THE DEVELOPMENT OF NUMBER CONCEPTS

IN PRESCHOOL CHILDREN

Thesis Approved:

Frances Stromberg
Thesis Adviser

Josephine Hoffer

Dean of the Graduate College

ACKNOWLEDGMENTS

I wish to take this opportunity to express appreciation to my adviser, Dr. Frances Stromberg, for her interest, encouragement, and assistance throughout this study. I wish to thank the other members of my committee, Dr. Josephine Hoffer and Dr. James Walters, for their critical reading of the manuscript and for their helpful suggestions. Special appreciation is given to Mrs. Judy Powell for her personal interest and assistance.

I am sincerely appreciative of the support given throughout my graduate studies by the members of the faculty in the Department of Family Relations and Child Development. Appreciation is expressed to all the nursery school teachers and children for their cooperation while data were being collected.

Finally, I am extremely grateful to my husband, Jim. He has provided me with supportive understanding and encouragement thoughout this study.

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CHAPTER I

INTRODUCTION

Problem

Research studies (Dutton, 1963; Josephina, 1965; Williams, 1965; Paschal, 1967; Heimgarter, 1968; and Suydam and Riedesel, 1969) indicate that young children display an early interest in numbers, yet by the time these children reach adulthood many will have limited mathematical ability and poor attitudes towards mathematics. Due to the coming of the Space Age and the increased need for scientists and technicians there has been an increased emphasis on cognitive development, including the development of number concepts, reaching down even to the preschool age child. Therefore, it seems increasingly important that parents and teachers learn more about the growth of mathematical understandings in order to help young children develop basic number concepts and positive attitudes toward mathematics.

Both Josephina (1965) and Paschal (1967) agree that the preschool age child possesses quantitative ability to a degree which needs the attention of teachers and other educators involved in planning early childhood education curricula. Josephina (1965) and Burston (1966) observed that much of children's arithmetical knowledge is learned through incidental experiences and that these early mathematical experiences in the preschool build toward more complicated concepts later in the child's life. However, Josephina (1965), Bravo (1965), Burston (1966), and Paschal (1967) believe that teachers need to build upon this foundation of early mathematical experiences in order to add to children's basic knowledge of number and quantity. Too often teachers wait for children's mathematical readiness to manifest itself instead of planning experiences to encourage interest in mathematical concepts. Specific planning by teachers of informal number experiences is necessary in order for children to benefit from activities that build upon each other. If a teacher is to plan well she must be aware of how children develop skills and mathematical concepts.

Piaget's (Flavell, 1963) fundamental thesis is that intellectual growth takes place in a succession of stages in all children. If a child has not yet reached a certain level of understanding, it would be meaningless for him to go on to higher levels. In order to help a child develop number concepts he should be given additional activities at the level of understanding which he has attained (Flavell, 1963). There is, however, a lack of agreement concerning how and when children develop number concepts and whether there is a developmental, sequential pattern which children follow in acquiring the concept of number. According to

Russel (1956) most children appear to go through a somewhat similar sequence in the development of their mathematical ideas, although there are wide individual differences at any one age level. Suppes (1966) states that sequential ideas are essential to learning, and learning a concept depends more on previous experience and training than on the concept itself. Bidwell (1969), Gagne (1965), and Suppes (1966) agree that mathematics has a clear structure of ideas or concepts and one must learn dependent ideas before he can learn a new idea. Wohlwill (1960) found that children go through a developmental process in arriving at an abstract concept of number, and they mastered the problems in his study in an ordered developmental sequence. The results of a study by D'Mello and Willemsen (1969) indicate that there is a sequence in the order in which certain mathematical skills develop, and that these skills may be scaled so that mastery of the skills involved in one task presupposes success on all preceding tasks. Suppes (1962) found, however, that young children's learning tends to be very specific, and that prior training on one concept; i.e., order, identity, or equipollence, did not improve learning on a second of these concepts.

There is a need for further research concerning the question of whether there is a sequence in the order in which preschool children develop number concepts, and there is also a need for further information regarding what factors may be related to or influence the development of the concept of number.

Purposes of Study

The major purpose of this research was to explore the possibility of there being an order in which preschool children develop number concepts which would be scalable according to Guttman's criteria as discussed by Green (1954). The five tasks utilized required the children to (a) count or recite number words in sequence, (b) match visual arrays of similar objects according to visually perceived equality of quantity, (c) match spoken number words to absolute quantity, (d) match visual symbols (numerals) with absolute quantity, and (e) order a group of objects from the smallest number to the largest number. In addition, the researcher proposed to compare responses of three-, four-, and five-year-old boys and girls to determine if an order exists which is similar for different age groups and for both boys and girls. Another purpose was to investigate whether number concepts are generalized to different situations or related only to specific situations. Also, this investigation proposed to consider whether age or sex are related to patterns of responses.

The specific hypotheses to be tested in this study were:

1) There is an expected order in which children develop number concepts.

a. The order is the same for girls and boys.

b. The order is the same for each age group.

2) The concepts reflected in children's responses to the number tasks using dominoes will be generalized to other situations; i.e., the number story.

3) Older children will pass more number tasks than younger children.

4) There will be no differences between responses of boys and girls of similar ages to the number tasks.

CHAPTER II

REVIEW OF THE LITERATURE

The review of literature will include (1) a definition of a concept and a short explanation of mathematical concept development, (2) literature based on common observation or informal evidence, and (3) research findings pertaining to number concept development.

Concept Development

According to Lovell (1961) there is a sequence in concept development from perception to abstraction to generalization. He defines a concept as

> ... a generalization about data which are related; it enables one to respond to, or think about, specific stimuli or percepts in a particular way. Hence a concept is an act of judgment (page 13).

Gagne (1965) describes concept learning as a common response to a class of stimuli which determines the concept to be learned.

Mathematical concepts are generalizations about certain kinds of data and are one class of concepts. Numbers are only one part of mathematical concepts. A number is an entity in itself; however, a set of numbers becomes a mathematical system when operations are defined on the set and the laws these operations obey are listed (Dean, 1960).

Ultimately, the relevant mathematical concept has to exist as an abstract concept before it can become fully operational. The ability to maneuver concepts in the mind is built from using concrete materials; however, the concepts are, themselves, independent of the actual materials used (Lovell, 1961). According to Lovell (1961) concepts develop slowly, not in an "all or none" fashion, and at first are very vague. With maturation they gradually grow in clarity, breadth and depth. Children may have developed a concept sufficiently for working purposes but may not be able to verbalize the concept. On the other hand, they may often use the appropriate term and yet have little understanding of the related concept (Lovell, 1961; Deal, 1968). Suppes (1966) believes that children develop simple mathematical concepts in approximately an "all or none" fashion. He speculates that new concepts are formed by random choice and are not formed out of old concepts as many educators believe.

