THE UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

THE IDENTIFICATION OF CHARGED PARTICLES IN NUCLEAR EMULSIONS

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

degree of

DOCTOR OF PHILOSOPHY

BY

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Norman, Oklahoma

1956

THE IDENTIFICATION OF CHARGED PARTICLES

IN NUCLEAR EMULSIONS

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ACKNOWLEDGEMENTS

The writer believes it would be impossible to present a complete list of reasons why he wishes to express his appreciation to his adviser, Dr. Robert A. Howard. Special mention is made of his advice, his encouragement, and his cooperation during the past year, while his absence from the campus of the University of Oklahoma made such cooperation extremely difficult.

It would also be impossible to list individually each person to whom the writer feels indebted. A few of these are: Dr. P. M. Fitzpatrick, whose dissertation has been used extensively as a reference; Mr. C. H. Skeen, with whom many conferences and discussions were held; Mr. J. J. McClure, who applied the methods discussed in Chapter V, to light particles; Mr. W. Z. Osborne and Mr. H. E. Hoffman; Mr. Allen Rose, who performed many of the calculations; Mrs. Joan Crossland, who exercised skill, patience and perseverance in typing the manuscript.

Finally, the writer expresses his gratitude to his wife, Willard, for everything she has done.

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CHAPTER I

INTRODUCTION

This report is the result of investigations made by the writer over a period of approximately two years, 1953 to 1956, in the Nuclear Emulsions Laboratory of the University of Oklahoma. Since the facilities of this laboratory have been and are being continuously improved, some of the earlier work was rendered obsolete by later innovations.

During the earlier part of his experiences the writer collaborated closely with Fitzpatrick. Such work as has already been reported,[1] will be reviewed only briefly in this report and only where it bears directly upon the findings of this report. Specifically, this includes: (1) the attempt to "simulate the conditions which would be present during actual processing" and then determine the optimum warming and cooling times; (2) the actual processing of the emulsions; (3) the original technique of track evaluation of severely clogged tracks, by means of area measurements made with a planimeter; (4) track evaluation by grain

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[1] Phillip M. Fitzpatrick, "The Interaction of Extremely Energetic Cosmic Ray Particles with Matter" (unpublished Ph. D. dissertation, Department of Physics, University of Oklahoma), 1955.

counting; (5) corrections for distortion.

Largely due to the efforts of C. H. Powell and his coworkers at the University of Bristol, nuclear emulsions have really come into their own since World War II. Ilford Limited has contributed a great deal to this research field by the development and manufacture of the G5 emulsion which is sensitive to all types of charged particles. These emulsions are available in the form of unsupported pellicles which provide advantages over glass-backed emulsions as they furnish a larger volume and hence a greater probability of obtaining longer tracks.

At the University of Oklahoma, the use of pellicles was largely in an exploratory stage at the time the reported studies were made and so the writer has used the glass-backed emulsions which were available. Other experimenters will describe, in their dissertations, some of the difficulties encountered in attempting to develop and use the pellicles. Some of these difficulties have been satisfactorily resolved. Specifically, the matter of affixing the emulsions to glass plates with a minimum of distortion and blistering, is definitely under control. There still remains the problem of tracing tracks from one emulsion into the next.

Objectives

The specific objectives of the researches herein reported have been:

1) To calibrate a microscope which had not previously been used;

2) To re-evaluate the potentialities of such techniques for measuring fundamental track parameters as were already in use in this laboratory;

3) To develop and evaluate the potentialities of a technique, original with the writer, for determining, by means of scattering measurements alone, the mass and the residual range of a particle which has left a non-ending track;

4) To apply this technique to tracks which exhibit unusual characteristics.

Anticipation

The microscope to which reference has been made was acquired at about the time ~~these~~ researches were begun. It is a Leitz Ortholux, a photograph of which appears as Figure 1. The instrument and its accessories are described in Chapter II.

Since this was a new instrument, calibrations of the vertical drive micrometer, and the measurement of the "noise level", which is important in multiple scattering observations, were carefully made. Both the techniques already used on another microscope and new techniques were employed.

Chapter III is a resumé of the use of residual range and ionization density, as observables in nuclear emulsion techniques.

In Chapter IV, the method of Constant Sagitta Scattering is discussed. The writer believes he has been able to improve slightly upon the applications of this method of analysis.

Chapter V treats the original technique for analysis of non-ending tracks. The principle of the method, and the application to both known and unknown particles, are reported.

In Chapter VI, this method is applied toward the analysis of two tracks which are believed to be:

1) a decay in flight; and

2) an inelastic scatter.

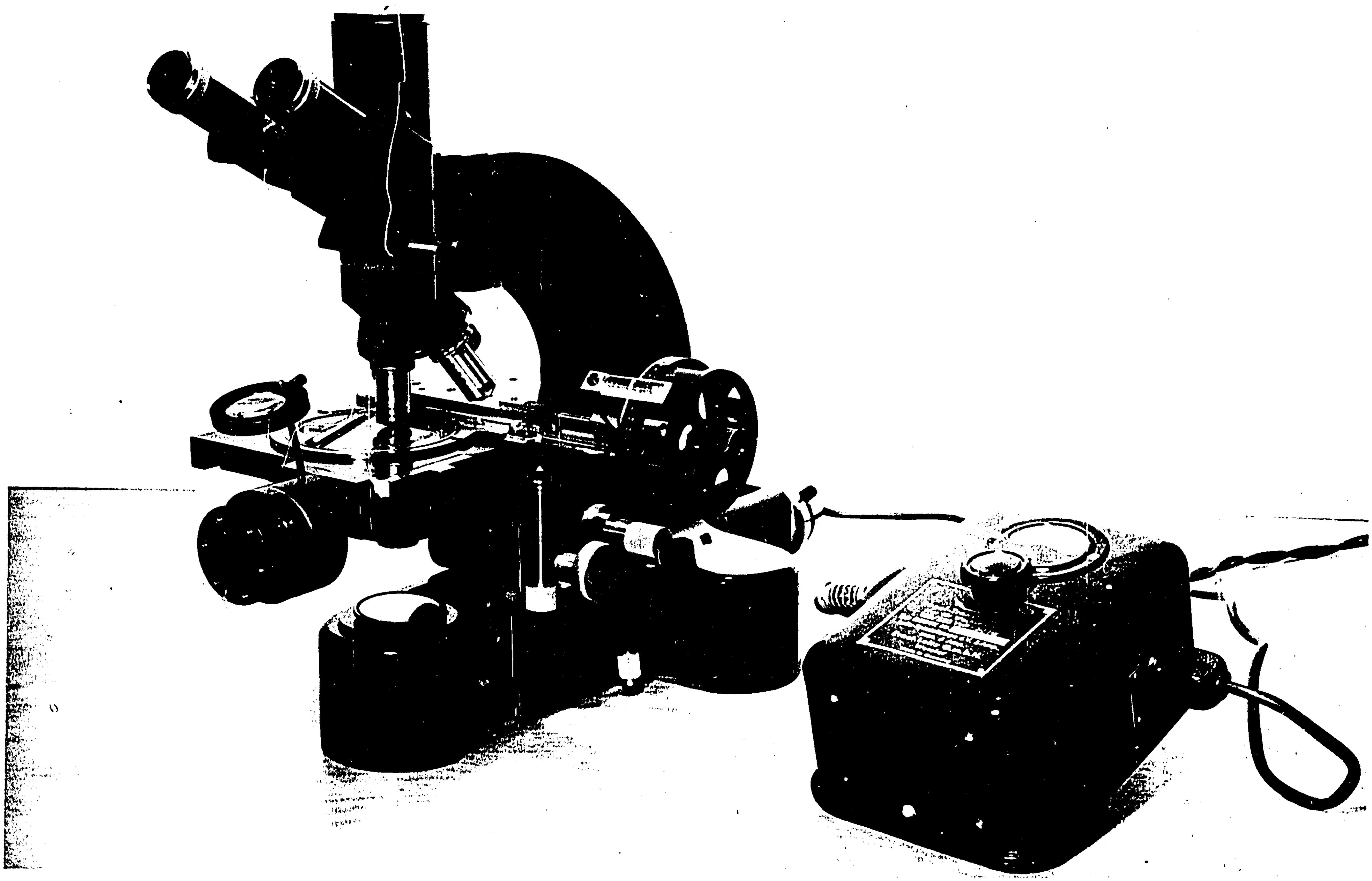

Figure 1 - The Leitz Ortholux Microscope

CHAPTER II

THE MICROSCOPE

General Description

Almost all of the work herein reported was done with the microscope shown in Figure 1. This instrument is a Leitz Ortholux, with built-in illuminator.

Two dry objectives of magnifications 3x and 47x, and three oil immersion objectives of magnifications 20x, 53x, and 100x are available. There are two pairs of eye-pieces of magnifications 12x and 25x. The lowest power oil objective is a flourite lens which makes possible a quick change from low power to high power, without change from "oil to dry". The two dry objectives and the highest power oil objective have proved most useful up to date. The lower power dry lens is employed for quick location of an event; the higher power dry lens for careful examination of an event; and the highest power oil lens for precision measurements.

The lower power eye-pieces have been used almost exclusively. With the higher power piece, lack of resolution is more apparent.

The built-in illuminator is a feature which is a great advantage. It is unnecessary for the observer to devote hours of his time to the accomplishment of a careful orientation of lamp and microscope.

Preliminary adjustments such as focusing and centering are done quickly and easily, and are permanent until it is necessary to replace the illuminator lamp.

This lamp is a spherical, 6-volt, 30-watt lamp manufactured by the General Electric Company.[1]

After the microscope was procured, it was twice modified in the local machine shop. The first improvement consisted of replacing the plate holder with a turn-table. A photograph of this turn-table appears as Figure 2. This feature was designed locally and the plans are on file. It will handle plates which are one inch by three inches. When a plate is carried by the turn-table it is possible to rotate the plate, so that a track may be driven parallel to its length. This facilitates both visual inspection and measurements.

A second local improvement was the addition of micrometer stage drives. The design of these drives was a modification of a design by Rosen.[2] These are shown in Figure 3. Thanks to this improvement, it

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[1] The manufacturer's number is FG 1119-CX. It has been necessary to order replacement lamps directly from the factory in Cleveland, Ohio. Any one who intends to use this microscope over an extended time might do well to be sure that replacements are always available. The transformer which energizes the lamp is equipped with a built-in rheostat and an ammeter. The ammeter scale shows a red region which presumably indicates the current which should not be exceeded. Unfortunately this "danger mark" occurs at six amperes rather than at five. If one keeps the current reading just below the red, he is still overloading the lamp and shortening its life. It has been learned that operating the lamp at five amperes, which results in a power level of thirty watts, provides sufficient illumination and prolongs the life of the lamp.

[2] Louis Rosen, Los Alamos National Laboratory, Los Alamos, New Mexico, private communication.

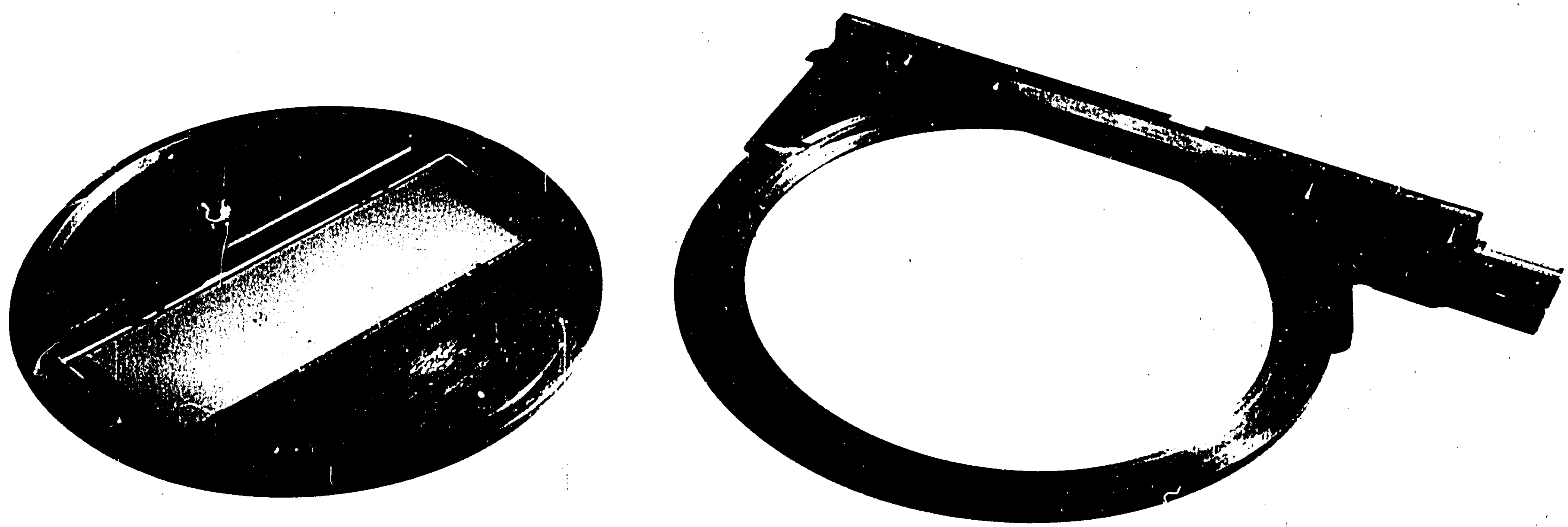

Figure 2 - The Turn-Table

is now possible to displace the plate a short distance, parallel to either of the horizontal scales (see Figure 3), and to read the displacement directly on the appropriate micrometer drum. Such displacement measurements have been found to be correct to within the reading error.[1]

Calibration of the Eye-Piece Scales

Several of the measurements required for track analysis involve the use of an eye-piece scale. The scale used in the work reported here is a circular glass disk upon which are inscribed two perpendicular scales. Each scale consists of one hundred scale divisions.

These scales have been calibrated by comparison with a standard in the form of a one-millimeter scale, subdivided into one hundred equal parts. The standard itself has been compared with other scales and found to agree with them to within one part in five hundred.

Comparison of the eye-piece scales with this standard, when using the 12x eye-pieces and the 100x objective, indicates the following. When the entire length of the eye-piece scale is used, thirty independent readings show one eye-piece scale division equal to 0.917 microns on the abscissa, with one scale division equal to 0.924 microns on the ordinate. When only the middle half of each eye-piece scale is used, one scale division is equal to 0.980 microns on either scale. It is believed that this discrepancy is caused by parallax and by distortion

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[1] One revolution of the micrometer drum displaces the microscope stage through half a millimeter, or five hundred microns. Thus one revolution is subdivided into five hundred scale divisions, so the displacement can be read directly in microns. Thus it is possible to interpolate to a few tenths of a micron.

Figure 3 - Close Up View of Micrometer Drives

at the edge of the field. It is recommended that only the center portion of the eye-piece scales be used for any precise work.[1]

Calibration of Vertical Drive

Focusing is accomplished, on the Leitz Ortholux, with a micrometer drive with which the stage, rather than the optical system, is raised or lowered. The vertical distance between two points in an emulsion can be measured by observing the difference between two readings of the dial on the control knob. This dial consists of a scale, which shows 100 divisions per revolution. One revolution is nominally equivalent to a vertical displacement of the stage through one hundred microns. If measurements of depth are to be relied upon, one must be informed as to both the accuracy and the linearity of the micrometer readings.

An attempt was made to use a small lucite "staircase" which had been constructed locally, for use in calibration of another microscope. Step-heights had already been measured with a micrometer caliper. These heights were remeasured with the same caliper, and an independent set of measurements was taken with a spherometer. Two different observers participated in taking the readings. The agreement between step-heights as measured by different observers, as well as that between those measured by the same person with different instruments, was highly

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[1] Measurements such as the noise-level measurements reported in this chapter, the gap-length measurements of Chapter III, and those required by the Coordinate Method of Multiple Scattering, have been taken by use of only the part of the scale between the 25-mark and the 75-mark.

unsatisfactory. Nevertheless, a series of step-height readings was taken on the microscope. The data were inconclusive, to say the least. A graphical representation indicated that the microscope readings were approximately linear for any one set of readings.[1] But for a different set of readings, the slope of the line might differ as much as three percent. It was believed that fluctuations might be occurring in the dimensions of the plastic calibration plate. Such fluctuations might well be caused by changes in the temperature or in the humidity or in both.

A technique was then used in which the plastic plate was eliminated. The same standard, as had been employed in calibrating the eye-piece scales, was set in a vertical position, on the microscope stage, as shown in Figure 4. An auxiliary microscope was placed in a stationary position, so that the standard could be viewed in its eye-piece. This second microscope is equipped with a cross-wire which may be focused upon the marks which make up the standard scale. Thus it was possible to raise or lower the standard scale as a whole, through a distance equal to that between two consecutive marks. At the same time observations were taken, of the number of scale divisions advanced by the micrometer dial. Data were taken with the microscope stage moving upward, and again with the stage moving downward.

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[1] The movement of the microscope drive consists of twenty eight turns of the dial, or nominally 2800 microns. The total height of the plastic step plate was about 25 microns. So it was necessary to take several different sets of readings, in order to include the entire movement.

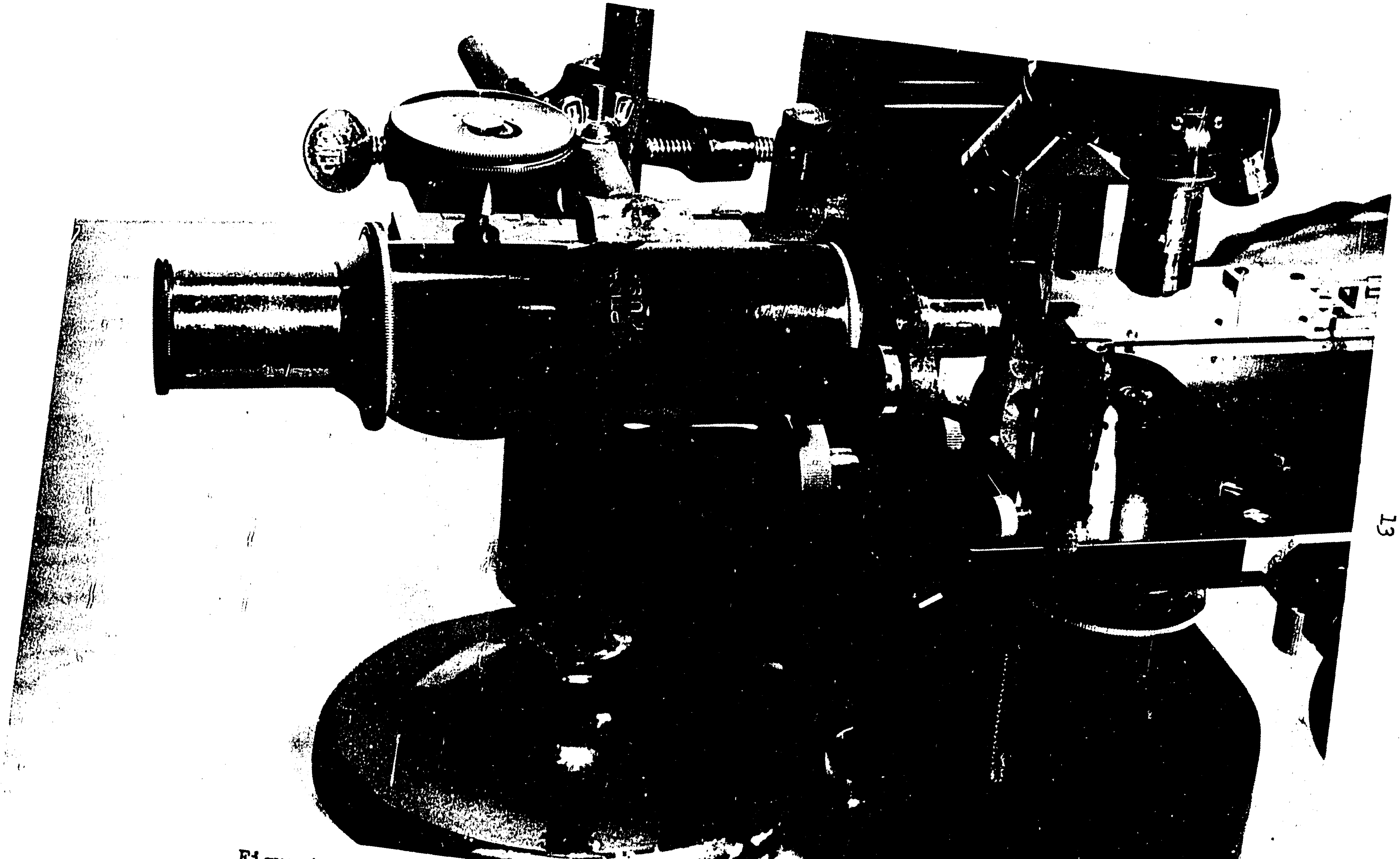

Figure 4 - Arrangement of Auxiliary Microscope and Standard Millimeter Scale for Calibration of Vertical Drive

This method has the following advantages over the use of the plastic step-plate:

1) there should be negligible fluctuations in the dimensions of the standard scale;

2) the standard scale itself may be more reliably calibrated;

3) readings may be taken more quickly and more easily;

4) the entire movement of the microscope stage can be covered in a single set of readings.

