Econometric Analysis

of the Beef and Pork Sectors of the Economy

Thomas D. Wallace and George G. Judge

Department of Agricultural Economics

OKLAHOMA STATE UNIVERSITY
INSTRUCTION RESEARCH EXTENSION
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Preface

This study is concerned with estimating the parameters of specified relations of the beef and pork sectors of the economy. As such, the presentation includes a technical discussion of the alternative methods of parameter estimation and a specification of alternative models to reflect the phenomena observed in the beef and pork sectors. Finally, the specified relationships are estimated by alternative estimation methods and the implications of the results for firm and government decision making are discussed.

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I. INTRODUCTION

A. The General Problem

With the many contributions of the physical sciences in the last few decades, and the increasing complexity of our socio-economic environment, decision making is a major activity at all structural levels. The most depressing situation that can confront the decision maker is one in which a multitude of alternatives exist with no means of assessing *a priori* the consequences of choice. At the opposite end of the continuum is that ideal state where out of a set of alternatives one action is clearly "best". Unfortunately this favorable condition is rare in most real problem situations. The problem is most often attacked by reducing the alternatives facing those responsible for choice to a finite set and predicting the consequences of a choice from that set in terms of likelihood.

As a first step, the body of logic known as economic theory provides the decision maker with a means of classification and qualitative estimation of the future course of economic variables under assumed conditions. However, it remains for the methods of economic measurement to provide quantitative prediction. Marshall *(24)*[1] anticipated the need for economic measurement in the statement:

> "Speaking generally the nineteenth century has in great measure achieved qualitative analysis in economics; but it has not gone further. It has felt the necessity of quantitative analysis, and has made some rough preliminary surveys of the way in which it is to be achieved: but the achievement itself stands over for you."

In recent years, much of our research efforts in applied economics has been oriented toward providing operational tools for the decision maker. From a practical standpoint, this work appears to be based on the premise that knowledge is useful if it helps to make the "best" decisions. Out of this research in economic measurement and related fields

1 Numbers in parentheses refer to the attached bibligraphy.

have stemmed such powerful tools as linear programming *(6, 7)*, game theory *(25)*, input-output analysis *(21)*, statistical decision functions *(31)*, and the compartmental simultaneous equations approach *(2, 17)*. The practical purpose of these research tools in quantitative economics is to provide the decision maker with "accurate" estimates of the future path, over time, of each variable of interest, corresponding to alternate levels of information and courses of action.

In the field of estimation, Stone *(29)*, views the objectives of quantitative economics as *(1)* settling questions of fact, *(2)* the testing of economic hypotheses, *(3)* parameter estimation, and *(4)* prediction. The first objective refers mainly to accounting and is not considered a research goal here. The methodology includes *(1)* constructing a model, *(2)* securing data to represent the variables included in the model, and *(3)* confronting the model with the data via estimation procedures. Constructing a model and choosing data necessitate choosing from a set of multiple, interdependent hypotheses with no criteria in many cases, other than the individual researcher's subjective discretion. In the model construction phase, Haavelmo *(11)* suggests autonomy as the guide-post. This emphasis is compatible with the recent trend away from such non-autonomous entities as market barometers as bases for decision making.

Recent work in the estimation of the parameters of economic relationships has stressed the importance of adapting statistical methods to the peculiarities of the data and the objectives of economic research. Recognition of the stochastic, dynamic and simultaneous nature of the generation of economic data has led to developments of techniques by Haavelmo *(12)* and other members of the Cowles Commission *(2, 17)* which are logically consistent with the interdependent characteristics of the economic system and the goals of structural estimation. The most recent contribution in estimation techniques is a method that may be attributed to Theil *(30)* and Basmann *(3)*.

Despite the advances made in estimating procedures there remain many unsolved problems in quantitative economic research. No really suitable methods have been discovered for eliminating many of the problems associated with the use of time series data. Model construction remains primarily a function of the knowledge and intuition of the individual researcher due to the lack of objective choice criteria and the absence of a generally accepted dynamic economic theory. A dearth of comprehensive, reliable data also troubles the econometrician. Many theoretically valid variables cannot be quantified properly or at all. The choice of algebraic form is another unsolved problem, again forcing the econometrician to use discretion rather than objective selection criteria. Current aggregation techniques are often unsuitable. Solutions to these problems will undoubtedly be an additive process—each piece of research contributing to the presumably unattainable Utopia of a completely objective methodology.

B. Specific Problem Area and Objectives

The beef segment of the economy is characterized by wide variations over time of such variables as price, production, consumption, and number of cattle on farms. Uncertainty as to the path over time that these variables will follow raises problems in regard to decisions relating to (1) resource use by the producing and processing firms, and (2) the consequences of alternative economic policies. Given this setting and the general purpose of providing information for decision making on the government and firm level, the major objective of this study is to employ certain techniques of statistical estimation in evaluating quantitatively those factors which appear to generate fluctuations in the quantity and price of beef produced and sold. In order to obtain quantitative approximations to some of the underlying relations, alternative models will be formulated to tie the many economic variables together in a complete, determinate system. The consequences of alternative model specification and methods on the attendant parameter estimates is a secondary objective of this study. In addition to the quantitative analysis of the beef sector of the economy, estimates of the underlying relations for pork will also be developed.

If the quantitative characteristics of the models, to be advanced, can be obtained, it will then be possible to predict with a specific level of probability, the future values of certain economic variables and the consequences of economic policies. Such knowledge of the structural relations for beef and pork is a prerequisite for intelligent formulation of government policy and for resource allocation by the firm.

As a corollary objective, this study should contribute to and stimulate improved methodological approaches in the general area of measurement in economics.

II. DEFINITIONS, ESTIMATION PROCEDURES AND TESTS

To avoid adumbrative references in the sections to follow, various terms and concepts currently used in the alternative estimation procedures will be briefly defined. Assumptions underlying the various methods of estimation will be made explicit and an expository account of the purposes of these methods, along with appropriate tests, will be presented.

A. Definitions and Concepts

1. The Economic Model

Construction of an economic model is an attempt to portray in a simplified way the underlying relations that reflect observable economic phenomena in some segment or the entirety of the economic system. The logic space of economic theory, augmented by special considerations of the sector to be depicted, acts as an aid in the task of constructing economic models. Since for many economic problems it is impractical to design an experiment, the model builder must choose a specification that will be reasonably consistent with the generation of the data. Obviously, abstraction is necessary in constructing a model—otherwise the model would be as cumbersome as the thing it represents, thus would offer no information not attainable by direct observation. Information gleaned from economic theory and from *a priori* knowledge of the sector being considered provides a foundation for (1) the kinds of relations to be considered, (2) the variables and their classification in each relation, and (3) certain qualitative restrictions on the relations. However, there exist many competing sets of *a priori* restrictions that can be imposed upon the structural equations without contradicting present knowledge of human behavior and environment. Therefore, several plausible variants of any descriptive effort are available. As viewed by Haavelmo (11), the building and choice of models is not a problem of pure logic, but of knowing something about real phenomena and making realistic assumptions about them. The construction of the economic model is perhaps the most important step in quantitative research. For this study, an economic model will be defined as that specification that results from considerations of economic theory and *a priori* knownedge of the phenomena to be explained.

2. Types of Equations

There are four types of equations that may be contained in an economic model. These are (a) behavior equations, (b) technical equations, (c) identities, and (d) institutional relationships. Behavior equations are so called because they are an attempt to depict man's reaction to economic stimuli. Examples are demand equations, supply equations, etc. Technical equations are used to express non-behavioral transformation relationships such as production functions. Identities are equations that express exact relationships. The remaining category, institutional equations, is postulated to represent the effect of institutional factors such as taxation upon the sector represented by a model.

3. Types of Variables

Variables are classified as (a) endogenous; those generated by the system characterized by the model, (b) exogenous; variables that affect the system but are in turn unaffected, (c) predetermined; observed values of variables determined independently of the current structural relations, and (d) disturbances or shocks; not directly observable random variables.

Predetermined variables are grouped with exogenous variables in terms of model specification. The classification of variables leaves considerable discretion to the individual researcher. For practical purposes, it is common to treat as exogenous those variables that may depend upon the system in part for their values but to an "insignificant" degree.

4. The Statistical Model

Preparing the economic model for estimation requires additional assumptions. For example, the algebraic form of the relations and the distributional properties of the variables must be specified. In completing these and other tasks, we are making a difficult transition from a theoretical model to one in which all variables are quantifiable and may be represented by real world counterparts. To make the result satisfactory, the resulting specification must be consistent with all of the assumptions of the investigator. To specify a statistical model, the researcher must choose among multiple, interdependent hypotheses, often with no better criteria than subjective discretion.

Given this formulation, a general statistical model may be written as

$$BY'_t + AZ'_t = U'_t \qquad t = 1, \ldots, T \tag{2.1}$$

where B is a G x G coefficient matrix of the Y's; Y'_t is the transpose of a 1 x G vector of endogenous variables; A is a G x H coefficient matrix of the Z's; Z'_t is the transpose of 1 x H vector of exogenous and/or predetermined variables; U'_t is the transpose of a 1 x G vector of disturbances (shocks).

A single equation, say the first, appearing in the general model may be expressed as

$$(\beta, 0)\ Y'_t + (\alpha, 0)\ Z'_t = U_{1t} \tag{2.2}$$

where $(\beta, 0)$ is 1 x G, the β partition being 1 x g and $(\alpha, 0)$ is 1 x H, the α partition being 1 x h.

5. Identification

Given the statistical model, the identification properties of the structural equations must then be considered prior to estimation. The problem of identification involves ascertaining (1) if each equation represents a definite, economic relationship, and (2) if the estimation of its structural parameters is possible. For the purposes of this study an equation is said to be identified if its parameters are uniquely determined from the specification of the model and the conditional distribution of the endogenous variables (13, *p.* 67). Conditions for the identification of equations in a model have been derived by Koopmans (18) and others (20).

Using the derivation of Koopmans and the notation developed in defining a statistical model, the necessary (but not sufficient) condition for a single equation to be just identified is

$$H - h = g - 1. \qquad (2.3)$$

Or in words, the number of exogenous variables not appearing in the equation (appearing with zero coefficients) must equal one less than the number of endogenous variables appearing in the equation with non-zero coefficients.

If an equation is underidentified, no method exists for estimating the parameters in that equation other than by ignoring the remainder of the system in which it appears. The underidentified case exists when

$$H - h < g - 1. \qquad (2.4)$$

The remaining case, where

$$H - h > g - 1 \qquad (2.5)$$

is termed overidentified. Methods have been developed that allow for estimation for cases (2.3) and (2.5,) and will be discussed later in the chapter.