Observations and Informal Evidence

In the past decade there have been many articles in textbooks and professional journals concerning the development of number concepts among preschool children. Much of this literature is based on observation or informal evidence and not research; however, it seems relevant to the present study, because it illustrates what teachers and parents have been reading concerning the concepts and skills which young children are developing. Therefore, a brief summary of this type of literature is included.

Recently, many new mathematical programs such as the Greater Cleveland Mathematics Program, Minnesota Mathematics and Science Teaching Project, and School Mathematics Study Group have been developed. In these programs the logical, sequential character of mathematics is stressed. These groups also emphasize the importance of teaching mathematics in a well-defined sequence. Glennon (1958) stresses the application of topical sequence, and Paschal (1967) states that the order in which children may best learn mathematical concepts is to some degree inherent in mathematics itself as a logical organization of ideas and relationships. Robison and Spodek (1965) made the following statement regarding the need to teach mathematical concepts in sequence.

> While no solid research supports the need to teach in sequence in order that the sequential character of the discipline be understood, the sheer logic of the position tends to support this assumption (page 108).

Discussion by educators suggests that preschool children have many number experiences through incidental contact in everyday life before they enter school. Bravo (1965) and Burston (1966) indicated that these early mathematical experiences and opportunities to manipulate concrete materials help the child build toward more complicated concepts later. Josephina (1965) concluded from a study of preschool children that since the children had some knowledge of number concepts and yet had not been taught formally that they gained their knowledge through incidental experience. Suppes (1962) found that incidental learning did not appear to be an effective method for kindergarten children in acquiring mathematical concepts after they were attending school. Other educators agree that experiences should be planned by the teacher, although the experiences may be very informal (Todd and Heffernan, 1968; Leeper et al., 1968; and Robison and Spodek, 1965).

Many authors attempted to identify the basic mathematical understandings or concepts which preschool children are able to develop (Ashlock, 1966, 1967; Deans, 1954; Todd and Heffernan, 1968; Robinson and Spodek, 1965; and Leeper et al., 1968). Some of the number concepts which these authors identified as important were sets and simple relations concerning sets, one-to-one correspondence, more or less than, counting and enumeration, cardinal number, ordinal number, beginning fractions, and seriation.

Number Concept Development

Studies by many authors indicated that the child's ability to count is not a reliable criterion of the extent to which he has developed the true concept of number. Number becomes a part of the child's repetitive language long before the true concepts for the number are meaningful to him (Deans, 1954). Brace (1965) studied 124 five- and six-year-old children's understanding of the concept of number as revealed by their manipulation of objects, rather than by their verbalization of numbers, names and combinations. He found that the large majority could count beyond twenty and yet still had almost complete lack of knowledge of the numeration system. There was a positive relationship between the children's knowledge of cardinal number and their ability to conserve number, but this relationship decreased with age. The relationship was significant for the lowest age group ($5\frac{1}{2}$ and under) but not significant for children above this age level. This would seem to suggest that for the younger children the ability to count was on a par with their knowledge of number which in each case is limited. The fact that the relationship was not significant with older children would seem to suggest that as children grow older the development of the ability to count is far greater than the development of the underlying concept of number.

In a study of 38 children who were entering first grade, Wheatley (1968) found that counting by one does not seem to be highly related to achievement for first grade students. However, Williams (1965) found different results in his study of 595 kindergarten entrants. He found that rote counting ability is substantially related to successful mathematical achievement with a correlation coefficient of .51. Bjonerud (1960) found a marked similarity in the ability to do rational counting by one and rote counting by one among 27 beginning kindergarten children.

Several investigators' studies included findings on "how far" children can count by rote. There was not specific agreement among the various findings. Bjonerud (1960) found that the kindergarten children in his study could do rote counting and half or more could count to nineteen. The findings of McDowell's (1962) study of the number concepts of preschool children indicated that the largest number name used by three-year-olds was nine. The results of a study by LeHew (1968) with 50 Head Start children between the ages of four years seven months to five years eleven months showed that 60 per cent of the four-year-olds and 77 per cent of the five-year-olds could count by rote over ten, and all could count to at least two.

In order to count rationally a child must pair each object in the collection to be counted with certain symbols (verbal or written) in proper sequence (Potter, 1968; Wilder, 1968). According to Potter three component skills underly the ability to count rationally. These are (1) knowledge of the names of numerals in correct order; (2) ability to take each item in an array one at a time, until all have been taken exactly once; and (3) ability to coordinate the first two skills. Potter's study was concerned with the second of these component skills, and her subjects ranged in age from two years to five years. Findings of a preliminary study indicated that children of two years of age may have difficulty in taking each item in an array only once if more than two items are present, but five-year-olds found counting nine items easy and some could count as many as twenty items. Potter's findings indicated that arrangement of items had a considerable effect on accuracy in pointing tasks. The results suggest that prior to development of a spatial strategy that takes account of an array as a whole, the child relies on features of individual items to distinguish the old from the new. This strategy is inefficient when there are too few or perhaps too many distinctive cues, and when there are many items in the array.

Several investigators were concerned with the ability of children to count rationally and to understand cardination. McDowell (1962) studied fourteen three-year-olds, fourteen four-year-olds and thirty five-year-olds, and found that it was difficult for many children to find three, four, or five candles in a group of five candles. Only 28 per cent of the three-year-olds, 42 per cent of the four-year-olds, and 60 per cent of the five-year-olds could do this. The results of a study by LeHew (1968) with 50 four- and five-year-olds indicated that when the children were asked to give the examiner a specific number of objects (5,7,8,10, and 6) from a group of ten items 65 per cent of the fours could give five objects; 45 per cent, six objects; 45 per cent, seven objects; 45 per cent, eight objects; and 40 per cent, ten objects. Seventy-seven per cent of the fives could give five objects; 67 per cent, six objects; 57 per cent, seven objects; and

47 per cent, eight and ten objects. In McDowell's (1962) study, when the children were required to count six pennies, two of the three-year-olds, ten of the four-year-olds, and twenty-two of the five-year-olds counted them correctly. In LeHew's (1968) study when the children were asked to enumerate 20 objects randomly placed on a table, 25 per cent of the four-year-olds and 33 per cent of the five-year-olds could count to fourteen. Twenty per cent of the four-year-olds and 23 per cent of the five-year-olds could count to twenty. When shown groups of objects and asked to name the specific number after only a hurried visual observation, the children had more difficulty than in the previous tasks which employed a motor as well as a visual sense. The findings of these two studies are supported by Brace (1965) who stated that preschool children have a very limited knowledge of cardinal number. Beckwith (1966) also agrees that children count better when allowed to use their motor skills.

