CONTRA-ROTATING DUCTED AXIAL FLOW COMPRESSOR

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1943

1947

Submitted to the Faculty of the Graduate School of the

Oklahoma Agricultural and Mechanical College

in Partial Fulfillment of the Requirements

for the Degree of

MASTER OF SCIENCE

1950

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PREFACE

In the advent of modern science there has been a concerted effort by many men to develop new ideas or recast old theories hoping they might make something better. The underlying impetus being man's desire to make his contribution to society in order that this may be a better world in which to live. Along this line of thought in the field of aeronautics there has been considerable research directed toward better design of compressors to ultimately obtain better efficiencies. Compressors of low efficiency substantially limit the power output of any engine of which it is an integral part and have a serious effect upon overall efficiency. This fact can be attested to by as much as three per cent increase in fuel consumption of a typical jet engine due to a reduction in compressor efficiency of one per cent for the same power output.[1] The jet engine exemplification cited above does not limit the application of the theory to just this phase but may be applied to any phase of application.

The theory of compressors is satisfactory only from an overall standpoint. That the theory is incomplete has been repeatedly proved by the fact that the theoretical losses based upon the most detailed design information are smaller than the actual losses. Naturally this means that the theory is used only as a starting point by designers to be supplemented by knowledge and experimental results.[2]

Of the two most common types of compressors, centrifugal and axial

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[1] Godsey and Young, Gas Turbines for Aircraft, pp.93.

[2] Ibid, pp. 96.

the former has the advantage of design in simplicity, low weight, and lower first cost, whereas the latter excells in smaller diameter for a large capacity and comparatively higher efficiency.[3] Even though the axial type is more efficient there is room for improvement along three lines: (1) more compact length thereby lower weight, (2) still higher efficiencies, and (3) greater pressure rise per stage.

It is the purpose of this thesis to develop a method of calculating performance of a contra-rotating ducted axial flow compressor.

It has been seen from previous analyses of single rotation axial flow machines that the several design variables are interrelated in such a manner as to complicate a complete theoretical analysis of the problem. This must be taken in stride if we are to accomplish our mission. An attempt to correlate all the readily determinable variables into one mathematical expression which can be interpreted in such a manner as to reveal just where we can make improvements and at the same time where it is not feasible must be forthcoming.

As stated previously relatively higher efficiencies can be expected from the axial compressor (i.e., single rotation with stator blades), than the centrifugal but at a much lower pressure rise per stage. Axial compressor design also indicates that at high pressure rises the efficiency is appreciably reduced by interference in stators induced by the greater slipstream rotation.

From this basic information it may be deduced that it is possible for the stator blades to be redesigned and given an opposite sense of rotation with respect to the rotor in a special way so that the slipstream rotation introduced by the rotor may be entirely removed. Thus high efficiency could be maintained while reducing the interference

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[3] M. J. Zucrow, Principles of Jet Propulsion and Gas Turbines, pp. 387.

of the stator and at the same time obtain high pressure rises.

The writer wishes to express his sincere appreciation to Mr. W. S. Burn for the original idea upon which this analysis was made, to Professor L. J. Fila for his many helpful suggestions and careful scrutiny of this material, also to Mrs. Ashley A. Brooks for the typing of this material.

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ASSUMPTIONS FOR THEORETICAL ANALYSIS

1. Flow is axial in front of and behind the system of blades.
2. Flow is steady, i.e., has a constant density, pressure, and velocity distribution in front of and behind the system of blades.
3. Flow does not have radial components, i.e., hub and duct form two concentric cylinders such that the streamlines coincide with the surface of a cylinder concentric with hub and duct.
4. The second rotor removes all of the whirl velocity, i.e., slipstream rotation, induced by the first rotor.
5. The velocity components arrange themselves such that the vortex theory of airfoils will apply in two dimensional form.
6. For preservation of flow continuity no axial component of whirl velocity occurs.
7. Peripheral velocity of the slipstream is uniformly distributed around the elementary annulus.
8. Profile characteristics are those of an airfoil of infinite aspect ratio, i.e., the solidity is such that no multiplane interference occurs.

NOTATION

Refers to blade element at radius r

b - Chord length, ft.

C - Axial velocity, ft./sec.

C_D - Drag coefficient - $D/\frac{1}{2}\rho V^2 b\, dr$

C_L - Lift coefficient - $L/\frac{1}{2}\rho V^2 b\, dr$

C_Q - Torque coefficient - $Q/\frac{1}{2}\rho C^2 \pi R^3$

C_S - Force coefficient on blade element acting perpendicular to plane of rotation-
$S/\frac{1}{2}\rho V^2 b\, dr$

C_T - Thrust coefficient - $T/\frac{1}{2}\rho C^2 \pi R^2$

C_U - Force coefficient on blade element acting in plane of rotation - $U/\frac{1}{2}\rho V^2 b\, dr$

D - Drag component of resultant force, lb.

ΔE - Increase in energy, lb./sq. ft.

E_1 - Energy before first rotor - $p_1 + \frac{1}{2}\rho C^2$, lb./sq. ft.

E_2 - Energy between rotors - $p_2 + \frac{1}{2}\rho C^2 + \frac{1}{2}\rho\omega^2 r^2$, lb./sq. ft.

E_3 - Energy after second rotor - $p_3 + \frac{1}{2}\rho C^2$, lb./sq. ft.

k - Energy increase coefficient - $\Delta E/\frac{1}{2}\rho C^2$

L - Lift component of resultant force, lb.

Δp - Pressure increase, lb./sq. ft.

p_1 - Pressure before first rotor, lb./sq. ft.

p_2 - Pressure between rotors, lb./sq. ft.

p_3 - Pressure after second rotor, lb./sq. ft.

Q - Torque, ft. lb.

r - Distance from compressor axis to elementary annulus, ft.

R - Blade tip radius, ft.

S - Force on blade element acting perpendicular to the plane of rotation, lb.

