ANALYSIS OF CONTINUOUS RECTANGULAR PLATE SYSTEMS BY THE SLOPE DEFLECTION METHOD

by

LAWRENCE NORWOOD DALLAM

Bachelor of Science
University of Missouri
Columbia, Missouri
1954

Master of Science
University of Missouri
Columbia, Missouri
1955

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ANALYSIS OF CONTINUOUS RECTANGULAR

PLATE SYSTEMS BY THE SLOPE

DEFLECTION METHOD

Thesis Approved:

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Thesis Adviser

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Dean of the Graduate School

PREFACE

This investigation concerns the application of the slope-deflection method to continuous rectangular plate systems which conform to the classical thin plate theory. The slope-deflection constants or stiffness factors are calculated by the method of finite differences.

This research is the result of ideas expressed by Professor Jan J. Tuma. It is also the result of the author's desire to develop a more readily understandable, numerical approach to the solution of continuous plate systems supported in a general manner.

In this, the final phase of my formal academic study, I wish to express my sincere appreciation to the following persons and organizations:

To Professor Jan J. Tuma for his assistance and guidance and encouragement.

To the National Science Foundation and the School of Civil Engineering of Oklahoma State University for the opportunity to participate in the Summer Institute in Structures and Soil Mechanics during the summer of 1962.

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L. N. D.

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Columbia, Missouri

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LIST OF SYMBOLS

CK	Carry-over moment stiffness factor due to moment
CT	Carry-over reaction stiffness factor due to reaction
D	Flexural rigidity of plate, $\frac{Eh^3}{12(1-\nu^2)}$
Δ	Vertical deflection of edge point
EI	Flexural rigidity of beam
FM	Fixed edge moment
FR	Fixed edge reaction
GJ*	Torsional rigidity of beam
h	Plate thickness
K	Moment stiffness factor
M_x, M_y	Interior bending moments in plate
M_{xy}, M_{yx}	Twisting moments in plate
ν	Poisson's ratio
R_{xz}, R_{yz}	Edge reactions of plate
σ_x, σ_y	Normal stresses in plate
SM	Carry-over moment stiffness factor due to reaction
SR	Carry-over reaction stiffness factor due to moment
T	Reaction stiffness factor
T_{ij}	Twisting Moment in beam
t	Ratio of length of plate in x-direction to length of plate in y-direction

Θ	Slope of edge point
τ_{xy}, τ_{xz}	Shear stresses in plate
V_{xz}, V_{yz}	Shear forces in plate
V_{ij}	Shear force in beam
W	Vertical deflection in plate

CHAPTER I

INTRODUCTION

1-1 General

A numerical analysis of continuous rectangular plates on rigid or elastic supports is developed. The bending moments and shears acting along the edges of each plate are selected as the redundants and are expressed in terms of the edge slopes and deflections. The resulting equations are called slope-deflection equations.

The method of finite differences in network form is used to determine the slope-deflection constants. The equations are developed in general form and are independent of the size of the finite difference network. Similar expressions are developed for the supporting beams including both the flexural and torsional stiffness. The compatibility between the plates and beams is secured by means of static equilibrium. The procedure of analysis is described and illustrated with numerical examples.

1-2 Historical Background

The study of continuous plates has been conducted by many investigators. The following discussion is applicable to thin rectangular plate systems incorporating the small deflection theory.

Solutions for rectangular plates continuous in one direction (one-way continuous plates) over rigid supports have been presented by Timoshenko (1)*, Galerkin (2), Marcus (3), Habel (4) and others. Galerkin is credited with the first of these solutions in 1933.

The solution of one-way continuous plates over flexible beams was first presented in 1937 by Weber (5). One year later Jensen (6) published more general results. Also in 1938, Newmark (7) developed a distribution procedure for the analysis of one-way continuous plates over rigid supports or flexible beams with the side edges always simply supported.

An approximate analysis of rectangular plates continuous in two directions (two-way continuous plates) over rigid supports was first presented by Bittner (8). Girkmann (9), Engelbreth (10), Maugh and Pan (11), and Siess and Newmark (12) also contributed approximate solutions of this problem. Ying (13) presented a rigorous solution using the Rayleigh-Ritz energy approach.

Nadai (14) and Galerkin (15) both considered the problem of a plate composed of an infinite number of identical panels supported by point columns and uniformly loaded. Sutherland, Goodman and Newmark (16) treated the same problem with beams of equal flexural stiffness framed between the point columns. Nielsen (17) made analyses of these same problems using finite differences.

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*Numbers in parenthesis refer to entries in the bibliography.

The solution of two-way continuous rectangular plates supported by flexible beams having both flexural and torsional stiffnesses has thus far apparently defied a rigorous mathematical treatment. Ang (18) used finite differences and a numerical procedure analogous to moment distribution to analyze this type of structure with the restriction that infinitely stiff rectangular columns of dimension 1/10 of the appropriate span be present at the corner of each panel. Ang with Newmark (19) and Prescott (20) published solutions using this approach. The use of Ang's method requires that a digital computer with a large storage capacity be available. Oden (21) also presented an approximate approach using trigonometric series.

In 1956, an investigation of floor slabs was initiated at the University of Illinois in order to better understand the structural behavior of two-way and flat slab reinforced concrete floor systems. The investigation included both theoretical and experimental studies. The theoretical studies considered the effects of openings in slabs (22) and the effects of varying column stiffnesses on moments and deflections of slabs (23). The experimental studies included the testing of five nine-panel reinforced concrete floor slabs and the results of analyses for moments have been reported (24,25,26,27,28,29). The effects of beam and column stiffnesses on moments and a correlation of computed and measured moments have also been reported (30). A study of deflections of reinforced concrete floor slabs was presented by

Vanderbilt, Sozen and Siess (31). This study discusses current building code requirements, design practice and introduces an approximate method for determining center span slab deflections.

In summary, solutions are available for the analysis of one-way continuous plates over rigid and flexible supports with certain limitations as to the type of load functions and type of edge support conditions. Most solutions use a uniform load, hydrostatic load or a point load at the center of a panel and the support conditions are either all rigid, or the interior is rigid with the sides flexible, or the sides are rigid with the interior flexible (3,8,9). Also, the torsional rigidity of the flexible support is usually neglected. There are not "exact" or approximate solutions available for two-way continuous plate systems subjected to a general loading condition and supported in a general manner. Ang's (19) approximate solution is the best available at this writing but it is limited as to the type of support condition and requires the use of a large digital computer.

This investigation was therefore undertaken to develop an approximate solution for continuous plate systems without restriction as to the loading or edge support conditions, and its application should not require the use of a large computer.

1-3 Statement of the Problem

This investigation is concerned with the analysis of continuous plate systems. The plates are rectangular, thin and of constant thickness, loaded perpendicular to the plate surface and supported by beams which are symmetrical with respect to the middle plane of the plate. The beams may have a constant or variable cross section.

The exterior edges of the plates are free, fixed, simply or elastically supported. The loads are stationary and of constant or variable magnitude.

1-4 Limitation of the Problem

The study is subject to the following limitations:

1. The material is linearly elastic, homogeneous, isotropic and continuous.
2. The plate thickness is constant and small in comparison with the other dimensions.
3. Stresses normal to the middle surface of the plate are negligible.
4. Planes normal to the middle surface of the plate before deformation, remain normal after deformation.
5. All deformations in the plate are small in comparison with the plate thickness.
6. Loads are applied normal to the middle surface of the plate.
7. There is no deformation in the middle plane of the plate.
8. Deformations due to shearing force are negligible.

1-5 Mathematical Model

The plate is represented by a network of straight lines as shown in Figure 1-1. The intersections of the lines are called nodes. In determining the slope-deflection constants, equilibrium and compatibility are satisfied only at each node.

1-6 Sign Convention

The positive directions of the edge moments and reactions are shown in Figure 1-2. Positive edge slopes and deflections are in the direction of the respective positive edge moments and reactions. Positive loads are taken as downward.

Positive directions of moments and shears acting on interior elements are shown in Figure 1-3, and positive stresses are shown in Figure 1-4.

1-7 Nomenclature

All symbols are defined when they first appear and are arranged alphabetically in the list of symbols.

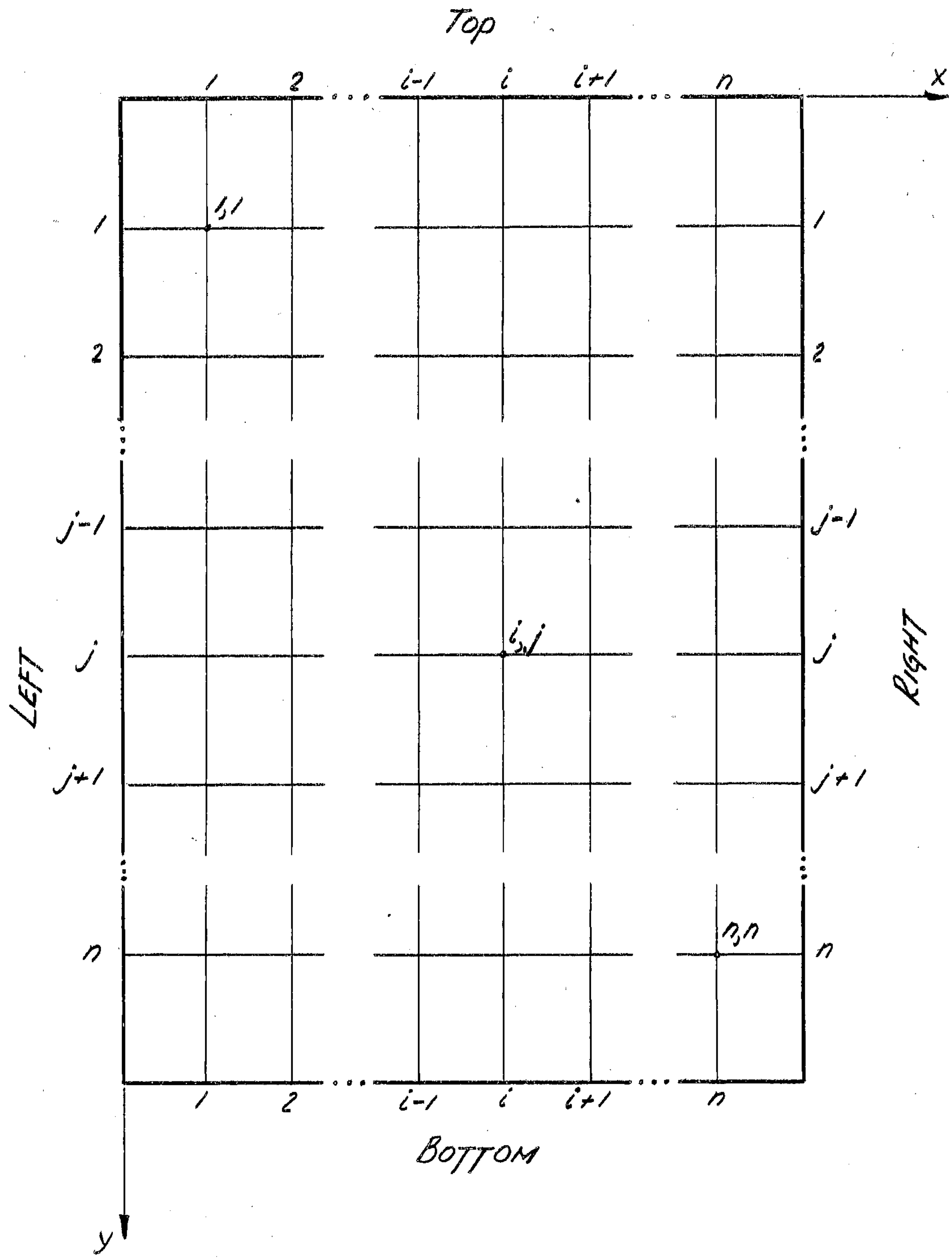


Figure 1-1 Plate Model

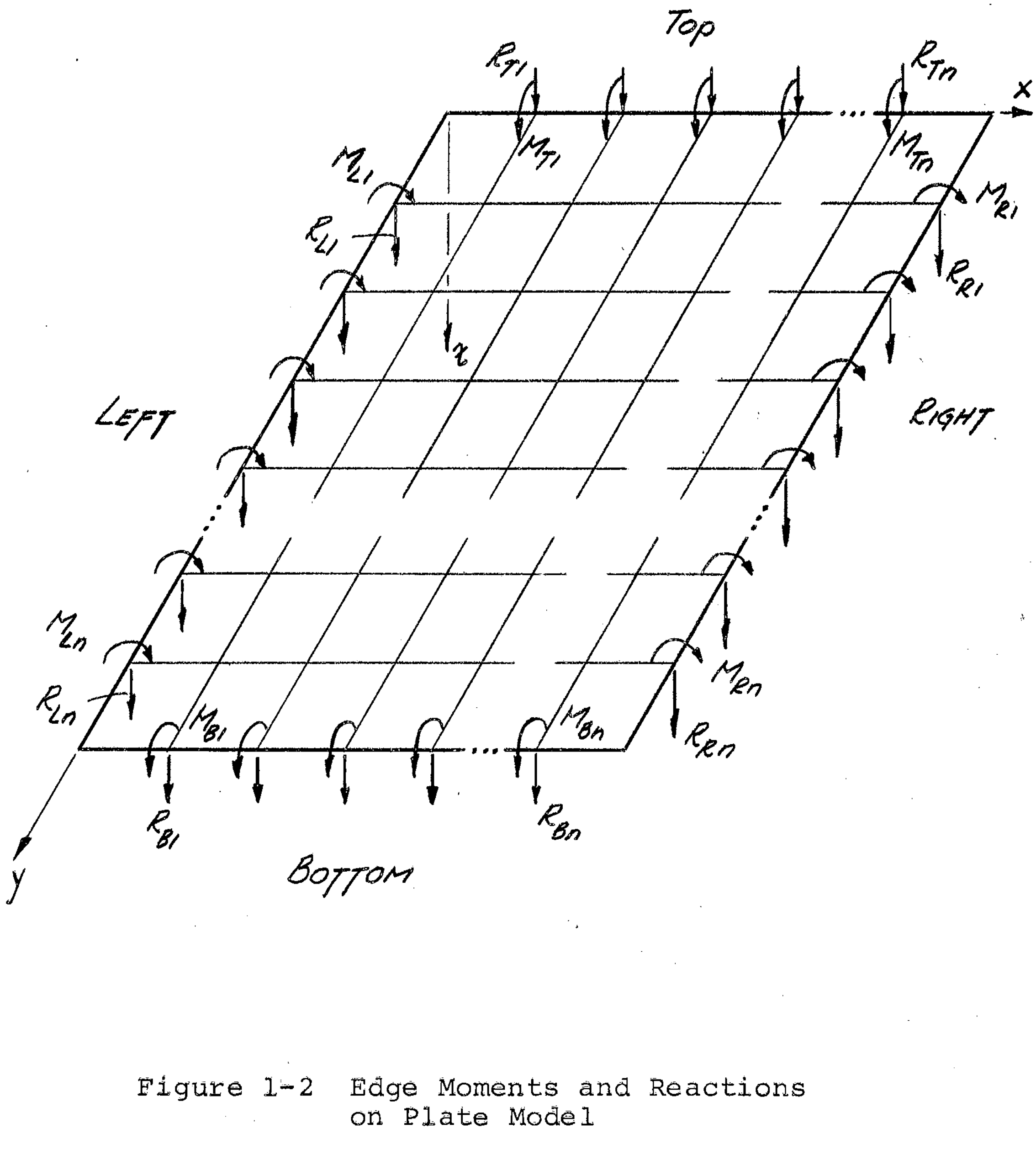


Figure 1-2 Edge Moments and Reactions on Plate Model

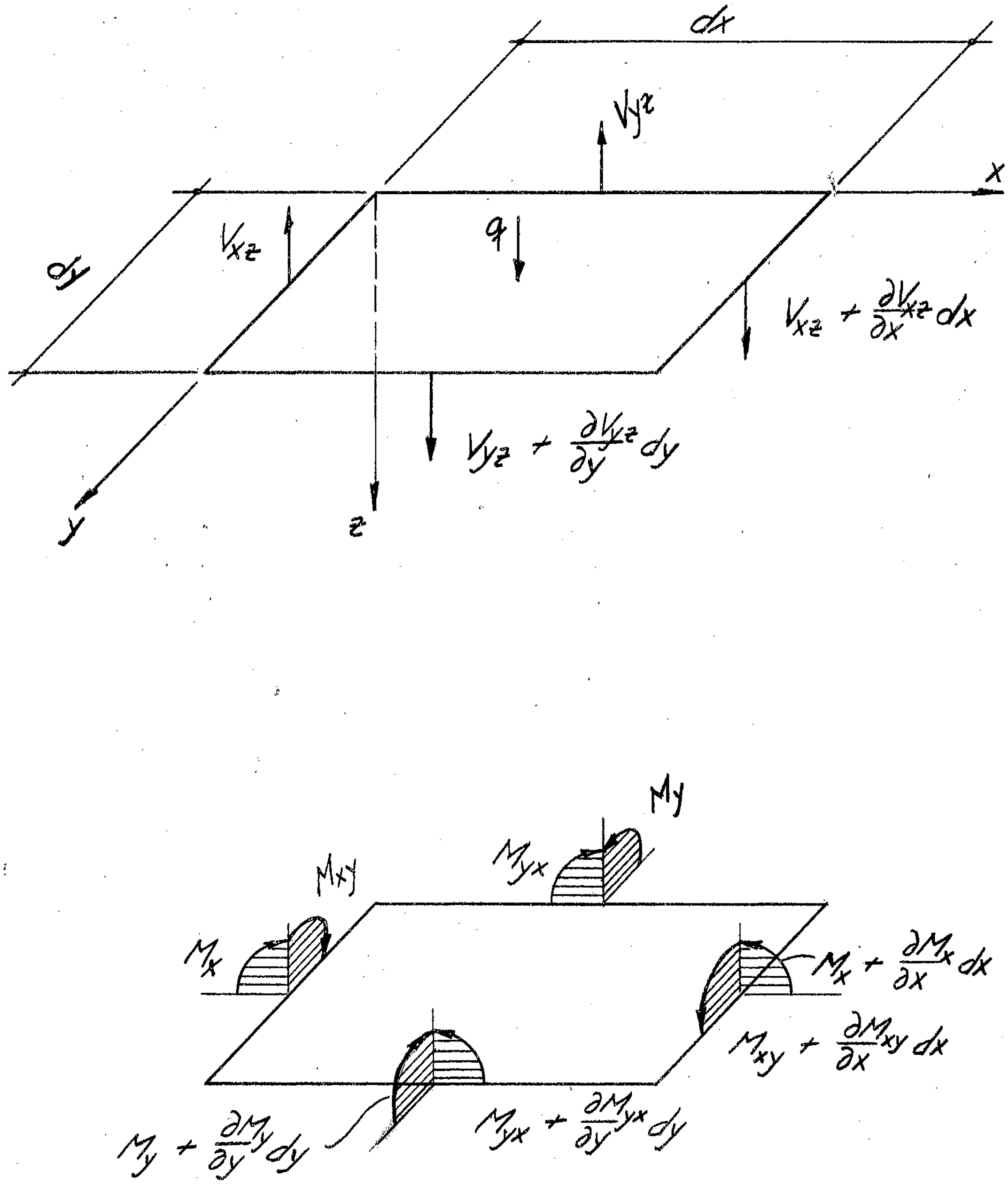


Figure 1-3 Intensity of Forces and Moments on a Typical Interior Element

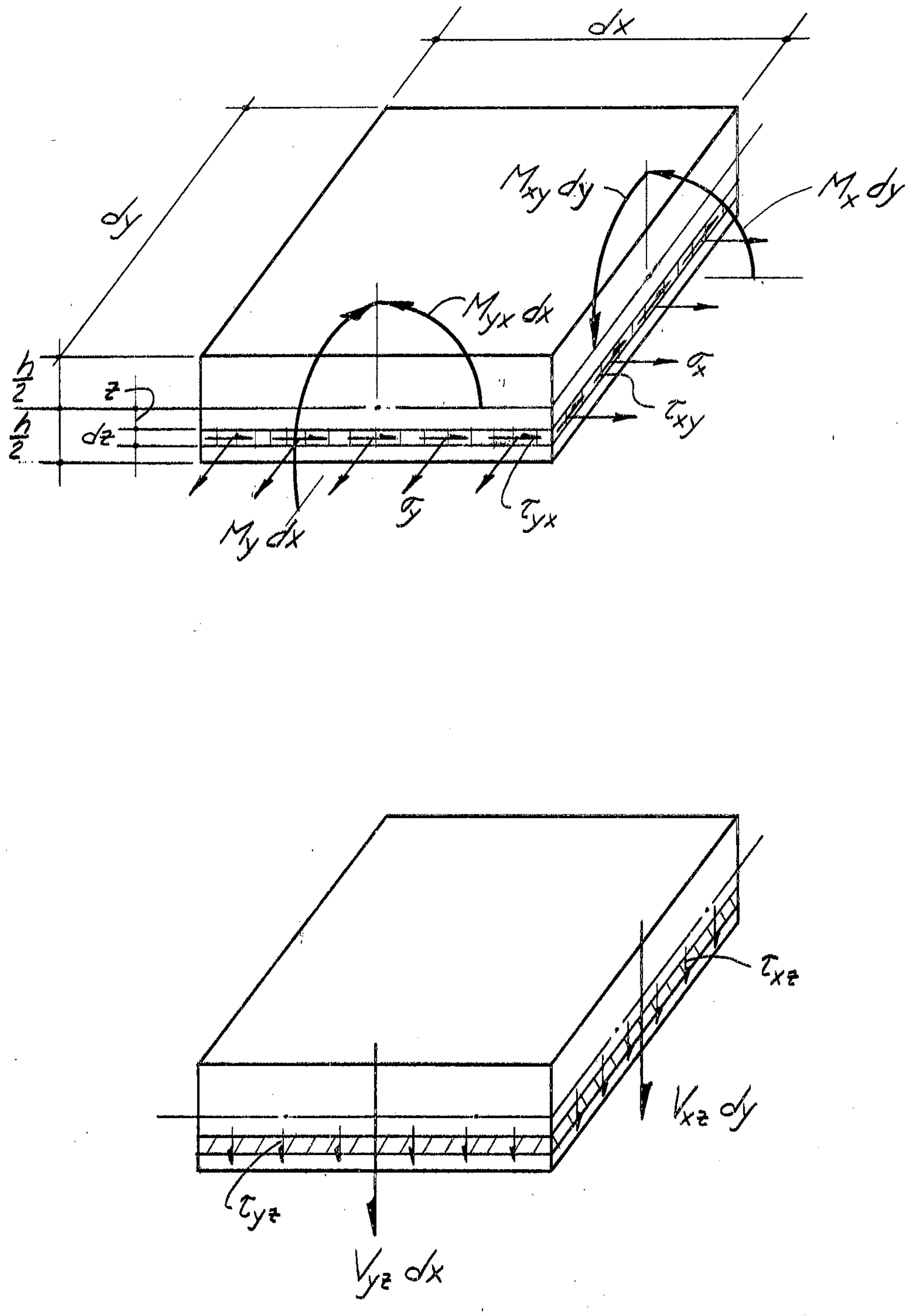


Figure 1-4 Sign Convention of Stresses

CHAPTER II

DIFFERENTIAL EQUATIONS

2-1 Deflection Surface

The differential equation of the deflection surface which was first obtained by Lagrange in 1811 is available in the literature and is restated here for convenience:

$$\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial^2 x \partial^2 y} + \frac{\partial^4 W}{\partial y^4} = \frac{q}{D} \tag{2-1}$$

where W is the deflection in the z-direction, q is the intensity of load distributed over the upper plate surface and D = $\frac{Eh^3}{12(1-\nu^2)}$, with h the plate thickness, ν is Poisson's ratio and E is the modulus of elasticity of the material.

2-2 Moments and Shears

The moments and shears acting on a typical interior element (see Fig. 1-3) are expressible in terms of the deflection surface as follows:

$$V_{xz} = -D\left(\frac{\partial^3 W}{\partial x^3} + \frac{\partial^3 W}{\partial x \partial y^2}\right) \tag{2-2}$$

$$V_{yz} = -D\left(\frac{\partial^3 W}{\partial y^3} + \frac{\partial^3 W}{\partial x^2 \partial y}\right) \tag{2-3}$$

$$M_x = -D\left(\frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2}\right) \tag{2-4}$$

$$M_y = -D\left(\frac{\partial^2 W}{\partial y^2} + \nu \frac{\partial^2 W}{\partial x^2}\right) \quad (2\text{-}5)$$

$$M_{xy} = -M_{yx} = D(1-\nu) \frac{\partial^2 W}{\partial x \partial y} \quad (2\text{-}6)$$

The development of Eqs. (2-1) through (2-6) can be found in reference (1), page 79. The units of the moments and shears are the intensity of moment and shear per unit length, respectively.

2-3 Boundary Values

The edge moments (bending and twisting) and edge shears can be reduced to only two unknowns at a specified edge point by imposing the Kirchhoff boundary conditions. This boundary treatment may be found in Timoshenko (1), p. 84 or Tuma (33), p. 15. Using the sign convention of Fig. 1-2, the expressions for edge moment and edge reaction are as follows:

Top Edge

$$M_y = -D\left(\frac{\partial^2 W}{\partial y^2} + \nu \frac{\partial^2 W}{\partial x^2}\right) \quad (2\text{-}7)$$

$$R_{yz} = D\left\{\frac{\partial^3 W}{\partial y^3} + (2-\nu)\frac{\partial^3 W}{\partial y \partial x^2}\right\} \quad (2\text{-}8)$$

Right Edge

$$M_x = +D\left(\frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2}\right) \quad (2\text{-}9)$$

$$R_{xz} = -D\left\{\frac{\partial^3 W}{\partial x^3} + (2-\nu) \frac{\partial^3 W}{\partial x \partial y^2}\right\} \quad (2\text{-}10)$$

Bottom Edge

$$M_y = +D\left(\frac{\partial^2 W}{\partial y^2} + \nu \frac{\partial^2 W}{\partial x^2}\right) \quad (2\text{-}11)$$

$$R_{yz} = -D\left\{\frac{\partial^3 W}{\partial y^3} + (2-\nu) \frac{\partial^3 W}{\partial y \partial x^2}\right\} \quad (2\text{-}12)$$

Left Edge

$$M_x = -D\left(\frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2}\right) \tag{2-13}$$

$$R_{xz} = D\left\{\frac{\partial^3 W}{\partial x^3} + (2-\nu)\frac{\partial^3 W}{\partial x \partial y^2}\right\} \tag{2-14}$$

The types of edge support of the plate model are simple, fixed and free. In terms of the deflection surface, these become (for the top edge):

Simply Supported Edge

$$W = 0$$

$$M_y = 0 \qquad \frac{\partial^2 W}{\partial y^2} = 0 \tag{2-15}$$

Fixed Edge

$$W = 0 \qquad \frac{\partial W}{\partial y} = 0 \tag{2-16}$$

Free Edge

$$M_y = 0 \quad , \quad \frac{\partial^2 W}{\partial y^2} + \nu \frac{\partial^2 W}{\partial x^2} = 0 \tag{2-17}$$

$$R_{yz} = 0 \quad , \quad \frac{\partial^3 W}{\partial y^3} + (2-\nu)\frac{\partial^3 W}{\partial y \partial x^2} = 0 \tag{2-18}$$

The above conditions must be satisfied at each point on the edge of the plate.

2-4 Stresses

When the shears and moments are determined, the maximum stresses can be calculated from the following equations, (1), pp. 42, 82:

$$(\sigma_x)_{max.} = \frac{6M_x}{h^2} , \quad (\sigma_y)_{max.} = \frac{6M_y}{h^2}$$

$$(\tau_{xy})_{max.} = \frac{6M_{xy}}{h^2} \tag{2-19}$$

$$(\tau_{xz})_{max.} = \frac{3}{2}\frac{V_{xz}}{h} , \quad (\tau_{yz})_{max.} = \frac{3}{2}\frac{V_{yz}}{h}$$

CHAPTER III

FINITE DIFFERENCE EQUATIONS

3-1 Deflection Surface

The application of the finite difference method in the solution of differential equations is available in the literature and will not be restated (32). Central differences are used wherever possible because they are more accurate than backward or forward differences in boundary value problems, (32), p. 63.

