Transformations

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TRANSFORMATIONS

Florida Association of Mathematics Teacher Educators

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From the President's Desk . . .

Dear Mathematics Educators:

I am excited that the Winter issue of the Transformations Journal is ready for your use. This journal is made available online through NSUWorks. I encourage you to submit your research articles so that we can share with the mathematics educators around the country. I also invite you to nominate a colleague or self nominate to serve on our Board so that we can help make a difference in the K-22 mathematics education community in the state of Florida and throughout the country.

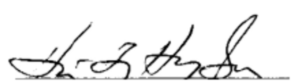
As an affiliate of the Florida Council of Teachers of Mathematics (FCTM), I am looking forward to achieve the following goals over the next two years:

1. Annual FAMTE Conference to promote the improvement of Florida's mathematics instructional programs and to promote cooperation and communication among the teachers of mathematicscs and mathematics teacher educators in Florida.

2. FAMTE Board represented by at least one K-12 Mathematics Teacher educators

3. Promote scholarly publications

With Warm Regards,



Hui Fang Huang "Angie " Su,

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Manuscript 1024

Empowering Teacher Leadership to Address Math Anxiety in Today's Schools

Joseph M. Furner

Christine Higgins

Empowering Teacher Leadership to Address Math Anxiety in Today's Schools

Abstract

Math anxiety is an uneasiness or worry when dealing with doing mathematics, ranging from slight nervousness, nausea, to complete panic. It prevents students from learning math and makes them more likely to give up. Math anxiety is common in many math classrooms today. Teachers can work together and take a real leadership role in addressing this problem to build math confidence. Solutions range from undergoing therapy to changing teaching styles to being more inclusive of students with math anxiety. This paper looks at ways teachers as leaders can work together to address math anxiety in their schools. School leaders can empower math teachers to work together to prevent and reduce math anxiety with a goal of improved math achievement school-wide while addressing math anxiety by teachers taking on active leadership roles in their schools and classrooms. This paper gives many recommendations to address math anxiety.

KEYWORD: Math Anxiety, Teacher Leadership, Best Practices, STEM, K-12

Introduction

An elementary school principal from the school that this data in this paper is from said once that she always interviews all new students coming into the school and always asks students, "What is your favorite subject?" She said that most of the younger children always say to her, "math." The same school had decided as part of their Southern Association of Colleges and Schools (SACS) accreditation as a K-12 international school (USA based curriculum) in Latin/South America, to survey 25% of their students at each grade level (they have approximately 100 students per grade), Grades 1-12, and administer the Abbreviated Version of the Mathematics Anxiety Rating Scale(MARS) (Alexander and Martray, 1989) to see how their students feel about their math attitudes (See Figures 1 and 2). The results are somewhat inconclusive, but the graph shows, primarily, that as students increase in grade level, their degree of math anxiety increases for the most part (See Figures 1 and 2). This may not be a complete surprise and seems consistent with the Third International Mathematics and Science Study (TIMSS) math results in the USA, where students increase in grade level but their degree of math achievement drops significantly from elementary, to middle, then to high school (Schmidt, 1998). Ingersoll, Sirinides, & Dougherty (2018) contend that teachers' role in improving schools is critical and they play an important leadership role in making decisions and for improving a schools' performance. When teachers are given opportunities for collaboration and sharing best practices, they can plan for and incorporate better teaching strategies during instruction in order to improve student performance. This paper explores how teachers as leaders together with the principal can address math anxiety issues in school to work toward improving math performance and confidence.

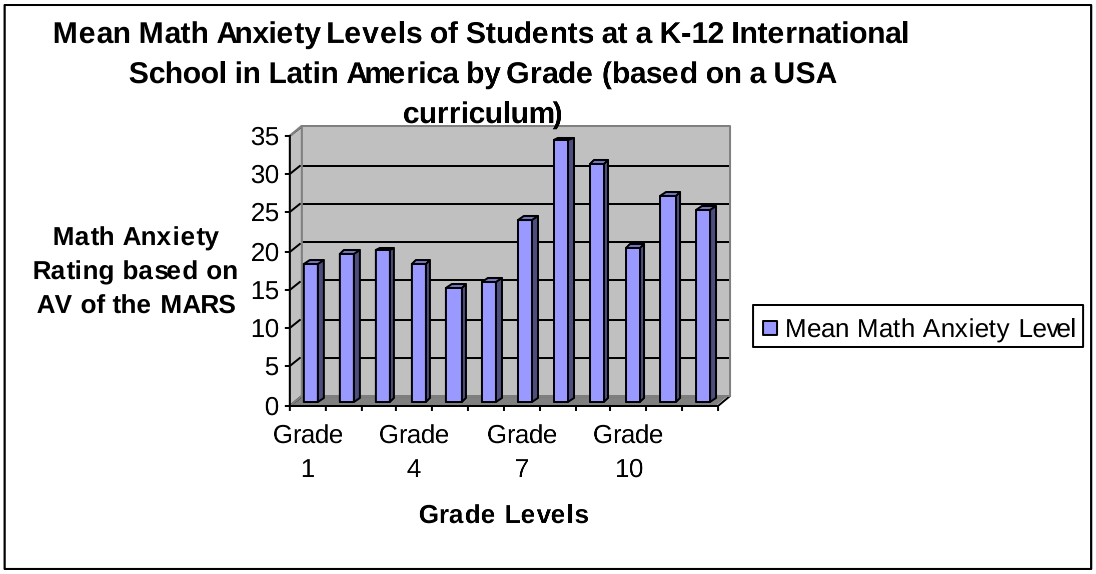

Figure 1. Mean Math Anxiety Levels by Grade

Grade	Mean Math Anxiety Level from AV of the MARS
Grade 1	18
Grade 2	19.3
Grade 3	19.7
Grade 4	18
Grade 5	15
Grade 6	15.7
Grade 7	23.8
Grade 8	34
Grade 9	31
Grade 10	20
Grade 11	26.8
Grade 12	25

Figure 2. Raw Data of Math Anxiety Levels by Grade

According to Reuters (2007) and the American Association for the Advancement of Science in San Francisco, math anxiety depletes working memory to do mathematics. Often times, worrying about doing math takes up a large part of a student's working memory which then spells disaster for the anxious student who is taking high-stakes tests. Today math teachers from around the world almost have to take on the role of counselors in their classrooms to address the many students who dislike or are fearful of mathematics. Mathematics teachers are encouraged to work with school counselors in helping to address the many math anxious students in today's schools. It has become a pandemic in our society where so many young people and adults have negative feelings and poor experiences with mathematics instruction. Metje, Frank, & Croft, (2007) believe that math anxiety is a worldwide phenomenon and that many people are not going into math fields, including engineering, and that more and more math instructors at the university level are not prepared to deal with the increased number of students who are unsuccessful at math due to this increased fear which has crippled their confidence. Addressing math anxiety has become one of the largest challenges for college professors.

Anyone can easily take an informal poll on the street or classroom and find that most respondents will not report positive experiences, feelings, or attitudes toward mathematics. However, we are now living in an age that depends so heavily on one being good at mathematics and problem solving. We are living in a world in which our students will soon be competing with young people from all parts of the globe for jobs. It is imperative that our students develop positive dispositions toward mathematics and the sciences in an information age of which has become so technologically oriented. Young people today need to be well prepared in the areas of math, science, and technology for all career choices. Nurses, engineers, architects, lawyers, teachers, along with many other fields will continue to use more advanced forms of technology that require one to know more mathematics and problem solving to perform their jobs effectively. Sequencing, ordering, patterning, logic, spatial sense, and problem solving are some of the basic skills that all careers require (NCTM, 2000). By the time our young people reach middle school they have developed certain dispositions toward mathematics. Students' confidence and ability to do mathematics and apply these skills in many diverse settings is essential for success; therefore, our young people need to be well prepared to do the mathematics of the 21st century.

Steen (1999) found that "national and international studies show that most U.S. students leave high school with far below even minimum expectations for mathematical and quantitative literacy." Neunzert (2000) contends that we have to understand ourselves as MINT-professionals, where MINT is M=mathematics, I=informatics, N=natural sciences, T=technology. Neunzert (2000) believes that mathematics is critical for people living in the 21st Century for them to be successful. Neunzert feels as educators we need to encourage our students in all countries to study more mathematics and to see it as a tool for success in life.

What is Math Anxiety?

What is math anxiety? Simply put, it is a fear when confronted with math, especially about one's own performance in solving math problems. It can range from slight nervousness to all-out panic. This anxiety makes it more difficult for students to focus in class, learn math, and solve math problems. Frequently, students would rather give up than have to face their fears. This means that they never get better at math and can, therefore, never overcome their anxiety. If this anxiety is not overcome, the student may suffer their entire life, even beyond their time in school. Math anxiety is a well-documented phenomenon that has affected our society for over sixty years, and not enough is being done to address it in our classrooms or in the way we teach math (Beilock & Willingham, 2014; Boaler, 2008; Dowker, Sarkar, & Looi, 2016; Geist, 2010; Metje, Frank, & Croft, 2007). Negative attitudes toward mathematics and math anxiety are serious obstacles for students in all levels of schooling today (Geist, 2010). Beilock and Willingham (2014) state that "Because math anxiety is widespread and tied to poor math skills, we must understand what we can do to alleviate it" (p. 29).

What Causes Math Anxiety? Math anxiety is caused by a combination of external and internal factors; however, we cannot change internal factors within the student, so as teachers it makes more sense to focus on what we can control (Chernoff & Stone, 2014). Studies show that math anxiety is caused primarily by the way the student learns math: the type of authority the teacher uses, an emphasis on right answers and fear of getting wrong answers, requirements that the student respond with an answer sooner than he or she might be ready, and exposure to the rest of the class and their potential condemnation of a student who responds poorly, in short the traditional way of teaching math (Chernoff & Stone, 2014, Finlayson, 2014). Traditional teaching emphasizes:

- "Basic skills
- Strict adherence to fixed curriculum
- Textbooks and workbooks
- Instructor gives/students receive
- Instructor assumes directive, authoritative role
- Assessment via testing/correct answers
- Knowledge is inert
- Students work individually." (Finlayson, 2014)

Unfortunately, these methods can cause and increase math anxiety in the classroom (Finlayson, 2014).

Math anxiety can also be transmitted and learned from others, usually from parent to child or teacher to student, but occasionally student to student. If someone teaching math, whether to their own child or to a class, experiences math anxiety, they are more likely to rush through things in order to "get it over with". They wouldn't be sure of their methods, so they would focus more on the correct answer. Like the student with math anxiety, they are also likely to become exasperated and give up rather than continue helping the student. This teaches the student that math is something to be afraid of and that, if they are not good at it, their parent or teacher may become upset with them and potentially leave. They also learn in class that, if their peers realize they are bad at math, they may be ridiculed publicly. Embarrassment is an enormous concern for students, especially in middle and high school.

Another problem for those who suffer from math anxiety is the nature of anxiety itself. According to Rubinstein et al (2015), anxious individuals tend to focus on negative stimuli more than positive stimuli, essentially making themselves more anxious. The same thing is true of individuals with math anxiety; the only difference is that for people with math anxiety, math is the negative stimuli (Rubinstein et al, 2015). This suggests that math anxiety could be handled through therapies designed to lessen anxiety, such as cognitive behavioral therapy and exposure therapy (exposing a person little by little to the thing that they are afraid of) (Rubinstein et al, 2015). While this is not something that a teacher could do with a full class to manage, it is something that tutors could be trained to help with; naturally, a licensed therapist would be the best option, but not all therapists are trained to help students with math. A combination of the two fields would be optimal.

Math anxiety remains a perplexing, persistent, and only partially understood problem from which many people suffer, NCTM (1991, p. 6) says, "Classrooms should be mathematics communities that thrive on conjecturing, inventing, and problem solving, and that build mathematical confidence. Unfortunately, many kids and adults today do not feel confident in their ability to do math. Math anxiety in students has become a concern for our high-tech world. Is it possible that only about seven percent of Americans have positive experiences with math classes from kindergarten through college study (Jackson, C. D. & Leffingwell, 1999)? Burns (1998) in her book *Math: Facing an American Phobia* tackles an interesting subject and has found that two-thirds of American adults fear or loathe math. Whether it is 93% or two-thirds of

Americans experiencing negative math experiences it is clear that there is a problem and we need to do something about it as educators. If math anxiety is such a problem, one has to wonder, what is being done about it in our schools today?

Evidence of students' poor attitudes and high levels of anxiety toward math is abundant. In the midst of a technological era, declining mathematics (math) scores on the Scholastic Aptitude Test (SAT) have been widely publicized. Some reports have shown that American students rank last when compared with students from all other industrialized countries on 19 different assessments. The TIMSS study has shown a trend in U. S. students' math scores as they decline as students increase in age group from grade four to grade twelve (Schmidt, 1998). What is happening to our students that so many of them lose interest in math and lack the confidence to do and take more math classes?

How Do We Fix Math Anxiety in our Schools?

To put it simply: better teaching. Finlayson suggests the constructivist style of teaching which emphasizes these ideas:

- "Begin with the whole – expanding to parts
- Pursuit of student questions/interests
- Primary sources/manipulative materials
- Learning is interaction – building on what students already know
- Instructor interacts/negotiates with students
- Assessment via student works, observations, points of view, tests. Process is as important as product
- Knowledge is dynamic/change with experiences
- Students work in groups" (2014)

This style of teaching is very different from the traditional style which can cause and increase math anxiety. The constructivist style is much less intimidating and doesn't emphasize timed assessments or correct answers; instead it focuses on the process of doing mathematics. Students are also likely to feel more engaged in class due to the more participatory style of teaching, making them want to work harder, instead of "getting it over with" heedless of how this affects their performance.

However, frequently the problems in the classroom that cause math anxiety are due to a teacher with math anxiety (Chernoff & Stone, 2014). These teachers choose the easiest ways of teaching (rote memorization of formulas, practice using one method to get one right answer, timed tests, etc.) in order to minimize their own math anxiety, not realizing that they are passing their anxiety onto their students (Chernoff & Stone, 2014). Therefore, we must first remove math anxiety from teachers, so they may teach their students not to experience math anxiety. Math is not inherently frightening, but that is the message that is told to many children, even from their parents and teachers.

