1969

A Comparative Investigation of the Effects of Precision of Response in Two Ninth Grade Algebra Classes

James Henry Krack
Central Washington University

331

A COMPARATIVE INVESTIGATION OF THE EFFECTS OF PRECISION OF RESPONSE IN TWO NINTH GRADE ALGEBRA CLASSES

A Thesis

Presented to

the Graduate Faculty

Central Washington State College

In Partial Fulfillment

of the Requirements for the Degree

Master of Education

---

by

James Henry Krack

August, 1969

APPROVED FOR THE GRADUATE FACULTY

Bernard L. Martin, COMMITTEE CHAIRMAN

Dale R. Comstock

Daryl Basler

ACKNOWLEDGMENT

The writer wishes to acknowledge gratefully the assistance and guidance of the committee chairman, Dr. Martin. His unselfish outpouring of time and energy, his cooperation, his understanding, and his criticism proved to be invaluable in bringing this paper to its final form. The writer is also indebted to Dr. Comstock and Dr. Basler for their criticisms which helped immensely to clarify the information and make the report more cohesive.

TABLE OF CONTENTS

LIST OF TABLES

CHAPTER I

THE PROBLEM, HYPOTHESES, AND PROCEDURES

The clear and definite structure that is characteristic of most parts of mathematics can be misleading when problems of mathematics instruction are considered. The very clarity of the structure itself can lead to the mistaken conclusion that nothing beyond this structure need be considered in analyzing and deciding how mathematics should be taught.

Yet anyone who has taught mathematics knows how far from the truth this type of thinking is. It is not a simple matter for the average student to learn mathematics. In fact most educators would agree that the average student experiences more difficulty in learning mathematics than in learning most of the other subjects in the curriculum. Thus, although mathematics itself fits into a clear and definite structure, the learning of mathematics does not share this same attribute. For this reason mathematics education research is aimed not at altering the structure of mathematics to adapt to the student's thought processes, but rather at altering the teaching methods and approaches so that the student can more easily become cognizant of the inherent beauty and orderliness found in the study of mathematics.

I. THE PROBLEM

Statement of the problem. One specific area of concern in mathematics education research is that of communication of mathematical concepts and processes. Brune, in reflecting on the role of language in the communication process, commented that language can either help or hinder the process. Brune said that:

> Intelligent living demands that we transmit thoughts; we communicate by means of language. Through language man has shared his discoveries, preserved his learnings, developed his civilizations, and educated his children. Thus language has benefitted mankind. Yet, because at best it reveals meanings imperfectly, language has produced misunderstandings, bred dissentions, and even formented wars. The power of language, like the force of fire, can effect good or ill in human affairs.
>
> In the teaching of mathematics, language has also succeeded and failed. Whenever it has led students to enjoy the thinking through of a mathematical situation, language has helped. Whenever it has engendered lack of clarity as pupils seek to solve problems, language has hindered . . . . In the drama of thinking, language plays the lead (5:156).

Clearly then, one of the problems in mathematics education is that of language. In analyzing this problem, Page differentiated the three basic forms of language involved in the mathematics learning situation. First, there is the everyday, common, though ambiguous, language that is in no way peculiar to the mathematics classroom. This form of language includes such words as set, square, line, et cetera. The second form of language is comprised of words that are used only when dealing with numeric quantities and ideas. Examples of this second, or middle, language are: divisor, subtrahend, commutative property, additive inverse, et cetera. The third and highest form of language is that of symbolization. This form deals with variables, quantifiers and logical constants such as: x, $+$, $=$, Σ, $\forall x$, $\ni y$, $\in$, $\exists$, et cetera.

In his comments, Page posed the question whether or not the student of mathematics would experience as much difficulty in understanding and expressing ideas if he were encouraged to utilize as much of the second and third forms of language as possible. He noted that it is the second and particularly the third form of language that enables one to communicate clearly, concisely, and precisely (8:1026).

It was the discussion of Brune and the question raised by Page that provided the impetus and direction for the research that is described in this paper. The problem investigated concerned itself with the type and effects of language variations. More specifically, this paper reports an investigation that was undertaken to study the effects that the precise use of words and symbols had on the acquisition and retention of mathematical material. Borrowing heavily from Brune and Page's comments, the writer operated on the assumption that the less precise a student was the greater was the possibility of ambiguity and misunderstanding. The question to be answered was whether or not the precision required of a mathematics student had any significant effect on his ability to acquire and retain concepts and processes.

Importance of the study. There are numerous statements in the literature that could be used as justification of the importance of research of the nature described in this paper. One source recognized the general need for the acquisition of more knowledge about the relationship between teaching and learning (16:113). Another recorded need is that of determining methods that contribute most to retention of mathematical ideas and information (16:113). Johnson, former president of the National Council of Teachers of Mathematics, stated that the relative effectiveness of different teaching methods on problem solving needed further study (17:425). Somewhat more closely related to the problem under consideration, Gagne posed the question, "If words are used in mediating problem solving, what conditions determine the precise meanings for these words so that they do not lead to errors in performance?" (9:52). Also, Henderson expressed the need for research aimed at determining whether or not the kind of language used in verbalizing was important (8:1026). The need was listed in a slightly different way when it was asked what effect the precise use of variables, quantifiers, and logical constants would have in helping students state generalizations correctly (8:1027). Thus, need recorded in the literature appears to justify research on the effects of precision in the use of mathematical language.

II. HYPOTHESES

After perusing the literature, the writer decided to design an experiment that hopefully would provide clues to answers of some of the aforementioned questions. Using an introductory unit on equation solving in ninth grade algebra classes, an attempt was made to test the hypothesis that requiring a student to answer precisely enhances his ability to solve equations. Also tested was the hypothesis that requiring precision in response would assist the student in retaining the ability to solve algebraic equations.

To test these hypotheses the following null hypotheses were established:

Hypothesis I. As measured by a specified achievement test, there is no significant difference in the ability to solve algebraic equations between students who are required to respond precisely, both in written and oral discourse, and students who are not required to respond precisely, both in written and oral discourse.

Hypothesis II. As measured by a specified achievement test, there is no significant difference in retention of the ability to solve algebraic equations between students who are required to respond precisely, both in written and oral discourse, and students who are not required to respond precisely, both in written and oral discourse.

III. PROCEDURES

Limitations of the study. Any study such as this has very definite limitations, the most obvious one being that the findings can be generalized only to the school and age group from which the particular sample was derived. The most serious limitation, however, was the lack of a sound psychological theory underlying the learning of mathematics. Although several noteworthy educators, such as Suppes and others, are currently engaged in this endeavor of theory development, much more work remains to be done before the various findings of studies similar to this can be woven into a meaningful network of correlated findings (28:5-21).

In addition to these general limitations, this study had a rather unique limitation. In an experiment such as this, the normal procedure is to have one instructor teach one of the methods and a second teacher teach the alternate method. For this study the writer was unsuccessful in his efforts to solicit the assistance of a cooperating instructor to teach either of the two classes. Hence, one of the serious but unavoidable limitations of this study was the use of one teacher teaching both the experimental and control methods.

Another limitation of the study was the impossibility of control over the prior level of precision to which the class as a whole was accustomed and the level to which the subjects were individually accustomed. That is, it was possible that certain individuals in the class with no external demands on precision could have possessed internal motivation on a par with that expected of the experimental class.

Assumptions of the study. With these limitations and other possible sources of bias in mind, the writer developed a plan of research that incorporated the following assumptions:

1. The previous training of the subjects did not unduly influence the performance of either class

2. The classes were not biased with respect to:

    a. class size

    b. sex ratio

3. The instructional procedures were not biased toward either of the two methodologies.

4. The measuring instruments were valid and reliable indicators of performance levels.

Brief description of procedures used. Ninth grade algebra students were chosen to be the subjects for the particular research described in this report. The subjects were involved in an experiment that lasted a total of fourteen weeks. Three of these weeks comprised a training period, while the remaining eleven weeks were used as a retention period. The three week training period was designed so as to coincide with an introductory unit on equation solving that came as a normal topic of instruction in the writer's three algebra classes.

Of the three classes taught, only two were involved in the study. One class was taught using an experimental method, referred to as method "A", and the other class was taught using a conventional, control method, method "B".

Throughout the fourteen week span, four tests were administered: a pre-test, a post-test, and two retention tests, denoted KR I and KR II respectively. At the conclusion of the experimental period, the results of these tests were statistically analyzed to determine the presence or absence of significant differences between the mean levels of performance of the two classes. It was at this point in the study that interpretation of the data was possible, and consequently it was possible to provide justifiable answers in response to the two proposed null hypotheses.

