled value-added estimates move towards the gross output estimates. However, in some cases, the estimates of dispersion and the relationship between productivity and other dimensions of firm heterogeneity move only slightly towards the gross output estimates, and remain very close to the original value-added estimates. Moreover, in many cases the estimates overshoot the gross output estimates. Even worse, in some cases the re-scaling moves them in the opposite direction and leads to estimates that are even further from the gross output estimates than the original value-added estimates. Finally, in several cases, the re-scaled estimates actually lead to a sign-reversal compared to both the value-added and gross output estimates. Overall, while in some cases the re-scaling applied to the value-added estimates moves them closer to the gross output estimates, it does a poor job of replicating the gross output estimates, and in many cases moves them even further away.

5 Conclusion

In this paper we show that the use of value-added production functions can generate substantially different patterns of productivity heterogeneity as compared to gross output. This suggests that empirical studies of productivity based on value added may lead to fundamentally different policy implications compared to those based on gross output. To illustrate this possibility, consider the recent literature that uses productivity dispersion to explain cross-country differences in output per worker through resource misallocation. As an example, the influential paper by Hsieh and Klenow (2009) finds substantial heterogeneity in productivity dispersion (defined as the variance of log productivity) across countries as measured using value added. In particular, when they compare the United States with China and India, the variance of log productivity ranges from 0.40-0.55 for China and 0.45-0.48 for India, but only from 0.17-0.24 for the United States. They then use this estimated dispersion to measure the degree of misallocation of resources in the respective economies. In their main counterfactual they find that, by reducing the degree of misallocation in China and India to that of the United States, aggregate TFP would increase by 30%-50% in China and 40%-60% in India. In our datasets for Colombia and Chile the corresponding estimates of the variance in log productivity using a value-added specification are 0.43 and 0.94, respectively. Thus their analysis applied to our data would suggest that there is similar room for improvement in aggregate TFP in Colombia, and much more in Chile.

However, when productivity is measured using a gross output framework, our empirical findings suggest a much different result. The variance of log productivity using gross output is 0.08 in Colombia and 0.15 in Chile. These significantly smaller dispersion measures could imply that there is much less room for improvement in aggregate productivity for Colombia and Chile. Since the 90/10 ratios we obtain for Colombia and Chile using gross output are quantitatively very similar to the estimates obtained by Syverson (2004) for the United States (who also employed gross output but in an index number framework), this also suggests that the differences in misallocation of resources between developed and developing countries may not be as large as the analysis of Hsieh and Klenow (2009) implies.[11]

Exploring the role of gross output production functions for policy problems such as the one above could be a fruitful direction for future research. A key message of this paper is that insights derived under value added, compared to gross output, could lead to significantly different policy conclusions.

[11]Hsieh and Klenow note that their estimate of log productivity dispersion for the United States is larger than previous estimates by Foster, Haltiwanger, and Syverson (2008) by a factor of almost 4. They attribute this to the fact that Foster, Haltiwanger, and Syverson use a selected set of homogeneous industries. However, Foster, Haltiwanger, and Syverson use gross output measures of productivity. Given our results in Section 4, it is likely that a large part of this difference is due to Hsieh and Klenow's use of value added, rather than their selection of industries.

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Table 1: Average Input Elasticities of Output

(Structural Estimates: Value Added vs. Gross Ouput)

	Industry (ISIC Code)											
	Food Products (311)		**Textiles (321)**		**Apparel (322)**		**Wood Products (331)**		**Fabricated Metals (381)**		**All**	
	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)
Colombia												
Labor	0.70	0.22	0.65	0.32	0.83	0.42	0.86	0.44	0.89	0.43	0.78	0.35
	(0.04)	(0.02)	(0.06)	(0.03)	(0.03)	(0.02)	(0.06)	(0.05)	(0.04)	(0.02)	(0.01)	(0.01)
Capital	0.33	0.12	0.36	0.16	0.16	0.05	0.12	0.04	0.25	0.10	0.31	0.14
	(0.02)	(0.01)	(0.04)	(0.02)	(0.02)	(0.01)	(0.04)	(0.02)	(0.03)	(0.01)	(0.01)	(0.01)
Intermediates	--	0.67	--	0.54	--	0.52	--	0.51	--	0.53	--	0.54
		(0.01)		(0.01)		(0.01)		(0.01)		(0.01)		(0.00)
Sum	1.03	1.01	1.01	1.01	0.99	0.99	0.98	0.99	1.14	1.06	1.09	1.04
	(0.03)	(0.01)	(0.04)	(0.02)	(0.02)	(0.01)	(0.07)	(0.04)	(0.02)	(0.01)	(0.01)	(0.00)
Mean(Capital) / Mean(Labor)	0.47	0.55	0.55	0.49	0.19	0.12	0.14	0.08	0.28	0.23	0.39	0.40
	(0.06)	(0.08)	(0.10)	(0.09)	(0.03)	(0.04)	(0.05)	(0.05)	(0.04)	(0.04)	(0.02)	(0.03)
Chile												
Labor	0.77	0.28	0.93	0.45	0.95	0.45	0.92	0.40	0.96	0.52	0.77	0.38
	(0.02)	(0.01)	(0.04)	(0.03)	(0.04)	(0.02)	(0.04)	(0.02)	(0.04)	(0.03)	(0.01)	(0.01)
Capital	0.33	0.11	0.24	0.11	0.20	0.06	0.19	0.07	0.25	0.13	0.37	0.16
	(0.01)	(0.01)	(0.02)	(0.01)	(0.03)	(0.01)	(0.02)	(0.01)	(0.02)	(0.01)	(0.01)	(0.00)
Intermediates	--	0.67	--	0.54	--	0.56	--	0.59	--	0.50	--	0.55
		(0.00)		(0.01)		(0.01)		(0.01)		(0.01)		(0.00)
Sum	1.10	1.05	1.17	1.10	1.14	1.08	1.11	1.06	1.22	1.15	1.13	1.09
	(0.02)	(0.01)	(0.03)	(0.02)	(0.03)	(0.02)	(0.03)	(0.01)	(0.03)	(0.02)	(0.01)	(0.01)
Mean(Capital) / Mean(Labor)	0.43	0.39	0.26	0.24	0.21	0.14	0.21	0.18	0.26	0.25	0.48	0.43
	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers in the first column are based on a value-added specification and are estimated using a complete polynomial series of degree 2 with the method from Ackerberg, Caves, and Frazer (2015). The numbers in the second column are based on a gross output specification and are estimated using a complete polynomial series of degree 2 for each of the two nonparametric functions (D and $\mathscr{C}$) of the Gandhi, Navarro, and Rivers (2017) approach.

