A STUDY OF MIDDLE SCHOOL STUDENTS' UNDERSTANDING OF NUMBER SENSE RELATED TO PERCENT

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CHAPTER I

THE RESEARCH PROBLEM

Introduction

In today's society, mathematical literacy is as essential as verbal literacy (National Research Council, 1989). Daily newspapers and television news programs frequently contain information presented using mathematical terms and concepts which citizens need to understand and apply in making reasoned decisions. One of those mathematical concepts in frequent use is number sense related to percent. Information about, and communicated to, the general public is often in the form of comparison data expressed as a percent. Examples include the rise in consumer prices, the decline in interest rates, results of opinion polls, unemployment rates, stock gains, and sale prices. Persons often draw conclusions based on their perception of information. If those perceptions are erroneous, the conclusions drawn are likely to be inaccurate.

Number sense is a term which encompasses several skills related to common sense about numbers. Those skills include: (1) having well-understood number meanings; (2) having developed multiple relationships among numbers; (3) recognizing the relative magnitudes of numbers; and (4) knowing the relative effect of operating on numbers (National Council of Teachers of

Mathematics, 1989). Understanding a number as a quantity of a specific magnitude and being able to judge how it compares to another number is basic to number sense (Sowder, 1988). In addition to having number sense about whole numbers, fractions, and decimals, students should develop number sense about percent. This includes understanding of the meaning of numbers expressed as percents, developing equivalent expressions for percents, comparing quantities expressed as percents, and recognizing the relative effect of finding a percent of a number.

Conceptual and procedural knowledge are two types of mathematical knowledge mentioned by Hiebert and Lefevre (1986). Researchers have indicated that current school programs place primary emphasis on learning procedures (Hiebert and Lefevre, 1986; Hoffer, 1988; Payne, 1984; Dewar, 1984). Thus, it is not surprising that test scores provide evidence that students are learning computations without having a basic conceptual understanding of fractions, decimals, and percents (Behr and Post, 1988; Kouba, Carpenter, and Swafford, 1989; Allinger and Payne, 1986).

The study of percent in the school curriculum is concentrated in the middle grades. Students are taught to find equivalent expressions among fractions, decimals, and percents. The emphasis of this study in the curriculum is the three types of percent problems and some applications of percent. Allinger and Payne (1986) suggest that the proportion method and the factor-factor-product method are the two common teaching approaches to percent, and today's textbooks often present both methods (Orfan et al., 1987;

Quast et al., 1987). Allinger and Payne (1986) claim that the way percent is taught encourages students to rely exclusively on rules and procedures to solve percent problems. This would imply that little attention is being given to helping students develop a number sense about percent.

Percent is a difficult topic in the middle grades' mathematics curriculum (Wiebe, 1986; McGivney and Nitschke, 1988; Hart, 1981b; Allinger and Payne, 1986). This perception and the claim that the current school curriculum emphasizes rules and procedures rather than concepts (Allinger and Payne, 1986) led to this investigation of what middle school students understand about the concept of percent, focusing on number sense skills rather than computational and application skills.

Statement of the Problem

The problem under investigation is a study of middle school students' number sense related to percent. To provide some insight into students' understanding of percent, three research questions were proposed:

1. Can students interpret a quantity expressed as a percent given a pictorial discrete set or continuous region with part or all of the area shaded?
2. Do students understand the meaning of a quantity expressed as a percent of a number?
3. What strategies do students use to make comparisons about percent quantities in both pictorial and abstract settings?

These questions are best answered using more than one method of data collection. Until recently, the statistical model dominated the research methodology in mathematics education (Kilpatrick and Greeno, 1989). Since the late 1960s, qualitative methods have gained popularity, even among mathematics educators. Today, studies which produce descriptions of student knowledge are considered a first step toward a better understanding of teaching and learning which will guide the development of effective instructional programs (Kilpatrick and Greeno, 1989; Hiebert and Lefevre, 1986).

For this study, a quantitative assessment instrument was developed by the researcher and used multiple-choice questions in both pictorial and abstract settings to determine whether students could make comparisons about percent quantities. The qualitative assessments included one open-ended test item which asked the students to explain the solution strategy used to determine if "87% of 10 was greater than, less than, or equal to 10," and the research interviews with selected students.

The population for this study was composed of the students in grades seven and eight enrolled in average mathematics classes in Putnam City Schools in Oklahoma during the 1990-91 school year. The sample included seventh-grade students (n=106) and eighth-grade students (n=93).

Importance of the Study

One of the twelve components of essential mathematics identified by the National Council of Supervisors of Mathematics (1988) was appropriate computational skills. Included as a skill was "the ability to recognize, use, and estimate with percents" (p. 2). In the Curriculum and Evaluation Standards for School Mathematics, the National Council of Teachers of Mathematics (1989) suggested increased attention in the middle grades on developing number sense and an understanding of percent. However, studies have documented that students do not perform well on questions dealing with percent (Kouba, Carpenter, and Swafford, 1989; Allinger and Payne, 1986; Comstock and Demana, 1987; Hart, 1981). In particular, students did not perform well on questions which focused on the concept of percent during the National Assessment of Educational Progress conducted in 1986. On a comparison question, such as "76% of 20 is greater than, less than, or equal to 20," only 37% of seventh graders and 69% of eleventh graders responded correctly (Kouba, Carpenter, and Swafford, 1989).

Researchers have produced some insight into how students learn rational number concepts (Post, Behr, and Lesh, 1986; Behr, Post, and Wachsmuth, 1986; Peck and Jencks, 1981). Written tests and interviews have been used to determine the conceptual understanding students have about fractions, including the cognitive strategies used in responding to fraction tasks (Post, Behr, and Lesh, 1986; Behr, Lesh, Post, and Silver, 1983; Behr, Post, and Wachsmuth, 1986; Peck and Jencks, 1981). Hiebert and Wearne (1986) have combined performance and interview data to determine the conceptual and procedural knowledges students have about decimal numbers. The combination of both quantitative and qualitative data has provided a rich description of the status of student learning in these areas.

Early work by Kircher (1926), Edwards (1930), and Guiler (1946a, 1946b) focused on students' understanding of percent by studying student errors on percent computation problems. A number of researchers have investigated approaches to teaching the three cases of percent problems (McCarty, 1967; Wynn, 1966; Bidwell, 1969; McMahon, 1960; May, 1965; Kenney and Stockton, 1958; Tredway and Hollister, 1963; Maxim, 1982). Additional work has been done proposing techniques or procedures to help students work the three types of percent problems (Dollins, 1981; Osiecki, 1988; McGivney and Nitschke, 1988; Wiebe, 1986; Allinger, 1985; Dewar, 1984). Little research has focused on any aspects of number sense related to percent.

The Pilot Study

Before the main study was conducted, a pilot study was used to investigate the feasibility of exploring the research problem and to refine the research design to be used. Students participating in the pilot study were enrolled in average mathematics classes in Putnam City Schools during May, 1990, and included sixth graders (n=27), seventh graders (n=26) and eighth graders (n=27). The performance of the students varied on the three sections of the test. They did best on those questions which utilized a continuous rectangular region as a whole, and poorest on those questions presented in a non-pictorial, abstract format.

Various explanations were given by the students to support a conclusion selected in comparing "87% of 10 as greater than, less than, or equal to 10." Of the students who correctly responded that "87% of 10 was less than 10," 36% provided an appropriate explanation, while another 12.5% gave an incorrect explanation. Forty percent of the students in the pilot study incorrectly responded to the question comparing 87% of 10. Of these unsuccessful students, approximately 16% gave an explanation that showed a lack of understanding of the problem. Among all of the students in the pilot study, more than 30% had no satisfactory explanation to support their conclusions.

The focus of the individual student interviews was the thinking strategies the students used to answer multiple-choice questions from the written test. Students employed a variety of strategies, including computation, estimation, pictorial and linear models, and comparison of numerical quantities. The use of strategies often varied depending on the quantities in the question. Several students demonstrated a good understanding of 100%, but did not use it as a reference in answering other questions.

Responses received during the pilot study from both the written test and the research interviews indicated varied levels of student understanding and the use of a variety of strategies when answering questions about percent quantities. These results and the lack of previous research combining quantitative and qualitative methods to explore students' understanding of percent supported the need for this study.

Definitions

For the purposes of this study, each of the following terms is used with the meaning described.

Percent is a ratio in which the second number is 100 (Hoffer, 1988).

Fraction is a non-negative rational number, having the form *a/b*, where *a* is a whole number and *b* is a natural number (Underhill, 1972).

Decimal is a numeral in the decimal system of numeration which is based on powers of ten.

Ratio is an ordered pair of measurements used to compare one quantity to another (Hoffer, 1988).

Proportion is a statement of equality between two ratios (Hoffer, 1988).

Cases of percent problems involve the relationship, percentage equals base times rate, where the rate is expressed as a percent (Underhill, 1972).

Case one percent problems are characterized by finding the percentage as the product of the base and the rate (Schminke, Maertens, and Arnold, 1973).

Case two percent problems are characterized by finding the rate using the percentage and the base (Schminke, Maertens, and Arnold, 1973).

Case three percent problems involve finding the base given the percentage and the rate (Schminke, Maertens, and Arnold, 1973).

The ratio or proportion method for solving percent problems establishes a proportion using the percent expressed as a ratio and constructing another ratio of the percentage to the base (Allinger and Payne, 1986).

The factor-factor-product method for solving percent problems uses the relationship, percentage equals base times rate, writing the percent as a decimal or a fraction, if given, and creating an equation in one unknown (Allinger and Payne, 1986).

The second National Assessment refers to the National Assessment of Educational Progress national testing in mathematics conducted in 1978.

The fourth National Assessment refers to the National Assessment of Educational Progress national testing in mathematics conducted in 1986.

Assumptions

It was assumed that the students in the sample would respond to the testing instrument with integrity and that the students would respond truthfully during the interview. It was assumed that the information on grade level, age, sex, ethnic origin, mathematics grade and self-assessment rating of mathematics ability were reliable, as they were self-reported by the students.

Limitations

This study is limited in scope as the sample was drawn from Putnam City Schools' students. The results of the study will generalize only to students in school districts of similar characteristics. Additionally, intact classes were used in selecting the students to participate in the study.

Another limitation of the study results from the use of a maximum variation sampling strategy, rather than random sampling, to determine the students to be selected for individual interviews. However, the purpose of the study was to provide insight into students' understanding of the concept of percent, not to produce findings that would generalize to all middle school students. The use of this sampling strategy can generate data among a diverse group of students and produce important patterns which emerge out of this heterogeneity (Patton, 1987).

The students participating in the pilot study were directed to indicate an order for the percents used in the questions. The percents were ranked from most familiar to least familiar. In each section of the test, the order of the questions using the percents was the same as that established through the pilot study. This ordering of the test items is also a limitation in the study.

One form of qualitative assessment used in the investigation was an open-ended question on the written test. Patton (1987) noted limitations to such test items which included the writing skills of the persons completing the instrument. The lack of skill or experience in answering similar open-ended questions by some of the students participating in this study may have affected the responses to the test item.

One possible limitation in all qualitative research is potential bias and subjectivity of the researcher. Even though the researcher strived for objectivity, previous experiences and anticipated outcomes may be reflected in the interpretation of student responses.

Overview

This study is divided into five chapters, the first presenting the statement of the problem under consideration. In Chapter II, a review of the literature includes work pertaining to number sense in the areas of fractions, decimals, ratio and proportion, and percent, estimation and mental computation, Piaget's constructivist theory, aspects of error analysis, as well as work on the theory of representations and translations and the contrast between conceptual and procedural knowledge. Chapter III presents the discussion of the research design including results of the pilot study, the measuring techniques used in the study, and the process of collecting and analyzing the data. The results of the analysis of both the quantitative and qualitative data gathered during the study is reported in Chapter IV. Chapter V presents a summary, conclusions, and recommendations for future study.

CHAPTER II

REVIEW OF THE LITERATURE

Introduction

Previous research on percent has centered on students' ability to solve percent computation problems and investigated the best approaches to teaching the three cases of percent. Little research has included aspects of number sense related to percent. Studies have been conducted to investigate how students learn rational number concepts including the cognitive strategies they use in working fraction and decimal tasks. These recent studies have provided insight into the thinking strategies students are using in responding to particular mathematical situations. Because percent is often viewed as a difficult topic, information on students' understanding of percent and the strategies they use to approach percent questions may provide a step toward better teaching and learning.

This study focuses on number sense with percent. A description of students' number sense related to percent should include aspects of the conceptual knowledge students have about percent and the strategies they use in thinking about percent questions. Research questions for this study include:

1. Can students interpret a quantity expressed as a percent

given a pictorial discrete set or continuous region with part or all of the area shaded?

2. Do students understand the meaning of a quantity expressed as a percent of a number?
3. What strategies do students use to make comparisons about percent quantities in both pictorial and abstract settings?

Students are usually introduced to percent in the middle grades after studying fractions and decimals and practice writing equivalent expressions in all three forms. They may view a 10 x 10 grid as a model for representing percents while the teacher defines percent as *per-hundred* or *out of one hundred.* The rest of the study of percent in the middle grades deals with the three types of percent problems and applications of percent, such as discount, taxes, and interest. One of two methods, proportion or factor-factor-product is usually presented as the means to solve problems (Allinger and Payne, 1986).

Understanding percent is related to the understanding of fractions, decimals, ratio and proportion. Number sense is associated with skills in estimation and mental computation (Reys and Reys, 1990). In addition, Ross (1989) claimed that to be successful teaching number sense, instruction must be founded in the theory that students construct their own knowledge. Student errors are one means of understanding the knowledge students have constructed, providing insight into their level of understanding and how they are thinking about mathematical problems (Wadsworth, 1989). Additional insight into students' mathematical thinking results from work on the theory of mathematical representations and translations and the contrasts between conceptual and procedural knowledge. The literature related to percent and number sense, as described, will be reviewed.

Fractions

Students in early grades become comfortable with the meaning of whole numbers, comparisons and operations with them (Hart, 1981a). When students encounter fractions, they find a very different set of numbers. There is no "next" number as there is with whole numbers (Skypek, 1984; Behr and Post, 1988). The effect of operating with fractions is not consistent with whole numbers because, as one example, multiplying sometimes produces a smaller product than either factor (Hart, 1981a). For any one fraction, there are several interpretations or meanings. A fraction can represent a part-whole relationship in either a continuous region or discrete set, a decimal, an indicated division, an operator, and a ratio (Post, Behr, and Lesh, 1982). Students usually do not see a relationship between the different interpretations. Research has shown that students need experience with all of the ways in which fractions can be interpreted (Driscoll, 1984). Various assessments and research projects have confirmed that learning fraction concepts is a complex task that students find difficult (Post, Behr, and Lesh, 1986; Driscoll, 1984; Peck and Jencks, 1981).

In the fourth National Assessment, more than half of the third graders could identify a shaded region that represented a common fraction (Kouba et al., 1988). Most seventh graders could compare two fractions, but had difficulty when the number of fractions to be ordered increased to four (Kouba et al., 1988). Results from the second National Assessment indicated that most 13-year-olds could identify fraction terms, such as denominator, improper fraction, and mixed numeral, and could reduce fractions, write equivalent fractions, and relate improper fractions and mixed numerals (Post, 1981). Even though students in the fourth National Assessment also did well on routine procedures, such as changing a mixed numeral to an improper fraction, they did not exhibit an understanding of basic concepts of equivalent representations, such as, 5 1/4 can be expressed as a sum, 5 + 1/4.

When comparing or ordering fractions, it is essential to have an understanding of a fraction as a single quantity. Behr, Post, and Wachsmuth (1986) and Sowder (1988) suggested that many children do not have this understanding of a fraction. Instead they viewed a fraction as two separate numbers without also considering the relationship between the numerator and the denominator (Behr, Post, and Wachsmuth, 1986; Hart, 1981a).

When comparing fractions, students are initially heavily influenced by the techniques of comparing whole numbers (Post, Behr, and Lesh, 1986). Post, Behr, and Lesh (1986) contrasted the use of two strategies to order whole numbers, with the need for three strategies to compare fractions. The two strategies to compare whole numbers involve counting or matching elements of finite sets. The three strategies to compare fractions result from the three situations where the fractions have the same numerators, the same denominators, or neither in common (Post and Cramer, 1987). Students need to develop strong internal images of fractions through numerous experiences with concrete representations to be successful in comparing fractions (Post and Cramer, 1987; Driscoll, 1984). These mental images are the foundation for the development of quantitative understanding of fractions (Bezuk and Cramer, 1989).

Post and Cramer (1987) also noted that some students are able to create their own strategies for comparing fractions. Students can develop the use of reference points in ordering fractions (Behr and Post, 1988). Common reference points are one-half and one. By comparing each fraction to the reference point, the order of the fractions is determined. It also appears that children's strategies are locally defined, created for a particular problem (Post, Behr, and Lesh, 1986). Leutzinger and Bertheau (1989) suggested that students should learn how numbers are related to one another and advocate the use of benchmarks or reference points by students to judge the relative size of numbers.

