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A STATISTICAL ANALYSIS OF THE GROWTH OF UNDERSTANDING MATHEMATICAL CONCEPTS BY THE PROSPECTIVE ELEMENTARY TEACHER

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CHAPTER I

INTRODUCTION TO THE PROBLEM

Today one need read only a small portion of the educational literature in order to observe a concern for the mathematical competence of those persons preparing for positions as elementary school teachers. Leaders in education are becoming concerned because many studies have shown that prospective elementary teachers lack an understanding of arithmetic concepts.

The Problem

Statement of the Problem

It is the purpose of this study to conduct a statistical analysis to determine the growth in understanding of certain basic mathematical concepts of the prospective elementary teachers at the University of Oklahoma at different stages in their training program. There are three main projects involved in a study of this nature: (1) to devise an objective instrument to measure the understanding of the prospective elementary teachers on some basic mathematical concepts; (2) to show the relationship between the understanding of mathematical concepts measured by this instrument and some measurable factors; and (3) to evaluate the effect of certain education and mathematics courses on the growth and mastery of the basic mathematical concepts.

Importance of the Study

A firm grasp of basic mathematical concepts is essential in order to teach mathematics meaningfully. Teachers cannot teach what they themselves do not understand.

Preparing students to teach mathematics in elementary schools became a special project of the Department of Mathematics and the College of Education at the University of Oklahoma as early as 1955. In the fall of 1962, Oklahoma was selected as the location for the first of ten conferences held by the Committee on the Undergraduate Program in Mathematics (CUPM). The problems discussed were associated with establishing in Oklahoma colleges and universities the curriculum recommended by the teacher training panel of CUPM for the preparation of elementary school teachers. As a result of this and subsequent conferences, an outline for two semesters (six hours) of mathematics courses for elementary teachers was developed. Professor Eunice M. Lewis and Professor Dora McFarland published a book entitled Introduction to Modern Mathematics for Elementary Teachers which covers, according to the authors, the subject matter suggested for the first six hours of the Level I recommendation of the CUPM.

The first eight chapters in the book are concerned primarily with a systematic development of the real numbers from the concept of set through the system of natural and whole numbers. Included also is a historical approach to the study of numeration systems and an introduction to sets of points. These chapters constitute a one-semester course offered at the University of Oklahoma as Mathematics 70, which is required for the prospective elementary teacher. Chapters eight through fourteen comprise another three-hour course offered as Mathematics 170. This course, however, is not required.

The College of Education at the University of Oklahoma requires the following courses in mathematics and in methods as part of their program to prepare teachers for elementary schools. The following courses are included:

1. Mathematics 70 (Mathematics for Elementary Teachers), described above. Mathematics 70 is the prerequisite for Education 243 and Mathematics 170. This course is required.
2. Education 243 (Arithmetic in Elementary Schools) is a two-hour course emphasizing scope and sequence in arithmetic as related to curriculum programs; the meaning and development of arithmetic concepts as related to classroom teaching, and the evaluation of arithmetic materials in the curriculum. This course is required.
3. Mathematics 170 (Mathematics for Elementary Teachers), described above. Although this course is not required it is recommended, as it is the continuation of Mathematics 70.

Since evaluation is an integral part of any educational program, a study is needed to evaluate the effectiveness of Mathematics 70 and the other courses in developing basic mathematical concepts. Such evaluation of a program can serve to improve its effectiveness in several ways:

1. Evaluation can establish levels of learning and locate a student at a level suitable for his current status in mathematics.

2. Evaluation is useful in improving the mathematics program in terms of curriculum content and organization, selection of materials for learning, and modes of construction and learning. It can furnish data which should be used in making value judgments.

Definition of Terms

TUCM - A test, designed by the writer, for measuring the understanding of concepts taught in Mathematics 70. The higher the score the better the understanding.

ACT - American College Testing Program.

ACTC - Composite score of American College Testing Program.

ACTM - Mathematics subset of the American College Testing Program.

HSGPA - High School Grade Point Average.

CGPA - College Grade Point Average.

CHCM - Credit Hours in College Mathematics.

M_{G_i} - M = mean, G = group, i = the number of the course.

G_o - Group which does not have Mathematics 70.

G_{70} - Group which does have Mathematics 70.

G_{170} - Group which does have Mathematics 70 and Mathematics 170.

G_{243} - Group which does have Education 243 and Mathematics 70.

G_s - The seniors in the prospective elementary teachers' program, in the spring of 1968, who had Education 243 at least one semester before the spring of 1968.

Dutton scale - The attitude scale prepared by Dutton in 1951 according to technique developed by Thurston and Chane.[1] While the original scale was 27 items ranging in value from 1.0 (extreme dislike) to 10.5 (extreme like), the scale was reduced to 15 items by Dutton. The reduced scale provided adequate coverage of scale values at each point between 1 (dislike) and 11 (like).

Review of the Literature

Little has been written in regard to the determination of the growth and mastery of the mathematical under-

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[1]L. L. Thurston and E. J. Chane, The Measurement of Attitude (Chicago: University of Chicago Press, 1929).

standing possessed by prospective elementary teachers at different stages of their training program.

Glennon[2] has reported pertinent research on basic mathematical understanding in teacher education programs. He concluded that there is no significant difference in the achievement of basic mathematical understanding between:

1. Teachers' college freshmen and teachers' college seniors.
2. Teachers' college seniors who have taken a course in the Psychology and Teaching of Arithmetic and those who have not taken such a course.
3. Teachers in service who have done graduate work in Psychology and Teaching of Arithmetic and those who have not done such work.

Further, near-zero correlation was found between achievement in mathematical understanding and the length of teaching experience of in-service teachers.

Weaver[3] administered Glennon's test of basic understandings to the following groups of students as a pretest

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[2]J. Vincent Glennon, "A Study in Needed Redirection in the Preparation of Teachers of Arithmetic," Mathematics Teachers, XLII (December, 1949), pp. 389-96.

[3]J. F. Weaver, "A Crucial Problem in the Preparation of Elementary School Teachers," Elementary School Journal 56 (February, 1956), pp. 255-61.

at the outset of their undergraduate professional course in "Methods of Teaching Arithmetic": Group A, 92 students tested in the fall of 1953; Group B, 92 students tested in the fall of 1954; Group C, 63 students tested in the spring of 1955; Group D, 101 students tested in the fall of 1955. The majority of these 348 students were juniors; the rest were sophomores and specials.

On the average, students responded correctly to less than 75 per cent of items dealing with the decimal system of notation and the basic understandings of integers and processes. Furthermore, on the average they responded to less than 50 per cent of the items dealing with basic understandings of fractions and processes, decimals and processes, and rational computation. Their mean score of 44.60 on the test as a whole represents only slightly more than 55 per cent of the test of items. "These findings tend to substantiate those reported by Glennon regarding the lack of background in mathematical understanding on the part of undergraduates."[4]

Fulkerson[5] administered a 40 item arithmetic test to

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[4]Ibid., p. 257.

[5]E. Fulkerson, "How Well Do 158 Prospective Elementary Teachers Know Arithmetic?" The Arithmetic Teachers 7 (March, 1960), pp. 141-64.

158 students majoring in elementary education at Southern Illinois University. He found that "the mean number of items answered correctly is 20.0, or just one half the total number in the test."[6] He concluded that the prospective teachers studied in his sample had insufficient knowledge in arithmetic to teach effectively. He found that performance becomes progressively better as the level of classification becomes higher. The 5 freshmen answered correctly a mean number of 17.0 items; the 69 sophomores 19.1; the 51 juniors, 20.1 and the 30 seniors, 22.4. Also those students with more than two years of high school mathematics performed significantly better than did those with two years or less.

Buswell[7] administered a slightly modified version of a British Arithmetic test to 102 prospective elementary teachers in California. The mean score in the 70-item test of these college students was 44, or approximately 63 per cent of the total number of items. Buswell found that 34 per cent of the prospective elementary teachers made scores below the top one-third of eleven year old English students

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[6] Ibid., p. 145.

[7] G. T. Buswell, "The Content and Organization of Arithmetic," The Arithmetic Teacher 7 (March, 1960), pp. 77-83.

and 10 per cent made scores below the mean of California eleven-year-olds.

Bean[8] tested 450 Utah elementary school teachers and found that the teachers' scores in an 80-item test ranged from 18 to 78, with a mean score of 52.46 or 65.58 per cent of the total items. These teachers could do the computations very well, but were unable to explain the principles involved for 40 per cent of the items.

Creswell[9] reported a study involving 313 prospective elementary teachers who were to be graduated from eight teacher-training colleges in Georgia in June and August, 1963. These students were administered the Metropolitan Achievement Test, Advanced Arithmetic Form AM, 1959 Edition. The test consists of 93 items, 45 of which are concerned exclusively with computation, and 48 with concepts and problem solving.

The results of this study compare quite favorably with the results of the studies by Buswell and Fulkerson concerning the computations. Also, prospective elementary

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[8] J. E. Bean, "Arithmetical Understanding of Elementary Teachers," Elementary School Journal 59 (May, 1959), pp. 450-77.

[9] J. L. Creswell, "The Competence in Arithmetic of Prospective Georgia Elementary Teachers," The Arithmetic Teacher (April, 1964), pp. 248-50.

teachers in Georgia scored better in arithmetic concepts and problem solving than they did in arithmetic computation. "These results are in rather sharp contrast"[10] with those of Bean's study. Creswell[11] stated,

> The prospective teachers in Georgia did better in concepts and problem solving than they did on computation. Bean's group had much more trouble with concepts. The fact that the sample used in Bean's study were experienced teachers may or may not account for the contrast.

Creswell and Kowitz[12] undertook research to examine the retraining of teachers. This examination was twofold: namely, the investigation of the results of planned retraining programs and the results of teaching modern mathematics for one year (for the new teacher).

Concerning the planned retraining program, Creswell and Kowitz found the greatest increase in mean scores was among the group who participated in graduate courses designed for in-service teachers.

Concerning the results of new teachers who had taught modern mathematics for one year, they found this teaching

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[10] Ibid.

[11] Ibid.

[12] J. L. Creswell and G. T. Kowitz, Modern Mathematics: A Problem in Retraining, Bureau of Education Research and Services (Houston, Texas, August, 1966).

experience insignificant in comparison with background factors, such as hours of credit earned in mathematics.

Creswell and Kowitz recommended establishing workshops which included both content and methods. They found that the teacher with a greater number of credit hours in mathematics benefitted more from teaching experience.

Survey of the literature thus shows that prospective elementary teachers lack an understanding of arithmetic concepts and processes.

Many reasons for these results have been given, such as:[13]

> 1. Few of our present prospective elementary teachers have had the benefit of truly meaningful arithmetic instruction throughout their own elementary schooldays.
>
> 2. Many of these same students received little or no additional mathematics instruction as part of their secondary school work. Furthermore, those who did study mathematics at the secondary level generally pursued work which contributed little to their existing background of mathematical understanding.
>
> 3. Few students preparing to teach in the elementary school elect courses in the field of mathematics, and numerous institutions have no required work in background mathematics of any type as

[13]J. R. Weaver, "A Crucial Problem in the Preparation of Elementary School Teachers," Elementary School Journal 56 (February, 1956), p. 257.

a part of their teaching-training curriculum for elementary school.[14]

Recent studies such as Creswell's, however, show that there is some improvement in the understanding of mathematical concepts on the part of elementary teachers. This may be the result of increasing emphasis on the importance of arithmetic and mathematics in high schools and colleges today.

Also, these studies show that strong background in high school mathematics, teachers' experiences, and college credit hours in mathematics contribute significantly to the understanding of mathematical concepts on the part of prospective teachers.[15]

Selection and Description of Sample Data

The Sample

The population sample was selected at the end of the following semesters: Summer 1967, Fall 1967, Spring 1968. The sample was taken from students in the Elementary Education program, as follows:

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[14] A. K. Ruddell, W. Dutton, and J. Reckzeh, "Background Mathematics for Elementary Teachers," Instruction in Arithmetic, National Council of Teachers of Mathematics 15 (1960), pp. 296-320.

[15] Fulkerson, op. cit., p. 146.

1. (G_0) Students who have not taken Mathematics 70 and who are enrolled in Education 120.

2. (G_{70}) Students who are just finishing Mathematics 70.

3. (G_{243}) Students who are just finishing Education 243 and who have completed Mathematics 70.

4. (G_{170}) Students who are just finishing Mathematics 170 and who have completed Mathematics 70.

The number of each group (n) was chosen at random according to the central limit theory technique. This technique provides us with the formula:[16]

$$n = \frac{(Z\sigma)^2}{(\bar{X} - \mu)^2} = \frac{(Z\sigma)^2}{d^2}$$

such that σ is the standard deviation of the population, μ is the mean of the population, $\bar{X}$ is the mean of the Sample, and Z score will be determined according to the level of significance (5%, 1%).

The Data

Two types of data were collected. The primary data were taken from test scores of the TUCM in the spring of 1968.

J. P. Guilford, Fundamental Statistics in Psychology and Education (New York: McGraw-Hill Book Co., 1965), p. 215.

The TUCM is a test developed by the writer in the summer of 1967 for the purpose of measuring the understanding of concepts taught in Mathematics 70. It was administered during the summer of 1967 to a small sample. After some improvement of the test items it was administered in the fall of 1968 to 150 students who had taken Mathematics 70. The scores of the TUCM were collected to establish statistical information on the test.

This study is organized, in addition to the present section, into three additional chapters.

The second chapter will provide descriptions of planning, item writing, organizing, administering, and scoring the test. Also, it will provide statistical information about the test such as mean, variance, standard error, reliability, and validity.

The third chapter will provide the mathematical development of the multiple regression analysis, and its use in eliminating certain environmental effects from our estimate of understanding of the mathematical concepts.

The fourth chapter will provide the mathematical development of the analysis of the covariance program in Fortran 1 V, and use this to evaluate the growth and mastery of certain basic mathematical concepts.

Chapter five will be the summary of the information gathered in this study. Based on that information, recommendations for a program for training elementary teachers at the University of Oklahoma will be made.

CHAPTER II

CONSTRUCTION OF THE TEST

Test Construction

The achievement test was developed through the following stages:[1]

1. Planning the test; making decisions about the course objectives to be covered, the type of item to be constructed, and the number of items to be used for sampling each of the objectives.
2. Preparing the situation to be presented to the students, or item writing.
3. Organizing, administering, and scoring the test.

The above three steps were designed so as to yield reliable, relevant, and objective information about the understanding of the student's mathematical concepts. We shall consider, therefore, the procedures that were used for each of the steps to help obtain such information.

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[1]Jack C. Merwin, "Constructing Achievement Tests and Interpreting Scores," Evaluation in Mathematics, National Council of Teachers of Mathematics, 26th Yearbook (Washington, D. C., 1961), pp. 43-70.

Planning the Test

In developing the test the objectives to be covered were established. These objectives are the following:

1. The student should have an understanding of the concept of sets, the concept of number, and the relationship between sets and number.
2. The student should have an understanding of the historical background of numeration systems.
3. The student should have an understanding of the number system with bases other than ten.
4. The student should have an understanding of the concept of sets of points.
5. The student should have an understanding of the natural number system.
6. The student should have an understanding of the system of whole numbers.
7. The student should have an understanding of the algorithms used in the computation with whole numbers.

Types of Items

Many mathematicians emphasize the subjective-type items, usually identified by such words as solve, add, prove, and name. These items, however, are evaluated in terms of the final answer produced, with scant attention given to the processes involved.[2] Often, however, a great deal can be learned through the use of objective questions (multiple-choice, matching, or true or false) measuring individual steps in process.[3] Furthermore, items of the objective type permit a much wider sampling of the knowledge and understanding possessed by a student than those in which a premium is placed only upon obtaining the correct answer to a problem.

The multiple-choice type of item is usually regarded as the most valuable and most generally applicable of all test forms.[4] Lindquist asserted that it is definitely superior to all other types for measuring such educational objectives as "inferential reasoning, reasoned understanding, or sound judgment and discrimination on the part of the pupil."[5]

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[2]Max Sobel and Donovan J. Johnson, Analysis of Illustrative Test Items, National Council of Teachers of Mathematics, 26th Yearbook (1961), p. 74.

[3]Ibid.

[4]Julian C. Stanley, Measurement in Today's School (Englewood Cliffs, N. J.: Prentice Hall, Inc., 1964), p. 222.

[5]H. E. Hawkes, E. F. Lindquist, and C. R. Mann, The Construction and Uses of Achievement Examination (Boston: Houghton Mifflin, 1963), p. 138.

There are several forms of multiple-choice items. Walter D. Cook[6] in the Encyclopedia of Educational Research states them as follows:

a. A test exercise of this type may consist of a direct question, followed by two or more statements, one of which is a correct answer.

b. It may consist of a direct question, followed by two or more statements, one of which is an incorrect answer.

c. It may consist of a direct question, followed by two or more statements, none of which is an entirely correct answer, but one of which is definitely a better answer than the other one.

d. It may consist of an incomplete statement, followed by two or more possible completions, one of which is correct or definitely a better answer than the other.

e. It may consist of a paragraph, diagram, chart, graph, map, or specifications, followed by statements, one of which is correct and one incorrect.

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[6] W. D. Cook, "Achievement Tests," Encyclopedia of Educational Research, Rev. ed. (New York: MacMillan, 1952), p. 1292.

The student may be directed to indicate his response by underlining or checking the correct response, or by writing or blackening its number in the space provided.