c. Since the input elasticities are heterogeneous across firms, we report the average input elasticities within each given industry.

d. The row titled "Sum" reports the sum of the average labor, capital, and intermediate input elasticities, and the row titled "Mean(Capital)/Mean(Labor)" reports the ratio of the average capital elasticity to the average labor elasticity.

Table 2: Heterogeneity in Productivity
(Structural Estimates)

	Industry (ISIC Code)											
	Food Products (311)		**Textiles (321)**		**Apparel (322)**		**Wood Products (331)**		**Fabricated Metals (381)**		**All**	
	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)	Value Added (ACF)	Gross Output (GNR)
Colombia												
75/25 ratio	2.20	1.33	1.97	1.35	1.66	1.29	1.73	1.30	1.78	1.31	1.95	1.37
	(0.07)	(0.02)	(0.09)	(0.03)	(0.03)	(0.01)	(0.08)	(0.04)	(0.04)	(0.02)	(0.17)	(0.01)
90/10 ratio	5.17	1.77	3.71	1.83	2.87	1.66	3.08	1.80	3.33	1.74	4.01	1.86
	(0.27)	(0.05)	(0.30)	(0.07)	(0.09)	(0.03)	(0.38)	(0.12)	(0.13)	(0.03)	(0.07)	(0.02)
95/5 ratio	11.01	2.24	6.36	2.38	4.36	2.02	4.58	2.24	5.31	2.16	6.86	2.36
	(1.11)	(0.08)	(0.76)	(0.14)	(0.22)	(0.05)	(1.01)	(0.22)	(0.34)	(0.06)	(0.02)	(0.03)
Exporter	3.62	0.14	0.20	0.02	0.16	0.05	0.26	0.15	0.20	0.08	0.51	0.06
	(0.99)	(0.05)	(0.10)	(0.03)	(0.07)	(0.03)	(0.63)	(0.14)	(0.05)	(0.03)	(0.12)	(0.01)
Importer	-0.25	0.04	0.27	0.05	0.29	0.12	0.06	0.05	0.26	0.10	0.20	0.11
	(0.08)	(0.02)	(0.10)	(0.04)	(0.08)	(0.03)	(0.53)	(0.08)	(0.06)	(0.02)	(0.05)	(0.01)
Advertiser	-0.46	-0.03	0.20	0.08	0.13	0.05	0.02	0.04	0.15	0.05	-0.13	0.03
	(0.10)	(0.02)	(0.07)	(0.03)	(0.04)	(0.02)	(0.09)	(0.04)	(0.04)	(0.02)	(0.06)	(0.01)
Wages > Median	0.59	0.09	0.60	0.18	0.41	0.18	0.34	0.15	0.55	0.22	0.63	0.20
	(0.19)	(0.02)	(0.09)	(0.03)	(0.03)	(0.02)	(0.17)	(0.04)	(0.06)	(0.02)	(0.05)	(0.01)
Chile												
75/25 ratio	2.92	1.37	2.56	1.48	2.58	1.43	3.06	1.50	2.45	1.53	3.00	1.55
	(0.05)	(0.01)	(0.07)	(0.02)	(0.07)	(0.02)	(0.08)	(0.02)	(0.06)	(0.02)	(0.03)	(0.01)
90/10 ratio	9.02	1.90	6.77	2.16	6.76	2.11	10.12	2.32	6.27	2.33	9.19	2.39
	(0.30)	(0.02)	(0.30)	(0.05)	(0.33)	(0.05)	(0.60)	(0.05)	(0.27)	(0.05)	(0.15)	(0.02)
95/5 ratio	21.29	2.48	13.56	2.91	14.21	2.77	25.08	3.11	12.52	3.13	20.90	3.31
	(0.99)	(0.05)	(0.84)	(0.09)	(0.77)	(0.09)	(2.05)	(0.11)	(0.78)	(0.10)	(0.47)	(0.04)
Exporter	0.27	0.02	0.07	0.02	0.18	0.09	0.12	0.00	0.03	-0.01	0.20	0.03
	(0.10)	(0.02)	(0.07)	(0.03)	(0.08)	(0.03)	(0.12)	(0.03)	(0.06)	(0.03)	(0.04)	(0.01)
Importer	0.71	0.14	0.22	0.10	0.31	0.14	0.44	0.15	0.30	0.11	0.46	0.15
	(0.11)	(0.02)	(0.05)	(0.02)	(0.05)	(0.02)	(0.10)	(0.03)	(0.05)	(0.02)	(0.03)	(0.01)
Advertiser	0.18	0.04	0.09	0.04	0.15	0.06	0.04	0.03	0.07	0.01	0.14	0.06
	(0.05)	(0.01)	(0.04)	(0.02)	(0.04)	(0.02)	(0.04)	(0.01)	(0.04)	(0.02)	(0.02)	(0.01)
Wages > Median	1.23	0.21	0.47	0.19	0.62	0.22	0.68	0.21	0.56	0.22	0.99	0.30
	(0.09)	(0.01)	(0.06)	(0.02)	(0.06)	(0.02)	(0.08)	(0.02)	(0.06)	(0.02)	(0.04)	(0.01)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers in the first column are based on a value-added specification and are estimated using a complete polynomial series of degree 2 with the method from Ackerberg, Caves, and Frazer (2015). The numbers in the second column are based on a gross output specification and are estimated using a complete polynomial series of degree 2 for each of the nonparametric functions (D and $\mathcal{C}$) of the Gandhi, Navarro, and Rivers (2017) approach.