Behr and Post (1988) claimed that the equivalence of fractions is fundamental to other fraction tasks, including comparing fractions and learning addition and subtraction. However, Payne (1984) suggested that equivalent fractions is a major weakness for students. Just as students develop their own strategies for comparing fractions, they develop strategies for determining fraction equivalence. One type of incorrect additive strategy would conclude that 2/5 is equal to 4/7 because 2 + 2 = 4 and 5 + 2 = 7. Students may use additive strategies when the concept of multiplication is not fully developed (Post, Behr, and Lesh, 1986).

Decimals

A decimal is one of the interpretations of a fraction. Relationships between fractions and decimals are built on the fact that they are two different symbols representing the same numerical concept (Hiebert and Wearne, 1986). However, many students fail to see the fundamental relationship and instead see the two systems as different sets of symbols and procedures.

Results from the second National Assessment on items concerned with equivalent decimal and fraction expressions, indicated that students were unable to use either an algorithm for changing any fraction to a decimal, or reasoning involving equivalent fractions with a denominator as a power of ten (Carpenter et al., 1981). However, in the fourth National Assessment about 60% of seventh-grade students could write simple fractions as decimals (Kouba et al., 1988). Behr and Post (1988) emphasized the importance of students having a variety of experiences with order and equivalence within and between the two symbol systems. Other results from the fourth National Assessment provided some evidence that students are learning decimal computation procedures before learning basic decimal concepts (Kouba, Carpenter, and Swafford, 1989). About 60% of seventh graders could add and multiply decimals, while performance on subtraction and division was lower. Less than one-half of the seventh-grade students were successful in expressing an improper fraction as a decimal, writing a decimal as a common fraction, and ordering decimals.

Several researchers have concluded that students lack an understanding of basic decimal concepts (Hiebert and Wearne, 1986; Kouba, Carpenter, and Swafford, 1989; Payne, 1984; Carpenter et al., 1981). Understanding decimals comes from both an understanding of the extension of the place value system to include tenths, hundredths, and so forth, and the fraction concept of part-whole, where the parts are equal to a multiple of ten (Hiebert and Wearne, 1986; Behr and Post, 1988). Students appeared to lack a firm understanding of the place value in decimals because they often viewed a decimal number as two different numbers, one on either side of the decimal point (Brown, 1981; Carpenter et al., 1981).

In understanding decimals, students must recognize the features of whole numbers that are appropriate for decimal numbers, as well as those characteristics that are not generalizable to decimals (Hiebert and Wearne, 1986). One area where children inappropriately generalize is in developing strategies to compare decimals. Students use the whole number rule that the number with more digits is the bigger number (Lichtenberg and Lichtenberg, 1982; Resnick et al., 1989). In comparing decimals, students often ignore the decimal point and apply the whole number rule (Hiebert and Wearne, 1986; Carpenter et al., 1981). A different inappropriate generalization used by older students, may occur when students indicate that the number with the most digits to the right of the decimal point is the smallest (Hiebert and Wearne, 1986; Grossman, 1983; Resnick et al., 1989). This strategy may be a result of students learning that the digits farther to the right of the decimal point represent smaller magnitudes (Hiebert and Wearne, 1986; Resnick et al., 1989). An application of this strategy can be a result of a student's effort to interpret decimals as fractions (Resnick et al., 1989).

The use of the incorrect rules for comparing decimals may be influenced by the curriculum sequence students experience (Resnick et al., 1989). In a study conducted by Resnick, Nesher, Leonard, Magone, Omanson, and Peled (1989), students who had not been taught fractions in school made mistakes which reflected the use of the whole number strategy. Students who had been taught fractions before decimals made mistakes which more frequently showed an attempt to interpret decimals with respect to fraction concepts.

Ratio and Proportion

A ratio is one of the interpretations of a fraction. Even though a ratio may look like a fraction, it differs from a fraction in significant ways. Some of those differences cited by Hoffer (1988) included: (1) components of ratios are not always rational numbers; (2) ratios can be represented by symbols other than fractions, such as with the use of the colon; (3) ratios can compare objects with different units; (4) ratios can have zero as the second component; and (5) ratios can combine in a way that is a common error used by students when adding fractions, i.e., 2:5+3:7=5:12. Students need to recognize the features of fractions which will generalize to ratios and those that will not.

Researchers have viewed proportional reasoning as a complex, but critical concept in the development of students' learning of mathematics (Hoffer, 1988; Lesh, Post, and Behr, 1988; Karplus, Pulos, and Stage, 1983). Among the several interpretations of fractions, it takes longer for students to develop an understanding of ratios (Driscoll, 1984). The study of proportional reasoning is concentrated in the middle grades and is related to some of the concepts which are difficult for students to acquire, such as equivalent fractions, place value, and percents (Lesh, Post, and Behr, 1988; Hoffer, 1988).

Students move through stages in the evolution of their proportional reasoning capabilities (Lesh, Post, and Behr, 1988). Evidence of these stages comes from varied experiments using different types of proportional reasoning tasks where students were asked to respond and explain their answers (Hart, 1981b; Lesh, Post, and Behr, 1988; Karplus, Pulos, and Stage, 1983). The first stage is one of incomplete or illogical responses. Students tend to ignore parts of the data and may focus on only certain numbers, such as the numerators, in the proportion (Lesh, Post, and Behr, 1988). The next stage involves a qualitative comparison, where students use terms such as *more* or *less.* Relationships among the four factors in a proportion may be noticed, but only in a qualitative, not quantitative, manner (Lesh, Post, and Behr, 1988).

The third stage is characterized by the use of an additive strategy where the difference between numbers is the focus rather than any multiplicative relationship. In Hart's (1981b) study, she found that students made a consistent effort to solve problems by using addition in some form and that there was little evidence that the method taught, $a/b=c/d$, is used. Several researchers have determined that students vary the method they use to solve proportion problems depending on characteristics of the problem (Hart, 1981b; Lesh, Post, and Behr, 1988; Karplus, Pulos, and Stage, 1983).

The last stage is the use of proportional reasoning. In solving problems with proportions, students found it easiest to work with doubling or halving ratios, and some children saw all questions of proportional enlargement as solved by doubling and all reductions were accomplished by halving (Hart, 1981b). Lesh, Post, and Behr (1988) split the last stage into two stages creating a fourth stage where multiplicative reasoning is based on pattern recognition and a fifth stage of proportional reasoning. In this fourth stage before proportional reasoning, students frequently used a "build-up " strategy (Hart, 1981b) making use of iterative doubling or halving.

One important component in understanding proportion is the recognition that it is a statement of equality between two items that are structurally similar, i.e. ratios (Lesh, Post, and Behr, 1988). The study of proportions can be an introduction to mathematics as a study of structure, invariant properties, equivalence and nonequivalence under a variety of transformations (Lesh, Post, and Behr, 1988). In this way, proportional reasoning is a foundation for the study of algebra and other higher mathematics (Hoffer, 1988; Lesh, Post, and Behr, 1988).

Estimation

One aspect of number sense is the ability to produce reasonable estimates (Hope, 1989). Estimation is defined as the "process of producing an answer that is sufficiently close to allow decisions to be made" (Reys, B. J., 1986a, p. 22). The study of estimation can be a means to help students develop an understanding of concepts and procedures, a flexibility in working with numbers, and an awareness of the reasonableness of results (National Council of Teachers of Mathematics, 1989).

For many teachers and students, estimation is synonymous with rounding (Reys, B. J., 1986b). However, there are other strategies useful in estimating that should be presented to students (Reys, R. E., 1985; Reys and Reys, 1990). Front-end estimation focuses on the left-most digit of a number to provide an initial estimate followed by a mental adjustment to determine a better estimate (Reys, B. J., 1986b). Averaging or clustering can be used when numbers cluster about a particular value (Reys, B. J., 1986b; Reys, R. E., 1985). The compatible numbers strategy refers to using a set of numbers that when estimated can easily be manipulated mentally (Reys, B. J., 1986b). The choice and use of these strategies develops a flexibility in thinking about and using numbers that fit a particular situation (Reys, B. J., 1986a).

Problems aimed at testing estimation skills were included on the fourth National Assessment. However, the items did not clearly assess estimation skills, because it appeared that student errors were sometimes a result of misinterpreting the problem (Kouba, Carpenter, and Swafford, 1989). For other questions, one correct solution strategy could involve doing the actual computation and selecting the choice closest to the computed result. Based on the results, there was some evidence that students may not judge the reasonableness of their answers and may have difficulty with the relative size of numbers larger than 100. Students generally did poorly estimating percents, square roots, and the sum and product of mixed numbers. In her studies, Sowder (1988) found that errors on estimation problems could be attributed to a lack of understanding of number size which led students to make poor approximations.

On the second National Assessment, a question asked students to estimate a 15% tip for a bill of $28.75 (Reys, R. E. et al., 1982). Only 23% of 17-year-olds gave an acceptable estimate. In an interview situation, other students in grades 7-12 as well as adults, were asked to explain the strategy used to answer the question. Their strategies included the use of a fraction or decimal equivalent to the percent with a compatible estimate for the bill, or an approximation for 15% and $28.75. Almost half of the participants used a distributive strategy, taking 10% of the dollar amount and adding one-half of the amount.

Levine (1982) investigated the strategies college students used to mentally estimate products and quotients of whole numbers and decimal numbers. Her strategy classification scheme included strategies which involved fractional relationships, powers of ten, exponents, and rounding. One of the strategies most frequently used was rounding both numbers in the problem. The other frequently used strategy was proceeding algorithmically, where a form of a standard algorithm was used to calculate, estimate, and then combine partial products or quotients. Students of lower quantitative ability used an algorithmic procedure for estimation more frequently than students of higher quantitative ability, who were more likely to use a variety of different estimation strategies. Levine (1982) noted that an algorithmic procedure did not require a student to apply any number sense during its use.

The compatible numbers strategy is especially useful in working percent problems. As an example, to estimate the savings of a bicycle originally priced as $152.98 that is now marked 30% off, one set of compatible numbers would involve estimating 30% to be one-third and $152.98 to be $150 (Rubenstein, 1987).

A strategy related to compatible numbers is estimation with "nice" numbers. Numbers that are easy to work with are considered "nice" (Trafton et al., 1986). "Nice fractions" include 1/2, 1/3, 1/4, 1/5, 1/6, 2/3, and 3/4, which are more easily understood (Trafton et al., 1986). An analogous strategy proposed by Allinger and Payne (1986) is the "EZ%" (p. 151). The "easy percents (EZ%)" are 1%, 10%, 50%, and 100%. As an example, to estimate 13% of 800, the EZ% closest to 13% would be used, so that an estimate would be 10% of 800 (Allinger and Payne, 1986).

Estimation involves the concept of comparing quantities. Children need to develop an understanding of terms such as *about, near, close,* and *between* (Hope, 1989). This understanding needs to extend to numbers expressed as decimals, fractions and percents. Estimating with these types of numbers requires not only an understanding of relative size, but also an ability to convert from one form of number to another (Hope, 1989).

Success in estimating fractions is related to understanding the size of fractions and an ability to compare and order fractions (Behr, Post, and Wachsmuth, 1986). The three abilities reinforce each other. Two strategies which students employed in estimating a sum of two fractions were mental computation using a common denominator and the use of a self-identified reference point to which other fractions were compared (Behr, Post, and Wachsmuth, 1986).

An understanding of place value is essential to being able to estimate decimals (Kindig, 1986; Vance, 1986). Students should recognize that the leading nonzero digit is the important one in determining the relative size of a decimal number. The leading-digit estimate is one strategy to use in estimating decimals (Vance, 1986). Other strategies include the use of common fraction equivalents and compatible numbers. Kindig (1986) suggested that students think of money estimates when computing with decimals.

Mental Computation

Mental computation is defined as "the process of producing an exact answer to a computational problem without any external computational aid" (Reys, B. J., 1986a, p. 22). While both mental computation and estimation can be done mentally, the process of estimation produces a response that is close to the exact answer, which would be the result of the process of mental computation. The thinking skills needed for mental computation help develop a sense of number and computational routines (Reys, B. J., 1985), and promote a greater understanding of the structure of numbers and their properties (Reys, R. E., 1984).

One aspect of number sense is the ability to choose the most efficient calculating procedure. Many times those efficient procedures are mental ones. Sowder (1990) cites Plunkett who described mental algorithms as variable, flexible, active, holistic, constructive, requiring understanding all along, and often generating an early approximation for the correct answer.

Hope and Sherrill (1987) identified characteristics of unskilled and skilled mental calculators. Students who were juniors and seniors in high school participated in the study which focused on mental multiplication. Strategies used by the students were grouped under four methods. Unskilled students most often used the pencil-and-paper mental analogue, frequently calculating each partial product digit by digit. The use of this method was often accompanied by an imaginary writing instrument used to perform calculations either in the air or on a table.

Skilled mental calculators, who infrequently used mental paper-and-pencil, employed distribution and factoring strategies (Hope and Sherrill, 1987). The distribution strategy involved transforming one or more factors into a series of sums or differences. The factoring method transformed one or more factors into a series of products or quotients. Hope and Sherrill (1987) noted that efficient mental calculation strategies eliminate the need to carry digits, proceed left-to-right, and continuously update a sum or product with each successive interim calculation.

Sowder (1988) claimed that a focus of mental computation is a reformulation of numbers to produce a basic facts computation. Among the skills needed to do this are an understanding of place value and the distributive property, and the ability to operate with multiples of ten. In mental computation, numerical concepts and skills are related with an emphasis on how numbers and their operations function (Sowder, 1988).

Various properties and relationships among numbers are used in mental computation. Hope (1986) cited factoring as useful in mental multiplication. An example using percents would rearrange 12 1/2% of 32 to be (25% of 32) ÷ 2, because 12 1/2% can be thought of as 25% ÷ 2. Other strategies for mental multiplication include the use of special products, front-end approach, and compensation (Hazekamp, 1986). These strategies require an understanding of the distributive property and inverse operations, as well as, basic facts. According to B. J. Reys (1985), mental computation problems using fractions, decimals, and percents can help students develop a better understanding of important basic concepts with these types of numbers.

Representations and Translations

According to Kaput (1987), mathematics is the study of the representation of one structure by another. Modes of representation encountered by middle grades students include pictorial, manipulative, oral, and written symbols (Post, Behr, and Lesh, 1986). A translation involves establishing an association between different representational systems (Lesh, Post, and Behr, 1987). It is important for students to be able to translate between and within different modes of representation (Post, Behr, and Lesh, 1986).

Seven representational translations which can be assessed with a paper-and-pencil instrument were used as a part of the Rational Number Project (Lesh, Landau, and Hamilton, 1983). An example of a symbol to picture translation is selecting the pictorial representation of the written symbol, 1/3 (Lesh, Post, and Behr, 1987). The most difficult translation for students in the project was picture to symbol, given one picture in the item stem and a response set of symbol choices (Lesh, Landau, and Hamilton, 1983). Ashlock (1986) discusses the introduction of numerals as a written record or representation of observations and manipulations of objects. Students who were able to translate between objects and symbols demonstrated a developing understanding of numerals (Ashlock, 1986).

Kaput (1987) claimed that some representations convey certain quantitative relationships more efficiently than others. With respect to fractions, common pictorial representations are continuous regions, discrete sets, and number lines. Students have the most difficulty with number lines (Payne, 1984; Behr et al., 1983). Some researchers have claimed that children have more difficulty conceptualizing a discrete set as a whole than viewing a continuous region as a whole (Behr and Post, 1988; Payne, 1984).

Some representations inherently carry perceptual distractors. At other times, the use of perceptual distractors can help assess the depth of student understanding (Behr and Post, 1988). An example shows one rectangle divided into three equal parts and another divided into six equal parts. In each case, a student was asked to shade two-thirds. The student may have difficulty pretending that the extra line in the second rectangle is not there.

Conceptual and Procedural Knowledge

Hiebert and Lefevre (1986) discussed the differences between conceptual and procedural knowledge, noting that not all knowledge is of one type or the other, but that the distinction can be useful in thinking about mathematics learning. Conceptual knowledge is characterized as a knowledge of relationships and must be learned meaningfully by recognizing relationships between units of knowledge.

Procedural knowledge consists of knowledge about the language or symbol system of mathematics and the rules, algorithms, and procedures used in mathematics. Many procedures learned in school mathematics are a sequence of steps to manipulate symbols. Such procedures may be learned without meaning. Hiebert and Lefevre (1986) suggested that procedures that are meaningful are linked to conceptual knowledge. Building such links gives mathematical symbols meaning. It also develops the ability to recall procedures, select an appropriate procedure for a given task, and judge the reasonableness of results.

Ginsburg (1977) claimed that it is very difficult for children to understand the written symbolism of mathematics. One link that is important is relating the symbolism to the mathematical knowledge children already possess (Ginsburg, 1977). Students often appear to search for rules which may be meaningless, but which they believe may work in a problem (Peck and Jencks, 1981). In their work with children in the area of fractions, Peck and Jencks (1981) found that children did not connect mathematical symbolism for operations with the physical manipulation of materials. Additionally, students were not able to determine whether their results of computation were reasonable.