T - Thrust, lb.

U - Force on blade element acting in plane of rotation, lb.

V - Resultant effective velocity - $C/\sin\beta$

w - Function of solidity - $Zb/\pi R$

x - Radius ratio - r/R

Z - Number of blades in rotor

α - Angle the chord line makes with resultant effective velocity, degrees.

β - Angle resultant effective velocity makes with plane of rotation, degrees.

γ - Angle chord line makes with plane of rotation, degrees.

δ - Blade element lift/drag ratio.

ϵ - Ratio of the whirl velocity or rotational velocity of the stream to the axial velocity - $\omega r/C$

η - Efficiency

θ - Ratio of axial velocity to circumferential geometric velocity of blade element - $C/\Omega r$

π - The number 3.14159265+ ; the ratio of the circumference of a circle to its diameter.

ρ - Density of air.

ω - Angular whirl velocity or rotational velocity of the stream.

Ω - Angular velocity of the blade element.

Subscripts:

1 - Applies to first rotor blade or blade element (except where noted).

2 - Applies to second rotor blade or blade element (except where noted).

r - Values that concern the root of the blade.

R - Values that concern the tip of the blade.

Symbols:

$\approx$ - Approximately equal.

$\neq$ - Is not equal.

APPLICATION OF COMBINED MOMENTUM, ENERGY, AND VORTEX THEORIES

Referring to figure 1 we have from the geometry of the figure:

$$V_1 = C/\sin\beta_1, \; V_2 = C/\sin\beta_2,$$

$$\tan\beta_1 = \frac{C}{\Omega_1 r - \frac{1}{2}\omega r} = \frac{\frac{C}{\Omega_1 r}}{1 - \frac{1}{2}\frac{\omega r C}{\Omega_1 r C}} = \frac{\theta_1}{1 - \frac{\theta_1 \epsilon}{2}} \tag{1}$$

$$\tan\beta_2 = \frac{C}{\Omega_2 r + \frac{1}{2}\omega r} = \frac{\frac{C}{\Omega_2 r}}{1 + \frac{1}{2}\frac{\omega r C}{\Omega_2 r C}} = \frac{\theta_2}{1 + \frac{\theta_2 \epsilon}{2}}$$

Resolving the lift and drag into components perpendicular and parallel to the plane of rotation we have (see fig. 2);

$$U_1 = L_1 \sin\beta_1 + D_1 \cos\beta_1,$$

$$S_1 = L_1 \cos\beta_1 - D_1 \sin\beta_1$$

which when written in coefficient form renders;

$$C_{U_1} = C_{L_1} \sin\beta_1 + C_{D_1} \cos\beta_1,$$

$$C_{S_1} = C_{L_1} \cos\beta_1 - C_{D_1} \sin\beta_1.$$

Let dT_1 be the thrust developed in the annulus between r and r + dr due to Z_1 blades of the first rotor then

$$dT_1 = C_{S_1} \tfrac{1}{2}\rho V_1^2 \, b_1 \, dr \, Z_1$$

which when written in coefficient form gives

$$dC_{T_1} = \frac{dT_1}{\frac{1}{2}\rho C^2 \pi R^2} = \frac{C_{S_1}\frac{1}{2}\rho V_1^2 b_1 dr Z_1}{\frac{1}{2}\rho C^2 \pi R^2} = \frac{C_{S_1} C^2 w_1 \pi R dx}{\sin^2\beta_1 C^2 \pi R} = \frac{C_{S_1} w_1 dx}{\sin^2\beta_1}$$

where $C_{T_1} = T_1/\frac{1}{2}\rho C^2\pi R^2$, $x = r/R$, $dx = dr/R$, $w_1 = \dfrac{Z\,b_1}{\pi\,R}$

or

$$\frac{dC_{T_1}}{dx} = \frac{C_{S_1} w_1}{\sin^2\beta_1} \tag{2}$$

In the same manner let $dQ_1 = C_{U_1}\frac{1}{2}\rho V_1^2 b_1 dr\, r\, Z_1$ then the coefficient form is

$$dC_{Q_1} = \frac{dQ_1}{\frac{1}{2}\rho C^2 \pi R^3} = \frac{C_{U_1}\frac{1}{2}\rho V_1^2 b_1 dr\, r\, Z_1}{\frac{1}{2}\rho C^2 \pi R^3} = \frac{C_{U_1} C^2 w_1 \pi R r dx}{\sin^2\beta_1 C^2 \pi R^2} = \frac{C_{U_1} w_1 x dx}{\sin^2\beta_1}$$

where $C_{Q_1} = \dfrac{Q_1}{\frac{1}{2}\rho C^2 \pi R^3}$, $x = \dfrac{r}{R}$, $dx = \dfrac{dr}{R}$, $w_1 = \dfrac{Zb_1}{\pi R}$, $V_1^2 = \dfrac{C^2}{\sin^2\beta_1}$

or

$$\frac{dC_{Q_1}}{dx} = \frac{C_{U_1} w_1 x}{\sin^2\beta_1} \tag{3}$$

Now we may write the efficiency, η_1, for the annulus of the first rotor based on C_T and C_S. By definition and basic assumption

$E_1 = p_1 + \frac{1}{2}\rho C^2$ and $E_2 = p_2 + \frac{1}{2}\rho C^2 + \frac{1}{2}\rho\omega^2 r^2$;

therefore,

$$\Delta E_1 = \Delta p_1 + \tfrac{1}{2}\rho\omega^2 r^2 .$$

Multiplying both sides by $2\pi r$ dr C we have

$$\Delta E_1 2\pi r \, dr \, C = \Delta p_1 2\pi r \, dr \, C + \tfrac{1}{2}\rho\omega^2 r^2 \, 2\pi r \, dr \, C$$

which becomes

$$\Delta E_1 2\pi r \, dr \, C = C \, dT_1 + \frac{1}{2} \omega dQ_1 \qquad (4)$$

where

$$dQ = \rho C 2\pi r \, dr \, \omega r \, r \qquad (5)$$

(an element of thrust is equal to the rate of change of angular momentum of the fluid) and $dT = \Delta p 2\pi r \, dr$ (6)