Over the first five turns, numerous readings were taken in order to establish the deviation for a single reading. From a set of fourteen readings on the first 100 microns of the standard scale, the probable deviation of a single reading was evaluated as 0.77 microns.

Figure 5 shows the cumulative height as measured on the standard scale as a function of the height as measured on the microscope micrometer. The departure from linearity near the top of the drive is slight but definite. Over the middle portion of the drive, from the tenth to the twentieth turn, the linearity is satisfactory. It is recommended that only this part of the drive be used for precision measurements.

Figure 5 also shows a broken line graph of the number of microns per scale division for each successive, two-turn interval of the micrometer drum. This was done because it was believed that the micrometer readings are linear, to within the reading error, over any interval of two hundred scale divisions.

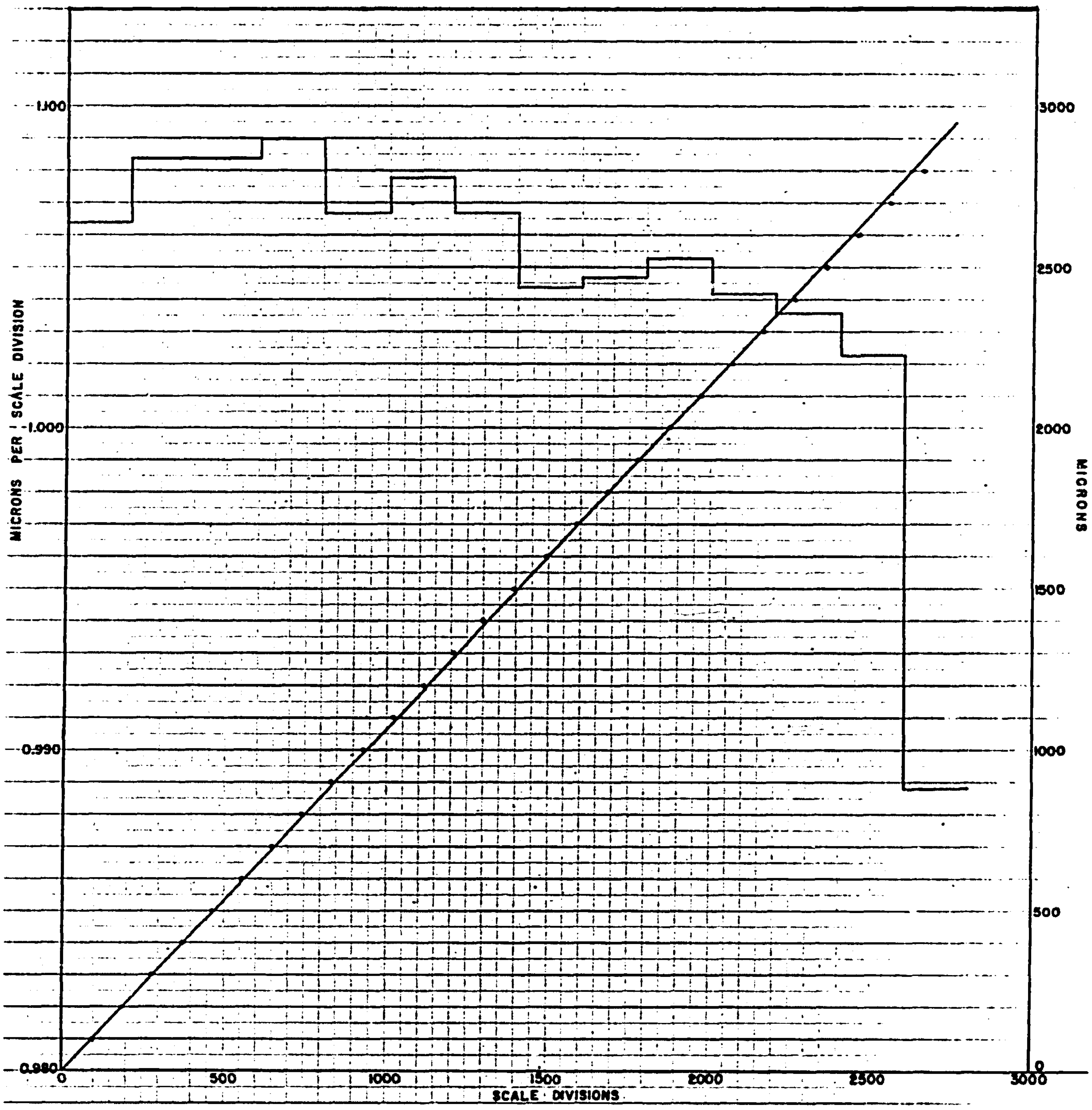

Figure 5 - Calibration of Vertical Micrometer Scale. Ordinates at right refer to the straight line. Ordinates at left refer to the broken line. Abscissae are micrometer readings.

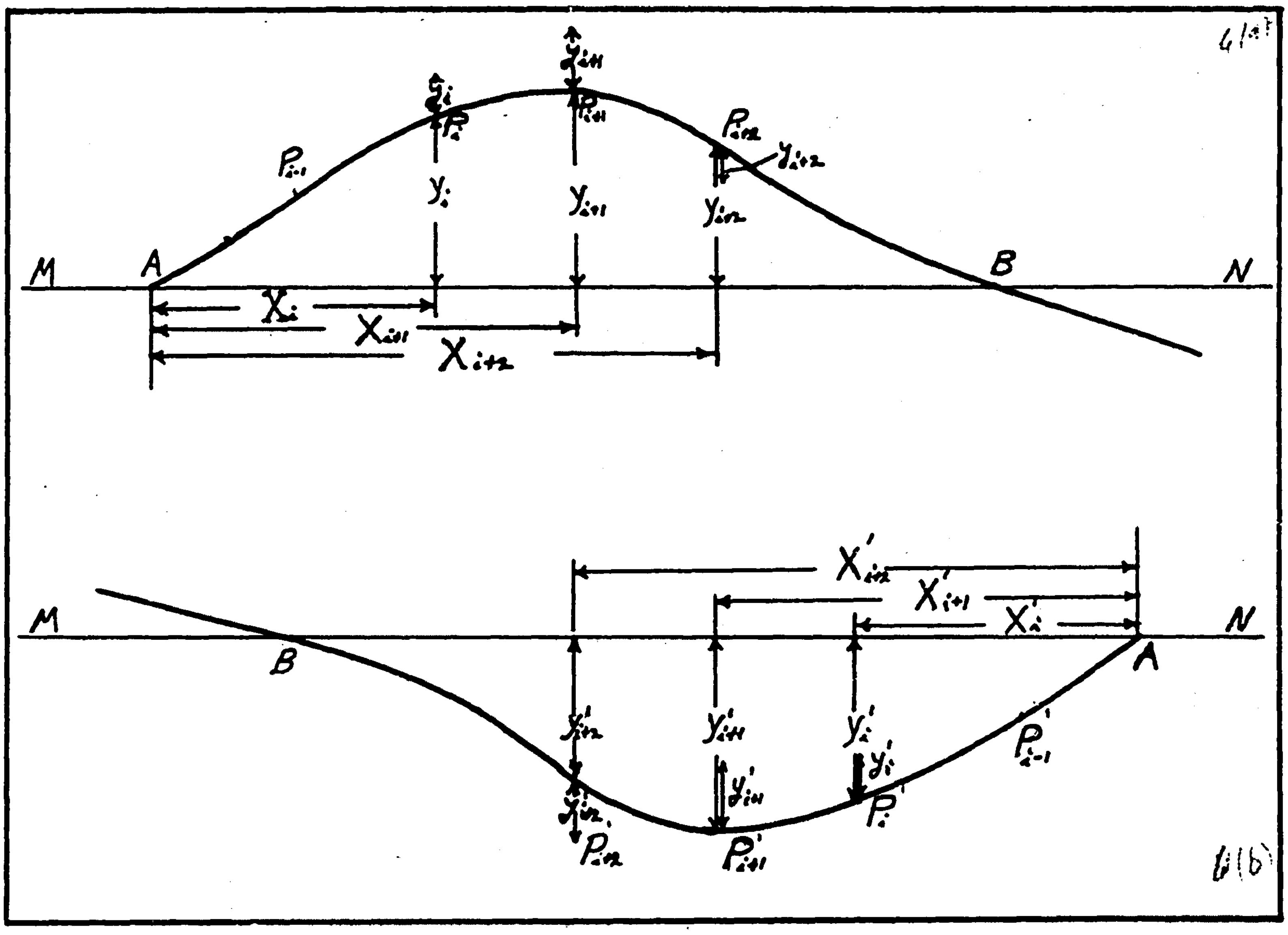

Figure 6 - A track which has been aligned with the microscope, and the same track after it has been rotated through one hundred eighty degrees.

Measurement of Noise Level

The "Coordinate Method of Measurement" of the effects of multiple scattering, is a very useful method for analysis of tracks in nuclear emulsions. Chapters IV, V, and VI of this writing are primarily concerned with this method.

Consider Figure 6a, in which the line MN represents that axis of the microscope stage, parallel to which the plate may be made to move. This displacement may be accomplished by turning the appropriate stage drive micrometer drum. The axis, MN, will henceforth be called the "microscope abscissa".

The track under observation includes the points

$$\ldots\ P_{i-1},\ P_i,\ P_{i+1},\ P_{i+2},\ \ldots\quad ,$$

which, at the moment, are to be considered as equally spaced along MN. The line AB is so constructed that

$$\sum_{j=1}^{N} y_j = 0\ ,$$

where y_j is the displacement from AB of any point, P_j, on the track. The algebraic sign of y_j is to be taken positive when the point lies above AB and negative when the point is below AB. The line, AB, will henceforth be called the "axis of the track". As an approximation, it is assumed that the axis of the track coincides with the microscope abscissa, and that the entire track lies in a plane. In Figure 6 then, AB and MN are shown as the same line.

The Coordinate Method of making multiple scattering measurements, consists of displacing the track, parallel to MN, so that each of the points, P_{i-1}, P_i, and so forth, is made to coincide, in turn, with the ordinate of the eye-piece scale. The apparent displacement from MN, of each of the points, P_i, is recorded [1] as Y_i. The algebraic sign of Y_i is taken positive when P_i appears to be above the abscissa of the eye-piece scale, and vice versa.

Now Y_i may be considered to be the algebraic sum of two parts. So

$$Y_i = y_i + \gamma_i , \qquad (2\text{-}1)$$

where y_i is the "true displacement due to scattering"[2] of the particle trajectory from the axis of the track, and γ_i is the apparent incremental displacement due to what is called "noise".

This noise is considered to be the resultant of three contributions:

1) deviations in reading the eye-piece scale,

2) deviations due to random departures of the microscope stage from rectilinear motion, and

3) deviations due to the distribution of grains about the particle trajectory.

For each of the points, P_i, then, one may observe a value of Y_i, and for any three consecutive Y_i's, one can compute the apparent second difference, D_i, defined by

$$D_i = Y_i - 2\,Y_{i+1} + Y_{i+2} . \qquad (2\text{-}2)$$

Over a segment of track which includes $N+2$ points, one may determine the mean of the absolute values of the D_i's,

$$D = \frac{\sum_{i=1}^{N} |D_i|}{N} .$$

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[1] Before any observations are made, the eye-piece scale must be oriented so that its abscissa is parallel to MN.

[2] An operational definition of "true displacement due to scattering" appears in the Appendices.

Equations (2-1) and (2-2) may be combined to give

$$D_i = d_i + \delta_i \quad , \qquad (2\text{-}3)$$

where $$d_i = y_i - 2y_{i+1} + y_{i+2} \quad ,$$

and $$\delta_i = \gamma_i - 2\gamma_{i+1} + \gamma_{i+2} \quad .$$

Now the mean of the absolute values of the second differences due to true scattering, Δ, and the mean of the absolute values of the second differences due to noise, ϵ, are respectively defined by

$$\Delta = \frac{\sum |d_i|}{N} \quad , \quad \text{and} \quad \epsilon = \frac{\sum |\delta_i|}{N} \quad .$$

Statistical arguments[1], show that

$$D^2 = \Delta^2 + \epsilon^2 \quad . \qquad (2\text{-}4)$$

It is assumed, as in the argument given above, that the members of each set, namely, the D_i's, the d_i's and the δ_i's follow normal distribution laws.

Nearly all applications of the Coordinate Method require that D be determined experimentally, and then be corrected by means of this last equation. This requires that ϵ , often called the "noise level", be known. It is commonly accepted as being a function of the microscope-observer system.

The usual method of determining the noise level, for a given system, is to select a long, straight, level track, which has been made by a lightly ionizing particle, and to neglect the contribution of the term, y_i, in each of the equations of the form of (2-1). Then the observed D for this track is substantially the noise level, ϵ .

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[1] Wm. Schriever, Professor of Physics, University of Oklahoma, has suggested a rather simple proof of Equation (2-4). This proof appears as one of the Appendices.

But one can never ~~be sure~~ that the true displacement due to scattering is negligible. So a technique has been devised which eliminates the effect of any such contribution to the measurements made upon the calibration track. This technique is described below.

Figure 6b shows the same track as Figure 6a, after it has been turned through 180 degrees, by means of the microscope turn-table. Now one can observe a set of Y'_i's, each of which is the apparent displacement from MN, of the corresponding point P'_i. Each X'_i is taken numerically equal to the corresponding X_i. Then it follows that each y'_i is, by definition, equal in numerical value but opposite in sign to the corresponding y_i. There will be N equations of the form (2-1), and for each of these there will be a primed equation of the form

$$Y'_i = y'_i + \gamma'_i \quad .$$

From this it follows that for each of the equations of the form (2-3), there will be an equation of the form

$$D'_i = d'_i + \delta'_i \quad ,$$

and that each d'_i is numerically equal but opposite in sign to the corresponding d_i. Then one can write N equations of the form

$$D_i + D'_i = \delta_i + \delta'_i \quad . \qquad (2\text{-}5)$$

One may square both sides of each of the N equations represented by Equation (2-5), and form the sum

$$\sum_{i=1}^{N} (D_i + D'_i)^2 = \sum_{i=1}^{N} (\delta_i^2 + \delta'^2_i) + 2\sum_{i=1}^{N} \delta_i \delta'_i \quad .$$

Since the δ_i's, the δ'_i's, the D_i's and the D'_i's have a normal distribution, the second term on the right hand side of this last equation will vanish, if N is a large number. Also

$$\sum_{i=1}^{N} \delta_i^2 = \sum_{i=1}^{N} \delta'^2_i \quad .$$

Therefore

$$\sum_{i=1}^{N}(D_i + D_i')^2 = 2\sum_{i=1}^{N}d_i^2 .$$

So

$$\sqrt{\frac{\sum_{i=1}^{N} d_i^2}{N}} = 0.707\sqrt{\frac{\sum_{i=1}^{N}(D_i + D_i')^2}{N}} .$$

Scarborough[1] shows that the root-mean-square value of an observable which has a normal distribution about a zero mean, is equal to 1.2533 times the mean of the absolute values of that observable. Therefore

$$\frac{\sum_{i=1}^{N}|d_i|}{N} = 0.707\ \frac{\sum_{i=1}^{N}|D_i + D_i'|}{N} . \qquad (2\text{-}6)$$

The left-hand member of Equation (2-6) is, by definition, ϵ , the mean of the absolute values of the second differences due to noise. So

$$\epsilon = 0.707\ \frac{\sum_{i=1}^{N}|D_i + D_i'|}{N} . \qquad (2\text{-}7)$$

<u>Alignment of the Track With the Abscissa</u>

With care, the alignment of the track, so that its axis coincides with the microscope abscissa, can be accomplished with fair accuracy. This is also true of the rotation through one hundred eighty degrees, which is required for the measurement of the noise level as described.

The following argument, involving Figure 7, shows that the angle between AB and MN may be as much as three degrees, before the error in D, which results from this systematic error, becomes as great as 0.75 percent.

If θ is the angle between AB and MN as shown in Figure 7, then Y^o

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[1] James C. Scarborough, <u>Numerical</u> <u>Mathematical</u> <u>Analysis</u> (2d ed.; The Johns Hopkins Press, Baltimore, 1950), p. 415.

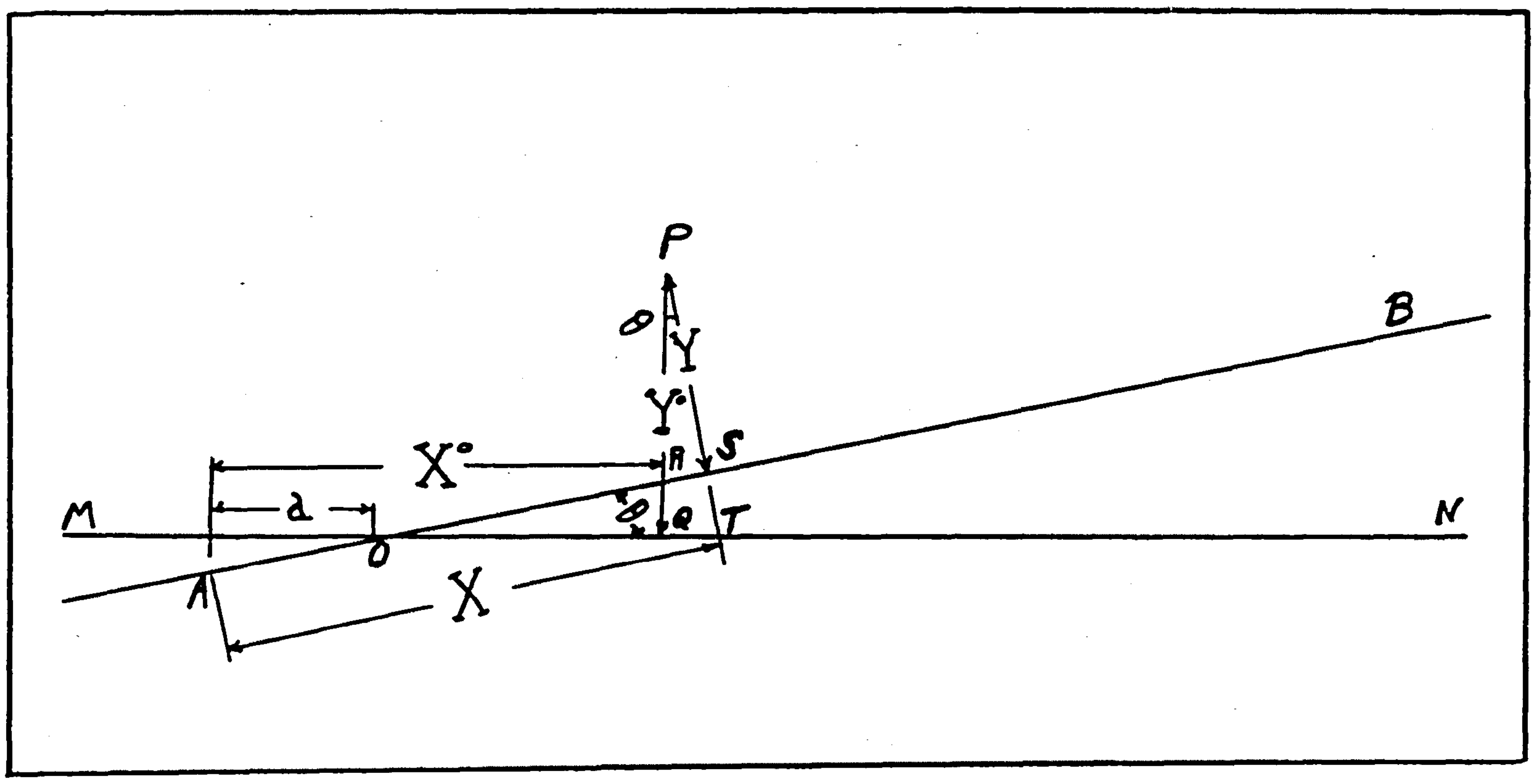

Figure 7 - The axis of the track not coincident with the abscissa of the microscope.

is the observed displacement of P and Y is the true displacement.

$$Y = (Y^{o} - \overline{QR}) \cos \theta .$$

$$\overline{QR} = (X^{o} - a) \tan \theta .$$

So

$$Y = Y^{o} \cos \theta - (X^{o} - a) \sin \theta .$$

Now P is any point on the track, so Y may be considered to be any one of the Y_i's, which are required for the computation of the D_i's. Hence

$$D_i = (Y_i^{o} - 2Y_{i+1}^{o} + Y_{i+2}^{o}) \cos \theta - \left[(X_i^{o} - X_{i+1}^{o}) - (X_{i+1}^{o} - X_{i+2}^{o})\right] \sin \theta .$$

The quantity in brackets is identically zero, since the X_i^{o}'s are equally spaced.[1] Therefore

$$D_i = (Y_i^{o} - 2Y_{i+1}^{o} + Y_{i+2}^{o}) \cos \theta ,$$

and

$$D_i^{o} = D_i \sec \theta ,$$

where D_i^{o} is the observed second difference. Suppose $\theta \leq 3$ degrees. Then $\sec \theta$ is between one and 1.0014, and the relative error in D_i is

$$\frac{|D_i^{o} - D_i|}{D_i} = \frac{D_i (\sec \theta - 1)}{D_i} \leq 0.0014 .$$

It is also true that the observed cell length, t^{o}, will be slightly different from the true cell length, t, if the axis of the track is not coincident with the axis of the microscope. In Figure 7

$$X = \overline{AO} + \overline{OS} ,$$

[1] This is strictly true only for what is known as "constant cell-length scattering observations". With "constant sagitta scattering observations", discussed in Chapter IV, the cell lengths vary with the residual range. The difference between two consecutive cell lengths is, however, very small.

$$\overline{OS} = \overline{OT}\cos\theta ,$$

$$\overline{OT} = X^o - a + \overline{QT} ,$$

and

$$\overline{QT} = Y^o \tan\theta .$$

So

$$\overline{OS} = (X^o - a + Y^o \tan\theta)\cos\theta ,$$

while

$$\overline{AO} = \frac{a}{\cos\theta} ,$$

since a is, by definition, the projection of AO upon MN.