At a typical interior node i,j, Eq. (2-1) becomes (see Fig. 3-1):

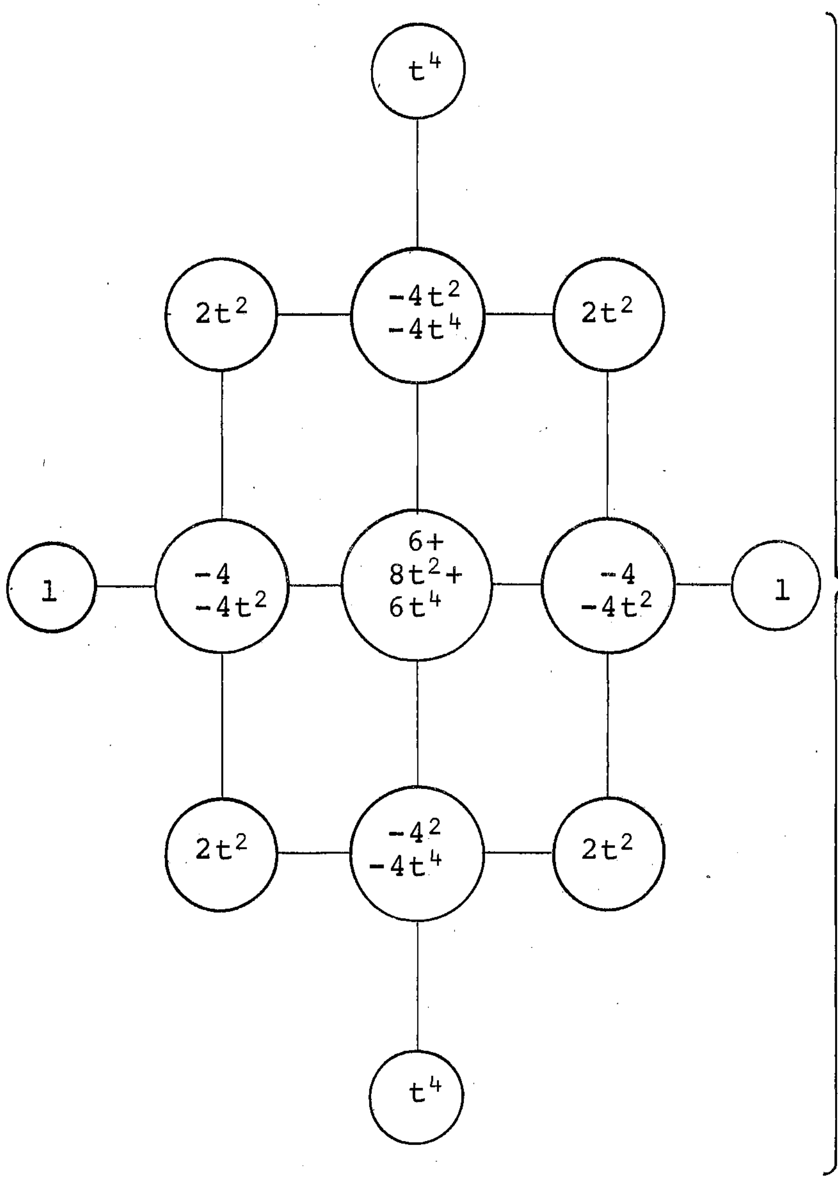
$$\frac{1}{(\Delta x)^4}(W_{i-2,j}-4W_{i-1,j}+6W_{ij}-4W_{i+1,j}+W_{i+2,j}) +$$

$$\frac{2}{(\Delta x)^2(\Delta y)^2}(W_{i-1,j-1}-2W_{i-1,j}+W_{i-1,j+1}-2W_{i,j-1}+4W_{ij}-$$

$$2W_{i,j+1}+W_{i+1,j-1}-2W_{i+1,j}+W_{i+1,j+1}) +$$

$$\frac{1}{(\Delta y)^4}(W_{i,j-2}-4W_{i,j-1}+6W_{ij}-4W_{i,j+1}+W_{i,j+2}) = \frac{q_{ij}}{D} \qquad (3\text{-}1)$$

Multiplying Eq. (3-1) by $(\Delta x)^4$ and letting $t = \frac{\Delta x}{\Delta y}$ and putting in operator form, then



$$W_{ij} = q_{ij} \frac{(\Delta x)^4}{D} \qquad (3\text{-}2)$$

where q_{ij} is the equivalent uniform load acting in the domain of node i,j . For a concentrated load Q_{ij} applied at node i,j ; q_{ij} becomes

$$q_{ij} = \frac{Q_{ij}}{\Delta x \Delta y} = Q_{ij} \frac{t}{(\Delta x)^2} \qquad (3\text{-}3)$$

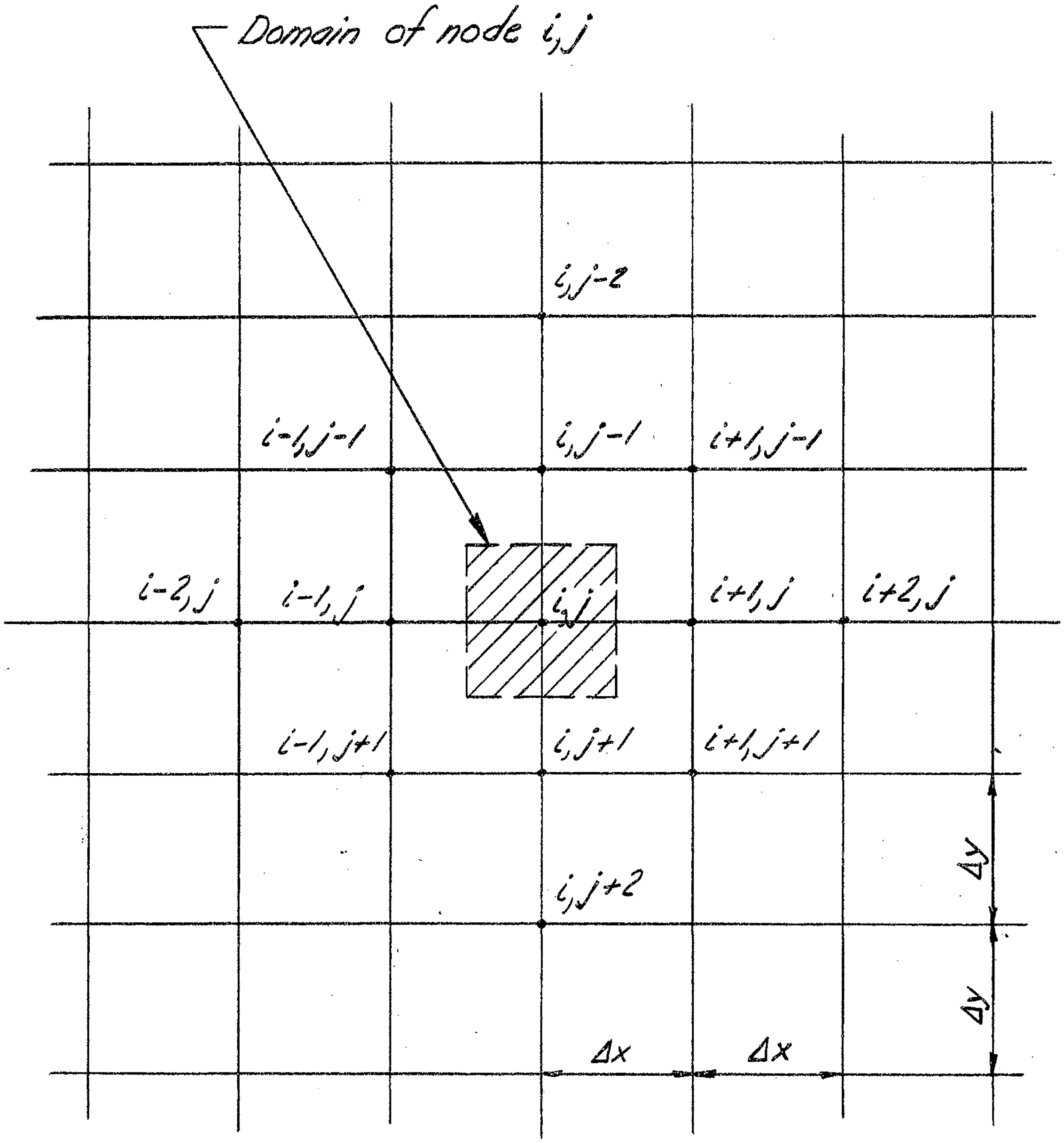


Figure 3-1 Typical Interior Network

3-2 Moments and Shears

The moments and shears at a typical interior node i,j are expressed in finite differences.

$$(V_{xz})_{ij} = \frac{-D}{2(\Delta x)^3}\left[-t^2W_{i-1,j-1} + t^2W_{i+1,j-1} - W_{i-2,j} + (2+2t^2)W_{i-1,j} - (2+2t^2)W_{i+1,j} + W_{i+2,j} - t^2W_{i-1,j+1} + t^2W_{i+1,j+1} \right] \tag{3-4}$$

$$(V_{yz})_{ij} = \frac{-D}{2(\Delta x)^3}\left[-t^3W_{i,j-2} - tW_{i-1,j-1} + (2t^3+2t)W_{i,j-1} - tW_{i+1,j-1} - (2t^3+2t)W_{i,j+1} + tW_{i-1,j+1} + t^3W_{i,j+2} + tW_{i+1,j+1} \right] \tag{3-5}$$

$$(M_x)_{ij} = -\frac{D}{(\Delta x)^2}\left[\nu t^2W_{i,j-1} + W_{i-1,j} - (2+2\nu t^2)W_{ij} + W_{i+1,j} + \nu t^2W_{i,j+1} \right] \tag{3-6}$$

$$(M_y)_{ij} = \frac{-D}{(\Delta x)^2}\left[t^2W_{i,j-1} + \nu W_{i-1,j} - (2t^2+2\nu)W_{ij} + \nu W_{i+1,j} + t^2W_{i,j+1} \right] \tag{3-7}$$

$$(M_{xy})_{ij} = + \frac{Dt}{(\Delta x)} \begin{bmatrix} W_{i-1,j-1} & - & W_{i+1,j-1} \\ -W_{i-1,j+1} & + & W_{i+1,j+1} \end{bmatrix} (1-\nu) \tag{3-8}$$

$$(M_{yx})_{ij} = - (M_{xy})_{ij}$$

3-3 Boundary Values

The finite difference expression for the bending moment at point T,i on the top edge, referring to Fig. (3-2) is

$$(M_y)_{T,i} = - \frac{D}{(\Delta x)^2} \left[\begin{array}{c} t^2 W_{TT,i} + \\ \nu W_{T,i-1} - (2t^2+2\nu) W_{T,i} + \nu W_{T,i+1} \\ + t^2 W_{1,i} \end{array} \right] \tag{3-9}$$

The analytical expression for the top edge reaction is

$$R_{yz} = D\{\frac{\partial^3 W}{\partial y^3} + (2-\nu) \frac{\partial^3 W}{\partial y \partial x^2}\} \tag{2-8}$$

If Eq. (2-8) is approximated by central differences, node points twice removed from the boundary would be introduced. The fictitious deflections of these nodes have no relation to the actual deflections in the plate for a fixed or simply supported edge. It was therefore decided to use central differences on the second derivative of the first term in Eq. (2-8) and then a forward difference to obtain the third derivative. Using central differences exclusively on the second term, the finite difference expression for Eq. (2-8) at point T,i is

$$(R_{yz})_{T,i} = \frac{Dt}{(\Delta x)^3}\left[\begin{matrix} -\frac{2-\nu}{2} W_{TT,i-1} - (t^2-2+\nu) W_{TT,i} - \frac{2-\nu}{2} W_{TT,i+1} \\ + 3t^2 W_{T,i} \\ + \frac{2-\nu}{2} W_{1,i-1} - (3t^2+2-\nu) W_{1,i} + \frac{2-\nu}{2} W_{1,i+1} \\ + t^2 W_{2,i} \end{matrix}\right] \quad (3\text{-}10)$$

<u>Right Edge</u> (Fig. 3-3)

$$(M_x)_{R,j} = + \frac{D}{(\Delta x)^2}\left[\begin{matrix} \nu t^2 W_{R,j-1} + \\ W_{n,j} - (2+2\nu t^2) W_{R,j} + W_{RR,j} \\ + \nu t^2 W_{R,j+1} \end{matrix}\right] \quad (3\text{-}11)$$

$$(R_{xz})_{R,j} = \frac{-D}{(\Delta x)^3}\left[\begin{matrix} -\frac{2t^2-\nu t^2}{2} W_{n,j-1} \qquad + \frac{2t^2-\nu t^2}{2} W_{RR,j-1} \\ -W_{n-2,j} + (3+2t^2-\nu t^2) W_{n,j} - 3W_{R,j} + (1-2t^2+\nu t^2) W_{RR,j} \\ -\frac{2t^2-\nu t^2}{2} W_{n,j+1} \qquad + \frac{2t^2-\nu t^2}{2} W_{RR,j+1} \end{matrix}\right]$$

(3-12)

The finite difference representation of Eqs. (2-11) through (2-14) for the bottom and left edges of the plate are similar to Eqs. (3-8) through (3-12).

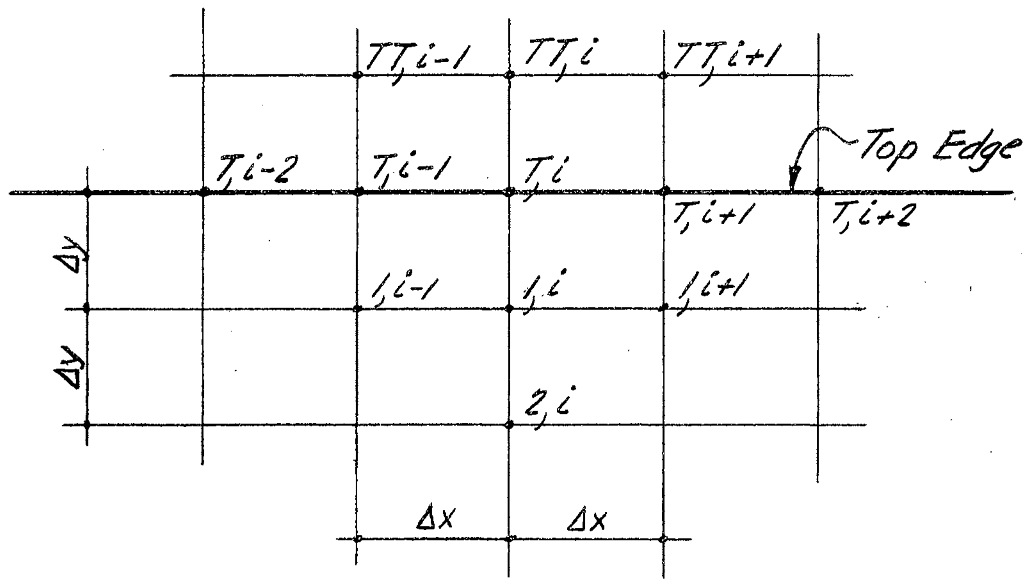


Figure 3-2 Network Nomenclature at General Point on Top Edge of Plate

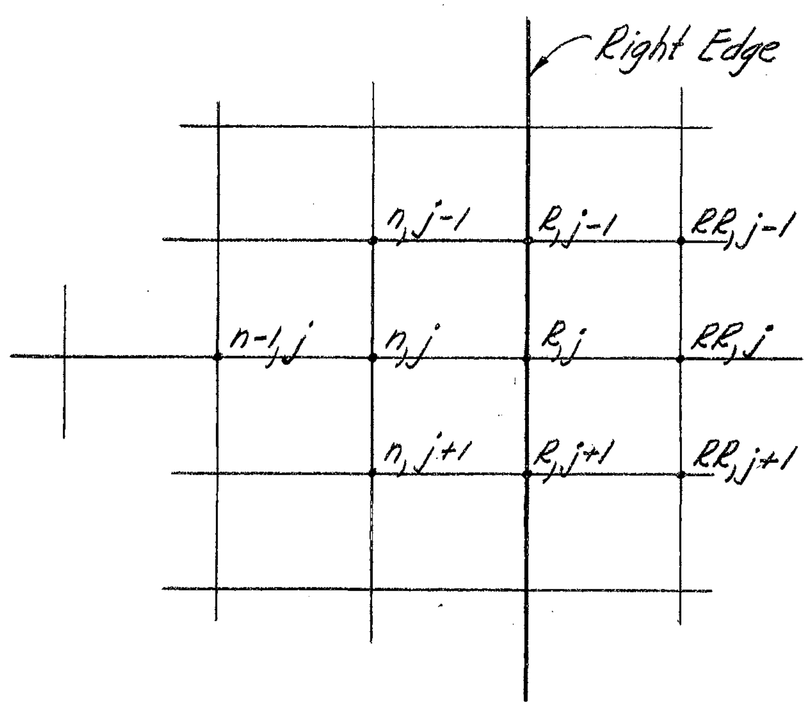


Figure 3-3 Network Nomenclature at General Point on Right Edge of Plate

The boundary requirements of a simple, fixed or free edge are expressible in finite differences and become (for the top edge, Fig. 3-2):

Simply Supported Edge

$$W_{T,i} = 0 \; ; \; i = 1,2,\ldots,n \qquad (3\text{-}13)$$

$$W_{TT,i} = -W_{1,i} \; ; \; i = 1,2,\ldots,n$$

Fixed Edge

$$W_{T,i} = 0 \; ; \; i = 1,2,\ldots n \qquad (3\text{-}14)$$

$$W_{TT,i} = +W_{1,i} \; ; \; i = 1,2,\ldots,n$$

Free Edge

$$\nu W_{T,i-1} - (2\nu+2t^2)W_{T,i} + \nu W_{T,i+1} + t^2 W_{TT,i} + t^2 W_{1,i} = 0 \; ; \; i = 1,2,\ldots,n \qquad (3\text{-}15)$$

$$\frac{2-\nu}{2}W_{TT,i-1} + (t^2-2+\nu)W_{TT,i} + \frac{2-\nu}{2}W_{TT,i+1} - 3t^2 W_{T,i} - \frac{2-\nu}{2}W_{1,i-1} + (3t^2+2-\nu)W_{1,i} - \frac{2-\nu}{2}W_{1,i+1} = 0 \qquad (3\text{-}16)$$

$$i=1,2,\ldots,n$$

3-4 Application

Consider the rectangular plate shown in Fig. (3-4) loaded with a uniform pressure, q. The plate is arbitrarily divided into six horizontal and six vertical strips.

Eq. (3-2) is applied at each interior node of the plate. For example, at node (3,3), if $a = b$ ($t=1$), Eq. (3-2) gives:

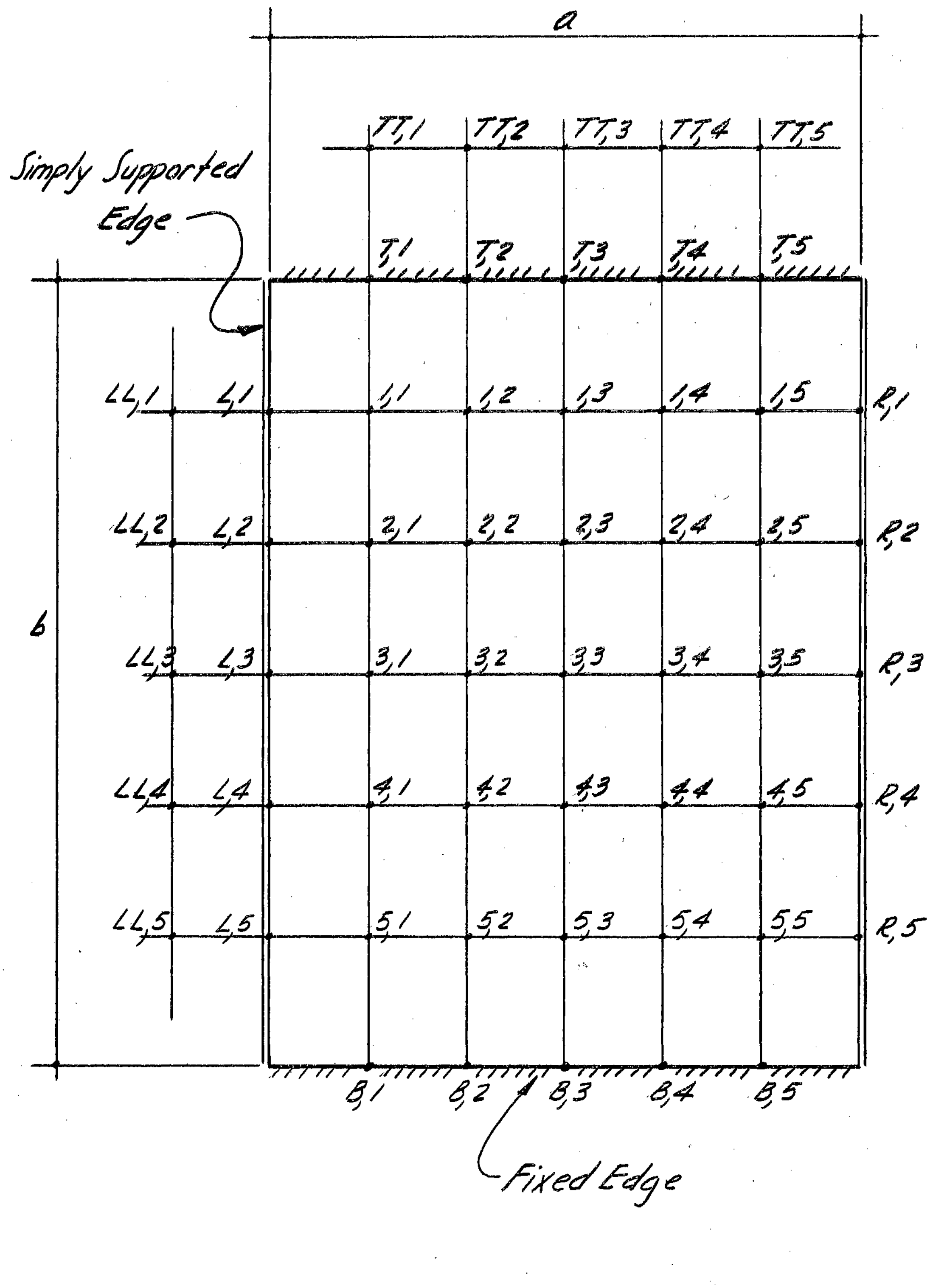


Figure 3-4 Plate Layout and Notation

$$\begin{array}{c} W_{13} \\ + \\ 2W_{22} - 8W_{23} + 2W_{24} \\ + \\ W_{31} - 8W_{32} + 20W_{33} - 8W_{34} + W_{35} = \frac{q_{33}}{D} (\frac{a}{6})^4 \\ + \\ 2W_{42} - 8W_{43} + 2W_{44} \\ + \\ W_{53} \end{array} \qquad (3\text{-}17)$$

at node (1,1), Eq. (3-2) becomes

$$\begin{array}{c} W_{TT1} \\ + \\ 0 + 0 + 0 \\ + \\ W_{LL1} - 0 + 20W_{11} - 8W_{12} + W_{13} = \frac{q_{11}}{D} (\frac{a}{6})^4 \\ + \\ 0 - 8W_{21} + 2W_{22} \\ + \\ W_{31} \end{array} \qquad (3\text{-}18)$$

From Eq. (3-13), $W_{LL1} = -W_{11}$ and from Eq. (3-14), $W_{TT1} = +W_{11}$. Substituting into Eq. (3-18):

$$\begin{aligned} 20W_{11} - 8W_{12} + W_{13} &= \frac{q_{11}}{D} (\frac{a}{6})^4 \\ -8W_{21} + 2W_{22} & \\ + W_{31} & \end{aligned} \qquad (3\text{-}19)$$

where $q_{11} = q_{33} = q$ in this problem.

Eq. (3-2) is written for each of the 25 interior nodes which results in 25 equations with 25 unknowns. Matrically, these equations are

$$[A] \{W\} = \frac{q}{D} (\frac{a}{6})^4 \{1\} \qquad (3\text{-}20)$$

where [A] is a 25 x 25 coefficient matrix, {W} is a 25 x 1 column matrix of the unknown deflections and $\{1\}$ is a unit column matrix. The solution of Eq. (3-20) is

$$\{W\} = [A]^{-1}\{1\}\frac{q}{D}\left(\frac{a}{6}\right)^4 \tag{3-21}$$

where $[A]^{-1}$ is the inverse of $[A]$.

With the deflection of each node now known, the moments, shears and reactions may be determined from their appropriate finite difference expressions.

CHAPTER IV

SLOPE DEFLECTION EQUATIONS - PLATE

4-1 General Form

Since at each point on the edge of a plate there is one slope and one deflection in the direction of the reactive forces, the general form of the slope deflection equations is (for any point Ti on the top edge)

$$M_{Ti} = CK_{TiT1}\Theta_{T1} + CK_{TiT2}\Theta_{T2} + \ldots + K_{TiTi}\Theta_{Ti} + \ldots$$
$$+ CK_{TiTn}\Theta_{Tn} + CK_{TiR1}\Theta_{R1} + \ldots + CK_{TiRn}\Theta_{Rn} + \ldots$$
$$+ CK_{TiBn}\Theta_{Bn} + \ldots + CK_{TiLn}\Theta_{Ln} +$$
$$+ SM_{TiT1}\Delta_{T1} + \ldots + SM_{TiTn}\Delta_{Tn} + \ldots + SM_{TiRn}\Delta_{Rn}$$
$$+ \ldots + SM_{TiBn}\Delta_{Bn} + \ldots + SM_{TiLn}\Delta_{Ln} + FM_{Ti} \qquad (4\text{-}1)$$

$$R_{Ti} = SR_{TiT1}\Theta_{T1} + \ldots + SR_{TiTn}\Theta_{Tn} + \ldots SR_{TiRn}\Theta_{Tn} + \ldots$$
$$+ SR_{TiBn}\Theta_{Bn} + \ldots + SR_{TiLn}\Theta_{Ln} + CT_{TiT1}\Delta_{T1} + \ldots$$
$$+ T_{TiTi}\Delta_{Ti} + \ldots + CT_{TiTn}\Delta_{Tn} + \ldots + CT_{TiRn}\Delta_{Rn}$$
$$+ \ldots + CT_{TiBn}\Delta_{Bn} + \ldots + CT_{TiLn}\Delta_{Ln} + FR_{Ti} \qquad (4\text{-}2)$$

where

M_{Ti} = the bending moment at point Ti on the top edge of the plate

R_{Ti} = the reaction at point Ti on the top edge of the plate

θ_{Ti} = the slope at point Ti in the direction of K_{TiTi}

Δ_{Ti} = the deflection at point Ti in the direction of T_{TiTi}

K_{TiTi} = the moment at Ti necessary to produce a unit slope at Ti with all other edge θ's and all edge Δ's equal to zero

T_{TiTi} = the reaction at Ti necessary to produce a unit deflection at Ti with all other edge Δ's and all edge θ's equal to zero.

CK_{TiRn} = the bending moment induced at Ti by K_{RnRn}

SM_{TiBn} = the bending moment induced at Ti by T_{BnBn}

CT_{TiLn} = the reaction at Ti induced by T_{LnLn}

SR_{TiTn} = the reaction at Ti induced by K_{TnTn}

FM_{Ti} = the bending moment at Ti due to loads with all edge θ's and Δ's equal to zero (fixed-edge moment)

FR_{Ti} = the reaction at Ti due to loads with all edge θ's and Δ's equal to zero (fixed-edge reaction)

4-2 Load Functions

For a given condition of loading, the value of the moment and reaction at each edge point of the plate model is determined. The edges of the plate model are fixed against translation and rotation.

Eq. (3-2) is applied at each interior node of the plate model subjected to the given loading condition, which results in n^2 equations with n^2 unknowns. The deflections of the node points outside the boundary are expressed in terms of the deflections of the interior nodes by Eq. (3-14). The set of n^2 equations in matrix form are

$$[A_1]\{W\} = \{q\}\frac{(\Delta x)^4}{D} \tag{4-3}$$

whose solution is

$$\{W\} = [A_1]^{-1}\{q\}\frac{(\Delta x)^4}{D} \tag{4-4}$$

where $[A_1]$ is the $n^2 \times n^2$ coefficient matrix and $[A_1]^{-1}$ is its inverse. With the deflection surface, $\{W\}$, known, the fixed-edge moments and reactions are determined from Eqs. (3-9) through (3-12). Figure 4-1 illustrates the load functions for an arbitrary loading if n is taken as four.

4-3 Stiffnesses

The moment stiffness factor, K_{TiTi}, at the point Ti on the top edge of the plate is defined as the moment per unit length necessary to cause a unit rotation at Ti with all other edge points fixed against rotation and displacement. The carry-over stiffness factors CK_{RjTi} and SR_{RjTi}, for example, are the values of the moment and reaction per unit length induced at the fixed edge point Rj on the right edge due to K_{TiTi}. Figure 4-2a shows the deflection surface and the carry-over stiffnesses caused by the moment K_{TiTi}. Figure 4-2b illustrates how K_{TiTi} is applied to the plate model.