Hiebert (1988) suggested a theory of developing competence in dealing with written symbol systems of mathematics. Students often generate answers by manipulating written symbols using memorized rules, because they may not have established connections between the symbols and procedures they use and meaningful referents (Hiebert, 1988; Ginsburg, 1977). The first processes in Hiebert's (1988) theory involve connecting symbols and symbolic procedures with concrete familiar referents, thereby providing the symbol system with meaning. Once this foundation of relationships between symbols and referents has been established, procedures can be extended and routinized.

Competency in mathematics involves having both conceptual and procedural knowledge and knowing how concepts, symbols, and procedures are related. Students often fail to develop such relationships. Hiebert and Lefevre (1986) suggested that school instruction emphasizing symbols and rules for manipulating those symbols contributes to this failure. Stress on procedural knowledge through high school develops students' reliance on symbol manipulation rules which are often not connected to conceptual understanding.

Researchers have indicated that current school programs place an emphasis on learning procedures (Hiebert and Lefevre, 1986; Hoffer, 1988; Payne, 1984; Dewar, 1984). Students' knowledge of fractions and decimals has indicated a strong reliance on procedures and little understanding of basic concepts (Payne, 1984; Kouba, Carpenter, and Swafford, 1989; Hiebert and Wearne, 1986; Behr and Post, 1988; Sowder, 1988; Carpenter, 1986). One aspect of these difficulties is students' use of routine symbol procedures without attention to connecting symbols with referents (Hiebert, 1988; Bezuk and Cramer, 1989). Current teaching strategies are blamed for students' poor understanding of proportional thinking and percent (Dewar, 1984; Allinger and Payne, 1986; Hoffer, 1988). More instructional time on establishing meanings and basic conceptual understanding is recommended (Sowder, 1988; Payne, 1984). Others suggest an emphasis on estimation and mental arithmetic can help students build concepts and reduce a heavy reliance on rules and procedures (Allinger and Payne, 1986; Reys, B. J., 1985).

Students who have developed links between conceptual and procedural knowledge remember procedures better and use them more effectively (Hiebert and Lefevre, 1986). Symbols are then used in a meaningful way and procedures seem to be reasonable ways of approaching problems. Conceptual knowledge, when linked to procedural knowledge, can monitor procedural outcomes in checking the reasonableness of an answer.

Piaget's Constructivist Theory

Piaget's theory of intellectual development involves the four basic concepts of schemata, assimilation, accommodation and equilibration (Wadsworth, 1989). Schemata are the mental structures developed by an individual which organize the environment. Schemata adapt and change as mental development occurs. Assimilation is the cognitive process used by an individual to place a new stimulus into existing schemata. When a new stimulus does not fit into existing schemata, accommodation occurs through the modification of the existing schemata or the creation of a new schema. After accommodation occurs, assimilation is possible because the new stimulus will fit into the current schemata. Equilibrium is achieved cognitively when assimilation has occurred. Equilibration is the "process of moving from disequilibrium to equilibrium" (Wadsworth, 1989, p. 16).

Through the processes of assimilation and accommodation, schemata never stop changing. In this way, individuals construct their personal knowledge of the world. This knowledge is shaped by experience and social interaction where the individual is actively involved (Clements and Battista, 1990; Wadsworth, 1989). Piaget was firm in his belief that active experiences with objects or people are fundamental to the construction of accurate knowledge (Wadsworth, 1989).

A student will behave in a manner which reflects the schemata which have been constructed (Wadsworth, 1989). The schemata reflect a student's current level of understanding and contain prior knowledge the student may bring to a particular task or situation. Wadsworth (1989) stated that children will strive to understand and make sense out of things. Hiebert and Lefevre (1986) suggested that the process of developing understanding involves assimilating new information into appropriate existing knowledge structures. When the necessary schemata for understanding are not present, children may use memorization to develop skills without comprehension (Wadsworth, 1989).

The role of the teacher applying Piagetian concepts may be viewed as provoking the construction of new personal knowledge by students (Ross, 1989). Teachers may do so by presenting tasks that bring about appropriate conceptual reorganizations and the invention or adoption of more sophisticated techniques for solving problems (Clements and Battista, 1990). Students persist in using their current ideas until they are presented with a reason to change. Reasons may include situations where their old ideas do not work or are inefficient (National Council of Teachers of Mathematics, 1989). Wadsworth (1989) described this situation as disequilibrium or cognitive conflict. Methods that encourage cognitive conflict motivate students to restructure their knowledge.

Critical exploration is a method of questioning used by the teacher to lead students into productive cognitive conflict (Wadsworth, 1989). The purpose of the method is to promote appropriate generalizations. After a student has solved a problem using a particular method, a second problem may be presented that when solved by the same method, results in an incorrect response. In dealing with the conflict created, the student's method is refined and an appropriate generalization can result.

Error Analysis

As students construct their knowledge of the world, errors and misconceptions will occur (Hart, 1983; Wadsworth, 1989; Resnick et al., 1989). Erroneous as well as correct concepts and procedures are learned in the same way (Ashlock, 1990). From a set of encounters with a concept or process, a student looks for commonalities and selects those common characteristics from which to form an idea of the procedure or concept (Ashlock, 1990). Children are motivated to make sense of situations and thus, the rules and procedures they create are meaningful to them (Wadsworth, 1989; Ginsburg, 1977; Radatz, 1979; Movshovitz-Hadar, Zaslavsky, and Inbar, 1987). Overgeneralization may result in incomplete or distorted procedures (Ashlock, 1990; Ginsburg, 1977).

Errors can reflect the level of understanding students have and the way they are processing information (Wadsworth, 1989; Hart, 1983; Borasi, 1985; Resnick et al., 1989). Cox (1975) focused on systematic errors noting that these errors are potentially remediable. Ashlock (1990) presented a number of error patterns in computation with suggestions for remediation. Radatz (1979) suggested that errors result from very complex processes and that one error may result from a close interaction among more than one cause.

It is important for teachers to use accurate information from student errors to choose appropriate remediation (Ginsburg, 1977; Roberts, 1968; Ashlock, 1986). Some information can be gained from analysis of written work but it needs to be supplemented with information gained from student interviews (Ashlock, 1990; Pincus et al., 1975). During an interview, the focus should be on a student's description of what he or she is thinking and doing (Cox, 1975; Hart, 1983; Ashlock, 1990).

Much work has been done in the area of classification of errors made by students doing whole number computation (e.g. Engelhardt, 1982; Brumfield and Moore, 1985; Kilian et al., 1980; Ginsburg, 1977; Pincus et al., 1975; Roberts, 1968; Cox, 1975; Ashlock, 1990). Many studies have suggested a classification scheme for errors that usually includes categories of procedural errors, computational errors, conceptual errors, careless errors, and random errors.

Early work in error analysis with fractions and decimals was done by Brueckner (1928a, 1928b) in the 1920s. In 1928, he reported the results of a study of errors students made working computation problems with fractions. More than 21,000 errors were analyzed and classified into categories indicating the nature or source of the error. Major difficulties for students with all four types of computational problems were a lack of comprehension of the computational process involved, errors in reducing fractions to lowest terms, and difficulty changing improper fractions to whole or mixed numbers. More errors were made with subtraction problems than with problems in the other three computational processes. Student errors due to computation were more frequent in multiplication than in the other three processes. With both subtraction and division problems, a significant number of students used the wrong process in working the problems.

Brueckner's (1928b) study of errors with decimals concerned decimal concepts as well as the four computational processes. The analysis involved more than 8,700 errors, which were classified into 114 categories. His findings indicated that students lack an understanding of the numerical values of decimals. The major difficulty in addition, multiplication and division problems was misplacing the decimal point. Overall, there were more errors with division problems than with other computational processes.

A study of difficulties in arithmetic essentials was conducted by Arthur (1950). Areas of arithmetic included concepts of whole numbers, fractions, decimals, and percents, and computation with whole numbers, fractions, decimals, and percents. One part of the test, given to high school freshmen, measured a student's ability to compute. A second part of the test consisted of verbal problems. The results found that students made more errors in the second part of the test. Throughout the entire test more errors were made finding what percent one number is of another, finding the whole when either a fractional or decimal part is given, and finding the whole when a percentage is given. Among the types of errors cited were performing the wrong operation, misplacing the decimal point, procedural errors, computational errors, and failure to correctly interpret results of verbal problems.

The results of a study of errors associated with the concept of percent were presented by Edwards in 1930. Seventh-grade students answered 113 test items which included questions of identification of the percent of the area of a figure that was shaded, changing fractions and decimals to equivalent expressions as percents and vice versa, and computation problems with the three types of percent problems.

Edwards (1930) cited four major categories of errors including, arithmetical errors, wrong responses which are closely connected to correct ones, consistent use of an easier, more familiar, though incorrect method of solution, and total confusion. He was particularly concerned that students seemed to give impossible and unreasonable answers to questions. In his conclusions, Edwards (1930) stated that "bright and dull" students make the same types of errors in approximately the same proportion, though "the dull make them more frequently" (1930, p 640)

In 1946, Guiler (1946a, 1946b) published the results of two studies analyzing difficulties students have with percent computation problems One study involved ninth-grade students and the other used college freshmen The test items were identical and the results of both studies highlighted students' poor performance with percent Guiler (1946a, 1946b) established a classification scheme of errors Most of the difficulties centered on a lack of understanding of the procedures involved and an inability to use the meaning of percent as hundredths Faulty computation and errors in working with decimals were other primary sources of difficulty

Percent

There seems to be agreement that percent is a hard concept for students to understand (Wiebe, 1986, McGivney and Nitschke, 1988, Hart, 1981b, Allinger and Payne, 1986) However, there is also agreement that percent is an important practical concept for students to learn because of the many areas in which it is used (Wiebe, 1986, Allinger and Payne, 1986)

One focus of the study of mathematics in the middle grades is on understanding multiple representations for a number, such as a fraction, decimal, and percent (Payne, 1984, National Council of Teachers of Mathematics, 1989) Results from the fourth National Assessment showed that about 70% of seventh graders are able to write a decimal expressed in hundredths, as a percent, and a two-digit whole number percent as a decimal (Kouba, Carpenter, and

Swafford, 1989) These problems could be easily solved by moving the decimal point and adding or deleting a percent sign Students were less successful changing 9 to a percent or 0 9% to a decimal

Researchers consider the understanding of a percent as a fractional part of a whole using hundredths to be fundamental before working percent problems (Allinger and Payne, 1986, Schminke, Maertens, and Arnold, 1973) However, a reliance on rules and procedures in studying multiple representations for a percent is being promoted by textbooks In the small space allowed on a textbook page for the presentation of new material, there is little emphasis on meaning but strong emphasis on rules, such as, move the decimal point two places to the left and drop the percent symbol (Orfan et al , 1987, Quast et al , 1987)

Students participating in the fourth National Assessment were more successful working with percents, such as 25% and 50%, that were familiar and for which they knew the fraction equivalents (Kouba et al , 1988) Most seventh graders recognized that when a whole is partitioned into parts represented by percents, the sum of the percents must be 100% On a comparison question, such as "76% of 20 is greater than, less than, or equal to 20," 37% of seventh graders and 69% of eleventh graders responded correctly (Kouba, Carpenter, and Swafford, 1989) In general on the 1986 National Assessment, about one-third of seventh graders and slightly more than one-half of eleventh graders demonstrated an understanding of basic percent concepts

An emphasis in the current textbooks is on solving the three types or cases of percent problems A study conducted by McCarty

(1967) investigated the effectiveness of introducing the study of percent problems in grades four, five, and six His results suggested that percent problems can be successfully introduced during the latter part of the fourth grade when the ratio method is used

Results from the fourth National Assessment showed that students are more successful calculating a percent of a number, case one problems, than the other two types (Kouba, Carpenter, and Swafford, 1989) Montgomery (1959) explored the difficulties seventh-grade students had mastering the cases of percent problems He found the most correct responses in case one problems and the fewest in case three problems Students also had the least difficulty with whole number percents that were familiar, such as 20%, 50%, and 75%

In her work, Hart (1981b) found that some students always divided by 100 when working any percent problem Dewar (1984) stated that even after students have been taught the three cases, they have trouble recognizing the appropriate case for a specific problem According to Allinger and Payne (1986), students rely heavily on a set of rules to do percent problems, may not apply the rule correctly, and often fail to choose the appropriate rule

Kircher (1926) studied eighth grade students' understanding of percent He found that students knew little about percent and were often confused The students used formulas with little understanding and accepted the answer from the formula procedure without considering its reasonableness Kircher (1926) blamed the instruction in the classroom for this situation He recommended increased attention to concepts, teaching the three cases of percent problems simultaneously emphasizing their interrelation, and requiring students to estimate answers

Edwards (1930) studied errors made by seventh-grade students with problems involving the concept of percent In addition to looking at errors for different aspects of percent problems, he criticized the methodology of teaching percent Students in the study gave answers that were senseless and impossible for problems involving percent Edwards (1930) blamed teachers for this failure to help students develop a sufficient sense for the problem

Several studies have been done comparing approaches for teaching percent Wynn's (1966) study of seventh graders found no significant difference in the effectiveness of the unitary analysis, formula and decimal methods The method of unitary analysis stresses finding the value of 1% of the whole and then solving for the total percent The formula method utilizes the relationship, rate times base equals percentage, while the decimal method has students convert the percent to a decimal and then either multiply or divide (Schminke, Maertens, and Arnold, 1973) Bidwell's (1969) study of fifth graders found no significant difference comparing the effectiveness of the ratio method and the equation method The equation method uses the relationship, rate times base equals percentage, to create an equation in one variable to be solved (Underhill, 1972)

McMahon (1960) compared two methods of teaching percent to seventh-grade students The two methods were the ratio method and a conventional method which focused on learning the rules for solving the three types of percent problems No significant difference was found among the two groups of students on tests of interpreting statements about percent, but students using the ratio method showed greater skill in computation and greater retention.

A comparison of three methods of teaching percent was conducted by May (1965). The traditional method focused on memorizing rules for the solution of the three types of percent problems. The discovery method involved students in the development of generalizations to solve percent problems. The third method was the ratio method which was the most effective for retention. May (1965) suggested that teachers use either the discovery method or ratio method for both immediate learning and retention.

A study among seventh graders was conducted by Kenney and Stockton (1958) using three methods of teaching percent. One method was characterized as drill or rote teaching emphasizing rules and repetition. A second method emphasized understandings while no rules were specifically taught. The third method used a composite of the first two methods so that procedures to develop understanding were used along with drill to "fix and facilitate learning" (Kenney and Stockton, 1958, p. 295). For students of middle ability, those in the third method group did better than students in either of the other two groups. Kenney and Stockton (1958) found some indication that the third method was more effective than the first method with all students and that students in the second method group had some advantage in problem solving.

The study conducted by Tredway and Hollister (1963) focused on teaching the three types of percent problems with two different teaching methods. The experimental groups were taught the three types of percent problems during which continual emphasis was placed on the interrelationships among the three types. The control groups were taught according to the normal fashion, following the text in their school. Results showed that the experimental method produced better results in learning at all levels of intelligence. Students of average intelligence exhibited better retention under the experimental method.

Maxim (1982) compared two sequences for teaching percent which differed in their approach to the solution of case one problems. One of the teaching sequences used the proportion method and the other the factor-factor-product method. Students in seventh grade participated in the study. Though there were few differences between treatment groups, where differences were found, they favored the students taught with the factor-factor-product method.

The way many propose to remedy the lack of understanding in working percent problems is the use of various procedures to help students work the three types of percent problems. Dollins (1981) symbolized the types of percent problems as: *a* is *b* percent of *c* and proposed the proportion method of solution with $b/100 = a/c$. In every problem, the percent will be one of the ratios, $b/100$, and the number after the *of* will be the bottom number in the other ratio. Osiecki (1988) used money as a comparison for percent, suggesting to her students that they interpret percent always in terms of cents. McGivney and Nitschke (1988) noted that most percent problems use the words, *is* and *of*. If a problem is not worded in this manner, it can be restated. They suggested identifying the *is number*, *of* *number*, and *percent number* and using the proportion *is number : of number = percent number :* 100 to solve the problem. Boling (1985) used the same three numbers, *is number*, *of number*, and *percent number*, in his triangular method. He divided a triangle into three compartments: top, left, and right. In filling the three compartments with two known quantities and a question mark, a product was formed to solve case one problems, while fractions, indicating division, are formed to solve case two and case three problems.

Others have used concrete models or calculators to enhance students' understanding. Wiebe (1986) supported the need to teach percent with the use of physical models. The 10 x 10 grid of 100 squares is often used (Wiebe, 1986; Allinger, 1985). Dewar's (1984) model established two vertical linear scales, one for the percent and the other for the quantities in the problem, to visually illustrate the proportion to be used to solve the problem. Allinger (1985) suggested activities integrating the calculator can help general mathematics students learn to solve case one and case two percent problems. He developed a sequence of calculator steps, first without the use of the percent key, which model the solution of the proportion. Coburn (1986) suggested the use of the calculator to develop skills in estimating and mentally computing percentages. Mellon (1985) found that her calculator-based units on decimals and percent were effective when used with seventh-grade students.

