

ANALYSIS OF TRUSS-PLATE STRUCTURES,
BY PLATE ANALOGY

By

CARL CLAYTON HOLLOWAY, JR.

Bachelor of Civil Engineering

Oklahoma State University

Stillwater, Oklahoma

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NOMENCLATURE

N_m	Axial Force in Member m .
P_j	Load at Joint j .
ϕ_j	Slope of Truss ij .
α_j	Angle Between P_j and Vertical Direction.
P_{ji}	Component of Load in the Plane of Truss ij .
Q	Edge Shearing Force.
F	Joint Force.
R	External Reaction.

CHAPTER I

INTRODUCTION

1.1 General

Space trusses may be analyzed by several methods. Some of these are:

1. Method of Joints
2. Method of Sections
3. Method of Moments
4. Henneberg's Method
5. Tension Coefficients
6. Method of Plate Analogy

It is the purpose of this thesis to discuss the application of the method of plate analogy to framed structures of the same general shape as barrel shells and folded plates. Cases of internal and external indeterminacy and the effect of instability are also discussed.

The method of plate analogy reduces the analysis of space trusses to a two dimensional problem. As applied to space trusses, this method consists of separating the space truss into planar trusses, resolving the loads on the structure into loads in the plane of the planar trusses, and completing the problem as an analysis of several planar trusses.

Thus, besides being convenient and practical, the method of plate analogy provides a simpler approach to the analysis of complicated space structures composed of planar trusses.

In addition to the advantages in the method of analysis, it is felt that roof structures of this type composed of steel framing have two main advantages over roofs formed of concrete. The first and primary advantage is cost. A concrete roof poured over complicated formwork can seldom be constructed more economically than a framework of straight steel members with simple connections. The second advantage is simplicity. Provisions for overhangs, concentrated loads, discontinuities and skylight openings, which are complicating factors in a concrete roof, can be readily handled in the engineering office and at the construction site. (1) *

1.2 Historical

According to historical information presented by Kinney⁽²⁾ and Gillespie⁽³⁾, the Italian architect, Andrea Palladio (1518 - 1580) is believed to have first used trusses of any great span, although the Romans used trusses in their wooden bridges and roofs. Two centuries later (1758) a Swiss carpenter, Ulric Grubenmann, built a 170 foot timber truss across the Rhine. Other timber bridges were built after this, but they combined the truss and the arch in a single structure. Then, in 1778, Ulric Grubenmann and his brother, Jean, built a longer timber bridge with a span of 390 feet. All these trusses were built by rule of thumb rather than by design based on a rational analysis.

The first all-metal trusses were built in the United States in 1840. In 1847, Squire Whipple published the first rational approach to the theory of structures and the analysis of trusses in the United States.

* This and subsequent numerals in parentheses refer to the list of references in the Selected Bibliography.

The general theory of three dimensional systems was first formulated by Mobius⁽⁴⁾ in 1837, but his work remained unknown to engineers, and the theory of space trusses was developed independently. August Föppl's book⁽⁵⁾ considered many important topics concerning space trusses for the first time (1892), and has been the basis of much of the later work in this field.

The method of plate analogy was first utilized by Schwyzer⁽⁶⁾ in his dissertation (1920). The work of Schwyzer was extended and published by Stüssi^(7, 8, 9). The method of plate analogy was summarized and published in the United States by Anderson and Nordby⁽¹⁰⁾.

In addition to this historical development, the method of plate analogy is briefly described by Niles and Newell⁽¹¹⁾ and Timoshenko and Young⁽¹²⁾. Also, Gillespie⁽³⁾ utilizes the method of plate analogy in his dissertation.

CHAPTER II

GENERAL THEORY

2.1 General

The justification of the method of plate analogy is presented in the order in which the analysis would be carried out in a typical problem. First, the stability of the structure must be considered. Then the structure must be separated into individual coplanar plate elements, and the effects on adjacent plate elements determined. After this, the loads on the structure must be resolved into loads in the plane of the plate elements. Finally, after the analysis of the planar trusses, the plate elements must be joined together again and the final axial forces in the members determined. At this point, with the unknown quantities and equations of equilibrium established, the determinancy of the method of analysis is discussed.

The usual assumptions, that the individual members of the structure transmit only axial forces, that all joints are frictionless spherical hinges, and that all loads are applied at the joints, are made in this investigation.

2.2 Stability Requirements

The first consideration is the stability of the entire structure. A typical roof composed of plate elements resting on walls, as in Fig. 2.1, is considered. For a vertical system of loads, the end ties will take

the horizontal thrust of the inclined trusses, and the horizontal reactions on the walls will be zero. However, under wind loading these reactions will exist, and the walls must be capable of transmitting the horizontal forces to the foundation.

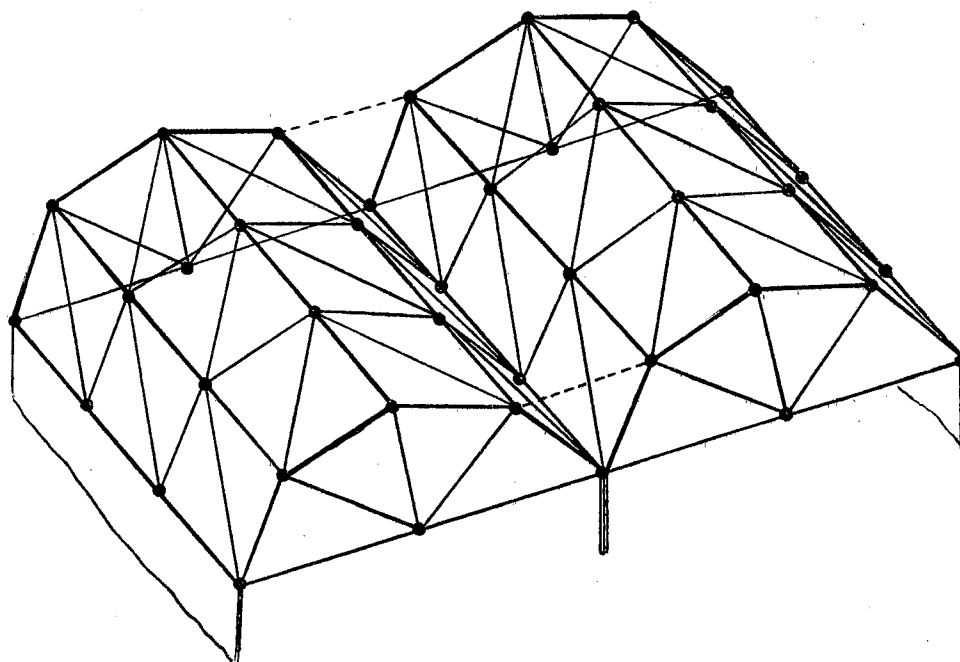


Fig. 2.1

Typical Roof Composed of Planar Trusses

The main requirement for stability in the vertical direction is that the center line of joints, or "valley" be restrained from moving. This may be accomplished by supporting the line of joints by the center posts as shown (Fig. 2.1). An alternative might be to fix the joints against rotation by providing ties, as shown by the dotted lines.

The second consideration is the stability of the individual planar trusses. When a coplanar truss is separated as a basic unit and the edge forces and loads applied to it, it is subject to the principles of coplanar truss analysis. If this coplanar truss is unstable in its own plane, then it is not rigid and its shape will be distorted. A simple example of this is shown in Fig. 2.2. The interior panel which does not have a diagonal member will not transmit shear and allows the distortion.