CHAPTER II

REVIEW OF THE LITERATURE

The initial step in developing the plan for this study was to peruse the literature, paying particular attention to those studies that were addressed to the problems of this particular research. Although not much has been done in the area of verbalization and precision of verbal response per se, there were several articles that provided interesting observations on related topics.

One such topic was the question of whether or not it is of importance for a student to be verbal. That is, is it possible for a student to grasp a concept without ever identifying the concept with a written or verbal label? Another question was concerned with the degree of direction that a teacher should provide in assigning labels to concepts. Specifically, in the process of acquiring knowledge, is it best to conduct a teacher-directed class, a student discovery-directed class, or a combination of these two. The writer also hoped to find some studies that dealt specifically with advantages or disadvantages of emphasizing precision in response.

I. STUDIES IN VERBALIZATION AND RETENTION

In response to the question dealing with the importance of a student being able to verbally communicate his comprehension of a concept, Hendrix suggested that verbalization is probably not as important as most educators believe. Her studies indicated that before it does any good to verbalize concepts, there must first of all be a prerequisite of meaning for the terms used. She said that the possession of the concept really comes in a sub-verbal, organic, dynamic state of awareness. This view is in conflict with the view of many psychologists, linguists, and philosophers who do not consider the concept complete until a person has attached a verbal or written symbol to it. Because of her position, Hendrix emphasized the need of teaching that developed the sub-verbal meaning of a concept, rather than emphasizing the verbal label of that concept (12:334).

How much direction should a teacher provide? Kittell's studies implied that the teacher should simply organize the materials for learning, then sit back and let the pupil discover the pattern, idea, or concept. He suggested that the teacher might assist the discovery process by suggesting meaningful relationships on which the pupil could base his discovery, but the teacher should not be too specific or precise in his answers. By providing statements of underlying relationships without giving specific, precise answers, the teacher can, according to Kittell, foster learning, retention, and transfer to other situations (20:403).

Wittrock disagreed with this approach. He said that educators who refrain from introducing and labeling any and all concepts when the student has adequate background to make the label meaningful, do so in a vain attempt to enhance transfer. He says that this probably only reduces transfer and wastes time during acquisition. In contrast to Kittell, Wittrock recommended that the teacher take a more active role, providing meaningful direction that guides the student in such a manner that his responses are in the desired fashion. By remaining passive in his direction, the teacher is allowing responses that could very easily produce negative transfer. In Wittrock's opinion the only sure way of enhancing correct responses and eliminating alternate responses is to carefully identify concepts with verbal labels and to use these labels to help weed out undesirable and incorrect responses (32:190).

What level of precision should a teacher expect in the verbal responses of his students? According to the Committee on the Analysis of Experimental Mathematics Programs, there is little common agreement of opinion on this question. Each program has a different philosophy on how rapidly a student should be led into sophisticated use of the mathematician's language. Part of the issue rests in determining what constitutes sophistication; part of it pertains to the changing meanings of non-changing words (2:3).

The University of Illinois Committee on School Mathematics produced a series of courses that was genuinely concerned with developing precision in the use of the language of mathematics. The UICSM believed that early in the course a student was ready and able to be led carefully from the general language of mathematics to a very precise and sophisticated use of it (2:60).

In contrast, the School Mathematics Study Group produced a series of texts that were somewhat less concerned with the level of precision in the use of the language of mathematics. The Committee on the Analysis of the Experimental Mathematics Programs commented that there was no evidence in the texts that the SMSG authors regarded precise and sophisticated use of language on the part of the student as an objective of prime importance. Material was presented in an intuitive fashion, so as to not unduly tax the student. The committee reports that in the "Introduction to Algebra" text, new terms were introduced rather casually and often without the benefit of a firm, decisive definition.

In another place the committee reports that the authors were careful in their selection of words but not to the point of being picky in the proper use of terms and symbolism (2:37).

This committee reported that most programs place a high premium on the precise use of mathematical language, but the programs differ mainly in the grade level and the degree of precision expected. Perhaps one of the reasons why there is no common agreement as to the degree of precision that should be expected is because of the lack of research in this area. Prior to 1967 there was virtually nothing in the literature dealing specifically with precision of response. However, in 1967 the National Council of Teachers of Mathematics reported that there were three research projects in progress that were concerned with verbalization and precision in mathematics:

1. The Effect of Teaching Certain Concepts of Logic on Verbalization of Discovered Mathematics.

2. A Study of the Role of Symbols in Learning Mathematical Principles.

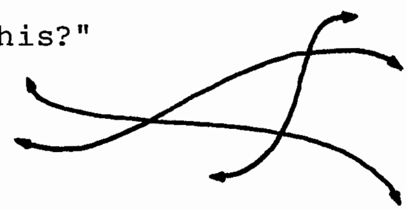
3. The Development of a Scientific (Theoretical) Language for the Precise Formulation of Basic Research on Mathematics Learning (16:111).

Only one study reported in the literature even came close to the type of research recorded in this paper. Gurau in 1967 reported a teaching method that emphasized precision of response in a geometry class. To illustrate how his sessions progressed, Gurau reconstructed one of the sessions for his article.

After he had introduced the rules of the game, Gurau encouraged students to offer definitions of a geometric object, for example, a triangle.

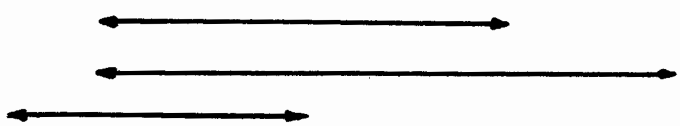
"A triangle is . . . well, it has three lines."

"Like this?"



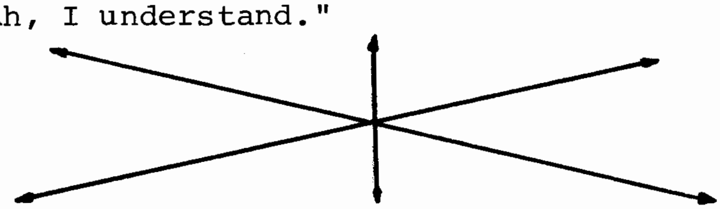
"No. They have to be straight."

"Oh, like this?"



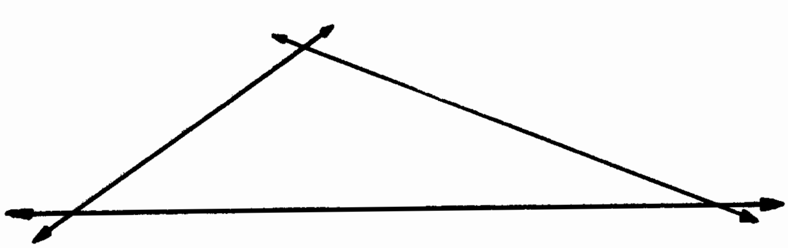
"Of course not. The lines have to meet."

"Ah, I understand."



"No, no, each one has to meet the next one."

"Oh, I see."

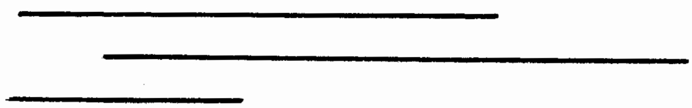


"Well, yes, a little bit like that, but you should cut off the leftovers."

"Can't anyone be more precise?"

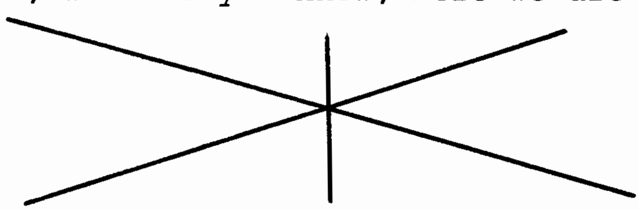
"Well, we don't want the whole line, we only want a piece of a line. I mean we want three pieces of three lines."

"Good. Here."



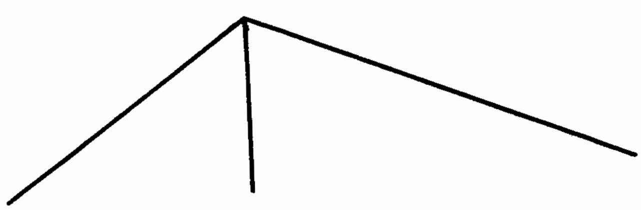
"No, but these three pieces of lines have to meet."

"Well, what do you know, here we are again."