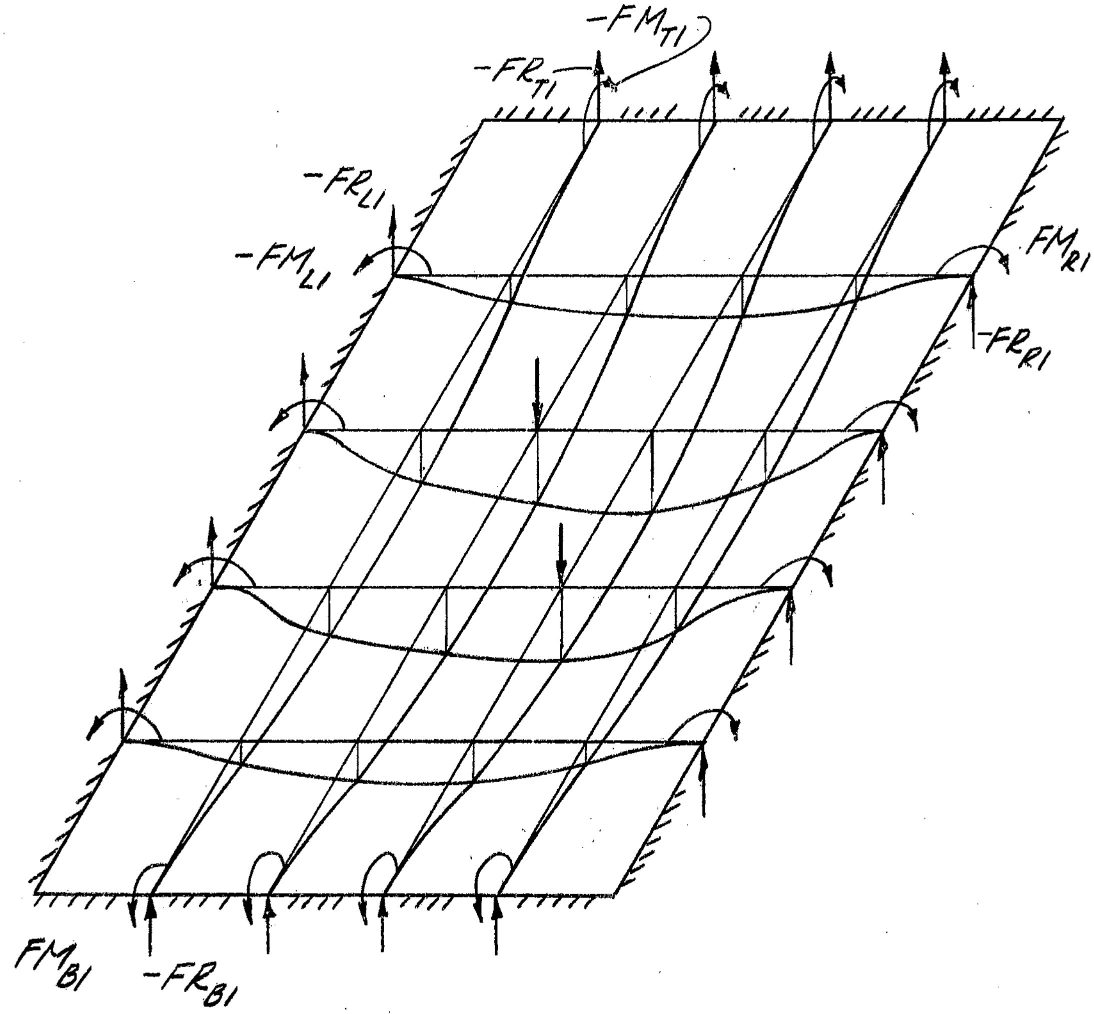


Figure 4-1 Load Functions

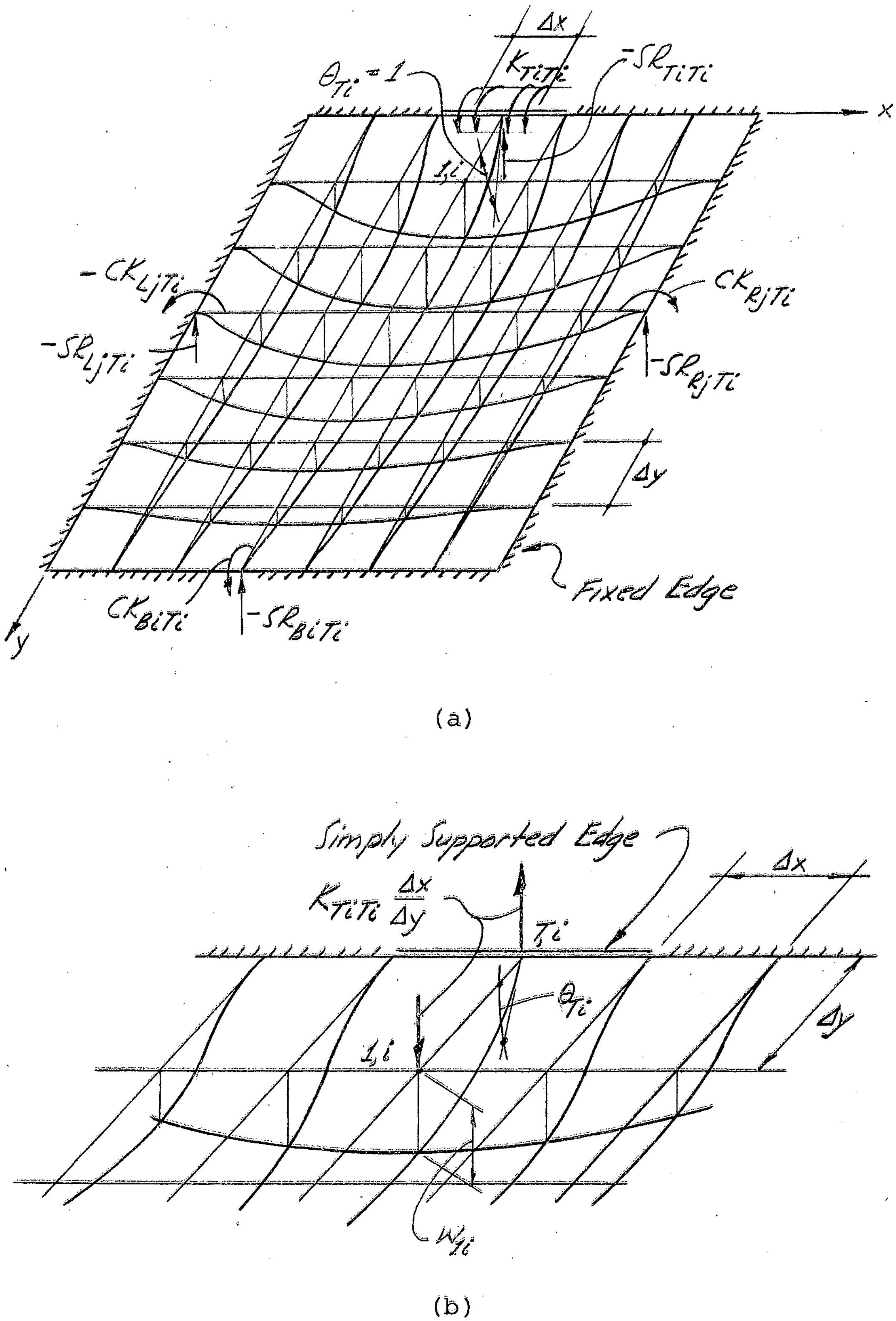


Figure 4-2 Plate Model for Moment Stiffness Factor

The resultant moment is K_{TiTi} (Δx) and the magnitude of each force in the equivalent couple is K_{TiTi} $(\Delta x)\frac{1}{\Delta y}$ or $t\ K_{TiTi}$. When Eq. (3-2) is applied to node (1i), the equivalent uniform load q_{1i} is

$$q_{1i} = \frac{tK_{TiTi}}{(\Delta x)\Delta y} = \frac{t^2 K_{TiTi}}{(\Delta x)^2} \tag{4-5}$$

Upon application of Eq. (3-2) to each interior node along with the boundary requirements, the following equations are obtained:

$$[A_2]\{W\} = \{1\}_{Ti} \frac{t^2 K_{TiTi} (\Delta x)^2}{D} \tag{4-6}$$

where $\{1\}_{Ti}$ is a column matrix such that each element is zero except the one corresponding to the node (1i) which is unity. The equations of Eq. (4-6) are solved for the unit value $\{1\}_{Ti}$ and K_{TiTi} is determined from the condition that Θ_{Ti} be unity. Now

$$\Theta_{Ti} = \frac{W_{1i}}{\Delta y} = 1\ ,\ \therefore\ W_{1i} = \Delta y \tag{4-7}$$

If W'_{1i} is the deflection at node (1i) from the solution of Eqs. (4-6), then

$$W_{1i} = W'_{1i} \frac{t^2 K_{TiTi} (\Delta x)^2}{D} = \Delta y \tag{4-8}$$

and

$$K_{TiTi} = \frac{D}{t^2} \frac{\Delta y}{(\Delta x)^2} \frac{1}{W'_{1i}} = \frac{D}{t^3 (\Delta x) W'_{1i}} \tag{4-9}$$

The deflection surface due to K_{TiTi} is now known and the carry-over stiffnesses are determined from Eqs. (3-9) through (3-12).

The reaction stiffness factor, T_{TiTi}, at the point Ti on the top edge of the plate is defined as the force per unit length necessary to cause a unit displacement at Ti with all other edge points fixed against rotation and displacement. The carry-over stiffnesses, CT_{RjTi} and SM_{RjTi}, for example, are the values of the reactions and moments per unit length induced at the fixed edge point Rj on the right edge due to T_{TiTi}.

Figure 4-3 shows the deflection surface and the carry-over stiffnesses caused by the reaction T_{TiTi}. To determine this deflection surface, the biharmonic plate equation, Eq. (3-2), is applied to each interior node and such that all q's are zero. The boundary requirements are that all edge slopes be zero and all edge displacements be zero except W_{Ti}, which is unity. In matrix form, the set of equations is

$$[A_3]\ \{W\} = -\{X\} \qquad (4\text{-}10)$$

where

$$\{X\} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 2t^2 \\ -4t^2-4t^4 \\ 2t^2 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ t^4 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \begin{matrix} 1,1 \\ 1,2 \\ \cdot \\ \cdot \\ \cdot \\ 1,i-2 \\ 1,i-1 \\ 1,i \\ 1,i+1 \\ 1,i+2 \\ \cdot \\ \cdot \\ \cdot \\ 2,i-1 \\ 2,i \\ 2,i+1 \\ \cdot \\ \cdot \\ \cdot \\ n,n \end{matrix} \qquad (4\text{-}11)$$

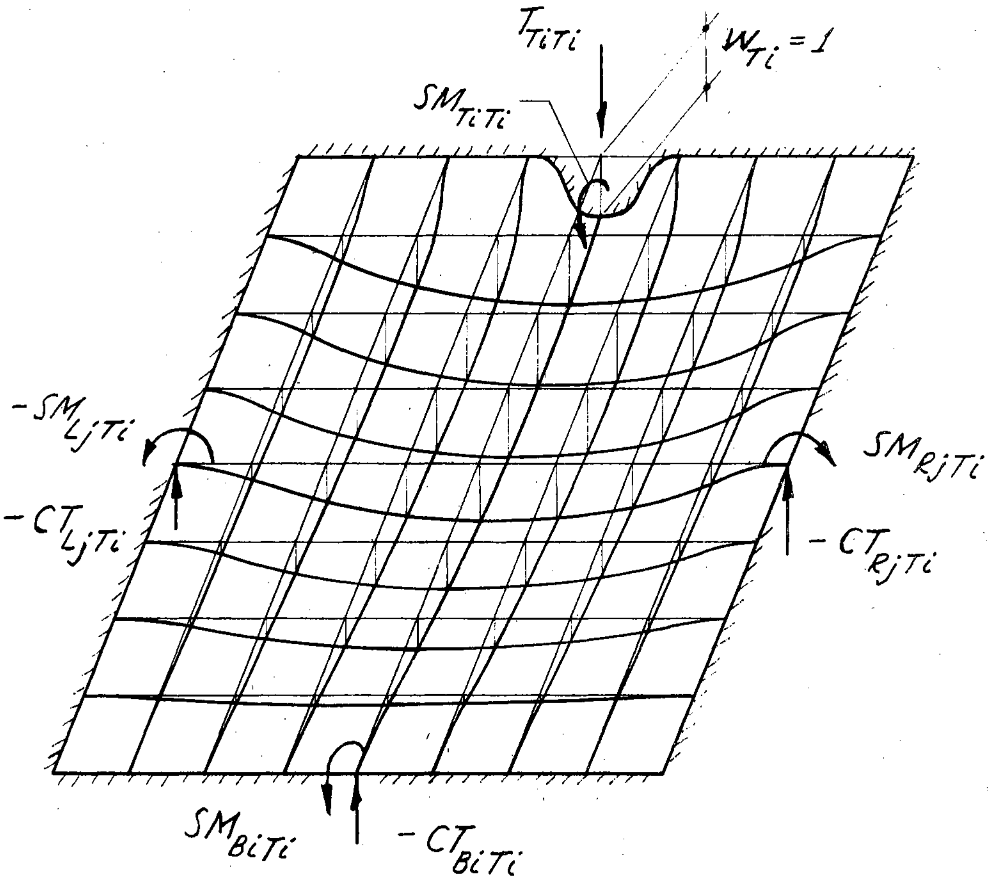


Figure 4-3 Plate Model for Reaction Stiffness Factor, T_{TiTi}

With the solution of Eq. (4-10), T_{TiTi} is calculated from Eq. (3-10) and the carry-over stiffnesses from Eqs. (3-9) through (3-12).

4-4 Stiffness Matrices

The slope deflection equations of the edge moments and reactions for a typical plate are, in matrix form,

$$\{M\} = [K]\{\theta\} + [SM]\{\Delta\} + \{FM\} \tag{4-12}$$

$$\{R\} = [SR]\{\theta\} + [T]\{\Delta\} + \{FR\} \tag{4-13}$$

where [K] is the moment stiffness matrix

$$[K]=\begin{bmatrix} K_{T1T1} & \cdots & CK_{T1Tn} & \cdots & CK_{T1Rn} & \cdots & CK_{T1Bn} & \cdots & CK_{T1Ln} \\ CK_{T2T1} & \cdots & CK_{T2Tn} & \cdots & CK_{T2Rn} & \cdots & CK_{T2Bn} & \cdots & CK_{T2Ln} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ CK_{TnT1} & \cdots & K_{TnTn} & \cdots & CK_{TnRn} & \cdots & CK_{TnBn} & \cdots & CK_{TnLn} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ CK_{RnT1} & \cdots & CK_{RnTn} & \cdots & K_{RnRn} & \cdots & CK_{RnBn} & \cdots & CK_{RnLn} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ CK_{BnT1} & \cdots & CK_{BnTn} & \cdots & CK_{BnRn} & \cdots & K_{BnBn} & \cdots & CK_{BnLn} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ CK_{LnT1} & \cdots & CK_{LnTn} & \cdots & CK_{LnRn} & \cdots & CK_{LnBn} & \cdots & K_{LnLn} \end{bmatrix}$$

[SM] is the carry-over moment stiffness matrix,

$$[SM]=\begin{bmatrix} SM_{T1T1} & SM_{T1T2} & \cdots & SM_{T1Tn} & \cdots & SM_{T1Rn} & \cdots & SM_{T1Bn} & \cdots & SM_{T1Ln} \\ SM_{T2T1} & SM_{T2T2} & \cdots & SM_{T2Tn} & \cdots & SM_{T2Rn} & \cdots & SM_{T2Bn} & \cdots & SM_{T2Ln} \\ \vdots & \vdots & & & & & & & & \\ SM_{TnT1} & SM_{TnT2} & \cdots & SM_{TnTn} & \cdots & & & & & \\ SM_{RnT1} & & & & & SM_{RnRn} & \cdots & & & \\ \vdots & & & & & & & & & \\ SM_{BnT1} & & & & & & \cdots & SM_{BnBn} & \cdots & \\ \vdots & & & & & & & & & \\ SM_{LnT1} & & & & & & & & \cdots & SM_{LnLn} \end{bmatrix} \qquad (4\text{-}15)$$

[SR] is the carry-over reaction stiffness matrix similar to Eq. (4-15), and [T] is the reaction stiffness matrix.

$$[T]=\begin{bmatrix} T_{T1T1} & CT_{T1T2} & \cdots & CT_{T1Tn} & \cdots & CT_{T1Rn} & \cdots & CT_{T1Bn} & \cdots & CT_{T1Ln} \\ CT_{T2T1} & T_{T2T2} & \cdots & CT_{T2Tn} & \cdots & CT_{T2Rn} & \cdots & CT_{T2Bn} & \cdots & CT_{T2Ln} \\ \vdots & \vdots & & & & & & & & \\ CT_{TnT1} & CT_{TnT2} & \cdots & T_{TnTn} & \cdots & & & & & \\ \vdots & & & & & & & & & \\ CT_{RnT1} & CT_{RnT2} & \cdots & & & T_{RnRn} & \cdots & & & \\ \vdots & \vdots & & & & & & & & \\ CT_{BnT1} & CT_{BnT2} & & & & & \cdots & T_{BnBn} & \cdots & \\ \vdots & \vdots & & & & & & & & \\ CT_{LnT1} & CT_{LnT2} & \cdots & & & & & & \cdots & T_{LnLn} \end{bmatrix} \qquad (4\text{-}16)$$

Each stiffness matrix is a 4n x 4n square matrix.

For a continuous plate model ($n\to\infty$) the moment stiffness matrix, [K], and the reaction stiffness matrix, [T], are symmetrical about the main diagonal. The carry-over stiffness matrices have a transpose relationship, that is

$$[SR] = [SM]^T, \text{ or } SR_{ij} = SM_{ji} \tag{4-17}$$

However, for a finite value of n, the plate model is not a continuous media and the above relationships will not necessarily be true.

4-5 Modified Stiffnesses

Loading conditions occur such that there is symmetry and/or anti-symmetry about either or both of the major axes of the plate. It is advantageous for the analyst to take this into account. Referring to Fig. 4-4:

Case 1 Symmetry About 0-0 Axis

$$\Theta_{Rj} = -\Theta_{Lj};\ \Delta_{Rj} = \Delta_{Lj};\ \Theta_{T,m+k} = \Theta_{T,m-k};\ \Delta_{T,m+k} = \Delta_{t,m-k} \tag{4-18}$$

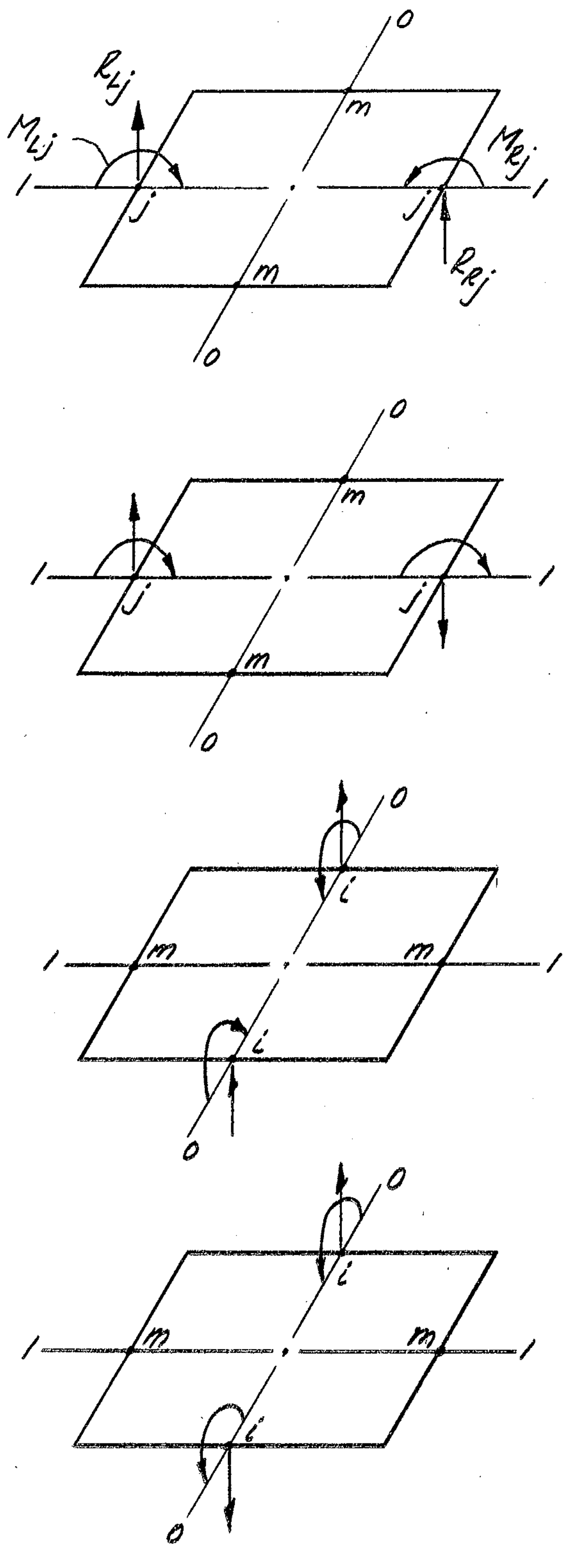
$$\Theta_{B,m+k} = \Theta_{B,m-k};\ \Delta_{B,m+k} = \Delta_{B,m-k};\ m = \frac{n+1}{2},\ k = 1,2,\ldots,n/2 \text{ for n odd}$$

$$j = 1,2,\ldots,n;\ m = \frac{n+1}{2},\ k = \frac{1}{2},\frac{3}{2},\ldots,\frac{n-1}{2} \text{ for n even}$$

Substituting into Eqs. (4-12) and (4-13):

$$\{M\} = [K'] \{\Theta\} + [SM'] \{\Delta\} + \{FM\} \tag{4-19}$$

$$\{R\} = [SR'] \{\Theta\} + [T'] \{\Delta\} + \{FR\} \tag{4-20}$$



Case 1

Symmetry About

0-0 Axis

Case 2

Anti-Symmetry About

0-0 Axis

Case 3

Symmetry About

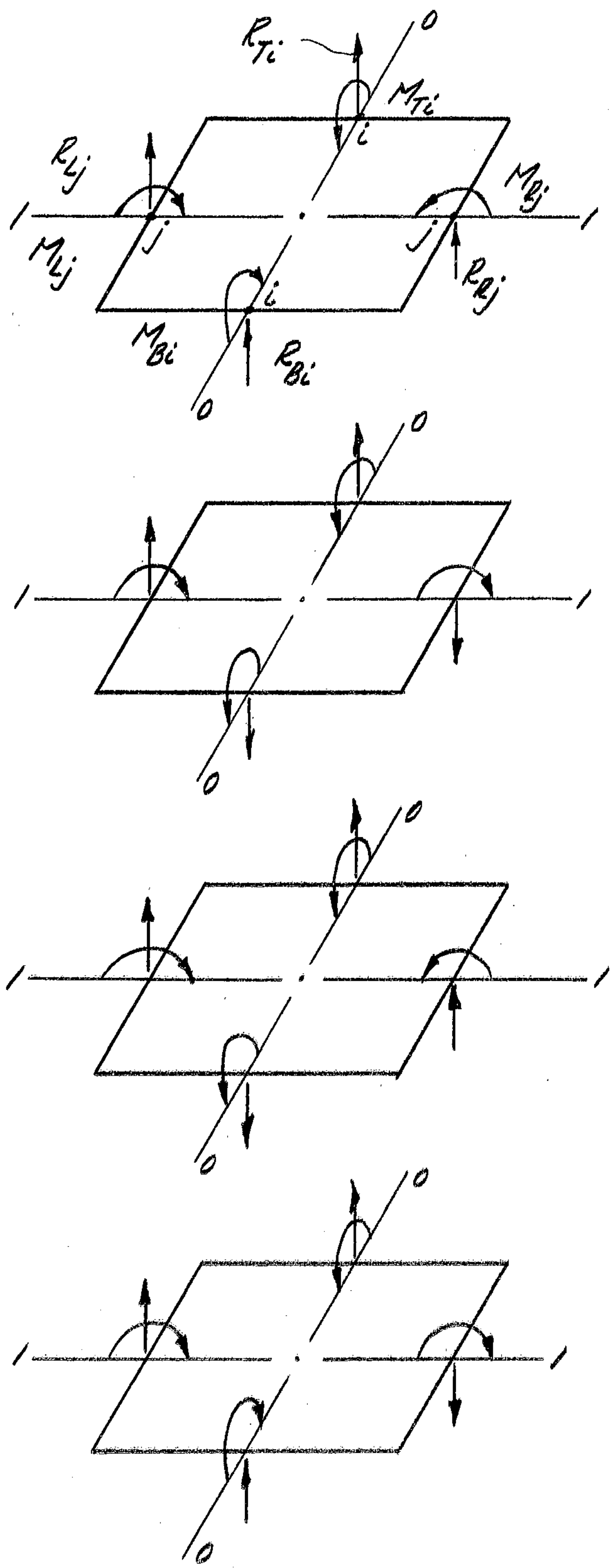
1-1 Axis

Case 4

Anti-Symmetry About

1-1 Axis

Figure 4-4 Special Cases of Basic Plate



Case 5

Symmetry About

0-0 and 1-1 Axes

Case 6

Anti-Symmetry About

0-0 and 1-1 Axes

Case 7

Sym. About 0-0 Axis

Anti-Sym. About 1-1 Axis

Case 8

Anti-Sym. About 0-0 Axis

Sym. About 1-1 Axis

Figure 4-5 Combined Special Cases of Basic Plate

Eq. (4-19) is

$$
\begin{bmatrix} M_{T1} \\ \vdots \\ M_{Tm} \\ M_{B1} \\ \vdots \\ M_{Bm} \\ M_{L1} \\ \vdots \\ M_{Lj} \\ \vdots \\ M_{1n} \end{bmatrix}
=
\begin{bmatrix}
K'_{T1T1} & CK'_{T1T2}\cdots & CK_{T1Tm} & CK'_{T1B1}\cdots CK_{T1Bm} & CK'_{T1L1}\cdots CK'_{T1L1}\cdots CK'_{T1Lj}\cdots CK'_{T1Ln} \\
\vdots & & & & \\
CK'_{TmT1} & CK_{TmT2}\cdots & K_{TmTm} & CK'_{TmB1}\cdots CK_{TmBm} & CK'_{TmL1}\cdots CK'_{TmL1}\cdots CK'_{TmLj}\cdots CK'_{TmLn} \\
CK'_{B1T1} & & & & \\
\vdots & & & & \\
CK'_{BmT1} & & & & \\
CK'_{L1T1} & & & & \\
\vdots & & & & \\
CK'_{LjT1} & & & & \\
\vdots & & & & \\
CK'_{LnT1} & & & & \cdots K'_{LnLn}
\end{bmatrix}
\begin{bmatrix} \Theta_{T1} \\ \vdots \\ \Theta_{Tm} \\ \Theta_{B1} \\ \vdots \\ \Theta_{Bm} \\ \Theta_{L1} \\ \vdots \\ \Theta_{Lj} \\ \vdots \\ \Theta_{Ln} \end{bmatrix}
$$

$$
+\begin{bmatrix}
SM'_{T1T1} & SM'_{T1T2}\cdots & SM_{T1Tm} & SM'_{T1B1}\cdots & SM_{T1Bm} & SM'_{T1L1}\cdots SM'_{T1Lj}\cdots SM'_{T1Ln} \\
\vdots & & & & & \\
SM'_{TmT1} & SM'_{TmT2}\cdots & SM_{TmTm} & SM'_{TmB1}\cdots & SM_{TmBm} & SM'_{TmL1}\cdots SM'_{TmLj}\cdots SM'_{TmLj} \\
Sm'_{B1T1} & & & & & \\
\vdots & & & & & \\
SM'_{BmT1} & & & & & \\
SM'_{L1T1} & & & & & \\
\vdots & & & & & \\
SM'_{LjT1} & & & & & \\
\vdots & & & & & \\
SM'_{LnT1} & & & & & \cdots SM'_{LnLn}
\end{bmatrix}
\begin{bmatrix}
\Delta_{T1} \\ \vdots \\ \Delta_{Tm} \\ \Delta_{B1} \\ \vdots \\ \Delta_{Bm} \\ \Delta_{L1} \\ \vdots \\ \Delta_{Lj} \\ \vdots \\ \Delta_{Ln}
\end{bmatrix}
+
\begin{bmatrix}
FM_{T1} \\ \vdots \\ FM_{Tm} \\ RM_{B1} \\ \vdots \\ FM_{Bm} \\ FM_{L1} \\ \vdots \\ FM_{Lj} \\ \vdots \\ FM_{Ln}
\end{bmatrix}
$$

(4-21)

where $K'_{T1T1} = K_{T1T1} + CK_{T1Tn}$

$CK'_{T1Ti} = CK_{T1Ti} + CK_{T1Tn-i}, \; i<m$

$CK'_{LjTi} = CK_{LjTi} + CK_{LjTn-i}, \; i<m$

$$CK'_{LjLi} = CK_{LjLi} - CK_{RjRi}, \; i = 1, \ldots, n \qquad (4\text{-}22)$$

etc.

These are called modified stiffnesses. Eq. (4-20) is similar to Eq. (4-21).

Case 2 Anti-Symmetry About o-o Axis

$\Theta_{Rj} = \Theta_{Lj}; \; \Delta_{Rj} = -\Delta_{Lj}; \; \Theta_{T,m+k} = -\Theta_{T,m-k}; \; \Delta_{T,m+k} = -\Delta_{T,m-k}$

$$\Theta_{B,m+k} = -\Theta_{B,m-k}; \; \Delta_{B,m+k} = -\Delta_{B,m-k}; \; j = 1,2,\ldots,n \qquad (4\text{-}13)$$

$m = \frac{n+1}{2}; \; k = 1,2,\ldots,\frac{n-1}{2}$ for n odd

$m = \frac{n+1}{2}; \; k = \frac{1}{2}, \frac{3}{2}, \ldots, \frac{n-1}{2}$ for n even

Substituting into Eqs. (4-12) and (4-13)

$$\{M\} = [K''] \{\Theta\} + [SM''] \{\Delta\} + \{FR\} \qquad (4\text{-}24)$$

$$\{R\} = [SR''] \{\Theta\} + [T''] \{\Delta\} + \{FR\} \qquad (4\text{-}25)$$

where the elements of the modified stiffness matrices are analogous to Eq. (4-22).

Cases 3 and 4 are similar to Cases 1 and 2.

Case 5 Symmetry About Axes 0-0 and 1-1

$$\Theta_{Bi} = -\Theta_{Ti}; \; \Delta_{Bi} = \Delta_{Ti}; \; \Theta_{Rj} = -\Theta_{Lj}; \; \Delta_{Rj} = \Delta_{Lj}; \; i,j = 1,2,\ldots,n \qquad (4\text{-}26)$$

$\Theta_{T,i+k} = \Theta_{T,i-k}; \; \Delta_{T,i+k} = \Delta_{T,i-k}; \; \Theta_{L,j+k} = \Theta_{L,j-k}$

$\Delta_{L,j+k} = \Delta_{L,j-k}; \; i,j = m = \frac{n+1}{2}, \; k = 1,\ldots,\frac{n-1}{2}$ for n odd

$i,j = m = \frac{n+1}{2}, \; k = \frac{1}{2},\frac{3}{2},\ldots,\frac{n-1}{2}$ for n even

$\Theta_{Lj} = \Theta_{Tj}; \; \Delta_{Lj} = \Delta_{Tj}; \; j = 1,2,\ldots,n$

Substituting into Eqs. (4-12) and (4-13):

$$\{M\} = [K^V]\{\Theta\} + [SM^V]\{\Delta\} + \{FM\} \tag{4-27}$$

$$\{R\} = [SR^V]\{\Theta\} + [T^V]\{\Delta\} + \{FR\} \tag{4-28}$$

Eq. (4-27) is

$$\begin{bmatrix} M_{T1} \\ M_{T2} \\ \vdots \\ M_{Tm} \end{bmatrix} = \begin{bmatrix} K^V_{T1T1} & CK^V_{T1T2} & \cdots & CK^V_{T1Tm} \\ CK^V_{T2T1} & K^V_{T2T2} & \cdots & CK^V_{T2Tm} \\ \vdots & \vdots & & \vdots \\ CK^V_{TmT1} & CK_{TmT2} & \cdots & K^V_{TmTm} \end{bmatrix} \begin{bmatrix} \Theta_{T1} \\ \Theta_{T2} \\ \vdots \\ \Theta_{Tm} \end{bmatrix} + \begin{bmatrix} SM^V_{T1T1} & SM^V_{T1T2} & \cdots & SM^V_{T1Tm} \\ SM^V_{T2T1} & SM^V_{T2T2} & \cdots & SM^V_{T2Tm} \\ \vdots & \vdots & & \vdots \\ SM^V_{TmT1} & SM^V_{TmT2} & \cdots & SM^V_{TmTm} \end{bmatrix} \begin{bmatrix} \Delta_{T1} \\ \Delta_{T2} \\ \vdots \\ \Delta_{Tm} \end{bmatrix} + \begin{bmatrix} FM_{T1} \\ FM_{T2} \\ \vdots \\ FM_{Tm} \end{bmatrix} \tag{4-29}$$

where

$$K^V_{T1T1} = K_{T1T1} + CK_{T1Tn} - CK_{T1R1} - CK_{T1Rn} - CK_{T1B1} - CK_{T1Bn} + CK_{T1L1} + CK_{T1Ln} \tag{4-30}$$

$$CK^V_{T1T2} = CK_{T1T2} + CK_{T1Tn-1} - CK_{T1R2} - CK_{T1Rn-2} - CK_{T1B2} - CK_{T1Bn-2} + CK_{T1L2} + CK_{T1Ln-2} \tag{4-31}$$

etc.

