RELIABILITY, MAINTAINABILITY, SUPPLIABILITY, AVAILABILITY AND COSTS FOR A MULTIGROUP FLEET

By

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NOMENCLATURE

Symbols:

N	is the number of systems.
C	is the value of the critical performance characteristic.
$f(x\|t)$	is the conditional probability of "x" given "t".
x	may be either C_R or C_O .
$\psi(x\|t)$	is the reliability (cumulative probability) of the design value relative to the required value if x is C_R and is the reliability of the operating relative to the design value if x is C_O .
$\Phi^{**}(x\|t)$	is the unreliability (cumulative probability).
$\Phi^{*}(x\|t)$	is the dubiety (cumulative probability).
R	is the joint probability that both the design value and the operating values of a composite (representative) system are reliable.
U	is the overall unreliability (joint probability) that both the design value and the operating values of a composite (representative) system are unreliable; it is, therefore, the "group unreliability".
D_y	is the dubiety of the group. If y is r, the dubiety is the joint probability that the composite design value is reliable relative to the required value and the composite operating value is unreliable relative to the composite design value. If y is m, the overall dubiety is the probability that either the design value is not reliable and the composite operating value is not unreliable or the composite design value is dubious and the composite operating value is unreliable.
L	is the allowed "turn-around" time (lead time).

μ is the service rate.

δ is the technological parameter which is state-of-the-art dependent and which determines the amount by which the design value or the operating value is decreased or improved by environment or maintenance.

\$ is the cost per time interval; i.e., payment due per time interval.

¢ is the unit cost per system per time interval.

P_z is the present worth of future costs of all associated activities at that time when z became of current interest. If z = g, then the time for which P_z is calculated is the birthdate of the group. If z = F, then the time for which P_z is calculated is the beginning of the current planning horizon; i.e., for time I_A.

B_g is the birthdate of group "g"; it is the time at which that group enters the fleet. Its Fortran symbol will be "j".

F_g is the final date of the group "g"; it is the time at which the last member of that group leaves the fleet.

I is the time at which the current planning horizon begins.

H is the time at which the current planning horizon ends.

$N_{e;T}$ is the total number of systems from the same group which have been exited from the fleet to date.

G_a is a function of C_D selected by management such that, if the true required value of the critical performance characteristic were to fall below it, then the composite design value would be considered reliable with respect to the requirements.

G_b is a function of C_D selected by management such that, if the true required value of the critical performance characteristic were to fall above it, then the composite design value would be considered unreliable with respect to the requirements.

G_2 is a function of C_D selected by management such that, if the operating value of the critical performance characteristic were to fall above it, the operating value would be considered reliable with respect to the design value.

G_3 is a function of C_D selected by management such that, if the operating value of the critical performance characteristic were to fall below it, the operating value would be considered unreliable with respect to the design value.

M is the probability that maintenance of a batch of systems from common group will be completed during the allowed turn-around time (L) in the proper type maintenance facility when standard tools, processes, knowledge and personnel are used under expected environmental conditions. It is the current "maintainability" of the batch in the facility.

S is the probability that the number of systems ordered "L_k" time intervals prior to need will be delivered to the fleet inventory at the required time if the source uses the standard information, design methods, manufacturing methods and personnel under expected environmental conditions. "Source" means a particular combination of design and manufacturing sources.

A is the ratio of the number of system-hours during which the forecasted required value for the critical performance characteristic are met to the total number of system-hours which could exist during the same time period if each system in the group were always reliable. It is the "availability".

U.C.A. is the cost per reliable system per time interval; it is the amount of payment required per reliable system during one time interval.

G.L.E.R. is the "group lifetime evaluation ratio" and is defined as the ratio of the cost of buying, systaining and exiting a typical system of a group over the lifetime of the group to the average number of reliable systems in the group during its time in the fleet.

W_p is the "procurement reaction control number" which is a management decision and seeks to assure the proper number of systems which will meet future requirements by taking into account both the decreasing number of currently reliable systems as time progresses and the suppliability of the new-group sources when ordering systems.

Subscripts:

t	is the time of occurrence. All subscripts involving time intervals will be written to the left of the associated symbol. All other subscripts will be written to the right of the associated symbol.
R	means "required" to meet the job specification.
g	stands for "group" and can be expressed in terms of its "j" and "k".
j	is the birthdate of the group; i.e., the date on which the group entered the fleet from some supply facility.
k	designates the group's type of design (standard, up-dated, modified) and the group's sources of design and manufacture.
D	means "design".
O	means "operational".
i	means "the fleet inventory".
r	means "repair facility" or "requiring repair".
ir	means from "i" to "r".
m	means "modification facility" or "requiring modification".
e	means "scrap pile — natural causes".
E	means "scrap pile — wrecked or arbitrarily forced".
F	refers to the whole fleet.
I	refers to the fleet inventory.
C	means "associated overhead".
I;C	shows that both the "I" and the "C" subscripts are necessary to fully designate the costs involved.

q	means "queuing" or "waiting".
A	means "performing the required activity".
Sh	means "shortage".
v	means "salvage value".
p	means "procured" or "procurement facility".
S	means supplied.
T	means "total".
RR	means "reliable in both design and operating value".

CHAPTER I

INTRODUCTION

This monograph is concerned with the activities of a fleet, not a system. To distinguish between a system and a fleet the terms "compound" and "mixture" will be borrowed from the field of chemistry. The constituents of a compound must be combined in fixed proportion and do not maintain their individual characteristics in the presence of each other. The constituents of a mixture are not combined in fixed proportions and commonly do maintain their individual characteristics in spite of the presence and actions of each other. A system is a "compound" of components. A fleet is a "mixture" of components. The components of a fleet are systems. Fleet systems may be planes, taxicabs, professors, engineers, machinists, booster rockets, turnapulls, milling machines, cold chisels, autonomous divisions of a company, companies, monies, etcetera; the term "fleet systems" is wide.

Job requirements for a fleet at any time is expressed in terms of

(1) the number of reliable systems required, and

(2) the value required for the predesignated critical performance characteristic.

Job specifications are those job requirements which must be equaled or exceeded during the time interval in question in order that the fleet be able to accomplish its preassigned task.

Forecasting the future job requirements for fleets is one of the jobs of high level management. The staffs of each manager involved in the forecasting will present to the managers their opinions and reasons for the various predictions. From these staff predictions and other knowledge the managers will form their own opinions concerning future job requirements for each fleet. Usually the opinions of the committee of managers who gather to solidify the forecasts of job requirements will differ somewhat. However, compromises regarding (1) the numbers of reliable systems required for each fleet over the planning horizon, (2) the designation of some systems performance characteristics for use as the critical ones for the expected mission types, and (3) the required maximum and the required minimum values of the critical performance characteristics over the planning horizons are eventually reached and the job specifications for each fleet are stated.

As shown in Figures 1 and 2, the resulting job specifications for a fleet do vary with time and are expressed in terms of (1) the estimated maximum and minimum values of the management-designated critical performance characteristic and (2) the required number of reliable systems. For example, the job specifications for a fleet of trans-

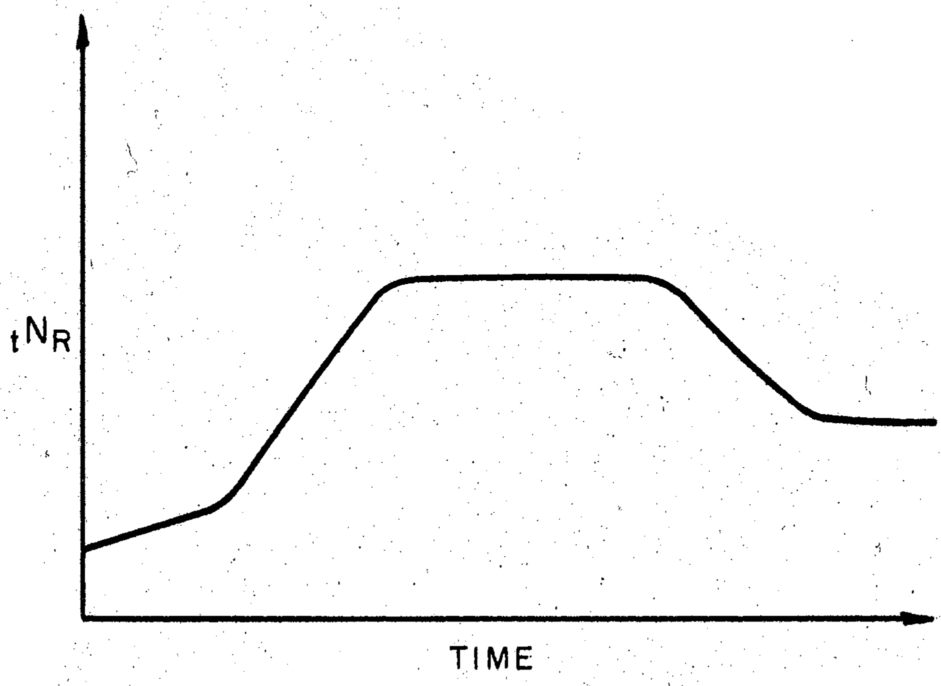

Figure 1. A Job Specification for a Fleet: The Forecasted Required Number of Reliable Systems

ports four years from now might require 100 planes with cruising speeds of 1100 to 1500 miles per hour. Job specifications for senior engineering professors seven years from now might be theoretical and applied knowledge as evidenced by a Ph.D. in engineering, a Master's degree in statistics or business administration and three to five years of progressive work in industry. The expected values for the extremes of the required critical performance characteristic (namely, ${}_tC_{R\ max}$ and ${}_tC_{R\ min}$) are based upon the expected types of missions and environments. Because the missions and environments change with time, the forecasted required values also change. Obviously the further into the future a forecaster must project, the less certain are the values. The range between the expected maximum and minimum values of ${}_tC_R$ increases as time progresses and, as shown in Figure 3, the management-estimated distribution of the C_R values between these extremes may flatten.

A fleet inventory is defined as being composed of those systems which are currently operational; i.e., those fleet systems which are neither in the scrap pile nor currently in the maintenance facilities. The function of a fleet inventory is to meet the projected job specifications for that fleet. At any moment in time a fleet inventory may experience simultaneously the following gains and losses which are illustrated in Figure 4:

(1) the permanent loss of aging and wrecked systems by transition to the unreliable state and there-

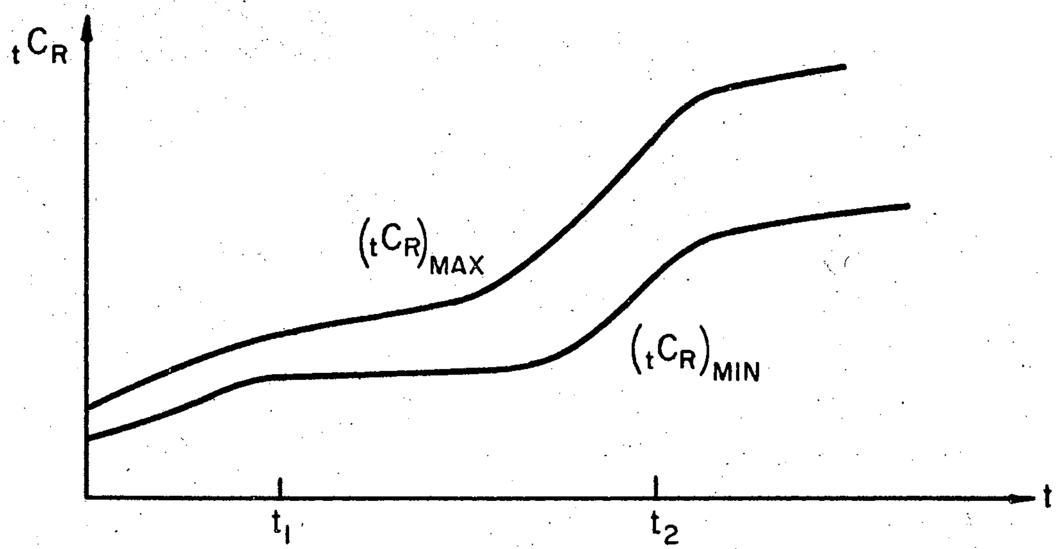

Figure 2. A Job Specification for a Fleet: The Estimated Maximum and Minimum Values of Management-Designed Performance Characteristic

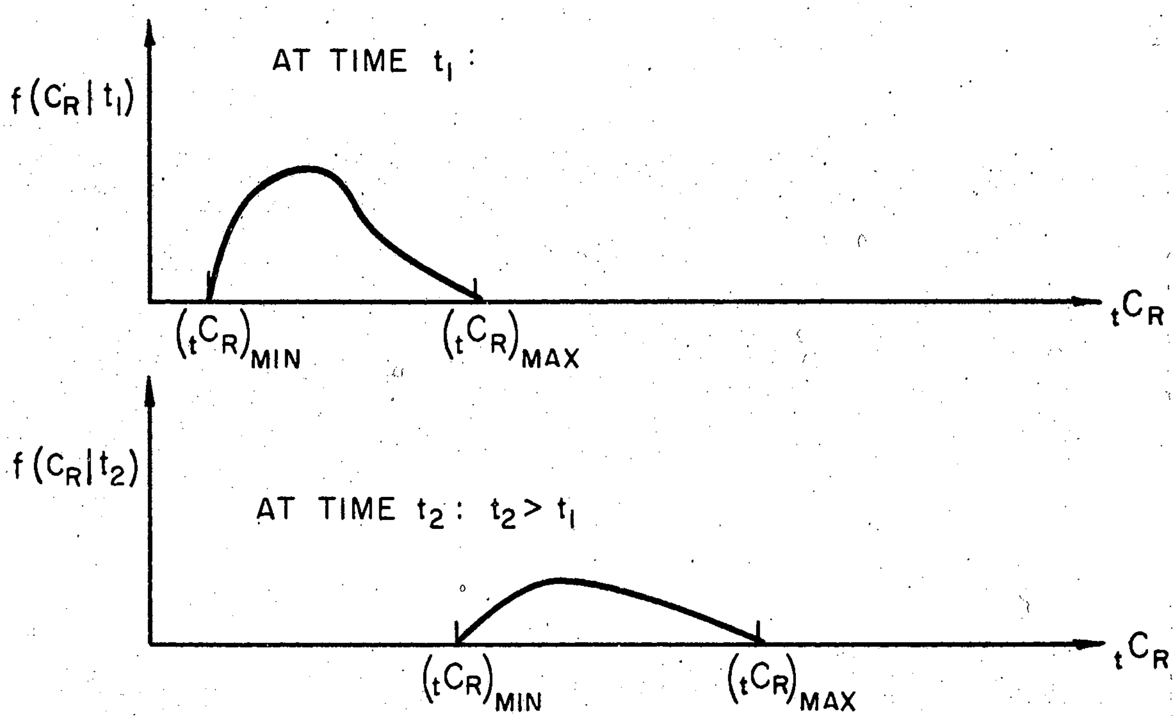

Figure 3. Postulated Distribution of the Required Value of the Critical Performance Characteristic at Different Times

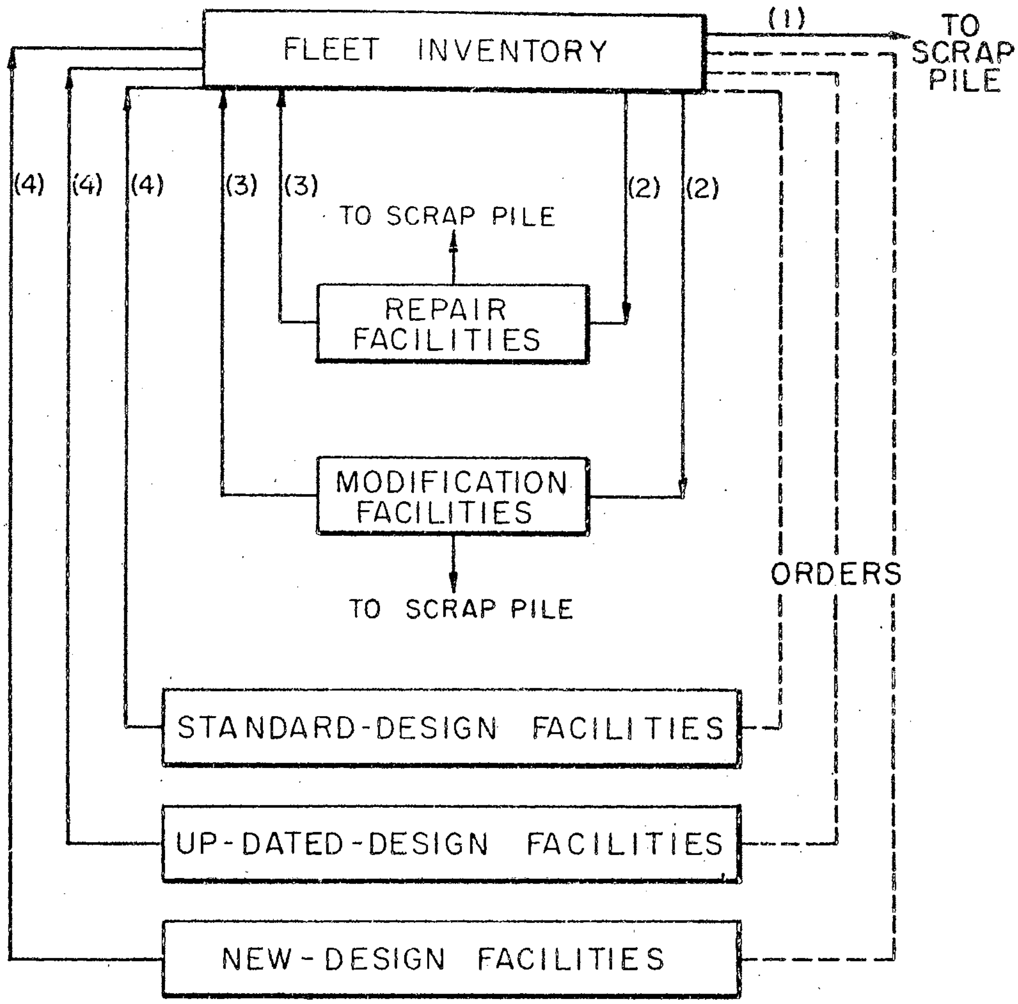

Figure 4. The Simultaneous-Gains-and-Losses Cycle of a Fleet Inventory

fore to the scrap pile,

(2) the temporary loss of aging systems by transition to the maintenance states and therefore to the repair and the modification facilities,

(3) the gain of renewed and modified systems from the maintenance (repair and modification) facilities,

(4) the gain of new systems from the supply facilities.

Repair is distinguished from modification by the fact that repair seeks to bring the operating value closer to the design value of the critical performance characteristic and hence improves only the operating value. Modification seeks to improve the design value of the critical performance characteristic and hence improves the operating value as well as the design value. For a plane, repair would be exemplified by both the installation of new turbine blades in the place of pitted ones in the jet engine and the installation of a new engine of the same design. Such repairs on a plane would increase the operating speed of the plane. Modification of a plane would be exemplified by the installation of a new engine of an improved type. Such a modification to a plane would increase the design value as well as the operating value of the speed of the plane.

The purpose of this monograph is to present a method for calculating the costs and other measures of fleet inventory effectiveness while continually trying to meet the time-dependent job specification for the fleet over a planning horizon. Based upon the theory developed several measures of effectiveness ("figures of merit") may be calculated in terms of the reliabilities, maintainabilities and suppliabilities of groups; namely,

(1) the average shortage of reliable systems over the planning horizon,

(2) the present worth of the cost of buying, operating and maintaining the fleet under the assumption that the purpose of the fleet inventory is to try to meet the changing job specifications,

(3) the availability of the fleet at any time "t" within the planning horizon,

(4) the cost per reliable system in the fleet at any time "t".

Several measures of effectiveness of the individual groups also may be calculated; namely,

(1) the availability of the group at any time "t" within its "lifetime" (the length of time any member of the group is in the fleet),

(2) the cost per reliable system in the group at any time "t" within the group's lifetime.

Taken into account for all measures of effectiveness will be design obsolesence, decay of critical performance characteristics, repair and modification, the entering of new systems of both standard and improved types of design into the fleet and the exiting of aged and other systems from the fleet.

Theoretical relationships among reliability, maintainability, availability, suppliability and costs for a fleet which must try to meet job requirements are developed in Chapters II through VII. Chapter VIII presents a sample problem in order to enhance the reader's awareness of the interplay among the state vector of the fleet inventory, the management-dictated decision variables, the parameters of the problem and the costs involved. Chapter IX states the summary, conclusions and suggestions for future research. For the convenience of the reader the nomenclature was placed at the beginning of the thesis and the symbols were listed approximately in order of occurrence.

CHAPTER II

DESIGN AND OPERATING VALUES OF THE CRITICAL PERFORMANCE CHARACTERISTIC

The design of a system is planned to meet or exceed the expected value of the then-current performance requirements; in other words, the design value of the performance characteristic at time "t", ${}_tC_D$, must equal to or greater than the required value, ${}_tC_R$, at that time. There is always a probability that a past design, a current design or even a just-firmed-up design may be obsolete. Fortunately most systems are designed for "growth" and hence are initially designed with ${}_tC_D$ greater than the forecasted maximum of the required value of the performance characteristic at time "t". When the C_D of a system is less than some future desired value, modification may be used to increase it from the initial production value to a somewhat larger value. Repair, however, does not change C_D.

Although a system is designed to have specific performance characteristics, it is well known that the operating value of the critical performance characteristic of a system, ${}_{t_2}C_O$, is generally less than that system's design value, ${}_{t_1}C_D$, when $t_2 > t_1$. The deterioration of the operating value is the result of usage and environmental conditions. The actual amount of decay which will take place is probabilistic, but the expected maximum and minimum values of ${}_tC_O$ may be predicted from historical data. Both repair and modification will increase ${}_tC_O$.

A group is a collection of systems which have the same design value and which were procured from the same source at the same time. Thus a group is designated by

(1) its "birthdate", which is the time it entered the fleet,

(2) its kind of design, which is either standard or up-dated or new,

(3) its source of design,

(4) its source of manufacture, which may be the same as its source of design.

In this monograph the first designation has been assigned the Fortran subscript "j". The last three designations have been combined in the assigned Fortran subscript "k" as illustrated by the array in Table I which arbitrarily assumes three sources of design and the same three sources of manufacture. Thus, group "g" is designated by a particular combination (j,k). For example, Boeing, Lockheed and North American could each design a plane for different speeds and could be forced by the government to manufacture the others design.

A batch is a collection of systems which are in the same group and which have the same operating value. Thus, a batch is characterized by its size (the number of systems

TABLE I

THE "k" DESIGNATION FOR A GROUP

The "k" Designation (Illustrated)		B/B	B/L	B/N	L/B	L/L	L/N	N/B	N/L	N/N
		Source of Design/Source of Manufacture								
Design Type	standard	1	4	7	10	13	16	19	22	25
	up-dated	2	5	8	11	14	17	20	23	26
	new	3	6	9	12	15	18	21	24	27

composing it) and by the design and the operating values of the critical performance characteristic common to the systems composing it.

When first procured, all systems in a group will belong to the same batch; but as time progresses the group will be fractured into increasingly smaller sized batches by the useage and the maintenance received by each system. Some systems will be scrapped. Eventually each batch remaining in the fleet would consist of only one system. To keep track of the history of each system in a fleet is time consuming or impossible, but the ability of the fleet to meet job specifications at any time is determined by the design and operating values of each system in the fleet inventory. The problem is attacked in the following way which permits calculations for all current fleet inventory systems belonging to the same group to be made as if the systems had a common design value and common maximum and minimum operating values and as if they had previously belonged in only three batches.

A "composite system", which represents in action the fleet inventory members of the group, is formed by calculating composite design values and composite operating values for those members. The composite design value and the composite operating values are calculated by weighting the design values and the operating values, respectively, of a typical system in each of the group's batches by the number of systems composing the batch the typical system represents. Figure 5 demonstrates the concurrent activities of the batches from a single group: One batch of reliable systems remain in the fleet inventory; two batches of non-reliable systems are sent to the two types of maintenance facilities; one batch of unreliable systems is sent to the scrap pile and two earlier batches of systems are returned to the fleet inventory from the maintenance facilities in renewed condition. The three batches of the group which determine the fleet inventory composite design value and the composite maximum and minimum operating values for that group during the current time interval are the two batches (N_{ri} and N_{mi}) which re-entered the fleet inventory from the maintenance facilities just prior to the beginning of the current time interval and the batch (N_{RR}) which remained reliable throughout that time interval. When members of the group are forced to leave the inventory, assume that

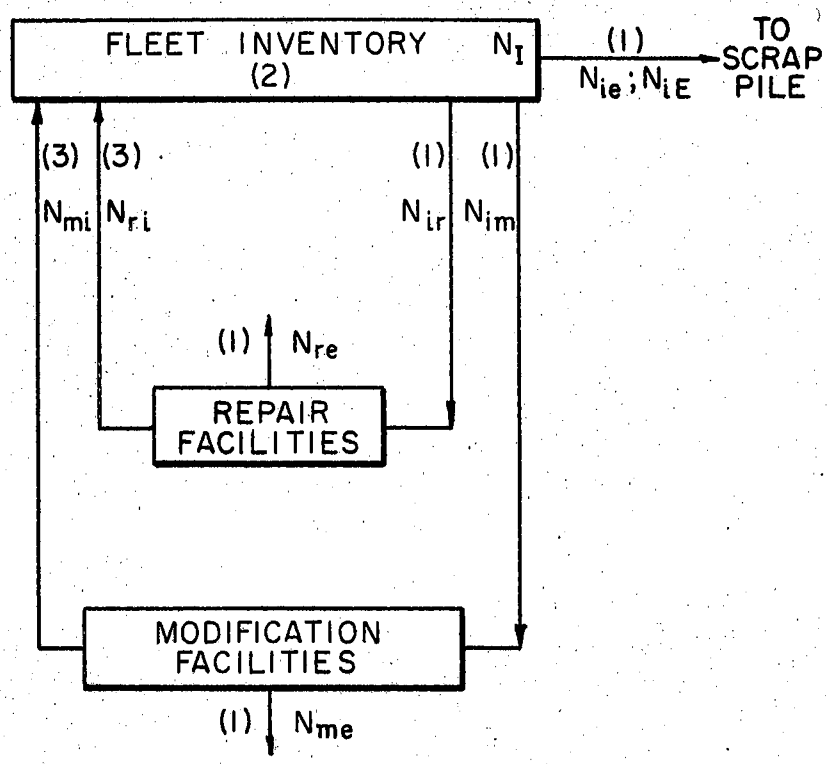

Figure 5. Concurrent Activities of the Batches of Group "g" at Time "t"

they (N_{iE}) belonged to the reliable batch. Obviously, those batches which have left the inventory have no effect on the composition of the fleet inventory during the current time interval. Let the composite design value and the composite maximum and minimum operating values for those member systems of group "g" which are in the fleet inventory at time "t" be calculated thusly:

$$
{}_{t}\bar{C}_{Dg} = \Big\{[{}_{t-1}\bar{C}_{Dg}][{}_{t-1}N_{RRg} - {}_{t-1}N_{ieg}] + [{}_{t-L_r-1}\bar{C}_{Dg}][{}_{t-1}N_{rig}] + [{}_{t-L_m-1}\bar{C}_{Dg} + {}_{t-L_m-1}\delta_{Dg}][{}_{t-1}N_{mig}]\Big\} \Big/ \Big\{{}_{t-1}N_{RRg} + {}_{t-1}N_{rig} + {}_{t-1}N_{mig} - {}_{t-1}N_{iEg}\Big\} \qquad (2.1)
$$

$$
{}_{t}\bar{C}_{\theta\ max;g} = \Big\{[{}_{t-1}\bar{C}_{\theta\ max;g} - {}_{t-1}\delta_{01g}][{}_{t-1}N_{RRg} - {}_{t-1}N_{iEg}] + [{}_{t-L_r-1}\bar{C}_{\theta\ max;g} + {}_{t-L_r-1}\delta_{03g}][{}_{t-1}N_{rig}] + [{}_{t-L_m-1}C_{\theta\ max;g} + {}_{t-L_m-1}\delta_{05g}][{}_{t-1}N_{mig}]\Big\} \Big/ \Big\{{}_{t-1}N_{RRg} + {}_{t-1}N_{rig} + {}_{t-1}N_{mig} - {}_{t-1}N_{iEg}\Big\} \qquad (2.2)
$$

$$
{}_{t}\bar{C}_{\theta\ min;g} = \Big\{[{}_{t-1}\bar{C}_{\theta\ min;g} - {}_{t-1}\delta_{02g}][{}_{t-1}N_{RRg} - {}_{t-1}N_{iEg}] + [{}_{t-L_r-1}\bar{C}_{\theta\ min;g} + {}_{t-L_r-1}\delta_{04g}][{}_{t-1}N_{rig}]
$$

$$+ \quad [{}_{t-L_m-1}\bar{C}_{\theta\ min;g} + {}_{t-L_m-1}\ \delta_{06g}][{}_{t-1}N_{mig}]\Big\} \Big/$$

$$\Big\{{}_{t-1}N_{RRg} + {}_{t-1}N_{rig} \pm {}_{t-1}N_{mig} - {}_{t-1}N_{iEg}\Big\} \qquad (2.3)$$

where

(1) the technological parameters ${}_{t-1}\ \delta_{01g}$ and ${}_{t-1}\ \delta_{02g}$ are the amounts by which deterioration reduces the ${}_{t-1}\bar{C}_{0g}$ maximum and minimum values of the systems which were in the fleet inventory during the previous interval of time;

(2) ${}_{t-L_r}\ \delta_{03g}$ and ${}_{t-L_r}\ \delta_{04g}$ are the amounts by which repair improves the ${}_{t-L_r}\bar{C}_{0g}$ maximum and minimum values of the systems which were in the repair facilities during the previous L_r intervals of time;

(3) ${}_{t-L_m}\delta\ _{05g}$ and ${}_{t-L_m}\ \delta_{06g}$ are the amounts by which the modification did improve the ${}_{t-L_m}\bar{C}_{0g}$ maximum and minimum values of those systems which were in the modification facilities during the previous L_m intervals of time; and

(4) ${}_{t-L_m}\delta\ _{Dg}$ is the amount by which the modification was designed to improve the ${}_{t-L}\bar{C}_{Dg}$ of those systems which were in the modification facilities during the previous L_m intervals of time.

The technological parameters are so-named because their values depend upon the state-of-the-arts involved and may improve as time progresses. The values assigned to them must be based on historical data and/or informed judgement.

Methods for calculating the number of systems in each of the batches will be given in Chapter V.

Equation (2.1) points out the fact that the design value of the characteristic which the composite system for group "g" will exhibit during the early part of the current time interval "t", which ends at time "t", is dependent upon the following:

(1) the composite design value of the previous time interval (${}_{t-1}\overline{C}_{Dg}$) which is weighted by the number of systems in the reliable batch held over from that time interval (${}_{t-1}N_{RR}$);

(2) the composite design value of a time interval which occurred L_r time intervals prior to the current time interval (${}_{t-L_r-1}C_{Dg}$) when that batch which was returned to the fleet inventory from the repair facility during the previous time interval had been sent to the repair facility from the fleet inventory; that design value is weighted by the number of systems in the returned batch (${}_{t-1}N_{rig}$);

(3) the composite design value of a time interval which occurred L_m time intervals prior to the current time interval (${}_{t-L_m-1}C_{Dg}$) when that batch which was returned to the fleet inventory from

the modification facility during the previous time interval had been sent to the modification facility from the fleet inventory; that design value is weighted by the number of systems in the returned batch ($_{t-1}N_{mig}$) and improved by the amount which the state-of-the-art of modification will provide.

For a plane which has speed defined as its critical performance characteristic, Equation (2.1) states that the design speed which the fleet inventory will seem to have during the current time interval "t" is the equal to the sum of

(1) the product of the unchanged design speed of the batch of reliable planes, which were left over from the previous time interval, and the number of planes which were in that batch;

(2) the product of the unchanged design speed of the batch of planes, which had just returned from the repair facility where it had spent L_r time intervals, and the number of the planes in the returned batch; and

(3) the product of the increased design speed of the batch of planes, which had just returned from the modification facilities where it had spent L_m time intervals, and the number of planes in the returned batch.

Figure 6 shows the effects of modification on the

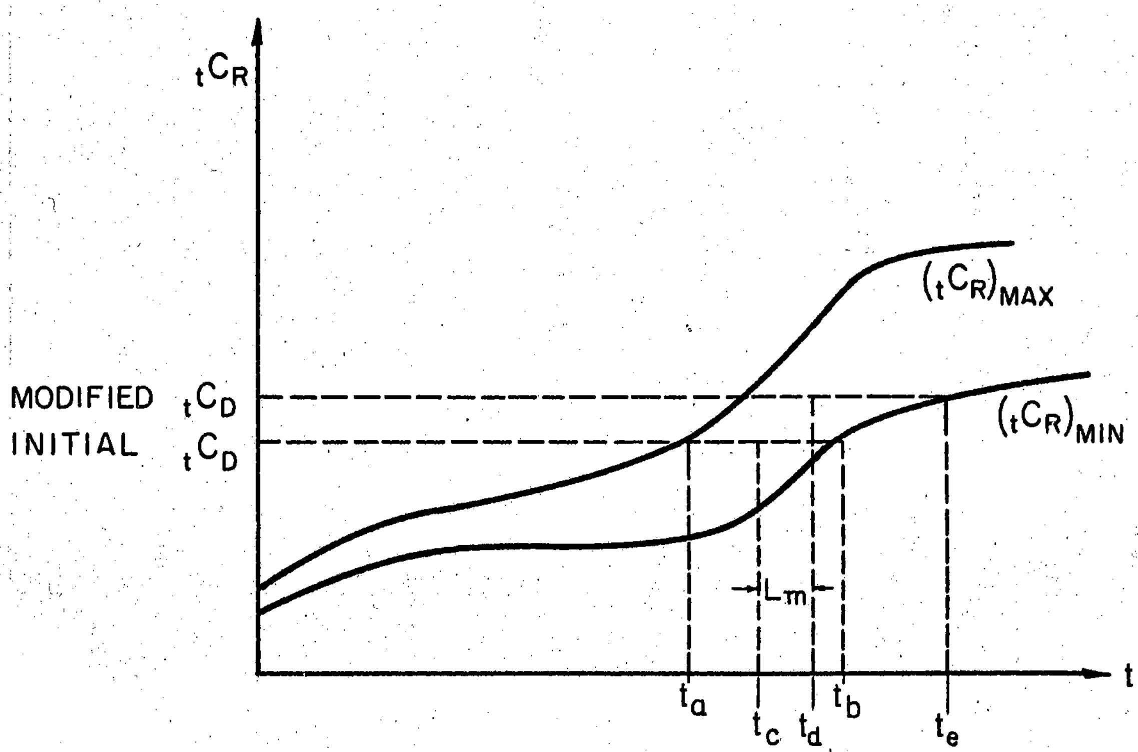

Figure 6. Effects of Modification on the Design Value of the Critical Performance Characteristic of a Group of Standard-Design Systems

design value of the critical performance characteristic of a group of standard-design systems. A standard design system is one whose design value of the critical performance characteristic (${}_tC_D$) barely meets the maximum forecasted required value of the characteristic (${}_tC_{R\ max}$) at the time it is delivered to the fleet. A design is obsolete when its ${}_tC_D \leq {}_tC_{R\ min}$. In Figure 5 "$t_a$" is the time at which the system enters the fleet; "t_b" is the time at which the design would become obsolete if modification did not occur; "t_c" is the time at which the system leaves the fleet inventory and goes to the modification facility; "t_d" is the time at which the modified system is returned to the fleet inventory; and "t_e" is the time at which the modified design becomes obsolete. $(t_c - t_d)$ is equal to the allowed modification "turn-around" time. "Turn-around" time includes the time to discover the need for maintenance, the administrative time, the two-way transportaion time and the actual time required for maintenance.

Figure 7 shows the estimated effects of decay, repair and modification on the expected maximum and minimum operating values of the critical performance characteristic of a group of systems. Modification takes longer to perform than does repair but it improves the operating values by greater amounts than does repair. In the figure, "t_a" is the time at which a group of systems entered the fleet; "t_k" is the time at which two of the systems were sent to the maintenance facilities; "t_c" is the time at which the

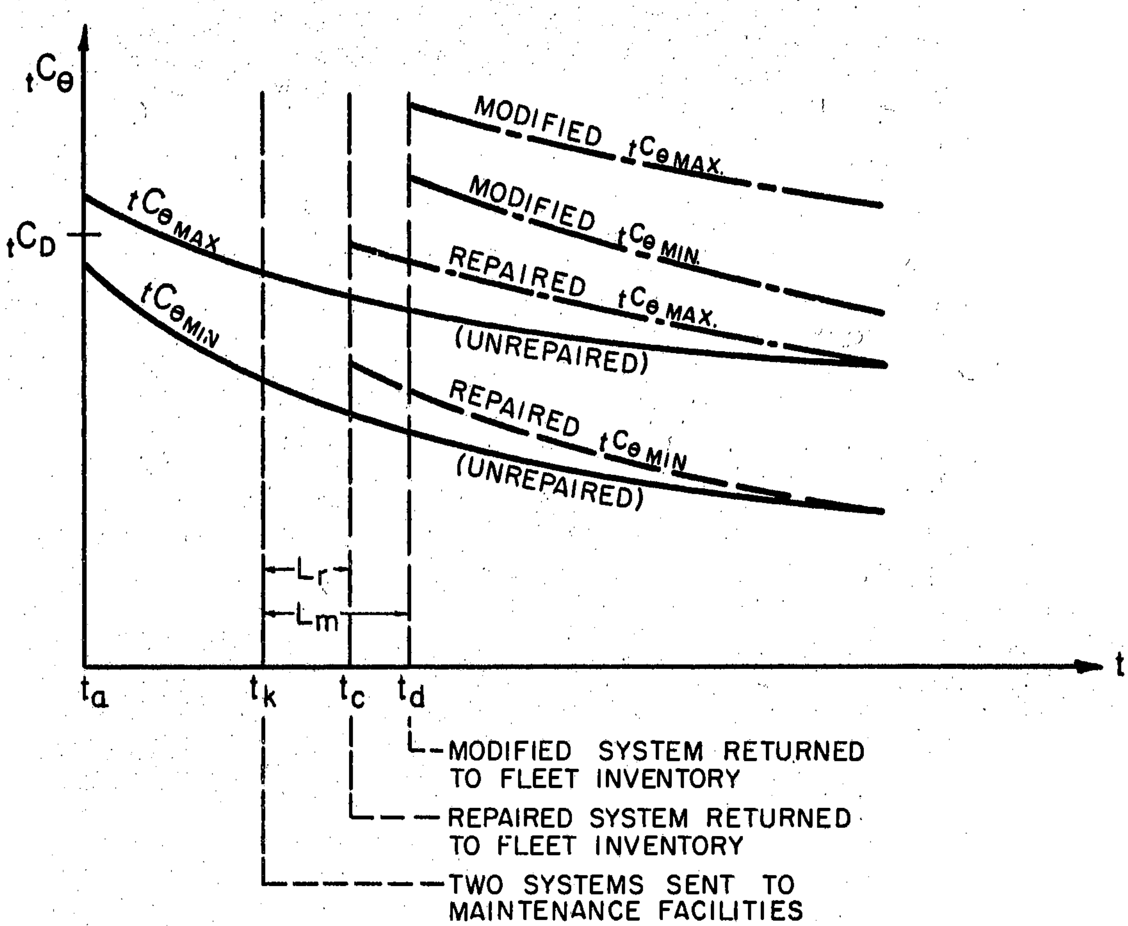

Figure 7. Estimated Effects of Decay, Repair and Modification on the Expected Maximum and Minimum Operating Values of the Critical Performance Characteristics of a Group of Systems

repaired system returned to the fleet inventory; and "t_d" is the time at which the modified system was returned to the fleet inventory.

Figure 8 shows the estimated effects of decay of operating values and maintenance on the operating and design values of the critical performance characteristic of a group of systems of new or up-dated design. Lines (ak) and (cl) show the forecasted decay patterns of the maximum and minimum operating values of the critical characteristic for a group when the original design has greater value than the maximum required at the time (B_g) when the group entered the fleet with the new or up-dated design and when no maintenance occurs. The range of operating values (point "a" to point "c") is the initial expected range of the group's operating value but the range is expected to widen as time progresses. Because a design is probably obsolete when its value is equal to the minimum required value of the critical performance characteristic, the greatest amount of elapsed time which could be expected prior to the design obsolescence is ($t_h - B_g$) if repair were continuous and always brought back the operating value of the critical performance characteristic to the design value of the characteristic; i.e., if there were ideal repair. Lines (or) and (pq) show the more usual results of actual repair on the maximum and minimum operating values of the critical characteristic; that is, usually repair does not make the operating values completely equal again the values which they had prior to

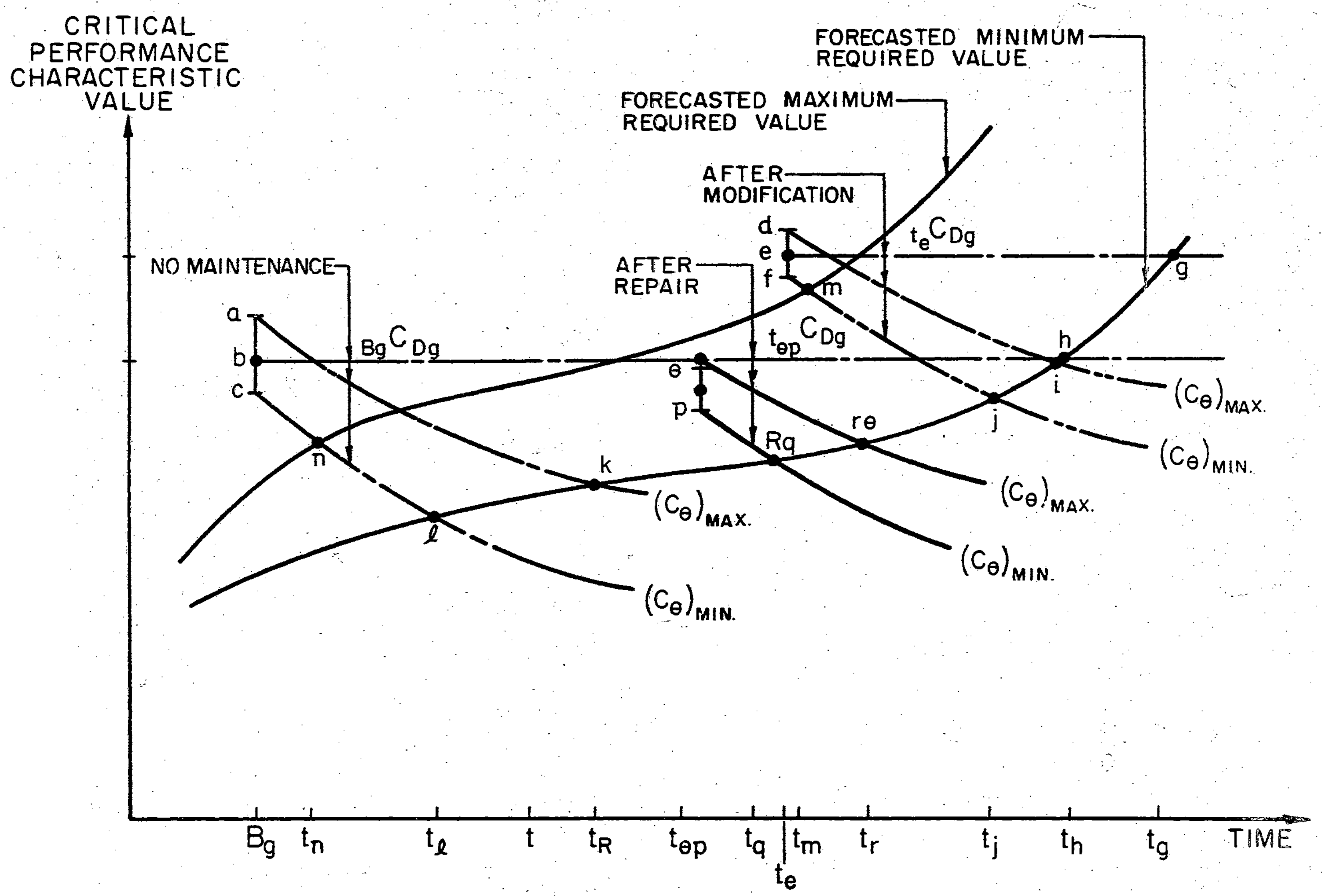

Figure 8. Estimated Effects of Decay and Maintenance on Both the Design Value and the Operating Value of the Critical Performance Characteristic of a Group of Up-Dated-Design Systems

decay nor equal to the design value. Lines (di) and (fj) are the operating values which might occur as a result of modification and line (eg) may be the resulting new design value of the critical performance characteristic which might occur from modification. Notice that modification has increased the time to probable design obsolescence from "t_h" to "t_g".