Therefore

$$X = \frac{a}{\cos\theta} + (X^o - a + Y^o \tan\theta)\cos\theta ,$$

and since X is any one of the X_i's,

$$X_i = a(\sec\theta - \cos\theta) + X_i^o \cos\theta + Y_i^o \sin\theta ,$$

and a similar expression holds for X_{i+1}.

So

$$X_{i+1} - X_i = (X_{i+1}^o - X_i^o)\cos\theta + (Y_{i+1}^o - Y_i^o)\sin\theta .$$

or

$$t = t^o \cos\theta - S_i^o \sin\theta ,$$

where S_i^o is the observed first difference, defined by

$$S_i^o = Y_i^o - Y_{i+1}^o .$$

For $t^o = 100$ microns, and $\theta \leq 3$ degrees, $S_i^o \cong 5$ microns. These data yield a relative error in t of about 0.004. If D is considered to be proportional to $t^{3/2}$,[1] the relative error in D is approximately 0.006.

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[1] Scattering theory - see Ch. IV, Equation (4-1) - predicts that $\Delta \sim t^{3/2}$.

The total relative error in D, then, resulting from misalignment of the track by three degrees should never exceed 0.0075.

Experimental Results

A long, straight track[1] which had previously been used for determining the noise level for another microscope, was carefully aligned, and a set of data observed. Both Y-readings, and Y'-readings were taken. Values of the noise level were computed, by means of Equation (2-7) for cell lengths of 25, 50, 75, 100, 150, 200, and 250 microns.

Figure 8 shows the logarithm of ϵ as a function of the logarithm of t. With the exception of the value for $t = 200$ microns, the points lie close to a smooth curve.

The standard deviation may be computed by[2]

$$\sigma(\epsilon) = \frac{0.75\,\epsilon}{N^{\frac{1}{2}}} \qquad (2\text{-}8)$$

For this "worst point", at $t = 200$ microns,

$$\sigma(\epsilon) = 0.05 \text{ microns.}$$

So the point lies off the curve by less than its standard deviation.

Levi-Setti[3] has found that for a microscope in which there is

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[1] For each of the tracks which is discussed in this report, the writer has compiled a separate folder. These folders are on file in the Nuclear Emulsions Laboratory at the University of Oklahoma, Room B24, Research Institute Building. A typical folder contains information regarding the location of the track, including the file number of the emulsion in which it appears, and the horizontal and vertical microscope coordinates which are required in order to locate the track in the emulsion. Such a folder also contains the experimental data as initially recorded and the details of the calculations performed in making the analyses.

[2] A verification of this equation appears in the Appendices.

[3] R. Levi-Setti, Nuovo Cimento, ser. 8, 8, 994, (1951).

very little stage noise, the noise level is independent of cell length. Voyvodic [1] states that increase in noise level with cell length is due to stage noise which varies approximately as $t^{\frac{1}{2}}$, for conventional microscopes with ball bearing stages.

Before this microscope was modified by the addition of the micrometer stage drives, noise level measurements had been made by the same observer, on the same track. These were definitely smaller. For example, for 100-micron cell lengths:

$$\epsilon \text{ (before modification)} = 0.15 \pm 0.01 \text{ microns;}$$

$$\epsilon \text{ (after modification)} = 0.24 \pm 0.02 \text{ microns.}$$

It is also true that before modification, almost equal noise levels were obtained for cell lengths of 50, 75, and 100 microns, and that for 25-micron cell lengths was only slightly less.

So it seems that when the microscope was improved, by the addition of the stage drives, some stage noise was added. Voyvodic's statement as well as the conclusions of Levi-Setti seem to be consistent with the findings for this instrument.

In later work, described in Chapters IV, V, and VI, values of the noise level, ϵ , were required in order to compute the true scattering, Δ. Such values have been taken from Figure 8.

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[1] L. Voyvodic, "Particle Identification with Photographic Emulsions and Related Problems", Progress in Cosmic Ray Physics, ed. J. G. Wilson, (Interscience Publishers Inc., New York, 1954), II, 728.

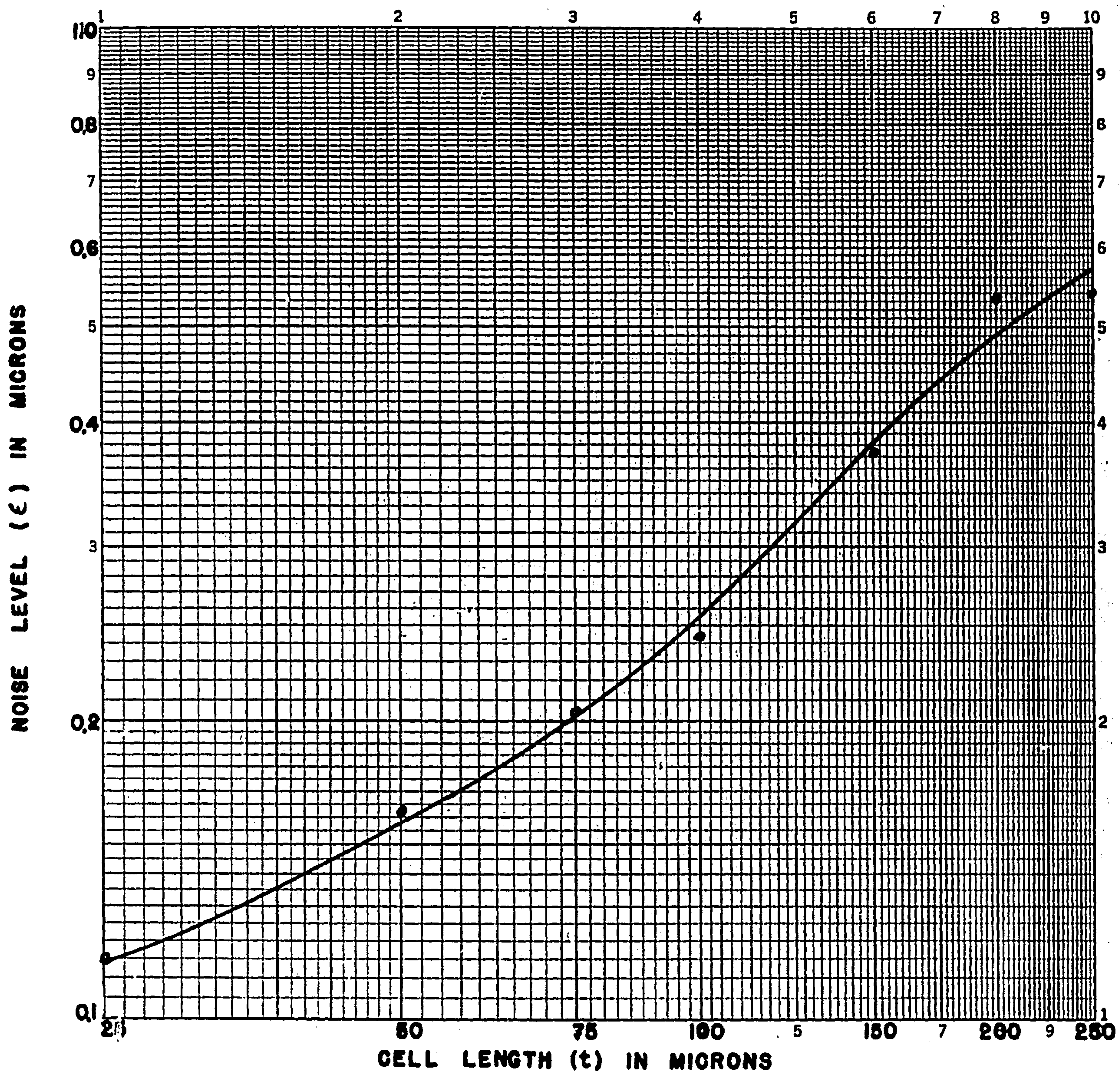

Figure 8 - The noise level, ϵ, shown as a function of the cell length, t, for the Leitz Ortholux microscope.

CHAPTER III

IONIZATION DENSITY AND RESIDUAL RANGE

Introduction

A highly energetic, charged particle passing through a nuclear emulsion, leaves behind it silver grains which have been rendered developable. When the emulsion has been developed, the resulting silver grains constitute a track. In favorable cases the mass of the particle can be evaluated from observations made upon the track.

A fundamental assumption, widely accepted, is that for a charged particle passing through matter, the space-rate of energy loss is directly proportional to the square of the charge and to some function of the velocity. That is

$$\frac{dE}{dR} = z^2 \; f_1(v) = z^2 \, f_2\left(\frac{E}{M}\right) , \qquad (3\text{-}1)$$

where M is the mass of the particle whose charge is z, and for which the residual range is R, at the point where the kinetic energy is E.

Then

$$z^2 \, dR = F'\left(\frac{E}{M}\right) dE \; ,$$

and non-relativistically,

$$z^2 R = M \int_0^{E/M} F'\left(\frac{E}{M}\right) d\left(\frac{E}{M}\right) \; .$$

So

$$\frac{z^2R}{M} = F\left(\frac{E}{M}\right) \quad . \qquad (3\text{-}2)$$

The function, F, is believed to follow a power law, since a graphical representation of the logarithm of E as a function of R, is a straight line. This has been established with machine-made particles for which the energy is known.[1]

Equation (3-2) may then be ~~written~~

$$\frac{z^2R}{M} = h\left(\frac{E}{M}\right)^{\nu} \quad , \qquad (3\text{-}3)$$

where h and ν are constants.

To evaluate these constants, Fitzpatrick[2] has used a curve due to Brown, and others,[3] which shows the logarithm of $\frac{dE}{dR}$ as a function of the logarithm of $\frac{R}{M}$, for Ilford, G5 emulsions.

If Equation (3-3) is differentiated with respect to R, the result is, for the special case of singly charged particles

$$\frac{dE}{dR} = \frac{1}{h\nu} \left(\frac{E}{M}\right)^{1-\nu} \quad , \qquad (3\text{-}4)$$

or

$$\log\left(\frac{dE}{dR}\right) = (1-\nu)\ \log\left(\frac{E}{M}\right) - \log\,(h\nu)\ .$$

The curve of Brown, and others, closely approximates a straight line in the interval in which $\frac{E}{M}$ varies from 0.01 to 0.10. Fitzpatrick

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[1] Arthur Beiser, Revs. Modern Phys., 24, 283, (1953).

[2] Fitzpatrick, op. cit., 38-40.

[3] Brown, Camerini, Fowler, Muirhead and Powell, Nature, 163, 83, (1949).

has treated this portion of the curve as if it were straight, and measured the slope at the mid-point. Thus he has determined

$$\nu = 1.761$$

It is well known that the kinetic energy of a mu-meson, at the point where it originates from the decay of a pi-meson, is unique and equal to 4.085 mev. This value, together with the rest-mass of a mu-meson, 206.6 m_e, and the measured range of such a decay meson, 590 microns, has been substituted into Equation (3-3). In this manner h is determined as equal to 2859.7, in units which are consistent with those of R, M, and E.

Equation (3-3) becomes, then, for a singly charged particle,

$$R = 2860 \ M^{-0.761} \ E^{1.761} \quad . \qquad (3\text{-}5)$$

Given a track made by a singly charged particle which came to rest, one may observe R directly. This leaves the two quantities, M and E, unknown. If E can be determined, indirectly, Equation (3-5) can be solved for M.

Equation (3-4) suggests that if $\frac{dE}{dR}$ can be determined, the ratio $\frac{E}{M}$ becomes known.

Ionization Density

To evaluate $\frac{dE}{dR}$, a widely accepted postulate is employed, namely that the space-rate of energy loss, $\frac{dE}{dR}$, is some function of the number of grains rendered developable per unit track length. That is

$$\frac{dN}{dR} = \psi\left(\frac{dE}{dR}\right) , \qquad (3\text{-}6)$$

where N is the total number of silver grains between the point where the residual range is R and the point where the particle came to rest.

Since the explicit form of ψ is neither known, nor required, Equation (3-6) may be generalized to

$$\frac{dI}{dR} = \varphi\left(\frac{dE}{dR}\right) , \qquad (3\text{-}7)$$

where I is any observable which is a function of N. The observable, I, may take one of the following forms:

1) the total projected track area,

2) the total number of blobs,

3) the total gap length,[1]

4) the total number of gaps, greater than some arbitrarily chosen minimum length.

The ionization density corresponding to each of the above forms of I, is the slope of the curve constructed by plotting I as a function of R.

The form of I which has been used in these researches, is third in the above list, the total gap length in range, R. This total gap length is determined by aligning the track, or a straight section of the track, with the microscope abscissa, as in measuring noise level, subdividing the segment into cells, and then determining the sum of the lengths of all the gaps in a cell. Then G, the total gap length is given by

$$G = \sum_{i=1}^{K} g_i \quad , \qquad (3\text{-}8)$$

where g_i is the sum of the individual gap-lengths in the ith cell, and K is the total number of cells in range, R.

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[1] By a gap is meant the vacant space between two adjacent blobs or clusters of silver grains. By a gap length is meant the distance between two adjacent blobs, measured parallel to the trajectory.

Equation (3-7) then becomes

$$\frac{dG}{dR} = \varphi\left(\frac{dE}{dR}\right) \quad ,$$

which together with (3-1) and (3-2) yields

$$\frac{dG}{dR} = G'\left(\frac{R}{M}\right) \quad , \qquad (3\text{-}9)$$

and

$$G = M \int_0^{R/M} G' \, d\left(\frac{R}{M}\right) \quad . \qquad (3\text{-}10)$$

Consider the two expressions for G, the operational definition, Equation (3-8), and Equation (3-10). As the number of cells in the sum becomes large, the value of the right hand side of Equation (3-8) approaches the value of the right hand side of Equation (3-10).

Thus it is concluded that, where a large number of observations is made, the total gap length in range, R, is, for a given mass, an implicit function of the range, R.

Calibration Curves

Suppose one has available a number of tracks made by "known" particles.[1] For each such track, a curve which shows G as a function of R can be constructed. Then an unknown particle can be tentatively identified by visual comparison of its "G-R" curve with the calibration curves of the known particles.

A more satisfactory calibration curve may be made by utilizing Equation (3-9). At several predetermined values of $\frac{R}{M}$, evaluations of G' are made by measurement of the slope of the G-R curve of a known

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[1] By a "known" particle is meant one for which an independent mass estimate is available.

particle. If several such curves are available, such measurements of the slopes are made upon each, and the mean of all such measurements is taken as the value of G' for the appropriate $\frac{R}{M}$. Then G' is shown graphically as a function of $\frac{R}{M}$. The resulting curve is a universal one since G' is a function only of the ratio $\frac{R}{M}$, and is not an explicit function of the mass.

Such a calibration curve is shown as Figure 9. Curves showing G as a function of R were constructed for each of six calibration tracks. Four of these calibration tracks had been established as having been made by protons, and two as having been made by deuterons.[1]

Gap Density Applied to Ending Tracks

For a track made by a particle which has come to rest, then, two observables, residual range and gap-density may be used to determine the mass. For the unknown particle, a G-R curve is constructed. At some point, or at several points, on this curve, the gap-density is measured. Reference to Figure 9 yields $\frac{R}{M}$, for this point. Since R is known, for this point, M may be computed.

As an example, the numerical values for the two deuterons which were eventually used as calibration particles, are quoted. The G-R curves for these particles were taken to be identical since the experimental points corresponded so closely to each other. From this curve it was found that at R = 1000 microns, G' = 0.0659.

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[1] The protons had been identified by constant sagitta scattering measurements. This is a technique which will be described at length, in Chapter IV. The deuterons were found by the method of the following paragraph to have masses approximately twice that of a proton.

From the mean G'-R curve for the four protons

$$\text{at } G' = 0.0659, \frac{R}{M} = 0.0247, \text{ so } M = \frac{1000}{0.247} = 4048 \ m_e.$$

A similar determination of M at R = 2000 microns gave M = 4016 m_e. So these two particles were accepted as deuterons and their values used together with those of the four protons in plotting Figure 9.

Gap-Density Applied to Non-Ending Tracks

As will be discussed at length in Chapter VI, a method for determining the mass of a particle, which has left the emulsion, has been originated as part of this research. A track which has been identified as "Track 5", has been used to test this method. At this time, Track 5 will be discussed and a mass estimate, obtained by gap-density observations, will be quoted.

There are about 14,000 microns of track available, from the three-prong star where the particle originated to the point where it left the top of the emulsion. Gap-densities were determined at two points on the track, 8000 microns apart. These gap-densities were respectively

$$G_1' = 0.133, \text{ and } G_2' = 0.299 \ .$$

Figure 9 yields

$$\frac{R_1}{M} = 1.42 \ \frac{\text{microns}}{m_e}, \quad \frac{R_2}{M} = 6.20 \ \frac{\text{microns}}{m_e} \ .$$

So

$$\frac{R_2 - R_1}{M} = 6.20 - 1.42 = 4.78 \ \frac{\text{microns}}{m_e} \ .$$

But

$$R_2 - R_1 = 8000 \text{ microns.}$$

Hence

$$M = \frac{8000}{4.78} = (1670 \pm 490)\ m_e \ .$$

A modified form of a method devised by Fitzpatrick[1] has been used to estimate the standard deviation in M. This method is explained in the Appendices.

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[1] Fitzpatrick, op. cit., 137.

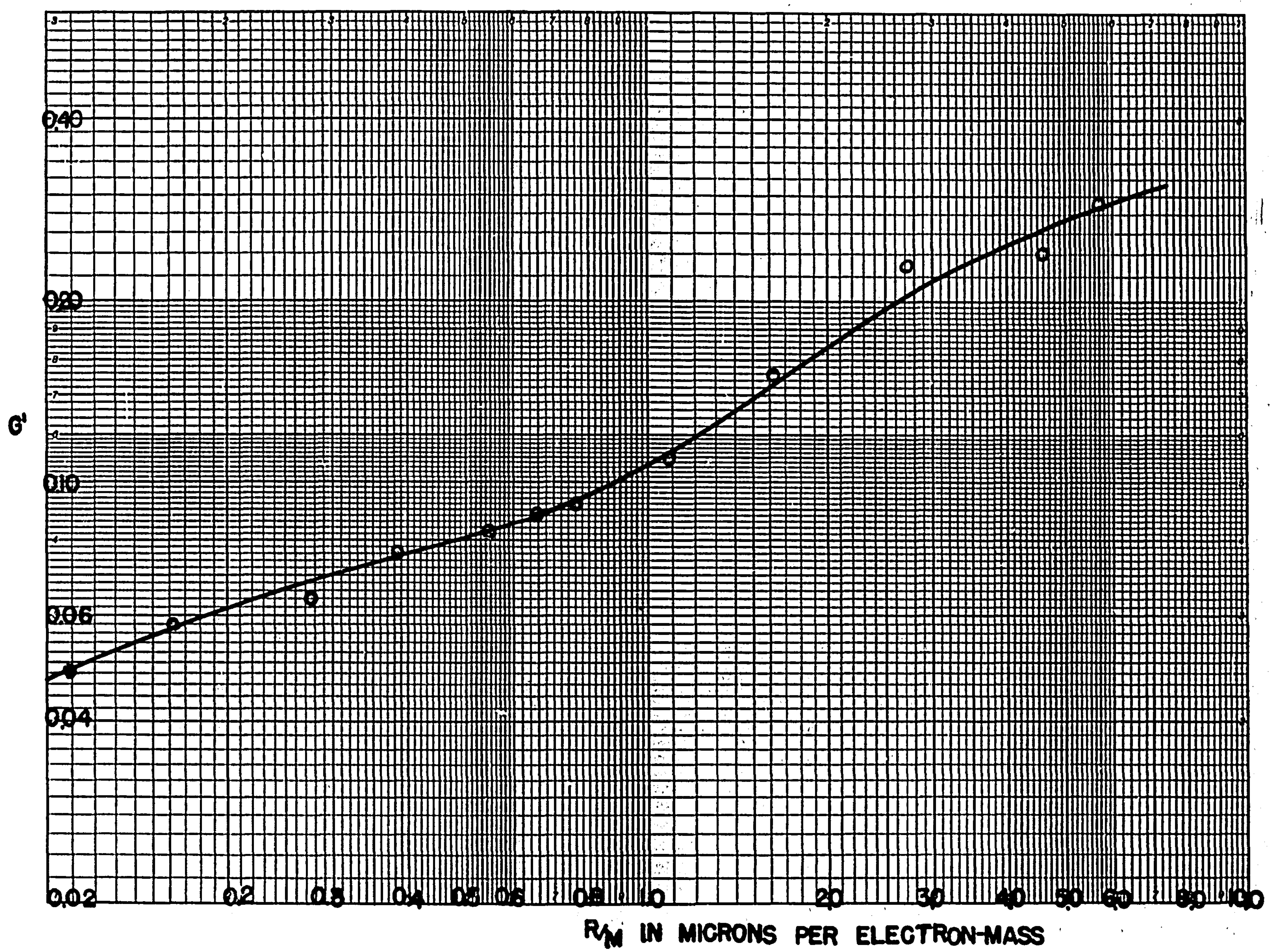

Figure 9 – A Universal Curve, for singly charged particles, showing gap density as a function of the ratio between the residual range and the mass.