(an element of thrust is equal to the increase in pressure produced by the blade elements times the area). Thus equation (4) gives the output of the elementary annulus. The input is

$$\Omega_1 dQ_1 \qquad (7)$$

Then the efficiency is

$$\eta_1 = \frac{CdT_1 + \tfrac{1}{2}\omega dQ_1}{\Omega_1 dQ_1} = \frac{C}{\Omega_1}\left[\frac{dT_1}{dQ_1}\right] + \frac{1}{2}\frac{\omega}{\Omega_1}$$

$$\text{or} \quad \eta_1 = r\theta_1 \left[\frac{dC_{T_1}}{dx} \bigg/ \frac{dC_{Q_1}R}{dx}\right] + \frac{1}{2} \epsilon \, \theta_1$$

where $\epsilon = \dfrac{\omega r}{C}$, $\theta_1 = \dfrac{C}{\Omega_1 r}$, $\dfrac{dT_1}{dQ_1} = \dfrac{dC_{T_1}}{dC_{Q_1}R}$

Using equations (2) and (3) above this becomes

$$\eta_1 = r\theta_1\left[\frac{C_{S_1}}{C_{U_1}xR}\right] + \frac{1}{2}\epsilon\theta_1\ , \text{ but } \quad x = r/R$$

therefore
$$\eta_1 = \theta_1\left[\frac{C_{S_1}}{C_{U_1}} + \frac{1}{2}\epsilon\right]$$

Now
$$\frac{C_{S_1}}{C_{U_1}} = \frac{C_{L_1}\cos\beta_1 - C_{D_1}\sin\beta_1}{C_{L_1}\sin\beta_1 + C_{D_1}\cos\beta_1}$$

dividing numerator and denominator by $C_{D_1}\cos\beta_1$ we have

$$\frac{C_{S_1}}{C_{U_1}} = \frac{\frac{C_{L_1}}{C_{D_1}} - \tan\beta_1}{\frac{C_{L_1}}{C_{D_1}}\tan\beta_1 + 1} = \frac{\delta_1 - \tan\beta_1}{\delta_1\tan\beta_1 + 1}$$

where $C_{L_1}/C_{D_1} = \delta_1$

but from (1) above $\tan\beta_1 = \dfrac{\theta_1}{1 - \dfrac{\theta_1\epsilon}{2}} = \dfrac{2\theta_1}{2 - \theta_1\epsilon}$

then
$$\frac{C_{S_1}}{C_{U_1}} = \frac{\delta_1 - \frac{2\theta_1}{2-\theta_1\epsilon}}{\frac{2\delta_1\theta_1}{2-\theta_1\epsilon} + 1}$$

and the efficiency for the first rotor becomes

$$\eta_1 = \theta_1 \left[\frac{\delta_1 - \dfrac{2\theta_1}{2+\theta_1 \epsilon}}{\dfrac{2\delta_1 \theta_1}{2+\theta_1 \epsilon} + 1} + \frac{1}{2}\epsilon \right] \tag{8}$$

Using the same analysis on the second rotor the only difference is as follows:

$$\Delta E_2 = \Delta p_2 - \tfrac{1}{2}\rho\omega^2 r^2$$

$$\eta_2 = \theta_2 \left[\frac{C_{S_2}}{C_{U_2}} - \frac{1}{2}\epsilon \right]$$

$$\tan\beta_2 = \frac{\theta_2}{1 + \dfrac{\theta_2 \epsilon}{2}} = \frac{2\theta_2}{2 + \theta_2 \epsilon}$$

which gives for η_2

$$\eta_2 = \theta_2 \left[\frac{\delta_2 - \dfrac{2\theta_2}{2+\theta_2 \epsilon}}{\dfrac{2\delta_2 \theta_2}{2+\theta_2 \epsilon} + 1} - \frac{1}{2}\epsilon \right] \tag{9}$$

Now as previously stated in equations (4) and (7)

$$\eta_1 = \frac{\Delta E_1 2\pi r \ dr \ C}{\Omega_1 dQ_1} \qquad \text{where} \quad dQ_1 = \rho C 2\pi r \ dr \ \omega r \ r$$

expressed in terms of k, θ, and ϵ this becomes

$$\eta_1 = \frac{k_1\theta_1}{2\epsilon} \tag{10}$$

where $\epsilon = \frac{\omega r}{C}$, $\theta = \frac{C}{\Omega r}$, $k = \frac{\Delta E}{\frac{1}{2}\rho C^2}$

likewise

$$\eta_2 = \frac{k_2\theta_2}{2\epsilon} \tag{11}$$

Combining equations (8) and (10) we have

$$k_1 = 2\epsilon\left[\frac{\delta_1 - \frac{2\theta_1}{2-\theta_1\epsilon}}{\frac{2\delta_1\theta_1}{2-\theta_1\epsilon} + 1}\right] + \epsilon^2 \tag{12}$$

and similarly from equations (9) and (11) we have

$$k_2 = 2\epsilon\left[\frac{\delta_2 - \frac{2\theta_2}{2-\theta_2\epsilon}}{\frac{2\delta_2\theta_2}{2-\theta_2\epsilon} + 1}\right] - \epsilon^2 \tag{13}$$

Since $k = k_1 + k_2$ equations (12) and (13) can be combined to give the slipstream rotation factor, ϵ, in terms of the basic design variables, namely k, δ, and θ. k is usually known from the output requirements and θ_1, θ_2, δ_1, δ_2, are a matter of choice only to effect certain conditions while obtaining high efficiency.