Building on information gathered by Maxim (1982) on the interrelationships among students' knowledge of fractions, decimals, and percents, Doerr (1985) investigated a hierarchy of prerequisite skills for the concept of percent. She designed a Fraction-Decimal Subskill Test and a Percent Concept Test. Analysis of the test data revealed a moderate relationship between the percent concept and selected subskills in fractions and decimals.

Targeting her research to help the classroom teacher, Gucken (1986) developed the High School Percent Diagnostic Test which can be successfully used as a test of achievement of percent concepts. The test included items addressing fractions, decimals, ratios and proportions, as well as relations among the numerical representations, and used both the proportion and factor-factor-product method to solve percent problems.

Summary

Percent is an important but difficult concept for students to learn. Previous research on students' knowledge of percent has centered on students' computational skills, with little attention given to aspects of number sense with percent. However, studies have been conducted recently which focused on students' understanding of fractions and decimals and included investigations of the cognitive strategies students used in responding to fraction and decimal tasks.

From the review of the literature, there exists a need to conduct an investigation to examine students' understanding of percent, including an analysis of thinking strategies used with percent quantities. This study of students' conceptual knowledge related to percent incorporated different representations of percent quantities and an analysis of both correct and incorrect strategies used by students to compare percent quantities. The results of the study provided insight into what students understand about percent, thereby increasing the knowledge base on student learning.

CHAPTER III

THE RESEARCH DESIGN

Introduction

Descriptive research systematically and accurately describes existing phenomena (Isaac and Michael, 1981). This study utilized both quantitative and qualitative methods to provide insight into the understanding students have about the concept of percent. The study focused on these research questions:

1. Can students interpret a quantity expressed as a percent given a pictorial discrete set or continuous region with part or all of the area shaded?
2. Do students understand the meaning of a quantity expressed as a percent of a number?
3. What strategies do students use to make comparisons about percent quantities in both pictorial and abstract settings?

Qualitative and quantitative assessments were prepared by the investigator and piloted with middle school students. The data from the pilot study was used to refine the research design, instrument and techniques used. These assessments were then administered to students in grades seven and eight. The investigator developed a unique method for analyzing the data from both assessment measures. The data collected in this study extended the knowledge base related to students' understanding of percent.

The Pilot Study

During May, 1990, a pilot study including sixth grade (n=27), seventh grade (n=26), and eighth grade (n=27) students was conducted in Putnam City Schools. One class from each grade level participated, and all students were of average or above average ability.

A longer written test was used in the pilot study to gather information to refine the instrument used in the main study. A copy of the pilot study instrument is included in Appendix C. The test was administered in each of the three classes by the mathematics coordinator for Putnam City Schools. During the week following the administration of the test, seven seventh-grade students participated in individual interviews with the researcher.

Questions answered by the pilot study included:

1. What is the reliability of the written instrument?
2. Does the researcher's order of test items, i.e., familiar to unfamiliar, agree with the students' perception of familiarity?
3. Are students able to meaningfully use the phrases, "greater than" and "less than" to determine an order of familiar quantities?
4. Is there a difference in students' performance in comparing a quantity expressed as a percent with a shaded region

when pictorial discrete sets and pictorial continuous regions are used?

5. What classification scheme can be used to analyze the responses to the open-ended question?
6. What are some strategies students used to answer these kinds of questions on percent?
7. What probing questions were useful in the interview?

Included in the pilot study instrument were three questions designed to determine if students were able to meaningfully use comparison terms to order familiar quantities. The percent of correct responses over all three questions and for all three grade levels was 89%. Additional information was gained from the responses to the open-ended question where students meaningfully used the comparison terms indicating quantities that were greater than, less than, or equal to other quantities. For a high percent of the students, it was determined that the terminology used in the response choices of the testing instrument would be understood.

The results from the pilot study showed differences in the level of student performance when pictorial discrete sets and pictorial continuous regions were used. Therefore, it was decided to retain both sections of the test for the main study to further study these differences.

Pilot study data can help a researcher decide on the advisability of proceeding with the main study (Isaac and Michael, 1981). Responses from the students to the open-ended question on the test and during the interviews indicated varied levels of student understanding and the use of a variety of strategies when answering questions about percent quantities. It was decided to continue the investigation with the main study, implementing the refinements suggested by the pilot study results. More detailed results of the pilot study are included in Appendix D. Since the main study was scheduled to be conducted in September, it was decided to use only seventh and eighth graders.

The Measuring Techniques

The Written Instrument

The written test used in the study was developed by the researcher and revised following the pilot study (Appendix A). Demographic information, including grade, sex, age, and ethnic origin, was collected as a part of this assessment. In addition, students were asked to choose a response indicating a self-assessment of mathematics ability and to state their grade in mathematics last year.

The purpose of the written instrument was to measure students' performance on a sample of questions focusing on percent. The three parts of the test which were developed dealing with the concept of percent used the same percent quantities in the same order. Students participating in the pilot study ordered the seven percent quantities from most familiar to least familiar as 50%, 25%, 100%, 60%, 110%, 33 1/3%, and 87%.

The questions in part one of the written test were designed to determine students' ability to compare a percent quantity with a quantity represented as a shaded part of a pictorial discrete set.

Post (1981) indicated that even though students were less proficient working with fractions using the discrete set representation of a whole than using a continuous region to represent a whole, it was important for them to be able to use both representations. Therefore, both representations were included in the written test for this study. The multiple-choice questions in part one used a discrete set of circles to represent a whole and assessed the following objective:

> Given a picture showing a set of discrete circles, part or all of which are shaded, the student will determine if a percent quantity is less than, greater than, or equal to the quantity represented by the shaded part of the set of circles.

Even though Post (1981) noted that students did better naming the shaded fractional part of a continuous region than the fractional shaded part of a discrete set, Hiebert and Wearne (1986) found that some ninth-grade students had difficulty writing a decimal fraction to represent the shaded part of a unit rectangular region. To determine students' ability to compare a shaded part of a continuous region with a given percent quantity, the questions in part two of the test were designed to assess the following objective:

> Given a picture showing a rectangle, part or all of which is shaded, the student will determine if a percent quantity is less than, greater than, or equal to the quantity represented by the shaded part of the rectangle

Questions in part three of the test were based on a released exercise used by the National Assessment of Educational Progress during its fourth assessment (Dossey et al., 1988). This item is characteristic of Level 300: Moderately Complex Procedures and Reasoning. In the 1986 NAEP, less than 16% of the 13-year-olds performed at or above this level. On a question similar to the released exercise, only 37% of seventh graders responded correctly (Kouba, Carpenter, and Swafford, 1989).

The multiple-choice questions in part three were designed to assess the following objective:

> Given a quantity expressed as a percent of a number, the student will determine if that quantity is less than, greater than, or equal to the given number.

An open-ended test item in part three asked each student to explain how he or she decided on a response to the test question comparing 87% of 10 to 10.

The Kuder-Richardson Formula 21 was used on the total group and grade subgroups of the pilot study (Isaac and Michael, 1981). A reliability coefficient of .855 was calculated for the total group. Reliability coefficients for each grade subgroup in the pilot study were .915 for eighth grade, .863 for seventh grade, and .737 for sixth grade.

Content validity is especially important for this type of test to show how well the content of the test samples the subject matter about which conclusions are to be drawn (Isaac and Michael, 1981). A panel of four middle school teachers rated a sample of test items from each of the three sections of the test as to whether the items measured the objective stated for that section. A majority of the teachers rated each question as assessing the stated objective.

The Research Interview

The research interview permits the gathering of data of greater depth and in more detail than may be provided through a questionnaire or other written instrument (Isaac and Michael, 1981). Dialogue during the interview gives the interviewer the flexibility to pursue student statements, probing for more detail and clarification (Isaac and Michael, 1981; Shaw and Pelosi, 1983; Rudnitsky, Drickamer, and Handy, 1981). Oral interviews have been effectively used by teachers to determine the strategies a student is using and the reasons for using particular steps in a procedure (Shaw and Pelosi, 1983; Rudnitsky, Drickamer, and Handy, 1981; Ashlock, 1986). Peck, Jencks, and Connell (1989) claimed that brief interviews used in conjunction with written tests yield more information about children's understanding than can be obtained from paper-and-pencil tests alone.

Individual student interviews were conducted to gather information on the thinking strategies students used to answer questions from the test. One of the main purposes of the research interview, which is especially appropriate for children, is to pursue unexpected results and to discover the reasons respondents answer as they do (Isaac and Michael, 1981). A structured interview is one where the interviewer follows a well-defined structure in asking a set of questions and allowing clarification and elaboration within narrow limits (Isaac and Michael, 1981). Each interview with a student followed a planned structure with the purpose of identifying the strategy or strategies a student used to determine the answer to a question. Even though each interview was structured as an attempt to standardize some questions, it is important to permit flexibility during the interview (Kirk and Miller, 1986). This flexibility allows the interviewer to explore particular remarks made by individual students in an attempt to gain as much information as possible.

During each interview, two types of items were used. Fixed-alternative items used in the interview were selected multiple-choice questions from the written test. A student was presented with a test item on a card and asked to select an answer to the question. Fixed-alternative items provide easy to code responses and more uniformity than open-ended questions.

After the student selected one of the multiple-choice responses to a test question, the interviewer asked the student to explain what he or she thought about to determine the answer. Probing follow-up questions were used with some students for clarification of their responses. Open-ended interview items also allow unexpected responses which may reveal unanticipated significant information (Isaac and Michael, 1981). The interview plan was tested during the pilot study to reduce weakness in the method. An outline of the interview structure with some examples of probing questions is found in Appendix B.

The purpose of the qualitative assessments in this study was to provide insight into students' understanding of percent which could not be gained with a quantitative multiple-choice assessment instrument. In the use of qualitative methods, it is assumed that the researcher is the main tool for gathering data and brings to the data a subjective interpretation (Emerson, 1983). Thus, "no description is independent of the describer and his or her actual methods for making and reporting observations" (Emerson, 1983, p. 22).

Credibility and applicability are criteria proposed in place of validity, by which qualitative research can be judged (Worthen and Sanders, 1987; Marshall and Rossman, 1989). The goal of credibility is the demonstration that the inquiry was conducted in a manner to ensure that the problem was accurately identified and described (Marshall and Rossman, 1989). Corroboration of data through the use of more than one method is one way to establish credibility (Worthen and Sanders, 1987). In addition to the use of more than one method of data collection in this study, the use of a third person who took notes during each interview provided a more thorough record of the interview and a point of view different from that of the researcher.

Applicability refers to the generalizability of the evaluation findings to other settings. Marshall and Rossman (1989) noted that the responsibility of "demonstrating the applicability of one set of findings to another context rests more with the investigator who would make that transfer than with the original investigator" (p. 145). The use of thick description in this study, which entails the problem being studied, the characteristics of the subjects and the techniques used, was one way of enhancing its applicability (Worthen and Sanders, 1987). Another means of developing applicability was the use of more than one source of data, which was incorporated into the research design of this study.

The subsample of students participating in the research interviews was not a representative sample. The information gathered during the interviews was not intended to generalize to the entire sample or population. Instead, the subsample was a diverse group of students from whom information about their individual approaches to questions could be used to further the insight educators have about student learning.

The Sample

The subjects in this study were students in grades seven and eight in the Putnam City school district during the school year, 1990-91. Putnam City Schools is a large urban district located in the northwest quadrant of the metropolitan Oklahoma City area. Putnam City, with an enrollment of 18,000 students, has the fourth largest student population in the state of Oklahoma. Students in the district represent diverse socio-economic and ethnic backgrounds. The district population is approximately 49.46% female and 50.54% male. The ethnic distribution in the district is approximately 86.61% White, 5.64% Black, 4.88% Native American, 1.14% Hispanic and 1.73% Asian. Seventh and eighth graders attend one of the district's four junior high schools.

It was arranged to use one seventh-grade class and one eighth-grade class from each of the four junior high schools. Each class had mathematics students of average ability. Honors and remedial classes were not included in the population. Participating students included 106 seventh graders, which is 10% of the population of students in average seventh-grade classes, and 93 eighth graders, which is 9% of the population of students in average eighth-grade classes.

Demographic data collected from seventh graders participating in the study showed that 50% were male and 50% were female, with ages ranging from 11 to 14 years old. Ethnic information showed that the sample of seventh-grade students was 73.5% White, 13% Black, 7.5% Native American, 1% Hispanic, 2% Asian, and 3% Other.

Demographic data collected from eighth graders participating in the study showed that 58% were male and 42% were female, with ages ranging from 13 to 15 years old. Ethnic information showed that the sample of eighth-grade students was 81% White, 9% Black, 5% Native American, 1% Hispanic, 2% Asian, and 2% Other.

Collection of the Data

One strength of this study was the use of methodological triangulation, which is defined as the process of using more than one method to gather data (Marshall and Rossman, 1989; Patton, 1987; Isaac and Michael, 1981). Qualitative data can complement data collected using quantitative methods (Reichardt and Cook, 1979; Patton, 1987). The quantitative approach uses statistics to compile information from a large number of subjects, generating data which can be used to make comparisons (Patton, 1987). Qualitative data can provide depth and detail of description focusing on diversity as well as commonalities, through the study of selected individuals (Patton, 1987; Marshall and Rossman, 1989). Used together, both approaches can provide a more comprehensive description of students' understanding. The responses to the multiple-choice questions on the written test are the quantitative data gathered in this study. The responses to the open-ended question on the written test and the data gathered during the student interviews are types of qualitative data collected in the study.

The written test was administered on September 25-26, 1990, by the mathematics coordinator for Putnam City Schools in the students' mathematics classrooms under normal school conditions. The tests were scored by the researcher during September, 1990. On the basis of the written test results, 14 seventh-grade students, 13% of the subjects, and 14 eighth grade students, 15% of the subjects, were selected for individual interviews. The strategy of maximum variation sampling focuses on selecting participants who vary greatly in particular characteristics (Patton, 1987). For this study, maximum variation sampling was applied to select students to be interviewed who represented a wide range of ability on the written test. Test scores of those students interviewed ranged from 9 to 21 out of a possible 21.

On September 27-28, 1990, selected students participated in individual interviews with the researcher. The mathematics coordinator for Putnam City Schools took notes during each interview. Questions, answers, and comments were recorded. Supplemental notes were made by the researcher during the interviews. The focus of the discussion during each interview was the strategy or strategies the student used in answering questions from the test.

Each student answered from four to eight multiple-choice questions from the test. Questions had been prepared individually on cards. After the student had selected an answer to a question, the student was asked to explain how he or she decided on the answer. At times when it was obvious the student was processing information to arrive at a response, the interviewer would request that the student explain what he or she was thinking about or doing to answer the question. In a few instances, when the student was obviously struggling with a question, the interviewer suggested the use of a model such as the continuous region or the set of circles with which the student had already been successful.

Analysis of the Data

Three measures of central tendency and variability were computed for the scores on the multiple-choice questions of the written test. For each grade level, the arithmetic mean for each part of the test as well as the overall mean, was calculated. The range of individual student scores for each section of the test and for the entire test provides one measure of dispersion and information on extreme scores. The standard deviation associated with each computed arithmetic mean provides another measure of the distribution of the data. For each grade level, the overall mean score for the test and the standard deviation was calculated for the grades in mathematics last year reported by the students. For each grade level, the overall mean score for the test and standard deviation was computed for the self-assessment ratings of mathematical ability given by the students.

The written test included one open-ended test item which asked the student to explain how he or she decided on a response to the test question which compared 87% of 10 with 10. The explanations given by both successful and unsuccessful performers on this question were classified using a category system, which resulted from a process of content analysis involving organizing data by identifying important examples and patterns. Through this process of content analysis, the data was organized into manageable categories which form a classification scheme (Patton, 1987). In developing the scheme, continual adjustments were made in verifying the accuracy of the categories and the placement of data into categories.

Guiler's (1946a) classification scheme for student errors with percent problems was used to provide guidance in the development and naming of some pertinent categories. Some of the students' explanations were classified as computation, estimation, and comparison. The pilot study responses to the open-ended question were presented to four middle school teachers along with a possible classification scheme. Their suggestions for improvement and clarification in the categories were implemented when the students' explanations were organized from the main study.

The data gathered from student interviews is presented in narrative form. An inductive analysis was conducted to locate patterns and themes of the data. These patterns were not established prior to data collection and analysis.

The use of the research interview to gather data adds an important dimension to this study. However, the interview technique also introduces the problem of subjectivity and bias on the part of the researcher (Isaac and Michael, 1981; Marshall and Rossman, 1989). Sources of bias may include antagonism, frustration, or eagerness to please on the part of the respondent (Isaac and Michael, 1981). The researcher may bring preconceived notions, such as answers that are expected, into the interview. Actions by the researcher, such as where attention is directed, what is ignored or forgotten, as well as what is remembered and recorded, may have an impact on the data.