CHAPTER IV

CONSTANT SAGITTA SCATTERING

Introduction

In addition to the observables already considered, namely, residual range and ionization density, the effects of multiple scattering may be used. These effects are caused by Coulomb type interactions between charged particles passing through an emulsion and the nuclei of the atoms of the emulsion itself. This chapter will deal with mass estimates from observations made upon ending tracks, by use of a convenient technique known as constant sagitta scattering.

Fitzpatrick[1] has thoroughly reviewed the literature pertaining to multiple scattering. He states that the theory of Williams[2], or that of Moliere[3], or a combination of the two, is most frequently used to study scattering of charged particles in photographic emulsions.

His discussion includes a description of Fowler's[4] Coordinate Method of measurement. Following Williams and Möliere, Fitzpatrick arrives

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[1] Fitzpatrick, op. cit., pp. 76-122.

[2] E. J. Williams, Proc. Roy. Soc. (London) A169, 531, (1939).

[3] G. Moliere, Z. Naturforsch, 3a, 78, (1945).

[4] P. H. Fowler, Phil. Mag. 41, 169, (1950).

at the scattering equation for singly charged particles. This equation is

$$\bar{\alpha} = \frac{K\, t^{\frac{1}{2}}}{10\, p\beta c} \quad , \qquad (4\text{-}1)$$

where $\bar{\alpha}$ is the mean of the absolute values of the angles between successive chords drawn between points along the track, p is the momentum and βc is the velocity of the primary particle. K is called the "scattering constant", but is really a slowly varying function of t and β .

Slow Particles

The authorities do not agree on a simple, explicit form for K, which is valid over the entire range of velocities. Rather, the application of Equation (4-1) is specialized to: (1) extremely relativistic particles; (2) fast particles, for which the energy is substantially constant over the segment of track used; and (3) slow, or subrelativistic particles.

Biswas, George, and Peters[1] quote Voyvodic and Pickup[2], as having calculated

$$\frac{K}{10} = (0.096)\left(\frac{2}{3}\right)^{\frac{1}{2}} (1.006) \left\{ 0.80 \left[\ln \frac{0.94t}{0.3+\beta^2} \right]^{\frac{1}{2}} + 1.45 \right\} ,$$

for Ilford G5 emulsions.

It is not clear that Biswas and others have intended their equation to be used only for non-relativistic particles. They have used the relativistic form for $p\beta c$, which may imply that they believe

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[1] S. Biswas, E. C. George, and B. Peters, Proc. Indian Acad. Sci. 38A, 423 (1953).

[2] L. Voyvodic and E. Pickup, Phys. Rev. 85, 91 (1952).

their expression to be valid into the relativistic region.

The Coordinate Method of Making Observations

Since $\bar{\alpha}$ does not lend itself readily to direct measurement, the "Coordinate Method", which has been anticipated[1], is utilized.

It will be recalled that

$$D_i = Y_i - 2Y_{i+1} + Y_{i+2} \quad ,$$

and that

$$\Delta = (D^2 - \epsilon^2)^{\frac{1}{2}} \quad , \qquad (4\text{-}2)$$

where

$$D = \frac{\sum_{i=1}^{N} |D_i|}{N} \quad ,$$

ϵ is the mean of the absolute values of the second differences due to noise, and Δ is the mean of the absolute values of the second differences due to true scattering.

It may be shown that

$$\bar{\alpha} = \frac{\Delta}{t} \frac{180}{\pi} \text{ degrees.}$$

When this value for $\bar{\alpha}$ is substituted into (4-1), the result is

$$\Delta = \frac{0.0137\ (t)^{3/2}}{p\beta c} \left\{ 1.45 + 0.8 \left[\ln \frac{0.94t}{\beta^2 + 0.3} \right]^{\frac{1}{2}} \right\} \qquad (4\text{-}3)$$

Range-Energy and Scattering

In principle, Equation (4-3) can be used to obtain an estimate for the mass of any non-relativistic particle which produces an ending track.

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[1] Above, pp. 17-19.

The Range Energy equation[1]

$$R = h\ M^{1-\nu} E^{\nu} ,$$

may be written in the form

$$E = j\ M^{1-\eta} R^{\eta} ,$$

where $j = h^{-\frac{1}{\nu}}$ and $\eta = \frac{1}{\nu}$. Since h and ν have been determined, j and η are known.

Then,

$$p\beta c = 2E = 2jM^{1-\eta} R^{\eta} ,$$

and Equation (4-3) becomes

$$\Delta = A\ t^{3/2}\ M^{\eta-1}\ R^{-\eta}\ F(\beta,t) , \qquad (4\text{-}4)$$

where A is a known constant, and $F(\beta,t)$ is given explicitly by the quantity in the brackets in (4-3).

One might conclude that here is an expression for the "scattering"[2], Δ, at a point on the track where the residual range is R. This implies that one can measure Δ and R, and then everything in Equation (4-4) is known except M.

Operationally, Δ cannot be observed at a single point, but must be computed by means of

$$\Delta = (D^2 - \epsilon^2)^{\frac{1}{2}} ,$$

which is derived from statistical considerations. If a segment of track is available over which Δ may be considered to be a linear function of R, then the above equation can be used to compute Δ for the mid-

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[1] Above, p. 30.

[2] To avoid writing and re-writing the rather cumbersome expression "mean of the absolute values of the second differences due to true scattering", the simplet term "scattering" will hereafter be used for Δ.

point of this segment. This idea will be considered further in a later chapter. A modification of this method has been used by Menon and Rochat[1].

Constant Sagitta Methods

In Chapter III, it was stated that the particles which made the tracks used in constructing the ionization-density calibration curves had been identified as protons. This identification was made on the basis of mass estimates made by constant sagitta scattering. This technique was originated independently by Biswas, George and Peters,[2] at Bombay, and by Dilworth, Goldsack and Hirschberg,[3] at Brussels. It is a method in which allowance is made for the change of the particle's energy with range; hence, for the change of β with R.

In Equation (4-4), the value to be used for the cell length, t, is presumably chosen with reference to the noise level, ϵ . It will be shown that statistically, Δ should be 2.4 times ϵ. The technique used for fast particles, where t is kept constant over the track segment, is to choose t, by a cut-and-try process. The observer records values of Y at short intervals, say every ten microns along the track. Second differences are computed for t = 10, 20, 30,.. etc. microns. The value of t which yields a Δ, that most nearly satisfies the criterion

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[1] M. G. K. Menon and O. Rochat, Phil. Mag., 42, 1232, (1951).

[2] Biswas, George, and Peters, loc. cit., p. 418.

[3] C. C. Dilworth, S. J. Goldsack and L. Hirschberg, Nuovo Cimento, ser. 9, 11, 113, (1954).

for $\frac{\Delta}{\epsilon}$, is the one finally used.

With slow particles, Δ changes rapidly with R.[1] The optimum cell length, then, decreases as R decreases. This presents a dilemma. When the cell length is kept constant, over a long segment of track, either t is too long, so that statistics are poor near one end of the segment, or t is too short so that the scattering-to-noise ratio is small near the other end.

The originators of the constant sagitta method proposed to use cell lengths which vary continuously along the track, so that the scattering remains constant. This means that before a set of measurements can be made, the observer must have available what is called a "scattering scheme".

Scattering Schemes

A scattering scheme may be defined as a set of R_i's (R_1, R_2, ... R_i, ... R_N) so determined that $R_{i+1} = R_i + t_i$, the t_i being so determined that when substituted into Equation (4-4), together with the corresponding R_i, the Δ_i's which result have a constant, predetermined value. This is possible only if the mass, M, is known or assumed. This seems, at first glance, to be a paradox, for it implies

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[1] More precisely, the scattering equation, (4-3) predicts the scattering which would be observed if a large number of observations were made, at the same range, on different tracks made by singly charged particles of the same mass. This means, then, that if one takes as the value for Δ, a mean of several values, obtained from the track of a single particle, the observations having been made at various ranges, the result is an approximation. The error introduced is greater at low velocities than at high velocities, if track segments of equal length are used.

that the mass must be known before the mass can be determined. But if a mass is assumed, which is of the correct order of magnitude,

$$M' = M\left(\frac{\Delta'}{\Delta}\right)^{\frac{1}{n-1}} ,$$

where M is the assumed mass, Δ is the desired scattering, M' the observed mass, and Δ' the observed scattering.

The details of calculating the R_i's which constitute a scattering scheme, are slightly different as reported by the two separate groups. The Dilworth group writes down the scattering equation, in the form of Equation (4-4), and proceeds to calculate scattering schemes with K treated as a constant. Three objections to the results are listed:

1) at low energies, the range energy equation is not a simple power law;

2) at high velocities, $p\beta c \neq 2E$, but is given by

$$p\beta c = E\left[1 + \sum_{n=0}^{\infty} \left(\frac{E}{M_6^{2}}\right)^{n}\right] ;$$

3) the "scattering constant" is in reality a slowly varying function of R and t.

Each of these may be shown to contribute a systematic error which in itself is small. The Dilworth group applies an over-all correction factor which is a function of R, M, and t.

The Biswas group has apparently not worried about either of the first two objections. The third objection is not applicable, since they actually calculate values of K as a function of β and t.

In this laboratory, Equation (4-4) has been used as is, to calculate scattering schemes. To minimize the imperfections in the range energy equation at low energy, a portion of the track near the end is discarded.

Although it is true that the range energy equation is a non-relativistic expression, it is believed that it can be used in the low part of the relativistic region with small error. Barkas[1] shows evidence that momentum as a function of range follows a power law well into the region of high velocities. For a proton whose residual range is 10,000 microns, the relative error introduced by taking $p\beta c = 2E$ may be shown to be less than three percent.

Choice of an Optimum Value of

As stated in Chapter II, and shown in the Appendix,

$$\sigma(D) = 0.75\,(N)^{-\frac{1}{2}}D$$

and

$$\sigma(\epsilon) = 0.75\,(N)^{-\frac{1}{2}} \quad ,$$

where N is the number of independent second differences, and $\sigma(D)$ and $\sigma(\epsilon)$ are the respective standard deviations in D and in ϵ.

The standard deviation in Δ will then be given by

$$\sigma(\Delta) = \left\{ \left[\frac{\partial\Delta}{\partial D}\right]^2 \left[\sigma(D)\right]^2 + \left[\frac{\partial\Delta}{\partial\epsilon}\right]^2 \left[\sigma(\epsilon)\right]^2 \right\}^{\frac{1}{2}} . \quad (4\text{-}5)$$

The necessary partial derivatives are calculated from

$$\Delta = (D^2 - \epsilon^2)^{\frac{1}{2}} .$$

Equation (4-5) then becomes

$$\sigma(\Delta) = 0.75(N)^{-\frac{1}{2}} \frac{(D^4 + \epsilon^4)^{\frac{1}{2}}}{\Delta} \quad ,$$

or

$$\frac{\sigma(\Delta)}{\Delta} = 0.75N^{-\frac{1}{2}}\Delta^{-2}(\Delta^4 + 2\Delta^2\epsilon^2 + 2\epsilon^4)^{\frac{1}{2}} . \quad (4\text{-}6)$$

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[1] Walter W. Barkas, Am. J. Phys. 20, 7, (1952).

For a given length of track, N is approximately proportional to $\Delta^{-2/3}$.[1] So

$$0.75\ N^{-\frac{1}{2}} = k\ \Delta^{1/3}$$

and

$$\frac{\sigma(\Delta)}{\Delta} = k\Delta^{-5/3}\left(\Delta^4 + 2\Delta^2\epsilon^2 + 2\epsilon^4\right)^{\frac{1}{2}} . \qquad (4\text{-}7)$$

For this relative deviation in Δ to be a minimum, the partial derivative with respect to Δ, of the right hand side of Equation (4-7) must vanish. Then

$$\Delta^{-8/3}\left(\Delta^4 + 2\Delta^2\epsilon^2 + 2\epsilon^4\right)^{-\frac{1}{2}}\left(\Delta^4 - 4\Delta^2\epsilon^2 - 10\epsilon^4\right) = 0.$$

For this to be true with both Δ and ϵ finite, the second quantity in parenthesis must vanish. This requires that

$$\Delta = \left[\frac{4\epsilon^2 \pm \sqrt{16\epsilon^4 + 40\epsilon^4}}{2}\right]^{\frac{1}{2}} .$$

Since Δ must be real,

$$\Delta = \epsilon\left(2 + \sqrt{14}\right)^{\frac{1}{2}} = 2.40\ \epsilon .$$

The optimum scattering scheme, then, is one with which the corrected second difference is 2.40 times the noise level.

The minimum relative deviation is, by (4-6)

$$\left.\frac{\sigma(\Delta)}{\Delta}\right]_{Min} = 0.75(N)^{-\frac{1}{2}}(2.40\,\epsilon)^{-2}\left[(2.40)^4 + 2(2.40)^2 + 2\right]^{\frac{1}{2}}\epsilon^2$$
$$= 0.89(N)^{-\frac{1}{2}}.$$

Since $\qquad M' = M\left(\frac{\Delta}{\Delta'}\right)^{\frac{1}{n-1}}$.

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[1] Actually, $\Delta^{-2/3}$ is nearly proportional to an average cell length, $\bar{t}$, defined by

$$\bar{t} = \frac{\sum_{i=1}^{N} t_i}{N} .$$

Hence, N is approximately proportional to $\Delta^{-2/3}$.

and

$$\eta = \frac{1}{1.761} = 0.568$$

$$M' = M\Delta^{2.31}\Delta'^{-2.31}$$

$$\sigma(M') = 2.31 \quad M\Delta^{2.31}\Delta'^{-3.31}\sigma(\Delta')$$

$$\frac{\sigma(M')}{M'} = \frac{2.31\sigma(\Delta')}{\Delta'} = (2.31)(0.89)N^{-\frac{1}{2}}$$

$$\frac{\sigma(M')}{M'} = 2.06N^{-\frac{1}{2}}$$

This is the relative standard deviation in the measured mass, when the optimum scattering scheme is used. A computation for $\frac{\Delta}{\epsilon} = 4$, shows the relative standard deviation to be about eleven percent greater, and for $\frac{\Delta}{\epsilon} = 2$, about three percent greater.

Since the noise level, for the microscope used in these experiments, varies with cell length (see Figure 8), a desired Δ of 0.6 microns has been adopted for purposes of making up scattering schemes. This is not entirely consistent with the statistical argument given above, but is consistent with the commonly accepted procedure of attempting to keep the scattering between 4 and 6 times as great as the noise level.

Determination of the numerical values of the R_i's which make up a scattering scheme, is quite a formidable task.

As has been explained, a tentative value of M must be assumed. This may well be done on the basis of ionization density observations. Equation (4-3) may be written

$$M = \frac{0.0137\ t^{3/2}}{\frac{p\beta c}{M}} \quad F(\beta,t), \qquad (4\text{-}8)$$

where

$$F(\beta,t) = 1.45 + 0.8\left[\ln \frac{0.94\ t}{\beta^2} + 0.3\right]^{\frac{1}{2}}.$$

The entire right hand side of Equation (4-8) is now a function of $\frac{R}{M}$ and t. For the assumed mass, M, the variables are R and t. The problem, then, is to allow R and t to vary in such a manner that the right hand side of (4-8) remains constant and equal to $M\Delta$. It has been shown empirically that if this is done, a curve which exhibits the logarithm of t as a function of the logarithm of R will very closely approximate a straight line.

Let it be required to set up a "P-0.6" scattering scheme; that is, the assumed mass is 1837 m_e and the desired scattering is 0.6 microns. The product 1837 times 0.6 is 1102.2. It is necessary to determine several values of R and t, so that each R, together with the corresponding t, will, when substituted into (4-8), make the right hand side of the equation numerically equal to 1102.2. Then a plot of the logarithm of t as a function of the logarithm of R can be constructed and values of the R_is and the t_is taken from the graph.

To aid in the construction of scattering schemes, Tables I, II, and III have been compiled. Table I enables one to find the value of $\frac{p\beta c}{M}$, and that of $\frac{R}{M}$, for preassigned values of β. The relativistic expression for the momentum has been used, as suggested by the Biswas group. Table II gives $F(\beta, t)$ and Table III gives $M\Delta$, for preassigned values of β and t.

To locate the first point on the graph from which the P-0.6 scattering scheme is to be taken, refer to Table I. For $\beta = 0.1$, the range of a proton is given as 144.7 microns. Table III then shows that for $\beta = 0.1$, and $M\Delta = 1102.2$, t in microns is between 20 and 30.

Interpolation, by proportional parts, yields t = 26 microns. Table II shows that for a given β, F_2 changes almost linearly with t. By interpolation then, $F_2(0.10, 26) = 3.123$, while Table I yields $\frac{p\beta c}{M}$, for $\beta = 0.10$, so that

$$\frac{0.0137\ (26)^{3/2}}{0.05136} = 353.6$$

$$M\Delta(0.10,26) = 353.6 \times 3.123 = 1104.$$

More precise calculations show that the exact value of t is greater than 25.9 microns.

So the value, t = 26.0 microns is chosen, for the point where R = 144.7 microns. This locates one point on the line labeled "P-0.6", Figure 10. Five other points have been similarly located. These five points include values of R, almost equally spaced along the abscissa, up to R = 13,983 microns. No departure from linearity can be detected.

A line for a "T-0.6" scheme is also shown.

From these graphs, the scattering schemes can be written down.

Experimental Results

As was stated in Chapter III, four tracks, used as calibration tracks for the gap-density curve, were identified by constant sagitta scattering as having been made by protons. Since the noise level, for the microscope which was used, did not prove to be independent of cell length, see Figure 8, it was thought advisable to divide the observed D_i's into groups, according to cell length. Thus for cell lengths up to 37.5 microns, the value of noise for 25-micron cell length was used; for t between 37.5 and 62.5 microns, that for 50-micron cell lengths, and so on. For each group of D_i's there results a Δ_j, obtained by applying Equation (4-2). Then an over-all, mean value of Δ is found by evaluating

$$\Delta = \frac{\sum_{j=1}^{J} w_j \, \Delta_j}{\sum_{j=1}^{J} w_j} ,$$

where w_j is the number of D_i's in the j th group, and J is the number of such groups.

The results for the four particles used as protons are shown in Table IV. The first three were identified as part of these researches, the fourth had already been identified as a proton, by Fitzpatrick.

Several other tracks have also been analyzed by this technique. These will be discussed in Chapter V. The analysis of "Track 12" was carried out as a matter of routine.

Summary

Although it is a laborious task to set up a scattering scheme, once such a scheme is available, it is a simple matter to obtain a mass estimate for a particle which has come to rest in an emulsion. Both the observations and the computations can be made conveniently and quickly.

With the micrometer stage drive on the Leitz Ortholux microscope, range settings can be made to better than the nearest micron. So the cell lengths have been written to the nearest tenth of a micron when possible. This precision is limited by one's ability to read the values off the curve, Figure 10.

It is believed to be well worth while to rotate the track each time a large-angle bend is encountered. This writer has established the following rule of thumb. Whenever the track changes direction so that several consecutive, observed first differences become as much as one scale division - approximately one micron - per cell length, re-orient the track. For cell lengths of twenty microns this is about the same criterion as was established in Chapter II, that is, to keep the angle between the axis of the track and the abscissa of the microscope less than three degrees. For longer cell lengths this is a more strict criterion. Empirically, it seems that the statistical fluctuations in the observed second differences are smaller when the track is kept more nearly parallel to the direction of drive.

Constant sagitta scattering, as a method for determination of the mass of a particle which has come to rest, is believed to have been firmly established. The accuracy is believed to rival that of ionization density methods for the heavy particles.

The labor involved in setting up scattering schemes is no more than that involved in the construction of calibration curves.