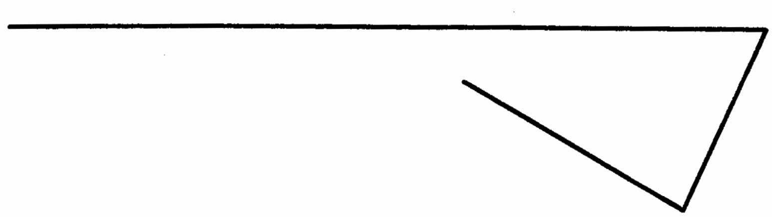
"No, no, they have to meet at their ends."

"Oh, I see."



"Now, make each one meet the other one, at their ends."

"But maybe they won't reach."



"Then we want three pieces of three lines that are long enough to meet each other, that meet at their ends, and that stop at that point."

"What was that last word you said?"

"Which one, point?"

"Yes, that sounds mathematical, doesn't it? Does anyone else know another name for pieces of lines?"

"Line segments."

"Good. Now can anyone try to define a triangle, using the words 'line segment' and 'point'?"

"A triangle is . . . a thing with three points, and the line segments that connect the points."

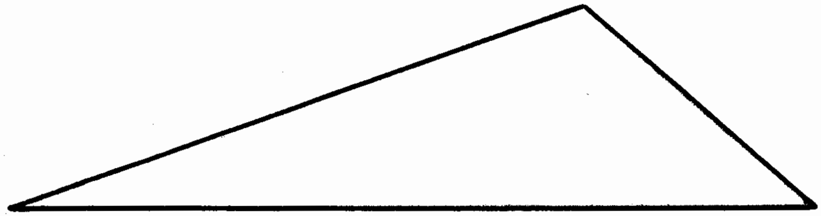
"How's this?"

"No, the three points shouldn't be straight. I mean they shouldn't be on the same line."

"Well, maybe we have something now. Can someone take all these pieces and put them together?"

"Well, a triangle is made up of three points, which shouldn't be on the same line, and the three line segments that connect the points."

"Very good, I think I'm trapped. I can't think of any way of drawing the figure without making it look like a triangle" (10:453-454).



Gurau reported that after such a session, the students were anxious to learn new words that describe a particular idea precisely. Words like "element" and "set" were not, the students found out, mere affectations on the part of the mathematician, but were ways of stating a particular concept in a compact and precise form.

Gurau noted that the students were fascinated with the idea of trying to be precise, and consequently he encouraged other educators to pursue this method of instruction.

II. ANALYSIS OF RESEARCH FINDINGS

Hendrix's writings revealed her fear that educators tend to place more emphasis on the label of a concept than on the comprehension of the concept. She pointed out that a good teaching methodology is one that allows abstractions to come first and the names for these abstractions later. Language, according to Hendrix, should always come as an insight (12:334-337). The writer agreed with the importance of teaching for meaning but thought it unwise to place all the emphasis at the sub-verbal state. Agreement was found with Wittrock's suggestion that as soon as a student can attach a meaningful label to a concept, be sure to attach it.

The writer disagreed with Kittell's suggestion that the teacher should remain passive in the instructional role. Kittell said that the teacher should refrain from giving specific, precise answers to question, but rather should provide statements only of the very basic underlying statements involved. Agreement was found with Wittrock's endorsement of the use of meaningful teacher direction, making full use of all verbal labels that were meaningful to the student (32:402-404). The writer was of the opinion that the teacher could provide the most meaningful direction in the discovery process if, like Gurau, he encouraged and rewarded responses that were meaningfully precise.

The writer found it hard to determine the level of precision that should be expected at different grade levels. However, in looking at evauluations of some of the experimental programs, the writer found evidence of texts that expected levels of precision that were artificially sophisticated (2:55). The conclusion that was drawn was that the best level of precision to expect would be the highest level that constituted meaningful precision but yet one that stopped short of artificial sophistication.

After perusing the literature, the writer had to develop a plan of research. Decisions had to be made. Who would the subjects be? Where would the experiment be conducted? Who would be the instructors? What would be the nature of the measuring instruments? How would the data from the experiment be analyzed? The answers to these questions and others were eventually determined and are reported in the following chapter.

CHAPTER III

PLAN OF RESEARCH

The basic design of this study was to teach two beginning algebra classes, using one class as an experimental class and the other as a control class.

I. PHYSICAL CHARACTERISTICS OF EXPERIMENT

Explanation of methods used. Of the two methods employed, the experimental method "A" was a method in which the student was required to give all responses in a manner that was explicit and precise. Ambiguity was not tolerated, and both the classmates and the teacher made it a point not to "know" what the responding student was trying to communicate unless his response was explicitly and precisely stated. These restrictions applied to both written and oral responses.

Method "B" was a control method in which the student could be quite careless in any and all responses. Ambiguity was accepted, and both the classmates and the teacher were satisfied with any response as long as they could decipher what the responding student was trying to communicate. Thus precisely correct words and symbols were not required in any written or oral response.

Subjects of study. The subjects involved in this experiment were sixty-five ninth grade algebra students at East Junior High School in Puyallup, Washington. The subjects comprised two out of the three algebra classes normally taught by the writer. Class size was virtually the same in the two classes used in this study, with the control class having thirty-two and the other thirty-three students. In one class the boy to girl ratio was 12:21, while the other class had a 15:17 ratio. Normal scheduling procedures provided classes that, for all practical purposes, were homogeneously grouped. As tested by the Otis Quick-score Primary Abilities Test, the I.Q. scores of the control class ranged from 84 to 128 with an average score of 103. The I.Q. range of the experimental class varied from 80 to 131 with an average score of 105. Thus both classes were of average intelligence.

Compensatory measures used. As previously mentioned, the normal procedure for an experiment such as this is to use two instructors, having each instructor teach one of the two classes. In this experiment, however, it was not possible to use two instructors. Hence, two important compensatory measures were incorporated in the design to counter the effects of this possible bias.

Recognizing the difficulty in using one teaching method in one section and another teaching method in another section of the same course five minutes later, the writer saw the need for a class which could be used for changing from one method to the other. Fortunately, normal scheduling procedures had provided the three classes taught by the writer to be held during the second, third, and fourth periods of the day. A decision was made to use the second and fourth periods for experimental purposes and use the third period class as a "change-over" class. Teaching the class that emphasized precision in response required more concentration on the part of the instructor. Therefore, it was decided that it would be best to teach the control class, method "B" class, first. During the next period, period three, the instructor became more demanding in the level of precision required in responses. By the time fourth period began the instructor was sufficiently prepared to adhere to the rigid demands of the experimental method, method "A".

The utilization of the third period as a "change-over" class was helpful, but in order to further assure consistency in each method, another compensatory device was incorporated into the experimental design. The instructor placed a portable tape recorder in his desk and recorded sessions at random. Each night during the training period the recordings were reviewed in order to evaluate and analyze his effectiveness during that particular day. During these nightly review sessions the instructor planned for the following day with respect to the following five areas:

(1) insights gained from reviewing the tape recordings; (2) general mathematical concepts necessary as background; (3) all mathematical principles to be used in the instructional and criterion measures; (4) use of the two specific methods of instruction; and (5) procedures for the administration and scoring of tests.

Although the instructor was trained for his role, the subjects were in no way conditioned for their contribution in the experiment. Also, without exception, the subjects remained unaware of the use of the recorder in the classroom.

Measuring instrument. The experimental period lasted fourteen weeks. During that time the subjects were given four tests: a pre-test, a post-test, and two retention tests, KR-I and KR-II respectively. All tests were alternate versions of the same test prepared by the experimenter. Of the forty items on each test, the following types of exercises were included:

a. three equations requiring the use of simple addition or subtraction to find the solution

b. three equations requiring the use of simple multiplication or division to find the solution

c. eight equations with decimal coefficients

d. ten equations with fractional coefficients

e. five equations making explicit use of the distributive property

f. fourteen equations containing variables in both members

g. five equations contained in word problems.

The test was an open-ended test, with no one at any time completing or even attempting all forty items. The time limit for each administration was held constant with thirty-five minutes elapsed time.

In scoring the tests, solutions were accepted if they were correct but in an unconventional form. That is, $\frac{13}{13}$ was accepted as 1 and $\frac{1.2}{.7}$ was accepted for the answer $\frac{12}{7}$. (Refer to the Appendix, pages 56-67.)

II. METHOD OF PROCEDURE

Instructional procedures. The text used in the training period was Johnson, Lindsey, and Slesnick's Modern Algebra, published by Addison-Wesley (18:55-165). The rate of instruction was held virtually constant with both classes covering the same material on the same days.