Cases 6, 7, and 8 are analogous to Case 5.

CHAPTER V

BEAM CONSTANTS

5-1 General

The supporting beam along the edge of the plate is divided into the same number of segments (n+1) as the plate model. The twisting moment and the shearing force in the beam are expressed in terms of the angle of twist, Θ, and the deflection, Δ.

5-2 Flexural Stiffness

Consider a segment of a beam of length λ where

$$\lambda = \frac{a}{n+1} \tag{5-1}$$

if a is the length of the edge of the plate and n is the number of edge points selected. Using the notation of Fig. 5-1, the shears are expressed in terms of the bending moments.

$$V_{jk} = \frac{1}{\lambda} (M_{jk} + M_{kj}) = -V_{kj} \tag{5-2}$$

From the elementary theory of beams,

$$M_{jk} = -EI\frac{d^2z}{dx^2} = -M_{kj} \tag{5-3}$$

where EI is the flexural rigidity of the beam segment. The finite difference approximation of Eq. (5-3) is (letting $\Delta = z$)

$$M_{jk} = \frac{-EI}{\lambda^2} (\Delta_i - 2\Delta_j + \Delta_k) \tag{5-4}$$

$$M_{kj} = + \frac{EI}{\lambda^2} (\Delta_j - 2\Delta_k + \Delta_l) \tag{5-5}$$

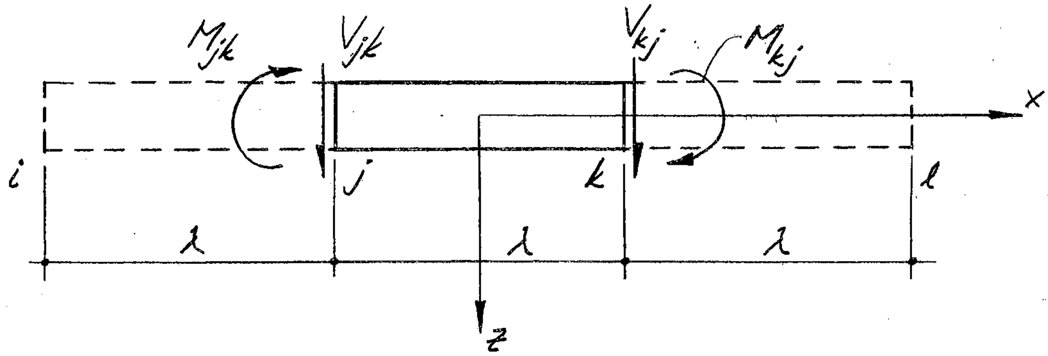


Figure 5-1 Segment of Supporting Beam in Bending

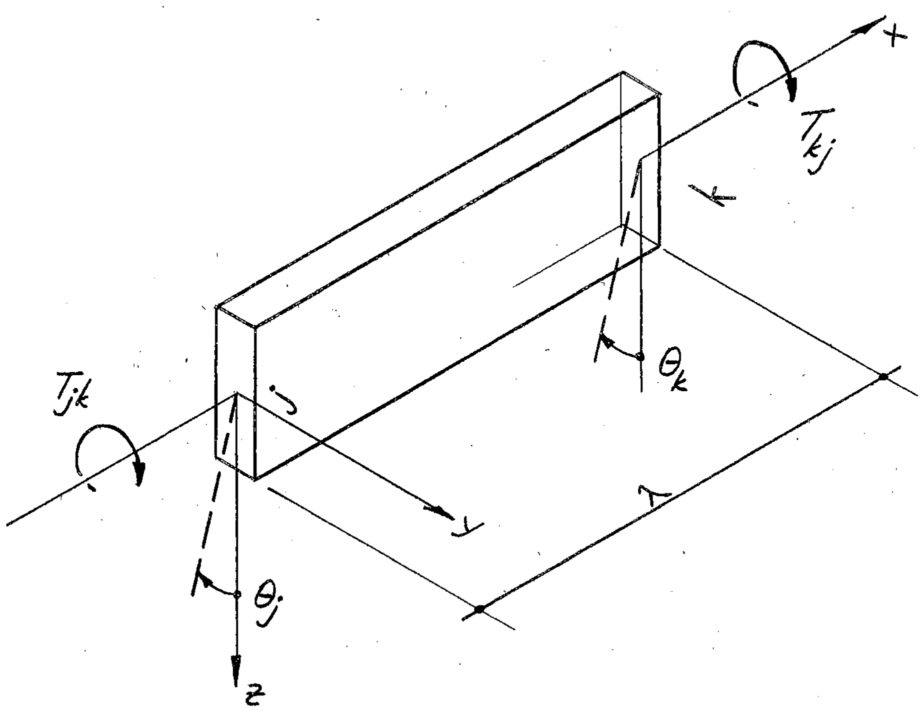


Figure 5-2 Segment of Supporting Beam in Torsion

Substituting into Eq. (5-2)

$$V_{jk} = \frac{EI}{\lambda^3} (-\Delta_i + 3\Delta_j - 3\Delta_k + \Delta_l) \quad (5\text{-}6)$$

$$V_{kj} = \frac{EI}{\lambda^3} (\Delta_i - 3\Delta_j + 3\Delta_k - \Delta_l) \quad (5\text{-}7)$$

Eqs. (5-6) and (5-7) are the slope-deflection equations of the beam segment for shear.

5-3 Torsional Stiffness

Referring to Figure 5-2, it is necessary to express the twisting moments in terms of the angles of twist, Θ_j and Θ_k. Assuming elastic behavior and no warping of the beam in the length λ, the elementary torsion theory gives

$$\frac{d\Theta}{dx} = \frac{T}{GJ^*} \quad (5\text{-}8)$$

where GJ* is the torsional rigidity of the beam.

Multiplying Eq. (5-8) by dx and integrating between j and k,

$$T_{kj} = \frac{GJ^*}{\lambda}(\Theta_k - \Theta_j) \quad (5\text{-}9)$$

$$T_{jk} = \frac{GJ^*}{\lambda}(\Theta_j - \Theta_k) \quad (5\text{-}10)$$

which are the slope-deflection equations of the beam segment for torsion.

5-4 Modified Stiffnesses

If symmetry exists in the edge beam of Fig. 5-3, then

$$\Delta_k = \Delta_i \ ; \ \Theta_k = \Theta_i \ ; \ \Delta_l = \Delta_h \ ; \ \Theta_l = \Theta_h$$

and Eqs. (5-6) and (5-7) become

$$V_{ij} = \frac{EI}{\lambda^3} (-\Delta_h + 4\Delta_i - 3\Delta_j) = V_{kj} \tag{5-11}$$

$$V_{ji} = \frac{EI}{\lambda^3} (\Delta_h - 4\Delta_i + 3\Delta_j) = 0 \tag{5-12}$$

If anti-symmetry exists, then

$$\Delta_k = -\Delta_1 \; ; \; \Theta_k = -\Theta_i \; ; \; \Delta_1 = -\Delta_h \; ; \; \Theta_1 = -\Theta_h$$

and

$$V_{ij} = \frac{EI}{\lambda^3} (-\Delta_h + 2\Delta_i - 3\Delta_j) = -V_{kj} \tag{5-13}$$

$$V_{ji} = \frac{EI}{\lambda^3} (\Delta_h - 2\Delta_i + 3\Delta_j) = -V_{jk} \tag{5-14}$$

It is noted that the equations for shear do not include the weight of the beam segment.

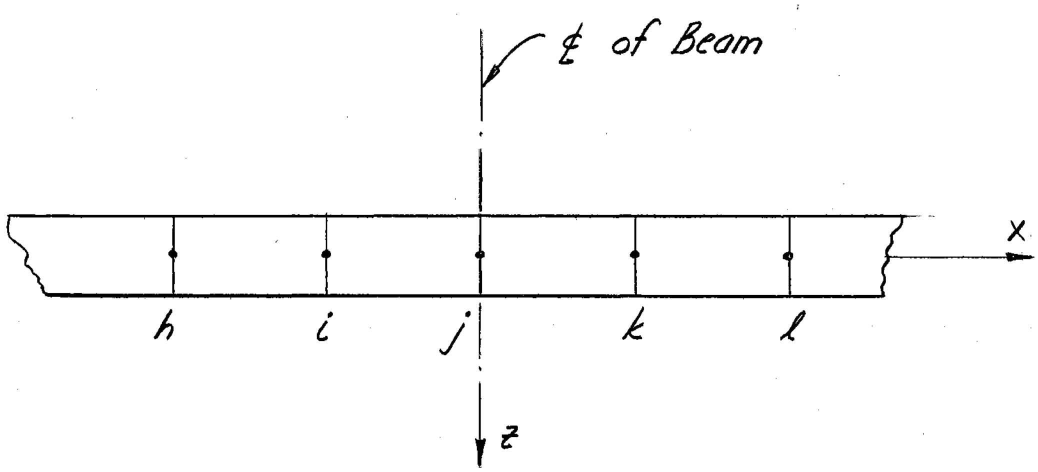


Figure 5-3 Portion of Edge Beam

CHAPTER VI

COMPATIBILITY RELATIONSHIPS

6-1 General

A two-way continuous plate system supported and loaded in a general manner is divided into basic plate elements and edge beams. A plate model is selected for the basic plates and the bending moment and reaction at each edge point of the basic plates are taken as the redundants. A slope deflection equation is written for each redundant and the continuity or compatibility requirements of the continuous plate system are determined by the moment and force equilibrium equation at each edge point or "joint." The resulting set of simultaneous equations in terms of the unknown Θ's and Δ's is expressed matrically and certain relationships are noted.

6-2 Joint Compatibility Equations

Consider the two-way continuous plate system of Figure 6-1 supported with beams along the edge of each plate element. A typical basic plate and the corresponding plate model are shown in Figure 6-2. Figure 6-3 shows the free body at "joint 3," which represents point 3 on the top edge of plate I, point 3 on the bottom edge of plate II and point 3 on the edge beam.

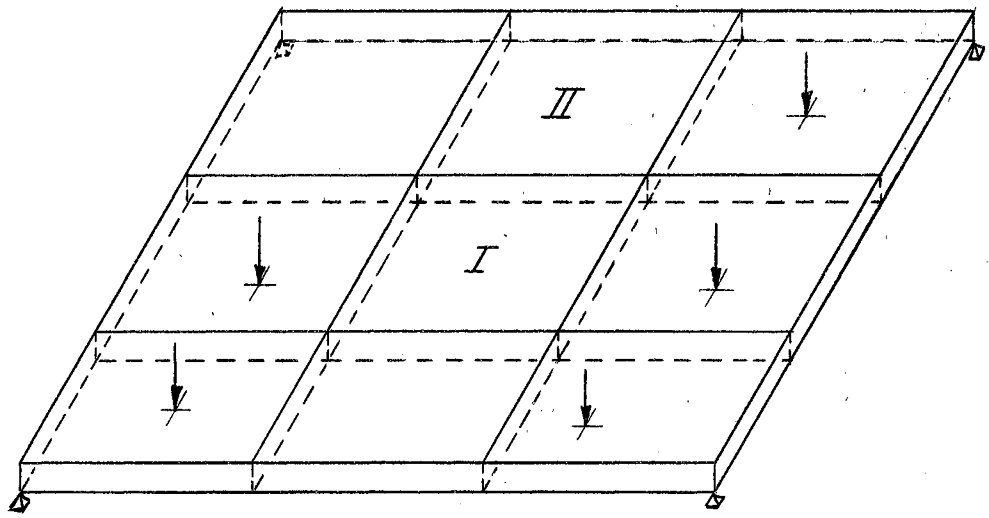


Figure 6-1 Two-Way Continuous Plate-Beam Structure

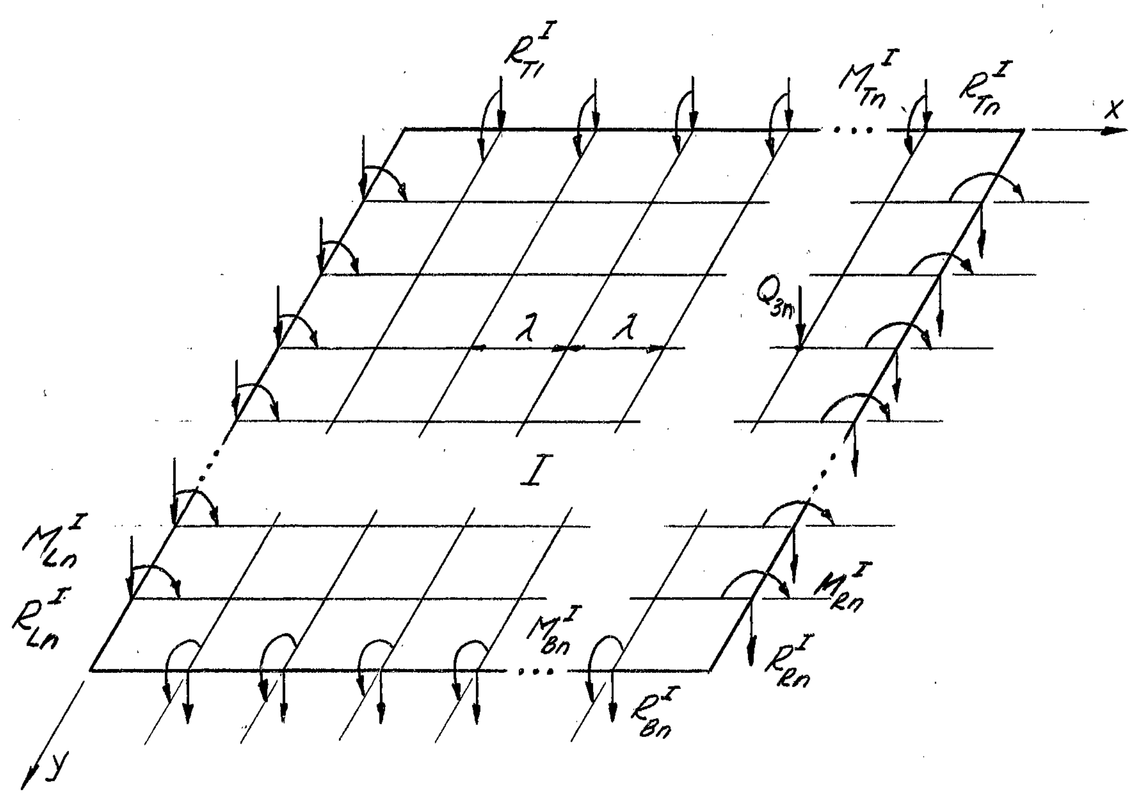


Figure 6-2 Basic Plate Element I

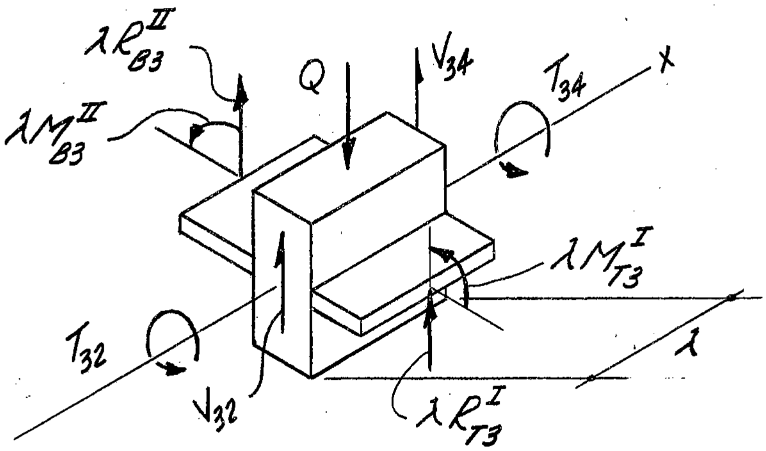


Figure 6-3 Freebody of Joint 3

The equilibrium requirements at joint 3 are

$$\sum F_z = 0 \quad \text{and} \quad \sum M_x = 0 \tag{6-1}$$

which gives

$$V_{32} + V_{34} + R^{I}_{T3} + R^{II}_{B3} - Q = 0 \tag{6-2}$$

$$T_{32} + T_{34} + M^{I}_{T3} + M^{II}_{B3} = 0 \tag{6-3}$$

It is noted that in Eq. (6-3) the rotational effect of the plate reactions is assumed negligible.

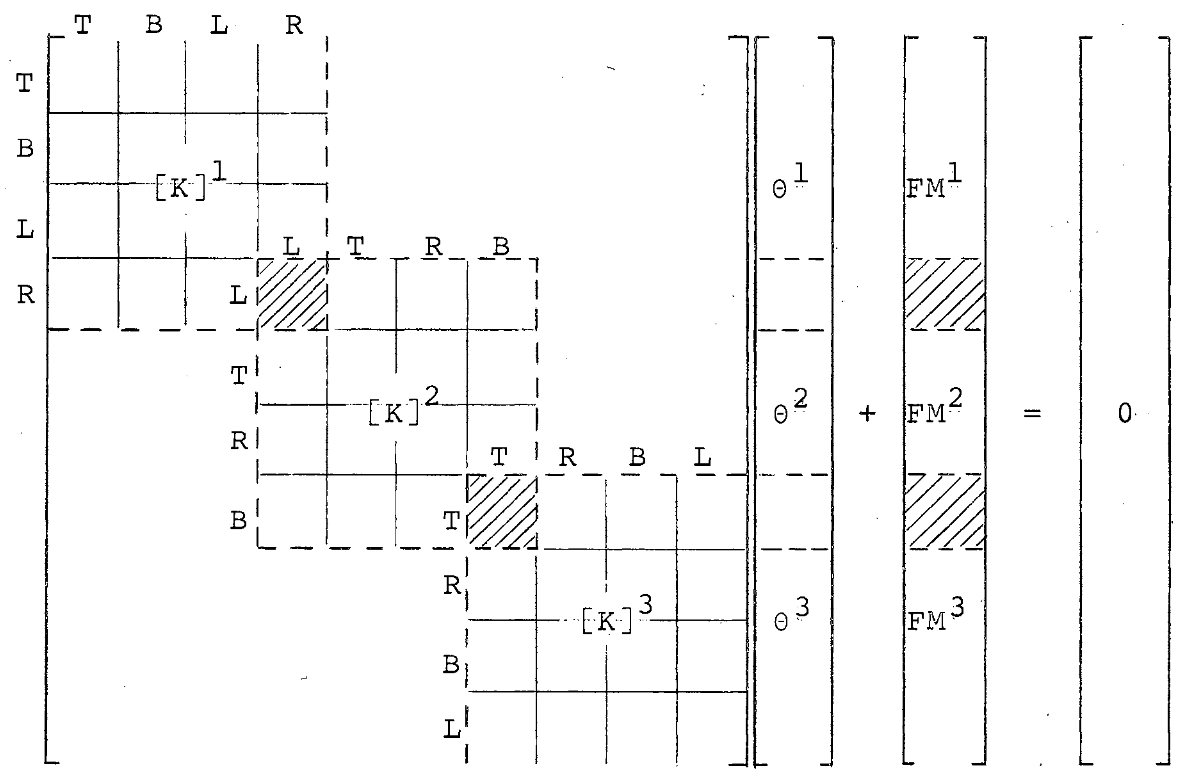
6-3 Compatibility Matrix

Equations (6-2) and (6-3) satisfy the compatibility and equilibrium requirements at joint 3 and when applied to each edge joint these requirements are satisfied for the entire system. The bending moments, shears and reactions are expressed in terms of the slopes and deflections of each joint, that is, the slope deflection equations for the moments, shears, and reactions are substituted into Eqs. (6-2) and (6-3). The resulting set of simultaneous equations when placed in matrix form is called the compatibility matrix.

As an example, consider the two-way plate system of Figure 6-4 supported rigidly along the edges of each plate element and loaded in a general manner. Since the supports are rigid, all Δ's are zero and the edge rotations, Θ_i, are the unknowns. The plate model of Figure 6-2 is selected for each basic plate and Eq. (6-3) is applied to each joint. There are n joints on each edge and ten edges, resulting in 10n equations

$$M_i^j + M_i^{j+1} = 0\ , \quad i = 1,2,\ldots,n;\ j = 1,2 \qquad (6\text{-}4)$$
$$M_i^k = 0\ , \quad k = 1,2,\ldots,8$$

Substituting Eqs. (4-12) and (4-14) into Eq. (6-4) the following matrix equation is obtained.



The shaded areas represent a summation of the respective elements of each stiffness or load matrix. All elements outside the dashed lines in the combined stiffness matrix are zero.

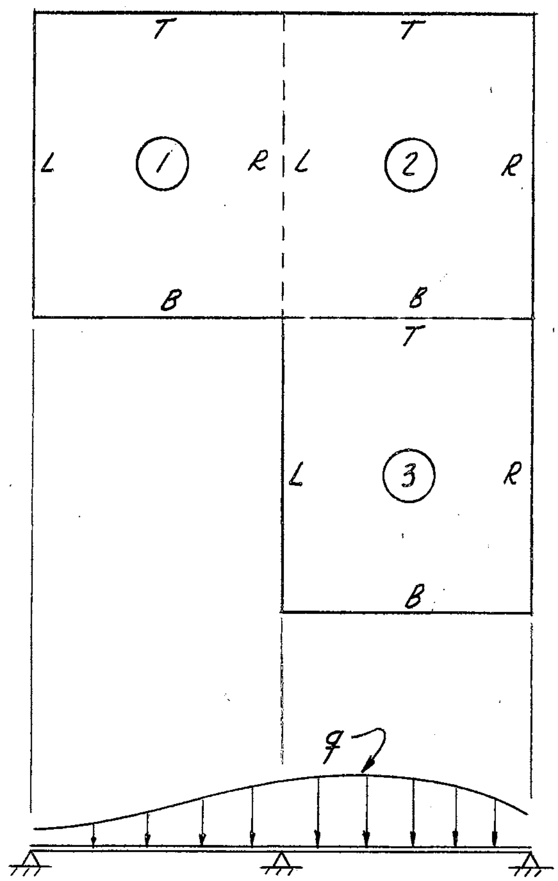


Figure 6-4 Two-Way Continuous Plate System on Rigid Supports

6-4 Incompatibility at Corners

Compatibility and equilibrium are not satisfied at the corners of the plate elements because a slope-deflection equation cannot be written for the bending moment at a corner. For the plate model adopted in this study it is impossible to determine the moment stiffness factor at a corner of the basic plate. The plate corner point is actually a point of discontinuity and the slope of that point is indeterminate. The deflection of the corner point may be included in the analysis but was omitted from this study. For a further discussion of the plate behavior at a corner refer to reference (1) p. 85.

CHAPTER VII

NUMERICAL APPLICATION OF THE THEORY

7-1 Plate Model

The mathematical model of the plate adopted for numerical application of the theory is shown in Figure 7-1, which is the general model of Figure 1-1 with n=5. This means that in a given plate system, equilibrium and compatibility are satisfied at only five points (exclusive of the corners) on each plate edge. The number five was selected to keep the number of unknowns at a minimum while assuring a "reasonable" accuracy of solution.

In calculating the elastic stiffness constants at the selected edge points, a finer gridwork was employed as shown by the solid and dashed lines in Figure 7-1. That is, 121 interior nodes were used to calculate the elastic constants at the five selected points on each edge.

7-2 Calculation of Stiffness Matrices

The procedure to follow in calculating the stiffnesses due to moment was outlined in Section 4-3 relative to the general plate model of Figure 4-2. For the numerical model of Figure 7-1 the plate edge is allowed to rotate freely between the points adjacent to the point in question. For

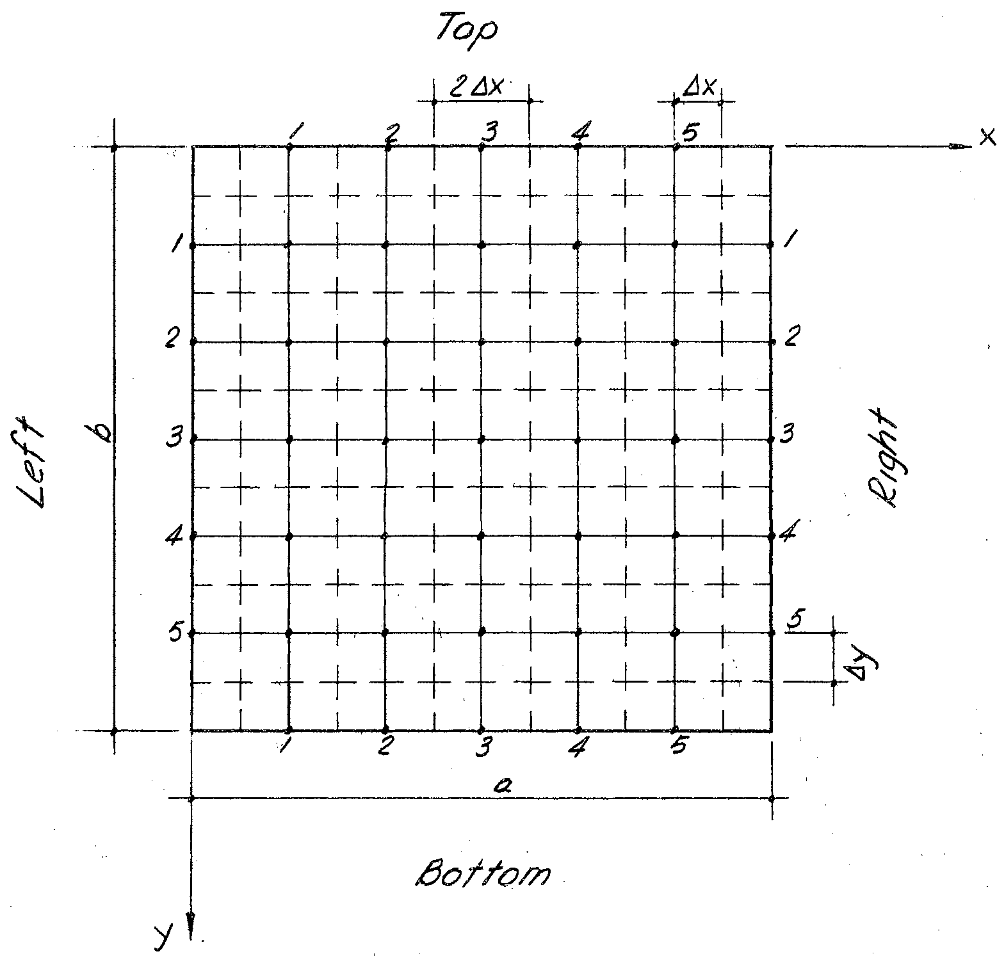


Figure 7-1 Plate Model

example, in determining K_{T3T3}, the edge between points 2 and 4 is simply supported and therefore the total applied moment at point 3 is $K_{T3T3}(2\Delta x)$. Eq. (4-6) then becomes

$$[A_2]\{W\} = \{1\}_{T3} \frac{2t^2 K_{T3T3}(\Delta x)^2}{D} \tag{7-1}$$

and Eq. (4-9) is

$$K_{T3T3} = \frac{D}{2t^3(\Delta x)W'_{13}} \tag{7-2}$$

The moment stiffness factors and the carry-over moment and reaction stiffnesses are tabulated in Appendix A for a square plate.

The plate model adopted to calculate the reaction stiffness factors is shown in Figure 7-2 for T_{T3T3}. The edge displacement between the points adjacent to the point in question is assumed linear. Applying Eq. (4-10) to Figure 7-2, then Eq. (4-11) becomes

$$\{X\} = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ t^2 \\ -2t^4 \\ -2t^2-4t^2 \\ -2t^2 \\ t^2 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ .5t^4 \\ t^4 \\ .5t^4 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \begin{matrix} 1,1 \\ \cdot \\ \cdot \\ \cdot \\ 1,3 \\ 1,4 \\ 1,5 \\ 1,6 \\ 1,7 \\ 1,8 \\ 1,9 \\ \cdot \\ \cdot \\ \cdot \\ 2,4 \\ 2,5 \\ 2,6 \\ 2,7 \\ 2,8 \\ \cdot \\ \cdot \\ \cdot \\ 121,121 \end{matrix} \qquad (7\text{-}3)$$

The reaction stiffness factors and the carry-over stiffnesses are tabulated in Appendix A for a square plate.