c. In the first three rows we report ratios of productivity for plants at various percentiles of the productivity distribution. In the remaining four rows we report estimates of the productivity differences between plants (as a fraction) based on whether they have exported some of their output, imported intermediate inputs, spent money on advertising, and paid wages above the industry median. For example, in industry 311 for Chile value added implies that a firm that advertises is, on average, 18% more productive than a firm that does not advertise.

Table 3: Heterogeneity in Productivity
(Uncorrected OLS Estimates)

Industry (ISIC Code)

	Food Products (311)		Textiles (321)		Apparel (322)		Wood Products (331)		Fabricated Metals (381)		All	
	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)	Value Added (OLS)	Gross Output (OLS)
Colombia												
75/25 ratio	2.17	1.16	1.86	1.21	1.65	1.17	1.72	1.23	1.78	1.23	1.93	1.24
	(0.06)	(0.01)	(0.06)	(0.01)	(0.03)	(0.01)	(0.06)	(0.02)	(0.04)	(0.01)	(0.02)	(0.00)
90/10 ratio	5.15	1.42	3.50	1.51	2.81	1.44	3.05	1.57	3.30	1.53	3.96	1.58
	(0.27)	(0.02)	(0.18)	(0.04)	(0.08)	(0.02)	(0.22)	(0.06)	(0.12)	(0.02)	(0.06)	(0.01)
95/5 ratio	10.86	1.74	5.77	1.82	4.23	1.74	4.67	2.01	5.22	1.82	6.81	1.94
	(0.94)	(0.05)	(0.55)	(0.08)	(0.20)	(0.04)	(0.72)	(0.15)	(0.31)	(0.04)	(0.15)	(0.02)
Exporter	3.42	0.09	-0.03	-0.01	0.10	0.00	0.21	0.10	0.12	0.03	0.45	0.01
	(0.99)	(0.04)	(0.04)	(0.01)	(0.05)	(0.01)	(0.19)	(0.09)	(0.04)	(0.02)	(0.12)	(0.01)
Importer	-0.23	-0.02	0.09	0.00	0.21	0.02	0.02	-0.03	0.20	0.05	0.14	0.04
	(0.07)	(0.01)	(0.06)	(0.01)	(0.06)	(0.01)	(0.06)	(0.02)	(0.05)	(0.01)	(0.04)	(0.01)
Advertiser	-0.46	-0.07	0.11	-0.04	0.10	-0.03	0.01	-0.02	0.08	0.00	-0.16	-0.02
	(0.11)	(0.02)	(0.05)	(0.02)	(0.03)	(0.01)	(0.07)	(0.03)	(0.04)	(0.01)	(0.06)	(0.01)
Wages > Median	0.51	0.06	0.49	0.10	0.39	0.13	0.33	0.11	0.50	0.13	0.56	0.13
	(0.15)	(0.02)	(0.07)	(0.02)	(0.03)	(0.01)	(0.08)	(0.03)	(0.04)	(0.01)	(0.05)	(0.01)
Chile												
75/25 ratio	2.91	1.30	2.57	1.40	2.56	1.36	3.07	1.39	2.47	1.46	3.01	1.45
	(0.05)	(0.00)	(0.07)	(0.01)	(0.07)	(0.01)	(0.08)	(0.01)	(0.06)	(0.01)	(0.03)	(0.00)
90/10 ratio	9.00	1.72	6.63	1.97	6.64	1.91	10.21	2.03	6.27	2.14	9.13	2.14
	(0.29)	(0.01)	(0.31)	(0.04)	(0.29)	(0.03)	(0.57)	(0.04)	(0.26)	(0.04)	(0.15)	(0.01)
95/5 ratio	20.93	2.15	13.49	2.57	14.20	2.45	25.26	2.77	12.18	2.80	20.64	2.86
	(0.96)	(0.02)	(0.83)	(0.07)	(0.80)	(0.05)	(2.05)	(0.07)	(0.77)	(0.06)	(0.47)	(0.03)
Exporter	0.17	-0.01	0.04	-0.02	0.12	0.01	0.12	-0.02	0.00	0.00	0.15	-0.01
	(0.09)	(0.02)	(0.06)	(0.02)	(0.08)	(0.02)	(0.09)	(0.02)	(0.06)	(0.02)	(0.04)	(0.01)
Importer	0.57	0.03	0.20	0.04	0.26	0.06	0.41	0.07	0.27	0.06	0.41	0.09
	(0.09)	(0.01)	(0.04)	(0.02)	(0.05)	(0.01)	(0.09)	(0.03)	(0.05)	(0.02)	(0.03)	(0.01)
Advertiser	0.12	0.00	0.07	0.01	0.11	0.02	0.02	0.01	0.05	0.01	0.10	0.04
	(0.04)	(0.01)	(0.04)	(0.01)	(0.04)	(0.01)	(0.04)	(0.01)	(0.04)	(0.02)	(0.02)	(0.01)
Wages > Median	1.11	0.12	0.45	0.15	0.58	0.16	0.66	0.13	0.53	0.16	0.94	0.24
	(0.07)	(0.01)	(0.05)	(0.02)	(0.06)	(0.02)	(0.07)	(0.02)	(0.06)	(0.02)	(0.03)	(0.01)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers in the first column are based on a value-added specification and are estimated using a complete polynomial series of degree 2 with OLS. The numbers in the second column are based on a gross output specification estimated using a complete polynomial series of degree 2 with OLS.

