DESIGN AND ANALYSIS OF A
TEMPERATURE CONTROL
SYSTEM

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PREFACE

Temperature control systems are as interesting as they are varied in purpose. Temperature control is used in large electric furnaces and for small ovens only large enough for a quartz crystal. Ovens may use simple thermostat control or a complex feedback network, keeping temperature constant within one-thousandth of a degree. Design of a temperature control system involves work with heat transfer fundamentals, electronic circuitry, and feedback theory. This thesis will present design considerations in all these areas. In addition, analysis of system operation using analog simulation techniques will be undertaken. This technique of system analysis has been especially rewarding. A complete control system was built to compare the theoretical design work and operation predictions with actual operation. The problem of measuring oven temperature stability was quite perplexing since the most sensitive means of temperature sensing available was used to control the oven in the first place.

I would like to express here my deep appreciation to my thesis adviser, Dr. Harold T. Fristoe, for his guidance in all the diversified fields of investigation undertaken. I also owe a debt of gratitude to Professor J. R. Norton for his help throughout my graduate endeavor, and to Mr. Freddie W. Wenninger for his unceasing encouragement and suggestions in writing the thesis.

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CHAPTER I

INTRODUCTION

The parameters of many important devices are dependent to some extent upon their temperature. Resonant frequency, the most important parameter of a piezoelectric quartz crystal, is remarkably independent of temperature, and is consequently used in applications utilizing this feature. It is this inherent independence of the frequency characteristic, however, which makes it worthwhile to stabilize this parameter still further by controlling temperature of the environment as accurately as possible. The dependence of the series resonant frequency upon temperature of a typical AT cut quartz crystal is indicated in Figure 1. (1).

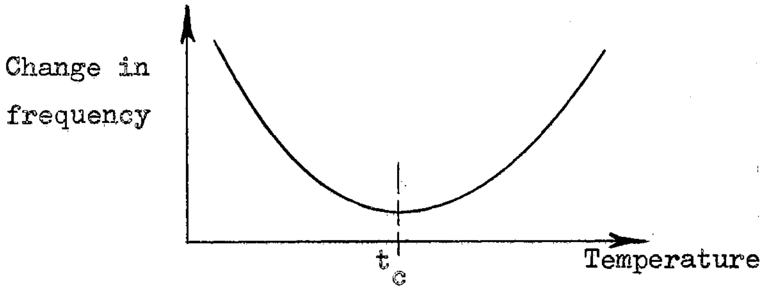


Figure 1. Frequency Dependence of a Typical AT Cut Quartz Crystal

This relationship of frequency and temperature suggests that its surroundings should not only be kept at as constant a temperature as possible, but at some particular "turnover" temperature, t_c, as well. This temperature, typically 50° C, should be kept in mind when an oven is designed for quartz crystal environment.

Piezoelectric crystals are by no means the only devices which benefit from a constant temperature environment. Another class of devices, having parameters which are monotonic functions of temperature, includes transistors, operational impedances for analog computation, and mechanical devices such as delay lines.

Achievement of a constant temperature has evolved through several stages of improvement. Temperature may be controlled with a thermostat, but in such a system the heater either delivers full power or is off. A temperature oscillation necessarily results and accuracy is limited to about 0.1° C. An important refinement was realized when power of the heater was caused to be proportional in some manner to the error in temperature.

In 1897, Gouy used a mechanical means of achieving proportional control. (2). A vibrating wire made contact with the top of a mercury column during some part of its cycle. The connection controlled a heater, its "on" period being a function of mercury column height. Ingenious as it was, this method and other mechanical means of proportional control have generally been replaced by electronic means of regulation.

The combination of two proportional units, one enclosing the other, represents the state of the art. Stabilities on the order of .001° C are presently being achieved using double proportionally controlled systems.

Various novel methods for controlling temperature, such as that depending on the change of state of a substance, have also been suggested, but are not generally used because of some limitation. (3).

The conventional method of designing the proportionally controlled oven is one involving mainly experience with such systems and very little theoretical consideration of the heat transfer characteristics of the oven. This paper will present a method of predicting the effects of heater placement, transfer characteristic of the feedback amplifier, etc., before building the system. This will be done primarily using analog simulation techniques. Theoretical aspects will also be discussed considering the relevant transfer functions.

Analysis will be confined to a single proportionally controlled system. Use of a nonlinear feedback amplifier for oven control in place of the usual linear amplifier will be shown to permit more desirable transient characteristics for a given steady-state temperature error.

Finally, operation of a system incorporating the desirable features suggested by theorectical considerations will be described. Effects of various changes of parameters predicted by the analog model will be verified using the real control system.

CHAPTER II

THE SYSTEM AND ITS COMPONENTS

Consideration of Temperature Control Systems

Almost all temperature control systems can be broken into block sections and represented as shown in Figure 2.

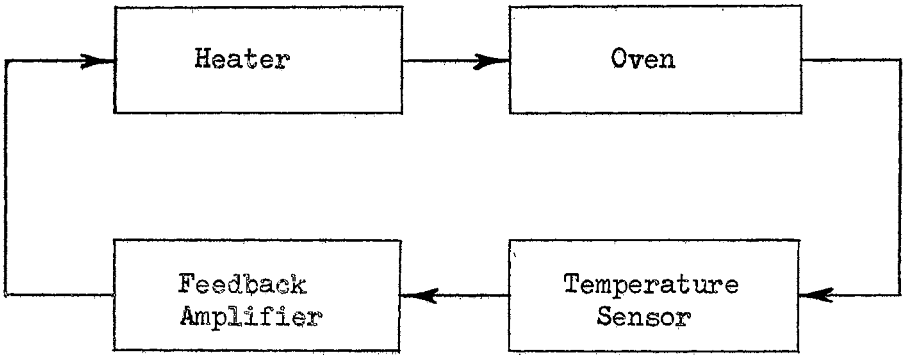


Figure 2. Block Diagram of a Conventional Temperature Control System

In the simple case of the thermostatically controlled oven, the feedback amplifier is not used; the temperature sensor itself controls the heater. More sophisticated systems, however, utilize a linear feedback amplifier which, for small temperature errors, applies heater control in the form of a heater voltage proportional to the error in temperature. Since heater power is proportional to the square of the control voltage, the over-all steady-state transfer function of heater power versus temperature error is shown in Figure 3.

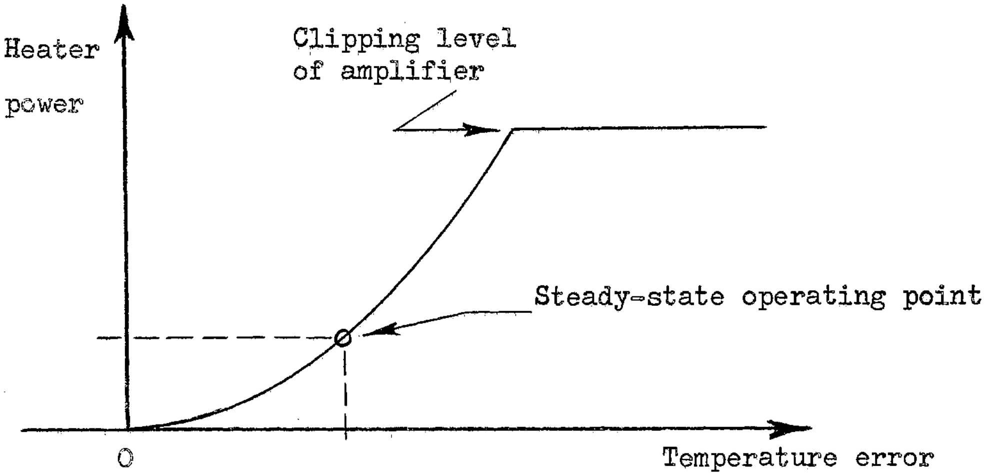


Figure 3. Heater Power of the Conventional System as a Function of Temperature Error

The steady-state operating point is indicated for some arbitrary heat power required for equilibrium. This characteristic leaves much to be desired, however, since maximum over-all gain is greatest in the region where it is not needed, the steady-state operating region having relatively low gain.

A more desirable steady-state transfer function is shown in Figure 4.

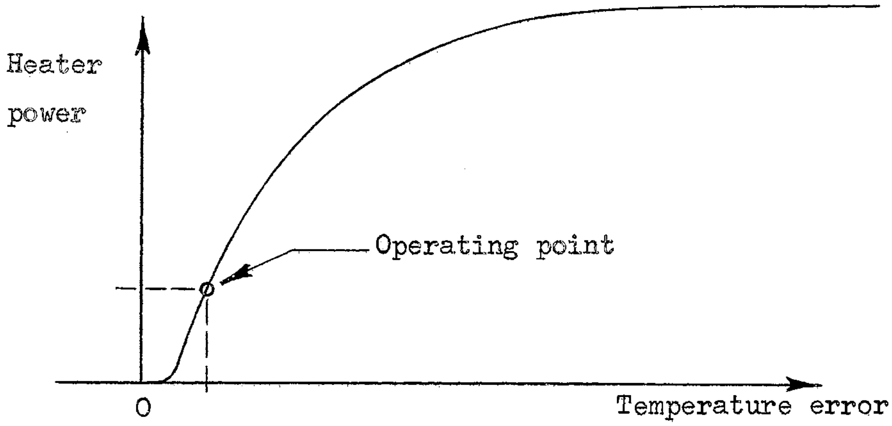


Figure 4. Heater Power as a Function of Temperature Error for the Improved System

It is seen that the highest gain available is in the region of steady-state operation. In the previous case, as the temperature approaches the steady-state value from a cool initial condition at the right (see Figure 3), the operating point is almost reached before the heater power begins to drop. In the improved case, however, the heater power is gradually lowered as the operating point is approached. It is evident that for identical gains at the operating point, use of the second system will result in less overshoot and possibly in fewer stability problems. If, on the other hand, the two systems were designed to satisfy stability and overshoot considerations, the second would yield a smaller steady-state temperature error.

At least one transfer characteristic other than the square function of Figure 3 has been used with some success. Malmberg and Matland have used compensating circuitry to obtain a truly proportional relationship between heater power and temperature error. (4).

A system will now be considered which provides the response depicted in Figure 4. Suppose that a nonlinear feedback system is used in place of the linear amplifier considered heretofore. This may be achieved as shown in Figure 5.

First, the operation of this system is analyzed beginning with the sensor. The temperature sensitive bridge shown is excited with a square wave source, providing a small square wave error voltage whose amplitude is proportional to the temperature error. This square wave is applied to the RC differentiating circuit shown, resulting in the familiar train of exponentially decaying pulses. This train is amplified by a linear amplifier, which may clip if it is driven beyond its range of linear operation. These clipped, exponentially decaying, pulses are then examined by a comparator which has some arbitrary output level when the pulse voltage exceeds the reference voltage shown. The train of output pulses from the comparator are rectangular in shape and their duration is always less than one-half cycle of the input error signal. Now suppose

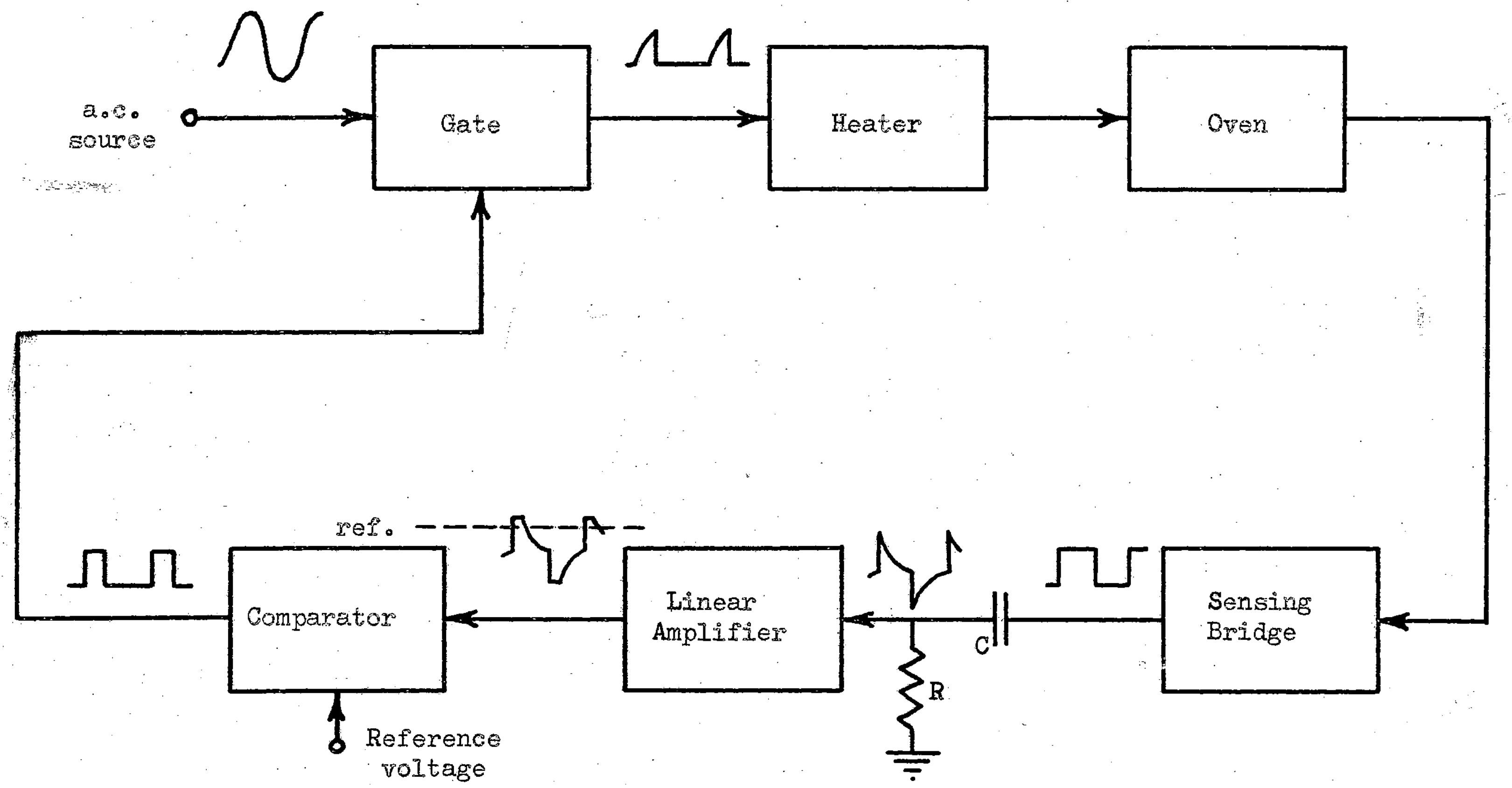


Figure 5. The Improved Temperature Control System

the bridge is supplied from an a.c. source synchronous with the a.c. power source, say the 60 cycle line. The output of the comparator is then synchronous with the power source and may be used to switch a particular part of each cycle of the power source voltage across the heating element, using the gate shown. It is seen that for large error signals, an entire half cycle of the supply voltage is impressed across the heater. Also, when there is no temperature error, the heater will not be turned on.