There are some disadvantages in the multiple-choice items, such as difficulty in construction, space consuming, and time consuming.[7] Although the multiple-choice item test has some disadvantages it is usually regarded as the most valuable of all test forms,[8] and superior to all test types[9] because of the following:

1. Objective.
2. Less open to guessing than the alternate response item (this type of test item requires the pupil to determine the truth or falsity of a statement, correctness or incorrectness).
3. Will adapt to measure understanding, discrimination, and judgment.[10]
4. Widely used and familiar to students.[11]

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[7]Dorothy A. Wood, _Test Construction_ (Columbus, Ohio: Merrill, 1960), Chapter 3.

[8]Stanley, _loc. cit._

[9]Hawkes, _et al._, _loc. cit._

[10]_Ibid._

[11]Wood, _loc. cit._

For the above reasons the Test of Understanding Concepts of Mathematics (TUCM) will be constructed as a multiple-choice items type.

Many of the published tests are in the form of multiple-choice items. For example, the American College Testing program (ACT), the College Entrance Examination, English items,[12] Kuhlmann-Fisch Intelligence Tests, test IV,[13] the Barrett-Ryan Literature test: Silas Marner,[14] Cooperative test of Social Studies Abilities: Experimental Form Q, Sequential Tests of Educational Progress (STEP): Science. Many multiple-choice tests also have been published in mathematics, such as Blyth Second-Year Algebra,[15] California Arithmetic Test,[16] Cooperative Elementary Algebra,[17] Cooperative General Mathematics Test for High School Classes,[18]

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[12]Stanley, op. cit., p. 226.

[13]Ibid., p. 224.

[14]Ibid., p. 225.

[15]Sheldon S. Meyers, Annotated Bibliography of Math Tests, The National Council of Teachers of Mathematics, Evaluation in Mathematics, 26th Yearbook (Washington, D. C., 1961), p. 183.

[16]Ibid., p. 185.

[17]Ibid., p. 189.

[18]Ibid., p. 189.

Cooperative Intermediate Algebra Test,[19] Cooperative Sequential test of Educational Progress (STEP): Mathematics,[20] Functional Evaluation in Mathematics,[21] Iowa Plane Geometry Aptitude Test (revised edition).[22] Some of these tests have four choices for every item such as STEP: Mathematics and Functional Evaluation in Mathematics. Others have five choices for every item, such as California Arithmetic Test and Cooperative Elementary Algebra Test. The use of four or five choices for every item will minimize chance success.[23]

For every item in the TUCM there were five choices. The use of five choices minimized chance success and also practically eliminated "response sets": the tendency for students to select a given option position, such as the second option, B, more often than would be predicted on the basis of chance alone. Position preferences of this kind tended to lower the validity of the test.[24]

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[19] Ibid., p. 190.

[20] Ibid., p. 194.

[21] Ibid., p. 196.

[22] Ibid., p. 200.

[23] Cook, op. cit., p. 1292.

[24] L. J. Cronbach, Educational and Psychological Measurement (1950), pp. 10, 3-31. Further evidence on response sets and test design.

Number of Items in the Test

There are no simple rules for determining the "right" number of items to use for each objective to be sampled.[25] In making such decisions the evaluator must consider how many items should be devoted to each objective. In writing the TUCM each of the seven objectives mentioned above should be covered; to do this, every one of the objectives will be divided into a sub-objective. At least one item will be written to cover every sub-objective.

Writing the Items

As an aid for writing test items, a number of rules have been established by testing specialists. Harry D. Berg[26] gives suggestions for increasing the thought content of objective items in social studies which he illustrates with four or five-option multiple choice items. Jason Millman offers 22 "multiple-choice item construction rules."[27] Merwin[28] considers seven basic rules that are generally applicable. Most of the rules are similar to those of Berg

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[25] Merwin, op. cit., p. 43.

[26] Stanley, op. cit., p. 231.

[27] Ibid.

[28] Merwin, op. cit., p. 43.

and Millman. Merwin's rules are as follows:

1. Make every effort to avoid ambiguities.
2. Avoid the use of excessive window dressing.
3. Avoid the use of long and involved statements.
4. Specify the degree of accuracy required for full credit when approximate answers are desired.
5. Avoid extraneous clues.
6. Avoid giving clues to one item in the statement of another.
7. Avoid the use of negative statements, whenever possible, and never use double negatives.

Merwin's rules will be followed in writing the items for the TUCM.

Also, in the choice of test questions the writer considered similar items from other published and unpublished tests which covered approximately the same material.

Test Items

I. Objective. The student should have an understanding of the concept of set and the concept of numbers and the relationship between the two.

Sub-objective 1. The student should understand the relationship between the concept of set and sub-set.

This sub-objective will be illustrated by the following item:

Let $X = \{a, e, i, o, u, x, y, z\}$ $V = \{e, i, o, u, y, z\}$
$Y = \{a, o, x, z\}$

1. Which of the following is true?

A) $V \subset X$ and $X \subset Y$
B) $V \subset X$ and $X \subset X$
C) $V \subset Y$ and $Y \subset X$
D) $Y \subset X$ and $X \subset V$
E) $Y \subset X$ and $X \subset Y$

<u>Sub-objective 2</u>. The student should have an understanding of the concepts of union and intersection.

This sub-objective will be illustrated by the following item:

Let $X = \{a, e, i, o, u, x, y, z\}$ $V = \{e, i, o, u, y, z\}$
$Y = \{a, o, x, z\}$

2. Which one of the following is equal to $\{X \cap V\} \cup Y$?

A) $\{o\}$
B) Y
C) V
D) X
E) $X \cap V$

<u>Sub-objective 3</u>. The student should have an understanding of the concept of the equivalent sets.

This sub-objective will be illustrated by the following item.

3. Considering these sets:
$U = \{a, b, x, y\}$ $V = \{3, 4, 6, 7, 2\}$ $X = \{\square, \triangle, \blacksquare, \circ\}$

Which of the following is true?

A) $U \simeq V$
B) $V \simeq X$
C) $U \not\simeq X$
D) $U \not\simeq V$
E) $U \not\simeq U$

<u>Sub-objective 4</u>. The student should have an understanding of the properties of any relation.

This sub-objective will be illustrated by the following item:

4. Considering the set A = $\{2,3,4,5\}$ and the "greater than" ($>$) relation on the set A; which of the following is true?

A) ($>$) relation on set A is reflexive only
B) ($>$) relation on set A is symmetric only
C) ($>$) relation on set A is transitive only
D) ($>$) relation on set A is equivalent
E) All the above are true.

<u>Sub-objective 5</u>. The student should have an understanding of the concept of ordinal numbers and cardinal numbers.

This sub-objective will be illustrated by the following item.

5. I was number 1 in line to buy 10, 5-cent stamps. What kind of numbers are used in this sentence?

A) 1 and 5 used as ordinal and 10 used as cardinal.
B) 1 and 10 used as cardinal and 5 used as ordinal.
C) 5 and 10 used as cardinal and 1 used as ordinal.
D) 1 and 10 used as ordinal and 5 used as cardinal.
E) 5 and 10 used as ordinal and 1 used as cardinal.

II. <u>Objective</u>: The student should have an understanding of the historical background of numeration systems.

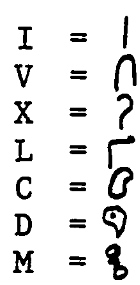
<u>Sub-objective 1</u>. The student should be familiar with the Egyptian numeration system.

This sub-objective will be illustrated by the following item.

6. Which characteristic of the Egyptian numeration system is true?
   A) It has place value.
   B) It has a symbol for zero.
   C) Position of symbols does not affect value.
   D) It is substractive.
   E) A and D are true.

<u>Sub-objective 2</u>. The student should have an understanding of the Roman numeration system.

This sub-objective will be illustrated by the following item.



Thousands are indicated by drawing a line over the symbol, for example:

 = 5,000

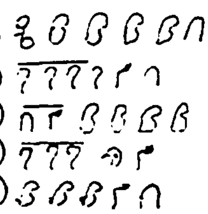
7.  equals?
   A) 1915
   B) 10315
   C) 10.915
   D) 10150
   E) 1015

Sub-objective 3. The student should have an understanding of the modern system with respect to the other, older systems.

This sub-objective will be illustrated by the following item.

8. $5(10^4) + 5(10^3) + 4(10^2)$ is represented by

A)
B)
C)
D)
E)



III. Objective: The student should have an understanding of the numeration system with bases other than ten.

Sub-objective 1. The student should have an understanding of the numeration system (base five).

This sub-objective will be illustrated by the following item.

9. Imagine a place where the inhabitants have only five fingers and numbers are written in a group of five. What symbol would be used in such a system to represent the number of marks in the following set:

* * * * * *
* * * * * *
* * * * * *
* * * * * *

A) 14
B) 24
C) 34
D) 44
E) 54

Sub-objective 2. The student should have an understanding of grouping in various bases.

This sub-objective will be illustrated in the following item.

10. The number of X's in the accompanying figure is written below in numerals in four different bases. Which number is correct?

XXXX	(I)	24_{five}
XXXX	(II)	14_{seven}
XXXX	(III)	12_{twelve}
XX	(IV)	1110_{two}

A) Only I is correct.
B) I, II are correct.
C) I, II, III are correct.
D) I, III, and IV are correct.
E) All of the above are correct.

Sub-objective 3. The student should have an understanding of the relationship between different numerals with different bases.

This sub-objective will be illustrated by the following item.

11. I $31_a = 16_c = 13$: then find a, c.

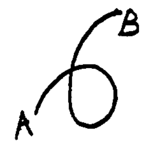
A) 2,4
B) 4,4
C) 7,4
D) 4,7
E) 7,7

IV. Objective: The student should have an understanding of the concept of sets of points.

Sub-objective 1. The student should be familiar with the meaning of the simple closed curve.

This sub-objective will be illustrated by the following item.

12. The following curve is:



A) a simple curve, but not closed.
B) a closed curve, but not a simple curve.
C) a curve that is not simple and not a closed curve.
D) a curve that is a simple curve and a closed curve.
E) none of the above is true.

Sub-objective 2. The student should be familiar with the meaning of the plane as a set of points.

This sub-objective will be illustrated by the following item.

13. The plane is:

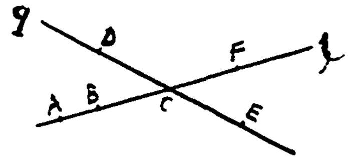
A) a finite set of points
B) the set of lines
C) the union of the finite sets of lines and finite sets of points
D) a particular set of points which is neither flat nor smooth
E) none of the above

Sub-objective 3. The student should have an understanding of the sets' operations as related to sets of points.

This sub-objective will be illustrated by the following item.

14. Which of the following is true?

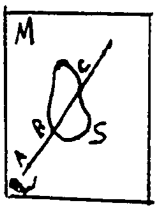
A) $C \notin \ell$
B) $C \notin q$
C) $\overrightarrow{FC} \cap \overrightarrow{AB} = \phi$
D) $\overrightarrow{ED} \cap \overrightarrow{FC} = \{B\}$
E) $\overrightarrow{EC} \cap \overline{FB} = \{C\}$



Sub-objective 4. The student should have an understanding of the set of interior points and exterior points as determined by a closed curve.

This sub-objective will be illustrated by the following item.

15. In this figure which of the following is true?
A) $\overline{BC} \cap \{\text{exterior of } S\} = BC$
B) $\ell \cap \{\text{region of } S\} = \phi$
C) $\{\text{interior of } S \cup \text{exterior of } S\} \cap \ell =$ all points on ℓ except B and C.
D) $M \cap S = M$
E) None of the above is true.



V. Objective: The student should have an understanding of the natural numbers.

Sub-objective 1. The student should have an understanding of the natural number and the cardinal number.

This sub-objective will be illustrated by the following item.

16. If X = {chair, stove, box, cat} and Y = {hat, coat} then which of the following is true?
A) N(X) = 2
B) N(Y) = 4
C) $N(X \cup Y) = 6$
D) N(X) + N(Y) = 8
E) N(AxB) = 12

<u>Sub-objective 2</u>. The student should have an understanding of the closure property for addition to the set of natural numbers.

This sub-objective will be illustrated by the following item.

17. Which one of the following sets is closed under addition?
    A) {1,2,3}
    B) {1,3,5,7,...}
    C) {10,5,15}
    D) {2,4,6,8,...}
    E) {1,3,4,6,8,...}

<u>Sub-objective 3</u>. The student should have an understanding of the relationship between the cartesian product of the sets and the multiplication operation in the set of natural numbers.

This sub-objective will be illustrated by the following item.

18. Given any two non-empty sets P = {s,t} and Q = {n,p,r} then P x Q is:
    A) {s,t,n,p,r}
    B) {(s,t) , (n,p,r)}
    C) {(s,n) , (s,p) , (s,r) , (t,n) , (t,s)}
    D) {(s,n) , (s,p) , (s,r) , (t,n) , (t,p) , (t,r)}
    E) {s x n, s x p, s x r, t x n, t x p, t x r}

<u>Sub-objective 4</u>. The student should be familiar with other relations defined in the natural numbers.

This sub-objective will be illustrated by the following item.

19. Which of the following statements is true?
    A) $18 < (3x5) + 3$
    B) $18 \ngtr (3x6)$
    C) $18 > (10x6) + 18$
    D) $18 \ngtr 2 \times 9$
    E) $18 \nleq 6 + (6+2)$

Sub-objective 5. The student should have an understanding of inequalities.

This sub-objective will be illustrated by the following item.

20. If $a > b$ and $c > d$ which of the following statements is always true?
    A) $a > c$
    B) $b > d$
    C) $a + d > b + c$
    D) $a + c > d + d$
    E $d + b < a + c$

VI. Objective: The student should have an understanding of the system of whole numbers.

Sub-objective 1. The student should have an understanding of operations other than addition and multiplication.

This sub-objective will be illustrated by the following item.

21. If a number system consisting of two elements and two binary operations and is completely defined by the tables shown: then which of the following statements is true?

$\oplus$	0	1
0	0	1
1	1	0

$\otimes$	0	1
0	0	0
1	0	1

    A) $0 \oplus (0 \otimes 1) = 1$
    B) $1 \oplus (1 \otimes 0) = 0$
    C) $1 \oplus (1 \otimes 1) = 1$
    D) $1 \otimes (1 \oplus 1) = 1$
    E) None of the above are true.

<u>Sub-objective 2</u>. The student should have an understanding of the properties of any defined operation on the whole numbers.

This sub-objective will be illustrated by the following item.

22. What property justifies [(9x6) + (2x6)] x 1 = (9x6) + (2x6)?

    A) Associative property for multiplication.
    B) Commutative property for multiplication.
    C) Distributive property over addition.
    D) Renaming 9 and 6 and 2.
    E) Multiplicative identity.

<u>Sub-objective 3</u>. The student should have an understanding of the addition property, as defined, for the whole number.

This sub-objective will be illustrated by the following item.

23. If we use Roman numerals we have X V + VI = X + V + V + I = X + (V+V) + I = X + X + I = X X I. Which of the following properties are illustrated?

    I commutative property for addition
    II associative property for addition
    III distributive property for addition

    A) none
    B) I only
    C) II only
    D) I and II only
    E) I, II, and III

<u>Sub-objective 4</u>. The student should have an understanding of the properties of the addition and multiplication operations, together.

The sub-objective will be illustrated by the following item.

24. If we have 8 + (8x4)
    = (8x1) + (8x4)
    = 8 x (1+4)

which of the following are used for justification?

I Commutative property for multiplication
II Distributive property for multiplication over addition
III Multiplicative identity

A) none
B) I and II
C) I and III
D) II and III
E) I and II and III

<u>Sub-objective</u> <u>5</u>. The student should have an understanding of the solution set of a sentence.

This sub-objective will be illustrated by the following item.

25. Which of the following is the solution set of the sentence: $\{b \in w: b > 5\} \cup \{b \in w: b < 9\}$?

A) $\{6,7,8\}$
B) $\{9,10,11,12,\ldots\}$
C) $\{8,7,6,5,4,3,2,1,0\}$
D) $\{6,7,8,9,\ldots\}$
E) $\{0,1,2,3,4,5,6,7,\ldots\}$

VIII. <u>Objective</u>: The student should have an understanding of arithmetic of the whole number.

<u>Sub-objective</u> <u>1</u>. The student should be familiar with the set of factors for any number of the whole numbers.

This sub-objective will be illustrated by the following item.

26. Which of the following sets represents F_{12} (F_{12} is the set of all factors of 12)?
    A) {1,2,6}
    B) {1,3,4}
    C) {1,2,3,4}
    D) {1,2,3,4,6}
    E) {1,2,3,4,6,12}

Sub-objective 2. The student should have an understanding of the nature of a prime number.

This sub-objective will be illustrated by the following item.

27. Which one of the following is a prime number?
    A) 51
    B) 55
    C) 57
    D) 59
    E) none of the above

Sub-objective 3. The student should be familiar with the fact that the only even prime number is 2.

The sub-objective will be illustrated by the following item.

28. How many even primes are in natural numbers?
    A) one
    B) infinite
    C) unknown
    D) equal to the number of odd primes
    E) none of the above is true

Sub-objective 4. The student should have an understanding of the concept of the greatest common divisor.

This sub-objective will be illustrated by the following item.

29. What is the greatest common factor of {65,420}?
 A) 35
 B) 7
 C) 15
 D) 5
 E) 65

Sub-objective 5. The student should have an understanding of the concept of the least common multiple.

This sub-objective will be illustrated by the following item.