The combined efficiency for the annulus will be

$$\eta = \frac{\Delta E\ 2\pi\ r\ dr\ C}{\Omega_1 dQ_1 + \Omega_2 dQ_2} = \frac{k}{2\epsilon}\left[\frac{\theta_1\theta_2}{\theta_1+\theta_2}\right] \tag{14}$$

The total overall efficiency for the stage is then

$$\eta_{Tot} = \frac{\int_r^R (\Delta E_1+\Delta E_2)\ 2\pi r\ dr\ C}{\int_r^R \Omega_1 dQ_1 + \int_r^R \Omega_2\ d\ Q_2} \tag{15}$$

Since from equation (5) $dQ = \rho C 2\pi r\ dr\ \omega r\ r$

or in coefficient form

$$dC_{Q_1} = dC_{Q_2} = \frac{dQ}{\frac{1}{2}\rho C^2 \pi R^3} = \frac{\rho C 2\pi r\ dr\ \omega r\ r}{\frac{1}{2}\rho C^2 \pi R^3} = 4\epsilon x^2 dx$$

where $\epsilon = \omega r/C$, $x = r/R$, $dx = dr/R$

Since dC_{Q_1} and dC_{Q_2} are oppositely directed and will probably be operated from the same power source, the total element of torque dC_Q for both rotors is equal to $dC_{Q_1} + dC_{Q_2}$

$$dC_Q = dC_{Q_1} + dC_{Q_2} = 4\epsilon x^2 dx + 4\epsilon x^2 dx = 8\epsilon x^2 dx \tag{16}$$

Now from equation (6) $dT_1 = \Delta p_1 2\pi r\ dr$

or in coefficient form

$$dC_{T_1} = \frac{dT_1}{\frac{1}{2}\rho C^2 \pi R^2} = \frac{\Delta p_1 2\pi r\ dr}{\frac{1}{2}\rho C^2 \pi R^2} = \frac{4\Delta p_1 x\ dx}{\rho C^2}$$

since by definition

$$E_1 = p_1 + \tfrac{1}{2}\rho C^2$$

$$E_2 = p_2 + \tfrac{1}{2}\rho C^2 + \tfrac{1}{2}\rho\omega^2 r^2$$

$$E_3 = p_3 + \tfrac{1}{2}\rho C^2$$

and $\Delta p_1 = p_2 - p_1 = E_2 - \tfrac{1}{2}\rho C^2 - \tfrac{1}{2}\rho\omega^2 r^2 - (E_1 - \tfrac{1}{2}\rho C^2) = \Delta E_1 - \tfrac{1}{2}\rho\omega^2 r^2$

therefore $$dC_{T_1} = \frac{4(\Delta E_1 - \tfrac{1}{2}\rho\omega^2 r^2)\,xdx}{\rho C^2} = 2(k_1 - \epsilon^2)\,xdx$$

where $k_1 = \Delta E_1/\tfrac{1}{2}\rho C^2$, $\epsilon = \dfrac{\omega r}{C}$

similarly $dT_2 = \Delta p_2 2\pi r\, dr$

or in coefficient form

$$dC_{T_2} = \frac{4\Delta p_2 xdx}{\rho C^2} = 2(k_2 + \epsilon^2)\,xdx$$

where $\Delta p_2 = \Delta E_2 + \tfrac{1}{2}\rho\omega^2 r^2$, $k_2 = \Delta E_2/\tfrac{1}{2}\rho C^2$, $\epsilon = \dfrac{\omega r}{C}$

since $\Delta p = \Delta p_1 + \Delta p_2 = (p_2 - p_1) + (p_3 - p_2)$

$$= (\Delta E_1 - \tfrac{1}{2}\rho\omega^2 r^2) + (\Delta E_2 + \tfrac{1}{2}\rho\omega^2 r^2) = \Delta E_1 + \Delta E_2$$

and $k = k_1 + k_2$

then $$dC_T = 2(k_1 - \epsilon^2)\,xdx + 2(k_2 + \epsilon^2)\,xdx = 2kxdx \qquad (17)$$

Now from equations (15), (16), and (17)

$$\eta_{Tot} = \frac{\int_r^R (\Delta E_1 + \Delta E_2)\ 2\pi r\ dr\ C}{\int_r^R \Omega_1 dQ_1 + \int_r^R \Omega_2 dQ_2}$$

$$= \frac{\int_r^R \Delta p 2\pi r dr C}{\Omega_1 \int_r^R (\rho C 2\pi r\ dr\ \omega r\ r) + \Omega_2 \int_r^R (\rho C 2\pi r\ dr\ \omega r\ r)} =$$

$$\frac{2C\int_r^R dT}{(\Omega_1+\Omega_2)\int_r^R dQ} = \frac{2C\int_r^R [dC_{T_1}(\frac{1}{2}\rho C^2\pi R^2) + dC_{T_2}(\frac{1}{2}\rho C^2\pi R^2)]}{(\Omega_1+\Omega_2)\int_r^R [dC_{Q_1}(\frac{1}{2}\rho C^2\pi R^3) + dC_{Q_2}(\frac{1}{2}\rho C^2\pi R^3)]} =$$

$$\frac{C\frac{1}{2}\rho C^2\pi R^2 2\int_r^R (dC_{T_1} + dC_{T_2})}{(\Omega_1+\Omega_2)\ \frac{1}{2}\rho C^2\pi R^3 \int_r^R (dC_{Q_1} + dC_{Q_2})} =$$

$$\frac{C}{(\Omega_1+\Omega_2)R} \cdot \frac{2\int_r^R dC_T}{\int_r^R dC_Q} = \frac{C}{(\Omega_1+\Omega_2)R} \cdot \frac{2C_T}{C_Q} =$$

$$\eta_{Tot} = \frac{\theta_1\theta_2}{\theta_1+\theta_2} \cdot \frac{2C_T}{C_Q} \qquad (18)$$

This equation is best utilized where it is designed to integrate the torque and thrust curves over the full blade length; however, the benefits of no slipstream rotation can be measured by a simpler equation as follows.