c. In the first three rows we report ratios of productivity for plants at various percentiles of the productivity distribution. In the remaining four rows we report estimates of the productivity differences between plants (as a fraction) based on whether they have exported some of their output, imported intermediate inputs, spent money on advertising, and paid wages above the industry median. For example, in industry 311 for Chile value added implies that a firm that advertises is, on average, 12% more productive than a firm that does not advertise.

Table 4: Average Input Elasticities of Output
(Gross Output: Structural vs. Uncorrected OLS Estimates)

	Industry (ISIC Code)											
	Food Products (311)		**Textiles (321)**		**Apparel (322)**		**Wood Products (331)**		**Fabricated Metals (381)**		**All**	
	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)	Gross Output (OLS)	Gross Output (GNR)
Colombia												
Labor	0.15	0.22	0.21	0.32	0.32	0.42	0.32	0.44	0.29	0.43	0.26	0.35
	(0.01)	(0.02)	(0.02)	(0.03)	(0.01)	(0.02)	(0.03)	(0.05)	(0.02)	(0.02)	(0.01)	(0.01)
Capital	0.04	0.12	0.06	0.16	0.01	0.05	0.03	0.04	0.03	0.10	0.06	0.14
	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.00)	(0.01)
Intermediates	0.82	0.67	0.76	0.54	0.68	0.52	0.65	0.51	0.73	0.53	0.72	0.54
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)
Sum	1.01	1.01	1.03	1.01	1.01	0.99	1.00	0.99	1.05	1.06	1.04	1.04
	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)	(0.04)	(0.01)	(0.01)	(0.00)	(0.00)
Mean(Capital) / Mean(Labor)	0.27	0.55	0.27	0.49	0.04	0.12	0.08	0.08	0.11	0.23	0.23	0.40
	(0.07)	(0.08)	(0.06)	(0.09)	(0.02)	(0.04)	(0.05)	(0.05)	(0.04)	(0.04)	(0.01)	(0.03)
Chile												
Labor	0.17	0.28	0.26	0.45	0.29	0.45	0.20	0.40	0.32	0.52	0.20	0.38
	(0.01)	(0.01)	(0.02)	(0.03)	(0.02)	(0.02)	(0.01)	(0.02)	(0.02)	(0.03)	(0.01)	(0.01)
Capital	0.05	0.11	0.06	0.11	0.03	0.06	0.02	0.07	0.07	0.13	0.09	0.16
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)
Intermediates	0.83	0.67	0.75	0.54	0.74	0.56	0.81	0.59	0.71	0.50	0.77	0.55
	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)
Sum	1.05	1.05	1.06	1.10	1.06	1.08	1.04	1.06	1.10	1.15	1.06	1.09
	(0.00)	(0.01)	(0.01)	(0.02)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.02)	(0.00)	(0.01)
Mean(Capital) / Mean(Labor)	0.28	0.39	0.22	0.24	0.12	0.14	0.12	0.18	0.21	0.25	0.42	0.43
	(0.03)	(0.03)	(0.04)	(0.04)	(0.03)	(0.03)	(0.05)	(0.03)	(0.04)	(0.03)	(0.02)	(0.02)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers in the first column are based on a gross output specification and are estimated using a complete polynomial series of degree 2 with OLS. The numbers in the second column are also based on a gross output specification using a complete polynomial series of degree 2 for each of the two nonparametric functions (D and $\mathscr{C}$) of the Gandhi, Navarro, and Rivers (2017) approach.

c. Since the input elasticities are heterogeneous across firms, we report the average input elasticities within each given industry.

d. The row titled "Sum" reports the sum of the average labor, capital, and intermediate input elasticities, and the row titled "Mean(Capital)/Mean(Labor)" reports the ratio of the average capital elasticity to the average labor elasticity.