CHAPTER III

RELIABILITY AND NON-RELIABILITY OF A GROUP

Only those systems which are in the fleet inventory at the moment can be used to meet current job specification. It is the reliability and the non-reliability of these fleet inventory systems which are the concern of this chapter. Group reliability at time "t" is defined as the probability that those systems which are in the fleet inventory during the time interval, which ends at time "t", will be able to perform their assigned tasks during that interval if they are used under the environmental conditions for which they were designed. Group reliability is dependent upon both the reliability of the composite design value of the group's fleet inventory systems and the reliability of the composite operating value of the group's fleet inventory systems.

Design Reliability, Dubiety and Unreliability

Whether a specific design value of the critical performance characteristic will be obsolete or reliable at some future time is dependent upon whether that design value is smaller or larger than the required value of the characteristic at that time. The future growth pattern of the required value of the critical performance characteristic can not be forecast with certainty. Instead, estimated maximum and minimum required values are stated for future dates. A probabilistic distribution of required values is assumed to exist between these extreme values. Hence there is a risk involved in declaring that at a specific future date the composite design value of a group's fleet inventory systems ($_t\bar{C}_{Dg}$) will become obsolete. Informed judgement should, however, allow management to select two functions of $_t\bar{C}_{Dg}$ such that, if the true required value were to fall below one of these functions—say, $_tG_{ag}$—then the composite design value would be considered reliable, and, if the true required value were to fall above the other of these functions—say, $_tG_{bg}$—then the composite design value of the group would be considered unreliable with respect to the concurrent required value. If the true required value were to fall inbetween $_tG_{ag}$ and $_tG_{bg}$, the status of the design value would be dubious. Because the true required value of the critical performance characteristic is not known at any time and only its probable maximum and minimum values and distribution are forecasted, the cumulative probabilities in the following equations will be used to define, respectively, the reliability, the unreliability and the dubiety of the composite design value for the group "g" at some time "t" with respect to the concurrent true required value ($_tC_R$) of the critical performance characteristic:

$$_t\Psi_{Dg} \equiv \int_{_t(C_R)_{min}}^{_t(G_a)_g} f(C_R | t)\; dC_R \qquad (3.1)$$

$$_t\Phi^{**}_{Dg} \equiv \int_{_t(G_b)_g}^{_t(C_R)_{max}} f(C_R|t)\ dC_R \qquad (3.2)$$

$$_t\Phi^{*}_{Dg} \equiv \int_{_t(G_a)_g}^{_t(G_b)_g} f(C_R|t)\ dC_R \qquad (3.3)$$

where

$_t(G_a)_g \equiv \text{function}_1 \text{ of } _t\bar{C}_{Dg}$

$_t(G_b)_g \equiv \text{function}_2 \text{ of } _t\bar{C}_{Dg}$

$_t(G_a)_g < {_t(G_b)_g}$.

Unless additional knowledge otherwise indicates, the following functions may be used to further define the G's:

$$_t(G_a)_g = [f_a][_t\bar{C}_{Dg}] \quad \text{and} \quad f_a < 1.000 \qquad (3.4)$$

$$_t(G_b)_g = [f_b][_t\bar{C}_{Dg}] \quad \text{and} \quad f_b > 1.000\ . \qquad (3.5)$$

As shown in Figure 9, $_t\Psi_{Dg} + {_t\Phi^{**}_{Dg}} + {_t\Phi^{*}_{Dg}} = 1$. (3.6)

For example, assume the design speed of the group "g's" composite fleet inventory plane is 800 miles per hour and that management stipulates that the composite design speed of the plane in the fleet inventory will be considered reliable with respect to the true required speed if the true required speed is less than 95 per cent of the design speed and will be considered unreliable if the true required speed is greater than 105 per cent of the design speed. Hence

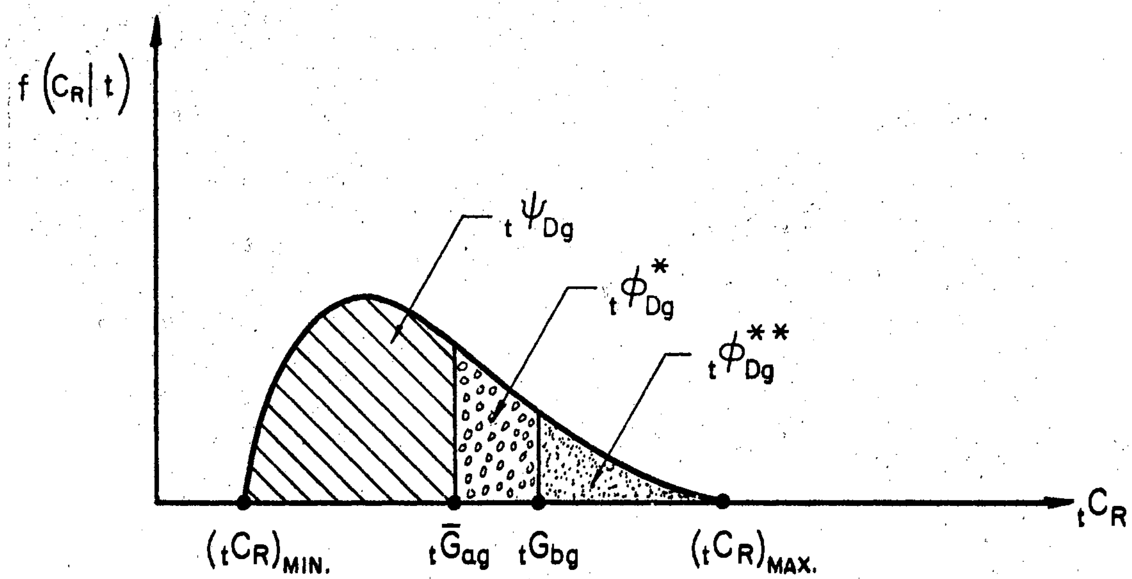

Figure 9. Design Reliability, Dubiety and Unreliability of the Current Fleet Inventory Systems of Group "g" Expressed in Terms of a Possible Distribution of the Systems Current Required Value of the Critical Performance Characteristic and Their Composite Design Value

${}_tG_{ag}$ = (.95)(800) = 760 miles per hour and ${}_tG_{bg}$ = (1.05)(800) = 840 miles per hour. However, the true required speed is not known with certainty as previously pointed out. Figure 10 illustrates the time dependency of the proportions of probability among the reliability, dubiety and unreliability of the design value with respect to the time-dependent estimated maximum and minimum required values of the critical performance characteristic (speed) when there has been no modification. At the time the group enters the fleet (time B_g), ${}_tG_{ag}$ is greater than all possible values of the true required speed. As time progresses the proportion of possible values of the true required speed which ${}_tG_{ag}$ is larger than becomes smaller and the proportion of possible values of the true required speed which ${}_tG_{bg}$ is less than becomes larger. The striped portions of the $f(C_R | t)$ versus C_R sections indicate the reliability (probability of being reliable) of the design speed with respect to the current true required speed. The smoky portions of the sections indicate the unreliability (probability of being unreliable) of the design speed and the dotted portions of the sections indicate the dubiety (probability of being of dubious worth) of the design speed.

Operating Reliability, Dubiety and Unreliability

The actual decay pattern of the composite operating value of the critical performance characteristic of the group "g" composite system can not be predicted exactly.

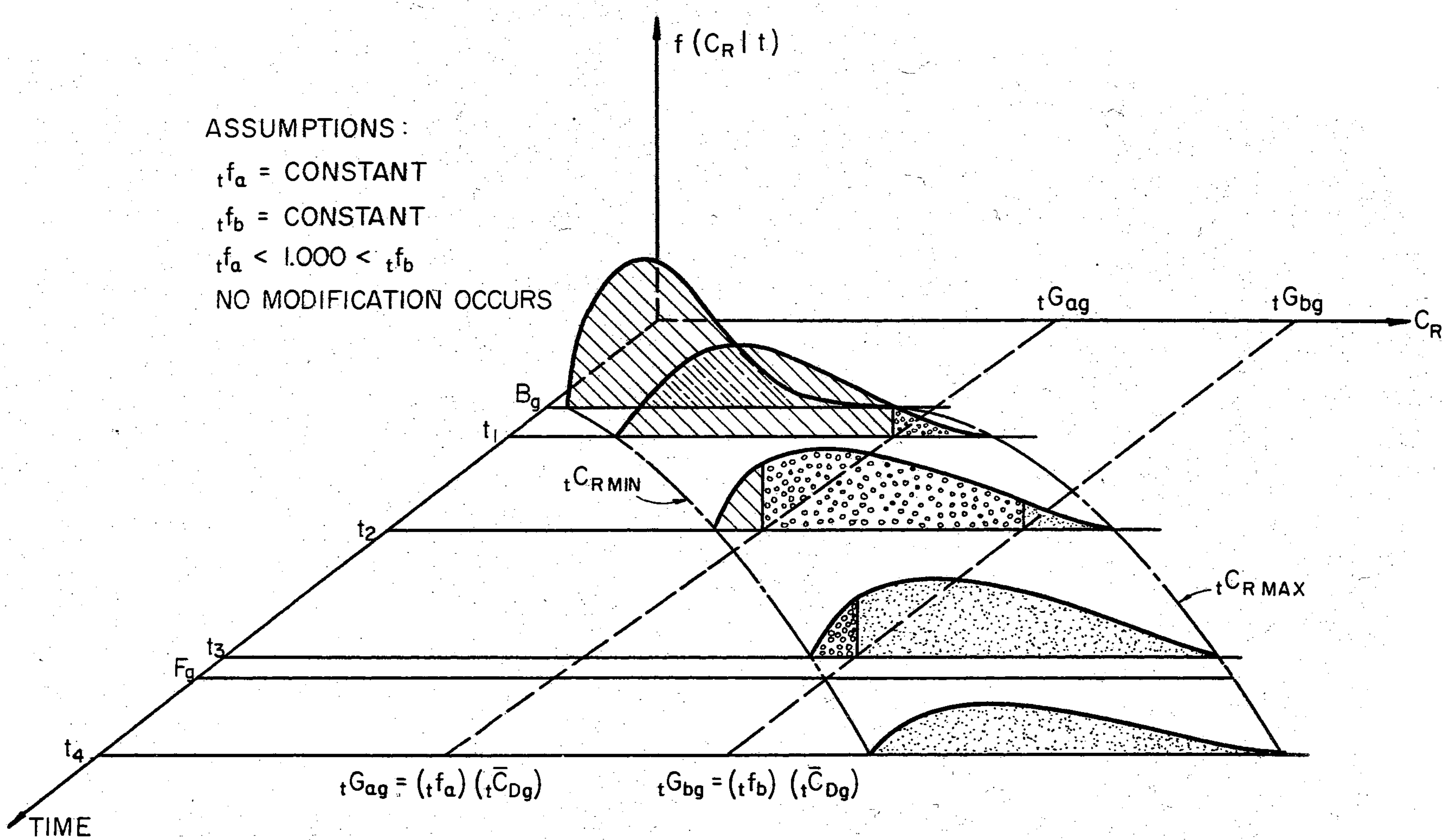

Figure 10. Time-Dependency of the Properties of Probability Among the Reliability, the Dubiety and the Unreliability of the Design Value With Respect to the Time-Dependent Estimated Maximum and Minimum Values of the Critical Performance Characteristics When There Is No Modification

Hence there is a risk involved in declaring the composite operating value of a group's fleet inventory systems, ${}_t\overline{C}_{0g}$, to be either reliable or unreliable with respect to its design value, ${}_t\overline{C}_{Dg}$. Informed judgement should, however, allow management to select some fraction of the design value (say, ${}_tG_{2g}$) above which the operating values should be classified as reliable and another fraction of the design value (say, ${}_tG_{3g}$) below which the operating values should be classified as unreliable. The status of those operating values which fall between the two management-designated values, ${}_tG_{2g}$ and ${}_tG_{3g}$, is dubious. Figure 11 illustrates these points graphically and the equations below illustrate them mathematically:

$$ {}_t\Psi_{0g} \equiv \int_{{}_tG_{2g}}^{({}_t\overline{C}_{0g})_{max}} f(C_C | t)\ dC_0 \qquad (3.7)$$

$$ {}_t\Phi^{**}_{0g} \equiv \int_{({}_t\overline{C}_{0g})_{min}}^{{}_tG_{3g}} f(C_0 / t)\ dC_0 \qquad (3.8)$$

$$ {}_t\Phi^{*}_{0g} \equiv \int_{{}_tG_{3g}}^{{}_tG_{2g}} f(C_0 / t)\ dC_0 \qquad (3.9)$$

where, unless additional knowledge indicates that different functions should be used,

$$ {}_tG_{2g} \equiv [{}_tf_2][{}_t\overline{C}_{Dg}] \qquad (3.10)$$

$$ {}_tG_{3g} \equiv [{}_tf_3][{}_t\overline{C}_{Dg}] \qquad (3.11)$$

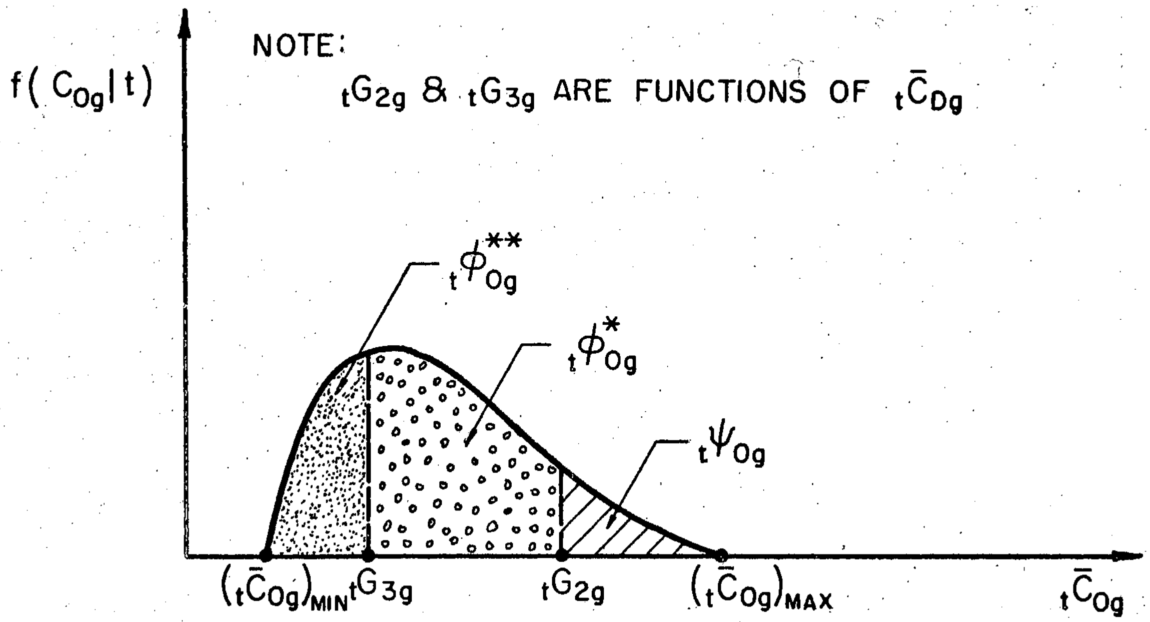

Figure 11. Operating Reliability, Dubiety and Unreliability of the Current Fleet Inventory Systems of Group "g" Expressed in Terms of Possible Distribution of the Systems Current Operating Values and Their Current Composite Design Value of the Critical Performance Characteristic

and ${}_tf_2$ and ${}_tf_3$ are fractions which are designated by current management policies.

As shown in Figure 11 for the composite system from group "g" at time "t":

$$ {}_t\Psi_{0g} + {}_t\Phi_{0g}^{**} + {}_t\Phi_{0g}^{*} = 1 \quad (3.12)$$

For example, assume that the composite design speed of group "g's" composite fleet inventory plane is 800 miles per hour and that management stipulates that the operating speeds of those planes which are greater than 90 per cent of the composite design speed will be classified as reliable and those operating speed which are less than 80 per cent of the composite design speed will be classified as unreliable. Hence ${}_tG_{2g}$ = (.90)(800) = 720 miles per hour and ${}_tG_{3g}$ = (.80)(800) = 640 miles per hour. Fleet inventory planes which belong to the same group have various operating speeds. Figure 12 illustrates the time-dependency of the proportions of probability among the reliability, dubiety and unreliability of the time-dependent operating values of the fleet inventory members of group "g" with respect to their composite design speed when there has been no repair or modification. At the time the group enter the fleet (time B_g) ${}_tG_{2g}$ is less than most or all of the operating speeds. As time progresses the operating speeds of the planes in the fleet inventory decay, the proportion of operating speeds greater than ${}_tG_{2g}$ decreases and the proportion of operating speeds less than ${}_tG_{3g}$ increases. The striped portions of

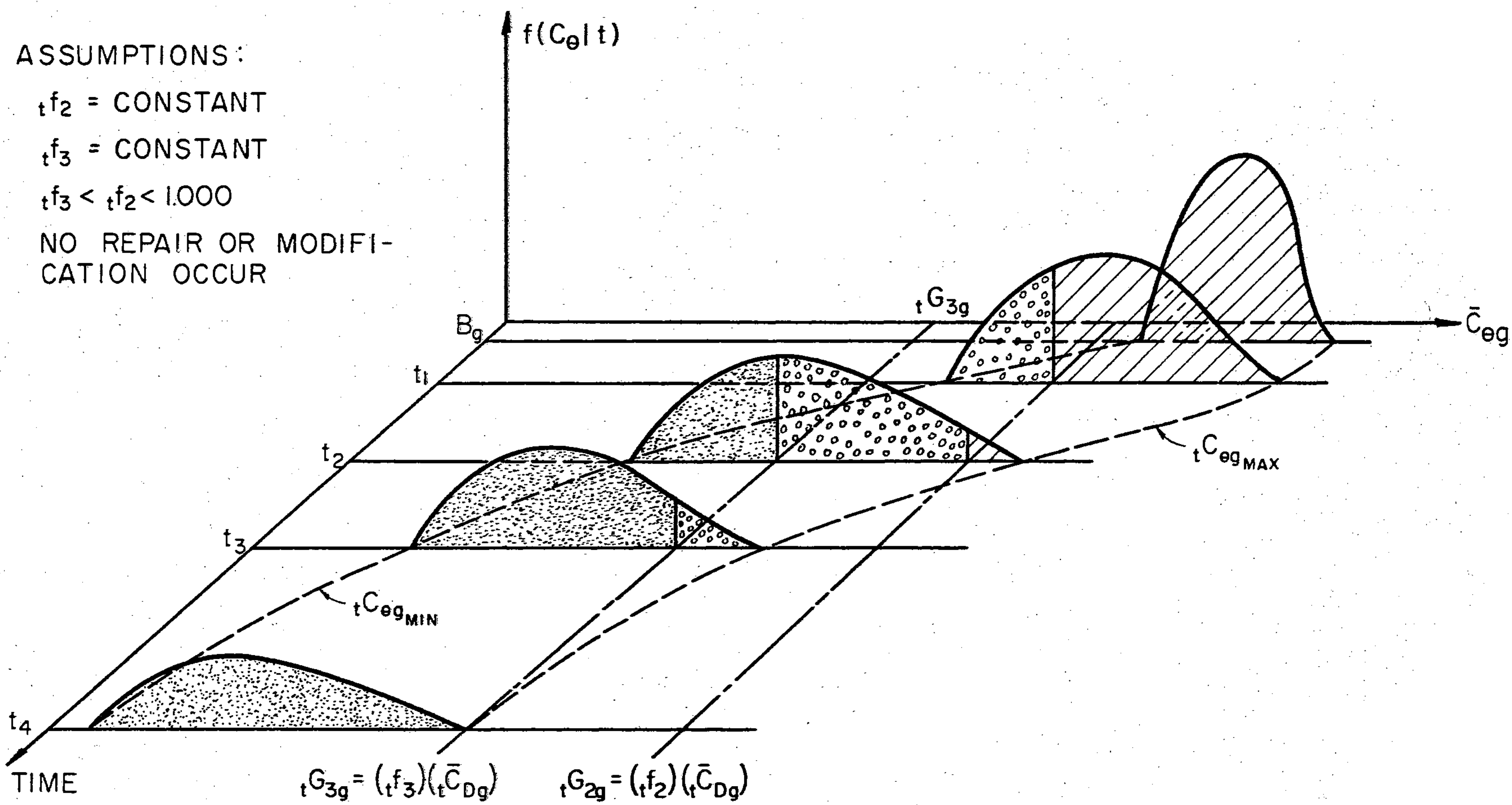

Figure 12. Time-Dependency of the Proportions of Probability Among the Reliability, the Dubiety and the Unreliability of the Time-Dependent Operating Values of the Fleet Inventory Members of Group "g" With Respect to Their Composite Design Value of the Critical Performance Characteristic When There Has Been No Repair or Modification

the $f(C_O|t)$ versus C_O sections indicate the reliability (probability of being reliable) of the composite operating speed with respect to the current composite design speed of the current fleet inventory members of group "g". The smoky portions of the sections indicate the unreliability (probability of being unreliable) of the operating speed and the dotted portions of the sections indicate the dubiety (probability of being of dubious worth) of the operating speed.

System Reliability, Repairability, Modifiability and Unreliability

A system is reliable at time "t" only if both its design and operating values are reliable at that time. Therefore the reliability of the composite system from group "g" at time "t" is the joint probability

$$ {}_tR_g = [{}_t\psi_{Dg}][{}_t\psi_{Og}] \qquad (3.13)$$

and determines the proportion of the fleet inventory systems from group "g" which will be able to meet the required job performance requirements at that time. In other words, ${}_tR_g$ is the conditional probability that those systems which belong to group "g" will be reliable during time interval "t", given that they were in the fleet inventory at time (t - 1).

A system is unreliable at time "t" if both its design and operating values are unreliable at that time. The unreliability of the composite system from group "g" at time "t" is the joint probability

$$_{t}U_{g} = [{}_{t}\Phi_{Dg}^{**}][{}_{t}\Phi_{Og}^{**}] \quad . \tag{3.14}$$

The unreliability of a composite system determines the proportion of fleet inventory systems from that group which will be unable to meet the job performance requirements at that time and will be sent to the scrap pile. Thereafter these systems are no longer classified as being part of the fleet.

If the composite system of group "g" at time "t" is reliable in design but non-reliable in operation, it is of dubious worth at time "t" because it requires repair. Hence, the repair-requiring dubiety of the composite system is the proportion of the fleet inventory systems from group "g" which at time "t" will be of dubious worth but will be repairable and is the joint probability

$$_{t}D_{rg} = [{}_{t}\Psi_{Dg}][1 - {}_{t}\Psi_{Og}] \quad . \tag{3.15}$$

If the composite system of group "g" at time "t" is non-reliable in design and not unreliable in operation, or, if its design is dubious and its operation is unreliable, then the composite system is of dubious worth because it requires modification. Hence the proportion of group "g's" fleet inventory systems which will be of dubious worth at time "t", because modification is required, is

$$_{t}D_{mg} = [1 - {}_{t}\Psi_{Dg}][1 - {}_{t}\Phi_{Og}^{**}] + [{}_{t}\Phi_{Dg}^{*}][{}_{t}\Phi_{Og}^{**}] . \tag{3.16}$$

The fact that

$$ {}_{t}R_{g} + {}_{t}U_{g} + {}_{t}D_{rg} + {}_{t}D_{mg} = 1 \qquad (3.17) $$

can be shown by realizing that a system must exist simultaneously in both a design state and an operating state and that

$$ [{}_{t}\Psi_{Dg} + {}_{t}\Phi^{**}_{Dg} + {}_{t}\Phi^{*}_{Dg}][{}_{t}\Psi_{Og} + {}_{t}\Phi^{**}_{Og} + {}_{t}\Phi^{*}_{Og}] = 1 \ . \qquad (3.18) $$

The Venn diagram in Figure 13 illustrates relationships among Equations (3.13), (3.14), (3.15), (3.16) and (3.18).

Henceforth the symbols ${}_{t}R_{g}$, ${}_{t}U_{g}$, ${}_{t}D_{rg}$ and ${}_{t}D_{mg}$ will be referred to, respectively, as the "reliability", "unreliability", "repair-dubiety" and "modification-dubiety" of group "g" during time interval "t".

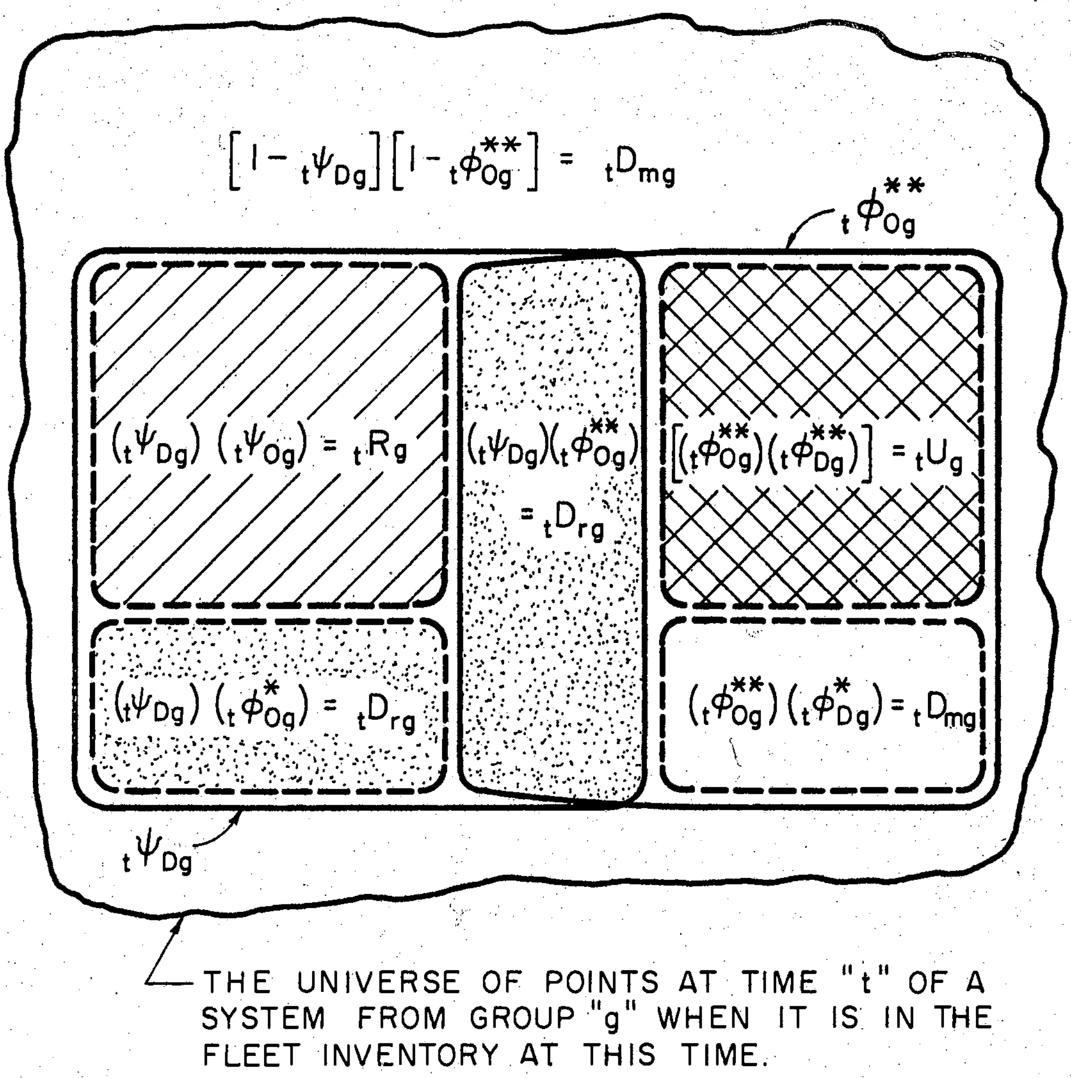

Figure 13. A Venn Diagram Illustrating the Relationships Among R, D_r, D_m, U and ψ_O, Φ_O^{**}, Φ_O^{*}, ψ_D, Φ_D^{**} and Φ^{*} For Group "g" at Time "t"

CHAPTER IV

MAINTAINABILITY AND SUPPLIABILITY

The purpose of a control function is to assure the attainment of objectives by limiting the amount of variation of the system's variables from the required values. Maintenance, in the forms of repair and modification, is the method by which the performance characteristics of the systems in the fleet inventory are controlled. Repair increases the operating value of the critical performance characteristic of a system by renewing faulty components. Modification increases both the design value and the operating value of the critical performance characteristic of a system by modernizing components of the system.

Maintainability is a function of the size of the incoming batches, the allocated lead time and the service rate of the maintenance facility. Batch design-maintainability $({}_{t}M_{Dg})$ is defined as the probability that modification of the batch from group "g" which entered the modification facilities at time "$(t - L_m)$" will be completed and the whole batch returned to the fleet inventory at time "t" when the modification is performed under the expected environmental circumstances. Environmental circumstances includes tools, techniques, personnel and physical environment and determines the service rate. In other words,

$$ {}_{t}M_{Dg} \equiv p(({}_{t}N_{mig} = {}_{t-L_m}N_{img}) | L_m, \mu_m) . \tag{4.1} $$

Batch operation-maintainability (${}_{t}M_{Og}$) is defined as the probability that repair of the batch from group "g" which entered the repair facilities at time "(t - L_r) will be completed and the whole batch returned to the fleet inventory at time "t" when the repair is performed under the expected environmental circumstances. In other words,

$$ {}_{t}M_{Og} \equiv p(({}_{t}N_{rig} = {}_{t-L_r}N_{irg}) | L_r, \mu_r) . \tag{4.2} $$

Design of the system will dictate the greatest ease with which and hence the minimum time in which repair and modification can be performed on the systems by an ideal repair or modification facility. The ranges of and the probabilities of maintenance times are functions of the types of equipment, the abilities of the maintenance personnel, the queuing classifications and the sequencing associated with the facility. Research to express the service rate and the maintenance times as functions of the elements mentioned in this paragraph is beyond the scope of this project but will be the next research project of the author.

For purposes of this research, the operation maintainability and the design maintainability will be determined from the general type of distribution curves illustrated in Figures 14 and 15. These curves demonstrate that the

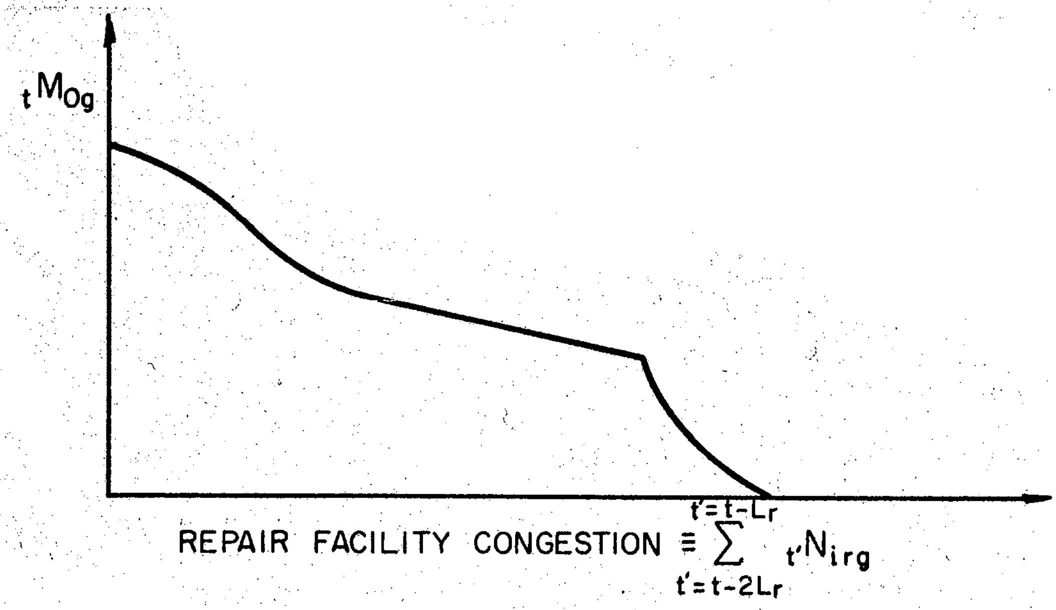

Figure 14. Operating Maintainability Versus Expected Congestion in the Repair Facility

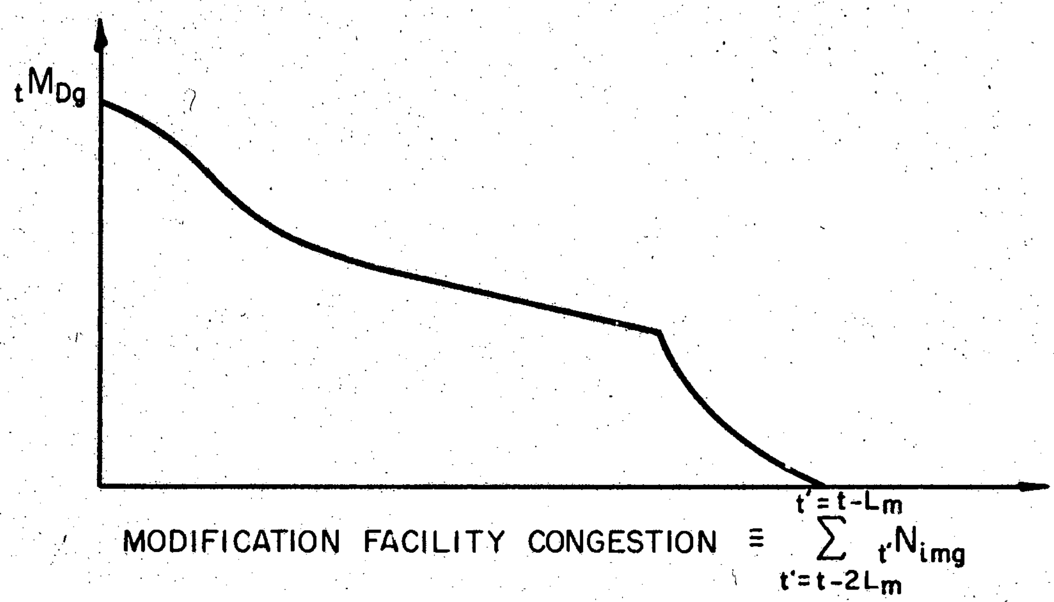

Figure 15. Design Maintainability Versus Expected Congestion in the Modification Facility

probability of returning to the fleet inventory at time "t" those systems which left exactly "L" time intervals previously is dependent upon

(1) the number of systems which were already in the maintenance facility at the time the recently arrived systems entered;

(2) the average maintenance time allocated, "L"; and

(3) the average maintenance service rate (μ) and its maximum value, which may be called its capacity.

The rate of change of maintainability curves with respect to congestions present in the facilities at the times the batches enter are bimodal because it is believed that the sense of urgency produced by the appearance of larger amounts of work will increase somewhat the service rates and hence will increase the probability that ${}_{t}N_{rig}$ will equal ${}_{t-L_r}N_{irg}$ and that ${}_{t}N_{mig}$ will equal ${}_{t-L_m}N_{img}$.

Suppliability

Supply is the method by which the number of systems in the fleet inventory can be limited and hence is a control function, too. An increase in the number of systems of up-dated design will be more costly and require a longer lead time than the same number of systems of standard design but usually the percentage of the up-dated design systems which will be reliable over a stated period of time is larger than the percentage of standard design systems which could enter the fleet at the same time.

Sources of systems may be either internal or external to the fleet inventory owner-company, which is the customer for the systems. Each source may require different lead times in order to assure delivery at a specific time for the standard and/or new or up-dated designs. Usually the fleet owner will specify a delivery date which, in turn will allocate the lead time. Thus the fleet owner is concerned with the ability of a source to deliver within the specified lead time the number and the type of systems ordered. Suppliability is a measure of the ability of a specific combination of designing and manufacturing sources to deliver a specific type (design) system in specific numbers within a specific amount of time. Suppliability is defined as the probability that the number of systems, ordered at time "t" which is "L_k" time intervals prior to need, will be delivered to the fleet inventory on time if the source uses standard information, design methods and manufacturing methods under expected environmental conditions. In other words,

$$ {}_{t+L_k}S_g \equiv p(({}_{t+L_k}N_{Sg} = {}_{t}N_p) \mid L_k, \mu_k) \ . \qquad (4.3) $$

For purposes of this research, suppliability will be expressed as a function of the congestion in the channels of the source which supplies systems of that design. Figure 16 illustrates the relationship between congestion and suppliability. As shown, congestion is measured by the number of systems involved in the unfilled orders. Notice that the less-standard the design the smaller is the probability that

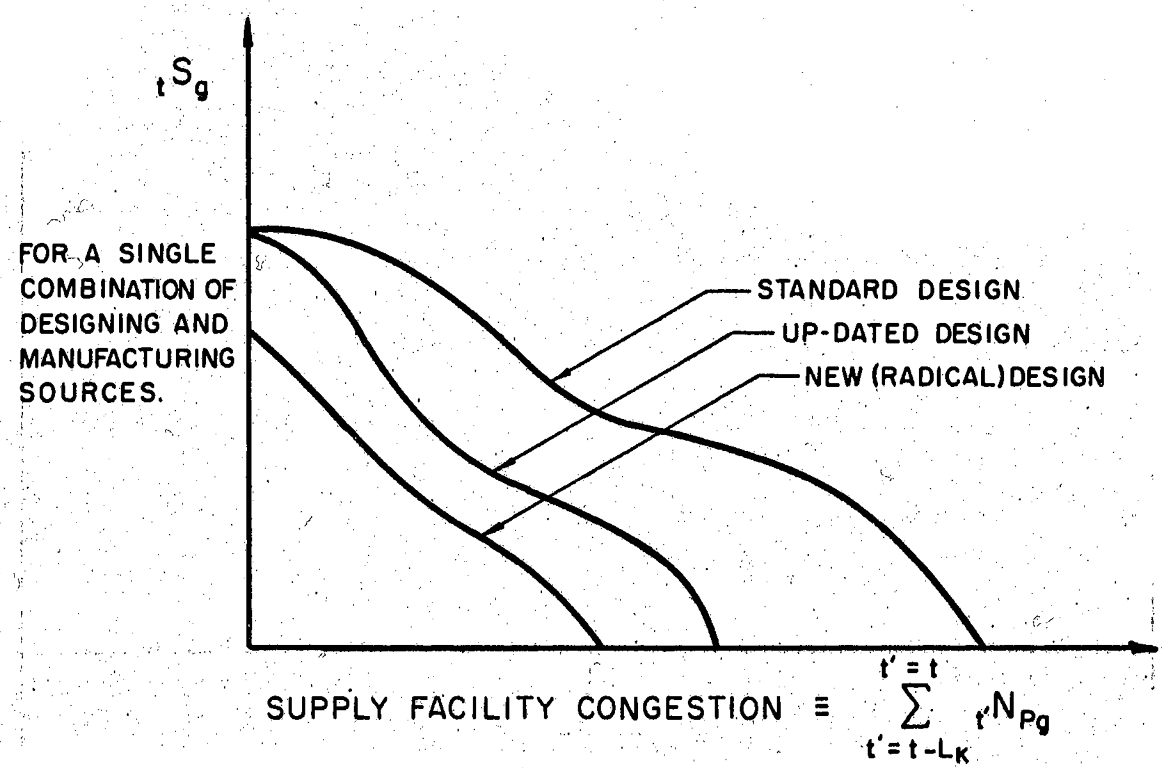

Figure 16. Suppliability Versus Expected Congestion in the Supply Facility

the design and manufacture of the group will be completed during the lead time (L_k) for that design. In other words a new design has much less chance of being completed during the lead time allocated to it than does a standard design because the "pitfalls" are better known in the latter case.

CHAPTER V

FLEET INVENTORY BATCHES

As shown in Figure 8 five batches of systems from group "g" simultaneously enter and leave the fleet inventory. The destinations of the three exiting batches are the scrap pile and the two types of maintenance facilities for repair and modification. The sources of the two entering batches are the two types of maintenance facilities.