B. Estimation Methods

The method used to estimate parameters associated with an equation contained in a statistical model should not violate the nature of the generation of the data and should be compatible with the assumptions basic to the model. Prior to the contribution by Haavelmo *(12)* the classical least squares method was the major technique used to accomplish estimation objectives. Use of the least squares technique requires one to ignore the simultaneous aspects of economic structures and violates the percept of interdependency of economic variables. This is not to say that all economic relationships are couched in a simultaneous and interdependent framework. In many instances the least squares method is compatible with the researcher's assumptions and his measurement objective. Indiscriminate use of any one estimation method has become obsolete, however, with the advances of the past decade in techniques of estimation. The purpose of this section is to briefly discuss the methods to be used in this study and to make explicit in each case the assumption upon which the estimation proceeds.

1. Single Equation-Maximum Likelihood

The single equation-maximum likelihood method is the classical least squares method with explicit assumptions that allow estimates to enjoy maximum likelihood properties. The general single equation model may be written

$$y_t = \sum_{i=1}^{h} \alpha_i z_{it} + u_t \qquad t = 1, \ldots, T \qquad (2.6)$$

where y_t is an endogenous variable, α_i are the coefficients of the z_{it}, the z_{it} are exogenous variables, and u_t is the equation disturbance.

The assumptions necessary to unbiased, consistent, and efficient estimates of the α_i are:

(a) The z_{it} are fixed and independent of u_t.

(b) The u_t are members of a fixed, normal distribution with zero mean and finite variance.

(c) The u_t are the result of incomplete specification and errors in observation on y_t only.

(d) The u_t are serially independent.

(e) The α_i are linear.

The method of estimating the α_i is to fit a geometric hyperplane that will minimize the sum of squares of error of the observations from that hyperplane parallel to the y axis.

2. Cowles Commission Methods

Again consider the simultaneous equation model of (2.1)

$$BY'_t + AZ'_t = U'_t \qquad t = 1, \ldots, T \qquad (2.7)$$

The general model may be written in reduced form as

$$Y'_t = \Pi Z'_t + V'_t \tag{2.8}$$

where $$\Pi = -B^{-1} A \tag{2.9}$$

and $$V'_t = B^{-1}U'_t \tag{2.10}$$

Maximum likelihood estimates of Π are the simple linear coefficients of all the y_{it} regressed on all the z_{it} as shown by Koopmans and Hood *(18, p. 147-50)*. The interest lies in estimating the parameters associated with a single equation, say the first, which can be written

$$(\beta, 0)\ Y'_t + (\alpha, 0)\ Z'_t = U_{1t} \tag{2.11}$$

Premultiplying the reduced form (2.8) by the vector $(\beta, 0)$ makes it identical with (2.11) and the following equivalence results by equating the coefficients of Z'_t.

$$(\beta, 0)\,\Pi = -(\alpha, 0) \tag{2.12}$$

The Π matrix may be partitioned as follows:

$$\Pi = \begin{Bmatrix} \Pi_1 & \Pi_2 \\ \Pi_3 & \Pi_4 \end{Bmatrix} \tag{2.13}$$

where Π_1 contains the coefficients resulting from regressing the y_{it} in the equation to be estimated on the z_{it} in the equation to be estimated. Π_2 results from fitting the y_{it} inside on the z_{it} outside; Π_3 is associated with fitting the y_{it} outside on the z_{it} inside and Π_4 results from fitting the y_{it} outside on the z_{it} outside.

Employing the partition and carrying out the multiplication results in the two equations

$$\beta\Pi_1 = -\alpha \tag{2.14}$$

$$\beta\Pi_2 = 0 \tag{2.15}$$

Maximum likelihood estimates of β and α are directly obtainable from (2.14) and (2.15) if the equation is just identified, due to the invariant properties of maximum likelihood estimates. The equation (2.15) represents a homogenous system of H — h equations in g unknowns. Since we normalize on one of the y_{it} a necessary and sufficient condition that a unique olution exists is that the rank of Π_2 must be g—1. If the rank is less than g — 1, the equation is underidentified and no solution exists and if the rank is greater than g — 1, the limited information method may be employed. The following paragraph gives a heuristic explanation of this method.

By ignoring part of the information contained in the model and maximizing the likelihood of a subset of the general model, estimates are obtainable for the parameters of an overidentified equation that are asymptotically unbiased (consistent) and as efficient as any other method using the same amount of information. The mathematical derivation (see Koopmans and Hood *(19)*) leads to a homogenous system of equations involving β and a g x g matrix that is forced in the overidentified case to meet the rank conditions by employing the smallest characteristic root of that matrix. Estimates of β can then be substituted into the relationship involving α to obtain estimates of α.

The assumptions underlying the Cowles Commission methods are:

(a) The z_{it} are fixed and independent of the U_{it}.

(b) The U_{it} are distributed normally with zero mean and finite variance.

(c) $E\ (U_{it}, U_{jt}) = \sigma_{ij}$

(d) $E\ (U_{it}, U_{it-\theta}) = 0;\ \theta \neq 0$

(e) The U_{it} are the results of incomplete equation specifications.

(f) The elements of B and A are linear.

3. Thiel-Basmann Method

A single equation of the general model may be written in normalized form as:

$$y_{1t} = \sum_{i=2}^{g} \beta_i y_{it} + \sum_{i=1}^{h} \alpha_i z_{it} + u_{it};\ t = 1, \ldots, T \qquad (2.16)$$

Given this formulation, Basmann (3) shows that consistent estimates are obtainable for β and α by replacing the y_{it} with $\hat{y}_{it}$ ($i = 2, \ldots, g$) in (2.16) where the vector

$$\hat{\bar{Y}}'_t = (M\bar{y}z\ Mzz^{-1})\ z'_t \qquad (2.17)^{1}$$

Or simply, the y_{it} are replaced by least squares estimates $\hat{y}_{it}$ where each y_{it}, $i = 2, \ldots, g$ is regressed on every exogenous variable in the system. Making the substitution, equation (2.16) may be written:

$$y_{1t} = \sum_{i=2}^{g} \beta_i \hat{y}_{it} + \sum_{i=1}^{h} \alpha_i z_{it} + e_t;\ t = 1, \ldots, T \qquad (2.18)$$

[1]M refers to second order moment matrix; the bar indicates $i = 2, \ldots, g$.

where e_t represents a new error term. The estimation of β_i and α_i proceeds then by applying the classical least squares method to the transformed equation (2.18). The least squares procedure for obtaining estimates of variance and standard errors of the coefficients also applies. The estimates obtained are consistent since it can be shown that:

$$\text{Plim } (\hat{y}_{it}, e_i) = 0 \text{ as T goes to infinity.} \quad (2.19)$$

However, the $\hat{y}_{it}$ and e_i being uncorrelated in the limit is not sufficient to insure best linear unbiased estimates of the β_i and α_i. Since the Theil-Basmann method uses information contained in all the exogenous variables in the complete system, it approaches the efficiency of the method of limited information. See Appendix B for a computational approach for this method.

C. Tests to be Employed

1. Non-StatisticalTests

In interpreting the empirical results of this study, an effort will be made to assess the validity of the estimates in terms of their theoretical counterparts, i.e. the extent to which the signs, magnitudes, or relative magnitudes of the parameter estimates agree with *a priori* expectation will be noted. This type of comparison can be called "testing" only in the loosest sense. When a parameter estimate conflicts with economic preconception, little can be said except that the divergence exists. One hesitates to cast aside accepted reasoning on the basis of one observation, but any attempt to "explain" a nonsensical estimate is often abortive since any of a number of interdependent causes is feasible. It proves irresistable, however, to offer "explanations" for aberrant estimates and this leads to a sequential type of analysis.

2. Statistical Tests

In order to judge the validity of certain *a priori* assumptions and restrictions imposed on a model, the following statistical tests will be employed.

a. *Durbin-Watson (8) Test for Serial Correlation*: As stated previously, one of the assumptions necessary to insure that estimates have certain desirable properties is that the disturbance variables are independent over time. To test this assumption, the following statistic will be calculated.

$$d^2/s^2 = \sum_{t=1}^{T} (u_t - u_{t-1})^2 / \sum_{t=1}^{T} u_t^2 \quad (2.20)$$

The distribution of d^2/s^2 has been partially tabulated by Durbin and Watson (8, p. 409).[1]

b. *Rubin-Anderson (2, p. 56) Test:* Identification depends on the restrictions imposed *a priori* concerning the appearance and non-appearance of variables in a particular equation. The Rubin-Anderson test was designed to test the assumption that certain coefficients are zero in a particular equation, given the validity of the other specifications. The statistic, $T \log_e (1 + 1/\lambda)$ has been shown to be distributed as X^2 (chi-square) with degrees of freedom corresponding to the number of overidentifying restrictions of the particular equation. T refers to the sample size and λ is the largest characteristic root of the matrix associated with final solution of β estimates in the limited information technique. If the computed statistic is larger than the corresponding X^2 value for a given level of confidence, the hypothesis that the overidentifying restrictions are valid is rejected.

c. *Test for Significance of Parameter Estimates:* Standard errors will be computed for all parameter estimates. The statistic $\beta/s(\hat{\beta})$ or $\hat{a}/s(a)$ where s indicates standard error is distributed as t with T—M—1 degrees of freedom. T refers to sample size and M refers to the number of coefficients estimated in the equation. A procedure for computing standard errors for limited information estimates is given in Appendix C.

III. THE MODELS

The economic models to be presented represent an attempt to portray in a simplified way the underlying relations that reflect observable economic phenomena in the beef and pork sectors of the economy. The role of the model is to provide a logical route to go from the characteristics of the real world to predictions about it. Economic theory, based on certain fundamental assumptions about maximization and the generation of economic variables, supplemented by knowledge of the special characteristics of an industry forms a basis for the make up of the structural equations and the classification of variables. However, there are no definitive set of rules for model construction, and for any particular problem several plausible variants of the model may be available.

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[1]For some of the shortcomings of using this test with the estimating methods employed in this study, see: C. Hildreth and F. G. Jarrett (13, pp. 77, 78).

Realizing that many competing sets of *a priori* restrictions can be imposed upon the structural equations, the procedure followed in this study was to first construct a relatively simple and static model of the beef and pork sectors of the economy. On the basis of this formulation, a second model was then constructed which included additional explanatory variables and recognized more completely the dynamic aspects of this sector of the economy. By utilizing this sequential scheme for model construction, it was felt that a more realistic model of the beef and pork sectors resulted. It is not maintained that the real world is actually described by the models to be presented. The desire and necessity for simplicity and, for example, the use of linear models with discrete time lags mean that the specifications are at best only approximations.

In presenting the models, the variables which are assumed to enter each structural equation will be specified and the logic for including these variables and their specification will be briefly discussed. Neither evidence like that alluded to (logic underlying certain formulation) nor any other can justify such a specification. However, the purpose is rather to show that observation does not render it absurd to suppose that a hypothesis embodying a certain specification can yield a fairly close approximation to observed behavior.

It has been noted that the special characteristics of the particular sector to be described is one important source of information in model building. Recognizing this source of information, a brief discussion of some descriptive models of the beef economy will, therefore, precede the presentation of the models.