Estes (1956) studied the ability of four-, five-, and six-year-old children to count objects and conserve number (maintain the invariance of number). One of her tasks required that the children count ten small green blocks which were arranged (1) in a straight line, (2) in a pattern, and (3) loosely piled. She found that if the children could count at all, they could count all three arrangements. She concluded that these children had grasped the idea of conservation of number because with a change of pattern they could still count correctly. Her conclusion is not supported by Dodwell (1960) who states that counting per se is no guarantee that a child grasps the concept of cardinal number or how it applies in concrete situations. He raises the question that if a child can count and point to objects and say numbers in correct sequence does this mean that he understands that the number will not change? Wohlwill (1962) seems to answer this question in his study of kindergarten children. Twenty-three children who maintained that the number of chips in two equal rows was different when one row was rearranged differently from the other were asked to count the chips in each row. Nineteen of the 23 continued to assert that there was a different number in the two rows immediately after counting seven chips in each. This gives evidence that counting is frequently a rote procedure for children and carries very little meaning.

A number of investigators have studied children's understanding of cardinal number, ordinal number and seriation. According to Coxford's (1964) interpretation of Piaget (1952), two operations basic to concept of number are cardinal number and ordinal number. A child understands cardinal number when he is able to construct a one-to-one correspondence between two sets of objects and conserve this correspondence when it is no longer perceptually obvious. One of the things a child must do to understand ordinal number is to arrange in a sequence a set of objects which differ in some aspect (seriation). Piaget (1952) concluded that the concepts of cardinal number and ordinal number are interdependent and develop together. According to Brace's (1965) study the concepts of cardinal and ordinal number do not develop concurrently. It would appear that a thorough understanding of cardinal number is necessary before the child can have real facility with ordinal number and before he can really appreciate the significance of the counting process. Dodwell (1960) in a study including 250 kindergarten, first- and second-graders disputed Piaget's (1952) thesis that ability to deal with serial relations and cardinal properties are necessary conditions of being able to deal with numbers. The results indicated that some children can deal operationally with cardinal-ordinal properties before they can deal with either classes or series separately, and the ability to deal with classes and series separately does not entail ability to deal with numbers or constructs combining ordinal and cardinal operations. Robinson's study (1968) of 99 first-grade children found that the relationship between the ability to seriate and mathematical achievement varies from "low to substantial," with no correlation exceeding .45.

A few investigators' studies were concerned with preschool children's knowledge of numerals. Dutton (1963) in a study of 236 kindergarten children in Los Angeles suggested that organized systematic instruction in writing numerals through nine or ten should be provided to keep pace with pupils' ability and need to read and identify numerals. Brace (1965), however, emphasized the fact that a thorough understanding of ordinal and cardinal numbers must be developed before operations involving number symbols are undertaken. Ashlock (1967) stated that

> Activities for some fours and fives probably should not involve recognition of numerals as names for numbers. The understanding of these mathematical ideas, including the cardinal number idea will not be developed as well if the teacher involves the children with symbolic representation too soon (page 415).

Several investigators tried to determine if sex were a factor in the development of number concepts. Crowder (1966) in a study of 425 first-graders found sex immaterial to the ability to learn arithmetic in the first grade. Baumann's (1966) study of 40 second- and fourth-graders found sex not significantly related to any of the concepts tested. Humphrey (1966) in a study of the ability of first-grade boys and girls to learn number concepts by playing games found that changes more often occurred for boys than girls. Heimgarter (1968) studied 224 kindergarten children in Denver with the purpose of determining selected mathematical abilities of beginning kindergarten children. He found no statistically significant difference between males and females except on knowledge of fractions one-half and one-fourth where females were significantly better. Although statistical tests were not performed on the data regarding sex differences in D'Mello and Willemsen's (1969) study due to the small number of children in each age group, the evidence suggested that at age three and four boys were better able to perform the number tasks, at age five and six girls performed better, and at age seven and eight the two sexes were equal.

With regard to age differences Potter (1968) found that the capacity to hold in mind an array of items that one has enumerated shows a steady and dramatic increase between the ages of two and one-half and four. Heimgarter's (1968) study showed statistically significant differences among age groups on development of number concepts of kindergarten children. Dodwell's (1960) study of kindergarten, first- and second-grade children showed considerable variations in types of responses given at any age level. Type of response may also vary from one test situation to another for a child. There are age trends with older children being more operational, but trends differ for different test situations. D'Mello and Willemsen (1969) found that older children were better able to perform the number tasks, with one exception. Four-year-old boys exceeded five- and six-year-old boys and four- and five-year-old girls. However, these results may be misleading, since only two four-year-old boys were included in the study. LeHew (1968) found that fours did less well than fives in counting ability. Long (1941) who studied the development of the ability to match and discriminate numbers found that performance varies in a regular and systematic manner with age. All of the average scores improved as age increased. The results of a survey by McDowell (1962) of number concepts of three-, four- and five-year-old children who had not attended any nursery school showed that today's fours have number concepts more nearly like those of fives of a generation ago.

> Tests of a generation ago indicate that yesterday's fours possessed concepts more like today's threes. The greatest number development seems to take place between the ages of three and four whereas formerly the greatest learning seemed to take place between the ages of four and five (page 443).

There have been relatively few studies concerning the nature of the process involved in the development of the number concept. Wohlwill (1960) did the first extensive study in this area. He used 72 children enrolled in kindergartens and various primary schools in Geneva, Switzerland. The basic question of this study concerned the sequence through which a child passes in his development of the concept of number. The concept of levels of abstractness, or degrees of symbolic mediation was the guiding principle in the selection of the individual tests. This was accomplished by utilizing tests which were used in his previous investigations and were assumed to vary in levels of abstractness. The method of scalogram analysis was used in which an analysis of successes and failures on these tasks by individuals at different developmental levels revealed whether these tasks constituted a scalable set. The results indicated that there were three fairly distinct stages in the development of the number: (1) an initial stage in which number is responded to wholly on a perceptual basis, (2) an intermediary one in which individual numbers are responded to in conceptual terms, and (3) a final one in which the relationship among the individual numbers is conceptualized.

The results of a study by D'Mello and Willemsen (1969) support the findings of Wohlwill (1960) that the number concept develops through increasing levels of abstraction. D'Mello and Willemsen tested 38 children in California between the ages of three and eight years on a set of four tasks which were selected primarily on intuitive grounds. The initial ordering of the tasks was tested by scalogram analysis developed by Guttman and discussed by Green (1954). Consideration of the actual patterns of successes and failures on the tasks caused D'Mello and Willemsen (1969) to rearrange the original order of the second and third tasks. After the reordering a coefficient of reproducibility of .940 resulted from analysis of first response data. Some subjects had been given second opportunities for responding when they seemed inattentive. Analysis of second try data resulted in a coefficient of reproducibility of .993 for all 38 subjects. These indices of reproducibility indicated that the number concept does develop through increasing levels of abstraction as defined by the four tasks used by these investigators. The four tasks in order of least to most difficult required the subjects to (a) recite number words in a sequence,

(b) match two visual displays of quantity, (c) match number words with absolute quantity, and (d) match the numerical symbol with a visual array of a certain quantity of dots. The results indicated that Task "a" is clearly easier than Task "d", with Tasks "b" and "c" of intermediate difficulty. The greatest variability of performance occurred in the nursery school group and by first grade all of the children had acquired the number concept as was defined in this study.