Several components of the research design were included to reduce bias in the results of the study. The use of a third person who took notes during each interview provided a more thorough record of the interview and a point of view different from that of the researcher. In classifying the open-ended responses on the written test, the paper of every tenth student was re-evaluated after all papers had been considered. While analyzing the interview data, the record of every fourth student was reconsidered.

The purpose of the analysis of both the quantitative and qualitative data was to produce a systematic and accurate description of some aspects of students' number sense related to percent. The results of the analysis are presented in Chapter IV. Conclusions and recommendations are discussed in Chapter V.

CHAPTER IV

ANALYSIS OF THE DATA

Introduction

The focus of this study is number sense with percent. A description of students' number sense related to percent should include aspects of the conceptual knowledge students have about percent and the strategies they use in thinking about some types of questions which contain percent quantities. Research questions for this study include:

1. Can students interpret a quantity expressed as a percent given a pictorial discrete set or continuous region with part or all of the area shaded?
2. Do students understand the meaning of a quantity expressed as a percent of a number?
3. What strategies do students use to make comparisons about percent quantities in both pictorial and abstract settings?

To answer the research questions, both quantitative and qualitative data gathered from students in grades seven and eight was analyzed. The quantitative data was generated from the multiple-choice questions on the written test. The mean score, standard deviation, and range of scores were determined for each grade level for each of the three sections of the test, as well as the overall test. Additionally, for each grade level, the mean score, standard deviation and range of scores were calculated for each group of students reporting each letter grade received in mathematics and for each self-assessment rating given by the students of their ability in mathematics.

The qualitative data was generated from the responses to the open-ended item on the written test and the research interviews. The process of content analysis was used on the responses to the open-ended question to organize the student explanations into categories. An inductive analysis was conducted on the interview data to locate patterns and themes around which the narrative discussion was organized.

Quantitative Data Analysis

Three measures of central tendency and variation were calculated on the results of the written test taken by the seventh-grade students. The range, mean score and standard deviation were determined for each of the three sections of the test and the entire test. Each of the three sections of the test contained seven multiple-choice questions, and thus the overall test data was based on a total of 21 multiple-choice questions. The mean scores, standard deviations and range of scores for the seventh-grade students are presented in Table I.

Students did best on questions in part two, which used the picture of a rectangular region. The students' mean score was lowest for part three, which also had the most variability in scores.

TABLE I

SEVENTH GRADE RESULTS
(n=106)

	Part 1 Discrete Sets	Part 2 Continuous Regions	Part 3 Abstract	Part 4 Overall
Mean	4.58	5.71	3.19	13.48
Standard Deviation	1.88	1.70	2.16	4.46
Range	0-7	0-7	0-7	1-21

The same measures of central tendency and variation were calculated on the results of the written test taken by the eighth-grade students. The results, including the mean scores, standard deviations, and range of scores, for the eighth-grade students are presented in Table II.

Generally, the eighth-grade students did well on questions in part two. The lowest mean score, as well as the most variability, occurred with part three of the test. For both seventh and eighth graders, their mean scores were highest for part two and lowest for part three. For each part of the test, as well as for the overall test, the mean scores of eighth-grade students were higher than the mean scores of seventh-grade students.

The students were asked to report the grade in mathematics received last school year. For each grade level, the overall mean score for the test and the standard deviation were calculated for each reported letter grade. Students in each grade level who listed two or more letter grades in mathematics were grouped with those who did not respond to the question. The mean scores, standard deviations and the range of scores are presented by grade level and by letter grade in Tables III and IV.

Students were asked to choose a self-assessment rating for their mathematics ability from among four multiple-choice responses. Students' scores were grouped according to the rating they chose, and for each grade level, the overall mean score for the test and the standard deviation was calculated. Students in seventh grade who chose two categories from among the multiple-choice responses were grouped with those who did not respond to the

TABLE II

EIGHTH GRADE RESULTS
(n=93)

	Part 1 Discrete Sets	Part 2 Continuous Regions	Part 3 Abstract	Part 4 Overall
Mean	5.73	6.38	4.09	16.20
Standard Deviation	1.44	0.87	2.52	3.99
Range	2-7	4-7	0-7	6-21

TABLE III

SEVENTH GRADE RESULTS BY MATHEMATICS
LETTER GRADE

Letter Grade	n	Mean	Standard Deviation	Range
A	24	14.42	4.50	3-21
B	44	14.45	3.91	3-21
C	21	11.90	4.23	1-19
D	2	16.00	2.83	14-18
F	4	15.25	2.63	13-19
Other	11			2-18

TABLE IV

EIGHTH GRADE RESULTS BY MATHEMATICS LETTER GRADE

Letter Grade	n	Mean	Standard Deviation	Range
A	24	17.25	3.64	8-21
B	28	16.71	3.70	9-21
C	18	16.33	3.83	10-21
D	10	15.10	4.72	6-21
F	2	11.50	3.54	9-14
Other	11			7-19

question. The mean scores, standard deviations and the range of scores are presented by grade level in Tables V and VI.

Qualitative Data Analysis

The Written Test

The written test included one open-ended test item which asked the student to explain how he or she decided on a response to the test question asking if "87% of 10 was greater than, less than, or equal to 10." For each grade level, the explanations given by both successful or unsuccessful performers on the multiple-choice test question were classified using a category system, which resulted from a process of content analysis. This process was used to identify important examples and patterns, which were then used to organize the data into categories. In developing the classification scheme for the data, explanations which conveyed the same concepts or procedures were grouped together and given appropriate labels. Other explanations, which were incomplete or of an unknown nature, were also grouped and labeled. Those explanations which indicated a lack of response were grouped with the different comments mentioned.

Of the students taking the test, 45% were successful answering the question. The explanations of those students who successfully answered the question were categorized and are presented in Table VII. For each category, an example of a student's explanation which was characteristic of that category, is presented as an illustration,

TABLE V

SEVENTH GRADE RESULTS BY SELF-ASSESSMENT OF MATHEMATICS ABILITY

Self Assessment	n	Mean	Standard Deviation	Range
Very Good	10	11.70	3.89	3-18
Good most of the time	52	14.69	4.63	1-21
Good some of the time	28	12.36	3.65	3-19
Not very good	11	12.00	4.12	4-18
Other	5			4-20

TABLE VI

EIGHTH GRADE RESULTS BY SELF-ASSESSMENT OF MATHEMATICS ABILITY

Self Assessment	n	Mean	Standard Deviation	Range
Very Good	11	18.36	3.14	11-21
Good most of the time	51	16.84	3.43	8-21
Good some of the time	21	15.52	3.79	9-21
Not very good	9	11.11	4.59	6-21
No response	1			19

TABLE VII

EXPLANATIONS PROVIDED BY SUCCESSFUL STUDENTS

	7th	8th	Percent
Understanding of Percent Situation Demonstrated			
Correct Comparison			
Using 100%: 100% is equal to 10 and 87% is less than 100%	6	19	27.89%
Using 87%: 87% of 10 is not quite the full amount of 10	4	7	12.20%
Correct Computation: 10 x .87 = 8.70	1	5	6.70%
Estimation: 87% of ten is about 9	2	1	3.30%
Incomplete Explanation: because when you do the percentage it's less	4	3	7.80%
Other: if you have 10 of something and 87% of that 10 is filled	1		1.10%
Lack of Understanding of Percent Situation			
Incorrect Comparison: .87 is smaller than 10		1	1.10%
Incorrect Computation			
Division: 87 ÷ 10	3		3.30%
Unknown Division: divide half of 87	1	1	2.20%
Other: there's eight as you count up to ten	1		1.10%
Unknown: 10 is a whole number	1	2	3.30%
Unknown: I used common sense	1	2	3.30%
Lack of Response			
I don't know	5	3	8.90%
I guessed	3	1	4.40%
Blank	6	6	13.40%

along with the number of students in each grade level and the overall percent of successful students who gave that explanation.

A variety of correct and incorrect explanations were given by the students who correctly stated that "87% of 10 was less than 10." Fifty percent of the students described a complete and appropriate solution process which indicated an understanding of the problem situation. More than one-fourth of the successful students compared 87% to 100% indicating that 100% of 10 would be 10 and therefore 87% of 10 would be less than 10. One student described the situation in some detail, noting that 50% of 10 is 5, 100% of 10 is 10, and thus 87% of 10 is between 5 and 10. Approximately 12% of the students did not specifically refer to 100%, but instead focused on the fact that 87% was a part of a whole to conclude that "87% of 10 was less than 10." Only 6.7% of the students computed to find the product, 8.7, and correctly judged this number to be less than 10. Three students used estimation to determine that 87% of 10 was about 9 and by comparing 9 to 10, concluded that 87% of 10 was less than 10.

Almost 60% of the successful students gave some explanation which showed an understanding of the percent situation. However, another one-fourth of the students had no explanation for their correct response to the question. An additional three students may have had an accurate understanding of the problem, but their explanations of "My brain" and "I decided on the most logical answer," do not provide any specific evidence to support such a conclusion.

The rest of the successful students, 11%, gave inappropriate explanations which may be due to a lack of understanding of the percent situation or an inability to describe the reasoning process used. A reliance on computation, especially division, was demonstrated by five students, three of whom divided the two numbers in the problem. Other incorrect explanations involved counting from eight to ten, or comparing the decimal representation of 87% to determine that it was less than 10. Three students gave explanations that did not provide enough detail to demonstrate an accurate understanding of the percent question.

Of the students taking the test, 63% of the seventh-grade students and 45% of the eighth-grade students were unsuccessful answering the question comparing 87% of 10 to 10. Many of the students, 61%, responded that "87% of 10 was greater than 10." Only one student responded that "87% of 10 was equal to 10." The rest of the students answering incorrectly, 38%, left the question blank or responded that they could not tell or did not know the answer. The explanations given by those students who were unsuccessful answering the test question were categorized and are presented in Table VIII. For each category, an example of a student's explanation which was characteristic of that category, is presented as an illustration, along with the number of students in each grade level and the overall percent of unsuccessful students who gave that explanation.

Of the students who were unsuccessful comparing 87% of 10 to 10, more than 65% were unable to provide an explanation for their responses. This figure reflects the fact that 38% of them had left the question blank or responded that they could not tell or did not know the answer.

TABLE VIII

EXPLANATIONS PROVIDED BY UNSUCCESSFUL STUDENTS

	7th	8th	Percent
Comparison of Two Numbers			
Neither as percents: 87 is higher than 10	8	3	10.10%
Both as percents: 87% is greater than 10%	3		2 80%
One number as a percent: 87% is larger than 10	3	3	5.50%
Unknown: 10% is too small		1	90%
Computation			
Subtraction			
Involving two numbers: 87-10=77	3		2.80%
Involving 100· 100-87=13; 13>10	1		.90%
Multiplication			
Involving 100%: change 87% to 100% and then multiplied by ten	1		.90%
Involving fractions: 87/100 x 10/1 = 87/5 = 17		1	90%
Division: divide 10 into 87%	1		.90%
Use of proportion: 87/100 = n/10; 870 ÷ 100 = 870		1	.90%
Unknown: It takes more for it to go into it	2		1 80%
Other			
Counting: I counted to 87 by 10's	1		.90%
Reference to a whole. 87% is almost all of a whole	2		1.80%
Unknown: I thought it was the best one	2	2	3.70%
Lack of Response			
I don't know	18	8	23.90%
I didn't understand		1	.90%
I guessed	2	6	7.30%
Blank	20	16	33 10%

Approximately 20% of the students used a comparison strategy which focused on the two numbers in the question. One-half of these students ignored the percent sign on 87 and noted that 87 was greater than 10. Another three students gave 10 a percent sign and compared 87% and 10%. Six students compared a percent quantity, 87%, with a number that was not a percent, 10, and stated that "87% was greater than 10."

Nine percent of the unsuccessful students incorrectly used a computational procedure to support their conclusions. Three students subtracted 10 from 87 and judged 77 to be larger than 10. Another student may have remembered that percent is related to 100 in that the solution process involved subtracting 87 from 100 and the difference was determined to be greater than 10. Using the proportion method, one student correctly wrote the proportion to solve the problem, but did not place a decimal point in the quotient.

About three percent of the students gave an explanation which used counting or made reference to a whole. One of the students sketched a rectangle showing most of it shaded, but concluded that he could not tell what the answer would be. An additional four percent of the unsuccessful students gave explanations, such as "I thought it would be greater" and "Very carefully using math," that provided no details of their reasoning process.

The Research Interview

Twenty-eight students were selected to participate in individual interviews with the researcher. The focus of the discussion during each interview was the strategy or strategies the student used in answering questions from the test. The research interview permitted the gathering of data of more depth because the interviewer was able to follow up student statements, probing for more detail and clarification, to gain as much information as possible. The students selected to participate in interviews were a diverse group from whom information about their individual approaches to questions could be used to provide insight into students' understanding of percent.

During each student interview, the student was asked to answer questions from the test by choosing from among the multiple-choice responses. Follow-up questions asked the student to explain the reasons behind the choice of answer. A variety of strategies and reasoning procedures were explained and demonstrated by the students.

Many students demonstrated a good understanding of the pictorial representation of a percent quantity. The use of 50% and 100% as reference points was common, as illustrated by the student who said that 50% was in the middle and the shaded part was more than 50%. Another common reference used the fraction, one-half, in that the student decided whether a percent quantity was greater than or less than one-half. Given the picture of a rectangle with more than one-half shaded, one eighth grader noted that 33 1/3% was less than one-half and more than one-half of the rectangle was shaded.

Students were frequently able to give a percent estimate for the quantity shaded and to show in a given figure what would be shaded to represent a particular percent. For a rectangle with one-half shaded, students would comment that the shaded part looked like

50%. One student noted that, if four out of six circles were shaded, the shaded part would be about 70%. Given six circles with three shaded, a student indicated that the shaded part was less than 87%. When the interviewer asked about how many more circles should be shaded to show 87%, the student replied that it would be one and one-half more. Some students showed how to shade 33 1/3% of a rectangle by pointing where the shading should stop.

Questions using a set of circles or a rectangle where fourths could be used as reference points, were relatively easy for students. Given four circles with three shaded, students often noted that 75% was shaded. For them, each circle represented 25%. This relationship helped one eighth-grade student estimate how much of four circles to shade to show 33 1/3%. He approached the problem saying that he would shade at least one circle, which was 25%, and a little of a second circle. Fourths were also used as a basis for estimating the amount shaded in a rectangle. One student mentally divided each rectangle into fourths, so that each represented 25%, and judged the shaded part by comparing it to the closest number of fourths.

Not all students successfully answered the questions comparing 33 1/3% with four circles, three of which were shaded. One eighth-grade student stated that since three circles were shaded, the shaded part was equal to 33 1/3%, and if only two circles had been shaded, then the shaded part would be 33 1/2%.

One question in part one showed six circles with only the last circle shaded. For one student, the location of the shaded circle seemed to be a distractor. She stated that if three of the six circles were shaded then 50% would be shaded. But when only the last circle was shaded, she said that the shaded part was greater than 60%. She seemed to focus on the first five circles rather than the last circle which was the only one shaded. Another student's theory indicated that the location of the shaded circle gave the percent for that circle. In the same question showing the six circles with the last shaded, this student gave each circle a value of ten, so that the total for the circles was 60. She then stated that the "shaded one tells where it stops." Therefore, her conclusion was that the amount shaded was equal to 60% because it was shaded in the sixth spot.

A different question showed six circles with the first five shaded. For one seventh grader, the number of shaded objects was the same as the percent. In this case, since five circles were shaded, 5% was shaded. When asked what percent would be shaded if all six circles were shaded, he stated that 6% would be shaded. The interviewer asked him to think about the six circles with only three shaded. In that case, he stated that one-half of the circles would be shaded, and that would be 3%.

The last question in part one pictured six circles with three shaded and asked the student to compare the shaded part with 87%. Two seventh graders chose "equal to 87%" from the multiple-choice responses. Their identical explanations noted that one-half or three circles were shaded while the other half or three of the circles were not shaded, and so it was equal. Neither student, in this explanation, mentioned 87%. One student continued in his explanation, indicating that the six circles made up the 87%, and each circle had a certain percentage in it. To support this statement, he divided six into 87 and took the whole number part of the quotient, 14, multiplied by three, then placed this product over 87 to form the fraction, 42/87.

The students had the most difficulty with part three, the abstract section of the test. Students' strategies for answering these questions included the use of visual models, fractional relationships, estimation, comparison, and a variety of computational procedures. Some students showed a solid understanding of the meaning of a quantity expressed as a percent of a number, while others revealed a state of confusion, as well as a number of inventive reasoning strategies.

Many students used 100% as a reference point, noting that the percent in the problem was greater than, less than, or equal to 100%, and drawing their conclusion based on those results. One eighth-grade student answered that 60% of 35 was less than 35, explaining that "If it's greater than 100%, it's greater than 35. So it's less than 100%, it's less than 35." Another eighth grader explained that "even if it's 99%, it would be less because it's not 100%."