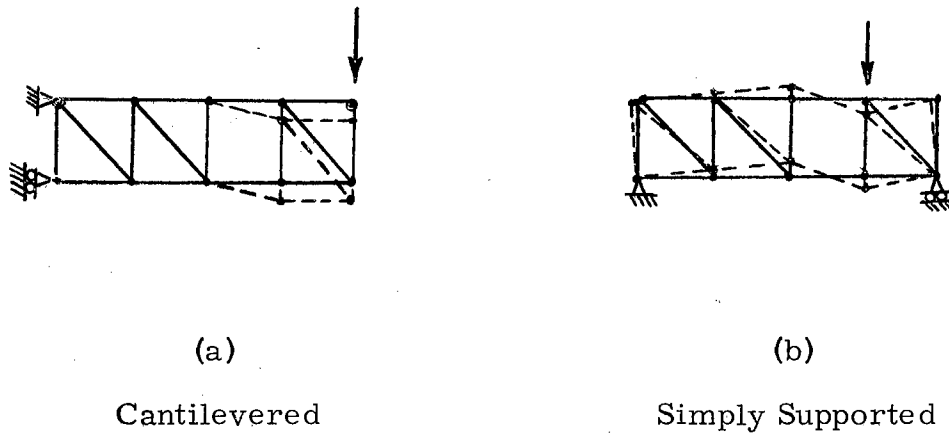


Fig. 2.2

Instability of Planar Truss

Regardless of the type of support, under any loading which causes a shearing force to exist at a panel which cannot resist shear, the planar truss will deform in order to resist the load. If the planar truss has to deform excessively to resist the load, it is a non-rigid form and is unstable.

Since the plate elements are planar trusses, they are subject to the same restrictions. However, in order for one plate element to distort, the plate elements adjacent to it must also distort in some manner

of compatibility. If two inclined stable trusses are joined together along a line of joints and both are supported on a rigid foundation, then the system as a whole cannot distort. Each coplanar truss is assumed to be flexible perpendicular to its own plane, but is prevented from movement in this direction by the adjacent trusses.

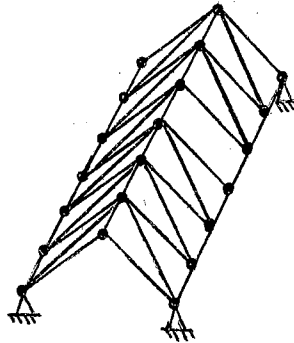


Fig. 2.3

Instability with Two Plate Elements

However, as in Fig. 2.3, if one of the trusses were unstable, it would not resist any out-of-plane movement by the other planar truss; and the structure, as a whole, would be unstable. In this case the stable planar truss could move in a direction perpendicular to its own plane and the unstable truss would not be able to resist this movement.

A roof structure composed of plate elements, one of which contains a section which will not resist shear in its own plane is considered (Fig. 2.4). The inclined truss 1-1, which is not rigid will allow the two lines of joints 1 to move in a direction perpendicular to the planes of trusses 1-2. The two plate elements to either side of the unstable inclined truss would not remain plane. If the displacement of the first lines of

joints, 1, are small, then the second lines of joints, 2, will not move.

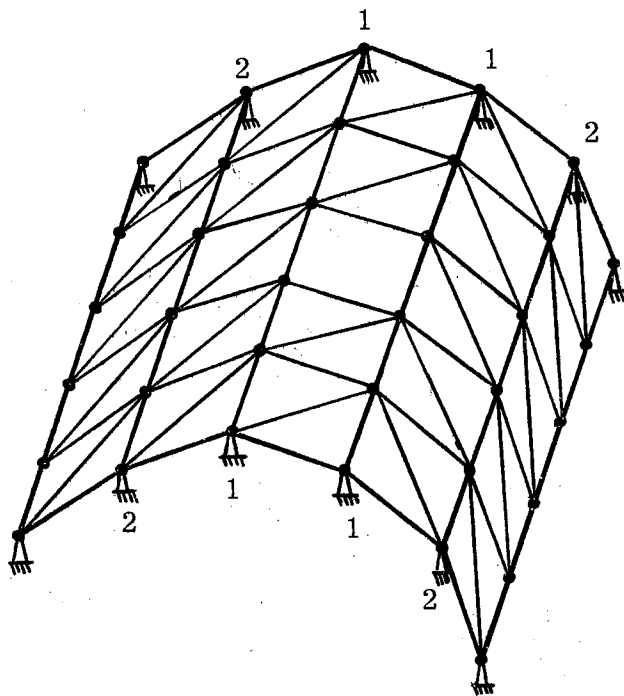


Fig. 2.4

Instability with Several Plate Elements

It may be concluded that if one plate element contains a section that will not transmit shear, it will distort and the two plate elements adjacent to it will not remain in one plane, but will be subject to out-of-plane deformations. The condition of deformation being that the line of joints on either side of the unstable truss must move perpendicular to the planes of the adjacent stable trusses and the unstable truss detrudes.

If an end plate element were to become unstable, the plate elements forming the roof would not offer any resistance to its distortion since each joint of the end truss corresponds to a line of joints in the roof.

2.3 Edge Forces

A roof formed of inclined planar trusses is shown in Fig. 2.1. In order to utilize the method of plate analogy in the analysis of this type of structure, the inclined trusses are separated from each other, or the structure is said to be "exploded", and each planar truss is treated as a plate element. In order to proceed in this manner, the forces exerted on one inclined truss by the inclined truss adjacent to it must be determined.

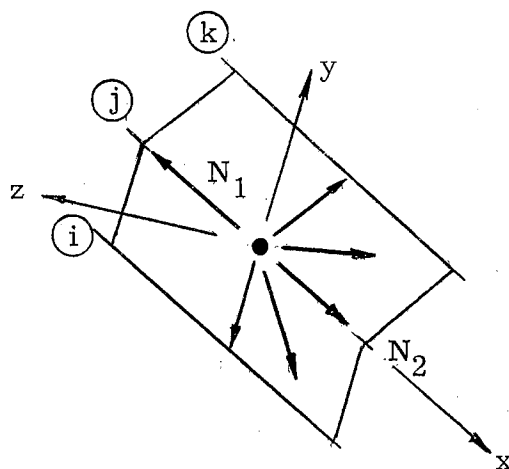


Fig. 2.5

Axial Forces at a Typical Joint

To illustrate this, any two adjacent inclined trusses ij and jk are considered. A typical joint along the line of joints j is removed (Fig. 2.5). The actual members framing into this joint are replaced by the axial forces they transmit. A coordinate system associated with the plane of truss ij is shown. The forces lie either in the plane of truss ij or jk . Forces N_1 and N_2 are common to both planes.

The basic requirement for equilibrium at a joint on any truss structure is:

$$\begin{aligned}\Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma F_z &= 0\end{aligned}\tag{2.1}$$

By considering equilibrium in the z-direction, it is evident that the resultant of all forces lying in the plane of truss jk , excluding N_1 and N_2 , must act in the x-direction. This resultant force is then an edge shearing force. Similarly, the influence of truss ij on truss jk is an edge shearing force and must be equal and opposite to the edge shear acting on truss ij .

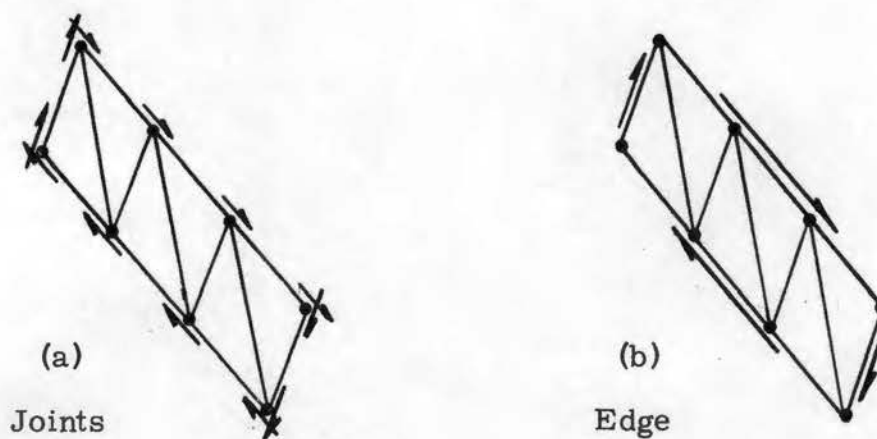


Fig. 2.6

Edge Shearing Forces

Taking truss ij as a plate element, the forces exerted on it by the adjacent trusses are as shown in Fig. 2.6(a). Since all these resultant forces have the same line of action, they may be considered as the edge shearing force acting on the plate element as in Fig. 2.6(b).