Summary

In summary the review of literature indicates:

1) A relevant mathematical concept has to exist as an abstract concept before it can become fully operational.

2) Some of the number concepts identified as important for preschool children were sets and simple relations concerning sets, one-to-one correspondence, more or less than, counting and enumeration, cardinal number, ordinal number, beginning fractions, and seriation.

3) Studies by many authors indicated that the child's ability to count is not a reliable criterion of the extent to which he has developed the true concept of number.

4) There was not specific agreement among investigators as to "how far" children can count by rote.

5) Studies seem to indicate that preschool children have a limited knowledge of cardinal number.

6) Literature revealed a disagreement among investigators as to whether the concepts of ordinal number and cardinal number develop concurrently or separately.

7) There is disagreement as to whether numerals should be taught in the preschool.

8) Sex did not seem to be a factor in the development of number concepts.

9) As age increases children have better developed number concepts.

10) The results of several studies indicated that there was an order in the development of the number concept.

11) At the present time, the greatest number development seems to be taking place between the ages of three and four, whereas in the past it seemed to be taking place between the ages of four and five.

CHAPTER III

PROCEDURE

Subjects

The subjects of this study were 60 preschool children, 29 boys and 31 girls ranging in age from three years one month to five years two months. These children were in attendance at the Oklahoma State University Preschool Child Development Laboratories. They were the children of faculty, students, or local businessmen and were primarily of middle socio-economic status.

Description of Instruments

Domino Number Tasks

Five number tasks similar to those developed by D'Mello and Willemsen (1969) were selected for use in the present research. One slight change was made in three of D'Mello and Willemsen's (1969) tasks. The order of the numerals within Tasks "b", "c", and "d" were changed from 3, 4, 5 to 3, 5, 4. The present investigator felt that using the numerals 3, 4, and 5 in the expected order would give children more opportunity for responding on the basis of imitation or idetic imagery while changing the order to

3, 5, 4 might give a more accurate picture of the child's understanding of the different numbers. The present investigator also added a final task, "e", because the results of D'Mello and Willemsen's (1969) study indicated that a more difficult task needed to be included in the study. The tasks were designed to determine whether there is an order or sequence in which preschool children develop number concepts. The basic proposition of this study is that the number concept develops for the young child through a process which amounts to a gradual increase in his ability to use number symbols abstractly. The underlying assumption is that mastery of a given task implies the necessary mastery of all tasks below its level of difficulty.

Four of the five tasks involved the use of a set of 28 dominoes with the number of dots ranging from double blanks to double sixes. The dominoes were 4 x 6 inches and were natural wood with black dots. A set of six 4 x 6 cards was also used. Each card had been lettered with one of the first six numerals with a black felt-tipped marking pen.

The five number tasks are described below.

<u>Task "a"</u> — S was to recite the first six numbers, that is, he was asked to count from one to six. If he did not respond he was asked to count, starting with one.

<u>Task "b"</u> — When E placed the blank-three domino in front of him, S was to match the three end with any domino from the set having a three on one end. Similarly, he was

to do this for the blank-five and the blank-four dominoes in that order.

Task "c" — S was to place in a box any three dominoes, then any five, then any four.

Task "d" — When presented with the blank-three domino, S was to point to one of the prominently displayed printed numerals corresponding to the number of dots on the domino; for instance, he should point to the "3" card. Similarly, he was to point to the correct numerals for five dots and four dots in that order.

Task "e" — E put the blank-one through blank-six dominoes in random order in front of S, and S was to point to the domino with the smallest number of dots. E removed this one and then asked S to point to the domino with the next smallest number of dots. This was repeated until all the dominoes were removed and placed in the order S had specified. S was then asked to look them over and see if he got them right. S could change the order if he wished.

The four tasks were ordered according to difficulty on the basis of D'Mello and Willemsen's (1969) findings. The results of their study indicated that Task "a" was the least difficult and Task "d" the most difficult. Tasks "b" and "c" were of intermediate difficulty. As a result of reviewing the literature related to the development of number concepts, the investigator selected seriation as a concept which might be somewhat more difficult than the concepts involved in Tasks "a" through "d", rote counting, matching or one-to-one correspondence, rational counting and cardination, and relating quantity to number symbol. Task "e" was designed to indicate the child's understanding of seriation through his ability to order quantities from smallest to largest.

Number Story

The number story test was developed from the story Two Lonely Ducks by Roger Duvoisin for the purpose of testing the validity of the previous number tasks. This number story test was given immediately after the five tasks in which the dominoes were used. A flannel board and felt characters were used in telling the story. The tasks were presented in the order "c", "d", "e", "a", and "b" instead of "a", "b", "c", "d", and "e" as in the previous test because of the format of the story. However, the items within the individual tasks were presented in the same order in both tests. The story and the domino number tasks were presented to six children, ages three, four, and five, who were not subjects for the major investigation. These six children seemed able to handle the domino number tasks comfortably but became restless before the end of the story tasks. The number story was shortened to retain all of the tasks and the plot of the story but to eliminate unnecessary aspects of the story. The complete text of the story as used in this investigation with directions for administering the test are presented in Appendix A.

Administration of Instruments

The same person served as E for all subjects. During the self-selected activity period of the regular preschool program E invited each S individually to accompany her from the playroom to a quiet room where S was seated on a blanket on which the materials were assembled. The six half-blank dominoes and the cards were on E's side of the blanket, and the remaining 22 dominoes were placed in a pile in front of S. After arriving in the room, E was seated opposite S and engaged him in a casual conversation of about one minute's duration to establish rapport.

The E then asked S to "Count from one to six," and if S did not respond, E said "Count for me starting with one." These requests constituted Task "a". No indication was given S that he was right or wrong, and there was no time limit. Similarly, on subsequent tasks, no positive or negative reinforcement was used and no time limits were imposed.

Next, E placed a blank-one domino in front of S and matched it with a domino from S's pile with one dot on an end. The use of the one-one domino was avoided and E emphasized verbally that any domino with a single dot on one of its ends was a match for the blank-one domino. The two dominoes were placed end to end with the ones next to each other and E said, "See, they match because this is a one and that is a one" (indicating the relevant ends of the two dominoes). Then S was asked to match a blank-three domino, a blank-five domino, and a blank-four domino one at a time, in that order. In each case E asked the subject to "Find one of your dominoes that matches this one" and placed the domino in front of S. The three requests constituted Task "b", the coordination of two visual displays of quantity.