As mentioned previously, math anxiety is a form of anxiety and therefore treatable through the same types of therapy we use to treat general anxiety and phobias (Rubinstein et al, 2015). This may prove especially helpful for adults with math anxiety, especially teachers; by working to handle their own math anxiety, adults would be able to prevent transmission of their anxiety to their children or students (Chernoff & Stone, 2014).

Discussing the Data from the K-12 School's Math Anxiety Levels Presented in this Study

The major trend from this data show a notable upward trend in math anxiety in students as students increase in grade level (See Figures 1 and 2). As students take more math classes and are exposed to more math teaching, unfortunately their level of math anxiety increased in this data set of a K-12 International School in South America with a US-based curriculum. In discussions with the administrators and teachers, little is often done year to year with students as they pass from grade to grade in respect to addressing a students' math anxiety. This math anxiety can fester and continue to pass on and increase as students continue through their studies. The author of this paper worked with this school for two years during this data collection in the school as part of the SACS accreditation. He also worked as the 9th Grade Geometry teacher for the first year prior to the data collection year and has extensive expertise in math anxiety research and implemented extensive math anxiety reduction and prevention techniques. The author employed these techniques with the 9th Grade mathematics students the year prior to the data collection. It is visible to see that the 10th Grade Students had reduced levels of math anxiety, likely due to the preventative and reductive math anxiety techniques used. Preventative strategies, like using "Best Practice" in mathematics, include using: manipulatives, cooperative groups, discussion of math, questioning and making conjectures, justification of thinking, writing about math in math journals, using a problem-solving approach to instruction, content integration, using technology Geometer's Sketchpad, and assessment as an integral part of instruction, such as homework, quizzes, and math portfolios. Along with math anxiety, reductive strategies include using: psychological techniques such as anxiety management, desensitization, counseling, support groups, bibliotherapy, and classroom discussions of how students feel about math and what they are learning. These insights can help us understand why the 10th Grade class had significantly lower math anxiety than the other middle school and high school grades. Students in elementary school often start out with little math anxiety, but this anxiety can increase as students go from grade to grade in their learning process. It is critical in an age of STEM (Science, Technology, Engineering, and Mathematics) that schools and teachers work to correct this trend of increased math anxiety as students advance from Grades K-12. More schools need to include affective aspects into their improvement plans, like checking for math anxiety, and then compare such data to their students' achievement levels. Unfortunately, like TIMSS revealed for US schools, the trend of math achievement went down as students increased in grade similar to this study which showed a likely correlation between math anxiety and student performance in math School leaders need to empower their teachers to take on a leadership role to start looking at both affective and cognitive aspects of learning to see the relationships and to better address achievement and performance of their students in mathematics and likely all subjects.

Math Teachers as Leaders

Higgins' (2013) research found that teacher leadership and professional learning are both present in the math department on formal and informal levels. While the level of leadership and learning may not be important, what is important to ask is, how do we sustain leadership and learning for teachers in schools today to impact student learning? Then more importantly, is it enough to sustain the organization? It has been recommended by Hargreaves (1999) that if you build the "intellectual capital, the knowledge and abilities of the staff," you will in turn create "organizational capital" based on the collective knowledge of the group (p. 124). Sustainability, according to Fullan (2002), is developed in a social environment where learning is vital and leadership at all levels is essential. Ingersoll, Sirinides, & Dougherty (2018) have found that when teachers are empowered and are allowed to be leaders, they can use their knowledge to make change and help to improve performance in their students. Today, math teachers not only are in charge of teaching math content, but it is also their job to check for positive mathematical dispositions in the students they teach.

Suggestions for building teacher's intellectual capital lie in affecting their daily practice when teaching mathematics. Drago-Severson (2007) suggested, "We need greater knowledge about practices that support teacher learning and growth by focusing on how teachers make sense of their experiences" (p. 71). Higgins' (2013) ethnographic study hoped to gain better knowledge into how a department of five teachers went about their daily routines and social experiences regarding learning and leadership. Underlying these experiences are the frameworks of Social and Adult Learning Theories which attempt to explain the social phenomenon that is inherently present in the learning process and what motivates adults to learn.

One endorsement considers both leadership and learning. A beam bridge consists of a horizontal beam supported at each end by piers. The weight of the beam is carried by these supports, in this case leadership and learning. Presently, the teachers are actively engaged in leadership roles, both inside and outside the department, and all of the teachers, are likewise engaged in some form of professional learning. The department members constitute a community of practice according to Wenger's (1998) definition. However, they seem to be going about their daily routines, experiencing much of the leadership and learning in isolation of the other. The further apart the piers in the beam bridge, the weaker the beam becomes. Therefore, in order to strengthen their community and the 'intellectual capital', which is necessary to change teachers' practices, we must bridge leadership and learning, thereby strengthening the beam. It is the practices of leadership-how leaders go about their work-during situations that call for their expertise that promotes a distributed leadership theory (Spillane, 2005), and it is the day-to-day discussions in the social setting that promote learning. Horn and Little (2010) posited that learning in the workplace is more likely to occur if the "talk" centers on "dilemmas and problems of practice" (p. 183). To effect teachers' practices leadership and learning must be bridged-they must not be isolated entities.

In order to accomplish bridging leadership and learning, another suggestion is the need for critical reflection in teaching. Taylor (1998) identified three themes that are central to Mezirow's transformational learning: experience, critical reflection, and rational discourse. The teachers are, again, engaged in experiences of leadership and learning, and discussion, or rational discourse, is evident. However, critical reflection is a missing element in their learning experience. According to Taylor (1998), critical reflection is the most important aspect needed to transform a person's way of thinking. The process of questioning our beliefs and assumptions and reconsidering new ways of thinking occurs when individuals self-reflect on experiences. An earlier discussion about the math teachers thinking in 'black and white' terms, as a barrier to professional learning, is a specific example of the need for critical reflection. Having time to reflect on their own practices may open their minds to other possibilities and make change easier; what works and what does not are all critical in the process of teaching effectively.

Higgins (2013) also found that the involvement of teachers in more aspects of their professional life is critical. Little (1982) showed that a characteristic of a successful school was the involvement of teachers in the curriculum planning, research, evaluation, and preparation of the material. This parallels with the recommendation for teachers to use critical reflection. Currently, the department chair receives and relays information to the teachers and, if necessary, a discussion took place. The recommendation is to strengthen that practice further, having teachers become involved together with the planning, research, data analysis, and program evaluations. This recommendation follows the research of Drago-Severson (2008) and the four pillar practices that support transformational learning.

In teaming, the first pillar consists of creating teams of colleagues to discuss, evaluate, and consider the other opinions regarding curriculum, student work, instructional strategies, and philosophies on teaching and learning. The second pillar allows transformational learning to take place when teachers are provided leadership roles that will offer new challenges and develop new perspectives. The idea of distributed leadership should not be confused with a distribution of duties, but instead should be viewed as a culture where leaders emerge as needed. The third pillar uses collegial inquiry to help teachers "become more aware of their assumptions, beliefs, and convictions…about their practice" so that they can improve their learning and improve the "overall organizational learning that may produce stronger student achievement outcomes" (Drago-Severson, 2008, p. 62). The final pillar, mentoring, provides support on an individual level.

The last recommendation is the consideration of time for professional learning and leadership. Higgins (2013) found in her study with interviews that a critical barrier to learning is time. Teachers in the department were concerned about student learning and knew of best practices that they would like to incorporate into their classrooms if only they had the time. Two teachers gave concrete examples of wanting more time to use data and the *GeoGebra* program to enhance student learning and understanding. School administration must be part of the solution for incorporating more time for teachers. While this is not a new discovery in education, time is one the keys to growing professional learning and leadership within an organization. In summary, there are four recommendations that may help strengthen professional learning and leadership:

1. Professional learning and teacher leadership should be bridged for any change to happen in teachers' practices.

2. Teachers should use critical reflection to understand their beliefs and to reconsider new ways of thinking.
3. Teachers should be involved with planning, research, evaluation, and preparation of their professional programs.
4. Time for learning must be a consideration.

For educators, professional learning and leadership are two areas that may be of high importance when we consider the impact of math anxiety on the quantitative development of our children. Consequently, the need for teachers to collaborate, share, and discuss their day-to-day practices with one another may never have a more meaningful time than now. Teacher leaders need to be prepared to help provide suggestions, strategies, and data driven solutions to their colleagues. With their content knowledge, cultural awareness of their department, and leadership skills, they become an invaluable piece of the educational puzzle. Harris (2003) reiterated that as work demands and distributed leadership practices increase in schools, it is also necessary for governments that are considering leadership accountability to bear in mind measures that will fairly assess a collective leadership style.

TIMSS data from 2011 show that we have not made statistical gains in mathematics achievement. Using data from Grade 8, there was a one-point difference between the U.S. average mathematics score in 2007 and the average score in 2011 (National Center for Educational Statistics, NCES). According to the NCES, a higher percentage of Florida fourth- and eighth-graders performed at or above the international benchmarks (2011). Looking at national data from the National Assessment of Educational Progress (NAEP, 2009) Florida's 12th graders are below the national average in mathematics. The *Foundations for Success: The Final Report of the National Mathematics Advisory Panel* (NMAP) was in reply to a "presidential executive order" to examine mathematics education in the United States (Spillane, 2008, p. 638). This document signified the on-going governmental influence in education policy and practice. Spillane (2008) argued that the Russians in 1957 have involved the government in educational matters since the launching of Sputnik. The federal government has used state and local governments to carry out many of its programs, including the Elementary and Secondary Education Act of 1965 and the No Child Left behind Act of 2001 (NCLB) (Spillane, 2008). While the federal government is behind the Race to the Top and the Common Core State Standards (CCSS), there seems to be more leeway for teachers to teach than previous programs. However, there are still many unanswered questions about the CCSS, including how will the standards be implemented and what part does the teacher play in the design of the curriculum and assessments? Also, are there affective factors teacher need to consider when teaching, things like checking for dispositions toward the subject and making connections and monitoring this throughout all their years in school? If educators are going to do more for increasing student achievement in mathematics, they are also going to have to take a key role in addressing math anxiety and factors that influence how students feel about the subject as well.

It has been said that to improve the quality of schools, the quality of teaching must also improve (Cochran-Smith, 2006; Stronge et al., 2007). There are many reasons that contribute to a student's success; these range from internal reasons connected to curriculum, leadership, and school structures, to external reasons involving families, support structures, and other resources. "Nonetheless, students' learning depends fundamentally on what happens inside the classroom as teachers and learners interact" (Ball & Forzani, 2011, p. 17). Therefore, understanding how teachers learn professionally and how, ultimately, that learning may influence student achievement is important in creating change and ultimately improving student achievement.

More explicitly, teachers' mathematical knowledge is an important component to teacher effectiveness and improved student achievement in mathematics. Researchers agree that teachers' knowledge of mathematics is a crucial component to improving mathematics education (Ball et al., 2001; Hill & Ball, 2004), as well as the leadership that is needed to move learning communities forward. The implications of this research are to improve the understanding of how teachers can learn professionally and the leadership opportunities that may affect learning. Since school administrators, department chairs, and lead teachers are a major factor of learning and leading in a school-by their utter words, they can influence, motivate, discourage or encourage.

Teachers should have time to work together, sharing, and talking about their practice. Elmore (2007) wrote that teachers who work in isolation create "self-sabotage." If school improvement is to happen, then schools must move towards a "culture of shared practices" (p. 32). Subject departments have the power to affect change-these "powerful subcultures" can provide professional learning and leadership opportunities (Heng & Marsh, 2009, p. 530). School leaders can use this research to realize the impact that having a conversation with the teachers and learning about their learning, and then taking steps to improve their learning is what Hargreaves (1999) said will build intellectual capital which will then sustain and improve the organization. Having teachers work together as a team of leaders, to learn about math anxiety research, and talk about what they see in their classrooms with their students and then making a plan to employee some of the math anxiety strategies here together in their classrooms to compare notes, talk about student success and feeling and confidence building can really impact student success when teachers take a more active leadership role to address issues like math anxiety in their department. If math teachers and school administration do not take an active leadership role in addressing math anxiety in a STEM world then we are inherently creating an injustice for our young people today.

Summary

Math anxiety is a concern facing students and teachers in today's classrooms. As educators, our goal should be to work together to minimize the effects of anxiety through utilizing better teaching strategies, as well as using teacher leadership and professional learning to advance teacher knowledge in this area. As adults, we need to be aware of our own anxiety in order to prevent it from being transmitted to our children and students; for those who are unduly impacted by math anxiety or for those who are more likely to transmit this anxiety to children, it may be helpful to receive assistance from a therapist. As teachers, we need to make our classrooms a safe haven for students with math anxiety by altering our teaching styles; this will help all students, not just those with math anxiety. In order to fix this problem, we need to go straight to the source, even if that source is in our own anxieties. Only then can we prevent future generations from becoming part of the pandemic of math anxiety. Teachers of mathematics need to take on the role of counselors to address the math anxious students they have in their classrooms. Ingersoll, Sirinides, & Dougherty (2018) state that "leadership matters, that good school leadership actively involves teachers in decision making, and that these are tied to higher student achievement." (P. 17).

School leaders have a responsibility to bridge the teacher leadership and professional learning so that it will impact student achievement and address the issues of math anxiety. Building confidence in math by using the many suggestions and recommendations mentioned in this article in classrooms/schools may help to prevent or reduce math anxiety. Teachers must take on leadership roles and use collegial inquiry to address this very serious concern in a STEM-enriched society. Additionally, helping teachers become aware of their own assumptions and beliefs of mathematics is important as more and more teachers realize the need to put on their educational psychologists' hats in their classrooms to help address the important issue of math anxiety. Teachers may also want to encourage learning opportunities for parents such as family math nights where parents and children can come together to "do math" and see its importance and value in life. As a society, we must work together to extinguish this discomfort that our students are having toward mathematics, especially as they increase in age. It is important that all students feel confident in their ability to do mathematics in an age that relies so heavily on problem solving, technology, science, and mathematics. Today's educators must make the difference in our children's attitudes toward math. Math teachers need to work together as a team of leaders who strive toward creating mathematically literate and confident young people for the new millennium. Math teachers should, not only, be teaching content but also checking for dispositions toward the subject. They need to ask their students how they feel about mathematics. It would be nice to hear more young people and adults say, "Math was my favorite subject" or "I am great at math!" We need to reverse the trend and not allow the data to create an escalating bar graph of increasing anxiety levels as students increase in grade level.

