AN EXAMINATION REGARDING THE CONCEALMENT OF WATERMARKS IN THE OSCILLATOR PHASE NOISE OF BINARY PHASE SHIFT KEYED SIGNALS

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Abstract: Watermarking provides an additional way to authenticate users in a stealthy way. This thesis investigates the possibility of hiding constellation dithered watermarks within oscillator phase noise of BPSK modulated signals. Message bit error rates and watermark bit error rates are compared for different phase noise standard deviations and watermark positions. Goodness of fit tests utilizing the Lilliefor's test, are conducted to determine if the investigated watermarking method can be considered stealthy.

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CHAPTER I

INTRODUCTION

Cyber security of wireless communication systems is a rising priority for many military operations. Encryption is a main source for protection, but can be costly in terms of money and other resources. Research in other cyber protection methods has grown over the last two decades. Physical layer authentication is a cyber security technique that adds an additional layer of security for wireless systems. This paper builds upon a physical layer authentication technique known as watermarking. Watermarking is a method of security that hides additional data through either multiplexing and/or embedding with a message signal [3-6].

1.1 Background

Cyber-attacks on wireless systems happen every day. With the advancement of technology and security, the types of cyber-attacks are continually changing. Denial of service (DoS) attacks aim at disrupting resources between networks and network users. Physical layer DoS attacks typically involve jamming or tampering of communication links [1]. A common attack on military networks involves a technique known as spoofing. In a spoofing attack scenario, adversaries disguise themselves as an authorized user. The main goal of a spoofing attack is to gain unauthorized access or advantages [2, 7, 8]. Once an adversary has access, they can eavesdrop on incoming and outgoing messages, manipulate and inject data to connected users, and also interfere or disrupt services to other users [7, 8].

The Department of Defense has three goals for cyber security assurance: confidentiality, integrity, and availability [9]. Confidentiality is the preventative aspect that keeps unauthorized users from gaining access to sensitive information. Integrity ensures that the data received has not been altered in any way outside of normal means. Availability is the ability to access systems or data when needed. Authenticity, a subsection of integrity, is an additional security requirement utilized by most network models [8]. The seven layers of the OSI model each have specific protocols and vulnerabilities. Past research has focused on higher layer cyber protection. Recently research has led to the focus on the physical layer, the lowest layer of the OSI model.

Physical Layer Authentication

Physical layer authentication is an emerging security measure that uses the properties of the physical layer in order to authenticate users as trusted sources [1-8]. Physical layer authentication can be used to provide an additional layer of security. Military systems could use this security measure in order to prevent and identify when an adversary is impersonating a trusted user. A number of physical layer authentication methods exist today. Physical layer authentication through fingerprinting is a method of authentication that uses characteristics of the wireless channel, or transmitter, to uniquely identify a user [12]. Another technique utilizes carrier frequency offsets and Kalman filters to determine whether or not an incoming signal is from an authorized user [27]. A method known as watermarking is the motivation for this paper.

Watermarking, a method traditionally used by media companies, adds a small signature to images or videos to prevent copyright infringement [4]. Watermarking can be defined by intentionally adding degradation to a signal with the purpose of adding a hidden signature. Historically, watermarking has been utilized in higher order layers, but has grown momentum for application on the physical layer [6]. An effective watermarking method can be characterized by stealth, security, and robustness [3-6]. Stealth is how well the watermarking method is hidden from adversaries or bystanders, security is how well a watermarking method prevents successful attacks from an adversary, and robustness is how well a watermarking method resists interference or disruption [3].

Physical Layer wireless watermarking has two main variations, constellation and baud dithering. Baud dithering involves superimposing a low power signal on top of the RF message. The superposition of the signal creates an offset in the time domain known as jitter. Constellation dithering maps a low power watermark signal to a M-ary symbol. The watermark adds or subtracts a phase and/or amplitude difference to the original message [4-6]. Ideally the watermark will appear to the adversary as typical signal noise. The research conducted for this paper focused on hiding a watermark within oscillator induced phase noise of a BPSK signal.

Oscillator Phase Noise

Oscillators are inherently imperfect. This imperfection causes signals to become noisy during modulation. Noise is typically referred to as an impairment, but has been researched to show that it is efficient at hiding messages and authenticating devices [26]. The noise added to the signal can be characterized as any undesired carrier modulation of the amplitude and phase [13-23]. An ideal oscillator distributes all of the power across one frequency. With a real world oscillator, phase noise causes the power to be distributed across several frequencies. Figure 1 illustrates the spectrum of a perfect oscillator with no phase noise, and a real world oscillator with phase noise [21].

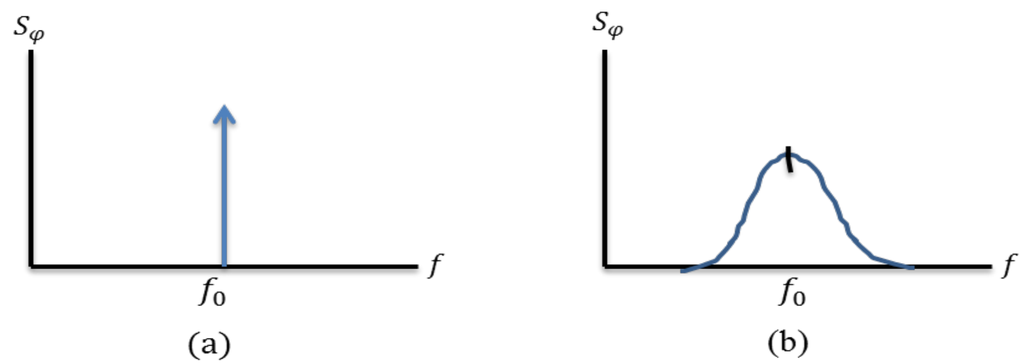

Figure 1: (a) Ideal Spectrum (b) Real World Spectrum

Phase noise causes degradation to modulated signals. Phase noise will impact each transmitted symbol equally [17,19]. The phase noise impairments cause the symbol to be shifted from its ideal symbol position. The bit error rate (BER) of the phase impacted signal will ultimately be worse than the BER of a signal with no phase noise.

Phase noise in oscillators has been known to have a cumulative nature [23]. Due to the cumulative nature of phase noise, past literature has accurately modeled phase noise as a zero-mean Gaussian process [13, 18, 20, 23-25, 29]. Figure 2 shows the constellation diagram of a QPSK modulated signal with Gaussian distributed phase noise. The transmitted symbols are distributed around the ideal symbol position. Reference [25] states that the transmitted symbols should theoretically not be shifted in the radial direction due to oscillator phase noise. Sampling imperfections of the receiver cause slight radial shifts in the constellation.

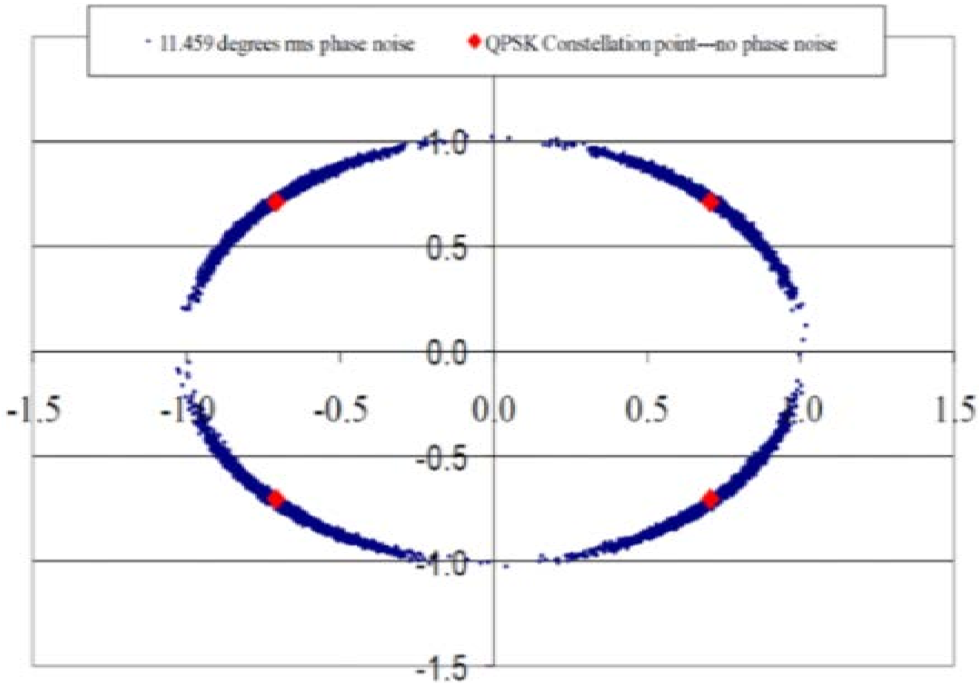

Figure 2: QPSK Constellation With Phase Noise [25]

Phase noise impact on the signal can depend on the symbol rate of the communication system. In [23], the author states the effect of phase noise on communication performance can be directly connected to the symbol rate of a system. Phase noise affects a system with a high symbol rate more than it does to a system with low symbol rate.

Amplitude noise can cause an increase or decrease to the amplitude of the signal. The power of the amplitude noise is much less than the power of the transmitted signal. According to [21] and [24], the amplitude variations, caused by oscillator noise, can be ignored due to technological characteristics of receivers.

Thermal Noise

Thermal noise at the receiver also affects the constellation and BER of the transmitted signal. Thermal noise is characterized by [31] as Additive Gaussian White Noise. Thermal noise has random properties that affect the constellation diagram of a signal. The amplitude and phase variations of the signal, at the receiver, are displayed in figure 3. The signal affected by thermal noise adds a random vector that is oriented in a random direction. The result is a received signal that is spread in amplitude and phase. Amplitude variations must be considered since the thermal noise affect on a signal at the receiver is much greater than the affect at the transmitter [14].

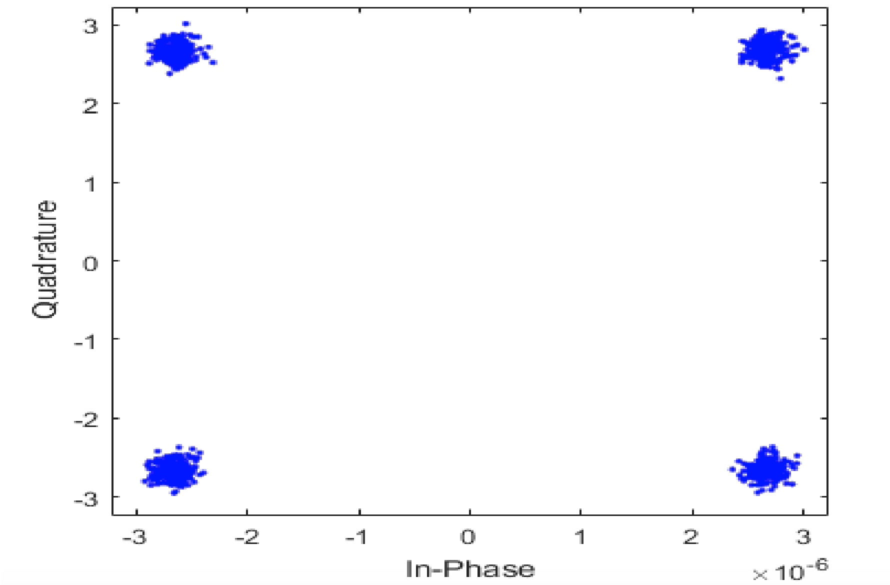

Figure 3: QPSK Signal in Thermal Noise [34]

The SNR of the signal will affect the spread of the received symbols from the ideal symbol location. When phase noise is added with thermal noise, the received signal will be spread similarly in phase as in figure 2, but will also be spread in amplitude.

1.2 Importance

The open architecture of wireless systems makes them prone to cyber attacks. Implementing traditional cyber security techniques, such as encryption, can be costly. Physical layer authentication through watermarking has recently grown to be economically feasible to implement [4]. When data is transmitted over a wireless channel, any one can access and see what information was transmitted. If a user encrypts their data being transmitted, other users may possibly see that something is transmitted, but cannot see the information within the transmission.

In many instances it may be useful to transmit data unencrypted. Watermarking could allow military users to transmit information with a hidden verification channel. The importance of hidden verification can be described in a variety of scenarios. For example, the military could have two users communicating over an unencrypted wireless channel without knowing an adversary is eavesdropping and manipulating data. If the military is using a watermark authentication scheme, as soon as information is manipulated and received by a watermark aware user, they will be able to identify that the information received is not authentic.

[4] describes a scenario in which watermarking would be useful in situations where a device is imitating an authentic TV broadcast signal. The device will mimic a TV signal to either protect itself from interference, or to prevent other Software Defined Radio (SDR) devices from utilizing other channels. If this becomes an issue, the TV broadcaster could use a watermarked signal that only their systems would be able to decode. The TV broadcast provider would be able to sense when an unauthorized device is impersonating a legitimate TV signal.

An additional way TV providers could possibly implement watermarking is for detecting unauthorized access of their Digital TV broadcast services [35,36]. The service provider could require the watermark aware set-top box (STB) receivers to send back a watermarked signal for verification. If an unauthorized user employing a universal STB sends back a signal that is not watermarked, the service provider could stop transmission to the unauthorized user.

Software Defined Radios allow the user to change frequencies on demand, without the loss of operation [32,33]. These devices analyze the environment and can decide what spectral band is best for transmission. A newer form of SDR known as cognitive radio (CR) uses spectral sensing in conjunction with communication with other CRs to estimate the best form of transmission. CR is more efficient using the spectrum and opening up frequencies for other users. With the opening of TV white spaces, which are unused spectral bands, CR can bounce from spectrum to spectrum while maintaining operation. The addition of watermarking with CR would allow for identification of unauthorized users in specific frequency bands [33].

1.3 Contribution

The purpose of this research is to examine a constellation dithering watermarking method that utilizes oscillator imperfections. One main goal of this research is to determine if it is possible to hide a usable watermark within oscillator phase noise. This paper will describe how well the watermark is hidden, and also compare this method of watermarking to previously researched methods.

In Chapter II a survey of past work will give more background on researched physical layer authentication methods. Chapter III discusses the development, testing setup, and implementation of this physical layer watermarking method. Chapter IV provides the expectations and results of the simulations conducted. Chapter V will conclude the paper with a recap and discussion of future implementations.

CHAPTER II

REVIEW OF LITERATURE

Previously, most cyber security protection primarily focused on higher protocol layers. Physical layer authentication has seen much growth in research over the past couple of years. Many variations of authentication have proved to be successful at adding an additional layer of security. Research in physical layer watermarking has recently gained momentum [4, 5]. Historically watermarking methods were utilized for copyright infringement protection. The research provided in [3] was a major contributor to the rise of physical layer watermarking interest, and added a new way for traditional watermarking techniques to be used on different types of data. The advancement of SDR technology has provided researchers with a cost efficient and practical approach to implement physical layer watermarking as a viable technique for cyber-security [4].

2.1 Previous Work

A novel approach to physical layer authentication is discussed in [3]. The author focuses on stealth, robustness, and security of the proposed method. The setup of the method utilizes a four-node scheme in a shared wireless environment. The author refers to the four nodes as Alice, Bob, Carol, and Eve. Alice is a watermark signal transmitter, Bob is a watermark aware receiver, Carol is an unaware receiver, and Eve is an active adversary that is aware of the watermark. The watermark method developed superimposes a watermark signal, known as a tag, on top of the message signal. Extra bandwidth and power are not required for this technique.

The authentication signal is sent within the same constraints as the message signal. Some of the energy is allocated from the message signal to the authentication signal. A secret key is shared between Alice and Bob. Eve tries to impersonate Alice, or disrupt the signal in some other fashion. The author uses the Lilliefors goodness of fit tests to test the stealthiness of the watermark method, by detecting anomalies between watermarked message signals and non-watermarked signals. Robustness is analyzed by determining how well the receiver can authenticate through channel and noise impairments. Security is examined by determining how resistant the scheme is to adversary attacks.

[6] discusses two watermarking schemes: constellation and baud dithering. The schemes were conducted using wireless baseband waveforms. For the constellation dithered technique, watermark data is mapped to QPSK symbols. A Gaussian distributed code is used to spread the symbols. The effect of the Gaussian distributed code on the symbol is similar to the effect of a low level AWGN signal. The result is a slight spread in the in-phase and quadrature direction. The baud dithering technique utilizes Manchester coding to map the watermarking information. The watermark appears as timing jitter. The methods are applied to an Orthogonal Frequency Division Multiplexed (OFDM) transmission. The methods discussed in [6] were developed to be used for IEEE 802.11 products. The results of [6] discovered using constellation dithering technique provided more flexibility, while baud dithering provides increased robustness and capacity.

[4] builds upon the constellation dithering technique discussed in [6]. The author employs a constellation dithering variation known as phase dithering. The method developed was tested using GNU radios. The phase dithering technique allows for more configurability with comparable performance to [6]. The author of [4] also added an additional goal of stealth and robustness. The results of [4] discovered that stealth can be increased by either lowering the

watermark bitrate, or by decreasing the phase offset of the watermark symbol. The main goal was to develop a plug and play watermarking method that did not require hardware modifications.

[27] exploits time varying carrier frequency offsets (CFO) between wireless devices for physical layer authentication. The authors describe two types of physical layer authentication approaches. Active schemes use the transmitted signal to embed identification information within the signal. The transmitted scheme is modified in order to add authentication information, similar to that proposed in [3]. The second scheme makes use of transmitter physical layer characteristics that are inherent in wireless systems. [27] indicates that transmitters and receivers each have uniquely individual carrier frequency offset values. Modeling combined CFOs as an auto-regressive random process, the author is able to integrate the time-varying CFOs in to the physical layer authentication framework. Kalman filters are used to track sequential CFO estimates which are used compare predicted values at the filter output. These values are then used as an identifier to determine whether or not the received signal is from an authorized transmitter [27].

2.2 Works Employed

The technique examined in this thesis is a phased dithering technique that utilizes inherent imperfections in oscillators to hide a watermark. The watermark technique used is comparable to the methods of [4- 6]. This paper focuses on single carriers that utilize BPSK modulation techniques. Research over oscillator imperfections conducted in [13-25] helped formulate the approach to hiding a watermark in phase noise.

Tests to determine stealthiness in this paper are similar to the tests utilized by [3]. Equations derived by [4] are used to help derive the required equations needed for estimating watermark error rates. Similarly to [5], the hope is that this method will provide additional means for authentication without the need to implement hardware changes. [32] is used to set a baseline for the amount of phase noise and symbol error rates allowed in a wireless environment.

As Mentioned in Chapter I, the main goal of this thesis is to investigate how well a watermarked signal may be hidden within oscillator phase noise. Chapter III gives an in depth description of the development, testing setup, and expectations of the watermarking method.

CHAPTER III

METHODOLOGY

Research in current and past authentication methods as well as research in to oscillator imperfections laid the groundwork for this thesis. The purpose of this section is to describe the objective, development, implementation, and testing setup, of the investigated physical layer watermarking technique.

3.1 Objective

The objective is to investigate a physical layer watermarking method that is stealthy. The need for alternative stealthy authentication methods guided the idea of hiding a phase dithered watermark within oscillator phase noise. Some past research focused on using oscillator imperfections as uniquely identifiable signatures. Practically no research exists for hiding watermarks within these oscillator imperfections [27]. Along with investigating a stealthy watermarking method, another objective is to keep the error rates for the message and watermark within range of the error rates described in [5] and [25].

Tools that could record the error rates, test for stealth, and produce a visual representation of the transmitted and received signals were needed in order to analyze the watermarking method. The need for these tools led to use of Matlab. Matlab allows the feasibility of creating customizable programs that meet the user's needs and objectives. Another aspect of Matlab that makes it very useful is the built in functions and toolboxes that allow for real world scenario simulations. In addition to Matlab, Mathcad will be used to help derive the bit error probabilities for the message signal and the watermark.

Since there is not any previous research over the specific method considered, it is intended that this watermarking technique be a model for future research and development. Similar tests and simulations conducted by [3-6] will be used to develop the results for this method.

3.2 Development

To help simplify the development of this technique, this thesis looks at implementation of the watermark technique with a single carrier. The amount of phase noise allowed by a system is important in regards to how it affects bit recovery. According to [28] the relative constellation error for a BPSK modulated signal, with a coding rate of 1/2, is -10dB. The relative constellation error is calculated for an OFDM transmission over the average of several subcarriers and is defined below. The relative constellation error, also known Error Vector Magnitude (EVM), is a data rate dependent measure of modulation accuracy. EVM encompasses phase noise, additive noise, and many other imperfections that may arise from modulation at the transmitter.

Relative Constellation Error (dB)	Modulation	Coding Rate (R)	Data Rate (Mb/s) (5MHz Channel spacing)
-5	BPSK	1/2	1.5, 2.25, 6, 9, 12, 13.5
-8	BPSK	3/4	
-10	QPSK	1/2	
-13	QPSK	3/4	

Table 1: Relative Constellation Error for Different Modulation Schemes [28}

EVM can be defined as the ratio of the amplitude of the RMS error vector to the amplitude of the ideal vector [29]. The units for EVM are typically defined in decibels or percentages. For a BPSK modulated signal, -5 dB is equivalent to 56.23%. The plot displayed in figure 4 gives the EVM percentage vs. RMS phase noise. As the phase noise increases, the EVM percentage also increases. Increasing the SNR will decrease the EVM percentage. The 56.23% EVM for a BPSK signal is relatively high but would still allow for recovery of the transmitted message [29]. The SNR and phase noise will be an important aspect of the testing setup, which will be described in section 3.3.