The interior framing in any of the inclined trusses does not affect the direction of the resultants.

A situation may exist where two plate elements are coplanar. An example of this is the end plate elements of the roof structure in Fig. 2.1. They have a common joint and a force in their plane may exist. The joint forces acting on these plate elements are shown in Fig. 2.7.

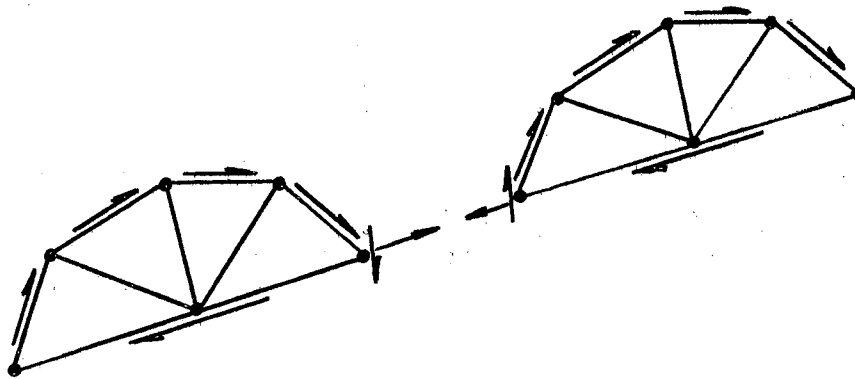


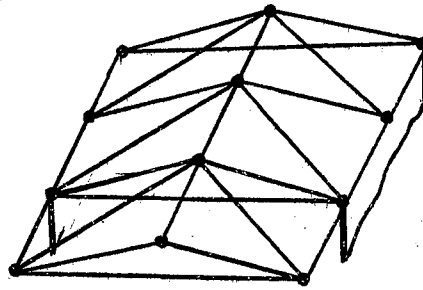
Fig. 2.7

Edge Forces on an End Plate Element

In addition to the end plate elements which are adjacent and coplanar, an overhang is another example in which joint forces may exist (Fig. 2.8).

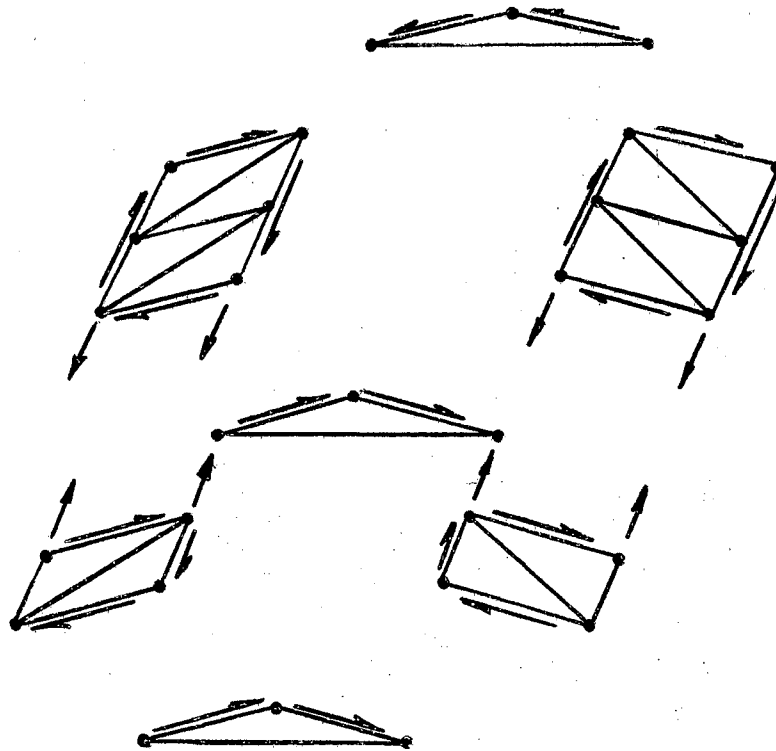
2.4 Loads

In the method of plate analogy, the planar trusses are separated from each other, loads and edge forces applied, and the analysis carried out as a two dimensional problem. As part of this process, the loads acting on the structure must be resolved into loads in the plane of the



(a)

Structure



(b)

Plate Elements

Fig. 2.8

Overhang

plate elements.

A typical joint is chosen from a line of joints j between two inclined trusses ij and jk . A load inclined at an angle, α_j , to the vertical is applied at this joint (Fig. 2.9).

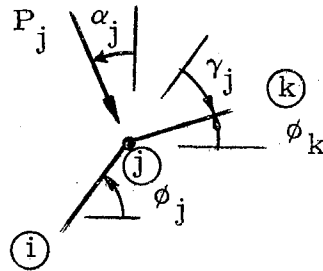


Fig. 2.9

Load Acting on a Typical Joint

The load P_j is resolved into components in the planes of the two inclined trusses by using the force polygon shown in Fig. 2.10. The values of the interior angles shown are:

$$\theta_1 = \phi_k + 90 - \alpha_j$$

$$\theta_2 = \gamma_j$$

$$\theta_3 = 90 + \alpha_j - \phi_j$$

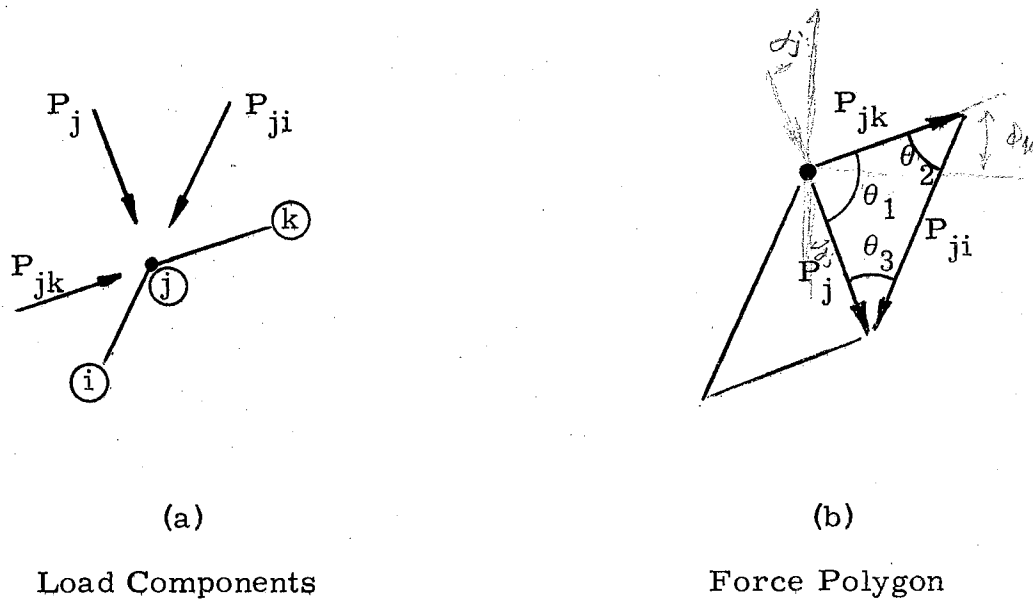


Fig. 2.10

Resolution of Loads

Then using the law of sines, the resolved loads are:

$$\begin{aligned}
 P_{ji} &= \frac{\sin \theta_1}{\sin \theta_2} P_j & P_{jk} &= \frac{\cos(\phi_k - \alpha_j)}{\sin \gamma_j} P_j \\
 P_{jk} &= \frac{\sin \theta_3}{\sin \theta_2} P_j & P_{ji} &= \frac{\cos(\phi_j - \alpha_j)}{\sin \gamma_j} P_j
 \end{aligned}
 \tag{2.2}$$

In the special case of a symmetrical roof composed of two inclined trusses with a vertical load, then;

$$\gamma_j = 2\phi_j$$

$$\alpha_j = 0$$

Then the loads become,

$$P_{ij} = P_{jk} = \frac{\cos \phi_i}{\sin 2\phi_i} P_j = \frac{P_j}{2 \sin \phi_i} \quad (2.3)$$

A typical plate element with all resultants acting on it is shown in Fig. 2.11. The superscript denotes the point at which the load acts.