Then E asked S to "Put one domino in the box ... like this" and picked up one domino (which was not one with one dot) to demonstrate what she wanted. Then E watched while S put one domino in the box. Subsequently, S was asked to put any three, then any five, then any four dominoes in the box. All dominoes placed in the box by S were returned to S's pile between requests. These three requests constituted Task "c" and required the S to coordinate number words with absolute quantities of objects.

For Task "d", the six cards with printed numerals were placed in front of S in a random selection from the possible arrangements. The cards were face up and in the same spatial arrangement for all S's. E put the blank-one domino in front of S and asked him to "Point to the card which tells how many dots there are." If S failed to point to the "1" card immediately, E demonstrated by pointing to the correct card and remarking, "This one does, doesn't it?" Then E presented in turn the blank-three, blank-five, and blank-four dominoes and requested that S show him which card in each case told the number of dots. Here, S must coordinate the numerical symbol with the visual array of a certain quantity of dots.

Then E put the blank-one through blank-six dominoes in random order in front of S, and S was asked to "Point to the domino with the smallest number of dots." This was repeated until all the dominoes were removed and placed in the order S had specified. Then E asked S to "Look them over and see if you got them right." S was allowed to change the order if he wished. These requests constituted Task "e" and required the S to place the dominoes in order from smallest number to largest number.

For the last portion of the testing situation the E led S to another blanket on which the flannel board and flannel objects were arranged and said, "Now I am going to tell you a story about Two Lonely Ducks, and I would like for you to help me put some things on the flannel board when I ask you to." The flannel characters that the E alone used were placed away from S, and the characters that both used were placed between E and S. A copy of the story with the examiner's directions may be found in Appendix A.

Scoring

Each number task was scored either pass (+) or fail (-) for each S. Task "a" was scored "+" if S recited the number words, one through six, in correct sequence the first or second time he was asked. Tasks "b", "c", and "d" were scored "+" if S gave a correct response on two out of the three trials within the task. Task "e" was scored "+" if S placed the dominoes in correct order either the first time or if he changed them to the right order when asked if he got them right. For the number story the S's responses were scored identically to the five number tasks in which dominoes were used and on the same score sheet. The score sheet for the number tasks and number story are found in Appendix B.

Validity and Reliability

Validity

The primary purpose of developing the number story was to check the validity of the five number tasks. Although the tasks were presented in a different order in the story ("c", "d", "e", "a", and "b" instead of "a", "b", "c", "d", and "e") and different materials were used, it was felt that the two instruments were measuring the same number concepts. Although it was not apparent in the pilot study, it later seemed that Task "b" of the number tasks and the number story were testing the concept of one-to-one correspondence in a different manner. Task "b" of the number tasks required the child to use visual skills in matching two dominoes, while Task "b" of the number story also required the child to use motor skills. Therefore, measures of validity were calculated both with and without the "b" tasks. Percentages of agreement between responses to the domino number tasks and responses to the tasks in the number story were calculated for the total group and found to be 87 per cent with Task "b" and 89 per cent without Task "b".

Reliability

A retest was given to 35 of the subjects within approximately a week of the first test. However, due to absence from nursery school several subjects were retested a couple of days after a week had passed. The purpose of the retest was to obtain a measure of reliability of the subjects' responses. The retest consisted of the five domino number tasks only and was administered and scored in an identical manner as the first test. Percentages of agreement between responses to the domino number tasks in the initial test and the retest were calculated for the total group and found to be 91 per cent for the scored tasks.

Collection of Data

The data were collected during the last part of the 1970 Fall semester and the first part of the 1971 Spring semester. The children were invited to play the special game and hear the story during the free play period of their regular nursery school, and were then escorted to a quiet room nearby. The total testing time was approximately 20 minutes for each subject.

The examiner was familar with most of the subjects already, as she had worked in the Child Development Laboratories at previous times when the same children were in attendance. Therefore, it was not necessary to engage in a lengthy conversation before testing to establish rapport. The examiner began testing with the three-year-old children and finished testing most of them before testing the four- and five-year-old children.

Analysis of Data

The purposes of this research were to examine the following hypotheses:

1) There is an expected order in which children develop number concepts.
    a) The order is the same for girls and boys.
    b) The order is the same for each age group.
2) The concepts reflected in children's responses to the number tasks using dominoes will be generalized to other situations; i.e., the number story.
3) Older children will pass more number tasks than younger children.
4) There will be no differences between responses of boys and girls of similar ages to the number tasks.

Guttman's coefficient of reproducibility as reported by Green (1954) was used to determine if there was an expected order in which children develop number concepts. The percentage of individual tasks passed and the mean number of tasks passed was calculated for each sex and age group.

Percentages of agreement between responses to the domino number tasks and number story tasks were calculated to determine if the understandings of the number concepts would be generalized to other situations. These percentages were calculated both for scored responses to the five tasks and for individual parts of each task. For example, the scores for Tasks "b", "c", and "d" were regarded as "pass" if the subject responded correctly to two of the three parts of each item.

The mean number of total tasks passed and the percentage of individual tasks passed were determined for each age group.

The same calculations were made for the responses of boys and girls.

CHAPTER IV

RESULTS AND DISCUSSION

The major purpose of this research was to explore the possibility of there being an order in which preschool children develop number concepts which would be scalable according to Guttman's criteria (Green, 1954). Five number tasks representing increasingly abstract levels of thinking were presented to 60 preschool children. A number story test with individual tasks approximately the same as the five number tasks was given immediately after the five domino number tasks. A retest of the domino number tasks was given to 35 subjects within approximately a week after the first test. Table I presents the number of subjects of each age and sex tested and retested.

TABLE I

SUBJECTS BY AGE FOR INITIAL TEST AND RETEST

	Initial Test N=60		Retest N=35	
Age	Boys	Girls	Boys	Girls
3	11	10	8	5
4	13	17	8	10
5	5	4	2	2

The responses of the subjects on the five domino number tasks were examined to determine if an order existed for the total group and if this order was similar for different age groups and for both boys and girls. The data were also treated to determine if older children passed more number tasks than younger children and if there were any differences between responses of boys and girls of similar age to the number tasks. The responses of the subjects to the domino number tasks and to the number story were compared to determine if the concepts reflected in the subjects' responses to the number tasks were generalized to the number story. In order to test the reliability of the domino number tasks, the responses of the subjects to the domino number tasks and to the retest were compared.

Five Domino Number Tasks

There were six permissible scale types for response records for the five number tasks. These are presented in Table II. If each subject produced a response record which fell into one of these six scale types, then the assertation could be made that the tasks were clearly ordered according to difficulty for that individual. A permissible scale type was obtained if there were no failures before a pass. Forty-nine of the subjects fell into one of the six possible scale types as may be seen in Table III.