30. Which of the following is the least common multiple of {45,15,10} ?
 A) 90
 B) 450
 C) 45
 D) 150
 E) 180

Organizing, Administering, and Scoring the Test

After the items were prepared, the next step was to organize them into a test. Two rules were followed: (a) "The ease with which the student can understand what he is to do, and where and how he recorded his answer, and (b) the ease with which the teacher will be able to locate and score the answer."[29] In consideration of these two rules, the following were considered:

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[29] Merwin, *loc. cit.*

Grouping the items: The items were grouped according to the objectives. Also, the items of every group were arranged so that the easier items came before the more difficult items.

Directions: The directions state clearly that the student will find the correct answer from the lettered choices and match it with the number, corresponding to the letter, on the machine answer sheet (I.B.M. form I.T.S. 1000A155). The directions also state that the student may use scratch paper.

After consideration of the above rules, the test for understanding of concepts of Mathematics 70 (TUCM) was written, a copy of which is included in Appendix A.

Administration: Every student worked from a separate copy of the test. All the materials, such as scratch papers, erasers, and pencils were available to the student. The best possible physical surroundings were provided: adequate light, ventilation, and desk space. The student was verbally instructed by the administrator to record on the answer sheet the time spent in finishing the test. The maximum time recorded was 55 minutes.

Speed was not a factor in the objectives covered by the test. Enough time was allowed so that each student had an opportunity to attempt all items. The symbols used in the test were explained by the administrator before the student began working the test. The symbols and definitions, such as subset($\subset$), were written on the blackboard by the administrator.

Scoring the Items: The answer key for the test was prepared and punched on test-scoring machine sheet I.B.M. Form I.T.S. 100A310.

Information about Achievement of the Student

The TUCM was administered to 150 students who had just finished Mathematics 70, at the University of Oklahoma, Summer 1968 and Fall 1968. The set of TUCM scores was collected. The measures of central tendency (means, median, and the mode) and some of the measure of dispersion were computed.

The following is a summary of the measure of central tendency.

Mean = 19.52 Median = 19

Mode = 19

The following is a summary of some of the measures of dispersion.

Range = 22 Variance = 25.806

Standard deviation = 5.080

Information about the Test: Results of a test provide information about reliability and validity. Reliability is consistency or stability of measurement.[30] Validity is usefulness for a given purpose, especially on outcome.[31]

Reliability: Reliability information is usually summarized in a reliability coefficient. Many procedures have been proposed for obtaining a reliability coefficient.[32] The most widely used is the Kuder-Richardson procedure,[33] expressed by the formula:[34]

$$r_{tt} = \frac{n}{n-1}\left(\frac{\sigma_t^2 - \Sigma pq}{\sigma_t^2}\right)$$

where n = number of items in the test.

p = proportion passing an item (or responding in some specified manner).

q = 1-p

σ_t = the standard deviation of the raw scores.

Another Kuder-Richardson procedure which gives a

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[30]Stanley, op. cit., p. 150.

[31]Ibid., p. 150.

[32]Ibid., p. 151-158.

[33]M. W. Richardson and G. F. Kuder, "The Calculation of Test Reliability Coefficient, Based Upon the Method of Rational Equivalence," Journal of Educational Psychology (1939), p. 681-687.

[34]Ibid.

coefficient that is generally a reasonable approximation of that described previously is expressed in the formula:[35]

$$r_{11} = \frac{n}{n-1}\left[1 - \frac{m\left(1-\frac{m}{n}\right)}{s^2}\right]$$

where m = the mean of the scores.

n = the number of items.

s = the standard deviation.

Using this formula the TUCM, with 30 items, a mean of 19.52 and a standard deviation of 5.080, has a reliability coefficient of:

$$r_{11} = \frac{30}{29}\left[1- \frac{19.520\ (30-19.52)}{30 \times 25.806}\right] = .76$$

The second procedure was used for the TUCM, for the following reasons:

1. The formula is set up for use with tests which are scored 1 for the right answer and 0 for the wrong answer, which is the procedure followed for scoring the TUCM.
2. Time is not a factor in determining the performance of the student. (Kuder-Richardson gives a "spuriously high coefficient for speeded tests".)[36]

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[35] Merwin, op. cit., p. 66.

[36] Ibid., p. 67.

3. The formula involves less computation than the other procedures.

The reliability coefficient for the TUCM is .76. On standardized achievement tests reliability coefficients are found to be above .75.[37]

Standard Error of Measurement. The possibility of errors of different magnitudes may be estimated with the standard error of measurement (SEM) which is defined by the following formula:[38]

$$SEM = S\sqrt{1-r_{11}}$$

where S = standard deviation

r = reliability coefficient of the test

Based on the assumption that errors of measurement are normally distributed. Using this formula:

$$\text{SEM for TUCM} = 5.080\sqrt{1 - .760}$$

$$= 2.41$$

It may be said that the odds are about 2 to 1 that a student's obtained score on the TUCM is no more than one standard error of measurement (2.41) from his true score and about 19 to 1 that this difference is no more than two standard errors

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[37] Ibid., p. 67.

[38] Ibid., p. 67.

of measurement (4,32).

Validity. Predictive validity information is usually summarized by a validity coefficient. The coefficient of predictive validity is the coefficient of correlation between the predictor and the criterion scores for a number of individuals. Since the TUCM covers the material in Mathematics 70, the criterion scores were the scores of the TUCM. The predictor was the grades of the same students in Mathematics 70.

In order to reduce the effect of other college work on the performance on the TUCM, the following criteria were applied to the scores of the 150 students:

1. The only college mathematics course completed by any of the students was Mathematics 70.
2. None of these students had enrolled in Education 243.

The number of students available who satisfied these two conditions, out of the 150 tested, was 82. Table 1 reflects the test scores against the grades in Mathematics 70.

In this table the number of cases was small. The distributions were dichotomous, and the prediction to be made was in one of two categories for each variable. In this case a phi coefficient for the correlation was preferred.[39]

[39]Guilford, op. cit., p. 499.

TABLE 1

CALCULATION OF Phi CORRELATION
BETWEEN THE TUCM SCORES AND
GRADE IN MATHEMATICS 70

	Grade A, B	Grade C, D
Above the mean in TUCM	48 (a)	6 (b)
Below the mean in TUCM	8 (c)	20 (d)

Since the frequence of two cells in the table was less than ten, the phi coefficient, with Yates correction, was applied to find the correlation between the TUCM scores and the corresponding grade in Mathematics 70. The phi coefficient was computed as follows:[40]

$$\text{phi} = \frac{\left| ad - bc \right| - n/2}{(a+b)(a+c)(b+d)(c+d)}$$

$$= \frac{\left| 20 \times 48 - 6 \times 8 \right| - 41}{56 \times 54 \times 26 \times 28}$$

$$= \frac{\left| 19 \times 48 \right| - 41}{56 \times 54 \times 26 \times 28}$$

$$= .587$$

$$\chi^2 = N(\text{phi})^2$$

$$= 82 \times .344569 = 28.25$$

[40]George A. Ferguson, Statistical Analysis in Psychology and Education (New York: McGraw-Hill Book Co., 1959), p. 196.

The critical value of χ^2 at the one per cent level of significance, with one degree of freedom, is 6.04. Hence, an χ^2 of 28.25 is significant beyond the one per cent level of significance; therefore, a phi of .587, likewise, is significant.[41]

The previous result showed that the TUCM scores correlated significantly with the grade in Mathematics 70.

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[41] *Ibid*., p. 199.

CHAPTER III

MULTIPLE REGRESSION ANALYSIS OF THE CRITERION VARIABLE WITH THE INDEPENDENT VARIABLES

The analysis of covariance which will be used in Chapter IV allows for adjustment in the TUCM scores of the four groups (G_o, G_{70}, G_{243}, and G_s), based on initial differences in certain independent variables. Thus it is necessary to determine the important measurable variables which aid in predicting success on the TUCM.

A survey of the literature indicates only a few studies dealing with the problem of measurable variables which predict the understanding or achivement of mathematics for elementary teachers. One of the studies indicated that a strong background in mathematics in high school and college seemed to contribute to the understanding of elementary teachers.

Numerous studies have reported the predicative variable for use as a placement guideline for freshmen in mathematics. Shana'a[1] reported that the best variable for use as a placement guide line for freshmen in mathematics appeared to be the American College Testing, Mathematics (ACTM) and that good predictive variables included High School Grade Point Average (HSGPA). Other measurable variables which were available were the American College Testing, Composite (ACTC) score and the Dutton Attitude Scale (DATS).

The independent variables which were considered in this study were: ACTC scores, ACTM scores, grade point average (CGPA), the number of credit hours in Mathematics (CHCM), the high school grade point average (HSGPA), and the score value on the Dutton scale (DATS).

The criterion variable was the student's score on the Test of the Understanding of Concepts of Mathematics. (TUCM).

In order to achieve the goal[2] of this part of the study, as stated previously, the multiple regression analysis

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[1]Joyce Adrin Shana's, "A Statistical Analysis of the Placement Program in Mathematics for Freshmen at the University of Oklahoma," Ph.D. Dissertation, University of Oklhaoma, 1966.

[2]Ostle, loc. cit.

was used. In the multiple regression analysis a linear relationship, of the form $Y = B_o + B_1X_1 + \ldots B_6X_6$, was assumed between the criterion variable and the independent variables. The B_j's were approximated from the data available by determining the best fitting curve to the data under the least square technique. The mathematical basis for this analysis can be found in Appendix II. The computer program for the multiple regression analysis can be found in Appendix II. The program was set up to find the following:

1. The mean of every variable.
2. Standard deviation of every variable.
3. Correlation coefficients between the independent variables and the dependent variable.
4. Partial regression coefficients (b_j, $j = 1,\ldots 6$) - the estimates for the unknown parameters (B_j, $j - 1,\ldots 6$) of the multiple regression equation $Y = B_1X_1 + \ldots B_6X_6$.
5. Standard error for regression coefficients.
6. Computed t-values.
7. Intercept (b_o) of the estimate of the unknown parameter B_o of the multiple regression equation $Y = B_o + B_1X_1 = \ldots B_6X_6$.
8. Multiple correlation coefficients.

9. Standard error of estimate

10. Analysis of variance for the multiple regression.

The Sample

The sample for the multiple regression analysis consisted of 74 students who took Mathematics 70 (Arithmetic for Elementary Teachers) in the spring of 1968. The TUCM was administered to this sample. The scores on this test were collected and considered in the multiple regression analysis subroutine as the dependent variable Y.

The six independent variables used in this study were:

variable X_1 - the ACTC scores

variable X_2 - the ACTM scores

variable X_3 - the CGPA

variable X_4 - the CHCM

variable X_5 - the HSGPA

variable X_6 - the DATS

Variables X_1 through X_5 were collected from the students' files. Variable X_6, the Dutton test, was given before the TUCM was administered and the value scale was recorded.

The seven measurable variables for every student were recorded and keypunched on separate cards. The following hypotheses were then tested, using the data (Table 2) resulting from the multiple regression analysis.

The analysis with respect to the American College Testing Composite (ACTC):

H_{0a} : 1. The correlation between the scores in the TUCM and the scores in the ACTC was not significantly different from zero.

The correlation coefficient (r) between the TUCM scores and the ACTC scores was .405, which was greater than the critical value for r (.303) at the one per cent level of significance.

Hence, this hypothesis was rejected at the one per cent level of significance. Thus the ACTC scores were positively correlated to the prospective elementary teacher's understanding of mathematical concepts.

H_{0a} : 2. The partial regression coefficient of the ACTC scores was not significantly different from zero.

The computed T value for the partial regression coefficient of the ACTC scores was .928, which was less than the critical value for T (1.990) at the five per cent level of significance for a two-tail test.

Hence, this hypothesis failed to be rejected at the five per cent level of significance. Thus the ACTC scores

TABLE 2

MULTIPLE REGRESSION ANALYSIS OF THE SAMPLE

Variable	Mean	Standard Deviation	Correlation Var. vs TUCM	Regression Coefficient	Std. Error of Reg. Coef.	Computed T Value
ACTC	20.04053	4.79850	0.40502**	0.11087	0.11942	0.92843
ACTM	22.16216	3.27307	0.37831**	0.06136	0.16422	0.37361
CGPA	2.7351	0.42865	0.36895**	1.94256	0.91636	2.11985*
CHCM	1.88513	3.02788	0.14768	-0.18325	0.13870	-1.32122
HSGPA	3.31594	0.48989	0.41909**	2.62029	0.88166	2.97199**
DATS	5.47024	2.15956	0.37737**	0.63284	0.20922	3.02483**
Dependent						
TUCM	22.03150	3.84812				

		Statistic	Critical value for 5% level of significance	Critical value for 1% level of significance
Intercept	1.31679	Correlation(r)	.232(two tail test)[a]	.303(two tail test)[a]
Multiple Correlation	0.63883**	Computed(T)	1.990(two tail test)[a]	2.640(two tail test)[a]
Std. Error of Est.	3.09027	F ratio	3.72[a]	7.02[a]

ANALYSIS OF VARIANCE FOR THE REGRESSION

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Squares	F Value
Attributable to Regression	6	441.15161	73.52527	7.69917**
Deviation from Regression	67	639.83472	9.54977	
Total	73	1080.98633		

* Indicates a difference that is significant beyond the 5% level of confidence.
** Indicates a difference that is significant beyond the 1% level of confidence.

[a]George A. Ferguson, <u>Statistical Analysis in Psychology and Education</u> (New York: McGraw-Hill Book Company, 1959), pp. 306-31.

Note: All numbers from this table rounded off to 3 places.

were not of great importance in estimating the criterion variable.[3]

The analysis with respect to the American College Testing Mathematics (ACTM):

H_{o_b}: 1. The correlation between the scores in the TUCM and the scores in the ACTM was not significantly different from zero.

The correlation coefficient (r) between the TUCM scores and the ACTM scores was .378, which was greater than the critical value of r (.303) at the one per cent level of significance.

Hence this hypothesis was rejected at the one per cent level of significance. Thus the ACTM scores were positively correlated to the prospective elementary teacher's understanding of mathematical concepts.

H_{o_b}: 2. The partial regression coefficient of the ACTM scores was not significantly different from zero.

The computed T value for the partial regression coefficient of the ACTM scores was .373, which was less than the critical value for T (1.990) at the five per cent level of significance for a two-tail test.

Hence, this hypothesis failed to be rejected at the

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[3] *Ibid.*, p. 218.

five per cent level of significance. Thus, the ACTM score was not of great importance in estimating the criterion variable.

The analysis with respect to the College Grade Point Average (CGPA):

H_{o_c}: 1. The correlation between the scores in the TUCM and the college grade point average was not significantly different from zero.

The correlation coefficient (r) between the TUCM scores and the CGPA was .368, which was greater than the critical value of r (.303) at the one per cent level of significance.

Hence, this hypothesis was rejected at the one per cent level of significance. Thus the CGPA was positively correlated to the prospective elementary teacher's understanding of mathematical concepts.

H_{o_c}: 2. The partial regression coefficient of CGPA was not significantly different from zero.

The computed t value for the partial regression coefficient of CGPA was 2.119, which was greater than the critical value for t (1.990) at the five per cent level of significance for a two-tail test.

Hence, this hypothesis was rejected at the five per cent level of significance. Thus CGPA was of great importance in estimating criterion variable.[4]

The analysis with respect to the number of Credit Hours in College Mathematics (CHCM):

H_{o_d}: 1. The correlation between the scores in TUCM and the number of college credit hours in mathematics was not significantly different from zero.

The correlation coefficient (r) between the TUCM scores and the CHCM was .147, which was less than the critical value of r (.232) at the five per cent level of significance.

Hence, this hypothesis failed to be rejected at the five per cent level of significance. Thus the CHCM did not correlate positively with the prospective elementary teacher's understanding of mathematical concepts.

H_{o_d}: 2. The partial regression coefficient of CHCM was not significantly different from zero.

The computed t value for the partial regression coefficient of the CHCM was -1.321, which was less than the critical value for t (1.990) at the five per cent level of significance for a two-tail test.

Hence, the hypothesis failed to be rejected at the

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[4] Ibid.

five per cent level of significance. Thus CHCM was not of great importance in estimating the criterion variable.

The analysis with respect to the High School Grade Point Average (HSGPA):

H_{O_e}: 1. The correlation between the scores of the TUCM and the high school grade point average was not significantly different from zero.

The correlation coefficient (r) between the TUCM and the HSGPA was .419, which was greater than the critical value of r (.303) at the one per cent level of significance.

Hence, this hypothesis was rejected at the one per cent level of significance. Thus the HSGPA was positively correlated to the prospective elementary teacher's understanding of mathematical concepts.

H_{O_e}: 2. The partial regression coefficient of HSGPA was not significantly different from zero.

The computed t for the partial regression coefficient of the HSGPA was 2.971, which was greater than the critical value of t (2.640) at the one per cent level of significance for a two-tail test.

Hence, this hypothesis was rejected at the one per cent level of significance. Thus the high school grade point average was of great importance in estimating the criterion variable.

The analysis with respect to the Dutton Attitude Test (DATS):

H_{0f}: 1. The correlation between the scores of the TUCM and the Dutton scale value of the DATS was not significantly different from zero.

The correlation coefficient (r) between the TUCM scores and the scale values of the DATS was .419, which was greater than the critical value of r (.303) at the one per cent level of significance.

Hence, this hypothesis was rejected at the one per cent level of significance. Thus the scale value of the Dutton Attitude Test is positively correlated to the prospective elementary teacher's understanding of mathematical concepts.