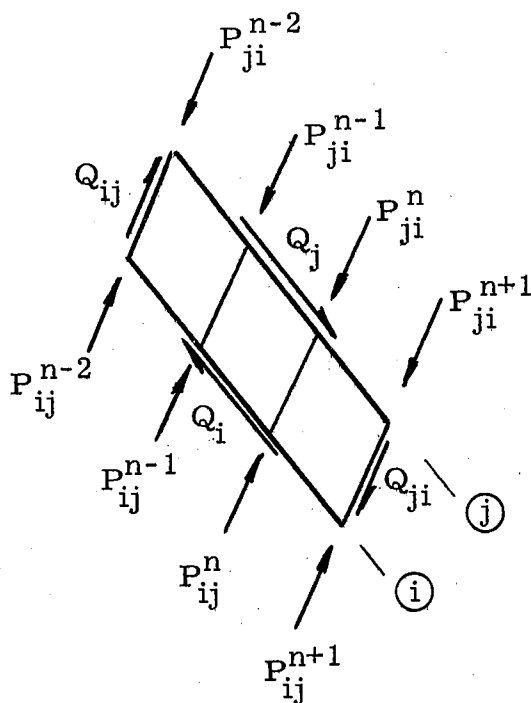


Fig. 2.11

Loads on a Typical Plate Element

2.5 Final Axial Forces

At this step in the process of analysis, the edge forces and loads acting on the plate elements have been calculated, and the axial forces in the members must be determined.

In Section 2.3, it was shown that an edge shearing force exists at each joint and, for purposes of equilibrium, can be considered as a single edge shear. It is this total edge shearing force that is now known and it must be applied somehow to the planar truss.

A force may be considered to act anywhere along its line of action to determine the conditions of equilibrium for a body, however, the point of application of the force affects the internal forces and stresses in the body.

The first consideration is the entire structure as the body and the edge shears as internal forces. In any body the internal forces act at a point and are equal and opposite. Therefore, the edge shearing force between adjacent plate elements must act at the same joint on the plate elements or the effect would be the same as an external load.

The second consideration is the plate element as a body and the axial forces of the members as internal forces. In this case, the edge shear may be applied to any joint along its line of action. The location of the point of application affects only the force in the members associated with the edge shear, i. e., the members common to adjacent plate elements.

This can be seen by taking a section normal to the line of action of the shearing force. If an end plate element is taken as an example because of its irregular shape, the edge shears are as shown in Fig. 2.12. If a section normal to the line of action of Q_3 is taken, Q_3 does not affect the shear on this section. The points of application of the other edge shears, except Q_5 , do not affect the shear on this section. There will be different values for the shear on this section corresponding to different points of application of Q_5 , but these differences will be balanced by different values for the axial force in the member associated with Q_5 .

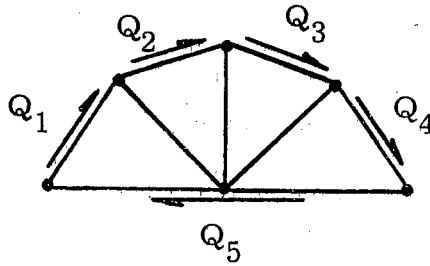


Fig. 2.12

Edge Shears on a Plate Element

Thus, it can be concluded that the axial forces in the interior members are independent of the points of application of the edge shears. This is readily apparent in a plate element in which the top and bottom chords are parallel.

Since the member which is common to the adjacent plate elements is considered as a part of each plate element, there will be a value of the axial force in this member for each plate element. The interior members of the plate element have the same axial forces as if they had been analyzed as a part of the three-dimension structure without the use of the plate analogy.

Thus, the only axial force remaining to be determined is the axial force in the member along which the edge shear acts. To maintain equilibrium with the interior bars, the axial force in this member must be the algebraic sum of the axial forces determined when the member is considered as a part of each plate element.

In this manner, the axial forces in all bars may be found. The determination of the axial forces in the members in each plate element

is by any conventional two-dimensional analysis. In this discussion, it is assumed that the plate elements are internally determinate. If they are internally indeterminate, the same procedure applies but further development must be made.

2.6 Determinancy

Once the influence of the plate elements on one another has been established, the determinancy of the method of analysis may be considered. The unknowns are the edge forces acting on the plate elements and the external reactions. Since the unknowns are taken as the edge forces and not the axial forces of the truss members, this discussion considers only the determinancy of the analysis by the method of plate analogy.

The first consideration is to establish the number of equations available. The plate elements are two dimensional trusses, therefore, there are three equations of statics available for each plate element. Also, there is an equality of the edge forces at a common edge.

Since the entire structure is in three dimensions, there will be six equations of statics for it. Also there are special conditions available when the summation of moments about a line of joints or "valley" must be equal to zero. However, these relationships are contained in the equations for the plate elements.

Thus a general expression may be written for the total number of equations available.

$$n = 3p + q \quad (2.4)$$

where:

$$n = \text{total number of equations available}$$

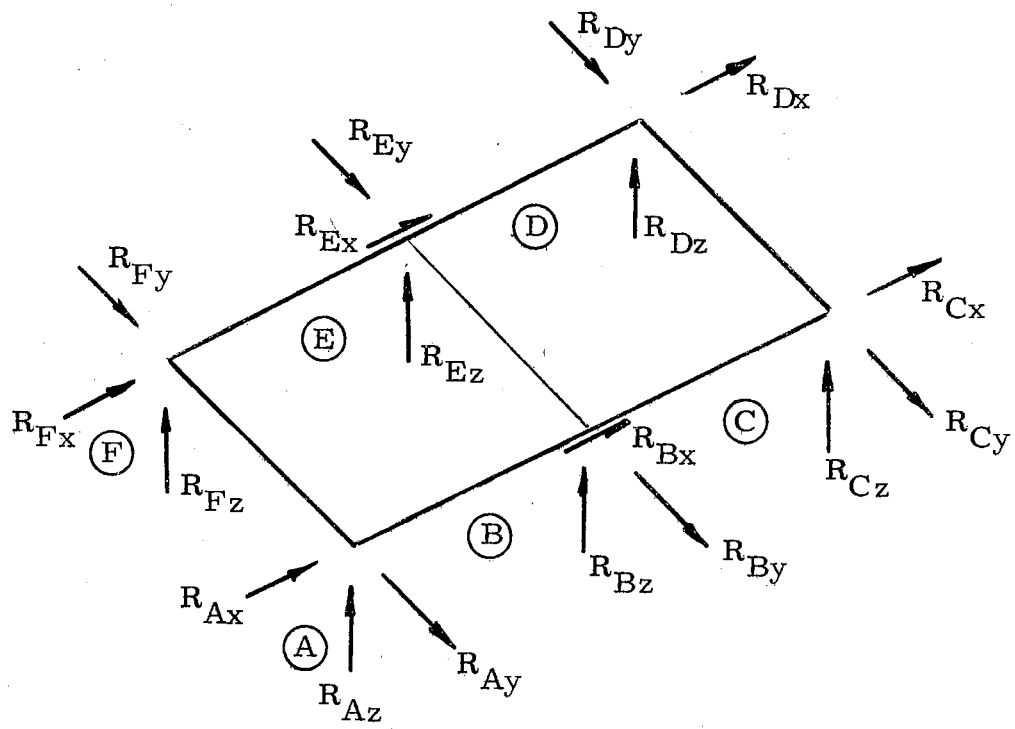
p = number of plate elements

q = number of common forces.

The second consideration in this discussion is to establish the number of unknowns. One group of unknowns is the edge forces. A second group is the external reactions. In the most general case there could be three reactions at each point of support. The outline of the base of a two span structure is shown in Fig. 2.13. The center posts are assumed to support a "valley" or line of joints which require support for stability. The roof framing may be general and the general set of reactions is shown in Fig. 2.13 (a). Each additional span required supports at the "valley", thus introducing six more reactions.