Those students with a good concept of 100% as the whole often used this fact to explain why "110% of 145 was greater than 145." Several students explained that 110% of 145 is 10% more than 145. When asked to estimate how much 110% of 145 would be, a common response was ten more or 155. Others responded that it would be a little bit more than 145, or gave a range of 10 to 20 more than 145. One student who had demonstrated an understanding of 100% and 110% was asked what 200% of 145 would be. Her prompt response was 145 times two.

While some students showed an understanding of 100% and 110%, others demonstrated a lack of understanding. Stating that 110% of 145 was less than 145, one seventh-grade student drew a rectangle to use as a model, labeled the middle of the rectangle as 150% and noted that 110% was less than 150%. Some students who claimed that "110% of 145 was less than 145," were asked what percent of 145 would be equal to 145. Two students responded that 145% of 145 would be 145.

Fractional relationships were used by some students in problems in part three involving 50% and 25%. Several students specifically noted that 50% was 1/2 and 25% was 1/4. One student commented that one quarter is 25% of one dollar. In determining that 25% of 15 was less than 15, one seventh grader said "there's four 25 percents to be 100, so take four into 15." This same student decided that 33 1/3% of 30 was less than 30 because "you could get about three of those into 30."

A common strategy given as an explanation by 18% of the unsuccessful students on the open-ended item on the test compared the sizes of the two numbers in the question. This strategy was used at least once by 25% of the students interviewed. Students using this strategy would explain that 50% of 20 was greater than 20 because 50% was greater than the number 20 or because 50% was greater than 20%. One student explained that 33 1/3% of 30 was greater than 30 because it was 3 1/3% over 30. An eighth-grade student responded that 25% of 15 was greater than 15. To illustrate, she drew a rectangle, correctly marked where 25% of the rectangle would be, and marked 15 to the left of 25%, explaining that

25% was greater than 15. While most students retained the order of the numbers as presented in the problem, one eighth-grade student responded that "110% of 145 was greater than 145" because 145 was greater than 110. He also concluded that "100% of 145 was less than 145" because 100 was less than 110.

For students who were confused or had difficulty answering the questions in part three of the test, the use of a visual model was often helpful. Some students selected their own models, while the interviewer offered a possibility for others. One student used the rectangle as a model for the problems he answered in the abstract section. A seventh-grade student used his ten fingers as a model for 100% and illustrated 33 1/3% by showing three fingers and part of a fourth finger. For other students, a suggested model was something familiar and real to them, such as soccer balls, football passes, or a number of test questions. Using these models, the students could often correctly answer questions to which they had previously given no answer.

Even though the questions in the abstract section of the test could be answered without computing, 36% of the students who were interviewed used some form of computational procedure on some of the questions. One student estimated 87% of 10 to be about 8, while another mentally computed 87% of 10 to be 8.7. An eighth-grade student found 25% of 15 to be 3.75, explaining that 50% of 15 was one-half of 15 which was 7.5, and one-half of 7.5 was 3.75. As a clarification, she explained that she knew that 25% was one-half of 50%. To find 60% of 35, one seventh grader noted that one-half of 35 was 17 and that 10% more was needed.

Students who used subtraction to answer these types of abstract questions would respond that 50% of 20 was greater than 20 because 50% minus 20 was 30, which was greater than 20. This method, however, would result in a correct selection from among the multiple-choice responses for 25% of 15 and 100% of 50. One student remarked that 87% of 10 was a lot greater than 10. He noted that "87% goes to 90, so it's 80% more than 10."

A few students demonstrated individual computational strategies to support their response to questions in this section of the test. One eighth-grade student answered that "50% of 20 was greater than 20" and "60% of 35 was greater than 35." To support each conclusion, he constructed fractions, 20/50 = 2/5 and 35/60 = 7/12, respectively. He also answered that "25% of 15 was less than 15." When asked to explain his reason, he said "because it was 25%."

A seventh grader did not choose a multiple-choice response to the question comparing 50% of 20 to 20. Instead, she used paper and pencil to compute by first dividing 100 by 20 and multiplying the quotient, 5, by 50 which gave a product, 250. She stopped to think about her answer, explaining that it did not seem to be correct, but was not able to determine what was wrong. The same student said she did not know the answer to the question comparing 25% of 15 to 15. The interviewer asked her to consider 50% of 15 and she responded that it would be about 7 1/2. When asked again about 25% of 15, she answered that 25% split 15 in half and seven times three was equal to 21. When the interviewer asked her to explain the relevance of seven times three, she promptly answered showing 25 divided by seven, generating a quotient of three and ignoring the remainder, four.

Another eighth-grade student consistently used a division algorithm to support her selection from the multiple-choice responses. To the first question, she initially answered that 50% of 20 was greater than 20. The procedure she used involved dividing the percent number into the other number in the question. The use of this algorithm prompted her to change her answer to less than 20, because her quotient was 0.4. Her consistent use of the algorithm, led her to reply that "100% of 50 was less than 50" because 50 divided by 100 was 0.5, and "87% of 10 was less than 10" because 10 divided by 87 was 0.114.

During the interviews, some students were asked about the relationship of fractions and decimals to percents. One student stated that they probably would be related "somewhere down the line, maybe in algebra." Other students gave examples which commonly included 50% as 1/2, 75% as 3/4, and 25% as 1/4. Decimal equivalents were mentioned by one student who stated that 1/2 was 0.5. Other fractions mentioned included 20/100 and 1/10. One eighth-grader explained the relationship between percents and fractions with examples. He began by noting that the fraction, 51/50, would be greater than 100%, and continued indicating 150% would be 75/50 because you need one-half of 50 more than 50.

After naming several examples of fractions with equivalent percents, an eighth-grade student demonstrated the Z method, which was used to convert a fraction, without 100 as a denominator, to a percent. An example equation was written using 5/8 as one fraction and a second fraction with 100 as the denominator and a blank space for its numerator. He proceeded to divide 100 by 8 and multiplied the quotient, 12.5, by 5 generating the product, 62.5, which he named as 62.5%. In tracing his steps, he drew a line segment from 8 to 100, then from 100 to 5, and over to the blank numerator space, creating a letter, Z.

Summary

This study combined quantitative and qualitative methods to provide insight into the understanding students have about the concept of percent. The results of the study provided a description of some aspects of students' number sense related to percent, including their ability to interpret a quantity expressed as a percent given a pictorial set, their level of understanding of a quantity expressed as a percent of a number, and the strategies they used to make comparisons about percent quantities.

The quantitative data from the written test provided statistical information from which comparisons could be made. On each part of the test, as well as on the overall test, the mean scores of eighth-grade students were higher than the mean scores of seventh-grade students. As a group, students in both grade levels did best on questions in part two, where they were asked to compare a percent quantity with the shaded part of a rectangle. The students' performance was poorer on questions in part one where they were asked to compare a percent quantity with the shaded part of a discrete set of circles. The students had the most difficulty with questions in part three, interpreting a quantity expressed as a percent of a number.

One open-ended test item asked each student to write an explanation of how he or she decided on the response to the test question comparing 87% of 10 to 10. Of the students taking the test, 45% successfully answered that "87% of 10 was less than 10." Fifty percent of these successful students gave written explanations which revealed an understanding of the percent situation and the application of an appropriate reasoning process. However, 11% of the successful students gave explanations which showed a lack of understanding of the percent question. The students who were unsuccessful in comparing 87% of 10 to 10 gave a variety of inappropriate written explanations which included comparison and computation strategies. More than 65% of the unsuccessful students provided no explanation for their responses.

Additional information on the strategies students used in making comparisons about percent quantities was gathered during individual interviews with 28 students. Many of the students demonstrated a good understanding of percent quantities represented pictorially in a continuous region or a discrete set. Some students also used pictorial models when answering abstract questions. The students were especially comfortable using 50% and 100% as reference points in making comparisons.

Several students presented theories and relationships they had developed about percent concepts. There were attempts by some students to use or develop an inappropriate computational procedure to answer the questions in part three of the test. As one seventh grader announced "If you don't do anything, it looks weird."

Combining the results of the quantitative and qualitative data analysis gives a description of some aspects of students' number sense related to percent. The findings of this study contribute to a better understanding of students' knowledge of percent and may be used to provide guidance in the development of effective instructional programs.

CHAPTER V

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary

Percent is an important practical mathematical concept because of the many everyday areas in which it is used. An understanding of the concept of percent would incorporate some aspects of number sense which would include understanding the meaning of numbers expressed as percents, comparing quantities expressed as percents, and recognizing the relative effect of finding a percent of another number. The purpose of this study was to investigate middle school students' number sense related to percent.

The sample for the study included seventh-grade students (n = 106) and eighth-grade students (n = 93) enrolled in average mathematics classes in Putnam City Schools in the metropolitan Oklahoma City area during the school year, 1990-91. This study used both quantitative and qualitative methods to answer the following research questions:

1. Can students interpret a quantity expressed as a percent given a pictorial discrete set or continuous region with part or all of the area shaded?
2. Do students understand the meaning of a quantity expressed as a percent of a number?

3. What strategies do students use to make comparisons about percent quantities in both pictorial and abstract settings?

The written instrument used in the study was developed by the researcher. Questions in part one of the test were designed to determine if students could compare a percent quantity with the shaded part of a discrete set of circles. Questions in part two of the test were designed to investigate students' ability to compare the shaded part of a continuous rectangular region with a given percent quantity. Questions in part three, based on a released exercise used by the fourth National Assessment, asked students to determine if a quantity expressed as a percent of a number was less than, greater than, or equal to the given number. The mean scores, standard deviations and range of scores were determined for each grade level for each section of the test as well as the overall test.

One open-ended question on the written test asked the student to explain how he or she decided on the multiple-choice response to the test item comparing 87% of 10 to 10. A content analysis of the open-ended responses generated categories which were used to organize the data provided by the students' explanations. Individual interviews were conducted with 28 selected students to identify strategies students used to make comparisons about percent quantities. Inductive analysis was used to locate patterns and themes in the interview data around which the narrative discussion was organized. The data from the written test and the interviews was collected in September, 1990.

The results of the data analysis provide a description of some aspects of students' understanding of the concept of percent.

Students did best on questions comparing a percent quantity with the shaded area of a continuous rectangular region and poorest on questions interpreting a quantity expressed as a percent of a number. Approximately one-fourth of the students provided an appropriate explanation to support the conclusion that "87% of 10 was less than 10." Another 48% of the students had no explanation for their responses, whether correct or incorrect. Eleven percent of those students who responded correctly that "87% of 10 was less than 10," gave an explanation that demonstrated a lack of understanding of the question. A variety of computational and comparison strategies were proposed by students who responded that "87% of 10 was greater than or equal to 10."

Additional information on the strategies students used was provided during the individual interviews. Some students demonstrated a thorough understanding of percent, as it was used in the situations presented in the written test. Other students have an understanding of familiar percents such as 50%, 25% and 100% but are uncertain when working with other percent quantities. The variety of procedures, theories, and relationships which the students had developed and demonstrated during the interviews and in their written explanations indicated a wide range of levels of understanding about percent concepts.

Conclusions

The following conclusions address the three research questions of the study and provide some additional insight into students' understanding of percent These conclusions are presented in the context of the limitations cited for this study

(1) Students did better interpreting a quantity expressed as a percent when a pictorial continuous region was used than when a pictorial discrete set of circles was used Students did better on both types of questions when the percent was 50%, 100% or 25%, than when 60%, 110%, 33 1/3% or 87% was used Students who were interviewed were frequently able to give an estimate expressed as a percent for the area of the figure or set that was shaded

(2) Students had difficulty interpreting a quantity expressed as a percent of a number Students were more successful with questions using 50% and 100% than with other percent quantities Many students demonstrated understanding of 100% as the whole and would use 100% as a reference point, noting that the percent in the problem was greater than, less than, or equal to 100%

(3) The strategies used by students to make comparisons about percent quantities represent a wide range of correct and incorrect approaches to the questions In addition to the use of 50% and 100% as common reference points, students successfully applied fractional relationships, estimation and mental computation to answer the questions A variety of inappropriate strategies which included computational procedures and numerical comparisons were also employed, some of which resulted in the correct multiple-choice response

(4) Students exhibited the use of internal mental images of some percent quantities Many of them could describe or pictorially demonstrate the quantity represented by a percent Students

demonstrated skills in translating between pictorial representations and written symbols One student used a rectangle to represent the quantity 35, marking one side of the rectangle as zero and the opposite side as 35, and showing 60% of 35 as slightly to the right of the middle of the rectangle Other students used four circles to represent 100%, noting that 87% would be represented by three shaded circles and part of the fourth circle shaded

(5) Even though students are taught the relationship among fractions, decimals, and percents, and students could cite specific examples of fractions and decimals which were equivalent to percents, they did not seem to use the interrelationships among numerical representations with confidence Montgomery (1958), in his investigation of difficulties students had in mastering the cases of percent problems, found similar results He noted that pupils had difficulty applying knowledge about fractions and decimals when solving percent problems

(6) Students have constructed some erroneous relationships in their knowledge of percent Students often used their incorrect rules and procedures with confidence These erroneous relationships have been shaped by a students' experience and reflect each student's current level of understanding of percent (Wadsworth, 1989) Some of these relationships and procedures are a result of students' attempts to make sense of percent and the instruction they have experienced

(7) Some student-developed incorrect strategies resulted in a correct selection from among the multiple-choice responses Some

students who chose the correct response supported it with an inappropriate reasoning process

(8) For some students, the percent symbol had no meaning They responded as if it was not there or simply added it on to any other number in the problem Students who computed with the numbers in the question, such as those who subtracted, often just dropped the percent symbol from the number

(9) Judging a student's knowledge of percent using only the written test results could generate misleading conclusions For example, a student who consistently and accurately used subtraction of the two numbers in the problem, would have four out of seven questions answered correctly The teacher may conclude that the student understands the concept of 100%, is able to work with 25%, 60%, and 33 1/3%, has difficulty with 110%, and perhaps made careless errors in the problems with 50% and 87% If the student has not shown any work on the problems, the teacher may never know what this student understands about percent, unless the teacher uses an individual interview which gives the student the opportunity to discuss thinking strategies being used

During this study, the use of the individual interview during which the focus was on a student's description of what he or she was thinking and doing, provided valuable information on the level of student understanding The variety of both correct and incorrect explanations was surprising Of particular interest was the use of number comparisons, where students focused on the sizes of the two numbers in the problem, and the use of inappropriate computational procedures Many students have some understanding of percent quantities, especially in pictorial settings, while other concepts, including finding the percent of a number, are not well-understood.

(10) A significant number of students were not able to provide a written explanation to support their conclusions in comparing 87% of 10 with 10. It is likely that students do not encounter questions asking for a written explanation during their mathematics classes. It is also possible that students had no explanation and simply guessed by selecting an answer to the question.

Recommendations

Five recommendations are offered for mathematics educators who teach percent and for others interested in further research.

(1) The use of pictorial representations of percent quantities is important and students should be encouraged to use visual models as they encounter abstract problems to help them estimate a reasonable answer. Hiebert and Wearne (1986) noted that an instructional priority is to help students create meaning for symbols indicating the use of appropriate conceptual referents as one possibility. Driscoll (1983) cited the importance of helping students see how visual and algorithmic approaches to fractions are related and suggested that instruction on fractions integrate the use of diagrams with the use of algorithms.

(2) More instructional time should be spent helping students develop an understanding of percent quantities. Translations among representations including pictorial to symbolic, as well as, among the numerical representations as a fraction, decimal and percent,

and the relative effect of finding a percent of another number, should be emphasized.

(3) Teachers should encourage all students to verbalize solution and reasoning processes for both correct and incorrect responses. Students should experience providing explanations both orally and in writing.

(4) Teachers should use and create more opportunities for individual discussions with students where the focus is on each student's thinking strategies. Teachers need to ask students how they arrive at conclusions, focusing on the process and relationships they employed in answering a question. Enough time should be allowed for students to formulate explanations. Discussions with students can reveal which procedures and concepts are wrong, those that are correct, as well as the fundamental supporting ideas.

(5) Because percent is an important practical mathematical concept used in many everyday settings, students need number sense about percent to make informed decisions. In this study middle school students demonstrated a wide range of ability and levels of understanding of the concepts of percent and used some erroneous strategies in reasoning about percent quantities. Further research is recommended to identity effective instructional practices to help students develop number sense and a firm understanding of percent.

Since the early 1900s, researchers have been concerned that students give unreasonable answers to questions about percent. In particular Edwards (1930) noted that students gave impossible, senseless answers to problems involving percent, and he blamed teachers for this failure to help students develop a sufficient sense for the problems. This study indicated that students still give senseless answers to questions about percent which reflect little use of number sense. It is time to bring this focus on number sense to the middle grades' classrooms to enhance students' mathematical literacy.