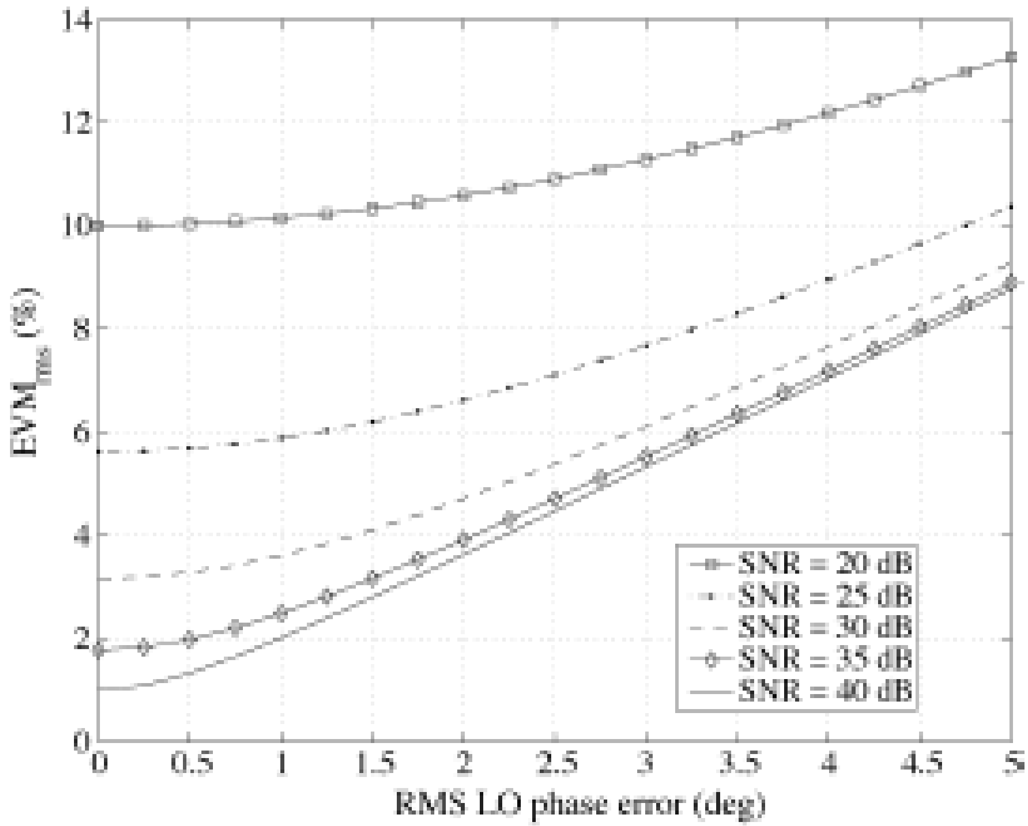

Figure 4: EVM(%) vs. RMS Phase Error (Noise) [29].

Software radios frequently use OFDM in conjunction with a variety of modulation techniques. The ability to switch between modulation techniques allows for SDR to be widely configurable and less prone to interference [4]. The development of SDR technology also makes it possible to easily implement watermarking schemes.

Watermarking Method

This thesis examines how well a phase dithered watermarking method can mask itself within oscillator phase noise. Equation 3.1 gives a general form for the phase dithered watermarking method. The message m_i is composed of a bi-polar bit stream, of amplitude +A and –A. The bit stream is modulated with a sinusoid carrier wave. The carrier wave has a phase that varies due to oscillator imperfections which is represented by $\theta(t)$. The variation of phase noise for this investigation will assume to be constant for the duration of the symbol, and will vary symbol to symbol. The watermark will also contain bits of information that only the transmitter and watermark aware receiver should be aware of. The watermark bits are set at a certain degree away from the ideal transmitted symbol, and are represented by ϕ_w.

$$x(t) = m_i * \cos(2\pi f t + \theta(t) + \phi_w) \tag{3.1}$$

A phasor vector representation of the watermark method is shown in figure 5. The addition of the watermark will slightly expand the total degree of phase noise effect on the constellation diagram. In order to keep the watermark hidden, the amount of signal degradation must be small enough that no anomalies can be detected. This thesis focuses on exploring this watermarking technique with BPSK modulated signals. Watermarking has shown in the past that it can potentially be used with higher order modulation schemes [3-6].

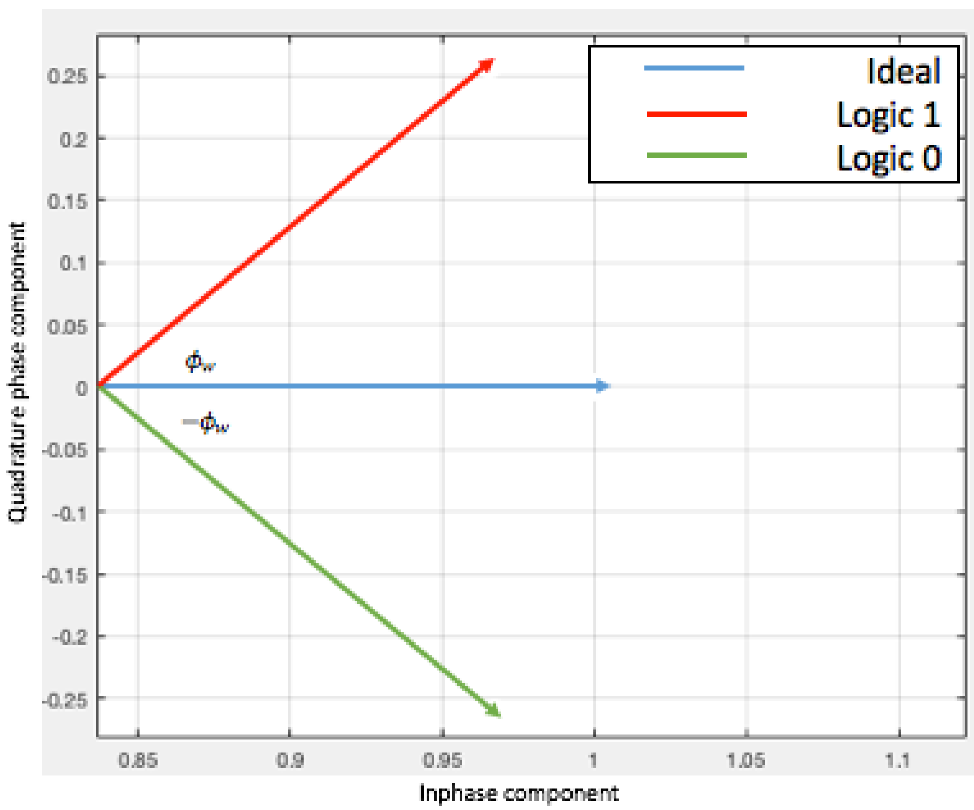

Figure 5: Watermark Phasor Representation

Binary Phase Shift Keying (BPSK)

BPSK is a modulation technique that uses two symbols to define data. The message signal is multiplied by an oscillator-produced sinusoid. A logic 1 symbol can be defined as data with real components greater than zero, while a logic zero can be defined as the real components that are less than zero. Each symbol contains one bit. The carrier wave should ideally have a phase that, in a diagram such as figure 5, does not vary. The constellation diagram of a BPSK signal is represented in figure 6. The introduction of phase noise causes the phase of the carrier wave to vary about the ideal phases. A result of the carrier phase offset causes a slight change in the symbol position. If the transmitted symbol is shifted greater than 90 degrees away from the ideal symbol, it could cause the receiver to make a decision error, depending on the effect of system noise. The message signal BER equation has been derived by several sources and can be calculated with the Q function.

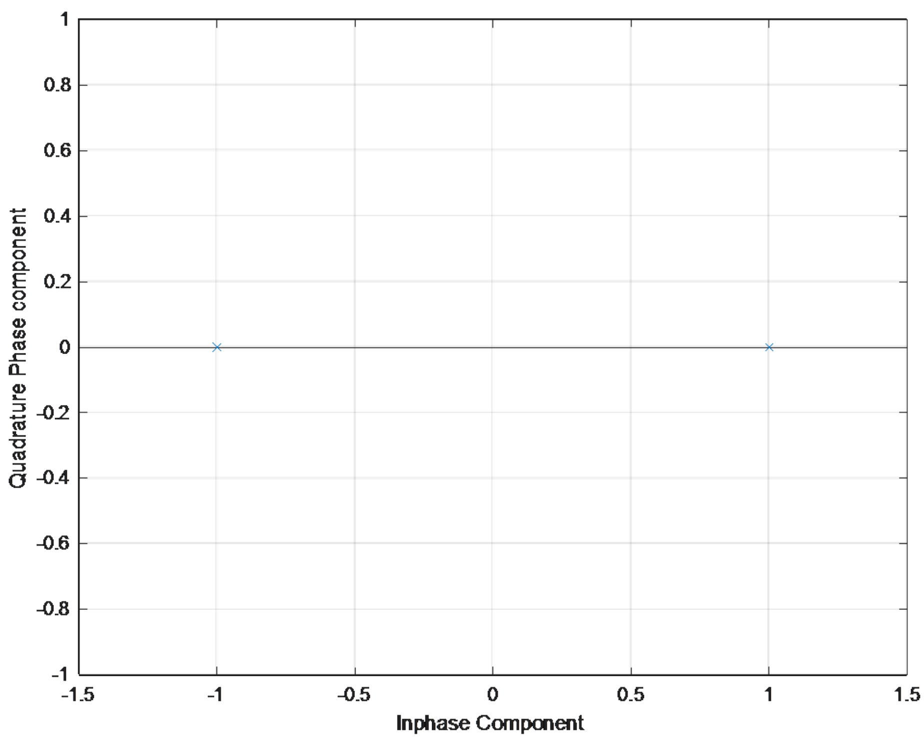

Figure 6: BPSK Constellation Diagram

Equation 3.2 is the bit error probability for a non-watermarked BPSK signal in thermal noise. The equation is derived by utilizing a Matched Filter with coherent detection at the receiver. A matched filter is used in order to maximize the SNR at the bit detector. SNR is a factor in determining the amount of signal degradation, caused by noise, that will occur during transmission. The Qfunction calculates the tail probability of a Gaussian distribution. E_b represents the energy per bit transmitted. N_0 is the noise power spectral density of the thermal noise. For coherent Matched Filter detection, SNR relates to $\frac{E_b}{N_0}$, by a factor of 2, meaning that the SNR is equal to the ratio of twice the energy per bit to noise power spectral density. The lower the SNR the more degradation occurs, which will affect the receiver's ability to determine the correct bit, and ultimately lead to a higher bit error rate.

$$P_b\left(\frac{E_b}{N_0}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{3.2}$$

Watermarked BPSK

The watermarked message signal will contain a logic one or a logic zero for each BPSK bit transmitted. A logic one for a watermarked signal could be any quadrature component greater than zero while a logic zero could be any quadrature component less that zero. Figure 7 gives an example of a constellation diagram for a watermarked BPSK signal with the watermark bits included.

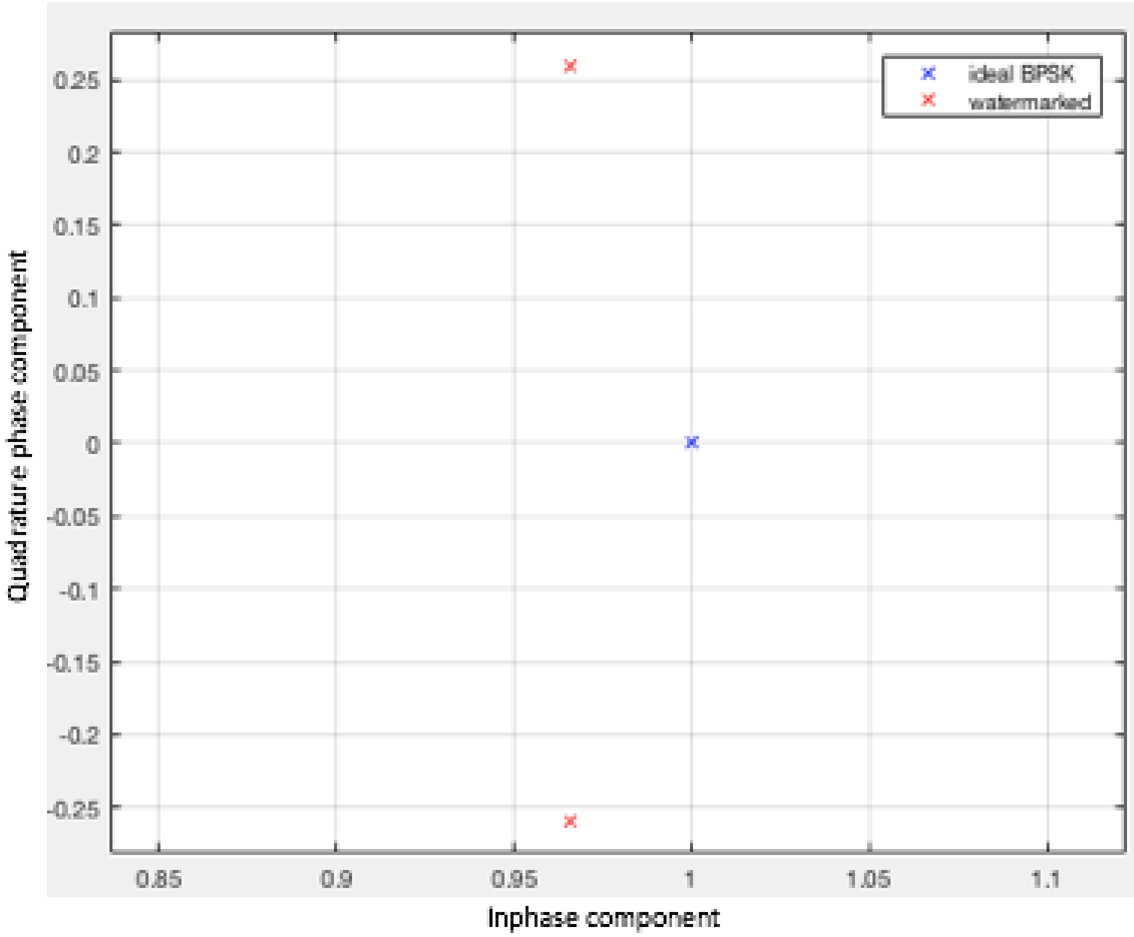

Figure 7: Watermarked BPSK Constellation Diagram

The watermark will ideally be hidden within the transmitter oscillator phase noise. The distribution of phase noise with respect to the watermark will be Gaussian with zero-mean. The transmitted watermarked signal will look similar to a transmitted signal with just phase noise, but will have symbols transmitted further away from the ideal signal. Figure 8 gives a comparison of constellation diagrams of a standard BPSK signal with phase noise only, and watermarked BPSK signal with combined phase noise.

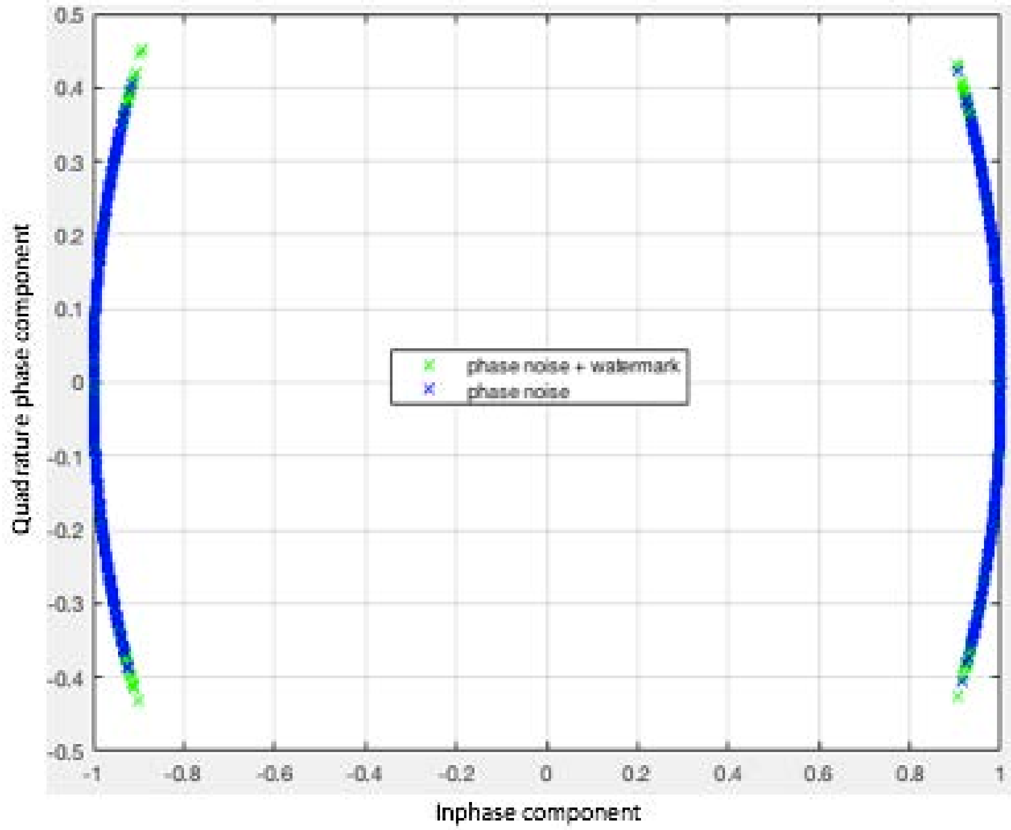

Figure 8: BPSK Constellation Diagram Comparison

The watermark will give the appearance of additional phase noise. The added degradation caused by the watermarking method will cause the message signal BER to be slightly worse than the message signal with just phase noise. Chapter 4 will provide expectations and estimations used to calculate the watermark error probability and the message error probability of the watermarked signal.

3.3 Implementation

The following section depicts the implementation of the watermarking method. Matlab will be used to simulate a transmitter and receiver.

The block diagram shown below gives a quick overview of the watermarking method simulated in Matlab. Random equally likely positive and negative ones will be generated to simulate the message signal. Gaussian distributed random numbers will be used to generate the phase noise and thermal noise. An array will be composed to generate the watermark for the simulation. Standard functions within Matlab will be used in order to formulate the necessary equations for simulation. Bit errors will be calculated and stored by Matlab. All results will be depicted and illustrated through Matlab. Built in Lilliefor's test functions will be used to determine the stealthiness of the proposed watermarking method.

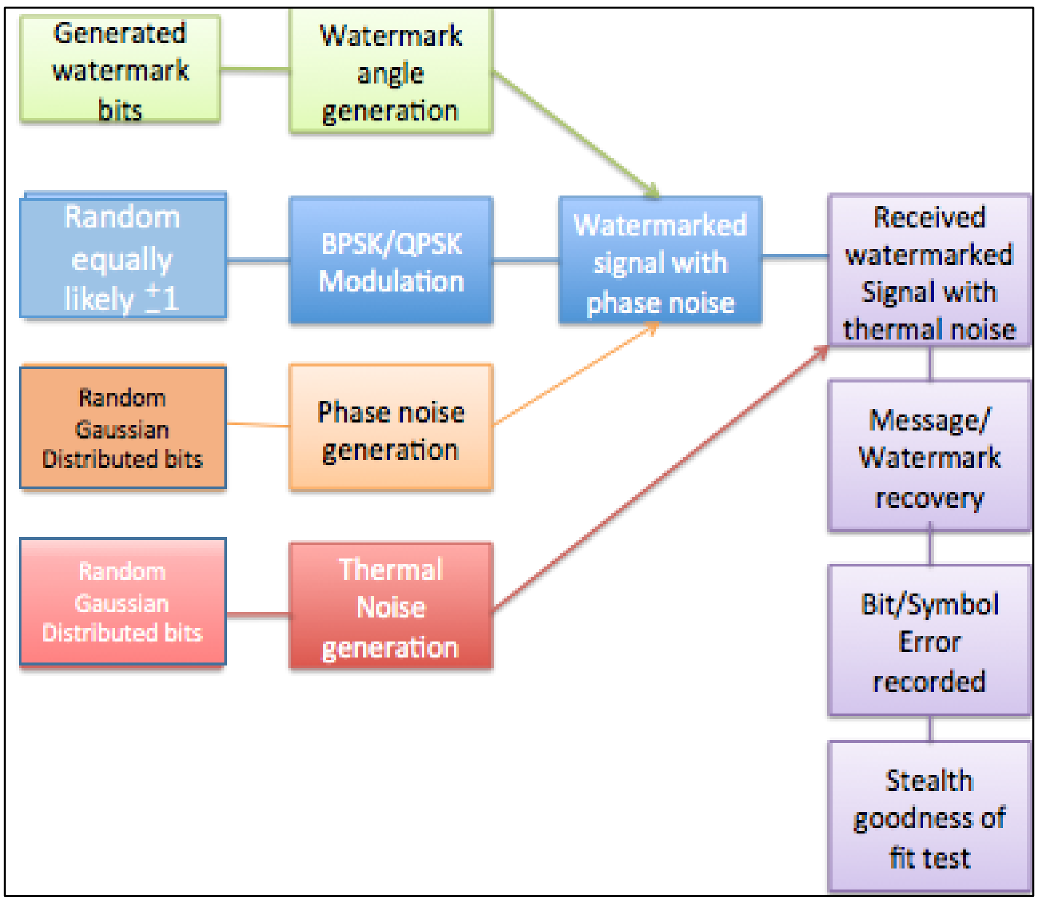

Figure 9: Block Diagram of Simulation

The random message signal will be modulated with a BPSK scheme. Phase noise will be introduced to the modulated signal. The watermark will be hidden within the phase noise and will be set a certain distance from the ideal symbol. Once the message signal has been multiplied by the carrier signal, thermal noise will be inserted. The signal will be represented as the received signal. A matched filter will be used to maximize the signal to noise ratio. An algorithm will be used to simulate a bit detector. The bit errors will be recorded and plotted against the SNR at the matched filter output. Another algorithm will be used to do the bit detection for the watermark bits. The next section goes more in to detail of simulations for these experiments.

3.4 Testing Setup

As mentioned in section 3.3 Matlab will be used to generate $2 * 10^6$ random bits, of equally likely positive and negative ones. The simulation will use a 1:1 ratio for the watermark bits meaning there will be one watermark bit per bit for BPSK message signal. The investigated watermarking method will simulate with different watermarking angles. The watermark angle ϕ_w, will be simulated at 1, 5, 10, and 15 degrees away from the ideal symbol. These watermark angles will also be simulated with different phase noise standard deviations. The Phase noise standard deviation will be tested using 3, 5, 11, and 18 degrees, similarly to the simulations in [25].

The range of SNR maximized by the matched filter, $\frac{E_b}{N_0}$ will be controlled from 0 to 15 dB. The bit errors will be recorded for each simulation conducted and compared to theoretical values. The BER plots will help determine the amount of degradation caused by the inserted watermark. The plots will also determine the probability of recovering the watermark and the message signal.

Noise has been described as coming from certain distributions with both known and unknown properties [3]. Statistical methods may be used to predict most likely outcomes for Noise. Statistical methods can also be used to determine whether or not there are anomalies within a signal, by looking at the signal's noise properties. In order to test the stealth of the proposed watermarking method, the cumulative density function of the combined phase noise and watermark will be compared to cumulative distribution function of just the phase noise. The built in Lilliefor's function in Matlab will be used to conduct a goodness of fit test. In order to pass the goodness of fit test, the CDF of the combined noise and watermark must show no obvious anomalies when compared to the CDF of just the phase noise. If the watermarked signal passes the goodness of fit test, the method proposed can be considered stealthy. Testing the method with different degrees of phase noise and watermark positions will allow for a better understanding of the interplay of these two parameters.