A. Alternative, Qualitative Models

The cattle economy has been characterized by the cyclical nature of such factors as cattle numbers, production and price. The most clearly defined cycle more often referred to is that of numbers of cattle on farms and ranches. The length of the cycle in cattle numbers has varied from ten to twelve years over the past several decades. Lorie (22) suggests that production leads numbers by approximately two or three years and because of inverse correlation with production, price lags number by about two or three years.

By observation and synthesis, those interested in the industry have developed theoretical explanations of the cyclical phenomena by classifying and grouping obvious variables associated with the cattle sector. These explanations offer an additional source of material for purposes of model construction, and therefore, should be reviewed. A logical, if rather arbitrary, framework will be used to present these explanations along the lines used by Breimeyer (4).

1. The Internal Approach

Assume a trough in numbers of cattle. Inventories of breeding stock have been depleted; production is retroceding. Market adjustments to the receding production result in increased prices. Cattle producers, anticipating continuing price increases, retain young heifers and cows past prime productivity that would ordinarily be marketed in an attempt to restock depleted inventories. As a result of this action, production continues downward, thus stimulating further increases in price. The spiral continues until production from expanded inventories satisfies demand. Then price levels off. Cattlemen interpret the leveling off of price as an incentive to reduce inventory. Marketing of breed stock, in addition to regular marketings, causes market saturation, decreases prices and the downward spiral continues until a new low is reached.

The length of the upswing of the cycle is attributed to the reproductive biology of cattle. A new-born heifer takes three to five years to become productive of market size beef, hence the decision to expand production is not reflected for five or six years. Once the decision to reduce inventory is made, young heifers and breed stock past prime are marketed. With few young heifers being held for replacement purposes and the old stock being culled, the length of the downswing is also about five or six years. Among those responsible for the internal explanation are Lorie (*22*) and Ezekial (*9*).

2. The External Approach

Some individuals interested in the cyclical nature of factors characterizing the cattle industry have stressed exogenous causation as most important. Among these persons, Burmeister (*5*) and Pearson (*27*) may be included. Variables such as rainfall, feed supplies and pasture conditions are postulated as shifting the supply schedule for beef in a manner so as to generate periodic fluctuations.

A natural synthesis of the two diverse theoretical approaches is to consider all factors, both exogenous and endogenous, as determining the course of the relevant variables associated with the cattle industry. The empirical results of this research will be compared with the theoretical constructs of both approaches in an effort to assess the likelihood of their validity.

B. Economic Model I

For this, as well as the second model, the exogenous and predetermined variables will be designated by z_{it} and the endogenous variables will be represented by y_{it}. β_{ij} and α_{ij} will be used to denote unknown constants to be estimated and the U_{it} will represent random disturbances.

1. The Demand for Beef at Retail

From the theory of consumer choice and market equilibrium, demand for a commodity is postulated as being a function of the price of the commodity, prices of all substitutes and complements, and consumer income. Employing this conceptual framework, pork will be postulated as the most important substitute for beef. Such items as fish, mutton, veal, eggs, etc. will be omitted. To include all possible substitutes and complement separately would increase the model to unmanageable proportions, and to include an index of all prices would be so aggregative that little information would be gained.

Population logically should be included to reflect shifts in aggregate demand due to changes in this factor. Lagged consumption of beef is included on the basis that consumers may be slow to respond to changing market conditions in a dynamic setting. A trend variable is included to reflect possible changes in consumption over time and to adjust for any other economic factors that change at a constant rate per unit of time.

Following the reasoning as outlined, the aggregate demand for beef at retail is postulated as:

$$\beta_{11}y_{1t} + \beta_{12}y_{2t} + \beta_{13}y_{3t} + \alpha_{11}z_{1t} + \alpha_{12}\, z_{2t} + \alpha_{13}z_{3t} + \alpha_{14}z_{4t} = u_{1t} \qquad (3.1)$$

where y_{1t} is total consumption of beef; y_{2t} is the retail price of beef; y_{3t} is the retail price of pork; z_{1t} is disposable income; z_{2t} is time; z_{3t} is consumption of beef, lagged one year; z_{4t} is population as of July 1 each year, and u_{1t} is a random disturbance. Income and population are considered exogenous by employing the causal principle. Prior to estimating this equation, the variables y_{1t}, z_{1t} and z_{3t} were deflated by population and, therefore, z_{4t} was omitted.

2. Supply of Beef at Retail

The supply of beef at retail is a derivation of supply of beef at the farm. Given farm production, the remaining factors specified to enter this equation are assumed to affect the farmers decision to sell or hold stock during time period t, or are assumed to affect the average weight of cattle marketed during the t^{th} time period, or both. These considerations lead to the following postulation for the supply of beef at retail:

$$\beta_{21}y_{1t} + \beta_{24}y_{4t} + \beta_{25}y_{5t} + \beta_{26}y_{6t} + \beta_{27}y_{7t} + \beta_{28}y_{8t} + a_{25}z_{5t} + a_{26t}z_{6t} = u_{2t} \qquad (3.2)$$

where y_{4t} is the farm price of beef; y_{5t} represents the farm price of pork; y_{6t} is the farm price of feed grains; y_{7t} is farm production of beef; y_{8t} is price of feeders and stockers at Kansas City; z_{5t} represents beef calves and steers on farm, January 1; and z_{6t} represents range conditions as of July 1.

3. Supply of Beef at the Farm

Farm production of beef, a physical relationship, is postulated as being determined by beginning inventory, range conditions, supply of feed grains and time. A trend variable in a supply equation is used to reflect possible technological change over time. Inclusion of other feed variables, such as protein feeds and supply of hay, was deemed inadvisable because of high intercorrelation among all feed variables. Hildreth and Jarrett *(13)* sought to combine all feeds on total digestible nutrient bases to avoid this difficulty in their aggregate livestock study, but following the original plans of keeping aggregation to a minimum, representative variables were employed in constructing this equation, while seemingly obvious ones were omitted. These considerations led to the following construction.

$$\beta_{37t}y_{7t} + a_{32}z_{2t} + a_{36t}z_{6t} + a_{37t}z_{7t} + a_{38}z_{8t} = u_{3t} \qquad (3.3)$$

where z_{7t} is the total inventory of beef cattle plus dairy cows two years old and older on farms and ranches, January 1; z_{8t} represents the availability of feed grains during the crop year beginning October 1 lagged one year. All other variables have been defined prior to the specification of this equation.

4. Demand for Beef at the Farm

The demand for beef at the farm is a derived demand. The farm price of beef is postulated as being interdependently related with retail price of beef, farm price of pork and farm production of beef. Wage rates in the slaughtering industry are included to reflect marketing costs. Time is interpreted here similar to the previous usage in the demand at retail equation.

$$\beta_{42}y_{2t} + \beta_{46}y_{4t} + \beta_{45}y_{5t} + \beta_{47}y_{7t} + a_{42}z_{2t} + a_{49}z_{9t} = u_{4t} \qquad (3.4)$$

where z_{9t} represents average hourly wage rates in the slaughtering industries and the other variables are reflected by previous definition.

5. **Demand for Pork at Retail**

The same logic used in specifying the demand for beef at retail was used in the construction of this equation.

$$\beta_{52}y_{2t} + \beta_{53}y_{3t} + \beta_{59}y_{9t} + a_{51}z_{1t} + a_{52}z_{2t} + a_{54}z_{4t} = u_{5t} \qquad (3.5)$$

where y_{9t} represents retail production of pork and all other variables have been defined by previous usage. Again, the assumption is made that consumption and retail supply are equivalent. Similar to the demand for beef at retail, the variables y_{9t} and z_{1t} were deflated by z_{4t} and z_{4t} was omitted prior to the estimation of this equation.

6. **Supply of Pork at Retail**

Analogous to the postulated retail supply of beef, the retail supply of pork is specified as being functionally related with farm price of beef and pork, farm price of feed grains, farm production of pork and time.

$$\beta_{64}y_{4t} + \beta_{65}y_{5t} + \beta_{66}y_{6t} + \beta_{69}y_{9t} + \beta_{6,10}y_{10t} + a_{62}z_{2t} = u_{6t} \qquad (3.6)$$

y_{10t} represents farm production of pork. All other variables have been defined prior to the specification of this equation.

7. **Supply of Pork at the Farm**

Farm production is viewed as a transformation resulting from beginning inventory, the availability of feed and time. Here the trend variable, as in all supply relationships in which it appears, is purported to reflect possible changes in technology over time. Two separate variables are postulated as reflecting beginning inventory.

$$\beta_{7,10}y_{10t} + a_{72}z_{2t} + a_{78}z_{8t} + a_{7,10}z_{10t} + a_{7,11}z_{11t} = u_{7t} \qquad (3.7)$$

z_{10t} represents the total number of gilts and sows six months old and older on farm, January 1, while z_{11t} reflects the total number of pigs zero to six months old on farm, January 1. All other variables appearing in the farm supply of pork relationship have been defined previously.

8. **Demand for Pork at the Farm**

Following much the same reasoning as used in specifying the farm demand for beef relationship, farm price of pork is postulated as being related with the retail price of pork, farm production of pork, time, and wage rates in the slaughtering industries.

$$\beta_{83}y_{3t} + \beta_{85}y_{5t} + \beta_{8,10}y_{10t} + a_{82}z_{2t} + a_{89}z_{9t} = u_{8t} \qquad (3.8)$$

All variables in the above equation are reflected by previous definition.

9. Demand for Feeder Cattle

The demand for feeder cattle is a reflection or derivation of the retail demand for beef. The price of feeder cattle was thought to be associated with the retail price of beef, the farm price of pork, the farm price of feed grains and the total number of feeder cattle bought. The number of beef steers and calves on farm, January 1, should indicate the size of the resource pool from which feeder cattle originate.

$$\beta_{92}y_{2t} + \beta_{95}y_{5t} + \beta_{96}y_{6t} + \beta_{98}y_{8t} + \beta_{9,11}y_{11t} + a_{92}z_{2t} + a_{95}z_{5t} = u_{9t} \qquad (3.9)$$

y_{11t} reflects the number of feeder cattle bought. No attempt was made to gather data to represent y_{11t} since this equation will not be estimated.

10. Supply of Feeder Cattle

The supply of feeder cattle is postulated as functionally related with the retail price of beef, the price of feeder cattle, the number of calves and steers on farms, January 1, and range conditions. Range feeding represents an alternative process, thus, range conditions should indicate the favorability of this alternative.

$$\beta_{10,2}y_{2t} + \beta_{10,8}y_{8t} + \beta_{10,11}y_{11t} + a_{10,5}z_{5t} + a_{10,6}z_{6t} = u_{10t} \qquad (3.10)$$

All variables have been previously defined.