The instructional procedures fell into two major categories: class discussion and class written work. Subjects in both classes "A" and "B" were expected to participate in class discussions on a random basis, with the instructor soliciting responses in a random manner. Classmate correction of responses was expected and tolerated in both classes.

Class "B" responses could be ambiguous and still be accepted. Any response that could be construed as being correct was tolerated. If necessary the teacher reworded the response to make it discernible, although attention was not drawn to the correction. Correct answers, not correct processes, characterized method "B". To better describe the method used, a sample of the method in practice is included:

(Teacher writing equation on chalkboard) "Let's look at this next equation."

$$2a + 3 = 7$$

"What is the first step in solving this equation?"

"Take 3 from both sides."

"Right. That makes the equation become . . . ?"

"$2a = 4$."

"Fine, now what is the next step?"

"You take $\frac{1}{2}a$ to both sides."

"$\frac{1}{2}a$?"

"Well, $\frac{1}{2}$ then, you know what I meant."

"Okay, after you take $\frac{1}{2}$ to both sides, what do you do?"

"Now you put $a = \frac{4}{2}$ and you get $a = 2$."

"Good. Now let's look at another problem."

$$4x = \frac{1}{2}$$

"Try to solve this one. What should I do?"

"You should put the additive inverse of 4 on both sides."

"What is the additive inverse of 4?"

"$\frac{1}{4}$."

"Oh, then you mean the multiplicative inverse of 4. Right. You put $\frac{1}{4}$ on each side which gives us what?"

"$x = \frac{1}{16}$."

"Fine. Now let's look at problem number 16 . . . ."

Thus subject "B"'s answers could lack clarity and precision and still be accepted. If the teacher could perceive that the subject knew what he wanted to say or write, the response was accepted.

Both groups were taught in a semi-discovery manner, with the teacher interjecting knowledge whenever required, or requested. Method "A" responses, as opposed to method "B" responses, had to always be precise and explicit before they were accepted. No ambiguity was tolerated and nothing was left to the assumption of either the teacher or other classmates.

To better illustrate method "A" an example of the method in practice is included:

(Teacher writing equation on chalkboard) "Now let's take a look at this next equation."

$$2a + 3 = 7$$

"What is the first step in solving this equation?"

"Well, you take . . . "

"I what?"

"You take . . . "

"I don't know how to take anything. What do you really mean?"

"Alright, then, you add -3 to both sides of the equation."

"Why -3?"

"Because -3 is the inverse of 3."

"Be specific. What kind of inverse?"

"I don't know."

"-3 is the additive inverse of 3. Remember that the sum of a number and its additive inverse equals the identity element, which is "0" for the operation of addition. Our equation, then, is changed to what?"

"$2a + 3 + -3 = 7 + -3$ or $2a = 4$."

"Good. What is the next step? Anyone."

"Put $\frac{1}{2}$ on both sides."

"Wait a minute. How can I just 'put' $\frac{1}{2}$ on both sides?"

"You're right, $\frac{1}{2}$ is the correct number to use, but why?"

"You use $\frac{1}{2}$ because it is the inverse . . . "

"Stop! What kind of inverse? Be specific."

"Well, $\frac{1}{2}$ is the multiplicative inverse of 2, so you multiply both sides by it. Then $2 . \frac{1}{2} = 1$, and $1a = 2$."

"Good. If you say exactly what you mean, it is much easier to understand you."

"But you know what I mean."

"Do I? How do you know? Be precise; then there is no doubt as to what you mean. Okay, let's look at another problem."

$$4x = \frac{1}{2}$$

"Try to use the properties to solve this one. What should I do?"

"You put the additive inverse of 4 on both sides."

"Put? I can't put anything anywhere without a reason. What do you really want to say?"

"I really can't see why you are so fussy, but anyway you multiply the ad. . . . I mean you multiply both sides of the equation by the multiplicative inverse of 4, which is $\frac{1}{4}$."

"Right. So what is the solution to this equation?"

"$\frac{1}{4} \cdot 4x = \frac{1}{4} \cdot \frac{1}{2}$ or $1x = \frac{1}{8}$."

"Fine. Now let's look at problem number 16 . . . ."

Clearly, subject "A"'s responses had to be void of any element of ambiguity and had to reflect a working knowledge of the properties of the real number system.

The same distinction of methods applied with respect to written work. Subject "A" had to have his problems correct with respect to sign placement, parentheses placement, sequence of steps, et cetera. Again, correct answers, not correct processes, characterised the demands on class "B".

All subjects worked independently in both classes and all work was done during class time. No homework was assigned during the experimentation period. All classwork papers were corrected and returned to the subjects, with subject "A"'s work being scrutinized with respect to the criteria mentioned.

Time schedule. The entire experimentation period lasted fourteen weeks, with a three week training period and an eleven week retention period, as shown in Table I.

Test results of all the subjects in the three classes were withheld until the end of the entire experimentation period. At that time the scores of the subjects in classes "A" and "B" were sent to the computer facility at Central Washington State College for statistical analysis. A description of the subsequent analysis follows.

TABLE I

TIME SCHEDULE

Date	Week	Event
		Training Period
10/13/67	0	Pre-test administration
	0	Start of training period
	1	Training period
	2	Training period
11/ 3/67	3	Post-test administration
	3	End of training period
		Retention Period
	3	Start of retention period
	4	Retention period
	5	Retention period
	6	Retention period
	7	Retention period
12/ 8/67	8	Retention test, KR I, administration
	9	Retention period
	10	Retention period
	11	Retention period
	12	Retention period
	13	Retention period
1/19/68	14	Retention test, KR II, administration
1/19/68	14	End of retention period

CHAPTER IV

ANALYSIS OF DATA

After the number of correct solutions to the equations on all the tests had been compiled and recorded, a copy of the scores was sent to Central Washington State College for statistical analysis. Prior to that time, however, plans had been made as to how to best obtain the maximum information from the sets of test scores.

I. COMPENSATORY MEASURES USED

As recorded earlier the experimentation period lasted for a period of fourteen weeks. As might be expected, not all of the students were present for all four test administrations during such a lengthy span. Some subjects were ill, some were dropped from the class, and for a variety of reasons, it was not possible to have the same number of subjects participate in each test administration. For those who were absent or deleted from the rolls on one or more occasion, a dummy score was substituted. The formula used to provide such a score for those tests missed was found in Li's text, Introduction to Statistical Analysis. The formula for the dummy value, d, is as follows (22:210):

$$d = \frac{kT + nR - S}{(k - 1)\ (n - 1)}, \text{ where}$$

k = number of subjects in class

n = number of tests given to the class

T = sum of the n - 1 scores of the test with the missing test score

R = sum of the k - 1 subjects taking the test with the missing test score

S = sum of the kn - 1 scores

Utilizing the dummy score, it was possible to keep a larger sample size and to make possible a more meaningful analysis of the data obtained.

II. BRIEF EXPLANATION OF t-TEST

Since it was only the numbers of correct solutions to the equations on the tests that provided the sources of data, the main objective of the statistical analysis was to determine if a significant difference was present between the two classes as indicated by the numbers of correct solutions. That is, was the difference of the mean performance levels of both classes a significant difference, or was it a difference that was generated by error, chance, or bias?

The statistical device used to help answer this question was Fisher's t-test for unpaired variates. The formula for t is as follows (15:217):

$$t = \frac{\bar{X}_B - \bar{X}_A}{\sqrt{\dfrac{\sum_{i=1}^{N_A}(X_{Ai} - \bar{X}_A)^2 + \sum_{i=1}^{N_B}(X_{Bi} - \bar{X}_B)^2}{N_A + N_B - 2}\left(\frac{1}{N_A} + \frac{1}{N_B}\right)}}$$

with $N_A + N_B - 2$ degrees of freedom,

where $\bar{X}_A$: mean of the test for class "A"

and N_A : size of class "A".

The importance of the t value is that, in general, the further $|t|$ is from zero, the more confident one can be that if there is a difference between the means, $\bar{X}_B$ and $\bar{X}_A$, that difference is a significant difference. Referring to Fisher's formula, the importance of the means of each class becomes readily apparent, for it is the sign of $\bar{X}_B - \bar{X}_A$ that necessarily determines the sign of t. Because of the importance of the difference of the means in this respect, subsequent reference will be made to it to help explain the sign of various t-scores.

Numerous comparisons between the two classes were made. Tables II and III summarize the data used for the comparisons, while the following section provides an interpretation and analysis of the significance of those various test comparisons.