The next chapter will present the expectations and results of the simulations conducted for this watermarking method

CHAPTER IV

FINDINGS

The following chapter discusses the expectations, results, and analysis of the investigated watermarking method.

4.1 Expectations

Methods for calculating error rates for BPSK signals have been extensively researched in the past [5]. The Bit Error Rate will be used to determine the effect the watermarking method has on the transmitted signal. The watermark error rate will be used to determine the robustness of the watermarking method. The purpose of this section is to describe the experimental and theoretical outcomes of the explored technique.

Theoretical Watermarked BPSK

From the previous chapter, we know the theoretical bit error rate of a non-watermarked BPSK message can be described by the expression:

$$P_b\left(\frac{E_b}{N_0}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{4.1}$$

The addition of phase noise adds more degradation to the transmitted signal. A BPSK signal with phase noise that has little variance will follow closely to the ideal approximation, but a signal disturbed by phase noise with high variance will be much worse than the ideal case [25].

Including a watermark within phase noise adds slightly more degradation to the signal than with just phase noise alone. Closed form expressions for the BER of a watermarked BPSK signal were derived in [5]. In order to derive the bit error probability for a watermarked signal disturbed by phase noise and thermal noise, a PDF transformation must be derived with respect to the distance from the threshold. Mathcad was used to perform the necessary derivations. Equation 4.2 is the bit error probability equation of a watermarked BPSK signal with additional phase noise. The equation can also be considered as the message bit error probability equation for a non-watermark aware receiver.

$$P_b\left(\frac{E_b}{N_0}\right) = Q\left(\frac{\sqrt{2}\,\mu_{tot}}{\sqrt{2\sigma_{tot}^2}}\right) \tag{4.2}$$

Where μ_{tot} is the sum of the mean of thermal noise and the mean of the mapped PDF composed of the watermark and phase noise. The variance, σ^2_{tot}, is the sum of the variance of the thermal noise and the variance of the PDF composed of the watermark and phase noise. The derivations of the PDF mapping are found in appendix A.

The theoretical message BER plot for a BPSK signal can be found in figure 10. Included in the plot are theoretical values for a watermark 1,5,10, and 15 degrees^2 away from the ideal signal with a phase noise standard deviation of 5 degrees. Also included in the plot is the message BER of the ideal BPSK signal with no watermark or noise. From the theoretical plot, we can expect the BER to worsen as the watermark position moves further away from the ideal bit. This is expected due to the phase noise and watermark spreading of the transmitted signal.

The watermark recovery is dependent on the watermark bit error rate. To derive the watermark error probability, another PDF transformation must be derived. The equation for the watermarked error rate is the similar to equation 4.2, but the PDF is mapped with respect to the distance from the real axis. The average distance away from the real axis is found by combining the mean of the mapped PDF of the combined phase noise and watermark with the mean of the thermal noise. Equation 4.3 is used to describe the watermark error probability.

$$P_w\left(\frac{E_b}{N_0}\right) = Q\left(\frac{\sqrt{2}\,\mu_{wmtot}}{\sqrt{2\sigma_{wmtot}^2}}\right) \tag{4.3}$$

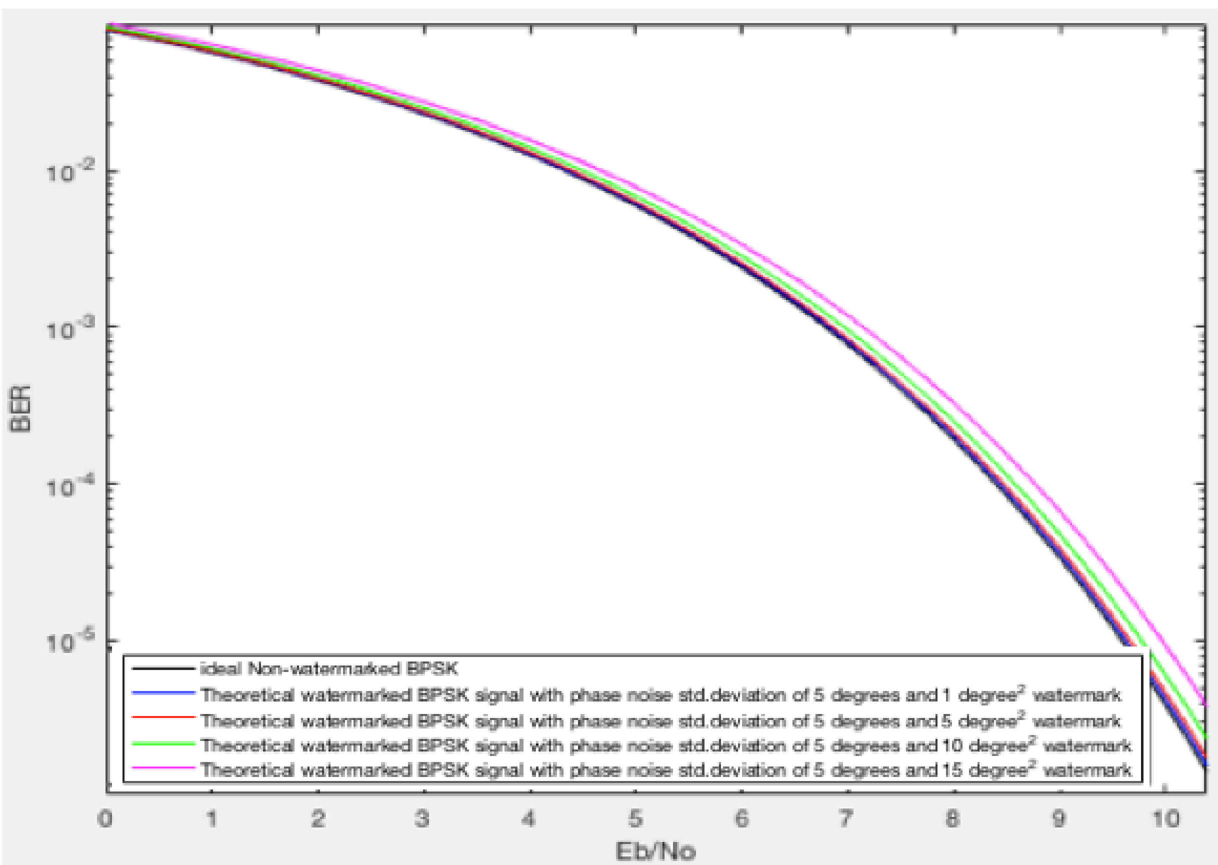

Figure 10: Theoretical BPSK Message BER For Non-watermark Aware Receiver

As stated in section 3.2, the real axis will be the decision boundary for whether the watermark is a logic 1 or a logic 0. The theoretical watermark error probability plots for a watermark aware receiver with a phase noise standard deviation of 5 degrees is illustrated in figure 11. The plots include watermarks at 1,5,10, and 15 degrees^2 away from the ideal signal.

From the theoretical plots, there is a distinct relation to watermark position and watermark error rate. The further the watermark is from the ideal symbol, the better chance of watermark recovery. This relationship is inversely related to the message bit error rate, meaning the message BER will worsen as the watermark vector increases.

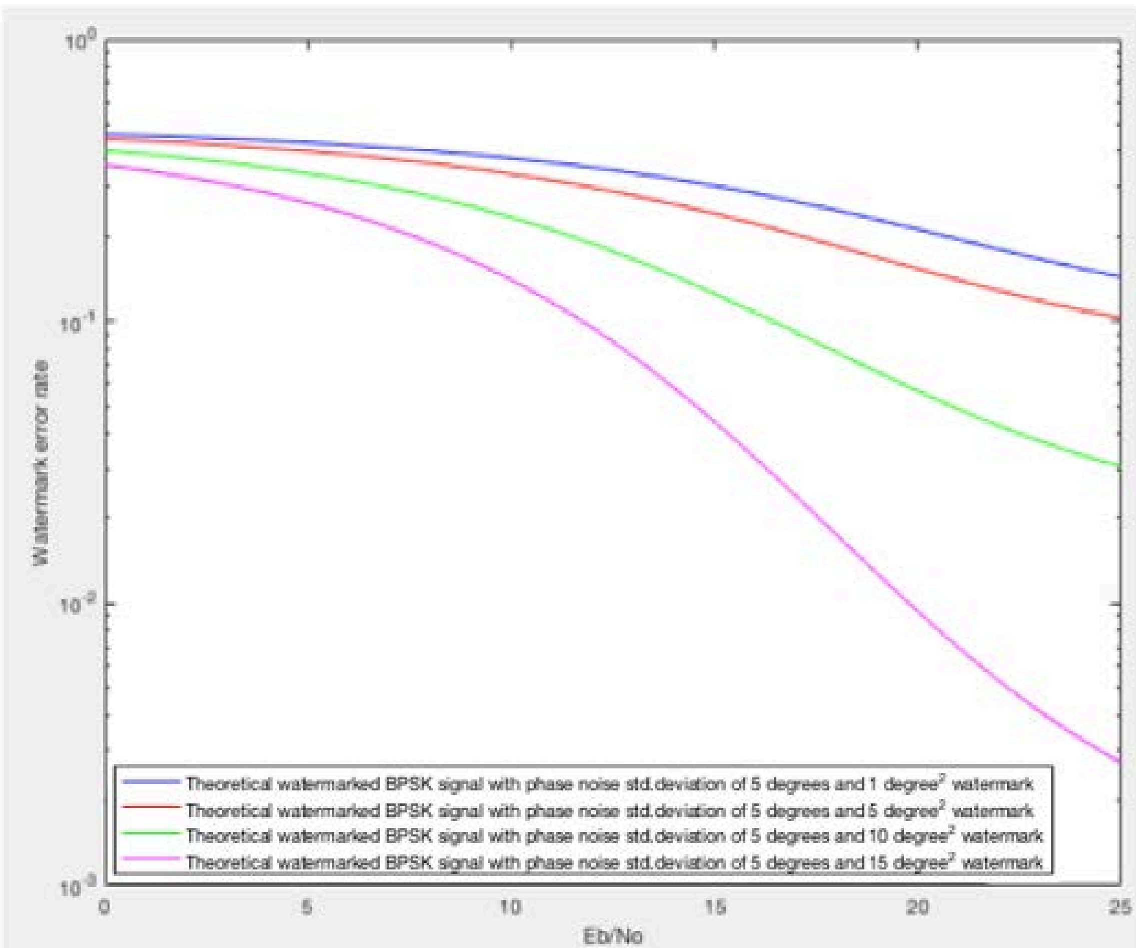

Figure 11: BPSK Theoretical Watermark Error Rate For Watermark Aware Receiver

Figure 12 and 13 theoretically depict how different oscillator phase noise standard deviation affects a single watermark value. A watermark value of 5 degrees^2 is tested with phase noise standard deviation values of 3, 5, 11, and 18. From the theoretical plots we can expect that increasing phase noise should decrease the simulated message BER while at the same time degrading the watermark error rates. To generate these curves, the same PDF mapping derivations used for the previous message and watermark error rate curves were used for the following plots.

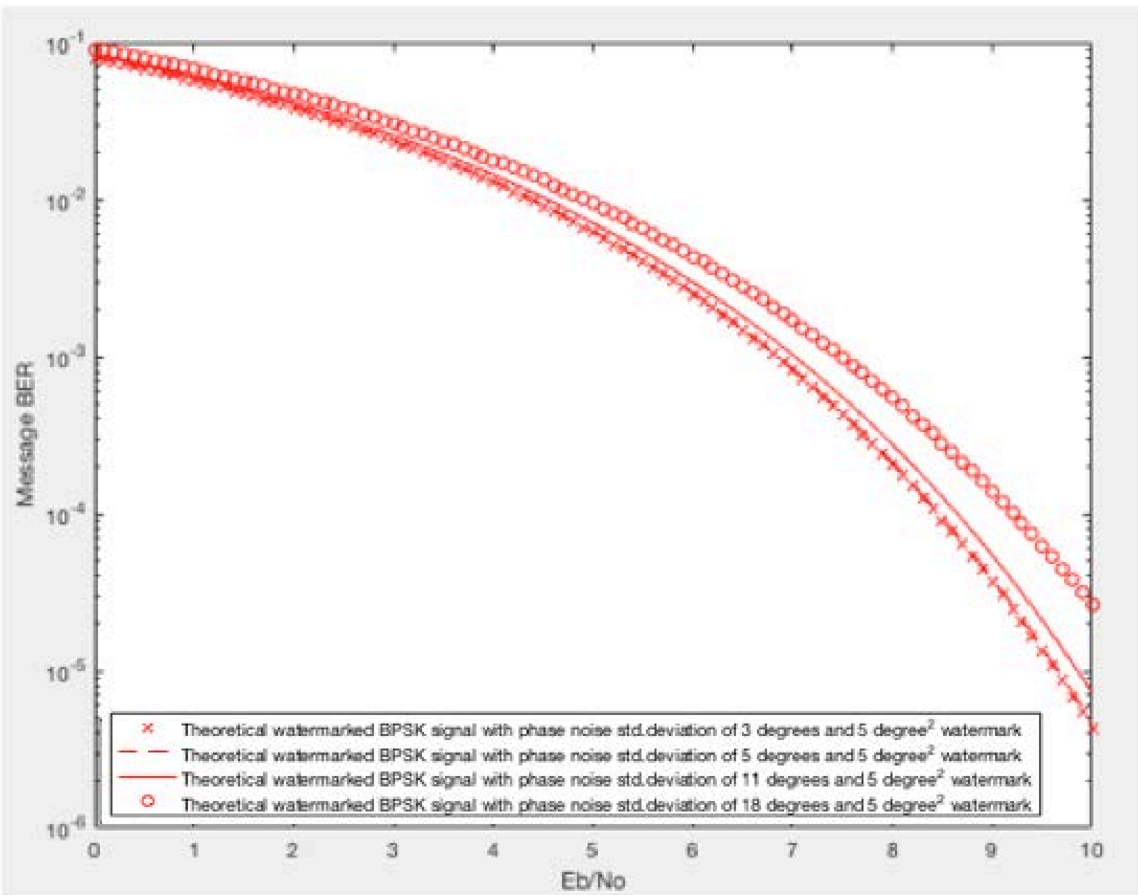

Figure 12: BPSK Theoretical Message BER For Non-Watermark Aware Receiver with varying phase noise standard deviation

We can see that theoretically the message BER will increase from a factor of 10^{-6} to a factor 10^{-5} ,at a SNR of 10 dB, as the phase noise standard deviation increases from 3 to 18 degrees.

The watermark error rate will also degrade at 10 dB when the phase noise standard deviation increases.

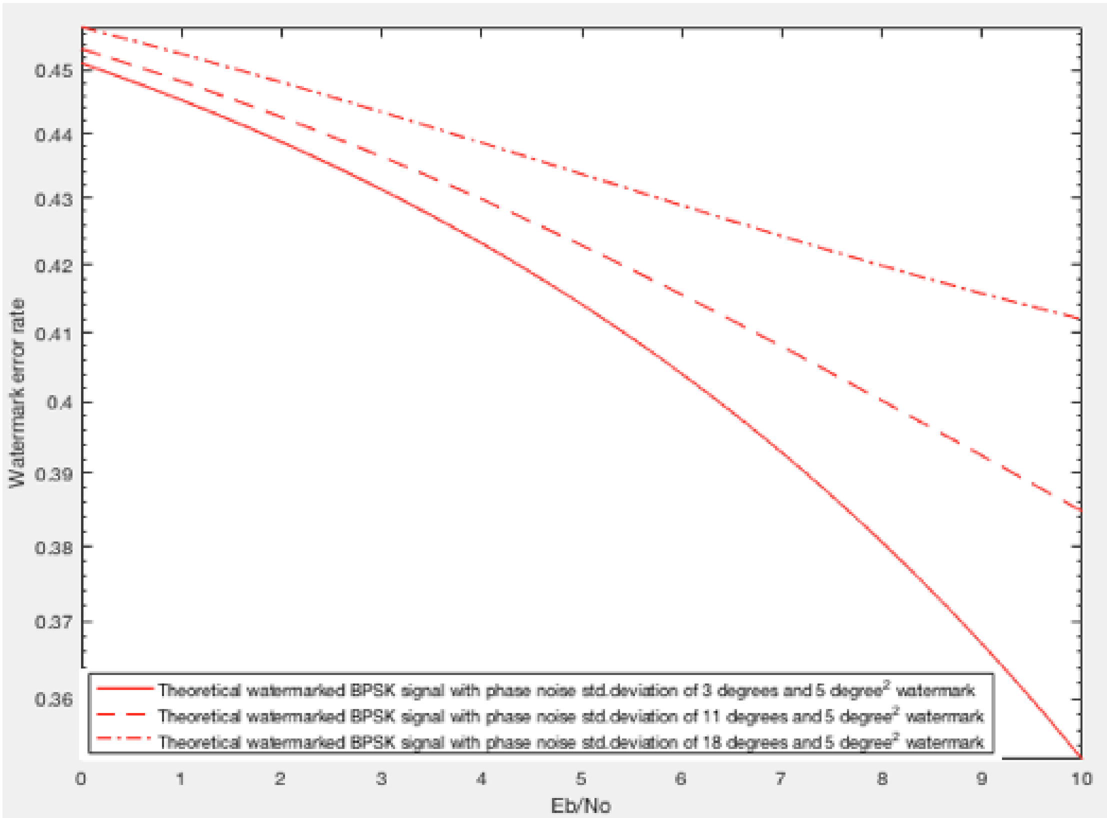

Figure 13: BPSK Theoretical Watermark Error Rate For Watermark Aware Receiver with varying phase noise standard deviation

4.2 Results

The following section illustrates the results of the conducted simulations. As mentioned in section 3.4, Matlab is used to generate the BPSK watermarked signals. The simulations are compared with the theoretical plots composed from section 4.1.

Watermarked BPSK Results

Using $2 * 10^6$ bits, the transmitted signal affected by oscillator noise and the watermark is shown in the following figures. Figure 14 displays transmitted BPSK constellation diagram for different watermark positions combined with a phase noise standard deviation of 5 degrees.

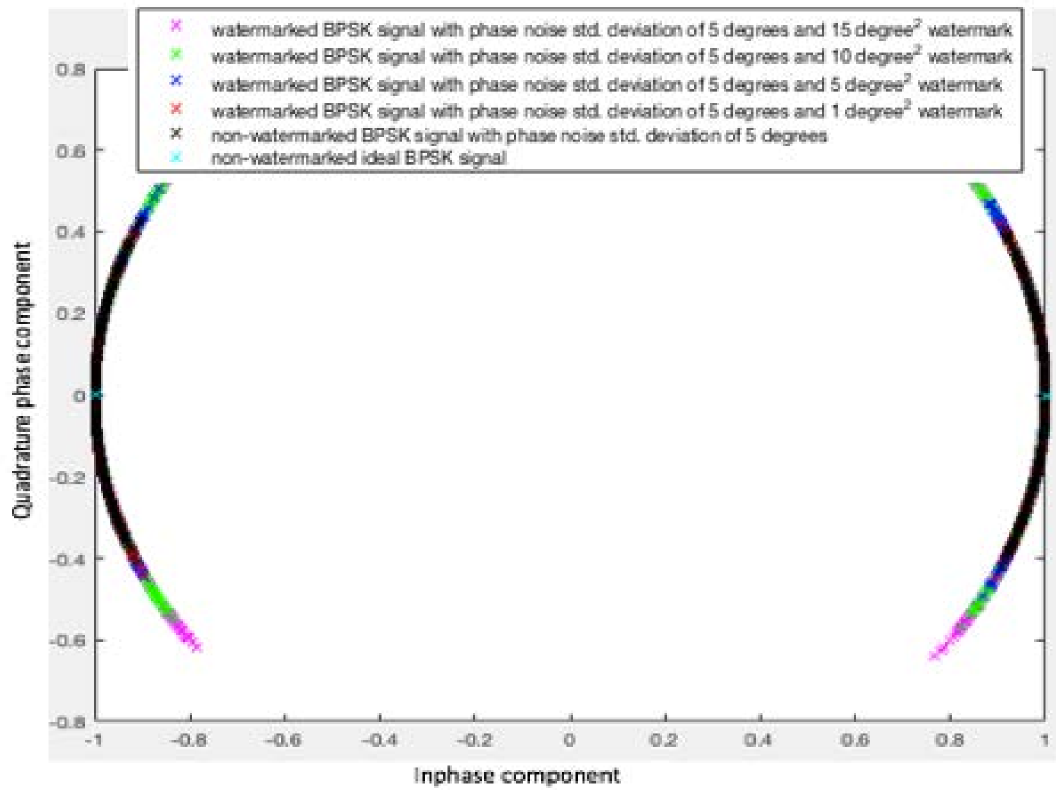

Figure 14: Transmitted BPSK Constellation Diagram with non-watermarked ideal transmission, non-watermarked transmission with phase noise, and watermarked transmission combined with phase noise at different watermark positions.

Figure 15 illustrates the received BPSK constellation diagram with different watermark positions combined with a phase noise standard deviation of 5 degrees and thermal noise. From the constellation diagrams, it is obvious that the combined watermark and phase noise causes additional degradation to the BPSK signal.