Since no such calibration curves, as must be available when ionization density methods are used, are necessary, scattering provides a relatively absolute method. This in itself is an argument in favor of the extended use of such a method.

TABLE I

QUANTITIES WHICH ARE USEFUL IN THE CALCULATION OF SCATTERING SCHEMES

β	$\frac{p\beta c}{M}$ $\frac{(\text{mev.})}{m_e}$	$\frac{E}{M}$ $\frac{(\text{mev.})}{m_e}$	$\frac{R}{M}$ $\frac{(\text{microns})}{m_e}$	R_P (microns)	R (microns)	R_π (microns)
0.05	0.001279	0.000639	0.0068	12.44	6.6	1.8
0.06	0.001843	0.000920	0.0134	24.65	13.0	3.7
0.07	0.002510	0.001257	0.0223	41.0	21.6	6.1
0.08	0.003377	0.001643	0.0357	65.7	34.6	9.8
0.09	0.04156	0.002070	0.0536	98.6	52.0	14.7
0.10	0.005136	0.002555	0.0787	144.7	76.2	21.5
0.11	0.006219	0.003120	0.1105	203.1	107.0	30.2
0.12	0.007410	0.003719	0.1507	276.7	145.8	41.1
0.13	0.009707	0.004374	0.2004	368.2	194.0	54.7
0.14	0.01011	0.000583	0.2611	479.6	252.8	71.3
0.15	0.01163	0.005848	0.3342	613.9	323.5	91.2
0.16	0.01326	0.006694	0.4240	778.9	410.4	115.8
0.17	0.01499	0.007548	0.5237	962.0	506.9	143.0
0.18	0.01683	0.008485	0.6437	1182.5	623.1	176.7
0.19	0.01879	0.009481	0.7826	1437.6	757.6	213.6
0.20	0.02087	0.010540	0.9426	1731.6	912.4	257.3
0.21	0.03205	0.011654	1.1256	2067.7	1089.6	307.3
0.22	0.02535	0.012834	1.3341	2450.7	1291.4	364.2
0.23	0.02777	0.014077	1.5698	2883.7	1519.6	428.5
0.24	0.03032	0.015385	1.8360	3372.7	1777.2	501.2
0.25	0.03299	0.016759	2.1343	3920.7	2066.0	582.7
0.26	0.03577	0.018200	2.4677	4533.2	2388.7	673.7
0.27	0.03869	0.019710	2.8396	5216.4	2748.7	775.2
0.28	0.04173	0.021292	3.2532	5976.1	3149.1	888.1
0.29	0.04491	0.022945	3.7110	6817.1	3592.2	1013.1

TABLE I
(Continued)

QUANTITIES WHICH ARE USEFUL IN THE CALCULATION OF SCATTERING SCHEMES

β	$\frac{p\beta c}{M}$ $\frac{(mev.)}{m_e}$	$\frac{E}{M}$ $\frac{(mev.)}{m_e}$	$\frac{R}{M}$ $\frac{(microns)}{m_e}$	R_P (microns)	R (microns)	R_π (microns)
0.30	0.04821	0.024674	4.2174	7747.4	4082.4	1151.4
0.31	0.05165	0.026478	4.7756	8772.8	4622.8	1303.7
0.32	0.05523	0.028361	5.3895	9900.5	5217.0	1471.3
0.33	0.05895	0.030324	6.0637	11139	5869.1	1655.4
0.34	0.06281	0.032371	6.8030	12497	6585.3	1857.2
0.35	0.06682	0.034503	7.6117	13983	7368.1	2078.0
0.36	0.07099	0.036723	8.4961	15608	8224.2	2319.4
0.37	0.07527	0.039030	9.4574	17373	9154.8	2581.9
0.38	0.07563	0.041441	10.510	19307	9147.4	2869.2
0.39	0.08440	0.043943	11.653	21407	1128.0	3181.3
0.40	0.08920	0.046547	12.896	23690	1248.3	3520.6
0.41	0.09418	0.049250	14.244	26166	1378.8	3888.6
0.42	0.09933	0.052071	15.712	28863	1520.9	4289.4
0.43	0.10465	0.055030	17.318	31813	1676.4	4727.8
0.44	0.11017	0.058177	19.093	35074	1848.2	5212.4
0.45	0.11587	0.061210	20.891	38377	20222	5703.2
0.46	0.12178	0.064503	22.908	42082	22175	6253.9
0.47	0.12789	0.067928	25.093	46096	24290	6850.4
0.48	0.13986	0.071490	27.456	50437	26577	7495.5
0.49	0.14075	0.075196	30.012	55132	29052	8193.3
0.50	0.14751	0.079052	32.774	60206	31725	8947.3

TABLE II

VALUES OF $F(\beta,t)$ FOR VARIOUS VALUES OF β AND t

t (microns)	β									
	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
20	3.076	3.071	3.063	3.052	3.039	3.024	3.010	2.991	2.972	2.953
30	3.154	3.149	3.142	3.131	3.119	3.106	3.093	3.073	3.055	3.037
40	3.207	3.186	3.195	3.185	3.173	3.160	3.147	3.129	3.112	3.094
50	3.247	3.242	3.236	3.225	3.215	3.201	3.186	3.171	3.154	3.137
60	3.279	3.275	3.268	3.258	3.247	3.234	3.220	3.204	3.188	3.171
70	3.306	3.302	3.295	3.285	3.274	3.262	3.247	3.232	3.216	3.200
80	3.329	3.325	3.318	3.308	3.298	3.285	3.271	3.256	3.230	3.224
90	3.349	3.345	3.338	3.329	3.318	3.305	3.292	3.277	3.261	3.245
100	3.366	3.362	3.356	3.347	3.336	3.323	3.309	3.295	3.280	3.264
110	3.382	3.378	3.372	3.363	3.352	3.340	3.326	3.312	3.296	3.380
120	3.397	3.393	3.386	3.377	3.366	3.354	3.340	3.326	3.311	3.296
130	3.410	3.405	3.399	3.391	3.380	3.368	3.354	3.340	3.325	3.310
140	3.422	3.418	3.411	3.403	3.392	3.380	3.367	3.353	3.338	3.322

TABLE III

VALUES OF MΔ FOR VARIOUS VALUES OF β AND t

t (microns)	β 0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
20	2946.2	732.4	322.5	179.2	115.3	76.8	55.2	41.1	31.4	24.5
30	5551.2	1608.3	607.3	337.8	218.4	145.0	104.1	77.5	59.3	46.3
40	8687.8	1380.3	935.7	528.8	340.5	227.1	163.1	121.5	93.1	72.7
50	12293.1	3056.0	1346.8	748.2	482.2	321.5	230.9	172.1	131.8	102.9
60	16319	4058.7	1788.2	993.7	640.0	427.0	306.7	228.6	175.1	136.8
70	20735	5156.7	2271.9	1263.1	813.3	542.8	389.7	290.6	222.6	173.9
80	25508	6344.1	2795.1	1553.4	1100.9	667.8	479.5	357.5	274.0	214.1
90	30620	7615.6	3355.4	1865.2	1206.8	801.8	576.1	429.6	329.0	257.1
100	36043	8964.8	3751.0	2196.6	1415.1	944.1	678.0	505.8	387.7	302.9
110	41781	10392	4580.0	2546.1	1640.6	1094.8	786.3	586.6	449.2	351.3
120	47816	11893	5240.2	2913.3	1876.9	1252.4	899.8	671.2	514.2	402.1
130	54124	13458	5931.2	3298.8	2125.0	1418.3	1018.9	759.8	582.5	455.1
140	60703	15098	6652.1	3699.4	2383.6	1590.6	1143.1	852.7	653.6	510.6

TABLE IV

MASS ESTIMATES BY CONSTANT SAGITTA SCATTERING

Track No.	Length (microns)	Mass Estimate and Standard Deviation (electron masses)	Scattering Scheme	Assumed Identification
(1)	4,200	1881 ± 250	P-0.6[c]	Proton[e]
(2)	2,800	2035 ± 480	P-0.5[a]	Proton[e]
(3)	3,500	2294 ± 410	P-1.0[b]	Hyperon
(3)	3,500	1910 ± 240	P-0.6[c]	Proton[e]
(4)	14,610	1656 ± 229	P-0.5[a]	Proton[e]
(12)	2,170	1717 ± 540	P-0.6[c]	Proton

[a] Scattering scheme calculated by Fitzpatrick.

[b] Scattering scheme calculated by Biswas and Others.

[c] Scattering scheme taken from Figure 10.

[d] Mass estimate determined by Fitzpatrick.

[e] Used to plot gap density calibration curve, Figure 9.

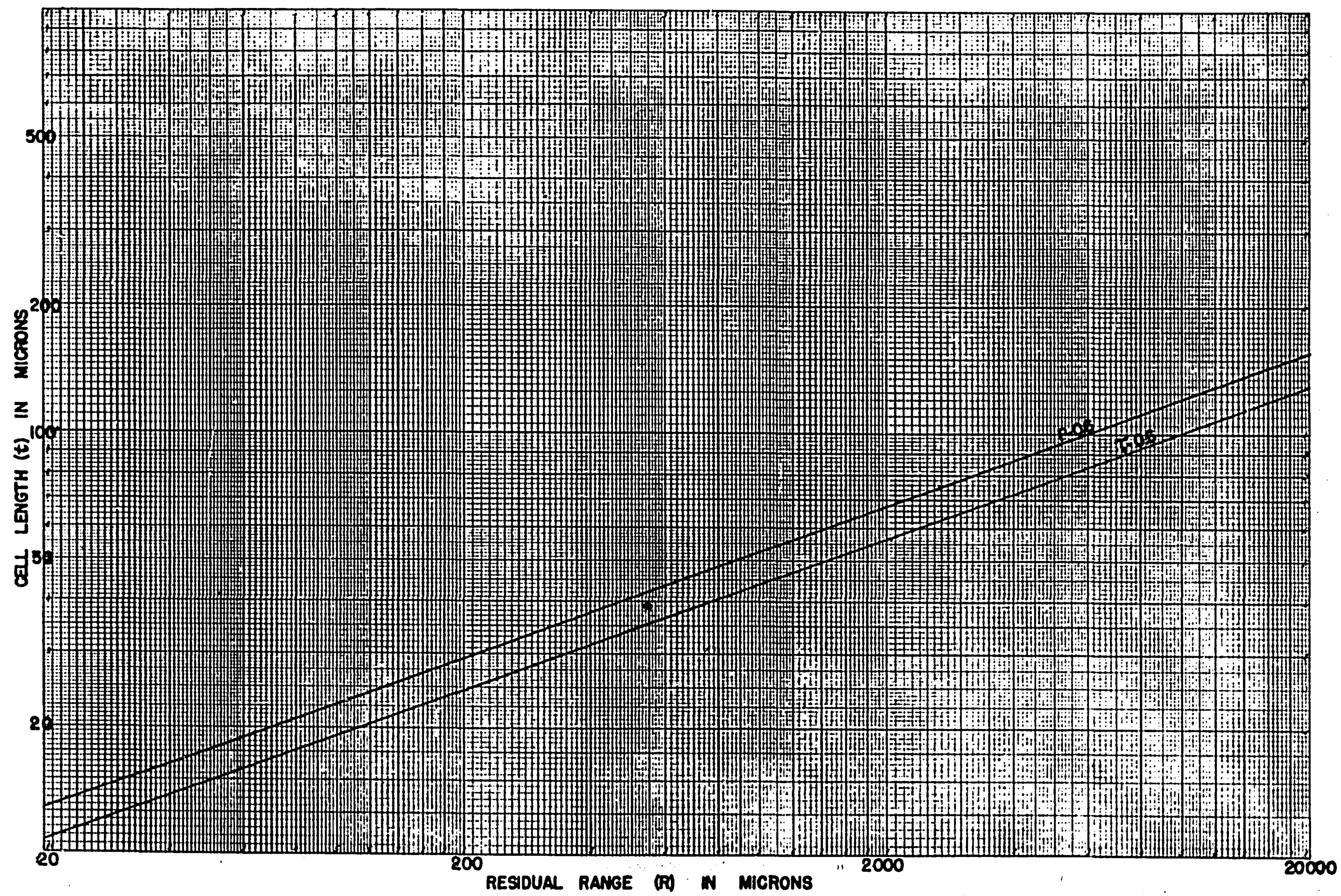

Figure 10 - Curves from which scattering schemes can be determined.

CHAPTER V

CHANGE OF SCATTERING WITH RANGE

Introduction

With a single exception, the methods of track analysis, which have been discussed in these writings, have been applied to ending tracks. This exception was a track described in Chapter III[1], for which a mass estimate was made by determining the difference in the gap density at two points. At first this mass estimate was considered to be a means to an end; namely, the analysis of the star from which the particle was emitted. When it became apparent that the star was uninteresting, the mass of the particle became an end in itself.

Another technique which is useful for estimating the mass of a particle, which has left the emulsion, is to combine the results of ionization density with scattering measurements. This method is particularly useful for fast particles.

It had occurred to this writer, that for the slower particles, for which the scattering changes appreciably with range, the space-rate of change of Δ as a function of R might be employed as a track observable. The idea was to plot Δ as a function of R, measure the slope of the

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[1] Above, p. 35. The mass estimate obtained was $(1670 \pm 490)\ m_e$.

curve, at several points along the curve, and thus obtain $\frac{d\Delta}{dR}$ as a function of R. For a non-ending track, then, the scattering equation

$$\Delta = a\, t^{3/2}\, M^{\eta -1}\, F(\beta,t)\, R^{-\eta} \tag{5-1}$$

and the equation obtained by differentiating with respect to R,

$$\frac{d\Delta}{dR} = A(-\eta)\, t^{3/2}\, M^{\eta -1}\, F(\beta,t)\, R^{-\eta -1} ,$$

provide two equations, which, in principle, can be solved simultaneously for M and for R.

The following assumptions are made:

(1) constant cell length scattering measurements are made;

(2) the particle has non-relativistic velocities;

(3) the dependence of F upon β can be ignored;

(4) the value of F can be determined.

While this idea may have merit, it has not been pursued. In the process of plotting Δ as a function of R another idea suggested itself; namely, that of determining the mass of the particle from a graphical representation of Equation (5-1).

Consider Equation (5-1), rewritten in the form

$$\log \quad = k - \eta \log R , \tag{5-2}$$

where

$$k = \log A + \frac{3}{2} \log t + (\eta - 1) \log M + \log F.$$

F changes slowly with R, as is clear from an examination of Table II, Chapter IV. A and η are known constants. For a non-relativistic particle, then, constant cell length scattering measurements should show the logarithm of Δ to approximate closely a linear function of the logarithm of R.

Suppose the track does not end in the emulsion. Then Equation (5-2) may be written

$$\log \Delta = k - 0.568 \quad \log (\mathcal{R} + R_o) \quad , \qquad (5\text{-}3)$$

where

$$R = R_o + \mathcal{R} \quad ,$$

and R_o is the residual range of the first point on the track at which scattering observations are made. The numerical value for n, +0.568, has been substituted into (5-3) in order to emphasize the fact that this value is known.

Equation (5-3) is not linear in $\log \mathcal{R}$ but approaches linearity as the ratio, $\frac{R_o}{\mathcal{R}}$, becomes small. This becomes clear when the equation is rewritten as

$$\log \Delta = k - 0.568 \quad \left[\log \mathcal{R} + \log \left(1 + \frac{R_o}{\mathcal{R}}\right)\right] \quad .$$

This is shown in Figure 11, for which values of Δ have been computed by means of Equation (5-3). The mass of the hypothetical particle has been taken as that of a proton, 1837 electron masses. R_o has been taken as 1000 microns and t as 100 microns. An average value for $F(\beta, 100)$ has been used.

The figure also shows the asymptote,

$$\log \Delta_o = k - 0.568 \log \mathcal{R} \; . \qquad (5\text{-}4)$$

Graphical Approximation of R_o and k

Figure 12 is the experimental counterpart of Figure 11. The observed values of Δ were obtained by the Coordinate Method, which has been described previously. These data were obtained by observations made on a track, to be discussed presently as "Track 9". The particle which made the track is assumed to have been a proton, because of the indicated mass estimate. This estimate was made by constant sagitta scattering, employing a P-0.6 scattering scheme.

The values of $\mathcal{R}$ were computed by subtracting 1000 microns from the residual range of each point at which a value of Δ was available. This amounts to ignoring that segment of the track for which the residual range was less than 1000 microns.

Assume, for purposes of the present discussion, that neither the mass of the particle, nor the residual range at the point where $\mathcal{R}$ is equal to zero, were known. It is possible to obtain approximations for these quantities in the following way. The experimental points are located on the graph. Then, the "best-fit" curve is constructed visually. The asymptote, whose slope is known, is also located visually. A value for Δ_o can be taken from the asymptote at some chosen value of $\mathcal{R}$. Then k can be calculated from Equation (5-4). At this same value of $\mathcal{R}$, a value for Δ can be taken from the curve. Then R_o can be calculated by means of Equation (5-3).

From the definition of k and the known values of A and η, it may be shown that

$$0.432 \log M = \frac{3}{2} \log t + \log F - k - 0.19955 \qquad (5\text{-}5)$$

The numerical values of t and F are known for any particular set of data.

From the value of R_o, the kinetic energy of the particle at the point where $\mathcal{R}$ is equal to zero can be determined.

So, in principle, both the mass of the particle, and its energy at

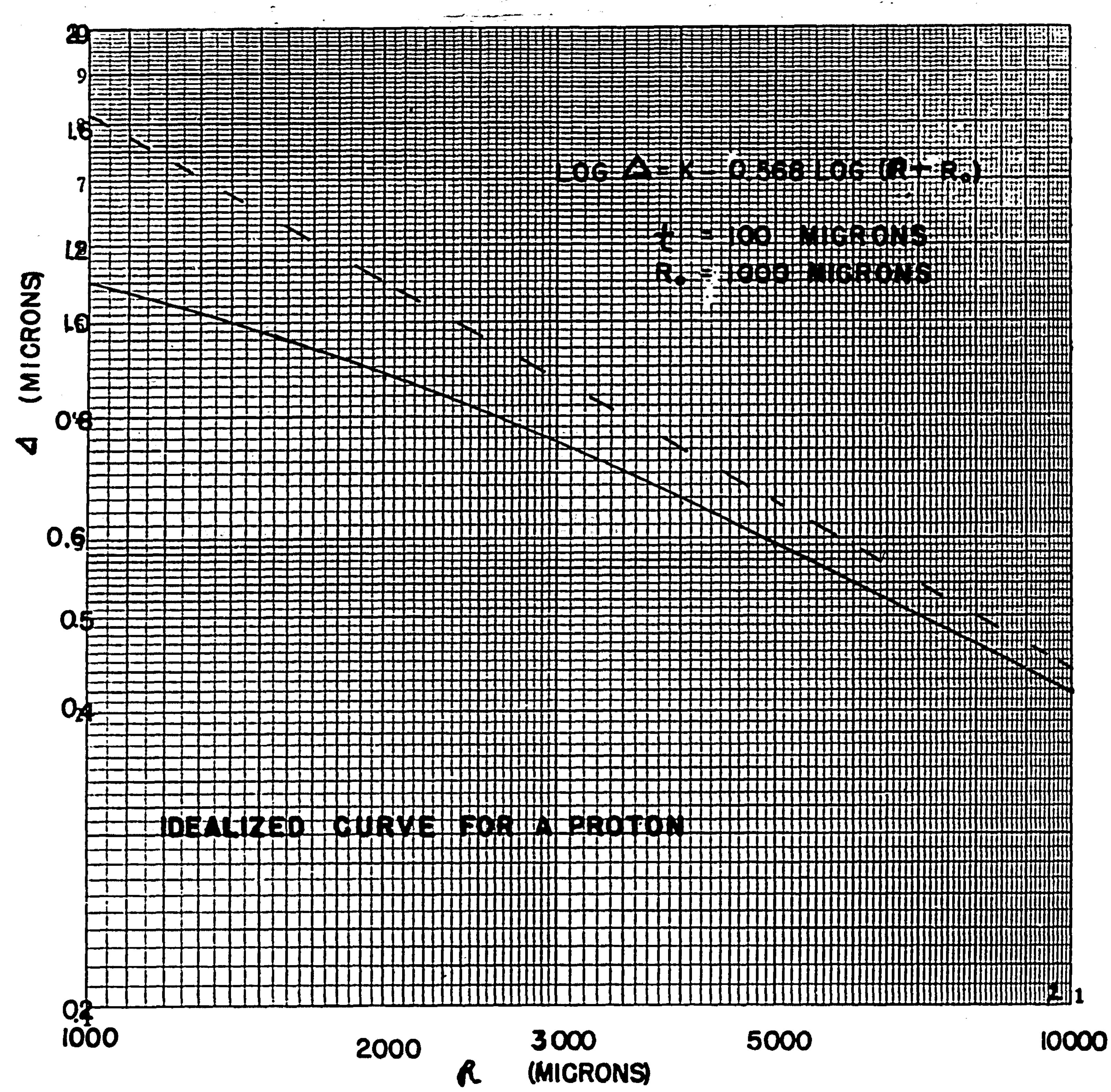

Figure 11 - A plot of the equation, $\log \Delta = k - 0.568 \log (R + R_0)$, where k has been evaluated for a proton, $M_p = 1837\ m_e$.

