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# A Fuzzy-based Inference Mechanism of Trust for Improved Social Recommenders

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## ABSTRACT

There have been various definitions, representations and derivations of trust in the context of recommender systems. This article presents a recommender predictive model based on collaborative filtering techniques that incorporate a fuzzy-driven quantifier, which includes two utmost relevant social phenomena parameters to address the vagueness inherent in the assessment of trust in social networks relationships. An experimental evaluation procedure utilizing a case study is conducted to analyze the overall predictive accuracy. These results show that the proposed methodology improves the performance of classical recommender approaches. Possible extensions are then outlined.

## Author Keywords

Collaborative Filtering, Recommender Systems, Fuzzy Linguistic, Similarity, Homophily, Small World Problem.

## ACM Classification Keywords

H.3.3 [Information Storage and Retrieval]: Information Search and Retrieval – *Collaborative Filtering*.

## INTRODUCTION

Recommender systems [1] have helped fuel the success of social computing-based applications by eliciting preferred user contributed contents such as movies, books and items alike. Content filtering techniques such as Collaborative Filtering [2] have been the most commonly employed technique of choice for recommendation algorithms. This filtering procedure tries to identify users that have common interests and preferences by calculating similarities among them in attempts to suggest items that are most likely to be of mutual interest. The recommendation quality of such systems depends on several factors. In recent years, concepts of social trust are being used to deal with the similarity conditions between pairs of users as additional information to potentially increase recommendation accuracy.

Although many different studies have shown that using trust from social networks can improve recommendations, there have been some limitations and gaps which this work proposes to bridge. To begin, the notion of trust is herein formalized within the sociological domain of the term to derive a trust metric that is more practically significant and

intrinsically more realistic and refined than previous efforts. In this respect, two factors of interest synthesize the underlined social phenomena that occur in social networks. First, while Lazarsfeld and Merton [3] first demonstrated how similar individuals have the tendency to associate and bond to one another, it was with McPherson et al. [4] that the *homophily* principle gained broader attention. The “Birds of a Feather” study cited over one hundred studies showing that similarity with regard to many socio-demographic, behavioral and intrapersonal characteristics breeds connection, creates strong divides in our personal environments, and impacts choice overall. Second, the degree to which connected people are separated one to another seems to have fascinated many. Karinyth [5] wrote a play portraying a shrinking modern world due to the ever-increasing connectedness of human beings which later led Milgram [6] to conduct a seminal empirical investigation aimed at measuring this connectedness in his “Small World” experiment. The experiment was designed to measure the path lengths between any two people by developing a procedure, in which random mailed packages were asked to be returned to a specific and unknown person through the network of friends to count the number of ties among origin and destination. However, it was with the urban myth “six degree of separation” popularized in a play written by John Guare [7] that the concept of *separation* gained enormous currency and paved the way for many other studies [8] relating the probability that two randomly selected people would know each other with the average number of ties needed. Lastly, in personal social networks of any size it is very difficult to accurately measure a trustworthiness value between individuals due to the ambiguity and vagueness associated with decisions in assessing such knowledge of (or, friendship with) someone. Based on these two governing social phenomena dimensions in particular, human-like reasoning rather than probabilistic algorithms may offer many advantages, including the use of linguistics expressions such as “strong/weak” (ties) for assessing such information to arrive at a trust decision.

In this paper, a discussion on the proposed trust metric and its application to improve recommendations is presented along with the empirical evaluation study that was used to

validate the assumptions.

## RELATED WORK

In the current literature, most popular approaches in the area of social trust-based recommenders include trust inference and propagation schemes. For instance, Guha et al. [9] developed a formal framework of trust propagation in which explicitly stated trust values were used to predict unknown trust values between any two people in the system with high accuracy. Golbeck et al. [10] proposed two algorithms for inferring trust relationships between individuals who are not directly connected in the networks in which known trust values were dependable on the ratings in the trust network application. DuBois et al. [11] employed a probabilistic trust inference algorithm and cluster methods in social network settings with trust ratings between users to generate recommendations more accurately. Massa et al. [12] introduced a trust-aware recommendation architecture with a trust propagation technique applied to the network of users which relies on explicit trust rates from one user to another. O'Donovan and Smyth [13] presented two computational models of trust which were created by estimating how correctly users made recommendations that have contributed to one another. Papagelis et al. [14] used the notion that interactions are based on the many ratings activities between users to feature a social network structures that were used to adopt a method of inferring trust between users that are not directly associated to each other.