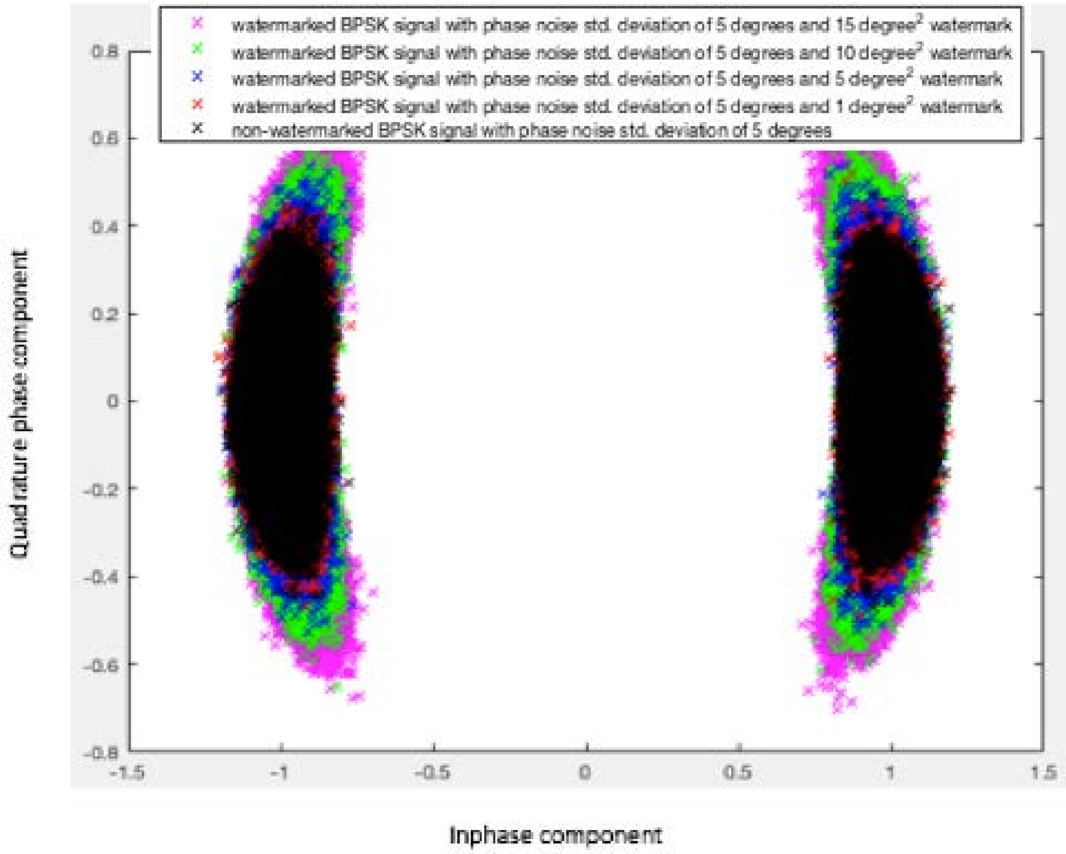

Figure 15: Received BPSK Constellation diagram of non-watermarked transmission with phase and thermal noise, and watermarked transmission combined with phase noise at different watermark positions.

The message bit error rate from simulations follow very closely to the approximations as shown in the figures below. The figures show the message BER curves of simulated watermarked signals with oscillator phase noise standard deviation of 5 degrees. In order to get a better idea of what is happening to the message error rate, the simulated and theoretical Message BER for watermarked signals with 1 and 15 degree^2 watermark combined with phase noise standard deviation of 5 degrees are plotted in figure 17. In terms of message BER, the simulations show that adding a 1 degree^2 watermark within oscillator phase noise does not significantly alter the message BER compared to a non-watermarked BPSK signal with phase and thermal noise alone.

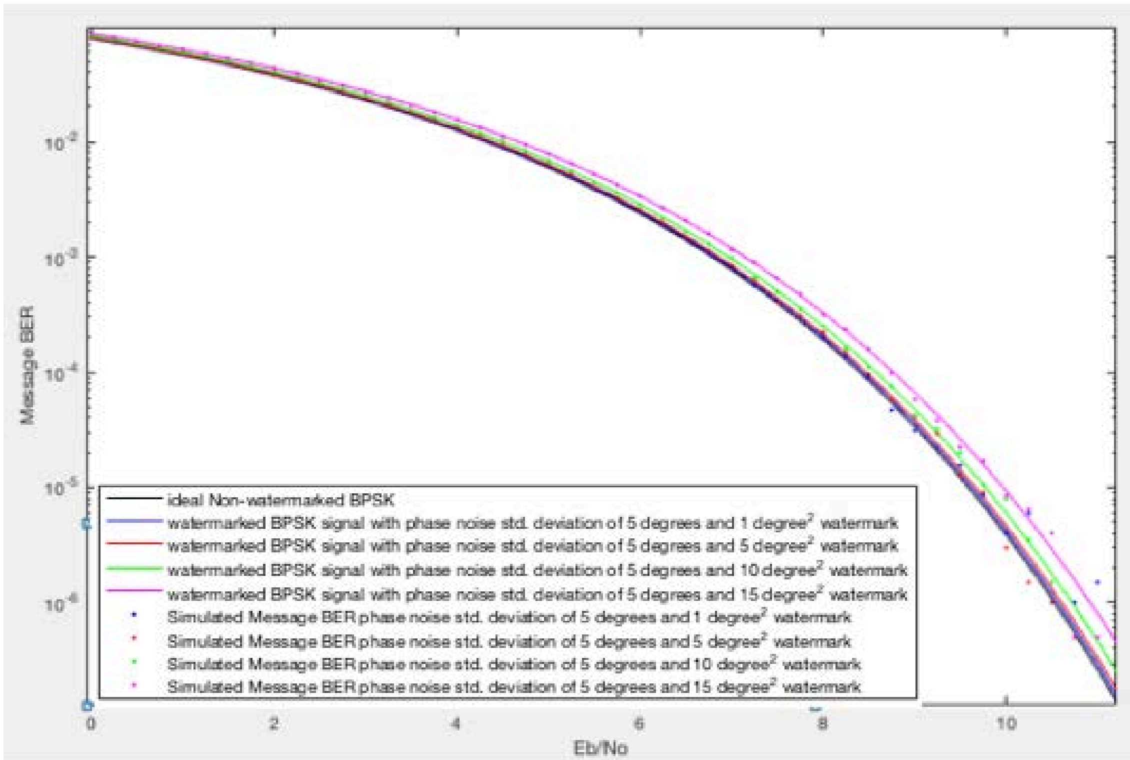

Figure 16: Simulated vs Theoretical watermarked BPSK Message BER for Non-Watermark Aware Receiver

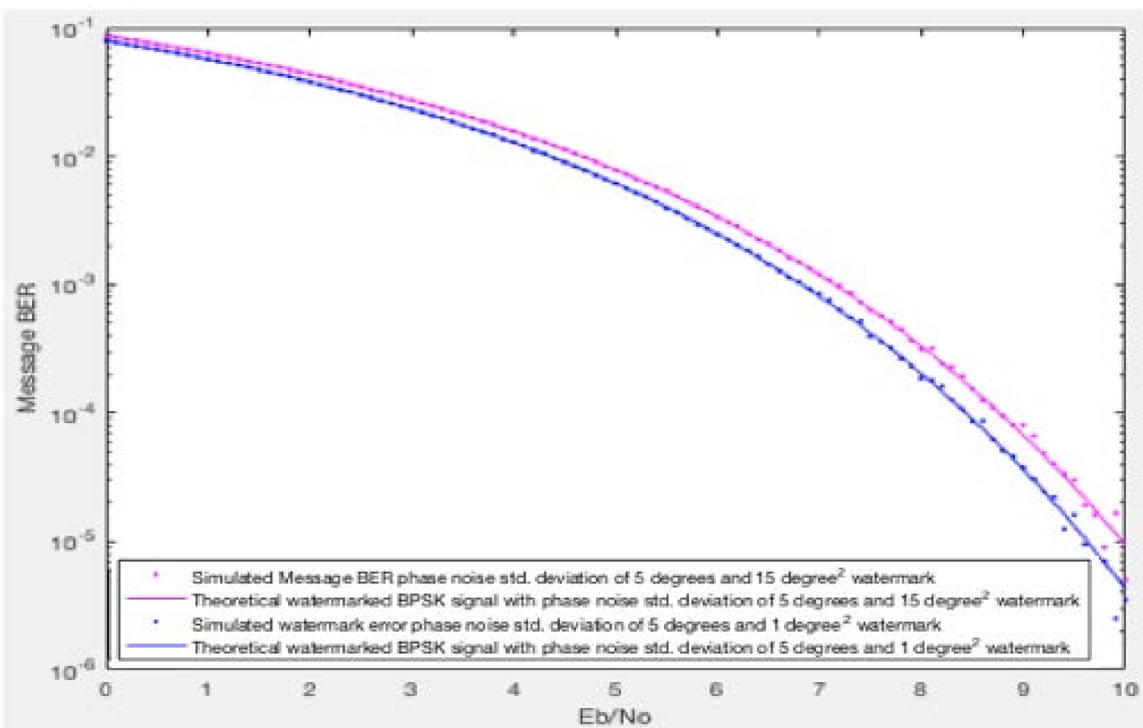

Figure 17: Simulated vs Theoretical 1 and 15 degree^2 watermarked BPSK Message BER for Non-Watermark Aware Receiver

Furthermore, it can be shown that the watermark placed 15 degrees^2 from the ideal symbol has more of a noticeable effect on the message BER, while increasing the watermark distance improves the watermark error rate.

Figure 18 displays the simulated watermark error rates. As mentioned above, we expected the watermark error rate to improve as the watermark angle increases. When the watermark angle increases, the overlap between the PDFs of the logic 1 and logic 0 watermark decreases. When this happens, the probability of an error decreases which improves the error rate.

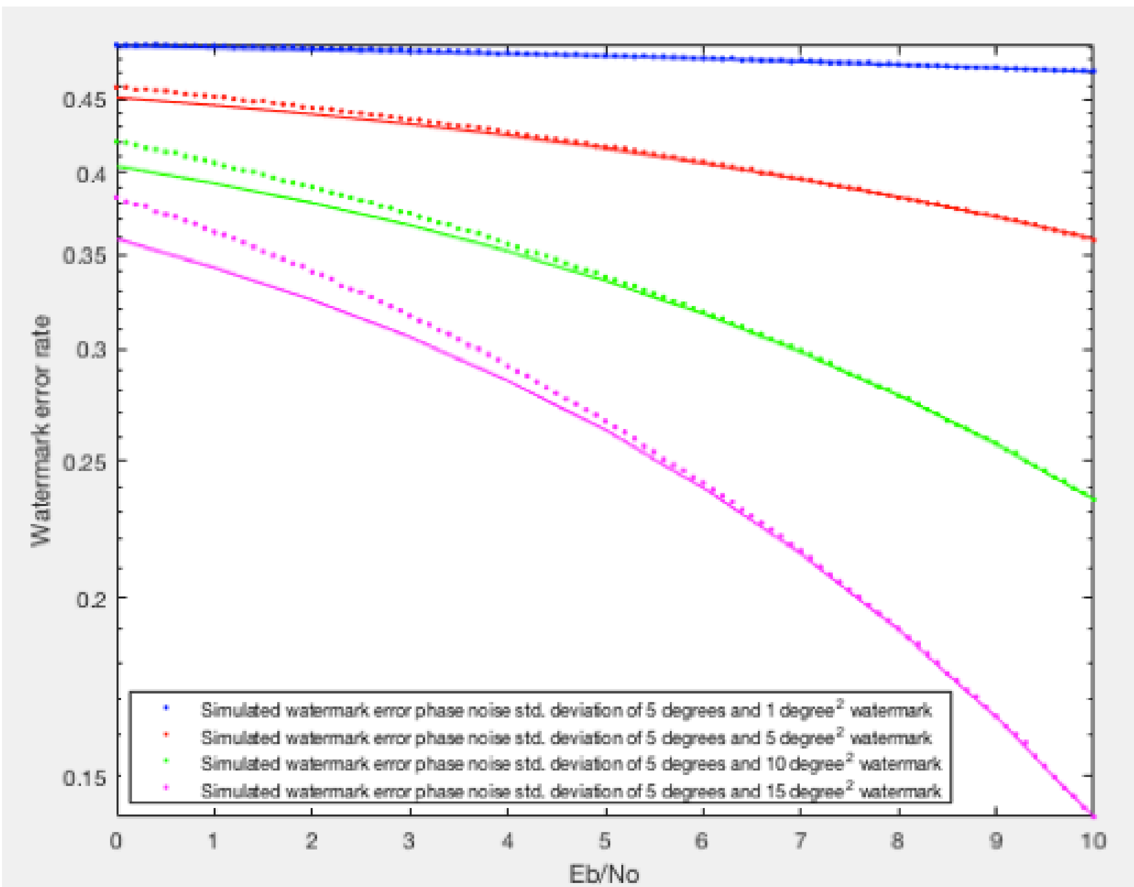

Figure 18: Simulated vs Theoretical Watermarked BPSK Watermark Error for Watermark Aware Receiver

Increasing the phase noise while keeping the watermark angle constant has a similar effect on the message BER as increasing the watermark distance while keeping the phase noise standard deviation constant. Figure 19 illustrates the simulated message error rate for phase noise standard deviation values of 3, 11, and 18 degrees combined with a constant watermark angle of 5 degrees^2.

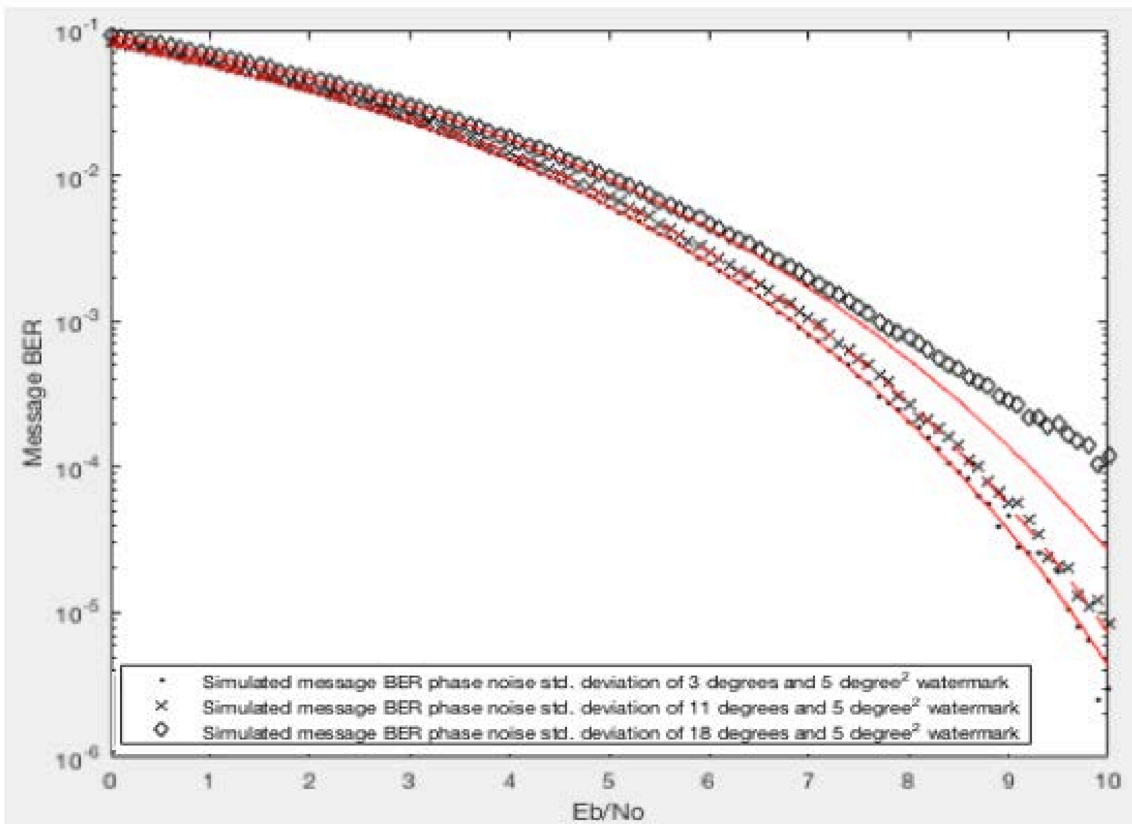

Figure 19: Simulated vs Theoretical 3,11, and 18 degree phase noise standard deviation with 5 degree^2 watermark for watermarked BPSK Message BER for Non-Watermark Aware Receiver

The impact of increasing the oscillator phase noise on the watermark error rate is illustrated in figure 20. The simulations agree fairly well with the theoretical plots and show that an increase in phase noise standard deviation affects watermark recovery.

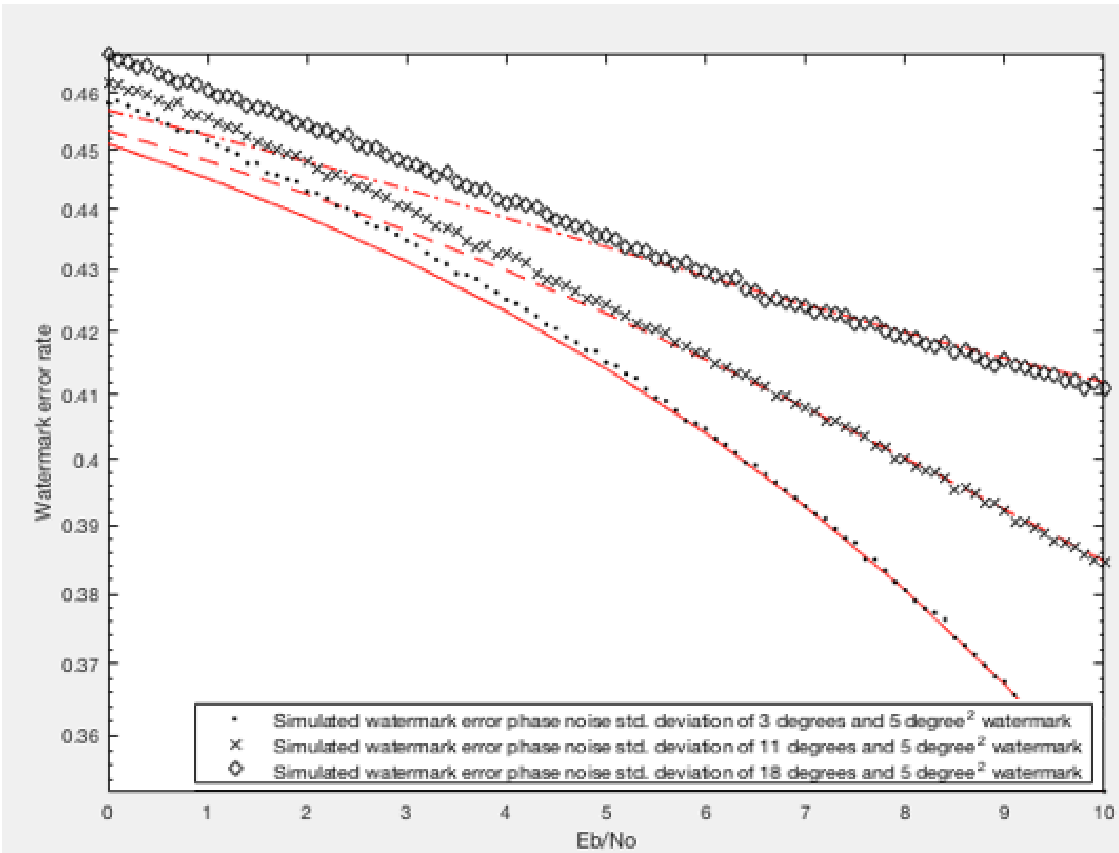

Figure 20: Simulated vs Theoretical 3, 11, and 18 degree phase noise standard deviation with 5 degree^2 watermark for BPSK Watermark error for watermark aware receiver

In order to determine whether or not hiding a watermark within oscillator noise can be stealthy, it must pass the Lilliefor's test, which compares the CDF of the total noise and the CDF of the phase noise. Anomalies must not be detected in order for the investigated method to be considered stealthy.

The figures below illustrate the CDF's at different oscillator phase noise standard deviation values, as well as different watermark positions. The built-in Lillietest() function is used in Matlab simulations to determine the stealthiness. If the CDF plot passes the Lilliefor's test, Matlab will return back a 0.

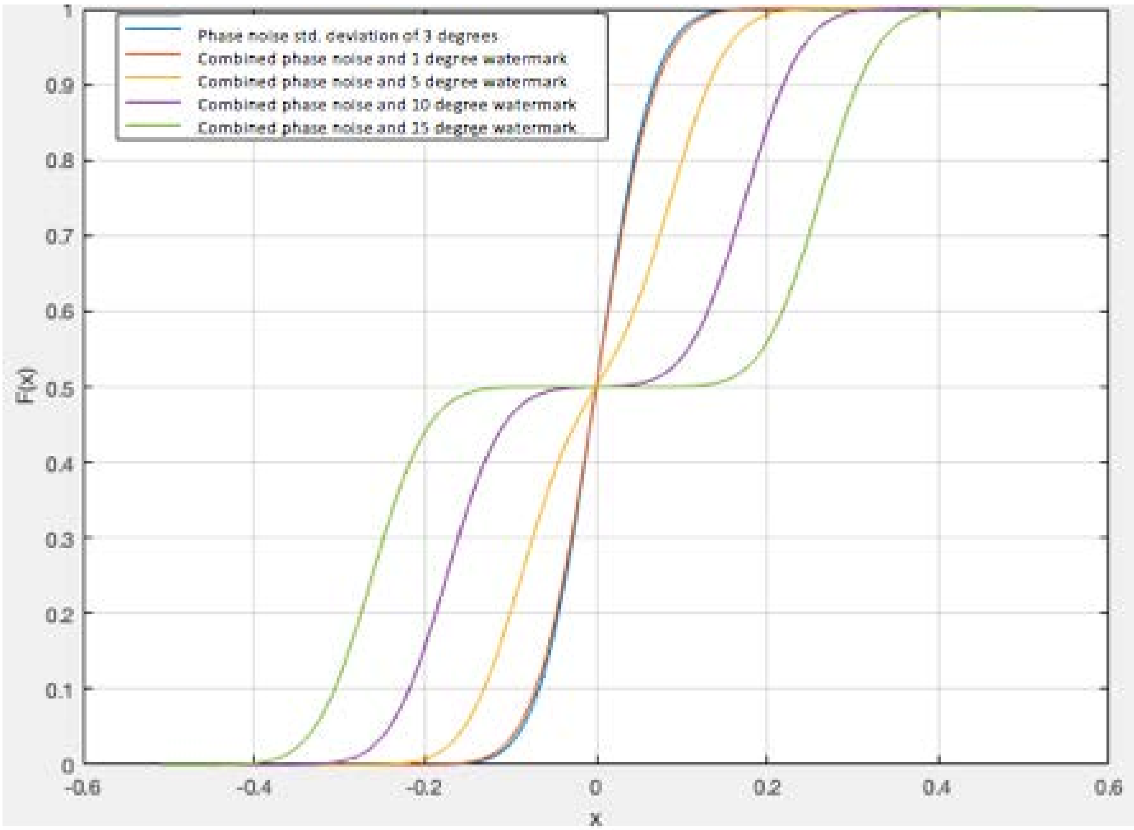

Figure 21: CDF Plots for Phase noise standard deviation of 3 degrees and varying Watermark positions

The CDF plots show how increasing the watermark angle affects the stealth of the watermark. For an oscillator phase noise standard deviation value of 3 degrees, only the watermark placed 1 degree away from the ideal symbol passes the Lilliefor's test.