The size of the exiting batches are dependent upon the total number of systems from group "g" which were in the fleet inventory at the beginning of the time interval and upon their reliability and non-reliability values during that interval. In other words, the number of "g" systems leaving the fleet inventory at time "t" to go to the scrap pile, to the repair facility and to the modification facility, respectively, are:

$$ {}_{t}N_{ieg} = ({}_{t-1}N_{Ig})({}_{t}U_{g}) \tag{5.1} $$

$$ {}_{t}N_{irg} = ({}_{t-1}N_{Ig})({}_{t}D_{rg}) \tag{5.2} $$

$$ {}_{t}N_{img} = ({}_{t-1}N_{Ig})({}_{t}D_{mg}) \; . \tag{5.3} $$

The sizes of the batches entering the fleet inventory from the maintenance facilities are dependent upon the number of systems which departed from the fleet inventory sufficiently long ago to have completed their required maintenance and upon the probability that the maintenance facilities were able to complete the repair or modification required by each. In other words, the number of systems entering the fleet inventory from the repair and modification facilities, respectively, are:

$$ {}_tN_{rig} = ({}_{t-L_r}N_{irg})({}_tM_{Og}) \tag{5.4} $$

$$ {}_tN_{mig} = ({}_{t-L_m}N_{img})({}_tM_{Dg}) \; . \tag{5.5} $$

Figure 17 shows the forward flows of the batches from the fleet inventory at one time to the fleet inventory after an elapse of the allowed lead times "L_r" and "L_m" and after passing through the repair and modification facilities which have maintainabilities of "M_O" and "M_D" respectively. Systems which were backlogged in the past in the maintenance facilities by not being completed within the allowed lead times will, in actuality, compose a part of every batch which leaves those facilities. To escape coping with the effects of backlogs it must be assumed that all systems which do not complete maintenance by the appropriate time are exited directly and immediately from the maintenance facility to the scrap pile. That is, the number of systems from group "g" which are sent to the scrap pile from the repair facility at time "t" is

$$ {}_tN_{reg} = ({}_{t-L_r}N_{irg} - {}_tN_{rig}) = ({}_{t-L_r}N_{irg})(1 - {}_tM_{Og}) \tag{5.6} $$

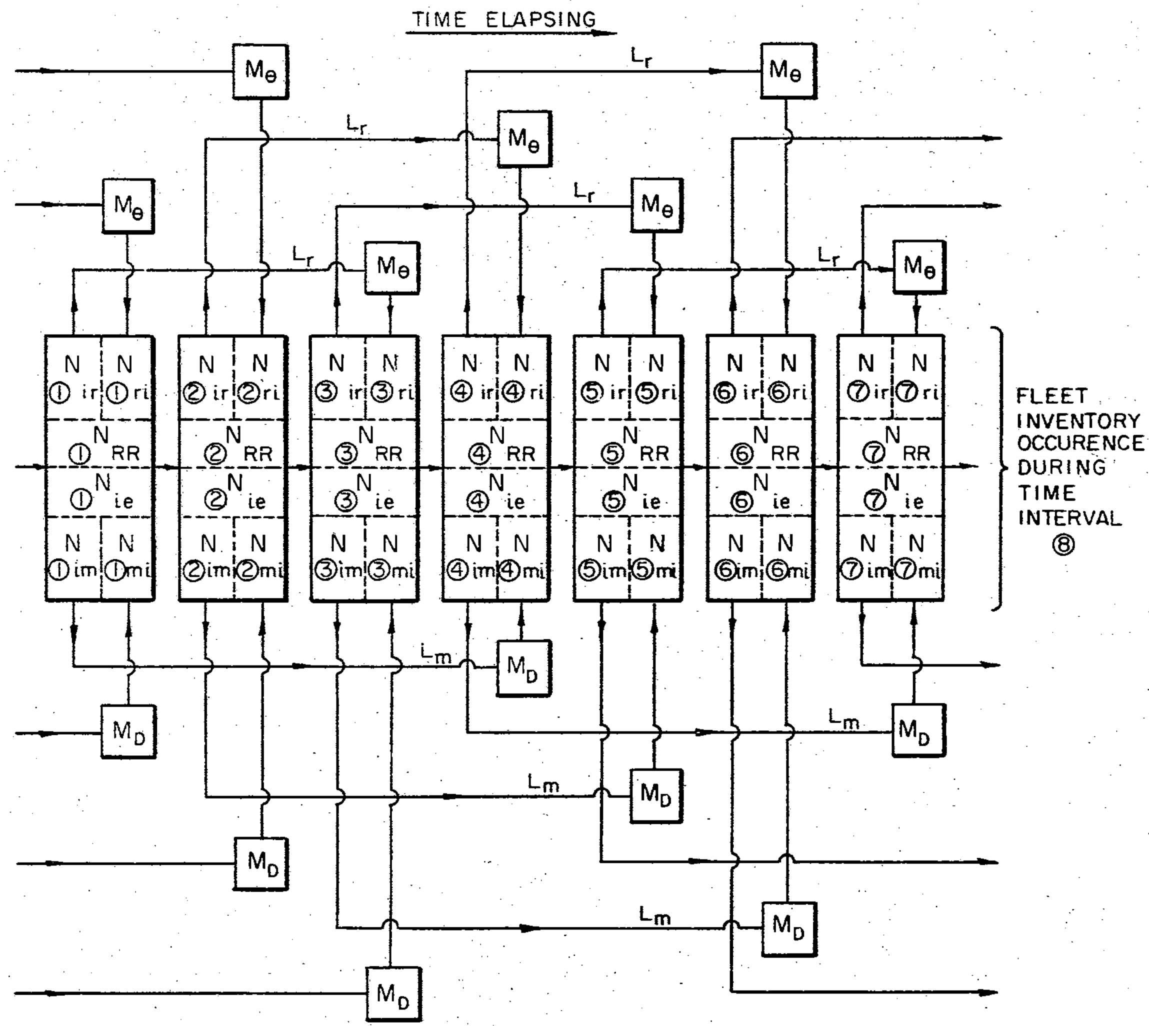

Figure 17. The Forward Flows of Batches From the Fleet Inventory to the Maintenance Facilities and Then to the Fleet Inventory Again as Time Elapses

and the number of systems from group "g" which are sent to the scrap pile from the modification facility at time "t" is

$$ {}_{t}N_{meg} = ({}_{t-L_m}N_{img} - {}_{t}N_{mig}) = ({}_{t-L_m}N_{img})(1 - {}_{t}M_{Dg}) \quad . \tag{5.7} $$

The effects of backlogs are neglected in this monograph but should be investigated.

The total number of systems sent to the scrap pile for all reasons at time "t" is

$$ {}_{t}N_{eg} = {}_{t}N_{ieg} + {}_{t}N_{iEg} + {}_{t}N_{reg} + {}_{t}N_{meg} \tag{5.8} $$

and the total number of group "g" systems which have been exited from the fleet to date is

$$ {}_{t}N_{e;Tg} = \left[\sum_{t'=j_g}^{t'=t-1} ({}_{t'}N_{eg}) \right] + \left[{}_{t}N_{eg} \right] \tag{5.9} $$

Substitution of Equations (5.2) and (5.3) into Equations (5.4) and (5.5) and adjustment of the time intervals appropriately shows that the numbers of systems from group "g" which are returned to the fleet inventory during interval "t" are dependent upon the product of the appropriate dubieties and maintainabilities; i.e.,

$$ {}_{t}N_{rig} = ({}_{t-L_r-1}N_{Ig})({}_{t-L_r}D_{rg})({}_{t}M_{Og}) \tag{5.10} $$

$$ {}_{t}N_{mig} = ({}_{t-L_m-1}N_{Ig})({}_{t-L_m}D_{mg})({}_{t}M_{Dg}) \quad . \tag{5.11} $$

The trade-offs between repair dubiety and operational maintainability and between modification dubiety and design maintainability exist. In Figure 18 is illustrated the tradeoffs required for various ratios of output of the maintenance facilities to the fleet inventory size. Thus, for example, out of every hundred group "g" planes in the fleet inventory at time "$t-L_m-1$", only 36 planes would have been to and returned from the modification facility by time "t" if any of the following sample combinations of design maintainability and modification-dubiety existed:

$$({}_{t-L_m}D_{mg},\ {}_tM_{Dg}) = (.40,\ .90) \text{ or } (.60,\ .60) \text{ or } (.90,\ .40).$$

A figure similar to Figure 18 could be made to show the tradeoffs between operational maintainability and repair dubiety which produce constant ratios of repair-facility output to fleet-inventory-size.

The number of fleet inventory systems from group "g" which meet or exceed the requirements for the critical performance characteristic during time interval "t" and hence help to satisfy the job specification quota at time "t" is

$$ {}_tN_{RRg} = ({}_{t-1}N_{Ig})({}_tR_g) \ . \qquad (5.12)$$

Essentially N_{RR} is the number of systems in the fleet inventory which did not have to be sent to the maintenance facilities or the scrap pile during the time interval (t-1) to (t), and which were operating during that time interval. However, reasons unrelated to the pre-determined critical performance characteristic may require that part of these

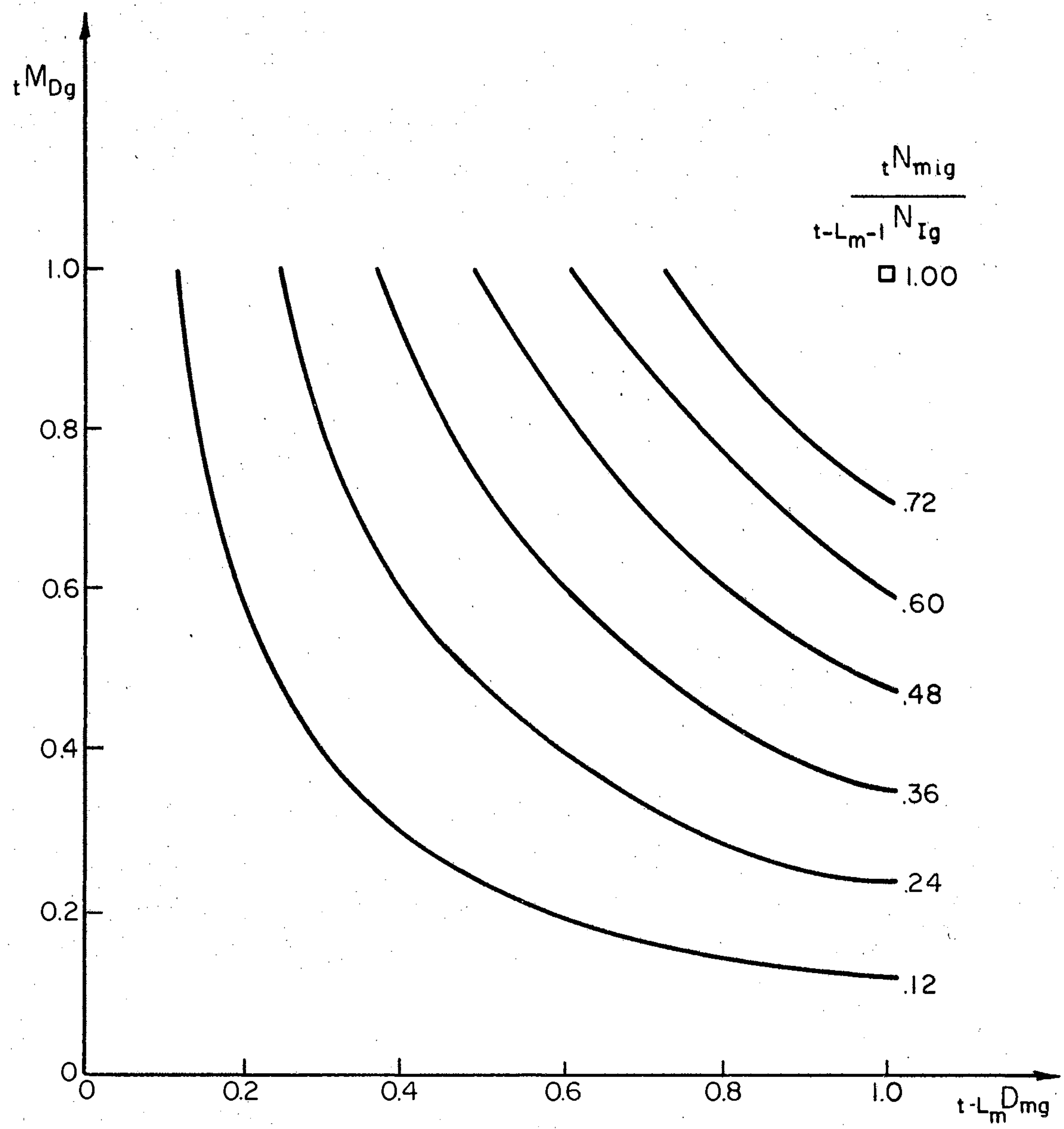

Figure 18. Trade-Offs Between Design-Maintainability and Modification Dubiety Which Produce Constant Ratios of Modification-Facility Outputs to Fleet Inventory Sizes

so-called "reliable" systems be scrapped. Some of the reasons in the case of freight-type airplanes would be dissatisfaction with the time required to load the planes, pilot dissatisfaction with flight characteristics of the planes, crashes, and development of fatigue cracks in longerons or other major structural components. These reasons will cause forced attrition of the fleet inventory. The number of systems in the batch which will be sent from group "g" at time "t" for these reasons is ${}_tN_{iEg}$.

The number of systems composing the fleet inventory at any time is a function of the number of systems previously in the fleet inventory and the numbers of systems which entered and which exited from the fleet inventory during time interval "t"; that is,

$$ {}_tN_{I;F} = \sum_{g} {}_tN_{Ig} \tag{5.13} $$

$$ \begin{aligned} {}_tN_{Ig} &= ({}_{t-1}N_{Ig}) - ({}_tN_{ieg}) + ({}_tN_{rig} + {}_tN_{mig}) - {}_tN_{iEg} \\ &= {}_{t-1}N_{Ig} - {}_tN_{ieg} - {}_tN_{iEg} - {}_tN_{irg} - {}_tN_{img} + {}_tN_{rig} \\ &\qquad + {}_tN_{mig} \end{aligned} \tag{5.14} $$

$$ = {}_tN_{RRg} + {}_tN_{irg} + {}_tN_{mig} - {}_tN_{iEg} \;. \tag{5.15} $$

When the number of systems in the fleet inventory does not meet the job specification quota, the amount of shortage which exists at time "t" is

$$ {}_{t}\text{Shortage}_{F} = [{}_{t}N_{R} - \sum_{g} {}_{t}N_{RRg}] \text{ .} \qquad (5.16) $$

Figure 19 illustrates graphically the history of three groups which entered the fleet inventory at times ($t = 0,4,9$). The first of the groups completed attrition at the time the third group entered the fleet ($t = 9$). The effect of time on the numbers of systems simultaneously in the scrap pile, in the repair facilities, in the modification facilities and in the reliable state is shown.

Figure 20 illustrates the effect of the number of reliable systems in each group on the total number of reliable systems which may help to meet the job requirement of reliable systems.

All systems in a group are assumed to have the same history on the date the group enters the fleet; i.e., on the group's "birth date". Hence all of the systems in the group then belong to a single batch. The size of the batch supplied at time "t" is dependent upon the size of the order sent to that source "L_k" time intervals previously and upon the current suppliability of that source for the number of systems of desired design; i.e.,

$$ {}_{t+L_k}N_{Sg} = [{}_{t}N_{pg}] \; [{}_{t+L_k}S_{g}] \text{ .} \qquad (5.17) $$

The question which now arises is, "What size batch should be ordered "L_k" time intervals prior to time of need?" Management's desires, influenced by costs, past

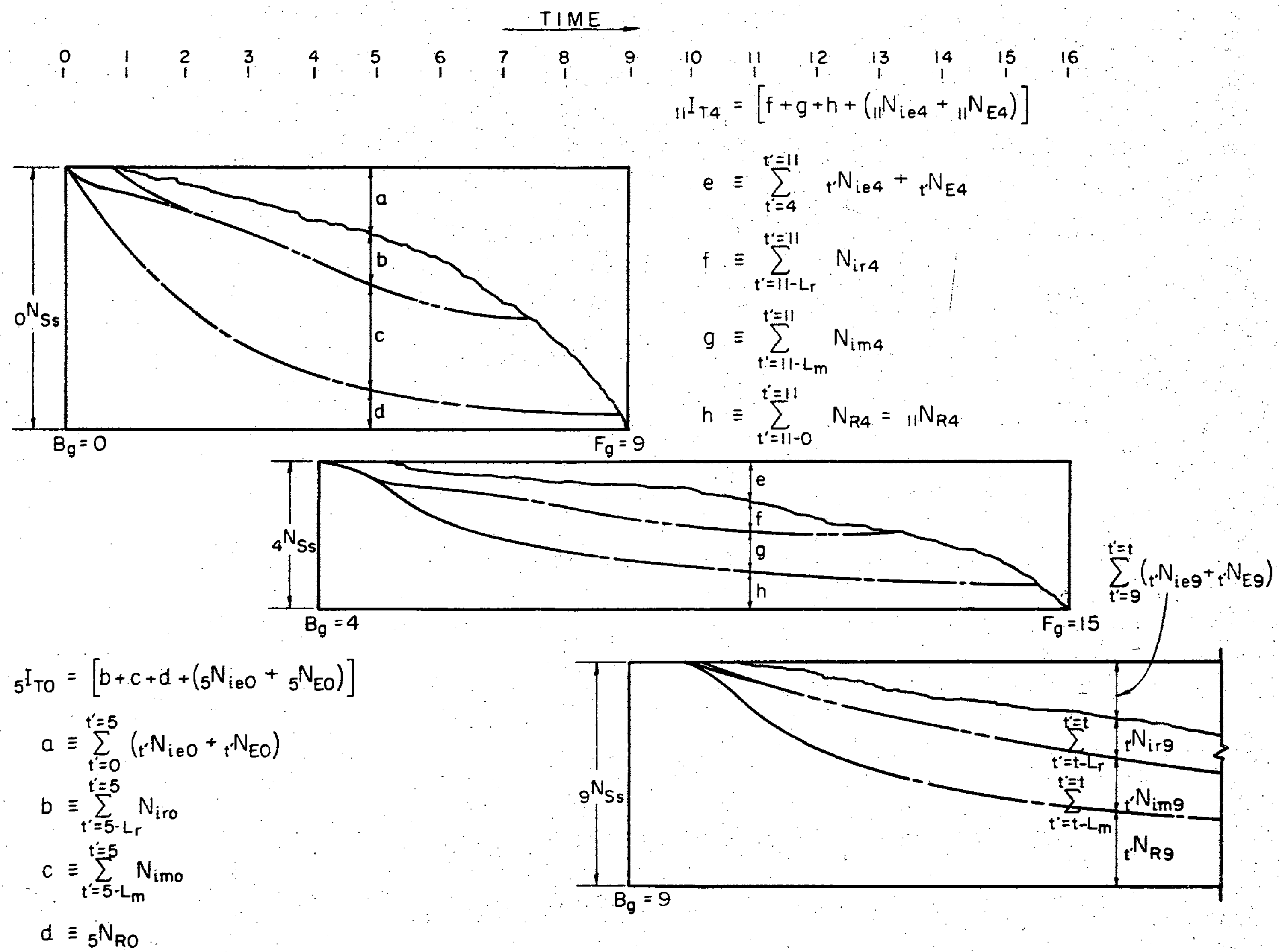

Figure 19. Distribution of Three Groups Versus Time

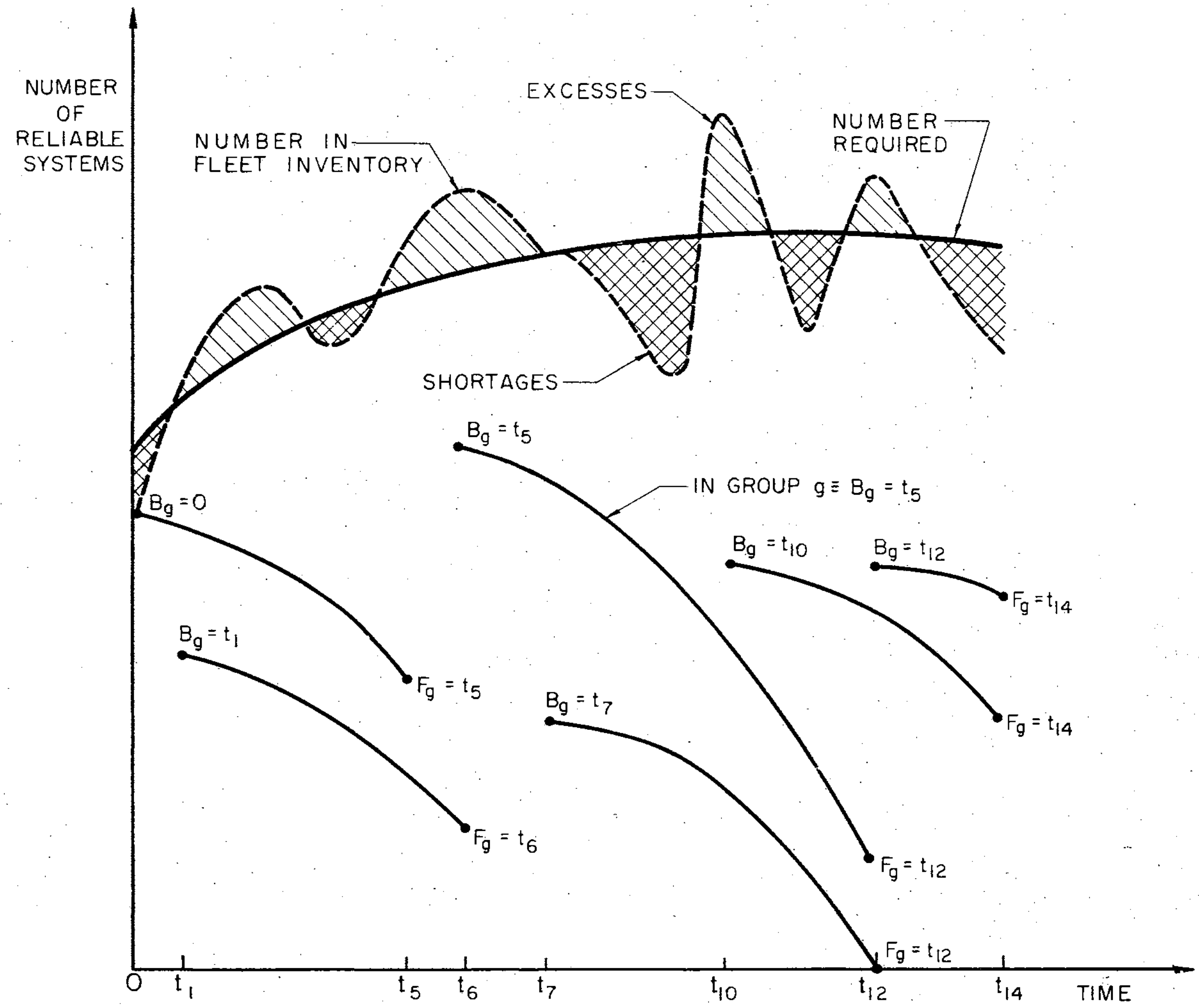

Figure 20. Number of Reliable Systems Required, the Actual Number in the Fleet Inventory and in the Individual Groups Versus Time

experience, forecasts and policies, dictate many different answers. In essence, however, all answers are dependent upon

(1) $_{t+L_k}N_R$ the required number of reliable systems fore-for the time at which the new systems will be needed;

(2) L_k the allowed time between sending the order and receiving the new systems of design type "k";

(3) $\sum_{g's@(t)} ({}_tN_{Rg})$ the total number of reliable systems present in the fleet inventory at the time the order must be sent;

(4) $_tW_P$ the procurement reaction control number which is a management decision at time "t", and is used to take into account both the decreasing number of reliable systems as time progresses and the suppliability of the new-group source.

Thus, the number of systems of design type "B" which should be ordered at time (t) from source "s" is

$$_tN_P = [{}_{t+L_k}N_R - \sum_{g's@(t)} ({}_tN_{RRg})][{}_tW_P] \ . \qquad (5.18)$$

CHAPTER VI

COSTS

There are five sources of costs for sustaining a fleet inventory; namely, fleet inventory operations, maintenance operations, supply operations, inventory shortage and scrapping costs. Each of these costs and their sum will be discussed in this chapter. A "pay-as-you-go" plan is assumed but costs may be adjusted to take into account the cost of financing if that is desired.

Fleet Inventory Operations Costs

Inventory cost is the sum of the costs of storing, operating and overhead for each group "g" and is payable at the end of each time interval. If 100 per cent of the systems in the fleet inventory operate during time intervals "t", the payment due at time "t", which occurs at the end of time interval "t", for each group is

$$_t\$_{Ig} = [_{t-1}N_{Ig}][_t\not{c}_{Hg} + {_t\not{c}_{IAg}}] + {_t\$_{I;Cg}} \qquad (6.1)$$

where

$_t\not{c}_{Hg}$ is the cost per inventory system from group "g" held at time "t";

$_{t-1}N_{Ig}$ is the number of group "g" systems in the fleet inventory at time "(t-1)";

$_t\phi_{I;Ag}$ is the cost per inventory system from group "g" operated during time interval "t";

$_t\$_{I;Cg}$ is that part of the operations costs which is not proportional to the number of group "g" systems in the inventory; it is called the overhead cost of the operations per time interval.

Maintenance Operations Costs

Maintenance operations costs are paid upon receipt of the renewed and modified systems and consists of the costs of storing systems awaiting repair and modification, operating repair and modification facilities and overhead. Part of the costs is proportional to the numbers of systems in the queues of the maintenance facilities. Another part of the cost is proportional to the number of items which are serviced. For each group "g" the repair operations costs which are payable at time "t" are

$$_t\$_{rg} = [(_t\phi_{r;qg})(_tL_{rg}) + {_t\phi_{r;Ag}}][_tN_{rig}] + {_t\$_{r;Cg}} \quad (6.2)$$

and the modification costs which are payable at that time are

$$_t\$_{mg} = [(_t\phi_{mqg})(_tL_{mg}) + {_t\phi_{mAg}}][_tN_{mig}] + {_t\$_{m;Cg}} \quad (6.3)$$

where

$_t\phi_{r;qg}$ is the waiting and storage cost per system in the repair queues per time interval;

$_tL_{rg}$ is the allowed repair, administrative and transportation lead time; it is called the repair "turnaround time";

$_t\phi_{m;qg}$ is the waiting and storage cost per system in the modification queues per time interval;

$_{t}L_{mg}$ is the allowed modification, administration and transportation lead time; it is called the modification "turnaround time";

$_{t}\not{c}_{r;Ag}$ is the cost of an average repair operation performed on the critical performance characteristic determinant of one system from group "g";

$_{t}\not{c}_{m;Ag}$ is the cost of an average modification operation performed on a critical performance characteristic determinant of one system from group "g";

$_{t}\$_{r;Cg}$ is the overhead cost of the repair facilities per time interval;

$_{t}\$_{m;Cg}$ is the overhead cost of modification facilities per time interval.

If the maintenance facilities are not owned by the same company which owns the fleet, the "overhead" costs must include the profit desired by the owners of the facilities.

Inventory Shortage Costs

Inventory shortage cost is evaluated on the basis of the consequences which could occur should the number of reliable systems in the fleet inventory be less than the required number. If this cost is proportional to the number of systems short, then

$$_{t}\$_{Sh} = [_{t}\not{c}_{Sh}][_{t}N_{R} - \sum_{g} {_{t}N_{RRg}}] \quad (6.4a)$$

$$= [_{t}N_{R}][_{t}\not{c}_{Sh}] - \sum_{g} {_{t}\$_{Sh\ g}} \quad (6.4b)$$

where

$_{t}\not{c}_{Sh}$ is the cost per system short at time "t";

$_{t}N_{R}$ is the forecasted number of reliable systems required at time "t";

${}_{t}N_{RRg}$ is the number of reliable "g" systems available during time interval "t".

$$ {}_{t}\$_{Sh;g} \equiv [{}_{t}\not{c}_{Sh}][{}_{t}N_{RRg}] \qquad (6.4c) $$

Exiting (Scrapping) Costs

Scrapping of systems is assumed to occur from all the fleet. Systems from a common group in the fleet are assumed to have a common salvage value and a common overhead "riddance" cost. The costs may differ for each group in the inventory. Hence for each group "g"

$$ {}_{t}\$_{e;Eg} = [{}_{t}\not{c}_{e;E;Cg} - {}_{t}\not{c}_{e;E;Vg}][{}_{t}N_{ieg} + {}_{t}N_{reg} + {}_{t}N_{meg} + {}_{t}N_{iEg}] \qquad (6.5) $$

where

${}_{t}\not{c}_{e;E;Cg}$ is the administration (overhead) cost of scrapping a system from group "g" at time "t";

${}_{t}\not{c}_{e;E;rg}$ is the salvage value of a system in group "g" at time "t".

Supply Operations Costs

The cost of supply operations is composed of the costs of purchasing raw materials, storing materials awaiting conversion into systems, operating the designing and manufacturing equipment, and overhead if the designing and manufacturing is done by the owner of the fleet inventory. If new systems are purchased from outside the company, set to zero all unit costs except the ${}_{t-L_k}\not{c}_{pg}$ which is then defined as the sum of the prorated procurement cost and the purchase price from source "s" per new system. In either case costs associated with new designs are generally higher than those associated with standard designs.

Supply costs are payable upon receipt of the new systems at time "t" from each source "s": Hence at time "t" the amount payable

$$_{t}\$_{Sg} = [_{t-L_k}\phi_{pg} + {}_{t}\phi_{Sqg}][_{t}N_{Sg}] + {}_{t}\$_{SCg} \qquad (6.6)$$

where

$_{t-L_k}\phi_{pg}$ is the cost of the raw materials purchased at time "$(t-L_k)$" to make one system;

$_{t}\phi_{S;qg}$ is the storage and manufacturing cost per future system accumulated by the components of the system when they were in the supply queues and channels during the time intervals covered by the supply lead time "L_k";

$_{t}\$_{S;Cg}$ is the overhead cost of supply operations per time interval for source "s";

$_{t}N_{Sg}$ is the number of new systems of either standard or newedesign which were delivered to the fleet inventory from supplier "s" during time interval "t".

Total Costs

All identical systems which entered the fleet inventory at time "Bg" ("birthdate of group 'g'") from supply source "s" are classified as members of group "g". The last member of group "g" will be exited from the fleet through the fleet inventory at time "Fg" ("finish date of group 'g'"). Therefore, at time "Bg" the present worth of the total cost which will be associated with a given group "g" during its future lifetime is

$$Pg = \$_{Sg} + \sum_{t'=Bg+1}^{t'=Fg-1} [{}_{t'}\$_{Tg}/(1 + i')^{t'-Bg}] \qquad (6.7)$$

where

$${}_{t'}\$_{Tg} \equiv [{}_{t'}\$_{Ig} + {}_{t'}\$_{rg} + {}_{t'}\$_{mg} + {}_{t'}\$_{e;Eg}] \qquad (6.8)$$

and where

i' is the annual rate of interest divided by the number of time intervals in a year.

The total payment which will be due at time "t" as a consequence of sustaining the fleet inventory during time interval "t" is

$${}_t\$_{T;F} = \sum_{g=(j=t;k=1)}^{g=(j=t;k=k_{max})} {}_t\$_{Sg} + \sum_{k=1}^{k=k_{max}} \sum_{g=(j\leq I\ \leq F_g;k)}^{g=(j<t\leq F_g;k)} {}_t\$_{Tg}$$

$$+ \left[{}_t\phi_{Sh}\right]\left[{}_tN_R - \sum_{k=1}^{k=k_{max}} \sum_{g=(j\leq I\ \leq F_g;k)}^{g=j<t\leq F_g;k)} {}_tN_{RRg}\right]. \qquad (6.9)$$

At time "I", which is the beginning of a planning horizon, the present worth of the future costs of buying, repairing, modifying and scrapping the members of the fleet inventory over the planning horizon which ends at time "H" is

$$P_F = \sum_{t'=I+1}^{t'=H} [{}_{t'}\$_{T;F}/(1 + i')^{t'-I}] . \qquad (6.10)$$

In the above equation the importance of ${}_t\$_{Tg}$ is noticed. The relationships among ${}_tR_g$, ${}_tM_{Og}$, ${}_tM_{Dg}$, ${}_tD_{rg}$, ${}_tD_{mg}$, ${}_tU_g$, N_{Sg}, ${}_tN_{iEg}$ and their associated costs is emphasized by the following equation which results from the combination of a large percentage of the equations in the previous chapters:

$$
\begin{aligned}
{}_t\$_{Tg} = {} & [{}_{t-1}N_{Ig}][{}_t\phi_{Hg} + {}_t\phi_{F;Ag} + ({}_tU_g)({}_t\phi_{e;E;Cg} - {}_t\phi_{e;E;vg})] \\
& + \left\{[{}_{t-L_r-1}N_{Ig}][{}_{t-L_r}D_{rg}]\right\} \left\{{}_t\phi_{e;E;Cg} - {}_t\phi_{e;E;Vg} \right. \\
& \qquad + [{}_tM_{Og}][{}_t\phi_{r;qg})(L_r) + {}_t\phi_{r;Ag} \\
& \qquad \left. - ({}_t\phi_{e;E;Cg} - {}_t\phi_{e;E;Vg})\right\} \\
& + \left\{[{}_{t-L_m-1}N_{Ig}][{}_{t-L_m}D_{mg}]\right\} \left\{{}_t\phi_{e;E;Cg} - {}_t\phi_{e;E;Vg} \right. \\
& \qquad + [{}_tM_{Dg}][({}_t\phi_{m;qg})(L_m) + {}_t\phi_{m;Ag} \\
& \qquad \left. - ({}_t\phi_{e;E;Cg} - {}_t\phi_{e;E;Vg})]\right\} \\
& + [{}_tN_{iEg}][{}_t\phi_{e;E;Cg} - {}_t\phi_{e;E;Vg}] + {}_t\$_{I;Cg} + {}_t\$_{r;Cg} \\
& + {}_t\$_{m;Cg} \,. \qquad (6.11)
\end{aligned}
$$

In Chapter VII this equation will be expressed in terms of availabilities, reliabilities, dubieties and maintainabilities.

CHAPTER VII

MEASURES OF EFFECTIVENESS

Based upon the theory developed in this monograph several measures of effectiveness ("figures of merit") of the fleet inventory exist; namely,

(1) the average shortage of reliable systems over the planning horizon,

(2) the present worth of the cost of buying and sustaining the fleet inventory under the assumption that it must try to meet the job requirements,

(3) the availability of the fleet at any time "t" within the planning horizon,

(4) the cost per reliable system in the fleet at any time "t" within the planning horizon.

Several measures of effectiveness of the individual groups also exist; namely,

(1) the availability of the group at any time "t" within its "lifetime",

(2) the cost per reliable system in the group at any time "t" within the group's lifetime.

Figures of merit are measures of performance. In the field of maintenance "availability", which is the ratio of "uptime" to "total time", is a common figure of merit used to indicate the effect of maintenance upon the reliability of a system. For a group the definition of "uptime" is the number of system-hours during which the forecasted required value for the critical performance characteristic are met. The definition of "total time" for a group is the total number of system-hours which could exist during the same time period of length "y" if each system in the group purchased were always reliable. The availability of group "g" during time interval number "t" which is "y" time units long is

$$ {}_{t}A_{g} \equiv \frac{(y)({}_{t}N_{RRg})}{(y)(N_{Sg})} = \frac{{}_{t}N_{RRg}}{N_{Sg}} = \frac{({}_{t}R_{g})({}_{t-1}N_{Ig})}{N_{Sg}} \; . \tag{7.1} $$

At any time "t" the availability of group "g" is dependent upon the past availabilities, reliabilities, dubieties, and maintainabilities of that group. Substitution of Equation (7.1) into the equivalent of Equation (5.15) allows the relationships among these probabilities to be developed as follows: From Equation (5.15),

$$ \frac{{}_{t-1}N_{Ig}}{N_{Sg}} = \frac{1}{N_{Sg}} \Big[({}_{t-2}N_{Ig})({}_{t-1}R_{g}) + ({}_{t-L_r-2}N_{Ig})({}_{t-L_r-1}D_{rg})({}_{t-1}M_{Og}) + ({}_{t-L_m-2}N_{Ig})({}_{t-L_m-1}D_{mg})({}_{t-1}M_{Dg}) - {}_{t-1}N_{iEg} \Big] \; . \tag{7.2} $$

Now substitute Equation (7.1) into Equation (7.2). Hence the availability at any time can be calculated in terms of former availabilities; i.e.,

$$ {}_{t}A_{g} = \left[{}_{t}R_{g}\right]\left[\frac{{}_{t-1}A_{g}}{1} + \frac{({}_{t-L_r-1}A_g)({}_{t-L_r-1}D_{rg})({}_{t-1}M_{Og})}{({}_{t-L_r-1}R_g)} + \frac{({}_{t-L_m-1}A_g)({}_{t-L_m-1}D_{mg})({}_{t-1}M_{Dg})}{({}_{t-L_m-1}R_g)} - \frac{{}_{t-1}N_{iEg}}{N_{Sg}}\right]. \tag{7.3} $$

Figures 21 through 24 illustrate the relationships among reliability, maintainability and availability given in Equation (7.3) when $\left[{}_{t-1}A_g - \left(\frac{{}_{t-1}N_{iEg}}{N_{Sg}}\right)\right]$ remains constant.

The availability of the fleet, which is composed of many groups, during time interval number "t" which is "y" time units long is calculated similarly to the group availability: i.e.,

$$ {}_{t}A_{F} = \frac{\left[\sum_{k=1}^{k=k_{max}} \sum_{g=(j\le t;k)}^{g=(j=t\le Fg;k)} (y)({}_{t}N_{RRg})\right] = \left[\sum_{k}\sum_{g}(y)({}_{t}A_g)(N_{Sg})\right]}{\sum_{k}\sum_{g}(y)(N_{Sg})} \tag{7.4} $$

It can be concluded that the availability of the fleet* at time "t" is equal to the weighted average of the availabilities of all the groups currently in existence in the fleet inventory at that time; i.e.,

$$ {}_{t}A_{F} = {}_{t}\bar{A}_{g} \quad . $$

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*Note: Suppliability helps to determine the number of systems added to the fleet during each time interval. Hence, effect of suppliability is implied in the value of the fleet's availability at any time.