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Appendix A: Standards and Strategies to Address Math Anxiety by Math Teacher Leaders

Mathematics teachers need to be leaders and work together to address math anxiety…

What NCTM says about Mathematics Anxiety and Dispositions Toward Mathematics

Standard 10: Mathematical Disposition (NCTM 1989)

As mathematics teachers it is our job to assess students' mathematical disposition regarding:

-confidence in using math to solve problems, communicate ideas, and reason;

-flexibility in exploring mathematical idea and trying a variety of methods when solving;

-willingness to persevere in mathematical tasks;

-interests, curiosity, and inventiveness in doing math;

-ability to reflect and monitor their own thinking and performance while doing math;

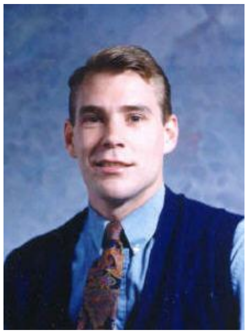
-value and appreciate math for its real-life application, connections to other disciplines and cultures and as a tool and language.

A Synthesis on How to Reduce Math Anxiety

1. Psychological Techniques like anxiety management, desensitization, counseling, support groups, bibliotherapy, and classroom discussions.
2. Once a student feels less fearful about math he/she may build their confidence by taking more mathematics classes.
3. Most research shows that until a person with math anxiety has confronted this anxiety by some form of discussion/counseling no "best practices" in math will help to overcome this fear.

A Synthesis on How to Prevent Math Anxiety

1. Using "Best Practice" in mathematics such as: manipulatives, cooperative groups, discussion of math, questioning and making conjectures, justification of thinking, writing about math, problem-solving approach to instruction, content integration, technology, assessment as an integral part of instruction, etc.
2. Incorporating the NCTM *Standards* and your State Standards into curriculum and instruction.
3. Discussing feelings, attitudes, and appreciation for mathematics with students regularly



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Transformations

Manuscript 1025

THE POWER OF A SELF-PACED, CHALLENGING, PROCESS-BASED, ONLINE MATHEMATICS CURRICULUM FOR TALENTED MIDDLE SCHOOL STUDENTS

Shari Stupp

Keith Nabb

Danielle Goodwin

Talented middle school students in most public schools and traditional private schools do not have access to a curriculum that allows them to go at their own pace, adequately challenges them, or presents mathematics in a process-based way so that students can become familiar with behaviors of mathematicians (Gentry, Gable, & Springer, 2000; Rogers, 2007). Many gifted students are subjected to materials written at levels that are inappropriate for them and teachers that are not adequately prepared to teach them (Rogers, 2007).

Talented students often become disengaged and may even become discipline problems when they are not given the chance to work at their own pace (Gentry, et al., 2000; Rogers, 2007). Many talented students see single-pace classes (even accelerated classes) as less productive and less enjoyable (Vialle, Ashton, & Carlton, 2001). Students in Vialle, et al.'s (2001) study remarked that they liked working independently "without the teacher interrupting" and that they don't "stare out the window so much" (p. 8). Rogers (2007) found that not only do gifted students prefer autonomy, they actually find it stressful to be in situations where the learning isn't progressing according to their individualized needs.

Neither acceleration nor testing out of content is the best solution. Talented students who are allowed to skip content or move up grade levels in school often report returning to the same level of boredom within 6-10 weeks of beginning the new level (Vialle et al., 2001). In addition to allowing students to work at their own pace, sufficient challenge must be provided and the environment must feel as if there are no ceilings on learning (Rogers, 2007).

Most mathematics textbooks provide little challenge to gifted students (Gentry, et al., 2000). Rogers (2007) found that half of the content in most mathematics curricula can be eliminated for talented learners with no negative effects on student achievement. Howley, Pendarvis, and Gholson (2005) state that schools "provide instruction that prepares few students for advanced" mathematics and therefore, few middle school students are "afforded experiences that position them to pursue...careers in mathematics or science" (p. 128). Talented learners require daily challenges that are authentic, increase motivation, and develop higher-order thinking skills (Diezmann & Watters, 2005).

Challenge is essential for deep understanding. Vygotsky's (1930-1934/1978) theory about the "Zone of Proximal Development" found that keeping instruction just ahead of a student's developmental level is of critical importance. Talented students also report more enjoyment of challenging curricula (Howley, et al., 2005).

Mathematics is more than just rules and procedures. According to NCTM (2000), understanding the processes of mathematics (problem-solving, representation, making connections, reasoning, and communication) is key to deep mathematical learning. It is critical for middle grades students to develop "habits of thinking" that mimic the habits of mind of practicing mathematicians

(Driscoll, 1999). Baroody and Niskayuna (1993) argue that a problem-solving approach which centers on the processes of mathematical inquiry is key to fostering the development of mathematical understanding. Using inquiry and problem-solving strategies leads to deeper understandings in talented students (Rogers, 2007). Moreover, students who use process-based curricula have more positive and diverse beliefs about the nature of mathematics (Howley, et al., 2005).

Hrina-Treharn (2011) states that the research of talented students' attitudes about mathematics is “limited and lacking” (p. 6). One of the few studies on the matter is that of Howley, Pendarvis, and Gholson (2005), which found that talented students do not have very positive or nuanced views of mathematics. “Far from seeing mathematics as a way of expressing ideas or as a method for characterizing relationships and patterns, these gifted children instead saw mathematics principally as a set of procedures with numbers – as calculations and algorithms” (Howley, et al., 2005, p. 138).

What students believe about the nature and role of mathematics is critical to their learning of mathematics, as negative beliefs stunt children's curiosity about mathematics and cause student motivations to learn mathematics to be based on grades and other extrinsic motivators (Howley, et al., 2005). Learning is not just an accumulation of facts. For meaningful learning to take place, students must have access to a challenging, process-based curriculum that is differentiated for their learning needs and exposes them to the authentic behaviors of a mathematician so that they can form deeper understandings of mathematics concepts and the nature of mathematics.

At this point in time, no controlled studies have been completed that explore the effects of the use of self-paced, challenging, process-based curricula on talented students’ beliefs about mathematics. If we expose gifted middle school students to self-paced, challenging, process-based curricula, we may change their beliefs about mathematics (Rogers, 2007). This research seeks to establish whether there is a relationship between students’ beliefs about mathematics and their images of a self-paced, challenging, process-based curriculum.

Background

In the 1960s a talented team of curriculum developers began to create the curriculum that is today known as *Elements of Mathematics: Foundations* (*EMF*) (IMACS, 2006). The founders of *EMF* began with the process-based view that doing mathematics involved reasoning, making connections, problem-solving, representation, and communication – the behaviors of a mathematician – and developed a curriculum that was consistent with what NCTM (2000) would later call the “Process Standards for Mathematics” and what the Common Core State Standards (CCSSI, 2010) would later term the “Standards for Mathematical Practice.” Through the Ford Foundation, U.S. Office of Education, and Central

Midwestern Regional Educational Laboratory (CEMREL) funding, the curriculum was developed and the first students started to use it in 1966 (IMACS, 2006). Today, there are classes using the paper textbook version of the curriculum with a face-to-face delivery model.

Starting in 2012, the self-paced, online version of the curriculum became available and is now in use by several school districts and by individual students from around the globe (IMACS, 2018). The mathematics content is presented in an integrated way and it addresses all of the Florida state and national standards for Prealgebra, Algebra 1, Algebra 2, Geometry, Trigonometry, and Precalculus; it also includes many topics that are not part of the traditional K-12 mathematics curriculum. There are 18 content courses as well as some supplemental courses that prepare students for standardized testing. Beginning in the summer of 2015, a large suburban district in Southeast Florida adopted the *EMF* curriculum for its most talented middle school mathematics students.

Research Questions & Methodology

The following research questions guided the research:

1. What beliefs do talented middle school students have about mathematics?
2. What beliefs do talented middle school students have about a self-paced, challenging, process-based curriculum?
3. What is the relationship between talented middle school students' beliefs about a self-paced, challenging, process-based curriculum and their images of mathematics?

To explore the research questions, a non-experimental, survey research design was employed. The respondents are 39 gifted sixth grade students from a large suburban district in Southeast Florida. The survey items about the nature of mathematics and mathematics education were used previously on studies about teachers' images of mathematics (Goodwin, 2010; Goodwin, Bowman, Wease, Keys, Fullwood, & Mowery, 2014).

Results

Demographics

Of the 39 student survey respondents, about 73% of the students were boys. Roughly 89% of the students are 12 years old. The average student reported that they were "confident" in their mathematical abilities. The average student reported that their grades in mathematics in the previous school year were "Straight As" and their grades in other subjects that year were also "Straight As."

The students reported that they spent 31-60 minutes on mathematics homework and watching TV each night, 61-90 minutes on reading and homework in all subjects daily, and 91-120 minutes on electronic devices each day. The average student agreed with the statement "I have enough time to complete all my homework each night."

About 84% of students declared mathematics as one of their favorite subjects. Science and music were also chosen as favorite subjects by about 63% and 55% of the students, respectively. Physical education and world languages were the least likely to be chosen as a favorite subject, with only about 21% of students choosing these subjects.

Beliefs about Mathematics

An overall image of mathematics item asks the student "Ideally, doing mathematics is like: cooking a meal, conducting an experiment, playing a game, doing a puzzle, doing a dance, or climbing a mountain." The most popular response by far was "doing a puzzle" with almost three-quarters of the responses. "Doing a dance" was not chosen by any students.

The students showed very positive beliefs about mathematics. Students most strongly agreed with the statements "Mathematics supports many different ways of looking at and solving the same problems," "Mathematics makes a unique contribution to human knowledge," "Math is thought provoking," and "Math is intricately connected to the real world." Students also agreed with the statements "Mathematics is fun," "In mathematics, you can be creative," and "The process of trying to prove a mathematical relationship can change your mind about it."

The age of the student was statistically significantly related to level of agreement with the statements "Mathematics supports many different ways of looking at and solving the same problems" ($t(35) = 2.94$, $p = .01$), "Math is intricately connected to the real world" ($t(35) = 3.81$, $p = .00$), and "In mathematics you can be creative" ($t(35) = 5.85$, $p = .00$). Those who were younger than 12 years old agreed more strongly with these statements than did their 12-year-old counterparts.

Whether mathematics is listed as a favorite subject or not was significantly related to agreement levels with the statements "Mathematics is fun" ($t(37) = -5.15$, $p = .00$) and "In mathematics, you can be creative" ($t(37) = -3.73$, $p = .00$). Those who listed mathematics as a favorite subject reported that they "strongly agreed" with the statement "Mathematics is fun" on average, while those who didn't "slightly disagreed" on average. Those who listed mathematics as a favorite subject reported that they "slightly agree" with the statement "In mathematics, you can be creative" on average, while those who didn't "slightly disagreed" on average. Reported level of confidence in mathematical abilities is significantly positively correlated with level of agreement with the statement "Mathematics is fun" ($r(38) = .45$, $p = .01$).

Responses on the item "Ideally, mathematics is like" were statistically significantly related to whether a student reported mathematics as a favorite subject ($\chi^2(1, N = 39) = 9.38$, $p = .00$). About 84% of those students who said one of their favorite subjects was mathematics selected "doing a puzzle" on the item

"Ideally, mathematics is like" while only roughly 29% of those students who did not say that mathematics was one of their favorite subjects picked "doing a puzzle."

The amount of time spent watching TV every night is significantly negatively correlated with their agreement with the statement "Math is thought provoking" ($r(39) = -.34$, $p = .03$).

Images of EMF

About 82% reported that their confidence in their mathematics abilities had improved since beginning *EMF*. The average student strongly agreed that "*EMF* is challenging," "*EMF* has made me more ready for high school than I would have been without *EMF*," and "*EMF* has made me more ready for college than I would have been without *EMF*."

Reported levels of enjoyment of *EMF* and other mathematics programs were significantly different ($t(38) = 2.66$, $p = .01$). The average student reported enjoying *EMF* at higher levels than they reported enjoying other mathematics programs. The students agreed that *EMF* has improved their ability to analyze complex problems, their self-study skills, their ability to focus for extended periods of time, and their confidence with respect to challenging programs.

Relationships between Beliefs about Mathematics and Images of EMF

Level of agreement with the statement "Mathematics is fun" is significantly positively correlated with level of agreement with the statements "I enjoy doing the *EMF* program" ($r(39) = .65$, $p = .00$) and "I enjoy doing other mathematics programs..." ($r(39) = .37$, $p = .02$).

Level of agreement with the statement "I enjoy doing the *EMF* program" is also significantly positively correlated with level of agreement with the statements "Mathematics supports many different ways of looking at and solving the same problems" ($r(39) = .44$, $p = .01$), "Mathematics makes a unique contribution to human knowledge" ($r(39) = .47$, $p = .00$), and "In mathematics, you can be creative" ($r(39) = .39$, $p = .02$).

Responses on the item that asked if *EMF* has improved confidence in mathematical abilities were significantly related to agreement levels with the statements "Mathematics is fun" ($t(36) = -4.56$, $p = .00$) and "In mathematics, you can be creative" ($t(36) = -3.73$, $p = .00$). Those who said that *EMF* improved their confidence in their mathematical abilities reported that they "strongly agreed" with the statement "Mathematics is fun" on average, while those who didn't "slightly disagreed" on average. Those who said that *EMF* improved their confidence in their mathematical abilities reported that they "slightly agree" with the statement "In mathematics, you can be creative" on average, while those who didn't "slightly disagreed" on average.

Responses on the item "Ideally, mathematics is like" were statistically significantly related to views about whether *EMF* improved confidence in mathematical abilities (χ^2 (1, N = 38) = 5.32, p = .02). Almost 90% of those who said "doing a puzzle" found *EMF* improved their confidence in their mathematical abilities while only about 56% of those who said something other than "doing a puzzle" reported that *EMF* improved their confidence in their mathematical abilities.

Perceived level of improvement in self-study skills due to *EMF* is significantly positively correlated with level of agreement with the statement "In mathematics, you can be creative" (r(37) = .36, p = .03). Perceived level of improvement in the ability to focus for extended periods of time due to *EMF* is significantly positively correlated with level of agreement with the statements "Mathematics is fun" (r(37) = .45, p = .01) and "In mathematics, you can be creative" (r(37) = .37, p = .03). Perceived level of improvement in confidence with respect to challenging programs due to *EMF* is significantly positively correlated with level of agreement with the statements "Mathematics is fun" (r(37) = .49, p = .00) and "In mathematics, you can be creative" (r(37) = .45, p = .01).