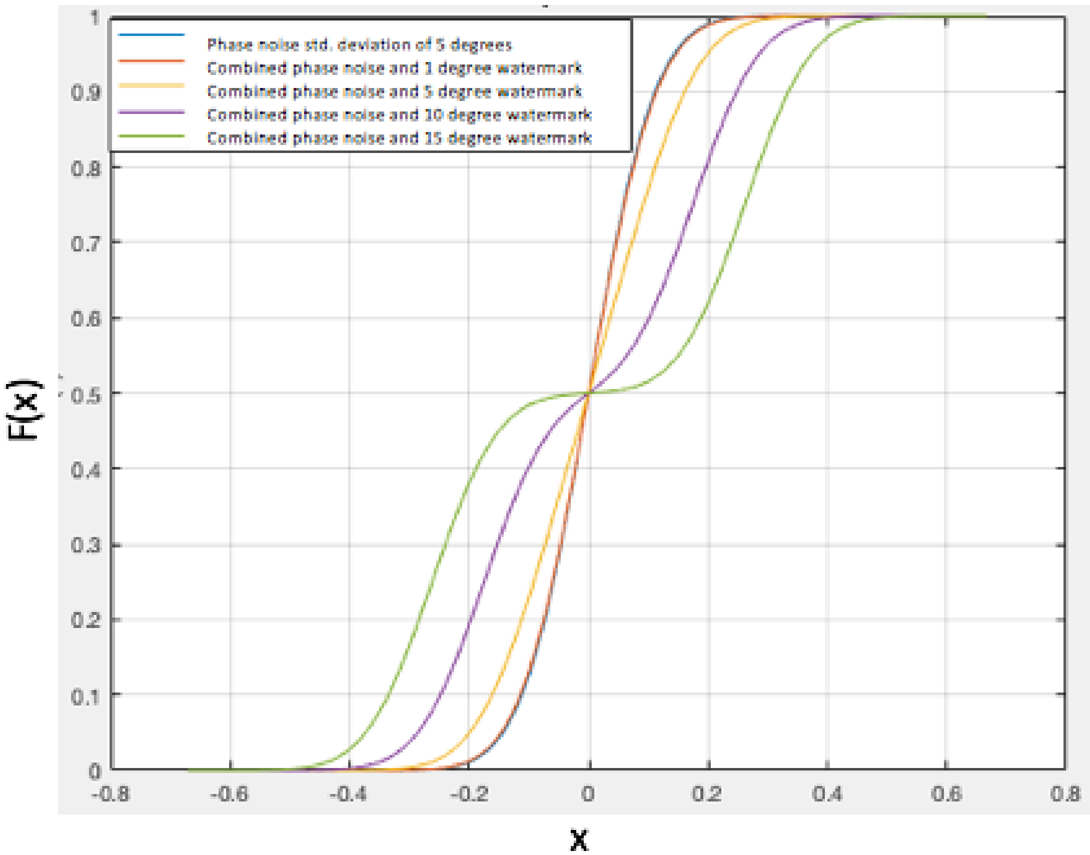

Figure 22: CDF Plots for Phase noise standard deviation of 5 degrees and varying Watermark positions

For an oscillator phase noise standard deviation of 5 degrees, the CDF plots move closer together. Although the plots move closer, once again only the 1 degree^2 watermark passes the Lilliefor's test.

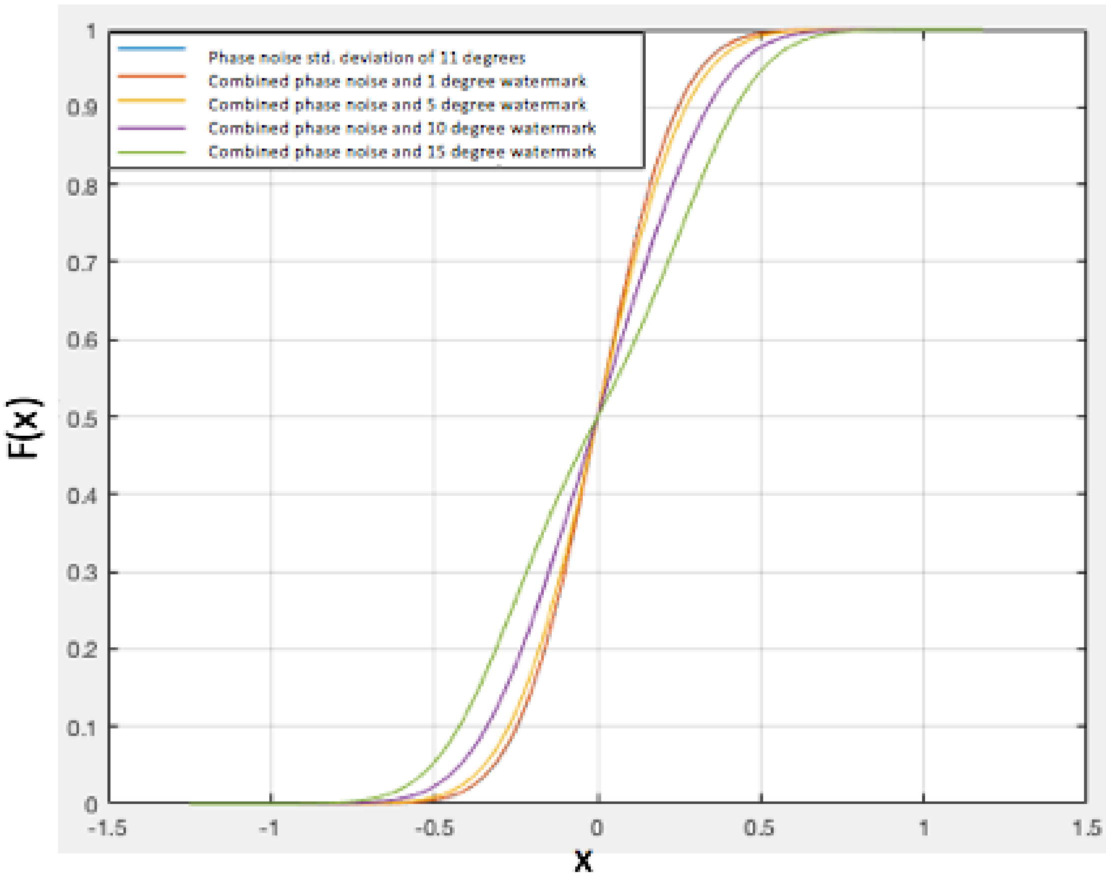

Figure 23: CDF Plots for 11 Degree Phase noise and Watermark

From figure 26, the 5 degreewatermark CDF is noticeably closer to the resemblance of the phase noise CDF. For the 11 degree oscillator phase noise standard deviation, the 1 degree^2 watermark CDF is the only one that passes the Lilliefor’s test.

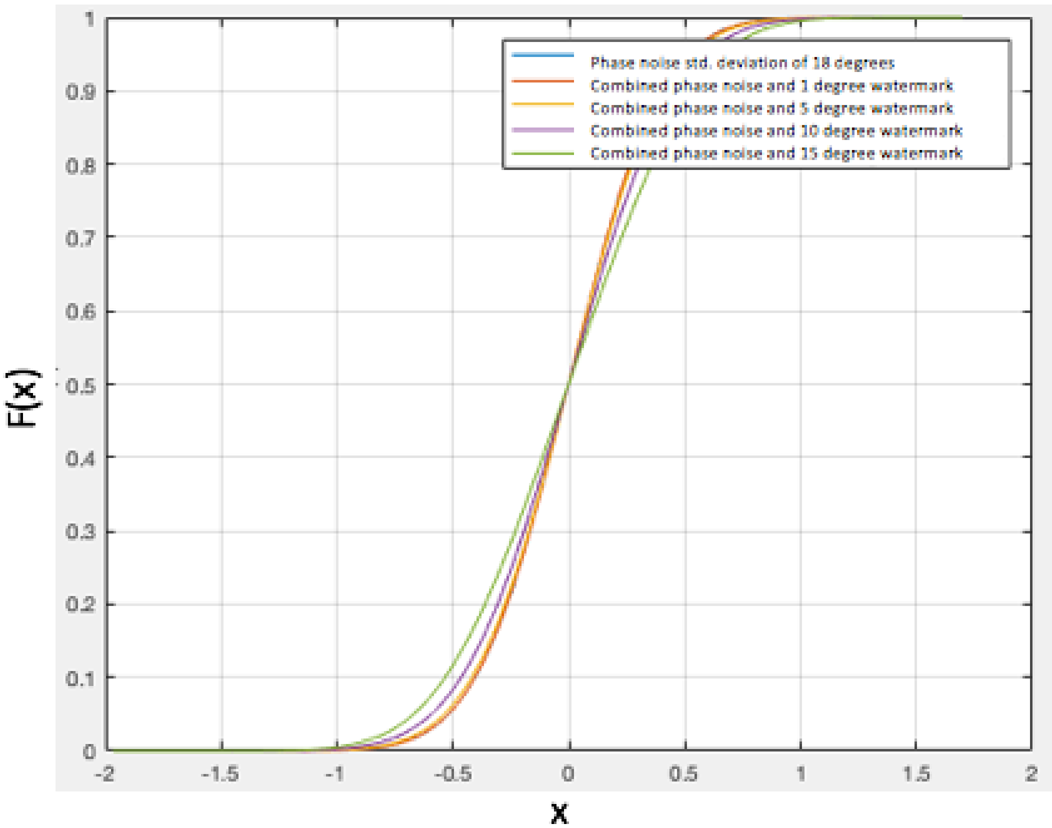

Figure 24: CDF Plots for 18 Degree Phase noise and Watermark

For an oscillator phase noise standard deviation of 18 degrees, the CDF plots are fairly close together for all watermark positions. At this position both the 1 degree^2 watermark and 5 degree^2 watermark pass the Lilliefor's test.

Figure 25 compares the CDF plots for a watermark of 1 degree^2 with different phase noise standard deviation values. The plot reinforces earlier statements of how phase noise combined with a constant watermark value causes slightly more spreading as the variance increases.

The next section provides analysis and the significance of the results from the simulated watermarked BPSK signals.

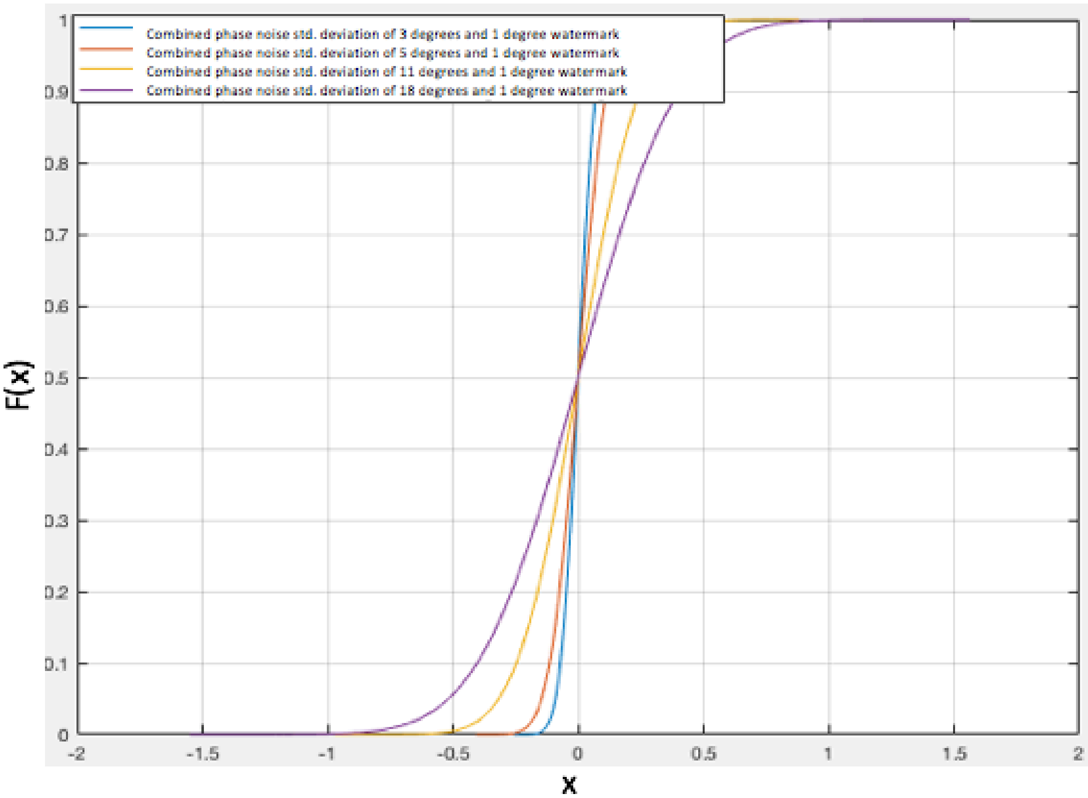

Figure 25: CDF Plots for different Phase noise standard deviation and a constant watermark position

4.3 Analysis

The purpose of this section is to elaborate further on the results of the simulations conducted for the watermarked BPSK signals. The simulations conducted indicate how stealthy and robust the investigated watermarking method is.

Message BER

From the theoretical plots, we expected the message BER to increase with respect to an increase in phase noise standard deviation and/or watermark position. The simulations aligned with the theoretical message BER showing a rise in degradation to the error rates as the imperfections increased. The increased deficiency was not as apparent for the 1 degree^2 and 5 degree^2 watermark positions as compared to the 10 and 15 degree^2 watermark. Increasing or decreasing the phase noise standard deviation and/or the watermark position also impacted the watermark error and stealth during the simulations.

Watermark Error Rate

From the research conducted in [5] and [6], the watermark error rate was expected to be worse than the message BER, which was illustrated in the simulations. The simulations conducted match theoretical plots well for high $\frac{E_b}{N_0}$ values. At low $\frac{E_b}{N_0}$ addition errors occur when spreading from thermal noise and phase noise causes the transmitted watermark to be received in the wrong quadrant. From the figure below, we can see that for a logic 1 watermark, there are two regions where the watermark will be received correctly, and two regions where the watermark will be received in error. By using probability equations in appendix B, and calculating the probability of the quadrant the watermark will be received, we can see that for a $\frac{E_b}{N_0}$ value of 0 dB, the percentage that the watermark will be received in error is 43%. The theoretical plots do not consider additional error quadrants which is the reason for the slight difference between the theoretical and simulation plots for low $\frac{E_b}{N_0}$ values.

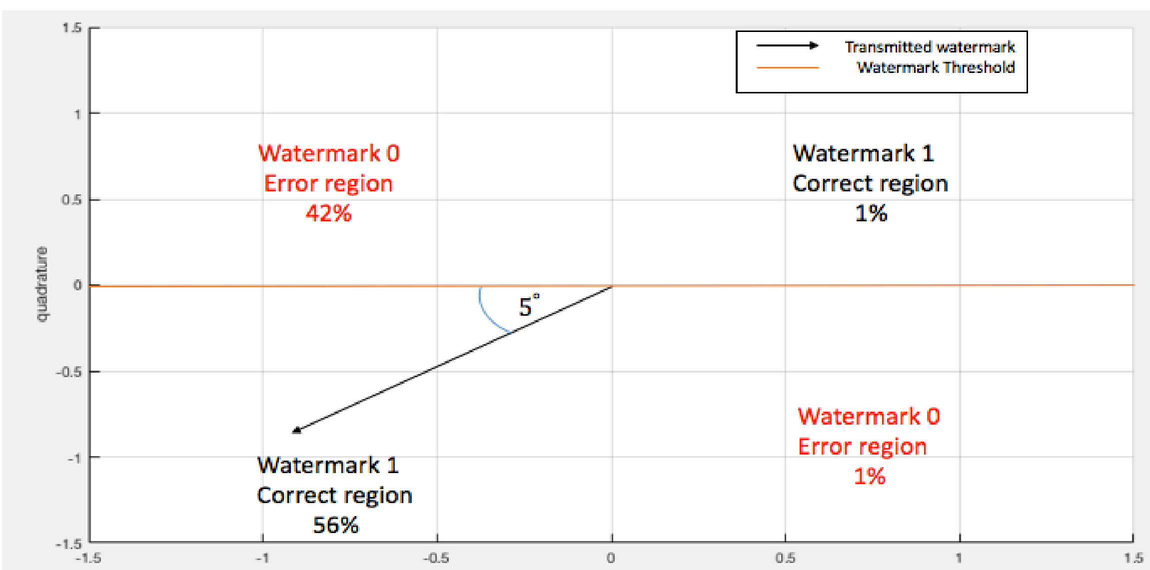

Figure 26: Watermark region representation

The watermark error rate improved as the watermark distance increased away from the ideal position. The significance of this is that the watermark recovery is related to the distance of the watermark from the ideal symbol. As the watermark moves further from the ideal symbol, the watermark detector is able to identify the watermark with less difficulty, thus improving watermark error rate. Additionally, as shown in previous watermarking research, reducing the effective bitrate of the watermark can improve the watermark recovery. Reducing the effective bitrate requires holding the watermark symbol in the same position over several message symbol intervals [5,6]. The closer the watermark falls to the decision boundary the more likely an error could occur. The position of the watermark is not only important to the outcome of the message BER and watermark error, but it also an important aspect of the stealthiness of the watermark.

Stealth

In order to test for stealth, the Lilliefor's goodness of fit test was conducted. The results of the test indicated that the watermark needs to be at most 1 degree^2 away for oscillator phase noise standard deviation less than or equal to 11 degrees. Testing conducted for 18 degree phase noise standard deviation indicates that an in increase in impairments allows for more flexibility in watermark positioning, and therefore signifies that watermark stealth can be improved as phase noise standard deviation increases.

It was represented in [25] that an increase in phase noise standard deviation above 5 degrees begins to have significant degradation to the error rate of a modulated signal in comparison to a signal with no phase noise. The next section describes the trade-offs between different values of phase noise standard deviation with different watermark positions.

Trade-offs

The trade-offs between message BER, watermark error rate, and stealthiness are factors that will have to be considered if developing a system that utilizes this investigated watermarking method.

For example, a system with a high value of phase noise with a low value watermark will have a better watermark error rate and will be stealthy, but will have a harder time detecting the received watermark bits of the main signal. Having a lower watermark value has been shown from past methods to increase the stealthiness of the watermarking method [6].

The simulations conducted showed that the watermark must remain close to the ideal symbol position in order to remain stealthy. A close watermark will not significantly degrade the message BER, but it will have a higher watermark error rate compared to a watermark positioned further away. If the oscillator phase noise has a high standard deviation value around 18 degrees, the watermark position will be able to move slightly further away from the ideal symbol, while maintaining stealthiness. This variation will also cause the watermark recovery error rate to decrease, but will significantly impact the received signal BER.

It was shown that an increase in phase noise standard deviation while keeping a specific watermark position will degrade the watermark error rate while at the same time allowing the watermark to remain stealthy. If the phase noise standard deviation is kept in the range of 3 to 5 degrees with a watermark of 1 degree^2, at most the degradation to the message BER for a specific watermark position will be around 1 dB compared to an non-watermarked signal. The significance of this is that for a system that wants to mask a watermark as oscillator phase noise, the best way to ensure integrity in message BER, watermark error, and stealth is to ensure oscillator phase noise is within a 3 to 5 degree standard deviation margin with a watermark position 1 degree^2 away from the ideal transmitted bit.

The next chapter provides future considerations over this topic and concludes this investigation of watermarking.

CHAPTER V

CONCLUSION

5.1 Conclusion

The investigation of whether or not it is possible to hide a watermark within oscillator phase noise has been discussed throughout this thesis. Simulations where conducted and compared to theory utilizing BPSK modulation. It should be noted that the theoretical values for the error rates were computed in Mathcad and transferred over to Matlab. Transferring these results by hand can lead to a round off error which can in some instances affect the theoretical plots as shown in figure 19. When the number of significant digits is increased, the simulations align more closely for the plot with a phase noise standard deviation of 18 degrees and a watermark of 5 degrees^2.

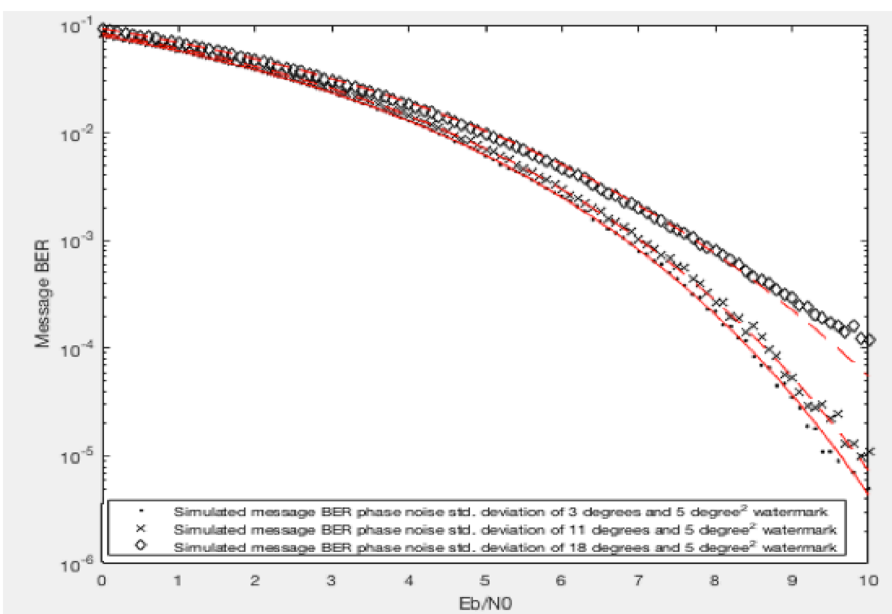

Figure 27: Message BER with increased significant figures

It was shown that for typical phase noise standard deviation between 3 and 11 degrees, a constellation dithered watermark can be successfully be stealthy with very little degradation to the message error rates as long as the dithering is within 1 degree of the ideal symbol position. In the situations examined, up to a 5 degree^2 watermark phase angle could be hidden quite well. This results in a high watermark BER that can be overcome by slowing the watermark symbol rate, i.e. by transmitting the same watermark symbol over several message symbol intervals.

Hiding a watermark within oscillator phase noise was determined to be stealthy by utilizing the Lilliefor's goodness of fit test. The Lilliefor's test compares the CDFs of a watermarked signal and non-watermarked signal in order to detect any anomalies [6]. It was illustrated that there exist trade-offs between stealth, watermark position, phase noise, and error rates.