Table 5: Average Input Elasticities of Output--Rescaled Value Added

(Structural Estimates: Rescaled Value Added vs. Gross Ouput)

Industry (ISIC Code)

	Food Products (311)			Textiles (321)			Apparel (322)			Wood Products (331)			Fabricated Metals (381)			All		
	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF--Rescaled)	Gross Output (GNR)
Colombia																		
Labor	0.70	0.20	0.22	0.65	0.28	0.32	0.83	0.38	0.42	0.86	0.40	0.44	0.89	0.40	0.43	0.78	0.33	0.35
	(0.04)	(0.01)	(0.02)	(0.06)	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)	(0.06)	(0.03)	(0.05)	(0.04)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)
Capital	0.33	0.08	0.12	0.36	0.15	0.16	0.16	0.07	0.05	0.12	0.05	0.04	0.25	0.11	0.10	0.31	0.13	0.14
	(0.02)	(0.01)	(0.01)	(0.04)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.04)	(0.02)	(0.02)	(0.03)	(0.01)	(0.01)	(0.01)	(0.00)	(0.01)
Intermediates	--	--	0.67	--	--	0.54	--	--	0.52	--	--	0.51	--	--	0.53	--	--	0.54
			(0.01)			(0.01)			(0.01)			(0.01)			(0.01)			(0.00)
Sum	1.03	1.01	1.01	1.01	1.00	1.01	0.99	0.99	0.99	0.98	0.99	0.99	1.14	1.06	1.06	1.09	1.04	1.04
	(0.03)	(0.01)	(0.01)	(0.04)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.07)	(0.03)	(0.04)	(0.02)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)
Mean(Capital) / Mean(Labor)	0.47	0.42	0.55	0.55	0.55	0.49	0.19	0.19	0.12	0.14	0.14	0.08	0.28	0.27	0.23	0.39	0.38	0.40
	(0.06)	(0.05)	(0.08)	(0.10)	(0.10)	(0.09)	(0.03)	(0.03)	(0.04)	(0.05)	(0.05)	(0.05)	(0.04)	(0.04)	(0.04)	(0.02)	(0.02)	(0.03)
Chile																		
Labor	0.77	0.21	0.28	0.93	0.37	0.45	0.95	0.37	0.45	0.92	0.32	0.40	0.96	0.43	0.52	0.77	0.29	0.38
	(0.02)	(0.01)	(0.01)	(0.04)	(0.02)	(0.03)	(0.04)	(0.02)	(0.02)	(0.04)	(0.01)	(0.02)	(0.04)	(0.02)	(0.03)	(0.01)	(0.01)	(0.01)
Capital	0.33	0.10	0.11	0.24	0.10	0.11	0.20	0.08	0.06	0.19	0.07	0.07	0.25	0.11	0.13	0.37	0.14	0.16
	(0.01)	(0.00)	(0.01)	(0.02)	(0.01)	(0.01)	(0.03)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)
Intermediates	--	--	0.67	--	--	0.54	--	--	0.56	--	--	0.59	--	--	0.50	--	--	0.55
			(0.00)			(0.01)			(0.01)			(0.01)			(0.01)			(0.00)
Sum	1.10	1.03	1.05	1.17	1.07	1.10	1.14	1.06	1.08	1.11	1.04	1.06	1.22	1.09	1.15	1.13	1.05	1.09
	(0.02)	(0.00)	(0.01)	(0.03)	(0.01)	(0.02)	(0.03)	(0.01)	(0.02)	(0.03)	(0.01)	(0.01)	(0.03)	(0.01)	(0.02)	(0.01)	(0.00)	(0.01)
Mean(Capital) / Mean(Labor)	0.43	0.46	0.39	0.26	0.26	0.24	0.21	0.21	0.14	0.21	0.22	0.18	0.26	0.27	0.25	0.48	0.49	0.43
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)	(0.04)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers in the first column are based on a value-added specification and are estimated using a complete polynomial series of degree 2 with the method from Ackerberg, Caves, and Frazer (2015). The numbers in the second column are obtained by raising the value-added estimates to the power of one minus the firm's share of intermediate inputs in total output. The numbers in the third column are based on a gross output specification and are estimated using a complete polynomial series of degree 2 for each of the two nonparametric functions (D and $\mathscr{E}$) of the Gandhi, Navarro, and Rivers (2017) approach.

c. Since the input elasticities are heterogeneous across firms, we report the average input elasticities within each given industry.

d. The row titled "Sum" reports the sum of the average labor, capital, and intermediate input elasticities, and the row titled "Mean(Capital)/Mean(Labor)" reports the ratio of the average capital elasticity to the average labor elasticity.

Table 6: Heterogeneity in Productivity--Rescaled Value Added

(Structural Estimates: Rescaled Value Added vs. Gross Ouput)

Industry (ISIC Code)