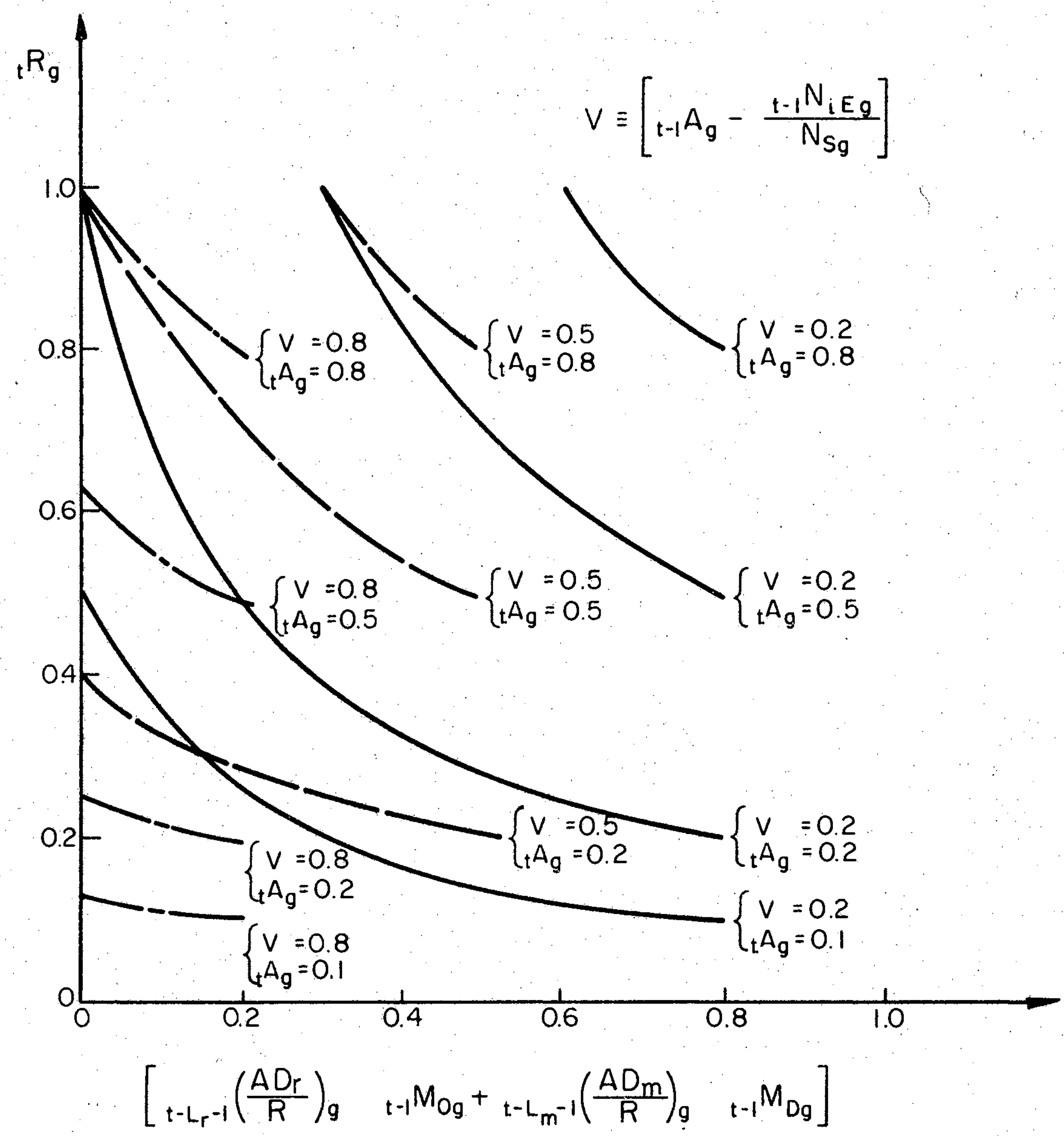


Figure 21. Group Reliability as a Function of Maintenance

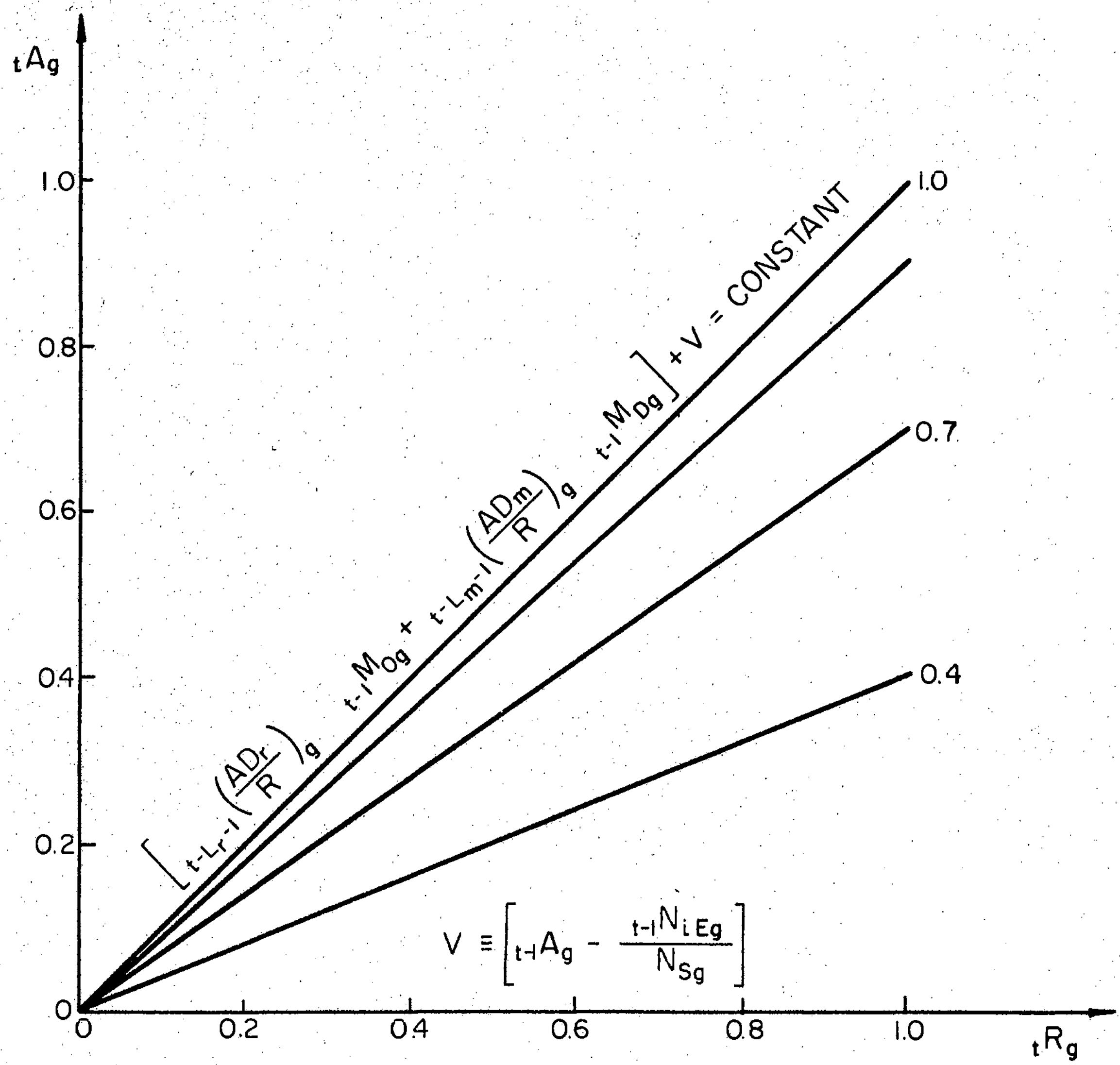


Figure 22. Group Availability as a Function of Group Reliability

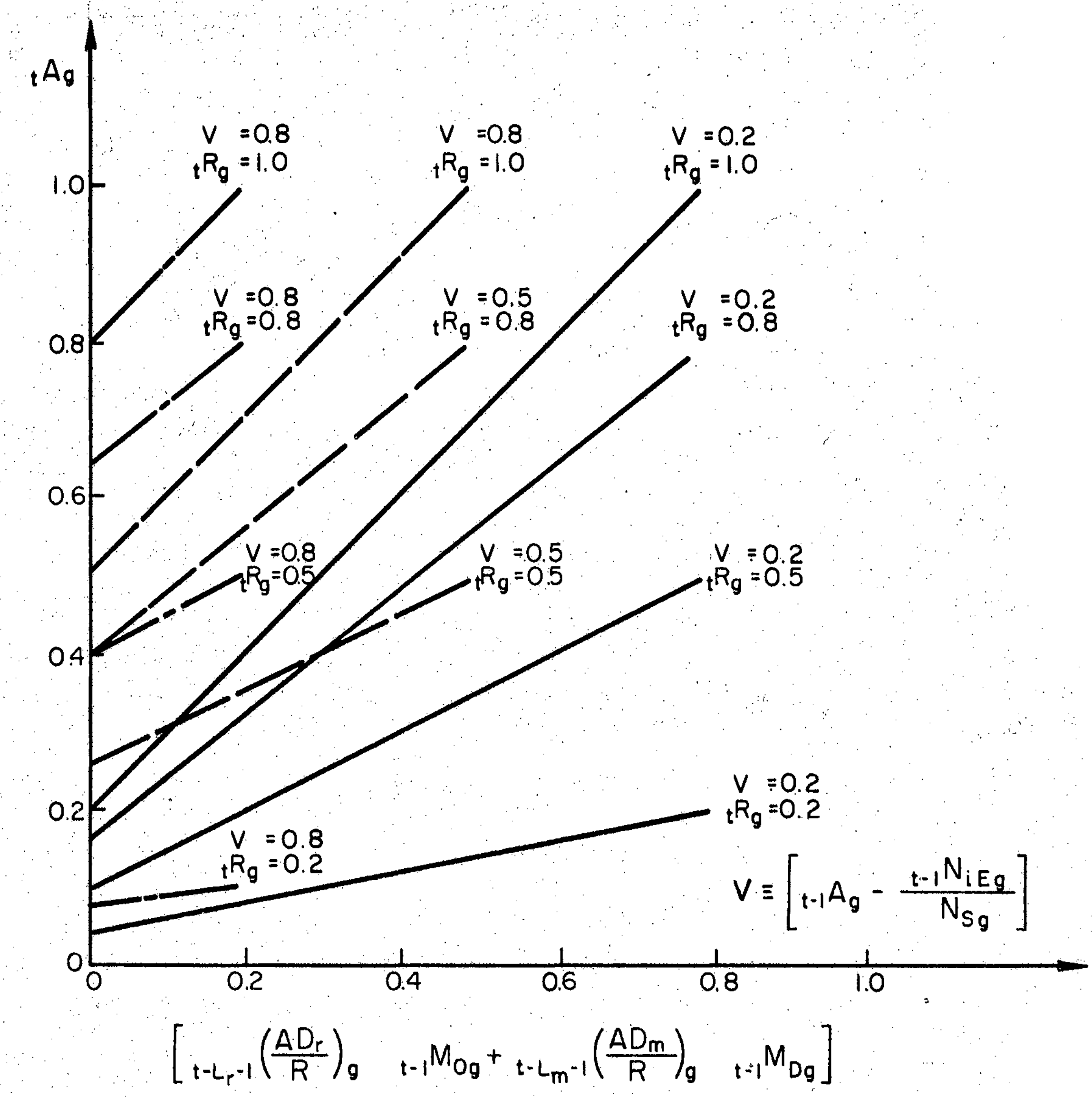


Figure 23. Group Availability as a Function of Maintainabilities

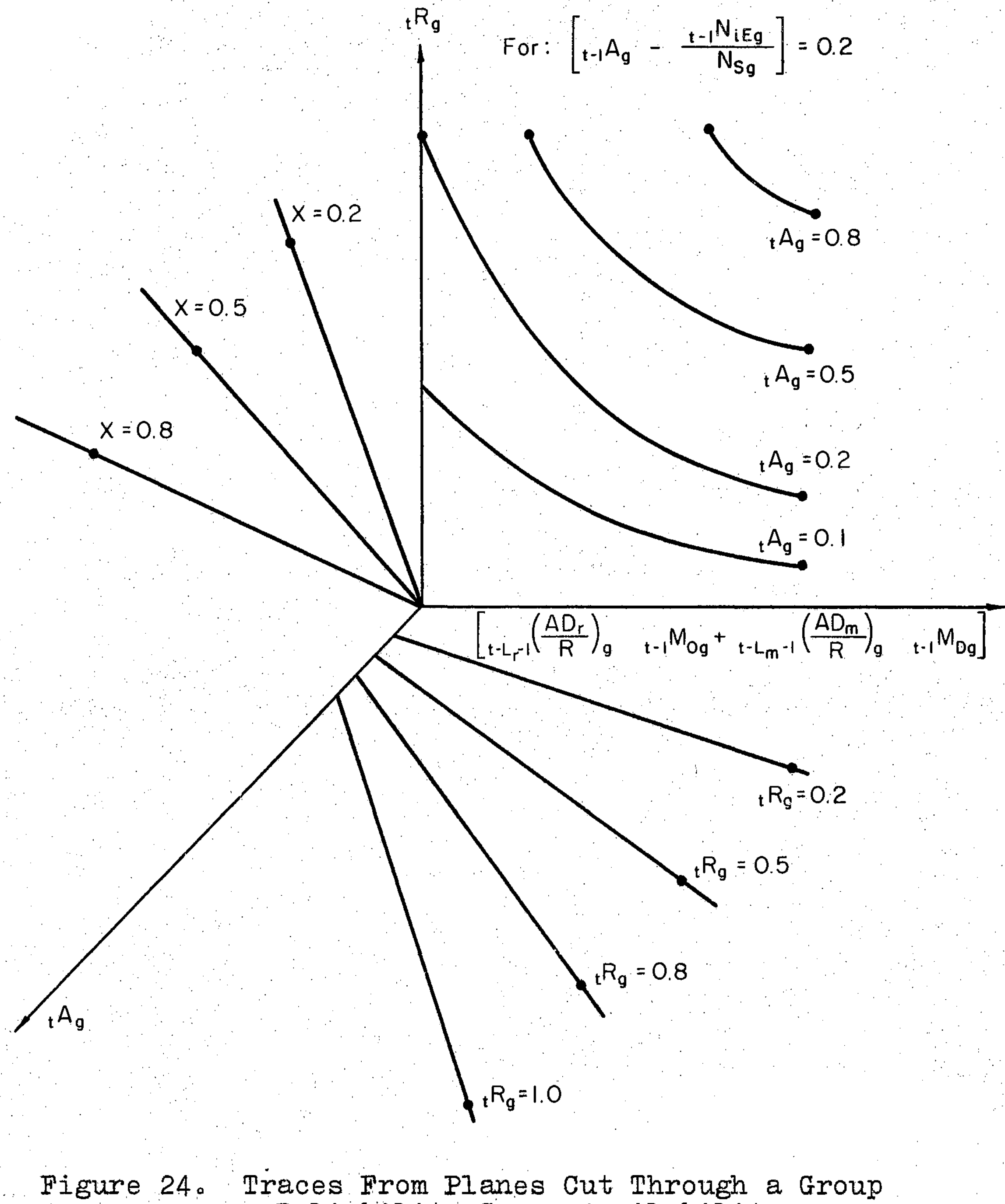


Figure 24. Traces From Planes Cut Through a Group Reliability-Group Availability-Maintainabilities Surface

A second figure of merit for a group is the average availability of the group over its lifetime. When the last of the systems from group "g", which were procured during time interval "Bg", is exited from the fleet by either natural or forced attrition during time interval "Fg", the "lifetime availability of group 'g'" can be calculated from the following equation:

$$_T\bar{A}_g = \frac{\sum_{t'=Bg}^{Fg-1} [_{t'}N_{RRg}]}{[Fg - Bg][N_{Sg}]} = \frac{\sum_{t'=Bg}^{Fg-1} [_{t'}A_g]}{[Fg - Bg]} . \qquad (7.6)$$

As mentioned in Chapter VI, the total payment due at time "t" to cover the costs of operating, maintaining and exiting systems from group "g" can be written in terms of availabilities, reliability, dubieties and unreliability. Substitution of Equation (7.3) into Equation (6.11) shows that

$$\left[\frac{_t\$_{Tg}}{N_{Sg}}\right] = \left[\frac{_tA_g}{_tR_g}\right]\left[(_tU_g)(_tC_1 + {_tC_2})\right]$$
$$+ \left[\frac{_{t-L_r}A_g}{_{t-L_r}R_g}\right]\left[_{t-L_r}D_{rg}\right]\left[(_tM_{Og})(_tC_3 - {_tC_1}) + {_tC_1}\right]$$
$$+ \left[\frac{_{t-L_m}A_g}{_{t-L_m}R_g}\right]\left[_{t-L_m}D_{mg}\right]\left[(_tM_{Dg})(_tC_4 - {_tC_1}) + {_tC_1}\right]$$
$$+ \left[\frac{_tN_{iEg}}{N_{Sg}}\right]\left[_tC_1\right] + {_tC_5} \qquad (7.7)$$

where

$${}_{t}C_{1} \equiv [{}_{t}\phi_{e;E;Cg} - {}_{t}\phi_{e;E;Vg}]$$

$${}_{t}C_{2} \equiv [{}_{t}\phi_{Hg} + {}_{t}\phi_{F;Ag}]$$

$${}_{t}C_{3} \equiv [({}_{t}\phi_{r;qg})(L_{r}) + {}_{t}\phi_{r;Ag}]$$

$${}_{t}C_{4} \equiv [({}_{t}\phi_{m;qg})(L_{m}) + {}_{t}\phi_{m;Ag}]$$

$${}_{t}C_{5} \equiv [{}_{t}\$_{I;Cg} + {}_{t}\$_{r;Cg} + {}_{t}\$_{m;Cg}] \; .$$

A third figure of merit for a group is the amount of payment for group "g" which will be required at time "t" for each system which is reliable during time interval "t". The cost per reliable system from group "g" during "t" is

$${}_{t}U.C.A._{g} \equiv [{}_{t}\$_{T;g} / {}_{t}N_{RRg}] \; . \tag{7.8}$$

This equation may be expressed in terms of the group availability during "t"; i.e.,

$${}_{t}U.C.A._{g} = [{}_{t}\$_{T;g} / ({}_{t}A_{g})(\;N_{Sg})] \tag{7.9}$$

and could be named the "system unit-cost-availability ratio" during "t".

Similarly, the cost per reliable system in the fleet during "t" is

$${}_{t}U.C.A._{F} \equiv [{}_{t}\$_{T;F} \Big/ \sum_{(g=Bg)\leq t}^{Fg>t} ({}_{t}A_{g})(\;N_{Sg})] \; . \tag{7.10}$$

The equation may be expressed in terms of the fleet availability during "t"; i.e.,

$$ {}_{t}U.C.A._{F} = \frac{{}_{t}\$_{TF}}{{}_{t}A_{F} \sum_{g} (N_{Sg})} \tag{7.11} $$

and could be called the "fleet unit-cost-availability ratio" during "t".

A fourth figure of merit, is the "group lifetime evaluation ratio" which is defined as the ratio of the cost of sustaining the group over its lifetime to the average number of reliable systems in the group over its lifetime; that is,

$$ G.L.E.R._{g} \equiv [P_{g}/({}_{T}\bar{A}_{g})(N_{Sg})] . \tag{7.12} $$

Each of the above figures of merit contribute to the evaluation of management decisions which were made during the planning horizon. Graphs of some of these figures of merit are shown on the following pages (Figures 25 and 26).

Optimization

Currently there is no way to optimize a fleet effectiveness function based upon the theory developed in this monograph except by trial and error through the use of simulation. The theory is written in terms of intervals of time ("stages") so that conversion to dynamic programming notation can be made when dynamic programming methods are developed to allow many non-Markovian non-ergodic counterflows to be taken into account. When this development

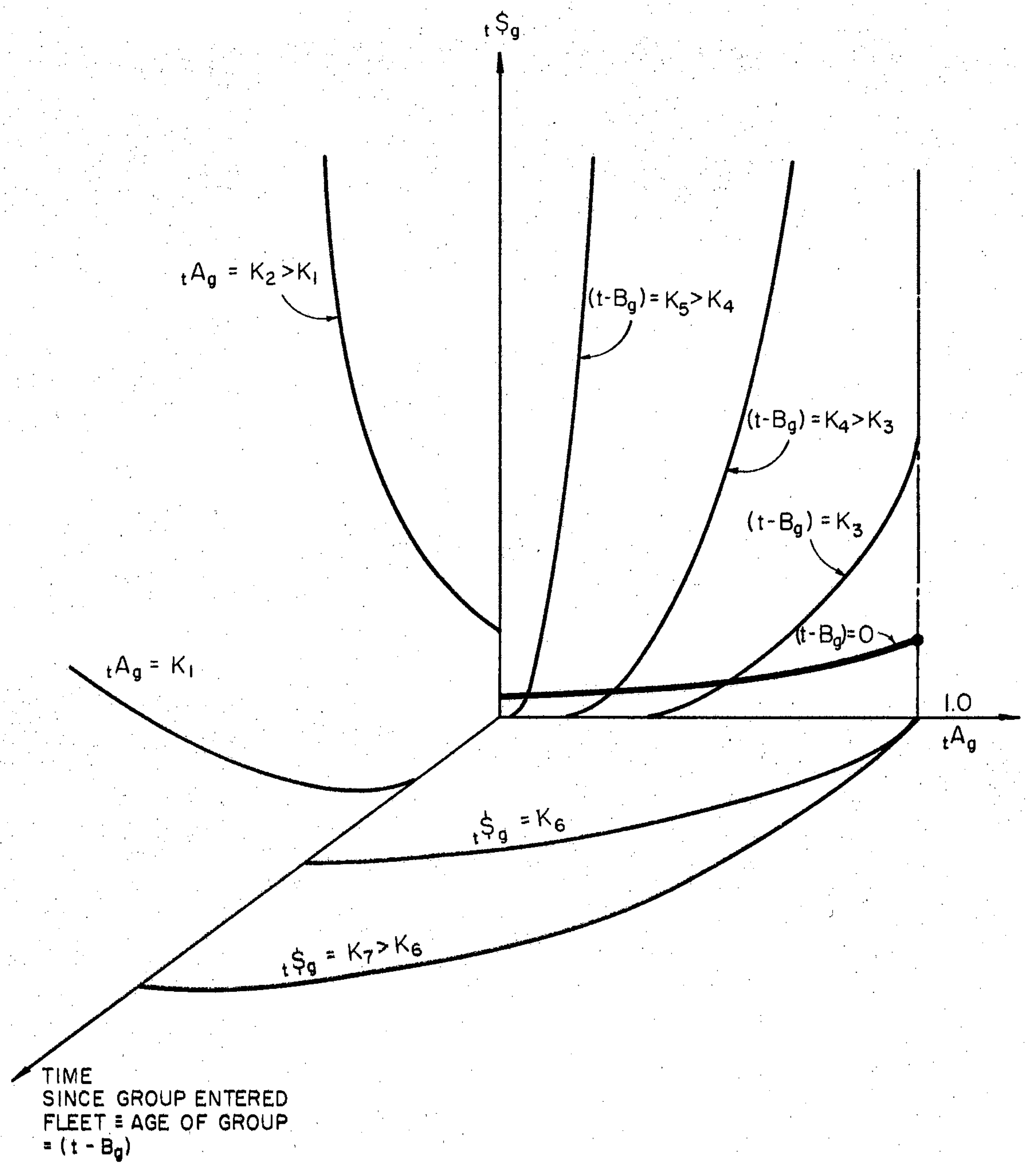


Figure 25. Payment Due For Operating, Maintaining and Exiting a Group When the Group Is $(t - B_g)$ Intervals Old as Functions of the Age of the Group and Corresponding Availability of the Group at That Age

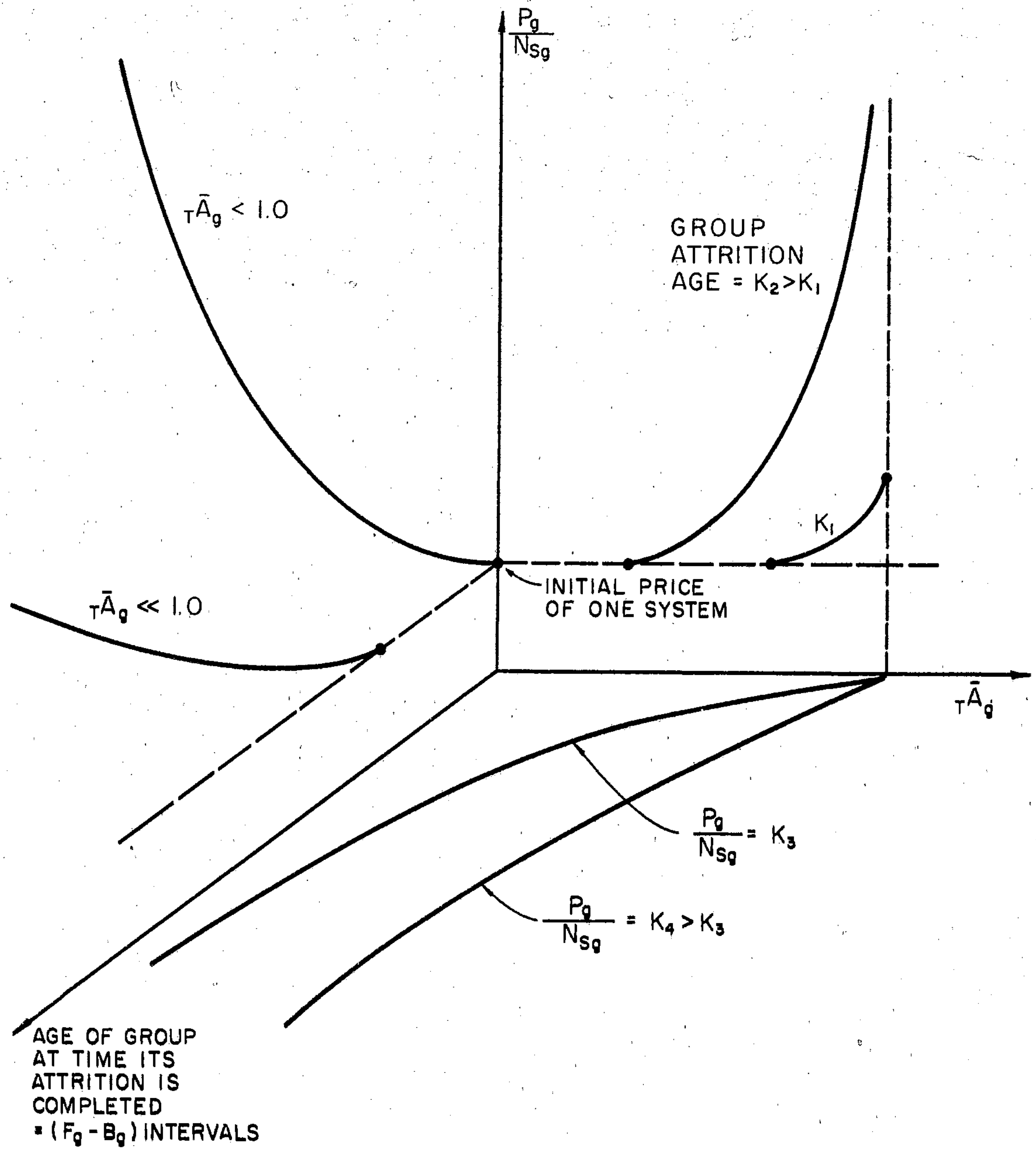


Figure 26. Present Worth of the Costs of Buying, Maintaining, Operating and Exiting a Group as Functions of the Age at Which the Group Attrition Is Completed and the Average Lifetime Availability of the Group

occurs, those values of the decision variables which will optimize the measures of fleet effectiveness should be found.

In terms of dynamic programming the objective is to optimize one of the measures of fleet effectiveness. The state vector of the fleet inventory at time "t" is composed of the state vectors of all the composite fleet inventory systems and the number of current fleet inventory systems from each group. The state vector of a composite fleet inventory system from group "g" at time "t" is (${}_tC_{Dg}$; ${}_tC_{Og;max}$; ${}_tC_{Dg;min}$). The decision variables are the management-decision factors ${}_tN_{iEg}$, ${}_tG_a$, ${}_tG_b$, ${}_tG_2$, ${}_tG_3$ and ${}_tW_p$. The parameters are the technological parameters, the forecasted frequency distributions of $(C_O|t)$ and $(C_R|t)$, and the forecasted job specifications, and the relationships between maintainabilities and suppliabilities of facilities and their congestions.

CHAPTER VIII

AN ILLUSTRATIVE PROBLEM

Introduction

The purpose of Chapter VIII of this monograph is to present a simple problem which is solvable by the theory developed previously in order to enhance the reader's awareness of the interplay among state vector of the fleet inventory, the management-dictated decision variables, the parameters of the problem and the costs involved.

The state vector of the fleet inventory at time "t" is composed of all the state vectors of the typical fleet inventory systems. The state vector of the typical fleet inventory system from group "g" at time "t" (Fortran subscript "i") is composed of $\left({}_tC_{Dg};\ {}_tC_{Ogmax};\ {}_tC_{Ogmin}\right)$ and ${}_tN_{Ig}$. The decision variables for the fleet inventory are the management-decision factors ${}_tG_a$, ${}_tG_b$, ${}_tG_2$, ${}_tG_3$, ${}_tW_p$ and ${}_tN_{iEg}$. The parameters are the technological parameters, the forecasted values of ${}_tN_R$, ${}_tC_{Rmax}$, ${}_tC_{Rmin}$, the frequency distributions of $(C_c|t)$ and $(C_R|t)$, and the relationships between the facilities congestions and their maintainabilities and suppliabilities.

A group is designated by its particular (j,k) combination. "j" is the time at which the group entered the fleet; that is, its "birthdate". "k" is an arbitrarily assigned number which indicates the design type of the systems in the group, the supply source of the design and the supply source of the manufacture of the group.

Eight types of input must be given for simulation; namely,

(1) job specifications data versus time from the beginning to the end of the planning horizon,

(2) data showing the previous states of each group born before the beginning of the horizon,

(3) data showing the state vectors, the costs and the lead times associated with new groups which could be bought during the horizon,

(4) data showing management decision factors, technological parameters and costs for the activity of each group in the fleet versus time,

(5) operation maintainabilities as functions of congestion in the repair facilities,

(6) design maintainabilities as functions of congestion in the modification facilities.

(7) shapes of the probability distribution curves for C_R and C_0 between their extreme values, and

(8) suppliabilities as functions of congestion in the supply sources.

Flow charts for simulation of the activities of the fleet in the problem are given and are rigidly followed in the example calculations which are shown step-by-step. To provide an "overview" of the order of the steps to be taken the general flow chart is expressed in terms of subroutines. Subroutines are numbered and given five-letter Fortran names which indicate their purposes.

Four major "DO loops" are used in the general flow chart. The two innermost "DO loops" assure that the series of required calculations will be performed for a single interval of time for all the groups currently in the fleet. The third loop decides what kind of design and how many systems will be needed in the new group which is ordered now for delivery "L_k" time intervals later. The outer loop assures that the series of calculations will be performed for the whole fleet for all times in the planning horizon.

Example

An aircraft repair and modification company which has been operating for almost 25 years decides to enter the international air freight field. The most critical performance factor is believed to be the speed of the transports. Forecasted job requirements over the planning horizon are shown in Table III. The distribution of the forecasted values for speed at any time is probably a skewed triangle like that shown in Table IX.

The initial group, which is composed of 50 new planes of up-dated design are scheduled to arrive during the quarter of the year which marks the 25th anniversary of the company. The design speed of these transports will be

700 mph, but the actual distribution of the values of the operating speeds at any time will be the skewed triangle as shown in Table IX.

The company intends only to buy, not make, additional planes during this ten year planning period. However, the company does intend to use its own facilities for repair and modification. Repair and modification facilities for the company's incoming freight fleet have been set up separate from the facilities allocated to servicing customer's aircraft. The Maintainability-Congestion curves stated in Tables VII and VIII for the freight fleet facilities are based upon the company's experience with standard and up-dated aircraft. Investigation has indicated that the Suppliability-Congestion curves of the sources with which the company must deal are as stated in Table X.

Among the policies stated by management to guide the activities of the fleet are those regarding the methods for estimating each of the following:

(1) the number of systems to be ordered at any time for future delivery,

(2) the type of design (standard up-dated) to be ordered, and

(3) the number of systems which will be forced out of the fleet at any time.

These methods are expressed in the flow charts of subroutines 13, 14, and 1, respectively.

Unit costs, interest and lead times are assumed constant over the planning period. Expected technological parameters, managerial decision parameters, etcetera, are shown in the remaining tables.

Flow Charts for Simulation

On the following pages are flow charts which show the subroutines and their usages in simulating the activities of the freight fleet. The purpose of the general flow chart is to provide the reader with an "overview" of the various paths between the subroutines and the tests which indicate the circumstances under which each path is to be followed. The list of subroutines indicates the output of each subroutine, its assigned Fortran name and subroutine number for ease of reference. Meanings of the flow chart symbols are given on a separate page.

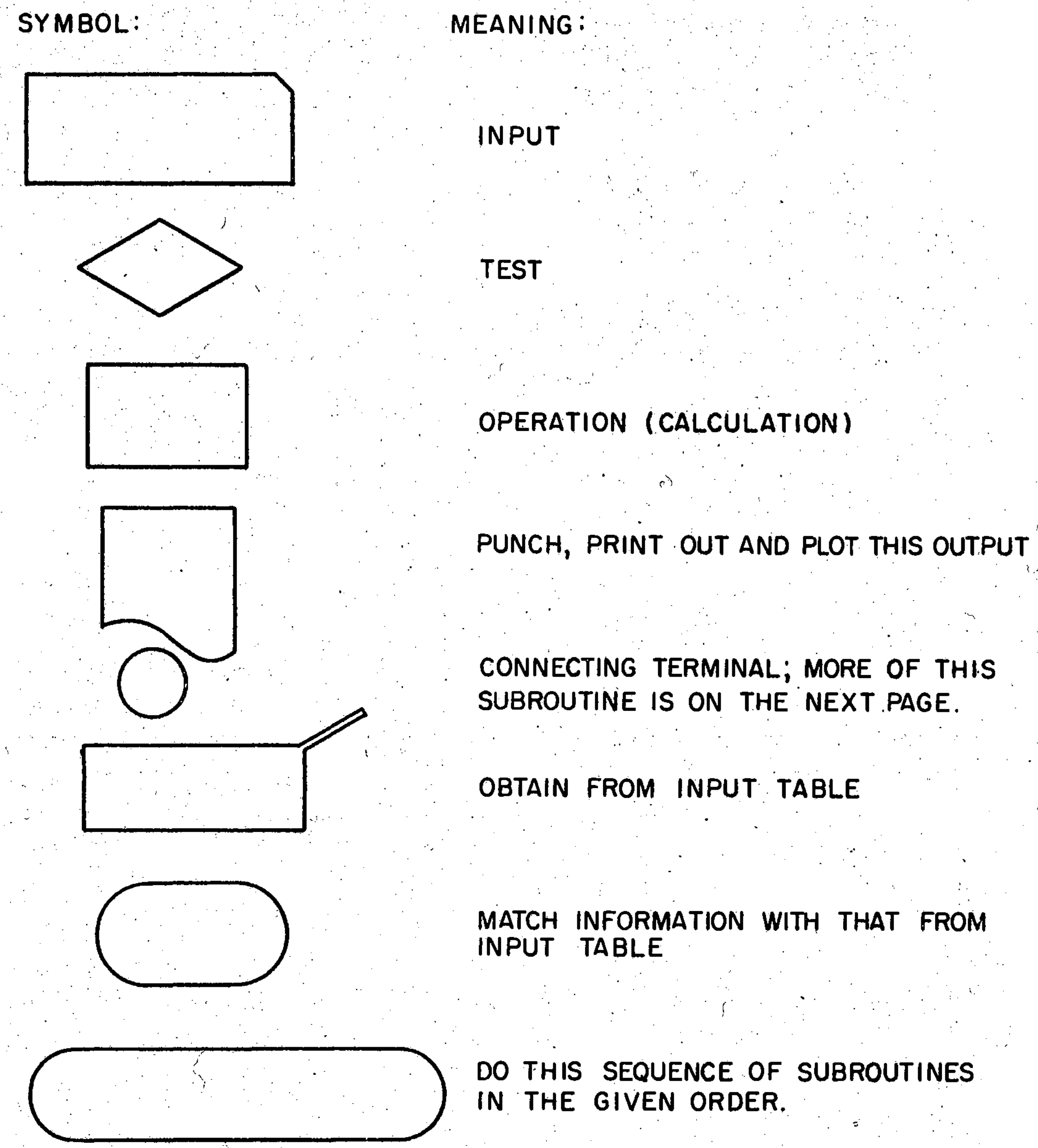


Figure 27. Flow Chart Symbols and Meanings

TABLE II

DESIGNATIONS OF SUBROUTINES

Subroutines No.	Name	Resulting Calculation
(1)	FCNEG	* ${}_tN_{iEg}$
(2)	FINVG	${}_tN_{Ig}$ (from Equation (5.11))
(3)	EXDTG	${}_tN_{e;Tg}$ (from Equation (5.9))
(4)	COCDG	${}_t\bar{C}_{Dg}$ (from Equation (2.1))
		${}_t\bar{C}_{Og\ max}$ (from Equation (2.2))
		${}_t\bar{C}_{Og\ min}$ (from Equation (2.3))
(5)	RDDUG	${}_tR_g$ (from Equation (3.13))
		${}_tU_g$ (from Equation (3.14))
		${}_tD_{rg}$ (from Equation (3.15))
(6)	BATCG	${}_tN_{RRg}$ (from Equation (5.8))
		${}_tN_{irg}$ (from Equation (5.2))
		${}_tN_{img}$ (from Equation (5.3))
		${}_tN_{ieg}$ (from Equation (5.1))
(7)	OPERM	${}_tM_{Og}$ (Input Table V)
(8)	DESNM	${}_tM_{Dg}$ (Input Table VI)
(9)	AVALG	${}_tA_g$ (from Equation (7.1))
(10)	COSTG	${}_t\$_{Tg}$ (from Equation (6.11))
(11)	COSTB	P_g (from Equation (6.7))
(12)	RAIOG	${}_tU.C.A._g$ (from Equation (6.7))

* Based upon management decision.

TABLE II (Continued)

No.	Name	Resulting Calculation
(13)	BUYNP	${}_tN_p$ (from Equation (5.14))
(14)	EVALS	* ${}_tN_{pg}$
(15)	AVALF	${}_tA_F$ (from Equation (7.4))
(16)	COSTF	${}_t\$_{TF}$ (from Equation (6.9))
(17)	COSTO	P_F (from Equation (6.10))
(18)	RAIOF	${}_tU.C.A._F$ (from Equation (7.10))
(19)	QUADR	intergration of a general curve between limits (IBM Program)
(20)	PLOTR	IBM, Calcomp routine.
(21)	AVLGL	${}_TA_g$ (from Equation (7.6))
(22)	GLERG	$G.L.E.R._g$ (from Equation (7.12))
(23)	AVLFL	A_F
(24)	SUPTS	${}_tS_g$ (Input Table VIII)
(25)	ARIVT	* N_{Sg}
(26)	AREAT	(from intergration of the figure shown in Input Table VII)

* Based upon management decision.

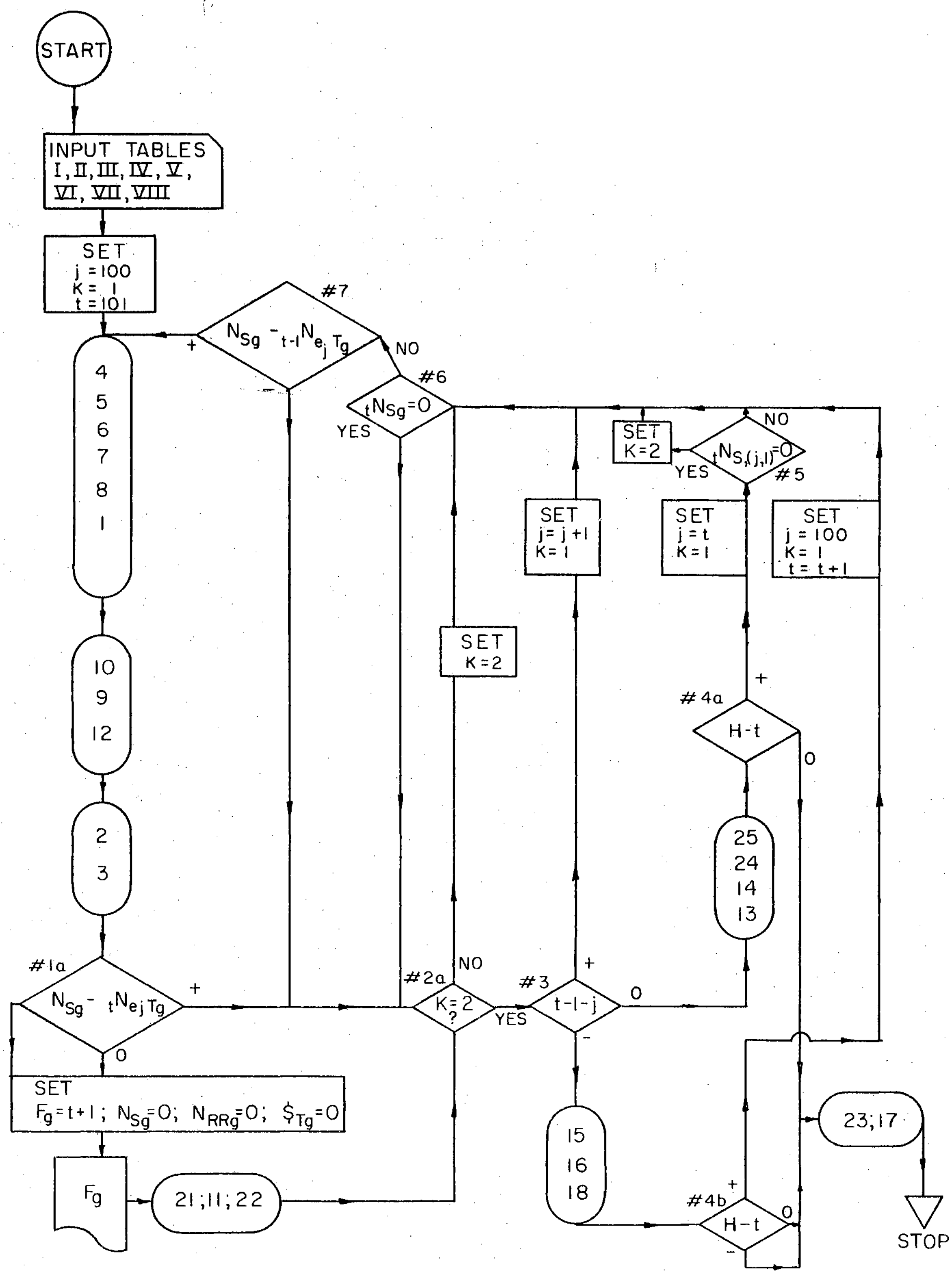


Figure 28. The General Flow Chart for the Problem

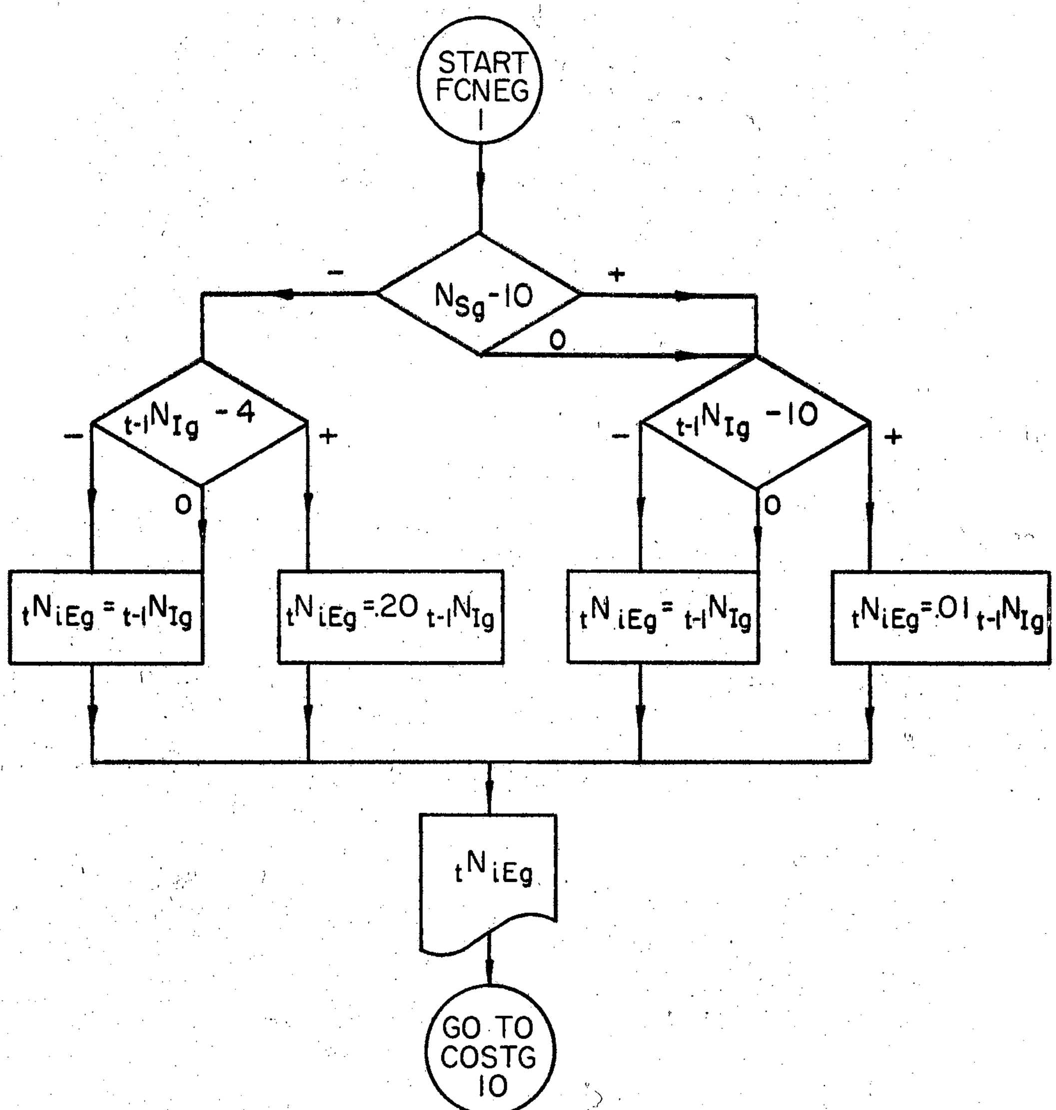


Figure 29. Subroutine # 1 (FCNEG)

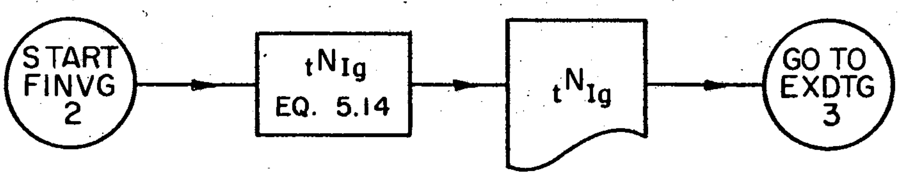


Figure 30. Subroutine # 2 (FINVG)

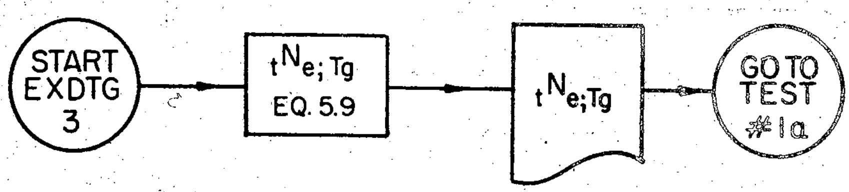


Figure 31. Subroutine # 3 (EXDTG)