The choice of the time constant of decay, RC, has a marked effect on system operation. If the rate of decay is chosen too small, corresponding to a large value of RC, drift of the comparator level will become a major source of error. If the decay rate is too large, resulting from a small value of RC, sensitivity of the system will be small unless the amplifier gain is made large, in which case noise becomes a problem. In view of these restrictions, the time constant was chosen to be 50° of the 180° half cycle of the a.c. line voltage. Each pulse will fall to about 2.5% of its initial value before the next pulse is initiated using this decay rate, which corresponds to a time constant of .00232 seconds.

The exact relationship between the error signal and power applied to the oven by the heater is not easily seen, so it is derived in Appendix A. The shape of the steady-state transfer function is that already shown in Figure 4. It will be considered in more detail in later chapters.

Design and Analysis of the Oven

In practice, when a system is sensitive to temperature, the size of the temperature sensitive section is usually small. Quartz crystals, solid state choppers, d.c. amplifier input stages, can all be contained in the volume of a few cubic inches. A single size of oven is therefore capable of handling the majority of circuit engineering problems.

The shape of an oven is not usually dictated by its contents, since components of the temperature sensitive circuit are relatively small and may be arranged to fit whatever configuration is available. The shape can, therefore, be chosen to satisfy other considerations. A spherical shape would be ideal from the theoretical point of view since there would be no corners to heat or cool more rapidly than the rest. Construction problems rise immediately, however. A rectangular configuration could be constructed easily, but has corners and a higher surface/volume ratio, resulting in more heat lost to the surroundings. A good compromise between these two extremes is a cylinder of about equal height and width. This shape is by no means critical, however, since long narrow cylinders are presently being used successfully.

Aluminum is usually chosen for the enclosure because of its rapid heat conduction and merits in fabrication. This heated enclosure may be surrounded by one of a number of insulating materials, cork being used extensively because of its low coefficient of thermal conduction and ease in handling.

Ideal placement of the heat source would be achieved by a distributed winding of resistance wire covering the entire surface. In practice, however, several lumped sources are used. In the case of the cylindrical oven, two heating windings, each placed near one end and wound around the cylinder, are found to be a good compromise.

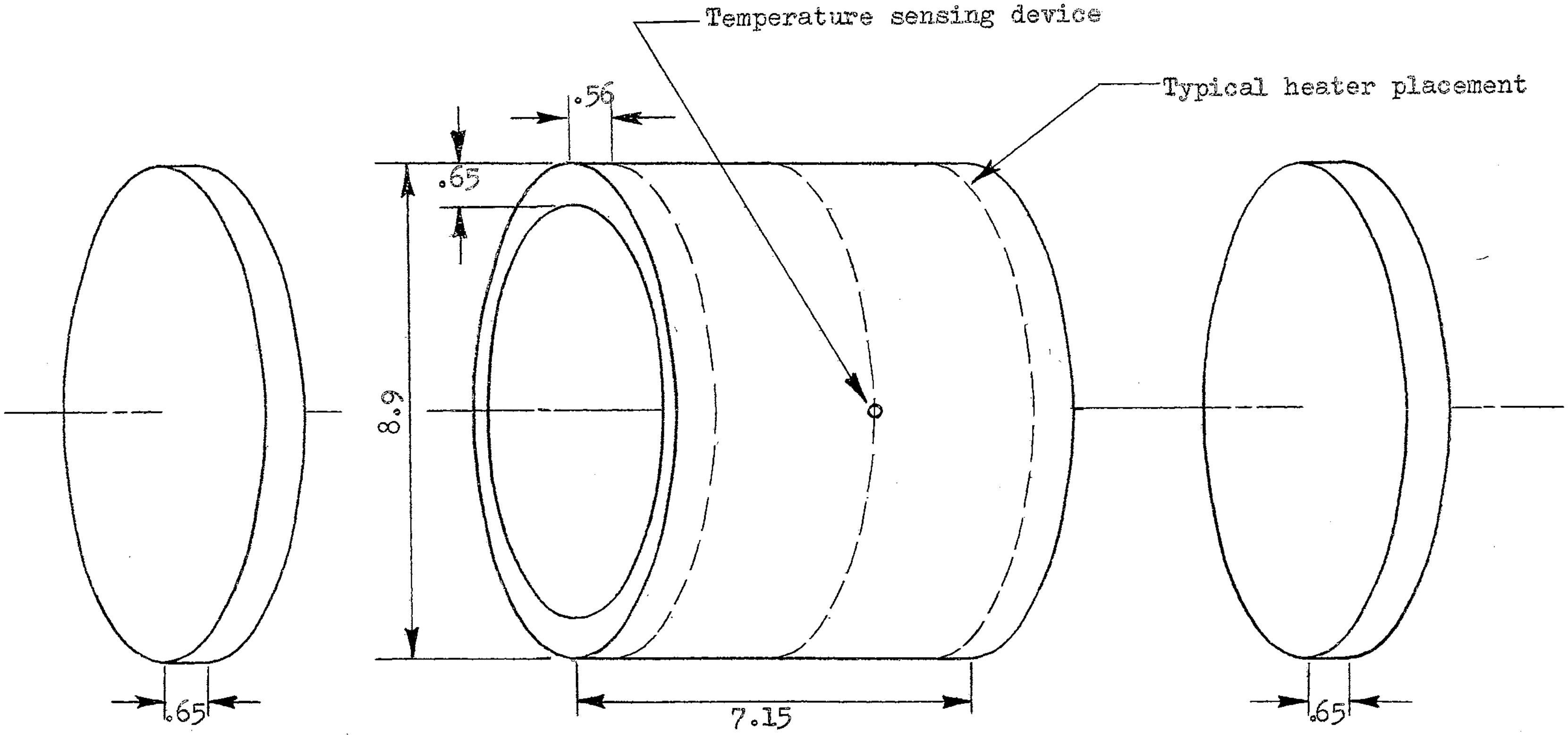
For the reasons given above, an aluminum oven, as shown in Figure 6, was machined and will be the configuration used for all investigations.

Primary investigation of operation will be done using the heater placement shown. The oven may be thought of as being broken into four sections, separated by the heaters and the central circumference. With the heater placement shown, the masses of these sections will be equal. This configuration has several important features including:

A. The ends will neither overheat nor assume the steady-state temperature too slowly in transient operation since their behavior will be similar to the point sampled by the temperature sensor.

B. Load is equally shared by the heaters, yielding symmetrical performance.

C. No heat flows across the central circumference, resulting in convenient theoretical analysis of the heat flow and resulting temperatures.



Note: Dimensions in cm.

Figure 6. The Oven

It is now to be established how the temperature at the sensing point changes when a step input of heat energy is supplied by the heaters. This is the case when a voltage is suddenly applied to the heaters, as in turn-on. Since, for each heater, equal amounts of heat will flow toward the center of the oven and toward the end, and no heat flows across the central circumference, the equivalent model in Figure 7 may be used.

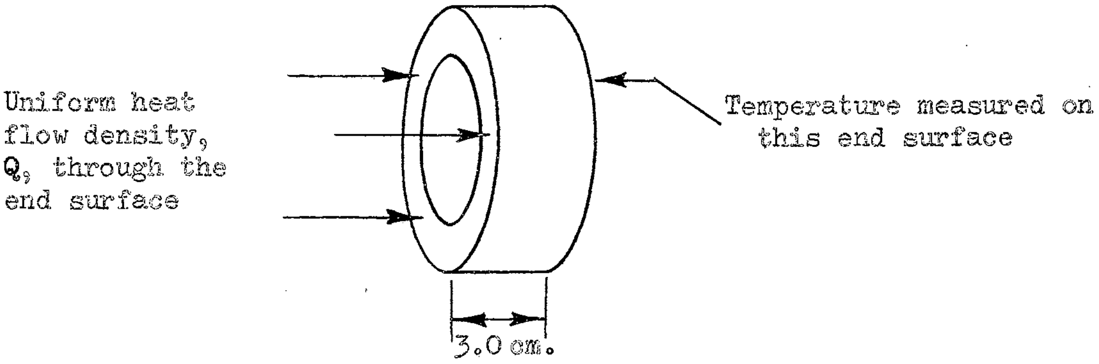


Figure 7. First Equivalent Model of the Oven

It is further assumed that the oven wall is thin and the generated heat is distributed evenly around the oven. It follows, then, that the heat flow is one dimensional. Because of this, the model can be simplified further, as shown in Figure 8, where the slab of constant thickness may be extended indefinitely in height and width.

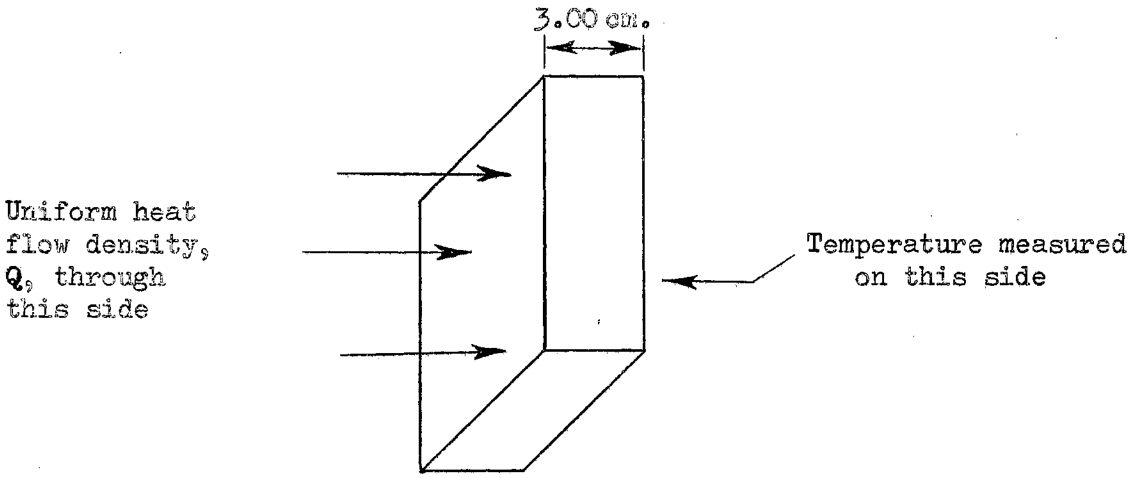


Figure 8. Second Equivalent Model of the Oven

In summary, Figure 8 shows solid matter bounded by two parallel planes. The initial condition is uniform zero reference temperature, and the boundary conditions are (a) constant flux, Q, into the solid at the left side, and (b) no heat flow from the solid at the right side. The temperature, v, at any time, t, under these conditions has been shown to be. (5).

$$v = \frac{QL}{K}\left\{\frac{kt}{L^2} - \frac{1}{6} - \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\,e^{-k\,n^2\,\pi^2\,t/L^2}\right\} \qquad (1.1)$$

where

Q is flux at the left boundary

K is the coefficient of thermal conductivity of the solid

k is the diffusivity constant of the solid

L is the thickness of the solid.

The summation term converges quite slowly for small values of time, so the equation was solved with the aid of a digital computer. Assuming a practical value of heater power, 15 watts total, and an aluminum oven of the configuration discussed, the solution of temperature at the sensing point as a function of time is given in Figure 9. It should be noted that this solution does not include heat losses from the oven. These will be insignificant throughout the time interval shown, but will become appreciable at a later time. A linear assymptote as shown, then, will not be valid except for small intervals of time, or small ranges of temperature.

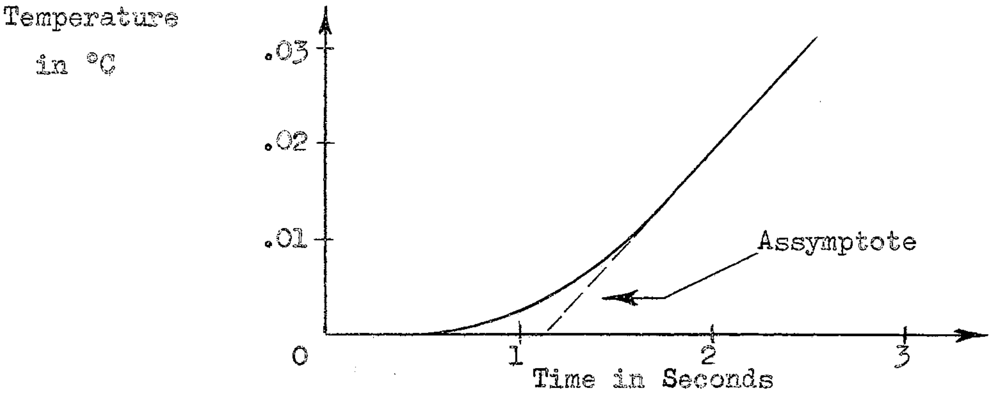


Figure 9. Temperature of the Oven as a Function of Time for Small Values of Elapsed Time

Considering operation on a much larger time scale, it is noted that heat lost from the oven through the insulation to the surroundings is proportional to the temperature difference between the oven and its surroundings. If ambient temperature is chosen to be zero as a reference, a basic heat transfer relationship may be used:

$$v = \int \frac{q_1 - q_2}{Mc}\, dt \qquad (1.2)$$

where

q_1 is the heat energy generated by the heater per unit time

q_2 is the rate of heat loss from the oven

M is the mass of the oven

c is the specific heat capacity of the oven material.

Realizing that q_2 is proportional to the temperature difference, the conduction coefficient of the insulating material, and the physical configuration, Equation (1.2) may be expressed as

$$v = \frac{1}{Mc}\int (q_1 - Bv)\, dt \qquad (1.3)$$

where B is a function of the insulation conduction coefficient and the physical configuration, which are constants. An equation explicit in v may be found from Equation (1.3) to be

$$v = \frac{q_1}{B}\left(1 - e^{-\frac{B}{Mc}t}\right) . \qquad (1.4)$$

This relationship is shown graphically in Figure 10.