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APPENDIXES

APPENDIX A

MAIN STUDY INSTRUMENT

September, 1990

Name:____

Grade: [ ] 7 [ ] 8 Sex: [ ] Male [ ] Female Age:________

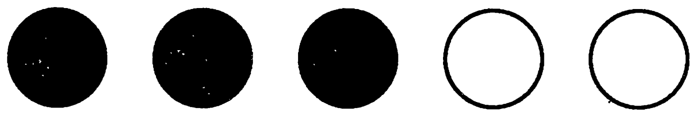
Ethnic Origin: [ ] White [ ] Hispanic
[ ] Black [ ] Asian
[ ] Native American [ ] Other

School:____

Teacher:____

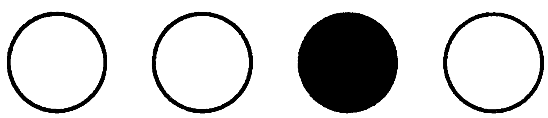
PART 1

Choose the best response for each question. Circle the letter of your choice.



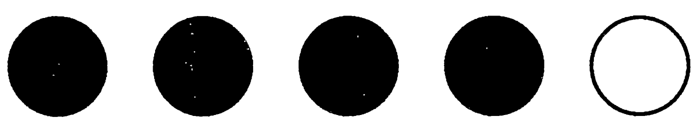
1. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 50%.
   B. It is less than 50%.
   C. It is equal to 50%.
   D. Can't tell.
   E. I don't know.



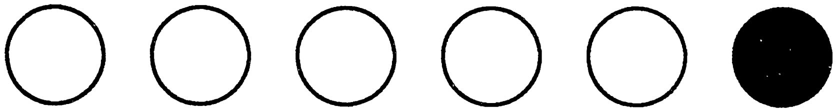
2. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 25%.
   B. It is less than 25%.
   C. It is equal to 25%.
   D. Can't tell.
   E. I don't know.



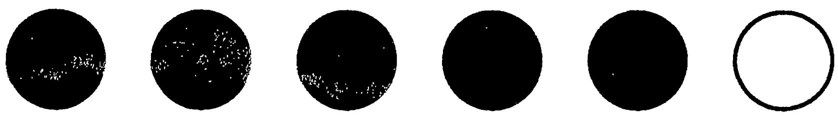
3. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 100%.
   B. It is less than 100%.
   C. It is equal to 100%.
   D. Can't tell.
   E. I don't know.



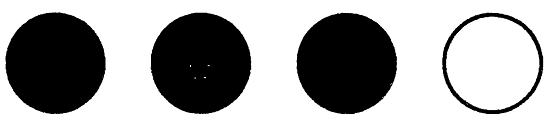
4. In the figure above, which of the following is true about the shaded part?

A. It is greater than 60%.
B. It is less than 60%.
C. It is equal to 60%.
D. Can't tell.
E. I don't know.



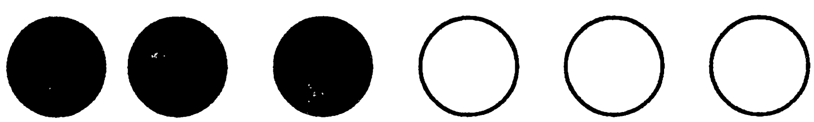
5. In the figure above, which of the following is true about the shaded part?

A. It is greater than 110%.
B. It is less than 110%.
C. It is equal to 110%.
D. Can't tell.
E. I don't know.



6. In the figure above, which of the following is true about the shaded part?

A. It is greater than 33 1/3%.
B. It is less than 33 1/3%.
C. It is equal to 33 1/3%.
D. Can't tell.
E. I don't know.

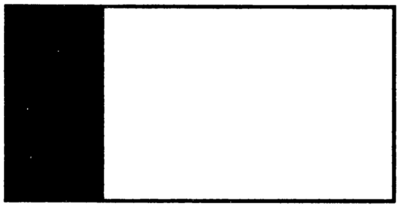


7. In the figure above, which of the following is true about the shaded part?

A. It is greater than 87%.
B. It is less than 87%.
C. It is equal to 87%.
D. Can't tell.
E. I don't know.

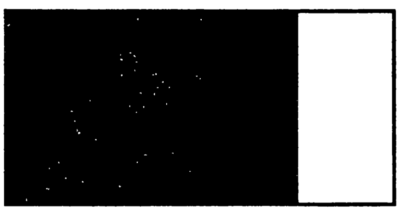
PART 2

Choose the best response for each question. Circle the letter of your choice.



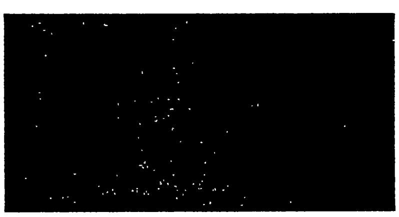
1. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 50%.
   B. It is less than 50%.
   C. It is equal to 50%.
   D. Can't tell.
   E. I don't know.



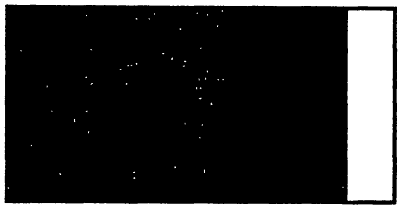
2. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 25%.
   B. It is less than 25%.
   C. It is equal to 25%.
   D. Can't tell.
   E. I don't know.



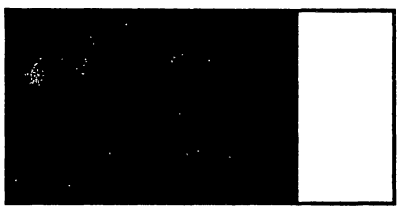
3. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 100%.
   B. It is less than 100%.
   C. It is equal to 100%.
   D. Can't tell.
   E. I don't know.



4. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 60%.
   B. It is less than 60%.
   C. It is equal to 60%.
   D. Can't tell.
   E. I don't know.



5. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 110%.
   B. It is less than 110%.
   C. It is equal to 110%.
   D. Can't tell.
   E. I don't know.

6. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 33 1/3%.
   B. It is less than 33 1/3%.
   C. It is equal to 33 1/3%.
   D. Can't tell.
   E. I don't know.

7. In the figure above, which of the following is true about the shaded part?

A. It is greater than 87%.
B. It is less than 87%.
C. It is equal to 87%.
D. Can't tell.
E. I don't know.

PART 3

Choose the best response for each question. Circle the letter of your choice. Use any of the space on the right if you want to show your work.

1. Which of the following is true about 50% of 20?

A. It is greater than 20.
B. It is less than 20.
C. It is equal to 20.
D. Can't tell.
E. I don't know.

2. Which of the following is true about 25% of 15?

A. It is greater than 15.
B. It is less than 15.
C. It is equal to 15.
D. Can't tell.
E. I don't know.

3. Which of the following is true about 100% of 50?

A. It is greater than 50.
B. It is less than 50.
C. It is equal to 50.
D. Can't tell.
E. I don't know.

4. Which of the following is true about 60% of 35?

A. It is greater than 35.
B. It is less than 35.
C. It is equal to 35.
D. Can't tell.
E. I don't know.

5. Which of the following is true about 110% of 145?

   A. It is greater than 145.
   B. It is less than 145.
   C. It is equal to 145.
   D. Can't tell.
   E. I don't know.

6. Which of the following is true about 33 1/3% of 30?

   A. It is greater than 30.
   B. It is less than 30.
   C. It is equal to 30.
   D. Can't tell.
   E. I don't know.

7. Which of the following is true about 87% of 10?

   A. It is greater than 10.
   B. It is less than 10.
   C. It is equal to 10.
   D. Can't tell.
   E. I don't know.

8. Explain how you decided on your answer to number 7.

PART 4

1. My grade in math last year was ____________.

   I don't remember exactly, but I think my grade in math last year was_________.

2. How good are you usually in math? Circle the letter of your answer.

   A. Very good
   B. Good most of the time
   C. Good some of the time
   D. Not very good

APPENDIX B

STRUCTURE OF THE RESEARCH INTERVIEW

Outlined Structure of the Student Interview

I. Establish rapport to help the student feel comfortable.

II. Briefly outline the purpose of the visit as learning more about how students think about percent so that I can help teachers help their students become better with percent.

III. Briefly outline the procedure for the visit.
   A. The student will be given a question from the percent test and asked to answer the question aloud.
   B. The student will be asked to share aloud how he or she decided on the answer.

IV. Hand the student one of the questions from the test. (Questions have been prepared individually so the student sees only one question at a time.) Ask the student to respond to the question.

V. If the student does not have an answer, suggestions may be offered to help prompt thinking, such as: "Perhaps it would help if you thought about 20 as $20."

VI. If the student struggles with a process, he or she will be asked to share aloud what he or she is thinking about in trying to answer the question. Suggestions may be offered, such as: "Perhaps it would help if you thought about objects."

VII. Once the student answers the question, the student will be asked to discuss how he or she decided on the answer.

VIII. Conclusion. After completing 4-8 questions, thank the student for participating and for his or her help with the project.

APPENDIX C

PILOT STUDY INSTRUMENT

May 17, 1990

Name:____

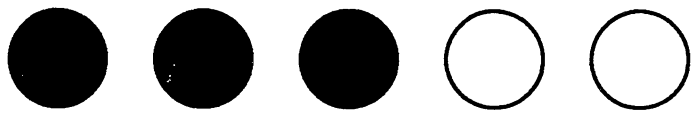
Grade:____

School:____

Teacher:____

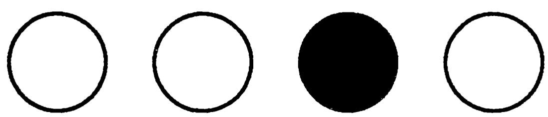
PART 1

Choose the best response for each question. Circle the letter of your choice.



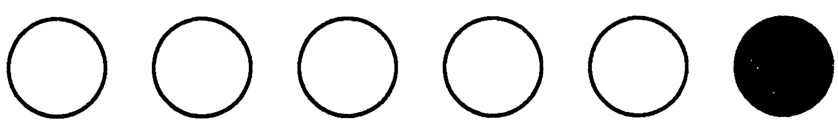
1. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 50%.
   B. It is less than 50%.
   C. It is equal to 50%.
   D. Can't tell.
   E. I don't know.



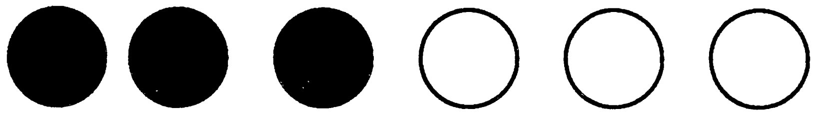
2. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 25%.
   B. It is less than 25%.
   C. It is equal to 25%.
   D. Can't tell.
   E. I don't know.



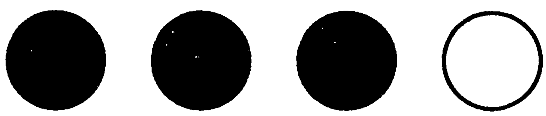
3. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 60%.
   B. It is less than 60%.
   C. It is equal to 60%.
   D. Can't tell.
   E. I don't know.



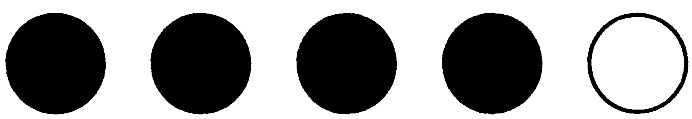
4. In the figure above, which of the following is true about the shaded part?

A. It is greater than 87%.
B. It is less than 87%.
C. It is equal to 87%.
D. Can't tell.
E. I don't know.



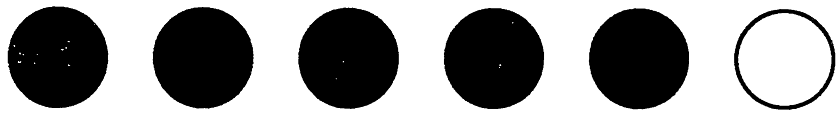
5. In the figure above, which of the following is true about the shaded part?

A. It is greater than 33 1/3%.
B. It is less than 33 1/3%.
C. It is equal to 33 1/3%.
D. Can't tell.
E. I don't know.



6. In the figure above, which of the following is true about the shaded part?

A. It is greater than 100%.
B. It is less than 100%.
C. It is equal to 100%.
D. Can't tell.
E. I don't know.



7. In the figure above, which of the following is true about the shaded part?

A. It is greater than 110%.
B. It is less than 110%.
C. It is equal to 110%.
D. Can't tell.
E. I don't know.

PART 2

Choose the best response for each question. Circle the letter of your choice.

1. Which of the following is true about 12?

A. It is greater than 5.
B. It is less than 5.
C. It is equal to 5.
D. Can't tell.
E. I don't know.

2. Which of the following is true about 1/2?

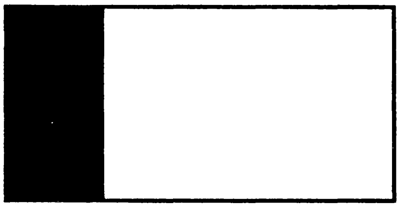
A. It is greater than 3.
B. It is less than 3.
C. It is equal to 3.
D. Can't tell.
E. I don't know.

3. Which of the following is true about 1.6?

A. It is greater than 3.4
B. It is less than 3.4
C. It is equal to 3.4
D. Can't tell.
E. I don't know.

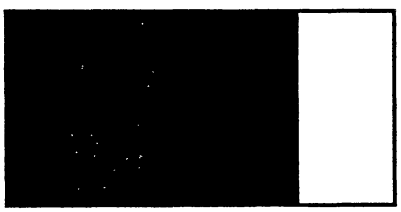
PART 3

Choose the best response for each question. Circle the letter of your choice.



1. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 50%.
   B. It is less than 50%.
   C. It is equal to 50%.
   D. Can't tell.
   E. I don't know.

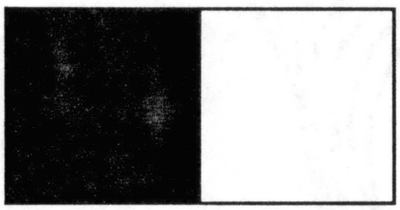


2. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 25%.
   B. It is less than 25%.
   C. It is equal to 25%.
   D. Can't tell.
   E. I don't know.

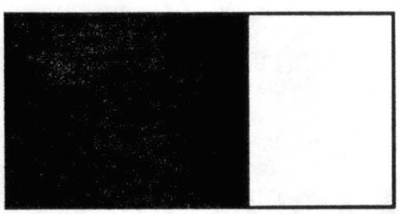
3. In the figure above, which of the following is true about the shaded part?

A. It is greater than 60%.
B. It is less than 60%.
C. It is equal to 60%.
D. Can't tell.
E. I don't know.



4. In the figure above, which of the following is true about the shaded part?

A. It is greater than 87%.
B. It is less than 87%.
C. It is equal to 87%.
D. Can't tell.
E. I don't know.

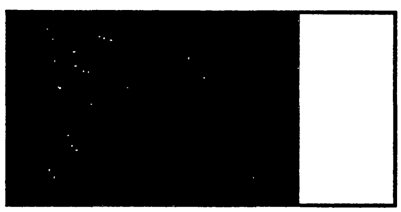


5. In the figure above, which of the following is true about the shaded part?

A. It is greater than 33 1/3%.
B. It is less than 33 1/3%.
C. It is equal to 33 1/3%.
D. Can't tell.
E. I don't know.

6. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 100%.
   B. It is less than 100%.
   C. It is equal to 100%.
   D. Can't tell.
   E. I don't know.



7. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 110%.
   B. It is less than 110%.
   C. It is equal to 110%.
   D. Can't tell.
   E. I don't know.

PART 4

Choose the best response for each question. Circle the letter of your choice. Use any of the space on the right if you want to show your work.

1. Which of the following is true about 50% of 20?

   A. It is greater than 20.
   B. It is less than 20.
   C. It is equal to 20.
   D. Can't tell.
   E. I don't know.

2. Which of the following is true about 25% of 15?

   A. It is greater than 15.
   B. It is less than 15.
   C. It is equal to 15.
   D. Can't tell.
   E. I don't know.

3. Which of the following is true about 60% of 35?

   A. It is greater than 35.
   B. It is less than 35.
   C. It is equal to 35.
   D. Can't tell.
   E. I don't know.

4. Which of the following is true about 87% of 10?

   A. It is greater than 10.
   B. It is less than 10.
   C. It is equal to 10.
   D. Can't tell.
   E. I don't know.

5. Explain how you decided on your answer to number 4.

6. Which of the following is true about 33 1/3% of 30?

   A. It is greater than 30.
   B. It is less than 30.
   C. It is equal to 30.
   D. Can't tell.
   E. I don't know.

7. Which of the following is true about 100% of 50?

   A. It is greater than 50.
   B. It is less than 50.
   C. It is equal to 50.
   D. Can't tell.
   E. I don't know.

8. Which of the following is true about 110% of 75?

A. It is greater than 75.
B. It is less than 75.
C. It is equal to 75.
D. Can't tell.
E. I don't know.

PART 5

1. What grade did you make in math last nine weeks? Circle the letter of your answer.

A B C D F

2. How good are you usually in math? Circle the letter of your answer.

A. Very good
B. Good most of the time
C. Good some of the time
D. Not very good

3. Rank these percents from easiest to hardest. Place a number in each blank. Put a 1 by the easiest, a 2 by the next easiest, and so on, with a 7 by the hardest.