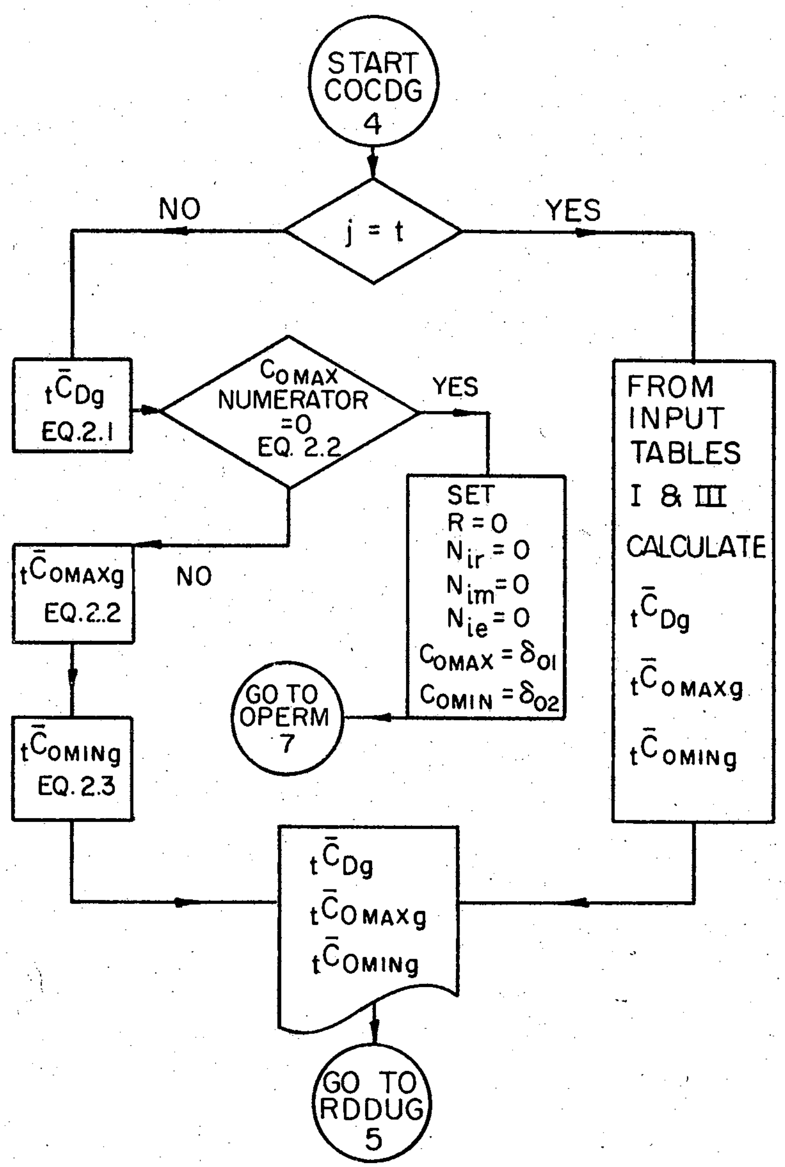


Figure 32. Subroutine # 4 (COCDG)

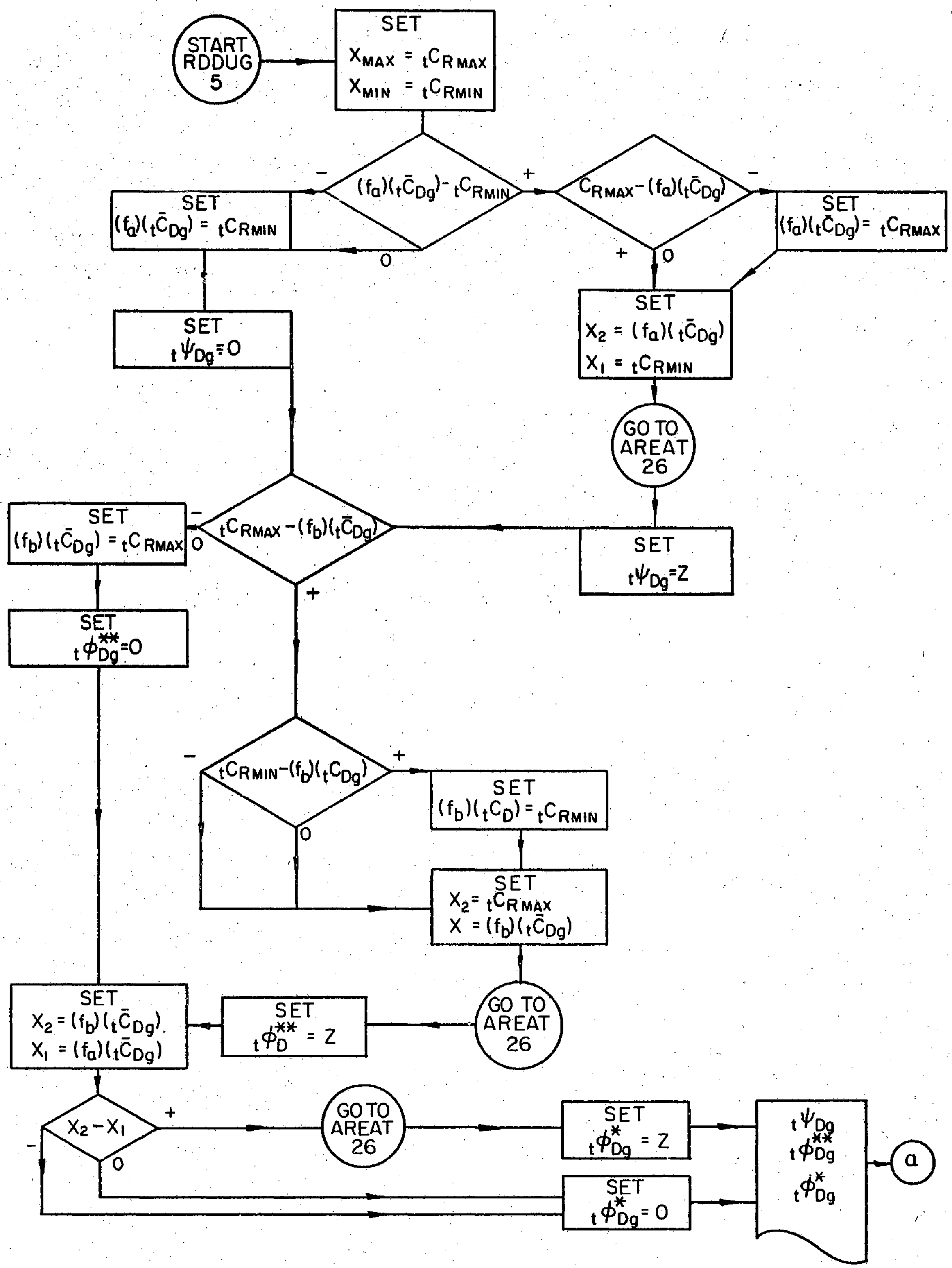


Figure 33. Subroutine # 5 (RDDUG)

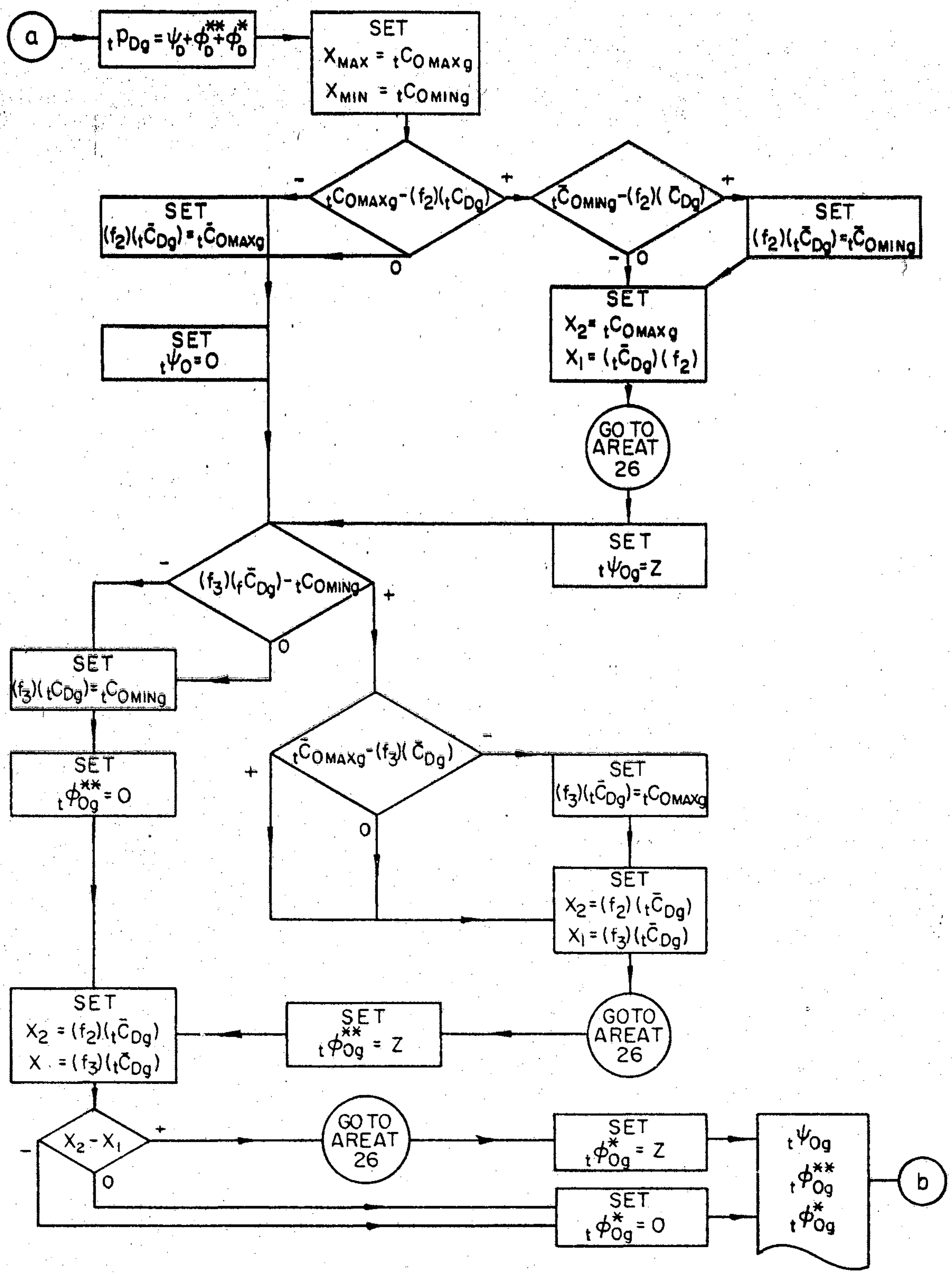


Figure 33. (Continued)

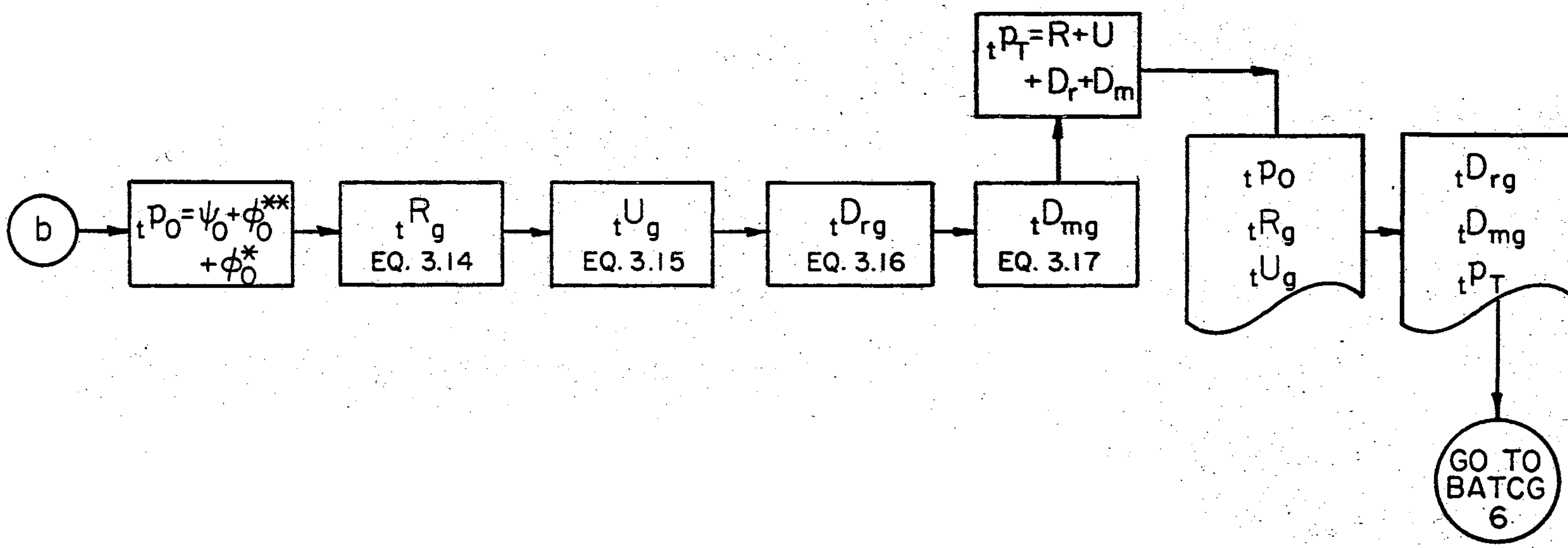


Figure 33. (Concluded)

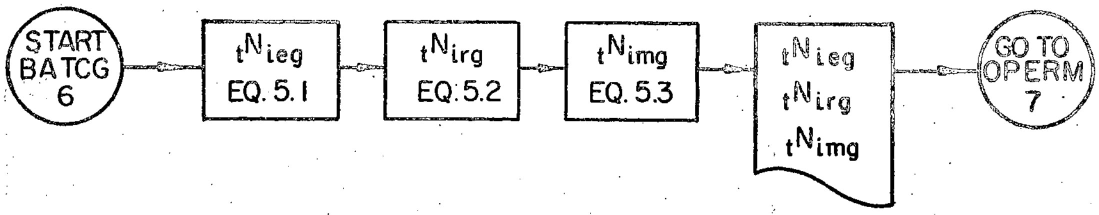


Figure 34. Subroutine # 6 (BATCG)

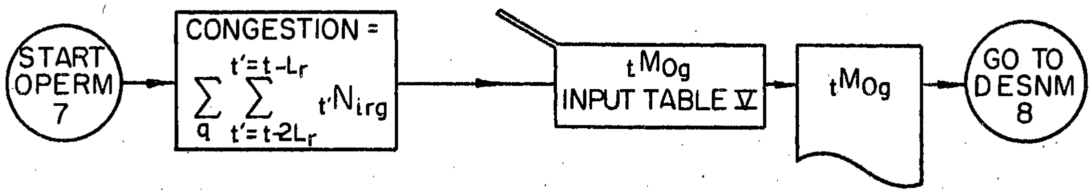


Figure 35. Subroutine # 7 (OPERM)

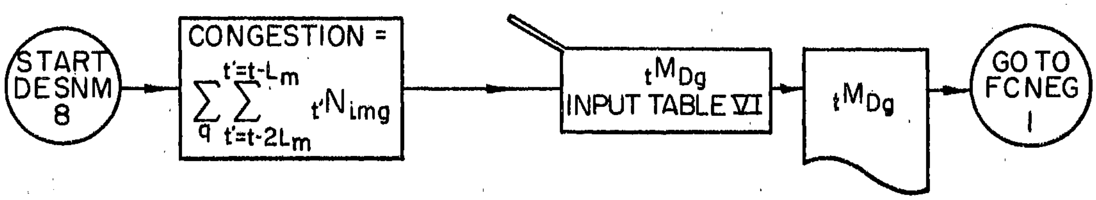


Figure 36. Subroutine # 8 (DESNM)

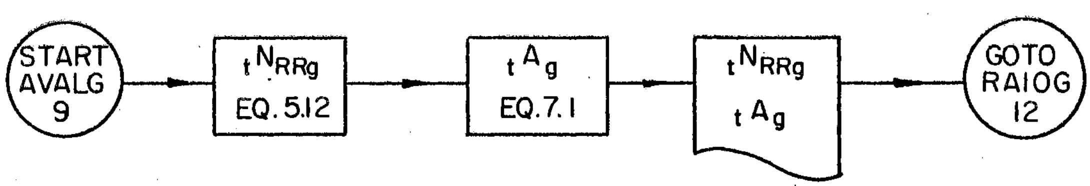


Figure 37. Subroutine # 9 (AVALG)

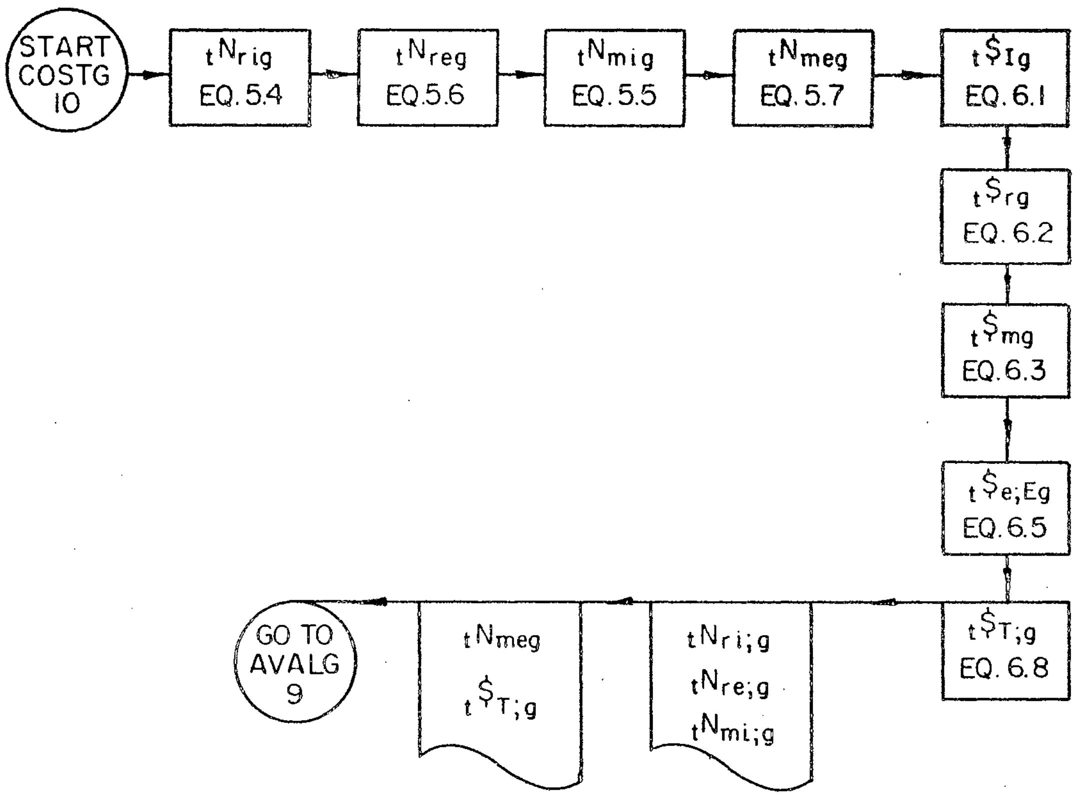


Figure 38. Subroutine # 10 (COSTG)

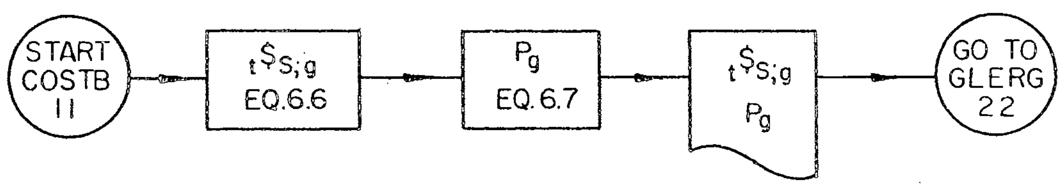


Figure 39. Subroutine # 11 (COSTB)

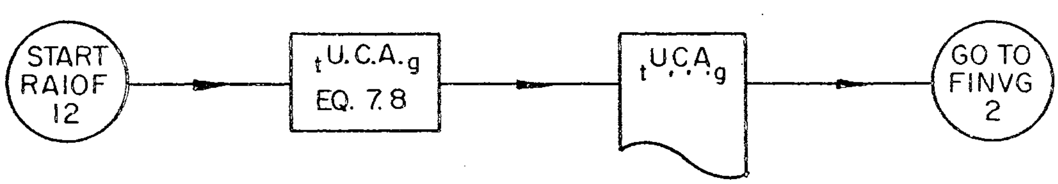


Figure 40. Subroutine # 12 (RAIOF)

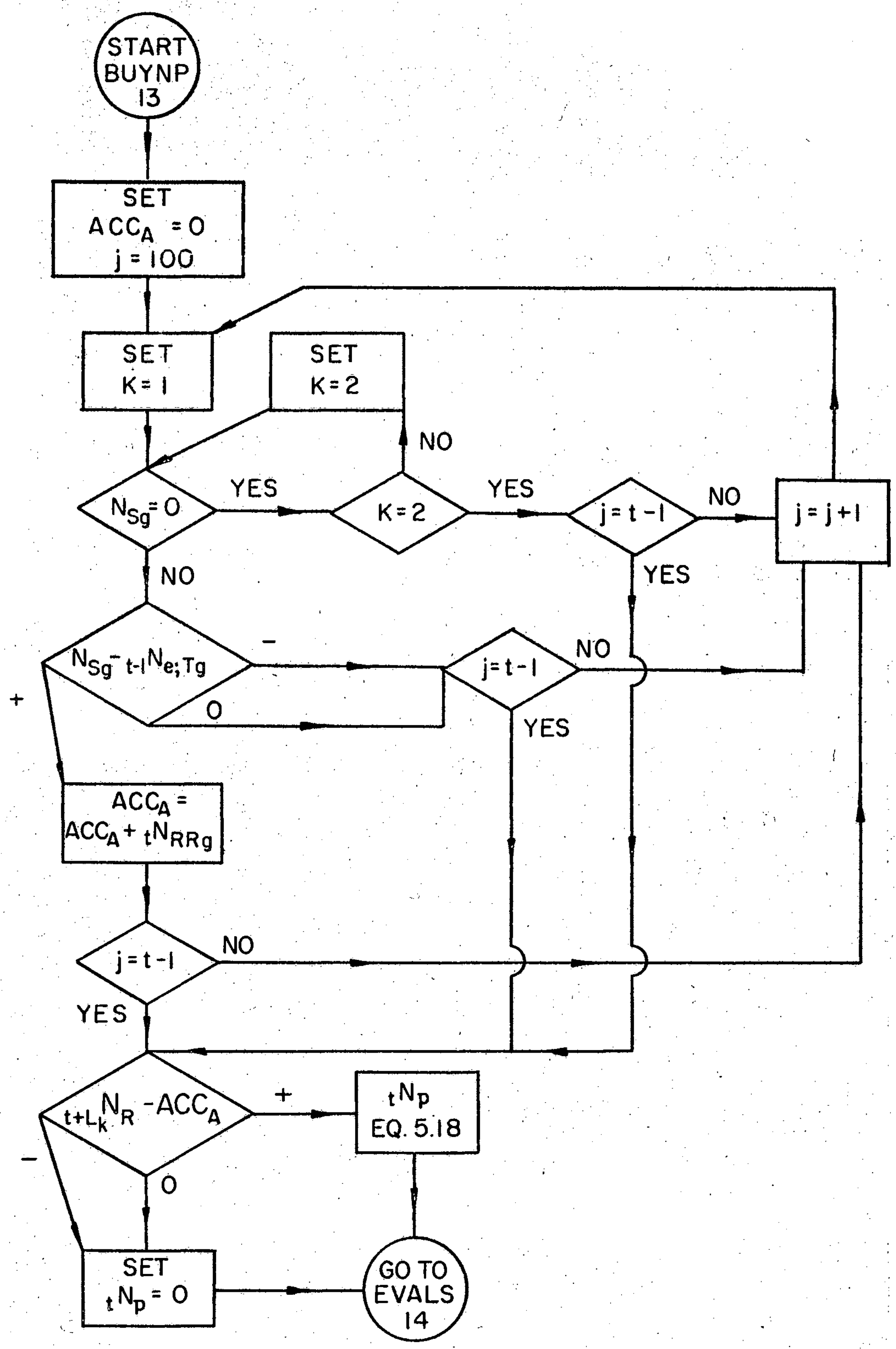


Figure 41. Subroutine # 13 (BUYNP)

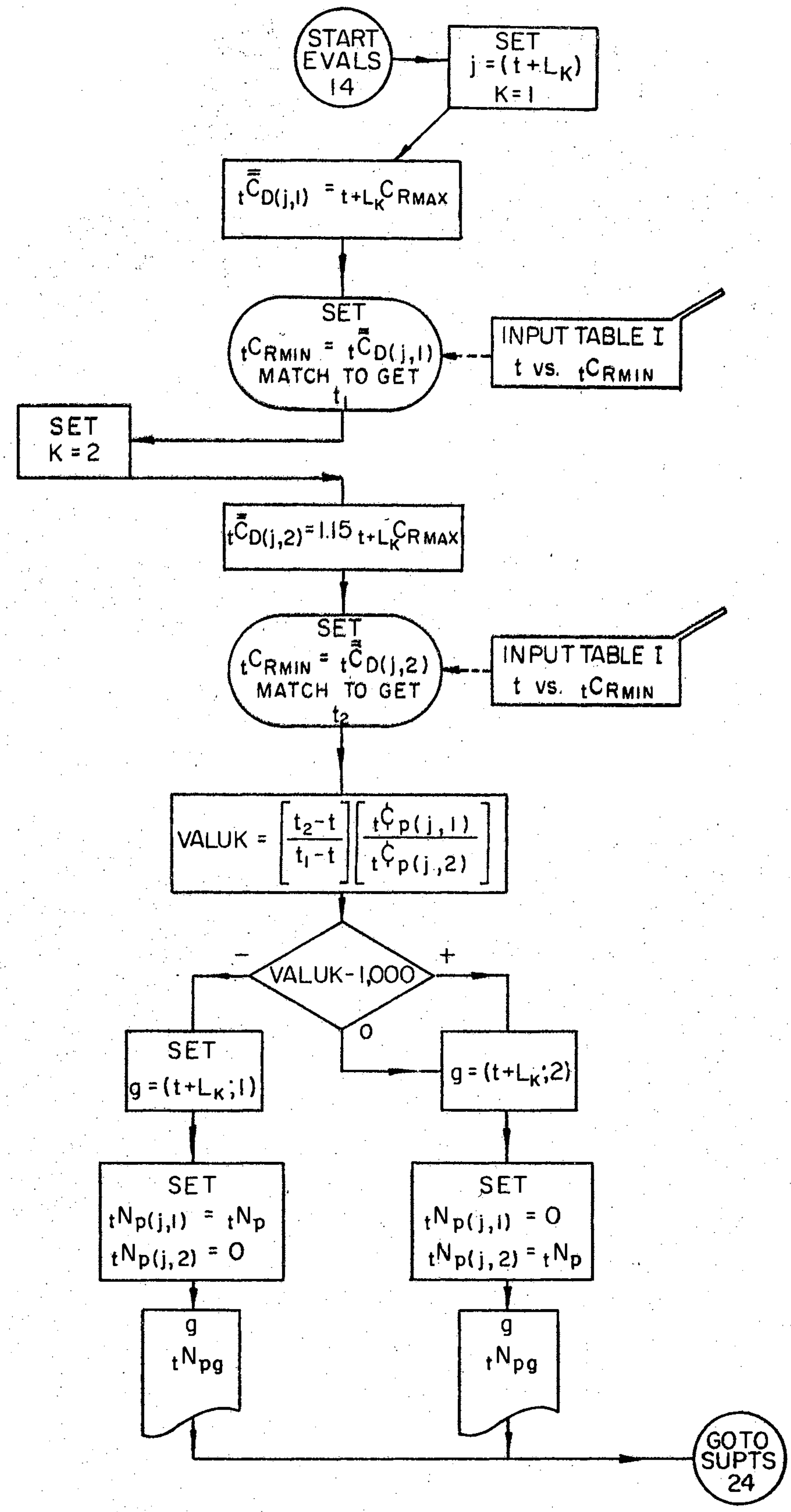


Figure 42. Subroutine # 14 (EVALS)

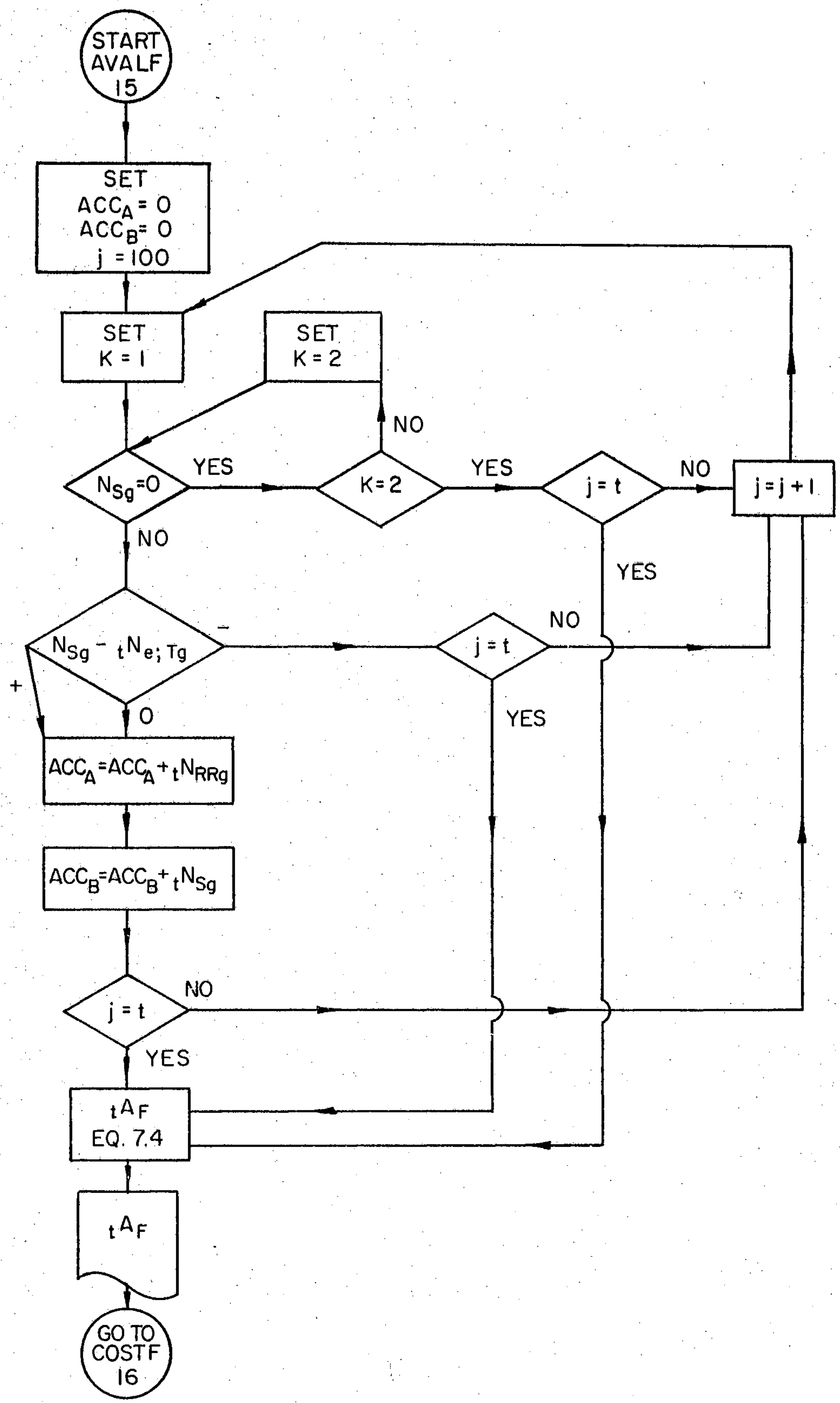


Figure 43. Subroutine # 15 (AVALF)

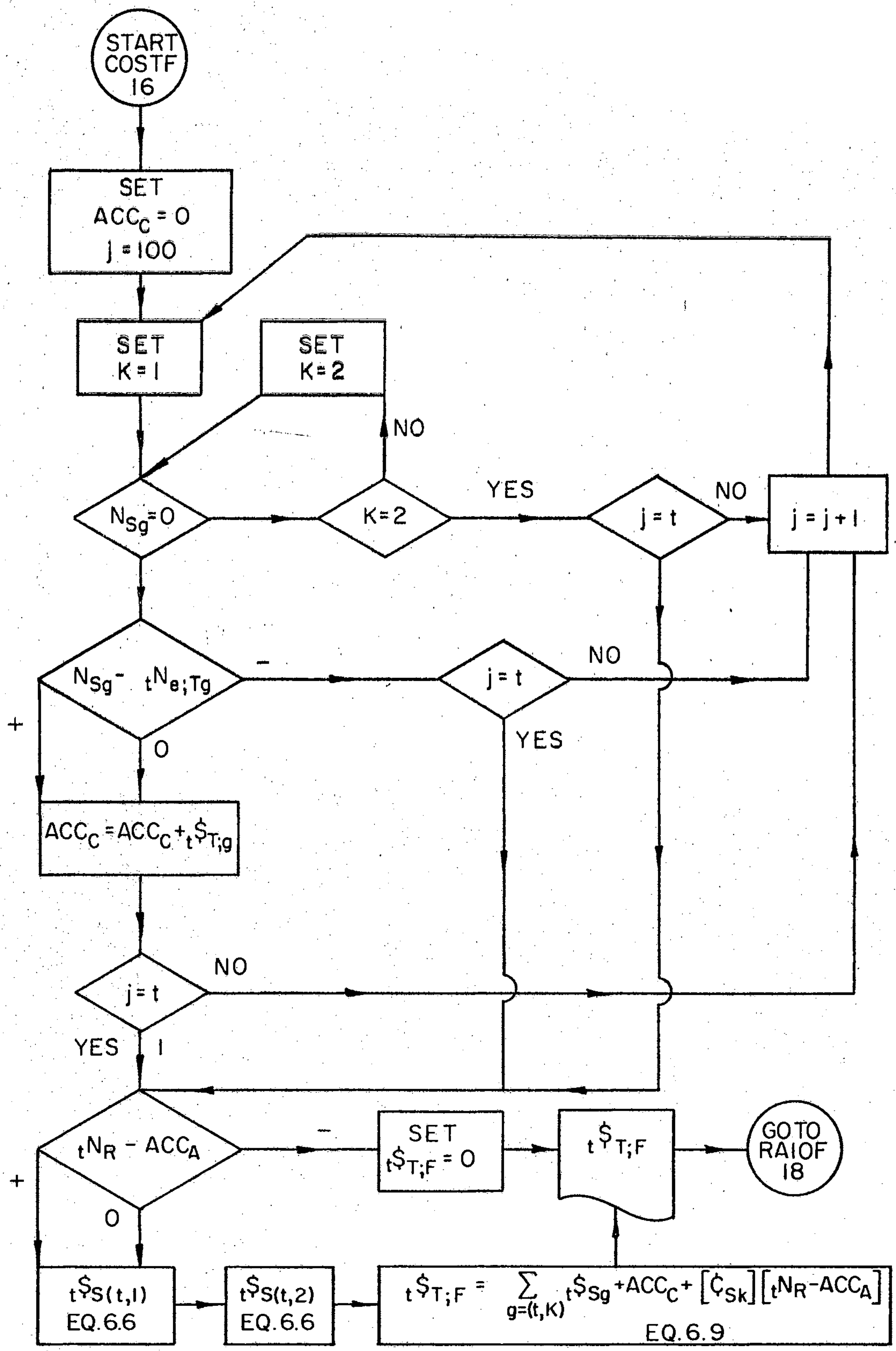


Figure 44. Subroutine # 16 (COSTF)

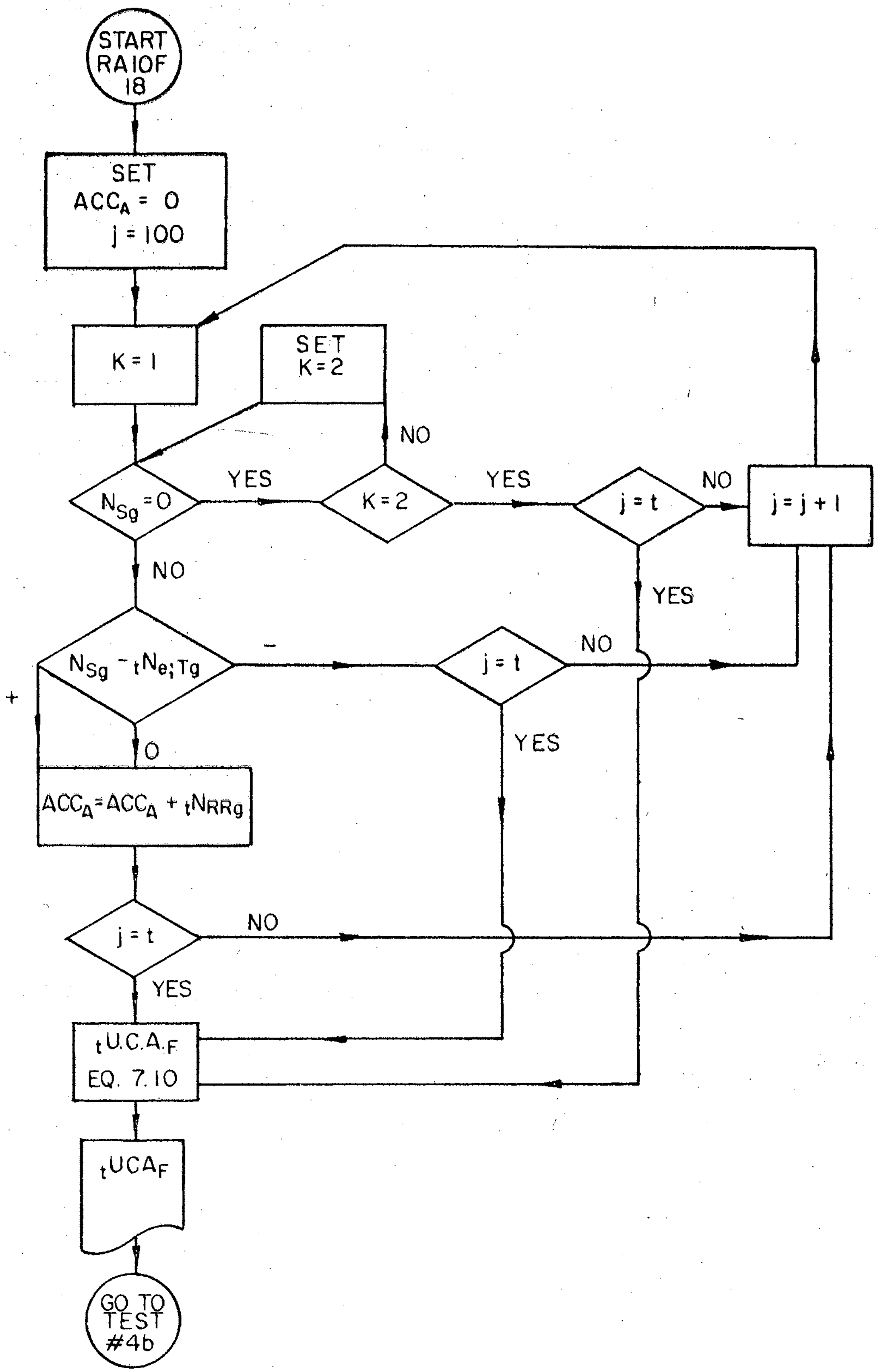


Figure 45. Subroutine # 18 (RAIOF)

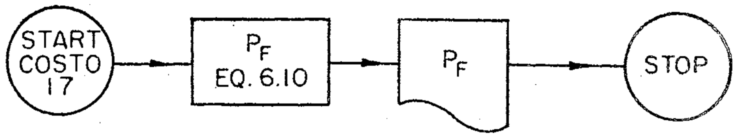


Figure 46. Subroutine # 17 (COSTO)

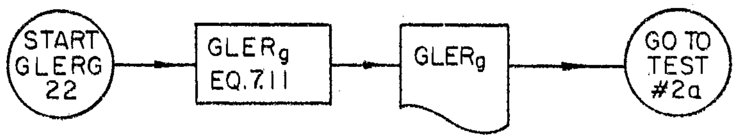


Figure 47. Subroutine # 22 (GLERG)

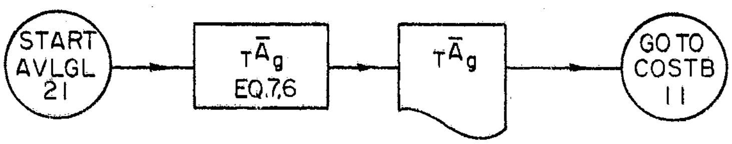


Figure 48. Subroutine # 21 (AVLGL)

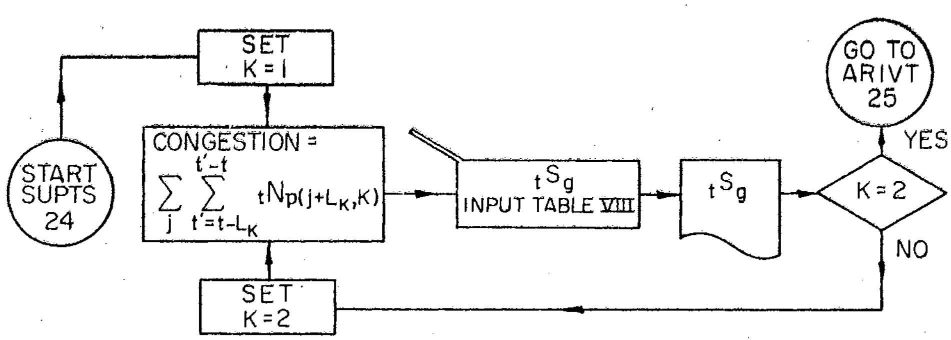


Figure 49. Subroutine # 24 (SUPTS)

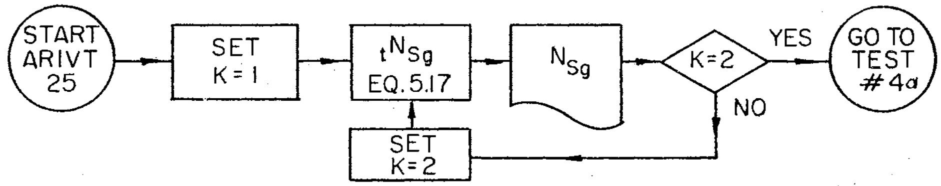


Figure 50. Subroutine # 25 (ARIVT)