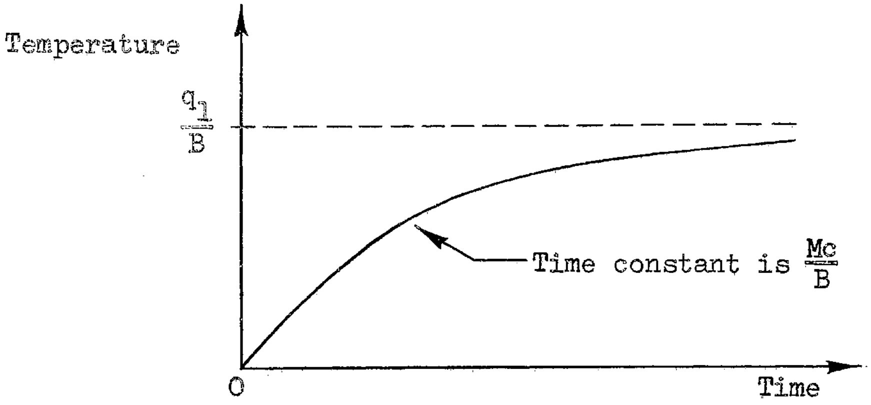


Figure 10. Temperature of the Oven, With Losses to the Surroundings, as a Function of Time

This plot indicates the maximum temperature, $\frac{q_1}{B}$, that can be reached by the oven when supplied with heat power at the rate q_1. It clearly shows the importance of using a heater capable of a much larger power than that power required for steady-state operation. This is especially true if the ambient temperature may assume an unexpected low value. It becomes evident, then, that the maximum temperature attainable is an important characteristic of an oven design and should be known before attempting to construct a practical system.

The problem of determining the temperature, $\frac{q_1}{B}$, is elementary if the insulation surrounding the oven can be considered a thin shell. In such a case, heat transfer is normal to the surfaces. In a practical case, using the configuration of the actual insulating jacket built, and shown in Figure 11, there are corner effects.

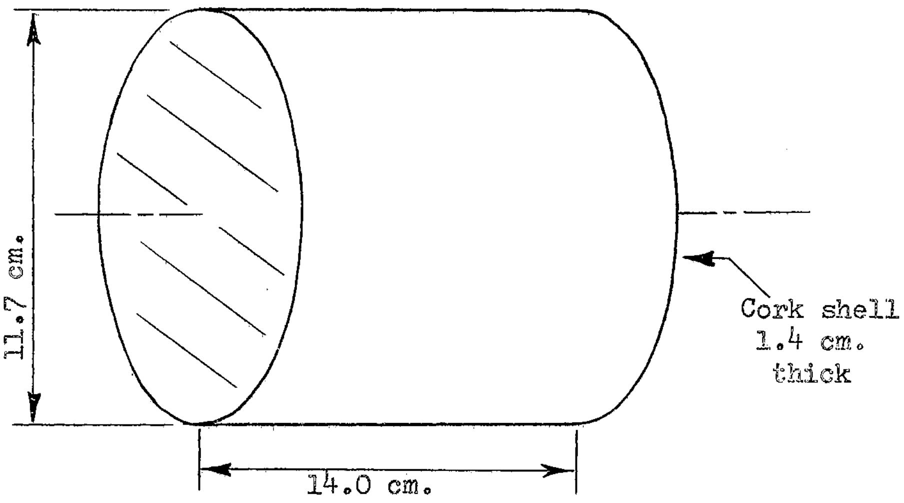


Figure 11. The Oven Insulating Jacket

Since the shell shown in Figure 11 is not too thick, a good approximation of $\frac{q_1}{B}$ may be determined by using mean values of its diameter and length, and its true thickness. The conduction coefficient of cork insulation can be found in the literature and ranges in value from .0001 to .00013 cal/sec cm °C. (2, 5).

The constant B, introduced in Equation (1.3) may be expressed more explicitly as $\frac{KA}{x}$, where A is the area of the shell using the mean dimensions, and x is shell thickness. A familiar relationship of steady-state heat conduction,

$$v = \frac{qx}{KA} \tag{1.5}$$

is used. (2). The steady-state temperature above zero reference, according to Equation (1.5), should lie between 68° and 88° C. The configuration and power of 15 watts already considered were used to arrive at these figures. The oven shown in Figure 12 assumed a steady-state temperature difference of 44° C using a heater power of 8.4 watts. This would correspond to a final temperature of 78° C if it were allowed to achieve equilibrium using the 15 watt heater power assumed in the calculation.

It is found that the approximations made regarding the cork shell appear to be satisfactory and a power of 15 watts is quite adequate, since the steady-state temperature of operation will be several times lower than the oven is capable of reaching.

Means of Temperature Sensing

Several devices have already been considered which show marked changes in parameters with temperature changes. This temperature dependence, however, undesirable in most applications, is necessary for a device to sense temperature of the oven, indicating what temperature correction is needed. The great majority of devices which have a

Figure 12. The Oven and Enclosure

temperature dependent parameter unfortunately will not serve the purpose well because of aging effects. The search for a suitable device inevitably leads to consideration of devices which are especially designed for temperature sensing or compensating applications. Solution of the sensing problem may be achieved using a sensistor or thermistor. The former is a small silicon bar whose resistance is a function of temperature. The latter is a ceramic device serving the same purpose. A sensistor was chosen because of its hermetically sealed, TO-5 case. Its resistance changes linearly with temperature over a sufficient temperature range with a coefficient of +0.7%/°C of the nominal, room temperature, resistance value.

To achieve a maximum sensitivity to temperature the sensistor is used in a bridge arrangement, preferably using wirewound resistances to avoid long term drift. The bridge supplies an error voltage to a following amplifier, which is used to correct the oven temperature in the direction indicated by the sensistor.

Specifications on the construction and materials of the sensistor used were not available. Its thermal characteristics, therefore, had to be found using experimental techniques. Appendix B describes a method of finding the resistance of the sensistor as a function of time when a step function of temperature is applied to the case. Figure 13 is the result of such an experiment using a temperature step of about 22° C.

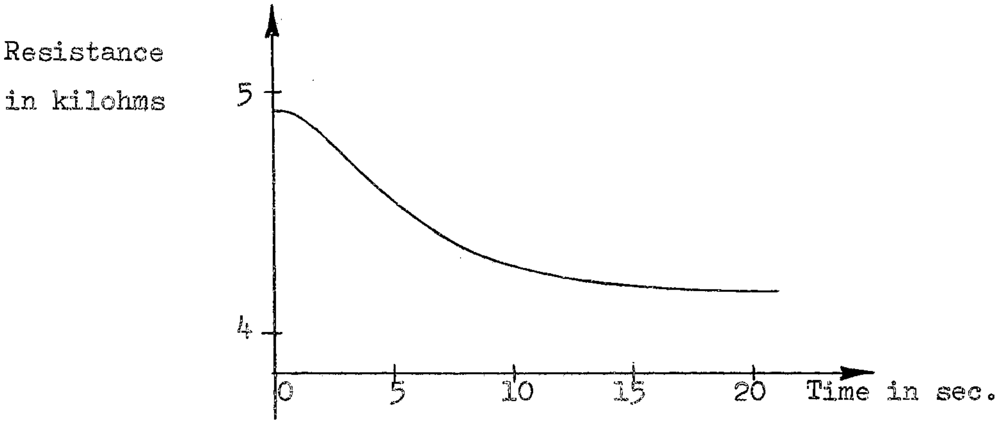


Figure 13. Transient Thermal Characteristic of the Sensistor

This response is somewhat slower than that expected and will be a very important consideration in system stability and the value of loop gain chosen.

The Feedback Amplifier

For accurate control of temperature, very small error signals must be detected and amplified to a level sufficient to control the heater. The linear amplifier shown in the system of Figure 5 (page 8) which serves this purpose must satisfy two main requirements. First, the amplifier must handle large overloads associated with the pulse peaks exceeding its linear operating range and recover quickly, amplifying the remainder of the exponential decay of the pulse linearly. This is not a condition easily achieved. Second, the gain of the amplifier must remain fairly constant, because gain changes will cause corresponding changes in the steady-state temperature error. It should be noted that temperature "error" is defined to be the temperature difference between operating temperature and that temperature causing zero error signal from the temperature sensor. If the steady-state error is constant, then, there is nothing more to be desired in the steady-state operating phase.

It was found that the most convenient means of achieving rapid recovery from overload was the use of a d.c. amplifier with heavy d.c. feedback to stabilize the operating point, and only enough a.c. feedback to provide a stable value of a chosen loop gain. In this solution, only one reactive element is needed; a single capacitor in the feedback path of the amplifier permits feedback only at d.c. and low frequencies. An arrangement achieving this selective feedback is shown in Figure 14.

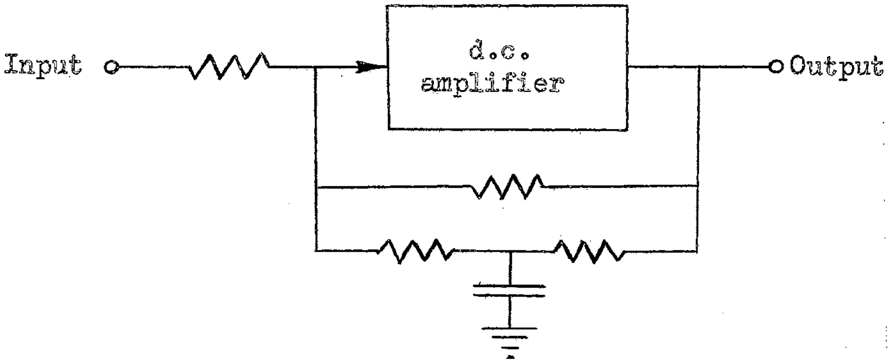


Figure 14. Feedback Amplifier Capable of Recovering Quickly From Transient Overload

The two resistances in the d.c. feedback path determine the amount of d.c. feedback. They also determine the closed loop overload recovery characteristics. If heavy d.c. feedback is used, the capacitor stores a greater charge through the smaller resistors than if light d.c. feedback were used. For smaller values of d.c. feedback, overload consequences, therefore, become progressively less disturbing. This indicates that the d.c. amplifier itself should be as drift free as possible, requiring a minimum of d.c. feedback to stabilize its operating point. An amplifier configuration meeting the requirements discussed is the differential amplifier. Design of a suitable circuit will be carried out in detail in Chapter III.

CHAPTER III

DESIGN OF THE FEEDBACK SYSTEM

The Sensing Bridge

In Chapter II, it was determined that a bridge arrangement using a sensistor in one of the arms was to be used as the temperature sensing device. In order to use the nonlinear feedback system proposed, bridge excitation must be in synchronism with the heater power source, and it must be a balanced square wave source. A convenient way of obtaining a synchronous square wave is by clipping a sine wave isolated from the line voltage, which is used as the heater supply. The balanced sine wave voltage needed can be obtained using transformers as shown in the diagram of the complete bridge network in Figure 15. As indicated, isolation from the 110 volt a.c. line is accomplished using two 6.3 volt filament transformers back-to-back, resulting in a balanced 110 volt source. To achieve as complete isolation as possible, the 6.3 volt windings are bypassed to ground. Balance of the isolated 110 volt sine wave is improved by using the two parallel RC networks shown. A single capacitance and resistance of the proper values, each placed on the correct line, would balance stray

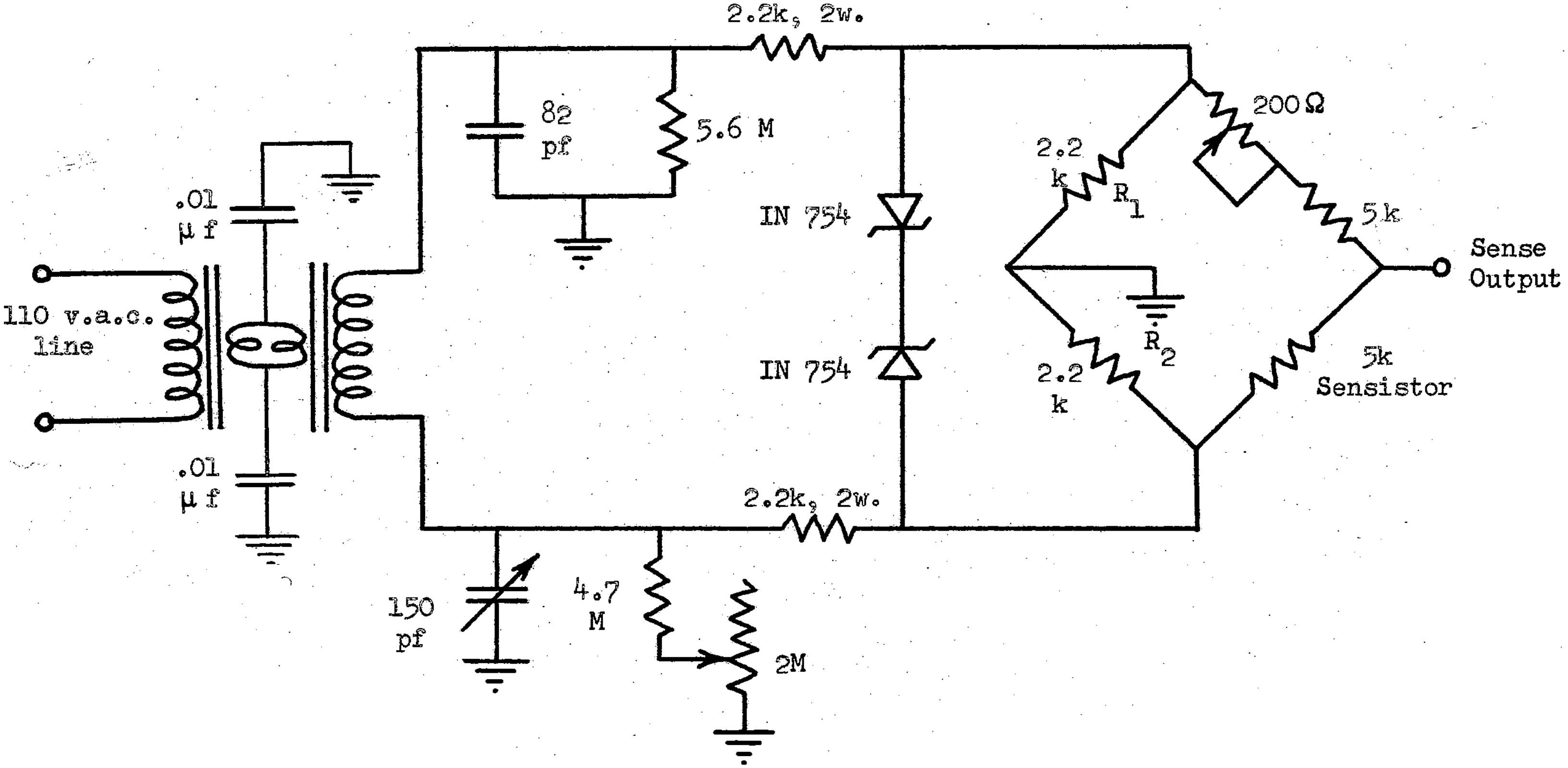


Figure 15. The Bridge Network

capacitance and leakage resistance. The two networks shown, one on each line, will achieve the best possible isolation if the fixed values are chosen correctly and the variable components are adjusted for balance.