5.2 Future Work

Expanding this investigated method to real world scenarios utilizing radios could be conducted in the future. It could also be worth expanding this method to higher order modulation schemes in order to determine how error rates and stealth may be affected.

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APPENDICES

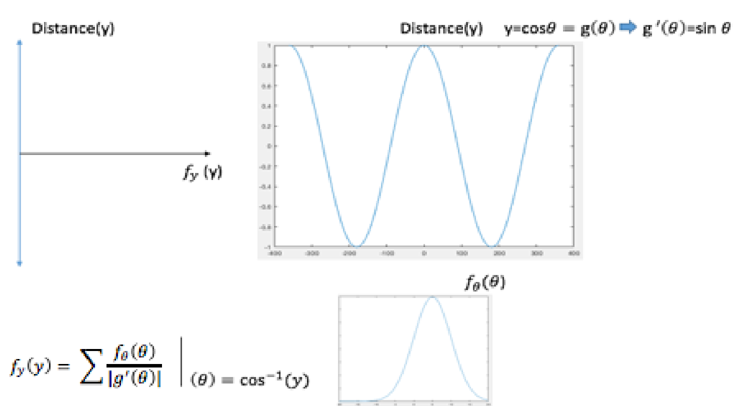
The following section contains derivations used in order to find the theoretical watermarked BPSK message/watermark error rates and the code used to conduct simulations.

APPENDIX A

In order to derive the Qfunctions for the Message error rates, a PDF mapping was conducted to find the mean and variance of the combined distribution composed of the watermark and phase noise. Mathcad was utilized in order to build and solve the necessary equations.

BPSK Message BER PDF Mapping

In order to map the phase noise PDF to distance from the threshold, a transformation of variables must be performed. The derivation below assumes θ never exceeds + or - 180 degrees, therefore, it is a two to one mapping over this interval. The derivation must be split into two separate derivations, one with phase noise + watermark angle >0 and one with phase noise + watermark angle <0. Since the threshold is the quadrature axis, the distance of the received bit can be found using geometric properties. The distance from the ideal transmitted position is a function of $\cos(\theta)$.



PDF Derivation: Phase Noise vs BPSK Distance from Threshold for the Message Symbol. Valid when the phase noise + watermark angle is > 0 degrees

watermark angle

$wmark := 5$

$$wmarkrad := wmark \cdot \frac{\pi}{180}$$

$wmarkrad = 0.087$

phase noise standard deviation

$sig\_pn := 5$

$$sig\_pnrad := sig\_pn \cdot \frac{\pi}{180}$$

$sig\_pnrad = 0.087$

Area under PDF f(y) when Watermark + Phase Noise is > 0 degrees

$$\int_0^1 \frac{e^{\frac{-(acos(y)-wmarkrad)^2}{2 \cdot sig\_pnrad^2}}}{\sqrt{2 \cdot \pi \cdot sig\_pnrad^2} \cdot \sin(acos(y))} \, dy = 0.841$$

Verifying total area under PDF = 1

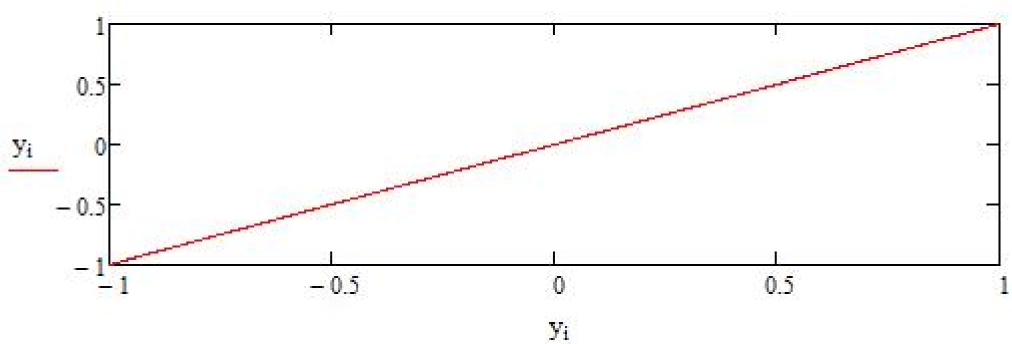
$$\int_{-\infty}^{\infty} \frac{e^{\frac{-(\theta-wmarkrad)^2}{2 \cdot sig\_pnrad^2}}}{\sqrt{2 \cdot \pi \cdot sig\_pnrad^2}} \, d\theta = 1$$

$i := 0..\, 1000$

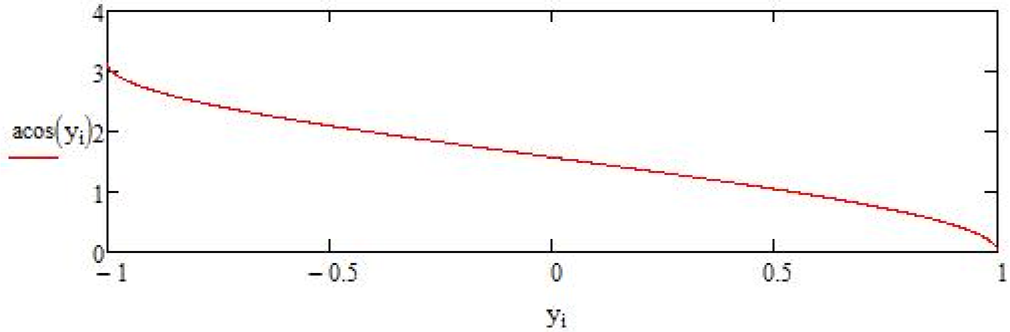
$$y_i := -1 + \frac{i}{500}$$

$z_i := acos(y_i)$

Plot of possible arc cosine values

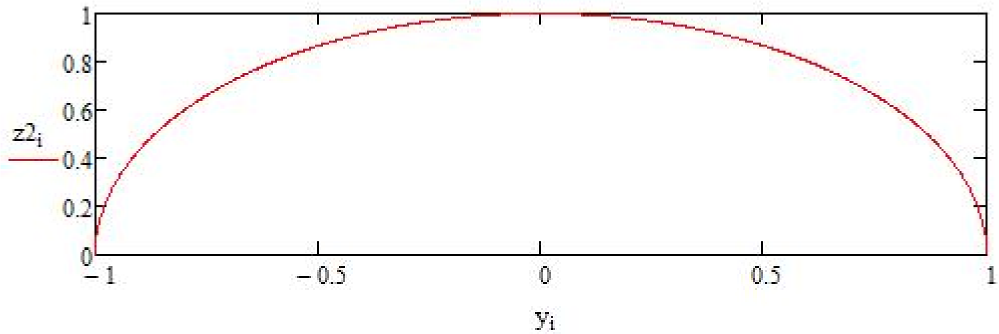


Plot of phase angle associated with each arc cosine value



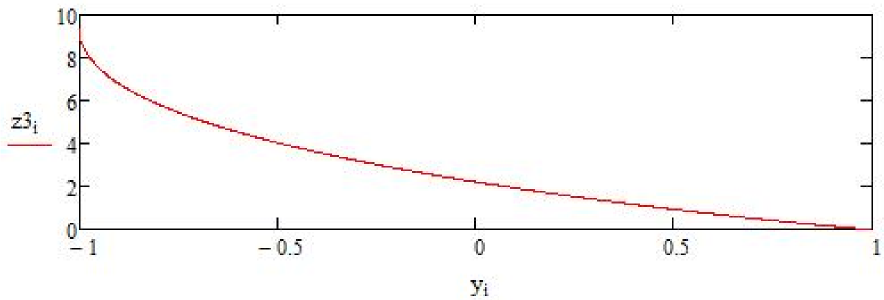
Plot of the sine of the arc cosine

$z2_i := \sin(\text{acos}(y_i))$



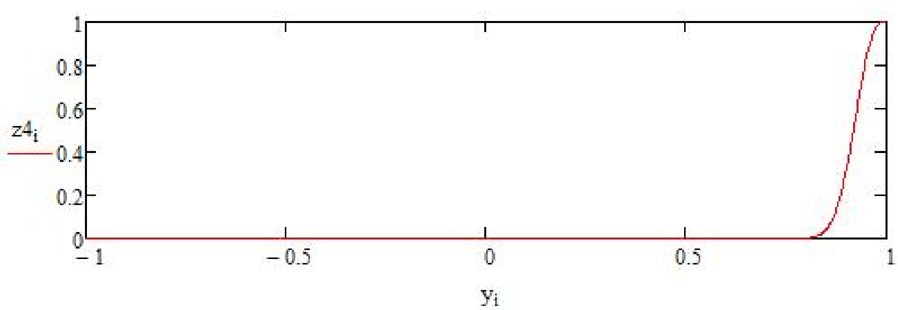
Plot of term in the exponent of "e"

$z3_i := (\text{acos}(y_i) - \text{wmarkrad})^2$



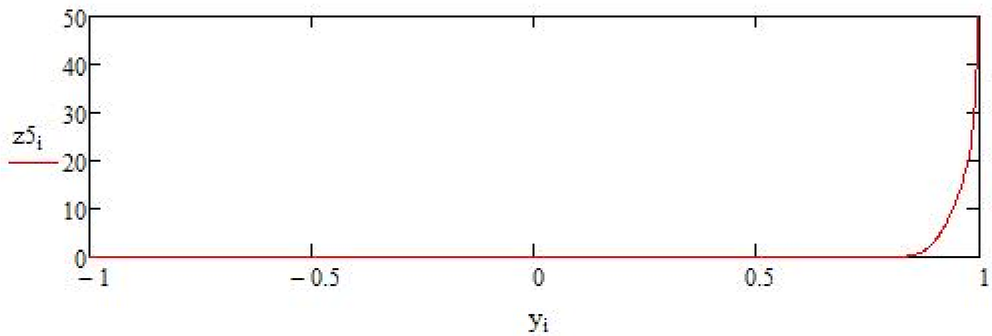
Plot of PDF numerator

$z4_i := e^{\left[\frac{-(z3_i)^2}{2 \cdot \text{sig\_pnrad}^2}\right]}$



$$z5_i := \frac{z4_i}{\left(\sqrt{2\cdot\pi\cdot sig\_pnrad^2}\cdot\left|\sin\left(acos\left(y_i\right)\right)\right| + 0.000001\right)}$$

Plot of output PDF associated with positive phase noise phase angles



Mean (distance from threshold assuming distance with no watermark or noise = 1)

$$\int_{-1}^{1} \frac{y\cdot e^{\frac{-(acos(y)-wmarkrad)^2}{2\cdot sig\_pnrad^2}}}{\sqrt{2\cdot\pi\cdot sig\_pnrad^2}\cdot\sin(acos(y))}\,dy = 0.83404$$

Second Moment

$$\int_{-1}^{1} \frac{y^2\cdot e^{\frac{-(acos(y)-wmarkrad)^2}{2\cdot sig\_pnrad^2}}}{\sqrt{2\cdot\pi\cdot sig\_pnrad^2}\cdot\sin(acos(y))}\,dy = 0.82688$$

Transformation of Variables results in the Distance from Threshold PDF = f(θ+) +f(θ-); i.e. equal to a 1 to 1 mapping of positive values of θ to output and a 1 to 1 mapping of negative values of θ to the output. Hence E[X] and E[X^2] can be found by summing these expected values of f(θ+) and f(θ-).

Negative side

PDF Derivation: Phase Noise vs BPSK Distance from Threshold for the Message Symbol
Valid when the phase noise + watermark angle is < 0 degrees
watermark angle

$wmark := 5$

$$wmarkrad := wmark\cdot\frac{\pi}{180}$$

$wmarkrad = 0.087$

phase noise standard deviation

$$sig\_pn := 5$$

$$sig\_pnrad := sig\_pn \cdot \frac{\pi}{180}$$

$$sig\_pnrad = 0.087$$

Verifying area under PDF = 1

$$\int_{-\infty}^{\infty} \frac{e^{\frac{-(\theta - wmarkrad)^2}{2 \cdot sig\_pnrad^2}}}{\sqrt{2 \cdot \pi \cdot sig\_pnrad^2}} \, d\theta = 1$$

Area under PDF when Phase Noise is < 0 degrees.

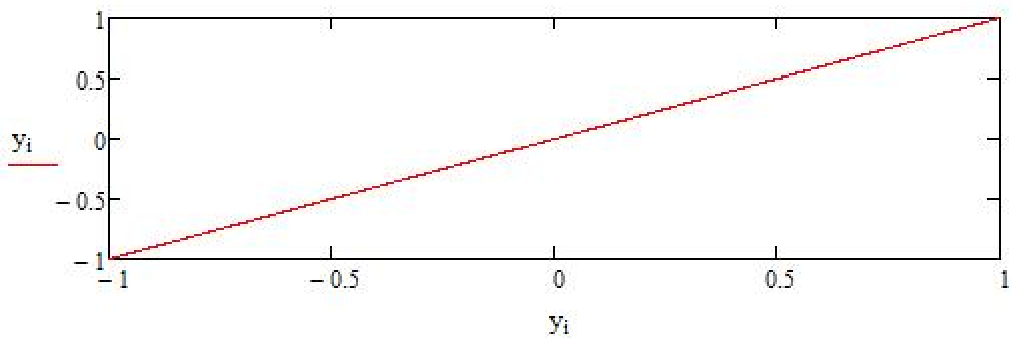
$$\int_{-1}^{1} \frac{e^{\frac{-(-acos(y) - wmarkrad)^2}{2 \cdot sig\_pnrad^2}}}{\sqrt{2 \cdot \pi \cdot sig\_pnrad^2} \cdot |sin(-acos(y))|} \, dy = 0.159$$

$$i := 0..1000$$

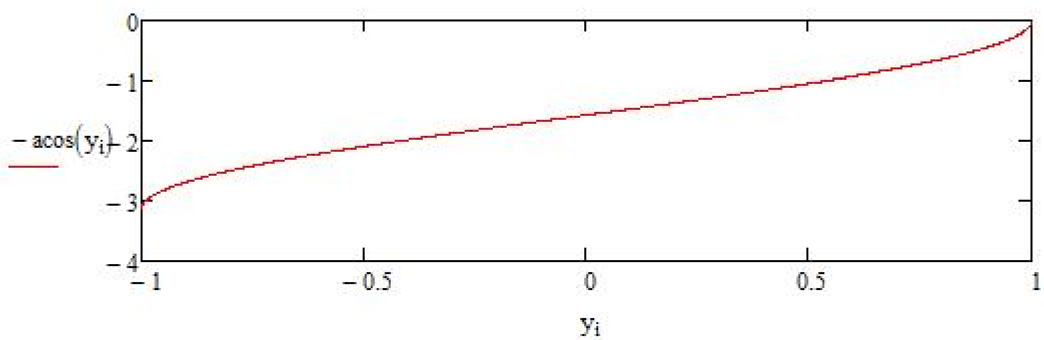
$$y_i := -1 + \frac{i}{500}$$

$$z_i := acos(y_i)$$

Plot of possible arc cosine values

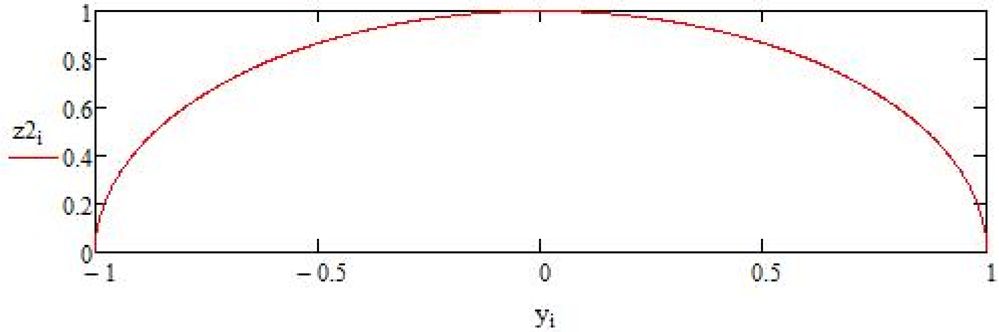


Plot of phase angle associated with each arc cosine value



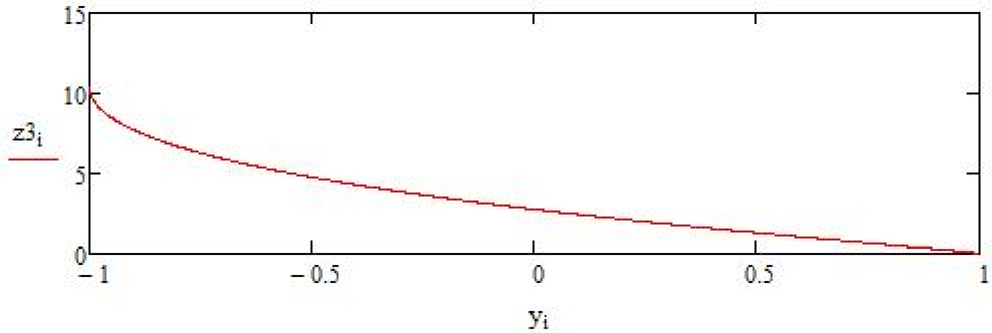
$$z2_i := |sin(-acos(y_i))|$$

Plot of the sine of the arc cosine



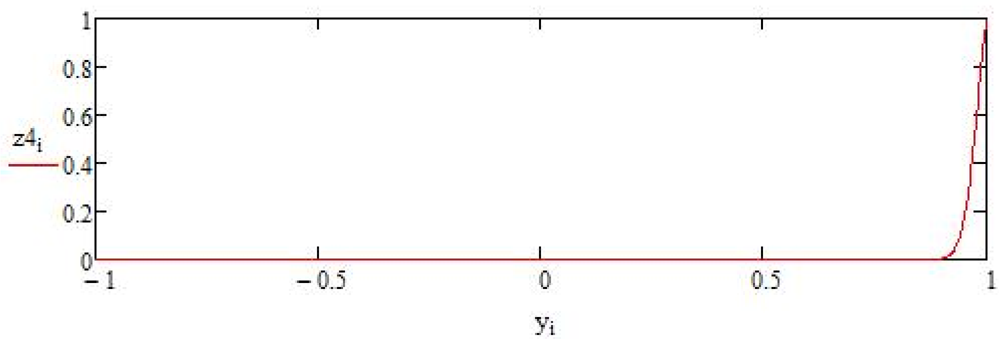
$$z3_i := \left(-\mathrm{acos}\left(y_i\right) - \mathrm{wmarkrad}\right)^2$$

Plot of term in the exponent of "e"



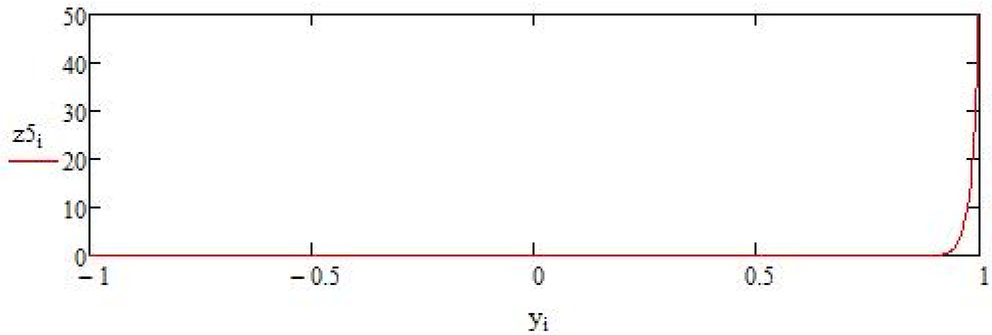
Plot of PDF numerator

$$z4_i := e^{\left[\frac{-\left(z3_i\right)^2}{2 \cdot \mathrm{sig\_pnrad}^2}\right]}$$



$$z5_i := \frac{z4_i}{\left(\sqrt{2 \cdot \pi \cdot \mathrm{sig\_pnrad}^2} \cdot \left|\sin\left(-\mathrm{acos}\left(y_i\right)\right)\right| + 0.000001\right)}$$

Plot of output PDF associated with negative phase noise phase angles



$$\cos\left(\frac{5\cdot\pi}{180}\right) = 0.996$$

Mean (add to previous mean to get the overall mean)

$$\int_{-1}^{1} \frac{y\cdot e^{\frac{-\left(-\operatorname{acos}(y)-\frac{0}{2}-\text{wmarkrad}\right)^2}{2\cdot \text{sig\_pnrad}^2}}}{\sqrt{2\cdot\pi\cdot \text{sig\_pnrad}^2}\cdot\sin(\operatorname{acos}(y))}\,dy = 0.15837$$

$$.07776\cdot 90 = 6.998$$

Second Moment (add to previous second moment to get overall 2nd moment)

$$\int_{-1}^{1} \frac{y^2\cdot e^{\frac{-\left(-\operatorname{acos}(y)-\frac{0}{2}-\text{wmarkrad}\right)^2}{2\cdot \text{sig\_pnrad}^2}}}{\sqrt{2\cdot\pi\cdot \text{sig\_pnrad}^2}\cdot\sin(\operatorname{acos}(y))}\,dy = 0.15808$$

Transformation of Variables results in the Distance from Threshold PDF = f(θ+) +f(θ-); i.e. equal to a 1 to 1 mapping of positive values of θ to output and a 1 to 1 mapping of negative values of θ to the output. Hence E[X] and E[X^2] can be found by summing these expected values of f(θ+) and f(θ-). The actual mean and standard deviation to use follow.

$$\text{Mean} := \text{Mean\_pos} + \text{Mean\_neg} \qquad \text{Mean} = 0.99241$$

$$\text{Moment2} := \text{Moment2\_pos} + \text{Moment2\_neg} \qquad \text{Moment2} = 0.98496$$

$$\text{Variance} := \text{Moment2} - \text{Mean}^2 \qquad \text{Variance} = 8.61347\times 10^{-5}$$

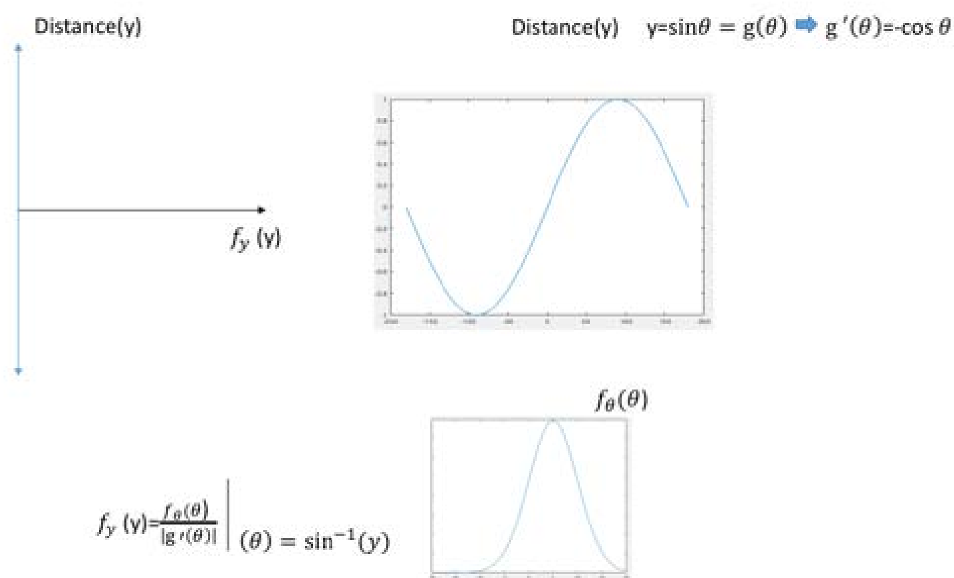
$$\text{Deviation} := \sqrt{\text{Variance}}$$

APPENDIX B

In order to derive the Qfunctions for the watermark error rates, a PDF mapping was conducted to find the mean and variance of the combined distribution composed of the watermark and phase noise. Mathcad was utilized in order to build and solve the necessary equations.