TABLE II

STATISTICAL DATA

Test	Mean	Variance	Standard Deviation
Class A (Experimental)			
Pre	9.696	7.717	2.778
Post	17.090	17.022	4.135
KR I	14.939	20.121	4.485
KR II	16.848	15.507	3.937
Post - Pre	7.393	17.433	4.175
KR II - KR I	1.909	1.079	1.039
Pre - KR I	-5.243	.843	.918
Post - KR I	2.151	1.125	1.060
Pre - KR II	-7.152	.703	.838
Post - KR II	.242	.985	.992
Class B (Control)			
Pre	10.031	10.805	3.287
Post	17.093	28.861	5.372
KR I	13.250	16.451	4.056
KR II	14.812	21.189	4.603
Post - Pre	7.062	20.963	4.578
KR II - KR I	1.562	1.176	1.084
Pre - KR I	-3.219	.851	.922
Post - KR I	3.843	1.416	1.189
Pre - KR II	-4.781	.999	.999
Post - KR II	2.281	1.564	1.250

TABLE III

COMPARISON OF MEANS AND t-SCORES OF VARIOUS TESTS

Class "B"		Class "A"	$\bar{X}_B - \bar{X}_A$	t-score
Pre	-	Pre	0.335	0.442
Post	-	Post	0.003	0.002
KR I	-	KR I	-1.689	-1.593
KR II	-	KR II	-2.036	-1.913
(Post - Pre)	-	(Post - Pre)	-0.331	-0.304
(KR II - KR I)	-	(KR II - KR I)	-0.347	-0.443
(Pre - KR I)	-	(Pre - KR I)	2.024	2.102
(Post - KR I)	-	(Post - KR I)	1.692	1.855
(Pre - KR II)	-	(Pre - KR II)	2.371	2.661
(Post - KR II)	-	(Post - KR II)	2.039	2.258

III. SIGNIFICANCE OF t-SCORES

Pre "B" - Pre "A" (t = .442). In this particular test, a t-score of this magnitude was anticipated since normal scheduling procedures provided classes that were relatively homogeneously grouped with respect to size, male-female ratio, and indicated ability.

The importance of such a small t-value is that it gave assurance that, for all intents and purposes, the two classes began the training period with virtually the same level of pre-training, competence, and ability.

Post "B" - Post "A" (t = .002). For this test, a t-score of such a small magnitude indicated that at the end of the three week training period, the performance levels of both classes were virtually equivalent. The mean score for class "B" was 17.093 correct responses as compared with class "A"'s average 17.090 correct responses.

The most apparent reason for this near equivalence was that once the initial introduction to the unit had been given, the simple algebraic equations could have been solved mechanically. Thus, left to their own method of solving the equations, the control class could have developed a set of stimulus-response patterns that allowed them to solve the equations simply by rote. In so doing, the control class exhibited performance levels on a par with the experimental class "A".

Thus, in answer to the first null hypothesis, it appeared evident that requiring a student to respond precisely in no way increased his competence level in solving simple algebraic equations. Although the experimental method was a more sophisticated and formal approach, it was apparently of no benefit in the process of acquiring knowledge. Hence, the first null hypothesis was accepted.

<u>$(\text{Post} - \text{Pre})_B - (\text{Post} - \text{Pre})_A$ ($t = -.304$).</u> Two general types of subjects were noted throughout the duration of the experimental period. First, there was the majority of the subjects who exerted varying degrees of enthusiasm and diligence in trying to adapt to the prescribed methods in each class. Secondly, there was the small group of subjects who tenaciously held to the less formal, half-forgotten methods of solving equations that they learned in the previous year. It was this type of subject who maintained low and relatively constant scores on all four tests. In the experimental class, this type of subject exhibited significantly more frustration and confusion than his counterparts in the other class. In one instance, a reluctant learner in the experimental class became so confused and frustrated that he performed even less well on the post test than on the pre test.

The mean increase of correct items, however, in the experimental class was 7.393 items as compared to 7.072 items in the control class. This small difference of means accounts for the small t-score of -.304. According to the previous discussion on the importance of the magnitude of t, the t for this comparison was too small to be considered significant.

KR I_B - KR I_A (t = -1.593). As anticipated, the mean level of performance of both classes dropped noticeably during the five week interval following the end of the training period. The mean score for the control class for this first retention test was 13.250, a decrease of 3.843 correct items. The mean score in the experimental class was 14.939, a decrease of only 2.151. Hence, t was a negative value. There was a definite difference between the performance levels, but once again, the difference was too slight to be considered significant.

KR II_B - KR II_A (t = -1.913). Six weeks after the the administration of the first retention test, a second and final retention test, KR II, was given. The mean score of the control class was 14.812, with a standard deviation of 4.603. The scores for the experimental class were higher and more tightly packed as evidenced by the mean of 16.848 and standard deviation of 3.937. In this comparison t (t = -1.913) was large enough to qualify for the 90 percent confidence interval, meaning that a difference of means this great would occur by chance alone only 10 percent of the time. Note that this was the first comparison to provide any indication that a significant difference was present.

In addition to these comparisons, various comparisons of "gain" scores were taken.

$\underline{(KR\ II - KR\ I)_B - (KR\ II - KR\ I)_A\ (t = -.443).}$ With a mean gain of 1.562 items in class "B", (KR II - KR I) = 1.562, and a corresponding mean gain of only 1.909 correct responses in class "A", it should be clear that $\overline{X}_B - \overline{X}_A$, and therefore t, was a small and negative number.

At this point, one might wonder why KR II - KR I was a gain instead of a loss that normally accompanies a time lapse in retention studies. This apparent discrepancy was caused by a three day assignment that involved the application of equation solving. This unit on linear equations came as a normal topic of discussion in a unit which followed the training period. Such an interjection was almost impossible to eliminate because of the extended length of the experimentation period and the building-block nature of algebra itself. One interesting observation was that the experimental class adapted somewhat more readily to the unit than did the control class.

<u>(Post - KR I)$_B$ - (Post - KR I)$_A$ (t = 1.855).</u> In the comparison of Post - KR I of both classes, a mean decrease in correctly solved equations was 3.843 for class "B", while class "A" showed an average decrease of only 2.151 items. Although the t-value was not large enough to qualify for the 95 percent confidence level, it did qualify for the 90 percent interval. The significance of this comparison is shown in the following comparison which is a combination of this particular comparison and the immediately preceeding comparison, (KR II - KR I)$_B$ - (KR II - KR I)$_A$.

<u>(Post - KR II)$_B$ - (Post - KR II)$_A$ (t = 2.258).</u> During this eleven week retention period, the mean decline in the number of correct solutions for class "A" was only .242 as compared to the mean decline of 2.281 for class "B". The magnitude of t for this comparison was large enough that it could be asserted with 95 percent confidence that the differences of the two mean declines in levels of performance was a result of the influence of the two differing methods of instruction. At this point it was theoretically possible to end the comparisons and conclude that the second hypothesis should be rejected. However, there was still more to learn from the following two comparisons.

$\underline{(\text{Pre - KR I})_B - (\text{Pre - KR I})_A \; (t = 2.102).}$ The (Pre - KR I) comparison is actually a combination of an earlier comparison (Post - Pre), and the comparison (Post - KR I). Reference to the t-score of both these comparisons makes it clear that the majority of the difference noted was exhibited after the cessation of the training period--that is, during the retention period. This fact gave further evidence to support the tentative conclusion that the experimental method produced results that were most noticeable during the period of retention.

During the eight week span extending from the pre-test to KR I, the mean level of performance in class "B" rose from 10.031 to 17.093 and dropped back to 13.250 correct equations. Class "A" had a pre-test mean of 9.696, a post-test mean of 17.090, and a KR I mean of 14.939 correct items. Hence, the total gain of "B" was 3.219 items for the eight week period, as compared to a total gain of 5.243 for "A". A t-score of this magnitude satisfied the desired 95 percent confidence level and gave further support to the tentative rejection of the second hypothesis. The most convincing evidence, however, was found in the most important comparison of all--Pre - KR II, the overall gain.

$\underline{(\text{Pre - KR II})_B - (\text{Pre - KR II})_A \; (t = 2.661).}$ The comparison of Pre - KR II was perhaps the most important comparison because it was the comparison of the entire span of the entire experimental period, from the initial test to the final retention test given fourteen weeks later. The mean gain in correct items for class "B" was 4.781 as compared to 7.152 items for class "A". That is, the experimental method assisted the subjects in retaining the ability to solve the equations to the extent that they could solve $1\frac{1}{2}$ times more equations correctly at the end of the fourteen weeks than could the subjects in the control class.