A square wave approximation is obtained by clipping the balanced sine voltage with two avalanche diodes. On each half cycle, one of the diodes is forward biased and the other is in the avalanche breakdown condition, their respective modes of operation changing for each successive half cycle. Choice of avalanche voltage is dependent upon several factors. It should be high enough to give sufficient bridge sensitivity, but low enough to yield a good square wave approximation from the clipped sine wave. Units having a 6.8 volt breakdown were found to be a good compromise. The value of the two 2.2 kilohm resistors shown in the diagram supplying the avalanche diodes with current were determined in consideration of the dissipation of the diodes. The value chosen results in a diode power dissipation well below the rated maximum value. The diodes also serve to eliminate line voltage variations as a source of error in operation of the bridge.

Choice of the bridge resistances for optimum performance is discussed extensively in the literature. (6). It can be shown that if the bridge is excited by a voltage source, the resistance of R_1 and R_2 in Figure 15 should be as low as possible. For the voltage source using avalanche diodes just discussed, however, low values of these resistors degrade the risetime of the approximated square wave, and for values lower than about 100 ohms the diodes quit clipping entirely. Again, a compromise must be made; values of 2.2 kilohm, as shown, were chosen. While greater sensitivity would be achieved by using a sensistor of much lower resistance than the input impedance of the following amplifier, a 5 kilohm unit was available and was used. Although the practical advantage would be small, a 2.2 kilohm unit could probably be used without excessively increasing the square wave risetime.

The feasibility of the sensing bridge in Figure 15 can be established by determining the output signal resulting from some temperature error, say 1° C. An equivalent circuit of the sensing bridge may be drawn as in Figure 16.

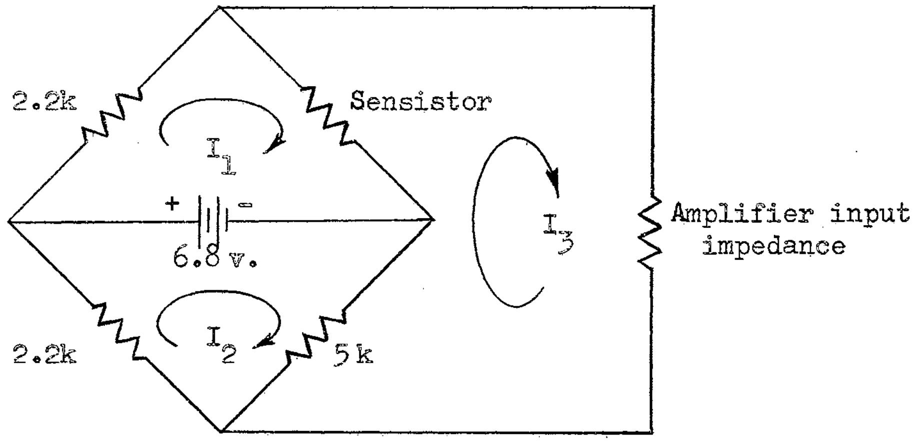


Figure 16. Equivalent Circuit of Bridge Network

The value of the amplifier's input impedance is anticipated considering that the input stage will be differential as already suggested. The input impedance is taken to be twice the common emitter input impedance of the transistors; 10 kilohm is the resulting typical value.[1] The loop equations associated with the three currents shown can be solved simultaneously, resulting in a sensitivity of 0.88 μa./°C. This value results from the bridge circuit being driven by a d.c. excitation source. If square wave excitation is applied as proposed, the sensitivity can be expressed as twice the value when d.c. excitation is used, or 1.76 μa. peak-to-peak/°C. It is seen that considerable gain will be required to drive a heater requiring several amperes when an error of one-hundredth of a degree or so is to be corrected.

The Linear Amplifier Circuits

Before the error signal from the sensing bridge is amplified, it is operated upon by an RC network, the result being a series of exponentially decaying pulses, already shown in Figure 5 (page 8). The time constant of the decay is established by the coupling capacitor and an

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[1] Although matching the input impedance of the amplifier to the equivalent bridge impedance would provide maximum power transfer, the advantage gained would be small since the signal level from the bridge is considerably higher than the equivalent noise input of the first amplifier stage.

equivalent resistance value consisting of the source resistance of the bridge in series with the amplifier input resistance. The value of the coupling capacitor will, therefore, depend upon the amount of voltage feedback used on the amplifier, since it affects its input impedance. If the amplifier is heavily fed back at signal frequencies, its input impedance is very low and the sensor's equivalent source impedance of 3.1 kilohm determines the value of the coupling capacitor. Using the value of RC selected in Chapter II of .00232 seconds, the value is found to be about .75 μf. If no feedback is used and the input impedance of the amplifier is taken to be 10 kilohm, the value is about 0.18 μf. In practice, the value will lie somewhere in this range unless the time constant is purposely chosen to be a different value.

The circuit of the differential amplifier to be considered is shown in Figure 17. Component values shown on the diagram are standard 10% tolerance values, chosen as the best values in consideration of the calculated values.

The first stage is designed to operate with a low noise figure since the error signal at this stage is very small. Optimum values of collector-emitter voltage and collector current for quietest operation are about 3 volts and 0.3 ma., respectively.[2] The base and emitter voltages

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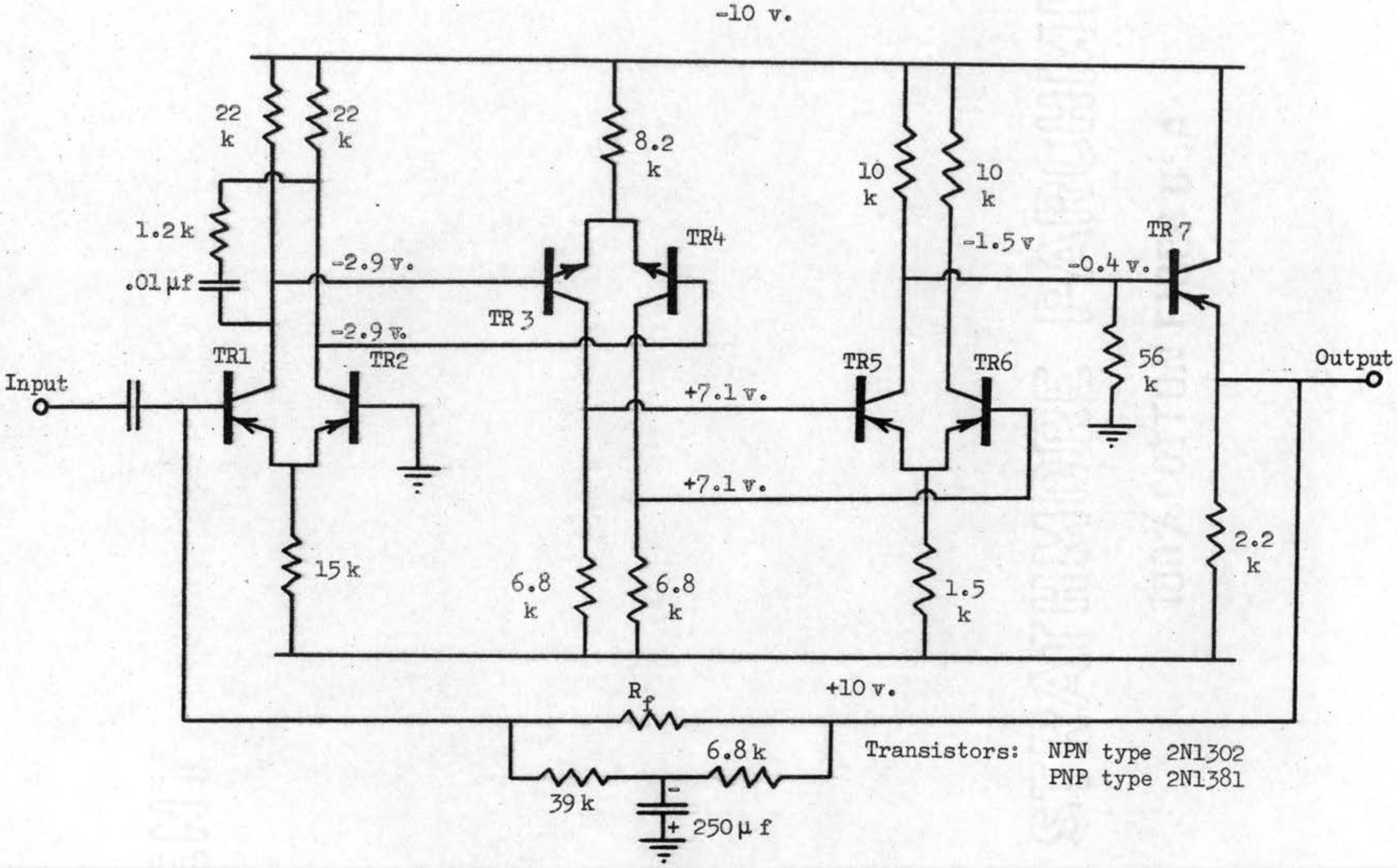
[2] This was determined in a seminar project under Professor Paul McCollum (1961).



Transistors: NPN type 2N1302
PNP type 2N1381

Figure 17. The Amplifier Circuit

of both transistors of the first stage are determined by the voltage reference applied to the base of TR2, in this case zero. The emitter and collector resistor values are found which determine the correct operating point; the closest standard values were actually used. A worst case calculation may be performed by assuming an emitter resistor 10% too low and collector resistors 10% too high. Such values result in a collector-emitter voltage of 1.0 volt. This is not a particularly good operating point, but it is perfectly safe.

Using the standard resistance values chosen in the design of the first stage, the collector voltages are determined. These must be very nearly equal when the amplifier is operating in its linear range because both transistors of stage two must be forward biased. This value of stage one collector voltage is used, along with a proposed operating point of stage two, to determine the values of resistances in the second stage.

Design of the third stage must be carried out in consideration of operation under overload. Ideally, under overload conditions, as when the system has been turned on but the oven has not approached steady-state operation, the output waveform should be a square wave synchronous with the heater power source. To accomplish this, the average value of the output voltage must be zero; otherwise, the stabilizing feedback path would apply extra bias current to the input transistor, producing an unsymmetrical rectangular waveform. For moderate overloads, it would be seen that this unsymmetrical waveform, as it appeared at the output, would indeed have an average value near zero.

When TR5 is bottomed in the overload condition, its mate is turned off, and its collector voltage is determined by the collector resistor, the emitter resistor, and the output impedance of the stage driving it. If this output impedance is somewhat higher than the parallel combination of the collector resistor and emitter resistor, the collector voltage is determined mainly by these two resistances acting as a voltage divider between the two power busses. The emitter resistor should be chosen considerably smaller than the collector resistor to permit large positive voltage swings before overload. The collectors of the second stage must rest at a fairly large positive voltage for the same reason. The second differential stage has been designed to operate at a collector voltage of +7.1 volts. If it is desired to use a collector current in the third stage of about 1 ma., the emitter current of 2 ma. and its voltage of +7.2 volts (allowing 0.1 volt for V_{be} drop) determine the value of the emitter resistor, 1.5 kilohm, as shown. Under no-signal conditions, the output voltage must rest near zero, this being the reference voltage at the base of TR2. The 10 kilohm collector resistor shown will effect the 10 volt drop when its equal share of the combined emitter current flows through it. The other collector resistor is chosen to be 10 kilohm also, although it could be left out of the circuit entirely if desired.

When TR5 is cut off, the output voltage is determined by the voltage divider consisting of its collector resistor and the resistor to ground shown. The latter is chosen to provide a negative output voltage equal in magnitude to the positive output voltage present when the transistor is saturated. The requirement of zero average voltage will then be met.

The output of the last differential stage is buffered with an emitter follower. The emitter resistor must be chosen low enough to provide from the +10 volt bus all load current needed when the output voltage assumes its most positive value. The value shown was determined after driving requirements of the following nonlinear circuits were determined.

Voltage gain of the complete amplifier circuit can be calculated if parameters of the transistors at their respective operating points are known. Using these parameters, which include common emitter current gains of 100 to 200, the over-all open loop voltage gain should be 4.5×10^5. Inclusion of the drop across the sensor's equivalent output impedance reduces this gain to an effective value of about 4×10^5 when only stabilizing feedback is used. Since it has been shown that the sensor's sensitivity is about 1.76 μa. peak-to-peak/°C when used with a 10 kilohm load, the amplifier will deliver a 7 volt peak-to-peak signal for .001 °C temperature error. If, after investigation of the entire system, this gain is too large, feedback may be applied to the amplifier via the feedback path, R_f.

D.C. and very low frequencies are fed back through the low pass section shown. The values of this section are not critical; those shown result in a d.c. loop voltage gain of 30, which is small enough to insure output drift of only a few tenths of a volt due to I_{cbo} and V_{be} changes in the first stage over a wide temperature range.