BPSK WATERMARK ERROR PDF Mapping

A PDF mapping is needed to find the mean and variance of the combined watermark and phase noise PDF. In order to map the PDF, a transformation of variables must be performed. The derivation below assumes θ never exceeds + or - 90 degrees, therefore, it is a one to one map over this interval. Since the threshold is the real axis, the distance of the received watermark can be found using geometric properties. The distance from the threshold is a function of $\sin(\theta)$.



PDF Derivation: Phase Noise vs BPSK Distance from Threshold for the Watermark Symbol

watermark angle >>>>>>>>

$wmark := 5$

$wmarkrad := wmark \cdot \frac{\pi}{180}$

$wmarkrad = 0.087$

phase noise standard deviation >>>>>>>>

$sig\_pn := 5$

$sig\_pnrad := sig\_pn \cdot \frac{\pi}{180}$

$sig\_pnrad = 0.087$

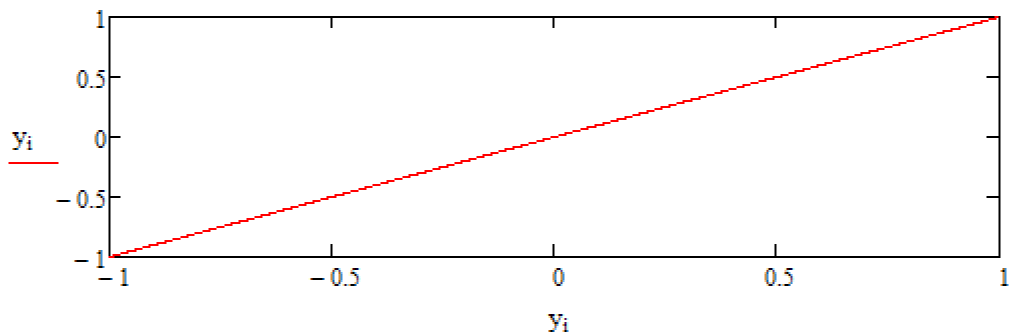
Verifying Area under PDF when Phase Noise is > 0 degrees = 1

$$\int_{-1}^{1} \frac{e^{\frac{-(\mathrm{asin}(y)-wmarkrad)^2}{2 \cdot sig\_pnrad^2}}}{\sqrt{2 \cdot \pi \cdot sig\_pnrad^2} \cdot \cos(\mathrm{asin}(y))} \, dy = 1$$

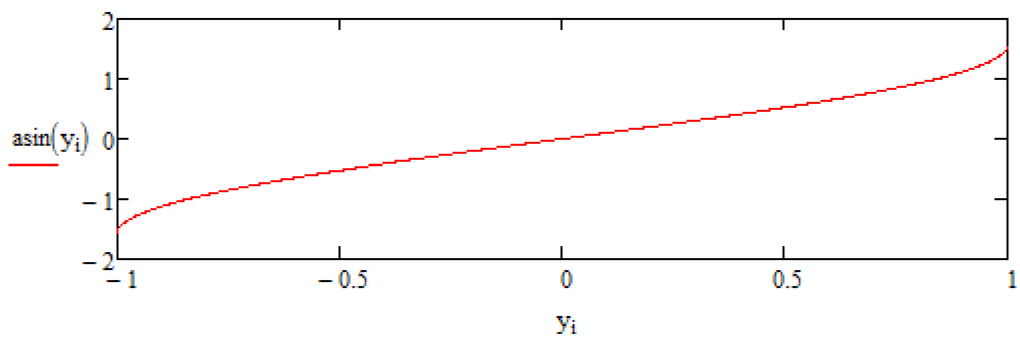
$i := 0..1000$

$y_i := -1 + \frac{i}{500}$

Plot of possible arc sine values

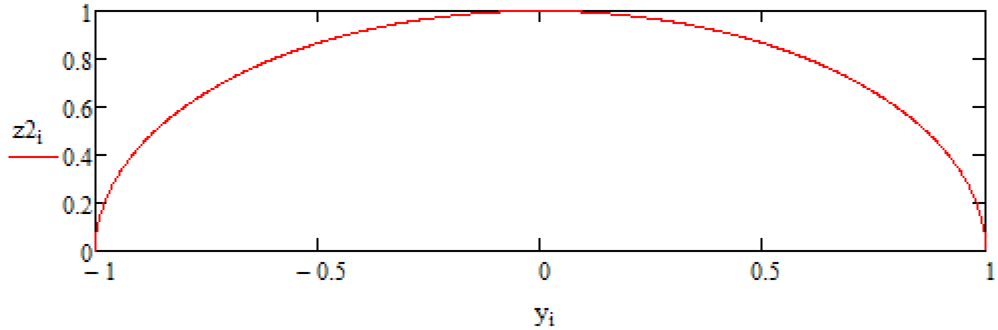


Plot of phase angle associated with each arc sine value



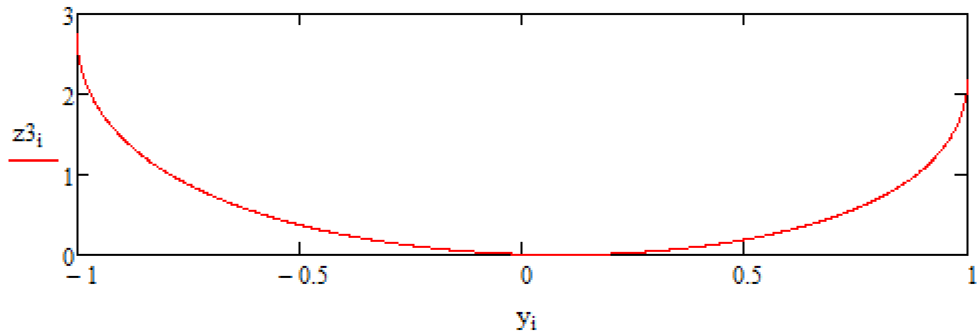
Plot of the cosine of the arc sine

$z2_i := \cos(\mathrm{asin}(y_i))$



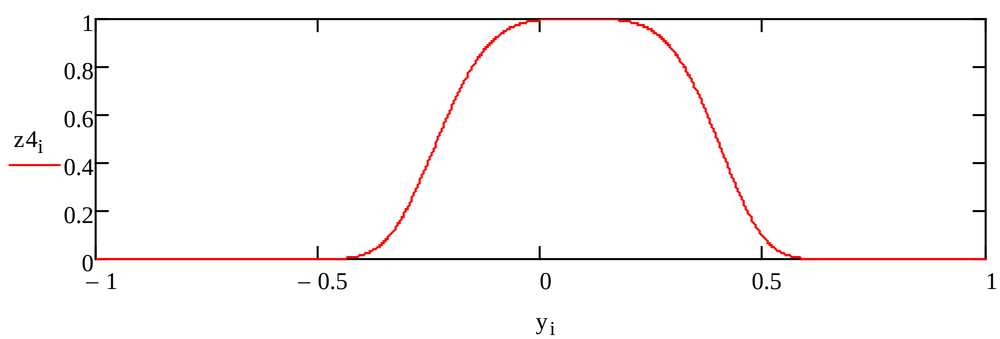
Plot of term in the exponent of "e"

$$z3_i := \left(\text{asin}(y_i) - \text{wmarkrad}\right)^2$$



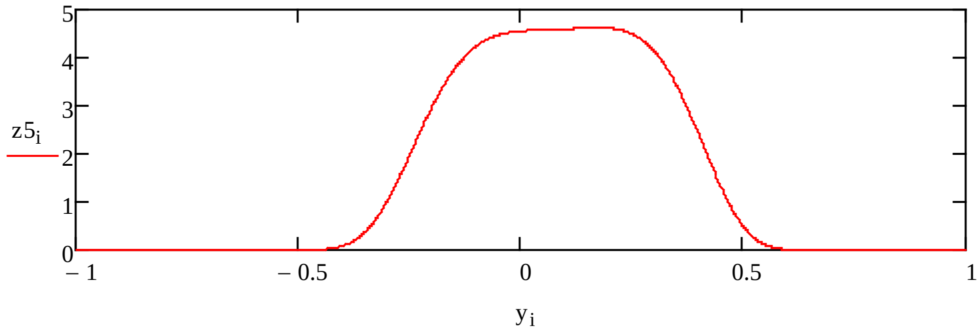
Plot of PDF numerator

$$z4_i := e^{\left[\frac{-\left(z3_i\right)^2}{2\cdot \text{sig\_pnrad}^2}\right]}$$



$$z5_i := \frac{z4_i}{\left(\sqrt{2\cdot\pi\cdot \text{sig\_pnrad}^2}\cdot\left|\cos\left(\text{asin}(y_i)\right)\right| + 0.000001\right)}$$

Plot of output PDF associated with positive phase noise phase angles



$$Mean := \int_{-1}^{1} \frac{y \cdot e^{\frac{-(asin(y)-wmarkrad)^2}{2 \cdot sig\_pnrad^2}}}{\sqrt{2 \cdot \pi \cdot sig\_pnrad^2} \cdot \cos(asin(y))} dy$$

Mean (increase in distance from threshold: 0 = no change)

$Mean = 0.08296$

Second Moment

$$Moment2 := \int_{-1}^{1} \frac{y^2 \cdot e^{\frac{-(asin(y)-wmarkrad)^2}{2 \cdot sig\_pnrad^2}}}{\sqrt{2 \cdot \pi \cdot sig\_pnrad^2} \cdot \cos(asin(y))} dy$$

Second Moment (in radians^2)

$Moment2 = 0.01504$

$Variance = 7.50054 \times 10^{-3}$

$Deviation := \sqrt{Variance}$

$Deviation = 0.08661$

BPSK WATERMARK RECEIVED QUADRANT PROBABILITY

Calculations for what quadrant the received symbol lands if a quadrant 3 logic 1 watermark is transmitted

$var := .25$ $\quad$ $meanx := -.9962$ $\quad$ $meany := -.087$

$$\int_{-\infty}^{0} \int_{0}^{\infty} \frac{e^{\frac{-(x-meanx)^2}{2 \cdot var}}}{\sqrt{2 \cdot \pi \cdot var}} \cdot \frac{e^{\frac{-(y-meany)^2}{2 \cdot var}}}{\sqrt{2 \cdot \pi \cdot var}} \, dx \, dy = 0.013182$$

probability 4th quadrant

$$\int_{-\infty}^{0}\int_{-\infty}^{0}\frac{e^{\frac{-(x-meanx)^2}{2\cdot var}}}{\sqrt{2\cdot\pi\cdot var}}\cdot\frac{e^{\frac{-(y-meany)^2}{2\cdot var}}}{\sqrt{2\cdot\pi\cdot var}}\,dx\,dy = 0.555886$$

probability 3rd quadrant

$$\int_{0}^{\infty}\int_{-\infty}^{0}\frac{e^{\frac{-(x-meanx)^2}{2\cdot var}}}{\sqrt{2\cdot\pi\cdot var}}\cdot\frac{e^{\frac{-(y-meany)^2}{2\cdot var}}}{\sqrt{2\cdot\pi\cdot var}}\,dx\,dy = 0.420945$$

probability 2nd quadrant

$$\int_{0}^{\infty}\int_{0}^{\infty}\frac{e^{\frac{-(x-meanx)^2}{2\cdot var}}}{\sqrt{2\cdot\pi\cdot var}}\cdot\frac{e^{\frac{-(y-meany)^2}{2\cdot var}}}{\sqrt{2\cdot\pi\cdot var}}\,dx\,dy = 9.981802\times 10^{-3}$$

probability 1st quadrant

APPENDIX C

The following appendix discusses the derivation of the Qfunction for the message and watermark error rates. The effect of the watermark the on the distance to the threshold and effect of phase noise and thermal noise on the mean distance from the threshold for both the message and watermark error rate will also be discussed.

Message error rate Qfunction Derivation

The combined random phase noise and watermark values are added to statistically independent random thermal noise voltages. The significance of this is that the PDF's of these two must be convolved. From the PDF plots above, the combined phase noise and watermark PDF is somewhat impulse like. Convolving a delta function with a bell shaped curve results in a bell shaped curve, therefor we can assume the resulting PDF will be approximately Gaussian. Since the resultant PDF is approximately Gaussian, we can use equation C.1 to define the total combined PDF. Where μ_{tot} is the sum of the mean of thermal noise (which is zero) and the mean of the mapped PDF composed of the watermark and phase noise. The variance, σ^2_{tot}, is the sum of the variance of the thermal noise and the variance of the PDF composed of the watermark and phase noise.

$$P(x) = \frac{1}{\sqrt{2\pi\sigma_{tot}^2}}\ exp^{-\left(\frac{(x-\mu_{tot})^2}{2\sigma_{tot}^2}\right)} \qquad \text{(C.1)}$$

For example, if we have a message vector, located on the in-phase axis, with a length of 1 from the quadrature axis, with a SNR value of 10db, we can find the variance of thermal noise. The noise power would be the ratio of the signal power over the signal to noise ratio which is equal to 0.1W. The thermal noise variance is found by dividing the noise power by 2, equating to 0.05. Using derivations in Appendix A, for watermark value of 5 degrees and phase noise standard deviation of 5 degrees, we get a mean vector length of .99241 units and a mean variance of 8.6135×10^{-5}. Therefore, the total mean distance from the threshold, μ_{tot}, is equal to .99241 with a total mean variance, σ^2_{tot}, of 0.050086135. The probability of error given a logic one was transmitted can be found by putting in terms of the complementary error function as shown in equation C.2.

$$p(e|logic\ 1) = \frac{1}{\sqrt{2\pi\sigma_{tot}^2}}\int_{-\infty}^{0} exp^{-\left(\frac{(x-\mu_{tot})^2}{2\sigma_{tot}^2}\right)}dy = \frac{1}{\sqrt{\pi}}\int_{\frac{\mu_{tot}}{\sqrt{2\sigma_{tot}^2}}}^{\infty} exp^{-z^2}dz = \frac{1}{2}erfc\left(\frac{\mu_{tot}}{\sqrt{2\sigma_{tot}^2}}\right) \qquad \text{(C.2)}$$

The Qfunction is related to the complementary error function from equation C.3 [37].

$$Q\left(\sqrt{2} * Z\right) = \frac{1}{2}erfc(z) \qquad \text{(C.3)}$$

From equation C.2 and C.3 we can derive the Qfunction as shown in C.4.

$$P_b\left(\frac{E_b}{N_0}\right) = \ Q\left(\frac{\sqrt{2}\ \mu_{tot}}{\sqrt{2\sigma_{tot}^2}}\right) \qquad \text{(C.4)}$$

Watermark error rate Qfunction Derivation

The combined random phase noise and watermark values are added to statistically independent random thermal noise voltages. The significance of this is that the PDF's of these two must be convolved. From the PDF plots above, the combined phase noise and watermark PDF is somewhat bell shaped. Convolving a bell shaped curve with a bell shaped curve results in a bell shaped curve, therefor we can assume the resulting PDF will be approximately Gaussian.

Since the resultant PDF is approximately Gaussian, we can use equation C.5 to define the total combined PDF. Where μ_{wmtot} is the sum of the mean of thermal noise and the mean of the mapped PDF composed of the watermark and phase noise. The variance, $\sigma^2_{\mathrm{wmtot}}$, is the sum of the variance of the thermal noise and the variance of the PDF composed of the watermark and phase noise.

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2_{wmtot}}} exp^{-\left(\frac{(x-\mu_{wmtot})^2}{2\sigma^2_{wmtot}}\right)} \quad \text{(C.5)}$$

For example, using the same parameters from above for the message vector, located on the in-phase axis, with a message vector length of one from the quadrature axis, a SNR of 10dB, thermal noise variance of 0.05, a watermark of 5 degrees^2, and phase noise standard deviation of 5 degrees, we can use derivations in Appendix B to find the vector length from the watermark threshold and the variance. From the derivations, we get a mean vector length, from the in-phase axis, of 0.08683 and a mean variance of 7.540054×10^{-3}. Therefore, the total mean distance from the threshold, μ_{wmtot}, is equal to 0.08683 with a total mean variance, $\sigma^2_{\mathrm{wmtot}}$, of 0.057540054. The probability of watermark error given a logic one was transmitted can be found by putting in terms of the complementary error function as shown in equation C.6.

$$p(we|logic\ 1) = \frac{1}{\sqrt{2\pi\sigma^2_{wmtot}}}\int_{-\infty}^{0} exp^{-\left(\frac{(x-\mu_{wmtot})^2}{2\sigma^2_{wmtot}}\right)} dy = \frac{1}{\sqrt{\pi}}\int_{\frac{\mu_{wmtot}}{\sqrt{2\sigma^2_{wmtot}}}}^{\infty} exp^{-z^2} dz = \frac{1}{2} erfc\left(\frac{\mu_{wmtot}}{\sqrt{2\sigma^2_{wmtot}}}\right) \quad \text{(C.6)}$$

The Qfunction is related to the complementary error function from equation C.7 [37].

$$Q\left(\sqrt{2} * Z\right) = \frac{1}{2} erfc(z) \quad \text{(C.7)}$$

From equation C.6 and C.7 we can derive the Qfunction as shown in C.8.

$$P_w\left(\frac{E_b}{N_0}\right) = Q\left(\frac{\sqrt{2}\,\mu_{wmtot}}{\sqrt{2\sigma_{wmtot}^2}}\right) \tag{C.8}$$

Watermark Effect

From the figure below, the watermark reduces the distance to the message threshold while at the same time increasing the distance from the watermark error threshold. The distance from the message threshold to the ideal position is 1. Looking at the 15 degree^2 watermark, the distance from the message threshold is about .965. To compare the watermark effect from the watermark threshold the difference in distance of each watermark must be analyzed. The 1 degree^2 watermark position is nearly on the decision boundary. As the watermark angle increases, the distance from the watermark threshold increases.

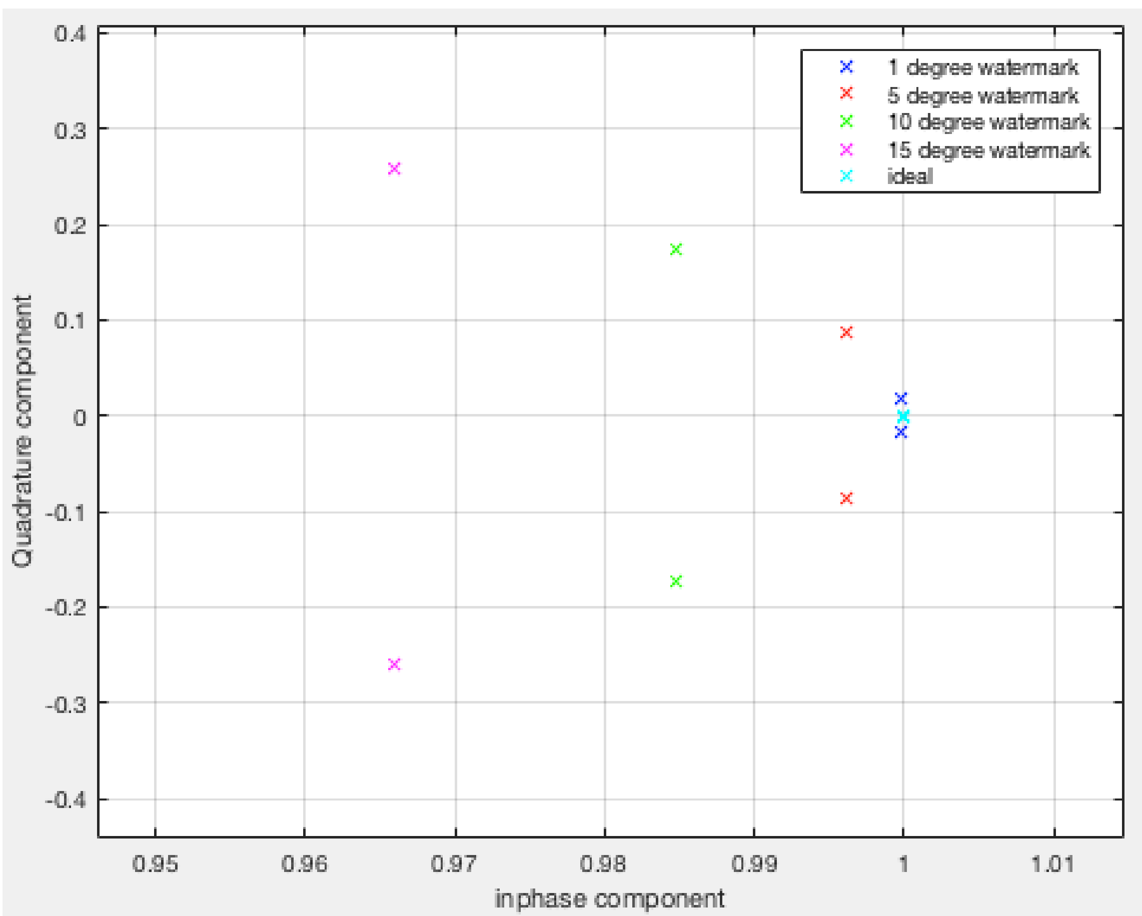


Figure 28: Constellation diagram with different watermark positions

Phase Noise Effect

The figure below is a plot of a signal affected by just phase noise. The distance to the message threshold, for this plot, for an ideal signal that is not affected by phase noise is 1. When phase noise is applied the distance to the message threshold is decreased, on average. The distance from the message threshold is never greater than 1.