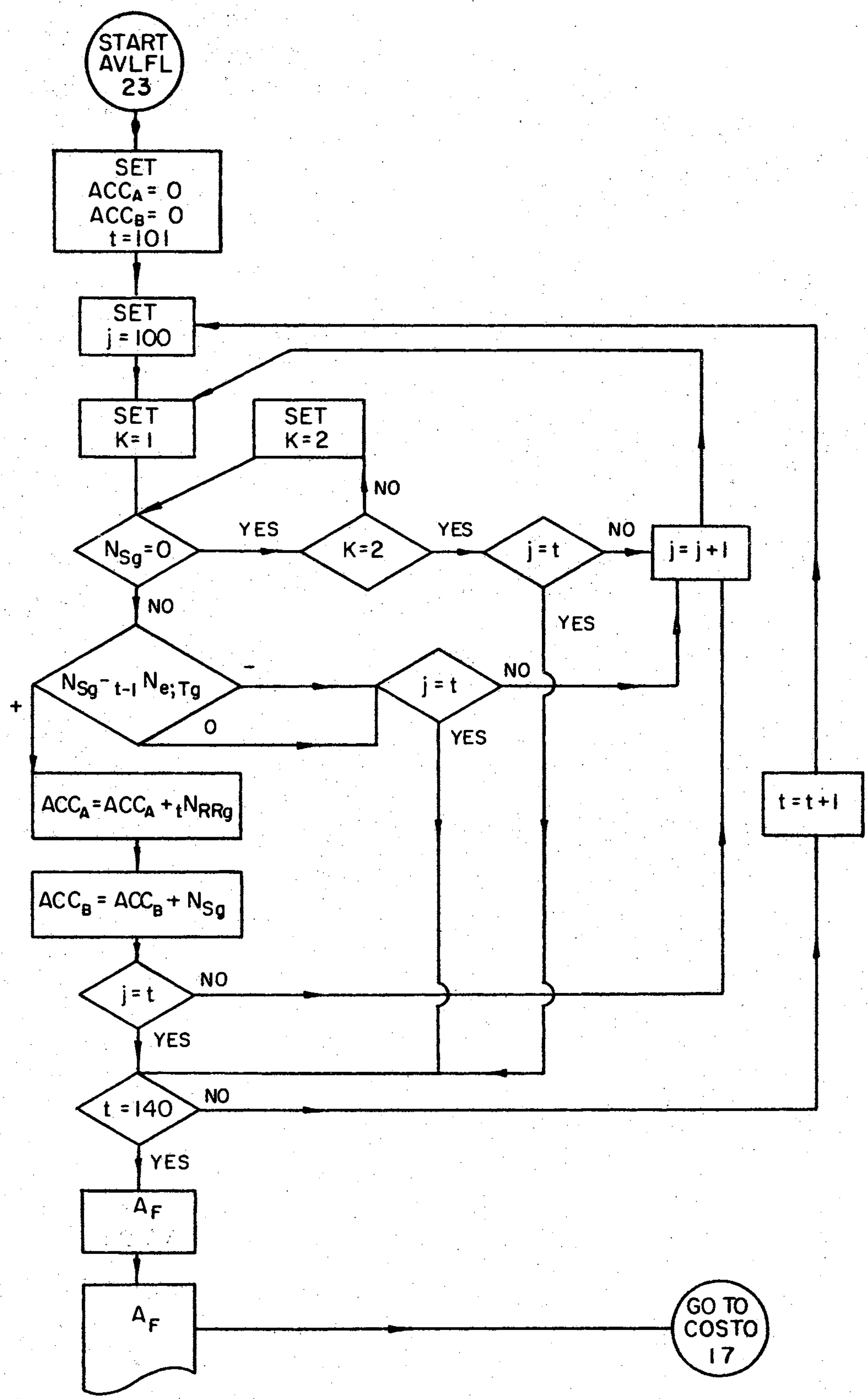


Figure 51. Subroutine # 23 (AVLFL)

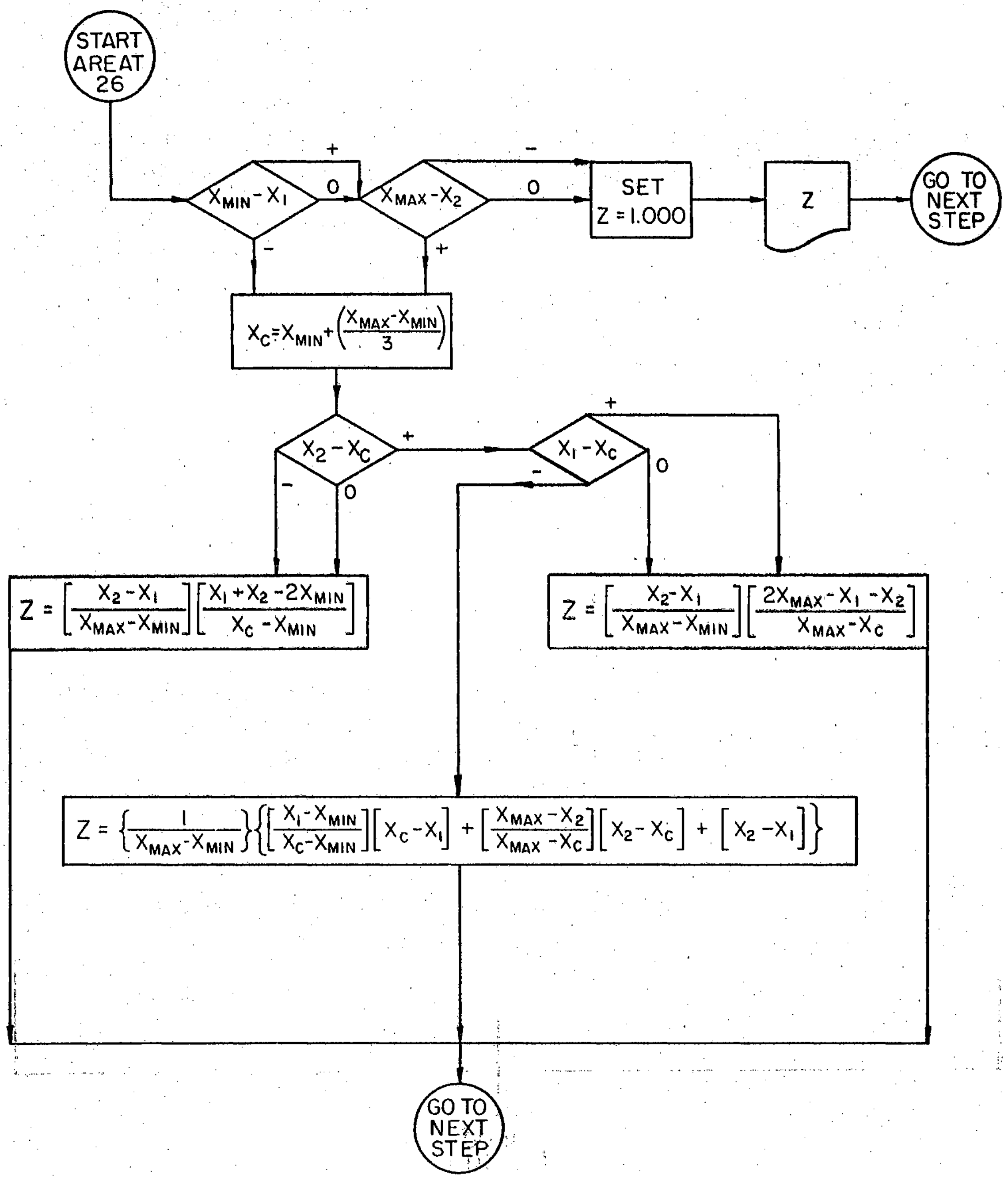


Figure 52. Subroutine # 26 (AREAT)

Input Tables

The inputs for the problem are expressed in the following eight input tables and in those subroutines which were previously indicated as being based upon management decisions.

Values associated with points lying between values given in the tables are calculated by assuming straight line functions between adjacent points.

TABLE III

INPUT TABLE I: JOB SPECIFICATIONS

TIME	t^N_R	t^C_R min	t^C_R max
Quarter	Systems		
100	50	700 m.p.h.	1200 m.p.h.
101	50	700	1200
102	70	717	1217
103	90	734	1234
104	110	750	1250
105	130	767	1267
106	150	784	1284
107	170	800	1300
108	190	817	1317
109	210	834	1334
110	235	850	1350
111	260	867	1367
112	285	884	1384
113	310	900	1400
114	335	912	1412
115	360	925	1424
116	385	937	1437
117	410	950	1450
118	435	962	1462
119	460	974	1474
120	485	987	1487
121	500	1000	1500
122		1037	1537
123		1074	1574
124		1112	1612
125		1150	1650
126		1187	1687
127		1224	1724
128		1262	1762
129		1300	1800
130		1312	1822
131		1325	1845
132		1337	1867
133		1350	1890
134		1356	1906
135		1362	1922
136		1368	1938
137		1375	1955
138		1384	1974
139		1392	1992
140	500	1400	2000
⋮	⋮	⋮	⋮
180	600	3000	4000

TABLE IV

INPUT TABLE II: NUMBER AND TIMING OF SYSTEMS ORDERED PRIOR TO THE START OF THE PRESENT PLANNING HORIZON

Time at which group "g" was ordered: t	Group "g": (j, k)	Number of systems which were ordered at time "t": Of design type k=1	k=2
85	(93,k)	0	0
86	(94,k)	|	|
87	(95,k)	|	|
88	(96,k)	|	|
89	(97,k)	↓	↓
90	(98,k)	0	0
91	(99,k)	25 |	25
92	(100,k)	|	25
93	(101,k)	|	9
94	(102,k)	↓	0
95	(103,k)	0	|
96	(104,k)	50	|
97	(105,k)	0	|
98	(106,k)	|	|
99	(107,k)	↓	↓
100	(108,k)	0	0

TABLE V

INPUT TABLE III: DATA FOR EACH NEWLY-BORN GROUP

i.e., for g = (j=t; k)		
k =	1	2
L_r	2 quarters	2 quarters
L_m	4 quarters	4 quarters
L_k	8 quarters	8 quarters
$\not{c}_p$	\$18,000,000	\$24,000,000
$\not{c}_q$	0	0
$\$_{S;C}$	0	0
i'	.03	.03
C_D	$_tC_{R\ max}$	$1.15\ _tC_{R\ max}$
$C_{O\ max}$	$_tC_{R\ max} + 40$	$1.15\ _tC_{R\ max} +80$
$C_{O\ min}$	$_tC_{R\ min} - 40$	$1.15\ _tC_{R\ max} -80$
$_{t-4}N_{ir}$	0	0
$_{t-3}$	↓	↓
$_{t-2}$	↓	↓
$_{t-8}N_{im}$	0	0
$_{t-7}$	↓	↓
$_{t-6}$	↓	↓
$_{t-5}$	↓	↓
$_{t-4}$	↓	↓
$_tN_{ie}$	0	0
$_tN_I$	$_tN_S$	$_tN_S$
$_tN_{RR}$	$_tN_S$	$_tN_S$

TABLE VI

INPUT TABLE IV: DATA FOR EACH CURRENTLY ACTIVE GROUP AT TIME "t"

g = (j;k)		
k =	1	2
f_a	.95	.95
f_b	1.05	1.05
f_2	.90	.90
f_3	.60	.60
W_p	.20	.20
δ_D	120 mph	120 mph
δ_{01}	35 mph	25 mph
δ_{02}	70 mph	50 mph
δ_{03}	60 mph	40 mph
δ_{04}	30 mph	30 mph
δ_{05}	150 mph	150 mph
δ_{06}	100 mph	100 mph
¢$_{H}$	$ 10,000	$ 10,000
¢$_{I;A}$	$ 50,000	$ 70,000
$\$_{I;C}$	$ 30,000	$ 40,000
¢$_{r;q}$	$ 6,000	$ 4,000
¢$_{r;A}$	$ 13,000	$ 10,000
$\$_{r;C}$	$ 50,000	$ 40,000
¢$_{m;q}$	$ 6,000	$ 4,000
¢$_{m;A}$	$ 26,000	$ 20,000
$\$_{m;C}$	$ 60,000	$ 50,000
¢$_{Sh}$	$440,000	$440,000
¢$_{e;a}$	$ 1,000	$ 1,000
¢$_{e;v}$	$ 0	$ 0

TABLE VII

INPUT TABLE V: OPERATION MAINTAINABILITY VERSUS CONGESTION

Repair Facility Congestion = $\sum_{g} \sum_{t'=t-4}^{t'=t-2} {}_{t'}N_{irg}$	M_O
0	1.00
50	0.95
1 00	0.90
3 00	0.50
4 00	0.30
5 00 and more	0.00

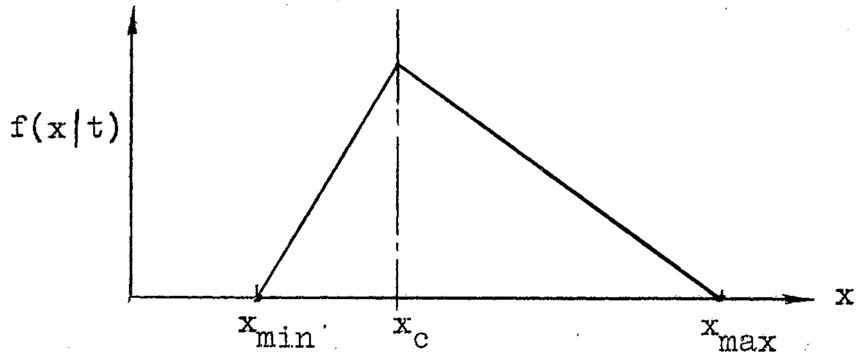
TABLE VIII

INPUT TABLE VI: DESIGN MAINTAINABILITY VERSUS CONGESTION

Modification Facility Congestion = $\sum_{g} \sum_{t'=t-8}^{t'=t-4} {}_{t'}N_{img}$	M_D
0	1.00
50	0.95
1 00	0.90
2 00	0.50
2 50	0.30
3 00 and more	0.00

TABLE IX

INPUT TABLE VII: CONDITIONAL FREQUENCY DISTRIBUTION OF x WHERE $x = C_O$ OR $x = C_R$



$$x_c \equiv x_{min} + \left(\frac{x_{max} - x_{min}}{3}\right)$$

$$f\,(x_{min}|t) = 0$$

$$f\,(x_{max}|t) = 0$$

$$f(x_c|t) = \left(\frac{2}{x_{max} - x_{min}}\right)$$

TABLE X

INPUT TABLE VIII: SUPPLIABILITY VERSUS CONGESTION

k=1 Supply Source Congestion = $\sum_{t'=t-8}^{t'=t} {}_{t'}N_p(t'+8;1)$	k=2 Supply Source Congestion = $\sum_{t'=t-8}^{t'=t} {}_{t'}N_p(t'+8;2)$	S
0	0	1.00
60	30	0.95
80	40	0.80
90 and more systems	60 and more systems	0.60

Output Tables and Conclusions

In the appendix is given the manual calculations involved in following precisely the flow charts for five complete cycles of time. On the following pages is recorded the output of the subroutines in the order in which they would occur in this example during those five cycles. Some of the output has been graphed for purposes of discussion.

Several conclusions about the illustrative freight fleet can be drawn from these five cycles; to wit:

(1) As shown in Figure 55, the number of shortages is large and continually occur. The maximum number of shortages possible at any time is equal to the number of reliable systems required. The difference between the actual and the maximum curves in Figure 55 is equal to the total number of reliable systems at that time. If the number of required systems is based upon contracts acquired, it is evident that a sound management policy would be to increase W_p so that the number of items ordered would be larger and hence would aid in decreasing the number of shortages and attendant costs.

(2) Input Table VIII indicates that the suppliability of the designer-manufacturer combination selected is rather low for the number being ordered. A second source of supply should be selected because the suppliability resulting from two designer-

manufacturer combinations will be higher than from one combination and will be needed when larger number of systems are ordered.

(3) Figure 56 shows that the availability of the fleet during time interval (104) is zero; in other words, between time (103) and time (104) no reliable systems are available. Figure 54 indicates that the causes of the shortage are the short lifetime of the group (101,2) and the fact that all fleet from group (100,2) are in the repair facilities.

Analysis of the calculations from which Figure 54 was plotted indicates that the extremely short life of group (101,2) is caused by the high forced-attrition rate which resulted from the management decision expressed in flow chart FCNEG (Subroutine No. 1) for groups having $N_S \leq 10$. If further investigation were to confirm that this part of FCNEG does express correctly the informed judgement of management, a larger number of systems should be ordered for delivery at time (101).

Figure 53 shows that for group (100,2) $C_{0\ max}$ becomes approximately equal to C_D within three time intervals after the birth of that group. Thus repair will be required by all fleet systems from that group within those three intervals. The rate at which $C_{0\ max}$ and $C_{0\ min}$ decrease with time and the range of C_0 should be checked to insure that they coincide with actual experience.

Although simulation of the problem has been performed for only five cycles, each possible pathway through the flow charts has been used and found to be logical. Since the problem was used only to illustrate as simply as possible the basic theory developed previously in this monograph and was not developed to generate usable data, no further calculations to acquire output is deemed necessary.

TABLE XI

OUTPUT FOR t = 101, g = (100,2)

Output for t = 101	g = (100,2)	N_S = 50
$\bar{C}_D$ = 1380 mph	$C_{\theta max}$ = 1435 mph	$C_{\theta min}$ = 1250 mph
ψ_D = 1.000	Φ_D^{**} = 0	Φ_D^{*} = 0
P_D = 1.000	ψ_θ = 1.000	Φ_θ^{**} = 0
Φ_θ^{*} = 0	P_θ = 1.000	R = 1.000
U = 0	D_r = 0	D_m = 0
P_T = 1.000	N_{ie} = 0	N_{ir} = 0
N_{im} = 0	M_θ = 1.000	M_D = 1.000
N_{iE} = 1 system	N_{ri} = 0	N_{re} = 0
N_{mi} = 0	N_{me} = 0	$\$_T$ = $,041,000
N_{RR} = 50 systems	A = 1.000	U.C.A.= $80,820 per system
N_I = 49 systems	$N_{e;T}$ = 1 system	$\$_S$ = $1,200,000,000

TABLE XII

OUTPUT FOR t = 101, g = (109,1)

Output for t = 101	g = (109,1)	
[14] $\bar{C}_D$ = 1334 mph		
[14] N_p = 32 systems	[24] S = .740	[29] N_S = 24

TABLE XIII

OUTPUT FOR t = 101, g = (109,2)

Output for t = 101	g = (109,2)	
[14] $\bar{C}_D$ = 1534 mph N_p = 0	[24] S = .98	[29] N_S = 0

TABLE XIV

OUTPUT FOR t = 101, g = (101,2)

Output for t = 101		g = (101,2)		N_S = 5	
$\bar{C}_D$	= 1380 mph	$\bar{C}_{\theta max}$	= 1460 mph	$\bar{C}_{\theta min}$	= 1300 mph
ψ_D	= 1.000	Φ_D^{**}	= 0	Φ_D^{*}	= 0
P_D	= 1.000	ψ_θ	= 1460 mph	Φ_θ^{**}	= 0
Φ_θ^{*}	= 0	P_θ	= 1.000	R	= 1.000
U	= 0	D_r	= 0	D_m	= 0
P_r	= 1.000	N_i	= 0	N_i	= 0
N_{im}	= 0	M_θ	= 1.000	M_D	= 1.000
N_{iE}	= 1	N_{ri}	= 0	N_{re}	= 0
N_{mi}	= 0	N_{me}	= 0	$\$_T$	= $441,000
N_{RR}	= 5	A	= 1.000	U.C.A.	= $ 88,200 per system
N_I	= 4	$N_{e;T}$	= 1	$\$_S$	= $120,000,000

TABLE XV

OUTPUT FOR FLEET AT t = 101

A_F = 1.000	$\$_{T.F}$ = \$124,481,000	$U.C.A._F$ = \$225,529 per system

TABLE XVI

OUTPUT FOR t = 102, g = (100,2)

Output for t = 102	g = (100,2)	N_S = 50
$\bar{C}_D$ = 1380 mph	$\bar{C}_{\theta max}$ = 1410 mph	$C_{\theta min}$ = 1200 mph
ψ_D = 1.000	Φ_D^{**} = 0	Φ_D^{*} = 0
P_D = 1.000	ψ_θ = .880	Φ_θ^{**} = 0
Φ_θ^{*} = .120	P_θ = 1.000	R = .880
U = 0	D_r = .120	D_m = 0
P_T = 1.000	N_{ie} = 0	N_{ir} = 6
N_{im} = 0	M_θ = 1.000	M_D = 1.000
N_{iE} = 0	N_{ri} = 50	N_{re} = 0
N_{mi} = 0	N_{me} = 0	$\$_T$ = \$3,960,000
N_{RR} = 43	A = .860	U.C.A.= \$92,093 per system
N_I = 43	$N_{e;T}$ = 1	

TABLE XVII

OUTPUT FOR t = 102, g = (101,2)

Output for t = 102	g = (101,2)	N_S = 5
$\bar{C}_D$ = 1380	$\bar{C}_{\theta max}$ = 1435	$\bar{C}_{\theta min}$ = 1250
ψ_D = 1.000	Φ_D^{**} = 0	Φ_D^{*} = 0
P_D = 1.000	ψ_θ = 1.000	Φ_θ^{**} = 0
Φ_θ^{*} = 0	P_θ = 1.000	R = 1.000
U = 0	D_r = 0	D_m = 0
P_T = 1.000	N_{ie} = 0	N_{ir} = 0
N_{im} = 0	M_θ = 1.000	M_D = 1.000
N_{iE} = 4	N_{ri} = 0	N_{re} = 0
N_{mi} = 0	N_{me} = 0	$\$_T$ = $284,000
N_{RR} = 4	A = .800	U.C.A.= $ 71,000 per system
N_I = 0	$N_{e;T}$ = 5	F_g = 103
$_T\bar{A}$ = .900	$\$_S$ = 120,000,000	P_g = $395,613
	G.L.E.R. = $87,914 per system	

TABLE XVIII

OUTPUT FOR t = 102, g = (110,1)

$\bar{\bar{C}}_D$ = 1350 mph	N_p = 0	S = .760	N_S = 0

TABLE XIX

OUTPUT FOR t = 102, g = (110,2)

$\bar{\bar{C}}_D$ = 1552	N_p = 38 systems	S = .830	N_S = 32

TABLE XX

OUTPUT FOR FLEET AT t = 102

A_F = .855	$\$_{T;F}$ = $14,364,000	U.C.A. = $290,000 per system

TABLE XXI

OUTPUT FOR t = 103, g = (100,2)

Output for t = 103	g = (100,2)	N_S = 50
$\bar{\bar{C}}_D$ = 1380	$\bar{C}_{\theta max}$ = 1385	$\bar{C}_{\theta min}$ = 1150
ψ_D = 1.000	Φ_D^{**} = 0	Φ_D^{*} = 0
P_D = 1.000	ψ_θ = 0	Φ_θ^{**} = 0
Φ_θ^{*} = 1.000	P_θ = 1.000	R = 0
U = 0	D_r = 1.000	D_m = 0
P_T = 1.000	N_{ie} = 0	N_{ir} = 43
N_{im} = 0	M_θ = 1.000	M_D = 1.000
N_{iE} = 0	N_{ri} = 0	N_{re} = 0
N_{mi} = 0	N_{me} = 0	$\$_T$ = $3,480,000
N_{RR} = 0	A = 0	U.C.A. = ∞
N_I = 0	$N_{e;T}$ = 1	

TABLE XXII

OUTPUT FOR $t = 103$, $g = (111,1)$

$N_p = 52$	$S = .600$	$N_S = 31$

TABLE XXIII

OUTPUT FOR $t = 103$, $g = (111,2)$

$N_p = 0$	$S = .830$	$N_S = 0$

TABLE XXIV

OUTPUT FOR FLEET AT $t = 103$

$A_F = 0$	$\$_{T;F} = \$43,080,000$	$U.C.A._F = \infty$

TABLE XXV

OUTPUT FOR $t = 104$, $g = (100,2)$

Output for $t = 104$	$g = (100,2)$	$N_S = 50$
$\bar{C}_D = 1380$		
$R = 0$ $N_{ie} = 0$	$N_{ir} = 0$ $N_{im} = 0$	
$M_\theta = .994$	$M_D = 1.000$	$N_{iE} = 0$
$N_{ri} = 6$	$N_{re} = 0$	$N_{mi} = 0$
$N_{me} = 0$	$\$_T = 238{,}000$	$N_{RR} = 0$
$A = 0$	U.C.A.= ∞	$N_I = 6$
$N_{e;T} = 1$		

TABLE XXVI

OUTPUT FOR $t = 104$, $g = (112,1)$

$\bar{\bar{C}}_D = 1367$	$N_p = 52$	$S = .600$	$N_S = 34$

TABLE XXVII

OUTPUT FOR $t = 104$, $g = (112,2)$

$\bar{\bar{C}}_D = 1572$	$N_p = 0$	$S = .830$	$N_S = 0$

TABLE XXVIII

OUTPUT FOR t = 104, g = (104,1)

Output for t = 104	g = (104,1)	N_S = 48
$\bar{\bar{C}}_D$ = 1250	$\bar{\bar{C}}_{\theta max}$ = 1290	$\bar{\bar{C}}_{\theta min}$ = 1210
ψ_D = .978	Φ_D^{**} = 0	Φ_D^* = .020
P_D = .998	ψ_θ = 1.000	Φ_θ^{**} = 0
Φ_D^* = 0	P_θ = 1.000	R = .978
U = 0	D_r = 0	D_m = .020
P_T = .998	N_{ie} = 0	N_{ir} = 0
N_{im} = 1	M_θ = .994	M_D = 1.000
N_{iE} = 0	N_{ri} = 0	N_{re} = 0
N_{mi} = 0	N_{me} = 0	$\$_T$ = \$140,000
N_{RR} = 47	A = .979	U.C.A.= \$ 29,787
N_I = 47	$N_{e;T}$ = 0	

TABLE XXIX

OUTPUT FOR FLEET AT t = 104

A_F = .480	$\$_{T;F}$ = \$28,962,000	U.C.A.$_F$ = \$616,213 per system

TABLE XXX

OUTPUT FOR t = 105, g = (100,2)

Output for t = 105	g = (100,2)	N_S = 50
$\bar{\bar{C}}_D$ = 1380	$\bar{\bar{C}}_{\theta max}$ = 1450	$\bar{\bar{C}}_{\theta min}$ = 1230
ψ_D = 1.000	Φ_D^{**} = 0	Φ_D^{*} = 0
P_D = 1.000	ψ_θ = .991	Φ_θ^{**} = 0
Φ_θ^{*} = .010	P_θ = 1.001	R = .991
U = 0	D_r = .010	D_m = 0
P_T = 1.001	N_{ie} = 0	N_{ir} = 0
N_{im} = 0	M_θ = .994	M_D = 1.000
N_{iE} = 6	N_{ri} = 43	N_{re} = 0
N_{mi} = 0	N_{me} = 0	$\$_T$ = 1,390,000
N_{RR} = 6	A = .120	U.C.A.= 231,667
N_I = 43	$N_{e;T}$ = 7	

TABLE XXXI

OUTPUT FOR t = 105, g = (104,1)

Output for t = 105	g = (104,1)	N_S = 48
$\bar{\bar{C}}_D$ = 1250	$\bar{\bar{C}}_{\theta max}$ = 1255	$\bar{\bar{C}}_{\theta min}$ = 1140
Ψ_D = .962	Φ_D^{**} = 0	Φ_D^{*} = .037
P_D = .999	Ψ_θ = 1.000	Φ_θ^{**} = 0
Φ_θ^{*} = 0	P_θ = 1.000	R = .962
U = 0	D_r = 0	D_m = .037
P_T = .999	N_{ie} = 0	N_{ir} = 0
N_{im} = 2	M_θ = .949	M_D = 1.000
N_{iE} = 0	N_{ri} = 0	N_{re} = 0
N_{mi} = 0	N_{me} = 0	$\$_T$ = $3,035,000
N_{RR} = 45	A = .938	U.C.A.= $ 67,444
N_I = 45	$N_{e;T}$ = 0	

TABLE XXXII

OUTPUT FOR t = 105, g = (113,1)

$\bar{\bar{C}}_D$ = 1400	N_p = 52	S = .600	N_S = 31

TABLE XXXIII

OUTPUT FOR t = 105, g = (113,2)

$\bar{\bar{C}}_D$ = 1610	N_p = 0	S = .830	N_S = 0

TABLE XXXIV

OUTPUT FOR FLEET AT t = 105

A_F = .520	$\$_{T;F}$ = $39,185,000	$U.C.A._F$ = $768,333

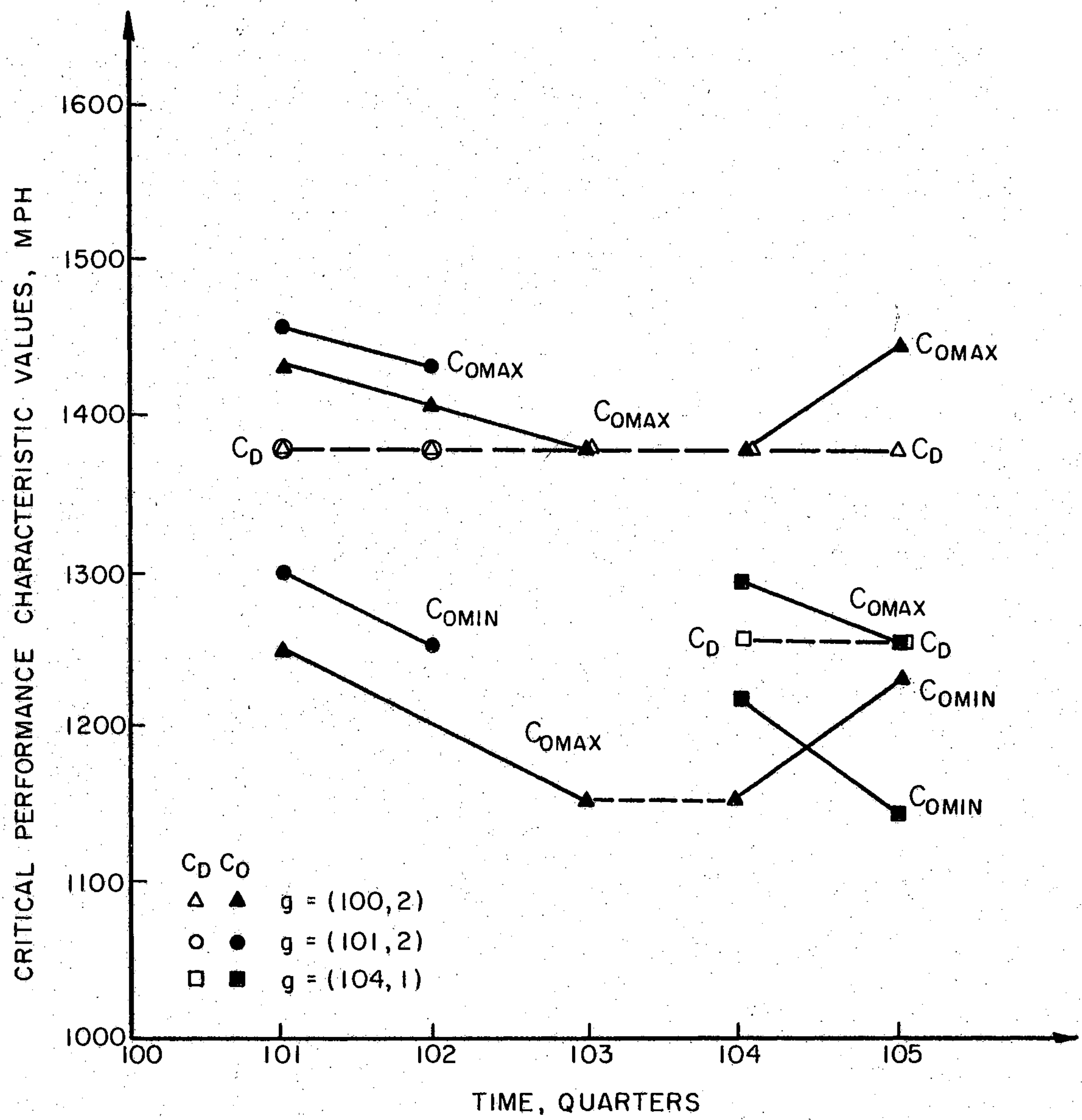


Figure 53. Design and Operating Values of the Critical Performance Characteristic Versus Time

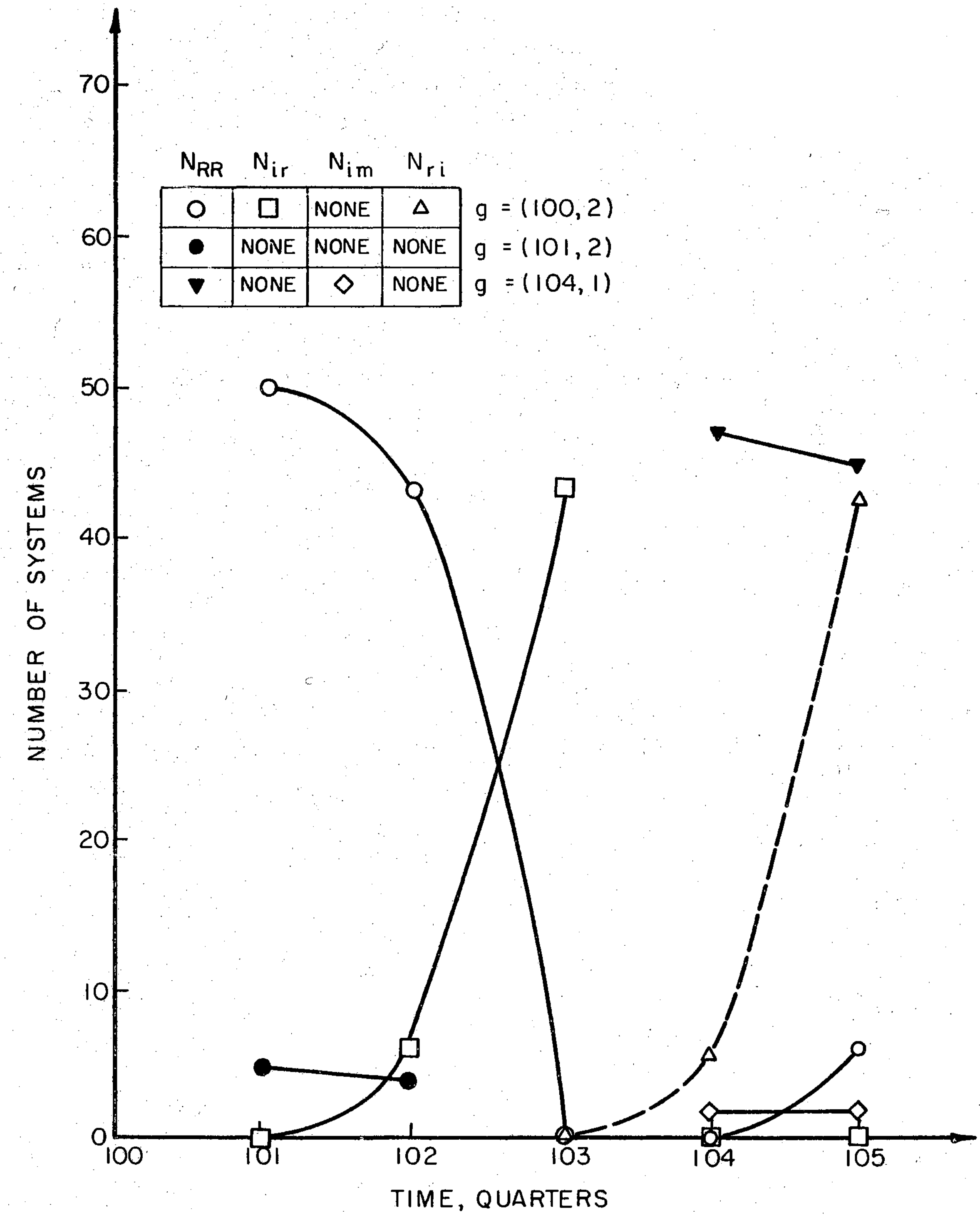


Figure 54. Fleet Activities of Each Group Versus Time

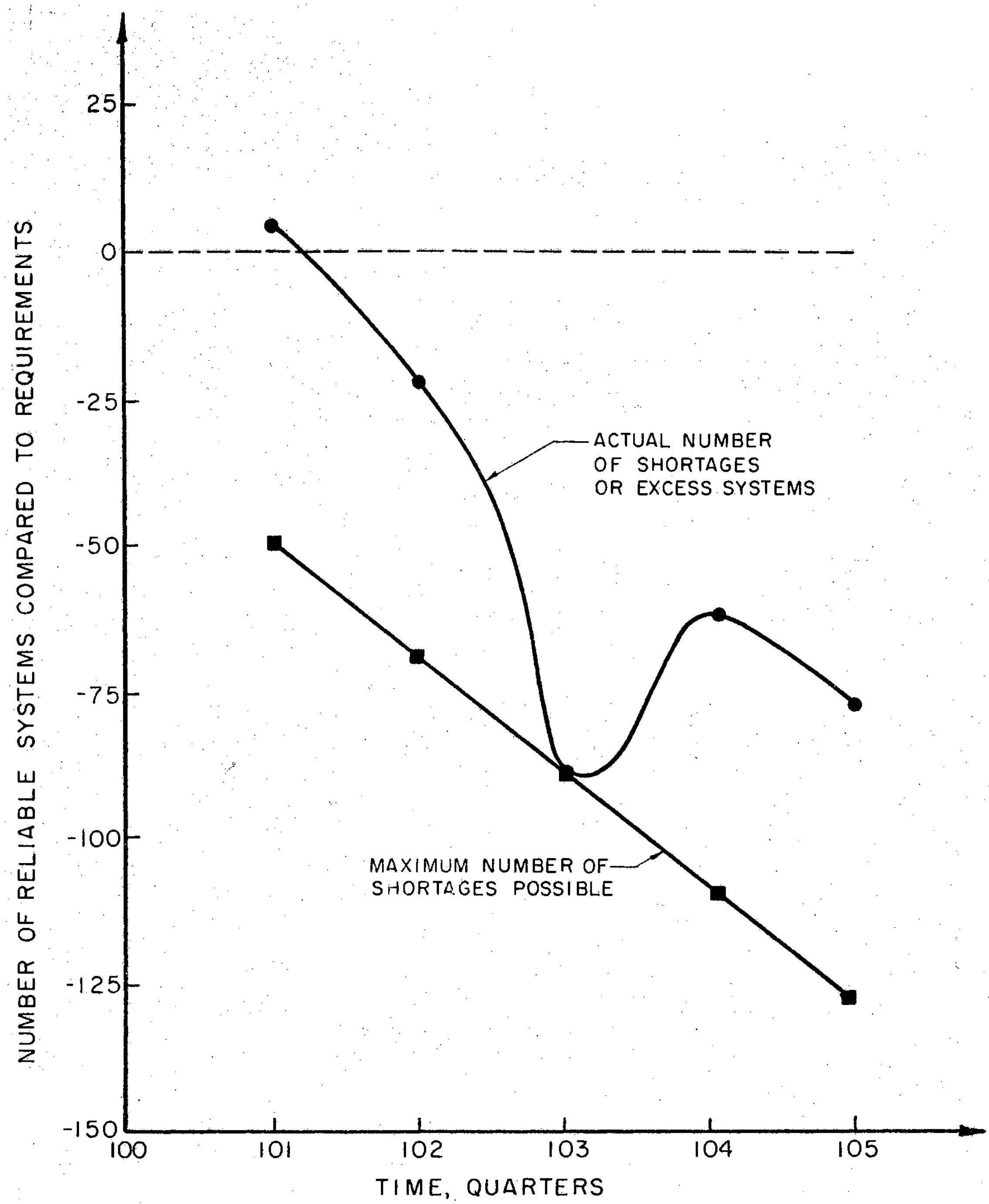


Figure 55. Number of Reliable Systems Compared to Requirements Versus Time

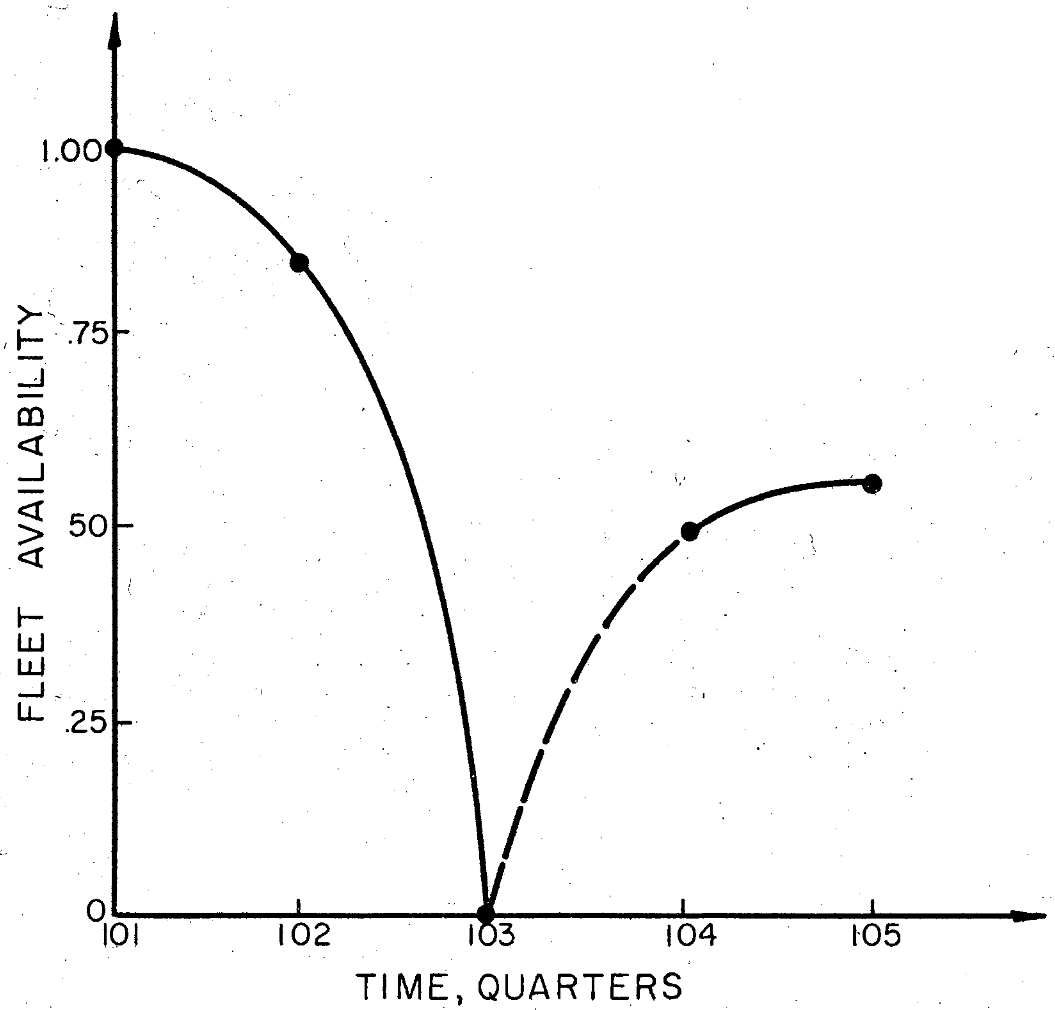


Figure 56. Fleet Availability Versus Time

CHAPTER IX

SUMMARY, CONCLUSIONS, AND SUGGESTIONS FOR FUTURE RESEARCH

Summary

The purpose of this study was to develop a model of the relationships among time-dependent job requirements for a fleet, the design and operating values of the management-designated critical performance characteristic of the heterogeneous groups composing the fleet, the types of maintenance required and the costs involved.

Briefly defined, a fleet is a "mixture" of components, whereas a system is a "compound" of components. The components of a fleet are systems. Examples of fleet systems are capital equipment items, divisions of companies, monies, and engineers; the term is very broad.