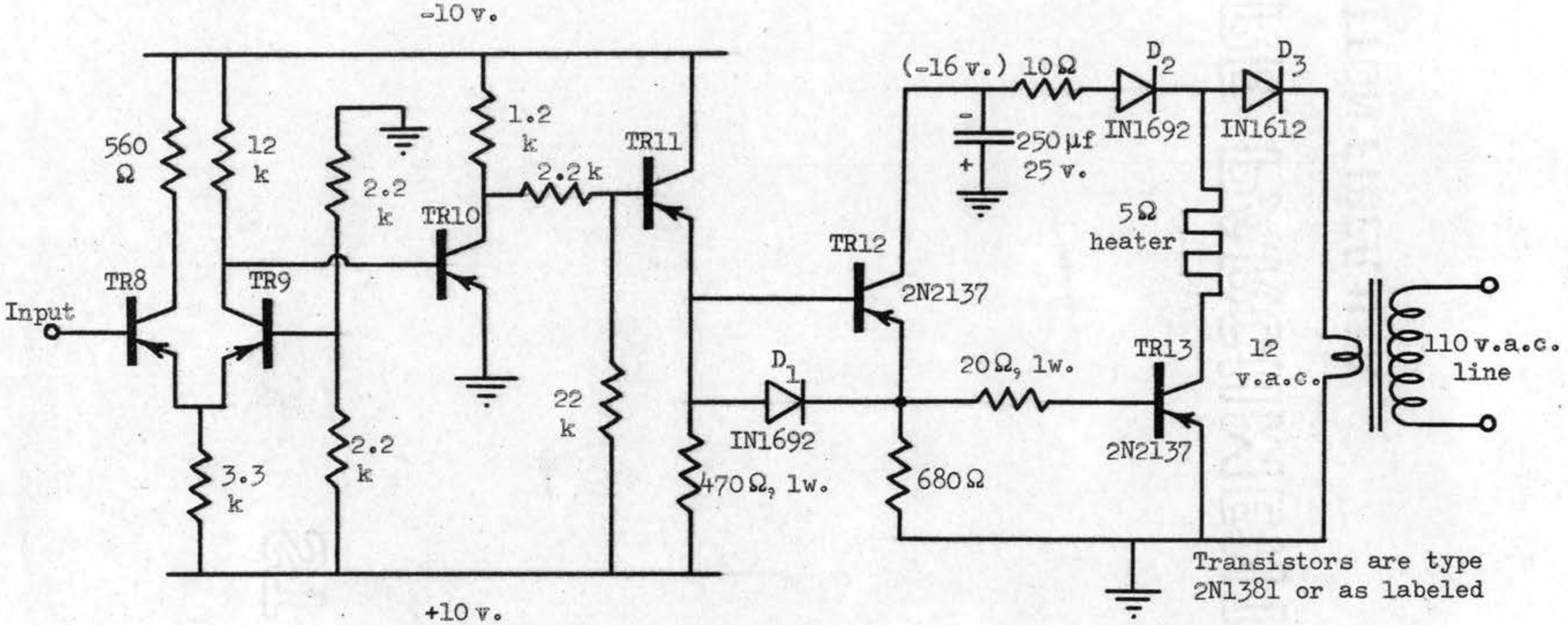
The gain of the amplifier is more stable than might be expected if only the common emitter current gain change with temperature were considered. As temperature rises, a transistor's current gain and input impedance also rise, the increased input impedance causing a larger proportion of the driving current to be lost in the collector resistor of the preceding stage. The compensation is less complete for the first stage, however, since it is driven from the relatively low impedance sensing bridge.

Gain may be reduced and made more stable by the addition of a non-selective feedback path. If feedback were applied, however, the uncompensated amplifier would probably not be stable because of distributed phase shifts of the four stages at high frequencies. To permit stable operation, the gain of the amplifier may be rolled off with the step network shown between the collectors of the first stage. The lower corner frequency will occur where the magnitude of the capacitor's reactance is equal to the sum of the two equivalent impedances at the collectors. Each of these impedances is determined by the load resistor, the transistor's output impedance, and the following stage's input impedance, which are respectively 22, 25, and 11 kilohm for the circuit and transistors actually used. The lower corner frequency is consequently 1.4 kc. Open loop gain will decrease 20 db. per decade as frequency increases to the upper corner frequency, which is determined by the step resistor. The value shown causes roll-off for one decade and is sufficient for light feedback.

The Switching Circuits

According to the system described in Chapter II, the train of exponentially decaying pulses from the feedback amplifier is to be compared to a reference voltage, and the heater turned on when a pulse exceeds this reference. A circuit accomplishing this, and subsequent circuitry driving the heater is shown in Figure 18.

The comparator function is accomplished using a differential pair arrangement. One base is referenced with the voltage divider shown; the signal to be compared with this reference voltage is applied to the other base. When the input voltage is more negative than the +5 volt reference, TR8 conducts and operates as an emitter follower. Possible input voltages lie between ±7 volts, being limited by clipping in the feedback amplifier. For the most negative voltage, about 17 volts will be dropped across



Transistors are type 2N1381 or as labeled

Figure 18. The Nonlinear Circuits

the emitter resistor. The collector resistor is chosen small enough to keep the transistor from bottoming, or it may be omitted entirely. TR8 operates in this emitter follower fashion until the output voltage reaches +5 volts. For inputs greater than +5 volts TR8 will be cut off, TR9 will be bottomed against the +5 volt reference, providing its collector resistor is chosen correctly, and TR10 will consequently be turned off.

Let us now investigate the "heater-on" condition which results when TR10 is turned off. TR11 and TR12 are emitter followers driving the power switch, TR13. The 20 ohm load presented at the emitter of the second follower appears multiplied by its current gain at its base. This value, in turn, appears at the base of the first follower multiplied by its own current gain. Using units with gains of about 50 for these two stages results in an input impedance for TR11 of about 50 kilohm. This value, along with the voltage divider values shown, results in a voltage of -6 volts at the base of the first follower since TR10 is cut off. This voltage, less about a volt drop across the three base-emitter junctions, appears across the 20 ohm resistor, resulting in a base drive of .25 ampere for the power switch. This value of drive will be quite sufficient considering a peak collector current of about 3 amperes is drawn and a switching transistor with gain of at least 12 is used. To insure good bottoming characteristics and switching reliability, a unit with current gain of 50 or 100 should actually be used.

When TR10 is turned on, the heater is turned off, so the only appreciable load resistance reflected to the base of TR11 is its emitter resistor multiplied by its gain. This value and the two resistors connected to its base determine its base voltage of about +1.6 volts. Adding the drops of two base-emitter junctions results in about 2 volts reverse bias on the power switching transistor. The diode D_1 provides a path for I_{cbo} of the power switch. Up to 17 ma. leakage current can be supplied by the 470 ohm emitter follower resistor. Diode D_2 and the filter capacitor provide a d.c. voltage source for the emitter follower TR12, so its current is not drawn from the supply used for lower level stages. Diode D_3 blocks positive voltage from the power switching transistor during positive half cycles.

Since the power switching transistor must handle up to 3 amperes of collector current, it must be a power type unit although power dissipated by it is small. On the other hand, although the last emitter follower draws only 250 ma. at most, the voltage appearing from emitter to collector results in an average dissipated power of more than a watt, requiring use of a power type here also.

CHAPTER IV

ANALYSIS OF THE SYSTEM USING ANALOG SIMULATION

Analog Representation of the Complete System

It would indeed be difficult to overemphasize the utility of electrical analog techniques in the analysis of heat flow problems. The parameters of a mechanical system, such as thermal conductivity, are difficult to change. Conditions such as heater placement are also varied with difficulty. Finally, the variables, such as temperature and heat flow, must be measured with specialized sensing devices. If an electrical analog of the system is used, however, the parameters are represented by component values, and conditions may be varied by merely changing electrical connections. The variables of temperature and heat flow are represented by voltage and current, respectively. Analog time may be chosen to be different from real system time, therefore, it is convenient in many heat flow problems to use a time scale such that a period of minutes or hours of real time may be represented by analog operation of only a few seconds. Many combinations of parameters can be investigated using an analog model in the time required for the mechanical system to respond to a single set of conditions.

An electrical analog model of the temperature control system already discussed is shown in Figure 19. The oven analog actually represents only one-half the oven, this procedure having been justified in Chapter II. The analog is versatile enough to analyze systems differing in heater and sensor placements, feedback system transfer function, and maximum heater power. Additional feedback paths to investigate the effects of rate feedback or perhaps first derivative compensation may be conveniently incorporated.

The analog system is magnitude scaled to operate within one degree Fahrenheit[1] of the steady-state operating temperature. Although this range is insufficient for the complete transition from the cold, or "off", state to the warm steady-state, it is adequate for investigation of recovery from complete overload conditions and is also sensitive enough to measure temperature changes as small as .01 °F easily.

The sensistor is small, so it is considered to sense the temperature of the oven without appreciable heat flow. The oven and sensor analogs are therefore buffered by an operational amplifier. The sensor analog actually represents the transient relationship between temperature of

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[1] The system of English units is used here because of its extensive use in the literature. The metric system adhered to in the previous chapters corresponds, in turn, to the units used in the field of thermodynamics and heat transfer.

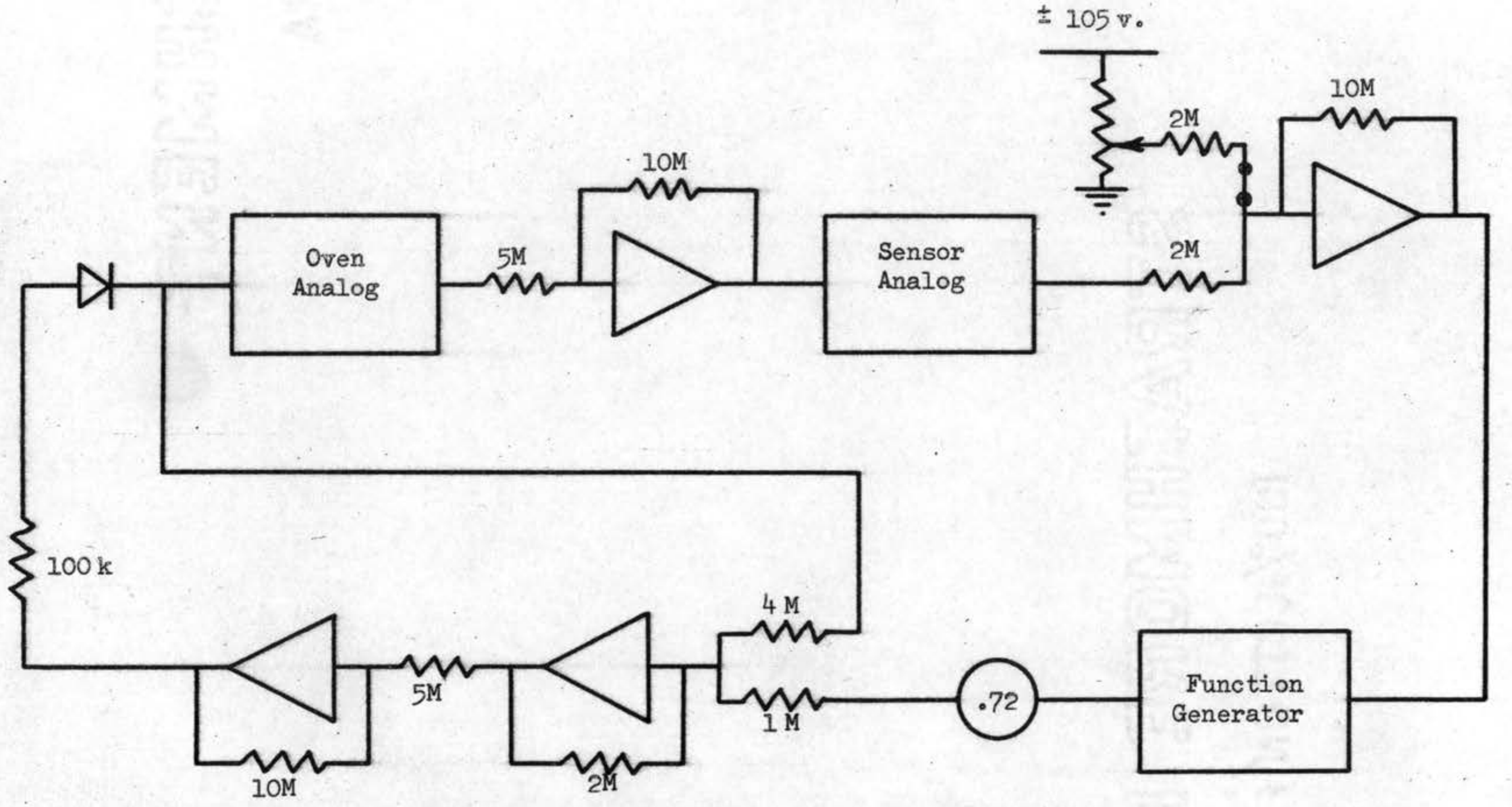

Figure 19. The Electrical Analog Model

the sensing element of the sensistor and its case temperature. The sensor analog output is therefore a voltage representing temperature of the sensing element. Sensitivity of the sensistor is included in the system's loop gain, which is treated in Appendix C. A step voltage can be added to the sensor analog voltage to represent the abrupt readjustment of the temperature at which the sensing bridge is balanced. Transient response of the system can be investigated using this means of disturbance. A function generator is used to achieve the desired relationship between this error signal and heater power.

The oven analog must be driven from a current source in accordance with the analogy between heat flow and electrical current. This is accomplished by driving it from a voltage source through some known, fixed, resistance. Change of voltage at the oven analog input is compensated for by adding an equal value to the voltage source through a feedback path of unity gain. Current flow into the oven analog is therefore seen to be proportional to the desired function of the error voltage created by the function generator.

Analog Circuits of the Oven and Sensistor

Analysis of steady-state heat flow in a solid can be carried out using an electrical analog circuit containing only resistors. The resistors represent, and are proportional to, the thermal resistivity of the solid. If transient response is also desired, an analog circuit incorporating capacitors representing heat capacity of the solid is used. Analog representations of both types are given excellent treatment by Karplus (7). The electrical analog is composed of several sections, each representing a corresponding section of the object, and each being of the form shown in Figure 20.

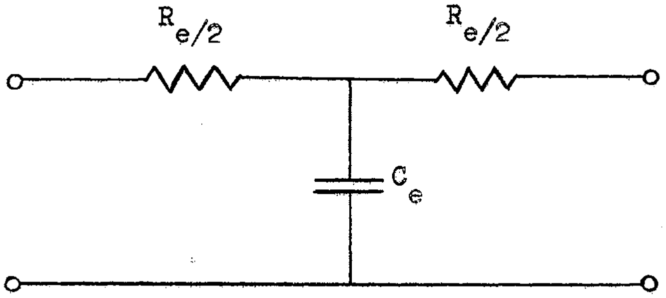

Figure 20. Electrical Analog for Representing Thermal Conduction in Solids

The sum of the two resistances represents the thermal resistance of the section, measured from one side of the section to the other in the direction of heat flow. The capacitor represents the heat capacity of the section. Values of the electrical analog are related to the thermal values by three scale factors:

L = voltage/°F

$$m = \frac{\text{Thermal capacity}}{\text{electrical capacitance}}$$

$$n = \frac{\text{electrical time}}{\text{thermal time}} \;.$$

Values of the electrical analog may be expressed in terms of the scale factors and the thermal values as

$$R_e = mnR_t \tag{4.1}$$

$$\text{and} \quad C_e = \frac{C_t}{m} \;. \tag{4.2}$$

The values of heat flow and temperature are measured as current and voltage respectively, and are related by the expressions

$$\text{Voltage} = (\text{temperature})(L) \tag{4.3}$$

$$\text{and} \quad \text{Current} = \left(\frac{L}{mn}\right)(\text{heat flow}) \;. \tag{4.4}$$

In choosing the scale factors, one should keep in mind a convenient analog time scale, practical restrictions on electrical components, especially the capacitors, and the maximum value of temperature which may be reached. Convenient values of the scale factors are chosen and analog values calculated in Appendix D.