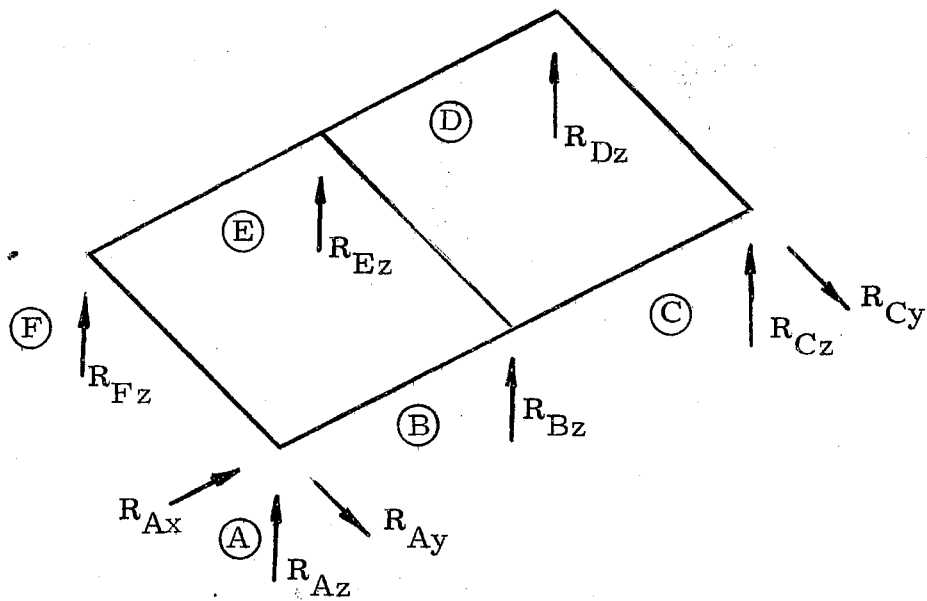
Due to the construction of the supports and the actual assumptions, the number of external reactions would probably be reduced. The first assumption that could be made is that the center posts resist only axial forces and do not provide horizontal reactions. Another simplification is that due to construction of the supports, certain horizontal reactions can be eliminated. The minimum number of external reactions normally provided is shown in Fig. 2.13 (b).

To establish the determinacy of a structure composed of planar trusses, it is necessary to separate the plate elements, apply all edge forces and reactions and determine the total number of unknowns. This must be equal to the number of equations available.



(a)

General



(b)

Minimum Required for Stability

Fig. 2.13

Reactions of a Typical Structure

CHAPTER III

INTERNAL AND EXTERNAL REDUNDANTS

3.1 General

The discussion in Chapter II assumed the structure to be internally and externally determinate. For a structure which is internally or externally indeterminate, the same procedure applies, but some compatibility conditions must be incorporated in the analysis.

3.2 Internal Redundants

Since the edge shears are independent of the arrangement of the bars in a plate element, the edge shearing forces will have the same value, under a given loading, regardless of the internal determinacy. However, if there is a redundant member in a plate element, a condition of compatibility must be satisfied. This condition is that the displacement of the joints which the redundant member connects, in the direction of the axis of the redundant member, must equal the axial deformation of the redundant member.

As an illustration, a structure with a redundant member in one panel of one inclined truss is considered (Fig. 3.1). The method of virtual work is used to evaluate the necessary deformations. First, a basic structure is selected with the redundant member cut and the loads are applied to it. All axial forces in the members of this basic structure are determined as described before. It should be noted that the

axial forces are those obtained after the plate elements have been joined together again.

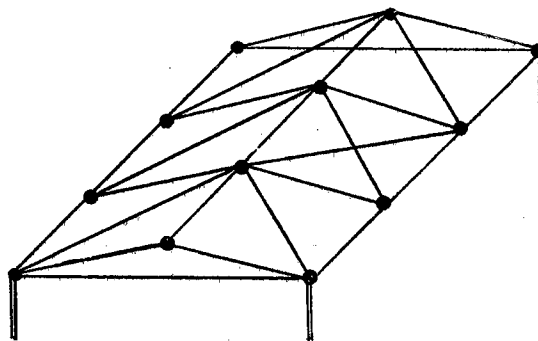


Fig. 3.1

Internally Indeterminate Structure

Next, the basic structure is loaded by the unknown redundant force equal to unity. The plate elements with the unit redundant load applied are shown in Fig. 3.2. The unit redundant load will not cause edge shears and causes only axial forces to exist in the panel in which it acts. For this reason, only that plate element need be considered in the calculation of the axial forces due to the unit redundant load.

The virtual work expression for the compatibility condition is given by Eq. 3.1, and the final axial forces by Eq. 3.2.

$$X = -\frac{a_{10}}{a_{11}} = -\frac{\sum B N_m n_m \lambda_m}{\sum n_m^2 \lambda_m} \quad (3.1)$$

$$N_m = B N_m + X n_m \quad (3.2)$$

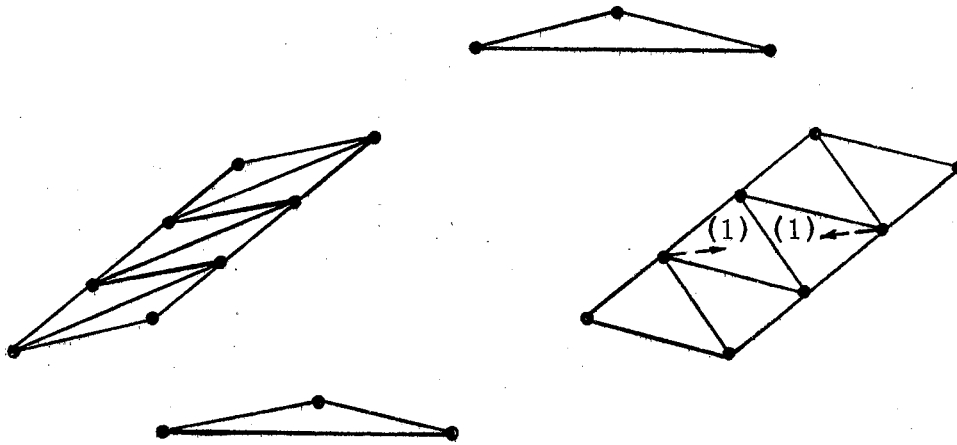


Fig. 3.2

Basic Structure with Unit Redundant Loads

where

X = redundant axial force

a_{10} = displacement between sections at the cut in the redundant member due to loads

a_{11} = displacement between sections at the cut in the redundant member due to $X = +1$

BN_m = axial force in member m of the basic structure due to loads

n_m = axial force in member m due to $X = +1$

$\lambda_m = \frac{L_m}{A_m E}$ = axial extensibility of member m

N_m = final axial force in member m .

The basic axial force n_m is zero for all members except those in the panel of the redundant member. Thus if there are two redundant members in panels that are not adjacent, two independent equations for the redundants will result. If the panels are adjacent, the redundants will influence each other and simultaneous equations will be obtained.

3.3 External Redundants

For a structure which is externally indeterminate, the total number of edge forces and external reactions is greater than the total number of equations of statics available. The additional equations must be obtained from conditions of deformation.

A structure is shown in Fig. 3.3, in which all plate elements are internally determinate. The redundants in this case are the horizontal thrusts of the end plate elements. A basic structure is chosen with one support on rollers, so that there is no horizontal reaction. All axial forces in the basic structure due to loads are then determined by the procedure described earlier.

Next the basic structure is loaded by the redundant horizontal thrust equal to unity and all axial forces determined. The unit redundant thrust causes axial forces only in the end plate element. The solution for the redundant horizontal thrust can be obtained by Eq. 3.1, where X is the redundant horizontal thrust. Again, Eq. 3.2 gives the final axial forces in the structure.

If the structure is both internally and externally indeterminate, then the basic structure for computing the external redundants is internally indeterminate. If the end plate element is internally indeterminate, then the internal and external redundants influence each other.