Armed with an overall t-value greater than 2.66, it was possible to conclude that there was only a 1 percent chance of getting such a great difference between the overall means by chance, error, or bias. Thus, with all traces of hesitancy removed, it was concluded that the second hypothesis should be rejected. That is, all evidence provided by this study indicated that rigidly requiring a subject to respond correctly, with respect to written and oral discourse, had a noticeable and positive effect on the ability of the subject to retain his competency in solving simple algebraic equations.

CHAPTER V

SUMMARY, DISCUSSION, AND RECOMMENDATIONS

I. SUMMARY

One of the areas of concern in mathematics education is that of communication of ideas and procedures. Since language plays the key role in the communication process, it is of value to examine its effect in the mathematics learning situation. The particular topic discussed in this paper concerned itself with various levels of precision in language usage in the mathematics classroom. The research was conducted to help shed light on the effect that rigidly requiring precision in written and oral response had on the ability to acquire and retain mathematical concepts and processes.

To accomplish this goal subjects from two ninth grade algebra classes were selected and taught a unit on elementary equation solving. One class was an experimental class, while the other served as a control class. In the experimental class all responses had to be precisely correct, while the subjects in the control class could respond in any manner that could be construed to be basically correct.

Two null hypotheses were offered, the first of which stated that the two differing methods of instruction would not produce significant differences between the mean levels of performance of the two classes by the end of the three week training period. The second hypotheses proposed that no significant differences would be evident at the end of the eleven week retention that followed the training period.

Before the training period was started, a pre-test was administered to the subjects. Three weeks later an alternate form of the pre-test, the post-test, was given. Two more alternate forms, called KR I and KR II, were administered during the retention period at spacings of five and eleven weeks, respectively. At the end of the fourteen week experimentation period, the scores from all the tests were sent to Central Washington State College for statistical analysis at the computer facility. The means and standard deviations of each test were determined, and at this point it was possible to make various comparisons between the tests.

Using Fisher's t-test for unpaired variates, the following comparisons were made between the mean performance levels of each class:

1. $Pre_B - Pre_A$
2. $Post_B - Post_A$

3. $(\text{Post} - \text{Pre})_B - (\text{Post} - \text{Pre})_A$

4. $\text{KR I}_B - \text{KR I}_A$

5. $\text{KR II}_B - \text{KR II}_A$

6. $(\text{KR II} - \text{KR I})_B - (\text{KR II} - \text{KR I})_A$

7. $(\text{Post} - \text{KR I})_B - (\text{Post} - \text{KR I})_A$

8. $(\text{Post} - \text{KR II})_B - (\text{Post} - \text{KR II})_A$

9. $(\text{Pre} - \text{KR I})_B - (\text{Pre} - \text{KR I})_A$

10. $(\text{Pre} - \text{KR II})_B - (\text{Pre} - \text{KR II})_A$

The Post - Pre and the overall comparison, Pre - KR II, proved to be the most interesting comparisons because they were specifically designed to either prove or disprove the two proposed null hypotheses. $(\text{Post} - \text{Pre})_B - (\text{Post} - \text{Pre})_A$ had a t-score of -.304, which was a value so small that the first null hypotheses was accepted. That is, it was concluded that neither method was clearly superior in helping the subject acquire the ability to solve the equations on the test.

The overall comparison $(\text{Pre} - \text{KR II})_B - (\text{Pre} - \text{KR II})_A$, was also an interesting comparison. For this comparison, t was 2.661 and large enough to qualify for the 99 percent level of confidence. Thus, the second null hypothesis was rejected, and it was concluded that the experimental method produced a definite and positive effect on the mean retention levels of the classes studied.

Simply stated, the study described in this paper indicated that emphasizing precision of response in the classes tested did not significantly affect the subjects' ability to acquire the ideas and processes presented to them during the training period. However, the study provided convincing evidence that requiring precision in both written and oral responses substantially increased the students' ability to remain cognizant of those same ideas and procedures.

II. DISCUSSION

Most of the variables inherent in such a study as this were isolated and compensated for. However, there were other variables that remained unchecked. One of these variables was the prior level of precision to which the class as a whole was accustomed and also the level to which the subjects were individually accustomed. As much as was possible, the experimenter de-emphasized precision of response prior to the actual experimentation period so as to not unduly influence the performance of either class. Nonetheless, there was one particular subject in the control class who, of her own volition, made her responses impeccably precise. Because of the experimental design for the control class, it was difficult to tactfully suppress her disproportionate number of verbal responses. As a result, some of the other subjects in the class followed her lead, trying to be somewhat more clear and definite in their responses. This unexpected occurrence constituted a limitation of the study as it definitely influenced the data, shrouding the true differences produced by the two methods.

Another uncontrolled variable was that the average student took longer on the problems attempted on the tests than did the subjects in the control class. The obvious reason for the increased time was that they took more pains in making every step meaningful and precise. In light of this time differential, it might also have been a meaningful comparison to have recorded each subject's ratio of correct versus attempted equations. Also, since sign errors accounted for the majority of the errors on the retention tests, it might have been meaningful to have tabulated the number of equations that were missed solely because of sign, incorrect addition, incorrect multiplication, et cetera. It was the opinion of the writer, however, that the comparisons reported in the paper would encompass the specific types of errors and would yield as much information as the specific comparisons. If a similar study were to be conducted, it might be worth considering delineating the various types of errors that led to the failure in performance.

Since the tests used in this research were instructor-made, both the validity and reliability of them are in question. So far as the writer was concerned, the test had face validity, but the instrument should perhaps have been analyzed item by item by a team of experts. Should this type of research ever be duplicated, the measuring instrument should be critically examined.

III. RECOMMENDATIONS

Although the experiment reported in this paper answered the proposed questions, there are other and new questions that have arisen as a by-product of this research. In particular, the following questions are raised by the writer and are open to further research:

1. Was the time lapse between the Pre-test, Post-test, KR I, and KR II adequate or disproportionate

2. Should the test scores have also been tabulated with respect to the number of incorrectly solved equations as a result of:

    a. Sign errors

    b. Arithmetic errors

    c. Incorrect application of real number properties

    d, Incorrect sequence of steps to solution

3. Would a duplication of the methodology described in this paper yield significantly different results if:

a. A different experimenter were used in the study

b. Two, rather than one, instructors were used

c. The study were conducted at different grade levels?

In conclusion, the writer hopes that the results of this study will assist mathematics educators in the training of their students. It is also hoped that the report will serve as an igniter of interest for continued research in the area of the effect of precise language usage. Hopefully some of the questions listed here can one day be answered.

BIBLIOGRAPHY

BIBLIOGRAPHY

1. Adelson, Joseph, and Joan Redmond. "Personality Differences in the Capacity for Verbal Recall," Journal of Abnormal and Social Psychology, 57:244-248, 1960.

2. An Analysis of New Mathematics Programs. Washington, D. C.: National Council of Teachers of Mathematics, Inc., 1963. 68 pp.

3. Baird, D. C. Experimentation: An Introduction to Measurement Theory and Experimental Design, New Jersey: Prentice-Hall, 1962. 191 pp.

4. Beberman, Max. An Emerging Program of Secondary School Mathematics. Cambridge, Massachusetts: Harvard University Press, 1958. 44 pp.

5. Brune, Irving H. "Language in Mathematics," The Learning of Mathematics: Its Theory and Practice, Twenty-first Yearbook, National Council of Teachers of Mathematics, New York, Bureau of Publications, Teachers College, Columbia University, 1953. 156-189 pp.

6. Bruner, Jerome S. The Process of Education, Cambridge: Harvard University Press, 1960. 92 pp.

7. Feierabend, Rosalind L. "Review of Research on Psychological Problems in Mathematics Education," Research Problems in Mathematics Education, U. S. Office of Education Cooperative Research Monograph Number 3, OE-12008, 1960. 130 pp.

8. Gage, N. L. (ed.). Handbook of Research on Teaching. Chicago: Rand McNally, 1963. 1218 pp.

9. Gagné, Robert M. "Implications of Some Doctrines of Mathematics Teaching for Research in Human Learning," Research Problems in Mathematics Education, U. S. Office of Education Cooperative Research Monograph Number 3, OE-12008, 1960. 130 pp.

10. Gurau, Peter. "Discovering Precision," Arithmetic Teacher, 13:453-456, October, 1966.

11. Hale, William T. "UICSM's Decade of Experimentation," The Mathematics Teacher, 54:613-619, December, 1961.

12. Hendrix, Gertrude. "Prerequisite to Meaning," The Mathematics Teacher, 43:334-339, November, 1950.

13. ________. "Nonverbal Awareness in the Learning of Mathematics," Research Problems in Mathematics Education, U. S. Office of Education Cooperative Research Monograph Number 3, OE-12008, 1960. 130 pp.