Heat flow through the cork insulation to a fixed ambient temperature from a section of the oven may be included in the analog as a resistance across the capacitor C_e. If this scheme were used, however, the lower, or common, bus would necessarily represent ambient temperature. In analog computation, the maximum machine voltage is generally 100 volts, so this voltage would represent the maximum temperature of some part of the system if properly scaled. A consequence would be that small changes about the steady-state operating temperature would be inconveniently small. This inconvenience can be remedied by considering operation only about the steady-state temperature. Use of the analog system in the condition of initial warm up is of little value because the heater is delivering peak power constantly. The resultant warm up characteristic has already been discussed in Chapter II. If interest is concentrated within a degree or two about the steady-state temperature, heat flow through the cork insulation may be assumed essentially constant. This approximation may be achieved by drawing a constant current from each of the analog capacitors, representing the heat lost from each of the oven sections. The required current sources may be formed by using two operational amplifiers for each source, the method of achieving constant current drive having been described in the first part of this chapter for use as the analog heat source. If the analog system uses many sections in the oven analog, a prodigious number of amplifiers is required. Another approximation can be made which reduces this number considerably. Although temperature of the oven may be caused to vary over a range of a degree or two by introducing disturbances, the temperatures of several adjacent sections vary similarly. Only two current sources are actually used, then, each draining from one section the correct leakage current. The leakage currents drawn from sections on each side of these two correctly driven ones are approximations to the correct values. The error is small, however, since the temperature differences between adjacent sections is small. Operation of these current sources is more clearly expressed in Figure 21, which shows the electrical analog of the oven, using the values of components found as described in Appendix D. The heater and sensor may effectively be placed at any one of eight positions along the oven by making the corresponding connections to any of the eight analog sections. The connections, as shown, result in an analog of the oven actually used.

An alternate approach was taken to determine the electrical analog of the sensistor. The internal structure and composition of the unit were not available, so the transient response determined in Appendix B was simulated using three sections arranged as shown in Figure 22.

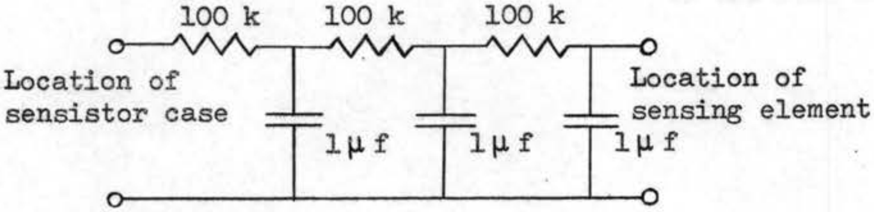

Figure 22. Sensistor Analog

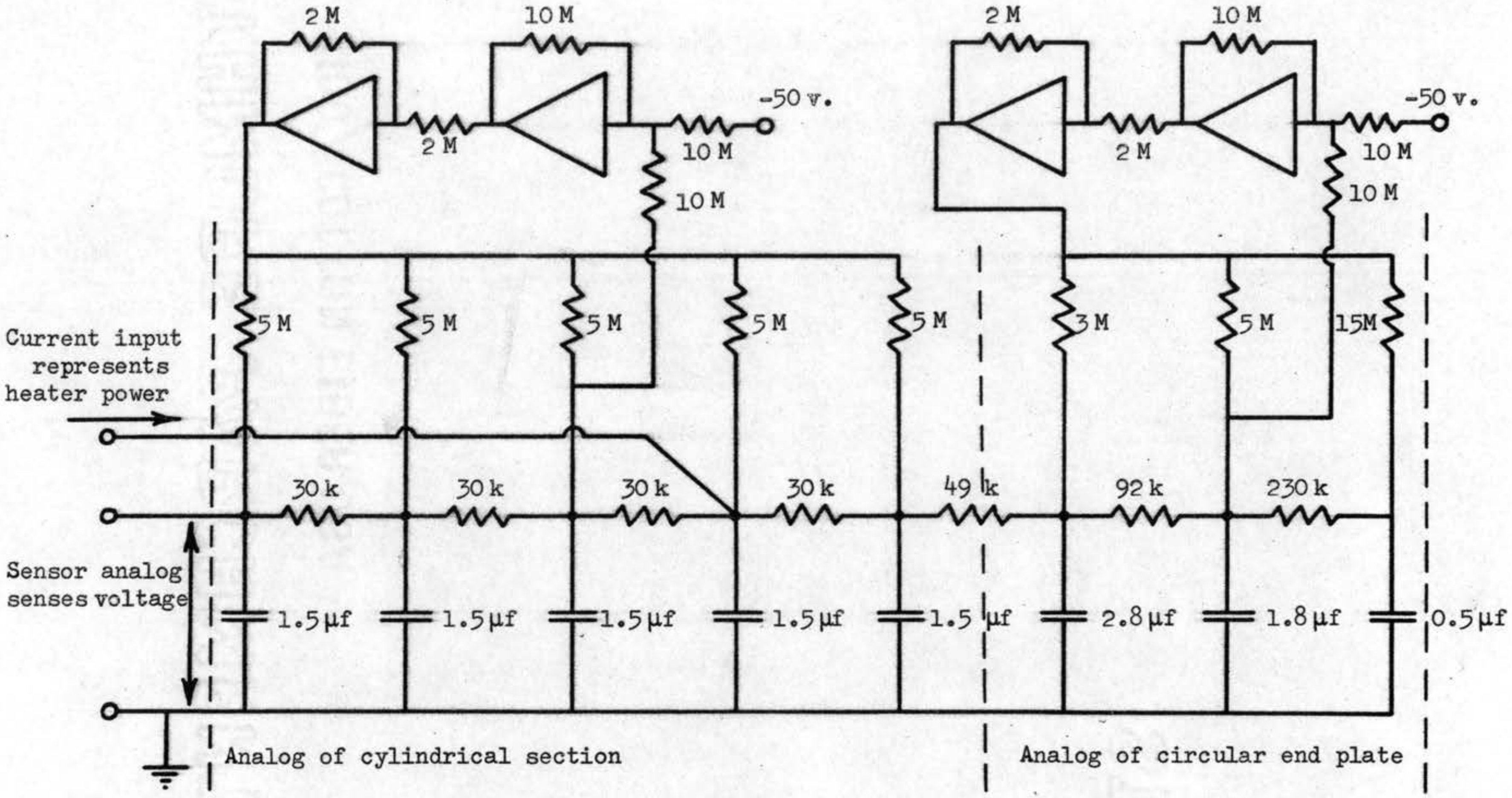

Figure 21. Analog of the Oven

The values shown were determined empirically and result in response to a step input which approximates the sensistor's response with a maximum error of about 5%. More sections would result in a closer approximation. This technique of determining the analog circuit is expedient when the transient response of the physical system is known. (8).

Using the Analog System

A prime application of the analog system is its use in determining a desirable value of gain for the amplifier in the feedback system. The maximum voltage gain that was considered useful in Chapter III was 4×10^5. The analog system of Figure 19 satisfies this gain requirement, but the system as shown is unstable, indicating the value of loop gain is too high. Oscillation of the oven and sensor temperatures is depicted in Figure 23. This strip chart recording was taken at a speed of 5 mm/second and a vertical sensitivity of 5 volts/cm. Scaling of the analog system results in the equivalent real system scales shown. The reference temperature corresponds to zero analog voltage and has little significance. Notice that the sensor temperature, measured at the sensor analog output, lags the oven temperature, measured at the heater location. This phase shift, necessary for the instability, is caused mainly by the sensistor; this can be affirmed by observing the phase relationship of the voltage changes on the input and output points of the sensor analog.

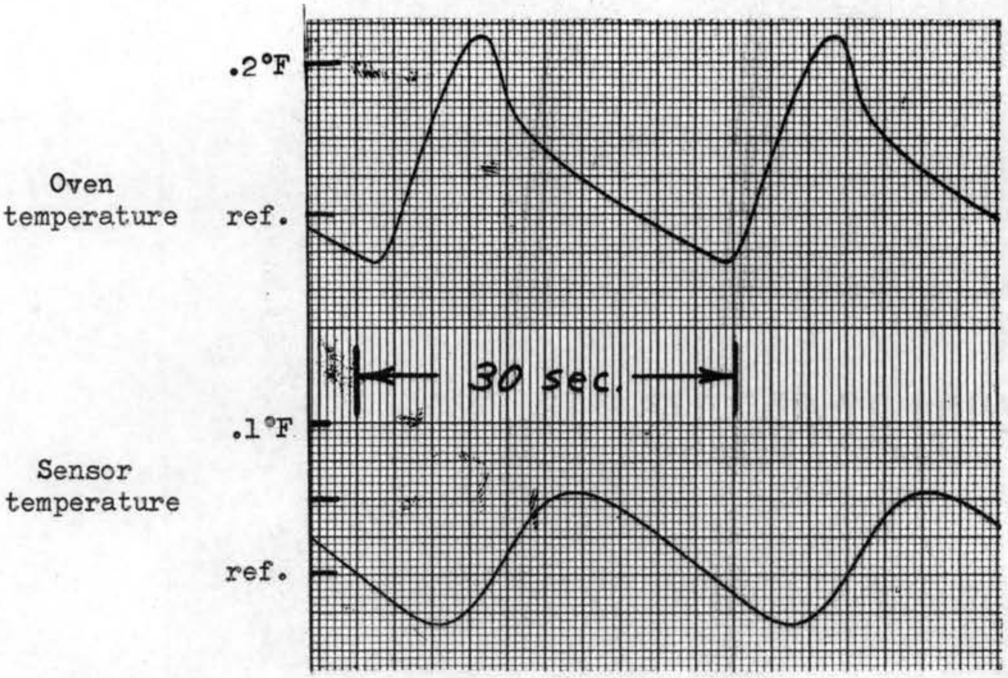

Figure 23. Unstable Oven Operation Using High Loop Gain

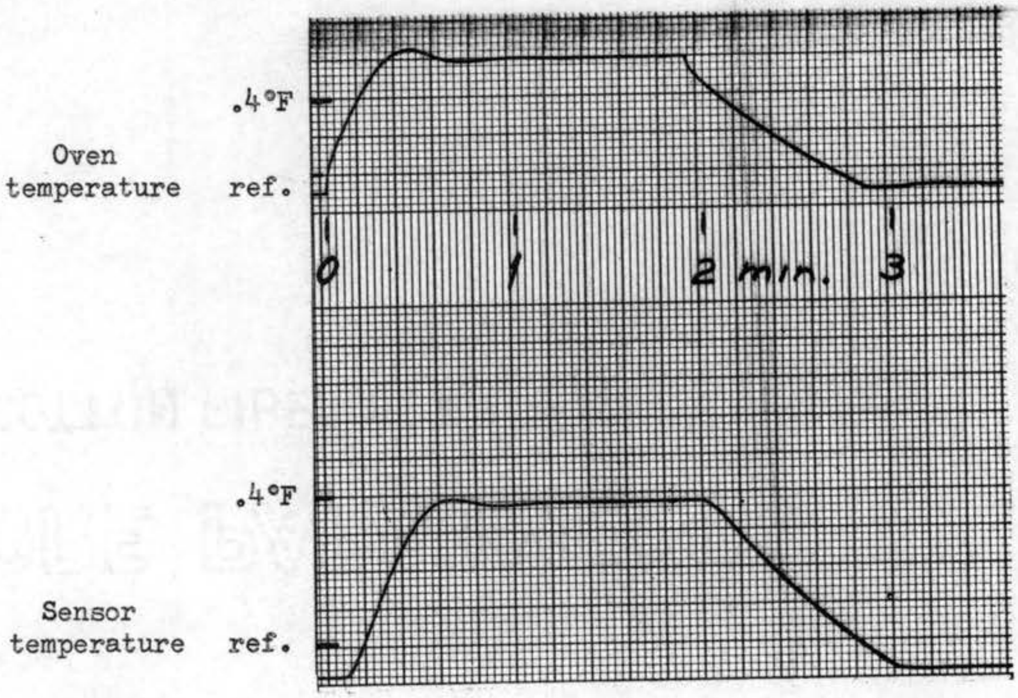

Figure 24. Transient Response Resulting From Moderate Loop Gain

For lower values of loop gain[2], the oscillation becomes smaller, and for values less than about 2×10^5, sustained oscillations no longer occur. A value somewhat lower than this gain is more desirable, however, to reduce overshoot and transient oscillations after system disturbances. A gain of 4×10^4 was found to yield desirable operation; oven and sensor temperatures for this gain are shown in Figure 24 (on preceding page) after a disturbance equivalent to .5° F is introduced. The sensor temperature corresponds to the sensor analog output, which is the temperature of the sensistor's sensing element. The oven temperature represented is the temperature at the heater position, although temperature at any of the other seven oven locations may be observed just as easily. The temperature disturbance is introduced into the system using the switch shown in Figure 19 (page 42). This corresponds to unbalancing the temperature sensing bridge, changing the steady-state operating temperature. The disturbance was switched into the system at the time reference zero and was removed at a later time apparent from the response plots.

Another fruitful field of investigation using the analog system is the investigation of heater and sensor placements and their effects on transient response.

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[2] Loop gain changes are effected by changing the feedback resistor of the summing amplifier following the sensor analog in Figure 19.

Figure 25 shows improvement of the transient response as a result of effectively distributing the heater winding evenly along the cylindrical portion of the oven surface. The disturbance was an equivalent .1° F step. Notice the temperature change at the heater is smaller for the distributed winding due to its proximity to the sensor. The overshoot is less for the same reason.

Operation of the heater while recovering from a .5° F disturbance is shown in Figure 26. Immediately after the disturbance is introduced, the heater begins to deliver its maximum power. This power drops, overshoots, and finally assumes a slightly higher value than originally to provide a higher oven temperature, as determined by the disturbance. The discontinuities of the slope of the power plot result from the function generator approximation of the transfer function of the feedback path. The sensor's temperature is shown on the lower chart. Notice that the heater is never cut completely off during the rise to a higher oven temperature. When the disturbance is removed, however, the heater is turned off and the oven slowly cools, loosing heat through the insulation until the sensor reaches its original temperature, when the heater is again turned on to maintain this steady-state temperature.