Responses on the item "Ideally, mathematics is like" were statistically significantly related to the reported level of challenge by *EMF* (t(37) = 2.04, p = .05). Those who said something other than "doing a puzzle" found *EMF* more challenging than those who said "doing a puzzle" The average "doing a puzzle" respondent was in the middle between "slightly agreeing" and "strongly agreeing" with the statement "*EMF* is challenging," while the average non-"doing a puzzle" respondent strongly agreed with the statement.

Responses on the item "Ideally, mathematics is like" were statistically significantly related to the reported change in confidence with respect to challenging programs (t(35) = -2.21, p = .03). Those who said "doing a puzzle" found *EMF* helped them gain more confidence with respect to challenging programs than those who said something other than "doing a puzzle." The average "doing a puzzle" respondent said their confidence with respect to challenging programs "is much stronger now," while the average non-"doing a puzzle" respondent said their confidence with respect to challenging programs "is slightly stronger now."

Level of agreement with the statement "*EMF* has made me more ready for high school than I would have been without *EMF*" is significantly positively correlated with level of agreement with the statements "Mathematics is fun" (r(39) = .48, p = .00), "Math is thought provoking" (r(39) = .47, p = .00), "Mathematics makes a unique contribution to human knowledge" (r(39) = .50, p = .00), and "Mathematics supports many different ways of looking at and solving the same problems" (r(39) = .50, p = .00).

Level of agreement with the statement "*EMF* has made me more ready for college than I would have been without *EMF*" is significantly positively correlated with level of agreement with the statements “Mathematics is fun” ($r(38) = .44$, $p = .01$), “Mathematics makes a unique contribution to human knowledge” ($r(38) = .52$, $p = .00$), and “Mathematics supports many different ways of looking at and solving the same problems” ($r(38) = .50$, $p = .00$).

Discussion of Findings

Overall, these talented students expressed positive views of mathematics. They strongly agreed that mathematics is thought-provoking, makes a unique contribution to human knowledge, connected to the real world, and supports many different ways of looking at and solving problems.

The students found *EMF* to be gratifying and challenging. Most reported that *EMF* better prepared them for high school and college mathematics courses than an accelerated, traditional curriculum and that *EMF* improved their self-study skills, ability to analyze complex problems and focus for extended periods of time, and confidence with respect to challenging programs.

Overall, the students who found *EMF* most enjoyable and rewarding, reported that *EMF* had improved their confidence, further developed their ability to focus and their self-study skills, and had made them more ready for future mathematics courses had more positive views of mathematics. Many statistically significant relationships between gifted middle school students’ images of mathematics and their opinions on a self-paced, challenging, process-based curriculum were uncovered.

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a schools' performance. When teachers are given opportunities for collaboration and sharing best practices, they can plan for and incorporate better teaching strategies during instruction in order to improve student performance. This paper explores how teachers as leaders together with the principal can address math anxiety issues in school to work toward improving math performance and confidence.

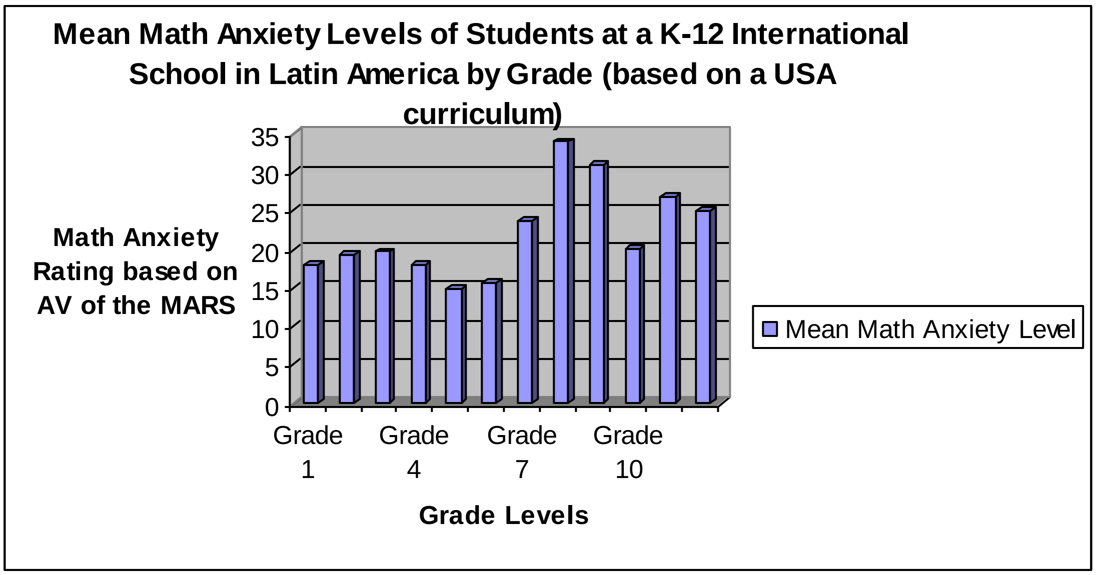


Figure 1. Mean Math Anxiety Levels by Grade

Grade	Mean Math Anxiety Level from AV of the MARS
Grade 1	18
Grade 2	19.3
Grade 3	19.7
Grade 4	18
Grade 5	15
Grade 6	15.7
Grade 7	23.8
Grade 8	34
Grade 9	31
Grade 10	20
Grade 11	26.8
Grade 12	25

Figure 2. Raw Data of Math Anxiety Levels by Grade

Using Number Properties to inspire teaching and learning in the K-12 Classroom

Abstract

Where would today's world be without the number system? Indeed, civilization would not have come to as far as in modern times. In fact, the software this paper is being written on is run by a computer that is embedded in numerical codes and commands– something that would not be possible if it were not for the advancement of technology using the modern number system. In this paper, we study the historical development of the modern-day number system. We investigate the classification of numbers and number properties for mathematical use; including standard functions, rules, and examples. As a culmination of our research, we provide a comprehensive activity for classroom use, accompanied by worksheets and a classroom-ready presentation.

Historical Development of Number

The discovery of "number" is unlikely to have been of any one individual or single ethnic group, but rather a plodding awareness in man's cultural advancement about 300,000 years ago. The historical development of number was a long and gradual progression influenced by many cultures, including the Greeks. Evidence shows that our early ancestors began by only counting to two and that anything beyond was designated as "many". This method is still used today, as most people count objects by an arrangement of twos. Unfortunately, due to poor preservation, few records remain but demonstrate that prehistoric man would sometimes record a number by scratching notches with a stick or a piece of animal bone. Such as, in Moravia, a wolf bone dated back to over 30,000 years old, was found with fifty-five cuts deeply incised. Two additional significant artifacts were found in Africa: a 35,000-year-old, baboon fibula with twenty-nine cuts and an Ishango bone with multiplicative entries, dating back to over 30,000 years ago. As technology has evolved and new research emerges, examiners are discovering that the idea of number is far older than we formerly recognized (Boyer, 1968).

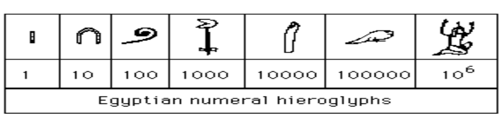
The Ishango bone, baboon fibula, ~30,000 years old, with notches, believed to represent early evidence of counting.

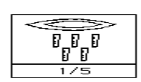
Aristotle wrote: "Or is it because we were born with ten fingers and so because they possess the equivalent of pebbles to the number of their fingers, come to use this number for counting everything else as well?" Yes, indeed, Artistole, counting developed from our nature of five fingers on each hand and foot. The historical development of number is dependent on several contributions from diverse cultures. The French used a base 20 number system and words such as quatre-vingt (four twenties), for the number eighty. The Old Khmer language used five as a point to secure, or "anchor" numbers. For example, after four comes five, five and one, five and two, and so on until ten, beginning at the next "anchor" number. As you can see, the Early European numbers, or more commonly known as Roman Numerals, follow this same trend with anchored numbers by the number five: IV, V, VI, VII, VIII...etc. Evidence from the decimal numbers early on in Ancient India shows us they used the number ten as their anchor. The largest known primary number, sixty, was found in the Babylonia sexagesimal system. The Babylonians cultivated their number system based on the number sixty rather than ten. Surprisingly, traces of their earliest system are alive today with sixty seconds in a minute/hour, and a circle is (6 X 60) (Aczel, 2015).

Number Systems

Numbers are classified into sets, called number systems, and there are five important number systems to consider: natural numbers, integers, rational numbers, real numbers, and complex numbers. The creation of the number system was a leading principal in the progression of our decadic system and many others. Although highly developed, the Mayan number system, one that uses basic twenty group, has an indiscretion with the second order: it's not 20 X 20 = 400 as expected, but 20 X 18 = 360. Theorists suggest that this is a correlation with the division of the Mayan year into eighteen months with each having twenty days, with five extra days. "Number Theory and It's History" illustrates the higher groups in the system as 360 X 20, 360 X 20^2 … etc. Many cultures did not use enormous numbers; in fact, many languages did not go beyond thousands or even hundreds. For example, the Greeks stopped at a myriad (10,000) and Romans, for an extensive period, did not go over 100,000. On the other hand, the Hindus were keen on large numbers, which ultimately resulted in the higher decadic groups to extremely high powers of ten (Ore, 1988).

Around 1800 BC, Egyptians used fractions within their number system with a base of ten. Their ancient Egyptian writing system contained illustrations, hieroglyphs, and they even used these pictures to represent numbers.



This is how they wrote 1/5: 

Ancient Babylonians came up with a sensible representation of fractions as well (around the base of ten), and did it well before the Romans method (Pumfrey, 2011).

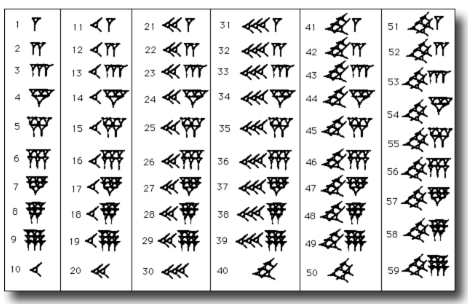
Negative numbers turned up in India around 620 CE, in the works of Brahmagupta (598-670). Brahmagupta symbolized fortunes and debts as positives and negatives. Notably, during this time, the place value system was already in effect in India and zero was used in the Indian Number System. Furthermore, to note the importance of the Greeks never really concentrated on negative numbers since they had a more geometrical approach. Greeks were more centered on numbers that had to be positive (lengths, areas, and volumes, all of which had to be positive) for their proofs of logical agreements (Rogers, 2011).

"A Brief History of Numbers" author Leo Corry, mentions that it was not until the mid-19th century that complex numbers were fully understood by mathematicians. Nonetheless, once complex numbers and their significance was discovered, they were incorporated in mathematics, and other areas, such as physics and engineering, for their countless uses (Corry, 2015). The evolution of complex numbers took almost three hundred years. In 1545, the book *Ars Magna* (The Great Art) by Jerome Cardan, an Italian mathematician, physician, and philosopher, was published. Cardan discusses various rules of algebra and other algebraic procedures for solving cubic and quartic equations. However, when using the cubic formula, or "The Cardan Formula," to solve the example, Cardan stated that the general formula was not applicable in this case (because of the root of -121). Nevertheless, it took Rafael Bombelli (1526-1572), a hydraulic engineer, approximately thirty years after Cardan's work was published, to figure out this problem. Bombelli justified Cardan's formula by introducing complex numbers, thus laying the groundwork. Bombelli's work demonstrated that sometimes the square roots of a negative number could be used to find real solutions. While Bombelli thought complex numbers were worthless, he unknowingly significantly influenced others with his work. In 1620, for instance, Albert Girard claimed an equation may have as many roots as its degree and shortly after, Rene' Descartes contributed the term "imaginary" for these numbers (Complex Numbers).

As outlined, it is evident that numbers and number systems have evolved over time. What follows is a thorough look at some of the number systems discussed.

The Babylonians used two cuneiform symbols and arranged them into fifty-nine base units using a base sixty number system. They used a positional number system like we have today; they organized their numbers into columns. As we know, the first column was the unit column, and it contained any of the fifty-nine base units. The next column contained multiples of sixty, for each of the fifty-nine base units. The third column was used to represent sixty squared or three thousand six hundred; each of the fifty-nine base

units could be placed in the third column (O'Connor & Robertson, 2000).



(*mathisgoodfor you.com)*

The Greeks used a number system that was based on the letters of their alphabet. The Greek alphabet consisted of twenty-four letters and three obsolete letters. Each of the letters was assigned a value from one to nine hundred. To distinct, the numbers between the letters, a special symbol called a keraia was used. The keraia was also used to make larger numbers; it was placed to the lower left of a letter to indicate that the value of the letter should be multiplied by one thousand. Furthermore, a myriad symbol (M) was used to indicate multiples of ten thousand, so even larger numbers could be created. Overall, the Greek number system was a base ten additive system; it added the numeric value of the letters to get a total (O'Connor & Robertson, 2000).

1	α	alpha	10	ι	iota	100	ρ	rho
2	β	beta	20	κ	kappa	200	σ	sigma
3	γ	gamma	30	λ	lambda	300	τ	tau
4	δ	delta	40	μ	mu	400	υ	upsilon
5	ϵ	epsilon	50	ν	nu	500	ϕ	phi
6	ϛ	vau*	60	ξ	xi	600	χ	chi
7	ζ	zeta	70	o	omicron	700	ψ	psi
8	η	eta	80	π	pi	800	ω	omega
9	θ	theta	90	ϟ	koppa*	900	ϡ	sampi

*vau, koppa, and sampi are obsolete characters

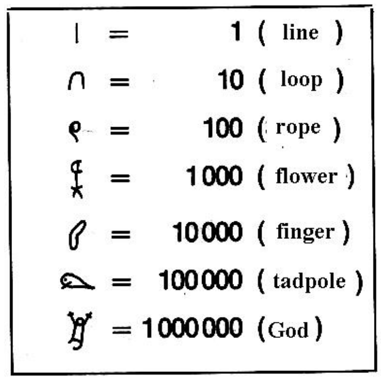
(www.simple-talk.com)

The Romans used seven letters from the Latin alphabet to represent the numbers one, five, ten, fifty, one hundred, five hundred, and one thousand. The Romans placed a line above a letter to multiply its value by one thousand (UNRV History, 2016).