11. Demand for Feed Grains

Feed grains are a major resource used in beef and pork production, thus, the demand for feed grains is a derived demand dependent upon the demand for meat products. The price of feed grains is postulated as being functionally related to farm price of pork, farm price of beef, pasture conditions, availability of feed grains, total grain-consuming animal units fed and time.

$$\beta_{11,4}y_{4t} + \beta_{11,5}y_{5t} + \beta_{11,6}y_{6t} + a_{11,2}z_{2t} + a_{11,6}z_{6t} + a_{11,8}z_{8t} + a_{11,12}z_{12t} = u_{11t} \qquad (3.11)$$

z_{12t} represents total grain-consuming animal units fed in year, beginning October 1, lagged one year. All other variables have been defined previously.

C. Economic Model II

Model II represents a reformulation of Model I to include additional explanatory variables and recognize more completely the dynamic aspects of the sectors under investigation.

1. **Demand for Beef at Retail**

This equation remains unchanged from Model I.

$$\beta_{11}y_{1t} + \beta_{12}y_{2t} + \beta_{13}y_{3t} + a_{11}z_{1t} + a_{12}z_{2t} + a_{13}z_{3t} + a_{14}z_{4t} = u_{1t}$$

2. **Supply of Beef at Retail**

$$y_1 = y_{12} \cdot y_{13} \qquad (3.12)$$

This equation is specified as an identity with y_{12} representing total numbers of mature cattle slaughtered annually and y_{13} the average weight of mature cattle slaughtered.

The following two equations represent an attempt to explain y_{12} and y_{13} for the t^{th} time period on the basis of predetermined variables. The assumption is that cattlemen are guided in their actions by conditions prevailing in a previous time period.

$$\beta_{2,12}y_{12t} + a_{22}z_{2t} + a_{2,13}z_{13t} + a_{2,14}z_{14t} + a_{2,15}z_{15t} + a_{2,16}z_{16t} = u_{2t} \qquad (3.13)$$

where z_{13t} is the number of feeder cattle on feed, January 1; z_{14t} represents number of beef cattle and calves not on feed plus number of dairy cows two years old and older on farm, January 1; z_{15t} is the real farm price of corn lagged one year and z_{16t} is the real farm price of beef lagged one year. All other variables are reflected by previous definition.

$$\beta_{3,13}y_{13t} + a_{32}z_{2t} + a_{3,13}z_{13t} + a_{3,16}z_{16t} + a_{3,17}z_{17t} + a_{3,18}z_{18t} = u_{3t} \qquad (3.14)$$

where z_{17t} is the production of corn for livestock plus stocks October 1, lagged one year; z_{18t} is y_{13t} lagged one year and the remaining variables are as defined previously.

3. **Supply of Beef at the Farm**

The farm supply of beef equation remains essentially the same as in Model I, except that the availability of corn is substituted for feed grains, the inventory variable is separated into cattle on feed and cattle not on feed, and range conditions are omitted. The only data available to reflect range conditions was felt to be so arbitrary that this variable was omitted in the reformulation of Model I even though it is a theoretically valid factor.

$$\beta_{47}y_{7t} + a_{42}z_{2t} + a_{4,13}z_{13t} + a_{4,14}z_{14t} + a_{4,17}z_{17t} = u_{4t} \qquad (3.15)$$

All variables are as defined previously.

4. **Demand for Beef at the Farm**

This demand for beef at the farm is again postulated as:

$$\beta_{52}y_{2t} + \beta_{54}y_{4t} + \beta_{55}y_{5t} + \beta_{57}y_{7t} + \alpha_{52}z_{2t} + \alpha_{59}z_{9t} = u_{5t} \quad (3.16)$$

5. **Demand for Pork at Retail**

The demand for pork equation was reformulated to include lagged consumption of pork.

$$\beta_{62}y_{2t} + \beta_{63}y_{3t} + \beta_{69}y_{9t} + \alpha_{61}z_{1t} + \alpha_{62}z_{2t} + \alpha_{64}z_{4t} + \alpha_{6,19}z_{19t} = u_{6t} \quad (3.17)$$

where z_{19t} represents consumption of pork lagged one year.

6. **Supply of Pork at Retail**

The price of feed grains is omitted in this model and the price of corn included in the supply of pork at retail equation. Otherwise, this equation is unchanged from Model I.

$$\beta_{74}y_{4t} + \beta_{75}y_{5t} + \beta_{79}y_{9t} + \beta_{7,10}y_{10t} + \beta_{7,14}y_{14t} + \alpha_{72}z_{2t} = u_{7t} \quad (3.18)$$

where y_{14t} repreesnts the farm price of corn.

7. **Supply of Pork at the Farm**

Again substituting corn for feed grains, the supply of pork at the farm is postulated as:

$$\beta_{8,10}y_{10t} + \alpha_{8,2}z_{2t} + \alpha_{8,10}z_{10t} + \alpha_{8,11}z_{11t} + \alpha_{8,17}z_{17t} = u_{8t} \quad (3.19)$$

8. **Demand for Pork at the Farm**

This equation is unchanged from Model I.

$$\beta_{93}y_{3t} + \beta_{95}y_{5t} + \beta_{9,10}y_{10t} + \alpha_{82}z_{2t} + \alpha_{89}z_{9t} = u_{9t} \quad (3.20)$$

9. **The Demand for Corn**

The demand for corn for grain depends to a large extent on the demand for meat products, thus, the farm price of beef and pork is postulated in the relationship along with the farm price of corn and the production of corn for grain. Total number of grain-consuming animal units eating out of feed supplies is included as a shift variable. Time is included to account for possible changes in feeding technology. Thus, the demand for corn is postulated as:

$$\beta_{10,4}y_{4t} + \beta_{10,5}y_{5t} + \beta_{10,14}y_{14t} + \beta_{10,15}y_{15t} + \alpha_{10,2}z_{2t} + \alpha_{10,12}z_{12t} = u_{10t} \quad (3.21)$$

when y_{15t} represents annual production of corn for grain.

10. The Supply of Corn

The supply of any grain depends to a great extent upon weather conditions. Lacking a variable to properly reflect this factor and others, unspecified factors are included in this relationship along with the price and production of corn, trend and number of acres planted annually in corn.

$$\beta_{11,14}y_{14t} + \beta_{11,15}y_{15t} + a_{11,2}z_{2t} + a_{11,20}z_{20t} + aK_t = u_{11t} \qquad (3.22)$$

where z_{20t} is total number of acres planted in corn (for grain) and K represents unspecified factors.

D. Discussion of the Models

Model I comprises a complete system of equations. This complete system involves 12 equations, 12 random residuals denoted by u_{it} and 12 endogenous or simultaneously observed variables denoted by y_{it}.

Model II as specified contains 12 equations, 12 endogenous variables and 12 random residuals. Equation (3.22) of this model is incompletely specified, i.e. the symbol K appearing in the equation represents unsecified variables. This means that the model is incomplete and thus may affect the efficiency of the estimates (1, 17, p. 393-409).

E. Algebraic Form and Data

Secondary data from various source publications of the Federal Government were used to reflect the variables included in both models. The sample time periods were 1925 through 1955 with 1934 omitted for estimating the supply relationships of both Model I and Model II. 1934 was omitted from the sample because of the large slaughter of that year induced by government programs. In estimating the demand relationships, the war years 1942-46, inclusive, were omitted because of the prevalence of rationing and price-setting policies in effect during most of that time period. All data were converted to a 1947-49 base index, and all price and income series were deflated by the consumer price index.

In regard to algebraic form of the structural equations, many alternatives exist, but only variables expressed in natural units or logarithms appear statistically tractable. There is a little *a priori* reason for choosing the results given by one rather than the other. The logarithmic form does have the advantage of flexibility and the resultant estimates can be interpreted directly as elasticities. Lacking an operational choice indicator, a functional form linear in the logarithms of the observed variables was used except for the variable z_{2t} (time). The time variable was specified as linear in the natural units. In the empirical estimates to follow, capital letters will be used to denote logarithms of the observed variables, i.e. $Y_{it} = \log y_{it}$, $Z_{it} = \log z_{it}$.

Having given the functional form of the equations and the sample time period used to reflect the postulated variables, it is now left to specify the assumptions relative to the way the relations are affected by unobserved influences. In this connection the unobserved random disturbances u_{it} are assumed to come from a multivariate normal distribution with zero means and a finite covariance matrix. They are assumed to be independent over time and their distribution is assumed to remain constant over the observation period. Since the joint distribution of the endogenous variables y_{it}, for any given t is given explicitly by the joint probability distributions of the disturbances u_{it} it is this specification that must form the basis for the estimation of the unknown parameters, the β_{ij} and α_{ij}.

It is the specifications given in this section that transforms the constructions from an economic to a statistical model.

IV. THE EMPIRICAL RESULTS

In this section, the results of confronting the models with the data via estimation procedures will be presented. By employing the counting condition, the retail demand equations for beef and pork were overidentified in both models. Therefore, for these equations the limited information method was used as the estimation technique. The Theil-Basmann method and the classical method of least squares were also employed for comparative purposes. The farm supply equations for beef and pork in both models were assumed to meet the requirements necessary for estimation by the single equation — maximum likelihood method. For Model II the equation representing retail supply of beef were also estimated by single equation—maximum likelihood methods.

The resulting estimates will be examined to determine their agreement or disagreement with theory in regard to sign and examples of the economic interpretation of the estimates will be given. Results of the statistical tests will be presented with standard errors of estimates directly below the coefficients and the other tests below the equation to which they apply. Comparisons between like equations for the two models will be made and the results of applying various estimation techniques to particular relationships will be discussed.

Since the postulated form of the relationships is linear in logarithms, the coefficients may be interpreted directly as elasticities. In the demand relationships, the quantity and income variables have been deflated by population.

Finally, an attempt will be made to point up economic implications of the results for both the firm and the policy planner.

A. The Demand Equations for Beef

Since retail demand equations for beef for each of the two models were overidentified, they were estimated by the limited information method. The Theil-Basmann method and the least squares technique were also employed for comparative purposes.

1. Equation 1—Model 1 (Demand for Beef at Retail)

(a) *Limited Information Estimates:* The parameters associated with this relationship were estimated by the method of limited information as:

$$Y_{1t} = \underset{(.2830)}{-1.3566Y_{2t}} + \underset{(.1888)}{.2015Y_{3t}} + \underset{(.1674)}{.9720Z_{1t}} + \underset{(.0147)}{.0654z_{2t}} + \underset{(.1434)}{.0804Z_{3t}} + 2.0188 + U_{1t} \qquad (4.1)$$

$$\hat{\sigma}^2 = .0202 \qquad d^2/s^2 = 1.10\dagger \qquad (4.2)$$

$$T \log_e (1 + v) = 21.5 \qquad (4.3)$$

Estimates of β and α do not conflict with theoretical preconception in regard to sign. The estimates may be interpreted in the form of the following *ceteris paribus* economic statements:

(1) A 1 per cent increase in the real retail price of beef would result in approximately a 1.36 per cent decrease in the per capita consumption of beef.

(2) A 1 per cent increase in the real retail price of pork would result in approximately a 0.20 per cent increase in the per capita consumption of beef.