	Food Products (311)			Textiles (321)			Apparel (322)			Wood Products (331)			Fabricated Metals (381)			All		
	Value Added (ACF)	Value Added (ACF-- Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF-- Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF-- Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF-- Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF-- Rescaled)	Gross Output (GNR)	Value Added (ACF)	Value Added (ACF-- Rescaled)	Gross Output (GNR)
Colombia																		
75/25 ratio	2.20	2.29	1.33	1.97	1.85	1.35	1.66	1.88	1.29	1.73	2.74	1.30	1.78	2.00	1.31	1.95	2.15	1.37
	(0.07)	(0.12)	(0.02)	(0.09)	(0.12)	(0.03)	(0.03)	(0.09)	(0.01)	(0.08)	(0.44)	(0.04)	(0.04)	(0.10)	(0.02)	(0.17)	(0.06)	(0.01)
90/10 ratio	5.17	4.99	1.77	3.71	3.28	1.83	2.87	3.60	1.66	3.08	6.27	1.80	3.33	3.81	1.74	4.01	4.68	1.86
	(0.27)	(0.51)	(0.05)	(0.30)	(0.39)	(0.07)	(0.09)	(0.34)	(0.03)	(0.38)	(1.51)	(0.12)	(0.13)	(0.35)	(0.03)	(0.07)	(0.24)	(0.02)
95/5 ratio	11.01	8.79	2.24	6.36	5.00	2.38	4.36	5.41	2.02	4.58	10.53	2.24	5.31	5.85	2.16	6.86	7.91	2.36
	(1.11)	(1.20)	(0.08)	(0.76)	(0.87)	(0.14)	(0.22)	(0.67)	(0.05)	(1.01)	(3.19)	(0.22)	(0.34)	(0.69)	(0.06)	(0.02)	(0.52)	(0.03)
Exporter	3.62	0.84	0.14	0.20	0.07	0.02	0.16	-0.04	0.05	0.26	0.55	0.15	0.20	0.06	0.08	0.51	0.09	0.06
	(0.99)	(0.30)	(0.05)	(0.10)	(0.08)	(0.03)	(0.07)	(0.07)	(0.03)	(0.63)	(0.60)	(0.14)	(0.05)	(0.07)	(0.03)	(0.12)	(0.06)	(0.01)
Importer	-0.25	-0.40	0.04	0.27	0.08	0.05	0.29	-0.09	0.12	0.06	-0.20	0.05	0.26	0.15	0.10	0.20	0.05	0.11
	(0.08)	(0.08)	(0.02)	(0.10)	(0.08)	(0.04)	(0.08)	(0.05)	(0.03)	(0.53)	(0.20)	(0.08)	(0.06)	(0.04)	(0.02)	(0.05)	(0.04)	(0.01)
Advertiser	-0.46	-0.42	-0.03	0.20	-0.16	0.08	0.13	-0.28	0.05	0.02	-0.24	0.04	0.15	-0.05	0.05	-0.13	-0.18	0.03
	(0.10)	(0.10)	(0.02)	(0.07)	(0.07)	(0.03)	(0.04)	(0.06)	(0.02)	(0.09)	(0.13)	(0.04)	(0.04)	(0.04)	(0.02)	(0.06)	(0.03)	(0.01)
Wages > Median	0.59	0.15	0.09	0.60	0.35	0.18	0.41	0.31	0.18	0.34	0.19	0.15	0.55	0.27	0.22	0.63	0.29	0.20
	(0.19)	(0.18)	(0.02)	(0.09)	(0.08)	(0.03)	(0.03)	(0.05)	(0.02)	(0.17)	(0.18)	(0.04)	(0.06)	(0.04)	(0.02)	(0.05)	(0.04)	(0.01)
Chile																		
75/25 ratio	2.92	2.36	1.37	2.56	2.06	1.48	2.58	2.94	1.43	3.06	3.31	1.50	2.45	2.47	1.53	3.00	2.92	1.55
	(0.05)	(0.11)	(0.01)	(0.07)	(0.17)	(0.02)	(0.07)	(0.27)	(0.02)	(0.08)	(0.36)	(0.02)	(0.06)	(0.19)	(0.02)	(0.03)	(0.10)	(0.01)
90/10 ratio	9.02	5.24	1.90	6.77	3.80	2.16	6.76	8.08	2.11	10.12	10.47	2.32	6.27	5.72	2.33	9.19	7.24	2.39
	(0.30)	(0.44)	(0.02)	(0.30)	(0.57)	(0.05)	(0.33)	(1.44)	(0.05)	(0.60)	(2.23)	(0.05)	(0.27)	(0.85)	(0.05)	(0.15)	(0.44)	(0.02)
95/5 ratio	21.29	8.91	2.48	13.56	5.54	2.91	14.21	13.89	2.77	25.08	19.83	3.11	12.52	9.40	3.13	20.90	12.23	3.31
	(0.99)	(0.96)	(0.05)	(0.84)	(1.05)	(0.09)	(0.77)	(2.94)	(0.09)	(2.05)	(5.41)	(0.11)	(0.78)	(1.84)	(0.10)	(0.47)	(0.91)	(0.04)
Exporter	0.27	0.34	0.02	0.07	-0.03	0.02	0.18	0.14	0.09	0.12	0.09	0.00	0.03	0.01	-0.01	0.20	0.12	0.03
	(0.10)	(0.12)	(0.02)	(0.07)	(0.06)	(0.03)	(0.08)	(0.11)	(0.03)	(0.12)	(0.11)	(0.03)	(0.06)	(0.07)	(0.03)	(0.04)	(0.05)	(0.01)
Importer	0.71	0.44	0.14	0.22	0.10	0.10	0.31	0.18	0.14	0.44	0.21	0.15	0.30	0.19	0.11	0.46	0.37	0.15
	(0.11)	(0.15)	(0.02)	(0.05)	(0.05)	(0.02)	(0.05)	(0.07)	(0.02)	(0.10)	(0.12)	(0.03)	(0.05)	(0.07)	(0.02)	(0.03)	(0.04)	(0.01)
Advertiser	0.18	0.05	0.04	0.09	0.00	0.04	0.15	0.06	0.06	0.04	-0.04	0.03	0.07	0.03	0.01	0.14	0.14	0.06
	(0.05)	(0.05)	(0.01)	(0.04)	(0.04)	(0.02)	(0.04)	(0.06)	(0.02)	(0.04)	(0.06)	(0.01)	(0.04)	(0.06)	(0.02)	(0.02)	(0.02)	(0.01)
Wages > Median	1.23	0.66	0.21	0.47	0.34	0.19	0.62	0.57	0.22	0.68	0.53	0.21	0.56	0.47	0.22	0.99	0.89	0.30
	(0.09)	(0.07)	(0.01)	(0.06)	(0.06)	(0.02)	(0.06)	(0.09)	(0.02)	(0.08)	(0.11)	(0.02)	(0.06)	(0.09)	(0.02)	(0.04)	(0.05)	(0.01)