Dotted line is plot of $\log \Delta_0 = K - 0.568 \log R$.

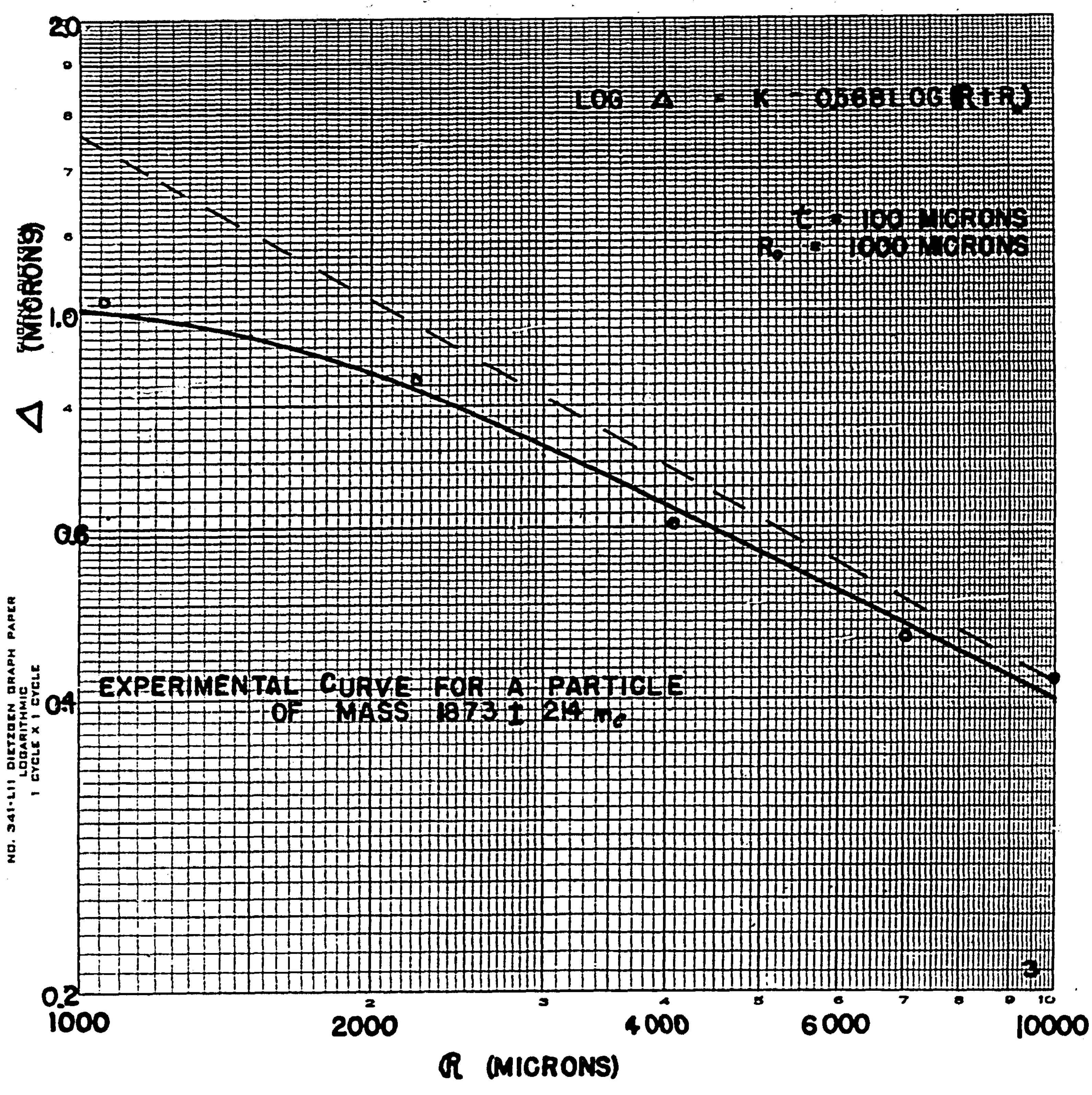

Figure 12 - The experimental counterpart of Figure 11.

any point on the track, can be obtained from observations made on the graph. Such evaluations can be made, but experience proves that there is so much room for choice in the construction of the curve and in the location of the line, that any estimates of mass or energy which result, are crude approximations.

An Analytical Evaluation

The well known Method of Least Squares is useful for the analysis of experimental data in order to determine the constants in an equation. When the equation is linear in its constants, the application is straight-forward. Thus Equation (5-3), specialized to the case of an ending track, becomes

$$\log \Delta = k - 0.568 \log R, \qquad (5\text{-}6)$$

which is linear in its one constant k. So for such a track, a mass estimate for the particle can be made by evaluating k by the Method of Least Squares, and then using Equation (5-5) to evaluate M.

But for the more general case of a particle which has left the emulsion, decayed in flight, or suffered a loss of energy, a more complicated method is required.

The method adopted in these studies is that described by Scarborough[1], as a "General Case Method". This is a modified form of the Method of Least Squares, in which the right-hand side of equation (5-3) is expanded in a Taylor's Series.

Suppose that approximate values of k and R_0 are available. Then,

[1] J. B. Scarborough, Numerical Mathematical Analysis, (The Johns Hopkins Press, Baltimore, 1950), pp. 463-469.

k and R_o may be treated as parameters. Suppose the approximations, as obtained graphically, are $k^{(1)}$ and $R_o^{(1)}$ respectively. Then

$$k = k^{(1)} + a \quad ,$$

and

$$R_o = R_o^{(1)} + b,$$

where a and b are corrections to be determined.

For a particular value of $\mathcal{R}$, Equation (5-3) becomes

$$y'(k,R_o) = y'(k^{(1)}, R_o^{(1)}) + a\frac{\partial y'}{\partial k}(k^{(1)}, R_o^{(1)}) \qquad (5\text{-}7)$$

$$+ b\frac{\partial y'}{\partial R}(k^{(1)}, R_o^{(1)}) + \text{Higher Order Terms} \quad .$$

where $y' = k - 0.568 \log (\mathcal{R} + R_o)$.

If the higher order terms are neglected, the equation is linear in a and in b. Then a and b can be evaluated by the Method of Least Squares.

Since the higher order terms are neglected, the values so obtained are themselves approximations and should be designated as $a^{(1)}$ and $b^{(1)}$. Then

$$k^{(2)} = k^{(1)} + a^{(1)} \ ,$$

and

$$R_o^{(2)} = R_o^{(1)} + b^{(1)} \ .$$

In a similar way, third approximations can be determined.

The Details of the Calculations

If the higher order terms in Equation (5-7) are neglected

$$y' \ (k^{(2)}, R_o^{(2)}) = A^{(1)} + B^{(1)}\ a^{(1)} + C^{(1)}\ b^{(1)} \quad ,$$

where

$$A^{(1)} = k^{(1)} - 0.568 \log \left[\mathcal{R} + R_o^{(1)}\right] ,$$

$$B^{(1)} = \frac{\partial y'}{\partial k}(k^{(1)}, R_o^{(1)}) \quad ,$$

$$C^{(1)} = \frac{\partial y'}{\partial R_o}(k^{(1)}, R_o^{(1)}) \quad .$$

In what follows, the rather cumbersome superscripts will be omitted, and it is to be understood that each of the quantities, except $\mathcal{R}$, is an approximation of the same order as that of k and of R_o.

Since the expansion is carried out for a particular value of $\mathcal{R}$, there are N equations of the form

$$y_i' = A_i + B_i a + C_i b \ ,$$

where N is the number of values of Δ available. Now let

$$y_i = \log \Delta_i \ ,$$

where Δ_i is any one of the N values of Δ.

The Principle of Least Squares requires that a and b be determined so that

$$\sum_{i=1}^{N}(y_i' - y_i)^2 \text{ is a minimum.}$$

Then

$$\frac{\partial}{\partial a} \sum_{i=1}^{N}(y_i' - y_i)^2 = 0,$$

and

$$\frac{\partial}{\partial b} \sum_{i=1}^{N}(y_i' - y_i)^2 = 0 \quad .$$

The first of these requirements yields

$$\sum_{i=1}^{N}(A_i + B_i a + C_i b - \log \Delta_i) B_i = 0 \ ,$$

or, since $B_i = 1$,

$$N a + b\sum_{i=1}^{N} C_i + \sum_{i=1}^{N}(A_i - \log \Delta_i) = 0. \qquad (5\text{-}8)$$

The second requirement yields

$$a\sum_{i=1}^{N} C_i + b \sum_{i=1}^{N} (C_i)^2 + \sum_{i=1}^{N}(A_i - \log \Delta_i)\, C_i = 0. \qquad (5\text{-}9)$$

Since each of the A_i's, the C_i's, and the Δ_i's can be evaluated, these last two equations can be solved simultaneously for a and for b.

The procedure should be repeated as many times as is consistent with the accuracy of the measured Δ_i's. Ideally, each value of a and each value of b will be numerically smaller than the last. In practice, it has been found that third approximations are usually sufficiently close.

Segment Length

The above discussion has tacitly ignored the fact that Δ_i is not directly observable at a single point on the track. In practice, the mean of the absolute values of the apparent differences, that is D_i, is the observable. Δ_i is then determined by means of the relation[1]

$$\Delta = (D^2 - \epsilon^2)^{\frac{1}{2}} .$$

D and ϵ are means, which are evaluated by averaging a large number of observations.

The method, presently being discussed, requires one to divide the available part of the track into segments, determine D for each segment, correct this value of D for noise, and then assume that the resulting value for Δ is the correct value at a point very close to the midpoint of the segment. That this last is, in general, true may be shown[2] by

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[1] Above, p. 19, and Appendix I, p. 103.

[2] Appendix III, p. 106.

an argument, somewhat similar to the Theorem of the Mean Value.

If a curve such as that shown in Figure 12 is to be constructed for N experimentally determined points, each point must represent a different segment of track. Each segment should be long enough that the value of Δ, for that segment, is statistically meaningful, yet short enough that Δ can be taken as the correct value for the midpoint of the segment.

When the first attempts to apply this technique were made, all segments were made of equal length. This practice was abandoned for three reasons. First, it was found that fluctuations in the observed scattering, from point to point, were enormous. Second, this seemed to weight the data heaviest where the residual range is greatest. Third, when more than eight segments were used the calculations became prohibitively long. The number of segments to be used in the analysis of a track has been arbitrarily set as not fewer than five nor more than eight.

The statistical deviation in the scattering, over any one segment, may be shown to be an increasing function of the ratio, $\frac{\epsilon}{\Delta}$, and to be inversely proportional to the square root of the number of cells which make up the segment. This implies that long segment lengths should be employed where the scattering is small, that is, where the range is great, and vice versa.

This can be conveniently accomplished by making each segment length greater than the last, so that the ratio between two consecutive segment lengths is constant.[1]

The Optimum Cell Length

McClure[2] has shown empirically that the same criterion should be applied for determining the optimum cell length as in the case of constant sagitta scattering. This criterion is to choose that cell length which makes the ratio, $\frac{\Delta}{\epsilon}$, most nearly equal to 2.4. In the present application the interpretation is that the ratio of the average scattering over the entire track length, to the noise level should be 2.4.

Application to Ending Tracks

For the special case of an ending track, $R_0 = 0$, and Equation (5-2),

$$\log\Delta = k - 0.568 \log R ,$$

applies. This equation is linear in its only unknown constant, k. The Method of Least Squares, proper, can therefore be applied. The result is

$$Nk = 0.568 \sum_{i=1}^{N} \log R_i + \sum_{i=1}^{N} \log\Delta_i \quad . \qquad (5\text{-}10)$$

This furnishes an auxiliary method for the evaluation of a mass estimate for a particle which has come to rest. Obviously the calculations are much less involved than for a particle which has left the emulsion.

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[1] As rule of thumb, one might make the first and shortest segment length equal to eight cell lengths. Then multiply each segment length by a factor of the order of 1.2 to obtain the next. With 100-micron cell lengths, this divides 10,000 microns of track into seven segments.

[2] John Jerry McClure, "A Scattering Method for The Analysis of Nuclear Emulsion Tracks", (Unpublished Master's Thesis, Dept. of Physics, University of Oklahoma, 1956), pp. 19-22.

There is no intention to imply that this method is superior to the method of constant sagitta scattering. But the fact that this method yields results which are consistent with those obtained by constant sagitta methods is considered to be significant.

Statistical Deviations

McClure[1] has derived the expressions for the deviations in the mass and in the residual range of a particle which has failed to come to rest. These will be considered in due time. Rather than specialize these to the case where R_o is equal to zero, it is simpler to derive the corresponding expression by fundamental methods.

In Equation (5-8), k may be considered to be a function of each of the indirectly observed Δ_is. The standard deviation for a single Δ_i is given by[2]

$$\sigma(\Delta_i) = \frac{0.75}{\Delta_i}\left[\frac{D_i^4 + \epsilon^4}{n_i}\right]^{\frac{1}{2}} .$$

By the Law of Propagation of Deviations,

$$\sigma(k) = \left\{\sum_{i=1}^{N}\left[\sigma(\Delta_i)\right]^2\left[\frac{\partial k}{\partial \Delta_i}\right]^2\right\}^{\frac{1}{2}} ,$$

and from Equation (5-8),

$$\frac{\partial k}{\partial \Delta_i} = \frac{\log e}{N\,\Delta_i} .$$

Then

$$\sigma(k) = \frac{0.75 \log e}{N}\left\{\sum_{i=1}^{N}\left[\frac{1}{\Delta_i}\right]^2 \frac{\Delta_i^4 + 2\Delta_i^2\epsilon^2 + 2\epsilon^4}{n_i\,\Delta_i^2}\right\}^{\frac{1}{2}}$$

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[1] Ibid, pp. 14-19.

[2] Above, p. 25.

or

$$\sigma(k) = \frac{0.75 \log e}{N}\left\{\sum_{i=1}^{N} \frac{1}{n_i} \left(1 + 2\gamma_i + 2\gamma_i^2\right)\right\}^{\frac{1}{2}},$$

where

$$\gamma_i = \left(\frac{\epsilon}{\Delta_i}\right)^2 .$$

The deviation in the estimated mass is given by

$$\sigma(M) = \sigma(k) \frac{dM}{dk} ,$$

and by Equation (5-5)

$$\frac{dM}{dk} = \frac{-M}{0.432 \log e} .$$

Thus

$$\sigma(M) = \frac{1.76\, M}{N}\left\{\sum_{i=1}^{N} \frac{1}{n_i} \left(1 + 2\gamma_i + 2\gamma_i^2\right)\right\}^{\frac{1}{2}} . \qquad (5\text{-}11)$$

When dealing with non-ending tracks, expressions are required for deviations in both M and R_o. Such expressions have been derived by McClure[1] and are

$$\sigma(M) = \frac{1.76\, M}{Q^2 - NS}\left\{\sum_{i=1}^{N} \frac{\left(Q \frac{\partial y_i'}{\partial R_o} - S\right)^2}{n_i} \left(1 + 2\gamma_i + 2\gamma_i^2\right)\right\}^{\frac{1}{2}}$$

and

$$\sigma(R_o) = \frac{0.34}{Q^2 - NS}\left\{\sum_{i=1}^{N} \frac{\left(Q - N \frac{\partial y_i'}{\partial R_o}\right)^2}{n_i} \left(1 + 2\gamma_i + 2\gamma_i^2\right)\right\}^{\frac{1}{2}} ,$$

where

$$Q = \sum_{i=1}^{N} \frac{\partial y_i'}{\partial R_o} ,$$

---

[1] The equations derived by McClure appear to be different, but it may be shown that they are the same as these. As presented here they seem to be more convenient for computation purposes.

and

$$S = \sum_{i=1}^{N} \left(\frac{\partial y_i}{\partial R_o} \right)^2 .$$

There is some question as to the correct value for n_i. In all experimental work herein reported, a technique involving overlapping cell lengths has been used. Readings of Y, the apparent displacement of the track trajectory from the axis of the track[1], were recorded every ten microns along the axis of the track. Since the optimum cell length can not be determined until after the data have been taken, it is almost necessary to record the data in this manner. Suppose, for example, an optimum cell length of 80 microns had been adopted. Then for a track segment which is 800 microns long, there would be ten values of D_i if no overlapping were employed. But there would be sufficient data available for eighty values of D_i, and all eighty would be used. In computing deviations, however, the number of values which would have been obtained had no overlapping been used, has been taken as the value for n_i.[2] In the opinion of this writer, this procedure overestimates the deviations.

Experimental Results with Ending Tracks

Table V indicates experimental results which have been obtained upon five tracks. Each of these has been treated as an ending track and one or more mass estimates obtained by constant sagitta scattering,

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[1] Figure 6, Chapter II, p. 16.

[2] This is consistent with the "Recommendations for Standardization of Measurements in Photographic Emulsions;" Varenna Summer School, 1953, and at the Meeting of the International Congress on Unstable Particles, Padua, 1954, as reported in Suppl. Nuovo Cimento, ser. 9, 12, 476 (1954).

as indicated. "Track 9" has already been utilized to construct the curve shown as Figure 12. It is considered to have been well established as the track of a proton. "Track 10" may very well also be that of a proton. An unsuccessful attempt has been made to make a case for a decay in flight out of the data for this particle. This will be discussed in a later paragraph. The analysis of the last 3330 microns was made from the point where the presumed decay took place. That this mass estimate agrees within 200 mass units with that made for the track as a whole is considered very significant. If one assumes that this particle was heavier than a proton, then it must have been one of the Y-particles. The Committee on Charged Hyperons, at the Padua Conference, 1954,[1] has reported " . . . evidence . . . from cloud chamber work may be interpreted with the assumption of a particle (until now known only in its negative charge state) with a mass of about 2570 m_e, which decays according to the scheme

$$Y_2^- \rightarrow \pi^- + \Lambda^0 + 60 \text{ mev.} \ldots "\ .$$

The Chicago group[2] indicates that a particle whose mass is 2590 m_e decays into another particle of mass 2185 m_e and a meson, with a Q of 67 mev. It seems reasonable to conclude that such particles might occasionally come to rest.

"Track 11" yields mass estimates which are consistently lower than the proton's mass. The writer hesitates to conclude that the particle

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[1] Astbury and others, Suppl. Nuovo Cimento, ser. 9, 12, 458(1954).

[2] "Events of Special Interest", (unpublished chart, prepared by the Nuclear Emulsions Group, University of Chicago, 1955).

TABLE V

MASS ESTIMATES FOR PARTICLES WHICH HAVE COME TO REST

Track No.	Length (microns)	Mass Estimate and Standard Deviation (electron masses) I[a]	II[b]	Particle is Assumed to Have Been
(9)	11,250	1873±210 (P-0.6)	1750±340 (80,6)	Proton
			1660±320 (100,6)	
			1820±390 (120,6)	
			1835±459 (150,6)	
(10)	8,100	3067±440 (P-0.6)	2890±700 (80,5)	Hyperon
			3270±890 (80,10)	
			2938±820 (80,7)	
			2830±780 (80,6)	
			2405±560 (100,6)	
	3,300	2900±530 (P-0.6)	2730±660 (100,7)	
(11)	14,250	1537±210 (P-0.6)	1492±280 (100,8)	Proton
		1471±180 (π-0.6)		
(7)	10,000	1616±250 (P-0.6)	1925±390 (100,7)	?
(7)	7,000	2259±380 (P-0.6)	2730±660 (100,7)	?
	3,190	1874±500 (P-0.6)	1790±600 (80,3)	

a. "I" indicates constant sagitta scattering methods have been used. The scattering scheme is shown in parenthesis.

b. "II" indicates the methods described in this chapter have been used. The cell length, in microns, and the number of segments are shown in parenthesis.

was anything other than a proton. It is true that the two independent estimates, obtained by means of constant sagitta scattering methods, agree with each other and with the single estimate made by means of the local technique, but it is thought probable that a systematic error due perhaps to large-angle scattering may have contributed approximately the same amount to each of these estimates. In other words, the agreement is thought to be more significant than the values themselves.

"Track 7" is possibly that of a proton. But there is evidence of something unusual in this example. More will be said in regard to this track, both in the next paragraph and in Chapter VI. "Track 8" also warrants special consideration.

Particles That Do Not Come to Rest

The first entry in Table VI is for the track which has already been mentioned as having motivated these studies. The mass estimate quoted here is consistent with that obtained by change in gap-density. The estimated residual range at the point where the particle left the top of the emulsion is thought to be a reasonable one. Visual inspection definitely leads to the conclusion that the particle was nearing the end of its range. It seems futile to attempt to identify this particle as anything other than a proton.

These estimates for Tracks 9 and 11 are interpreted as being in satisfactory agreement with those estimates already quoted in Table V. As is to be expected the deviations are appreciably greater when the track is treated as a non-ending one.