7-3 Load Functions

The fixed-edge moment and reaction are determined from the application of Eqs. (4-3) and (4-4) to the plate model of Figure 7-1 for a unit concentrated load applied at the

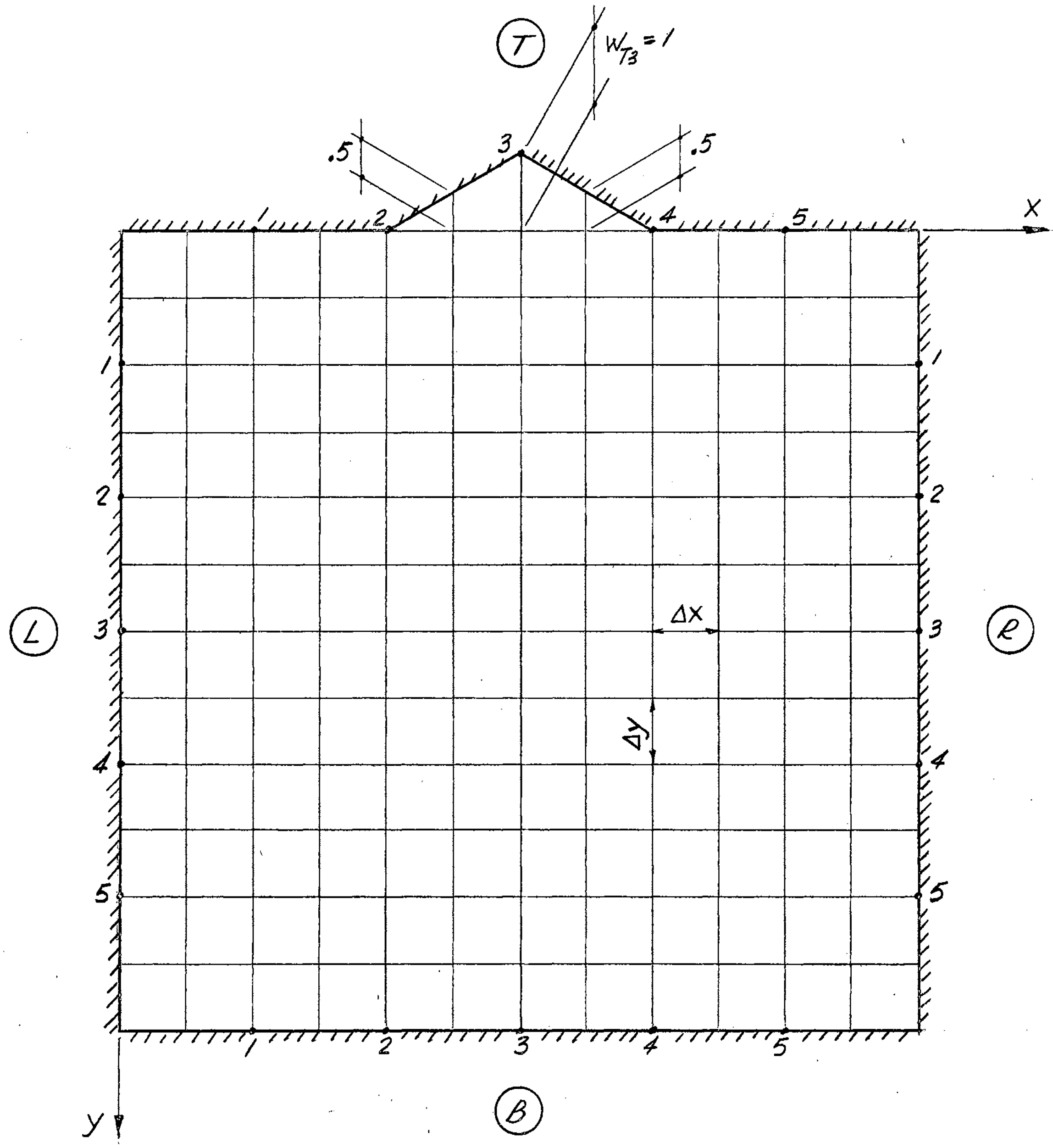


Figure 7-2 Plate Model for Reaction Stiffness Factor

darkened nodes shown. These values are tabulated in Appendix B for square plates.

7-4 Numerical Examples

Three numerical examples are presented to illustrate the method and to determine the accuracy of the plate model.

To improve the accuracy of the solution and to reduce the number of equations, the stiffness factors were calculated for three special plates which could occur as exterior plates in a continuous system. These factors are listed in Appendix A and are used in some of the example problems.

Example 1

It is desired to find the moments at the supports and the center moments and deflection of the continuous plate system shown in Figure 7-3.

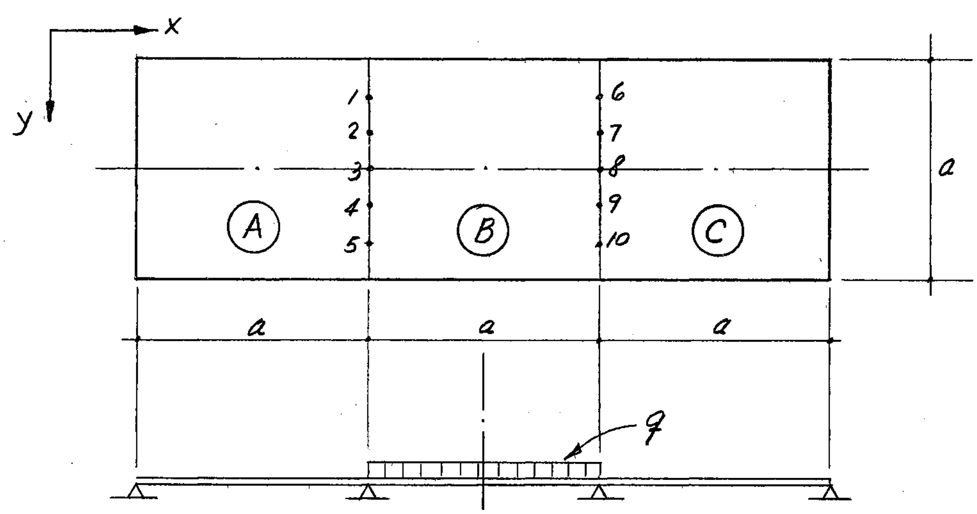


Figure 7-3 Example 1. Three Span Continuous Plate on Simple, Rigid Supports

Plate A

Taking advantage of symmetry, noting that all support deflections are zero, and applying the slope-deflection equation to edge points 1, 2, and 3 from Table A-X,

$$M_1^A = (31.29\theta_1 - 8.724\theta_2 - 2.214\theta_3 - 0.677\theta_4 - 0.218\theta_5)\frac{D}{a}$$

Since $\theta_5 = \theta_1$ and $\theta_4 = \theta_2$:

$$M_1^A = (31.07\theta_1 - 9.401\theta_2 - 2.214\theta_3)\frac{D}{a}$$

$$M_2^A = (-8.68\theta_1 + 23.89\theta_2 - 2.21\theta_3)\frac{D}{a}$$

$$M_3^A = (-4.08\theta_1 - 17.29\theta_2 + 25.68\theta_3)\frac{D}{a}$$

Plate B

From Tables A-IX and B-I:

$$M_1^B = (31.41\theta_1 - 8.652\theta_2 - 2.126\theta_3 - .601\theta_4 - .176\theta_5$$
$$+ .398\theta_6 + .555\theta_7 + .469\theta_8 + .290\theta_9 + .127\theta_{10})\frac{D}{a}$$
$$- .0393qa^2$$

From symmetry, $\theta_6 = -\theta_1$, $\theta_7 = -\theta_2$, $\theta_8 = -\theta_3$, ... , $\theta_5 = +\theta_1$, $\theta_4 = +\theta_2$.

Therefore,

$$M_1^B = (30.71\theta_1 - 10.10\theta_2 - 2.60\theta_3)\frac{D}{a} - 0.0393qa^2$$

$$M_2^B = (-9.35\theta_1 + 22.94\theta_2 - 9.38\theta_3)\frac{D}{a} - 0.0610aq^2$$

$$M_3^B = (-4.79\theta_1 - 18.64\theta_2 + 25.14\theta_3)\frac{D}{a} - 0.0677qa^2$$

Equilibrium

$$\sum M_i = 0 \; : \; M_i^A + M_i^B = 0 \; , \; i = 1,2,3$$

$$\therefore \quad 61.78\theta_1 - 19.50\theta_2 - 4.81\theta_3 = .0393\frac{qa^3}{D}$$

$$-18.03\theta_1 + 46.83\theta_2 - 18.09\theta_3 = .0610\ \frac{qa^3}{D}$$

$$-8.87\theta_1 - 35.93\theta_2 + 50.82\theta_3 = .0677\frac{qa^3}{D}$$

Solution

$$\theta_1 = +.00221\frac{qa^3}{D}\ ,\ \theta_2 = +.00388\frac{qa^3}{D}\ ,\ \theta_3 = +.00446\frac{qa^3}{D}$$

$$M_1^B = -.0223qa^2\ ,\ M_2^B = -.0345qa^2\ ,\ M_3^B = -.0384qa^2$$

Center Moments , $\nu = .2$

$$M_x^B = .1332\frac{D}{a}(\theta_1 + \theta_5 - \theta_6 - \theta_{10}) + .1573\frac{D}{a}(\theta_2 + \theta_4 - \theta_7 - \theta_9) + .1176\frac{D}{a}(\theta_3 - \theta_8) + .0307qa^2$$

$$M_y^B = .0517\frac{D}{a}(\theta_1 + \theta_5 - \theta_6 - \theta_{10}) + .250\frac{D}{a}(\theta_2 + \theta_4 - \theta_7 - \theta_9) + .4366\frac{D}{a}(\theta_3 - \theta_8) + .02312qa^2$$

$$M_x^B = +\ .0354qa^2\ ,\ M_y^B = +\ .0313qa^2$$

Center Deflection , Plate B

$$W = .01398\ a(\theta_1 + \theta_5 - \theta_6 - \theta_{10}) + .02491\ a(\theta_2 + \theta_4 - \theta_7 - \theta_9) + .03029\ a(\theta_3 - \theta_8) + .00200\frac{qa^4}{D}$$

$$W = .00278\frac{qa^4}{D}$$

Example 2

A square plate is elastically supported on each edge and is subjected to a uniform load of intensity q. It is required to find the moments and deflection at the center of the plate. The torsional stiffness of the beam is neglected.

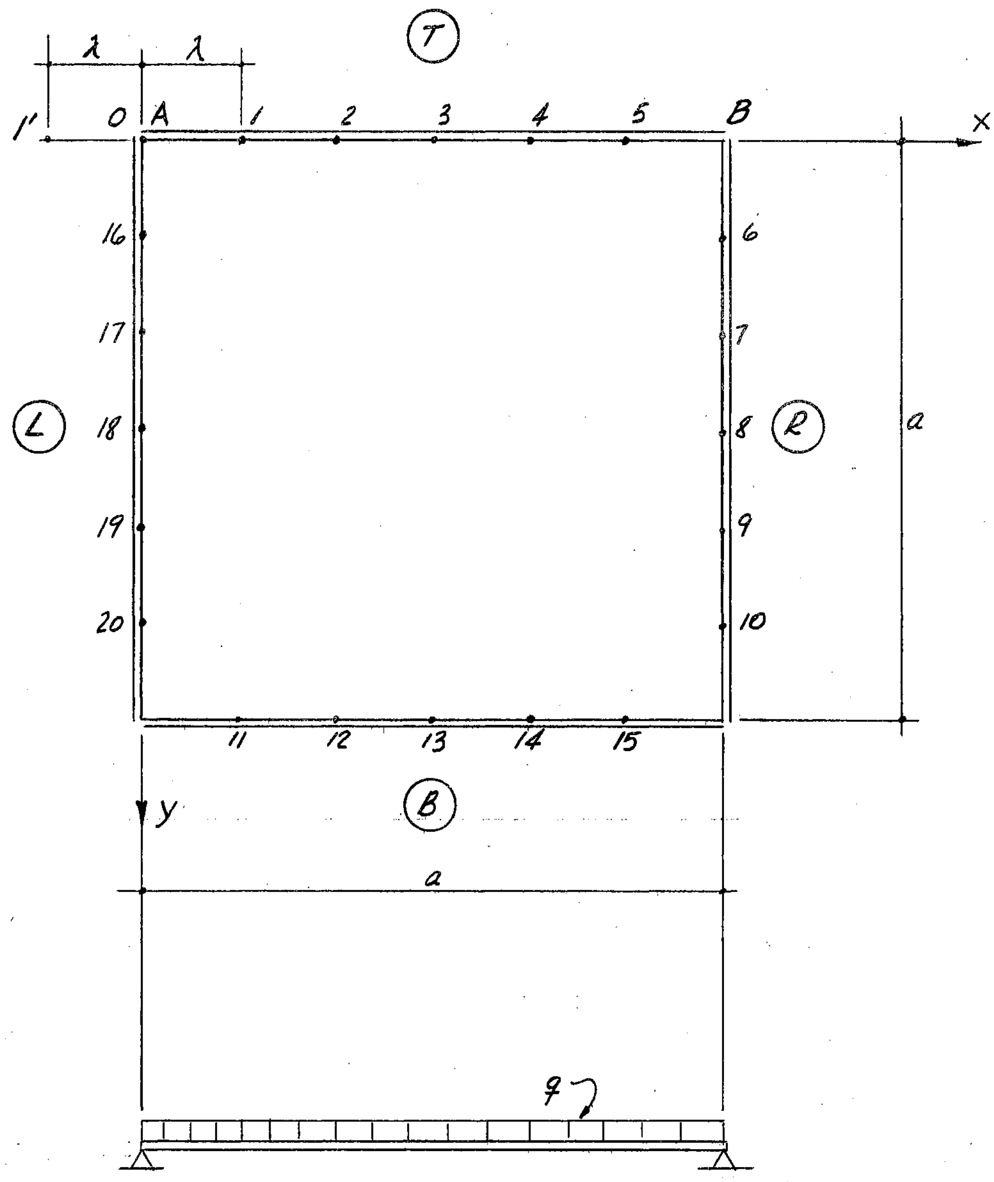


Figure 7-4 Example 2. Square Plate With Edge Beams

In matrix notation, the slope-deflection equations of the plate are:

$$\{M\} = [K]\{\Theta\} + [SM]\{\Delta\} + \{FM\}$$

$$\{R\} = [SR]\{\Theta\} + [T]\{\Delta\} + \{FR\}$$

Referring to Figure 7-4 and Tables A-I, II, III, IV and B-I:

$$\begin{aligned} M_1 = {} & \frac{D}{a}(39.64\Theta_1 - 6.690\Theta_2 - 1.347\Theta_3 - .279\Theta_4 - .042\Theta_5 \\ & + .038\Theta_6 + .106\Theta_7 + .112\Theta_8 + .066\Theta_9 + .017\Theta_{10} \\ & + .038\Theta_{11} + .103\Theta_{12} + .110\Theta_{13} + .065\Theta_{14} + .017\Theta_{15} \\ & - 9.312\Theta_{16} - 3.222\Theta_{17} - .906\Theta_{18} - .236\Theta_{19} - .041\Theta_{20}) \\ & + \frac{D}{a^2}(129.5\Delta_1 - 45.49\Delta_2 - 1.278\Delta_3 + .337\Delta_4 + .294\Delta_5 \\ & + .121\Delta_6 - .154\Delta_7 - .194\Delta_8 - .055\Delta_9 + .067\Delta_{10} \\ & + .104\Delta_{11} - .156\Delta_{12} - .196\Delta_{13} - .058\Delta_{14} + .066\Delta_{15} \\ & - 50.50\Delta_{16} - 9.890\Delta_{17} - .954\Delta_{18} + .150\Delta_{19} + .236\Delta_{20}) \\ & - .01915qa^2 \end{aligned}$$

From symmetry, $\Theta_5 = \Theta_1$, $\Theta_{15} = -\Theta_5$, $\Theta_6 = -\Theta_{16}$, $\Theta_{16} = \Theta_1$, $\Theta_{20} = \Theta_{16}$, $\Delta_5 = \Delta_1 = \Delta_6 = \Delta_{10} = \Delta_{15} = \Delta_{11} = \Delta_{20} = \Delta_{16}$, etc.

Therefore,

$$\begin{aligned} M_1 = {} & \frac{D}{a}(30.135\Theta_1 - 10.767\Theta_2 - 2.475\Theta_3) + \frac{D}{a^2}(79.888\Delta_1 \\ & - 55.316\Delta_2 - 2.622\Delta_3) - .01915qa^2 \end{aligned}$$

$$\begin{aligned} M_2 = {} & \frac{D}{a}(-12.511\Theta_1 + 21.942\Theta_2 - 10.504\Theta_3) + \frac{D}{a^2}(-49.204\Delta_1 \\ & + 96.248\Delta_2 - 53.517\Delta_3) - .04128qa^2 \end{aligned}$$

$$\begin{aligned} M_3 = {} & \frac{D}{a}(-5.912\Theta_1 - 21.430\Theta_2 + 24.424\Theta_3) + \frac{D}{a^2}(.290\Delta_1 \\ & - 106.708\Delta_2 + 101.671\Delta_3) - .04949qa^2 \end{aligned}$$

$$R_1 = \frac{D}{a^2}(-499.256\Theta_1 + 51.927\Theta_2 - 13.511\Theta_3) + \frac{D}{a^3}(694.316\Delta_1 - 469.216\Delta_2 + 20.126\Delta_3) - .14103qa$$

$$R_2 = \frac{D}{a^2}(20.397\Theta_1 - 347.244\Theta_2 + 55.509\Theta_3) + \frac{D}{a^3}(-537.268\Delta_1 + 1004.192\Delta_2 - 530.014\Delta_3) - .29377qa$$

$$R_3 = \frac{D}{a^2}(-46.740\Theta_1 + 103.56\Theta_2 - 304.492\Theta_3) + \frac{D}{a^3}(30.150\Delta_1 - 1076.408\Delta_2 + 1008.947\Delta_3) - .34617qa$$

In the supporting beam AB, referring to Eqs. (5-6) and (5-7),

$$V_{10} = \frac{EI}{\lambda^3}(\Delta_{1'} - 3\Delta_A + 3\Delta_1 - \Delta_2)$$

$V_{10} = \frac{EI}{\lambda^3}(2\Delta_1 - \Delta_2)$ since $\Delta_{1'} = -\Delta_1$ because M_{A1} is zero.

$$V_{12} = \frac{EI}{\lambda^3}(3\Delta_1 - 3\Delta_2 + \Delta_3)$$

$$V_{21} = \frac{EI}{\lambda^3}(-3\Delta_1 + 3\Delta_2 - \Delta_3)$$

$$V_{23} = \frac{EI}{\lambda^3}(-\Delta_1 + 3\Delta_2 - 3\Delta_2 + \Delta_4)$$

$V_{23} = \frac{EI}{\lambda^3}(-\Delta_1 + 4\Delta_2 - 3\Delta_3)$, since $\Delta_4 = \Delta_2$

$$V_{32} = \frac{EI}{\lambda^3}(\Delta_1 - 4\Delta_2 + 3\Delta_3)$$

$$V_{34} = \frac{EI}{\lambda^3}(-\Delta_2 + 3\Delta_3 - 3\Delta_4 + \Delta_5)$$

$$V_{34} = \frac{EI}{\lambda^3}(\Delta_1 - 4\Delta_2 + 3\Delta_3)$$

Equilibrium

Summing vertical forces at joint 1:

$$\lambda R_1 + V_{10} + V_{12} = 0$$

or

$$R_1 + \frac{1}{\lambda}(V_{10} + V_{12}) = 0 \quad , \quad \lambda = \frac{a}{6}$$

Letting EI = 100Da and substituting for R_1, V_{10} and V_{12}:

$$\frac{D}{a^2}(-499.256\Theta_1 + 51.927\Theta_2 - 13.511\Theta_3) + \frac{D}{a^3}(648,694.32\Delta_1 - 518,869.22\Delta_2 + 129,620.13\Delta_3) - .14103qa = 0$$

At joint 2, $R_2 + \frac{1}{\lambda}(V_{21} + V_{23}) = 0$ and

$$\frac{D}{a^2}(20.397\Theta_1 - 347.244\Theta_2 + 55.509\Theta_3) + \frac{D}{a^3}(-518,937.27\Delta_1 + 908,204.19\Delta_2 - 518,930.01\Delta_3) - .29377qa = 0$$

At joint 3, $R_3 + \frac{1}{\lambda}(V_{32} + V_{34}) = 0$ and

$$\frac{D}{a^2}(-46.740\Theta_1 + 103.560\Theta_2 - 304.492\Theta_3) + \frac{D}{a^3}(259,230.15\Delta_1 - 1,037,876.4\Delta_2 + 778,608.95\Delta_3) - .34617qa = 0$$

Since the torsional stiffness of the edge beams is neglected, equilibrium of moments at joints 1, 2, and 3 yields $M_1 = 0 = M_2 = M_3$. A simultaneous solution of these six equations gives:

$$\Theta_1 = .006171\frac{qa^3}{D} \qquad \Delta_1 = .000224\frac{qa^4}{D}$$

$$\Theta_2 = .011896\frac{qa^3}{D} \qquad \Delta_2 = .000386\frac{qa^4}{D}$$

$$\Theta_3 = .013792\frac{qa^3}{D} \qquad \Delta_3 = .000444\frac{qa^4}{D}$$

The moments and deflection at the center of the plate are obtained by multiplying the values of center moment and deflection due to unit edge slope and deflection with the above solution and summing along each edge. This gives

$$M_x = M_y = .0511qa^2 \ , \ \Delta = .00504\frac{qa^4}{D}$$

The solution of this same problem with $\frac{EI}{Da} = 0$ (corner supported plate) resulted in

$$\theta_1 = .00149\frac{qa^3}{D} \qquad \Delta_1 = .0113\frac{qa^4}{D}$$

$$\theta_2 = .00132\frac{qa^3}{D} \qquad \Delta_2 = .0157\frac{qa^4}{D}$$

$$\theta_3 = .00112\frac{qa^3}{D} \qquad \Delta_3 = .0170\frac{qa^4}{D}$$

At the center of the plate,

$$M_x = M_y = .0619qa^2 \;, \; \Delta = .0197\frac{qa^4}{D}$$

Example 3

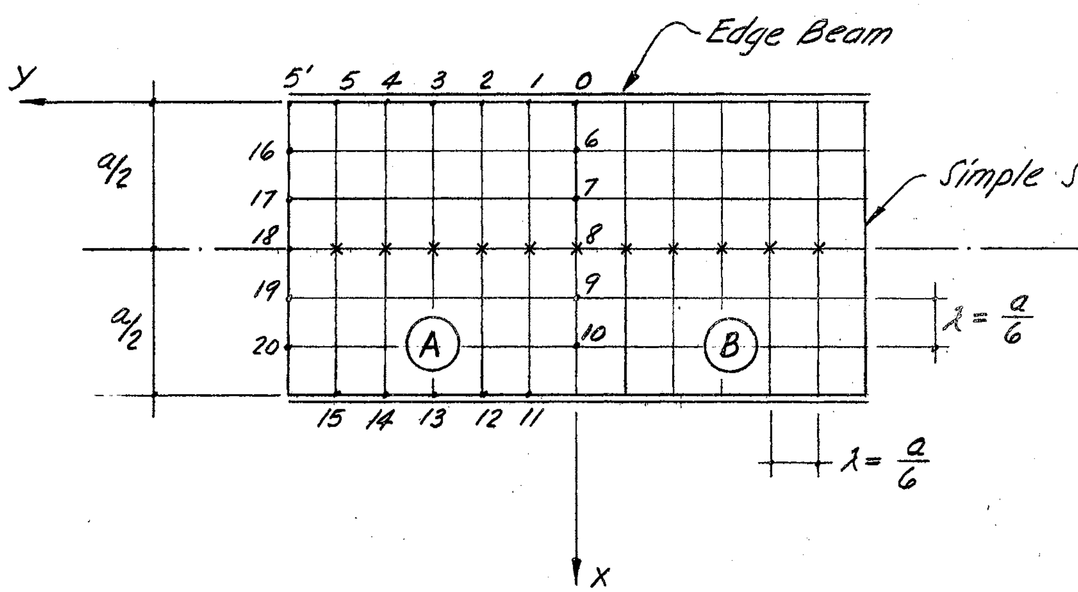
The two-span continuous plate system shown in Figure 7-5 is subjected to a line load of 100 lbs. per ft. along the longitudinal center line. The plates are rigidly supported transversely at their ends and elastically supported with beams along their longitudinal edges. The following properties are assumed:

$$\frac{EI}{Da} = 50 \;, \; \frac{GJ^*}{Da} = 12 \;, \; \nu = .20$$

The torsional and flexural stiffness of the edge beams are included in the analysis. From symmetry, $\theta_6 = 0 = \theta_7 = \theta_8 = \theta_9 = \theta_{10}$; $\theta_{11} = -\theta_1$, $\theta_{12} = -\theta_2$, $\Delta_{11} = \Delta_1$, $\Delta_{12} = \Delta_2$, etc. Therefore, the slope-deflection equations for plate A using Tables A-IV, V, VI, VII and B-IV and B-V are:

$$M_1 = \frac{D}{a}(39.60\theta_1 - 6.799\theta_2 - 1.482\theta_3 - .393\theta_4 - .122\theta_5)$$
$$+ \frac{D}{a^2}(-129.39\Delta_1 + 45.34\Delta_2 - 1.085\Delta_3 - .379\Delta_4$$
$$- .181\Delta_5) - 100(\frac{a}{6})(.0223 + .0369 + .0287 + .0159$$
$$+ .0064)$$

$$M_2 = \frac{D}{a}(-8.146\theta_1 + 28.17\theta_2 - 8.165\theta_3 - 2.317\theta_4 - .730\theta_5)$$



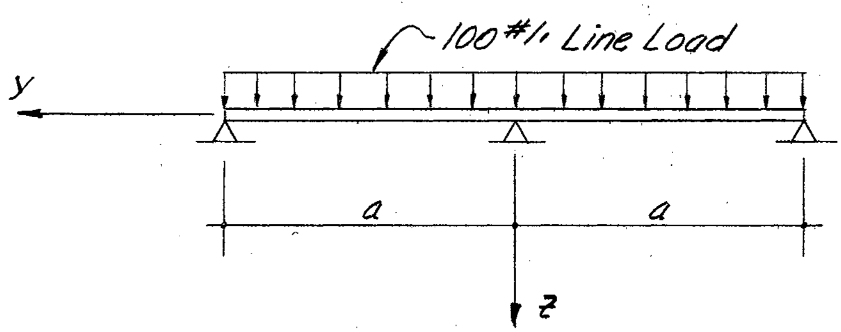


Figure 7-5 Example 3. Two Span Continuous Plate

$$+ \frac{D}{a^2}(42.25\Delta_1 - 109.65\Delta_2 + 47.97\Delta_3 + 1.257\Delta_4 - .393\Delta_5) - 100\frac{a}{6}(.3295)$$

$$M_3 = \frac{D}{a}(-1.840\Theta_1 - 8.891\Theta_2 + 25.91\Theta_3 - 9.160\Theta_4 - 2.496\Theta_5) + \frac{D}{a^2}(-.214\Delta_1 + 47.73\Delta_2 - 108.42\Delta_3 + 48.20\Delta_4 + 1.480\Delta_5) - \frac{100a}{6}(.4601)$$

$$M_4 = \frac{D}{a}(-.502\Theta_1 - 2.39\Theta_2 - 9.171\Theta_3 + 25.83\Theta_4 - 9.157\Theta_5) + \frac{D}{a^2}(-.799\Delta_1 + 1.130\Delta_2 + 48.17\Delta_3 - 108.12\Delta_4 + 48.61\Delta_5) - \frac{100a}{6}(.4420)$$

$$M_5 = \frac{D}{a}(-.149\Theta_1 - .704\Theta_2 - 2.325\Theta_3 - 8.491\Theta_4 + 31.04\Theta_5) + \frac{D}{a^2}(-.293\Delta_1 - .436\Delta_2 + 1.470\Delta_3 + 48.60\Delta_4 - 109.65\Delta_5) - \frac{100a}{6}(.2734)$$

$$R_1 = \frac{D}{a^2}(-414.3\Theta_1 + 68.00\Theta_2 - 11.90\Theta_3 - .634\Theta_4 + .334\Theta_5) + \frac{D}{a^3}(1295.6\Delta_1 - 496.5\Delta_2 + 6.275\Delta_3 + 9.965\Delta_4 + 3.307\Delta_5) - \frac{100a}{6a}(.2141)$$

$$R_2 = \frac{D}{a^2}(56.25\Theta_1 - 390.5\Theta_2 + 63.92\Theta_3 - 17.13\Theta_4 - 2.85\Theta_5) + \frac{D}{a^3}(-465.1\Delta_1 + 1035.1\Delta_2 - 525.7\Delta_3 + 1.498\Delta_4 + 7.961\Delta_5) - \frac{100}{6}(1.4032)$$

$$R_3 = \frac{D}{a^2}(-16.62\Theta_1 + 61.16\Theta_2 - 309.8\Theta_3 + 62.53\Theta_4 - 19.40\Theta_5) + \frac{D}{a^3}(17.57\Delta_1 - 523.3\Delta_2 + 1018.3\Delta_3 - 532.3\Delta_4 - 3.43\Delta_5) - \frac{100}{6}(2.1524)$$

$$R_4 = \frac{D}{a^2}(-2.241\Theta_1 - 18.31\Theta_2 + 62.21\Theta_3 - 310.6\Theta_4$$

$$+ 59.36\Theta_5) + \frac{D}{a^3}(14.05\Delta_1 + 3.042\Delta_2 - 531.96\Delta_3$$

$$+ 1011.8\Delta_4 - 540.9\Delta_5) - \frac{100}{6}(2.1160)$$

$$R_5 = \frac{D}{a^2}(-.311\Theta_1 - 3.103\Theta_2 - 18.23\Theta_3 + 63.04\Theta_4$$

$$-360.5\Theta_5) + \frac{D}{a^3}(4.725\Delta_1 + 8.6661\Delta_2 - 3.208\Delta_3$$

$$- 540.9\Delta_4 + 1015.3\Delta_5) - \frac{100}{6}(1.3185)$$

Considering the flexural stiffness of the upper edge beam of plate A, then, from Eq. (5-6) and Example 2:

$$V_{10} = \frac{EI}{\lambda^3}(4\Delta_1 - \Delta_2)$$

$$V_{12} = \frac{EI}{\lambda^3}(3\Delta_1 - 3\Delta_2 + \Delta_3)$$

$$V_{21} = \frac{EI}{\lambda^3}(-3\Delta_1 + 3\Delta_2 - \Delta_3)$$

$$V_{23} = \frac{EI}{\lambda^3}(-\Delta_1 + 3\Delta_2 - 3\Delta_3 + \Delta_4)$$

$$V_{32} = \frac{EI}{\lambda^3}(-\Delta_1 - 3\Delta_2 + 3\Delta_3 - \Delta_4)$$

$$V_{34} = \frac{EI}{\lambda^3}(-\Delta_2 + 3\Delta_3 - 3\Delta_4 + \Delta_5)$$

$$V_{43} = \frac{EI}{\lambda^3}(\Delta_2 - 3\Delta_3 + 3\Delta_4 - \Delta_5)$$

$$V_{45} = \frac{EI}{\lambda^3}(-\Delta_3 + 3\Delta_4 - 3\Delta_5)$$

$$V_{54} = \frac{EI}{\lambda^3}(\Delta_3 - 3\Delta_4 + 3\Delta_5)$$

$$V_{55'} = \frac{EI}{\lambda^3}(-\Delta_4 + 2\Delta_5)$$

From Eq. (5-9) the beam torsional moments are:

$$T_{10} = \frac{GJ^*}{\lambda}(\Theta_1 - \Theta_0) = \frac{GJ^*}{\lambda}\Theta_1$$

$$T_{21} = \frac{GJ^*}{\lambda}(\Theta_2 - \Theta_1) = -T_{12}$$

$$T_{32} = \frac{GJ^*}{\lambda}(\Theta_3 - \Theta_2) = -T_{23}$$

$$T_{43} = \frac{GJ^*}{\lambda}(\Theta_4 - \Theta_3) = -T_{34}$$

$$T_{54} = \frac{GJ^*}{\lambda}(\Theta_5 - \Theta_4) = -T_{45}$$

$$T_{55'} = \frac{GJ^*}{\lambda}\Theta_5$$

Equilibrium at joint 1 requires that

$$\Sigma F_z = 0 \ , \ \lambda R_1 + V_{10} + V_{12} = 0$$

$$\Sigma M_y = 0 \ , \ \lambda M_1 + T_{10} + T_{12} = 0$$

Substituting, with $\frac{EI}{\lambda^3} = \frac{64,800D}{a^3}$ and $\frac{GJ^*}{\lambda} = \frac{432D}{a}$:

$$\frac{D}{a^2}(-414.3\Theta_1 + 68.00\Theta_2 - 11.90\Theta_3 - .634\Theta_4 + .334\Theta_5)$$
$$+ \frac{D}{a^3}(454,895.7\Delta_1 - 259.696.5\Delta_2 + 64,806.3\Delta_3 + 9.965\Delta_4$$
$$+ 3.307\Delta_5) - 3.568 = 0$$

$$\frac{D}{a}(903.6\Theta_1 - 438.8\Theta_2 - 1.482\Theta_3 - .393\Theta_4 - .122\Theta_5)$$
$$+ \frac{D}{a^2}(-129.39\Delta_1 + 45.34\Delta_2 - 1.085\Delta_3 - .379\Delta_4 - .181\Delta_5)$$
$$- 1.853 = 0$$