Assuming constant efficiency along the blade length we may determine the overall efficiency of the stage from the preceding equation for the annulus, which if it is high dictates a high efficiency for the stage.

Combining equations (10), (11), and (14) we have

$$\eta = \frac{\theta_2\eta_1 + \theta_1\eta_2}{\theta_1 + \theta_2} \tag{19}$$

which upon substitution of equations (8) and (9) becomes

$$\eta = \frac{\theta_1\theta_2}{\theta_1 + \theta_2}\left[\frac{(\delta_1-\theta_1) + \frac{1}{2}\epsilon(1-\frac{1}{2}\epsilon\theta_1)}{\delta_1\theta_1+1-\frac{1}{2}\epsilon\theta_1} + \frac{(\delta_2-\theta_2) - \frac{1}{2}\epsilon(1+\frac{1}{2}\epsilon\theta_2)}{\delta_2\theta_2+1+\frac{1}{2}\epsilon\theta_2}\right] \tag{20}$$

Since the first term of the bracket increases as ϵ increases and the second term of the bracket decreases with an increase in ϵ, one term compensates for the other such that an approximate expression for η written from equation (20) neglecting ϵ will be

$$\eta \approx \frac{\theta_1\theta_2}{\theta_1+\theta_2}\left[\frac{\delta_1-\theta_1}{\delta_1\theta_1+1} + \frac{\delta_2-\theta_2}{\delta_2\theta_2+1}\right] \tag{21}$$

This can be easily verified by referring to Table I where over a range of ϵ the efficiency η is practically the same.

In the particular case for constant $\delta = \delta_1 = \delta_2$ where δ is greater than 50, which is easily attainable for a large number of airfoil sections (see fig. 3), and at a C_L below $C_{L\,max}$ the radial distribution of ϵ may be approximated by combining equation (14) and (21)

$$\epsilon \approx \frac{\frac{1}{2}k}{\frac{\delta - \theta_1}{\delta\theta_1+1} + \frac{\delta-\theta_2}{\delta\theta_2+1}} \qquad (22)$$

TABLE I

Special case of $\delta_1=\delta_2=50$

ϵ	θ_1	θ_2	k (22)	η (20)	η (21)
.25	1.0	1.0	.962	.9598	.9615
.50	1.0	1.0	1.923	.9595	.9615
.75	1.0	1.0	2.881	.9575	.9615
1.00	1.0	1.0	3.843	.9564	.9615

It is seen from this table that an efficiency of almost one is attained by converting all of the input energy into pressure rise rather than speeding up the flow.

CONCLUSIONS

A method has been developed herein for calculating the performance of a contra-rotating ducted axial flow compressor.

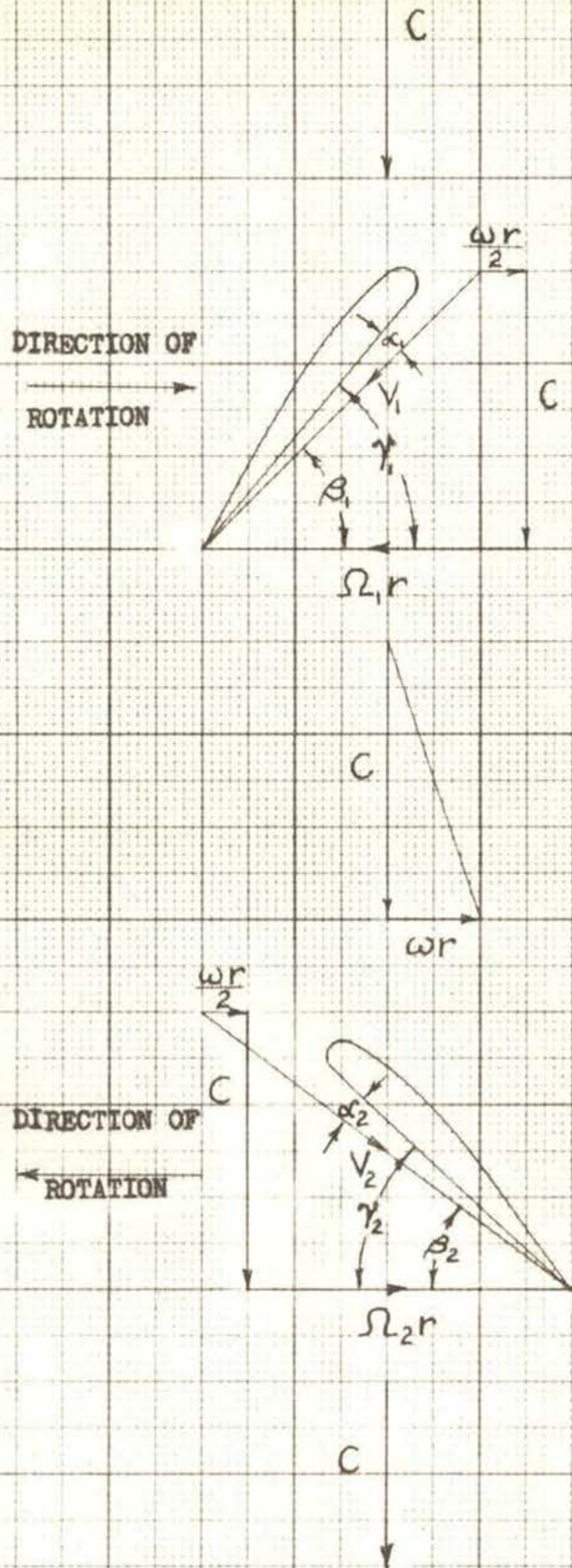
In this thesis only one method of design was presented and one range of values used as seen in Table I. This table could be extended to embrace numerous values and a better picture of the design method could be realized.