Notes:

a. Standard errors are estimated using the bootstrap with 200 replications and are reported in parentheses below the point estimates.

b. For each industry, the numbers in the first column are based on a value-added specification and are estimated using a complete polynomial series of degree 2 with the method from Ackerberg, Caves, and Frazer (2015). The numbers in the second column are obtained by raising the value-added estimates to the power of one minus the firm's share of intermediate inputs in total output. The numbers in the third column are based on a gross output specification and are estimated using a complete polynomial series of degree 2 for each of the two nonparametric functions (D and $\mathscr{C}$) of the Gandhi, Navarro, and Rivers (2017) approach.

c. In the first three rows we report ratios of productivity for plants at various percentiles of the productivity distribution. In the remaining four rows we report estimates of the productivity differences between plants (as a fraction) based on whether they have exported some of their output, imported intermediate inputs, spent money on advertising, and paid wages above the industry median. For example, in industry 311 for Chile value added implies that a firm that advertises is, on average, 18% more productive than a firm that does not advertise.

Appendix: Value Added

In this appendix we provide additional details regarding value added.

Restricted Profit Functions

Recall equation (5) in the main body:

$$VA_{jt}^{E} = F\left(k_{jt}, l_{jt}, m_{jt}\right) e^{\omega_{jt}} - M_{jt} \equiv \mathcal{V}_t\left(k_{jt}, l_{jt}, e^{\omega_{jt}}\right).$$

It can be shown that the total derivative of value added with respect to one of its inputs is equal to the partial derivative of gross output with respect to that input. For example, the total derivative of value added with respect to productivity is given by:

$$\begin{aligned}
\frac{dVA_{jt}^{E}}{de^{\omega_{jt}}} &= \frac{d\mathcal{V}\left(k_{jt}, l_{jt}, e^{\omega_{jt}}\right)}{de^{\omega_{jt}}} \\
&= \frac{\partial F\left(k_{jt}, l_{jt}, m_{jt}\right) e^{\omega_{jt}}}{\partial e^{\omega_{jt}}} - \left(\frac{\partial F\left(k_{jt}, l_{jt}, m_{jt}\right) e^{\omega_{jt}}}{\partial M_{jt}} - 1\right) \frac{\partial M_{jt}}{\partial e^{\omega_{jt}}} \\
&= \frac{\partial GO_{jt}}{\partial e^{\omega_{jt}}}.
\end{aligned}$$

Due to the first-order condition in equation (3) in the main text, the term inside the parentheses on the second line is equal to zero, where the relative price of output to intermediate inputs has been normalized to one via deflation. This implies that:

$$\underbrace{\frac{\partial GO_{jt}}{\partial e^{\omega_{jt}}} \frac{e^{\omega_{jt}}}{GO_{jt}}}_{elas_{e^{\omega_{jt}}}^{GO_{jt}}} = \underbrace{\frac{dVA_{jt}^{E}}{de^{\omega_{jt}}} \frac{e^{\omega_{jt}}}{VA_{jt}^{E}}}_{elas_{e^{\omega_{jt}}}^{VA_{jt}}} \frac{VA_{jt}^{E}}{GO_{jt}^{E}} = \underbrace{\frac{dVA_{jt}^{E}}{de^{\omega_{jt}}} \frac{e^{\omega_{jt}}}{VA_{jt}^{E}}}_{elas_{e^{\omega_{jt}}}^{VA_{jt}}} \left(1 - S_{jt}\right).$$

However, once we add back in the ex-post shocks we have the following:

$$\begin{aligned}
\frac{dVA_{jt}^{E}}{de^{\omega_{jt}}} &= \frac{d\mathcal{V}\left(k_{jt}, l_{jt}, e^{\omega_{jt}}, e^{\varepsilon_{jt}}\right)}{de^{\omega_{jt}}} \\
&= \frac{\partial F\left(k_{jt}, l_{jt}, m_{jt}\right) e^{\omega_{jt}+\varepsilon_{jt}}}{\partial e^{\omega_{jt}}} - \left(\frac{\partial F\left(k_{jt}, l_{jt}, m_{jt}\right) e^{\omega_{jt}+\varepsilon_{jt}}}{\partial M_{jt}} - 1\right) \frac{\partial M_{jt}}{\partial e^{\omega_{jt}}}.
\end{aligned}$$

Notice now that the term inside the parentheses is no longer equal to zero, due to the presence of the ex-post shock, ε_{jt}. The reason is that the first-order condition, which previously made that term equal to zero, is an *ex-ante* object, whereas what is inside the parentheses is *ex-post*. Therefore, we cannot simply transform the value-added elasticities into their gross output counterparts by re-scaling via the ratio of value added to gross output.

The first-order condition implies that $\frac{\partial F(k_{jt},l_{jt},m_{jt})e^{\omega_{jt}+\varepsilon_{jt}}}{\partial M_{jt}} = \frac{e^{\varepsilon_{jt}}}{\mathcal{E}}$. In turn, this implies that

$$\begin{aligned} \frac{dVA_{jt}^{E}}{de^{\omega_{jt}}} &= \frac{\partial F\left(k_{jt},l_{jt},m_{jt}\right)e^{\omega_{jt}+\varepsilon_{jt}}}{\partial e^{\omega_{jt}}} - \quad \frac{e^{\varepsilon_{jt}}}{\mathcal{E}} - 1 \quad \frac{\partial M_{jt}}{\partial e^{\omega_{jt}}} \\ \Rightarrow elas_{e^{\omega_{jt}}}^{VA_{jt}^{E}} &= elas_{e^{\omega_{jt}}}^{GO_{jt}} \frac{GO_{jt}}{VA_{jt}^{E}} - \frac{\partial M_{jt}}{\partial e^{\omega_{jt}+\varepsilon_{jt}}} \frac{e^{\omega_{jt}}}{VA_{jt}^{E}} \quad \frac{e^{\varepsilon_{jt}}}{\mathcal{E}} - 1 \quad . \end{aligned}$$