(3) A 1 per cent increase in real per capita disposable income would result in approximately a 0.97 per cent increase in the per capita consumption of beef.

(4) A 1 per cent increase in the per capita consumption of beef in time period t — 1 would result in approximately a 0.08 per cent increase

in the per capita consumption of beef for time period t.

The trend variable, z_{2t}, was entered as linear in natural units, therefore, its parameter estimate cannot be interpreted directly as an estimate of elasticity. The positive trend coefficient implies a constant rate of increase in beef consumption over the time period sampled, given certain levels of prices and incomes.

The Durbin Watson statistic, concerning the independence over time of the disturbances, was estimated at 1.10. This estimate fell in the inconclusive range of tabular values, therefore, no statement can be made concerning the non-autocorrelation assumption. For other equations this statistic will not be discussed. No symbol will indicate an unfavorable test, a (†) will indicate inconclusive results and a (*) will be used to indicate a favorable test result.

The overidentifying restrictions test statistic for this relationship was such that the hypothesis that the overidentifying restrictions were valid is rejected at the 99 per cent confidence level.

(b) *Theil-Basmann estimates:* Application of the Theil-Basmann method to the retail beef demand equation resulted in the following parameter estimates:

$$Y_{1t} = \underset{(.1266)}{-.7730\,Y_{2t}} + \underset{(.1108)}{.1960\,Y_{3t}} + \underset{(.1120)}{.6048\,Z_{1t}} + \underset{(.0114)}{.0472\,z_{2t}} + \underset{(.1184)}{.2142\,Z_{3t}} + 1.3917 + U_{1t} \quad (4.4)$$

$$\hat{\sigma}^2 = .00037 \qquad d^2/s^2 = 1.64^* \quad (4.5)$$

Again, estimates obtained using this method do not conflict with *a priori* reasoning regarding signs of the coefficients. Using the simple ordering principle for comparison, the Theil-Basmann estimates of price and income elasticity are considerably lower in magnitude than the same estimates obtained by using the limited information technique.

(c) *Least Squares estimates:* Results of the application of the least squares method to the retail demand for beef relation are as follows:

$$Y_{1t} = \underset{(.1041)}{-.7555\,Y_{2t}} + \underset{(.0976)}{.2277\,Y_{3t}} + \underset{(.1048)}{.5757\,Z_{1t}} + \underset{(.0106)}{.0482\,z_{2t}} + \underset{(.1108)}{.2004\,Z_{3t}} + 1.3787 + U_{1t} \quad (4.6)$$

$$R^2 = .9468 \qquad d^2/s^2 = 1.66^\dagger \quad (4.7)$$

The signs of all coefficients are again compatible with *a priori* reasoning. In terms of magnitude, the estimates obtained by least squares approximate more closely the Theil-Basmann estimates than coefficients obtained by employing the method of limited information.

2. Equation 1—Model II (Demand for Beef at Retail)

(a) *Limited Information estimates:* Applying the limited information technique to Equation 1—Model II resulted in the following estimates:

$$Y_{1t} = \underset{(.3001)}{-.8651\, Y_{2t}} + \underset{(.2868)}{.3231\, Y_{3t}} + \underset{(.1680)}{.5912\, Z_{1t}} + \underset{(.0201)}{.0560\, z_{2t}} + \underset{(.2055)}{.1226\, Z_{3t}} + 1.5140 + U_{1t} \tag{4.8}$$

$$\hat{\sigma}^2 = .0074 \qquad d^2/s^2 = 2.12^* \tag{4.9}$$

$$T \log_e (1 + v) = 4.35 \tag{4.10}$$

No divergence in sign is discerned between the empirical estimates and theory. There were 10 degrees of freedom associatd with the overidentifying restrictions test statistic and when compared to the appropriate X^2 values, the hypothesis that the overidentifying restrictions are valid, cannot be rejected.

(b) *Theil-Basmann estimates:* Results of applying the Theil-Basmann procedure to equation 1—Model II, are as follows:

$$Y_{1t} = \underset{(.1058)}{-.7707\, Y_{2t}} + \underset{(.1014)}{.2383\, Y_{3t}} + \underset{(.1052)}{.5793\, Z_{1t}} + \underset{(.0108)}{.0491\, z_{2t}} + \underset{(.1125)}{.1911\, Z_{3t}} + 1.3970 + U_{1t} \tag{4.11}$$

$$\hat{\sigma}^2 = .00036 \qquad d^2/s^2 = 1.63^* \tag{4.12}$$

It is again notable that Theil-Basmann estimates of price and income elasticities are smaller in magnitude than estimates determined by using the method of limited information.

3. Intermodel Comparisons of the Demand for Beef Equation[1]

The extent to which the signs, magnitudes, or relative magnitudes of the various estimated coefficients agree with the *a priori* expectations

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[1]Parameter estimates for beef from other selected studies are given in Appendix D.

advanced has been noted for each equation. In each equation, the estimated parameters conform to *a priori* expectation as to the appropriate sign. Also, in each case, the magnitude of the parameter estimates could be regarded as plausible from *a priori* knowledge of the underlying relationships of the models. Except for equation (4.1), differences among the parameter estimates resulting from the alternative methods and models do not seem unreasonable when compared to the indicated magnitude of sampling fluctuations. However, for equation (4.1), the price and income elasticity estimate differences are large enough to have practical consequences if their relaibility were firmly established. Although an invariant statement regarding estimate choice is not possible, it should be noted that the extreme value obtained for the overidentifying test statistic of equation (4.1) is an indication of difficulty somewhere in the statistical specification used. This observation, along with what is thought to be a superior specification for Model II, leads to the recommendation that Model II be employed by those interested in using the results of this study. Within this set of estimates, a choice should be conditional on the type of prediction that is to be made.

In regard to methods, there seems to be a gradation in terms of divergence of parameter estimates within models depending upon the technique used. For this equation, limited information estimates are more diverse between models than are the Theil-Basmann results. Thus, as a general observation, at least for this example, the limited information method seems more sensitive to model specification than does the Theil-Basmann approach. It should be noted that Theil-Basmann estimates are conditioned by the choice of the dependent variable. If the normalization choice for this relationship had been Y_{2t}, some difference in the estimates could logically be expected. In this regard, the Theil-Basmann procedure is similar to the least squares technique.

B. The Demand for Pork Equations

Although the primary emphasis of this study was directed toward obtaining parameter estimates of certain relationships in the beef sector of the economy, an analysis of the pork sector was also undertaken. In estimating the parameters of the demand relationship for pork, three alternative methods were employed. The results are discussed in this section.

1. Equation 5—Model I (The Demand for Pork at Retail)

(a) *Limited Information estimates:* The parameters associated with the retail demand for pork relationship were estimated by the limited information method as:

$$Y_{9t} = .3999\ Y_{2t} - .8234\ Y_{3t} + .8068\ Z_{1t} - .0826\ z_{2t} + 1.1598 + U_{5t} \qquad (4.13)$$
$$\quad (.1703) \qquad (.1442) \qquad (.1876) \qquad (.0178)$$

$$\hat{\sigma}^2 = .0353 \qquad d^2/s^2 = 1.78^* \qquad (4.14)$$

$$T \log_e (1 + v) = 17.95 \qquad (4.15)$$

The parameter estimates appear reasonable and the signs of the coefficients consistent with the underlying theory. It is interesting to note that the trend variable is estimated as having a negative coefficient, thus indicating a constant rate of decrease for pork consumption during the time period under consideration, given certain levels of prices and income. The overidentifying restrictions test statistic is such that when compared to the appropriate X^2 values, the hypothesis that the overidentifying restrictions are valid, is rejected at the 99 per cent confidence level.

(b) *Theil-Basmann estimates:* By employing the Theil-Basmann method to estimate the parameters associated with the retail demand for pork in Model I, the following results were obtained:

$$Y_{9t} = .5415\ Y_{2t} - .9519\ Y_{3t} + .8176\ Z_{1t} - .0924\ z_{2t} + 1.3609 + U_{5t} \qquad (4.16)$$
$$\quad (.2053) \qquad (.1595) \qquad (.2052) \qquad (.0188)$$

$$\hat{\sigma}^2 = .0013 \qquad d^2/s^2 = 2.54^* \qquad (4.17)$$

Again, as in the case of the demand for beef, the only divergence from limited information estimates is in magnitude. Estimated coefficients of the exogenous variables are nearly equal for both methods. The Theil-Basmann method again appears to underestimate price and income elasticities when compared to the limited information technique.

(c) *Least Squares estimates:* Estimation of the retail demand for pork relationship by least squares yielded:

$$Y_{9t} = .3999\ Y_{2t} - .8234\ Y_{3t} + .8068\ Z_{1t} - .0826\ z_{2t} + 1.3894 + U_{5t} \qquad (4.18)$$
$$\quad (.1703) \qquad (.1442) \qquad (.1876) \qquad (.0178)$$

$$R^2 = .6608 \qquad d^2/s^2 = 1.77^* \qquad (4.19)$$

The least squares estimates agree in sign with estimates obtained by using the alternative methods, but disagree in magnitude.

2. **Equation 5—Model II (The Demand for Pork at Retail)**

In the alternative model the demand for pork relationship was estimated by both the limited information and Theil-Basmann methods.

(a) *Limited Information estimates:*

$$Y_{9t} = \underset{(.1813)}{.3513\, Y_{2t}} - \underset{(.1698)}{.9770\, Y_{3t}} + \underset{(.1443)}{.8628\, Z_{1t}} - \underset{(.0136)}{.0782\, z_{2t}} + \underset{(.0606)}{.2101\, Z_{12t}} + 1.2436 + U_{7t} \quad (4.20)$$

$$\hat{\sigma}^2 = .0237 \qquad d^2/s^2 = 2.18^* \quad (4.21)$$

$$T \log_e (1 + v) = 4.16 \quad (4.22)$$

For model II the variable Z_{12t}, lagged per capita consumption of pork, was included in the retail demand for pork equation. All coefficients agree with *a priori* reasoning concerning their sign. The over-identifying restrictions statistic is such that the assumption that the over-identifying restrictions are met, cannot be rejected.

(b) *Theil-Basmann esimates:*

$$Y_{9t} = \underset{(.1665)}{.3016\, Y_{2t}} - \underset{(.1461)}{.7556\, Y_{3t}} + \underset{(.1902)}{.6813\, Z_{1t}} - \underset{(.0193)}{.0641\, z_{2t}} + \underset{(.1350)}{.2745\, Z_{12t}} + 1.1109 + U_{7t} \quad (4.23)$$

$$\hat{\sigma}^2 = .0011 \qquad d^2/s^2 = 1.83^* \quad (4.24)$$

The Theil-Basmann estimates agree with the estimates obtained by using the method of limited information in regard to sign. The Theil-Basmann estimates again appear to underestimate price and income elasticities relative to limited information estimates.