I	1	XXI	21	XLI	41	LXI	61	LXXXI	81
II	2	XXII	22	XLII	42	LXII	62	LXXXII	82
III	3	XXIII	23	XLIII	43	LXIII	63	LXXXIII	83
IV	4	XXIV	24	XLIV	44	LXIV	64	LXXXIV	84
V	5	XXV	25	XLV	45	LXV	65	LXXXV	85
VI	6	XXVI	26	XLVI	46	LXVI	66	LXXXVI	86
VII	7	XXVII	27	XLVII	47	LXVII	67	LXXXVII	87
VIII	8	XXVIII	28	XLVIII	48	LXVIII	68	LXXXVIII	88
IX	9	XXIX	29	XLIX	49	LXIX	69	LXXXIX	89
X	10	XXX	30	L	50	LXX	70	XC	90
XI	11	XXXI	31	LI	51	LXXI	71	XCI	91
XII	12	XXXII	32	LII	52	LXXII	72	XCII	92
XIII	13	XXXIII	33	LIII	53	LXXIII	73	XCIII	93
XIV	14	XXXIV	34	LIV	54	LXXIV	74	XCIV	94
XV	15	XXXV	35	LV	55	LXXV	75	XCV	95
XVI	16	XXXVI	36	LVI	56	LXXVI	76	XCVI	96
XVII	17	XXXVII	37	LVII	57	LXXVII	77	XCVII	97
XVIII	18	XXXVIII	38	LVIII	58	LXXVIII	78	XCVIII	98
XIX	19	XXXIX	39	LIX	59	LXXIX	79	XCIX	99
XX	20	XL	40	LX	60	LXXX	80	C	100
								D	500
								M	1000

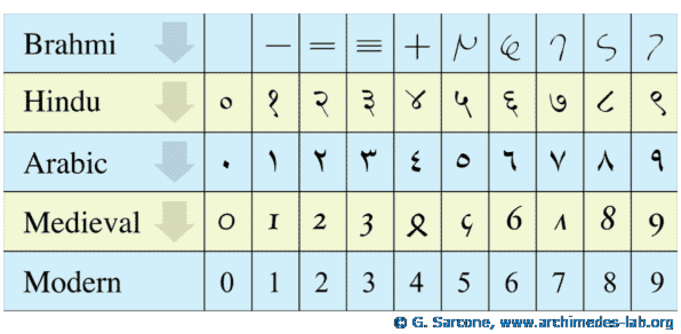
(kreannasandoval.wordpress.com)

The Egyptians used a base ten number system. It was an additive system in which numeric values were created by combining symbols. The symbols for one through nine contained single lines, or strokes of equal number for each symbol and the symbols for ten, one hundred, one thousand, ten thousand, one hundred thousand, and one million were made from objects from their everyday lives. They wrote their numeric symbols from left to right or from right to left; they would also write their numbers vertically in columns (Holt, 2016).



(mpec.sc.mahidol.ac.th)

The Hindu-Arabic is a numeration system much like the system that we use today. The Hindu-Arabic number system uses ten digits that can be utilized in any combination to represent any value. The digits in the number system are zero, one, two, three, four, five, six, seven, eight, and nine. The Hindu-Arabic number system groups by tens – whereas ten ones are replaced by a ten, ten tens are replaced by a hundred, ten hundreds are replaced by a thousand, ten one thousands are replaced by ten thousands, and so on. Also, this number system uses place value, from right to left: ones, tens, hundreds, thousands, and so on (O'Connor & Robertson, 2000).



(www.archimedes-lab.org)

Properties of Numbers

The properties of numbers are the basic rules of our number system. Understanding the properties of numbers is essential to one's ability to solve mathematical problems. First and foremost, let's look at the Properties of Integers. Integers are whole numbers and their opposites. The opposite of a whole number is the negative of the said whole number. The number 0 is also considered an integer, but 0 is the opposite of itself. There are five properties related to integers. The Commutative Property of Addition states that you can add numbers in any order. For example, adding negative two plus three is the same as adding three plus negative two. Therefore, a +b = b + a. The Commutative Property of Multiplication states that you can be able to multiply numbers in any order, without changing the result – the product. For example, four times negative five is the same as negative five times four. Therefore, ab=ba. The Associative Property of Addition states that numbers in a sum can be grouped in any way, with the resulting sum remaining the same. For example, the sum of three and four plus two is the same as the sum of four and two plus three. Therefore, (a +b) + c = (b + c) +a. The Associative Property of Multiplication states that you can group factors in any way and still get the same product. For example, you can multiply negative six and positive two and then multiply that product by two or you can multiply two and positive two and multiply that product by negative six and get the same result. Therefore, (ab)c = a (bc). The Distributive Property applies to a mathematical expression involving addition that is then multiplied by something. It states that you can add first and then multiply or multiply and then add; either way, the multiplication is distributed over all of the terms in the mathematical expression. For example, in the mathematical expression -5(4+2) = (-5 x 5) + (-5 x 2) you can either add the numbers within the parentheses first. Which is four plus two and then multiply the result by negative five or you can multiply negative five and each term of the expression separately and then add the two products together. Therefore, a (b + c) = ab + ac (Math.com, 2005).

In addition to Properties of Integers, there are also Properties of Rational Numbers. Rational numbers are real numbers that can be written as a fraction in the form of a/b, as a ratio. Rational numbers are associate and commutative and under addition and multiplication. The first property of rational numbers is that of the Closure Law. Rational numbers are considered closed under addition, subtraction, and multiplication. If a and b, in a ratio are rational numbers, then the sum, difference and product of the rational numbers are also a rational number. If the rational numbers of a particular ratio satisfy the conditions above, then they satisfy the Closure Law.

CLOSURE LAW

	Addition	Subtraction	Multiplication	Division
Whole Number	X		x	
Integers	X	X	x	
Rational Numbers	X	X	x	

Rational numbers are also commutative under addition and multiplication. If a and b in a ratio are rational numbers, then the Commutative Law under addition applies: $a + b = b + a$ and the Commutative Law under multiplication applies: $a \times b = b \times a$.

COMMUTATIVE LAW

	Addition	Subtraction	Multiplication	Division
Whole Numbers	X		X	
Integers	X		X	
Rational Numbers	X		x	

Rational numbers can also be associative under addition and multiplication. If a, b, and c are rational numbers, then the Associative Law under addition applies: $a + (b+c) = (a +b) + c$ and the Associative Law under multiplication: $a (bc) = (ab) c$.

ASSOCIATIVE LAW

	Addition	Subtraction	Multiplication	Division
Whole Numbers	X		X	
Integers	X		X	
Rational Numbers	X		x	

Furthermore, it is imperative to remember the following additional properties of rational numbers: one is the multiplicative identity for rational numbers, zero is the additive identity for rational numbers, the additive inverse of a rational number pq is –pq and the additive inverse of –pq is pq, and if pq times ab is equal to one, then ab is the reciprocal of pq (Pearson, 2014).

Not only do integers and real numbers have properties that apply to their use, but so do real numbers. Real numbers include whole numbers, rational numbers, or irrational numbers, and they can be positive, negative, or zero. The only types of numbers that are not considered real numbers are imaginary numbers and infinity. There are five properties of real numbers. The Commutative Property of Addition states that you can add numbers in any order and still get the same sum. For example, 8a + 9 = 9 + 8a. The Commutative Property of Multiplication states in any order, you can multiply and still get the same product. For example, 7 x 6 x 3a = 3a x 7 x 6. The Associative Property of Addition states that you can group numbers in an expression in any way and still get the same sum. For example, (3x + 1x) + 9x = 3x + (9x + 1x). The Associative Property of Multiplication states that you can group factors together in any way and still get the same product. For example, 8 x (2 x 3) = (3 x 8) x 2. The Distributive Property applies to mathematical expressions that involve both addition and multiplication. The Distributive Property states that if a term in an expression is multiplied by terms in parentheses, the multiplication needs to be distributed over all terms inside the parentheses. For example, 4x (7 + y) = 28x + 4xy. The Density Property states that you can find another real number that lies between any two real numbers.

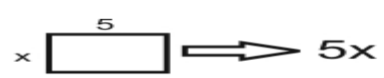
For example, between 7.51 and 7.52, there are the numbers 7.511, 7.512, 7.513, and so on. The Identity Property of Addition states that when zero is added to any number, the sum is the number itself. Zero is referred to as the additive identity. For example, 8y + 0 = 8y. The Identity Property of Multiplication states that the number one multiplied by any other number results in the other number. One is referred to as the multiplicative identity. For example, 8f x 1 = 8f (Brennan, 2002).

Just as there are properties that relate to integers, rationale's, and reals there are also properties that relate to complex numbers. Complex numbers are numbers that are a combination of real numbers and imaginary numbers. Complex numbers are in the form of a + ib, where a is the real numbers and bi are the imaginary number. Imaginary numbers are called imaginary because they lie in the imaginary plane; they arise from taking square roots of negative numbers. The (i) on an imaginary number is equal to the square root of negative one. Imaginary numbers behave like natural numbers when it comes to addition and subtraction. For example, 4i + 6i = 10i and 85i – 5i = 80i. In terms of multiplication, $\sqrt{a} \times \sqrt{a} = a$ Therefore, the following is also true: $\mathrm{i} \times \mathrm{i} = -1$ since **i** = -1, and $\sqrt{-1} \times \sqrt{-1} = -1$. In looking at the division, imaginary numbers can be divided just like any other number if there is only one term: i divided by i equals one or 3i divided by I equals three. If there are two terms divided by two terms, use the complex conjugate: a – bi (Stapel, 2016).

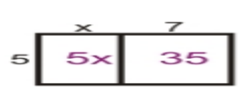
Description of the Activity

In this activity, the twenty-six students will be introduced to mathematical properties that form the foundation of computation. This task is aimed to help students understand the distributive property using area models. Students will build upon their prior knowledge of properties of operations as strategies to multiply and divide rational numbers. According to Lee E. Boyer (1967), clear are the distributive property of multiplication over addition. "The two forms of the distributive property: a (b + c) = ab + ac and (b + c) a = ba + ca. Clearly, if it was worthwhile (and the author thinks it was) to emphasize the two forms of the distributive property in the separate number systems of the natural numbers, the integers, the rational numbers, and the real numbers - in which both forms of the distributive property were always equivalent."

The first part of the Area Representative of the Distributive Property activity can be used to help student recall information regarding the area as a precursor for the distributive property. The first section introduces students to the idea of writing the area of a rectangle as an expression of length x width, even when a variable may represent one more dimension. For example:



According to Arlene Roberts and Jeffrey Chaffee (2010), Distributing and Factoring Using Area it is believed that "students usually grasp the concept of the expanding component rather quickly, but they struggle with the factoring component." The students will show how to model area tiles by multiplying the length and width of a figure together. Also, they explain not only how to complete the step-by-step method but also the reasoning behind it. This sector allows students to begin to piece together some of the fundamental concepts of the distributive property. The teacher suggests that students write the area of each of the figures within the corresponding boxes. For example:



In this key section of the activity, students represent the area of the individual rectangle in two forms to distribute the common factor among all parts of the expression in parentheses. Students love the opportunity to model different ways to solve problems. They also appreciate using the tile manipulatives to create an area model to draw the diagrams in their interactive notebook. The teacher can assign partners or allow students to select their partners. "It is difficult for students to learn to consider, evaluate, and build on the thinking of others, especially when their peers are still developing their own mathematical understandings" (NCTM 2000, p. 63).

It leads to the fundamental concepts of the distributive property. The area as a product section requires students to think about how to represent the area of the entire rectangle without using the area of each of the individual boxes. Area of a rectangle can be created by multiplying the length and width of a figure. The area as a sum division compels students to think about how to symbolize the area of the rectangle while using taking the sum of the areas to find the area of the whole rectangle. For instance, the students would be able to know that the area found by multiplying $(\boldsymbol{x} + \mathbf{7})\ \mathbf{5} = (\boldsymbol{x} + \mathbf{7})$ by the commutative property. The area as a sum of the first rectangle can be found by multiplying the length, x, and the width, 5, together. Thus, the area of the first rectangle is x (5) = 5x by the commutative property. The area of the second rectangle can be found by multiplying the length, 7, by the width, 5. Thus, the area of the second rectangle is 7(5) = 35. To find the total combined area, students must add together the areas of both figures. Therefore, the total combined area is identified as the expression 5x+35. It is helpful to have students to write in their interactive notebook; in fact, it would allow the teacher to determine the different levels student demonstrate understanding.

During the activity, the teacher is monitoring how students are pair-think-sharing responses from the problems and how they are justifying their reasoning. According to Ronald V. McDougall (1967), “the distributive property is one of the most effective instruments we have for achieving these worthwhile objectives”. To accommodate struggling students, the teacher will pose leading questions to help guide them with the lesson. Another accommodation for struggling students, the teacher will have extended the time to finish the task. Students are on different levels in the classroom; for this reason, "some of the students may be ready to move to a visual representation" using distributive property (Morelli 1992). Different demonstrations of the distributive property will reach all learners. According to Lynn Morelli (1992), "several more exercises can be conducted with the students moving between the verbal steps, the numerical example, the visual representation, and the algebraic symbols".

During the final activity, the teachers will help struggling students comprehend both expressions that they produced from the created questions are equivalent and symbolize the same information in different ways. According to Scott Beckett (1990), "The following activity helps take the mystery out of the distributive property for middle school students. It allows them to build a physical model of the property that they label with its symbolic name". Students who are working above level will be given more challenging problems to work on with the teacher as I walk around the room. English learner will have more area model to work on to determine lesson comprehension.

Addressing Different Learning Styles

There is strong evidence that proves that working with number tracks or lines helps students develop a better number sense, and being fluent in flexible counting strategies can lay the foundation for effective calculation strategies. Furthermore, recognition of real number systems and fractions, decimals, measuring, and division are all interrelated concepts and ideas (Askew, 2015). To address students' multiple intelligent and incorporate appropriate different learning styles, teaching real number systems will focus on addressing various avenues. Teaching will focus on the auditory presentation of material for the auditory learners, while assessments in the form of tests, quizzes, and written homework, will address the verbal learners/thinkers.