14. ________. "Learning by Discovery," The Mathematics Teacher, 54:290-299, May, 1961.

15. Hodgman, Charles D. Mathematical Tables. Cleveland, Ohio: Chemical Rubber Publishing Company, 1960. 445 pp.

16. Holtan, Boyd. "Some Ongoing Research and Suggested Research Problems in Mathematics Education," Research in Mathematics Education, Washington, D. C.: National Council of Teachers of Mathematics, Inc., 1967. 108-114 pp.

17. Johnson, Donavan A. "A Pattern for Research in the Mathematics Classroom," The Mathematics Teacher, 59:418-425, May, 1966.

18. Johnson, R. E., L. L. Lendsey and W. E. Slesnick. Modern Algebra First Course, Reading, Massachusetts: Addison-Wesley Publishing Company, 1961. 628 pp.

19. Karnano, D. K. and Janet Drew. "Selectivity in Memory of Personally Significant Material," Journal of General Psychology, 65:25-32, July, 1961.

20. Kittell, J. E. "An Experimental Study of the Effect of External Direction during Learning on Transfer and Retention of Principles," Journal of Educational Psychology, 48:391-405, November, 1957.

21. Levinger G., and J. Clark. "Emotional Factors in the Forgetting of Word Associations," Journal of Abnormal and Social Psychology, 62:99-105, 1961.

22. Li, Jerome C. R. Introduction to Statistical Inference. Ann Arbor, Michigan: Edward Bros., 1957. 568 pp.

23. Pederson, F., and D. Marlow. "Capacity and Emotional Differences in Verbal Recall," Journal of Clinical Psychology, 16:219-222, April, 1960.

24. Runquist, W. N., and V. H. Hunt. "Verbal Concept Learning in High School Students with Pictorial and Verbal Representations of Stimuli," Journal of Educational Psychology, 52:108-111, April, 1961.

25. Sobel, Max A. "A Comparison of Teaching Two Methods of Certain Topics in Ninth Grade Algebra," Dissertation Abstracts, 14:1647, 1954.

26. Sommer, R. "Sex Differences in the Retention of Quantitative Material," Journal of Educational Psychology, 49:187-192, August, 1958.

27. Suppes, Patrick. "The Case for Information-Oriented (Basic) Research in Mathematics Education," Research in Mathematics Education, Washington, D. C.: National Council of Teachers of Mathematics, Inc., 1967. 1-5 pp.

28. ________. "Some Theoretical Models for Mathematics Learning," Journal of Research and Development in Education, 1:5-22, Fall, 1967.

29. Underwood, B. J. "Speed of Learning and Amount Retained," Psychological Bulletin, 51:276-282, 1954.

30. Van Krevelen, Alice. "Relationships Between Recall and Meaningfulness of Motive-Related Words," Journal of General Psychology, 65:229-233, 1961.

31. Weaver, Fred. "Research on Mathematics Education, K-8, for 1966," Arithmetic Teacher, 14:509-517, October, 1967.

32. Wittrock, M. C. "Verbal Stimuli in Concept Formation: Learning by Discovery," Journal of Educational Psychology, 54:183-190, August, 1963.

33. Worthen, Blaine R. "A Comparison of Discovery and Expository Sequencing in Elementary Mathematics Instruction," _Research in Mathematics Education_, Washington, D. C.: National Council of Teachers of Mathematics, Inc., 1967. 44-59 pp.

34. Wright, E. Muriel J. "A Rationale for Direct Observation of Verbal Behaviors in the Classroom," _Research Problems in Mathematics Education_, U. S. Office of Education Cooperative Research Monograph Number 3, OE-12008, 1960. 62-71 pp.

APPENDIX

PRE-TEST

ALGEBRA

NAME________________

1) $x - 6 = 13$

2) $r + 1 = 10$

3) $d - 37 = 52$

4) $14x = -56$

5) $.6z - .1z = 7.2$

6) $-13x = -52$

7) $5\ y = 70$

8) $3a + 2a = 25$

9) $\frac{3}{2}x = \frac{48}{32}$

10) $2x + 9 = 65$

11) $1.03y + .97y = 38$

12) $.4x = 8.4$

13) $.03x = .03$

14) $n - .2 = 10.8$

15) $2x - .4 = 9.6$

16) $.08r + .4 = 1.56$

17) $,26g + .3g = 2$

18) $\frac{x}{3} + 2 = 8$

19) $\frac{r}{.2} + 7 = 5$

20) $.1x + .9x + 3x = 76 - 2x$

21) $15 - (a - 6) = 6 - (3a + 5)$

22) $\frac{h}{2} = 7 - \frac{2h}{3}$

23) $\frac{n}{9} - \frac{2}{3} = \frac{15}{3}$

24) $\frac{3 - x}{2} = \frac{-6 - 5x}{7}$

25) $8 - 7x + 3 = 2x - 151$

26) $\frac{3h + 75}{7} = 0$

27) $\frac{18}{2x + 5} - 5 = \frac{3}{2x + 5}$

28) $\frac{5}{x - 8} = 1 - \frac{2}{x - 8}$

29) $\frac{4}{x} + 7 = \frac{2}{3x} + 6$

30) $5(4h - 1) + 10h = 3(5 + 3 + h) - 2$

31) $3(z - 1.4) = .6(3 - z) + .8$

32) $3 - 2x + 3(3 - x) = 4(x - 2) - 4 + 4x$

33) $\frac{1}{3}(5q + 1) + \frac{1}{9}(19q + 7) = \frac{1}{2}(3q - 1) - \frac{1}{6}(7q - 1)$

34) $17r = 2r - 6 - 27 + r - 45 + 2r + 18$

35) $7(x + 3) - 2 - 2(5x + 2) = 11(x - 2) - 5 - 4(2x - 3)$

36) The difference between two-thirds of a number and one-sixth of the same number is 78. What is the number?

37) Find three consecutive integers such that the sum of the first and the third is 5 less than 44 times the second.

38) John has four more nickels than pennies. If he were to spend three of his nickels and one of his pennies, he would have 70 cents left. How many pennies and nickels does he have?

39) The perimeter of a triangle is 192". The length of the longest side is four times that of the shortest side, and the length of the third side is 6 inches less than that of the longest side. Find the length of each side.

40) The sum of three consecutive odd integers is 1503. What are the integers?

POST-TEST

ALGEBRA

NAME ____________________

PERIOD ________________

1) $x - 7 = 27$

2) $r + 7 = -16$

3) $d - 29 = -63$

4) $17x = -68$

5) $.9z - .4z = 7.2$

6) $-19g = -114$

7) $8f = 56$

8) $6z + a = 56$

9) $\frac{5x}{4} = \frac{65}{52}$

10) $5x + 99 = 44$

11) $3.26x + .74x = 36$

12) $.7x = 1.2$

13) $.34y = .34$

14) $m - .5 = 10.4$

15) $5x + .77 = 3.72$

16) $.05x + .44 = 1.59$

17) $.26g + .3g = 2$

18) $\frac{x}{3} + 2 = 8$

19) $\frac{r}{.3} + 6 = 18$

20) $.2x + 3x + .8x = -2x + 76$

21) $30 - 2(a - 6) = 12 - 2(6a + 5)$

22) $\frac{x}{4} = 33 - \frac{2x}{3}$

23) $\frac{n}{6} - \frac{2}{3} = \frac{15}{3}$

24) $\frac{3 - x}{2} = \frac{-6 - 5x}{7}$

25) $8 - 7x + 3 = 2x - 151$

26) $\frac{7x + 930}{6} = 0$

27) $\frac{18}{2x + 5} = 5 + \frac{3}{2x + 5}$

28) $\frac{5}{x - 5} = 1 - \frac{2}{x - 5}$

29) $\frac{4}{x} + 7 = \frac{2}{3x} + 6$

30) $5(4h - 1) + 10h = 3(5 + h + 3) - 2$

31) $5(z - 1.4) = .8(3 - z) + .8$

32) $6 - 4z + 6(3 - z) = 4(2z + 4) - 8 + 8z$

33) $\frac{1}{3}(5q + 1) + \frac{1}{9}(19q + 7) = \frac{1}{2}(3q - 1) - \frac{1}{6}(7q - 1)$

34) $17r + 2r - 6 - 27 + r = 45 + 2r + 18$

35) $7(x + 3) - 2 - 2(5x + 2) = 11(x - 2) - 5 - 4(2x - 3)$

36) The difference between two-thirds of a number and one-sixth of the same number is 57. What is the number?