3. **Inter-Model Comparisons for the Demand for Pork**[1]

All estimates agree as to sign between models and among methods. The coefficient of the price of beef shows the largest divergence in magnitude between models and methods. The remarks concerning the choice of model and method for the estimated beef demand relations are also relevant for the pork demand results.

C. Supply of Beef at the Farm

The farm supply of beef equations are assumed to meet the quali-

[1]Parameter estimates for po k from other selected studies are given in Appendix D.

fications necessary for estimation by least squares, with resultant estimates having full maximum likelihood properties.

1. **Equation 3—Model I (Supply of Beef at the Farm)**

$$Y_{7t} = \underset{(.0111)}{.0318}\, z_{2t} - \underset{(.1472)}{.4210}\, Z_{6t} + \underset{(.1351)}{.8512}\, Z_{7t} + \underset{(.0543)}{.2867}\, Z_{8t} + .5119 + U_{3t} \quad (4.25)$$

$$R^2 = .9754 \qquad d^2/s^2 = .95\dagger \quad (4.26)$$

The coefficient of Z_{6t}, range conditions, is obviously contrary to logic in terms of sign. This could be due to any of a number of reasons but the most obvious factor is the subjective nature of the data used to represent this variable. Range conditions are estimated in terms of "per cent of normal", and "normal" is likely to be a variable influenced by technology and other factors. Therefore, errors of observation for this variable may be unusually large. All other coefficients agree with *a priori* reasoning.

2. **Equation 3—Model II (Supply of Beef at the Farm)**

The range conditions variable was omitted, supply of corn substituted for feed grains and the inventory variable was separated into cattle on feed and cattle not on feed in the reformulation of Model I. Least-squares estimation of the reformulated equation resulted in the following parameter estimates:

$$Y_{7t} = \underset{(.0097)}{.0272}\, z_{2t} + \underset{(.1057)}{.2233}\, Z_{13t} + \underset{(.1113)}{.8081}\, Z_{14t} + \underset{(.0608)}{.1096}\, Z_{17t} - .3256 + U_{4t} \quad (4.27)$$

$$R^2 = .9825 \qquad d^2/s^2 = .76\dagger \quad (4.28)$$

All estimates agree with theory in sign. The coefficient of the trend variable, z_{2t}, was positive for this equation in both models, indicating a constant rate of increase in output from given inputs.

D. The Farm Supply Equations for Pork

1. **Equation 7—Model I (Supply of Pork at the Farm)**

The parameters associated with this relationship were estimated by the method of least squares. The results are as follows:

$$Y_{10t} = \underset{(.0094)}{.0547}\, z_{2t} - \underset{(.0867)}{.0343}\, Z_{8t} + \underset{(.1223)}{.7996}\, Z_{10t} + \underset{(.0831)}{.1332}\, Z_{11t} + .0743 + U_{7t} \quad (4.29)$$

$$R^2 = .9566 \qquad d^2/s^2 = 1.42\dagger \quad (4.30)$$

The coefficient of Z_{8t}, the availability of feed grains, is incompatible with *a priori* reasoning. However, the ratio of this estimate to its calculated standard error gives a rough idea of the reliability of the sign of this coefficient. The signs of the remaining coefficients are consistent with the theory underlying the equation specification.

2. **Equation 7—Model II (Supply of Pork at the Farm)**

Availability of corn was used to replace availability of feed grains in the reformulation. Estimation of this alternative specification by least squares resulted in:

$$Y_{10t} = \underset{(.0091)}{.0545\, z_{2t}} + \underset{(.1159)}{.7938\, Z_{10t}} + \underset{(.0861)}{.1350\, Z_{11t}} - \underset{(.0672)}{.0272\, Z_{17t}} + .0685 + U_{8t} \qquad (4.31)$$

$$R^2 = .9566 \qquad d^2/s^2 = 1.41\dagger \qquad (4.32)$$

Again, the coefficient of the feed variable is contrary to logic for this equation. The size of the standard error relative to the magnitude of the coefficient directs suspicion as to the sign of the estimate. All other estimates agree in sign with *a priori* considerations.

E. The Supply of Beef at Retail

Model II was formulated in part to permit a more detailed specification for the variable, retail supply of beef. This is perhaps the most important variable in the beef marketing economy in terms of affecting consumption, the price of beef, etc. An identity, retail production equals number of cattle slaughtered times their average weight, was employed and functional relationships postulated for the two endogenous variables, average weight and number slaughtered. The two relationships were assumed to meet all requisites necessary for estimation by the single equation—maximum likelihood method.

1. **Equation 2-a—Model II (The Function Involving Number of Cattle Slaughtered)**

$$Y_{12t} = \underset{(.0204)}{.0090\, z_{2t}} + \underset{(.1587)}{.5126\, Z_{13t}} + \underset{(.2366)}{.6908\, Z_{14t}} + \underset{(.0514)}{.2132\, Z_{15t}} - \underset{(.0970)}{.2535\, Z_{16t}} - .3906 + U_{2t} \qquad (4.33)$$

$$R^2 = .9265 \qquad d^2/s^2 = 1.67\dagger \qquad (4.34)$$

The coefficients of Z_{15t}, lagged price of corn, and Z_{16t}, lagged price of beef, seem opposed to theory. However, recalling the long production adjustment period for cattle producers, it seems logical that if the price of beef increased in time period t — 1, cattlemen could interpret this as a sign to increase inventories, thus reducing slaughter for time period t. Similar reasoning applies to the positive coefficient for the lagged price of corn—an increase in the price of corn for time period t — 1 might encourage cattle producers to reduce inventory for time period t, thus increasing current slaughter. All other estimates agree in sign with *a priori* considerations.

2. **Equation 2-b—Model II (The Function Involving Average Weight of Cattle Slaughtered)**

$$Y_{13t} = -\underset{(.0050)}{.0094}\, z_{2t} + \underset{(.0594)}{.0191}\, Z_{13t} + \underset{(.0236)}{.0431}\, Z_{16t} + \underset{(.0352)}{.0694}\, Z_{17t} + \underset{(.1229)}{.5351}\, Z_{18t} + .6983 + U_{3t} \qquad (4.35)$$

$$R^2 = .7636 \qquad d^2/s^2 = 1.57\dagger \qquad (4.36)$$

The magnitude of the coefficient of determination (R^2) and the relative size of the standard errors indicates a need for other specifications of this relationship. Unfortunately the results do not indicate a direction in which one might productively look for such alternatives. Incorrect algebraic form or other defects in the statistical specification and high correlation between certain predetermined variables are examples of possible causes of this result. The signs of the coefficients appear consistent with the underlying theory.

F. Implications of the Parameter Estimates

1. General Considerations

Knowledge of the structure of a particular segment of the economy is useful if it facilitates making the "best" decisions. Within this normative framework, knowledge of structural parameters, such as those estimated in this study, provide one basis for assessing in advance the probable quantitative impact of various economic policy actions on the time path of certain economic variables, given certain goals.

As viewed by Marschak (23), economic policy action consists of changing those elements of the structure and those exogenous variables that can be controlled. Given those factors that can be controlled along with those that cannot, it is the economic researchers' task to predict which stochastic processes will be generated by alternative policies. For purposes of policy, the decision-maker must then rank the alternative outcomes or consequences relative to some ordered preference field. Knowledge of the network of interrelations that describe the structure of an economy along with the attendant connecting parameter estimates is a necessary prerequisite for intelligent policy action. The nature and completeness of this information is of course conditional on the policy or policies being considered.

It is of paramount importance, for policy purposes, to realize that actions directed toward a particular segment of the economy may have repercussions and consequences within and between other sectors. Given this proposition, it becomes apparent that an analytical model reflecting the phenomena to be explained is a necessity if repercussions and consequences of certain actions are to be identified. By constructing an analytical model, the investigator is forced to formulate the assumptions about the phenomena to be studied more completely and precisely. These formulations then become available for criticism and discussion and yield information as to what we cannot do, in order that we do not fool ourselves. Thus, by formulating, identifying and estimating structural equations, the affect of certain policy actions may be estimated and the uncertainty as to the consequences of these actions reduced.

2. Implications for the Firm

Perhaps the greatest aid for the firm manager that could stem from a study of this nature is pragmatic estimates of the future values of such factors as the price of beef, the price of pork, etc. By having knowledge of future price and cost conditions, the firm could adjust production plans to more nearly meet profit and efficiency objectives. Knowledge by processors of the time flow of animals that would be forthcoming should make possible decisions which would lead to a reduction in processing costs. The equations that represent supply of beef at retail in Model II are such that short run prediction of the retail supply of beef is possible. Whether sufficiently accurate forecasting can be made from these relationships remains to be proven. It is hoped that as more comprehensive and reliable data become available and as the methodology of measurement research in economics becomes more objective, more accurate pragmatic prediction will become feasible.

3. Implications for the Policy Planner

The model approach has analytical advantages to the policy planner other than parameter estimation. As noted earlier, the construction of a hypothetical model serves to point up the dynamic, simultaneous, and interdependent nature of our economic society. In policy planning, it is all too easy to overlook far-reaching repercussions without some formal approach such as that offered by model building.

The models presented in this publication are not meant to answer all possible policy questions within the beef and pork segments of the economy. The models can, however, provide an objective means of analyzing such questions as what effect a beef price policy will have on (1) consumption of beef, (2) future production of beef, (3) the price and consumption of pork, etc. To the extent that large fluctuations in the number of cattle on farms and retail beef supply are considered an evil, policies can be suggested that will dampen such cycles. For purposes of illustration, consider the estimates obtained from Model II. If an economic policy action established a price of beef, ten percent higher than previously, the following consequences might be expected: (1) the consumption of beef could be expected to decrease by about 9 percent; (2) the consumption of pork would increase by approximately 4 percent; (3) next years' retail production of beef would decrease by about 3 per cent as cattlemen begin to build up inventories for future production. Implications of other policies could be traced through the models and the expected time path of certain variables estimated.

4. Conditional Restrictions on the Estimates

The youthful state of the field of econometrics suggests that caution should be employed when applying the results of this study. In the construction and estimation of econometric models, there are many limitations to which the estimates may be subject. Sampling variations alone are of a magnitude to give considerable dispersion of observed variables around the values that can be forecast from the estimated equations. This range of error associated with the forecasts at reasonable probability levels may be larger than required for many problems. In addition, there is a possibility that some of the assumptions on which the analysis is based are unrealistic, therefore, incurring certain specification errors. For example: (1) The formulations may over-simplify the dynamic phenomena whose generation is to be explained and (2) although assumed otherwise, the data are not free of errors of measurement and, in many cases, the data used may imperfectly reflect the variables as theoretically specified. In spite of these shortcomings, these estimates should provide a source of information over and above prevailing qualitative knowledge for estimating the potential impact of certain policy actions.

V. SUMMARY

Knowledge of the structural relationships for the beef and pork sectors is a prerequisite for intelligent decision-making relative to government economic policy and resource allocation by the firm. Given this goal and the restriction of incomplete knowledge, this study attemped to obtain quantitative approximations of the underlying relationships postulated for these sectors.