Eleven of the 60 subjects did not fall into any of the six permissible scale types. The response records of these

TABLE II

PERMISSIBLE RESPONSE RECORDS

Scale Type	Task a	Task b	Task c	Task d	Task e
1	−	−	−	−	−
2	+	−	−	−	−
3	+	+	−	−	−
4	+	+	+	−	−
5	+	+	+	+	−
6	+	+	+	+	+

TABLE III

NUMBER OF SUBJECTS RESPONDING IN EACH PERMISSIBLE SCALE TYPE

Age	Scale Type					
	1	2	3	4	5	6
3	5	2	3	1	4	0
4	1	0	2	7	13	2
5	0	0	0	0	5	4

eleven subjects contained a fail for one item followed by a pass for a more difficult item and were said to contain an error. Each of the individual records of these eleven subjects contained only one error, making a total of eleven errors out of a total of 300 item responses. Six of the eleven errors occurred between Tasks "a" and "b", and three of the eleven errors occurred between Tasks "b" and "c". Five of the six errors between Tasks "a" and "b" occurred with three-year-olds. The response patterns of the children whose records contained errors are found in Appendix C.

Task "a", counting from one to six, appeared to be easier than Task "e", ordering numbers from one to six. However, the results indicated no discernable difference between Task "a" and Task "b" for this group of subjects. All of the five-year-olds and 90 per cent of the four-year-olds passed both Tasks "a" and "b". Slightly over 50 per cent of the three-year-olds passed both Tasks "a" and "b". It would appear that five per cent more of the three-year-olds passed Task "b". However, upon closer inspection of the data one finds that among the three-year-old female subjects only 50 per cent passed Task "a" while 70 per cent passed Task "b", and among the four-year-old male subjects 77 per cent passed Task "a" while 85 per cent passed Task "b". These figures suggest that rote counting (Task "a") was not necessarily accomplished earlier than one-to-one correspondence as shown in Task "b". However, among the three-year-old girls two who did not pass Task "a" during the testing situation were observed to count past six during free play situations before the testing. Since the subjects included only ten three-year-old girls the "errors" in the responses of these two accounted for 20 per cent in the percentages of three-year-old females passing Task "a". Among the three-year-old male subjects and among the four-year-old female subjects the order followed the predicted pattern, suggesting that discrepancies in percentages of children passing Task "a" and Task "b" do not necessarily indicate that the concept of one-to-one correspondence develops earlier than rote counting.

Task "b" clearly appeared easier than Task "c" for three-year-olds, slightly easier for four-year-olds, and all five-year-olds were able to pass both Task "b" and Task "c". Task "c" appeared slightly easier than Task "d" for three-year-olds. Task "c" appeared clearly easier than Task "d" for four-year-olds, and all five-year-olds passed both Task "c" and Task "d". Task "d" clearly appeared easier than Task "e" for all age groups. Task "e" appeared to be much more difficult for all age groups than the other four tasks. Table IV presents the percentages of children passing each task.

The coefficient of reproducibility (Rep.) which corresponded to the proportion of responses of scale type obtained was calculated and found to be .963. Rep = 1 - (N errors/Nk) where N is the number of subjects and k the

TABLE IV

PERCENTAGES OF CHILDREN PASSING EACH TASK

Task	Age 3 N = 21	Age 4 N = 30	Age 5 N = 9	% of total males N = 29	% of total females N = 31	Total group N = 60
a	52	90	100	72	84	78
male	55	77	100			
female	50	100	100			
b	57	90	100	72	87	80
male	45	85	100			
female	70	94	100			
c	29	87	100	59	77	68
male	27	69	100			
female	30	100	100			
d	24	57	100	41	61	52
male	18	38	100			
female	30	71	100			
e	0	10	33	7	16	12
male	0	0	40			
female	0	18	50			

number of items (tasks); that is, Nk represents the total number of responses made by the 60 subjects. The number of errors is the number of fail-pass sequences in all the records of all the subjects. The coefficient of reproducibility of .963 corresponds to the coefficient of reproducibility of .913 as found by D'Mello and Willemsen (1969) for their subjects ages three through six. These results support the hypothesis that there is an expected order for developing the number concepts included in this study.

The number of passes on the five domino number tasks was calculated for each subject and the resulting data examined for age and sex differences. The mean number of tasks passed are presented in Table V. The mean number of tasks passed by older children was clearly greater than the mean number of tasks passed by younger children. The mean number of tasks passed by girls appeared to be slightly greater than the mean number of tasks passed by boys with the greatest difference appearing at age four. This is in contradiction to the findings of D'Mello and Willemsen (1969) who found that at ages three and four the mean number of tasks passed by boys was slightly greater than the mean number of tasks passed by girls with the greatest difference appearing at age four.

The data also seems to indicate that the order for developing the number concepts included in this study was similar for different age groups and for both boys and girls as can be seen by inspection of Table IV.

TABLE V

MEAN NUMBER OF TASKS PASSED IN DOMINO NUMBER TASKS

Age (Years)	Boys	Girls
3	1.5	1.8
4	2.7	3.8
5	4.4	4.5

The results of Tasks "b", "c", and "d" were analyzed to determine if there was any difference in the attainment of the concepts of the numbers three, four, and five. The results are presented in Table VI. The evidence suggests that the number three was better known than the numbers four and five. The number four was better known than the number five, but the difference is not as great. A greater percentage of five-year-olds knew all three numbers than did the four-year-olds, and a greater percentage of four-year-olds knew all three numbers than did three-year-olds.

Comparison of Domino Number Tasks and Number Story

The percentages of agreement between the domino number tasks and the number story were calculated for the five scored tasks (Table VII and Appendix D) and also for the entire 22 items comprising each test (Table VIII and

TABLE VI

UNDERSTANDING OF NUMBERS THREE, FOUR, AND FIVE AMONG AGE GROUPS AS REFLECTED IN PERCENTAGES

Age	Number 3			Number 4			Number 5		
	b_3	c_3	d_3	b_4	c_4	d_4	b_5	c_5	d_5
3	67	43	29	48	43	47	52	19	19
4	90	87	77	80	83	50	80	67	47
5	100	100	100	89	100	100	100	89	100
TOTAL	83	73	63	70	72	47	73	53	45
AVERAGE		74			63			58	

TABLE VII

PERCENTAGES OF AGREEMENT BETWEEN SCORED TASKS IN DOMINO NUMBER TASKS AND NUMBER STORY

Age	Task a	Task b	Task c	Task d	Task e
3	81	62	90	81	95
4	97	87	97	80	77
5	100	89	100	100	89
TOTAL	92	78	95	83	85

Appendix D). The results were examined for age differences. The percentage of agreement = 2N of agreements/N responses, where N responses equals the total number of possible responses.