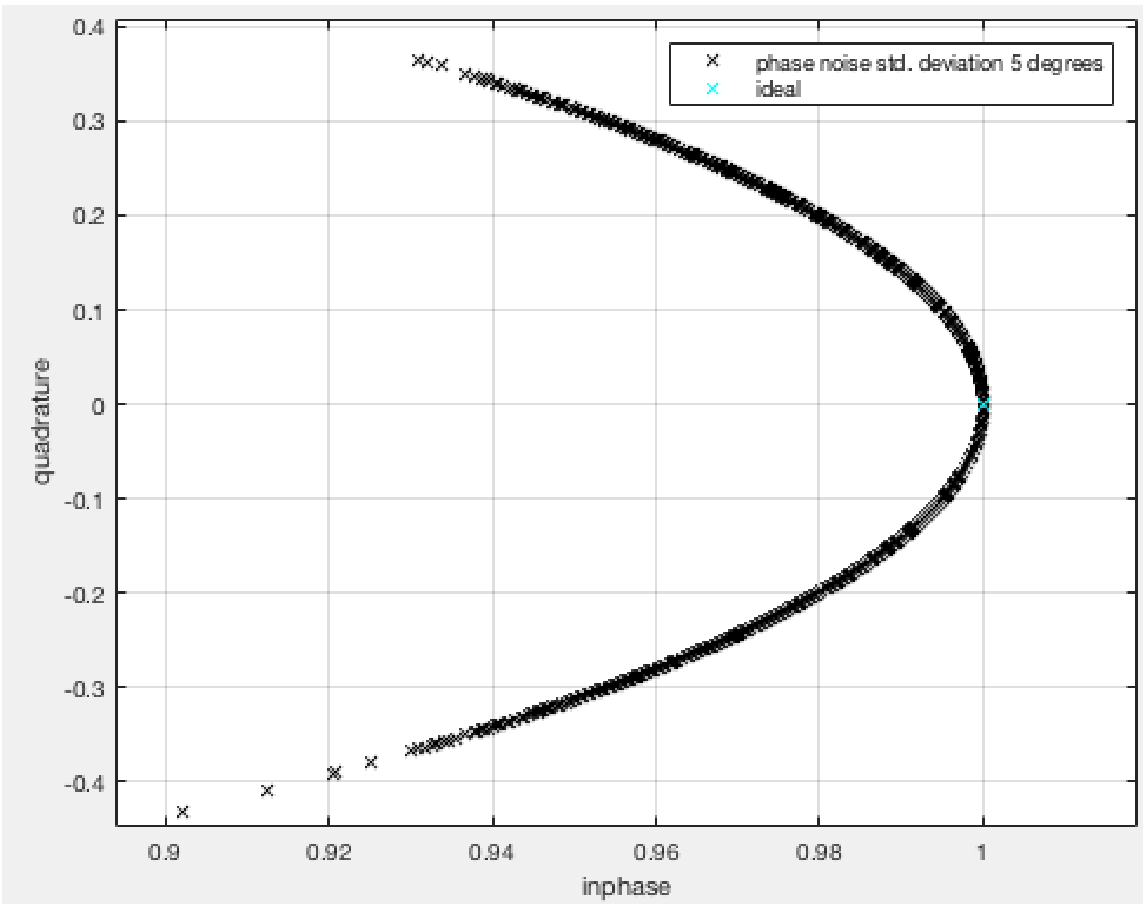


Figure 29: Constellation diagram with phase noise std. deviation of 5 degrees

Thermal Noise Effect

From previous sections it was mentioned that thermal noise can be considered as Additive white Gaussian noise. The noise will have zero mean, with a standard deviation that is influenced by the SNR. From figure 3 in chapter one, it is shown that thermal noise causes random spread in amplitude and phase with the spread centered around the ideal symbol. Since it is centered around the ideal symbol the mean distance to the threshold will be the distance to the ideal symbol [38]. Therefore, when phase noise and a watermark are included with thermal noise, the mean distance from the thresholds will be the distance found from the combined PDFs in appendix A and B.

APPENDIX D

```
%{
Jesse Venzor
Code for watermarked BPSK signals with phase and thermal noise
%}
N = 1*10^6; % % Number of bits
Eb_N0 = 0:1/10:10; % multiple Eb/N0 values in dB
phi_rms_deg =18;%RMS phase noise (variance)
nS = 1;
Tb=1;%1 micros bit duration
fc = 1e6; %carrier freq
os = 2; % oversampling factor
fs = 2/Tb; % sampling frequency in MHz
t = 0:(Tb/(nS)):N*Tb-(Tb/(nS)); %time
nt = round((N*t));

%% initialize Variables
var=5;
if var ==1
    nErr1 = zeros(1,length(Eb_N0));
    yhat1=zeros(1,N);
    y_err1= zeros(1,length(Eb_N0));
    ipHat1=zeros(1,N);
end

if var == 5
    nErr5 = zeros(1,length(Eb_N0));
    yhat5=zeros(1,N);
    y_err5= zeros(1,length(Eb_N0));
    ipHat5=zeros(1,N);
end

if var ==10
    nErr10 = zeros(1,length(Eb_N0));
    yhat10=zeros(1,N);
    y_err10= zeros(1,length(Eb_N0));
    ipHat10=zeros(1,N);
end
if var == 15
    nErr15 = zeros(1,length(Eb_N0));
    yhat15=zeros(1,N);
    y_err15= zeros(1,length(Eb_N0));
    ipHat15=zeros(1,N);
end
nErrpn = zeros(1,length(Eb_N0));
```

```
wm = [ones(1,N/2),-ones(1,N/2)]; %watermark bits

for ii = 1:length(Eb_N0)
    %% Random bit generation for signal
    quad_re_t = rand(1,length(nt))>0.5; % generated random bits from 1
to 0 with equal probability
    ip_t = (2*quad_re_t-1);%non return to zero bits 1 or -1 with equal
probability
    %% noise
    phi_rand = randn(1,length(nt)); %generats gaussian distributed
random numbers with mean 0 and variance of 1
    n = 1/sqrt(2)*(randn(1,N*os) + 1i*randn(1,N*os)); % white gaussian
noise, 0dB variance
    % we model as two randn() variables each with variance 1/2.
    %Hence the normalization by 1/sqrt(2)

    %% qpsk mod
    tx_mod = ip_t;
    osc = exp(1i*(2*pi*fc*t));

    %% phase addition at Transmitter
    phit =phi_rms_deg *(pi/180)*phi_rand; %gaussian distributed phase
noise with mean 0 and variance of 1 in radians
    %% Phase noise
    pn = tx_mod.*exp(1i*(2*pi*fc*t+phit)); % message signal with just
phase noise

    %% upsample 1
    sUpn = [pn;zeros(os-1,length(pn))];
    sUpn = sUpn(:).';
    sFiltpn = 1/sqrt(os)*conv(sUpn,ones(1,os));
    sFiltpn = sFiltpn(1:N*os);

    sUtx = [tx_mod;zeros(os-1,length(tx_mod))];
    sUtx = sUtx(:).';
    sFilttx = 1/sqrt(os)*conv(sUtx,ones(1,os));
    sFilttx = sFilttx(1:N*os);
    %% upsample osc
    sUosc = [osc;zeros(os-1,length(osc))];
    sUosc = sUosc(:).';
    sFiltosc = 1/sqrt(os)*conv(sUosc,ones(1,os));
    sFiltosc = sFiltosc(1:N*os);

    % RCVR Thermal Noise addition 1
    ypn = (sFiltpn + 10^(-Eb_N0(ii)/20)*n).*sFiltosc; % non-watermarked
signal with phase noise
    %
    if var == 1
        watermark1 = 1*(pi/180)*(wm); %generate watermark 1 degree away
from ideal
        t_wm1 = tx_mod.*exp(1i.*(2*pi*fc*t+watermark1));
        tot_1 =tx_mod.*exp(1i.*(2*pi*fc*t+watermark1+phit));
%watermarked signal in phasor notation with phase noise
        %-------------------upsample 5----------------------------------
        sU1 = [tot_1;zeros(os-1,length(tot_1))];
        sU1 = sU1(:).';
        sFilt1 = 1/sqrt(os)*conv(sU1,ones(1,os));
        sFilt1 = sFilt1(1:N*os);
        %------------ RCVR Thermal Noise addition 5---------------------
------
        y1 = (sFilt1 + 10^(-Eb_N0(ii)/20)*n).*sFiltosc; %Variance of
thermal noise decreases with respect to increasing SNR
        % mathched filter 5
```

```
        yFilt1 = conv(y1,ones(1,os)); % convolution
        ySamp1 = yFilt1(os:os:N*os);  % sampling at time T
        % find WM error 5
        y_in1 = real(ySamp1);
        y_q1 = imag(ySamp1);
        yhat1(y_in1>0&y_q1<0) = -1;
        yhat1(y_in1>0&y_q1>=0) = 1;
        %find bit error
        ipHat1 = real(ySamp1)>0;
        % BE count
        nErr1(ii) = size(find((quad_re_t- ipHat1)),2); % couting the
number of errors
        % Watermark error
        y_err1(ii) = size(find((wm-yhat1)),2); % counting the number of
errors
    end
    %
    %% RECEIVED WM 5 deg
    if var == 5
        watermark5 = 5*(pi/180)*(wm); %generate [1,-1]
        t_wm = tx_mod.*exp(1i.*(2*pi*fc*t+watermark5));
        tot_5 =tx_mod.*exp(1i.*(watermark5+phit)); %watermarked signal
in phasor notation
        %-------------------upsample 5---------------------------------
        sU5 = [tot_5;zeros(os-1,length(tot_5))];
        sU5 = sU5(:).';
        sFilt5 = 1/sqrt(os)*conv(sU5,ones(1,os));
        sFilt5 = sFilt5(1:N*os);
        %------------ RCVR Thermal Noise addition 5---------------------
------
        y5 = (sFilt5 + 10^(-Eb_N0(ii)/20)*n); %Variance of thermal noise
decreases with respect to increasing SNR
        % mathched filter 5
        yFilt5 = conv(y5,ones(1,os)); % convolution
        ySamp5 = yFilt5(os:os:N*os);  % sampling at time T
        % find WM error 5
        y_in5 = real(ySamp5);
        y_q5 = imag(ySamp5);
        yhat5(y_in5>0&y_q5<0) = -1;
        yhat5(y_in5>0&y_q5>=0) = 1;
        %find bit error
        ipHat5 = real(ySamp5)>0;
        % BE count
        nErr5(ii) = size(find((quad_re_t- ipHat5)),2); % couting the
number of errors
        % Watermark error
        y_err5(ii) = size(find((wm-yhat5)),2); % counting the number of
errors
    end
    %}
    %% RECEIVED WM 10 deg
    %{-
    if var == 10
        watermark10 = 10*(pi/180)*(wm); %generate [1,-1]
        t_wm10 = tx_mod.*exp(1i.*(2*pi*fc*t+watermark10));
        tot_10 =tx_mod.*exp(1i.*(2*pi*fc*t+watermark10+phit));
%watermarked signal in phasor notation
        %-------------------upsample 5---------------------------------
        sU10 = [tot_10;zeros(os-1,length(tot_10))];
        sU10 = sU10(:).';
        sFilt10 = 1/sqrt(os)*conv(sU10,ones(1,os));
        sFilt10 = sFilt10(1:N*os);
        %------------ RCVR Thermal Noise addition 5---------------------
------
```

```
        y10 = (sFilt10 + 10^(-Eb_N0(ii)/20)*n).*sFiltosc; %Variance of 
thermal noise decreases with respect to increasing SNR
        % mathched filter 5
        yFilt10 = conv(y10,ones(1,os)); % convolution
        ySamp10 = yFilt10(os:os:N*os);  % sampling at time T
        y_in10 = real(ySamp10);
        y_q10 = imag(ySamp10);
        yhat10(y_in10>0&y_q10<0) = -1;
        yhat10(y_in10>0&y_q10>=0) = 1;
        %find bit error
        ipHat10 = real(ySamp10)>0;
        % BE count
        nErr10(ii) = size(find((quad_re_t- ipHat10)),2); % couting the 
number of errors
        % Watermark error
        y_err10(ii) = size(find((wm-yhat10)),2); % counting the number 
of errors
    end
    %}
    %% RECEIVED WM 5 deg
    if var == 15
        watermark15 = 15*(pi/180)*(wm); %generate [1,-1]
        t_wm15 = tx_mod.*exp(1i.*(2*pi*fc*t+watermark15));
        tot_15 =tx_mod.*exp(1i.*(2*pi*fc*t+watermark15+phit)); 
%watermarked signal in phasor notation
        %-------------------upsample 5---------------------------------
        sU15 = [tot_15;zeros(os-1,length(tot_15))];
        sU15 = sU15(:).';
        sFilt15 = 1/sqrt(os)*conv(sU15,ones(1,os));
        sFilt15 = sFilt15(1:N*os);
        %------------ RCVR Thermal Noise addition 5---------------------
------
        y15 = (sFilt15 + 10^(-Eb_N0(ii)/20)*n).*sFiltosc; %Variance of 
thermal noise decreases with respect to increasing SNR
        % mathched filter 5
        yFilt15 = conv(y15,ones(1,os)); % convolution
        ySamp15 = yFilt15(os:os:N*os);  % sampling at time T
        % find WM error 5
        y_in15 = real(ySamp15);
        y_q15 = imag(ySamp15);
        yhat15(y_in15>0&y_q15<0) = -1;
        yhat15(y_in15>0&y_q15>=0) = 1;
        %find bit error
        ipHat15 = real(ySamp15)>0;
        % BE count
        nErr15(ii) = size(find((quad_re_t- ipHat15)),2); % couting the 
number of errors
        % Watermark error
        y_err15(ii) = size(find((wm-yhat15)),2); % counting the number 
of errors
    end
    %% mathched filter qp

    %% mathched filter 1
    yFiltpn = conv(ypn,ones(1,os)); % convolution
    ySamppn = yFiltpn(os:os:N*os);  % sampling at time T
    %% find bit error
    ipHatpn = real(ySamppn)>0;
    nErrpn(ii) = size(find((quad_re_t- ipHatpn)),2); % couting the 
number of errors
end

%% wm 1
if var == 1
```

```
    % wm err
    wm_ber_test = y_err1/N;
    % bpsk error
    bpsk_err = nErr1/N;
end
%% wm 5
if var == 5
    % wm err
    wm_ber_test_5 = y_err5/N;
    % bpsk error
    bpsk_err5 = nErr5/N;
    Pxx1 = pwelch(y5,[],[],1024,'twosided');
end
%% wm 10
if var == 10
    % wm err
    wm_ber_test_10 = y_err10/N;
    % bpsk error
    bpsk_err10 = nErr10/N;
end
%% wm 15
if var == 15
    % wm err
    wm_ber_test_15 = y_err15/N;
    % bpsk error
    bpsk_err15 = nErr15/N;
end
```

APPENDIX E

Code for theoretical error plots for watermarked BPSK signal with phase and thermal noise

```
%% set parameters
Eb_N0 = 0:1/10:10; %SNR in dB
snr=10.^(Eb_N0/10);
eb = 1; %bt energy
N0=eb./snr; %noise power
vartherm=N0/2; % noise variance

%% wm 1 degree
mu1=0.99605; % Mean of mapped PDF for combined phase noise and watermark
```

```
message error
var1 = 32.07666*10^-6;  % variance of mapped PDF for combined phase
noise and watermark message error
muwm1=0.01738608; % Mean of mapped PDF for combined phase noise and
watermark, watermark error
varwm1=7.55544*10^-3; % variance of mapped PDF for combined phase noise
and watermark, watermark error
vartot1 = vartherm+var1; %combined thermal and mapped pdf variance for
message error rate
varwmtot1 = vartherm+varwm1; %combined thermal and mapped pdf variance
for watermark error rate
bpsk1=qfunc(sqrt(2)*mu1./(sqrt(2*vartot1)));%qfunction eqn 4.2
watermarkerror1 = qfunc(sqrt(2)*muwm1./(sqrt(2*varwmtot1)));%qfunction
eqn 4.3
%% wm 5 degree
mupn5=  .99241;
moment2=.985;
var5=moment2-mupn5^2
mupwm5=0.08682451;
varwm5=7.50054*10^-3;
vartot5 = vartherm+var5;
varwmtot5 = vartherm+varwm5;
bpsk5 = qfunc(sqrt(2)*mupn5./(sqrt(2*vartot5)));
watermarkerror5 = qfunc(sqrt(2)*mupwm5./(sqrt(2*varwmtot5)));
%% wm 10 degree
mu10=0.98107;
var10 = 255.69624*10^-6;
muwm10= 0.1729882;
varwm10=7.33071*10^-3;
vartot10 = vartherm+var10;
varwmtot = vartherm+varwm10;
bpsk10=qfunc(sqrt(2)*mu10./(sqrt(2*vartot10)));
watermarkerror10 = qfunc(sqrt(2)*muwm10./(sqrt(2*varwmtot)));
%% wm 15 degree
mu15=0.96225;
var15 = 533.19567*10^-6;
muwm15=0.25783541;
varwm15=7.05339*10^-3;
vartot15 = vartherm+var15;
varwmtot15 = vartherm+varwm15;
bpsk15=qfunc(sqrt(2)*mu15./(sqrt(2*vartot15)));
watermarkerror15 = qfunc(sqrt(2)*muwm15./(sqrt(2*varwmtot15)));
%% BPSK MESSAGE ERROR THEORETICAL PLOTS
ideal = qfunc(sqrt(2*eb)./(sqrt(N0)));
%{
figure

semilogy(Eb_N0,ideal,'-k')
hold on
semilogy(Eb_N0,bpsk1,'-b')
hold on
semilogy(Eb_N0,bpsk5,'-r')
hold on
semilogy(Eb_N0,bpsk10,'-g')
hold on
semilogy(Eb_N0,bpsk15,'-m')


legend('ideal Non-watermarked BPSK','watermarked BPSK signal with phase
noise variance of 5 degrees and 1 degree watermark',...
'watermarked BPSK signal with phase noise variance of 5 degrees and 5
degree watermark',...
'watermarked BPSK signal with phase noise variance of 5 degrees and 10
degree watermark',...
```

```
'watermarked BPSK signal with phase noise variance of 5 degrees and 15
degree watermark',...
'Simulated Message BER phase noise variance of 5 degrees and 1 degree
watermark',...
'Simulated Message BER phase noise variance of 5 degrees and 5 degree
watermark',...
'Simulated Message BER phase noise variance of 5 degrees and 10 degree
watermark',...
'Simulated Message BER phase noise variance of 5 degrees and 15 degree
watermark')
xlabel('Eb/No')
ylabel('Message BER')
%}
% BPSK WATERMARK ERROR THEORETICAL PLOTS


semilogy(Eb_N0,watermarkerror1,'-b')
hold on
semilogy(Eb_N0,watermarkerror5,'-r')
hold on
semilogy(Eb_N0,watermarkerror10,'-g')
hold on
semilogy(Eb_N0,watermarkerror15,'-m')
%

legend('Theoretical watermarked BPSK signal with phase noise variance of
5 degrees and 1 degree watermark',...
'Theoretical watermarked BPSK signal with phase noise variance of 5
degrees and 5 degree watermark',...
'Theoretical watermarked BPSK signal with phase noise variance of 5
degrees and 10 degree watermark',...
'Theoretical watermarked BPSK signal with phase noise variance of 5
degrees and 15 degree watermark',...
'Simulated watermark error phase noise variance of 5 degrees and 1
degree watermark',...
'Simulated watermark error phase noise variance of 5 degrees and 5
degree watermark',...
'Simulated watermark error phase noise variance of 5 degrees and 10
degree watermark',...
'Simulated watermark error phase noise variance of 5 degrees and 15
degree watermark')

xlabel('Eb/No')
ylabel('Watermark error rate')
%}
%% different phase noise values with 5 degree watermark
mu3=.99483;
var3=24.4522*10^-6;
var3tot=var3+vartherm;
muwm3=0.08703635;
varwm3=2.71331*10^-3;
varwm3tot=varwm3+vartherm;
bp3=qfunc(sqrt(2)*mu3./(sqrt(2*var3tot)));
bpwm3=qfunc(sqrt(2)*muwm3./(sqrt(2*varwm3tot)));



mu11=.978;
var11=919.77563*10^-6;
var11tot=var11+vartherm;
muwm11= 0.08556423;
varwm11=0.03527;
varwm11tot=varwm11+vartherm;
```

```
bp11=qfunc(sqrt(2)*mu11./(sqrt(2*var11tot)));
bpwm11=qfunc(sqrt(2)*muwm11./(sqrt(2*varwm11tot)));


mu18=.948227796595367;
var18=.005062963309333;
var18tot=2*var18+vartherm;
muwm18=0.08295756;
varwm18=0.08892;
varwm18tot=varwm18+vartherm;
bp18=qfunc(sqrt(2)*mu18./(sqrt(2*var18tot)));
bpwm18=qfunc(sqrt(2)*muwm18./(sqrt(2*varwm18tot)));


%
% % figure
% semilogy(Eb_N0,bp3,'--r')
% % hold on
% % semilogy(Eb_N0,bpsk5,'--r')
% hold on
% semilogy(Eb_N0,bp11,'--r')
% hold on
% semilogy(Eb_N0,bp18,'--r')

figure
semilogy(Eb_N0,bpwm3,'-r')
hold on
semilogy(Eb_N0,bpwm11,'--r')
hold on
semilogy(Eb_N0,bpwm18,'-.r')
legend('Theoretical watermarked BPSK signal with phase noise
std.deviation of 3 degrees and 5 degree watermark',...
'Theoretical watermarked BPSK signal with phase noise std.deviation of
11 degrees and 5 degree watermark',...
'Theoretical watermarked BPSK signal with phase noise std.deviation of
15 degrees and 5 degree watermark',...
'simulated watermarked BPSK signal with phase noise std.deviation of 3
degrees and 5 degree watermark',...
'simulated watermarked BPSK signal with phase noise std.deviation of 11
degrees and 5 degree watermark',...
'simulated watermarked BPSK signal with phase noise std.deviation of 15
degrees and 5 degree watermark')


xlabel('Eb/No')
ylabel('Watermark error rate')
```

VITA

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