No satisfactory explanation is suggested for the rather low mass

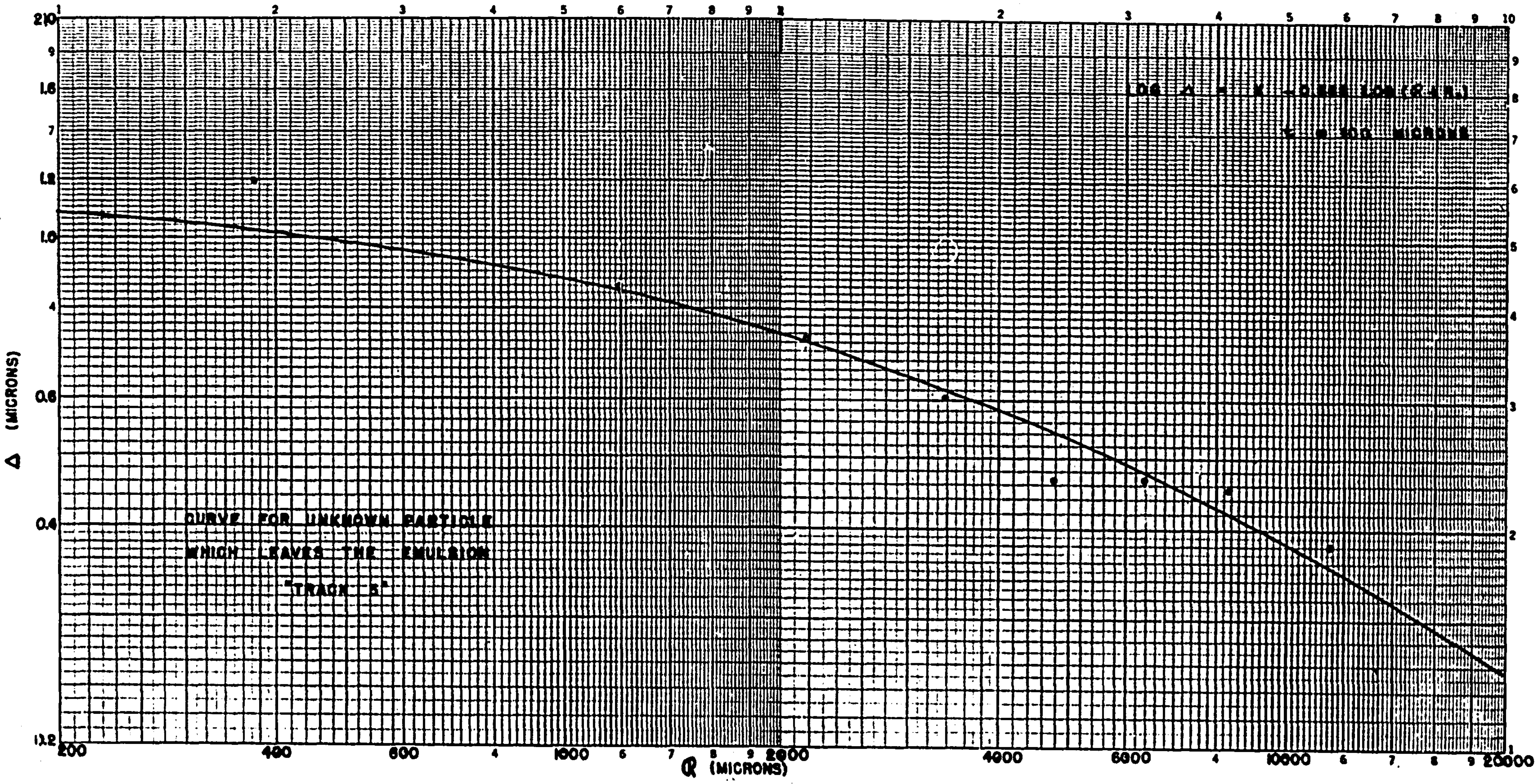

Figure 13 - The best-fit curve for particle, whose mass has been estimated - see Table VI - as (2052 ± 680) m_e and for which R_o = (1572 ± 1050) microns.

TABLE VI

RESULTS OF TRACK ANALYSES MADE BY THE METHOD FOR NON-ENDING TRACKS

Track No.	Length (microns)	Cell Length (microns)	No. of Segments	Mass Estimate and Standard Deviation (electron masses)	Residual Range and Standard Deviation (microns) I[a]	II[b]	Remarks
5	14,000	100	8	2052 ± 680	1572 ± 1050	?	Left the emulsion $M(G',R) = 1670 \pm 490 m_e$
9	11,250	100	6	2120 ± 680	822 ± 600	1000	Proton. See Table V.
		80	7	2140 ± 830	883 ± 875	1000	
		80	8	1960 ± 950	1012 ± 1100	1000	
		120	6	2130 ± 970	1204 ± 1150	1000	
10	8,100	100	6	2250 ± 1080	196 ± 710	100	Hyperon, See Table V.
11	14,250	100	7	1490 ± 300	-1 ± 400	0	Proton, See Table V.
		100	7	1490 ± 250	245 ± 330	250	
		100	7	1525 ± 380	402 ± 380	500	
		100	6	1590 ± 520	1215 ± 520	1450	
8	10,000	100	6	549 ± 540	5881 ± 6400	982	Discussed at length in Chapter VI.
7	7,000	100	5	2370 ± 7000	4540 ± 1300	3190	Discussed at length in Chapter VI.

[a] "I" indicates the method described in this chapter have been used to determine the residual range at a point on the track where $\mathcal{R}$ was arbitrarily taken as equal to zero.

[b] "II" indicates the residual range has been measured directly at this same point.

value obtained for the alleged hyperon. As has already been mentioned, an attempt has been made to analyze the data for this particle, by making the assumption that a decay in flight has occurred at the point 3330 microns from the point where the particle came to rest. This unsuccessful attempt has emphasized what is probably the weakest factor in this method of analysis, namely, that when R_o becomes greater than about 3,000 microns, the quantity

$$(NS - Q^2)$$

which appears in the denominator of the solutions of the two simultaneous equations, (5-8) and (5-9), becomes very small. It follows that there are enormous uncertainties in k and in R_o.

In this particular work with Track 11, it was possible to place a lower limit on the mass. This lower limit was of the same order of magnitude as the mass estimates for the part of the track which was ignored, so the attempt to make a case for a decay in flight was abandoned.

Summary

This original method of estimating the mass and the residual range of a particle which has failed to come to rest, is considered to have been shown to be valid. As in all nuclear emulsion measurements the statistical deviations are great. Especially is this true for the estimated ranges. If the missing part of the track is more than 3,000 microns long, such estimates are almost meaningless.

It is suggested that the method which has been described might be combined with constant sagitta scattering methods. This could be done by obtaining a value for R_o and a value for M, and then using these values to set up a scattering scheme. This should give a "better" mass estimate.

Baroni, Cortini, and Manfredini,[1] at the University of Rome, have devised and used a constant sagitta technique upon non-ending tracks. They apparently do not obtain a mass estimate but do obtain satisfactory range estimates. They quote statistical deviations of the same order of magnitude as those quoted here.

For most satisfactory results it is recommended that:

(1) overlapping cell lengths be employed;

(2) that cell length be chosen which makes the ratio of the mean of the absolute values of the second differences due to actual scattering, as nearly as is feasible, equal to 2.4 times the noise level;

(3) progressively increasing segment lengths be used;

(4) the plate be rotated by means of the microscope turn table, each time a large-angle bend occurs.

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[1] Baroni, Cortini, and Manfredini, Nuovo Cimento, ser. 10, 1, 473(1955).

CHAPTER VI

APPLICATIONS OF THE METHODS OF PRECEDING CHAPTERS TO SPECIAL CASES

Introduction

In Chapter V, Table VI, mass estimates are quoted for several particles. For two of these the results are and have been designated unsatisfactory. Track 7, which will be treated first in the detailed discussion, also shows a discontinuity in gap density. For the case of Track 8, the discrepancy between the two mass estimates, which were obtained by the method of scattering as a function of range, is sufficient reason for considering this track more carefully.

In this chapter, various possibilities will be considered as explanations for these two "unfavorable cases". As is often true, no one interpretation can be offered which is completely satisfactory. The writer will offer what he believes to be the most plausible explanations.

In addition to those methods of track analysis which have been explained, the Law of Conservation of Energy, and the Law of Conservation of Momentum will be employed as tools.

Track 7

General Description

This track was located while data were being collected for the gap density calibration curves discussed in Chapter III. Discontinuities in the gap density, and the presence of an unusual number of large angle bends were apparent. The particle which caused the track was tentatively identified by visual inspection as having been a proton. The particle had entered the emulsion from the top, at a point about two millimeters from the edge of the emulsion and traveled a distance of 3705 microns, to a point 322 microns below the surface.[1] From this point, the particle traveled an additional 3195 microns and came to rest at a point 289 microns below the surface. So over this last segment, the track is essentially parallel to the surface of the emulsion.

A "profile", that is a graphical representation of the projection of the track onto a plane parallel to the surface of the emulsion, was constructed. This profile shows large-angle bends, one degree or greater, at each of three points, where the residual range was 663 microns,[2] 978 microns, and 2309 microns, respectively.

There is no sign of a decay particle at the point where the particle came to rest. At the point where the residual range was 3195 microns, there is a rather long delta-ray, and the profile shows a change of direction through a projected angle of about two degrees.

Gap Density

The curve for this track which shows total gap length as a function

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[1] This depth is that in the unprocessed emulsion. The correction for shrinkage was made by the method described by Fitzpatrick, *op. cit.*, pp. 26-28.

[2] The initial gap length data indicated a discontinuity in the gap density at this point. Later measurements failed to confirm this. It has been decided the initial data were erroneous.

of residual range indicates that this particle is of slightly smaller mass than those particles in the proton group. In the range interval between 2800 and 3500 microns there is a discontinuity in gap density. The average of five mass estimates, from gap densities measured at five points for each of which R is less than 3000 microns, is 1608 electron masses. The average of five values for ranges greater than 3000 microns is 1348 electron masses. This is not a great difference and may not be significant. But it may indicate either a collision in which the delta-ray was produced, or an actual change in the mass. It seemed reasonable to proceed by treating the track as being composed of two different parts; one, a non-ending track 3075 microns long; and the other, an ending track 3195 microns long. In the following, the first part will be called "Part A", and the second, "Part B".[1]

The technique was employed to obtain a mass estimate in which gap densities were observed at two different points. For the particle which caused Part A, a value of 1833 electron masses was determined. That this number is so near the mass of the proton is thought to be of incidental significance, but the fact that it is greater than that value, already quoted as having been obtained over Part B, is thought to be important. The value obtained for the residual range of the initial point on Part B, was 4037 microns.

Constant Sagitta Scattering

A P-0.6 scattering scheme has been used to obtain mass values for

[1] The word "segment" rather than "part" might be preferable, except that segment has already been assigned a different meaning in the work of Chapter V.

this particle. First, the entire track was included. In this way a mass value of 2289 ± 480 was calculated. Since the track was divided into segments, it was possible to examine the data with regard to fluctuations in the second differences. For the six segments which comprise Part B, the mean value of Δ was 0.594 microns, with fluctuations from 0.35 to 0.72. For the remainder of the track, the segments were in general, longer. The Δ's varied from 0.24 to 0.97. This highest value, 0.97 should probably be eliminated, as it is for that segment where the particle was nearest the surface. If this were done the mass would be slightly higher than that quoted.

When only Part B was included in the analysis, constant sagitta scattering gave 1874 ± 500 electron masses.

Constant Cell Length Scattering

At this point it seemed safe to proceed on the assumption that Part B was made by a proton. An analysis was made of Part A, which employed the technique described in Chapter V for non-ending tracks. The length of track available was admittedly too short for precise determinations.

A graphical approximation and two successive applications of the methods of Chapter V, gave

$$M = 2370 \, m_e, \text{ and}$$

$$R_0 = 4540 \text{ microns.}$$

The deviations are undoubtedly large.

Only five points were available for the graph. For the largest segment, the number of observed second differences which was available for calculation of the scattering was only sixty six, with overlapping cell lengths.

It was not feazible to employ the technique in which progressively increasing segment lengths are used, since it was necessary to rotate the plate several times, because of large-angle bends. Thus the track was effectively divided into segments in the actual process of recording the data.

Since R_o is not small relative to $\mathcal{R}$, it is clear that according to the conclusions reached in Chapter V, the calculated values for R_o and M must be thought of with skepticism. These values are qualitatively consistent with those obtained by gap density methods in two respects. The residual range of the particle at the end of Part A is greater than the residual range of the assumed proton at the beginning of Part B. The mass estimate for Part A is greater than that for Part B. The experimental results for Track 7 are summarized in Table VII.

Various Possibilities

One explanation which has been considered is the following. The track was made by a single particle, a proton, which lost energy at the point where the delta ray is observed. If this is the case, one can use the residual range as quoted above, that is, 4540 microns, to compute the kinetic energy of the proton, before the collision in which the delta-ray was generated.

When this was done, it was found that the electron would have to have a range of 81,000 microns to account for the 6.7 mev. lost by the proton. The length of the delta-ray is of the order of a few microns,

TABLE VII

EXPERIMENTAL RESULTS FOR TRACK 7

Mass Value (electron masses)	Mass Method		Assumed Particle	R_o (microns)
Entire Track Analyzed as a Whole, 6270 microns				
2259	css (P-0.6)	a		0
1478	G' vs $\frac{R}{M}$	e		0
Part A, 3075 microns				
2370	Δ vs $\mathcal{R}$	c	hyperon	4540
1833	δ G'	d		4037
Part B 3190 microns				
1874	css (P-0.6)	a	proton	0
1742	css (Υ-0.6)	a		0
1790	Δ vs R_R	b		0
1348	G' vs $\frac{R}{M}$	e		0

[a] Constant sagitta scattering, scattering scheme shown in parenthesis.

[b] Scattering v.s. residual range, ending track.

[c] Scattering v.s. range, non-ending track.

[d] Change in gap density between two points.

[e] Gap density measured at several points.

certainly less than twenty. If the range of the electron were twenty microns its energy would be only 0.06 mev.; about one percent of that to be accounted for.

It must be observed that the excess energy can be accounted for by proposing that it is taken on by a heavy nucleus which might easily not be observed. It is also only fair to point out that the estimated energy of the particle before the collision is quite likely too large, and that the loss of energy may have been much less than the 6.7 mev. which has been calculated.

So the possibility that the entire track was made by a proton can not be rejected.

It is interesting to speculate on the basis of another hypothesis, namely, the decay of a hyperon into a proton. Two facts about the experimental data are taken as grounds for the following analysis. First, the mass estimates, by change of gap density and by constant cell length scattering, each indicate a larger mass before the event which separates the two parts of the track. Second, constant sagitta scattering indicates that the particle had either a greater range or a greater mass, or both, over the first part of the track, than over the second.

The following analysis is presented largely in order to exemplify an application of the methods which have been presented. It does yield a plausible explanation of the descrepancies which have been observed.

The Committee on Charged Hyperons[1] at the International Congress on

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[1] Astbury, Bonetti, Ceccarelli, Dallaporta, Franzinetti, Friedlander and Tomasini, Suppl. Nuovo Cimento, ser. 9, 12, 448 (1954).

Unstable Particles at Padua give as one of the accepted decay schemes

$$Y_P^{\pm} \rightarrow P^{+} + \pi^{o} \quad ,$$

where the symbol, Y_P, represents a particle whose mass is greater than that of a proton. P represents a proton, and ^{o} represents a pi meson. Five of seven cases reported are consistent with a unique Q-value of 116 $\pm$ 2 mev. In particular, this committee mentions a particle of mass 2300 $\pm$ 800 m_e which has decayed according to the above scheme. A group of nuclear emulsions physicists at the University of Chicago[1] has indicated that the decay of a particle whose mass is 2335 m_e, into a proton and a neutral pi meson, has been established.

A decay in flight, in which the primary particle is a hyperon which yields a proton and a neutral particle has been assumed. The Laws of Conservation of Energy and Conservation of Momentum make possible the calculation of the mass of any one of the three particles, if the mass, the total energy, and the direction of each of the other particles are known.

Thus the mass of the unknown neutral particle in the assumed decay scheme is given by Equation (6-1), a derivation of which appears in the Appendices.

$$M_X^2 = M_Y^2 + M_P^2 - 2\frac{\mathcal{E}_Y\,\mathcal{E}_P}{c^4} + 2\,\frac{\cos\,\theta}{c^4}\left[(\mathcal{E}_Y - M_Y^2c^4)(\mathcal{E}_P - M_P^2c^4)\right]^{\frac{1}{2}} \quad (6\text{-}1).$$

In the above equation, $\mathcal{E}$ is the total energy; that is,

$$\mathcal{E} = Mc^2\,(1 - \beta^2)^{-\frac{1}{2}} \,,$$

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[1] "Events of Special Interest", loc. cit.

and the subscripts, Y, P, and X refer to the hyperon, proton, and unknown particle, respectively. θ is the angle between the direction of the hyperon and the direction of the proton.

There is great uncertainty in the measurement of the angle, θ. Fortunately, for this application, the cosine of an angle is nearly constant for angles between zero and five degrees.

The second entry in Table VIII, shows the results of a calculation in which the experimentally determined mass value has been used for the hyperon, together with the experimentally determined residual range at the point of decay. The angle, θ, has been observed as two degrees. The respective residual ranges of the hyperon, and the proton have been used to compute the kinetic energies, hence the total energies, of these particles. Clearly the mass which is obtained for the neutral particle is far from that of any well known particle.

One may assume the decay scheme which has been mentioned, namely, that of a 2335-m_e hyperon into a proton and a pi meson. Then everything in Equation (6-1) is known except $\mathcal{E}_Y$. Since the equation is quadratic in $\mathcal{E}_Y$, two values result from the solution. Physically, this is because the direction of the pi meson is not unique. When one has a numerical value for $\mathcal{E}_P$ he can determine the kinetic energy and finally the residual range of the assumed decaying particle.

The third and fourth entries in Table VIII show the results of such calculations. The fourth entry is for an accepted decay scheme[1] in which the Λ^+-particle is taken as the primary particle. In each of these

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[1] Ibid.

TABLE VIII

SUMMARY OF RESULTS OBTAINED BY MAKING VARIOUS ASSUMPTIONS REGARDING TRACK 7

Assumed Event	Mass of Primary (m_e)	Mass of Secondary (m_e)	R_o (microns)	Q (mev)	δE (mev)	Mass of Neutral (m_e)
Collision	1837 (a)	1837 (a)	4540 (c)	0	6.7 (d)	x
Decay in flight	2370 (m)	1837 (a)	4570 (c)	5.1(d)	x	523 (d)
Decay in flight	2335 (b)	1837 (b)	9830 (d)	120 (b)	x	263 (b)
Decay in flight	2185 (b)	1837 (b)	8216 (d)	43 (b)	x	263 (b)

(a) indicates the value has been assumed as shown in Table VII.

(b) indicates the value has been assumed with the decay scheme.

(c) indicates the value has been computed as shown in Table VII.

(d) indicates tha value has been computed as explained in the text.

x indicates the quantity has no significance in the event.

solutions only one root has been obtained which is physically meaningful; the other value for $\mathcal{E}_Y$ being smaller than the rest energy.

Conclusions Regarding Track 7

In the opinion of the writer, the explanation which was suggested first, is most satisfactory. That is, the particle was a proton which lost energy in a collision with another particle. The following are his reasons:

(1) the observed delta-ray implies that a collision has taken place;

(2) the occurrence of such a collision is believed to be much more probable than that of a decay in flight;

(3) since the estimated residual range is uncertain, the loss of energy may well have been much less than that which was computed;

(4) the small angle between the direction of the particle before the event and that after the event seems indicative of a collision rather than a decay.

On the other hand, one may accept any one of the suggested decay schemes. This may be done on the basis of the following reasons:

(1) the constant sagitta scattering data indicate an abrupt increase in the scattering at the point under consideration, as would be expected from a decrease in the mass, or in the range, or in both;

(2) both gap density data and scattering data indicate a decrease in the mass;

(3) the change in the gap density is much less emphatic than the change in the scattering, as is to be expected, since gap density is a function of $\frac{R}{M}$.

It is considered to be remarkable that any plausible explanation can be made from data on such a short track.

Track 8

General Description

This track was made by a particle which entered the emulsion through one edge. Since this point at which the particle entered the emulsion was near a corner of the plate, no observations were taken on the first two millimeters of the track. At the first point where observations were made, the depth in the undeveloped emulsion was 215 microns. The angle of dip, that is, the angle between the track and the plane of the emulsion was estimated, at points where the direction of the particle changed abruptly. This angle is never greater than 1.5 degrees.

As for Track 7, a profile was constructed. This representation indicates large angle bends at sixteen points, which are included in the entire length of 10,000 microns, over which observations were made.

Visual inspection suggests the track of a proton.

Table VI[1] indicates that if the track is considered to be that of a single particle, with no discontinuities, then the particle was probably a proton. Constant sagitta scattering, and the method of Chapter V for ending tracks have been employed.