While these studies provide better recommendations, the idea that users have to explicitly state trust values to one another limits personalization applications, which require a more implicit calculation approach based on aspects of the actual social networking context. Moreover, the limited number of studies in integrating sociological components of trust to improve recommendations is another of the motivations behind this work, which is explained next.

## FUZZY-TRUST QUANTIFIER MODELING

Often, the individual's decisions on his or her social networks relationships involve some degree of fuzziness and ambiguity. In large, complicated social networks, the individual may be able to specify his or her acquaintances only in the form of linguistic expressions such as "important" or "ordinary" for rating such knowledge of or friendship with someone. It is difficult, therefore, to accurately quantify an acquaintanceship value between each two individuals. To address this problem, a fuzzy quantifier has been developed which expresses their social trust.

The development of this component uses the concepts of fuzzy-set theory originated by Zadeh [15] and the concepts of fuzzy control developed by Takagi-Sugeno [16]. The fuzzy formulation herein proposed has also been inspired by influential works that try to point out the sociological factors in determining how much of human behavior can be explained in terms of the individual's group affiliation, such as Simmel [17]. The consideration of these properties leads to the development of a computational model that permits

the determination of intransitive user similarities based on social parameters inferences for addressing the social trust problem.

## Fuzzy output

In social networks relationships, a fuzzy linguistic variable "trustworthiness rating" ( $T$ ) is a fuzzy variable that represents the output variable. This linguistic variable can be represented by a family of linguistic terms (fuzzy sets  $W$ ,  $M$ , and  $S$  as shown in Figure 1). These three fuzzy sets cover the space of trustworthiness-rating solutions ranging from "strong" for  $S$  to "weak" for  $W$ . Each of these three sets (e.g., set  $M$ ) has a triangular membership function, with some overlap among them, as shown in Figure 1. It is noted that the ranges shown in Figure 1 for the different membership functions (e.g., set  $M$  ranges from 20% to 80%) were designed to exhibit a linear increase in the trustworthiness values. This gives same weight to low and high relationships between a pair of individuals, thus enforcing this correlation later during prediction. These membership functions are used to quantify a crisp value for the trustworthiness relationship between each two individuals, as discussed in the following subsections.

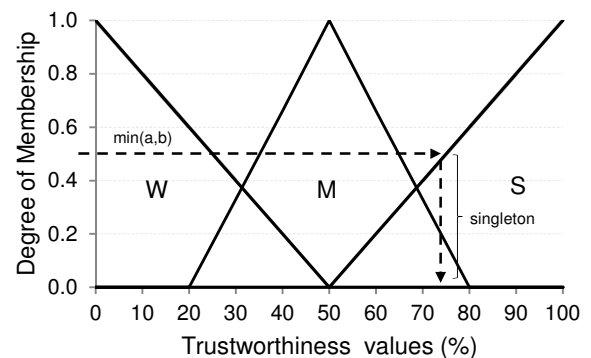


Figure 1. Fuzzy sets for the output variable "trustworthiness rating", with sample calculation indication

## Fuzzy inputs

In real-world sociology and psychology investigations, various factors have been considered by researchers to help describing the social phenomena that affects the trust perception of individuals one to another. In the context of the present development, the (a) homophily and (b) separation factors have been identified to be the most prevailing ones in determining the trustworthiness weights between each two users. Based on the preceding discussions, the problem at hand involves two fuzzy input variables: "degree of homophily" ( $DH$ ), and "degree of separation" ( $DS$ ). These two variables affect the "trustworthiness rating" fuzzy output variable identified earlier. A family of fuzzy sets has been formulated for the two fuzzy variables, and for simplicity, each variable was limited to three membership functions, being "low" ( $L$ ), "medium" ( $M$ ), and "high" ( $H$ ) for  $DH$  and "close" ( $C$ ), "medium" ( $M$ ), and "far" ( $F$ ) for  $DS$ . The shape and range of values of the six membership functions were determined through experimentation ( $DH$ ) and correlation with existing