A third remaining use of the analog system is the analysis of operation for systems using different transfer functions of heater power delivered versus temperature

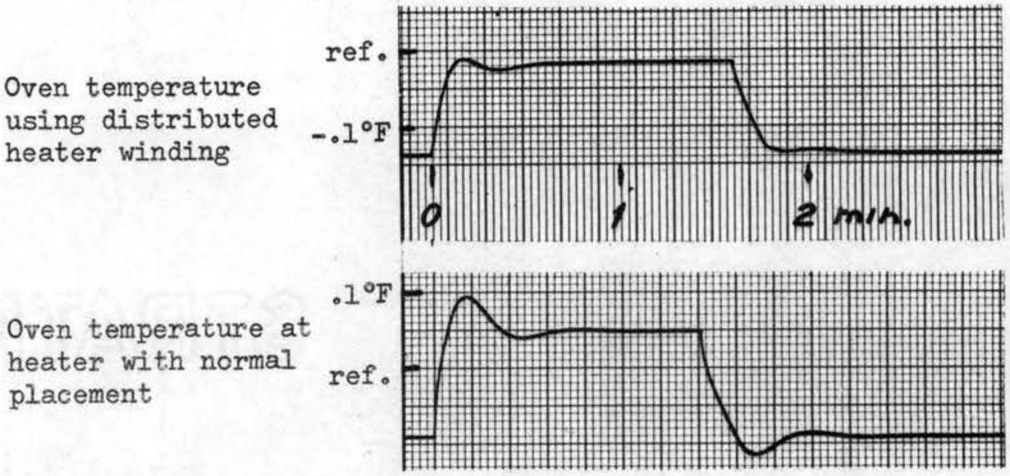

Figure 25. Effect of Heater Placement on Transient Response

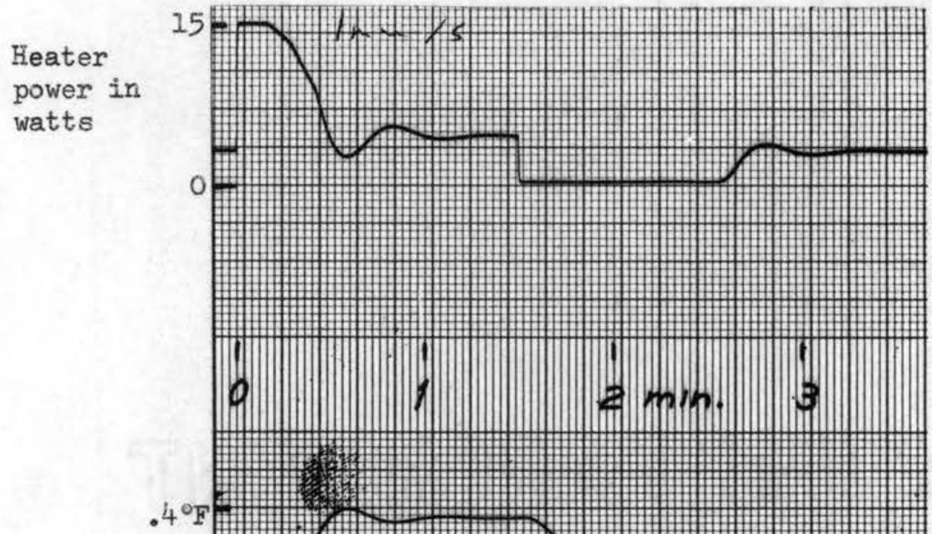

Figure 26. Heater Power in Transient Operation

error sensed. In at least one case, workers in the temperature control field have used a feedback path which causes the heater power to be proportional to the error signal, i.e., a linear feedback path. (4). The more common method of control is one already discussed in Chapter II, the heater power being proportional to the square of the error signal, shown in Figure 3 (page 5). Transient response of these two systems are shown in Figures 27 and 28, respectively. In each case, loop gain was normalized to be equal to the gain of the system used heretofore at the steady-state operating temperature. Operation of one system cannot be said to be definitely superior to another, each offering distinct advantages. Response of the two systems may be compared to that of the former system, shown in Figure 24 (page 50).

In addition to the three major fields of investigation described, the analog model is useful in determining the effects of such factors as increased maximum heater power, and alternate feedback paths. More valuable than any single result obtained from the analog model, however, is the insight into system operation as a whole, made possible by the rapid and completely accessible solutions on the electrical model.

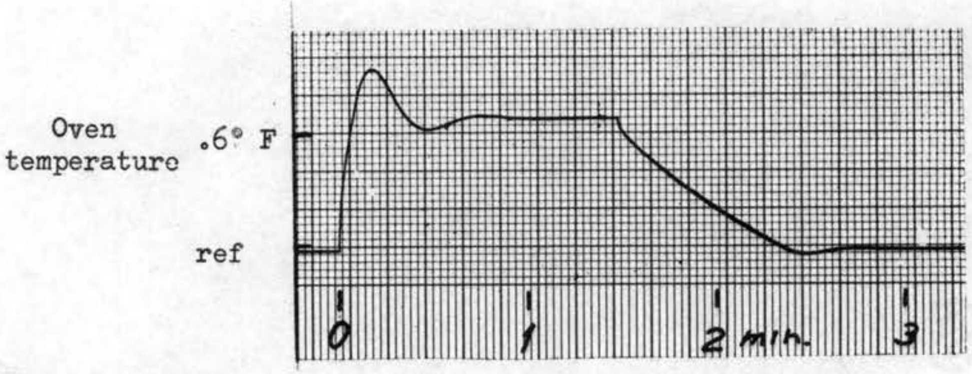

Figure 27. Response Resulting From a Linear Feedback Path

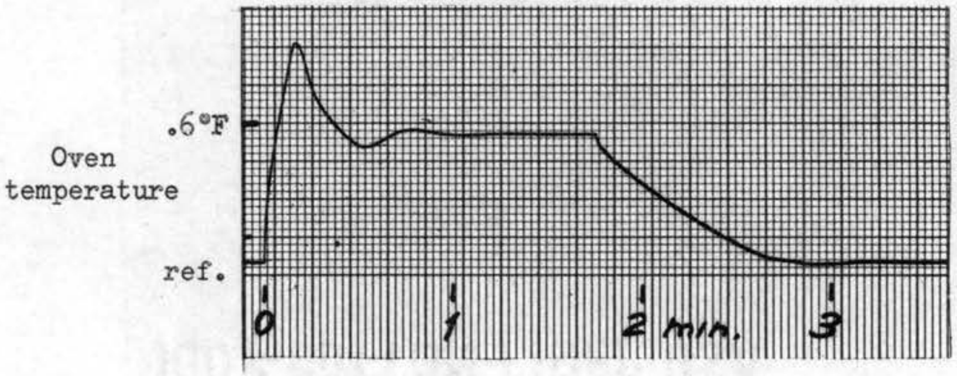

Figure 28. Response of System With Power Proportional to the Square of Error

CHAPTER V

CONCLUSION

In a temperature control system which does not employ integral feedback, there will be some small difference between the steady-state operating temperature and the temperature at which the sensor senses no error. This is the small signal which, when amplified, drives the heating element. Even though this error can be made small by using a high gain feedback path, it is still subject to changes from a number of sources. The most serious source is usually change of the temperature surrounding the oven. If this temperature rises, less power will be required from the heater, and a smaller error signal than before will be generated to produce the lower heater power. A figure of merit may be defined which indicates the extent of this error. For a certain rise in surrounding temperature, the temperature at the sensor will also rise, but on a much smaller scale. If the ratio of ambient change to oven change is formed, some large number results. The system using a feedback amplifier gain of 4×10^4 results in the figure of merit being about 100.[1] If transient response is not an important consideration, the analog system has indicated that gains up to 2 x 10^5 can be used before instability becomes a problem. The figure of merit would then have a maximum value of about 500 for this system. This means that for each degree the ambient temperature changes the oven temperature at the sensor will change 1/500 degree. This figure indicates why it is common practice to place the oven inside a second temperature controlled enclosure.

Another source of error results from changes in gain of the feedback system. This problem has been discussed earlier in consideration of gain stability of the feedback amplifier. In addition to gain changes originating in the circuitry, which can be reduced by proper design and use of feedback, gain may effectively be changed by changes in the a.c. line voltage, which supplies the heater winding. A rise in line voltage produces a greater heater power for the same temperature error, which is effectively an increased loop gain. An increase of 10% in line voltage results in a drop in the error required for steady-state operation of about .009° C. If a greater gain is used, this temperature change can be reduced to a few thousandths of a degree for normal line voltage fluctuations. Line

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[1] Steady-state system response to changes in conditions such as the ambient temperature can be determined using the feedback transfer function shown in Figure 30 of Appendix A.

voltage regulation with a regulator transformer does not offer a means of improvement since this device utilizes the saturation characteristic of the transformer core to limit the peak voltage. In steady-state operation, the heater draws power from the line only during the first 60° of a cycle and this portion of the line voltage cycle is altered very little by the regulator transformer. In addition to peak voltage regulation, the transformer introduces a phase shift which is dependent upon line voltage. A serious drift problem could result, completely obscuring any benefit obtained from the constant voltage feature, unless the sensing bridge were supplied from the transformer also.

Although the sensor is capable of controlling the temperature at its own location, the temperatures of other sections of the oven are just as important. The analog system revealed that a temperature gradient exists in the oven which causes the temperatures of different parts to differ from the sensor temperature by as much as .05° C in steady-state operation. This difference would be insignificant in almost any application, but if the heater power changes to compensate for ambient temperature changes, the temperature gradient through the oven also changes. The average temperature of the oven's surface will, therefore, be dependent to a marked extent upon the ambient temperature. Unlike the first source of error mentioned, which is a function of loop gain, this relationship is unrelated to gain; the temperature gradient is a function of the physical configuration and the heater power. Distribution of the heater winding appears to be the most expedient means for reducing the temperature gradient and, consequently, its changes. Theoretically, the gradient could be entirely eliminated in the steady-state condition by properly distributing the heater winding. In addition to the use of a distributed heater, placement of the oven within another enclosure which is temperature regulated would reduce error due to the temperature gradient. Another reason for the well-founded popularity of double proportional control has then been found.

It should finally be noted that while much of the analysis undertaken has been in consideration of transient operation, in many cases this characteristic is of little importance. The system developed, offering good transient operation, would be useful for enclosing temperature sensitive devices which themselves dissipate a variable amount of power. Ovens not enclosed within a second regulated enclosure also may have to operate under transient conditions, while double proportionally controlled systems enclosing circuits dissipating small or constant power would be less demanding of transient operation. For this type of system, more regard should be given to minimizing long term drift, achieving stability with high loop gain, etc. There appear to be several areas of investigation which might prove fruitful in achieving very accurate temperature control with excellent long term stability. A much higher d.c. loop gain might be used if gain were attenuated sufficiently at frequencies where phase lag resulting from the oven and sensor become appreciable. Integral feedback could be used to achieve more accurate regulation by reducing the steady-state error theoretically to zero. Undoubtedly, as new, more exacting applications are found for temperature controlled enclosures, these and other techniques will be used in achieving temperature stabilities not yet approached.

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APPENDIX A

THE FEEDBACK SYSTEM TRANSFER FUNCTION

APPENDIX A

A temperature control system which provides a desirable relationship between heater power and temperature error was described in Chapter II. The electronic section of the feedback system determined at that time is more fully described as shown in Figure 29, which includes the error signal supplied by the sensing bridge and subsequent circuits controlling the oven heater, shown as resistance R_L.

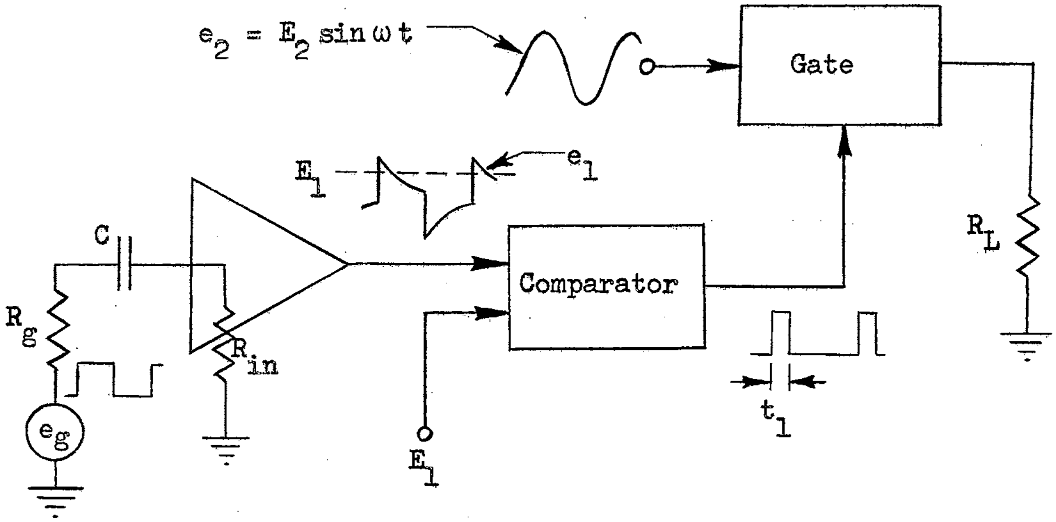

Figure 29. The Oven Control System

The voltage gain, K, is defined to include the voltage divider effect of the sensor impedance, R_g, and amplifier input impedance, R_{in}, on the error voltage, e_g. The voltage, e_1, may then be expressed as a time function.

$$e_1 = Ke_g \varepsilon^{-t/RC} \qquad (1)$$

where $R = R_g + R_{in}$

ε is the natural logarithm base

and time, t, is any value from $t = 0$, when the pulse begins, to its duration.

Taking the logarithm of both sides of Equation (1) yields:

$$\text{Ln } e_1 = \text{Ln } K + \text{Ln } e_g - t/RC \qquad (2)$$

or, solving for t,

$$t = RC(\text{Ln } K + \text{Ln } e_g - \text{Ln } e_1) = RC \text{ Ln}\left(\frac{Ke_g}{e_1}\right) . \qquad (3)$$

Now, let $e_1 = E_1$, a constant reference level used by the comparator. Then

$$t_1 = RC \text{ Ln}\left(\frac{K}{E_1} e_g\right) . \qquad (4)$$

In order to determine the part of the cycle of the power source voltage which is applied to the heater, Equation (4) may be stated as:

$$\omega t_1 = \omega RC \text{ Ln}\left(\frac{K}{E_1} e_g\right) \qquad (5)$$

This form yields the radian measure of the power cycle applied.