The assumptions made in the study were:

(1) Jobs for a fleet can be expressed in terms of the number of reliable systems required and the range of required values for the critical characteristic.

(2) Estimates of future values which will be required for the critical characteristic cannot be made accurately.

(3) System design values will become obsolete but modification will delay the occurrence.

(4) System operating values decay with time but repair will partially restore the ability of the system to operate reliability.

(5) The amount of decay which occurs per unit of time is uncertain.

(6) Systems may be forced out of the fleet by arbitrary action, by deliberate action and by becoming unreliable.

(7) The actions of one system has no effect on the actions of another system unless it increases congestion in a maintenance facility.

(8) The number of groups in the fleet can change with time.

(9) Systems which do not complete maintenance during the allocated lead time are exited from the fleet.

(10) Those parts of supply orders which are incomplete at the end of the allocated lead time never enter the fleet. No penalties are imposed upon the suppliers.

(11) Systems can be bought with design values which are standard or better.

(12) Management (a) estimates the future job requirements for the fleet, the effects of congestion in the maintenance and supply facilities, and the effects of time and maintenance on the operating

and design values of the critical performance characteristic of the systems of the fleet, (b) determines the policies under which systems will be bought and scrapped, and (c) selects four values which express management's beliefs regarding reliability of design and operation of a system.

Definitions of frequently used symbols are these:

"${}_tA_g$", the group availability, is the percentage of the time interval "t" that the group "g" is able to perform the job assigned.

"${}_tR_g$, the group reliability, is the probability that the fleet inventory members from group "g" will be able to perform the job assigned when operated under the expected conditions during interval "t".

"${}_tD_{rg}$", the group repair dubiety, is the probability that the fleet inventory members from group "g" will be sent to the repair facility during "t".

"${}_tD_{mg}$", the group modification dubiety, is the probability that the fleet inventory members from group "g" will be sent to the modification facility during "t".

"${}_tU_g$", the group unreliability, is the probability that they will be sent to the scrap pile.

"${}_tM_{Og}$", the group operation maintainability, is the probability that the whole batch of "g" systems which had been sent to the repair facility "L_r"

time intervals prior to "t" will complete repair and return to the fleet inventory by time "t".

"$_tM_{Dg}$", the group design maintainability, is the probability that the whole batch of "g" systems which had been sent to the modification facility "L" time intervals prior to "t" will complete modification and return to the fleet inventory by time "t".

Conclusions

Four measures of fleet effectiveness exist:

(1) the availability of the fleet at any time "t" within the planning horizon,

(2) the shortage of reliable systems over the planning horizon,

(3) the cost per reliable system in the fleet at any time "t" within the planning horizon, and

(4) the present worth of the cost of obtaining and sustaining the fleet inventory.

Evaluated at the start of the planning horizon, the present worth of all future payments made over the planning horizon to operate, exit, repair and modify the present and future fleet groups, to buy future groups, and to cover the costs of not having sufficient numbers of reliable systems available during some time intervals can be calculated by simulation. Currently simulation is the only means for studying the effects of management decisions upon the four measures of fleet effectiveness.

Evaluated at the proposed birthdate of the future group, the present worth of all future payments and purchase price made to buy, operate, repair, modify and exit systems of that group throughout its lifetime can be calculated so that "best-buy" comparisons may be made before a group is actually bought. The lifetimes of the groups are not estimated by management. Instead, the "death-date" of a group is the result of the impact of the management decision factors upon the reliability of design and operation of the systems in the group.

The payment which must be made at time "t" for activities of a group during the previous time interval is a function of the unit costs, the maintenance lead times, and the following ratios:

$$\left[\frac{\left({}_tA_g\right)\left({}_tU_g\right)}{{}_tR_g}\right], \left[\frac{\left({}_{t-L_r}A_g\right)\left({}_{t-L_r}D_r\right)\left({}_tM_{Og}\right)}{{}_{t-L_r}R_g}\right], \left[\frac{\left({}_{t-L_m}A_g\right)\left({}_{t-L_m}D_m \; {}_tD_{Dg}\right)}{{}_{t-L_m}R_g}\right],$$

$$\text{and} \qquad \left[\frac{{}_tN_{iEg}}{N_{Sg}}\right].$$

Group reliability, unreliability, repair dubiety, and modification dubiety can be expressed as functions of the following: (1) the current (original and/or modified) values of the design of its systems which are in the fleet inventory, (2) the current (original, decayed and/or repaired) operating values of the critical performance characteristic of its systems which are in the fleet inventory, (3) the frequency distribution of the job-requirement values of the critical characteristic, and (4) the frequency distribution of the operating value of the critical characteristic.

The proportions of group "g" systems which are in the fleet inventory (are operatable) at time "(t-1)" and which will become unreliable, or show need of repair, or show need of modification, or remain reliable during the time interval from "(t-1)" to "(t)" are equal to, respectively, ${}_tU_g$, ${}_tD_{rg}$, ${}_tD_{mg}$ and ${}_tR_g$.

The proportion of the original number of systems in group "g" which will be reliable and hence available during time interval "t" to help do the job of the fleet is equal to ${}_tA_g$. ${}_tA_g$ is found by use of a recursive equation which involves past values of A_g, U_g, D_{rg}, D_{mg}, M_{Og}, M_{Dg} and N_{iEg} and the present value of ${}_tR_g$.

Suggestions for Future Research

Lead Times:

Allowed lead times are determined by the expected total environment in which the fleet inventory operates. The event of war and other catastrophes shortens allowed lead times. The effects of sudden shortening of lead time should be investigated.

Renegging:

Calculations of the sizes of the batches leaving the fleet inventory for the maintenance facilities and the scrap pile and the size of the batch which helps to satisfy the job specifications are based upon the assumption that the true state of each system in the fleet can be determined and that all systems found to be in a given state will be allowed to go to the appropriate facility (scrap pile, fleet inventory, maintenance facilities). The effects of other assumptions, such as periodic, random and continual, and forced renegging, should be investigated.

Number of Required Systems:

The forecasted numbers of required systems over the planning horizon were assumed to be deterministic. This assumption should be lifted.

Multi-Maintenance Facilities:

At any time there may be several repair-maintenance facilities and several modification-maintenance facilities to which systems of dubious worth could be sent for the appropriate maintenance. Each facility of a stated maintenance type may have a Congestion versus Maintenance relationship and a set of associated costs which differ from those of the other facilities of the same type. A method for evaluating the facilities and optimizing the dispersal of batches among these facilities.

Backlogs:

In the actual operation of a fleet those systems which were backlogged in the past in the maintenance or supply facilities by not being completed within the allowed lead times will help to compose every batch which leaves those facilities and enters the fleet inventory for some number of time periods after their expected completion dates. These systems will differ from their on-time predecessors only in having a slightly later re-entry or birthdate and not in operating or design values. Effects of maintenance and supply backlogging should be investigated.

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APPENDIX

MANUAL SIMULATION OF THE PROBLEM FOR FIVE CYCLES

Start

Sub.Routine	$t = 101 \quad g = (100,2) \quad N_S = 50$
# 4	$100 \overset{?}{\neq} 101$ no; then
	(2.1) $\bar{C}_D = \dfrac{[(1.15)(1200)][50] + [80][0] + [1380 + 120][0]}{50 + 0 + 0}$
	$= 1380$ mph
	(2.2) $C_{\theta max} = \dfrac{[(1.15)(1200) + 80 - 25][50] + [1460 + 20][0]}{50 + 0 + 0}$
	$+ \dfrac{+ [1460 + 150][0]}{50 + 0 + 0}$
	$= 1460 - 25 = 1435$ mph
	(2.3) $C_{\theta min} = \dfrac{[1300 - 50][50] + [1300 + 40][0]}{50 + 0 + 0}$
	$+ \dfrac{+ [1300 + 100][0]}{50 + 0 + 0}$
	$+ 1250$ mph
# 5 a	$x_{max} = 1200$ mph
	$x_{min} = 700$ mph
	$[(f_a)]({}_t\bar{C}_{Dg}) - {}_tC_{Rmin}] = [(.95)(1380) - 700] = +$; then
	$[C_{Rmax} - (f_a)(\bar{C}_D)] = 1200 - 1311 = -$; then
	set $(f_a)(\bar{C}_D) = 1200$ mph
	$x_2 = 1200$ mph;
	$x_1 = 700 = 700$ mph;
26	$x_{min} - x_1 = 700 - 700 = 0$; then
	$x_{max} - x_2 = 1200 - 1200 = 0$; then $z = 1.000$
	$\psi_D = 1.000$

\# 5 a $[C_{Rmax} - (f_b)({}_t\bar{C}_{Dg})] = [1200 - (1.05)(1380)] = -;$

then set $(f_b)({}_t\bar{C}_{Dg}) = 1200$ mph

$\Phi_D^{**} = 0$

$x_2 = 1200$

$x_1 = 1200$

$x_2 - x_1 = 1200 - 1200 = 0;$ then

$\Phi_D^{*} = 0$

	$t = 101$ $g = (100,2)$ $N_S = 100$

\# 5 a $P_D = \Psi_D + \Phi_D^{**} + \Phi_D^{*} = 1.000$

$x_{max} = 1435$ mph

$x_{min} = 1250$ mph

$[C_{\theta max} - (f_2)({}_t\bar{C}_D)] = [1435 - (.90)(1380)]$

$= 1435 - 1242 = +;$ then

$[C_{\theta min} - (f_2)(\bar{C}_D)] = 1250 - 1242 = +;$ then

Set $(f_2)(C_D) = 1250$ mph

$x_2 = 1435$ mph

$x_1 = 1250$ mph

\#26 $x_{min} - x_1 = 1250 - 1250 = 0;$ then

$x_{max} - x_2 = 1435 - 1435 = 0;$ then $z = 1.000$

$\Psi_\theta = 1.000$

\# 5 $[(f_3)(\bar{C}_D) - \bar{C}_{\theta min}] = [(.60)(1380) - 1250]$

$= 828 - 1250 = -;$ then

set $(f_3)(\bar{C}_D) = 1250$ mph

$\Phi_{\theta\theta}^{**} = 0$

$x_2 = 1250$ mph

$x_1 = 1250$ mph

#26 $x_{min} - x_1 = 1250 - 1250 = 0$; then

$x_{max} - x_2 = 1435 - 1250 = +$; then

$$x_C = \left[1250 + \left(\frac{1435 - 1250}{3}\right)\right] = 1250 + 62 = 1312 \text{ mph}$$

$x_2 - x_C = 1250 - 1312 = -$; then

$$z = \frac{1250 - 1250}{1435 - 1250} \quad \frac{1250 + 1250 - 2(1250)}{1312 - 1250} = 0$$

$\Phi^*_\theta = 0$

---

5 b $P_\theta = \psi_\theta + \Phi^{**}_\theta + \Phi^*_\theta = 1.000$

(3.13) $R = \psi_\theta \psi_D = (1.000)(1.000) = 1.000$

(3.14) $U = \Phi^{**}_\theta \; \theta^{**}_D = (0)(0) = 0$

(3.15) $D_r = (\psi_D)(1 - \psi_\theta) = (1.000)(0) = 0$

(3.16) $D_m = (1 - \psi_D)(1 - \Phi^{**}_\theta) + (\Phi^*_D)(\Phi^{**}_\theta)$

$= (0)(1) + (0)(0) = 0$

$P_T = R + D_r + D_m + U = 1.000$

$t = 101$ $g = (100,2)$ $N_S = 50$

---

6 (5.1) $N_{ie} = (50)(0) = 0$

(5.2) $N_{ir} = (50)(0) = 0$

(5.3) $N_{im} = (50)(0) = 0$

---

7 Congestion $= {}_{101-4}N_{ir} + {}_{101-3}N_{ir} + {}_{101-2}N_{ir}$

$= 0 + 0 + 0 = 0$

Input Table V

$M_\theta = 1.000$

# 8	Congestion $= {}_{101-8}N_{im} + {}_{101-7}N_{im} + {}_{101-6}N_{im}$ $+ {}_{101-5}N_{im} + {}_{101-4}N_{im}$ $= 0 + 0 + 0 + 0 + 0 = 0$ Input Table VI $M_D = 1.000$
# 1	${}_{101-1}N_I - 10 = 50 - 10 = +$; then $N_{iE} = (.010)(50) = 0.5 \Rightarrow 1$
#10	(5.4) $N_{ri} = ({}_{101-2}N_{ir})(M_\theta) = (0)(1.000) = 0$ (5.6) $N_{re} = ({}_{101-2}N_{ir})(1 - M_\theta) = (0)(0) = 0$ (5.5) $N_{mi} = ({}_{101-4}N_{im})(M_D) = (0)(1.000) = 0$ (5.7) $N_{me} = ({}_{101-4}N_{im})(1 - M_D) = (0)(0) = 0$ (6.1) $\$_I = (50)(10{,}000 + 70{,}000) + 40{,}000$ $= \$4{,}040{,}000$ (6.2) $\$_r = [(4000)(2) + 10{,}000][0] = 0$ (6.3) $\$_m = [(4000)(4) + 20{,}000][0] = 0$ (6.5) $\$_{e;E} = [1000 - 0]\ [1] = \$1{,}000$ (6.8) $\$_T = 4{,}040{,}000 + 0 + 0 + 1{,}000 = \$4{,}041{,}000$
# 9	(5.12) $N_{RRg} = [50][1.000] = 50$ (7.1) $A = \frac{50}{50} = 1.000$
#12	(7.8) U.C.A. = \$4,041,000/50 = \$80,820/system
# 2	(5.14) $N_I = 50 - 0 - 0 - 0 - 1 + 0 + 0$ $= 49$ systems

# 3	(5.9) $N_{e;T} = 0 + 0 + 1 + 0 + 0 = 1$ system
Test $\#_{1a}$	$N_S - N_{e;T} = 50 - 1 = +$; then
Test $\#_{2a}$	$k \overset{?}{=} 2$ yes; then
Test $\#_3$	$(t - 1 - j) = (101 - 1 - 100) = 0$; then
#13	$j = J_A = 100$; Set $ACC_A = 0$; Set $k = 1$ $N_{S(100,1)} \overset{?}{=} 0$; yes; then $k \overset{?}{=} 2$ No; then set $k = 2$ $N_S = 50$; $N_S \overset{?}{=} 0$ No; then $[N_S - {}_{100-1}N_{e;T}] = 50 - 0 = +$; then $ACC_A = 0 + N_{RR} = 0 + 50 = 50$ $100 \overset{?}{=} (101 - 1)$ Yes; then ${}_{101+8}N_R - ACC_A = 210 - 50 = +$; then (5.14) $N_p = [{}_{101+8}N_R - ACC_A]W_p = [210 - 50][.20]$ $= 32$ systems
#14	${}_t\bar{C}_{D(100,1)} = (1.00)({}_{101+8}C_{Rmax}) = 1334$ mph $t_1 = 131^+$ quarter $\bar{C}_{D(100,2)} = (1.15)({}_{101+8}C_{Rmax}) = 1534$ mph $t_2 = 143^+$ quarter $VALUK = \left[\frac{143 - 131}{131 - 101}\right]\left[\frac{18,000,000}{24,000,000}\right] = .30$ $[VALUK - 1.000] = -$; then New $g = (101,1)$ ${}_{101}N_{p(101,1)} = 32$; ${}_{101}N_{p(101,2)} = 0$
# 24	Set $k = 1$

Supply Facility Congestion$_{(109,1)}$ =

$$= \sum_{t'=93}^{t'=101} {}_{t'}N_{p(t'+8,1)} = 82 \quad \text{(Table X)}$$

$S_{(109,1)} = .740$; Set k = 2 ; S.F.Congestion = 10

$S_{(109,2)} = .98$

#25 Set k = 1

(5.17) $N_{S(109,1)} = [{}_{101}N_{p(109,1)}][S_{(109,1)}]$

$= (32)(.74) = 24$ systems

$k \overset{?}{=} 2$ No ; Set k = 2 ;

(5.17) $N_{S(109,2)} = [{}_{101}N_{p(109,2)}][S_{(109,2)}]$

$= [0][.98] = 0$

Test #4 a $[H - t] = [140 - 101] = 39$

Set j = 101

Test #5 $N_{S(101,1)} \overset{?}{=} 0$ Yes ; then Set k = 2

	$t = 101 \overset{?}{=} 101$ Yes $g = (101,2)$ $N_S = 5$

---

4 $\bar{C}_D$ $101 \overset{?}{=} 101$ Yes ; then

$\bar{C}_D = 1.15$ $C_{Rmax} = (1.15)(1200) = 1380$ mph

$\bar{C}_{\theta max} = (1.15)(1200) + 80 = 1460$ mph

$\bar{C}_{\theta min} = (1.15)(1200) - 80 = 1300$ mph

---

5 $x_{max} = 1200$ mph

$x_{min} = 700$ mph

$[(f_a)({}_t\bar{C}_{Dg}) - {}_tC_{Rmin}] = [(.95)(1380) - 700] = +;$

then

$[C_{Rmax} - (f_a)({}_tC_{Dg})] = [1200 - (.95)(1380)]$

$= [1200 - 1311] = -;$ then

Set $(f_a)({}_tC_{Dg}) = 1200$ mph

$x_2 = 1200$ mph

$x_1 = 700$ mph

26 $x_{min} - x_1 = 700 - 700 = 0;$ then

$x_{max} - x_2 = 1200 - 1200 = 0.$ Then $z = 1.000$

5 $\psi_D = 1.000$

	t = 101 g = (101,2) $N_S = 5$

5 $\psi_D = 1.000$

$[({}_tC_{Rmax}) - (f_b)({}_tC_{Dg})] = [1200 - (1.09)(1380)]$

$= -;$ then

Set $(f_b)({}_t\bar{C}_{Dg}) = 1200$ mph

$\Phi_D^{**} = 0$

$x_2 = 1200$

$x_1 = 1200$

$x_2 - x_1 = 1200 - 1200 = 0;$ then

$\Phi_D^{*} = 0$

---

5 a $P_D = 1.000$

$x_{max} = 1460$ mph

$x_{min} = 1300$

$[\bar{C}_{\theta max} - (f_2)(\bar{C}_D)] = [1460 - (.90)(1380)]$

$= [1460 - 1242] = +;$ then

$[C_{\theta min} - (f_2)(\bar{C}_D)] = [1300 - 1242] = +$; then

Set $(f_2)(C_D) = 1300$ mph

$x_2 = 1460$ mph

$x_1 = 1300$ mph

#26 $[x_{min} - x_1] = 1300 - 1300 = 0$; then

$[x_{max} - x_2] = 1460 - 1460 = 0$; then $z = 1.000$

5 $\psi_\theta = 1.000$

$[(f_3)(\bar{C}_D) - C_{\theta min}] = [(.60)(1380) - 1300]$

$= [828 - 1300] = -$; then

Set $(f_3)(\bar{C}_D) = 1300$ mph

$\Phi_\theta^{**} = 0$

$x_2 = 1300$

$x_1 = 1300$

#26 $x_{min} - x_1 = 1300 - 1300 = 0$; then

$x_{max} - x_2 = 1460 - 1300 = +$

$x_c = \left[1300 + \left(\frac{1460 - 1300}{3}\right)\right] = 1353$ mph

$[x_2 - x_c] = 1300 - 1353 = -$; then

$z = \left[\frac{1300 - 1300}{1460 - 1300}\right]\left[\frac{1300 + 1300 - 2(1300)}{1353 - 1300}\right] = 0$

5 $\Phi_\theta^{*} = 0$

---

b $P_\theta = 1.000$

$R = 1.000$ $D_m = 0$ $D_r = 0$

$D_m = 0$

---

6 $N_{ie} = (5)(0) = 0$

$N_{ir} = (5)(0) = 0$

$N_{im} = (5)(0) = 0$

# 7	Congestion = $\sum_{t'=101-4}^{t'=101-2} {}_{t'}N_{ir} = 0$ $M_\theta = 1.000$
# 8	Congestion = $\sum_{t'=101-8}^{t'=101-4} {}_{t'}N_{im} = 0$ $M_D = 1.000$
# 1	$N_S - 10 = -$ $[{}_{101-1}N_I - 4] = 5 - 4 = +$; then $N_{iE} = (0.20)(5) = 1$
# 10	$N_{ri} = ({}_{101-2}N_{ir})(M_\theta) = (0)(1.000) = 0$ $N_{re} = ({}_{101-2}N_{ir})(1 - M_\theta) = (0)(0) = 0$ $N_{mi} = {}_{101-4}N_{im})(M_D) = (0)(1.000) = 0$ $N_{me} = ({}_{101-4}N_{im})(1 - M_D) = (0)(0) = 0$ $\$_I = (5)(10,000 + 70,000) + 40,000 = \$440,000$ $\$_r = [(4000)(2) + 10,000][0] = 0$ $\$_m = [(4000)(4) + 20,000][0] = 0$ $\$_{e;E} = [1000 - 0][\$] = \$1,000$ $\$_T = \$441,000$
# 9	$N_{RR} = [5][1.000] = 5$ $A = \frac{5}{5} = 1.000$
# 12	U.C.A. = 441,000/5 = \$88,200/system
# 2	$N_I = 5 - 0 - 0 - 0 - 1 + 0 + 0 = 4$

# 3	$N_{E;T} = 0 + 0 + 1 + 0 + 0 = 1$
Test # 1 a	$[N_S - N_{E;T}] = 5 - 1 = +$; then
Test # 2 a	$k \stackrel{?}{=} 2$ Yes; then
	$t - 1 - j = 101 - 1 - 101 = -$; then
#15	Set $ACC_A = 0$; Set $ACC_B = 0$; Set $j = 100$;
	Set $k = 1$
	$N_{S(100,1)} \stackrel{?}{=} 0$ Yes; then
	$k = 2$? No; then $k = 1 + 1 = 2$
	$N_{S(100,2)} = 0$? No; then
	$[N_{S(100,2)} - {}_{101}N_{e;T(100,2)}] = 50 - 1 = +$; then
	$ACC_A = 0 + 50 = 50$
	$ACC_B = 0 + 50 = 50$
	$100 = 101$? No; then $j = 100 + 1 = 101$;
	Set $k = 1$;
	$N_{S(101,1)} = 0$? Yes; then
	$k = 2$? No; then $k = 1 + 1 = 2$
	$N_{S(101,2)} = 0$? No; then
	$[N_{S(101,2)} - {}_{101}N_{e;T(101,2)}] = 5 - 1 = +$
	$ACC_A = 50 + {}_{101}N_{RR(101,2)} = 50 + 5 = 55$
	$ACC_B = 50 + 5 = 55$
	$(j \stackrel{?}{=} t) = (101 \stackrel{?}{=} 101)$ Yes; then
	(7.4) ${}_{101}A_F = \frac{55}{55} = 1.000$

	$t = 101$ Fleet

#16 Set $j = 100$; Set $ACC_C = 0$; Set $k = 1$

$N_{S(100,1)} = 0$? Yes; then

$k = 2$? No; then $k = 1 + 1 = 2$

$N_{S(100,2)} = 0$? No ; then

$[N_{S(100,2)} - {}_{101}N_{e;T(100,2)}] = 50 - 1 = +$; then

$ACC_C = 0 + 4{,}041{,}000 = \$4{,}041{,}000$

$(j = i) = (100 = 101\ ?)$ No; then $j = 100 + 1 = 101$;

Set $k = 1$

$N_{S(101,1)} = 0$? Yes ; then

$k = 2$? No ; then $k = 1 + 1 = 2$

$N_{S(101,2)} = 0$? No ; then

$[N_{S(101,2)} - {}_{101}N_{e;T(101,2)}] = 5 - 1 = +$; then

$ACC_C = \$4{,}041{,}000 + 440{,}000 = \$4{,}481{,}000$

$(j = t) = (101 = 101\ ?)$ Yes; then

(6.6) ${}_{101}\$_{S(101,1)} = [18{,}000{,}000 + 0][0] + 0 = 0$

(6.6) ${}_{101}\$_{S(101,2)} = [24{,}000{,}000 + 0][5] + 0$

$= \$120{,}000{,}000$

$[{}_{101}N_R - {}_{101}ACC_A] = [50 - 55] = -$; then

set $[{}_{101}N_R - {}_{101}ACC_A] = 0$

(6.9) $${}_{101}\$_{T;F} = {}_{101}\$_{S(101,1)} + {}_{101}\$_{S(101,2)} + \sum_{k=1}^{k=2} \sum_{g=(100;k)}^{g=(100;k)} {}_{101}\$_{Tg}$$

$$+ [{}_{101}¢_{Sh}] \left[{}_{101}N_R - \sum_{k=1}^{k=2} \sum_{g=(100;k)}^{g=(101;k)} {}_{101}N_{RRg} \right]$$

$${}_{101}\$_{T;F} = 0 + 120{,}000{,}000 + 0 + 4{,}481{,}000$$

$$+ [440{,}000][50 - 55]$$

$$= 120{,}000{,}000 + 4{,}481{,}000 + 0$$

$$\$124{,}481{,}000$$

#18 Set $j = 100$; Set $ACC_A = 0$; Set $j = 100$;

Set $k = 1$

$N_{S(100,1)} = 0$? Yes ; then

$k = 2$? No; then $k = 1 + 1 = 2$

$N_{S(100,2)} = 0$? No; then

$[N_{S(100,2)} - {}_{101}N_{e;T(100,2)}] = 50 - 1 = +$; then

$ACC_A = 0 + 50 = 50$

$(j = t\ ?) = (100 = 101\ ?)$ No; then $j = 100+1 = 101$

Set $k = 1$

$N_{S(101,1)} = 0$? Yes; then

$k = 2$? No; then $k = 1 + 1 = 2$

$N_{S(101,2)} = 0$? No; then

$[N_{S(101,2)} - {}_{101}N_{e;T(101,2)}] = 5 - 1 = +$; then

$ACC_A = 50 + 5 = 55$

$(j = t\ ?) = (101 = 101\ ?)$ Yes; then

(7.10) $${}_{101}U.C.A._F = {}_{101}\$_{T;F}/ACC_A = \frac{124{,}041{,}000}{55}$$

$$= \$225{,}529 \text{ per system}$$

Test # 4 b $[H - t] = [140 - 101] = +$; then

$i = 101 + 1 = 102$

Set $j = 100$; Set $k = 1$

$N_{S(100,1)} = 0$? Yes; then $k = 2$?; No; then

$k = 1 + 1 = 2$

$N_{S(100,2)} = 0$? No; then

$[N_{S(100,2)} - {}_{102-1}N_{e;Tg}] = [50 - 1] = +$; then

	$t = 102$ $\quad g = (100,2)$ $\quad N_S = 50$

\# 4 $[j = i\ ?] = [100 = 102\ ?]$ No; then

(2.1) $\bar{C}_D = \dfrac{[1380][49] + [1380][0] + 1380][0]}{50 + 0 + 0 - 1}$

$= 1380$ mph

(2.2) $C_{\theta max} = \dfrac{[(1435)-25][49]+[1480][0]+[1460+150][0]}{50 + 0 + 0 - 1}$

$= 1410$ mph

(2.3) $C_{\theta min} = \dfrac{[1250-50][49]+[1300+40][0]+[1300+100][0]}{50 + 0 + 0 - 1}$

$= 1200$ mph

\# 5 $x_{max} = 1217$

$x_{min} = 717$

$[(f_a)(\bar{C}_D) - C_{Rmin}] = [(.95)(1380) - 717] = +$; then

$[C_{Rmax} - (f_a)(\bar{C}_D)] = [1217 - 1312] = -$; then

Set $[(f_a)(\bar{C}_D)] = 1217$

$x_2 = 1217$

$x_1 = 717$

\#26 $x_{min} - x_1 = 717 - 717 = 0$; then

$x_{max} - x_2 = 1217 - 1217 = 0$; then $z = 1.000$

\# 5 $\psi_D = 1.000$

$[C_{Rmax} - (f_b)(\bar{C}_D)] = [1217 - (1.05)(1280)] = -$;

then Set $[(f_b)(\bar{C}_D)] = 1217$

$\Phi_D^{**} = 0$

$x_2 = 1217$

$x_1 = 1217$

$x_2 - x_1 = 1217 - 1217 = 0$; then

$\Phi^*_D = 0$

\# 5 a $P_D = 1.000 + 0 + 0 = 1.000$

$x_{max} = 1410$

$x_{min} = 1200$

$[C_{\theta max} - (f_2)(\bar{C}_D)] = [1410 - (.90)(1380)]$

$= [1410 - 1242] = +$; then

$[C_{\theta min} - (f_2)(\bar{C}_D)] = [1200 - 1242] = -$; then

$x_2 = 1410$

$x_1 = (.90)(1380) = 1242$

\#26 $[x_{min} - x_1] = [1200 - 1242] = -$; then

$x_c = 1200 + \dfrac{1410 - 1200}{3} = 1200 + 70 = 1270$

$[x_2 - x_c] = [1410 - 1270] = +$; then

$[x_1 - x_c] = [1242 - 1270] = -$; then

$$z = \left\{\frac{1}{1410 - 1200}\right\}\left\{\frac{[1242 - 1200]}{[1270 - 1200]}[1270 - 1242] + \frac{[1410 - 1410]}{[1410 - 1270]}[1410 - 1270] + [1410 - 1242]\right\}$$

$$= \left\{\frac{1}{210}\right\}\left\{\left[\frac{42}{70}\right][32] + 0 + 168\right\} = .880$$

\# 5 $\Psi_\theta = .890$

$[(f_3)(\bar{C}_D) - C_{\theta min}] = [(.60)(1380) - 1200]$

$= 828 - 1200 = -$; then

Set $[(f_3)(C_D)] = 1200$

$\Phi^{**}_\theta = 0$

$x_2 = 1242$

$x_1 = 1200$

#26 $[x_{min} - x_1] = [1200 - 1200] = 0$; then

$[x_{max} - x_2] = [1410 - 1242] = +$; then

$x_c = 1200 + \left[\frac{1410 - 1200}{3}\right] = 1270$

$[x_2 - x_c] = 1242 - 1270] = -$; then

$$z = \left[\frac{1242 - 1200}{1410 - 1200}\right] \left[\frac{1200 + 1242 - 2(1200)}{1270 - 1200}\right]$$

$$= \left[\frac{42}{210}\right] \left[\frac{42}{70}\right] = .120$$

$\Phi^*_\theta = .120$

b $P_\theta = .880 + 0 + .120 = 1.000$

$R = (1.000)(.880) = .880$

$U = (0)(0) = 0$

$D_r = [1.000][1 - .880] = .120$

$D_m = 0$

---

6 $N_{ie} = (50)(0) = 0$

$N_{ir} = (50)(.120) = 6$

$N_{im} = (50)(0) = 0$

---

7 Congestion $= {}_{102-4}N_{ir} + {}_{102-3}N_{ir} + {}_{102-2}N_{ir}$

$= 0 + 0 + 0 = 0$

$M_\theta = 1.000$

---

8 Congestion $= {}_{102-8}N_{im} + {}_{102-7}N_{im} + \cdots + {}_{102-5}N_{im}$

$+ {}_{102-4}N_{im} = 0$

$M_D = 1.000$

---

1 $[N_S - 10] = +$

$[{}_{102-1}N_I - 10] = [49 - 10] = +$; then

$N_{iE} = [.010][49] = .49 \Rightarrow 0$

#10 $N_{ri} = ({}_{102-2}N_{ir})(M_{\theta}) = (0)(1.00) = 0$

$N_{re} = ({}_{102-2}N_{ir})(1 - M_{\theta}) = (0)(0) = 0$

$N_{mi} = ({}_{102-4}N_{im})(M_D) = (0)(1.000) = 0$

$N_{me} = ({}_{102-4}N_{im})(1 - M_{\theta}) = (0)(0) = 0$

$\$_I = [(49)(10,000 + 70,000) + 40,000] = \$3,960,000$

$\$_r = [(4000)(2) + 10,000][0] = 0$

$\$_m = [(4000)(4) + 20,000][0] = 0$

$\$_{e;E} = [1000 - 0][0] = 0$

$\$_T = 3,960,000 + 0 + 0 + 0 = \$3,960,000$

---

9 $N_{RR} = [49][.880] = 43$

$A = \frac{43}{50} = .860$

#12 $U.C.A. = \frac{\$3,960,000}{43} = \$92,093$ per system

2 $N_I = 49 - 0 - 6 - 0 - 0 + 0 + 0 = 43$

3 $N_{e;T} = 1 + 0 + 0 + 0 + 0 = 1$

Test # 1 a $[N_S - N_{e;T}] = [50 - 1] = +$; then

$k = 2$? Yes; then $(t - 1 - j) = (102 - 1 - 100)$

$= +$; then

Set $j = 100 + 1 = 101$; Set $k = 1$

$[N_{S(101,1)} = 0?]$ Yes; then $k = 2$? No; then

Set $k = 2$

$[N_{S(101,2)} = 0?]$ No; then

$[N_{S(101,2)} - {}_{102-1}N_{e;T(101,2)}] = 5 - 1 = +$; then

---

4 $(j = t?) = (101 = 102?)$ No; then

$$\bar{C}_{D(101,2)} = \frac{[1380][4] + [1380][0] + 1380][0]}{5 + 0 + 0 - 1}$$

$$= 1380 \text{ mph}$$

$$\bar{C}_{\theta max(101,2)} = \frac{[1460-25][4]+[1460+20][0]+[1460+150][0]}{5 + 0 + 0 - 1}$$

$$= 1435 \text{ mph}$$

$$\bar{C}_{\theta min(101,2)} = \frac{[1300-50][4]+[1300+20][0]+[1300+150][0]}{5 + 0 + 0 - 1}$$

$$= 1250 \text{ mph}$$

	$t = 102$ $g = (101,2)$ $N_S = 5$

\# 5 $x_{max} = 1217$

$x_{min} = 717$

$[(f_a)(\bar{C}_D) = C_{Rmin}] = [(.95)(1380) - 1217]$

$= 1312 - 1217 = +$; then

$[C_{Rmax} - (f_a C_D)] = 1217 - 1312 = -$; then

Set $[f_a C_D] = 1217$

$x_2 = 1217$

$x_1 = 717$

\#26 $[x_{min} - x_1] = 717 - 717 = 0$; then

$[x_{max} - x_2] = 1217 - 1217 = 0$; then $z = 1.000$

\# 5 $\psi_D = 1.000$

$[C_{Rmax} - (f_b)(\bar{C}_D)] = 1217 - (1.05)(1380) = -$; then

Set $[f_b C_D] = 1217$

$\Phi_D^{**} = 0$

$x_2 = 1217$

$x_1 = 1217$

$[x_2 - x_1] = 1217 - 1217 = 0$

a $\Phi_D^{*} = 0$

5 a $P_D = 1.000 + 0 + 0$

$x_{max} = 1435$

$x_{min} = 1250$

$[C_{\theta max} - f_2 C_D] = [1435 - (.90)(1380)] = 1435 - 1242$
$= +$; then

$[C_{\theta min} - f_2 C_D] = [1252 - 1242] = +$; then

Set $[f_2 C_D] = 1250$

$x_2 = 1435$

$x_1 = 1250$

#26 $[x_{min} - x_1] = 1250 - 1250 = 0$; then

$[x_{max} - x_2] = 1435 - 1435 = 0$; then $z = 1.000$

5 $\Psi_\theta = 1.000$

$[(f_3)(\bar{C}_D) - \bar{C}_{\theta min}] = [(.60)(1380) - 1250]$
$= 828 - 1250 = -$; then

Set $[f_3 \bar{C}_D] = 1250$

$\Phi^{**}_\theta = 0$

$x_2 = 1250$; $x_1 = 1250$

#26 $[x_{min} - x_1] = 1250 - 1250 = 0$; then

$[x_{max} - x_2] = 1435 - 1250 = +$; then

$$x_c = 1250 + \left[\frac{1435 - 1250}{3}\right] = 1250 + 62 = 1312$$

$[x_2 - x_c] = 1250 - 1312 = -$; then

$$z = \left[\frac{1250 - 1250}{1435 - 1250}\right] \left[\frac{1250 + 1250 - 2(1250)}{1312 - 1250}\right] = 0$$

$\Phi^{*}_\theta = 0$

b $P_\theta = 1.000 + 0 + 0$

$R = (1.000)(1.000) = 1.000$

$U = (0)(0) = 0$

$D_r = (1.000)(0) = 0$

	$D_m = 0$ $P_T = 1.000 + 0 + 0 + 0 = 1.000$
# 6	$N_{ie} = (4)(0.000) = 0$ $N_{ir} = (4)(0) = 0$ $N_{im} = (4)(0) = 0$
# 7	$\text{Congestion} = \sum_{t'=102-4}^{t'=102-2} {}_{t'}N_{ir} = 0 + 0 + 0 = 0$ $M_\theta = 1.000$
# 8	$\text{Congestion} = \sum_{t'=102-8}^{t'=102-4} N_{im} = 0 + 0 + 0 + 0 = 0$ $M_D = 1.000$
# 1	$[N_S - 10] = [5 - 10] = -$; then $[{}_{102-1}N_I - 4] = 4 - 4 = 0$; then $N_{iE} = 4$
#10	$N_{ri} = [{}_{102-2}N_{ir}][M_\theta] = [0][1.000] = 0$ $N_{re} = [0][1 - 1.000] = 0$ $N_{mi} = [{}_{102-4}N_{im}][M_D] = [0][1.000] = 0$ $N_{me} = [0][1 - 1.000] = 0$ $\$_I = [(4)(10{,}000 + 70{,}000) + 40{,}000] = \$280{,}000$ $\$_r = [(4000)(2) + 10{,}000][0] = 0$ $\$_m = [(4000)(4) + 20{,}000][0] = 0$ $\$_{e;E} = [1000 - 0][0 + 4] = \$4{,}000$ $\$_T = \$284{,}000$

# 9	$N_{RR} = (4)(1) = 4$ $A = 4/5 = .800$
#12	U.C.A. = \$284,000/4 = \$71,000 per systems
# 2	$N_I = [4] - [0 + 0 + 0 + 4] + 0 + 0 = 0$
# 3	$N_{e;T} = 4 + 1 = 5$
Test # 1 a	$[N_S - N_{e;T}] = [5 - 5] = 0$; then
	Set $F_g = 102 + 1 = 103$
#21	$_T\bar{A} = \left[\sum_{t'=101}^{t'=103-1} {}_{t'}N_{RR}\right] \Big/ [103 - 101][5]$ $= [5 + 4]/10 = .900$
#11	$\$_S = [24{,}000{,}000 + 0][5] + 0 = \$120{,}000{,}000$ $P_g = \$_S + \sum_{t'=101+1}^{t'=103-1} [{}_{t'}\$_T/(1 + .03)^{t'-101}]$ $= 120{,}000 + [284{,}000/(1.03)^1]$ $= \$395{,}613$
#22	G.L.E.R. = [395,613/(.900)(5)] = \$87,914 per system
Test # 2 a	k = 2? Yes; then [102 - 1 - 101] = 0; then
#13	Set j = 100; Set $ACC_A = 0$; Set k = 1

$N_{S(100,1)} = 0$; Yes; then $k = 2$? No; then

Set $k = 2$

$N_{S(100,2)} = 0$? No; then

$[N_{S(100,2)} - (102-1)N_{e;T(100,2)}] = [50 - 1] = +$;

then $ACC_A = 0 + 43 = 43$

$[100 = (102 - 1)?] =$ No; then $j = 100 + 1 = 101$;

Set $k = 1$

$N_{S(101,1)} = 0$; Yes; then $k = 2$? No; then

Set $k = 2$

$N_{S(101,2)} = 0$? No; then

$[N_{S(101,2)} - (102-1)N_{e;T(101,2)}] = 5 - 1 = +$; then

$ACC_A = 43 + 4 = 47$

$[101 = (102 - 1)?]$ Yes; then

$[{}_{102+8}N_R - ACC_A] = [235 - 47] = +$; then

$N_P = [{}_{102+8}N_R - ACC_A][.20] = [188][.20] = 38$ systems

---

#14 $\overline{\overline{C}}_{D(102,1)} = 1.000[{}_{102+8}C_{Rmax}] = 1350$ mph

$t_1 = 133^{rd}$ quarter

$\overline{\overline{C}}_{D(102,2)} = 1.15[{}_{102+8}C_{Rmax}] = 1552$ mph

$t_2 = 144^{th}$ quarter

$$VALUK = \left[\frac{144 - 102}{133 - 102}\right] \left[\frac{18,000,000}{24,000,000}\right] = 1.008$$

$[VALUK - 1.000] = +$; then

$g = (102 + 8, 2) = (110, 2)$

Set $N_{P(100,1)} = 0$

Set $N_{P(100,2)} = 38$ systems

#24 Set $k = 1$

$$\text{Congestion} = \sum_{t'=102-8}^{t'=t=102} {}_{t'}N_{P(1)}$$

$$= {}_{94}N_{P(1)} + {}_{95}N_{P(-,1)} + \cdots + {}_{101}N_{P(-,1)}$$

$$+ {}_{102}N_{P(-,1)} = 50 + 32 = 82$$

$S_{(100,1)} = .74$

k = 2? No; then Set k = 2

$$\text{Congestion} = \sum_{t'=102-8}^{t'=102} N_{P(-,2)} = 38$$

$S_{(110,2)} = .83$

k = 2? Yes; then

#25	Set k = 1
	$N_{S(110,1)} = [0][.74] = 0$ systems
	k = 2? No; then Set k = 2
	$N_{S(110,2)} = [38][.83] = 32$ systems
	k = 2? Yes; then

Test # 4 a	[H - 5] = [140 - 102] = +; then
	Set j = 102
Test # 5	$N_{S(102,1)} = 0$? Yes; then Set k = 2
	$N_{S(102,2)} = 0$? Yes; then
Test # 2 a	k = 2? Yes; then
Test # 3	[102 - 1 - 102] = -; then

#15 Set $j = 100$; Set $ACC_A = 0$; Set $ACC_B = 0$;

Set $k = 1$

$N_{S(100,1)} = 0$? Yes; then [$k = 2$?] No; then

Set $k = 2$

$N_{S(100,2)} = 0$? No; then

$[N_{S(100,2)} - N_{e;T(100,2)}] = [50 - 7] = +$; then

$ACC_B = 0 + 50 = 50$

$ACC_A = 0 + 43 = 43$

[$100 = 102$?] No; then $j = 100 + 1 = 101$;

Set $k = 1$

$N_{S(101,1)} = 0$? Yes; then $k = 2$? No; Set $k = 2$

$N_{S(101,2)} = 0$? No; then

$[N_{S(101,2)} - N_{e;T(101,2)}] = 5 - 5 = 0$

$ACC_A = 43 + 4 = 47$

$ACC_B = 50 + 5 = 55$

Then $j = 101 + 1 = 102$

Set $k = 1$; [$N_{S(102,1)} = 0$?] Yes; then $k = 2$?