Applying the equilibrium equations at joints 2, 3, 4, and 5 and putting all equations in matrix form:

$$\begin{bmatrix} -414.3 & 68.00 & -11.90 & -.634 & .334 \\ 903.6 & -438.8 & -1.482 & -.393 & -.122 \\ 56.25 & -390.5 & 63.92 & -17.13 & -2.85 \\ -440.2 & 892.2 & -440.6 & -2.317 & -.730 \\ -16.62 & 61.16 & -309.8 & 62.53 & -19.40 \\ -1.840 & -440.9 & 889.9 & -441.2 & -2.496 \\ -2.241 & -18.31 & 62.21 & -310.6 & 59.36 \\ -.502 & -2.393 & -441.2 & 889.8 & -441.2 \\ -.311 & -3.103 & -18.23 & 63.04 & -360.5 \\ -.149 & -.704 & -2.325 & -440.5 & 895.0 \end{bmatrix} \begin{bmatrix} \Theta_1 \\ \Theta_2 \\ \Theta_3 \\ \Theta_4 \\ \Theta_5 \end{bmatrix} +$$

$$\begin{bmatrix} 454{,}896. & -259{,}696. & 64{,}806.3 & 9.965 & 3.307 \\ -129.390 & 45.34 & -1.085 & -.379 & -.181 \\ -259{,}665. & 389{,}835. & -259{,}726. & 64{,}801.5 & 7.961 \\ 42.25 & -109.65 & 47.97 & 1.257 & -.393 \\ 64{,}817.6 & -259{,}723. & 389{,}818. & -259{,}732. & 64{,}796.6 \\ -.214 & 47.73 & -108.42 & 48.20 & 1.480 \\ 14.05 & 64{,}803. & -259{,}732. & 389{,}811. & -259{,}741. \\ -.799 & 1.130 & 48.17 & -108.12 & 48.61 \\ 4.725 & 8.662 & 64{,}796.8 & -259{,}741. & 325{,}015. \\ -.293 & -.436 & 1.470 & 48.60 & -109.65 \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \\ \Delta_5 \end{bmatrix} = \begin{bmatrix} 3.568 \\ 1.853 \\ 23.39 \\ 5.492 \\ 35.87 \\ 7.668 \\ 35.27 \\ 7.367 \\ 21.97 \\ 4.557 \end{bmatrix}$$

The simultaneous solution of these equations gives

$$\theta_1 = .02586\frac{a^2}{D} \qquad \Delta_1 = .001233\frac{a^3}{D}$$
$$\theta_2 = .04874\frac{a^2}{D} \qquad \Delta_2 = .003288\frac{a^3}{D}$$
$$\theta_3 = .05987\frac{a^2}{D} \qquad \Delta_3 = .004702\frac{a^3}{D}$$
$$\theta_4 = .05409\frac{a^2}{D} \qquad \Delta_4 = .004593\frac{a^3}{D}$$
$$\theta_5 = .03196\frac{a^2}{D} \qquad \Delta_5 = .002930\frac{a^3}{D}$$

Substituting back into the respective slope-deflection equations,

$$M_1 = -1.276a \qquad M_6 = -2.381a$$
$$M_2 = -5.072a \qquad M_7 = -5.692a$$
$$M_3 = -7.301a \qquad M_8 = -7.092a$$
$$M_4 = -7.071a \qquad M_9 = -5.692a$$
$$M_5 = -4.283a \qquad M_{10} = -2.381a$$

The values of moment and deflection at the center of each plate are determined by multiplying the values at the center in Table A-V by the respective θ's and the values at the center in Table A-VII by the respective Δ's and summing with the center values in Table B-VI. Then,

$$M_x = 6.753a \quad , \quad M_y = 9.130a \quad , \quad \Delta = .3270\frac{a^3}{D}$$

7-5 Comparisons

The following is a summary of the results of the first and second example problems compared with some published solutions.

Plate Function	Solution (ν = .2) Thesis	Timoshenko (1)	Marcus (3)
<u>Example 1</u>			
Δ at center	.00278	--	--
M_x at center	.0354	.0375	--
M_y at center	.0313	.0317	--
M_3 at edge ℄	-.0384	-.0381	--
<u>Example 2</u>			
(a) EI/Da=100			
Δ at center	.00504	.00412	--
M_x,M_y at center	.0511	.0443	--
(b) EI/Da=0			
Δ at center	.0197	.0257	.0249
M_x,M_y at center	.0619	.1065	.1010
Δ_3 at edge ℄	.0170	--	.0171

Note: Moments are multiplied by qa^2 and Δ's by qa^4/D

Example problems 1 and 2 provide a basis for estimating the accuracy of the plate model used. In problem 1 the solution is quite close to the series solution reported by Timoshenko, (1) p. 231, because the stiffness factors were determined for plates which satisfied the boundary conditions of the simply supported edges. Therefore, this problem is quite accurate and shows the slope-deflection approach at its best. The difference in solutions varies from 1 percent to 5.6 percent.

Example problem 2 was intended to show the selected plate model at its worst, because only a relatively few number of edge boundary conditions are satisfied (<u>i.e.</u>, five on each edge, not counting the corners). This problem was worked twice, once with edge beams (neglecting torsional resistance) and once without edge beams (corner supported plate). The difference in solutions varied from 15 percent to 22 percent in the case of the edge beams. For the corner supported plate, the difference in the slope-deflection approach and a series solution was 23 percent and 42 percent. A comparison with another numerical solution resulted in a variance of 22 percent and 39 percent in the values at the plate center, and the same edge deflection (3).

CHAPTER VIII

SUMMARY AND CONCLUSIONS

8-1 Summary

The analysis of continuous rectangular plate systems loaded perpendicular to their surface was presented. The slope-deflection equations were developed for a general plate model using the finite difference method.

To apply the theory to numerical systems, a specific plate model was adopted. The plates were divided into six horizontal and six vertical strips and the slope deflection constants were calculated at the five points on each edge. The plate equation and the boundary conditions were approximated by central differences in order to obtain the solution. The assumptions and limitations of the solution are those incorporated in thin plate theory and the finite difference method of solving boundary value problems. Fixed edge moments and reactions were determined for a uniform load and concentrated loads at each of twenty-five points on the plate.

Three example problems were solved to illustrate the application of the slope-deflection equations to continuous plate systems. Elastic edge beams with flexural and torsional stiffnesses were included.

8-2 Conclusions

From the comparisons of section 7-5, it is concluded that the plate model selected and the resulting slope-deflection equations are sufficiently accurate for analyzing continuous plate systems.

Proper use of the tables of coefficients listed in the appendices will enable an engineer to analyze any one-way or two-way continuous plate system subject to the assumptions found in section 1-4. The advantage of this method is found in its "relative" simplicity and ease of application to plate systems supported either partially or continuously along its edges with elastic beams of varying stiffness (from zero to infinity). Few solutions are available for continuous plates having unsymmetrical and discontinuous edge support and loading conditions. This is because a displacement function must be found which satisfies the plate equation (2-1) and the boundary or edge conditions of the system and explains why there are few solutions available for loading conditions other than uniform, hydrostatic or a point load at the center of the plate.

The slope-deflection equation for square plates derived in this thesis can be used to obtain a solution for plate systems supported in any normal manner and loaded in any fashion perpendicular to the plate; and the accuracy of the solution should be within 20 percent of a more rigorous solution. The more restraints imposed upon the system, the more accurate the solution will be. The method presented in this thesis can also be used to analyze space frames composed of elastic beams, columns and plates which hitherto have not included the stiffnesses of the plate in the analysis.

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APPENDIX A

STIFFNESS MATRICES

The following tables contain the stiffness matrices for square plates with various edge conditions. A hash-marked line,  , denotes a fixed edge and a plain line denotes a simply supported edge. In all cases Poisson's ratio, ν , is taken as 0.20. The edge nomenclature is shown in Figure A-1.

When using the tables in a problem, the rule of matrix multiplication must be employed, that is, row by column multiplication. For example, in the general slope-deflection equations for moment

$$\{M\} = [K]\{\theta\} + [SM]\{\Delta\} + \{FM\} \quad ,$$

M_1 equals the sum of the products of the elements in the first row of the K matrix by the corresponding elements of the θ column matrix plus the same operations for the SM matrix times the Δ matrix plus FM_1. Equation (4-1) indicates the manner of expansion.

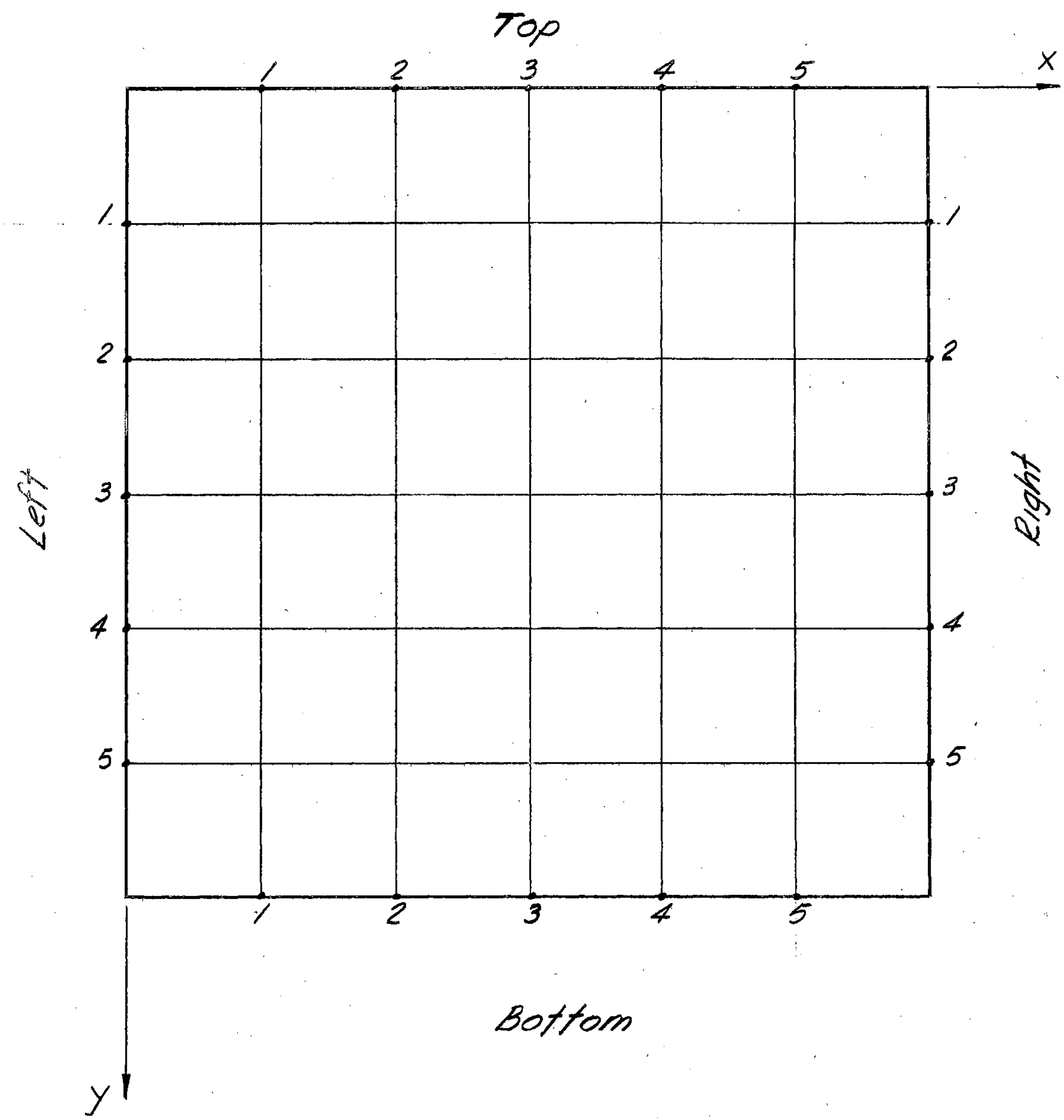
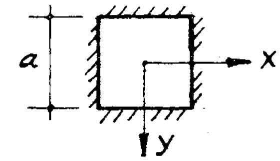


Figure A-1 Plate Notation for Stiffnesses

TABLE A-I

MOMENT STIFFNESS MATRIX (K)



EDGE	NODE	TOP 1	TOP 2	TOP 3	TOP 4	TOP 5	RIGHT 1	RIGHT 2	RIGHT 3	RIGHT 4	RIGHT 5
	1	39.64	-6.689	-1.347	-0.279	-0.042	0.038	0.106	0.112	0.066	0.017
	2	-7.994	28.65	-8.105	-1.822	-0.340	0.266	0.551	0.518	0.294	0.084
T	3	-1.649	-8.304	27.16	-8.304	-1.649	1.011	1.450	1.092	0.545	0.151
	4	-0.340	-1.822	-8.105	28.65	-7.994	3.470	2.842	1.477	0.584	0.142
	5	-0.042	-0.279	-1.347	-6.689	39.64	9.312	3.222	0.906	0.236	0.041
	1	0.041	0.236	0.906	3.222	9.312	39.64	-6.689	-1.347	-0.279	-0.042
	2	0.042	0.584	1.477	2.842	3.470	-7.994	28.65	-8.105	-1.822	-0.340
R	3	0.151	0.545	1.092	1.450	1.011	-1.649	-8.304	27.16	-8.304	-1.649
	4	0.084	0.294	0.518	0.551	0.266	-0.340	-1.822	-8.105	28.65	-7.994
	5	0.017	0.066	0.112	0.106	0.038	-0.042	-0.279	-1.347	-6.689	39.64
	1	0.038	0.103	0.110	0.065	0.017	-0.017	-0.066	-0.112	-0.106	-0.038
	2	0.132	0.348	0.404	0.267	0.083	-0.084	-0.294	-0.518	-0.551	-0.266
B	3	0.145	0.416	0.552	0.416	0.145	-0.151	-0.545	-1.092	-1.450	-1.011
	4	0.083	0.267	0.404	0.348	0.132	-0.142	-0.584	-1.477	-2.842	-3.470
	5	0.017	0.065	0.110	0.103	0.038	-0.041	-0.236	-0.906	-3.222	-9.312
	1	-9.312	-3.222	-0.906	-0.236	-0.041	0.038	0.103	0.110	0.065	0.017
	2	-3.740	-2.842	-1.477	-0.584	-0.142	0.132	0.348	0.404	0.267	0.083
L	3	-1.011	-1.450	-1.092	-0.545	-0.151	0.145	0.416	0.552	0.416	0.145
	4	-0.266	-0.551	-0.518	-0.294	-0.084	0.083	0.267	0.004	0.348	0.132
	5	-0.038	-0.106	-0.112	-0.066	-0.017	0.017	0.065	0.110	0.103	0.038

VALUES AT CENTER OF PLATE DUE TO UNIT ROTATION OF EDGE NODE

M(X)	.0424	.2467	.4339	.2467	.0424	-.0215	-.0355	.0007	-.0355	-.0215
M(Y)	.0215	.0355	-.0007	.0355	.0215	-.0424	-.2467	-.4339	-.2467	-.0424
W	.0066	.0174	.0230	.0174	.0066	-.0066	-.0174	-.0230	-.0174	-.0066

MULTIPLY ALL MOMENTS BY $\frac{D}{a}$ AND W BY a

TABLE A-I CONT.

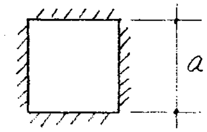
EDGE	NODE	BOTTOM 1	BOTTOM 2	BOTTOM 3	BOTTOM 4	BOTTOM 5	LEFT 1	LEFT 2	LEFT 3	LEFT 4	LEFT 5
	1	0.038	0.103	0.110	0.065	0.017	-9.312	-3.222	-0.906	-0.236	-0.041
	2	0.132	0.348	0.404	0.267	0.083	-3.470	-2.842	-1.477	-0.584	-0.142
T	3	0.145	0.416	0.552	0.416	0.145	-1.011	-1.450	-1.092	-0.545	-0.151
	4	0.083	0.267	0.404	0.348	0.132	-0.266	-0.551	-0.518	-0.294	-0.084
	5	0.017	0.065	0.110	0.103	0.038	-0.038	-0.106	-0.112	-0.066	-0.017
	1	-0.017	-0.066	-0.112	-0.106	-0.038	0.038	0.103	0.110	0.065	0.017
	2	-0.084	-0.294	-0.518	-0.551	-0.266	0.132	0.348	0.404	0.267	0.083
R	3	-0.151	-0.545	-1.092	-1.450	-1.011	0.145	0.416	0.552	0.416	0.145
	4	-0.142	-0.584	-1.477	-2.842	-3.470	0.083	0.267	0.404	0.348	0.132
	5	-0.041	-0.236	-0.906	-3.222	-9.312	0.017	0.065	0.110	0.103	0.038
	1	39.64	-6.689	-1.347	-0.279	-0.042	0.041	0.236	0.906	3.222	9.312
	2	-7.994	28.65	-8.105	-1.822	-0.340	0.142	0.584	1.477	2.842	3.470
B	3	-1.649	-8.304	27.16	-8.304	-1.649	0.151	0.545	1.092	1.450	1.011
	4	-0.340	-1.822	-8.105	28.65	-7.994	0.084	0.294	0.518	0.551	0.266
	5	-0.042	-0.279	-1.347	-6.689	39.64	0.017	0.066	0.112	0.106	0.038
	1	0.038	0.106	0.112	0.066	0.017	39.64	-6.689	-1.347	-0.279	-0.042
	2	0.266	0.551	0.518	0.294	0.084	-7.994	28.65	-8.105	-1.822	-0.340
L	3	1.011	1.450	1.092	0.545	0.151	-1.649	-8.304	27.16	-8.304	-1.649
	4	3.470	2.842	1.477	0.584	0.142	-0.340	-1.822	-8.105	28.65	-7.994
	5	9.312	3.222	0.906	0.236	0.041	-0.042	-0.279	-1.347	-6.689	39.64

VALUES AT CENTER OF PLATE DUE TO UNIT ROTATION OF EDGE NODE

	BOTTOM 1	BOTTOM 2	BOTTOM 3	BOTTOM 4	BOTTOM 5	LEFT 1	LEFT 2	LEFT 3	LEFT 4	LEFT 5
M(X)	-.0424	-.2467	-.4339	-.2467	-.0424	.0215	.0355	-.0007	.0355	.0215
M(Y)	-.0215	-.0355	.0007	-.0355	-.0215	.0424	.2467	.4339	.2467	.0424
W	-.0066	-.0174	-.0230	-.0174	-.0066	.0066	.0174	.0230	.0174	.0066

TABLE A-II

CARRY-OVER MOMENT STIFFNESS MATRIX (SM)



EDGE	NODE	TOP 1	TOP 2	TOP 3	TOP 4	TOP 5	RIGHT 1	RIGHT 2	RIGHT 3	RIGHT 4	RIGHT 5
	1	129.5	-45.49	-1.278	0.337	0.294	0.121	-0.154	-0.194	-0.055	0.067
	2	-42.04	109.0	-48.79	-1.510	0.964	0.201	1.119	-1.040	-0.341	0.221
T	3	0.459	-48.42	107.4	-48.42	0.459	-0.934	-3.631	-2.275	-0.612	0.366
	4	0.964	-1.510	-48.79	109.0	-42.04	-9.370	-8.359	-2.853	-0.437	0.413
	5	0.294	0.337	-1.278	-45.49	129.5	-50.50	-9.890	-0.954	0.150	0.236
	1	-0.236	-0.150	0.954	9.890	50.50	-129.5	45.49	1.278	-0.337	-0.294
	2	-0.413	0.437	2.853	8.359	9.370	42.04	-109.0	48.79	1.510	-0.964
R	3	-0.366	0.612	2.275	3.631	0.934	-0.459	48.42	-107.4	48.42	-0.459
	4	-0.221	0.341	1.040	1.119	-0.201	-0.964	1.510	48.79	-109.0	42.04
	5	-0.067	0.055	0.194	0.154	-0.121	-0.294	-0.337	1.278	45.49	-129.5
	1	-0.104	0.156	0.196	0.058	-0.066	-0.067	0.055	0.194	0.154	-0.121
	2	-0.212	0.616	0.834	0.370	-0.195	-0.221	0.341	1.040	1.119	-0.201
B	3	-0.254	0.691	1.179	0.691	-0.254	-0.366	0.612	2.275	3.631	0.934
	4	-0.195	0.370	0.834	0.616	-0.212	-0.413	0.437	2.853	8.359	9.370
	5	-0.066	0.057	0.196	0.156	-0.104	-0.236	-0.150	0.954	9.890	50.49
	1	-50.49	-9.890	-0.954	0.150	0.236	0.104	-0.156	-0.196	-0.057	0.066
	2	-9.370	-8.359	-2.853	-0.437	0.413	0.212	-0.616	-0.834	-0.370	0.195
L	3	-0.934	-3.631	-2.275	-0.612	0.366	0.254	-0.691	-1.179	-0.691	0.254
	4	0.201	-1.119	-1.040	-0.341	0.221	0.195	-0.370	-0.834	-0.616	0.212
	5	0.121	-0.154	-0.194	-0.055	0.067	0.066	-0.058	-0.196	-0.156	0.104

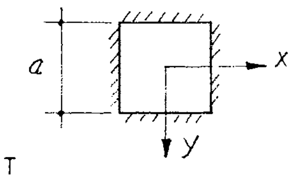
MULTIPLY ALL VALUES BY $\frac{D}{a^2}$

TABLE A-II CONT.

EDGE	NODE	BOTTOM 1	BOTTOM 2	BOTTOM 3	BOTTOM 4	BOTTOM 5	LEFT 1	LEFT 2	LEFT 3	LEFT 4	LEFT 5
	1	0.104	-0.156	-0.196	-0.058	0.066	-50.50	-9.890	-0.954	0.150	0.236
	2	0.212	-0.616	-0.834	-0.370	0.195	-9.370	-8.359	-2.853	-0.437	0.413
T	3	0.254	-0.691	-1.179	-0.691	0.254	-0.934	-6.631	-2.275	-0.612	0.366
	4	0.195	-0.370	-0.834	-0.616	0.212	0.201	-1.119	-1.040	-0.341	0.221
	5	0.066	-0.058	-0.196	-0.156	0.104	0.121	-0.154	-0.194	-0.055	0.067
	1	-0.067	0.055	0.194	0.154	-0.121	-0.104	0.156	0.196	0.058	-0.066
	2	-0.221	0.341	1.040	1.119	-0.201	-0.212	0.616	0.834	0.370	-0.195
R	3	-0.366	0.612	2.275	3.631	0.934	-0.254	0.691	1.179	0.691	-0.254
	4	-0.413	0.437	2.853	8.359	9.370	-0.195	0.370	0.834	0.616	-0.212
	5	-0.236	-0.150	0.954	9.890	50.50	-0.066	0.058	0.196	0.156	-0.104
	1	-129.5	45.49	1.278	-0.337	-0.294	-0.236	-0.150	0.954	9.890	50.50
	2	42.04	-109.0	48.79	1.510	-0.964	-0.413	0.437	2.853	8.359	9.370
B	3	-0.459	48.42	-107.4	48.42	-0.459	-0.366	0.612	2.275	3.631	0.934
	4	-0.964	1.510	48.79	-109.0	42.04	-0.221	0.341	1.040	1.119	-0.201
	5	-0.294	-0.337	1.278	45.49	-129.5	-0.067	0.055	0.194	0.154	-0.121
	1	0.121	-0.154	-0.194	-0.055	-0.067	129.5	-45.49	-1.278	0.337	0.294
	2	0.201	-1.119	-1.040	-0.341	0.221	-42.04	109.0	-48.79	-1.510	0.964
L	3	-0.934	-3.631	-2.275	-0.612	0.366	0.459	-48.42	107.4	-48.42	0.459
	4	-9.370	-8.359	-2.853	-0.437	0.413	0.964	-1.510	-48.79	109.0	-42.04
	5	-50.50	-9.890	-0.954	0.150	0.236	0.294	0.337	-1.278	-45.49	129.5

TABLE A-III

REACTION STIFFNESS MATRIX (T)



EDGE	NODE	TOP 1	TOP 2	TOP 3	TOP 4	TOP 5	RIGHT 1	RIGHT 2	RIGHT 3	RIGHT 4	RIGHT 5
	1	1295.	-497.5	5.470	9.200	2.560	1.142	0.978	0.765	0.615	0.216
	2	-466.8	1036.	-522.8	4.756	11.71	3.066	-2.787	-2.316	0.005	1.184
T	3	15.38	-521.2	1025.	-521.2	15.38	-4.072	-15.49	-6.438	-0.078	2.204
	4	11.71	4.756	-522.8	1036.	-466.8	-91.51	-35.14	-3.203	2.517	2.553
	5	2.560	9.200	5.470	-497.5	1295.	-606.7	11.28	13.23	4.712	1.168
	1	1.168	4.712	13.23	11.28	-606.7	1295.	-497.5	5.470	9.200	2.560
	2	2.553	2.517	-3.203	-35.14	-91.51	-466.8	1036.	-522.8	4.756	11.71
R	3	2.204	-0.078	-6.438	-15.49	-4.072	15.38	-521.2	1025.	-521.2	15.38
	4	1.184	0.005	-2.316	-2.787	3.066	11.71	4.756	-522.8	1036.	-466.8
	5	0.216	0.615	0.765	0.978	1.142	2.560	9.200	5.470	-497.5	1295.
	1	0.728	0.944	0.731	0.555	0.202	0.216	0.615	0.765	0.978	1.142
	2	1.509	-0.768	-1.695	-0.391	1.020	1.184	0.005	-2.316	-2.787	3.066
B	3	1.563	-1.436	-3.177	-1.436	1.563	2.204	-0.078	-6.438	-15.49	-4.072
	4	1.020	-0.391	-1.695	-0.768	1.509	2.553	2.517	-3.203	-35.14	-91.51
	5	0.202	0.555	0.731	0.944	0.728	1.168	4.712	13.23	11.28	-606.7
	1	-606.7	11.28	13.23	4.712	1.168	0.728	0.944	0.731	0.555	0.202
	2	-91.51	-35.14	-3.203	2.517	2.553	1.509	-0.768	-1.695	-0.391	1.020
L	3	-4.072	-15.49	-6.438	-0.078	2.204	1.563	-1.436	-3.177	-1.436	1.563
	4	3.066	-2.787	-2.316	0.005	1.184	1.020	-0.391	-1.695	-0.768	1.509
	5	1.142	0.978	0.765	0.615	0.216	0.202	0.555	0.731	0.944	0.728

VALUES AT CENTER OF PLATE DUE TO UNIT DEFLECTION OF EDGE NODE

	TOP 1	TOP 2	TOP 3	TOP 4	TOP 5	RIGHT 1	RIGHT 2	RIGHT 3	RIGHT 4	RIGHT 5
M(X)	-.4696	.8297	2.651	.8297	-.4696	-.3169	-.5243	-1.282	-.5243	-.3169
M(Y)	-.3169	-.5243	-1.282	-.5243	-.3169	-.4696	.8297	2.651	.8297	-.4696
W	-.0046	.0791	.1254	.0791	-.0046	-.0046	.0791	.1254	.0791	-.0046

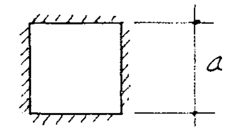
TABLE A-III CONT.