Even in the simplest case of one dimensional flow the efficiency is not one; however, it is very high. To achieve high efficiency in a contra-rotating compressor it is necessary to select very carefully the design variables so that the whirl velocity is a minimum. The figures taken from Dr. B. Eckert's A COLLECTION OF COMPRESSOR TEST RESULTS, Jan 1946, Stuttgart, FKFS Report No. 10 1276/7 show that generally higher efficiencies and higher pressure ratio rises are practically attainable with the contra-rotating compressor design.

Fertile fields of investigation beyond the scope of this thesis remain. The next step in the investigation should consider the three dimensional case and the effect of such things as Reynolds number and Mach number.

VELOCITY DIAGRAM FOR SINGLE STAGE CONTRA-ROTATING DUCTED AXIAL FLOW COMPRESSOR

FIG 1



FORCE DIAGRAM FOR SINGLE STAGE CONTRA-ROTATING AXIAL FLOW DUCTED COMPRESSOR

FIG 2

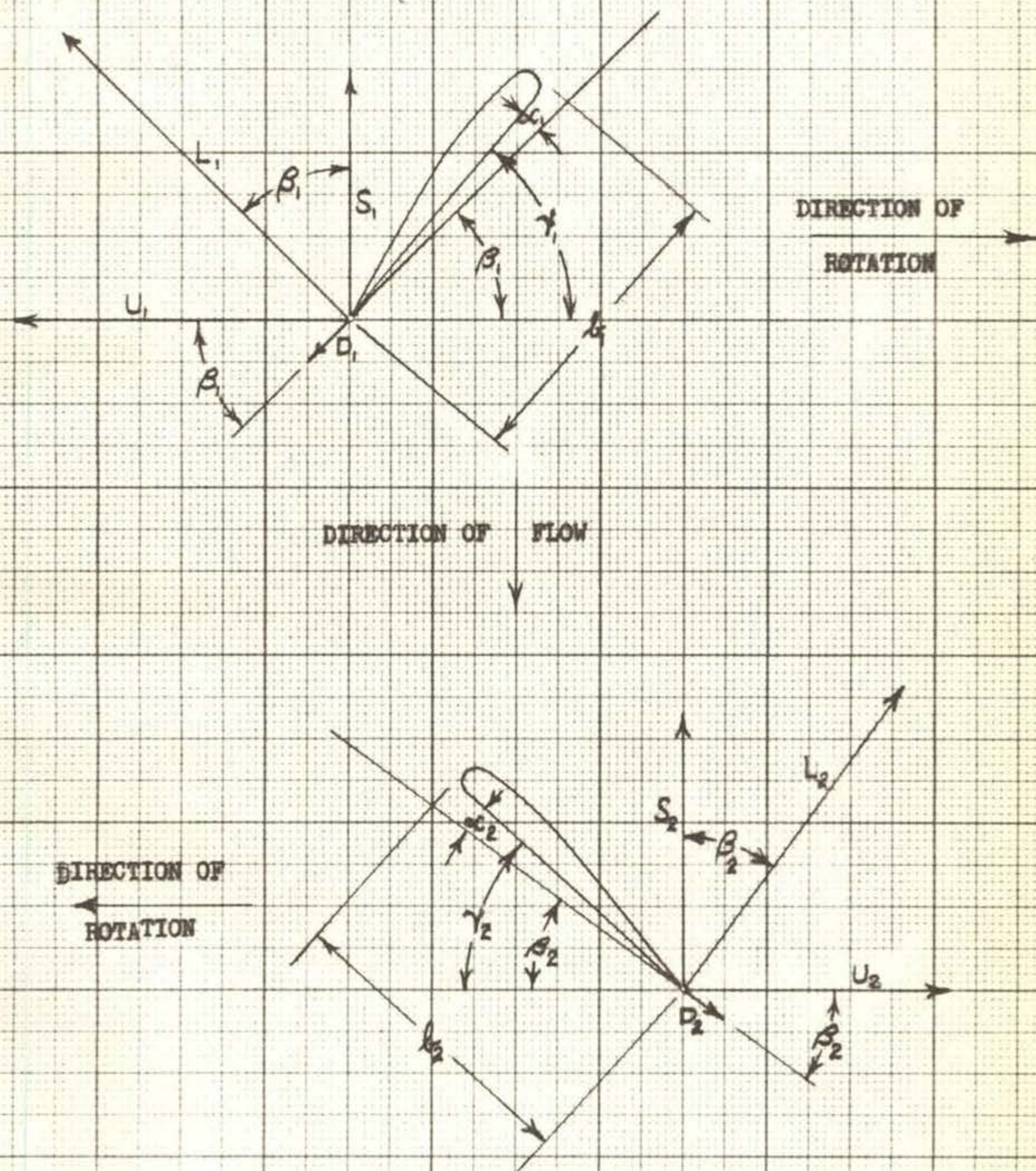
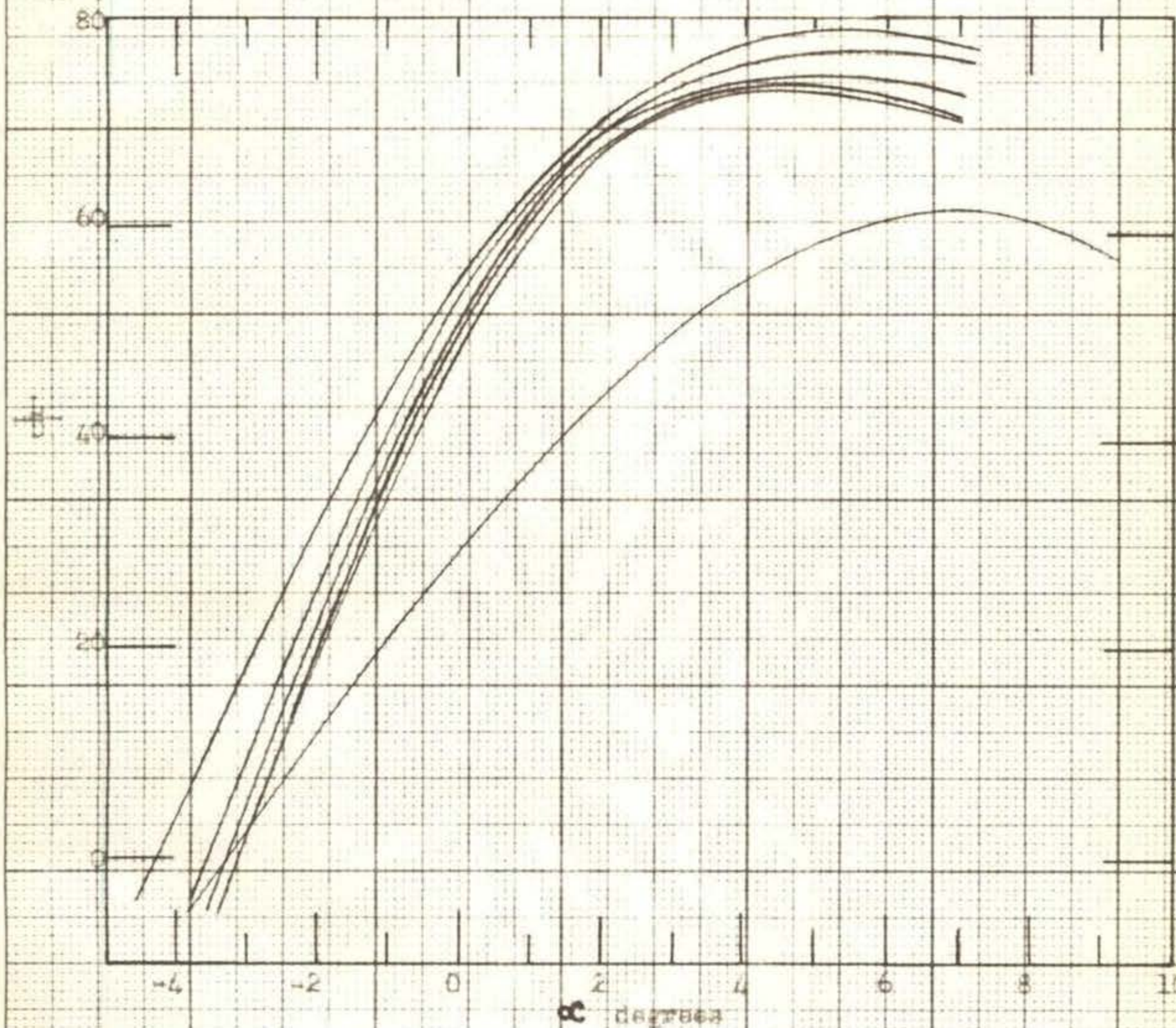