No; then Set $k = 2$

$N_{S(102,2)} = 0$? Yes; then $k = 2$? Yes; then

[$102 = 102$?] Yes; then

$A_F = \frac{47}{55} = .855$

	$t = 102$ Fleet

#16 Set $j = 100$; Set $ACC_C = 0$; Set $k = 1$

$N_{S(100,1)} = 0$? Yes; then $k = 2$? No; then

Set $k = 2$

$N_{S(100,2)} = 0?$ No; then

$[N_{S(100,2)} - N_{e;T(100,2)}] = 50 - 7 = +;$ then

$ACC_C = 0 + \$3,960,000 = \$3,960,000$

[100 = 102?] No; then j = 101; Set k = 1;

$N_{S(101,1)} = 0?$ Yes; then k = 2; No; then

Set k = 2

$N_{S(101,2)} = 0?$ No; then

$[N_{S(101,2)} - N_{e;T(101,2)}] = 5 - 5 = 0;$ then

$ACC_C = 3,960,000 + 284,000 = \$4,244,000$

[101 = 102?] No; then j = 102; Set k = 1;

$N_{S(102,1)} = 0?$ Yes; then k = 2? No; then

Set k = 2

$N_{S(102,2)} = 0?$ Yes; then k = 2? Yes; then

[102 = 102?] Yes; then

$\$_{S(102,1)} = 0$

$\$_{S(102,2)} = 0$

$[{}_{102}N_R - {}_{102}ACC_A] = +;$ then

$$\$_{T;F} = \sum_{g=(102,1)}^{g=(102,2)} {}_{102}\$_{Sg} + \sum_{k=1}^{k=2} \sum_{g=(j\leq 100\leq Fg;k)}^{g=(j<102\leq Fg;k)} {}_{102}\$_{T}$$

$$+ [{}_{102}\phi_{Sh}][{}_{102}N_R - \sum_{k} \sum_{g} {}_{102}N_{RRg}]$$

$$= 0 + 0 + ACC_C = [\phi_{Sh}][N_R - ACC_A]$$

$$= 0 + 0 + \$4,244,000 + [\$440,000][70 - 47]$$

$$= \$14,364,000$$

#18	Set $j = 100$; Set $ACC_A = 0$; Set $k = 1$
	$N_{S(100,1)} = 0$? Yes; then [k = 2]? No; then
	Set $k = 2$
	$N_{S(100,2)} = 0$? No; then
	$[N_{S(100,2)} - N_{e;T(100,2)}] = (50 - 7) = +$; then
	$ACC_A = 0 + 43 = 43$
	[100 = 102?] No; then $j = 100 + 1 = 101$;
	Set $k = 1$
	$N_{S(101,1)} = 0$? Yes; then $k = 2$? No; Set $k = 2$
	$N_{S(101,2)} = 0$? No; then
	$[N_{S(101,2)} - N_{e;T(101,2)}] = 0$; then
	$ACC_A = 43 + 4 = 47$; then $j = 101 + 1 = 102$
	Set $k = 1$; $[N_{S(102,1)} = 0?]$ Yes; then $k = 2$? No
	Then Set $k = 2$
	$N_{S(102,2)} = 0$? Yes; then $k = 2$? Yes; then
	[102 = 102?] Yes; then
	U.C.A. = [$14,364,000/47] = $290,000 per system
Test # 4 b	$[H - 5] = [140 - 102] = +$; then set $t = 102+1=103$;
Test # 6	Set $j = 100$; Set $k = 1$; $N_{S(100,1)} = 0$? Yes; then
Test # 6	$k = 2$? No; Set $k = 2$; $N_{S(100,2)} = 0$? No; then
Test # 7	$[N_{S(100,2)} - {}_{102}N_{e;T(100,2)}] = 50 - 7 = +$; then
#4	100 = 103? No; then
	$_{103}\bar{C}_{D(100,2)} = \frac{[1380][43-0] + 0 + 0}{43 + 0 + 0 - 0} = 1380$

$$103\bar{C}_{\theta max(100,2)} = \frac{[1410 - 25][43 - 0] + 0 + 0}{43 + 0 + 0 - 0} = 1385$$

$$103\bar{C}_{\theta min(100,2)} = \frac{[1200 - 50][43 - 0] + 0 + 0}{43 + 0 + 0 - 0} = 1150$$

	t = 103 g = (100,2) N_S = 50
# 5	$x_{max} = 1234$
	$x_{min} = 734$
	$[(f_a)(\bar{C}_D) = C_{Rmin}] = [(.95)(1380) - 734] = +;$
	then $[C_{Rmax} - (f_a)(\bar{C}_D)] = [1234 - 1311] = -;$
	then set $(f_a)(C_D) = 1234$
	$x_2 = 1234$
	$x_1 = 734$
#26	$[x_{min} - x_1] = 734 - 734 = 0;$ then
	$[x_{max} - x_2] = 1234 - 1234 = 0;$ then as previously done: $\Psi_D = 1.000$, $\Phi_D^{**} = 0$, $\Phi_D^{*} = 0$
# 5 a	$P_D = 1.000$
	$x_{max} = 1410$
	$x_{min} = 1200$
	$[C_{\theta max} - (f_2)(\bar{C}_D)] = [1410 - (.90)(1380)] = +;$
	then $[C_{\theta min} - (f_2)(\bar{C}_D)] = [1200 - 1242] = -;$
	then $x_2 = 1410$
	$x_1 = 1242$
#26	$[x_{min} - x_1] = 1200 - 1242 = -;$ then
	$x_c = 1200 - \left[\frac{1410 - 1200}{3}\right] = 1200 + 70 = 1270$
	$[x_2 - x_c] = 1410 - 1270 = +;$ then
	$[x_1 - x_c] = [1242 - 1270] = -;$ then

\# 5 $x_{max} = 1234$

$x_{min} = 734$

$[(f_a)(\bar{C}_D) - C_{Rmin}] = [(.95)(1380) - 734] = +;$

then $[C_{Rmax} - (f_a)(\bar{C}_D)] = [1234 - 1311] = -;$

then set $[(f_a)(\bar{C}_D)] = 1234$

\#26 $x_2 = 1234$

$x_1 = 734$

$[x_{min} - x_1] = 734 - 734 = 0;$ then

$[x_{max} - x_2] = 1234 - 1234 = 0;$ then as previously shown $\begin{cases} \psi_D = 1.000 \\ \Phi_D^{**} = 0 \\ \Phi_D^{*} = 0 \end{cases}$

\# 5 a $P_D = 1.000$

$x_{max} = 1385$

$x_{min} = 1150$

$[C_{\theta max} - (f_2)(\bar{C}_D)] = [1150 - (.90)(1380)]$

$= 1150 - 1242 = -;$

then Set $(f_2)(\bar{C}_D) = 1385$

$\psi_\theta = 0$

$[(f_3)(\bar{C}_D) - \bar{C}_{\theta min}] = [(.60)(1380) - 1150] = -;$

then Set $(f_3)(\bar{C}_D) = 1150$

$\Phi_\theta^{**} = 0$

$x_2 = 1385$

$x_1 = 1150$

\#26 $\Phi_\theta^{*} = 1.000$

\# 5 b $P_\theta = 1.000$

$R = 0$

$U = 0$

$D_r = [1.000][1 - 0] = 1.000$

$D_m = 0$

$P_r = 1.000$

---

6 $N_{ie} = [43][0] = 0$

$N_{ir} = [43][1.000] = 43$

$N_{im} = [43][0] = 0$

---

7 $$\sum_{g} \sum_{t'=103-4=99}^{t'=103-2=101} N_{irg} = \text{Congestion} = 0+0+0+0+0+0 = 0$$

$M_\theta = 1.000$

8 $$\text{Congestion} = \sum_{g} \sum_{t'=103-8=91}^{t'=103-4=99} N_{img} = 0+0+0+0+0+0=0$$

$M_D = 1.000$

1 $[N_S - 10] = [50 - 10] = +;$ then

$[{}_{102}N_I - 10] = [43 - 10] = +;$ then

$N_{iE} = (.010)(43) = 0$

#10 $N_{ri} = [{}_{103-2}N_{ir}][M_\theta] = [0][1.000] = 0$

$N_{re} = [0][1 - 1] = 0$

$N_{mi} = [{}_{103-2}N_{im}][M_D] = [0][1.000] = 0$

$N_{me} = [0][1 - 1] = 0$

$\$_I = \$3,480,000$

---

9 $N_{RR} = (43)(0) = 0$

$A = \frac{0}{50} = 0$

---

12 $U.C.A. = \frac{\$3,480,000}{0} = \infty$

---

2 $N_I = 43 - 0 - 43 - 0 + 0 + 0 = 0$

# 3	$N_{e;T} = 1 + 0 = 1$

Test #

1 a $[N_S - N_{e;T}] = 50 - 1 = +$; then

2 a $k = 2$? Yes; then $[103 - 1 - 100] = +$; then

Set $j = 100 + 1 = 101$

$k = 1$

	$t = 103$ $g = (101,1)$ $N_S = 0$

$N_S = 0$? Yes; then $k = 2$? No; then set $k = 2$

	$t = 103$ $g = (101,2)$ $N_S = 5$

$N_S = 0$? No; then $[N_S - {}_{t-1}N_{e;T}] = 5 - 5 = 0$;
then $k = 2$? Yes; then $[103 - 1 - 101] = +$;
then set $j = 101 + 1 = 102$; $k = 1$

	$t = 103$ $g = (102,1)$ $N_S = 0$

$N_S = 0$? Yes; then $k = 2$? No; set $k = 2$

	$t = 103$ $g = (102,2)$ $N_S = 0$

$N_S = 0$? Yes; then $k = 2$? Yes; then
$[103 - 1 - 102] = 0$; then

#13	Set $ACC_A = 0$; Set $j = 100$; Set $k = 1$ $N_{S(100,1)} = 0$; Yes; then $k = 2$? No; Set $k = 2$ $N_{S(100,2)} = 0$; No; then $[N_{S(100,2)} - {}_{102}N_{e;Tg}]$ $= 50 - 1 = +$; then $ACC_A = 0 + {}_{103}N_{RR(100,2)} = 0$ $j = 103 - 1 = 102$? No; then $j = 100 + 1 = 101$; Set $k = 1$ $N_{S(101,1)} = 0$? Yes; then $k = 2$? No; then Set $k = 2$ $N_{S(101,2)} = 0$? No; then $[N_S - {}_{102}N_{e;T(101,2)}]$ $= 5 - 5 = 0$ Then $j = 103 - 1 = 102$? No; set $j = 102$? Set $k = 1$ $N_{S(102,1)} = 0$? Yes; then $k = 2$? No; set $k = 2$; then $N_{S(102,2)} = 0$? Yes; then $k = 2$? Yes; then $j = 103 - 1 = 102$? Yes; then $[{}_{103+8}N_R - ACC_A] = [260 - 0] = +$; then $N_p = [260 - 0][.20] = 52$ systems
#14	Set $j = [103 + 8] = 111$; set $k = 1$ $\bar{\bar{C}}_{D(111,1)} = 1367$ $t_1 = 136$; Set $k = 2$ $\bar{\bar{C}}_{D(111,2)} = [1.15][1367] = 1572$ $t_2 = 143$ $VALUK = \frac{143-103}{136-103} \quad \frac{18,000,000}{24,000,000} = \frac{40}{33} \quad \frac{3}{4} = \frac{10}{11}$ $[VALUK - 1.000] = -$; then set $g = (111;1)$ set $N_{p(111,1)} = 52$ set $N_{p(111,2)} = 0$

#24	Set k = 1; Congestion = 82+52=134; $S_{(111,1)}$=.300 k = 2?; No; then set k = 2; congestion=38+0=38; $S_{(111,2)}$ = .83
#25	Set k = 1; $N_{S(111,1)}$ = (52)(.300) = 16; k = 2? No; set k = 2 $N_{S(111,2)}$ = (0)(.83) = 0
Test # 4 a Test # 6	[140 - 103] = +; then set j = 103, k = 1 $N_{S(103,1)}$ = 0? Yes; set k = 2 $N_{S(103,2)}$ = 0? Yes; then k = 2? Yes; [103 - 1 - 103] = -; then

#15	Set ACC_A = 0; ACC_B = 0; j = 100, k = 1
	$N_{S(100,1)}$ = 0? Yes; k = 2? No; set k = 2
	$N_{S(100,2)}$ = 0? No; [$N_{S(100,2)}$ $-_{103}N_{e;T(100,2)}$]
	= 50 - 1 = +
	ACC_A = 0 + 0 = 0; ACC_B = 0 + 50 = 50; j = 103?
	No; set j = 101; Set k = 1; $N_{S(101,1)}$ = 0; yes;
	then k = 2? No; set k = 2; $N_{S(101,2)}$ = 0? No;
	then [$N_{S(101,2)}$ $-_{103}N_{e;Tg}$] = 5 - 5 = 0
	ACC_A = 0 + 0 = 0; ACC_B = 50 + 0 = 50; j = 103?
	No; set j = 102; set k = 1; $N_{S(102,1)}$ = 0? Yes;
	k = 2? No; set k = 2; $N_{S(102,2)}$ = 0? Yes
	k = 2? Yes; j = 103; No; set j = 103; set k = 1;
	$N_{S(103,1)}$ = 0? Yes; k = 2? No; set k = 2;

	$N_{S(103,2)} = 0$? Yes; $k = 2$? Yes; $j = 103$? Yes $A_F = \frac{0}{50} = 0$
#16	$ACC_C =$ SR # 16 Has same "gates" as SR # 15; Hence $ACC_C = 0 + {}_{103}\$_{T(100,2)} + {}_{103}\$_{T(101,2)}$ $= 0 + 3,480,000 + 0 = \$3,480,000$ @ $j = 103$; then $[{}_{103}N_R - ACC_A] = 90 - 0 = +$ ${}_{103}\$_{T;F} = \$_{S(103,2)} + ACC_C + [{}_t\not{c}_{Sh}][N_R - ACC_A]$ $= 0 + 0 + 3,480,000 + [440,000][90 - 0]$ $= \$43,080,000$
#18	Set $ACC_A = 0$; $j = 100$; NOTE: Procedure is identical to part of S.R. 15. Hence ${}_{103}(ACC_A)_{SR15} = {}_{103}(ACC_A)_{SR18}$ ${}_{103}ACC_A = 0$; $j = 103$; Yes $U.C.A._F = [\$43,080,000]/[0] = \infty$
Test # 4 b	$[140 - 103] = +$; Set $j = 100$; $k = 1$; $t = 103+1 = 104$
# 6	$N_{S(100,1)} = 0$? Yes; $k = 2$? No; set $k = 2$;
# 7	$N_{S(100,2)} = 0$? No; $[N_{S(100,2)} - {}_{103}N_{e;Tg}]$ $= [50 - 1] = +$

	$t = 104$ $g = (100,2)$ $N_S = 50$

# 4	$100 = 104$? No $\overline{C}_D = 1380$ per previous calculation Numerator of $C_{\theta max} = 0$? Yes; $k = 2$?; Yes

Set j = 101, k = 1

# 7	Congestion $= \sum_{g} \sum_{t'=104-4}^{t'=104-2} {}_{t'}N_{irg} = 6$; $M_{\theta} = .994$
# 8	Congestion $= \sum_{g} \sum_{t'=104-8}^{t'=104-4} {}_{t'}N_{img} = 0$; $M_{D} = 1.000$
# 1	$[N_{Sg} - 10] = [50 - 10] = +$; $[{}_{103}N_{I(100,2)} - 10] = [0 - 10] = -$;
	$N_{iE} = 0$
#10	$N_{ri} = [{}_{104-2}N_{ir}][M_{\theta}] = [6][.994] = 6$
	$N_{re} = [6][1 - .994] = 0$
	$N_{mi} = [0][1.000] = 0$
	$N_{me} = [0][1 - 1.000] = 0$
	$\$_{I} = [0][10,000 + 70,000] + 40,000 = \$40,000$
	$\$_{r} = [(4000)(2) + 10,000][6] + 40,000 = \$148,000$
	$\$_{m} = [(4000)(4) + 20,000][0] + 50,000 = \$50,000$
	$\$_{e;E} = [100 - 0][0 + 0 + 0 + 0] = 0$
	$\$_{T} = 40,000 + 148,000 + 50,000 = \$238,000$
# 9	$N_{RR} = [0][0] = 0$
	$A = 0/50 = 0$
#12	U.C.A. $= \frac{238,000}{0} = \infty$
# 2	$N_{I} = 6$
# 3	$N_{e;T} = 7$

Test #	
1 a	$[N_{S} - {}_{104}N_{e;T}] = [50 - 1] = +$; k = 2? Yes;
	$[104 - 1 - 100] = +$;
	Set j = 100 + 1 = 101; Set k = 1; $N_{S(101,1)} = 0$? Yes

$k = 2$? No; set $k = 2$; $N_{S(101,2)} = 0$? No;

$[N_{S(101,2)} - {}_{103}N_{e;T(101,2)}] = [5 - 5] = 0$;

$k = 2$? Yes; $(104 - 1 - 101) = +$;

Set $j = 102$; $k = 1$; $N_{S(102,1)} = 0$? Yes; $k = 2$?

No; Set $k = 2$; $N_{S(102,2)} = 0$? Yes; $k = 2$?

Yes; $(104 - 1 - 102) = +$

Set $j = 103$; $k = 1$; $N_{S(103,1)} = 0$? Yes; $k = 2$?

No; set $k = 2$

Test # 3 — $N_{S(103,2)} = 0$? Yes; $k = 2$? Yes; $(104-1-103)=0$;

#13 — Set $ACC_A = 0$; $j = 100$; $k = 1$

$N_{S(100,1)} = 0$? Yes; $k = 2$? No; set $k = 2$;

$N_{S(100,2)} = 0$? No

$[N_{S(100,2)} - {}_{103}N_{e;T(100,2)}] = 50 - 1 = +$;

$ACC_A = 0 + {}_{104}N_{RR(100,2)} = 0$; $j = 104 - 1$? No;

Set $j = 101$, $k = 1$; $N_{S(101,1)} = 0$? Yes; $k = 2$? No;

Set $k = 2$; $N_{S(101,2)} = 0$? No;

$[N_{S(101,2)} - {}_{103}N_{e;Tg}] = 0$;

$j = 104 - 1$? No; Set $j = 102$, $k = 1$; $N_{S(102,1)} = 0$?

Yes; $k = 2$? No; set $k = 2$; $N_{S(102,1)} = 0$? Yes;

$k = 2$? No; set $k = 2$; $N_{S(102,2)} = 0$? Yes;

$k = 2$? Yes; $j = 104 - 1 = $? No; set $j = 103$,

$k = 1$; $N_{S(103,1)} = 0$? Yes;

$k = 2$? No; set $k - 2$; $N_{S(103,2)} = 0$? Yes;

$k = 2$? Yes; $j = 104 - 1$? Yes; $[{}_{104+8}N_R - ACC_A]$

$= 285 - 0 = 285 = +$

$N_p = [285][.20] = 57$

#14	Set $j = 104 + 8 = 112$
	$\bar{\bar{C}}_{D(112,1)} = {}_{112}C_{Rmax} = 1384$
	$t_1 = 138$
	$\bar{\bar{C}}_{D(112,2)} = (1.15)(1384) = 1592$
	$t_2 = 145$
	$VALUK = \left[\frac{145-104}{138-104}\right]\left[\frac{18,000,000}{24,000,000}\right] = \left[\frac{41}{34}\right]\left[\frac{3}{4}\right] = \frac{10+}{11+} < 1.000$
	$[VALUK - 1.000] = -$
	Set $g = (112,1)$, $N_{p(112,1)} = 57$; $N_{p(112,2)} = 0$

	$t = 104$ $g = (112,1)$

#24 &	$\text{Congestion} = \sum_j \sum_{t'=104-8}^{t'=104} t' N_{p(j,1)} = 191$
#25	$S_g = .600$
	$N_{S(112,1)} = (57)(.600) = 37$; $k = 2$? No
	Set $k = 2$
	$\text{Congestion} = \sum_j \sum_{t'=104-8}^{t'=104} t' N_{p(j,2)} = 38$
	$S_g = .830$
	$N_{S(112,2)} = [0][.830] = 0$

Test # 4 a	$[140 - 104] = +$; Set $j = 104$, $k = 1$; $N_{S(104,1)} = 0$?
	No; $N_{S(104,1)} = 0$? No;
	$[N_{S(104,1)} - {}_{103}N_{e;T(104,1)}] = [48-0]$

	$t = 104$ $g = (104,1)$ $N_S = 48$

# 4	$(104 = 104)$? Yes

$\bar{C}_D = (1.000)(C_{Rmax}) = 1250$

$\bar{C}_{\theta max} = 1250 + 40 = 1290$

$\bar{C}_{\theta min} = 1250 - 40 = 1210$

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5 $x_{max} = 1250$

$x_{min} = 750$

$[(f_a)(\bar{C}_D) - C_{Rmin}] = [(.95)(1250) - 750] = +$

$[C_{Rmax} - (f_a)(\bar{C}_D)] = [1250 - (.95)(1250)] = +$

$x_2 = 1186$

$x_1 = 750$

#26 $[x_{min} - x_1] = 0$

$[x_{max} - x_2] = +$

$x_c = 750 + \left[\frac{1250 - 750}{3}\right] = 917$

$x_2 - x_c = 1187 - 917 = +$

$x_1 - x_c = 750 - 917 = -$

$$z = \left\{\frac{1}{500}\right\}\left\{\left[\frac{750-750}{917-750}\right][917-750] + \left[\frac{1250-1187}{1250-917}\right][1187-917] + [1187-750]\right\} = .978$$

$\psi_D = .978$

$[C_{Rmax} - (f_b)(C_D)] = [1250 - (1.05)(1250)] = -;$

Set $(f_b)(C_D) = 1250$, $\Phi_D^{**} = 0$

$x_2 = 1250$

$x_1 = (.95)(1250) = 1187$

$x_2 - x_1 = +;$

$[x_{min} - x_1] = 750 - 1187 = -$

$x_c = 750 + \frac{500}{3} = 917$

$x_2 - x_c = 1250 - 917 = +$

$x_1 - x_c = 1187 - 917 = +$

$$z = \left[\frac{1250-1187}{1250-750}\right]\left[\frac{2(1250)-1187-1250}{1250-917}\right] = \left[\frac{63}{500}\right]\left[\frac{63}{333}\right] = .020$$

$\Phi_D^* = .020$

a $P_D = 0.998$

$x_{max} = 1290$

$x_{min} = 1210$

$[\bar{C}_{\theta max} - (f_2)(\bar{C}_D)] = [1290 - (.90)(1250)] = +;$

$[\bar{C}_{\theta min} - (f_2)(\bar{C}_D)] = [1210 - 1125] = +$

Set $(f_2)(C_D) = 1210$

$x_2 = 1290$

$x_1 = 1210$

$\Psi_\theta = 1.000$

$\Phi_\theta^{**} = 0$

$\Phi_\theta^* = 0$

b $P_\theta = 1.000$

$R = (.978)(1.000) = (.978)$

$U = (0)(0) = 0$

$D_r = (.978)(1 - 1.000) = 0$

$D_m = .020$

$P_T = 1.000$

\# 6 $N_{ie} = (48)(0) = 0$

$N_{ir} = (48)(0) = 0$

$N_{im} = (48)(.020) = .98 = 1$

\# 7 $\text{Congestion} = \sum_g \sum_{t'=104-4}^{t'=104-2} {}_{t'}N_{irg} = 6 \quad M_\theta = 1.000$

\# 8 $\text{Congestion} = \sum_g \sum_{t'=104-8}^{t'=104-4} {}_{t'}N_{img} = 0 \quad M_D = 1.000$

# 1	$[N_S - 10] = +$; $[{}_{103}N_I - 10] = +$ $N_{iE} = [.01\ {}_{103}N_I] = 0$
# 10	$N_{ri} = [{}_{104-2}N_{irg}][M_\theta] = [0][.994] = 0$ $N_{re} = 0$ $N_{mi} = [{}_{104-4}N_{img}][M_D] = 0$ $N_{me} = 0$ $\$_I = [{}_{103}N_I][10,000 + 50,000] + 30,000 = 30,000$ $\$_r = [(6,000)(2)+13,000][{}_{104}N_{ri}]+50,000 = 50,000$ $\$_m = [(6,000)(4)+26,000][{}_{104}N_{mi}]+60,000 = 60,000$ $\$_{e;E} = [1,000-0][{}_{104}N_{ie}+N_{re}+N_{me}+N_{iE}] = 0$ $\$_T = \$140,000$
# 9	$N_{RR} = (48)(.978) = 47$ $A = 47/48 = .979$
# 12	U.C.A. = \$140,000/47 = \$29,787
# 2	$N_I = 48 - 0 - 0 - 1 - 0 + 0 + 0 = 47$
# 3	$N_{e;T} = 0$
Test # 1 a	$[N_S - N_{e;T}] = 48 - 0 = +$; $k = 2$? No; Set $k = 2$ $N_{S(104,2)} = 0$? Yes; $k = 2$? Yes; $[104-1-104] = -$
# 15	Set $ACC_A = 0$, $ACC_B = 0$, $j = 100$, $k = 1$ $N_{S(100,1)} = 0$? Yes; $k = 2$? No; set $k = 2$; $N_{S(100,2)} = 0$? No $[N_{S(100,2)} - {}_{104}N_{e;T(100,2)}] = 50 - 1 = +$;

$ACC_A = 0 + {}_{104}N_{RR(100,2)} = 0$; $ACC_B = 0 + 50 = 50$;
$j = 104$? No; set $j = 101$, $k = 1$; $N_{S(101,1)} = 0$?
Yes; $k = 2$? No; set $k = 2$; $N_{S(101,2)} = 0$? Yes
$k = 2$? Yes; $j = 104$? No; set $j = 102$, $k = 1$;
$N_{S(102,1)} = 0$? Yes; $k = 2$? No; set $k = 2$;
$N_{S(102,2)} = 0$? Yes; $k = 2$? Yes; $j = 104$? No;
set $j = 103$, $k = 1$; $N_{S(103,1)} = 0$? Yes; $k = 2$?
No; set $k = 2$; $N_{S(103,2)} = 0$? Yes; $k = 2$? Yes;
$j = 104$? No; set $j = 104$, $k = 1$; $N_{S(104,1)} = 0$?
No; $[N_{S(104,1)} - {}_{104}N_{e;T(104,1)}] = 48 - 0 = +$;
$ACC_A = 0 + 47 = 47$
$ACC_B = 50 + 48 = 98$; $j = 104$? Yes; $A_F = 47/98 = .480$

#16 Set $ACC_C = 0$, $j = 100$, $k = 1$; $N_{S(100,1)} = 0$? Yes
$k = 2$?; No; set $k = 2$; $N_{S(100,2)} = 0$? No
$[N_{S(100,2)} - {}_{104}N_{e;T(100,2)}] = 50 - 1 = +$;
$ACC_C = 0 + {}_{104}\$_{T;(100,2)} = \$238,000$; $j = 104$? No;
Set $j = 101$, $k = 1$; $N_{S(101,1)} = 0$? Yes; $k = 2$?
No; set $k = 2$; $N_{S(101,2)} = 0$? Yes; etc. until
$N_{S(104,1)} = 0$? No; $[N_{S(104,1)} - {}_{104}N_{e;T(104,1)}]$
$= 48 - 0 = +$; then
$ACC_C = 238,000 + 140,000 = \$378,000$; $j = 104$? Yes;
$[{}_{104}N_R - ACC_A] = 110 - 47 = +$;
$\$_{S(104,1)} = [18,000,000][48] = \$864,000,000$
$\$_{S(104,2)} = [24,000,000][0] = 0$
$\$_{T;F} = 864,000,000 + 378,000 + [440,000][110-47]$
$= 1,242,000 + 27,720,000 = \$28,962,000$

#18	Set ACC_A = 0, j = 100, k = 1; $N_{S(100,1)}$ = 0? Yes; k = 2? No; set k = 2; $N_{S(100,2)}$ = 0? No; $[N_{S(100,2)} - {}_{104}N_{e;T(100,2)}] = 50 - 1 = +$; ACC_A = 0; j = 104? No; set j = 102, k = 1; $N_{S(102,1)}$ = 0? Yes; k = 2? No; set k = 2; $N_{S(102,2)}$ = 0? Yes; k = 2? Yes; set j = 103, k = 1; $N_{S(103,1)}$ = 0? Yes; k = 2? No; set k = 2; $N_{S(103,2)}$ = 0? Yes; k = 2? Yes; set j = 104, k = 1; $N_{S(104,1)}$ = 0? No; $[N_{S(104,1)} - {}_{104}N_{e;T(104,1)}] = 48 - 0 = +$; ACC_A = 47; j = 104? Yes $U.C.A._F$ = \$28,962,000/47 = \$616,213 / system
Test # 4 b	[140 - 104] = +; set j = 100, k = 1, t = 105 $N_{S(100,1)}$ = 0? Yes; k = 2? No; $N_{S(100,2)}$= 0? No; $[N_{S(100,2)} - {}_{104}N_{e;T(100,2)}] = 50 - 1 = +$
	t = 105 g = (100,2) N_S = 50
# 4	j = 105? No $\bar{C}_D = \frac{[1380][0-0]+[1380][6]+[1380+120][0]}{0 + 6 + 0 - 0} = 1380$ $\bar{C}_{\theta max} = \frac{[25-25][0-0]+[1410+40][6]+ 0}{0 + 6 + 0 - 0} = 1450$ $\bar{C}_{\theta min} = \frac{[50-50][0-0]+[1200+30][6] + 0}{0 + 6 + 0 - 0} = 1230$

5 $x_{max} = 1267$

$x_{min} = 767$

$[(f_a)(C_D) - C_{Rmin}] = [(.95)(1380) - 767] = +$

$[C_{Rmax} - (f_a)(C_D)] = [1267 - 1311] = -$

Set $(f_a)(C_D) = 1267$

$x_2 = 1267$

$x_1 = 767$

#26 $(x_{min} - x_1) = 767 - 767 = 0$

$(x_{max} - x_2) = 1267 - 1267 = 0 \qquad z = 1.000$

$\psi_D = 1.000$

5 a $\Phi_D^{**} = 0 \quad ; \quad \Phi_D^{*} = 0 \; ;$

$P_D = 1.000$

$x_{max} = 1450$

$x_{min} = 1230$

$[C_{\theta max} - (f_2)(C_D)] = 1450 - (.90)(1380) = +$

$[C_{\theta min} - (f_2)(C_D) = 1230 - 1242 = -$

$x_2 = 1450$

$x_1 = 1242$

#26 $(x_{min} - x_1) = 1230 - 1242 = -$

$x_c = 1230 + \dfrac{1450 - 1230}{3} = 1303$

$(x_2 - x_c) = 1450 - 1370 = +$

$(x_1 - x_c) = 1242 - 1370 = -$

$$z = \left\{\frac{1}{220}\right\} \left\{\left[\frac{1242-1230}{1303-1230}\right] [1303-1242] + \left[\frac{1450-1450}{1450-1303}\right]\left[80\right] + [208]\right\} = .991$$

$\psi_\theta = .991$

5 $[(f_3)(C_D) - C_{\theta min}] = [(.60)(1380) - 1230] = 828 - 1230 = -$

Set $(f_3)(C_D) = 1230$, $\Phi_\theta^{**} = 0$

$x_2 = 1242$

$x_1 = 1230$

#26 $[x_2 - x_1] = +$; $[x_{min} - x_1] = 1230 - 1230 = 0$

$[x_{max} - x_2] = 1450 - 1242 = +$

$x_c = 1230 + \frac{220}{3} = 1303$

$(x_2 - x_c) = 1242 - 1303 = -$

$$z = \left[\frac{1242-1230}{1450-1230}\right]\left[\frac{1230+1242-2(1230)}{1303 - 1230}\right] = \left[\frac{12}{220}\right]\left[\frac{12}{173}\right]$$

$$= \frac{144}{16,060}$$

$\Phi_\theta^* = .010$

5 $R = (1.000)(.991) = .991$

$U = (0)(0) = 0$

$D_r = (1.000)(.010) = .010$

$D_m = 0$

$P_T = 1.001$

---

6 $N_{ie} = (6)(0) = 0$

$N_{ir} = (6)(.010) = .06 \Rightarrow 0$

$N_{im} = (6)(0) = 0$

---

7 Congestion (Repair) = 0 + 6 + 43 = 49

$M_\theta = .994$

---

8 Congestion (MOD.) = 1.00

---

1 $(N_S - 10) = +$

$({}_{104}N_I - 10) = 6 - 10 = -$

$N_{iE} = 6$

#10	$N_{ri} = ({}_{103}N_{ir})(M_\theta) = (43)(.994) = 43$
	$N_{re} = (43)(1 - .994) = 0$
	$N_{mi} = ({}_{101}N_{im})(1.000) = 0$
	$N_{me} = 0$
	$\$_I = [6][10,000 + 70,000] + 40,000 = 520,000$
	$\$_r = [4,000(2) + 10,000][43] + 40,000 =$
	$= 774,000 + 40,000 = \$814,000$
	$\$_m = [4,000(4) + 20,000][0] + 50,000 = 50,000$
	$\$_{e;E} = [1000][0 + 0 + 0 + 6] = 6000$
	$\$_T = 520,000 + 814,000 + 50,000 + 6,000 = \$1,390,000$
# 9	$N_{RR} = (6)(.991) = 6$
	$A = 6/50 = .120$
#12	U.C.A. = 1,390,000/6 = \$231,667/system
# 2	$N_I = 6 - 0 - 6 - 0 - 0 + 43 + 0 = 43$
# 3	$N_{e;T} = 1 + 6 = 7$
Test # 1 a	$[N_S - N_{e;T}] = 50 - 7 = +$
# 2 a	k = 2? Yes
# 3	[105 - 1 - 100] = +; set j = 101, k = 1;
	$N_{S(101,1)} = 0$? Yes;
	k = 2? No; set k = 2; $N_{S(101,2)} = 0$? Yes;
	k = 2? Yes; [105 - 1 - 101] = +; set j = 102;
	k = 1; $N_{S(102,1)} = 0$? Yes; k = 2? No; set k = 2
	$N_{S(102,2)} = 0$? Yes; k = 2? Yes; (105-1-102) = +;

Set $j = 103$, $k = 1$; $N_{S(103,1)} = 0$? Yes; $k = 2$? No;

Set $k = 2$; $N_{S(103,2)} = 0$? Yes; $k = 2$? Yes;

$(105 - 1 - 103) = +$; set $j = 104$, $k = 1$;

$N_{S(104,1)} = 0$? No

	$t = 105$ $\quad g = (104,1)$ $\quad N_S = 48$
Test # 7	$[N_S - N_{e;T}] = 48 - 0 = +$
# 4	$j = 105$? No
	$\bar{C}_D = 1250$
	$\bar{C}_{\theta max} = 1290 - 35 = 1255$
	$\bar{C}_{\theta min} = 1210 - 70 = 1140$

\# 5 $\quad x_{max} = 1267$

$x_{min} = 767$

$[(.95)(1250) - (767)] = 1188 - 767 = +$

$[1267 - 1188] = +$; set $x_2 = 1188$, $x_1 = 767$

\#26 $\quad 767 - 767 = 0$; $\quad 1267 - 1188 = +$;

$$x_c = 767 + \frac{1267-767}{3} = 767 + 167 = 934$$

$[1188 - 934] = +$; $\quad [767 - 934] = -$;

$$z = \left\{\frac{1}{500}\right\}\left\{\left[\frac{767-767}{934-767}\right]\left[167\right] + \left[\frac{1267-1188}{1267-934}\right][1188-934] + [1188 - 767]\right\} = .962$$

\# 5 $\quad \psi_D = .962$

$[1267 - 1.05(1250)] = 1267 - 1312 = -$

Set $(f_b)(C_D) = 1267$, $\quad \Phi_D^{**} = 0$

$x_2 = 1267$; $\quad x_1 = 1188$

#26 $(x_2 - x_1) = +;\quad (767 - 1188) = -$

$x_c = 934$

$(x_2 - x_c) = 1267 - 934 = +;\ (x_1 - x_c) = 1188-934 = +$

$$z = \left[\frac{1267-1188}{1267-767}\right]\left[\frac{2(1267)-1188-1267}{1267 - 934}\right]$$

$$= \left(\frac{79}{500}\right)\left(\frac{79}{333}\right) = .037$$

$\Phi_D^* = .037$

5 b $P_D = 0.999;\ x_{max} = 1255;\ x_{min} = 1140$

$[1255 - .90(1250)] = 1255 - 1125 = +;$

$[1140 - 1125] = +;$ set $(f_2)(C_D) = 1140$

#26 $x_2 = 1255;\ x_1 = 1140$

$(1140 - 1140) = 0;\ (1255 - 1255) = 0;\ z = 1.000$

$\psi_\theta = 1.000$

$\Phi_\theta^{**} = 0$

$\Phi_\theta^* = 0$

5 $P_\theta = 1.000$

$R = (.962)(1.000) = .962$

$U = 0$

$D_r = (.962)(1.000) = .962$

$D_m = (.037)$

$P_T = 0.999$

---

6 $N_{ie} = (47)(0) = 0$

$N_{ir} = (47)(0) = 0$

$N_{im} = (47)(.037) = 2$

---

7 $$\text{Congestion(Repair)} = \sum_g \sum_{t'=105-4}^{t'=105-2} t' N_{irg} = 49$$

$M_\theta = .949$

#8 Congestion (MOD.) = $\sum_{g} \sum_{t'=105-8}^{t'=105-4} {}_{t'}N_{img} = 0$

$M_D = 1.000$

---

1 $[N_S - 10] = 48 - 10 = +$

$[{}_{104}N_I - 10] = 47 - 10 = +$

$N_{iE} = (.01)(47) = 0$

#10 $N_{ri} = ({}_{105-2}N_{ir})(M_\theta) = 0$

$N_{re} = (0)(1 - .949) = 0$

$N_{mi} = ({}_{105-4}N_{im})(M_D) = 0$

$N_{me} = (0)(1 - 1.000) = 0$

$\$_I = [{}_{104}N_I][10,000 + 50,000] + 30,000$

$= (47)(60,000) + 30,000 = \$2,850,000$

$\$_r = [(6000)(2) + 13,000] + 50,000 = \$75,000$

$\$_m = [(6000)(4) + 26,000] + 60,000 = \$110,000$

$\$_{e;E} = [1000 - 0][0 + 0 + 0 + 0] = 0$

$\$_T = \$3,035,000$

---

9 $N_{RR} = (47)(.962) = 45$

$A = 45/48 = .938$

#12 U.C.A. = 3,035,000/45 = $67,444

---

2 $N_I = 47 - 0 - 0 - 2 + 0 + 0 = 45$

---

3 $N_{e;T} = 0 + 0 = 0$

Test #	
1 a	$[N_S - N_{e;T}] = 48 - 0 = +$; $k = 2$? No;
	Set $k = 2$; $N_{S(104,2)} = 0$? Yes; $k = 2$? Yes;
# 3	$(105 - 1 - 104) = 0$
#13	Set $ACC_A = 0$, $j = 100$, $k = 1$; $N_{S(100,1)} = 0$? Yes;
	$k = 2$? No; set $k = 2$; $N_{S(100,2)} = 0$? No.
	$[N_{S(100,2)} - {}_{104}N_{e;T(100,2)}] = 50 - 1 = +$
	$ACC_A = 0 + {}_{105}N_{RR(100,2)} = 0 + 6 = 0$
	Similarly: When $j = 105 - 1$:
	$ACC_A = 0 + {}_{105}N_{RR(100,2)} + {}_{105}N_{RR(104,1)}$
	$= 0 + 6 + 45 = 51$
	$[{}_{105+8}N_R - ACC_A] = [310 - 51] = +$
	${}_{105}N_p = [310 - 51][.20] = 52$
#14	Set $j = (105 + 8) = 113$, $k = 1$
	${}_{105}\overline{\overline{C}}_{D(113,1)} = {}_{113}C_{Rmax} = 1400$
	$t_1 = 140$
	Set $k = 2$
	${}_{105}\overline{\overline{C}}_{D(113,2)} = 1.15(1400) = 1610$
	$t_2 = 145$
	$VALUK = \left[\frac{145-105}{140-105}\right]\left[\frac{18,000,000}{24,000,000}\right] = \left(\frac{40}{35}\right)\left(\frac{3}{4}\right) = \frac{10}{12} < 1$
	$[VALUK - 1.000] = -$
	Set $g = (105 + 8, 1) = (113, 1)$
	Set ${}_{105}N_{p(113,1)} = 52$
	${}_{105}N_{p(113,2)} = 0$

#24 Set $k = 1$

Congestion (Supply) $= \sum_{j} \sum_{t'=105-8}^{t'=105} {}_{t'}N_{p(j+8,1)} = 193$

$S_{(113,1)} = .600$

$k = 2?$; No; set $k = 2$

Congestion (Supply) $= \sum_{j} \sum_{t'=105-8}^{t'=105} {}_{t'}N_{p(j+8,2)} = 38$

$S_{(113,2)} = .830$

$k = 2?$ Yes

#25 Set $k = 1$

$N_{S(113,1)} = (52)(.600) = 31$

$k = 2?$ No; set $k = 2$

$N_{S(113,2)} = (0)(.830) = 0$

Test # 4 a

$[140 - 105] = +$; set $j = 105$, $k = 1$; $N_{S(105,1)} = 0?$ Yes; set $k = 2$; $N_{S(105,2)} = 0?$ Yes; $k = 2?$ Yes

$(105 - 1 - 105) = -$

#15 Set $ACC_A = 0$, $ACC_B = 0$, $j = 100$, $k = 1$

By the usual procedure the following are the only ones which are either + or 0:

$[N_{S(100,2)} - {}_{105}N_{e;T(100,2)}] = 50 - 7 = +$

$[N_{S(104,1)} - {}_{105}N_{e;T(104,1)}] = 48 - 0 = +$

Hence

$$ACC_A = 0 + {}_{105}N_{RR(100,2)} + {}_{105}N_{RR(104,1)}$$
$$= 0 + 6 + 45 = 51$$

$ACC_B = 0 + {}_{105}N_{S(100,2)} + {}_{105}N_{S(104,1)}$

$= 0 + 50 + 48 = 98$

Last j = 105? Yes; $A_F = 51/98 = .520$

#16 Like 15 to the place where

$ACC_A = 0 + {}_{105}N_{RR(100,2)} + {}_{105}N_{RR(104,1)} = 51$

Then S. R. 16 has

$ACC_C = 0 + {}_{105}\$_{T(100,2)} + {}_{105}\$_{T(104,1)}$

$= 0 + \$1,390,000 + \$3,035,000$

$= \$4,425,000$

j = 105? Yes

$[{}_{105}N_R - ACC_A] = [130 - 51] = 79 = +$

$\$_{S(105,1)} = 0$

$\$_{S(105,2)} = 0$

$\$_{T;F} = 0 + 0 + \$4,425,000 + [440,000][79]$

$= 4,425,000 - 34,760,000 = \$39,185,000$

#18 Like 15

$ACC_A = 51$

j = 105? Yes

$U.C.A._F = 39,185,000/51 = \$768,333$

Test # 4 b (140 - 105) = +

Set(j = 100; k = 1; t = 105 + 1 = 106), Then

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	t = 106 g = (100,2) $N_S = 50$
# 4	This is the start of the sixth cycle. Since methods of calculations have been shown several times for each subroutine, calculations will be stopped here.

VITA

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