________ 50%
________ 25%
________ 60%
________ 87%
________ 33 1/3%
________ 100%
________ 110%

APPENDIX D

PILOT STUDY RESULTS

Results of the Pilot Study

Students participating in the pilot study were enrolled in average mathematics classes in Putnam City Schools during May, 1990, and included sixth graders (n = 27), seventh graders (n = 26), and eighth graders (n = 27). One seventh-grade student refused to answer any of the content questions, but did respond to the questions in part five. His responses to all questions except the last one were not included in the tabulation of the data.

The mathematics coordinator for Putnam City Schools administered the test to students in their regular mathematics classrooms. Her instructions were similar to each class, though she provided some clarification on some questions to classes and to individual students. Most questions were related to items in part five.

One question to be answered by the pilot study concerned whether students understood the terms "greater than" and "less than." The questions in part two of the test were designed to answer this concern. The overall percent of correct responses to these three questions was 89%. Table IX reports the percent correct for each question. The one eighth-grade student who chose the wrong response to the first question missed none of the percent questions. It is possible the problem was simply misread. During the administration of the test, it was noted that some of the eighth graders were not responding conscientiously.

The overall reliability coefficient of the written instrument was .855. For each grade subgroup, the reliability coefficients were

TABLE IX

PERCENT CORRECT FOR QUESTIONS IN PART TWO
ON THE PILOT STUDY INSTRUMENT

	Sixth Grade	Seventh Grade	Eighth Grade
$12 > 5$	93%	100%	96%
$1/2 < 3$	78%	100%	85%
$1.6 < 3.4$	81%	84%	81%

.737 for sixth grade; .863 for seventh grade; and .915 for eighth grade.

The last question in part five was designed to establish an order of familiarity of the percents used in the test. Tabulating student rankings, the order from most familiar to least familiar was: 50%, 25%, 100%, 60%, 110%, 33 1/3%, and 87%.

Student performance on the three parts of the test dealing with percent concepts are shown in Table X. Each section had seven questions.

In part one, which involved pictures of discrete sets, students did best on the questions involving 50%, 60%, and 100%. On part three which used pictures of continuous regions, students did best on the questions involving 50%, 25%, 100%, and 110%. On part four where abstract questions compared the percent of a number, students did best on questions using 50%, 25%, and 110%. Seventh- and eighth-grade students did considerably better than sixth graders on the question about 100% in part four.

Students were asked for their grade in mathematics last nine weeks. The overall mean score out of 21 possible is reported in Table XI for each grade level and each mathematics grade.

Students also gave an indication of their self-confidence in mathematics by rating how they perceived themselves as mathematics students. The overall mean score is reported in Table XII for each grade level and each confidence rating.

The open-ended question in part four asked students to explain the choice made in question four judging the size of 87% of 10.

TABLE X

MEAN SCORES FROM THE PILOT STUDY

	Sixth Grade	Seventh Grade	Eighth Grade
Part one Discrete Sets	5.26	5.96	5.67
Part three Continuous Regions	5.96	6.60	6.26
Part four Abstract	3.41	4.92	5.30

TABLE XI

MEAN SCORES BY MATHEMATICS LETTER GRADE
IN THE PILOT STUDY

Letter Grade	Sixth Grade	Seventh Grade	Eighth Grade
A	13.00	18.80	17.40
B	15.76	18.00	18.60
C	14.00	16.00	17.50
D	12.00	21.00	15.70
F	10.00	17.00	16.50

TABLE XII

MEAN SCORES BY SELF-ASSESSMENT OF MATHEMATICS ABILITY IN THE PILOT STUDY

Self Assessment	Sixth Grade	Seventh Grade	Eighth Grade
Very Good	14.7	19.0	19.6
Good most of the time	14.0	18.1	17.2
Good some of the time	15.7	15.5	19.6
Not very good	None	15.7	12.6

Explanations were organized first into two groups, those responding correctly to question four and those responding incorrectly. A classification scheme of strategies used and the number of students giving each type of response are shown in Tables XIII and XIV.

Eight days after the administration of the test, seven seventh-grade students were interviewed as a part of the pilot study. Questions from each of the three parts of the test had been selected prior to the interviews because of the degree of success or difficulty experienced by students taking the written test. During each interview the student answered from four to eight questions. After selecting a response choice, the student was asked to explain how the choice was made. At times when it was obvious the student was processing information to arrive at a response, the interviewer would request that the student explain what he or she was thinking about and doing to answer the question.

Evidence of a reliance on procedure and a concern for remembering the correct procedure was seen in the responses of two students. One said she does not remember whether you multiply nor what to multiply. In working problems in part four, this student always wanted to multiply or divide. Responding correctly that "50% of 20 was less than 20," her reason was that $50 \div 20 = 2$, which was less than 20. Another student described changing decimals to percents as hard because she does not remember how to move the decimal point. One student used the proportion method, explaining that the percent over one hundred is always one fraction and the other fraction has the number after the *of* on the bottom. She calculated well in her head using the cross product method. Every

TABLE XIII

INCORRECT RESPONSES FROM THE PILOT STUDY

Strategy	Number of Responses
Comparison of two numbers, either both as percents or both without the percent sign	9
Division	2
Incorrect use of proportion	1
Illogical response	1
No explanation (includes I don't know; I guessed; I just did it)	19

TABLE XIV

CORRECT RESPONSES FROM THE PILOT STUDY

Strategy	Number of Responses
Correct conceptualization that compared 87% < 100%, the whole	14
Correct conceptualization that focused on 87% of a number is less than the number	10
Multiplication using percent as a decimal	3
Estimation	1
Thought of an application, like discount	1
Incomplete explanations that focus on a relationship to 100% or the size of the percent	2
Incorrect comparison of 87 and 10	2
Division (80 ÷ 10 < 10)	1
Comparison centered on the size of the base (i.e., it is only a percent of 10, not a percent of a number greater than 10)	1
Illogical response	6
No explanation (includes I don't know; I guessed)	6

problem in part four was solved by this calculating procedure. She used little, if any, reliance on the size of the percent in the problem, though she did correctly estimate 87% of $10 to be about $8.

Several students showed a good understanding of 100% of something, but did not use it as a reference to help answer other questions. Questions with 33 1/3% were difficult for students. Only one student showed an understanding of the quantity, 33 1/3%, when she estimated the size of the shaded area of a continuous region that would represent 33 1/3%. Another student decided that because 33 1/3% had a fraction that another circle was needed in the discrete set and that one-half of that circle should be shaded.

One student answered questions from part four with some inconsistency. He concluded that "60% of 35 was greater than 35," comparing the size of the numbers. His first response to 87% of 10 was that it was greater than 10. The interviewer suggested he think about $10 and asked about how much 87% of $10 would be. He responded very promptly that it would be "about eight or nine dollars." He changed his response on the question, choosing less than 10. When asked again about 60% of 35, thinking about 35 as $35, he insisted that 60% of 35 was still greater than 35.

Several of the students associated specific number values with each element of a discrete set. For example in part one, question four shows three of six circles shaded. One student explained that each circle was about 20 so the shaded part was less than 87%.

A version of this strategy was successfully applied to abstract questions in part four by one student. He had previously answered all of the questions from part one correctly. His first thoughtful response to the question of 50% of 20 was "I don't know." The interviewer suggested it might help if he thought about objects, like circles. He decided to think about two circles, each representing ten, so that 50% would be one circle. He chose the correct answer that "50% of 20 is less than 20." On the question of 60% of 35, he thought of 3 1/2 circles, each representing ten, and reasoned that 60% would be about two shaded circles.

With the next questions, his strategy changed to the use of a linear model, resembling a number line. For the question dealing with 33 1/3% of 30, he indicated where 30 was on the line and showed 33 1/3% as being to the right of 30. When asked for clarification, he said that 33 1/3% was three more than 30. For the question of 110% of 75, he correctly used the linear model, indicating where 75 would be, and mentioning that 110% of 75 is a little more than 75 and would be found just a little to the right of 75.

APPENDIX E

MIDDLE SCHOOL TEACHER SURVEY

September 11, 1990

Dear Middle School Teacher:

This semester I am working on my dissertation which focuses on what middle school students know about percent. The study is a descriptive one using both a written test and interviews to provide some insight into students' understanding of the concept of percent and strategies they use in thinking about percent questions. Descriptions of student knowledge, such as this one, can be used to guide the development of effective instructional programs.

A pilot study was conducted in May with one class each of sixth, seventh, and eighth grade students. The written test centered on three types of questions, two involving the use of pictorial representations of quantities and the third based on a sample question from the 1986 National Assessment of Educational Progress. One open-ended question asked the students to write an explanation of how they decided on the answer to one of the multiple-choice questions.

Since the test was written by me, an idea of its validity can be ascertained from the opinion of inservice middle school mathematics teachers. For each type of question, an objective is presented, followed by three sample questions from the test. Your opinion on whether each question assesses the stated objective would be very helpful to me.

The last part of this survey presents some of the students' answers to the open-ended question. A suggested classification scheme for correct as well as incorrect responses is also presented. Your opinion on how to classify the answers will be helpful. Each student's answer, even the spelling and grammatical errors, has been reported exactly as it appeared. Frankly, some of the answers puzzle me. I will appreciate your insight. Please feel free to add any comments you think will be helpful.

Enclosed please find the survey and a stamped, addressed envelope for its return to me. Since I hope to conduct the actual study during the last week of September, I would appreciate receiving your input as soon as possible. Thank you very much for your assistance.

Sincerely,

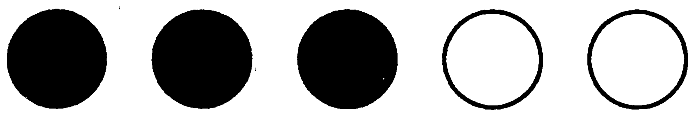
Susan Gay
Instructor

Enclosure

Middle School Teacher Survey September, 1990

Does each question below assess the following objective? Please answer yes or no by checking the appropriate box and add any comments.

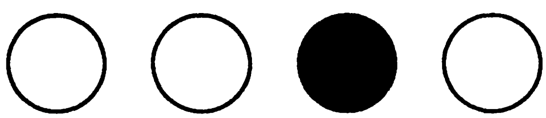
Objective 1: Given a picture showing a set of discrete circles, part or all of whom are shaded, the student will determine if a percent quantity is less than, greater than, or equal to the quantity represented by the shaded part of the set of circles.



1. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 50%.
   B. It is less than 50%.
   C. It is equal to 50%.
   D. Can't tell.
   E. I don't know.

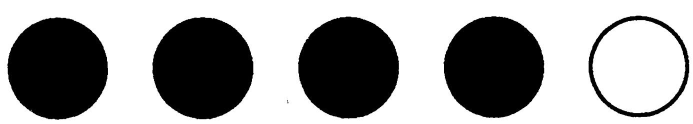
Meets objective?
[ ] yes [ ] no
Comments:



2. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 25%.
   B. It is less than 25%.
   C. It is equal to 25%.
   D. Can't tell.
   E. I don't know.

Meets objective?
[ ] yes [ ] no
Comments:



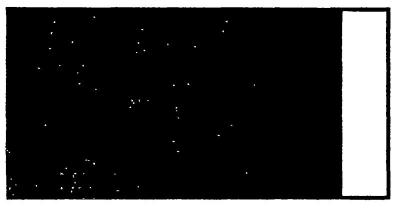
7. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 100%.
   B. It is less than 100%.
   C. It is equal to 100%.
   D. Can't tell.
   E. I don't know.

Meets objective?
[ ] yes [ ] no
Comments:

Does each question below assess the following objective? Please answer yes or no by checking the appropriate box and add any comments.

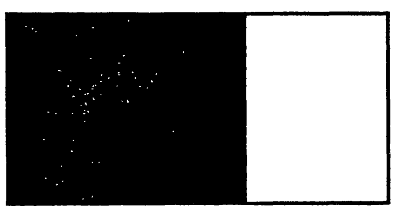
Objective 2: Given a picture showing a rectangle, part or all of which is shaded, the student will determine if a percent quantity is less than, greater than, or equal to the quantity represented by the shaded part of the rectangle.



4. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 60%.
   B. It is less than 60%.
   C. It is equal to 60%.
   D. Can't tell.
   E. I don't know.

Meets objective?
[ ] yes [ ] no
Comments:



6. In the figure above, which of the following is true about the shaded part?

   A. It is greater than 33 1/3%.

   B. It is less than 33 1/3%.

   C. It is equal to 33 1/3%.

   D. Can't tell.

   E. I don't know.

Meets objective?
[ ] yes [ ] no
Comments:

8. In the figure above, which of the following is true about the shaded part?

Meets objective?
[ ] yes [ ] no
Comments:

A. It is greater than 110%.
B. It is less than 110%.
C. It is equal to 110%.
D. Can't tell.
E. I don't know.

Does each question below assess the following objective? Please answer yes or no by checking the appropriate box and add any comments.

Objective 3: Given a quantity expressed as a percent of a number, the student will determine if that quantity is less than, greater than, or equal to the given number.

1. Which of the following is true about 50% of 20?

Meets objective?
[ ] yes [ ] no
Comments:

A. It is greater than 20.
B. It is less than 20.
C. It is equal to 20.
D. Can't tell.
E. I don't know.

4. Which of the following is true about 87% of 10?

Meets objective?
[ ] yes [ ] no
Comments:

A. It is greater than 10.
B. It is less than 10.
C. It is equal to 10.
D. Can't tell.
E. I don't know.

7. Which of the following is true about 100% of 50?

Meets objective?
[ ] yes [ ] no
Comments:

A. It is greater than 50.
B. It is less than 50.
C. It is equal to 50.
D. Can't tell.
E. I don't know.

On the pilot test, the students were asked to explain how they decided on their answer to:

Which of the following is true about 87% of 10?

This is question number 4 on the previous page of your survey. The following are some of their answers and part of a suggested classification scheme. The rest of the classification scheme categorizes the blank responses and those responding with "I don't know," "I guessed," etc.

Place the letter of the classification you would give in the blank for each answer provided. Any comments would be helpful in analyzing the answers or adjusting the classification scheme.

Students who made an *incorrect choice* to number 4, gave the following explanations.

Explanations	Classifications
_____ 1. 87 was greater than 10.	A. Compared two numbers, neither as percents
_____ 2. You can't tell because 10 is to small a number	B. Compared two numbers, both as percents
_____ 3. 87% is greater than 10	C. Compared two quantities, one a percent
_____ 4. I divided 87 ÷ 10	D. Divided the numbers
_____ 5. It is 87 out of 10: greater	E. Incorrect use of proportion
_____ 6. 87 is a bigger number than 10, so 87% if more than 10%	F. Incomplete explanation
_____ 7. 87/100 = n/100 n = 87	G. Unknown

Comments:

Students who made a *correct choice* in number 4,

87% of 10 is less than 10,

gave the following explanations.

Explanations

____ 1. If you take 87% out of 10 you can't come up with 10 even though 87 looks bigger than 10.

____ 2. A percent of a number less than 100% has to be less than the original number.

____ 3. It is only a percent of ten not a percent of a number greater than 10.

____ 4. It is more than half of ten some it makes it fewer.

____ 5. If there is 10 things and you have them all then you have 100% so you must have less than 10 to have 87%.

____ 6. There is a 100% in 10, 87% is only a part.

____ 7. 87 is bigger than 10.

____ 8. 87% of 10 would be like 2.3.

____ 9. 87 is less than 10.

____ 10. take 87% from 10.

____ 11. Because 87% is more than 10.

____ 12. 87% of 10 is about a dollar or two.

____ 13. I just counted to eight and figured it would be eight.

____ 14. Divide 10 into 80 you get 8 and that is less than 10.

____ 15. I multiplied in my head 0.87 x 10

Classifications

Understanding of percent situation demonstrated

A. Comparison of 87% to 100%, the whole

B. Comparison using 87% of a number is less than the number

C. Analogy to use of discount

D. Incomplete explanation, showing some correct understanding of the situation

Use of computation

E. Multiplied base by rate

F. Divided two numbers

Comparison of whole numbers

G. Incorrect comparison of whole numbers

Lack of understanding of percent situation

H. Compared two numbers, neither as percents

I. Comparison to the size of the base

J. Incomplete explanation, showing lack of understanding of the situation

K. Unknown

Comments:

THANK YOU!

VITA

Aleda Susan Gay

Candidate for the Degree of

Doctor of Education

Thesis: A STUDY OF MIDDLE SCHOOL STUDENTS' UNDERSTANDING OF NUMBER SENSE RELATED TO PERCENT

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Professional Experience: Graduate Assistant, Department of Curriculum and Instruction, Oklahoma State University, Stillwater, Oklahoma, 1988-1990; Mathematics Specialist and Computer Consultant, Oklahoma State Department of Education, Oklahoma City, Oklahoma, 1983-1988; Mathematics Teacher, Stillwater Public Schools, Stillwater, Oklahoma, 1973-1983.

Professional Organizations: National Council of Teachers of Mathematics, The Mathematical Association of America, National Council of Supervisors of Mathematics, Association of State Supervisors of Mathematics, Phi Delta Kappa.