The equation above can then be rearranged to form relationship between the elasticities as:

$$\underbrace{\frac{\partial GO_{jt}}{\partial e^{\omega_{jt}}}\frac{e^{\omega_{jt}}}{GO_{jt}}}_{elas_{e^{\omega_{jt}}}^{GO_{jt}}} = \underbrace{\frac{\partial VA_{jt}^{E}}{\partial e^{\omega_{jt}}}\frac{e^{\omega_{jt}}}{VA_{jt}^{E}}}_{elas_{e^{\omega_{jt}}}^{VA_{jt}^{E}}} \left(1 - S_{jt}\right) + \frac{\partial M_{jt}}{\partial e^{\omega_{jt}}}\frac{e^{\omega_{jt}}}{GO_{jt}} \quad \frac{e^{\varepsilon_{jt}}}{\mathcal{E}} - 1 \quad .$$

A similar result holds when we analyze the elasticities with respect to the entire productivity shock, $e^{\omega_{jt}+\varepsilon_{jt}}$, instead of just the persistent component, $e^{\omega_{jt}}$. In this case we have the following relationship:

$$\underbrace{\frac{\partial GO_{jt}}{\partial e^{\omega_{jt}+\varepsilon_{jt}}}\frac{e^{\omega_{jt}+\varepsilon_{jt}}}{GO_{jt}}}_{elas_{e^{\omega_{jt}+\varepsilon_{jt}}}^{GO_{jt}}} = \underbrace{\frac{\partial VA_{jt}^{E}}{\partial e^{\omega_{jt}+\varepsilon_{jt}}}\frac{e^{\omega_{jt}+\varepsilon_{jt}}}{VA_{jt}^{E}}}_{elas_{e^{\omega_{jt}+\varepsilon_{jt}}}^{VA_{jt}^{E}}} \left(1 - S_{jt}\right) + \frac{\partial M_{jt}}{\partial e^{\omega_{jt}+\varepsilon_{jt}}}\frac{e^{\omega_{jt}+\varepsilon_{jt}}}{GO_{jt}} \quad \frac{e^{\varepsilon_{jt}}}{\mathcal{E}} - 1 \quad .$$

"Structural" Value Added

As discussed in Section 3.2, for the Leontief case we have

$$Y_{jt} = \min\left[\mathcal{H}(k_{jt},l_{jt}),\mathcal{C}\left(m_{jt}\right)\right]e^{\omega_{jt}+\varepsilon_{jt}}. \tag{13}$$

The standard Leontief condition, $\mathcal{H}(k_{jt}, l_{jt}) = \mathcal{C}(m_{jt})$, will not generally hold unless $\mathcal{C}(m_{jt}) = aM_{jt}$. Even in this linear case, the value-added production function, $\mathcal{H}(k_{jt}, l_{jt})e^{\omega_{jt}+\varepsilon_{jt}}$, does not relate cleanly to the empirical measure of value added $VA_{jt}^{E} \equiv Y_{jt} - M_{jt}$, since $VA_{jt}^{E} = \mathcal{H}(k_{jt}, l_{jt})\left(e^{\omega_{jt}+\varepsilon_{jt}} - \frac{1}{a}\right)$. However, it does correspond directly to gross output since $Y_{jt} = \mathcal{H}(k_{jt}, l_{jt})e^{\omega_{jt}+\varepsilon_{jt}}$.

Neither of these issues is resolved by moving ω_{jt} inside the min function in equation (13). Suppose that instead of equation (13), one wrote the production function as: $Y_{jt} = \min[\mathcal{H}(k_{jt}, l_{jt})e^{\omega_{jt}}, \mathcal{C}(m_{jt})]e^{\varepsilon_{jt}}$. For similar reasons, the condition, $\mathcal{H}(k_{jt}, l_{jt})e^{\omega_{jt}} = \mathcal{C}(m_{jt})$, only holds when $\mathcal{C}(m_{jt}) = aM_{jt}$. Even when this is the case, the value-added production function will again not correspond to the empirical measure of value added as $VA_{jt}^{E} = \mathcal{H}(k_{jt}, l_{jt})e^{\omega_{jt}}\left(e^{\varepsilon_{jt}} - \frac{1}{a}\right)$. As was the case above, however, it directly corresponds to gross output: $Y_{jt} = \mathcal{H}(k_{jt}, l_{jt})e^{\omega_{jt}+\varepsilon_{jt}}$.

It is also the case that moving ε_{jt} inside the min function does not help. The problem is that the key condition, $\mathcal{H}(k_{jt}, l_{jt})e^{\varepsilon_{jt}} = aM_{jt}$, will not hold when ω_{jt} is outside the min because of the presence of the ex-post shock ε_{jt}. Since ε_{jt} is realized after input decisions are made, the key condition will generally not hold. Thus neither VA_{jt}^{E} nor gross output Y_{jt} correspond to the structural value-added production function $\mathcal{H}(k_{jt}, l_{jt})e^{\omega_{jt}+\varepsilon_{jt}}$. An analogous argument holds when ω_{jt} is inside the min function.