If e_2 is expressed as a time function, the average power dissipated by the heater when the voltage e_2 is applied for only a fixed part of each cycle may be expressed as:

$$P_L = \frac{\frac{1}{T}\int_0^{t_1} (e_2)^2 \, dt}{R_L} \quad . \tag{6}$$

If T, the period of e_2, is replaced by its value $2\pi/\omega$, Equation (6) may be expressed as:

$$P_L = \frac{1}{R_L} \frac{\omega}{2\pi} \int_0^{t_1} E_2^2 \sin^2 \omega t \, dt \tag{7}$$

which may be evaluated giving:

$$P_L = \frac{E_2^2}{8\pi R_L} \left[2\,\omega t - \sin 2\omega t \right] \tag{8}$$

$$\text{for } 0 \leq \omega t \leq \pi \text{ .}$$

Substituting the expression of Equation (4) for the value of time in Equation (8) yields:

$$P_L = \frac{E_2^2}{8\pi R_L} \left\{ 2\,\omega RC \, \mathrm{Ln} \left(\frac{K}{E_1} e_g \right) - \sin \left[2\,\omega RC \, \mathrm{Ln} \left(\frac{K}{E_1} e_g \right) \right] \right\} . \tag{9}$$

This value of heater power as a function of error signal is shown graphically in Figure 30.

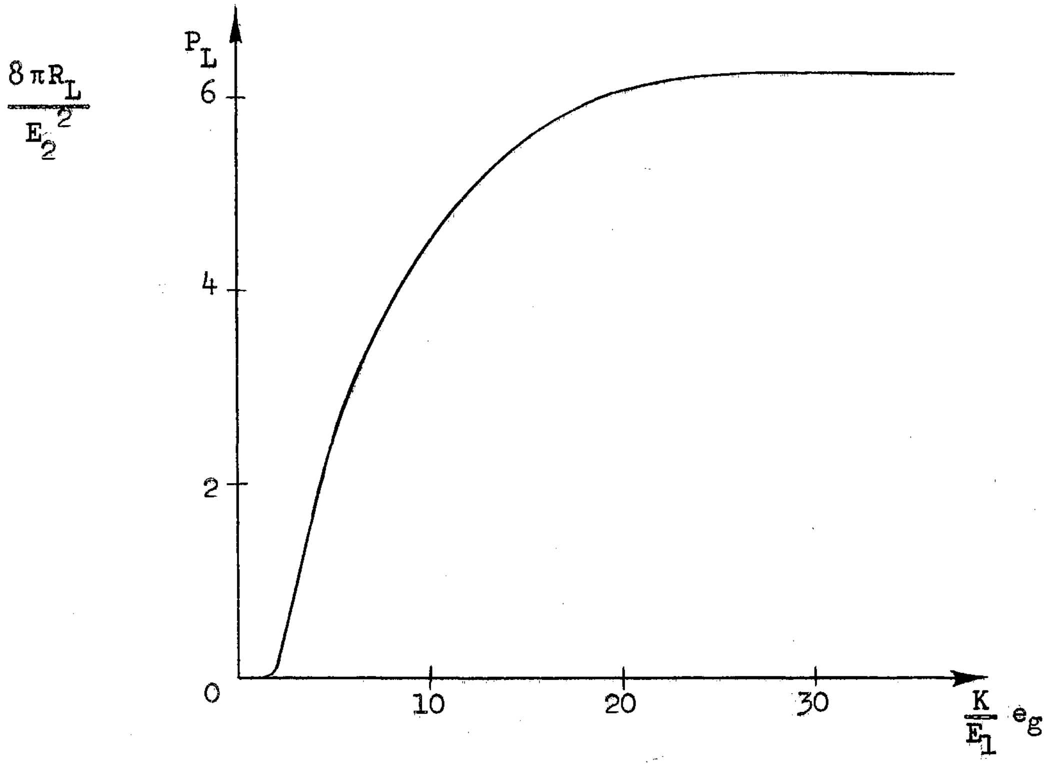

Figure 30. The Relationship Between Error Signal, e_g, and Heater Power, P_L

APPENDIX B

THERMAL CHARACTERISTICS OF THE SENSISTOR

APPENDIX B

It is necessary to know the thermal characteristics of the temperature sensor, in addition to those of the oven itself, in order to choose the gain of the control system wisely. The change of the temperature sensitive parameter of the device is usually available with a specified resistance change per unit temperature change. This is only a steady-state specification, however, and information concerning the transient behavior is not generally available. Such is the case with the sensistor used in the sensing bridge described in Chapter II.

In order to determine the resistance of the sensistor as a function of time when its temperature is suddenly changed (step input of temperature), the arrangement shown in Figure 31 was used.

The output voltage, e_o, of the circuit shown in Figure 31 is approximately 1 volt for each 100 ohm change in resistance of the sensistor.

The temperature step input was applied by suddenly immersing the sensistor body (but not the leads!) into a container of crushed, melting, ice and moving it continually to maintain a temperature of zero °C on its surface.

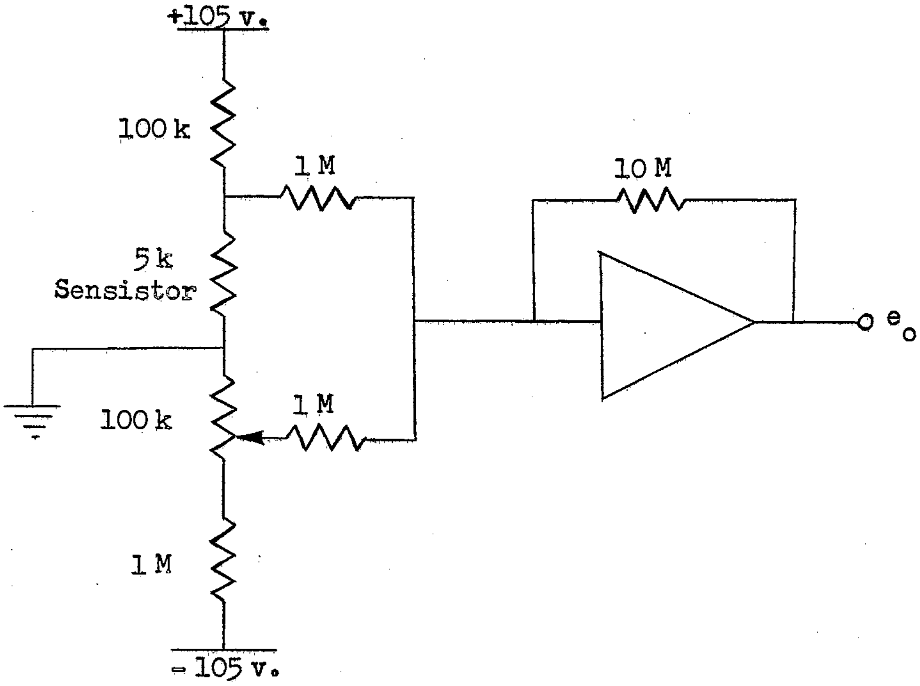

Figure 31. Arrangement for Determining the Thermal Properties of the Sensistor

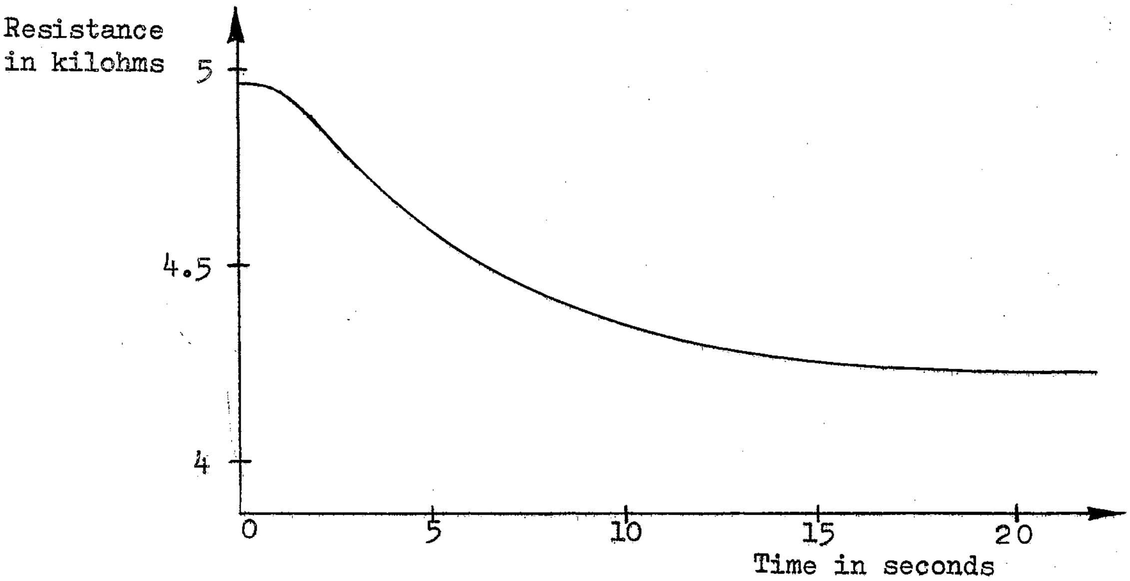

Figure 32. Transient Thermal Response of the Sensistor

The approximate relationship of output voltage and resistance change as already described is sufficiently accurate, considering the possible experimental errors in the procedure.

The transient thermal response found resulting from a 22° C temperature change is shown in Figure 32 (on the preceding page).

APPENDIX C

CHOICE OF LOOP GAIN OF THE ANALOG SYSTEM

APPENDIX C

It will be noticed that the feedback path of the analog system in Figure 19 of Chapter IV (page 42) does not stipulate how the steady-state transfer function is obtained in the real system. While the desired transfer function of the control system is actually obtained using the pulse techniques and switching circuits already described, the analog merely represents the over-all feedback transfer function using the function generator and the distributed gain of the feedback loop. The gain of the feedback loop is found by considering what gain is needed to just drive the heater to its maximum power.

Recalling the method used to achieve the transfer function, an error signal pulse which just reaches the reference voltage of the comparator has an initial value equal to the comparison voltage and decays with a time constant of .00232 second. Its instantaneous voltage one-half cycle later, then, will be

$$E_c \varepsilon^{-t/RC} = E_c \varepsilon^{-.00833/.00232} = .027\ E_c\ ,$$

E_c being the comparison voltage. It was determined in Chapter III that a .001° C temperature error would cause a 3.5 volt peak signal at the comparator. This corresponds to a .00257° F error producing a signal which just equals the comparator reference of 5 volts. The temperature error necessary to just cause the comparator to hold the oven on for a complete half cycle is therefore

$$.00257° \text{ F} / .0275 = .094° \text{ F}.$$

Loop gain of the analog system is chosen so that the oven analog is driven with the current representing full power drive when an error of .094° F is present. One point of the nonlinear transfer function is, therefore, determined. The function generator is then adjusted to yield the transfer function, using the known point as a reference.

APPENDIX D

CALCULATION OF ANALOG COMPONENT VALUES

APPENDIX D

In Figure 21 of Chapter IV, the oven half is shown represented by eight analog sections. Five of these represent the cylindrical portion as equal rings; the others represent the circular end plate as three concentric annuli. The thermal resistances and capacities of these sections can be found using available relationships. (7). For example, the thermal resistance, R_t, of one of the ring sections may be expressed as:

$$R_t = \frac{x}{kA} , \tag{1}$$

where:

x is the length of the section,

A is its cross sectional area,

and k is the thermal conductivity of the material. Using English units for dimensions, heat in BTU, and time in hours, as is the general practice, results in a value of

$$R_t = .010 \text{ hr.}^\circ\text{F} / \text{BTU}.$$

Also, the thermal capacity of the section is

$$C_t = \rho c V , \tag{2}$$

where:

ρ is density of the material,

c is specific heat of the material,

and V is volume of the section.

The resulting value of C_t for the ring is

$$C_t = .0150 \text{ BTU} / {}^\circ\text{F} .$$

Some considerations in the choice of scale factors relating thermal characteristics and the analogous electrical component values have been discussed. A convenient time scale may be chosen as five seconds analog time per minute real time. Expressing electrical time in seconds and real time in hours, as is generally done, results in the scale factor

$$n = 300 \text{ sec./hr.}$$

The value of m, defined as thermal capacity per farad of electrical capacity, is taken to be

$$m = 10^4 \text{ BTU} / {}^\circ\text{F farad}$$

because the resulting value of analogous capacitance for the ring sections is convenient. The value of L, as voltage of the analog per degree Fahrenheit, is fixed by the desired range of temperature to be covered and the maximum operating voltage of the analog. The oven analog is capable of a 50 volt negative swing, leaving 50 volts for operation of the current drivers. This voltage and the maximum temperature drop of 1°F determine the value of L:

$$L = 50 \text{ volts}/°F.$$

Having determined the scale factors, the electrical component values of the ring analog are expressed as:

$$R_e = m\,n\,R_t \tag{3}$$

or

$$R_e = 50 \text{ kilohm}$$

and

$$C_e = \frac{C_t}{m} \tag{4}$$

or

$$C_e = 1.5\ \mu f\ .$$

Equations (1) and (2) were also used to determine the thermal resistances and capacities of the three end sections. The resulting thermal resistance values are only approximate, an average value of the cross sectional area having been used for the value of A in Equation (1).

Values of the resistors drawing off the leakage currents are found in consideration of the heat lost through the cork insulation. Current in the analog is related to heat flow in the oven by the expression

$$I = \frac{L}{mn}\,q \tag{5}$$

where q is the heat flow in BTU/hr. Solution of this equation results in a scaled value of current,

$$I = .057 \text{ ma./watt} .$$

It was found that the oven reached a temperature 78° C higher than ambient temperature in steady-state operation with a power of 15 watts. Heat loss, when the oven is at its normal operating temperature difference of 15°C, is therefore (15)(15°C/78°C), or 2.88 watts. Since the temperature at all points on the oven surface are approximately equal, this loss will be distributed among the sections of the analog proportionate to the area each represents. For example, one of the rings represents 6.1% of the area of the oven. This portion of the total power loss is 0.175 watts, which corresponds to an analog current of .010 ma. The current source drawing this current is seen from Figure 21 to actually be a 50 volt voltage source causing a current to flow through the leakage resistor. This voltage and the leakage current just found fix the value of the resistor as 50 volts/.010 ma., or 5 megohms. Resistance values of the other leakage paths are found similarly.

VITA

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