H_{0f}: 2. The partial regression coefficient of DATS was not significantly different from zero.

The computed t for the partial regression coefficient of the scale values of the DATS was 3.024, which was greater than the critical value of t (2.640) at the one per cent level of significance for a two-tail test. Hence, this hypothesis was rejected at the one per cent level of significance. Thus, the Dutton value scale was of great importance.

variables together:

H_{o_g}: 1. The correlation between the TUCM and all the independent variables together was not significantly different as a whole from zero.

The multiple correlation coefficient (r) between the TUCM scores and all the independent variables together was .63883, which was greater than the critical value of r (.303) at the one per cent level of significance.

Hence, this hypothesis was rejected at the five per cent level of significance. Thus all the independent variables as a whole correlated positively with the prospective elementary teacher's understanding of mathematical concepts.

H_{o_g}: 2. The partial regression coefficient was not significantly different from zero.

The computed F value for the regression coefficient of all the independent variables was 7.699, which was greater than the critical value of F (7.02) at the one per cent level of significance.

Hence, this hypothesis was rejected at the one per cent level of significance. Thus the independent variables as a whole were of great importance in estimating the criterion variable.

CHAPTER IV

COVARIANCE ANALYSIS OF THE EFFECT OF CERTAIN EDUCATION AND MATHEMATICS COURSES ON THE GROWTH OF THE UNDERSTANDING OF MATHEMATICAL CONCEPTS

The evaluation of the effect of certain education and mathematics courses on the growth and mastery of the basic mathematical concepts of the prospective elementary teacher was explored in this chapter. In Chapter III it was shown that there were some independent variables which had a positive correlation with the criterion variable.

In order to examine the growth of the understanding of mathematical concepts in different stages in the elementary teacher's education program, taking into account the initial difference due to the previous independent variables, the analysis of covariance was applied.[1] In this technique six independent variables: ACTC, ACTM, HSGPA, CGPA, CHCM, and DATS were controlled, and consequently they were used to adjust the TUCM scores.

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[1]Fred N. Kerlinger, Foundations of Behavioral Research (New York: Holt, Rinehart and Winston, 1965, p. 348.

The mathematical basis for this analysis can be found in Appendix III. The computer program adapted from Cooley[2] was written in Fortran IV language, by the investigator, for the covariance analysis. This program can be found in Appendix III.

The covariance analysis computed the test of equality of experimental mean vectors with covariance control, where the total number of variables did not exceed 40. Some of the output of this program was as follows:

1. For each group KG = identification number of the groups, KN = identification number of the subjects in each group, adjusted group means, and group standard deviations.

2. Means for total sample.

3. Standard deviation for total sample.

4. Adjusted standard deviation on experimental variable for the total sample.

5. Coefficient for forming the adjusted means for each experimental variable.

In this study five groups were examined: prospective elementary teachers who had not had mathematics 70

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[2]Cooley and Lohnes, op. cit.

(Arithmetic for Elementary Teachers) (G_o); prospective elementary teachers who had had mathematics 70 (G_{70}); prospective elementary teachers who had had mathematics 70 and Education 243 (methods course) (G_{243}); prospective elementary teachers who had had mathematics 170 after completing mathematics 70 (G_{170}), and prospective elementary teachers who were seniors and had completed the program of training (G_s).

Due to the overlapping of the students of G_{170} with those of the other groups, it was difficult to find sufficient cases therein. However, the other groups covered such a range that it was deemed reasonable to discard group G_{170} in the covariance analysis, rather than to include them in any group.

The total group for the covariance analysis included 247 students, of which 87 were G_o, 74 were G_{70}, 47 were G_{243}, and 39 were G_s. The prospective elementary teachers were enrolled in the College of Education at the University of Oklahoma in the spring semester of 1968.

The data were collected through the administration of the Test of the Understanding of Concepts in Mathematics (TUCM) for the four groups at the end of the spring semester of 1968. The control variables, ACTC, ACTM, CGPA, CHCM, and HSGPA, were collected from the student's files. The DATS was given to each student before the TUCM was administered and the value scale also was recorded.

In order to determine the significance of the size of each group, the mean of the TUCM scores for each group used in the covariance analysis was computed and the following formula was applied.[3]

$$n = \frac{z^2 \sigma^2}{d^2}$$

n = size needed in order to achieve a significant deviation of the population mean.

z = the standard score at the level of significance.

σ = standard deviation of the population.

d = deviation from the mean of the sample to the mean of the population.

The calculation of the minimum as shown in Table 3 indicates that the minimum number of students (n) for any of the four groups, to be significant at the five per cent level of significance, was 33.69. Hence, the four groups had a significant number of students at the five per cent level of significance.

Before applying the covariance analysis the variances of the four groups were tested for homogeneity.

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[3]Guilford, op. cit., p. 215.

TABLE 3

MEANS OF THE GROUPS AND THE NUMBER OF STUDENTS

Group	G_o	G_{70}	G_{243}	G_s
n	78.	74.	47.	39.
Mean	8.3013	22.0135	17.829	16.2820

μ = The population mean is the mean score of TUCM = 19.52.
σ = The population standard deviation is the standard deviation of TUCM score = 5.08.
The differences between the mean of the groups $\bar{X}_{G_i}$ and the population mean $d = \bar{X}_{G_i} - \mu$, i = 0,70,243,s.

d_{G_o} = 11.2187 $d_{G_{70}}$ = 2.673

$d_{G_{243}}$ = 1.691 d_{G_s} = 3.2380

Minimum d = 1.691 $n = \frac{Z^2 \sigma^2}{d^2}$

$Z = 1.96$ at 5% level of significance.

$$\text{Minimum } n = \frac{(5.08)^2 x (1.96)^2}{(1.691)^2} = 33.96$$

Otherwise, a significant difference between the population means of the four groups might have been due to the non-homogeneity of their variance. In this case the conclusion about the means would have been in some doubt.[4] To show the homogeneity of these groups the following hypothesis was

[4] Ibid., p. 274.

tested.

H_{o_h}: 1. The four groups, G_o, G_{70}, G_{243}, and G_s, are from populations of like variance ($\sigma^2_{G_o} = \sigma^2_{G_{70}} = \sigma^2_{G_{243}} = \sigma^2_{G_s}$).

Table 4 shows the summary computation for Bartlett's Test for homogeneity of the variances. The null hypothesis failed to be rejected at the five per cent level of significance. Thus the four groups had homogeneous variables.

Analysis of Variance with Adjusted Covariance Control

The analysis of variance with adjusted covariance control was performed on the four independent groups G_o, G_{243}, G_{70}, and G_s where the scores earned in the TUCM were adjusted by the control variables ACTC, ACTM, CHCM, CGPA, HSGPA, and the value scales of the DATS. A summary of the analysis of variance with covariance control is represented in Table 5.

Referring to Table 5, the hypothesis can be tested.

H_{o_i}: 1. The adjusted scores on the TUCM of G_o, G_{70}, G_{243}, and G_s are from a population of like means [$\mu_{G_o} = \mu_{G_{70}} = \mu_{G_{243}} = \mu_{G_s}$]

H_{1_i}: 1. $\mu_{G_o} \neq \mu_{G_{70}} \neq \mu_{G_{243}} \neq \mu_{G_s}$.

TABLE 4

COMPUTATIONS FOR BARTLETT'S TEST FOR HOMOGENEITY OF VARIANCE

$$H_o: \sigma^2_{G_o} = \sigma^2_{G_{70}} = \sigma^2_{G_{243}} = \sigma^2_{G_s}$$

Sample	X_i^2	Degree of Freedom	1/8F	S_i^2	log S_i^2	
G_o	1258.63	86	.0116	10.3684	1.0170	87.4620
G_{70}	1080.98	73	.0169	10.9561	1.037	75.701
G_{243}	1311.898	38	.0263	10.6276	1.025	38.950
G_s	712.6385	46	.0217	10.7584	1.029	47.3341
Sum	4364.1463	243	.0765			249.4470

Pooled estimate of variance = S^2 = 17.96

$$B = \log_{10} S^2 \sum_{i=1}^{u} (n_i - 1) = 304.4547$$

$$\chi^2_3 = \log_e 10 \left[B - \sum_{i=1}^{n} (n_i - 1) \log_{10} S_i^2\right]$$

$$= \log 550.07 = 2.7404$$

$$\text{Correction factor} = 1 + \frac{1}{3x3} [\ .0765) - \frac{1}{243})] = 1.008$$

$$\text{Corrected}\ \chi^2_3 = \frac{2.7404}{1.008} = 2.71$$

At the 5% level of significance failed to reject the null hypothesis.

TABLE 5

SUMMARY OF THE ANALYSIS OF VARIANCE
WITH COVARIANCE CONTROL

Source	Degree of Freedom	Sum of Square	Mean of Sum of Square	F
Treatment (Among)	3	7915.5572	2638.5190	**
Error (Within)	243	4364.1528	17.96	146.91
Total	246	12279.7100		

Standard deviation for the total sample = 7.0652

TABLE OF THE MEANS OF THE GROUPS

Group	G_o	G_{70}	G_{243}	G_s	Total
Mean	8.3013	22.0135	17.829	16.2820	15.654

$F_1 = 3$ $F_2 = 237$

$F = 160.759$ **

Coefficients for adjusting means of dependent variable:

0.2529268 -0.570224 0.9020426 0.1732690 0.5676005 0.3077125

TABLE OF THE ADJUSTED MEANS OF THE GROUPS

Group	G_o	G_{70}	G_{243}	G_s
Mean	9.87	21.784	17.55	15.34

* indicates a difference that is significant beyond the 5% level of significance.

** indicates a difference that is significant beyond the 1% level of significance.

The computed F ratio for the analysis of variance with covariance control was 160.759, which was greater than the critical value of F (3.88) at the one per cent level of significance. The homogeneity of the groups has been shown by Bartlett's test for homogeneity. Hence, F must indicate significant differences somewhere among the means.[5]

Hence, this null hypothesis was rejected at the one per cent level of significance. Thus the alternative hypothesis, that the population means of the four were different, was accepted. Hence, the four groups, G_o, G_{70}, G_{243}, and G_s, differed in their understanding of the mathematical concepts.

The computed F ratio was very large; this was due possibly to the following:

1. The large differences between two or more means of the groups, such as G_{70} and G_o.

2. The unequal number of cases for each group always contributes to the bias of the value of the F ratio.[6]

Since F was significant, then the student t test was applied to compare between the population adjusted means of the groups, two at a time.[7]

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[5] Ibid.

[6] Ferguson, op. cit., p. 243.

[7] Ibid., p. 238.

Table 6 shows the matrix of the computed t between the population adjusted means of the group, two at a time, using the following formula:[8]

$$t = \frac{\bar{X}_{G_i} - \bar{X}_{G_j}}{\sqrt{S^2_{w}/n_{G_i} + S^2_{w}/n_{G_j}}}$$

i, j = 0, 70, 243, 2

$\bar{X}_{G_i}$ = the adjusted mean of the group G_i

i = 0,70,243,s

$\bar{X}_{G_j}$ = the adjusted mean of the group G_j

j = 0,70,243,s

S_w = the within group variance

n_{G_i} = the number of cases in G_i

n_{G_j} = the number of cases in G_j

Since an assertion was made (by the investigator) about the direction of the difference, the one-tail test was applied.[9]

Referring to Table 6, the following hypothesis can be tested:

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[8]*Ibid*.

[9]*Ibid*., p. 135.

TABLE 6

MATRIX OF THE STUDENTS' t TEST BETWEEN THE MEANS OF THE GROUPS

	G_o	G_{70}	G_{243}	G_s
G_{70}	17.01**			
G_{243}	10.00**	5.37**		
G_s	6.88**	7.70**	2.41**	

$$t = \frac{\bar{X}_{G_i} - \bar{X}_{G_j}}{\sqrt{s^2 w/_{n_{G_i}} + s^2 w/_{n_{G_i}}}}$$

Degree of freedom 243

* indicates a difference that is significant beyond the 5% level of significance.

** indicates a difference that is significant beyond the 1% level of significance.

H_{o_i}: 2. The scores on the TUCM of G_o and G_{70} are from populations of like means $[\mu_{G_o} = \mu_{G_{70}}]$

H_{1_i} : 2. $\mu_{G_{70}} > \mu_{G_o}$

The computed t value between the population means of

G_o and G_{70} was 17.01 , which was greater than the critical value of t (2.35) at the one per cent level of significance with 243 degrees of freedom.

Hence, this null hypothesis was rejected at the one per cent level of significance. Thus the alternative hypothesis that the population mean of G_{70} was significantly greater than the population mean of G_o was accepted. Hence, the prospective elementary teachers who had Mathematics 70 performed better on the TUCM than the prospective elementary teachers who had not had Mathematics 70.

H_{o_i}: 3. The scores on the TUCM of G_o and G_{243} are from populations of like means. [$\mu_{G_o} = \mu_{G_{243}}$]

H_{1_i}: 3. $\mu_{G_{243}} > \mu_{G_o}$

The computed t value between the population means of G_o and G_{243} was 10, which was greater than the critical value of t (2.35) at the one per cent level of significance with 243 degrees of freedom.

Hence, this null hypothesis was rejected at the one per cent level of significance. Thus the alternative hypothesis that the population mean of G_{243} was significantly greater than the population of G_o was accepted. Hence, the prospective elementary teachers who had had Mathematics 70 and Education 243 performed better on the TUCM than the prospective elementary teachers who had not had Mathematics 70.

H_{o_i}: 4. The scores on the TUCM of G_o and G_s are from populations of like means. [$\mu_{G_o} = \mu_{G_s}$]

H_{1_i}: 4. $\mu_{G_s} > \mu_{G_o}$

The computed t value between the population means of G_o and G_s was greater than the critical value of t (2.35) at the one per cent level of significance, with 243 degrees of freedom.

Hence, the null hypothesis was rejected at the one per cent level of significance. Thus the alternative hypothesis that the population mean of the senior prospective elementary teachers was greater than the population mean of the prospective elementary teachers who had not had Mathematics 70 was accepted. Hence, the prospective elementary teachers who were seniors performed better on the TUCM than the prospective elementary teachers who had not had Mathematics 70.

H_{o_i}: 5. The scores on the TUCM of G_{70} and G_{243} are from populations of like means. [$\mu_{G_{70}} = \mu_{G_{243}}$]

H_{1_i}: 5. $\mu_{G_{70}} > \mu_{G_{243}}$

The computed t value between the population means of G_{70} and G_{243} was 5.37, which was greater than the critical value of t (2.6) at the one per cent level of significance with 243 degrees of freedom.

Hence, this null hypothesis was rejected at the one per cent level of significance. Thus the alternative hypothesis that the mean of the population of G_{70} is greater than the mean of the population of G_{243} was accepted. Hence, the prospective elementary teachers who had taken Mathematics 70 performed better on the TUCM than the prospective elementary teachers who had taken Mathematics 70 and Education 243.

H_{o_i}: 6. The scores on the TUCM of G_{70} and G_s are from populations of like means. [$\mu_{G_{70}} = \mu_{G_s}$]

H_{1_i}: 6. $\mu_{G_{70}} > \mu_{G_s}$

The computed t value between the population means of G_{70} and G_s was 7.70, which was greater than the critical value of t (2.35) at the one per cent level of significance with 243 degrees of freedom.

Hence, this null hypothesis was rejected at the one per cent level of significance. Thus the alternative hypothesis that the mean of the population of G_{70} is greater than the mean of the population of G_s was accepted. Hence, the prospective elementary school teachers who had just finished Mathematics 70 performed better on the TUCM than the seniors.

H_{o_i}: 7. The scores on the TUCM of G_s and G_{243} are from populations of like means. [$\mu_{G_s} = \mu_{G_{243}}$]

H_{1_i}: 7. $\mu_{G_{243}} > \mu_{G_s}$

The computed t value between the population mean of G_{243} and G_s was 2.41, which was greater than the critical value of t (2.35) at the one per cent level of significance with 243 degrees of freedom.

Hence, this null hypothesis was rejected. Thus the alternative hypothesis that the mean of the population of G_{243} was significantly greater than the mean of the population of the seniors was accepted. Also, in this case the prospective elementary teachers who had taken Mathematics 70 and Education 243 performed better on the TUCM than the seniors.

CHAPTER V

SUMMARY AND CONCLUSION

This study was undertaken with the objective of conducting statistical analysis to determine the growth and understanding of certain basic mathematical concepts possessed by the prospective elementary teachers at the University of Oklahoma at different stages in their education program.

Summary and Conclusion

One of the ways in which the data were gathered for this study was through the administration of the Test of Understanding Concepts of Mathematics (TUCM). Most of the published tests available in the library were designed before the introduction of modern mathematics into the curriculum of elementary teacher education programs. An examination of these tests by the researcher revealed no test which would meet the needs of this study. Therefore, the construction of an instrument that could be used in this study for gathering the data became a necessity.

An objective test, TUCM, with 30 items and 5 choices for every item, was constructed, a copy of which is included in Appendix I. The items of the test covered the following seven areas of basic mathematical understanding: 1. The concept of number and the concept of set; 2. the historical background of numeration; 3. the numeration system with bases other than ten; 4. the concept of sets of points; 5. the natural number system; 6. the whole number system; and 7. arithmetic of whole numbers.