FIG 3

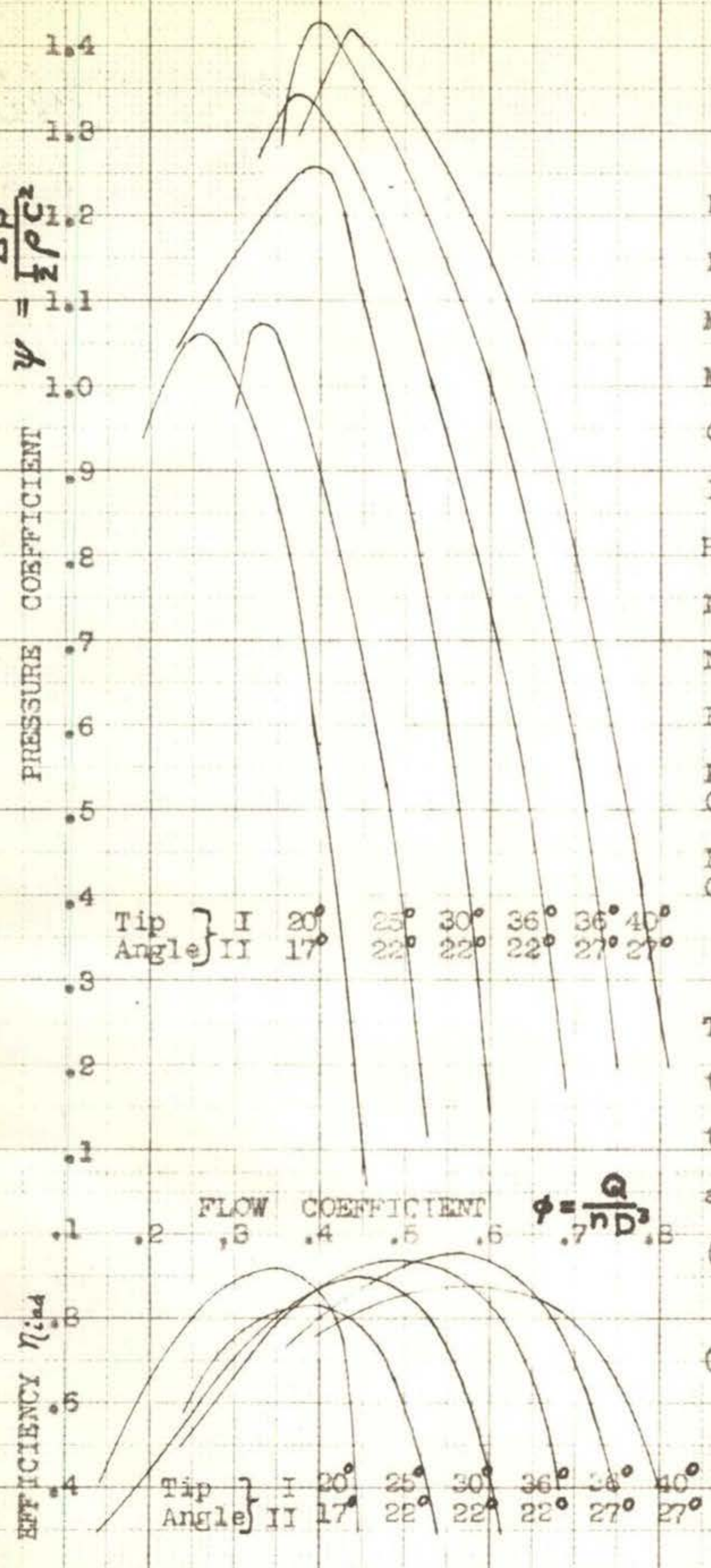
CHARACTERISTIC LIFT/DRAG CURVES FOR SEVERAL AIRFOILS