*See Attached Real Number System Worksheet Handout:

The Real Number System

I. Mark an X for each category that the number applies

II. Construct a number system on construction paper with the numbers below, in order, from least to greatest. Be sure to label your number line.

	Number	Real	Rational	Irrational	Integer	Whole	Natural
1	-5						
2	61%						
3	0						
4	$\pi/3$						
5	2.9						
6	2/9						
7	$\sqrt{6}$						
8	$\sqrt{64}$						
9	1						
10	¼						
11	-2						
12	4.79						
13	$3\pi/4$						
14	99%						
15	5/4						
16	6 ½						
17	13.5						
18	$7\sqrt{2}$						
19	$2\sqrt{3}/3$						
20	2/5						
21	13 5/8						
22	2,000,000						
23	-6982						
24	18.1						
25	-18.1						

Moreover, when teaching a lesson on the real number system, as an educator, it is imperative to ask several types of questions:

- A Comprehension Question—"What is this question asking me?"

  Look for a detailed rationalization that connects these number classifications to the real number system. Do the students understand the differences between real, rational, irrational, integer, whole and natural numbers?

- A Kinesthetic Question—Ask three students to model the first five number classifications, physically on paper. Stand up, demonstrate and explain to me what it looks like.

- A Visual Question—Ask three students to draw a picture of what the next five numbers look like. Draw the numbers, in order, on the board.

As an educator, for students to successfully master mathematical skills and concepts, covering all the various learning styles is vital. One can address different learning styles by incorporating an assortment of questions into the daily curriculum. As well, asking diverse questions achieves a few goals. First, it allows students to articulate a variety of learning methods. Secondly, integrating questions covers a broad range of various levels of thinking skills, such as critical thinking and analytical thinking (Buher).

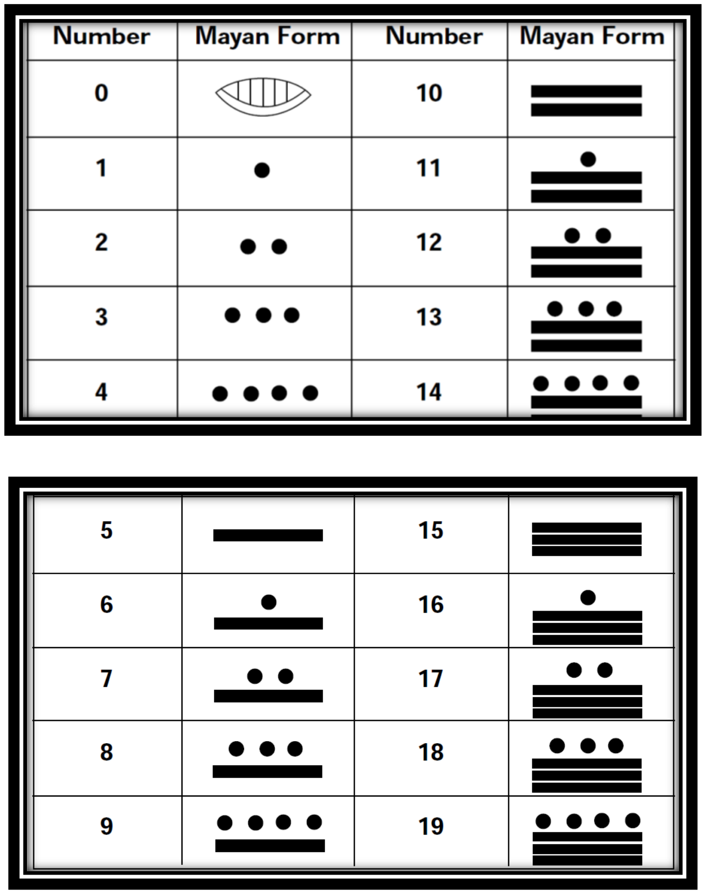
Classroom Use

According to the writer Julie Murgel, the following Mayan Numbers activity help students develop number sense, understand and use the correct vocabulary, and analyze the relationship between numbers and problem-solving situations. Moreover, students link concepts and procedures as they develop and use computational techniques, including mental arithmetic, estimation, paper-and-pencil, calculators, computers, and other manipulatives in problem-solving situations and communicate the reasoning used in solving these problems.

Also, to accommodate the needs of all students, the activity stimulates students to construct and to interpret number meaning through real-world experiences and the use of hands-on materials and relate these meanings to mathematical symbols and numbers. Students model, explain and use the four basic operations—addition, subtraction, multiplication, and division—in problem-solving and real-world situations. During the in-class activity, students are required to identify Mayan numbers, convert a base 10 number to a base 20 number (Mayan) and vice-versa, and use Mayan numbers to add, subtract, multiply, and divide.

The teacher will lecture for few minutes using PowerPoint to facilitate the understanding of visual learners that the Maya used a base 20 number system. They symbolized their numbers using dot and bars; where a dot equaled 1 and a bar equaled 5. After the teacher describes and illustrates the Mayan numeric system, students will practice identifying Mayan numbers from the problems on the board. Then, they will observe how to convert a Mayan number to our number system and vice-versa. After that, students will work in groups of two to complete section 1 and two from worksheet 1.

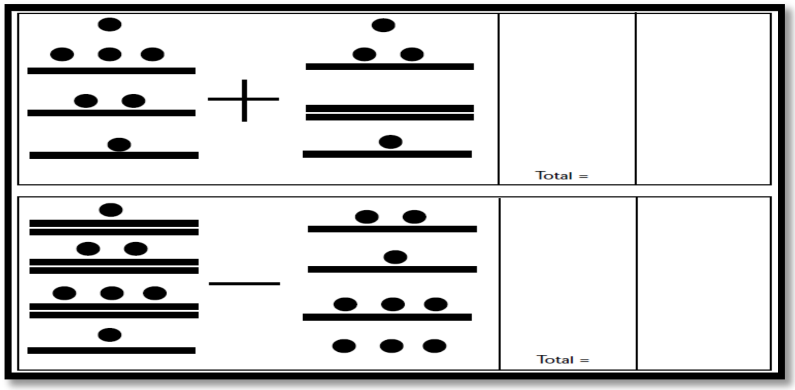
Mayan Number Chart

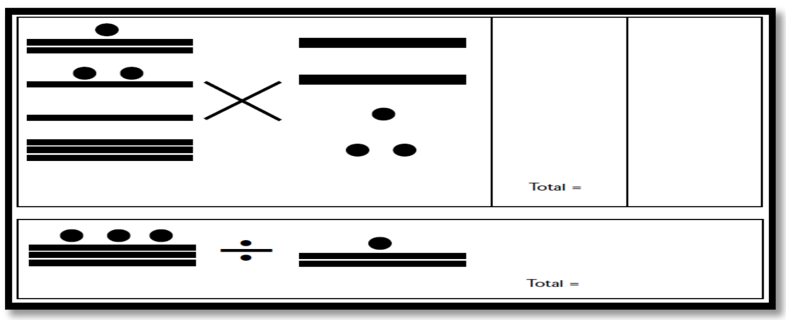


Mayan Numbers: Worksheet 1

Directions: By means of the Mayan numerical system, compute each problem. Be sure the final answer is in Mayan mathematical symbols.

Name: ___________________________________Date____________________





(Murgel, J., 2000).

Numerical System

According to the article “Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond,” the numerical system is categorized as natural numbers, whole numbers, integers, rational numbers, irrational numbers, real numbers, and complex numbers. Some of them have common elements, and some have no intersection at all such as whole and irrational numbers. Also, transcendental numbers, algebraic numbers, and quaternion numbers are considered part of the numerical system (Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond, 2014)

The Natural Numbers

The Natural Numbers is known as counting numbers such as 1, 2,3,4,5,6,7,8, etc. There are infinitely many natural numbers. The Whole Numbers differs from the Natural ones because is contain the zero elements. Also, the result of adding two natural numbers will as always be another natural number. For instance, 204 + 200 = 404, and by multiplying two natural numbers, the outcome will always be another natural number such as 200 x 3 = 600. On the other hand, for the division and subtraction cases the statement is not true (Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond, 2014).

The Integers

The integers are known as a set of infinite positive and negative numbers including the number zero like {…,-3,-2,-1, 0, 1, 2, 3…}. The integers number is represented by the capital letter Z. Also, whenever two integers are added, subtracted, or multiplied the answers will always be another integer number; however, this statement is no true for the division case such as 9 divides by 2 is equal to 2.5 which is a decimal number (Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond, 2014).

The Rational Numbers

The rational number can be defined as every number that is written in the form of a divided b with b different of zero. All elements of integers and natural numbers can be called as rational numbers because the denominator of an integer number are 1. Also, terminate decimals are rational numbers also like 3.245 = 3245/1000 because it can be expressed as a fraction. Repeating decimals can be converted to fraction also such as 0.444 = 4/9. To conclude, "The set of rational numbers is closed under all four basic operations, that is, given any two rational numbers, their sum, difference, product, and quotient is also a rational number (as long as we don't divide by 0)," (Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond, 2014).

The Irrational Numbers

On the other hand, the irrational number cannot be expressed in the form of a divided by b with b different of zero. The pattern of an irrational number can be identified as a decimal number that it never ends or repeats. Going back to the Ancient history, the Greeks made great discoveries regards the irrational numbers. They also proved that not all numbers are rational and that certain equations cannot be solved by ratios of integers. For instance, solving the equation $x^2 = 2$ the solution will be $x = 1.41422135624$ etc. By squaring the solution a number close to 2 can be found, but it will never hit exactly the number 2. So, the conclusion is that square root of 2 is an irrational number that is equal to decimal numbers that repeat without a pattern (Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond, 2014).

Also, "other famous irrational numbers are **the golden ratio**, a number with great importance to biology: $\frac{1+\sqrt{5}}{2} = 1.61803398874989$ *And,* π (pi), the ratio of the circumference of a circle to its diameter: $\pi = 3.14159265358979$. And e, the most important number in calculus: $e = 2.71828182845904...$ and e is the most important number in calculus," (Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond, 2014).

The Real Numbers

The real numbers are known as a set of natural numbers, integers, rational and irrational numbers. As we know, the number line contains all real numbers, and there are infinitely many real numbers. Also, "The "smaller", or a **countable** infinity of the integers and rationals is sometimes called $\aleph_0$ (alef-naught), and the **uncountable** infinity of the reals is called $\aleph_1$ (alef-one). There are even "bigger" infinities, but you should take a set theory class for that," (Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond, 2014).

The Complex Numbers

Also known as the imaginary numbers, the complex numbers are expressed as a set of a + b*i* which a and b stands for rational numbers. Also, *i* is the imaginary unit such as the square root of -1 is only possible to extract by substituting the negative sign by *i* square. The complex numbers can be written as a capital letter C, and it has a critical function regarding solving any polynomial p(x) with real number coefficients (Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond, 2014)

Conclusion

After examining the historical development of numbers and various number systems, it is evident that the use of numbers themselves and the individual number systems have significantly evolved over time. The evolution of the vast array of number systems has provided humanity with the opportunity to employ the use of the number systems and their corresponding properties. Furthermore, it is essential that students develop an understanding of the properties associated with the five important sets of numbers in our number system: natural numbers, integers, rational numbers, real numbers, and complex numbers. Students can develop an understanding of the five essential sets of numbers and their corresponding properties through the use of various learning strategies; activities that allow for individual and cooperative learning groups. Understanding the properties of different sets of numbers lays the foundation for higher order thinking in mathematics: critical thinking skills, analyzing and problem-solving skills, and application to everyday situations.

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According to Reuters (2007) and the American Association for the Advancement of Science in San Francisco, math anxiety depletes working memory to do mathematics. Often times, worrying about doing math takes up a large part of a student's working memory which then spells disaster for the anxious student who is taking high-stakes tests. Today math teachers from around the world almost have to take on the role of counselors in their classrooms to address the many students who dislike or are fearful of mathematics. Mathematics teachers are encouraged to work with school counselors in helping to address the many math anxious students in today's schools. It has become a pandemic in our society where so many young people and adults have negative feelings and poor experiences with mathematics instruction. Metje, Frank, & Croft, (2007) believe that math anxiety is a worldwide phenomenon and that many people are not going into math fields, including engineering, and that more and more math instructors at the university level are not prepared to deal with the increased number of students who are unsuccessful at math due to this increased fear which has crippled their confidence. Addressing math anxiety has become one of the largest challenges for college professors.

Anyone can easily take an informal poll on the street or classroom and find that most respondents will not report positive experiences, feelings, or attitudes toward mathematics. However, we are now living in an age that depends so heavily on one being good at mathematics and problem solving. We are living in a world in which our students will soon be competing with young people from all parts of the globe for jobs. It is imperative that our students develop positive dispositions toward mathematics and the sciences in an information age of which has become so technologically oriented. Young people today need to be well prepared in the areas of math, science, and technology for all career choices. Nurses, engineers, architects, lawyers, teachers, along with many other fields will continue to use more advanced forms of technology that require one to know more mathematics and problem solving to perform their jobs effectively. Sequencing, ordering, patterning, logic, spatial sense, and problem solving are some of the basic skills that all careers require (NCTM, 2000). By the time our young people reach middle school they have developed certain dispositions toward mathematics. Students' confidence and ability to do mathematics and apply these skills in many diverse settings is essential for success; therefore, our young people need to be well prepared to do the mathematics of the 21st century.

Steen (1999) found that "national and international studies show that most U.S. students leave high school with far below even minimum expectations for mathematical and quantitative literacy." Neunzert (2000) contends that we have to understand ourselves as MINT-professionals, where MINT is M=mathematics,

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Tackling Math Anxiety through Photography while using GeoGebra

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Abstract

Math teachers can insert photographs into GeoGebra software then explore various math objectives related to the Common Core Math Standards, the paper shows how to motivate students to learn math and minimize math anxiety while doing it. While covering the new Common Core State Standards, the topics will explore the math that surrounds us in the real world thus creating a connection between the abstract math and the life experiences. When math has a purpose, then students are willing to spend time in exploring and understanding new concepts. Real-life photographs that are inserted into GeoGebra will provide the basis to observe relationships with different and similar shapes. Technology like GeoGebra can help motivate young learners to enjoy learning mathematics while addressing math anxiety and attitudes. The presentation/paper will show educators how by importing photography into the GeoGebra software, teachers can explain math concepts and make the learning of math more real-world and relevant. In an age of STEM, it is critical that we motivate and turn young people onto math through technology. Online websites and resources for addressing math anxiety and attitudes are also shared.