TABLE VIII

PERCENTAGES OF AGREEMENT BETWEEN INDIVIDUAL ITEMS OF TASKS IN DOMINO NUMBER TASKS AND NUMBER STORY

Age	Task a	Task b	Task c	Task d	Task e
3	81	59	81	67	95
4	97	72	78	78	77
5	100	78	93	100	89
TOTAL	92	68	81	77	85

Since it appeared after the tests were begun that Task "b" of the domino number tasks and Task "b" of the number story were testing the concept of one-to-one correspondence in a different manner, the percentages of agreement for the total group of subjects were calculated with and without the "b" tasks. Task "b" of the domino number tasks required the children to use only visual skills while Task "b" of the number story required both visual and motor skills. Table IX presents the percentages of agreement with and without Task "b" for the individual items and the five scored tasks. The percentages of agreement in all cases are greater without Task "b". It appeared that Task "b" of the number story was slightly more difficult than Task "b" of the domino number tasks. Eighty per cent of the total group passed Task "b" of the domino number tasks while 75 per cent passed Task "b" of the number story.

TABLE IX

PERCENTAGES OF AGREEMENTS BETWEEN TESTS WITH AND WITHOUT TASK "b"

	With Task "b"	Without Task "b"
Domino Number Tasks & Number Story		
Scored Tasks	87	89
Individual Items	78	81
Domino Number Tasks & Retest		
Scored Tasks	88	91
Individual Items	81	86

Comparison of Initial Responses to Domino Number Tasks and Retest

The percentages of agreement between the initial responses to the domino number tasks and the retest were calculated for the five scored tasks and also for the total 22 items comprising each test. The results were examined for age differences. Tables X and XI present the percentages of agreement between the initial responses to the domino number tasks and to the retest. The percentages of agreement with and without Task "b" were also calculated between the domino number tasks and the retest and are found in Table IX.

Summary of Findings

1) Evidence was presented to suggest that there was an expected order in which children developed number concepts in this study. The order was "a" or "b", "c", "d", and "e".
2) The order appeared to be similar for three-, four-, and five-year-old children.
3) The order appeared to be similar for both boys and girls.
4) The concepts reflected in the subjects' responses to the number tasks appeared to generalize to the number story.
5) Older children passed more tasks than younger children.
6) Sex appeared to be a factor in pattern of responses in this study. Girls of each age group passed a greater mean number of tasks, and a greater percentage of girls passed each task than did boys.

TABLE X

PERCENTAGES OF AGREEMENT REFLECTING RELIABILITY OF TEST-RETEST RESPONSES TO SCORED TASKS OF DOMINO NUMBER TASKS

Age	Task a	Task b	Task c	Task d	Task e
3	62	62	92	92	100
4	100	83	94	83	100
5	75	75	100	100	100
TOTAL	83	74	94	89	100

TABLE XI

PERCENTAGES OF AGREEMENTS REFLECTING RELIABILITY OF TEST-RETEST RESPONSES TO INDIVIDUAL ITEMS OF TASKS OF DOMINO NUMBER TASKS

Age	Task a	Task b	Task c	Task d	Task e
3	62	62	85	82	100
4	100	72	85	81	100
5	75	92	92	100	100
TOTAL	83	70	86	84	100

CHAPTER V

SUMMARY

The main purpose of this research was to explore the possibility of there being an order in which preschool children develop number concepts which would be scalable according to Guttman's criteria. Five domino number tasks representing increasingly abstract levels of thinking were presented to 60 preschool children, 29 boys and 31 girls. The children ranged in age from three years one month to five years two months, and were in attendance at the Oklahoma State University Preschool Child Development Laboratories.

A number story test with individual tasks approximately the same as the five domino number tasks was given immediately after the domino number tasks. A retest of the domino number tasks was given to 35 subjects within approximately a week after the first test.

The data were examined to determine if an order existed for the total group and if this order was similar for different age groups and for both boys and girls. The data were also analyzed to determine if older children passed more number tasks than younger children and if there were any differences between responses of boys and girls of similar age to the number tasks. The responses of the subjects to the domino number tasks and to the number story were compared to determine if the concepts reflected in the subjects responses to the number tasks were generalized to the number story. In order to test the reliability of the domino number tasks, the responses of the subjects to the domino number tasks and to the retest were compared.

The findings of this research were as follows:

1) Evidence was presented to suggest that there was an expected order in which children developed number concepts in this study. The order was "a" or "b", "c", "d", and "e".

2) The order appeared to be similar for three-, four- and five-year-old children.

3) The order appeared to be similar for both boys and girls.

4) The concepts reflected in the subjects' responses to the number tasks appeared to generalize to the number story.

5) Older children passed more tasks than younger children.

6) Sex appeared to be a factor in pattern of responses in this study. Girls of each age group passed a greater mean number of tasks, and a greater percentage of girls passed each task than did boys.

Recommendation for Future Research

The greatest variability of responses to the number tasks seemed to occur with three- and four-year-old children. Since five-year-old children passed all of the tasks except Task "e", further study would be necessary to determine what number concepts five-year-olds seem to be concentrating on most. Then it would be possible to verify the scalability of the number concepts which this population is developing.

Since there was only five per cent difference between Task "b" of the number task and Task "b" of the number story, an investigation could be undertaken to determine how the usage of visual and motor skills affect the development of the concept of one-to-one correspondence.

Further study regarding sex differences in the development of number concepts would be necessary in order to determine if the findings of this study could be generalized to a population of three-, four-, and five-year-olds.

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APPENDIX A

NUMBER STORY

Two Lonely Ducks

One little white drake, quack, quack went to the pond, merrily shaking its tail. One little white duck went down behind; and so there were two ducks swimming in the blue pond. "Ah," said the duck to the drake, "How lonely it is, just the two of us swimming about in this blue pond. It is high time we raise a family and have ducklings swimming behind us."

The little duck built herself a nest in the corner of the barn and when it was finished, the duck laid the first egg in the middle of the nest. (E said to S, "Can you put one egg in the nest." After S was finished E removed the egg.) When she laid a second egg, there were two eggs in the nest. (E put two eggs in the nest, paused and then removed them.) When she laid the third egg, there were three eggs in the nest. (E said to S, "Can you put three eggs in the nest." After S was finished E removed the eggs.) When she laid the fourth egg, there were four eggs in the nest. (E put four eggs in the nest, paused and then removed them.) When she laid the fifth egg, there were five eggs in the nest. (E said to S, "Can you put five eggs in the nest." After S was finished E removed the eggs.) When she laid the sixth egg, there were six eggs in the nest. (E put six eggs in the nest, paused and then removed them.)

(E put the six numerals from one to six up on the flannel board in random order. Then E put groups of one, two, three, four, five, and six eggs in nests on the board in random order. E said to S, "If there are this many eggs in a nest, can you point to the numeral which tells how many eggs there are in this nest." E asked S to respond to groups of one, three five, and four eggs, in that order. E pointed to the group of one and if S failed to respond, E demonstrated by pointing to the correct numeral and remarked, "This one does, doesn't it." Then S was asked to respond to the other groups.)