The test was administered at the end of the fall semester of 1967 to 150 students who had just finished Mathematics 70 at the University of Oklahoma. The data were collected and the following information about the test was based on the collected data:

1. The reliability of the test as reflected by the Kuder-Richardson procedure was .76.

2. The standard error of measurement was estimated to be 2.41.

3. The mean score of the test was 19.52

4. The standard deviation was 5.08.

5. Validity is described in the phi correlation coefficient, which was .587, between the test (TUCM) and the grades earned in Mathematics 70; the value of phi was

significant.

The results from the analysis were the following:

1. The scores on the TUCM correlated significantly with the ACTM, ACTC, CGPA, HSGPA and the DATS. The scores on the TUCM did not correlate significantly with the number of Credit Hours in College Mathematics (CHCM).

The last result seemed to be negative in comparison with some studies reported in the literature, such as those of Fulkerson,[1] Creswell and Kowitz.[2]

In searching for the reasons for this result of the study, the researcher found that 27 per cent of the prospective elementary teachers had courses in mathematics other than Mathematics 70 (excluding the students who had Mathematics 170). About 85 per cent of the students who elected to take other mathematics courses had Mathematics 2 at the University of Oklahoma. The topics included in Mathematics 2, as stated by the Department of Mathematics at the University of Oklahoma, were the following: nature of mathematics, rational numbers, real numbers, complex numbers, equation and inequalities, function graphs and systems of

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[1]Fulkerson, _op_. _cit_., p. 141-64.

[2]Creswell and Kowitz, _op_. _cit_.

equations. The above list of topics for Mathematics 2 indicates that the contents of Mathematics 2 had little in common with the contents of the course of mathematics for elementary teachers, Mathematics 70.

2. The second result of the multiple regression analysis was that there were some variables, which were of great importance in estimating the criterion variable. These variables were College Grade Point Average (CGPA), High School Grade Point Average (HSGPA), and the value score on the Dutton Attitude Test (DATS). Even though the American College Testing, Mathematics and Composite (ACTM and ACTC) scores correlated significantly with TUCM scores they were not of great importance in estimating the TUCM scores, as shown by the regression analysis.

3. One of the most interesting results was that the values of the Dutton Attitude Scale not only correlated with the TUCM scores but also were of great importance in estimating the TUCM score. Therefore, the understanding of mathematical concepts correlated with the attitude toward arithmetic.

4. The last result indicated that the six independent variables as a whole not only correlated significantly with the TUCM scores but also were of great importance in

estimating the TUCM scores. The regression equation for estimating TUCM scores from the independent variable was:

$$Y = 1.31679 + 0.11087\ X_1 + 0.06136\ X_2 + 1.94256\ X_3 - 0.18325\ X_4 + 2.62029\ X_5 + 3.02483\ X_6$$

Y - the estimate of the TUCM scores

X_1 - the ACTC scores

X_2 - the ACTM scores

X_3 - the CGPA

X_4 - the CHCM

X_5 - the HSGPA

X_6 - the DATS

In this study analysis of covariance was performed on four groups to determine the significant difference between their population means with respect to their scores on the TUCM. These groups were the following:

1. Prospective elementary teachers who had not had Mathematics 70 (G_o).

2. Prospective elementary teachers who had Mathematics 70 (G_{70})

3. Prospective elementary teachers who had Mathematics 70 (G_{70}) and Education 243 (G_{243}).

4. Prospective elementary teachers who were seniors.

The scores on the TUCM were adjusted by the control variables ACTM, ACTC scores, CGPA, CHCM, HSGPA, and DATS.

The results from the analysis of covariance were the following:

1. The prospective elementary teachers who had only Mathematics 70 performed significantly better on the TUCM than the prospective elementary teachers who had not had Mathematics 70. Hence, the statistical evidence showed that Mathematics 70 contributed to the understanding of basic mathematical concepts.

2. The prospective elementary teachers who had just finished Mathematics 70 performed significantly better on the TUCM than the prospective elementary teachers who had previously had Mathematics 70 and Education 243. Hence, the statistical evidence showed that Education 243 did not contribute to the growth of the understanding of the mathematical concepts on the part of the elementary teachers.

3. The prospective elementary teachers who had Education 243 performed significantly better on the TUCM than the prospective elementary teachers who were seniors. Hence, the statistical evidence showed that there was a reduction of the understanding of mathematical concepts.

4. Those prospective elementary teachers who were

seniors performed on the TUCM significantly better than the prospective elementary teachers who had not started the sub-program of mathematics in their training.

Result number 4 seems to be in conflict with some studies, such as Glennon's. Considering the difference in the conditions of Glennon's study and this research, the above result seems to be in conflict with Glennon's conclusion that there is no significant difference between seniors and those students who are beginning the elementary teacher education program, on their understanding of the basic mathematical concepts. This apparent discrepancy may be attributed, in part, to the following:

1. The increasing emphasis put on the importance of mathematics and arithmetic in high schools and colleges today.

2. The development of new material which emphasizes the understanding of mathematical concepts.

Some of the conclusions of this study, although expected, were without statistical evidence, which has now been made available. As a result of the conclusions, other than those expected, the investigator submits the following recommendations for study by the Department of Mathematics and the College of Education at the University of Oklahoma.

The following recommendations are submitted for study to the Department of Mathematics, at the University of Oklahoma:

1. The TUCM can be used as an advanced test for Mathematics 70. The working time for this test does not exceed 55 minutes.

2. Mathematics 2 does not appear to be a good substitute for Mathematics 70.

3. The best single predictor of success in Mathematics 70 from all the six independent variables is the high school grade point average.

The following recommendations are submitted for study to the College of Education, at the University of Oklahoma:

1. The sample of students who had taken Mathematics 70, Education 243, and Mathematics 170 was 17 students, which was too small to include in the overall sample. But the mean of this sample was sufficiently high, 23.53, to suggest that because of the apparent decline in the understanding of mathematical concepts over a period of time, the addition of Mathematics 170 as a required course would be beneficial as a reinforcement tool.

2. The statistical evidence showed that Education 243 did not contribute significantly to the growth of the

understanding of mathematical concepts. It should be pointed out that the primary objective of Education 243 is not the development of mathematical concepts. The course emphasizes scope and sequence in arithmetic as related to curriculum programs; the meaning and development of arithmetic as related to classroom teaching and the evaluation of arithmetic material in the curriculum. However, one of the classes tested in the Education 243 group had a mean score of 22.50, which was close to the mean score of the group in Mathematics 70, which could indicate that individual teaching methods and possibly the mathematical background of the teacher has an effect upon the extent to which Education 243 reinforces the mathematical concepts learned in Mathematics 70. The investigator, therefore, recommends that greater consideration be given to the mathematical background of the teacher for the methods course, Education 243.

Recommendations for Future Research

The results of this study seem to indicate that more research is needed in the area of the growth of mathematical concepts.

1. Research is needed in the area of the effect of the psychological factors, such as goals, motivation,

readiness to learn and reinforcement on the growth of mathematical concepts.

2. Further research is needed in the area of the effect of the mathematical background of the teacher on the growth of the students' understanding of mathematical concepts.

3. Due to the dynamic nature of the teaching situation the researcher suggests that continual research is needed to obtain continued evaluation of the teacher education program.

4. Research is also needed to ascertain if Mathematics 170 should be required or if some other course is needed as a reinforcement tool.

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APPENDIX I

TEST FOR UNDERSTANDING CONCEPTS OF MATHEMATICS (TUCM)

AND THE ANSWERS

APPENDIX I

TEST FOR UNDERSTANDING CONCEPTS OF MATHEMATICS (TUCM)

THE UNIVERSITY OF OKLAHOMA

Name ____________________________

Directions: Choose the correct answer from the lettered choices. Use available space which you have on the page for your scratch work or extra paper.

In questions 1 and 2 consider: $X = \{a, e, i, o, u, x, y, z\}$ $V = \{e, i, o, u, y, z\}$ $Y = \{e, o, x, z\}$

1. Which of the following is true?
   A) $V \subset X$ and $X \subset Y$
   B) $V \subset X$ and $X \subset X$
   C) $V \subset Y$ and $Y \subset X$
   D) $Y \subset X$ and $X \subset V$
   E) $Y \subset X$ and $X \subset Y$

2. Which one of the following is equal to $\{X \cap V\} \cup Y$?
   A) $\{o\}$
   B) Y
   C) V
   D) X
   E) $X \cap V$

3. Considering these sets:

   $U = \{a, b, x, y\}$ $V = \{3, 4, 6, 7, 2\}$ X = {▱, △, □, O}

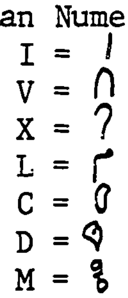
   Which of the following is true?
   A) $U \cong V$
   B) $V \cong X$
   C) $U \ncong X$
   D) $U \ncong V$
   E) $U \ncong U$

4. Considering the set A = {2,3,4,5} and the "greater than" (>) relation on the set A; which of the following is true?
   A) (>) relation on set A is reflexive only
   B) (>) relation on set A is symmetric only
   C) (>) relation on set A is transitive only
   D) (>) relation on set A is equivalent
   E) All the above are true.

5. I was number 1 in line to buy 10, 5-cent stamps. What kinds of numbers are used in this sentence?
   A) 1 and 5 used as ordinal and 10 used as cardinal.
   B) 1 and 10 used as cardinal and 5 used as ordinal.
   C) 5 and 10 used as cardinal and 1 used as ordinal.
   D) 1 and 10 used as ordinal and 5 used as cardinal.
   E) 5 and 10 used as ordinal and 1 used as cardinal.

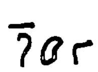
6. Which characteristic of the Egyptian numeration system is true?
   A) It has place value.
   B) It has a symbol for zero.
   C) Position of symbols does not affect value.
   D) It is substractive.
   E) A and D are true.

In questions 7 and 8 the following symbols are used in the same fashion as Roman Numerals:

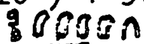


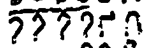
Thousands are indicated by drawing a line over the symbol, for example:

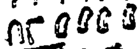
 = 5,000

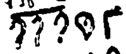
7.  equals?
   A) 1915
   B) 10315
   C) 10.915
   D) 10150
   E) 1015

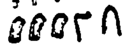
8. $5(10^4) + 5(10^3) + 4(100)$ is represented by

A) 

B) 

C) 

D) 

E) 

9. Imagine a place where the inhabitants have only five fingers and numbers are written in a group of five. What symbol would be used in such a system to represent the number of marks in the following set:

* * * * * *
* * * * * *
* * * * * *
* * * * * *

A) 14
B) 24
C) 34
D) 44
E) 54

10. The number of X's in the accompanying figure is written below in numerals in four different bases. Which number is correct?

XXXX
XXXX
XXXX
XX

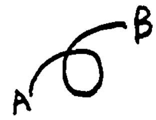
(I) 24_{five}
(II) 14_{seven}
(III) 12_{twelve}
(IV) 1110_{two}

A) only I is correct
B) I, II are correct
C) I, II, III are correct
D) I, III, IV are correct
E) All of the above are correct.

11. If $31_a = 16_c = 13$: then find a, c.

A) 2,4
B) 4,4
C) 7,4
D) 4,7
E) 7,7

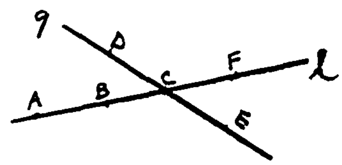
12. The following curve is:



A) a simple curve, but not closed
B) a closed curve, but not a simple curve
C) a curve that is not simple and not a closed curve
D) a curve that is a simple curve and a closed curve
E) none of the above is true.

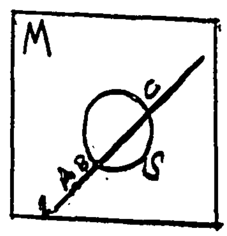
13. The plane is:
A) a finite set of points
B) the subset of lines
C) the union of the finite sets of lines and finite sets of points
D) a particular set of points which is neither flat nor smooth
E) none of the above is true.

14. Which of the following is true?



A) $c \in l$
B) $c \in q$
C) $\overrightarrow{FC} \cap \overrightarrow{AB} = \varphi$
D) $\overleftrightarrow{ED} \cap \overrightarrow{FA} = B$
E) $\overleftrightarrow{EC} \cap \overline{FB} = C$

15. In this figure which of the following is true?



A) $BC \cap$ {exterior of s} = BC
B) $l \cap$ {region of s} =
C) {(interior $\cup$ exterior of s)} $\cap\, l$ = all points on l except B and C
D) $M \cap S = M$
E) none of the above is true.

16. If X = {chair, stove, box, cat} and Y = {hat, coat} then which of the following is true?
A) N(X) = 2
B) N(Y) = 4
C) $N(X \cup Y) = 6$
D) N(AxB) = 12
E) N(X) + N(Y) = 8

17. Which one of the following sets is closed under addition?
A) {1,2,3}
B) {1,3,5,7,...}
C) {10,5,15}
D) {2,4,6,8,...}
E) {1,3,4,6,8,...}

18. Given any two non-empty sets P = {s,t} and Q = {n,p,r} the P x Q is:
A) { s,t,n,p,r}
B) {(s,t) , (n,p,r)}
C) {(s,n) , (s,p) , (s,r) , (t,n) , (t,s)}
D) {(s,n) , (s,p) , (s,r) , (t,n) , (t,p) , (t,r)}
E) {sxn , sxp , sxr , txn , txp , txr}

19. Which of the following statements is true?
A) $18 < (3x5) + 3$
B) $18 \ngtr (3x6)$
C) $18 > (10x0) + 18$
D) $18 \geqslant 2x9$
E) $18 \leqslant 6+(6+2)$

20. If a b and c d which of the following statements is always true?
A) $a > c$
B) $b > d$
C) $a+d > b+c$
D) $a+c > d+d$
E) $d+b < a+c$

21. If a number system consisting of two elements and two binary operations and is completely defined by the tables shown:

$\oplus$	0	1
0	0	1
1	1	0

$\otimes$	0	1
0	0	0
1	0	1

then which of the following statements is true?
A) $0 \oplus (0 \otimes 1) = 1$
B) $1 \oplus (1 \otimes 0) = 0$
C) $1 \oplus (1 \otimes 1) = 1$
D) $1 \otimes (1 \oplus 1) = 1$
E) none of the above are true.

22. What property justifies [(9x6) + (2x6)] x 1 = (9x6) + (2x6)?
A) Associative property for multiplication
B) Commutative property for multiplication
C) Distributive property over addition
D) Renaming 9 and 6 and 2
E) Multiplicative identity

23. If we use Roman numerals we have X V + VI = X+V+V+I = X+(V+V)+I = X+X+I = X X I
Which of the following properties are illustrated?
I commutative property for addition
II associative property for addition
III distributive property for addition

A) none
B) I only
C) II only
D) I and II only
E) I, II, and III

24. If we have 8 + (8x4)
= (8x1) + (8x4)
= 8x(1+4)
which of the following are used for justification?
I commutative property for multiplication
II distributive property for multiplication over addition
III multiplicative identity

A) none
B) I and II
C) I and III
D) II and III
E) I and II and III

25. Which of the following is the solution set of the sentence: $\{b \in w: b > 5\} \cup \{b \in w: b < 9\}$?
A) $\{6,7,8\}$
B) $\{9,10,11,12,...\}$
C) $\{8,7,6,5,4,3,2,1,0\}$
D) $\{6,7,8,9,...\}$
E) $\{0,1,2,3,4,5,6,7,...\}$

26. Which of the following sets represents F_{12} (F_{12} is the set of all factors of 12)?
A) $\{1,2,6\}$
B) $\{1,3,4\}$
C) $\{1,2,3,4,6\}$
D) $\{1,2,3,4\}$
E) $\{1,2,3,4,6,12\}$

27. Which one of the following is a prime number?
    A) 51
    B) 55
    C) 57
    D) 59
    E) none of the above

28. How many even primes are in natural numbers?
    A) one
    B) infinite
    C) unknown
    D) equal to the number of odd primes
    E) none of the above is true

29. What is the greatest common factor of $\{65, 420\}$?
    A) 35
    B) 7
    C) 15
    D) 5
    E) 65

30. Which of the following is the least common multiple of $\{45, 15, 10\}$?
    A) 90
    B) 450
    C) 45
    D) 150
    E) 180

****

Please fill in the following information:

COLLEGE ______________ CREDIT HRS. COLLEGE MATH. _____

ACT, MATH. ___________ HIGH SCHOOL G.P.A. ___________

ACT, TOTAL ___________ GRADE, MATH. _________________

G.P.A. _______________

ANSWERS FOR THE TEST FOR UNDERSTANDING CONCEPTS OF MATHEMATICS (TUCM)

THE UNIVERSITY OF OKLAHOMA

1.	-	B	16.	-	C
2.	-	D	17.	-	D
3.	-	D	18.	-	D
4.	-	C	19.	-	D
5.	-	A	20.	-	E
6.	-	C	21.	-	E
7.	-	D	22.	-	E
8.	-	C	23.	-	C
9.	-	D	24.	-	D
10.	-	D	25.	-	E
11.	-	D	26.	-	E
12.	-	C	27.	-	D
13.	-	E	28.	-	A
14.	-	E	29.	-	D
15.	-	C	30.	-	A

APPENDIX II

MULTIPLE LINEAR REGRESSION AND ITS PROGRAM ADJUSTED TO THIS STUDY

MULTIPLE LINEAR REGRESSION

To elicit information on the criterion measure characteristic (Y), by considering measurement (X_1, $X_2, \ldots X_k$), a relationship among the various characteristics under consideration should be assumed. Such a relationship is customarily expressed by some mathematical function.[1]

Assuming a functional relationship exists among the characteristics of Y and the characteristics of (X_1, $X_2, \ldots X_k$), this relationship might be expressed as

$$Y = \varphi (X_1, X_2, \ldots, X_k | B_o, B_1, \ldots B_k)$$

This reads that Y is the function of X_1 through X_k with parameters B_o through B_k. If φ is assumed to be linear, then the relationship is expressed as

$$Y = B_o + B_1X_1 + B_2X_2 + \ldots + B_kX_k$$

To determine the function it is necessary to know not only the form (which is assumed to be linear) but also the values of all parameters appearing in the previous equation. Since only sample observations on Y, X_1, $X_2, \ldots, X_k$ were available, estimate values for the unknown parameters were

[1]Bernard Ostle, Statistics in Research (Ames, Iowa: Iowa State College Press, 1954), p. 117.

given by the methods of least squares,[2] which were applied as follows:

Given a set of n observation of the k + 1 characteristic being measured:

$(Y_1, X_{11}, \ldots, X_{k_1})$

.