Key Words: Math Anxiety, GeoGebra, Photography, *Mathitudes Survey*

Introduction

Mathematics teachers can better reach their students and show them how math surrounds us by using photography and GeoGebra while teaching math. In today's high tech world, students need to be proficient in Science, Technology, Engineering, and Mathematics (STEM) fields. As endorsed by the National Council of Teachers of Mathematics (NCTM, 2000) and stressed in the new Common Core State Standards(CCSS) in Mathematics, it is critical that we teach using technology, address attitudes and anxiety toward math, and make the math that students are learning relevant and meaningful. Often, it may be best to start teaching young people geometry first as opposed to numbers, which are considered more abstract and difficult to learn. Geometry is one of the most concrete branches of mathematics and focusing on this first can benefit students' whole view of mathematics and their attitudes towards learning it. Today teachers also need to be cognizant and checking for attitudes and dispositions toward learning mathematics as math anxiety is an issue in today's classrooms. This paper looks at ideas for teaching mathematics with the use of technology and photography using the free dynamic mathematics software, GeoGebra, to help teachers create mathematically confident young people.

Checking for Mathematical Dispositions in our Classrooms

Today math anxiety is a common problem in many classrooms. Richardson and Suinn (1972) originally defined math anxiety as "a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (p.551). Mathematics anxiety is the "irrational dread of mathematics that interferes with manipulating numbers and solving mathematical problems within a variety of everyday life and academic situations" (Buckley and Ribordy, 1982, p. 1).

Research on math anxiety has been around since the 1970s (Richardson & Suinn, 1972). Math anxiety still continues to plague our society and affects our young peoples' success and achievement with the subject (Finlayson, 2014; Quander, 2013). Quander feels that elementary teachers need to help prepare students to be lifelong learners and develop a productive mathematical disposition so that they are prepared for future schooling and eventual careers. Math anxiety can impede not only mathematical performance but also interest and then career choice and many decisions in life. The idea of looking closely at math anxiety levels, motivation to learn mathematics, and using technology like GeoGebra to teach and motivate students is critical today in a world of STEM and also can impact achievement goals of the learners (Gonzalez-DeHass, Furner, Vásquez-Colina, & Morris (2017).

As part of the NCTM Standards (1989), the NCTM believe that mathematics teachers need to assess students' mathematical disposition regularly regarding: checking for confidence in using math to solve problems, communicate ideas, and reason; flexibility in exploring mathematical ideas and trying a variety of methods when solving problems; willingness to persevere in mathematical tasks; interests, curiosity, and inventiveness in doing math; student ability to reflect and monitor their own thinking and performance while doing math; and value and appreciate math for its real-life application, connections to other disciplines and cultures and as a tool and language. NCTM has set the stage since the late 1980’s in making educators check for dispositions and attitudes toward mathematics part of the assessment of the learner.

In research from Jackson and Leffingwell (1999) they found that in their study only seven percent of the population reported having positive experiences with mathematics from kindergarten through college. Their study cited that there are many covert (veiled or implied) and overt (apparent and definite) behaviors exhibited by the math instructor in creating math anxiety in students. Things like difficulty of material, hostile instructor behavior, gender bias, perceptions of uncaring teacher, angry behavior, unrealistic expectations, embarrassing students in front of peers, communication and language barriers, quality of instruction, and evaluation methods of the teacher. Math instructors' behaviors and teaching methods can be hurtful and negative to students learning math. Students often say: "I like the class because of the teacher" because the teacher knows how to present developmentally the subject matter, creates a learning environment conducive to learning with compassion, has high expectations for all students without regard to gender, race, or language barriers, and uses a variety of assessment methods and teaching styles to better reach all students to address math anxiety (Chernoff & Stone, 2014; Dowker, Sarkar, & Looi, 2016).

Research by Furner (2007) in synthesizing math anxiety treatments, it was found that there are two distinctions to math anxiety: prevention and reduction and there are distinct strategies and methods to address each is different ways. It has been found that there are three ways to prevent math anxiety: 1). Using “Best Practice” in mathematics such as: manipulatives, cooperative groups, discussion of math, questioning and making conjectures, justification of thinking, writing about math, problem-solving approach to instruction, content integration, technology, assessment as an integral part of instruction, etc.; 2). Incorporating the NCTM and State/Common Core Math Standards into the curriculum and instruction; and lastly, the importance of discussing feelings, attitudes, and appreciation of mathematics with students. This same research found that there are three methods to reduce math anxiety: 1). Psychological Techniques like anxiety management, desensitization, counseling, support groups, bibliotherapy, and discussions; 2).Once a student feels less fearful about math, he/she may build their confidence by taking more mathematics classes; and 3). It has been found that most research shows that until a person with math anxiety has confronted this anxiety by some form of discussion/counseling no “best practices” in math will help to overcome this fear.

It may also be beneficial to provide students with a math attitude surveys at the beginning of each school year or course and also to read the book, *Math Curse* (Scieszka & Smith,1995), to get students to talk about their true feelings toward math, surveys and biblio-therapy are both effective forms of starting the process of opening up and getting inner feelings out young people may have about mathematics or unpleasant past experiences. It is recommended that mathematics teachers survey their students at the beginning of a school year to check for their students' dispositions toward mathematics. See Appendix C Attached for a possible survey to use called *Mathitudes Survey*. There are also two good online surveys that test for math anxiety and may be useful to classroom teachers as follows are: Mathpower (http://mathpower.com/anxtest.htm) and Mathipedia (http://www.mathipedia.com/student-math-anxiety-test.html), both of these websites offer online tools for teachers ad students to be able to take a short survey to assess their overall dispositions toward mathematics.

Math Teachers during the school year while teaching mathematics should use some advantageous instructional methods which are advocated now for teaching mathematics using the Concrete-Representational-Abstract (CRA) Model for teaching mathematics as follows: First educators need to tart with the Concrete using hands-on manipulatives like Geoboards, then secondly, they must move to Representational models in diagrams (or use Virtual Manipulatives like NLVM at: http://nlvm.usu.edu/), and lastly, connect to the Abstract symbolism where student understand and function at an abstract level completely (GeoGebra software works well at: http://www.geogebra.org/cms/en/). The CRA Model is really the bases for the best practices pedagogy for teaching mathematics starting with young people, but should also be used at all levels of math instruction.

Today learning connections can be made when we teach math using such things as technology like GeoGebra and photography. Munakata and Vaidya (2012) based on their research found that students do not consider mathematics and science to be creative endeavors, although the traditional artistic disciplines rank high in this regard. To address this problem in perception, the authors used photography as a means to encourage students to find the deep-rooted connections between science and mathematics and the arts. The photography project was used in a formal classroom setting as well as an outside activity, i.e. in a more informal setting. The project found student interest and motivation were peaked when photography was part of the instructional strategies to teach new material while making meaningful connections to the math using the photography. Jones (2012) also in her book, *Visualizing Mathematics*, discusses how teachers need to help students visualize and create representations of their math understanding so to turn them on to the subject. Beilock & Willingham (2014) feel math teachers can help to address and reduce math anxiety. The author believes by using technology like GeoGebra along with the photography teachers can make better connections and students are going to be more highly motivated to learn math.

Common Core State Standards (CCSS) as They Relate to using GeoGebra

Most schools and states today are adhering to the new Common Core Math Standards (National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO), 2010) found at: http://www.corestandards.org/

See Appendix B for examples on using GeoGebra and photography to create meaning and understanding of geometry for the students.

When math teachers relate real-world problems through the use of dynamic technology like GeoGebra and connecting them to photography to make important connections in math, our learners recognize that geometry and shapes/mathematics surround us! Many Common Core Math Standards can all be taught using GeoGebra and Photographs an done in a way that is less anxiety provoking for students when learning mathematics. See Appendix B several examples.

Using Technology like GeoGebra in the Teaching of Mathematics in Today's Classrooms

Using technological tools is critical in today's world. Our students need to learn to excel at higher levels of generalization, model and solve complex problems, and focus on decision-making and reasoning (National Council of Teachers of Mathematics (NCTM) 1989, 2000, 2006). NCTM believes that mathematical power can arise from technology that includes: increased opportunity for learning, increased opportunities for real-life social contexts, and orientation to the future. The President's Council of Advisors on Science and Technology (PCAST) (Holdren, Lander, & Varmus, 2010) released an executive report in November 2010 where specific recommendations to the administration are given to ensure that the United States is a leader in Science, Technology, Engineering, and Mathematics (STEM) education in the coming decades. One recommendation is to recruit and train 100,000 new STEM middle and high school teachers over the next decade that are able to prepare and inspire students to have strong majors in STEM fields and strong content-specific pedagogical preparation. PCAST regards teachers as the most important factor in ensuring excellence in STEM education. Despite the ongoing efforts to promote the use of technology in education (e.g., National Council of Teachers of Mathematics [NCTM], 2000; National Educational Technology Standards for Teachers [NETS.T], 2008), teachers' ineffective use of technology has been reported in the literature. One reason frequently cited is that teachers are not trained in utilizing technology in the classroom within the subject context. Hwang, Su, Huang, & Dong, (2009) found that by combining virtual manipulatives and software like GeoGebra along with whiteboard, teachers can better model problems, help students understand and solve the problems while reaching higher levels in the teaching of many mathematical ideas in the curriculum.

Using Emerging Technology like GeoGebra

The software, GeoGebra, is a multi-platform dynamic mathematics software for all levels of education from elementary through university that joins dynamically geometry, algebra, tables, graphing, spreadsheets, statistics and calculus in one easy-to-use package (Hewson, 2009; Hohenwarter, Hohenwarter, & Lavicza, 2009). This open-source dynamic mathematics software can be downloaded for free and accessed at: http://www.geogebra.org/cms/en/info. There are no licensing issues associated with its use, allowing students and teachers freedom to use it both within the classroom and at home. GeoGebra has a large international user and developer community with users from 190 countries is currently translated into 55 languages.

Some research by Fahlberg-Stojanovska, & Stojanovski (2009), they discovered that using GeoGebra is motivating for students and helps them learn at a higher level while exploring and conjecturing as they draw and measure. Rosen & Hoffman (2009) established the importance to integrate both concrete and virtual manipulatives into the math classroom, such as representational models like GeoGebra. Furner & Marinas (2007,2014) found that young people can easily transition from the concrete when using manipulatives like geoboards to the abstract when using geometry sketching software like GeoGebra. Although GeoGebra has been primarily intended for mathematics instruction in secondary schools, it certainly has uses in higher education and even now introduced in the elementary math levels. The Appendix A provides online websites and resources related to GeoGebra.

GeoGebra may be used to show how mathematical equations can be applied to everyday objects. Aydin & Monaghan (2011) in their research feel that math teachers need to explore the potential for students to "see" mathematics in the real world through "marking" mathematical features of digital images using a dynamic geometry system like GeoGebra. Mathematics teachers may find the following videos (Mathematics and Multimedia, n.d.) of basic training for GeoGebra at: http://mathandmultimedia.com/2011/01/01/geogebra-essentials-series/ useful as they provide great resources for how to quickly use GeoGebra in their classrooms.

Research using GeoGebra was described as raising the enthusiasm for the effective and wise application of technology to the teaching/learning enterprise (Fahlberg-Stojanovska and Stojanovski, 2009; Hewson, 2009). Observations of participants in schools and during the summer workshops are also cited as evidence. GeoGebra was also credited with changing teacher habits. Two features were specifically referenced as causing this change: 1) that it is an award winning software system, and therefore has admirable features, and 2) that it provides an effective pedagogical model for teachers.

Mishra and Koehler (2006) found that Technological Pedagogical Content Knowledge (TPCK) is the basis of good teaching with technology and requires not only content knowledge or pedagogical knowledge but an understanding of the representation of concepts using technologies, how to teach these math concepts using technology, knowledge on the challenges their students will face when presented with this new pedagogy, and how technology can be used to build on existing knowledge and develop new knowledge. Scandrett (2008) feels that math teachers need to always start by using concrete models in geometry using manipulatives like geoboards which provide a concrete model of understanding. Rosen & Hoffman (2009) have found that teachers need to connect students understanding from the concrete to abstract and using virtual manipulatives and software like GeoGebra better help make those connections to representational models connecting the concrete using geoboards to something oven more abstract in understanding. With the availability of GeoGebra, teachers are able to make graphical representations of math concepts. As the concepts are introduced with pictorial representations, teachers and their students are able to make the connections between the pictures, the math concepts, and the symbolic representation. When presented with a new concept, students need to think, visualize and explore relationships and patterns. This is consistent with the CRA (Concrete, Representational, and Abstract) Model for teaching mathematics currently in better reaching students as they learn and understand mathematical concepts. Technology makes all of this possible for them in a short amount of time.

So Why Should Math Teachers use GeoGebra as part of their instruction?

In reviewing the research on GeoGebra, there are many reasons to use GeoGebra some of which are: that it is free to download and use from GeoGebra.org; it is an up and coming dynamic teaching tool in our schools today, dynamic for learners; it is user-friendly for students and teachers; it lends itself well to connection from the hands-on Geoboards to virtual Geoboards to something even more abstract; it is a a software that provides many resources and teaching tools at its wiki for educators at: http://www.GeoGebra.org/en/wiki/index.php/Main_Page; GeoGebra may be used for primary-aged students through college: and lastly it is fun, easy to use, and students learn a lot about geometry, algebra, measurement and beyond by using this dynamic learning tool.

Math educators may ask why it is important to make connections and excite students about learning math while using GeoGebra? To answer this educators will find that when using GeoGebra, educators will be able to: show a purpose for math; develop relationships between math concepts and shapes and ideas; the software will show practical applications to math in life; it employs innovative teaching in the classroom; it stimulates through photography/modeling; it employs emerging technologies in math with the real world application; and it can address math anxiety so students feel confident for any STEM field when they graduate from school.