EDGE	NODE	BOTTOM 1	BOTTOM 2	BOTTOM 3	BOTTOM 4	BOTTOM 5	LEFT 1	LEFT 2	LEFT 3	LEFT 4	LEFT 5
	1	0.728	0.944	0.731	0.555	0.202	-606.7	11.28	13.23	4.712	1.168
	2	1.509	-0.768	-1.695	-0.391	1.020	-91.51	-35.14	-3.023	2.517	2.553
T	3	1.563	-1.436	-3.177	-1.436	1.563	-4.072	-15.49	-6.438	-0.078	2.204
	4	1.020	-0.391	-1.695	-0.768	1.509	3.066	-2.787	-2.316	0.005	1.184
	5	0.202	0.555	0.731	0.944	0.728	1.142	0.978	0.765	0.615	0.216
	1	0.216	0.615	0.765	0.978	1.142	0.728	0.944	0.731	0.555	0.202
	2	1.184	0.005	-2.316	-2.787	3.066	1.509	-0.768	-1.695	-0.391	1.020
R	3	2.204	-0.078	-6.438	-15.49	-4.072	1.563	-1.436	-3.177	-1.436	1.563
	4	2.553	2.517	-3.203	-35.14	-91.51	1.020	-0.391	-1.695	-0.768	1.509
	5	1.168	4.712	13.23	11.28	-606.7	0.202	0.555	0.731	0.944	0.728
	1	1295.	-497.5	5.470	9.200	2.560	1.168	4.712	13.23	11.28	-606.7
	2	-466.8	1036.	-522.8	4.756	11.71	2.553	2.517	-3.203	-35.14	-91.51
B	3	15.38	-521.2	1025.	-521.2	15.38	2.204	-0.078	-6.438	-15.49	-4.072
	4	11.71	4.756	-522.8	1036.	-466.8	1.184	0.005	-2.316	-2.787	3.066
	5	2.560	9.200	5.470	-497.5	1295.	0.216	0.615	0.765	0.978	1.172
	1	1.142	0.978	0.765	0.615	0.216	1295.	-497.5	5.470	9.200	2.560
	2	3.066	-2.787	-2.316	0.005	1.184	-466.8	1036.	-522.8	4.756	11.71
L	3	-4.072	-15.49	-6.438	-0.078	2.204	15.38	-521.2	1025.	-521.2	15.38
	4	-91.51	-35.14	-3.203	2.517	2.553	11.71	4.756	-522.8	1036.	-466.8
	5	-606.7	11.28	13.23	4.712	1.168	2.560	9.200	5.470	-497.5	1295.
VALUES AT CENTER OF PLATE DUE TO UNIT DEFLECTION OF EDGE NODE											
	M(X)	-.4696	.8297	2.651	.8297	-.4696	-.3169	-.5243	-1.282	-.5243	-.3169
	M(Y)	-.3169	-.5243	-1.282	-.5243	-.3169	-.4696	.8297	2.651	.8297	-.4696
	W	-.0046	.0791	.1254	.0791	-.0046	-.0046	.0791	.1254	.0791	-.0046

MULTIPLY ALL MOMENTS BY $\frac{D}{a^2}$ AND W BY 1

TABLE A-IV

CARRY-OVER REACTION STIFFNESS MATRIX (SR)



EDGE	NODE	TOP 1	TOP 2	TOP 3	TOP 4	TOP 5	RIGHT 1	RIGHT 2	RIGHT 3	RIGHT 4	RIGHT 5
	1	-398.8	67.70	-12.15	-0.835	0.190	-0.150	-0.084	-0.085	-0.106	-0.068
	2	56.51	-308.0	64.63	-15.83	-1.621	0.985	2.151	1.811	0.867	0.177
T	3	-16.00	62.77	-293.2	62.77	-16.00	6.764	7.687	4.712	1.940	0.408
	4	-1.621	-15.83	64.63	-308.0	56.51	32.66	16.98	6.148	1.746	0.248
	5	0.190	-0.835	-12.15	67.70	-398.8	101.3	15.60	1.573	-0.202	-0.209
	1	0.209	0.202	-1.573	-15.60	-101.3	398.8	-67.70	12.15	0.835	-0.190
	2	-0.248	-1.746	-6.148	-16.98	-32.66	-56.51	308.0	-64.63	15.83	1.621
R	3	-0.408	-1.940	-4.712	-7.687	-6.764	16.00	-62.77	293.2	-62.77	16.00
	4	-0.177	-0.867	-1.811	-2.151	-0.985	1.621	15.83	-64.63	308.0	-56.51
	5	0.068	0.106	0.085	0.084	0.150	-0.190	0.835	12.15	-67.70	398.8
	1	0.160	0.158	0.127	0.112	0.067	-0.068	-0.106	-0.085	-0.084	-0.150
	2	-0.246	-0.936	-1.162	-0.734	-0.176	0.177	0.867	1.811	2.151	0.985
B	3	-0.396	-1.363	-1.868	-1.363	-0.396	0.408	1.940	4.712	7.687	6.764
	4	-0.176	-0.734	-1.162	-0.936	-0.246	0.248	1.746	6.148	16.98	32.66
	5	0.067	0.112	0.127	0.158	0.160	-0.209	-0.202	1.573	15.60	101.3
	1	-101.3	-15.60	-1.573	0.202	0.209	-0.160	-0.158	-0.127	-0.112	-0.067
	2	-32.66	-16.98	-6.148	-1.746	-0.248	0.246	0.936	1.162	0.734	0.176
L	3	-6.764	-7.687	-4.712	-1.940	-0.408	0.396	1.363	1.868	1.363	0.396
	4	-0.985	-2.151	-1.811	-0.867	-0.177	0.176	0.734	1.162	0.936	0.246
	5	0.150	0.084	0.085	0.106	0.068	-0.067	-0.112	-0.127	-0.158	-0.160

MULTIPLY ALL VALUES BY $\frac{D}{a^2}$

TABLE A-IV CONT.

EDGE	NODE	BOTTOM 1	BOTTOM 2	BOTTOM 3	BOTTOM 4	BOTTOM 5	LEFT 1	LEFT 2	LEFT 3	LEFT 4	LEFT 5
	1	-0.160	-0.158	-0.127	-0.112	-0.067	-101.3	-15.60	-1.573	0.202	0.209
	2	0.246	0.936	1.162	0.734	0.176	-32.66	-16.98	-6.148	-1.746	-0.248
T	3	0.396	1.363	1.868	1.363	0.396	-6.764	-7.687	-4.712	-1.940	-0.408
	4	0.176	0.734	1.162	0.936	0.246	-0.985	-2.151	-1.811	-0.867	-0.177
	5	-0.067	-0.112	-0.127	-0.158	-0.160	0.150	0.084	0.085	0.106	0.068
	1	-0.068	-0.106	-0.085	-0.084	-0.150	0.160	0.158	0.127	0.112	0.067
	2	0.177	0.867	1.811	2.151	0.985	-0.246	-0.936	-1.162	-0.734	-0.176
R	3	0.408	1.940	4.712	7.687	6.764	-0.396	-1.363	-1.868	-1.363	-0.396
	4	0.248	1.746	6.148	16.98	32.66	-0.176	-0.734	-1.162	-0.936	-0.246
	5	-0.209	-0.202	1.573	15.60	101.3	0.067	0.112	0.127	0.158	0.160
	1	398.8	-67.70	12.15	0.835	-0.190	0.209	0.202	-1.573	-15.60	-101.3
	2	-56.51	308.0	-64.63	15.83	1.621	-0.248	-1.746	-6.148	-16.98	-32.66
B	3	16.00	-62.77	293.2	-62.77	16.00	-0.408	-1.940	-4.712	-7.687	-6.764
	4	1.621	15.83	-64.63	308.0	-56.51	-0.177	-0.867	-1.811	-2.151	-0.985
	5	-0.190	0.835	12.15	-67.70	398.8	0.068	0.106	0.085	0.084	0.150
	1	-0.150	-0.084	-0.085	-0.106	-0.068	-398.8	67.70	-12.15	-0.835	0.190
	2	0.985	2.151	1.811	0.867	0.177	56.51	-308.0	64.63	-15.83	-1.621
L	3	6.764	7.687	4.712	1.940	0.408	-16.00	62.77	-293.2	62.77	-16.00
	4	32.66	16.98	6.148	1.746	0.248	-1.621	-15.83	64.63	-308.0	56.51
	5	101.3	15.60	1.573	-0.202	-0.209	0.190	-0.835	-12.15	67.70	-398.8

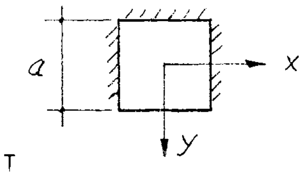


TABLE A-V

MOMENT STIFFNESS MATRIX (K)

EDGE	NODE	TOP 1	TOP 2	TOP 3	TOP 4	TOP 5	RIGHT 1	RIGHT 2	RIGHT 3	RIGHT 4	RIGHT 5
	1	39.64	-6.690	-1.353	-0.284	-0.044	0.040	0.114	0.128	0.090	0.045
	2	-8.001	28.60	-8.123	-1.841	-0.347	0.273	0.578	0.575	0.385	0.190
T	3	-1.659	-8.327	27.08	-8.327	-1.659	1.021	1.487	1.169	0.672	0.304
	4	-0.347	-1.841	-8.123	28.60	-8.001	3.478	2.867	1.531	0.678	0.263
	5	-0.044	-0.284	-1.353	-6.690	39.64	9.313	3.226	0.918	0.260	0.075
	1	0.043	0.241	0.913	3.226	9.313	39.64	-6.688	-1.356	-0.303	-0.077
	2	0.152	0.611	1.512	2.868	3.480	-8.004	28.55	-8.135	-1.932	-0.516
R	3	0.172	0.607	1.175	1.515	1.034	-1.673	-8.393	26.63	-8.484	-2.075
	4	0.119	0.397	0.660	0.664	0.306	-0.383	-1.993	-8.493	26.58	-8.627
	5	0.055	0.180	0.275	0.239	0.086	-0.093	-0.497	-1.933	-7.999	31.43
	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
B	3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	1	-9.313	-3.226	-0.913	-0.241	-0.043	0.040	0.111	0.126	0.090	0.045
	2	-3.480	-2.868	-1.512	-0.611	-0.152	0.142	0.384	0.480	0.385	0.214
L	3	-1.034	-1.515	-1.175	-0.607	-0.172	0.167	0.498	0.723	0.676	0.421
	4	-0.306	-0.664	-0.660	-0.397	-0.119	0.119	0.400	0.678	0.749	0.530
	5	-0.086	-0.239	-0.275	-0.180	-0.055	0.056	0.207	0.392	0.492	0.388

VALUES AT CENTER OF PLATE DUE TO UNIT ROTATION OF EDGE NODE

	TOP 1	TOP 2	TOP 3	TOP 4	TOP 5	RIGHT 1	RIGHT 2	RIGHT 3	RIGHT 4	RIGHT 5
M(X)	.0488	.2649	.4575	.2649	.0488	-.0282	-.0608	-.0549	-.1245	-.1188
M(Y)	.0220	.0369	.0012	.0369	.0220	-.0429	-.2484	-.4352	-.2477	-.0506
W	.0070	.0185	.0245	.0185	.0070	-.0070	-.0190	-.0264	-.0224	-.0131

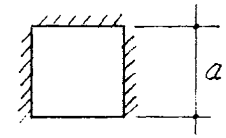
MULTIPLY ALL MOMENTS BY $\frac{D}{a}$ AND W BY a

TABLE A-V CONT.

EDGE	NODE	LEFT 1	2	3	4	5
	1	-9.313	-3.226	-0.918	-0.260	-0.075
	2	-3.478	-2.867	-1.531	-0.678	-0.263
T	3	-1.021	-1.487	-1.169	-0.672	-0.304
	4	-0.273	-0.578	-0.575	-0.385	-0.190
	5	-0.040	-0.114	-0.128	-0.090	-0.045
	1	-0.040	-0.111	-0.126	-0.090	-0.045
	2	-0.142	-0.384	-0.480	-0.385	-0.214
R	3	-0.167	-0.498	-0.723	-0.676	-0.421
	4	-0.119	-0.400	-0.678	-0.749	-0.530
	5	-0.056	-0.207	-0.392	-0.492	-0.388
	1	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0
B	3	0.0	0.0	0.0	0.0	0.0
	4	0.0	0.0	0.0	0.0	0.0
	5	0.0	0.0	0.0	0.0	0.0
	1	39.64	-6.688	-1.356	-0.303	-0.077
	2	-8.004	28.55	-8.135	-1.932	-0.516
L	3	-1.673	-8.393	26.63	-8.484	-2.075
	4	-0.383	-1.993	-8.493	26.58	-8.627
	5	-0.093	-0.497	-1.933	-7.999	31.43
	M(X)	.0282	.0608	.0549	.1245	.1188
	M(Y)	.0429	.2484	.4352	.2477	.0506
	W	.0070	.0190	.0264	.0224	.0131

TABLE A-VI

CARRY-OVER MOMENT STIFFNESS MATRIX (SM)



EDGE	NODE	TOP 1	TOP 2	TOP 3	TOP 4	TOP 5	RIGHT 1	RIGHT 2	RIGHT 3	RIGHT 4	RIGHT 5
T	1	-129.5	45.50	1.293	-0.328	-0.297	0.127	-0.161	-0.226	-0.128	-0.042
	2	42.03	-109.0	48.85	1.543	-0.977	0.221	-1.145	-1.159	-0.622	-0.218
	3	-0.477	48.46	-107.3	48.46	-0.477	-0.906	-3.666	-2.441	-1.016	-0.301
	4	-0.977	1.543	48.85	-109.0	42.03	-9.349	-8.385	-2.978	-0.752	-0.132
	5	-0.297	-0.328	1.293	45.50	-129.5	-50.49	-9.897	-0.989	0.061	0.082
R	1	0.239	0.141	-0.970	-9.899	-50.49	-129.5	45.50	1.313	-0.247	-0.138
	2	0.431	-0.480	-2.929	-8.404	-9.352	42.01	-109.0	48.96	1.977	-0.072
	3	0.407	-0.709	-2.449	-3.738	-0.891	-0.528	48.50	-106.9	49.64	2.274
	4	0.291	-0.498	-1.334	-1.305	0.277	-1.092	1.632	49.55	-106.4	49.79
	5	0.149	-0.220	-0.525	-0.373	0.215	-0.465	-0.237	2.220	49.77	-108.7
B	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
L	1	50.49	9.899	0.970	-0.141	-0.239	0.110	-0.163	-0.228	-0.132	-0.043
	2	9.352	8.404	2.929	0.480	-0.431	0.238	-0.651	-0.988	-0.720	-0.321
	3	0.891	3.738	2.449	0.709	-0.407	0.314	-0.770	-1.523	-1.439	-0.794
	4	-0.277	1.305	1.334	0.498	-0.291	0.293	-0.502	-1.378	-1.720	-1.177
	5	-0.215	0.373	0.525	0.220	-0.149	0.172	-0.199	-0.750	-1.167	-0.952

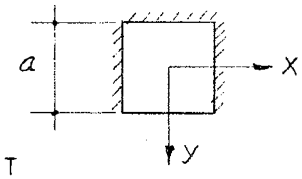
MULTIPLY ALL VALUES BY $\frac{D}{a^2}$

TABLE A-VI CONT.

EDGE	NODE	LEFT 1	2	3	4	5
	1	-50.49	-9.897	-0.989	0.061	0.082
	2	-9.349	-8.385	-2.978	-0.752	-0.132
T	3	-0.906	-3.666	-2.441	-1.016	-0.301
	4	0.221	-1.145	-1.159	-0.622	-0.218
	5	0.127	-0.161	-0.226	-0.128	-0.042
	1	-0.110	0.163	0.228	0.132	0.043
	2	-0.238	0.651	0.988	0.720	0.321
R	3	-0.314	0.770	1.523	1.439	0.794
	4	-0.293	0.502	1.378	1.720	1.177
	5	-0.172	0.199	0.750	1.167	0.952
	1	0.0	0.0	0.0	0.0	0.0
	2	0.0	0.0	0.0	0.0	0.0
B	3	0.0	0.0	0.0	0.0	0.0
	4	0.0	0.0	0.0	0.0	0.0
	5	0.0	0.0	0.0	0.0	0.0
	1	-129.5	45.50	1.313	-0.247	-0.138
	2	42.01	-109.0	48.96	1.977	-0.072
L	3	-0.528	48.50	-106.9	49.64	2.274
	4	-1.092	1.632	49.55	-106.4	49.79
	5	-0.465	-0.237	2.220	49.77	-108.7

TABLE A-VII

REACTION STIFFNESS MATRIX (T)



EDGE NODE		TOP 1	TOP 2	TOP 3	TOP 4	TOP 5	RIGHT 1	RIGHT 2	RIGHT 3	RIGHT 4	RIGHT 5
	1	1295.	-497.5	5.493	9.213	2.555	1.134	0.988	0.813	0.735	0.443
	2	-466.8	1036.	-523.0	4.666	11.75	3.121	-2.859	-2.643	-0.757	0.054
T	3	15.44	-521.3	1025.	-521.3	15.44	-3.981	-15.61	-6.984	-1.392	0.132
	4	11.75	4.666	-523.0	1036.	-466.8	-91.46	-35.21	-3.544	1.681	1.243
	5	2.555	9.213	5.493	-497.5	1295.	-606.7	11.29	13.29	4.689	1.564
	1	1.164	4.722	13.25	11.29	-606.7	1295.	-497.5	5.507	9.312	2.902
	2	2.616	2.368	-3.469	-35.30	-91.45	-466.7	1036.	-523.5	3.043	8.497
R	3	2.400	-0.527	-7.262	-16.01	-3.866	15.73	-521.5	1023.	-527.6	-0.862
	4	1.586	-0.841	-3.964	-3.864	3.510	12.50	4.155	-527.4	1018.	-536.5
	5	0.759	-0.354	-1.314	-0.440	1.814	3.856	8.953	-0.724	-536.5	1019.
	1	0.251	2.589	3.016	1.645	-0.305	-0.397	1.580	3.938	5.633	4.674
	2	1.117	-0.127	-0.345	0.464	0.709	0.722	0.781	0.577	1.310	2.662
B	3	1.219	-0.690	-1.971	-0.690	1.219	1.606	0.472	-2.539	-4.659	1.192
	4	0.709	0.464	-0.345	-0.127	1.117	1.971	2.497	-1.317	-16.44	-15.09
	5	-0.305	1.645	3.016	2.589	0.251	0.337	6.355	18.89	13.65	-167.6
	1	-606.7	11.29	13.25	4.722	1.164	0.722	0.951	0.768	0.653	0.405
	2	-91.45	-35.30	-3.469	2.368	2.616	1.560	-0.891	-2.225	-1.545	-0.536
L	3	-3.866	-16.01	-7.262	-0.527	2.400	1.838	-1.809	-4.741	-4.678	-2.571
	4	3.510	-3.864	-3.964	-0.841	1.586	1.552	-1.113	-4.560	-6.166	-4.369
	5	1.814	-0.440	1.314	-0.354	0.759	0.869	-0.291	-2.484	-4.350	-3.670

VALUES AT CENTER OF PLATE DUE TO UNIT DEFLECTION OF EDGE NODE

M(X)	-.4934	.8891	2.752	.8891	-.4934	-.3531	-.4767	-1.064	-.0189	.4122
M(Y)	-.3189	-.5199	-1.274	-.5199	-.3189	-.4726	.8329	2.668	.8803	-.3647
W	-.0062	.0828	.1317	.0828	-.0062	-.0070	.0820	.1392	.1131	.0530

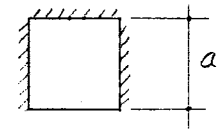
MULTIPLY ALL MOMENTS BY $\frac{D}{a^2}$ AND W BY 1

TABLE A-VII CONT.

EDGE	NODE	LEFT 1	2	3	4	5
T	1	-606.7	11.29	13.29	4.869	1.564
	2	-91.46	-35.21	-3.544	1.681	1.243
	3	-3.981	-15.61	-6.984	-1.392	0.132
	4	3.121	-2.859	-2.643	-0.757	0.054
	5	1.134	0.988	0.813	0.735	0.443
R	1	0.722	0.951	0.768	0.653	0.405
	2	1.600	-0.891	-2.225	-1.545	-0.536
	3	1.838	-1.809	4.741	4.678	-2.571
	4	1.552	-1.113	-4.560	-6.166	-4.369
	5	3.856	-0.291	-2.484	-4.350	-3.670
B	1	0.337	6.355	18.89	13.65	-167.6
	2	1.971	2.497	-1.317	-16.44	-15.09
	3	1.606	0.472	-2.539	-4.659	1.192
	4	0.722	0.781	0.577	1.310	2.662
	5	-0.397	1.580	3.938	5.633	4.674
L	1	1295.	-497.5	5.507	9.312	2.902
	2	-466.7	1036.	-523.5	3.043	8.497
	3	15.73	-521.5	1023.	-527.6	-0.862
	4	12.50	4.155	-527.4	1018.	-536.5
	5	3.856	8.953	-0.724	-536.5	1019.
	M(X)	-.3531	-.4767	-1.064	-.0189	.4122
	M(Y)	-.4726	.8329	2.668	.8803	-.3647
	W	-.0070	.0820	.1392	.1131	.0530

TABLE A-VIII

CARRY-OVER REACTION STIFFNESS MATRIX (SR)



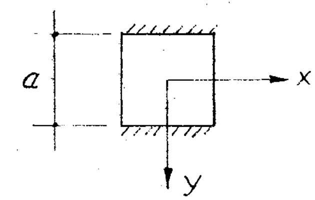
EDGE	NODE	TOP 1	TOP 2	TOP 3	TOP 4	TOP 5	RIGHT 1	RIGHT 2	RIGHT 3	RIGHT 4	RIGHT 5
	1	-398.7	67.80	-12.13	-0.826	0.193	-0.153	-0.095	-0.109	-0.146	-0.114
	2	56.54	-307.4	64.83	-15.88	-1.641	1.005	2.225	1.964	1.112	0.465
T	3	-16.04	62.91	-292.3	62.91	-16.04	6.797	7.803	4.956	2.347	0.908
	4	-1.641	-15.88	64.83	-307.4	56.54	32.68	17.04	6.277	1.990	0.569
	5	0.193	-0.826	-12.13	67.80	-398.7	101.3	15.57	1.533	-0.249	-0.265
	1	0.211	0.208	-1.564	-15.59	-101.3	414.5	-67.83	12.05	0.777	-0.229
	2	-0.280	-1.840	-6.269	-17.07	-32.69	-56.53	389.4	-65.34	16.00	2.249
R	3	-0.506	-2.229	-5.104	-7.995	-6.875	16.12	-62.90	307.2	-65.06	17.83
	4	-0.365	-1.437	-2.613	-2.800	-1.219	1.871	16.86	-64.82	307.6	-61.50
	5	-0.157	-0.605	-0.959	-0.791	-0.165	0.147	2.365	16.71	-65.04	358.9
	1	0.504	1.065	1.207	0.849	0.304	-0.309	-0.956	-1.662	-2.018	-1.604
	2	-0.106	-0.480	-0.515	-0.226	0.004	-0.011	0.180	0.440	0.434	0.074
B	3	-0.222	-0.878	-1.244	-0.878	-0.222	0.220	1.150	2.699	3.884	2.826
	4	0.004	-0.226	-0.515	-0.480	-0.106	0.106	1.129	4.274	10.58	14.10
	5	0.304	0.849	1.207	1.065	0.504	-0.575	-1.545	-1.058	12.17	71.28
	1	-101.3	-15.59	-1.564	0.208	0.211	-0.163	-0.166	-0.146	-0.143	-0.105
	2	-32.69	-17.07	-6.269	-1.840	-0.280	0.280	1.061	1.423	1.131	0.601
L	3	-6.875	-7.995	-5.104	-2.229	-0.506	0.498	1.741	2.648	2.519	1.572
	4	-1.219	-2.800	-2.613	-1.437	-0.365	0.370	1.451	2.606	2.968	2.138
	5	-0.165	-0.791	-0.959	-0.605	-0.157	0.164	0.738	1.523	1.999	1.605

MULTIPLY ALL VALUES BY $\frac{D}{a^2}$

TABLE A-VIII CONT.

	EDGE NODE	LEFT 1	2	3	4	5
	1	-101.3	-15.57	-1.533	0.249	0.265
	2	-32.68	-17.04	-6.277	-1.990	-0.569
T	3	-6.797	-7.803	-4.956	-2.347	-0.908
	4	-1.005	-2.225	-1.964	-1.112	-0.465
	5	0.153	0.095	0.109	0.146	0.114
	1	0.163	0.166	0.146	0.143	0.105
	2	-0.280	-1.061	-1.423	-1.131	-0.601
R	3	-0.498	-1.741	-2.648	-2.519	-1.572
	4	-0.370	-1.451	-2.606	-2.968	-2.138
	5	-0.164	-0.738	-1.523	-1.999	-1.605
	1	0.575	1.545	1.058	-12.17	-71.28
	2	-0.106	-1.129	-4.274	-10.58	-14.10
B	3	-0.220	-1.150	-2.699	-3.884	-2.826
	4	0.011	-0.180	-0.440	-0.434	-0.074
	5	0.309	0.956	1.662	2.018	1.604
	1	-414.5	67.83	-12.05	-0.777	0.229
	2	56.53	-389.4	65.34	-16.00	-2.249
L	3	-16.12	62.90	-307.2	65.06	-17.83
	4	-1.871	-16.86	64.82	-307.6	61.50
	5	-0.147	-2.365	-16.71	65.04	-358.9

TABLE A-IX



MOMENT STIFFNESS MATRIX (K)

EDGE	NODE	TOP 1	TOP 2	TOP 3	TOP 4	TOP 5	BOTTOM 1	BOTTOM 2	BOTTOM 3	BOTTOM 4	BOTTOM 5
	1	31.41	-8.007	-1.964	-0.559	-0.176	0.398	0.515	0.433	0.269	0.128
	2	-8.652	26.44	-8.543	-2.139	-0.601	0.555	0.804	0.779	0.554	0.290
T	3	-2.126	-8.595	26.05	-8.595	-2.126	0.469	0.784	0.918	0.784	0.469
	4	-0.601	-2.139	-8.543	26.44	-8.652	0.290	0.554	0.779	0.804	0.555
	5	-0.176	-0.559	-1.964	-8.007	31.41	0.128	0.269	0.433	0.515	0.398
	1	0.398	0.515	0.433	0.269	0.128	31.41	-8.007	-1.964	-0.559	-0.176
	2	0.555	0.804	0.779	0.554	0.290	-8.652	26.44	-8.543	-2.139	-0.601
B	3	0.469	0.784	0.918	0.784	0.469	-2.126	-8.595	26.05	-8.595	-2.126
	4	0.290	0.554	0.779	0.804	0.555	-0.601	-2.139	-8.543	26.44	-8.652
	5	0.128	0.269	0.433	0.515	0.398	-0.176	-0.559	-1.964	-8.007	31.41
VALUES AT CENTER OF PLATE DUE TO UNIT ROTATION OF EDGE NODE											
	M(X)	.0517	.2500	.4366	.2500	.0517	-.0517	-.2500	-.4366	-.2500	-.0517
	M(Y)	.1332	.1573	.1176	.1573	.1332	-.1332	-.1573	-.1176	-.1573	-.1332
	W	.0140	.0249	.0303	.0249	.0140	-.0140	-.0249	-.0303	-.0249	-.0140

MULTIPLY ALL MOMENTS BY $\frac{D}{a}$ AND W BY a

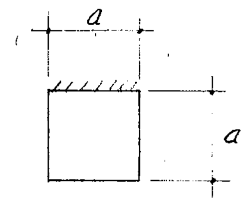


TABLE A-X

MOMENT STIFFNESS MATRIX(K)

	EDGE NODE	TOP 1	2	3	4	5
	1	31.29	-8.724	-2.214	-.6765	-.2182
	2	-8.050	26.15	-8.711	-2.255	-.6267
T	3	-2.038	-8.646	25.68	-8.646	-2.038
	4	-.6267	-2.255	-8.711	26.15	-8.050
	5	-.2182	-.6765	-2.214	-8.724	31.29

MULTIPLY ALL VALUES BY $\frac{D}{a}$

APPENDIX B

LOAD FUNCTIONS

The following tables give the values of fixed edge moments and reactions and center moments and deflection for various loading conditions. Figure B-1 illustrates the plate notation used in the tables.