(E removed the numerals. S was then asked to "Point to the nest with the smallest number of eggs." E removed this one and then asked S to "Point to the nest with the smallest number of eggs now." This was repeated until all the nests were removed and placed in the order S had specified. S was then asked to "Look them over and see if you got them in the right order." S was allowed to change the order if he wanted to. E then replaced the duck and nest.)

Now the drake came in and said to the duck: "Please, please, not another egg. With food so dear, six ducklings -- that's all we can afford to raise." So said the drake. But I think the reason was he could not count over six. (E said to S, "Can you count from one to six.")

Now the little duck sat on her eggs, and she sat, and sat, and sat. She sat one day. She sat two days, she sat three days. She sat four days. (E said to S, "Can you put four suns up in the sky for the four days that she sat." After S was finished E removed the suns.) She sat five days. She sat six days. She sat seven days, and that was one whole week. Wasn't she a patient little white duck! Altogether she sat one whole month. But now she sat no more, because . . . she heard a twitt under her, and a knock, and a crack, and she saw a baby duckling peeping out of its broken shell: and he said, "Twitt, twitt." And so it was that the little white duck became the mother of a baby duckling. (E put up a broken shell and a baby duckling and said, "See they match, because there is one broken egg and one baby duckling." Then E took them down.) Then there was another baby duckling. (E put up two ducklings and shells and then took them down.) And then there was another baby duckling. (E put up three ducklings and said to S, "Can you match the shells with the ducklings." After S was finished E removed them.) And another baby duckling. (E put up four ducklings and matched them with their shells, then took them down.) And another baby duckling. (E put up five ducklings and said to S, "Can you match the shells with the ducklings." After S was finished E removed them.) Another baby duckling. (E put up six ducklings and matched them with their shells, then removed them.)

Now all the egg shells lay broken at the bottom of the nest. And there were so many twitt, twitts no one could count them. The drake came in, and he was so proud. He counted this many that were girls. (E put up two ducklings, matched them with shells, and then took them down.) He counted this many that were boys. (E put up four ducklings and said to S, "Can you match the shells with the boy ducklings." After S was finished E removed them.)

Then the little white duck left her nest, and all the ducklings tumbled out after her. And the little white drake, and the little white duck, and their six little ducklings all walked down to the pond. No one was lonely any more, because now there were eight ducks swimming in the blue pond.

APPENDIX B

SCORE SHEET

SCORE SHEET

Name ____________________

Date ____________________

Age ____________________

DOMINOES			STORY		
Task		Comments	Task		Comments
a			c_3		
b_3			5		
5			4		
4			d_3		
c_3			5		
5			4		
4			e		
d_3			a		
5			b_3		
4			5		
e			4		
Total Passes			Total Passes		

APPENDIX C

RESPONSE RECORDS CONTAINING ERRORS

TABLE XII

RESPONSE RECORDS CONTAINING ERRORS

Child	a	b	c	d	e
12	−	+	+	−	−
18	+	+	+	−	+
24	+	−	+	+	−
32	+	−	+	+	−
34	−	+	−	−	−
42	−	+	−	−	−
43	−	+	−	−	−
44	+	−	+	−	−
52	−	+	−	−	−
53	−	+	−	−	−
54	−	−	−	+	−

APPENDIX D

PERCENTAGES OF AGREEMENTS

TABLE XIII

PERCENTAGES OF AGREEMENTS FOR INDIVIDUAL SUBJECTS ON ALL TASKS

Child	Sex	Age	Number Tasks and Total Items	Number Story Five Tasks	Number Tasks and Total Items	Retest Five Tasks
1	F	5.0	91	100	91	100
2	M	5.0	91	100		
3	M	5.0	91	100		
4	F	5.0	100	100		
5	F	5.0	100	100		
6	M	5.1	82	100	82	60
7	M	5.1	91	100	100	100
8	F	5.1	91	80	100	100
9	M	5.2	82	80		
10	M	4.0	82	80		
11	F	4.2	73	100	73	60
12	M	4.2	54	80		
13	M	4.3	100	100	54	80
14	M	4.3	73	100	73	80
15	F	4.3	82	80	100	100
16	F	4.3	68	80		
17	F	4.3	91	100		
18	F	4.3	54	60		
19	M	4.3	91	100	82	100
20	M	4.5	68	40	82	80
21	F	4.5	73	80	100	100
22	F	4.5	91	100		
23	F	4.5	91	80		
24	F	4.5	91	80	82	80
25	F	4.5	68	80		
26	F	4.6	73	100	91	100
27	F	4.7	82	60		
28	F	4.7	73	80	73	100
29	M	4.8	82	100		
30	M	4.8	68	80	82	100
31	M	4.9	73	100		
32	M	4.9	54	60	73	80
33	F	4.10	100	100	100	100
34	M	4.10	73	100	82	100
35	F	4.10	73	80	82	100
36	F	4.10	82	100	82	100
37	M	4.11	91	100	100	100
38	M	4.11	91	100		
39	F	4.11	82	100	91	100

TABLE XIII (Continued)

Child	Sex	Age	Number Tasks and Total Items	Number Story Five Tasks	Number Tasks and Total Items	Retest Five Tasks
40	M	3.1	91	100	91	100
41	M	3.2	91	80		
42	F	3.2	68	80	91	100
43	F	3.3	82	100	82	80
44	F	3.4	100	100	82	80
45	F	3.5	82	80		
46	M	3.5	100	100	68	80
47	M	3.5	68	80	68	60
48	M	3.5	54	60	100	100
49	M	3.6	73	80	54	60
50	F	3.7	82	100		
51	M	3.7	73	80	91	100
52	M	3.7	68	60		
53	F	3.7	54	60	91	100
54	F	3.8	54	80	27	40
55	F	3.9	68	80		
56	F	3.10	45	80		
57	F	3.10	73	100		
58	M	3.11	73	80		
59	M	3.11	82	80	100	100
60	M	3.11	54	60	68	60

VITA

Christine Lanie McDonald

Candidate for the Degree of

Master of Science

Thesis: THE DEVELOPMENT OF NUMBER CONCEPTS IN PRESCHOOL CHILDREN

Major Field: Family Relations and Child Development

Biographical:

Personal Data: Born in Hawthorne, California, October 11, 1946, the daughter of Mr. and Mrs. Paul Lanie.

Education: Graduated from Wakita High School, Wakita, Oklahoma, in May, 1964. Attended Northwestern State College, Alva, Oklahoma, from 1964 to 1965; attended Oklahoma State University from 1965 to 1969, received the Bachelor of Science degree in Family Relations and Child Development in May, 1969. Completed requirements for the Master of Science degree in May, 1971.

Professional Organizations: Omicron Nu, Phi Kappa Phi, Phi Delta Kappa.