Additional help with math anxiety and its research can be found at: Professor Freedman Provides Math Help at: http://www.mathpower.com and Mathitudes Online website at: http://www.coe.fau.edu/centersandprograms/mathitudes/

A famous quote from W. V. Williams (1988) is a reminder of how critical it is to teach for understanding making things as hands-on and real-world as possible: “Tell me mathematics, and I will forget; show me mathematics and I may remember; involve me...and I will understand mathematics. If I understand mathematics, I will be less likely to have math anxiety. And if I become a teacher of mathematics, I can thus begin a cycle that will produce less math-anxious students for generations to come.” Today math teachers need to break the cycle of math anxiety and address it, and by using GeoGebra and making connections with photography teachers can better connect the math they teach to students and their understanding while using emerging technologies.

Furner (1999) also made these related observations as they relate to the importance of mathematics confidence: "If math teachers do something about helping their students to develop their confidence and ability to do math, we can impact their lives in a positive way forever." And "Our students' careers and ultimately many of their decisions they will make in life could rest upon how we decide to teach math. We must make the difference for the future of our kids in an ever growing, high-tech, competitive, global world which depends so heavily on mathematics."

Final Thoughts

Young learners intrigued by technology will construct and investigate geometric shapes and many math ideas with GeoGebra and will start enjoying math and have less math anxiety in our STEM World that we now live in. By using technology like GeoGebra and incorporating photographs, our young learners who are often intrigued by technology will construct and investigate geometric shapes; when using photography inserted in the GeoGebra software students can start enjoying math more and will hopefully be less math anxious in the years to come so to pursue any STEM field of their liking. Students will see math in the real world more when using photography inserted into the GeoGebra to learning the mathematics in today's curriculum to cover many standards. There are many free resources for math teachers Grades K-12 to download which are in Appendix A.

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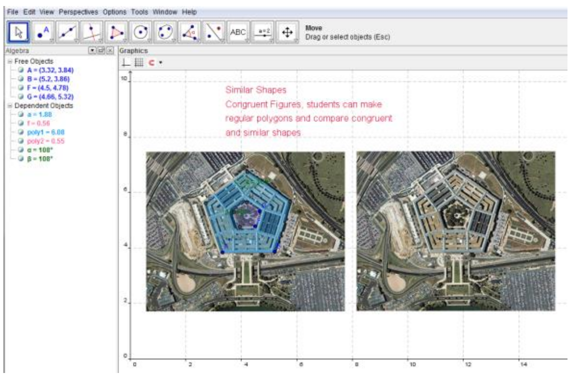
Appendix A: GeoGebra Websites and Resources for the Mathematics Classroom

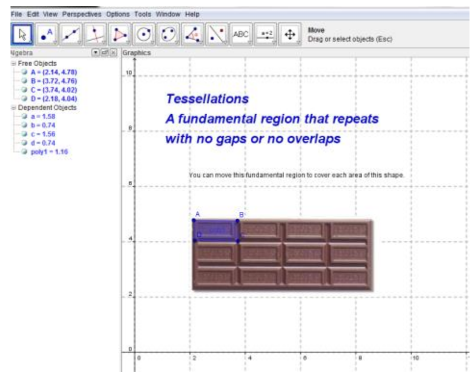
Geoboard Resources	http://msteacher.org/epubs/math/QuickTakes/geoBoard.aspx
GeoGebra	http://GeoGebra.org
GeoGebra Wiki Forum	http://www.GeoGebra.org/en/wiki/index.php/Main_Page
GeoGebra Data Files	http://matharoundus.com
Math Academy	http://www.mathacademy.com/pr/minitext/anxiety/
Mathitudes Online	http://www.fau.edu/education/centersandprograms/mathitudes/

Appendix B: K-6 Math Topics Covered with GeoGebra and Photography

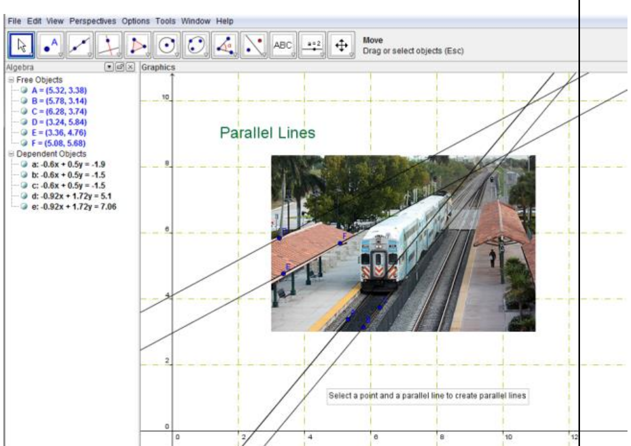
Similar Shapes



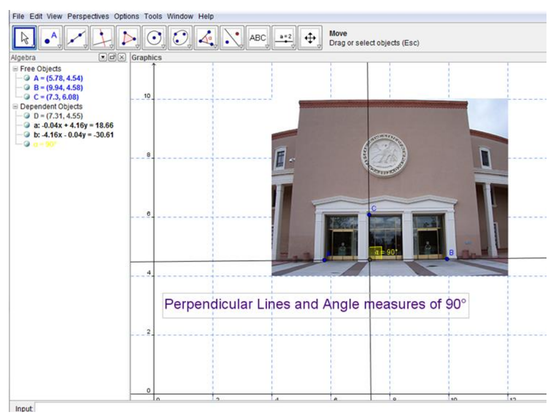
Tessellations



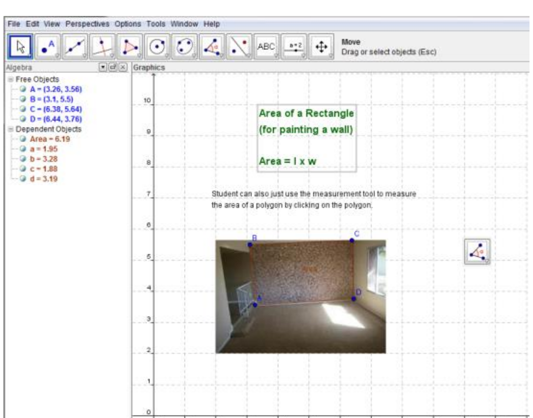
Parallel Lines



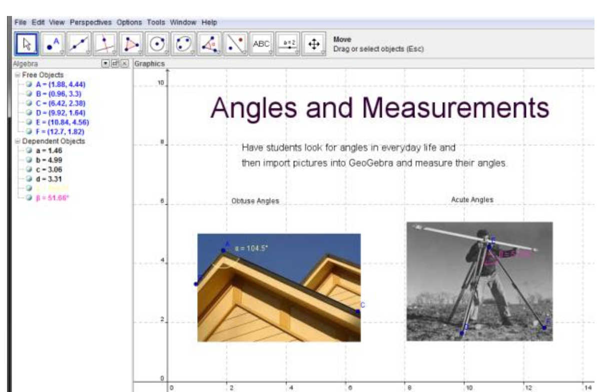
Perpendicular Lines



Area



Angles and Measures



Appendix C

Mathitudes Survey

Name___________________________________
Grade___________________________________
Math Class_______________________________
Age_____________________________________
Career or Career Interests____________________

<u>Mathitudes Survey</u>

1. When I hear the word math I.......

2. My favorite thing in math is......

3. My least favorite thing in math is.......

4. If I could ask for one thing in math it would be............

5. My favorite teacher for math is______________because______________

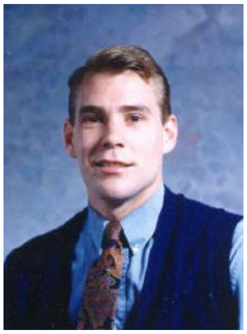
6. If math were a color it would be.....

7. If math were an animal it would be.....

8. My favorite subject is________________ because___________________

9. Math stresses me out: True or False Explain if you can.

10. I am a good math problem-solver: True or False Explain if you can.



Author Bio

Joseph M. Furner, Ph.D., is a Professor of Mathematics Education in the Department of Teaching and Learning at Florida Atlantic University in Jupiter, Florida. He received his Bachelor's degree in Education from the State University of New York at Oneonta and his Masters and Ph.D. in Curriculum and Instruction and Mathematics Education from the University of Alabama. His scholarly research relates to math anxiety, the implementation of the national and state standards, English language issues as they relate to math instruction, the use of technology in mathematics instruction, math manipulatives, family math, and children's literature in the teaching of mathematics. Dr. Furner is the founding editor of *Mathitudes Online* at: http://www.coe.fau.edu/centersandprograms/mathitudes/ He is the author of more than 80+ peer-reviewed papers. Dr. Furner has worked as an educator in New York, Florida, Mexico, and Colombia. He is concerned with peace on earth and humans doing more to unite, live in Spirit, and to care for our Mother Earth and each other. He is the author of *Living Well: Caring Enough to Do What's Right*. Dr. Furner currently lives with his family in Palm Beach, Florida. He enjoys his job, family, civic and church involvement and the beach. Please feel free to write to him at: jfurner@fau.edu.

FAMTE's MATHEMATICS TEACHER EDUCATOR OF THE YEAR AWARD

The Board of Directors of the Florida Association of Mathematics Teacher Educators (FAMTE) has established the **Mathematics Teacher Educator of the Year Award**. The Award will be given on an annual basis with recognition of the recipient at the annual meeting of FAMTE during the Florida Council of Teachers of Mathematics (FCTM) annual conference. The purpose of this award is to recognize excellence in the areas of teaching, research and service.

Eligibility
All mathematics educators who have been employed in a public or private university in the State of Florida for the past two consecutive years and who are members of FAMTE are eligible to apply. Applicants must not have received this award within the past 3 years prior to their application.

Criteria
Nominations are invited that highlight a nominee's involvement at the university, state, and national levels regarding teaching, research, and service. Examples of contributions within each area are included below.

Teaching: For example…
a. Implementation of effective and innovative teaching practices
b. Demonstration of innovative teaching methods (e.g. publications, materials, video)
c. Recipient of awards in teaching from department, college, university, state, and/or national entities
d. Support of doctoral student development
e. Textbook authorship

Service: For example…
a. Active participation in advancing the development and improvement of mathematics teacher education (e.g., membership and leadership roles in state, national, and international organizations)
b. Unusual commitment to the support of mathematics teachers in the filed (e.g., distinctive mentoring experiences)
c. Participation in editorial boards and/or editorial review of journal manuscripts

Scholarship: For example…
a. Dissemination of research findings offering unique perspectives on the preparation or professional development of mathematics teachers
b. Publications useful in the preparation or continuing professional development of mathematics teachers
c. Acquisition of state and/or nationally funded training and/or research grants
d. Contribution of theoretical perspectives that have pushed the field of mathematics education forward
e. Recipient of awards in research from department, college, university, state, and/or national entities

Required Documentation (Maximum of 3 items)
1. A current vita of the nominee
2. A letter of nomination from a FAMTE member documenting evidence related to the indicated criteria that supports the nomination.
3. An (**one**) additional letter of support from an individual active in the educational community (or individuals if letter is co-authored) knowledgeable of the nominee's contributions to mathematics education.

Nomination Process/Deadline: No self-nominations will be accepted. The nomination materials should be sent to FAMTE president, Angie Su at shuifang@gmail.com. Complete nomination packets should be submitted by Friday, October 4, 2016.

FAMTE Mathematics Teacher Educator of the Year
Award Revised 1/18/19

FAMTE's DOCTORAL STUDENT OF THE YEAR AWARD

The Board of Directors of the Florida Association of Mathematics Teacher Educators (FAMTE) has established the **Doctoral Student of the Year Award**. The Award will be given on an annual basis with recognition of the recipient at the annual meeting of FAMTE during the Florida Council of Teachers of Mathematics (FCTM) annual conference. The purpose of this award is to acknowledge a mathematics education doctoral student who has shown active involvement in mathematics education at the university, state, and/or national level and who shows potential for success in the field across the areas of teaching, research and service.

Eligibility
All mathematics education doctoral students enrolled in a public or private university in the State of Florida in good academic standing and who are members of FAMTE are eligible for the award. Applicants must not have received this award within the past 2 years prior to their application. Nominees must be enrolled in a doctoral program at the time the award is given.

Criteria
Nominations are invited that highlight a nominee's involvement at the university, state or national level in regards to:

Teaching: For example…
a. Supervised planning and teaching of undergraduate students
b. Preparation and delivery of professional development sessions
c. Assistance to faculty delivery of courses

Service: For example…
a. Membership in state and/or national professional organizations
b. Committee participation and/or leadership in state and/or national professional organizations
c. Participation in editorial review of journal manuscripts or other significant documents

Scholarship: For example…
a. Collaboration on one or more research projects with peers and/or faculty
b. Dissemination of research or practitioner-oriented data in journals
c. Dissemination of research or practitioner-oriented data via conference presentations

Required Documentation (Maximum of 3 items)
1. A current vita of the nominee (specifically indicating institution and doctoral advisor or committee chair)
2. A letter of nomination from a FAMTE member documenting evidence related to the indicated criteria that supports the nomination.
3. An (**one**) additional letter of support from an individual active in the educational community (or individuals if letter is co-authored) knowledgeable of the nominee's contributions to mathematics education.

Nomination Process/Deadline
No self-nominations will be accepted. The nomination materials should be sent to FAMTE president, Angie Su at shuifang@gmail.com. Complete nomination packets should be submitted by Friday, October 4, 2016.

FAMTE Doctoral Student of the Year
Award Revised 1/18/19

Florida Associaiton of Mathematics Teacher Educators
Membership Application
(Individual or Affiliate Group)

Florida Association of Mathematics Teacher Educators (FAMTE) – Check membership option and amount.

__________ **One** Year Membership - $25.00

__________ **Two** Year Membership - $45.00

__________ **Five** Year Membership - $100.00

Write your check for the appropriate amount, payable to **FAMTE**, and mail the check and this form to:

Angie Su
FAMTE Membership
2150 Areca Palm Road
Boca Raton, FL 33432-7994

Please complete the following; help us keep our records up to date.

NAME ____

MAILING ADDRESS ______________________________________

CITY ________________________ **STATE** __________ **ZIP** ________

PREFERRED TELEPHONE NUMBER ________________________

PREFERRED EMAIL ____________________________________

UNIVERSITY / ORGANIZATION ___________________________

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Transformations
c/o Angie Su,
2150 Areca Palm Road,
Boca Raton, FL 33432