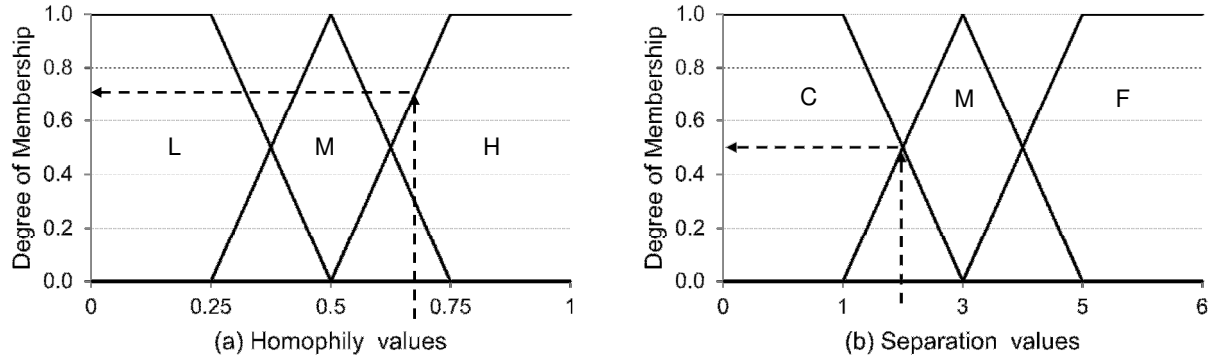


Figure 2. Fuzzy sets for the input variables, with sample calculation indication

axioms (DS). Accordingly, triangular and trapezoidal shapes were adopted (Figure 1 and Figure 2). These two shapes are the most frequently used in the literature ([18], [19]). All values shown on the membership functions are proposed based on writers' experience or testing. Homophily is assumed to vary from 0 to 1, where these extremes denote all characteristics are unequal or equal, respectively. The shape of the DH membership function is symmetrical and centers around 0.5, that is, half of the characteristics are equal. Modifying the membership functions values requires an intensive survey among practitioners, which is the subject of future research. The second membership function *DS* is similar to that of *DH*, but with a different scale, ranging from 0 to 6.

#### Fuzzy decision rules

So far, the "trustworthiness rating" desired to be determined is governed by two fuzzy variables, *DH* and *DS*. Since each of these fuzzy parameters has three membership functions *Low/Medium/High*, and *Close/Medium/Far*, there could be a total of  $3^2$  (9) distinct combinations of preconditions that affect the trustworthiness rating. These preconditions have to be stored in the form of rules (i.e., *fuzzy rules*) along with the decision maker's preference in their associated trustworthiness rating. An example rule is

Rule #8:

IF Degree of Homophily (*DF*) is High (*H*)  
 AND Degree of Separation (*DS*) is Medium (*M*) (1)  
 THEN Trustworthiness rating (*T*) is Strong (*S*)

As shown in this rule, the THEN part refers to one of the three membership functions associated with the fuzzy output variable "trustworthiness rating".

In developing the fuzzy rules for the problem at hand, a pragmatic approach was used to determine the appropriate membership function (*W*, *M*, or *S*) to associate with the three preconditions of each rule. For each input variable, a score of 3, 2 or 1 was given to the "high", "medium", or "low" linguistic term, respectively, of the *DH* parameter. Similarly, a score of 1, 2, or 3 was given to the "far", "medium", or "close" linguistic term, respectively, of the *DS* variable. For instance, considering the fuzzy rule #8, the two preconditions of the rule have a total score of 5 (3 for *DH* + 2 for *DS*). Once the total score was calculated, it was

compared with a present value of 2, 3, 4, 5 and 6 that relates to the use of the membership functions *W* (2, and 3), *M* (4), and *S* (5, and 6) respectively. Following this process, the fuzzy rules are formulated as shown in Table 1.

| Trustworthiness<br>[0 - 100%] |        | Separation [1 - 6] |             |           |
|-------------------------------|--------|--------------------|-------------|-----------|
|                               |        | CLOSE              | MEDIUM      | FAR       |
| Homophily [0 - 1]             | LOW    | Mild (#1)          | Weak (#2)   | Weak (#3) |
|                               | MEDIUM | Strong (#4)        | Mild (#5)   | Weak (#6) |
|                               | HIGH   | Strong (#7)        | Strong (#8) | Mild (#9) |

Table 1. Fuzzy decision rules

#### Fuzzy inference mechanism

With the membership functions and fuzzy rules formulated, it is possible to use them with specific values of the input variables (numeric, not linguistic) to compute a numeric value of the output variable. This process is known as *fuzzy rule-based inferencing*. It first requires the recommender algorithm to retrieve input numeric values for the *DH* and *DS* variables between each two individuals. This is made possible by calculating two indexes, which provide practicality and conformity to the sociological origins of the terms. In terms of *DH*, a homophily index *H* is computed by using the binary Jaccard coefficient [20], as follows:

$$H_{x,y} = \frac{C}{X + Y - C} \quad (2)$$

where, the homophily index between two individuals *x* and *y* is defined as the ratio of the number of shared attributes such as age, gender, ethnicity, religion, profession, education, kinship and so forth to the overall number of attributes, that is, *C* represents the total number of attributes both individuals have equally, *X* represents the total number of attributes recorded for one individual, *Y* represents the total number of attributes recorded for the other individual. In terms of *DS*, a separation index *S* is calculated as the number of sequenced links in the shortest path connecting two individuals *x* and *y*:

$$S_{x,y} = \sum_{i|x \vee y} \text{Min}(x_i \rightarrow y_i) \quad (3)$$

The process then fuzzifies these values through the membership functions of the input variables. For instance, let's assume the recommender inputs *DH* and *DS* values of 0.7 and 2 respectively between two given individuals. These values are applied on the 9 rules, one by one, to determine the firing strength of each rule and how much it contributes to the output value. According to an intermediate rule (rule #8, for example), the *DH* value of 0.7 was applied to its *H* membership function, and the *DS* value of 2 was applied to its *M* membership function. The intersection of these values with the corresponding membership functions provided membership values of 1 and 0.5, respectively (see Figure 2). The firing strength of that rule is then calculated using the minimal (AND) operator, which is the smallest of the two membership values of the rule (0.5).

The determined firing strength of 0.5 was used to needle the membership function for the output at this value giving 75% as a result (see Figure 1), thus forming a unitary output called singleton that defines the contribution of this rule to the overall output. Once these calculations are completed for all rules, the union operator is used to aggregate the consequences (Output1 to Output9) of the 9 rules to form an overall membership function whose values are the firing strength itself at a one particular point and zero everywhere else. This overall membership function, which is a collection of several singletons, is then converted into a crisp (non-fuzzy) value through a defuzzification process. Various methods can be employed to defuzzify the overall membership function, among which in this case the center-of-gravity-for-singletons method is used. Using this method, the final output of 78% (0.78) is computed by the weighted average of all nine rules' singleton-based outputs. In a similar fashion, a trustworthiness rating between any two individuals in the social network can be calculated based on recommender input of *DH* and *DS* values.

## EMPIRICAL STUDY

This section presents the experimental evaluation procedure that was derived in order to compare the algorithms and the results of the evaluation are discussed.

### Dataset

In order to address the data needs of this research work, three aspects need to be considered to carry out the case study experimentation: a (1) network structure made up of individuals and the many dyadic ties between them; (2) set of social-demographic characteristics of each individual; and, (3) set of ratings of individuals in the network with reference to items of choice.

Widely used standard datasets such as MovieLens [21] for movies do not include explicit social relationships nor arrange for the means to generate a reliable entity-relation model between people with social-demographics attributes. To circumvent these limitations, a mutual friend network experiment for rating movies has been devised using a

commercially available social network service provider<sup>1</sup>. The database currently consists of 27 interconnected friends of a friend who deliberately provided 46 ratings in the range of 1(min) to 5(max) to 25 movies. The lowest sparsity level is therefore  $(27 \times 25) - 46 / (27 \times 25) \approx 0.93$ . The prediction algorithms are tested over a pre-selected 26-ratings set extracted by the set of actual ratings. For the purpose of this work, friends' favorite movies, ratings, ties and socio-demographics features were explored using a third-party interactive graph visualization application<sup>2</sup> integrated with the social networks environment. Three matrices were then generated as input for the recommender construct: an (1) user-item matrix with explicit ratings on movies, (2) user-user matrix that has as elements values that show the degree of homophily, and an (3) user-user matrix whose elements contain the number of shortest paths between two individuals. The interested user is strongly encouraged to visit the website, obtain a more detailed view, and connect to participate in the experiment.

### Fuzzy Trust-Based Recommendation

The proposed prediction model adopts Resnick's prediction strategy [22] since it is the most widely used. In addition, it considers two adaptations to incorporate fuzzy trust-level metrics into the traditional recommendation process: inference of fuzzy-trust values and their aggregation with similarity values.

#### Similarity Metric

The computation of similarity metric produces the output User Similarity matrix of size  $m \times m$  in which  $m$  is the total number of individuals. It is computed using Pearson's correlation coefficient:

$$pSim(x, y) = \frac{\sum_{i=1}^n (x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \cdot \sum_{i=1}^n (y_i - \bar{y})^2}} \quad (4)$$

where,  $n$  is the total number of commonly rated items,  $x_i$  and  $y_i$  represent the current rate of a pair of items of two individuals  $x$  and  $y$  ( $i = 1$  to  $m$ ), and  $\bar{x}$  and  $\bar{y}$  represent the average of all of those rates.