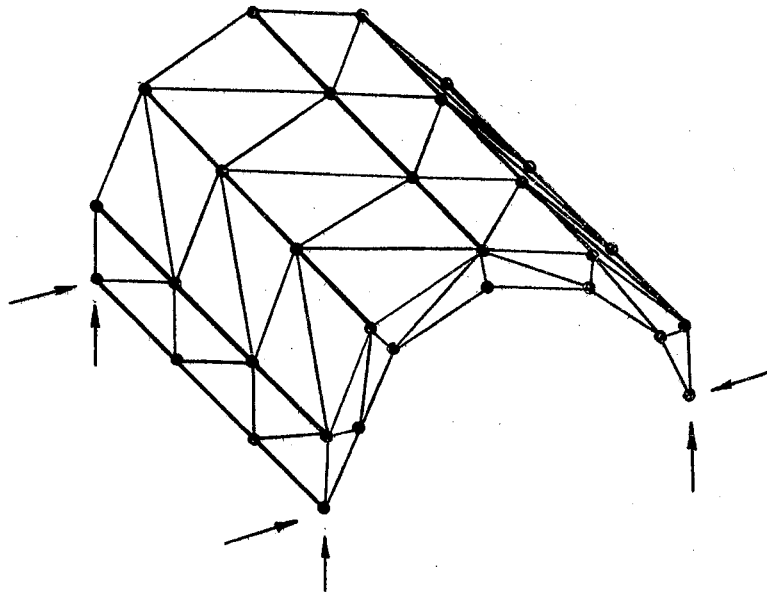


Fig. 3.3

Externally Indeterminate Structure

If a redundant panel in a roof plate element is adjacent to a redundant panel in the end plate element, the redundant forces influence each other. Then when the unit redundant thrust is applied to the basic structure, the members in the redundant panel in the roof truss will have axial forces.

If a roof plate element has redundant members in all panels, then all panels will be influenced by the redundant horizontal thrust. However, if there is one panel that is internally determinate, then the influence will only carry to that point (Fig. 3.4).

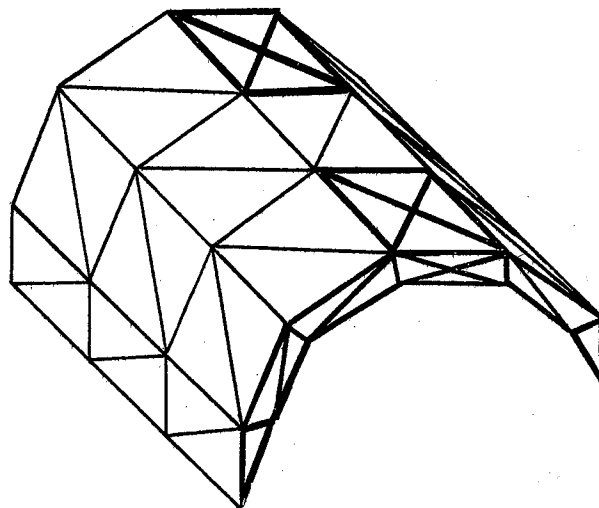


Fig. 3.4

Influence of Internal Redundants

CHAPTER IV

APPLICATION

4.1 General

The two span truss-plate structure shown in Fig. 4.1 is analyzed for a uniformly distributed load. The overhang, two end plate elements and redundant member illustrate topics discussed in Chapters II and III.

The center supports are assumed to provide vertical reactions only. The external reactions are shown in the figure.

The defining geometry of the structure is as follows:

slope length = 15.81 ft.

$\sin \phi_2 = \sin \phi_4 = .316$

length of diagonals = 21.79 ft.

The structure is assumed to support a uniformly distributed load of 75 psf. The concentrated loads applied at the joints and the loads applied to the plate elements are listed in Table 4.1.

TABLE 4.1 LOADS

	P_j	P_{ji}
Interior Joint	16.90 ^k	26.75 ^k
Edge Joint	8.45 ^k	13.38 ^k
Corner Joint	4.23 ^k	-

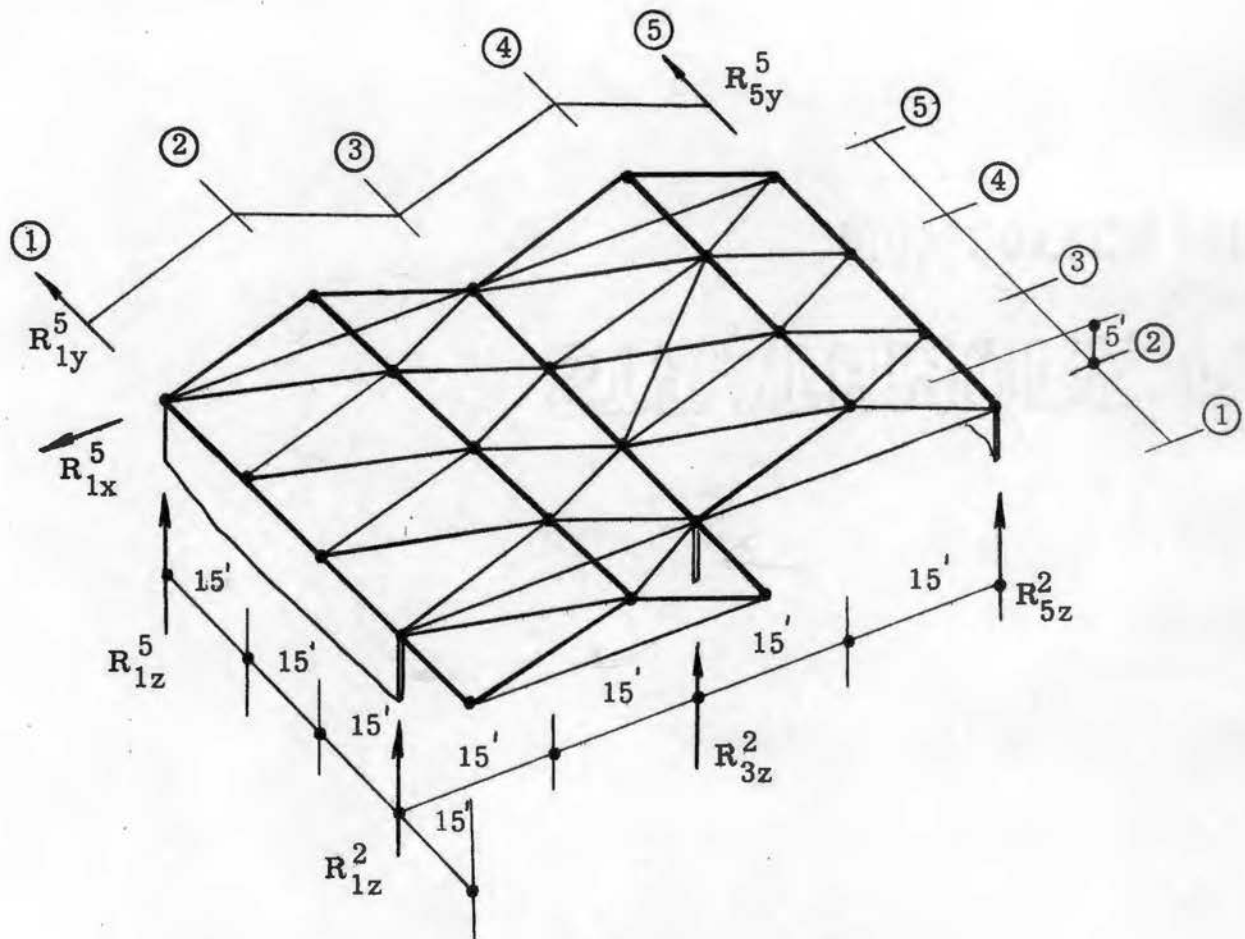


Fig. 4.1

Two Span Truss-Plate Structure

4.2 Edge Forces

The plate elements are separated and loads and edge forces applied in Fig. 4.2. There are nine external reactions. Also, there are sixteen unknown edge shears and eight unknown joint forces. Thus, the total number of unknowns is 33. There are eleven plate elements, therefore, the total number of equations available is:

$$n = 3p = 33.$$

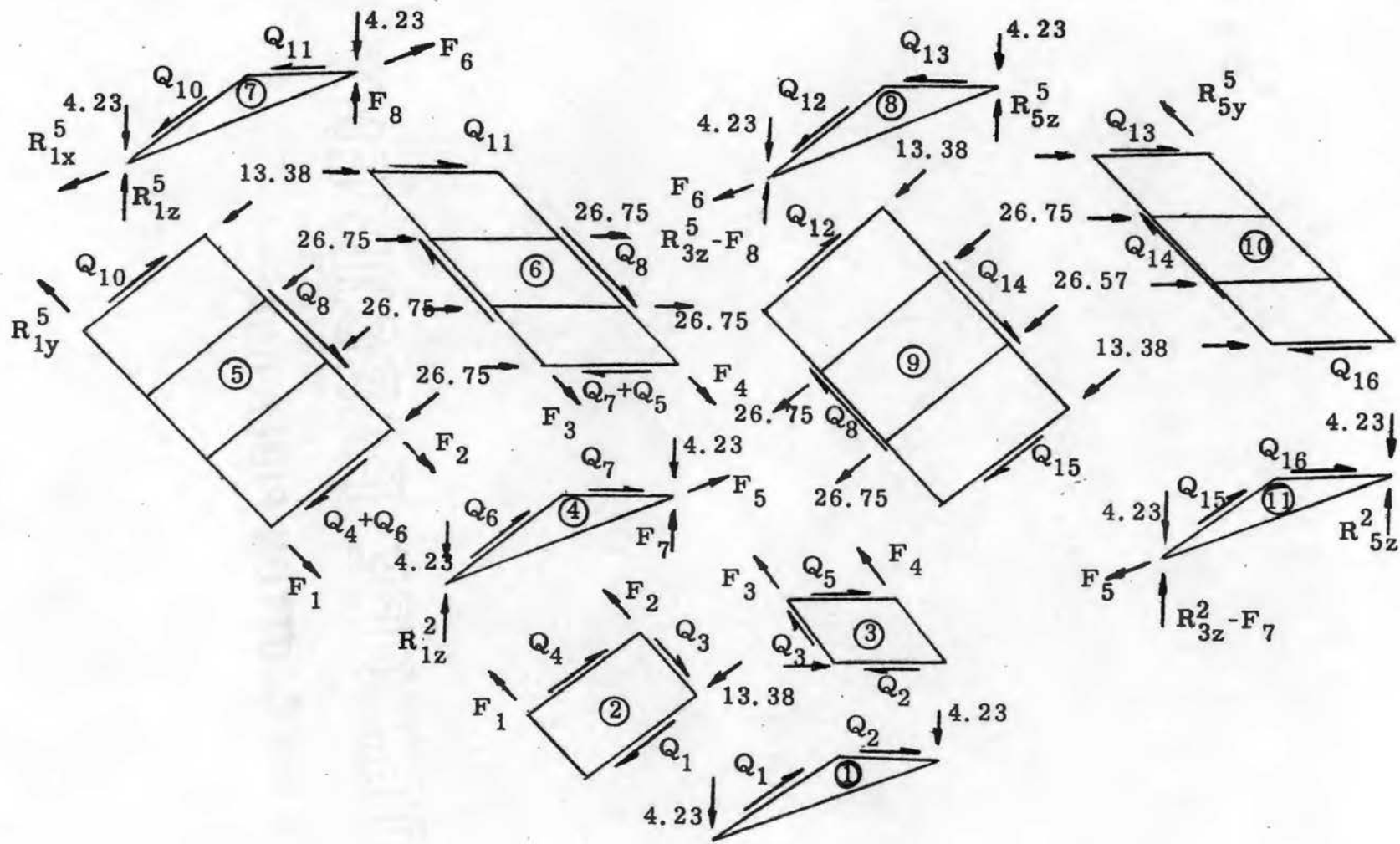


Fig. 4.2

Two Span Truss-Plate Structure

The equality of forces at a common edge was incorporated in the notation of the sketch. Thus the structure is externally determinate. However, the X-bracing in one plate element makes the structure internally indeterminate.

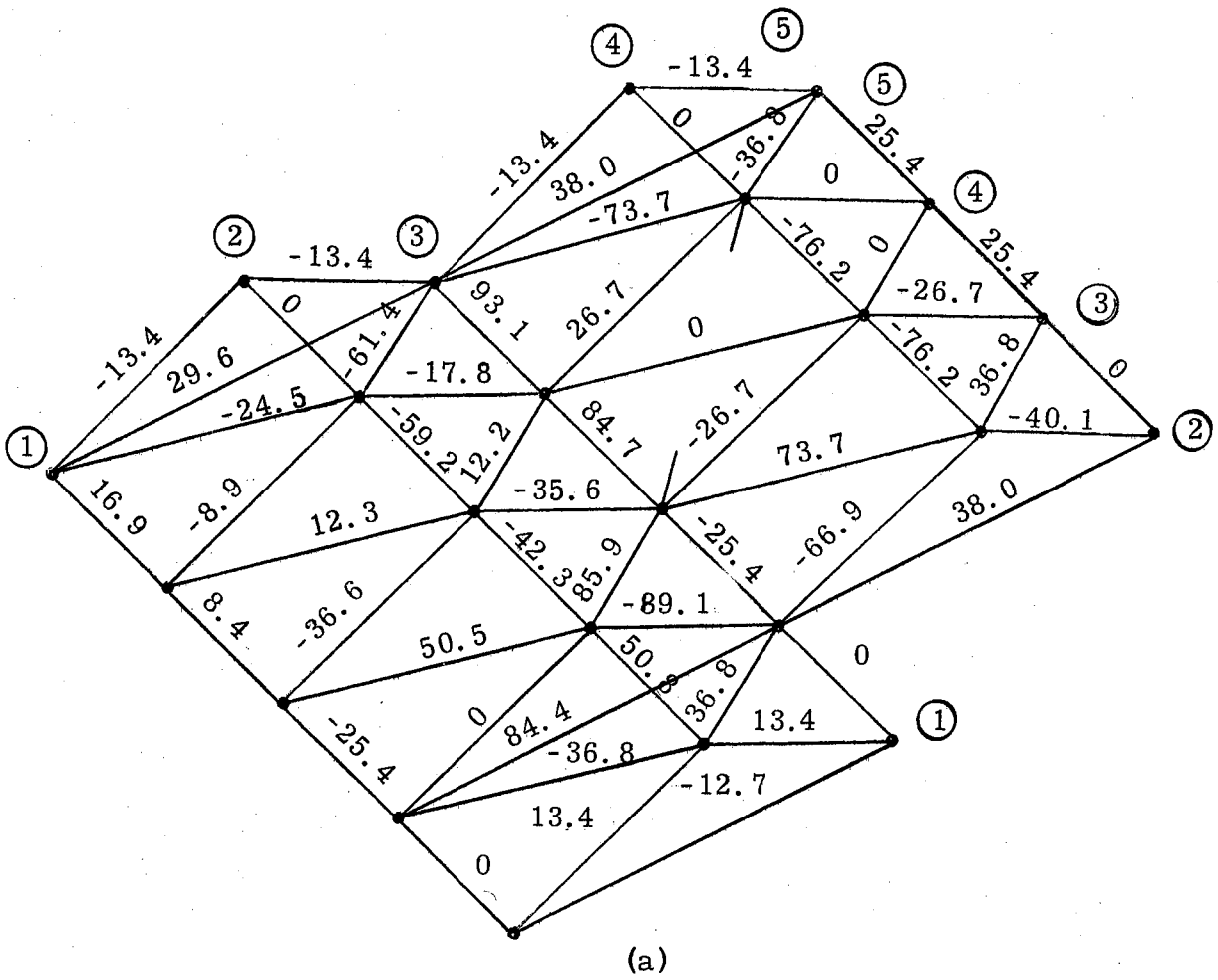
The unknown reactions and edge forces were found by applying the equations of statics to the plate elements starting with elements 1 and 10. The results are listed in Table 4.2.