.

.

$(Y_n, X_{in}, \ldots X_{kn})$

and denoting the estimator of B_i by b_i $(i = 1, \ldots, m)$ the n differences were:

$Y_1 - \quad (X_{11}, \ldots X_{k1} | b_1, \ldots b_m)$

.

.

.

$Y_r - \quad (X_{1n}) \ldots X_{kn} | b_1, \ldots b_m)$

and then find the estimator by minimizing the sum of the squares of the above deviations, that is, calling $Q(X_{1i}, \ldots X_{k1} | b_1, \ldots b_m) = \varphi_i$ minimizing the function

$$Q = \sum_{n=1}^{n} (Y_i - \varphi_i)^2$$

---

[2] <u>Ibid</u>., p. 120.

This can be done by differentiating Q with respect to each of the estimators (partial derivative) and each time the partial derivative is set equal to zero. This formally appears as

$$\frac{\partial Q}{\partial b_i} = 0 \quad i = 1, 2 \ldots, m$$

The resulting system must be solved to give the required estimates.

Multiple linear regression. If there is a relation between Y and (X_1, $X_2 \ldots X_k$ $B_o, \ldots B_k$) then it may be expressed as:

$$Y = B_o = B_1 X_1 + B_2 X_2 + \ldots + B_k X_k$$

where B_o, $B_1, \ldots B_k$ are unknown parameters, or constants. The estimates b_o, $b_1, \ldots, b_k$ for these unknown parameters were found by using the available data. The procedure is to apply the method of the least squares to find values of the estimates which will minimize Q:

$$Q = \sum_{n=1}^{n} (Y_i - b_o - b_1 X_{1i} - \ldots - b_k X_{ki})^2$$

Differentiating with respect to b_o, $b_1, \ldots, b_k$ the following system of equations was obtained which was solved to give the required estimates:

$$Y = n\, b_o + b_1\, X_1 + b_2\, X_2 + \ldots + b_k\, X_k$$

$$X_1 Y = b_o\, X_1 + b_1\, X_1^2 + b_1\, X_1 X_2 + \ldots + b_k\, X_1 X_k$$

.

.

$$X_kY = b_o X_k + b_1 X_1 X_k + b_2 X_2X_k + \ldots + b_k X_k^2$$

equation no. 1

All summations extend from 1 to n, the number of observational units in the sample (the subscript of summation has been left off).

There are many procedures for solving systems of linear equations to find b_o, b_1, ... b_k. For example, if the following matrix (X):[3]

$$X = \begin{pmatrix} n & X_{1_2} & X_2 \ldots & X_k \\ X_1 & X_1 & X_1X_2 \ldots & X_1X_k \\ . & & & \\ . & & & \\ . & & & \\ X_k & X_1X_k & X_2X_k \ldots & X_k^2 \end{pmatrix}$$

If the determinant, say $\Delta = 1x1$, is not equal to zero, then the value b_i $(i=o,\ldots,k)$ may be found by the equation:[4]

$$b_i = \frac{1}{\Delta} \left| \begin{array}{l} \text{matrix X with the elements} \\ \text{of the (j+1) the column replaced} \\ \text{by the corresponding elements} \\ \text{from the left-hand side of the} \\ \text{equation described above} \end{array} \right| \quad j=o,\ldots,K$$

Other methods of solving systems of linear equations such as

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[3] *Ibid.*, p. 203.

[4] *Ibid.*

pivoting, inverse matrix, augmented matrix, Guass-Jordan method, etc. can be found in many algebra textbooks such as Finkbeiner's Introduction to Matrix and Linear Transformation.[5]

The procedure which will be adopted in this study is described in System/360 Scientific Subroutine Package (360-CM-03X) Version II Programmer's Manual 1967 as multiple linear regression program. Appendix II contains the main program for multiple regression--ReGRe--and the Subroutine Data adjusted for this study. The output of this program will give useful information such as means, standard deviation, correlation coefficients between the independent variables and the dependent variables, regression coefficients, computed t-values, intercept, multiple correlation coefficients, standard error of estimate, and analysis of variance for multiple regression.

Before discussing this procedure in detail, the problem was simplified somewhat. The first equation (1) was written as

$$\overline{Y} = b_o + b_1\overline{X}_1 + \ldots + b_k\overline{X}_k \qquad \text{equation no. 2}$$

---

[5]Daniel T. Finkbeiner II, Introduction to Matrix and Linear Transformation (San Francisco: W. H. Freeman and Co., 1960).

or $b_o = \bar{Y} - b_1\bar{X}_1 - \ldots - b_k\bar{X}_k$ equation no. 3

Thus the problem was reduced to one of obtaining $b_1, b_2, \ldots, b_k$ since b_o was defined as the function of remaining b's by equation (3).

By substituting for b_o in equation (1) this left the problem of finding the values of $b_1, b_2, \ldots b_k$ which minimizes Q'.

$$Q' = \sum_{i=1}^{n} [(Y_i - Y) - b_1 (X_{1i} - \bar{X}_1) - \ldots - b_k(X_{ki} - \bar{X}_k)]$$

$$= \sum_{i=1}^{n} (\hat{Y}_i - b_1\hat{X}_{1i} - \ldots - b_k\hat{X}_{ki})^2 \qquad \text{equation no. 4}$$

where $\hat{Y}_i = Y_i - \bar{Y}$ and $\hat{X}_{ji} = X_{ji} - \bar{X}_1$ for $i = 1, 2, \ldots, k$
equation no. 5

The values of $b_1, \ldots, b_k$ which minimized Q' were of course the same as those which minimized Q but now the system contained one less equation and appeared as

$$b_1 \sum \hat{X}^2 + b_2 \sum \hat{X}_1\hat{X}_2 + \ldots + b_k \sum \hat{X}_1\hat{X}_k = \sum \hat{X}_1\hat{Y}$$

$$b_1 \sum \hat{X}_1\hat{X}_2 + b_2 \sum \hat{X}_2^{\,2} + \ldots + b_k \sum \hat{X}_2\hat{X}_k = \sum \hat{X}_2\hat{Y}$$

$$\vdots$$

$$b_1 \sum \hat{X}_1\hat{X}_k + b_1 \sum \hat{X}_2\hat{X}_k + \ldots + b_k \sum \hat{X}_k^{\,2} = \sum \hat{X}_k\hat{Y}.$$

equation no. 6

where all variables were deviations from their respective means and where the subscript of summation had been dropped.

If both sides of the first equation in (6) were divided by $\sqrt{\sum \hat{X}_1^2}\sqrt{\sum \hat{Y}^2}$; both sides of the second equation in (6) by $\sqrt{\sum \hat{X}_2^2}\sqrt{\sum \hat{Y}_2^2}$, and so forth to the kth equation, which was divided by $\sqrt{\sum \hat{X}_k^2}\sqrt{\sum \hat{Y}^2}$ at the same time introducing $b_j = b_i\sqrt{\sum \hat{X}_j^2} \;/\sqrt{\sum \hat{Y}^2}$ for j=1,...K, then equation (6) becomes:

$$\begin{aligned} b_1 + r_{12}b_2 + \ldots + r_{1k}b_k &= r_{iy} \\ r_{21}b_1 + b_2 + \ldots + r_2 b_k &= r_{2y} \\ &\vdots \\ r_{k1}b_1 + r_{k2}b_2 + \ldots + b_k &= r_{ky} \end{aligned} \qquad \text{equation no. 7}$$

where r_{iy} was the product moment correlation coefficient between X_{ji} and Y_i that is:[6]

$$r_{jy} = r_{yi} = \frac{\sum_{i=1}^{n}(X_{ji}-\bar{X}_j)(Y_i-\bar{Y})}{\sum_{i=1}^{n}(X_{ji}-\bar{X}_j)^2 \sum_{i=1}^{n}(Y_i-\bar{Y})^2}$$

$$\frac{\sum_{i=1}^{n} X_{ji}Y_i}{\sqrt{\sum_{i=1}^{n} X_{ji}^2}\sqrt{\sum_{i=1}^{n} Y_i^2}}$$

<u>Multiple</u> <u>Linear</u> <u>Regression</u>. In order to solve equation (7),

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[6] Ostle, *loc. cit.*

the scientific subroutine package,[7] multiple linear regression--using the Guass-Jordan method--was used. This subroutine is normally performed by calling four subroutines in sequence:

1. CORRE: This subroutine calculates the means, standard deviation, sums of cross-products of deviation from means, and the product moment correlation coefficient from input data X_{ij}, where i=1,...n are observations and j=1,2, ...,m are variables.

The following equations are used to calculate these statistics.

Sums of cross-products of deviations:

$$S_{jk} = \sum_{i=1}^{n} (X_{ij} - T_j) \quad (X_{ik} - T_k) - \frac{\sum_{i=1}^{n} (X_{ij} - T_j) \sum_{i=1}^{n} (X_{ik} - T_k)}{n} \qquad (1)$$

where j=1,2,...,m; k=1,2,...,m

$$T_j = \frac{\sum_{i=1}^{m} X_{ij}}{m} \qquad (2)$$

(These temporary means T_j are subtracted from data in the equation (1) to obtain computational accuracy.)

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[7]I.B.M. System/360 Scientific Subroutine Package, I.B.M., White Plains, N. Y., 1967.

$$\text{Means:} \quad \bar{X}_j = \frac{\sum_{i=1}^{n} X_{ij}}{n} \qquad (3)$$

where j=1,2,...,m

Correlation coefficients:

$$r_{jk} = \frac{S_{jk}}{\sqrt{S_{jj}}\ \sqrt{S_{jj}}} \qquad (4)$$

where j=1,2,...,m; k=1,2,...,m

Standard deviations:

$$s_j = \frac{\sqrt{S_{jj}}}{\sqrt{n-1}} \qquad (5)$$

where j=1,2,...,m

2. Order: This subroutine is to choose a dependent variable and a subset of independent variables, from a larger set of variables.

3. MINV: This subroutine is to invert the correlation matrix of subset, selected by order.

4. MULTR: This subroutine is to perform a multiple regression analysis for a dependent variable and a set of independent variables.

Beta weights (B_j) are calculated using the following equation:

$$\beta_j = \sum_{i=1}^{k} riy \cdot rij^{-1} \qquad (1)$$

where r_{iy} = intercorrelation of i^{th} independent variable with dependent variable

r_{ij}^{-1} = the inverse of intercorrelation r_{ij}

$i = j = 1,2,\ldots,k$ are independent variables

r_{iy} and r_{ij}^{-1} are input to this subroutine.

Then, the regression coefficients are calculated as follows:

$$b_j = \beta_j \cdot \frac{s_y}{s_j} \qquad (2)$$

where s_y = standard deviation of dependent variable

s_j = standard deviation of j^{th} independent variable

$j = 1,2,\ldots,k$

s_y and s_j are input to this subroutine.

The intercept is found by the following equation:

$$b_o = \overline{Y} - \sum_{j=1}^{k} b_j \cdot \overline{X}_j \qquad (3)$$

where $\overline{Y}$ = mean of dependent variable

$\overline{X}_j$ = mean of j^{th} independent variable

$\overline{Y}$ and $\overline{X}_j$ are input to this subroutine.

Multiple correlation coefficient, R, is found first by calculating the coefficient of determination by the following equation:

$$R^2 = \sum_{i=1}^{k} B_i r_{iy} \qquad (4)$$

and taking the square root of R^2:

$$R = \sqrt{R^2} \qquad (5)$$

The sum of squares attributable to the regression is found by:

$$SSAR = R^2 \cdot D_{yy} \qquad (6)$$

where D_{yy} = sum of squares of deviations from mean for dependent variable

D_{yy} is input to this subroutine.

The sum of squares of deviations from the regression is obtained by:

$$SSDR = D_{yy} - SSAR \qquad (7)$$

Then, the F-value for the analysis of variance is calculated as follows:

$$F = \frac{SSAR/k}{SSDR/\ (n-k-1)} = \frac{SSAR(n-k-1)}{SSDR(k)} \qquad (8)$$

Certain other statistics are calculated as follows:

Variance and standard error of estimate:

$$S^2_{y.12\ldots k} = \frac{SSDR}{n-k-1} \qquad (9)$$

where n = number of observations

$$S_{y.12\ldots k} = \sqrt{S^2_{y.12\ldots k}} \qquad (10)$$

Standard deviations of regression coefficients:

$$S_{b_j} = \sqrt{\frac{r_{jj}^{-1}}{D_{jj}} \cdot S^2_{y.12\ldots k}} \tag{11}$$

where D_{jj} = sum of squares of deviations from mean for j^{th} independent variable. D_{jj} is input to this subroutine.

$j = 1,2,\ldots,k$

Computed t:

$$t_j = \frac{b_j}{S_{b_j}} \tag{12}$$

$j = 1,2,\ldots,k$

```
//  EXEC 6567GCLG
//FORT.SYSIN DD *
C
C      .......................................................................
C
C         SAMPLE MAIN PROGRAM FOR MULTIPLE REGRESSION - REGRE
C
C         PURPOSE
C            (1) READ THE PROBLEM PARAMETER CARD FOR A MULTIPLE REGRES-
C            SION, (2) READ SUBSET SELECTION CARDS, (3) CALL THE SUB-
C            ROUTINES TO CALCULATE MEANS, STANDARD DEVIATIONS, SIMPLE
C            AND MULTIPLE CORRELATION COEFFICIENTS, REGRESSION COEFFI-
C            CIENTS, T-VALUES, AND ANALYSIS OF VARIANCE FOR MULTIPLE
C            REGRESSION, AND (4) PRINT THE RESULTS.
C
C         REMARKS
C            THE NUMBER OF OBSERVATIONS, N, MUST BE GREATER THAN M+1,
C            WHERE M IS THE NUMBER OF VARIABLES.  IF SUBSET SELECTION
C            CARDS ARE NOT PRESENT, THE PROGRAM CAN NOT PERFORM MULTIPLE
C            REGRESSION.
C            AFTER RETURNING FROM SUBROUTINE MINV, THE VALUE OF DETER-
C            MINANT (DET) IS TESTED TO CHECK WHETHER THE CORRELATION
C            MATRIX IS SINGULAR.  IF DET IS COMPARED AGAINST A SMALL
C            CONSTANT, THIS TEST MAY ALSO BE USED TO CHECK NEAR-
C            SINGULARITY.
C
C         SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C            CORRE  (WHICH, IN TURN, CALLS THE SUBROUTINE NAMED DATA)
C            ORDER
C            MINV
C            MULTR
C
C         METHOD
C            REFER TO B. OSTLE, 'STATISTICS IN RESEARCH', THE IOWA STATE
C            COLLEGE PRESS', 1954, CHAPTER 8.
C
```

```
C     .................................................................REGRE035
C                                                                       REGRE036
C     THE FOLLOWING  DIMENSIONS MUST BE GREATER THAN OR EQUAL TO THE    REGRE037
C     NUMBER OF VARIABLES, M..                                          REGRE038
C                                                                       REGRE039
      DIMENSION XBAR(40),STD(40),D(40),RY(40),ISAVE(40),B(40),          REGRE040
     1          SB(40),T(40),W(40)                                      REGRE041
C                                                                       REGRE042
C     THE FOLLOWING DIMENSION MUST BE GREATER THAN OR EQUAL TO THE      REGRE043
C     PRODUCT OF M*M..                                                  REGRE044
C                                                                       REGRE045
      DIMENSION RX(1600)                                                REGRE046
C                                                                       REGRE047
C     THE FOLLOWING DIMENSION MUST BE GREATER THAN OR EQUAL TO          REGRE048
C     (M+1)*M/2..                                                       REGRE049
C                                                                       REGRE050
      DIMENSION R(820)                                                  REGRE051
C                                                                       REGRE052
C     THE FOLLOWING DIMENSION MUST BE GREATER THAN OR EQUAL TO 10..     REGRE053
C                                                                       REGRE054
      DIMENSION ANS(10)                                                 REGRE055
C                                                                       REGRE056
C     .................................................................REGRE057
C                                                                       REGRE058
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE  REGRE059
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION      REGRE060
C        STATEMENT WHICH FOLLOWS.                                       REGRE061
C                                                                       REGRE062
C     DOUBLE PRECISION XBAR,STD,RX,R,D,B,T,RY,DET,SB,ANS,SUM            REGRE063
C                                                                       REGRE064
C        THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS    REGRE065
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS      REGRE066
C        ROUTINE.                                                       REGRE067
C                                                                       REGRE068
C        ..............................................................REGRE069
C                                                                       REGRE070
```

```
    1 FORMAT(A4,A2,I5,2I2)                                                 REGRE071
    2 FORMAT(25H1MULTIPLE REGRESSION.....A4,A2//6X,14HSELECTION.....I2//REGRE072
     1)                                                                    REGRE073
    3 FORMAT(9H0VARIABLE,5X,4HMEAN,6X,8HSTANDARD,6X,11HCORRELATION,4X,10REGRE074
     1HREGRESSION,4X,10HSTD. ERROR,5X,8HCOMPUTED/6H    NO.,18X,9HDEVIATIOREGRE075
     2N,7X,6HX VS Y,7X,11HCOEFFICIENT,3X,12HOF REG.COEF.,3X,7HT VALUE)  REGRE076
    4 FORMAT(1H ,I4,6F14.5)                                                REGRE077
    5 FORMAT(10H DEPENDENT)                                                REGRE078
    6 FORMAT(1H0/10H INTERCEPT,13X,F13.5//23H MULTIPLE CORRELATION  ,F13REGRE079
     1.5//23H STD. ERROR OF ESTIMATE,F13.5//)                             REGRE080
    7 FORMAT(1H0,21X,39HANALYSIS OF VARIANCE FOR THE REGRESSION//5X,19HSREGRE081
     1OURCE OF VARIATION,7X,7HDEGREES,7X,6HSUM OF,10X,4HMEAN,12X,7HF VALREGRE082
     2UE/30X,10HOF FREEDOM,4X,7HSQUARES,9X,7HSQUARES)                    REGRE083
    8 FORMAT(30H ATTRIBUTABLE TO REGRESSION    ,I6,3F16.5/30H DEVIATION FREGRE084
     1ROM REGRESSION     ,I6,2F16.5)                                       REGRE085
    9 FORMAT(1H ,5X,5HTOTAL,19X,I6,F16.5)                                  REGRE086
   10 FORMAT(36I2)                                                         REGRE087
   11 FORMAT(1H ,15X,18HTABLE OF RESIDUALS//9H CASE NO.,5X,7HY VALUE,5X,REGRE088
     110HY ESTIMATE,6X,8HRESIDUAL)                                        REGRE089
   12 FORMAT(1H ,I6,F15.5,2F14.5)                                          REGRE090
   13 FORMAT(53H1NUMBER OF SELECTIONS NOT SPECIFIED.  JOB TERMINATED.)   REGRE091
   14 FORMAT(52H0THE MATRIX IS SINGULAR.  THIS SELECTION IS SKIPPED.)    REGRE092
C                                                                          REGRE093
C     .....................................................................REGRE094
C                                                                          REGRE095
C     READ PROBLEM PARAMETER CARD                                          REGRE096
C                                                                          REGRE097
  100 READ (5,1) PR,PR1,N,M,NS                                             REGRE098
C        PR.........PROBLEM NUMBER (MAY BE ALPHAMERIC)                     REGRE099
C        PR1........PROBLEM NUMBER (CONTINUED)                             REGRE100
C        N..........NUMBER OF OBSERVATIONS                                 REGRE101
C        M..........NUMBER OF VARIABLES                                    REGRE102
C        NS.........NUMBER OF SELECTIONS                                   REGRE103
C                                                                          REGRE104
C     LOGICAL TAPE 13 IS USED AS INTERMEDIATE STORAGE TO HOLD INPUT       REGRE105
C     DATA.  THE INPUT DATA ARE WRITTEN ON LOGICAL TAPE 13 BY THE         REGRE106
```

```
C        SPECIAL INPUT SUBROUTINE NAMED DATA.  THE STORED DATA MAY BE USED
C        FOR RESIDUAL ANALYSIS.
C
         REWIND 13
C
         IO=0
         X=0.0
C
         CALL CORRE (N,M,IO,X,XBAR,STD,RX,R,D,B,T)
C
         REWIND 13
C
C        TEST NUMBER OF SELECTIONS
C
         IF(NS) 108, 108, 109
  108 WRITE (6,13)
         GO TO 300
C
  109 DO 200 I=1,NS
         WRITE (6,2) PR,PR1,I
C
C        READ SUBSET SELECTION CARD
C
         READ (5,10) NRESI,NDEP,K,(ISAVE(J),J=1,K)
C           NRESI......OPTION CODE FOR TABLE OF RESIDUALS
C                           0   IF IT IS NOT DESIRED.
C                           1   IF IT IS DESIRED.
C           NDEP.......DEPENDENT VARIABLE
C           K..........NUMBER OF INDEPENDENT VARIABLES INCLUDED
C           ISAVE......A VECTOR CONTAINING THE INDEPENDENT VARIABLES
C                           INCLUDED
C
         CALL ORDER (M,R,NDEP,K,ISAVE,RX,RY)
C
         CALL MINV (RX,K,DET,B,T)
C
```

```
C     TEST SINGULARITY OF THE MATRIX INVERTED                          REGRE143
C                                                                      REGRE144
      IF(DET) 112, 110, 112                                            REGRE145
  110 WRITE (6,14)                                                     REGRE146
      GO TO 200                                                        REGRE147
C                                                                      REGRE148
  112 CALL MULTR (N,K,XBAR,STD,D,RX,RY,ISAVE,B,SB,T,ANS)               REGRE149
C                                                                      REGRE150
C     PRINT MEANS, STANDARD DEVIATIONS, INTERCORRELATIONS BETWEEN      REGRE151
C     X AND Y, REGRESSION COEFFICIENTS, STANDARD DEVIATIONS OF         REGRE152
C     REGRESSION COEFFICIENTS, AND COMPUTED T-VALUES                   REGRE153
C                                                                      REGRE154
      MM=K+1                                                           REGRE155
      WRITE (6,3)                                                      REGRE156
      DO 115 J=1,K                                                     REGRE157
      L=ISAVE(J)                                                       REGRE158
  115 WRITE (6,4) L,XBAR(L),STD(L),RY(J),B(J),SB(J),T(J)               REGRE159
      WRITE (6,5)                                                      REGRE160
      L=ISAVE(MM)                                                      REGRE161
      WRITE (6,4) L,XBAR(L),STD(L)                                     REGRE162
C                                                                      REGRE163
C     PRINT INTERCEPT, MULTIPLE CORRELATION COEFFICIENT, AND STANDARD  REGRE164
C     ERROR OF ESTIMATE                                                REGRE165
C                                                                      REGRE166
      WRITE (6,6) ANS(1),ANS(2),ANS(3)                                 REGRE167
C                                                                      REGRE168
C     PRINT ANALYSIS OF VARIANCE FOR THE REGRESSION                    REGRE169
C                                                                      REGRE170
      WRITE (6,7)                                                      REGRE171
      L=ANS(8)                                                         REGRE172
      WRITE (6,8) K,ANS(4),ANS(6),ANS(10),L,ANS(7),ANS(9)              REGRE173
      L=N-1                                                            REGRE174
      SUM=ANS(4)+ANS(7)                                                REGRE175
      WRITE (6,9) L,SUM                                                REGRE176
      IF(NRESI) 200, 200, 120                                          REGRE177
C                                                                      REGRE178
```

```
C     PRINT TABLE OF RESIDUALS
C
  120 WRITE (6,2) PR,PR1,I
      WRITE (6,11)
      MM=ISAVE(K+1)
      DO 140 II=1,N
      READ (13) (W(J),J=1,N)
      SUM=ANS(1)
      DO 130 J=1,K
      L=ISAVE(J)
  130 SUM=SUM+W(L)*B(J)
      RESI=W(MM)-SUM
  140 WRITE (6,12) II,W(MM),SUM,RESI
      REWIND 13
  200 CONTINUE
      GO TO 100
  300 CONTINUE
      END
      SUBROUTINE DATA (M,D)
      DIMENSIOND(1)
      READ(5,1)(D(I),I=1,M)
    1 FORMAT( 7F6.0)
      WRITE(13)(D(I),I=1,M)
      RETURN
      END
```

APPENDIX III

GENERALIZED ANALYSIS OF VARIANCE WITH COVARIANCE CONTROL INCLUDING ALL REQUIRED SUBROUTINES

ANALYSIS OF COVARIANCE

The analysis of covariance is a statistical technique in which analysis of variance and regression are combined. Analysis of variance is used in testing the hypothesis H_2: $\mu_1 = \mu_2 = \ldots = \mu_g$ where the μ's are population centroids (i.e., mean vectors) for the groups. Rao defines Wilkes' lambda criterion as follows:[1]

$$\Lambda = |W|/|T|$$

where W is the pooled within-groups deviation score cross product matrix and T is the total sample deviation score cross products matrix. The elements of the W and T matrix are defined as follows:[2]

$$W_{ij} = \sum_{k=1}^{g} \left[\sum_{k=1}^{N_g} (X_{ikn} - \bar{X}_{ik})(X_{jkn} - \bar{X}_{jk}) \right]$$

$$t_{ij} = \sum_{n=1}^{N} (X_{in} - \bar{X}_i)(X_{in} - \bar{X}_j)$$

where g is the number of groups, N_g = number of subjects in every group g, N = total number of subjects, and i and j run from 1 to p where p = the number of variables.

---

[1] C. R. Rao, Advanced Statistical Methods in Biometric Research (New York: John Wiley and Sons, 1952), pp. 258-275.

[2] Ibid.

Rao has described a method of performing covariance adjustments on the W and T matrices prior to forming Λ for any number of control variables, with appropriate adjustment of the degrees of freedom.

The first step in the computations for the generalized covariance analysis is to form W and T matrix for all the variables, the p experimental variables and the C control variable, the influence of which is to be removed by regression.

These matrices are then partitioned as follows:[3]

$$W = \left[\begin{array}{c|c} W_{pp} & W_{pc} \\ \hline W_{cp} & W_{cc} \end{array}\right] \quad T = \left[\begin{array}{c|c} T_{pp} & T_{pc} \\ \hline T_{cp} & T_{cc} \end{array}\right]$$

Then two adjusted matrices are formed:[4]

$$W_{p\cdot c} = W_{pp} - W_{pc} W_{cc}^{-1} W_{cp}$$

$$T_{p\cdot c} = T_{pp} - T_{pc} T_{cc}^{-1} T_{cp}$$

$$= \frac{|W_{p\cdot c}|}{|T_{p\cdot c}|}$$

and where p number of experimental variables, c = number of control variables, n = N-1 and q = g-1. The Rao F

---

[3]Wm. W. Cooley and Paul R. Lohnes, Multivariate Procedures for the Behavioral Sciences (New York: John Wiley and Sons, 1962), p. 64.

[4]Ibid.

transformation:[5]

$$F^{2r}_{ms+2\lambda} = \left(\frac{1-y}{y}\right)\left(\frac{ms+2d}{2r}\right)$$

where $y = \Lambda^{1/s}$, $q = g-1$, $\lambda = -(pq-2)/4$, $r = pq/2$, $S = \sqrt{(p^2q^2-4)/(p^2+q^2-5)}$, and $m = (n-c)-p+q)/2$. To obtain the adjusted means for the experimental variables the regression coefficients are computed using two partitions of W. This involves multiplying W^{-1}_{cc} times each of the column vector W_{cp}. The product vectors are the coefficients b_c used in computing the adjustment for each of the p·q experimental means $\bar{X}_{jg}$.

The adjusted mean for group g on experimental variable X is:[6]

$$\bar{\bar{X}}_{jg} = \bar{X}_{ig} - [b_1(\bar{Y}_{ig}-\bar{Y}_{1.}) + b_2(\bar{Y}_{2g}-\bar{Y}_{2.}) + \ldots + b_c(\bar{Y}_{cg}-\bar{Y}_{c})]$$

where $\bar{Y}_{cg}$ is the mean of group g on the control variable Y_c and $\bar{Y}_{c.}$ is the grand mean on Y_c.

Following the above mathematical procedure, a program was adapted from Cooley[7] and written in the Fortran IV language by the writer for the analysis of covariance (see Appendix III).

---

[5] *Ibid*., p. 62.

[6] *Ibid*., p. 64.

[7] Cooley and Lohnes, *op. cit*.

This program computes the test of equality of experimental mean vectors with covariance control where the total number of variables does not exceed 40.

The output of the program includes:

1. Program title, and K=Number of the groups.

2. M = number of experimental variables, and N = number of control variables.

3. For each group KG = identification number of the groups, KN = identification number of the subject in each group, group means, and group standard deviations.