37) Find three consecutive odd positive such that their sum is 381.

38) Find three consecutive integers such that the sum of the first and three times the second is 44 less than five times the third.

39) John has four more nickels than pennies. If he were to spend three of his nickels and one of his pennies, he would have 70 cents left. How many pennies and nickels does he have?

40) The perimeter of a triangle is 192". The length of the longest side is four times that of the shortest side, and the length of the third side is 6 inches less than that of the longest side. Find the length of each side.

KR I

ALGEBRA

NAME____________________

PERIOD____________________

1) $x - 5 = 14$

2) $r + 7 = -16$

3) $d - 37 = -52$

4) $16x = -80$

5) $.7z - .3z = 7.2$

6) $-17z = -119$

7) $8f = 56$

8) $2a + 5a = 56$

9) $\frac{5a}{9} = \frac{45}{81}$

10) $2x + 9 = 65$

11) $3.26x + .74x = 36$

12) $.05x = .05$

13) $.4x = 9.2$

14) $m - 7 = 10.4$

15) $2x + .77 = 3.75$

16) $.08r + .4 = 1.56$

17) $.26g + .3g = 2$

18) $\frac{x}{3} + 2 = 8$

19) $\frac{r}{.5} + 7 = -3$

20) $.1x + .9x + 3x = 76 - 2x$

21) $8 - (2a - 4) = 6 - (2a + 7)$

22) $\frac{h}{2} = 7 - \frac{2h}{3}$

23) $\frac{n}{9} - \frac{2}{3} = \frac{-15}{3}$

24) $\frac{5 - x}{2} = \frac{-4 - 5x}{7}$

25) $4 - 7x - 7 + 2x = 151$

26) $\frac{5h - 85}{19} = 0$

27) $\frac{7}{x - 7} = 1 - \frac{2}{x - 7}$

28) $\frac{4}{x} + 7 = \frac{2}{3x} + 6$

29) $5(4h - 1) + 10h = 3(5 + h + 3) - 2$

30) $10(z - 1.4) = 1.6(3 - z) + 1.6$

31) $3 - 2x + 3(3 - x) = 4(x - 2) - 4 + 4x$

32) $\frac{1}{3}(5q + 1) + \frac{1}{9}(19q + 7) = \frac{1}{2}(3q - 1) - \frac{1}{6}(7q - 1)$

33) $17r = 2r - 6 - 27 + r + 45 + 2r + 18$

34) $14(x + 3) - 4 - 4(5x + 2) = 22(x - 2) - 10 + -8(2x - 3)$

35) The difference between five-sevenths of a number and one-seventh of the same number is 32. Find the number.

36) Find three consecutive integers such that the sum of the first and the third is 500 less than 43 times the second.

37) Bill has four more nickels than pennies. If he were to spend three of his nickels and one of his pennies, he would have 70¢ left. How many pennies and nickels does he have?

38) The perimeter of a triangle is 192". The length of the longest side is four times that of the shortest side, and the length of the third side is 6" less than that of the longest side. Find the length of each side.

39) The sum of three consecutive odd integers is 381. What are the integers?

40) $\frac{18}{2x + 5} - 5 = \frac{3}{2x + 5}$

KR II

ALGEBRA

NAME________________

PERIOD______________

1) $x - 7 = 27$

2) $r + 7 = -16$

3) $d - 29 = -63$

4) $17x = -68$

5) $.9z - .4z = 7.2$

6) $-13x = -52$

7) $5y = 70$

8) $3a + 2a = 25$

9) $\frac{3}{2}x = \frac{48}{32}$

10) $2x + 9 = 65$

11) $3.26 + .74x = 36$

12) $.7x = 1.2$

13) $.34y = .34$

14) $m - .5 = 10.4$

15) $5x + .77 = 3.72$

16) $.08r + .4 = 1.56$

17) $.26g + .3g = 2$

18) $\frac{x}{3} + 2 = 8$

19) $\frac{r}{.2} + 7 = 5$

20) $.1x + .9x + 3x = 76 - 2x$

21) $30 - 2(a - 6) = 12 - 2(6a + 5)$

22) $\frac{x}{4} = 33 - \frac{2x}{3}$

23) $\frac{n}{6} - \frac{2}{3} = - \frac{15}{3}$

24) $\frac{3 - x}{2} = \frac{-6 - 5x}{7}$

25) $8 - 7x - 3 = 2x - 151$

26) $\frac{3h + 75}{7} = 0$

27) $\frac{18}{2x + 5} - 5 = \frac{3}{2x + 5}$

28) $\frac{5}{x - 5} = 1 - \frac{2}{x - 5}$

29) $\frac{4}{x} = -7 + \frac{2}{3} x + 6$

30) $5(4h - 1) + 10h = 3(5 + h + 3) -2$

31) $3(z - 1.4) = .6(3 -z) + .8$

32) $6 - 4z + 6(3 - z) = 4(2z + 4) - 8 + 8z$

33) $\frac{1}{3}(5q + 1) + \frac{1}{9}(19q + 7) = \frac{1}{2}(3q - 1) - \frac{1}{6}(7q - 1)$

34) $17r = 2r - 6 - 27 + r + 45 + 2r + 18$

35) $7(x + 3) - 2 - 2(5x + 2) = 11(x - 2) - 5 - 4(2x - 3)$

36) The difference between two-thirds a number and one-sixth of the same number is 57. What is the number?

37) Find three consecutive odd positive integers such that their sum is 381.

38) Find three consecutive integers such that the sum of the first and three times the second is 44 less than five times the third.

39) John has four more nickels than pennies. If he were to spend three of his nickels and one of his pennies, he would have 70¢ left. How many pennies and nickels does he have?

40) The perimeter of a triangle is 192". The length of the longest side is four times that of the shortest side, and the length of the third side is 6" less than that of the longest side. Find the length of each side of the triangle.

ANSWER SHEET

ALGEBRA

NAME ____________________

PERIOD ____________________

1) ____________________
2) ____________________
3) ____________________
4) ____________________
5) ____________________
6) ____________________
7) ____________________
8) ____________________
9) ____________________
10) ____________________
11) ____________________
12) ____________________
13) ____________________
14) ____________________
15) ____________________
16) ____________________
17) ____________________
18) ____________________
19) ____________________
20) ____________________
21) ____________________
22) ____________________
23) ____________________
24) ____________________
25) ____________________
26) ____________________
27) ____________________
28) ____________________
29) ____________________
30) ____________________
31) ____________________
32) ____________________
33) ____________________
34) ____________________
35) ____________________
36) ____________________
37) ____________________
38) ____________________
39) ____________________
40) ____________________

RAW SCORES OF CONTROL CLASS

Subject Number	Pre	Post	KR I	KR II
M1	13	18	15	17
M2	10	20	16	16
F1	15	20	13	17
M3	2	8	-	-
F2	12	21	18	21
F3	9	13	13	12
F4	-	16	12	15
F5	14	16	14	17
M4	10	23	17	14
M5	11	20	15	20
M6	12	-	11	17
F6	8	16	11	9
F7	10	17	10	14
M7	12	15	10	14
F8	11	12	9	8
F9	9	14	7	10
F10	4	19	11	13
M8	9	14	13	13
M9	9	9	10	14
M10	3	3	5	-
M11	13	26	23	26
F11	7	22	19	18
M12	7	16	13	16
F12	6	-	9	11
M13	9	15	16	12
F13	14	19	15	13
F14	8	25	14	18
F15	12	16	17	12
M14	14	28	18	19
F16	11	22	17	22
F17	12	21	16	20
M15	-	19	12	15

RAW SCORES OF EXPERIMENTAL CLASS

Subject Number	Pre	Post	KR I	KR II
M1	9	19	7	11
F1	7	12	9	11
M2	15	15	18	19
F2	9	11	13	20
M3	10	22	23	26
F3	4	19	17	15
M4	8	15	15	12
F4	9	19	15	17
M5	11	15	15	14
F5	12	19	18	21
M6	7	11	9	13
M7	12	-	13	15
F6	7	17	13	16
F7	-	17	15	18
F8	11	16	12	14
F9	11	18	16	20
F10	9	19	9	13
F11	12	21	15	21
M8	9	17	12	14
F12	6	18	14	16
F13	14	25	20	20
M9	11	15	15	20
F14	11	19	9	20
F15	11	8	12	11
F16	2	14	9	11
F17	10	16	12	20
F18	5	12	15	12
F19	12	23	26	21
M10	12	-	21	18
M11	11	28	22	20
M12	12	17	19	23
F20	10	20	19	17
F21	10	13	16	-