In order to obtain these parameter estimates, alternative economic models, which sought to portray in a simplified way the underlying relations that reflect observable economic phenomena in the beef and pork sectors of the economy, were formulated. Alternative methods of estimating structural relationships and the relevant economic and statistical tests were reviewed. Time series data were employed as the sampling observations for reflecting the variables specified in the models. Parameter estimates for certain specified relations were then obtained by using alternative models and methods.[1] The estimates were then subjected to certain economic and statistical tests and the implications of the results for firm and government action were reviewed.

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[1]See Appendix A for summary of the parameter estimates obtained.

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VII. APPENDICES

Appendix A—Summary of Empirical Results

1. **Equation 1—Model I** (Demand for Beef at Retail)

(Limited Information)

$$Y_{1t} = -1.3566\, Y_{2t} + .2015\, Y_{3t} + .9720\, Z_{1t} + .0654\, z_{2t} +$$
$$(.2830) \quad (.1888) \quad (.1674) \quad (.0147)$$
$$.0804\, Z_{3t} + 2.0188 + U_{1t} \tag{A.1}$$
$$(.1434)$$

$$\hat{\sigma}^2 = .0202 \qquad d^2/s^2 = 1.10\dagger \tag{A.2}$$

$$T \log_e (1 + v) = 21.5 \tag{A.3}$$

(Theil-Basmann)

$$Y_{1t} = -\, .7730\, Y_{2t} + .1960\, Y_{3t} + .6048\, Z_{1t} + .0472\, z_{2t} +$$
$$(.1266) \quad (.1108) \quad (.1120) \quad (.0114)$$
$$.2142\, Z_{3t} + 1.3917 + U_{1t} \tag{A.4}$$
$$(.1184)$$

$$\hat{\sigma}^2 = .00037 \qquad d^2/s^2 + 1.64^* \tag{A.5}$$

(Least Squares)

$$Y_{1t} = -.7555\, Y_{2t} + .2277\, Y_{3t} + .5757\, Z_{1t} + 0.482\, z_{2t} +$$
$$(.1041) \quad (.0976) \quad (.1048) \quad (.0106)$$
$$.2004\, Z_{3t} + 1.3787 + U_{1t} \tag{A.6}$$
$$(.1108)$$

$$R^2 = .9468 \qquad d^2/s^2 = 1.66\dagger \tag{A.7}$$

2. **Equation 1—Model II** (Demand for Beef at Retail)

(Limited Information)

$$Y_{1t} = -.8651\, Y_{2t} + .3231\, Y_{3t} + .5912\, Z_{1t} + .0560\, z_{2t} +$$
$$(.3001) \quad (.2868) \quad (.1680) \quad (.0201)$$
$$.1226\, Z_{3t} + 1.5140 + U_{1t} \tag{A.8}$$
$$(.2055)$$

$$\hat{\sigma}^2 = .0074 \qquad d^2/s^2 = 2.12^* \tag{A.9}$$

$$T \log_e (1 + v) = 4.35 \tag{A.10}$$

(Theil-Basmann)

$$Y_{1t} = -.7707\, Y_{2t} + .2383\, Y_{3t} + .5793\, Z_{1t} + .0491\, z_{2t} +$$
$$(.1058) \quad (.1014) \quad (.1052) \quad (.0108)$$
$$.1911\, Z_{3t} + 1.3970 + U_{1t} \tag{A.11}$$
$$(.1125)$$

$$\hat{\sigma}^2 = .00036 \qquad d^2/s^2 = 1.63^* \tag{A.12}$$

3. **Equation 5—Model I** (Demand for Pork at Retail)

(Limited Information)

$$Y_{9t} = \underset{(.2523)}{.8404}\, Y_{2t} - \underset{(.1895)}{1.1312}\, Y_{3t} + \underset{(.1694)}{.8192}\, Z_{1t} - \underset{(.0128)}{.1091}\, z_{2t} + 1.1598 + U_{5t} \quad \text{(A.13)}$$

$$\hat{\sigma}^2 = .0353 \qquad d^2/s^2 = 1.78 \quad \text{(A.14)}$$

$$T \log_e (1 + v) = 17.95 \quad \text{(A.15)}$$

(Theil-Basmann)

$$Y_{9t} = \underset{(.2053)}{.5415}\, Y_{2t} - \underset{(.1595)}{.9519}\, Y_{3t} + \underset{(.2052)}{.8176}\, Z_{1t} - \underset{(.0188)}{.0924}\, z_{2t} + 1.3609 + U_{5t} \quad \text{(A.16)}$$

$$\hat{\sigma}^2 = .0013 \qquad d^2/s^2 = 2.54^* \quad \text{(A.17)}$$

(Least Squares)

$$Y_{9t} = \underset{(.1703)}{.3999}\, Y_{2t} - \underset{(.1442)}{.8234}\, Y_{3t} + \underset{(.1876)}{.8068}\, Z_{1t} - \underset{(.0178)}{.0826}\, z_{2t} + 1.3894 + U_{5t} \quad \text{(A.18)}$$

$$R^2 = .6608 \qquad d^2/s^2 = 1.77^* \quad \text{(A.19)}$$

4. **Equation 5—Model II** (Demand for Pork at Retail)

(Limited Information)

$$Y_{9t} = \underset{(.1813)}{.3513}\, Y_{2t} - \underset{(.1698)}{.9770}\, Y_{3t} + \underset{(.1443)}{.8628}\, Z_{1t} - \underset{(.0136)}{.0782}\, z_{2t} + \underset{(.0606)}{.2101}\, Z_{12t} + 1.2436 + U_{7t} \quad \text{(A.20)}$$

$$\hat{\sigma}^2 = .0237 \qquad d^2/s^2 = 2.18^* \quad \text{(A.21)}$$

$$T \log_e (1 + v) = 4.16 \quad \text{(A.22)}$$

(Theil-Basmann)

$$Y_{9t} = \underset{(.1655)}{.3016}\, Y_{2t} - \underset{(.1461)}{.7556}\, Y_{3t} + \underset{(.1902)}{.6813}\, Z_{1t} - \underset{(.0193)}{.0641}\, z_{2t} + \underset{(.1350)}{.2745}\, Z_{12t} + 1.1109 + U_{7t} \quad \text{(A.23)}$$

$$\hat{\sigma}^2 = .0011 \qquad d^2/s^2 = 1.83^* \quad \text{(A.24)}$$

5. **Equation 3—Model I** (Supply of Beef at the Farm)

(Single Equation)

$$Y_{7t} = \underset{(.0111)}{.0318}\, z_{2t} - \underset{(.1472)}{.4210}\, Z_{6t} + \underset{(.1351)}{.8512}\, Z_{7t} + \underset{(.0543)}{.2867}\, Z_{8t} + .5119 + U_{3t} \quad \text{(A.25)}$$

$$R^2 = .9754 \qquad d^2/s^2 = .95^\dagger \quad \text{(A.26)}$$

6. **Equation 3—Model II** (Supply of Beef at the Farm)
(Single Equation)

$$Y_{7t} = \underset{(.0097)}{.0272}\, z_{2t} + \underset{(.1057)}{.2233}\, Z_{13t} + \underset{(.1113)}{.8081}\, Z_{14t} + \underset{(.0608)}{.1096}\, Z_{17t} - .3256 + U_{4t} \qquad \text{(A.27)}$$

$$R^2 = .9825 \qquad d^2/s^2 = .76\dagger \qquad \text{(A.28)}$$

7. **Equation 7—Model I** (Supply of Pork at the Farm)
(Single Equation)

$$Y_{10t} = \underset{(.0094)}{.0547}\, z_{2t} - \underset{(.0867)}{.0343}\, Z_{8t} + \underset{(.1223)}{.7996}\, Z_{10t} + \underset{(.0831)}{.1332}\, Z_{11t} + .0743 + U_{7t} \qquad \text{(A.29)}$$

$$R^2 = .9566 \qquad d^2/s^2 = 1.42\dagger \qquad \text{(A.30)}$$

8. **Equation 7—Model II** (Supply of Pork at the Farm)
(Single Equation)

$$Y_{10t} = \underset{(.0091)}{.0545}\, z_{2t} + \underset{(.1159)}{.7938}\, Z_{10t} + \underset{(.0861)}{.1350}\, Z_{11t} - \underset{(.0672)}{.0272}\, Z_{17t} + .0685 + U_{8t} \qquad \text{(A.31)}$$

$$R^2 = .9566 \qquad d^2/s^2 = 1.41\dagger \qquad \text{(A.32)}$$

9. **Equations 2-a and 2-b—Model II** (Supply of Beef at Retail)
(Single Equation)

$$Y_{12t} = \underset{(.0204)}{.0090}\, z_{2t} + \underset{(.1587)}{.5126}\, Z_{13t} + \underset{(.2366)}{.6908}\, Z_{14t} + \underset{(.0514)}{.2132}\, Z_{15t} - \underset{(.0970)}{.2535}\, Z_{16t} - .3906 + U_{2t} \qquad \text{(A.33)}$$

$$R^2 = .9265 \qquad d^2/s^2 = 1.67\dagger \qquad \text{(A.34)}$$

(Single Equation)

$$Y_{13t} = -\underset{(.0050)}{.0094}\, z_{2t} + \underset{(.0594)}{.0191}\, Z_{13t} + \underset{(.0236)}{.0431}\, Z_{16t} + \underset{(.0352)}{.0694}\, Z_{17t} + \underset{(.1229)}{.5351}\, Z_{18t} + .6983 + U_{3t} \qquad \text{(A.35)}$$

$$R^2 = .7636 \qquad d^2/s^2 = 1.57\dagger \qquad \text{(A.36)}$$

Appendix B—Computation of Theil-Basmann Estimates

An alternative estimation procedure, presented by Radner and Bobkoski (28) and Klein (16), which is computationally preferable for obtaining estimates that are equivalent to the Basmann formulation (Chapter II), is as follows:

(1) Compute the second order moments My*z and Mzz, where the asterisk refers to those variables appearing in the relationship to be estimated.

(2) Invert the Mzz matrix

(3) Compute $My^*zMzz^{-1}Mzy^*$, where Mzy* is the transpose of My*z.

(4) Define an equation

$$D\ (\beta,a)' = d, \tag{B.1}$$

where:

$$D = \begin{Bmatrix} M\bar{y}zMzz^{-1}Mz\bar{y} & M\bar{y}z^* \\ Mz^*\bar{y} & Mz^*z^* \end{Bmatrix} \tag{B.2}$$

and

$$d = \begin{Bmatrix} M\bar{y}zMzz^{-1}Mzy_1 \\ Mz^*y_1 \end{Bmatrix} \tag{B.3}$$

The bar over the y indicates i = 2, ..., g. (i.e., $M\bar{y}z$ indicates the second order moments of all endogenous variables in the equation to be estimated except y_{1t} on all the z_{it} in the complete system.) β and a are the unknown parameters to be estimated.