#### Fuzzy Trust Metric

This step calculates the degree of trustworthiness for each pair of individuals  $x$  and  $y$  in the network, producing the output  $m \times m$  in which the  $i$ -th of  $m$  rows of individuals contains the fuzzified homophily and separation indexes of  $i$ -th individual against every other individual.

#### Rating Predictor

The rating prediction calculation is comprised of two steps. First, fuzzy trust metrics are combined with similarity values to produce a compound weighting to be used further in step two. The *Case Amplification* formulation [23] is used for this transformation, as follows:

<sup>1</sup> [www.facebook.com/rate-a-movie.experiment](http://www.facebook.com/rate-a-movie.experiment)

<sup>2</sup> TouchGraph Facebook Browser

$$tSim(x, y) = pSim(x, y) \cdot |pSim(x, y)|^{\rho - 1} \quad (5)$$

where,  $\rho$  is the amplification power given by the *Degree of Trustworthiness*  $\leq 1$  output of a pair  $x$  and  $y$  of  $m$  total individuals, and  $pSim(x, y)$  is the similarity coefficient given by one of the traditional collaborative filtering techniques [Eq. (4)]. In this example, an amplification transformation over addition, subtraction and multiplication methods such as harmonic or geometric means was chosen as it performed best in preliminary optimization tests. Step two computes the final prediction using the classic last step of Collaborative Filtering, as follows:

$$Pred(x, y) = \bar{x} + \frac{\sum_{i=1}^k tSim(x, y) \cdot (y_i - \bar{y})}{\sum_{i=1}^k tSim(x, y)} \quad (6)$$

where, the predicted rating of item  $i$  for the current individual  $x$  is the weighted sum of the ratings given to item  $i$  by  $k$  neighbours  $y$  of  $x$ ; in the proposed algorithm, all  $y$  neighbours of individual  $x$  are considered, that is,  $k = n$ .

### Evaluation Metrics

Two classes of metrics can be used to evaluate recommender algorithms: error metrics and classification metrics. This research work focuses on statistical accuracy metrics that are used to compare the numerical deviation of the predicted ratings from the respective actual individual rating. The most commonly used and accepted metrics are the Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE). These are calculated as follows:

$$MAE = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i| \quad (7)$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (8)$$

The MAE and the RMSE can be used together to diagnose the variation in the errors in a set of forecasts. The RMSE will always be larger or equal to the MAE; the greater difference between them, the greater the variance in the individual errors in the sample. If the RMSE equals to MAE, then all the errors are of the same magnitude. The lower the errors, the better are the performances. An MAE or RMSE equals to zero means that the estimator predicts observations of the parameter with perfect accuracy.

### Results

Table 2 summarizes the prediction accuracy for the baseline and proposed methodologies. From the results, it can be seen that the proposed methodology outperformed the traditional approach by about 4%. There is not a huge improvement in prediction accuracy, but this is an expected result. First, one of the known drawbacks of using similarity values is that sparse data implies in unreliable neighborhood formation. Another factor that can be attributed to the modest performance improvement relates to the fact that certain collaborative filtering algorithms

perform better or worse, depending on the chosen similarity computation formulae.

| Strategy           | MAE          | RMSE         |
|--------------------|--------------|--------------|
| Traditional        | 0.7819       | 0.9706       |
| Proposed           | 0.7503       | 0.9302       |
| <b>Improvement</b> | <b>4.04%</b> | <b>4.16%</b> |

**Table 2. Average prediction error and relative benefit**

### CONCLUSION

Trust is a concept in social recommenders that has received increasing attention by researchers and practitioners. In this work, a new trust model based on soft computer techniques has been devised. The proposed fuzzy logic quantifier effectively translated the vagueness in social network relationships into crisp numbers that account for the degrees of homophily and separation of one individual to another. A new trust-based recommendation strategy which incorporates the new model into the typical collaborative filtering recommender systems was derived. Through an experimental study, the prediction performance of both approaches was evaluated. The empirical results indicate that the proposed methodology reduces prediction errors compared to the traditional baseline, suggesting that the newly developed social trust metric can be an effective way of recommending user-generated content. A future challenge is to extend the recommender strategy to very large social networks. This may be possible by improving the fuzzy quantifier's ability to use linguistic variables, such as "Too Far" or "Too Close" to describe very distant or near individuals, respectively. More experimentation with alternative similarity algorithms, such as cosine computation, is also being pursued to verify its capabilities in the current formulation.

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