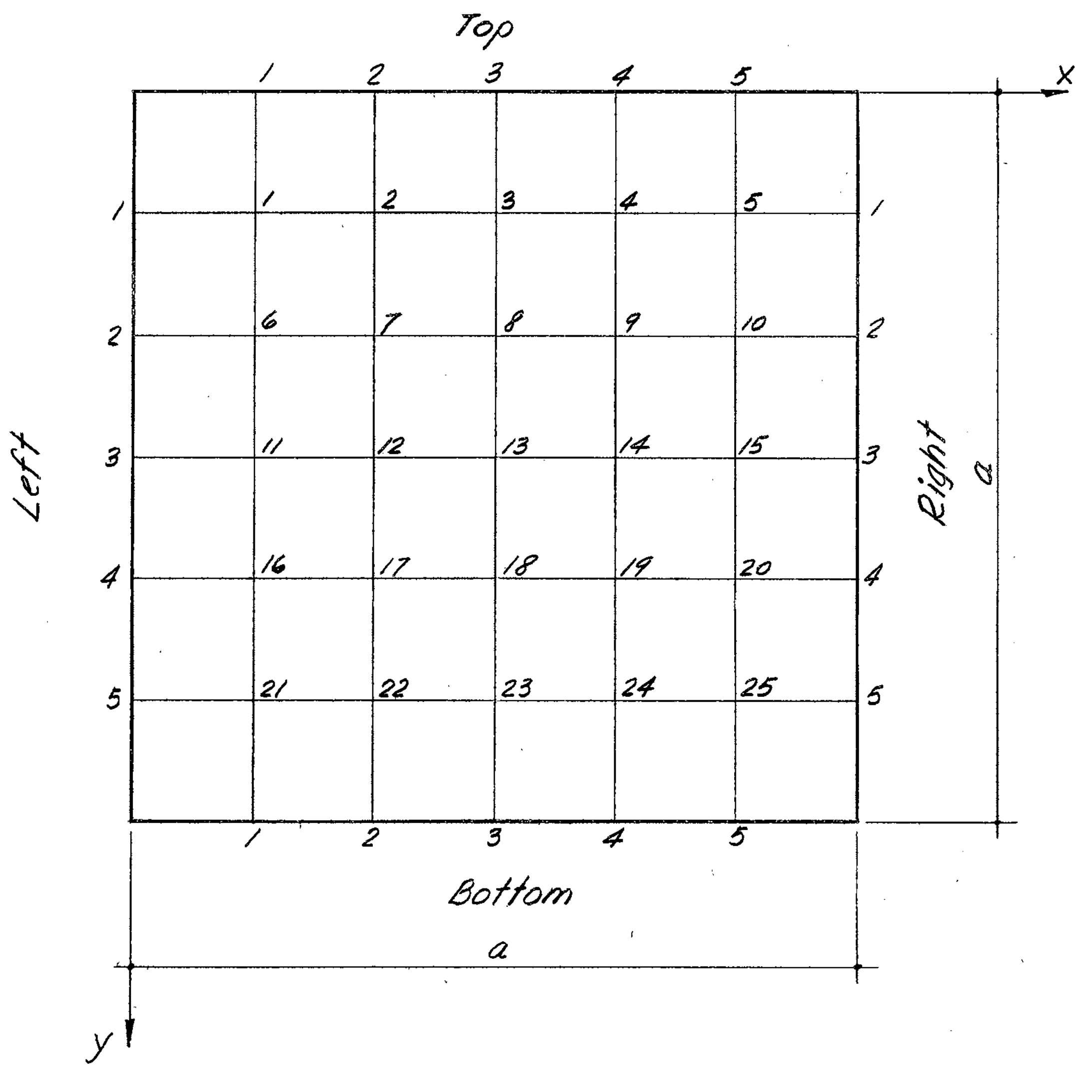
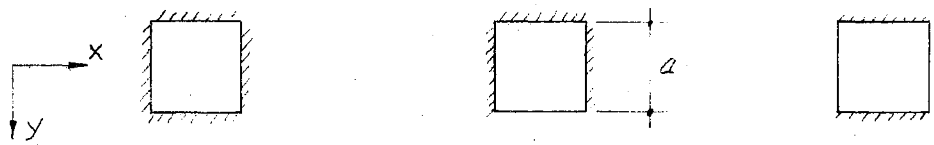


Figure B-1 Plate Notation for Loads

TABLE B-I

FIXED EDGE MOMENTS AND REACTIONS AND CENTER MOMENTS AND DEFLECTION FOR UNIFORMLY LOADED SQUARE PLATES

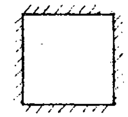
$\nu = .2$



		FM	FR	FM	FR	FM	FR
	1	-.01915	-.14103	-.01988	-.13562	-.03929	-.32308
	2	-.04128	-.29377	-.04397	-.34357	-.06103	-.43797
T	3	-.04949	-.34617	-.05315	-.40858	-.06771	-.46688
	4	-.04128	-.29377	-.04397	-.34357	-.06103	-.43797
	5	-.01915	-.14103	-.01988	-.13562	-.03929	-.32308
	1	.01915	-.14103	.01989	-.13368		
	2	.04128	-.29377	.04492	-.34075		
R	3	.04949	-.34617	.05800	-.42736		
	4	.04128	-.29377	.05612	-.42582		
	5	.01915	-.14103	.03733	-.32202		
	1	.01915	-.14103			.03929	-.32308
	2	.04128	-.29377			.06103	-.43797
B	3	.04949	-.34617			.06771	-.46688
	4	.04128	-.29377			.06103	-.43797
	5	.01915	-.14103			.03929	-.32308
	1	-.01915	-.14103	-.01989	-.13368		
	2	-.04128	-.29377	-.04492	-.34075		
L	3	-.04949	-.34617	-.05800	-.42736		
	4	-.04128	-.29377	-.05612	-.42582		
	5	-.01915	-.14103	-.03733	-.32202		

M(X)=.02137	M(X)=.02615	M(X)=.02312 qa^2
M(Y)=.02137	M(Y)=.02177	M(Y)=.03072 qa^2
W=.00134	W=.00165	W=.00200 qa^4/D

TABLE B-II



FIXED EDGE MOMENTS DUE TO CONCENTRATED LOAD,P

	EDGE NODE	TOP 1	TOP 2	TOP 3	TOP 4	TOP 5	RIGHT 1	RIGHT 2	RIGHT 3	RIGHT 4	RIGHT 5
	1	-.1679	-.0896	-.0289	-.0081	-.0013	.0013	.0040	.0043	.0025	.0006
	2	-.0875	-.2466	-.1233	-.0391	-.0074	.0066	.0167	.0162	.0091	.0021
U	3	-.0272	-.1227	-.2625	-.1227	-.0272	.0220	.0410	.0326	.0162	.0037
N	4	-.0074	-.0391	-.1233	-.2466	-.0875	.0662	.0770	.0437	.0176	.0036
I	5	-.0013	-.0081	-.0289	-.0896	-.1679	.1679	.0896	.0289	.0081	.0013
T	6	-.0662	-.0770	-.0437	-.0176	-.0036	.0035	.0111	.0131	.0084	.0021
	7	-.0715	-.1671	-.1353	-.0639	-.0150	.0141	.0408	.0454	.0283	.0073
L	8	-.0383	-.1335	-.1988	-.1335	-.0383	.0361	.0919	.0914	.0521	.0133
O	9	-.0150	-.0639	-.1353	-.1671	-.0715	.0715	.1671	.1353	.0639	.0150
A	10	-.0036	-.0176	-.0437	-.0770	-.0662	.0875	.2466	.1233	.0391	.0074
D	11	-.0220	-.0410	-.0326	-.0162	-.0037	.0036	.0129	.0175	.0129	.0036
	12	-.0361	-.0919	-.0914	-.0521	-.0133	.0131	.0441	.0597	.0441	.0131
A	13	-.0271	-.0893	-.1224	-.0893	-.0271	.0271	.0893	.1224	.0893	.0271
T	14	-.0133	-.0521	-.0914	-.0919	-.0361	.0383	.1335	.1988	.1335	.0383
	15	-.0037	-.0162	-.0326	-.0410	-.0220	.0272	.1227	.2625	.1227	.0272
P	16	-.0066	-.0167	-.0162	-.0091	-.0021	.0021	.0084	.0131	.0111	.0035
O	17	-.0141	-.0408	-.0454	-.0283	-.0073	.0073	.0283	.0454	.0408	.0141
I	18	-.0131	-.0441	-.0597	-.0441	-.0131	.0133	.0521	.0914	.0919	.0361
N	19	-.0073	-.0283	-.0454	-.0408	-.0141	.0150	.0639	.1353	.1671	.0715
T	20	-.0021	-.0091	-.0162	-.0167	-.0066	.0074	.0391	.1233	.2466	.0875
	21	-.0013	-.0040	-.0043	-.0025	-.0006	.0006	.0025	.0043	.0040	.0013
	22	-.0035	-.0111	-.0131	-.0084	-.0021	.0021	.0091	.0162	.0167	.0066
	23	-.0036	-.0129	-.0175	-.0129	-.0036	.0037	.0162	.0326	.0410	.0220
	24	-.0021	-.0084	-.0131	-.0111	-.0035	.0036	.0176	.0437	.0770	.0662
	25	-.0006	-.0025	-.0043	-.0040	-.0013	.0013	.0081	.0289	.0896	.1679

MULTIPLY ALL VALUES BY P

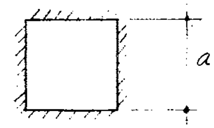
TABLE B-II CONTINUED

	EDGE NODE	BOTTOM 1	BOTTOM 2	BOTTOM 3	BOTTOM 4	BOTTOM 5	LEFT 1	LEFT 2	LEFT 3	LEFT 4	LEFT 5
	1	.0013	.0040	.0043	.0025	.0006	-.1679	-.0896	-.0289	-.0081	-.0013
	2	.0035	.0111	.0131	.0084	.0021	-.0662	-.0770	-.0437	-.0176	-.0036
U	3	.0036	.0129	.0175	.0129	.0036	-.0220	-.0410	-.0326	-.0162	-.0037
N	4	.0021	.0084	.0131	.0111	.0035	-.0066	-.0167	-.0162	-.0091	-.0021
I	5	.0006	.0025	.0043	.0040	.0013	-.0013	-.0040	-.0043	-.0025	-.0066
T	6	.0066	.0167	.0162	.0091	.0021	-.0875	-.2466	-.1233	-.0391	-.0074
	7	.0141	.0408	.0454	.0283	.0073	-.0715	-.1671	-.1353	-.0639	-.0150
L	8	.0131	.0441	.0597	.0441	.0131	-.0361	-.0919	-.0914	-.0521	-.0133
O	9	.0073	.0283	.0454	.0408	.0141	-.0141	-.0408	-.0454	-.0283	-.0073
A	10	.0021	.0091	.0162	.0167	.0066	-.0035	-.0111	-.0131	-.0084	-.0021
D	11	.0220	.0410	.0326	.0162	.0037	-.0272	-.1227	-.2625	-.1227	-.0272
	12	.0361	.0919	.0914	.0521	.0133	-.0383	-.1335	-.1988	-.1335	-.0383
A	13	.0271	.0893	.1224	.0893	.0271	-.0271	-.0893	-.1224	-.0893	-.0271
T	14	.0133	.0521	.0914	.0919	.0361	-.0131	-.0441	-.0597	-.0441	-.0131
	15	.0037	.0162	.0326	.0410	.0220	-.0036	-.0129	-.0175	-.0129	-.0036
P	16	.0662	.0770	.0437	.0176	.0036	-.0074	-.0391	-.1233	-.2466	-.0875
O	17	.0715	.1671	.1353	.0639	.0150	-.0150	-.0639	-.1353	-.1671	-.0715
I	18	.0383	.1335	.1988	.1335	.0383	-.0133	-.0521	-.0914	-.0919	-.0361
N	19	.0150	.0639	.1353	.1671	.0715	-.0073	-.0283	-.0454	-.0408	-.0141
T	20	.0036	.0176	.0437	.0770	.0662	-.0021	-.0084	-.0131	-.0111	-.0035
	21	.1679	.0896	.0289	.0081	.0013	-.0013	-.0081	-.0289	-.0896	-.1679
	22	.0875	.2466	.1233	.0391	.0074	-.0036	-.0176	-.0437	-.0770	-.0662
	23	.0272	.1227	.2625	.1227	.0272	-.0037	-.0162	-.0326	-.0410	-.0220
	24	.0074	.0391	.1233	.2466	.0875	-.0021	-.0091	-.0162	-.0167	-.0066
	25	.0013	.0081	.0289	.0896	.1679	-.0006	-.0025	-.0043	-.0040	-.0013

MULTIPLY ALL VALUES BY P

TABLE B-III

FIXED EDGE REACTIONS DUE TO CONCENTRATED LOAD, P



	EDGE NODE	TOP 1	TOP 2	TOP 3	TOP 4	TOP 5	RIGHT 1	RIGHT 2	RIGHT 3	RIGHT 4	RIGHT 5
	1	-1.629	-.8808	-.2154	-.0382	.0015	.0029	-.0096	-.0134	-.0064	.0013
	2	-.8299	-2.332	-1.114	-.2630	-.0206	-.0008	-.0553	-.0611	-.0289	.0020
U	3	-.1751	-1.102	-2.437	-1.102	-.1751	-.0505	-.1781	-.1463	-.0607	.0003
N	4	-.0206	-.2630	-1.114	-2.332	-.8299	-.3185	-.4650	-.2430	-.0758	-.0009
I	5	.0015	-.0382	-.2154	-.8808	-1.629	-1.629	-.8808	-.2154	-.0382	.0015
T	6	-.3185	-.4650	-.2430	-.0758	-.0009	.0025	-.0332	-.0459	-.0252	.0023
	7	-.3686	-.9825	-.8037	-.3274	-.0321	-.0124	-.1473	-.1809	-.0993	-.0003
L	8	-.1522	-.7740	-1.172	-.7740	-.1522	-.0985	-.4120	-.4288	-.2175	-.0150
O	9	-.0321	-.3274	-.8037	-.9825	-.3686	-.3686	-.9825	-.8037	-.3274	-.0321
A	10	-.0009	-.0758	-.2430	-.4650	-.3185	-.8299	-2.332	-1.114	-.2630	-.0206
D	11	-.0505	-.1781	-.1463	-.0607	.0003	.0019	-.0401	-.0626	-.0401	.0019
	12	-.0985	-.4120	-.4288	-.2175	-.0150	-.0092	-.1616	-.2410	-.1616	-.0092
A	13	-.0590	-.3979	-.5775	-.3979	-.0590	-.0590	-.3979	-.5775	-.3979	-.0590
T	14	-.0150	-.2175	-.4288	-.4120	-.0985	-.1522	-.7740	-1.172	-.7740	-.1522
	15	.0003	-.0607	-.1463	-.1781	-.0505	-.1751	-1.102	-2.437	-1.102	-.1751
P	16	-.0008	-.0553	-.0611	-.0289	.0020	.0023	-.0252	-.0459	-.0332	.0025
O	17	-.0124	-.1473	-.1809	-.0993	-.0003	-.0003	-.0993	-.1809	-.1473	-.0124
I	18	-.0092	-.1616	-.2410	-.1616	-.0092	-.0150	-.2175	-.4288	-.4120	-.0985
N	19	-.0003	-.0993	-.1809	-.1473	-.0124	-.0321	-.3274	-.8037	-.9825	-.3686
T	20	.0020	-.0289	-.0611	-.0553	-.0008	-.0206	-.2630	-1.114	-2.332	-.8299
	21	.0029	-.0097	-.0134	-.0064	.0013	.0013	-.0064	-.0134	-.0096	.0029
	22	.0025	-.0332	-.0459	-.0252	.0023	.0020	-.0289	-.0611	-.0553	-.0008
	23	.0019	-.0401	-.0626	-.0401	.0019	.0003	-.0607	-.1463	-.1781	-.0505
	24	.0023	-.0252	-.0459	-.0332	.0025	-.0009	-.0758	-.2430	-.4650	-.3185
	25	.0013	-.0064	-.0134	-.0097	.0029	.0015	.0382	-.2154	-.8808	-1.629

MULTIPLY ALL VALUES BY $\frac{P}{a}$

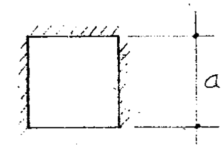
TABLE B-III CONTINUED

UNIT LOAD AT POINT	EDGE NODE	BOTTOM 1	BOTTOM 2	BOTTOM 3	BOTTOM 4	BOTTOM 5	LEFT 1	LEFT 2	LEFT 3	LEFT 4	LEFT 5
	1	.0029	-.0096	-.0134	-.0064	.0013	-1.629	-.8808	-.2154	-.0382	.0015
	2	.0025	-.0332	-.0459	-.0252	.0023	-.3185	-.4650	-.2430	-.0758	-.0009
U	3	.0019	-.0401	-.0626	-.0401	.0019	-.0505	-.1781	-.1463	-.0607	.0003
N	4	.0023	-.0252	-.0459	-.0332	.0025	-.0008	-.0553	-.0611	-.0289	.0020
I	5	.0013	-.0064	-.0134	-.0097	.0029	.0029	-.0096	-.0134	-.0064	.0013
T	6	-.0008	-.0553	-.0611	-.0289	.0020	-.8299	-2.332	-1.114	-.2630	-.0206
	7	-.0124	-.1473	-.1809	-.0993	-.0003	-.3686	-.9825	-.8037	-.3274	-.0321
L	8	-.0092	-.1616	-.2410	-.1616	-.0092	-.0985	-.4120	-.4288	-.2175	-.0150
O	9	-.0003	-.0993	-.1809	-.1473	-.0124	-.0124	-.1473	-.1809	-.0993	-.0003
A	10	.0020	-.0289	-.0611	-.0553	-.0008	.0025	-.0332	-.0459	-.0252	.0023
D	11	-.0505	-.1781	-.1463	-.0607	.0003	-.1751	-1.102	-2.437	-1.102	-.1751
	12	-.0985	-.4120	-.4288	-.2175	-.0150	-.1522	-.7740	-1.172	-.7740	-.1522
A	13	-.0590	-.3979	-.5775	-.3979	-.0590	-.0590	-.3979	-.5775	-.3979	-.0590
T	14	-.0150	-.2175	-.4288	-.4120	-.0985	-.0092	-.1616	-.2410	-.1616	-.0092
	15	.0003	-.0607	-.1463	-.1781	-.0505	.0019	-.0401	-.0626	-.0401	.0019
P	16	-.3185	-.4650	-.2430	-.0758	-.0009	-.0206	-.2630	-1.114	-2.332	-.8299
O	17	-.3686	-.9825	-.8037	-.3274	-.0321	-.0321	-.3274	-.8037	-.9825	-.3686
I	18	-.1522	-.7740	-1.172	-.7740	-.1522	-.0150	-.2175	-.4288	-.4120	-.0985
N	19	-.0321	-.3274	-.8037	-.9825	-.3686	-.0003	-.0993	-.1809	-.1473	-.0124
T	20	-.0009	-.0758	-.2430	-.4650	-.3185	.0023	-.0252	-.0459	-.0332	.0025
	21	-1.629	-.8808	-.2154	-.0382	.0015	.0015	-.0382	-.2154	-.8808	-1.629
	22	-.8299	-2.332	-1.114	-.2630	-.0206	-.0009	-.0758	-.2430	-.4650	-.3185
	23	-.1751	-1.102	-2.437	-1.102	-.1751	.0003	-.0607	-.1463	-.1781	-.0505
	24	-.0206	-.2630	-1.114	-2.332	-.8299	.0020	-.0289	-.0611	-.0553	-.0008
	25	.0015	-.0382	-.2154	-.8808	-1.629	.0013	-.0064	-.0134	-.0096	.0029

MULTIPLY ALL VALUES BY $\frac{P}{a}$

TABLE B-IV

FIXED EDGE REACTIONS DUE TO CONCENTRATED LOAD,P



UNIT LOAD AT POINT

EDGE NODE	TOP 1	TOP 2	TOP 3	TOP 4	TOP 5	RIGHT 1	RIGHT 2	RIGHT 3	RIGHT 4	RIGHT 5
1	-.1680	-.0898	-.0292	-.0084	-.0014	.0013	.0043	.0049	.0035	.0017
2	-.0877	-.2472	-.1241	-.0398	-.0076	.0068	.0176	.0182	.0123	.0057
3	-.0275	-.1236	-.2637	-.1236	-.0275	.0223	.0422	.0353	.0208	.0089
4	-.0076	-.0398	-.1241	-.2472	-.0877	.0664	.0779	.0458	.0212	.0079
5	-.0014	-.0084	-.0292	-.0898	-.1680	.1680	.0899	.0296	.0094	.0028
6	-.0664	-.0779	-.0449	-.0184	-.0038	.0037	.0122	.0156	.0124	.0064
7	-.0721	-.1694	-.1384	-.0661	-.0156	.0148	.0438	.0522	.0396	.0198
8	-.0391	-.1364	-.2028	-.1364	-.0391	.0369	.0958	.1005	.0677	.0313
9	-.0156	-.0661	-.1384	-.1694	-.0721	.0721	.1703	.1427	.0769	.0307
10	-.0038	-.0184	-.0449	-.0779	-.0664	.0877	.2478	.1261	.0441	.0137
11	-.0226	-.0429	-.0351	-.0180	-.0042	.0041	.0152	.0226	.0211	.0121
12	-.0374	-.0966	-.0977	-.0568	-.0145	.0143	.0503	.0735	.0668	.0375
13	-.0287	-.0953	-.1305	-.0953	-.0287	.0287	.0974	.1411	.1211	.0638
14	-.0145	-.0568	-.0977	-.0966	-.0374	.0397	.1401	.2145	.1615	.0733
15	-.0042	-.0180	-.0351	-.0429	-.0226	.0278	.1254	.2689	.1347	.0433
16	-.0075	-.0198	-.0202	-.0119	-.0029	.0029	.0120	.0209	.0231	.0152
17	-.0162	-.0483	-.0553	-.0354	-.0092	.0093	.0376	.0661	.0739	.0484
18	-.0156	-.0534	-.0723	-.0534	-.0156	.0159	.0646	.1201	.1407	.0919
19	-.0092	-.0354	-.0553	-.0483	-.0162	.0171	.0744	.1607	.2139	.1325
20	-.0029	-.0119	-.0202	-.0198	-.0075	.0083	.0436	.1345	.2691	.1216
21	-.0022	-.0074	-.0086	-.0055	-.0013	.0013	.0062	.0120	.0152	.0112
22	-.0058	-.0194	-.0238	-.0160	-.0041	.0042	.0187	.0370	.0483	.0368
23	-.0063	-.0228	-.0310	-.0228	-.0063	.0064	.0295	.0631	.0917	.0775
24	-.0041	-.0160	-.0238	-.0194	-.0058	.0060	.0293	.0728	.1324	.1411
25	-.0013	-.0055	-.0086	-.0074	-.0022	.0023	.0134	.0432	.1216	.2260

MULTIPLY ALL VALUES BY **P**

TABLE B-IV CONT.

	EDGE NODE	LEFT 1	2	3	4	5
	1	-.1680	-.0899	-.0296	-.0094	-.0028
	2	-.0664	-.0779	-.0458	-.0212	-.0079
U	3	-.0223	-.0422	-.0353	-.0208	-.0089
N	4	-.0068	-.0176	-.0182	-.0123	-.0057
I	5	-.0013	-.0043	-.0049	-.0035	-.0017
T	6	-.0877	-.2478	-.1261	-.0441	-.0137
	7	-.0721	-.1703	-.1427	-.0769	-.0307
L	8	-.0369	-.0958	-.1005	-.0677	-.0313
O	9	-.0148	-.0438	-.0522	-.0396	-.0198
A	10	-.0037	-.0122	-.0156	-.0124	-.0064
D	11	-.0278	-.1254	-.2689	-.1347	-.0433
	12	-.0397	-.1401	-.2145	-.1615	-.0733
A	13	-.0287	-.0974	-.1411	-.1211	-.0638
T	14	-.0143	-.0503	-.0735	-.0668	-.0375
	15	-.0041	-.0152	-.0226	-.0211	-.0121
P	16	-.0083	-.0436	-.1345	-.2691	-.1216
O	17	-.0171	-.0744	-.1607	-.2139	-.1325
I	18	-.0159	-.0646	-.1201	-.1407	-.0919
N	19	-.0093	-.0376	-.0661	-.0739	-.0484
T	20	-.0029	-.0120	-.0209	-.0231	-.0152
	21	-.0023	-.0134	-.0432	-.1216	-.2260
	22	-.0060	-.0293	-.0728	-.1324	-.1411
	23	-.0064	-.0295	-.0631	-.0917	-.0775
	24	-.0042	-.0187	-.0370	-.0483	-.0368
	25	-.0013	-.0062	-.0120	-.0152	-.0112

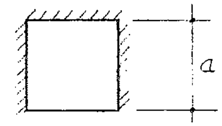


TABLE B-V

FIXED EDGE REACTIONS DUE TO CONCENTRATED LOAD, P

	EDGE	TOP					RIGHT				
	NODE	1	2	3	4	5	1	2	3	4	5
	1	-1.628	-.8814	-.2163	-.0388	.0015	.0030	-.0106	-.0163	-.0120	-.0054
	2	-.8296	-2.334	-1.117	-.2648	-.0203	-.0006	-.0583	-.0703	-.0470	-.0206
U	3	-.1748	-1.104	-2.441	-1.104	-.1748	-.0503	-.1822	-.1589	-.0863	-.0332
N	4	-.0203	-.2648	-1.117	-2.334	-.8296	-.3183	-.4681	-.2530	-.0967	-.0293
I	5	.0015	-.0388	-.2163	-.8814	-1.628	-1.629	-.8818	-.2187	-.0453	-.0084
T	6	-.3182	-.4673	-.2467	-.0781	-.0006	.0027	-.0370	-.0572	-.0468	-.0237
	7	-.3676	-.9888	-.8139	-.3336	-.0312	-.0117	-.1578	-.2128	-.1618	-.0784
L	8	-.1509	-.7820	-1.185	-.7820	-.1509	-.0976	-.4259	-.4724	-.3060	-.1320
O	9	-.0312	-.3336	-.8139	-.9888	-.3676	-.3679	-.9937	-.8395	-.4032	-.1387
A	10	-.0006	-.0781	-.2467	-.4673	-.3182	-.8296	-2.336	-1.128	-.2930	-.0646
D	11	-.0497	-.1832	-.1544	-.0656	.0011	.0025	-.0480	-.0861	-.0836	-.0485
	12	-.0964	-.4250	-.4499	-.2302	-.0130	-.0078	-.1828	-.3051	-.2848	-.1600
A	13	-.0565	-.4143	-.6043	-.4143	-.0565	-.0572	-.4263	-.6662	-.5785	-.2991
T	14	-.0130	-.2302	-.4499	-.4250	-.0964	-.1507	-.7975	-1.249	-.9404	-.3976
	15	.0011	-.0656	-.1544	-.1832	-.0497	-.1745	-1.111	-2.469	-1.176	-.2954
P	16	.0006	-.0636	-.0742	-.0366	.0032	.0032	-.0373	-.0807	-.0944	-.0633
O	17	-.0091	-.1678	-.2137	-.1189	.0027	.0019	-.1312	-.2755	-.3227	-.2162
I	18	-.0054	-.1870	-.2825	-.1870	-.0054	-.0122	-.2613	-.5653	-.6874	-.4599
N	19	.0027	-.1189	-.2137	-.1678	-.0091	-.0298	-.3653	-.9297	-1.268	-.8132
T	20	.0032	-.0366	-.0742	-.0636	.0006	-.0195	-.2793	-1.172	-2.481	-1.116
	21	.0048	-.0187	-.0275	-.0145	.0027	.0025	-.0183	-.0464	-.0628	-.0473
	22	.0061	-.0555	-.0810	-.0459	.0054	.0045	-.0612	-.1533	-.2146	-.1670
	23	.0059	-.0675	-.1072	-.0675	.0059	.0032	-.1075	-.2896	-.4578	-.3943
	24	.0054	-.0459	-.0810	-.0555	.0061	.0016	-.1188	-.3901	-.8115	-.8571
	25	.0027	-.0145	-.0275	-.0187	.0048	.0028	-.0576	-.2928	-1.115	-2.188

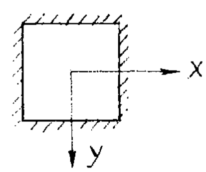
MULTIPLY ALL VALUES BY $\frac{P}{a}$

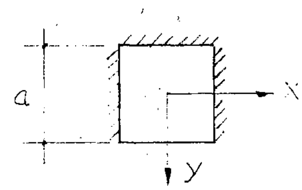
TABLE B-V CONTINUED

	EDGE NODE	LEFT 1	2	3	4	5
	1	-1.629	-.8818	-.2187	-.0453	-.0084
	2	-.3183	-.4681	-.2530	-.0967	-.0293
U	3	-.0503	-.1822	-.1589	-.0863	-.0332
N	4	-.0006	-.0583	-.0703	-.0470	-.0206
I	5	.0030	-.0106	-.0163	-.0120	-.0054
T	6	-.8296	-2.336	-1.128	-.2930	-.0646
	7	-.3679	-.9937	-.8395	-.4032	-.1387
L	8	-.0976	-.4259	-.4724	-.3060	-.1320
O	9	-.0117	-.1578	-.2128	-.1618	-.0784
A	10	.0027	-.0370	-.0572	-.0468	-.0237
D	11	-.1745	-1.111	-2.469	-1.176	-.2954
	12	-.1507	-.7975	-1.249	-.9404	-.3976
A	13	-.0572	-.4263	-.6662	-.5785	-.2991
T	14	-.0078	-.1828	-.3051	-.2848	-.1600
	15	.0025	-.0480	-.0861	-.0836	-.0485
P	16	-.0195	-.2793	-1.172	-2.481	-1.116
O	17	-.0298	-.3653	-.9297	-1.268	-.8132
I	18	-.0122	-.2613	-.5653	-.6874	-.4599
N	19	.0019	-.1312	-.2755	-.3227	-.2162
T	20	.0032	-.0373	-.0807	-.0944	-.0633
	21	.0028	-.0576	-.2928	-1.115	-2.188
	22	.0016	-.1188	-.3901	-.8115	-.8751
	23	.0032	-.1075	-.2896	-.4578	-.3943
	24	.0045	-.0612	-.1533	-.2146	-.1670
	25	.0025	-.0183	-.0464	-.0628	-.0473

TABLE B-VI

CENTER MOMENTS AND DEFLECTION DUE TO CONCENTRATED LOAD, P





		$W \times Pa^2/D$	M x P	M x P	$W \times Pa^2/D$	M x P	M x P
	1	.000342	.00230	.00230	.000366	.00268	.00233
	2	.001010	.01429	.00514	.001083	.01546	.00523
U	3	.001366	.02721	.00337	.001463	.02875	.00349
N	4	.001010	.01429	.00514	.001083	.01546	.00523
I	5	.000342	.00230	.00230	.000366	.00268	.00233
T	6	.001010	.00514	.01429	.001105	.00664	.01441
	7	.002866	.03824	.03824	.003125	.04233	.03856
L	8	.003971	.09952	.03997	.004305	.10480	.04038
O	9	.002866	.03824	.03824	.003125	.04233	.03856
A	10	.001010	.00514	.01429	.001105	.00664	.01441
D	11	.001366	.00337	.02721	.001574	.00661	.02747
	12	.003971	.03997	.09952	.004506	.04838	.10018
A	13	.006069	.28303	.28303	.006751	.29379	.28385
T	14	.003971	.03997	.09952	.004506	.04838	.10018
	15	.001366	.00337	.02721	.001574	.00661	.02747
P	16	.001010	.00514	.01429	.001347	.01027	.01476
O	17	.002866	.03824	.03824	.003701	.05130	.03930
I	18	.003971	.09952	.03997	.005025	.11623	.04122
N	19	.002866	.03824	.03824	.003701	.05130	.03930
T	20	.001010	.00514	.01429	.001347	.01027	.1476
	21	.000342	.00230	.00230	.000713	.00763	.00290
	22	.001010	.01429	.00514	.001907	.02819	.00633
	23	.001366	.02721	.00337	.002496	.04541	.00461
	24	.001010	.01429	.00514	.001907	.02819	.00633
	25	.000342	.00230	.00230	.000713	.00763	.00290

VITA

Lawrence Norwood Dallam

Candidate for the Degree of

Doctor of Philosophy

Thesis: ANALYSIS OF CONTINUOUS RECTANGULAR PLATE SYSTEMS BY THE SLOPE DEFLECTION METHOD

Major Field: Engineering

Biographical:

Personal Data: Born July 29, 1929, in Kansas City, Missouri, the son of Ira L. and Mildred A. Dallam.

Education: Graduated from Paseo High School, Kansas City, Missouri, in June, 1946. The degrees of Bachelor of Science and Master of Science in Civil Engineering were received from the University of Missouri in February, 1954, and June, 1955, respectively. Completed the requirements for the degree of Doctor of Philosophy, January, 1966.

Professional Experience: Designer, Missouri State Highway Department, summers 1954, 1955. Designer, Howard, Needles, Tammen & Bergendoff, Consulting Engineers, 1955-57. Assistant Professor of Civil Engineering, University of Missouri, 1957-1966. Registered Professional Engineer, State of Missouri. Member, American Concrete Institute, American Society of Civil Engineers, the Society of the Sigma Xi. Project research director, "Study of Composite Bridge Stringers," sponsored by Missouri State Highway Commission and U. S. Bureau of Public Roads, 1962 and 1965.