INFINITE ASPECT RATIO[4]



[4] Naiman, Irven, METHOD OF CALCULATING PERFORMANCE OF DUAL-ROTATING PROPELLERS FROM AIRFOIL CHARACTERISTICS, NACA, ARR No. 3E24, May 1943, fig 4.

FIG.4



PURPOSE-RESEARCH

Reynolds No I-95,000

Reynolds No II-76,700

Mach No I 0.26

Mach No II 0.26

O.D. Rotors 11.82"

I.D. Rotors 7.48"

Hub-Tip Ratio 0.634

No Blades I 12

No Blades II 12

RPM (Design) 5,400

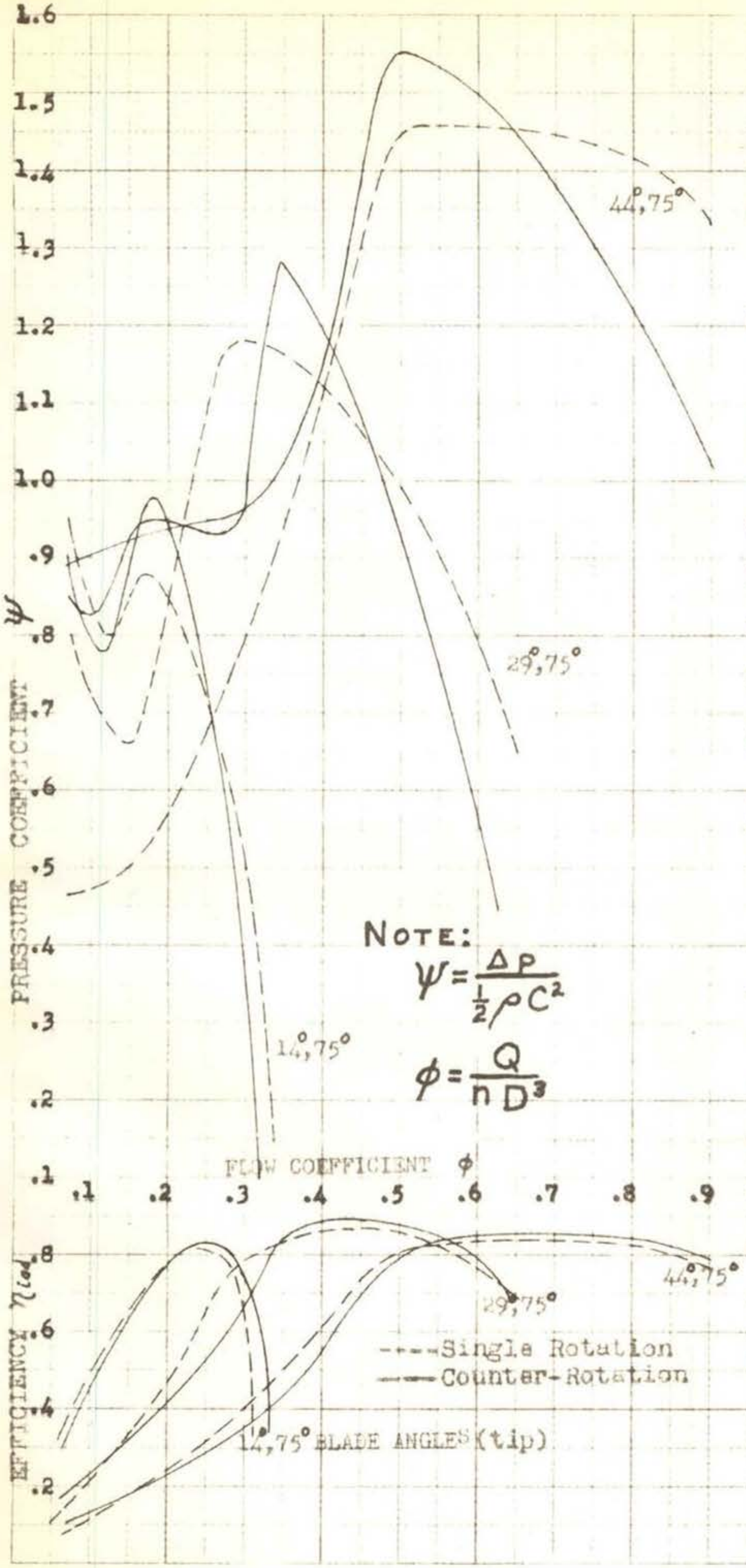
Profile Thickness to Chord Ratio 0.1

Radial Clearance to O.D. Ratio 0.5×10^{-2}

REMARKS

This blower was used to determine volumetric efficiency in axial compressors (High Pressure, Counter-Rotating) (Single Stage with no inlet or outlet guide vanes)

FIG. 5



PURPOSE-RESEARCH

Re No- 50,000 to 120,000

Mach No- 0.17 to 0.22

O.D. Rotors 11.82"

I.D. Rotots 5.91"

Hub-Tip Ratio 0.5

No Blades Rot 12

No Blades Stat 13

RPM (Design) 3,000

Profile Thickness to Chord Ratio .1

Radial Clearance to O.D. Ratio 0.001

REMARKS

An experiment on two two stage axial blowers with single and counter-rotation of the rotors, respectively. (13 inlet and outlet guide vanes)

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