4. NT = total number of subjects.

5. Means for total sample.

6. Standard deviation for total sample.

7. Correlation matrix for total sample.

8. Adjusted standard deviation on experimental variable for the total sample.

9. Logarithm base e determinant of adjusted T matrix, and corresponding determinant.

10. The base e of determinant of adjusted W matrix, and corresponding determinant.

11. Wilks Lambda, the degree of freedom (F_1 and F_2) and F for test of the equality of the experimental mean vectors.

12. Coefficient for forming the adjusted means, for each experimental variable.

```
// EXEC F123GCLG
//FORT.SYSIN DD *
      DIMENSION  A(40),B(40),C(40),D(40),E(40),G(40),H(40),
     1           U(40,40),V(40,40),W(40,40),X(40,40),Y(40,40),Z(40,40)
      READ(1,1)JOBS
  111 READ(1,1)K,M,L
    1 FORMAT  (3I2)
      ML = M+L
      WRITE(3,2)K
    2 FORMAT (51H1GENERALIZED ANALYSIS OF VARIANCE. NO. OF GROUPS = I2)
      WRITE(3,3)M
    3 FORMAT(26H0NO. EXPERIMENTAL TESTS = I2)
      WRITE(3,4)L
    4 FORMAT (21H0NO. CONTROL TESTS = I2)
      DO 8  I=1,ML
    7 B(I) = 0.0
      DO 8 J=1,ML
      W(I,J) = 0.0
    8 X(I,J) = 0.0
      NT = 0
      GROUPS = K
      XM = M
      XL = L
      Q = GROUPS - 1.0
    9 DO 10 I=1,ML
   10 A(I) = 0.0
      DO 11 I=1,ML
      DO 11 J=1,ML
   11 U(I,J) = 0.0
      READ(1,12)KG,KN
   12 FORMAT (I2,I5)
      ENK = KN
      WRITE(3,13)KG,KN
   13 FORMAT(15H1THIS IS GROUP I2,14H NO.SUBJECTS= I5)
      CASES = ENK
   14 READ(1,666)(C(I),I=1,ML)
```

```
666 FORMAT(7F6.0)
    DO 16 I=1,ML
 15 A(I) = A(I)+C(I)
    DO 16 J=1,ML
 16 U(I,J) = U(I,J)+C(I)*C(J)
    CASES = CASES-1.0
    IF (CASES) 17,17,14
 17 DO 18 I=1,ML
    DO 18 J=1,ML
    V(I,J) = U(I,J)-A(I)*A(J)/ENK
    U(J,I) = U(I,J)
 18 V(J,I) = V(I,J)
    DO 19 I=1,ML
    D(I) = A(I)/ENK
 19 B(I) = SQRT  (V(I,I)/(ENK-1.0))
    WRITE(3,20)
 20 FORMAT (20HOMEANS OF THIS GROUP)
    WRITE(3,21)(D(I),I=1,ML)
 21 FORMAT (5F14.7)
    WRITE(3,22)
 22 FORMAT (34HOSTANDARD DEVIATIONS OF THIS GROUP)
    WRITE(3,21)(E(I),I=1,ML)
    DO 23 I=1,ML
 23 B(I) = B(I)+A(I)
    DO 24 I=1,ML
    DO 24 J=1,ML
    W(I,J) = W(I,J) + U(I,J)
 24 X(I,J) = X(I,J) + V(I,J)
    NT = NT + KN
    GROUPS = GROUPS-1.0
    IF (GROURS) 25,25,9
C   X IS NOW RULON W.
 25 WRITE(3,251)NT
251 FORMAT (25HOTOTAL NO. OF SUBJECTS = I5)
    ENT = NT
    DO 26 I=1,ML
```

```
      DO 26 J=1,ML
   26 Y(I,J) = W(I,J)-(B(I)*B(J)/ENT)
C     Y IS NOW RULON T.
      DO 27 I=1,ML
      G(I) = B(I)/ENT
   27 H(I) = SQRT  (Y(I,I)/(ENT-1.0))
      WRITE(3,28)
   28 FORMAT (22HOMEANS OF TOTAL SAMPLE)
      WRITE(3,21)(G(I),I=1,ML)
      WRITE(3,29)
   29 FORMAT (37HOSTANDARD DEVIATIONS FOR TOTAL SAMPLE)
      WRITE(3,21)(H(I),I=1,ML)
      DO 291 I=1,ML
      DO 291 J=1,ML
      Z(I,J) = Y(I,J)/ SQRT  (Y(I,I)*Y(J,J))
  291 Z(J,I) = Z(I,J)
      WRITE(3,292)
  292 FORMAT(36H1CORRELATION MATRIX FOR TOTAL SAMPLE)
      CALL MPRINT (Z,ML,2,6HTOT R )
      DO 30 I=1,L
      DO 30 J=1,L
      IL = I+M
      JL = J+M
   30 U(I,J) = Y(IL,JL)
C     U IS NOW RULON T(C,C).
      CALL MATINV (U,L,B,0,DETERM)
      DO 31 I=1,M
      DO 31 J=1,L
      JL = J+M
   31 V(I,J) = Y(I,JL)
C     V IS NOW RULON T(T,C).
      DO 32 I=1,M
      DO 32 J=1,L
      W(I,J) = 0.0
      DO 32 K=1,L
   32 W(I,J) = W(I,J)+V(I,K)*U(K,J)
```

```
      DO 33 I=1,L
      DO 33 J=1,M
   33 U(I,J) = V(J,I)
C     U IS NOW RULON T(C,T).
      DO 34 I=1,M
      DO 34 J=1,M
      V(I,J) = 0.0
      DO 34 K=1,L
   34 V(I,J) = V(I,J)+W(I,K)*U(K,J)
      DO 35 I=1,M
      DO 35 J=1,M
   35 W(I,J) = Y(I,J) - V(I,J)
C     W IS NOW RULON T(T.C).
      DO 355 I=1,M
  355 A(I) = SQRT (W(I,I)/(ENT-1.0))
      WRITE(3,48)
   48 FORMAT (46HOADJUSTED STANDARD DEVIATIONS FOR TOTAL SAMPLE)
      WRITE(3,21)(A(I),I=1,M)
      CALL HDIAG (W,M,1,V,NR)
      DETERT = 0.0
      DO 36 I=1,M
   36 DETERT = DETERT + ALOG(W(I,I))
      WRITE(3,47)DETERT
   47 FORMAT (30H1LOG DETERMINANT ADJUSTED T = F14.7)
      DO 37 I=1,L
      DO 37 J=1,L
      IL = I+M
      JL = J+M
   37 U(I,J) = X(IL,JL)
      CALL MATINV (U,L,B,0,DETERM)
      DO 38 I=1,M
      DO 38 J=1,L
      JL = J+M
   38 V(I,J) = X(I,JL)
      DO 39 I=1,M
      DO 39 J=1,L
```

```
      W(I,J) = 0.0
      DO 39 K=1,L
   39 W(I,J) = W(I,J) + V(I,K)*U(K,J)
      DO 40  I=1,L
      DO 40 J=1,M
   40 Z(I,J) = V(J,I)
      DO 41 I=1,M
      DO 41 J=1,M
      V(I,J) = 0.0
      DO 41 K=1,L
   41 V(I,J) = V(I,J) + W(I,K)*Z(K,J)
      DO 42 I=1,M
      DO 42 J=1,M
   42 W(I,J) = X(I,J) - V(I,J)
C     W IS NOW RULON W(T.G).
      CALL HDIAG (W,M,1,V,NR)
      DETERW = 0.0
      DO 44 I=1,M
   44 DETERW = DETERW + ALOG(W(I,I))
      WRITE(3,46)DETERW
   46 FORMAT (30HOLOG DETERMINANT ADJUSTED W = F14.7)
      XLAMBD = DETERW - DETERT
      YLAMBD = EXP  (XLAMBD)
      WRITE(3,45)YLAMBD
   45 FORMAT (16HOWILKS LAMBDA = F14.7)
      IF (XM-2.0) 453,451,453
  451 F1 = 2.0* Q
      F2 = 2.0 * (ENT - Q - XL - 2.0)
      XY = SQRT  (YLAMBD)
      F = (1.0 - XY) * F2 / (XY * F1)
      GO TO 52
  453 S = SQRT (((XM**2)*(Q**2) - 4.0) / (XM**2 + Q**2 - 5.0))
      IF (S) 999,49,51
   49 F1 = Q
      F2 = ENT - XL -(Q + 1.0)
      F = ((1.0 - YLAMBD)/YLAMBD)*(F2/F1)
```

```
      GO TO 52
   51 XLAN = - (XM * Q - 2.0)/2.0
      XY = YLANBD**(1.0/S)
      XMM=(ENT-1.0-XL)-(XM+Q+1.0)/2.0
      F1 = XM*Q
      F2 = (XMM*S+XLAM)
      F=(1.0-XY) * F2/(XY*F1)
   52 WRITE(3,53)F1
   53 FORMAT (6H0F1 = F14.7)
      WRITE(3,54)F2
   54 FORMAT (6H0F2 = F14.7)
      WRITE(3,55)F
   55 FORMAT(20H0FOR TEST OF H2, F= F14.7)
      DO 60 I=1,L
      DO 60 J=1,M
      V(I,J) = 0.0
      DO 60 K = 1,L
   60 V(I,J)=V(I,J)+U(I,K)*Z(K,J)
      DO 61 J = 1,M
   61 WRITE(3,64)J,(V(I,J),I=1,L)
   64 FORMAT (47H0COEF FOR ADJUSTING MEANS OF DEPENDENT VARIABLE I2 /
     1 6F14.7)
  999 JOBS = JOBS - 1
      IF (JOBS) 9999, 9999, 111
 9999 CALL EXIT
      END
      SUBROUTINE   HDIAG (H,N,IEGEN,U,NR)
      DIMENSION H(40,40), U(40,40), X(40), IQ(40)
      CALL GFM
      IF (IEGEN) 15,10,15
   10 DO 14 I=1,N
      DO 14 J=1,N
      IF(I-J)12,11,12
   11 U(I,J)=1.0
      GO TO 14
   12 U(I,J)=0.
```

```
   14 CONTINUE
   15 NR = 0
      IF (N-1) 1000,1000,17
C     SCAN FOR LARGEST OFF-DIAGONAL ELEMENT IN EACH ROW
C     X(I) CONTAINS LARGEST ELEMENT IN ITH ROW
C     IQ(I) HOLDS SECOND SUBSCRIPT DEFINING POSITION OF ELEMENT
   17 NMI1=N-1
      DO 30 I=1,NMI1
      X(I) = 0.
      IPL1=I+1
      DO 30 J=IPL1,N
      IF ( X(I) - ABS ( H(I,J))) 20,20,30
   20 X(I)=ABS (H(I,J))
      IQ(I)=J
   30 CONTINUE
C     SET INDICATOR FOR SHUT-OFF,RAP=2**-27,NR=NO. OF ROTATIONS
      RAP=7.450580596E-9
      HDTEST=1.0E38
C     FIND MAXIMUM OF X(I) S FOR PIVOT ELEMENT AND
C     TEST FOR END OF PROBLEM
   40 DO 70 I=1,NMI1
      IF (I-1) 60,60,45
   45 IF ( XMAX- X(I)) 60,70,70
   60 XMAX=X(I)
      IPIV=I
      JPIV=IQ(I)
   70 CONTINUE
C     IS MAX, X(I) EQUAL TO ZERO, IF LESS THAN HDTEST, REVISE HDTEST
      IF ( XMAX) 1000,1000,80
   80 IF (HDTEST) 90,90,85
   85 IF (XMAX - HDTEST) 90,90,148
   90 HDIMIN = ABS ( H(1,1) )
      DO 110 I= 2,N
      IF (HDIMIN- ABS ( H(I,I))) 110,110,100
  100 HDIMIN=ABS (H(I,I))
  110 CONTINUE
```

```
      HDTEST=HDIMIN*RAP
C     RETURN IF MAX.H(I,J)LESS THAN(2**-27)ABSF(H(K,K)-MIN)
      IF (HDTEST- XMAX) 148,1000,1000
  148 NR = NR+1
C     COMPUTE TANGENT, SINE AND COSINE,H(I,I),H(J,J)
  150 TANG=SIGN (2.0,(H(IPIV,IPIV)-H(JPIV,JPIV)))*H(IPIV,JPIV)/(ABS (H(I
     1PIV,IPIV)-H(JPIV,JPIV))+SQRT ((H(IPIV,IPIV)-H(JPIV,JPIV))**2+4.0*H
     2(IPIV,JPIV)**2))
      COSINE=1.0/SQRT (1.0+TANG**2)
      SINE=TANG*COSINE
      HII=H(IPIV,IPIV)
      H(IPIV,IPIV)=COSINE**2*(HII+TANG*(2.*H(IPIV,JPIV)+TANG*H(JPIV,JPIV
     1)))
      H(JPIV,JPIV)=COSINE**2*(H(JPIV,JPIV)-TANG*(2.*H(IPIV,JPIV)-TANG*H
     1II))
      H(IPIV,JPIV)=0.
C       PSEUDO RANK THE EIGENVALUES
C       ADJUST SINE AND COS FOR COMPUTATION OF H(IK) AND U(IK)
      IF ( H(IPIV,IPIV) - H(JPIV,JPIV)) 152,153,153
  152 HTEMP = H(IPIV,IPIV)
      H(IPIV,IPIV) = H(JPIV,JPIV)
      H(JPIV,JPIV) = HTEMP
C        RECOMPUTE SINE AND COS
      HTEMP = SIGN  (1.0, -SINE) * COSINE
      COSINE = ABS  (SINE)
      SINE = HTEMP
  153 CONTINUE
C     INSPECT THE IQS BETWEEN I+1 AND N-1 TO DETERMINE
C     WHETHER A NEW MAXIMUM VALUE SHOULD BE COMPUTED SINCE
C     THE PRESENT MAXIMUM IS IN THE I OR J ROW.
      DO 350 I=1,NMI1
      IF(I-IPIV)210,350,200
  200 IF(I-JPIV)210,350,210
  210 IF(IQ(I)-IPIV)230,240,230
  230 IF(IQ(I)-JPIV)350,240,350
  240 K=IQ(I)
```

```
  250 HTEMP=H(I,K)
      H(I,K)=0.
      IPL1=I+1
      X(I) =0.
C     SEARCH IN DEPLETED ROW FOR NEW MAXINUM
      DO 320 J=IPL1,N
      IF ( X(I)- ABS ( H(I,J)) ) 300,300,320
  300 X(I) = ABS (H(I,J))
      IQ(I)=J
  320 CONTINUE
      H(I,K)=HTEMP
  350 CONTINUE
      X(IPIV) =0.
      X(JPIV) =0.
C     CHANGE THE OTHER ELEMENTS OF H
      DO 530 I=1,N
      IF(I-IPIV)370,530,420
  370 HTEMP = H(I,IPIV)
      H(I,IPIV) = COSINE*HTEMP + SINE*H(I,JPIV)
      IF ( X(I) - ABS ( H(I,IPIV)) )380,390,390
  380 X(I) = ABS (H(I,IPIV))
      IQ(I) = IPIV
  390 H(I,JPIV) = -SINE*HTEMP + COSINE*H(I,JPIV)
      IF ( X(I) - ABS ( H(I,JPIV)) ) 400,530,530
  400 X(I) = ABS (H(I,JPIV))
      IQ(I) = JPIV
      GO TO 530
  420 IF(I-JPIV)430,530,480
  430 HTEMP = H(IPIV,I)
      H(IPIV,I) = COSINE*HTEMP + SINE*H(I,JPIV)
      IF ( X(IPIV) - ABS ( H(IPIV,I)) ) 440,450,450
  440 X(IPIV) = ABS (H(IPIV,I))
      IQ(IPIV) = I
  450 H(I,JPIV) = -SINE*HTEMP + COSINE*H(I,JPIV)
      IF ( X(I) - ABS ( H(I,JPIV)) ) 400,530,530
  480 HTEMP = H(IPIV,I)
```

```
      H(IPIV,I) = COSINE*HTEMP + SINE*H(JPIV,I)
      IF ( X(IPIV) - ABS ( H(IPIV,I)) ) 490,500,500
  490 X(IPIV) = ABS (H(IPIV,I))
      IQ(IPIV) = I
  500 H(JPIV,I) = -SINE*HTEMP + COSINE*H(JPIV,I)
      IF ( X(JPIV) - ABS ( H(JPIV,I)) ) 510,530,530
  510 X(JPIV) = ABS (H(JPIV,I))
      IQ(JPIV) = I
  530 CONTINUE
C     TEST FOR COMPUTATION OF EIGENVECTORS
      IF(IEGEN)40,540,40
  540 DO 550 I=1,N
      HTEMP=U(I,IPIV)
      U(I,IPIV)=COSINE*HTEMP+SINE*U(I,JPIV)
  550 U(I,JPIV)=-SINE*HTEMP+COSINE*U(I,JPIV)
      GO TO 40
 1000 RETURN
      END
      SUBROUTINE EFM
      RETURN
      END
      SUBROUTINE MATINV(A,N,B,M,DETERM)
      DIMENSION IPIVOT(40), A(40,40), B(40,1), INDEX(40,2), PIVOT(40)
      COMMON PIVOT, INDEX, IPIVOT
      EQUIVALENCE (IROW,JROW), (ICOLUM,JCOLUM), (AMAX, T, SWAP)
C     INITIALIZATION
   10 DETERM=1.0
   15 DO 20 J=1,N
   20 IPIVOT(J)=0
   30 DO 550 I=1,N
C     SEARCH FOR PIVOT ELEMENT
   40 AMAX=0.0
   45 DO 105 J=1,N
   50 IF (IPIVOT(J)-1) 60, 105, 60
   60 DO 100 K=1,N
   70 IF (IPIVOT(K)-1) 80, 100, 740
```

```
 80 IF (ABS (AMAX)-ABS (A(J,K))) 85, 100, 100
 85 IROW=J
 90 ICOLUM=K
 95 AMAX=A(J,K)
100 CONTINUE
105 CONTINUE
110 IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
C      INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
130 IF (IROW-ICOLUM) 140, 260, 140
140 DETERM=-DETERM
150 DO 200 L=1,N
160 SWAP=A(IROW,L)
170 A(IROW,L)=A(ICOLUN,L)
200 A(ICOLUM,L)=SWAP
205 IF(M) 260, 260, 210
210 DO 250 L=1, M
220 SWAP=B(IROW,L)
230 B(IROW,L)=B(ICOLUM,L)
250 B(ICOLUM,L)=SWAP
260 INDEX(I,1)=IROW
270 INDEX(I,2)=ICOLUM
    WRITE (3,36)(A(ICOLUN,ICOLUM))
 36 FORMAT(7F6.0)
310 PIVOT(I)=A(ICOLUM,ICOLUM)
320 DETERM=DETERM*PIVOT(I)
C      DIVIDE PIVOT ROW BY PIVOT ELEMENT
330 A(ICOLUM,ICOLUM)=1.0
340 DO 350 L=1,N
350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)
355 IF(M) 380, 380, 360
360 DO 370 L=1,M
370 B(ICOLUN,L)=B(ICOLUM,L)/PIVOT(I)
C      REDUCE NON-PIVOT ROWS
380 DO 550 L1=1,N
390 IF(L1-ICOLUM) 400, 550, 400
400 T=A(L1,ICOLUM)
```

```
420 A(L1,ICOLUM)=0.0
430 DO 450 L=1,N
450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
455 IF(N) 550, 550, 460
460 DO 600 L=1,M
500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
550 CONTINUE
C     INTERCHANGE COLUMNS
600 DO 710 I=1,N
610 L=N+1-I
620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
630 JROW=INDEX(L,1)
640 JCOLUM=INDEX(L,2)
650 DO 705 K=1,N
660 SWAP=A(K,JROW)
670 A(K,JROW)=A(K,JCOLUM)
700 A(K,JCOLUM)=SWAP
705 CONTINUE
710 CONTINUE
740 RETURN
    END
    SUBROUTINE MPRINT (R, M, L, T1)
    DIMENSION R(40,40), J(40)
    IF (L-1) 2,2,4
  2 L1=19
    GO TO 5
  4 L1=9
  5 J1=0
    J2=0
    JSEC = 0
    DO 8  I= 1,M
  8 J(I) = I
  9 J1 = J2 +1
    J2 = J1 +L1
    IF (J2 - M) 13,13,12
 12 J2=M
```

```
13 JSEC = JSEC + 1
   IF (JSEC -1) 18, 18, 19
18 WRITE(3,17)TI,JSEC
17 FORMAT (1H0, A6, 9H  SECTION I3/)
   GO TO 201
19 WRITE(3,20)TI,JSEC
20 FORMAT (1H1, A6, 9H  SECTION I3/)
201 IF(L-1) 21, 21, 26
21 WRITE(3,22)(J(I),I=J1,J2)
22 FORMAT(6H0 0ROW 3X20I5)
   DO 23 I=1,M
23 WRITE(3,24) I,(R(I,JJ),JJ=J1,J2)
24 FORMAT(I6,4X,20F5.2)
   GO TO 31
26 WRITE(3,27) (J(I),I=J1,J2)
27 FORMAT(6H  ROW ,3X,10I11)
   DO 29 I=1,M
29 WRITE(3,30)I,(R(I,JJ),JJ=J1,J2)
30 FORMAT (I6, 4X 10F11.2)
31 IF (J2-M) 9,32,32
32 RETURN
   END
   SUBROUTINE PUNCH
   DIMENSION R(40,40),J(40)
   IF(M-1) 8, 8, 22
 8 J1 = 0
   J2 = 0
   JSEC = 0
 9 J1 = J2 + 1
   J2 = J1 + 9
   IF (J2 - M) 13, 13, 12
12 J2 = M
13 JSEC = JSEC + 1
   DO 15 I = 1,M
15 WRITE(2,16)IPROB,ATYP,I,JSEC,(R(I,JJ),JJ=J1,J2)
16 FORMAT (I3, 1XA1, I3, I2, 10F7.2)
```

```
      IF (J2 - M) 9, 19, 19
   19 CONTINUE
      GO TO 32
   22 DO 30 I = 1,M
      J1 = 0
      J2 = 0
      JSEC = 0
   24 J1 = J2 + 1
      J2 = J1 + 4
      IF (J2 - M) 26,26,25
   25 J2 = M
   26 JSEC = JSEC +1
      WRITE(2,28)IPROB,ATYP,I,JSEC,(R(I,JJ),JJ=J1,J2)
   28 FORMAT (I3, 1XA1, I3, I2, 5E14.7)
      IF (J2- M) 24,30,30
   30 CONTINUE
   32 RETURN
      END
//GO.SYSIN DD *
```