(5) Invert D and compute

$$(\beta,a)' = D^{-1}d \tag{B.4}$$

This completes estimation of the β_i and a_i.

(6) To determine $\hat{\sigma}^2$, an estimate of total variance, calculate U_t for all t = 1, ..., T, then compute:

$$\hat{\sigma}^2 = \sum_{t=1}^{T} U_t^2/T - g - h \tag{B.5}$$

(7) The variance covariance matrix is estimated by:

$$\hat{\Sigma} = \hat{\sigma}^2 D^{-1} \tag{B.6}$$

(8) Standard errors for $\hat{\beta}$, $\hat{a}$, are estimated by obtaining square roots of the main diagonal elements of $\hat{\Sigma}$.

The following is a numerical example of the computations necessary to estimate equation 1, Model I by the Theil-Basmann method.

(1) Assuming that the reader is familiar with methods of obtaining second order moments and matrix inversion and multiplication, we

may begin by presenting the matrix resulting from all operations through step 3 of the process. This matrix was computed as:

$$My^*zMzz^{-1}Mzy^* = \begin{Bmatrix} .121890 & .095734 & .070532 \\ .095734 & .199282 & .097084 \\ .070532 & .097084 & .118693 \end{Bmatrix} \quad (B.7)$$

(2) The matrix of second order moments of the y_{it} in the equation taken on the z_{it} in the equation, a submatrix of My*z, was computed as:

$$My^*z^* = \begin{Bmatrix} .152312 & 1.355271 & .097251 \\ .205097 & 1.861864 & .087711 \\ .116475 & .728781 & .081494 \end{Bmatrix} \quad (B.8)$$

(3) The matrix of second order moments of the z_{it} in the equation taken on themeslves, a sub-matrix of Mzz, was found to be:

$$Mz^*z^* = \begin{Bmatrix} .262121 & 2.175328 & .124973 \\ 2.175328 & 23.461600 & 1.064903 \\ .124973 & 1.064903 & .108352 \end{Bmatrix}$$

(4) The matrix D as defined by (B.2) is as follows:

$$D = \left\{\begin{array}{ccccc} .199282 & .097084 & .205097 & 1.861864 & .087711 \\ .097084 & .118693 & .116475 & .728781 & .081494 \\ \hline .205097 & .116475 & .262121 & 2.175328 & .124973 \\ 1.861864 & .728781 & 2.175328 & 23.461600 & 1.064903 \\ .087711 & .081494 & .124973 & 1.064903 & .108352 \end{array}\right\} \quad (B.9)$$

The 2 x 2 partition in the upper left portion of D is the $M\bar{y}zMzz^{-1}$ $Mz\bar{y}$ matrix, obtained by deleting the first row and first column of $My^*zMzz^{-1}Mzy^*$ (B.7). The 2 x 3 partition in the upper right portion of D is the $M\bar{y}z^*$ matrix, obtained by deleting the first row of My*z* (B.8). The 3 x 2 lower left partition of D is $Mz^*\bar{y}$, the transpose of $M\bar{y}z^*$. The lower right partition of D is the 3 x 3 matrix, Mz*z* (B.9).

(5) The vector d, defined by (B.3) is as follows:

$$d = \left\{\begin{array}{c} .095734 \\ .070532 \\ \hline .152312 \\ 1.355271 \\ .097251 \end{array}\right\} \quad (B.10)$$

The 2 x 1 upper partition of d is $M\bar{y}zMzz^{-1}Mzy_1$, obtained by deleting the first row and all except the first column of $My^*zMzz^{-1}Mzy^*$. The 3 x 1 lower partition is the transpose of the first row of My*z*.

(6) The D matrix was inverted on an electronic computer and the result was:

$$D^{-1} = \begin{Bmatrix} 43.7840 & -20.2189 & -16.3287 & -2.2909 & 21.1122 \\ -20.2189 & 33.5599 & -6.1839 & 2.1926 & -23.2904 \\ -16.3287 & -6.1839 & 34.2910 & -1.2771 & -9.1306 \\ -2.2909 & 2.1926 & -1.2771 & .3584 & -1.8445 \\ 21.1122 & -23.2904 & -9.1306 & -1.8445 & 38.3151 \end{Bmatrix} \quad (B.11)$$

(7) The multiplication, $D^{-1}d$, resulted in a vector of estimates for β_i and α_i (B.4)

$$D^{-1}d = \begin{Bmatrix} -.7730 \\ .1960 \\ .6048 \\ .0472 \\ .2142 \end{Bmatrix} \quad (B.12)$$

Where the upper two elements are estimates of β_{12} and β_{13} and the lower three elements are estimates of α_{11}, α_{12}, and α_{13}.

(8) The constant term associated with the equation was computed in the usual manner, then residuals computed for all t = 1, ..., T.

(9) The sum of squares of the residuals divided by appropriate degrees of freedom yields an estimate of total variance as:

$$\hat{\sigma}^2 = .00037 \quad (B.13)$$

(10) To obtain error variance for the $\hat{\beta}_i$ and $\hat{\alpha}_i$, the estimated variance was multiplied times all main diagonal elements of D^{-}.[1]

Square roots of the error variance yielded standard errors for $\hat{\beta}_i$ and $\hat{\alpha}_i$

Appendix C—Estimation of Standard Errors for the Limited Information Method (15)

The computational procedure in symbolic form is as follows:[1]

Step 1: Compute $(\hat{\beta}\, Wy^*y^*)' \, (\hat{\beta}\, Wy^*y^*)$ (C.1)

where $\hat{\beta}$ is the normalized vector of coefficients of the endogenous variables in the single equation and

$$Wy^*y^* = My^*y^* - My^*zMzz^{-1}Mzy^* \quad (C.2)$$

(Refer to Appendix B for a definition of the symbology.)

Step 2: Compute $\dfrac{1}{\lambda\hat{\beta}Wy^*y^*\hat{\beta}'}$, where λ is the largest characteristic root, used in obtaining the $\hat{\beta}$'s.

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[1]For a computational procedure for the limited information method, see Judge (14).

Step 3: Compute

$$R - \frac{1}{\lambda\hat{\beta}Wy^*y^*\hat{\beta}'} (\hat{\beta}Wy^*y^*)' \, (\hat{\beta}\, Wy^*y^*) \qquad (C.3)$$

where

$$R = My^*zMzz^{-1}Mzy^* - My^*z^*Mz^*z^{*-1}Mz^*y^* \qquad (C.4)$$

This results in a G* x G* matrix. Delete the first row and column from the matrix of (C.3) and call the G* — 1 x G* — 1 resultant sub-matrix $F_{\beta\beta}^{-1}$.

Step 4: Invert $F_{\beta\beta}^{-1}$ by any appropriate method of matrix inversion. Call the result $F_{\beta\beta}$.

Step 5: Calculate $Mz^*z^{*-1}Mz^*y^*$ and delete the first column of the resultant product matrix. Call the matrix obtained by this deletion P.

Step 6: Compute

$$F_{\beta}'\alpha = P\ F_{\beta\beta} \qquad (C.5)$$

where P is the matrix obtained in step 5 and $F_{\beta\beta}$ is the matrix calculated in step 4.

Step 7: Pre-multiply the transpose of $F_{\beta}'\alpha$ (the transpose of the matrix obtained in step six) by P. Call the resultant matrix $F_{\alpha\alpha}$. In symbols this operation is

$$P\ F_{\beta\alpha} = F_{\alpha\alpha} \qquad (C.6)$$

Step 8: Multiply all elements of $F_{\beta\beta}$, $F_{\beta\alpha}$ and $F_{\alpha\alpha}$ by the factor $\frac{\hat{\sigma}^2 (u)}{n - m}$

where

$\hat{\sigma}^2$ (u) is the estimated variance of the equation, n is the number of observations and m is the number of variables in the equation in question.

$$\frac{\hat{\sigma}^2 (u)}{n - m} F_{\beta\beta} = V_{\beta\beta} \qquad (C.7)$$

$$\frac{\hat{\sigma}^2 (u)}{n - m} F_{\beta\alpha} = V_{\beta\alpha} \qquad (C.8)$$

$$\frac{\hat{\sigma}^2 (u)}{n - m} F_{\alpha\alpha} = V_{\alpha\alpha} \qquad (C.9)$$

Step 9: $V_{\beta\beta}$ is the estimated variance-covariance matrix of the y_{it} in the equation; $V_{\beta\alpha}$ is the estimated covariance matrix of the y_{it} on the z_{it} in the equation and $V_{\alpha\alpha}$ is the estimated variance-covariance matrix of the z_{it} in the equation. To obtain standard errors of the $\hat{\beta}$'s, pick off the main diagonal elements of $V_{\beta\beta}$, and take square roots of those elements. To obtain standard errors of the $\hat{\alpha}$'s, perform a similar operation on the main diagonal elements of $V_{\alpha\alpha}$.

Appendix D—A Comparison of Selected Results With Other Studies

The following table (Table 1) presents a comparison of the estimates of price and income elasticities for beef and pork from this study and other similar research. Besides obvious differences of model construction, methods, etc., this study differs from the others reported in that post war data were used.

Table 1.—A Comparison of Estimated Price and Income Elasticities for Pork and Beef for Various Studies.

BEEF

Source	Postulated Algebraic Form	Sample Period	Method	Estimated Price Elasticity	Estimated Income Elasticity
Fox (10)	△ logarithms	1922-41	Least squares	—0.79	0.73
Nordin, Judge and Wahby (26)	Linear logs	”	”	—0.96	0.33
Working (32)	”	”	”	—0.90	- --
This Study	”	1925-41 1947-55	”	—0.76	0.58
Nordin, Judge and Wahby (26)	”	1922-41	Simultaneous Equation-Maximum Likelihood	—0.77	0.65
This Study Model I	”	1925-41 1947-55	Limited Information	—1.36	0.97
This Study Model II	”	”	”	—0.87	0.59
This Study Model I	”	”	Theil-Basmann	—0.77	0.60
This Study Model II	”	”	”	—0.77	0.58

Table 1.—Continued

PORK

Source	Postulated Algebraic Form	Sample Period	Method	Estimated Price Elasticity	Estimated Income Elasticity
Fox (10)	△ logarithms	1922-41	Least Squares	—0.81	0.72
Nordin, Judge and Wahby (26)	Linear logs	”	”	—0.78	0.43
Working (32)	”	”	”	—0.99	0.48
This Study	”	1925-41 1947-55	”	—0.82	0.81
Nordin, Judge and Wahby (26)	”	1922-41	Simultaneous Equation-Maximum Likelihood	—0.91	0.76
This Study Model I	”	1925-41 1947-55	Limited Information	—1.13	0.82
This Study Model II	”	”	”	—0.98	0.86
This Study Model I	”	”	Theil-Basmann	—0.95	0.82
This Study Model II	”	”	”	—0.76	0.68