Table VI[1], however, indicates that when a point, 982 microns from where the particle came to rest, was treated as a terminus, completely unsatisfactory results were obtained. This is also true when points

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[1] Above, p. 76.

which are respectively, 585 microns, 1203 microns, and 1781 microns are treated as termini. At these points large angle bends occur.

Constant sagitta scattering indicates no obvious discontinuity as was indicated with Track 7.

A Possible Explanation

As will be explained, the particle is believed to have been a proton at the point where it came to rest. At a point whose measured range was 2720 microns, a change in direction of seven degrees was observed. When the track, made by the particle before it reached this point, was analyzed, the mass estimate obtained was 2050 m_e, and the evaluated residual range at this point was 1205 microns.

The interesting, and confusing part about these evaluations, is that the estimated residual range is less than the measured range. As before, that part of the track which was made before this point, will be called "Part A", and that part made afterward, "Part B". If Part A and Part B were made by the same particle, the particle must have gained energy at the point of separation. Of course this is possible.

Table IX shows various mass estimates obtained for Track 8. The values obtained by constant sagitta scattering are very close to the mass of the proton.

If one assumes a decay, and calculates, by means of Equation (6-1) the mass of the neutral particle, he obtains 175 m_e, which is within one hundred mass units of that of a neutral pi meson, 263 m_e.

As was done with Track 7, one may assume that an established decay has occurred and compute the residual range of the primary particle.

TABLE IX

EXPERIMENTAL RESULTS FOR TRACK 8

Mass Value (m_e)	Mass Method	Assumed Particle	R_o (microns)
Entire Track Analyzed as a Whole, 10,000 microns			
1616 1530 1925	css (P-0.6) [a] css (γ-0.6) [a] Δ vs R [b]	proton	0 0 0
Part A, 7280 microns			
2050	Δ vs $\mathcal{R}$ [c]	hyperon	1205
Part B, 2720 microns			
1776 1890 1550	css (P-0.6) [a] css (γ-0.6) [a] Δ vs R [b]	proton proton meson	0 0 0

[a] Constant sagitta scattering, scattering scheme shown in parenthesis.

[b] Scattering v.s. residual range, ending track.

[c] Scattering v.s. range, non-ending track.

TABLE X

SUMMARY OF RESULTS OBTAINED BY MAKING VARIOUS ASSUMPTIONS REGARDING TRACK 8

Assumed Event	Mass of Primary (m_e)	Mass of Secondary (m_e)	R_o (microns)	Q (mev)	δE (mev)	Mass of Neutral
Collision	1837 (a)	1837 (a)	1205 (c)	0	9.2	x
Decay in flight	2050 (a)	1837 (a)	1205 (c)	19.4 (d)	x	175 (d)
Decay in flight	2185 (b)	1837 (b)	680 (d) or 16,670	120 (b)	x	263 (b)
Decay in flight	2335 (b)	1837 (b)	50 (d) or 10,300	43 (b)	x	263 (b)

(a) indicates the value has been assumed as shown in Table IX.

(b) indicates the value has been assumed with the decay scheme.

(c) indicates the value has been computed as shown in Table IX.

(d) indicates the value has been computed as explained in the text.

x indicates the value has no significance in the event.

This has been done for the two decay schemes already described, namely, the decay of a particle whose mass is 2185 m_e, and that of a particle whose mass is 2335 m_e. The first computation gave a residual range of 680 microns or 16,670 microns. The second computation gave a residual range of less than fifty microns or of 10,200 microns. Of these, the first is the only one which is not entirely unsatisfactory.

As has been stated, the angle, between the track of the assumed primary particle and that of the assumed proton, is not capable of precision measurement. If this were eight degrees, rather than seven degrees, one would find for the assumed decay scheme of a 2185-m_e particle into a proton, and a pi meson, that the residual range of the hyperon at the point of decay is 611 microns. So this confirms the observation, which was implied earlier in the discussion, to the effect that the value of this angle is not critical in its effect upon the final results. The results, obtained by making the various assumptions, are summarized in Table X.

Conclusions Regarding Track 8

It is believed that the arguments as presented make a reasonably strong case for the decay in flight of a hyperon, whose mass is of the order of that of the Λ-particle, into a proton and a neutral particle, whose mass is of the order of a pi meson.

If Part A, of the track, were made by a proton, then one must assume a gain in energy of 9.3 mev. This is thought to be improbable.

On the other hand, if respective deviations of seven percent in the estimated mass, and of forty-five percent in the estimated residual range are assumed then the results are consistent with the proposed decay.

Summary

The objectives of the work reported in this Chapter were:

(1) to try to suggest plausible explanations for the inconsistencies in two special cases, and

(2) to give further evidence that the methods described in Chapter V yield realistic results. It is believed that these have been accomplished.

There has been no intention to suggest that the events themselves are extraordinary. The decay in flight has long been a source of interest to the writer and it is gratifying to believe that very possibly one has been observed.

CHAPTER VII

DISCUSSION AND SUMMARY

As the title of this dissertation suggests, the emphasis throughout has been upon particle identification rather than upon analysis of unique events.

As is usually the case, much has been left undone. In particular, the writer would have been interested in the following: the analysis of an apparently interesting event, in which a neutral particle seems to have decayed into a proton and a meson; further investigation into the possibility of employing total number of delta rays as a track parameter; the application of constant sagitta scattering methods to non-ending tracks; the use of the scattering-range technique with pellicles; further investigation of the merits of using overlapping cell lengths in making scattering observations.

Preliminary investigation of the event mentioned in the preceding paragraph indicates that the alleged neutral particle was emitted from a star which is in the same plane as the point at which the two charged particles originate. One particle has been shown to have had a mass of the order of that of a proton. The track of the other particle is too short for positive identification. The general appearance of the track suggests that it was a pi meson. The track terminates in what is believed to be a three-prong sigma star. An attempt to conserve momentum shows that the assumption that this lighter particle was a pi meson is inconsistent with the assumption of the decay of a neutral particle which originated in the manner indicated. It is believed that a case might be made for a tau meson rather than a pi.

Experimental work with delta rays has indicated that the total number of delta rays in a given residual range, is an observable which is subject to large statistical deviations. The writer believes that tracks made by doubly charged particles, such as alpha particles, can be distinguished from those made by singly charged particles, if there is available a residual range of not less than two thousand microns. He also believes that if the residual range is 3500 microns or more, one may be able to distinguish between tracks made by deuterons and tracks made by protons.

A track made by a particle which failed to come to rest might well be analyzed by the method which employs scattering as a function of range. The estimated mass and the estimated residual range could then be used to determine a constant sagitta scattering scheme. Then an auxiliary mass estimate could be obtained, which should be better than the first. Any observations, which one makes using constant cell length, are inconsistent with scattering theory. The theory is based upon the constancy of the probability that a particle will be scattered through an angle between α and $\alpha + d\alpha$. This assumption is not valid if a comparison is made between a fast particle and a slow particle whose masses are equal. Therefore, in principle, one must use a variable cell length. In practice, the use of a constant cell length may be an acceptably good approximation.

Summary

Chapters II through VI of this report consist of detailed discussions which were anticipated in Chapter I. In some cases the methods used were methods which had been well established in this laboratory, for example, the determination of gap density by measuring the slope of the curve which shows total gap length as a function of residual range. In other cases the methods of previous workers have been modified slightly. Examples are: the use of a universal curve of gap density as a function of the ratio between residual range and mass; and the technique of allowing for the change in noise level with cell length, in constant sagitta scattering analyses. These modifications are believed to yield better results. In still other cases, the methods used have been fundamentally different from those of previous workers. The "reversing sagitta" method, described in Chapter II, has been shown to yield a more accurate noise level calibration than that which results when the true scattering in the calibration track is neglected. The use of scattering as a function of range to obtain a mass estimate for a particle which has failed to come to rest, is another technique which is fundamentally new. This method is discussed at length in Chapter V. Previously the tracks of such particles were analyzed by using gap density and scattering as observables. The method proposed in this report yields mass estimates and range estimates which are as good or better than the "G' vs R" determinations. The results for the two special cases, discussed in Chapter VI, provide further evidence that this method is both valid and useful.

While it is true that the calculations required are both long and tedious, it is believed that the labor required is less than that required to make and analyze the additional set of gap density measurements.

Unlike procedures involving measurements of ionization density, this method which employs scattering measurements only, is not subject to the errors introduced by differential development.

As has been pointed out, the statistical deviations which result from the use of this method, are large. When the unavailable part of the track is longer than three thousand microns the deviations become extremely large. This had been anticipated. In many cases it is surprising that such realistic mass estimates have been obtained together with such enormous statistical deviations.

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APPENDIX I

PROOF OF EQUATION (2-4)

Equation (2-3), page 19, reads

$$D_i = d_i + \delta_i$$

in which D_i is any one of the N observed second differences, d_i is the corresponding second difference due to scattering, and δ_i the corresponding second difference due to noise.

One may square both sides of the above equation and form the sum

$$\sum_{i=1}^{N} D_i^2 = \sum_{i=1}^{N} (d_i^2 + \delta_i^2) + 2 \sum^{N} d_i \delta_i \quad .$$

The second term on the right vanishes, and therefore

$$\frac{1}{N} \sum_{i=1}^{N} D_i^2 = \frac{1}{N} \left[\sum^{N} d_i^2 + \sum_{i=1}^{N} \delta_i \right]$$

By the well known theorem[1] which says that the root mean square value is proportional to the mean of the absolute values

$$\left[\frac{\sum_{i=1}^{N} |D_i|}{N} \right]^2 = \left[\frac{\sum_{i=1}^{N} |d_i|}{N} \right]^2 + \left[\frac{\sum_{i=1}^{N} |\delta_i|}{N} \right]^2 \quad .$$

So, by definition of D, Δ and ϵ,

$$D^2 = \Delta^2 + \epsilon^2 \ .$$

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[1] Scarborough, op. cit., p. 415.

APPENDIX II. THE STATISTICAL DEVIATION IN THE MASS WHEN DETERMINED FROM THE GAP DENSITY

Consider the case of a non-ending track, where the mass is determined by

$$M = \frac{R_2 - R_1}{\varphi_2 - \varphi_1} ,$$

where $R_2 - R_1$ is the directly observed distance along the track between the two points where the gap densities are G_2' and G_1', and φ_2 and φ_1 are the respective ratios $\frac{R_2}{M}$ and $\frac{R_1}{M}$ taken from the curve which shows log G' as a function of $\log \frac{R}{M}$. M may be considered to be a function of the two φs, so that

$$(M) = \left\{ \left[\frac{\partial M}{\partial \varphi_1}\right]^2 \left[\sigma(\varphi_1)\right]^2 + \left[\frac{\partial M}{\partial \varphi_2}\right]^2 \left[\sigma(\varphi_2)\right]^2 \right\}^{\frac{1}{2}} .$$

Now φ may be considered to be a function of G', therefore

$$\sigma(\varphi) = \frac{d\varphi}{dG'} \sigma(G') .$$

$\sigma(G')$ can be determined experimentally by measuring a number of values of the slopes of the curves constructed by plotting G as a function of R for known particles. $\frac{d\varphi}{dG'}$ can be determined for any given value of by measuring the slope of the curve which shows log G' as a function of log φ, at the appropriate point. If m is the slope of the curve,

$$\frac{d\varphi}{dG'} = \frac{\varphi}{mG'} ,$$

so the expression for $\sigma(M)$ becomes

$$\sigma(M) = \frac{R_2 - R_1}{(\varphi_2 - \varphi_1)^2} \left\{ \left[\frac{\varphi_1}{m_1 G_1'} \sigma(G_1') \right]^2 + \left[\frac{\varphi_2}{m_2 G_2'} \sigma(G_2') \right]^2 \right\}^{\frac{1}{2}} .$$

For the special case of an ending track,

$$\sigma(M) = \frac{R}{\varphi m G'} \sigma(G') .$$

APPENDIX III. JUSTIFICATION FOR TAKING THE MEAN VALUE OF Δ OVER A SEGMENT AS THE VALUE AT THE MID-POINT OF THE SEGMENT

Consider the scattering equation in the form

$$\Delta = A R^{-n} ,$$

where the value of A depends upon the mass of the particle, upon the cell length, upon the emulsions and upon the "scattering constant".

The mean value of Δ, over a segment of length s, is given by

$$\bar{\Delta} = \frac{A \int_{R_1}^{R_1+s} R^{-n} \, dR}{s}$$

where R_1 is the residual range of that point on the segment, nearest the point where the particle came to rest. At some point, the residual range of which is $R_1 + \theta s$, the value of Δ will be the same as the above value for $\bar{\Delta}$. So

$$A \frac{(R_1 + s)^{1-n} - R_1^{1-n}}{(1-n)s} = A(R_1 + \theta s)^{-n} ,$$

and

$$\theta s + R_1 = \left[\frac{(R_1 + s)^{1-n} - R_1^{1-n}}{(1-\eta)s} \right]^{-\frac{1}{n}} .$$

Hence,

$$\theta = \left[\frac{(R_1 + s)^{1-n} - R_1^{1-n}}{(1-\eta)s^{1-n}} \right]^{-1/n} - \frac{R_1}{s}$$

or

$$\theta = \left\{ \frac{R_1^{1-n} \left[(1 + \frac{s}{R_1})^{1-n} - 1 \right]}{(1-\eta)s^{1-n}} \right\}^{-\frac{1}{n}} - \frac{R_1}{s}$$

Now if the substitutions,

$$q = \frac{s}{R_1} \qquad \text{and} \qquad \eta = 0.568 \quad ,$$

are made, the result is

$$\phi = q^{0.761} \; \frac{\left[(1 + q)^{0.432} - 1\right]^{-1.761}}{4.38} - \frac{1}{q} \quad .$$

So the fractional part of the segment length, which gives the distance from the initial point to that point at which Δ is equal to the mean value of Δ over the segment, is a function of the ratio of the segment length to the residual range of the initial point on the segment.

Values of ϕ for various values of q are shown below.

q	ϕ
0.10	0.52
.12	.49
.15	.48
.20	.49
.30	.49
.35	.50
.40	.50
.50	.49
.60	.47
.70	.46
.80	.46
.90	.46
1.00	.46

It is concluded that, unless the segment length is well over half the residual range at the initial point on the segment, no correction need be made for this small systematic error.

APPENDIX IV. CONSERVATION OF ENERGY AND MOMENTUM APPLIED TO A DECAY IN FLIGHT

Suppose a heavy particle such as a hyperon decays into two lighter particles, and suppose mass estimates and estimates of the energies are available for the decaying particle and for one of the other particles. If the <u>total</u> energy is called $\mathcal{E}$, and if the subscripts, Y, P, and X refer to the primary particle, the known secondary and the unknown secondary, respectively, then

$$\mathcal{E}_Y = \mathcal{E}_P + \mathcal{E}_X \quad . \tag{1}$$

If Θ is the angle between the direction of the primary particle and that of the known decay particle, then the Law of Conservation of Momentum yields

$$p_X^2 = p_Y^2 + p_P^2 - 2p_Y p_P \cos\Theta . \tag{2}$$

The relativistic expression for the total energy, $\mathcal{E}$, of any one of the particles is

$$\mathcal{E} = Mc^2 (1 - \beta^2)^{-\frac{1}{2}} ,$$

in which M is the mass, and βc the velocity of the particle under consideration. The momentum of a particular particle is given by

$$p = M\beta c(1 - \beta^2)^{-\frac{1}{2}} .$$

If β is eliminated between these equations, the result is

$$p^2 = \frac{1}{c^2} (\mathcal{E}^2 - M^2c^4) \tag{3}$$

or

$$\mathcal{E}^2 = p^2c^2 + M^2c^4 \qquad (4)$$

Equation (1) may be rewritten in the form

$$\mathcal{E}_X^2 = \mathcal{E}_Y^2 - 2\mathcal{E}_Y\mathcal{E}_P + \mathcal{E}_P^2 \quad ,$$

or

$$M_X^2c^4 + p_X^2c^2 = \mathcal{E}_Y^2 - 2\,\mathcal{E}_Y\mathcal{E}_P + \mathcal{E}_P^2 \qquad (5)$$

Equations (2) and (3) give

$$p_X^2c^2 = (\mathcal{E}_Y^2 - M_Y^2c^4) + (\mathcal{E}_P^2 - M_P^2c^4) - 2\cos\theta\left[(\mathcal{E}_Y^2 - M_Y^2c^4)(\mathcal{E}_P^2 - M_P^2c^4)\right]^{\frac{1}{2}}.$$

This expression together with that of Equation 4 for the momentum of the unknown particle gives

$$M_X^2 = M_Y^2 + M_P^2 - 2\,\frac{\mathcal{E}_Y\mathcal{E}_P}{c^4} + 2\,\frac{\cos\theta}{c^4}\left[(\mathcal{E}_Y^2 - M_Y^2c^4)(\mathcal{E}_P^2 - M_P^2c^4)\right]^{\frac{1}{2}}.$$

APPENDIX V. THE DEVIATION IN APPARENT SCATTERING

The expressions used in Chapters II, IV, and V, for the standard deviations in D and ϵ, are

$$\sigma(D) = 0.75\, N^{-\frac{1}{2}} D \quad ,$$

and

$$\sigma(\epsilon) = 0.75\, N^{-\frac{1}{2}} \epsilon \quad ,$$

where N is the number of independent values of D or ϵ. This expression, which is widely used, has its origin in a statement by Kendall[1] to the effect that the variance in the mean deviation (from the mean), is given by

$$\mathrm{Var}\ (D) = \frac{[\sigma(x)]^2}{N} \quad (1 - \frac{2}{\pi})$$

where $\sigma(x)$ is the standard deviation in the observable, x, and N is the number of independent observations. The observable, x, is presumed to follow a normal distribution law.

The following definitions are taken from the Mathematical Tables from the Handbook of Chemistry and Physics:[2]

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[1] M. G. Kendall, The Advanced Theory of Statistics, (Charles Griffin and Co. Ltd., London), I, 215.

[2] Chemical Rubber Company, Mathematical Tables from the Handbook of Chemistry and Physics, comp. Chas. D. Hodgeman (8th ed., Chemical Rubber Publishing Co., Cleveland, 1946), 260.

$$\sigma\text{, standard deviation} = \sqrt{\mu_2} \quad ;$$

$$\mu_2 = \text{variance} \; ;$$

D, <u>mean deviation</u> (from the mean) $= \frac{1}{N}\sum_{i=1}^{N} f_i \; |x_i - \bar{x}| \; ;$

where f_i is the weighting factor assigned to the value, x_i, and $\bar{x}$ is given by

$$\bar{x} = \frac{\sum_{i=1}^{N} f_i x_i}{\sum_{i=1}^{N} f_i} \quad .$$

Scarborough[1] shows that the standard deviation is equal to 1.2533 times the arithmetic deviation.

In this application

$$f_i = 1 \quad ,$$

$$x_i = D_i \quad ,$$

and

$$\bar{x} = 0 \quad ,$$

Therefore,

$$\sigma(D) = \sqrt{\text{Var (D)}} = \sigma(D_i) \left[\frac{1 - \frac{2}{\pi}}{N} \right]^{\frac{1}{2}} \quad ,$$

$$\sigma(D) = (1.2533)\;(0.603)\;N^{-\frac{1}{2}}D = 0.75\;N^{-\frac{1}{2}}D \quad .$$

A committee of twenty eight eminent physicists[2], has recommended that the statistical deviation be

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[1] James B. Scarborough, Numerical Mathematical Analysis, (The Johns Hopkins Press, Baltimore, 1950), 415.

[2] "Recommendations for The Standardization of Measurements in Photographic Emulsions", International Congress on Unstable Particles, (Padua, 1954), Suppl. Nuovo Cimento, ser. 9, <u>12</u>, 476, (1954).

"... taken as

$$\frac{0.75}{\sqrt{N_2}} \quad ,$$

where N_2 is the number of measurements on non-overlapping long cells".

APPENDIX VI

AN OPERATIONAL DEFINITION OF "TRUE DISPLACEMENT" DUE TO SCATTERING

Consider Equation (2-1), page 18, and Figure 6, page 16. Suppose a very large number of observed Y_i's is recorded, and suppose that by the Method of Least Squares the equation of the best fit curve is determined. This equation may be written down as

$$y = Y(X) .$$

The function on the right can, in principle, be approximated as closely as is desired. Then suppose another set of Y_i's is observed and again the equation of the best-fit curve is determined. It is assumed that the second set of observed Y_i's, within small fluctuations, yields the same equation of best-fit curve as the first set. That is, it is assumed that this equation, as determined from a single set of observed values, is reproducible.

The "true trajectory of the track" is defined as the locus of this equation. At a point where $X = X_i$, then, $y = y_i$ is, by definition, the "true displacement due to scattering", of the trajectory from the axis of the track. Now Equation (2-1), written in the form

$$\mathcal{Y}_i = Y_i - y_i ,$$

defines the contribution to the observed displacement, which is due to noise. It follows that

$$d_i = y_i - 2\, y_{i+1} + y_{i+2} \quad ,$$

and

$$\delta_i = \gamma_i - 2\gamma_{i+1} + \gamma_{i+2}$$

define the respective ith second differences due to true displacement and to noise.