TABLE 4.2 EDGE FORCES AND REACTIONS

Q_1	13.4	Q_{14}	0	R_{3z}^2	70.5	Q_{10}	31.2
Q_2	-13.4	R_{5y}^5	0	R_{5z}^2	16.9	Q_6	-89.0
Q_4	26.8	Q_{16}	40.1	F_6	- 25.4	F_2+Q_9	25.4
F_1	-25.4	Q_{13}	-40.1	R_{3z}^5	48.0	F_8	22.6
F_2-Q_3	25.4	Q_8	0	R_{5z}^5	16.9	R_{1z}^5	14.1
Q_5	-26.8	Q_{12}	66.9	Q_{11}	- 58.0	F_7	45.1
F_4	-25.4	Q_{15}	-66.9	Q_7	115.9	R_{1z}^2	36.6
F_3+Q_3	25.4	F_5	-25.4	F_3-Q_9	25.4		

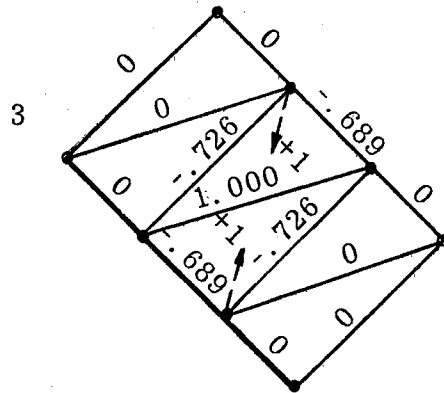
4.3 Axial Forces

In order to solve for the final axial forces, the redundant member must be cut and the axial forces in the basic structure determined. The axial forces of the basic structure under loads are shown in Fig. 4.3(a). The axial forces in the redundant panel due to a unit redundant load are shown in Fig. 4.3(b).



Due to Loads

4



Due to Unit Redundant

Fig. 4.3

Axial Forces in Basic Structure

Assuming the members which are stressed by the unit redundant load have the same cross-sectional area and modulus of elasticity, the redundant is found using Eq. 3.1. The calculations are tabulated in Table 4.3.

TABLE 4.3 CALCULATION OF REDUNDANT

m	L_m	n_m	BN_m	$BN_m n_m L_m$	$n_m^2 L_m$
1	15.81	- .726	26.7	-307	8.35
2	15.81	- .726	26.7	307	8.35
3	15.00	- .689	-76.2	786	7.13
4	15.00	- .689	84.7	-874	7.13
5	21.79	1.000	0	0	21.79
6	21.79	1.000	0	0	21.79
Σ				- 88	37.27

Then

$$X = - \frac{-88}{37.27} = 2.36$$

Using Eq. 3.2, the final axial forces are shown in Fig. 4.4.

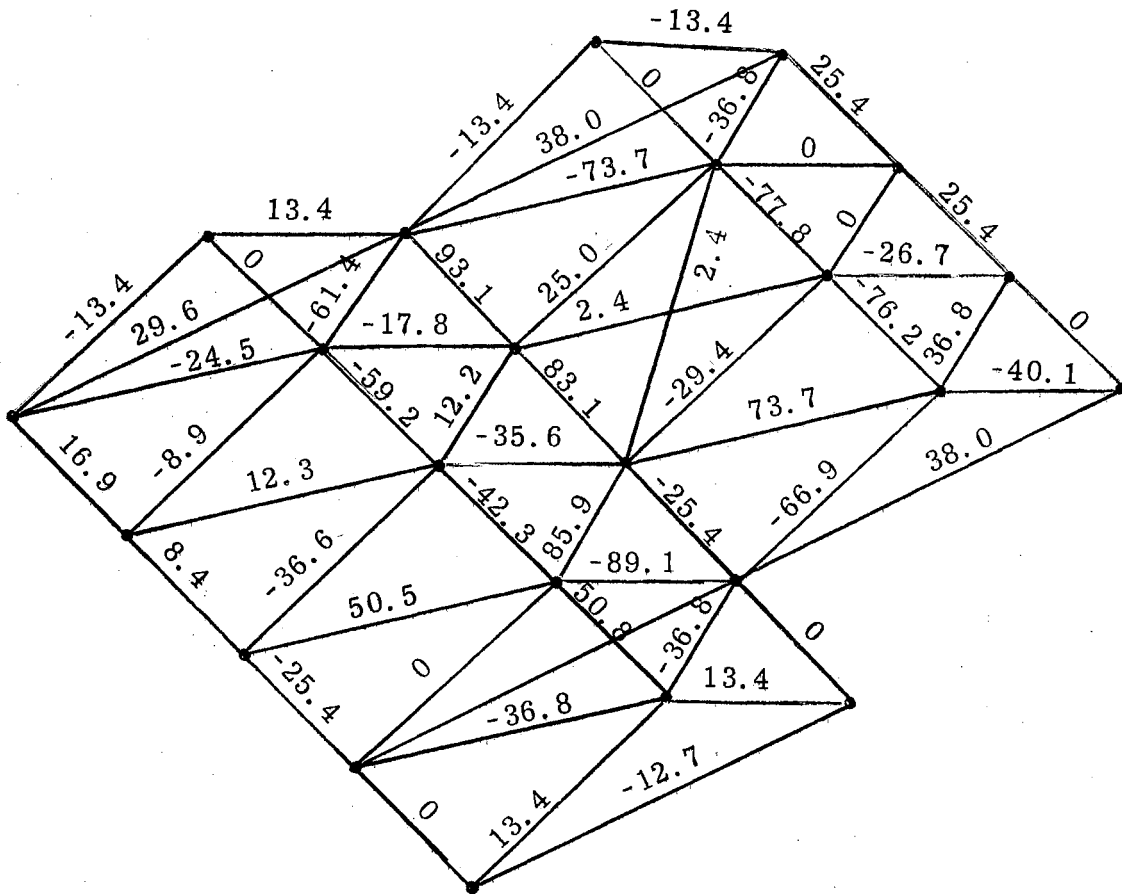


Fig. 4.4

Final Axial Forces

CHAPTER V

SUMMARY AND CONCLUSIONS

5.1 Summary

The method of plate analogy in the analysis of roof structures composed of planar truss elements was presented in this thesis. By considering the equilibrium of a typical joint, the influence of adjacent inclined trusses on each other was established. Also, the proportion of the load at a common joint which is resisted by each planar truss was determined. Then it was shown that the plate elements may be separated, loads and edge forces applied, and the planar trusses analyzed as a two dimensional problem. The manner in which the known edge shears must be applied to the plate elements to determine the axial member forces was shown. The condition for statical determinacy was discussed, and a procedure for the evaluation of internal and external redundants was outlined.

5.2 Conclusions

The method of plate analogy reduces the analysis of three dimensional structures composed of planar trusses to a two dimensional problem. Because the equations used in solving for the edge forces are equations of statics, the concept involved in this method is elementary. Since many of the plate elements in a structure are of the same or similar shape, the process of solving the equations of equilibrium becomes systematic.

In cases where joint forces exist, an unknown joint force may be combined with an edge shear if they are colinear. Then when the axial forces in the members are determined, the edge shear is applied at the joint acted upon by the joint force.

With a clear system of notation, the method of plate analogy provides a simplified method of analysis of three dimensional structures composed of planar trusses.

5.3 Extension

The material presented in this theses can be directly extended to more complex and involved forms of truss structures. The analysis of truss domes could be investigated. The method of plate analogy might be extended to include a more general and complete study of internally and externally redundant structures.

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VITA

Carl Clayton Holloway, Jr.

Candidate for the Degree of

Master of Science

Thesis: ANALYSIS OF TRUSS PLATE STRUCTURES BY
PLATE ANALOGY

Major Field: Civil Engineering

Biographical:

Personal Data: Born August 6, 1939, in Richland, Nebraska,
the son of Carl C. and Helen M. Holloway.

Education: Graduated from Kearney High School, Kearney,
Nebraska in May, 1957. Received the degree of Bachelor
of Science in Civil Engineering from Oklahoma State Univer-
sity in May, 1962. Member of Sigma Tau and Chi Epsilon.
Completed requirements for the degree of Master of Science
in August, 1963.