ANALYSIS OF CONTINUOUS RECTANGULAR PLATES ON FLEXIBLE BEAM SUPPORTS BY FLEXIBILITY METHODS

By

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NOMENCLATURE

a, b	Carry-Over Factors
c, c', d, d'	Dimensionless Quantities
i, j, k	Network Points
p	Intensity of Load
t	Dimensionless Quantity
x, y	Rectangular Co-ordinates, Co-ordinate Axes
D	Flexural Rigidity of Plate
D_{ii}	Displacement Flexibility
E	Young's Modulus of Elasticity
F_{ii}	Angular Flexibility
G_{ij}	Angular Carry-Over
H_{ij}	Displacement Carry-Over
I	Flexural Rigidity of Beam
M	Sum of Bending Moments x $\frac{1}{(1+\nu)}$
O	Origin of Co-ordinate System
P	Concentrated Load
Q_{ij}	Angular-Displacement Carry-Over
R	Vertical Edge Reaction Per Unit Length
w	Vertical Deflection
δ	Displacement Load Function
η	Deflection Influence Coefficient
θ	Angle of Rotation of Plate

λ	Dimensionless Quantity
ν	Poisson's Ratio
τ	Angular Load Function
ΔA	Area of the Domain of Point ij
Δx, Δy	Dimensions of Plate Element

CHAPTER I

INTRODUCTION

1.1. Preliminary Remarks. Continuous rectangular plates can broadly be classified into two groups depending upon the number of directions of continuity. "One-way" continuous plates are those that are continuous over supports in only one direction, and "two-way" continuous plates are those that are continuous in two mutually orthogonal directions.

One-way rectangular plates continous over rigid supports have been treated by Marcus (1), Jensen (2) and Hawk (3). Newmark (4) extended the distribution method to one-way continuous plates over flexible supports.

Rigorous solutions for two-way continuous plates are available for limited special cases only. Southerland, et. al. (5) and Neilson (6) treated the problem of a plate consisting of a number of identical panels and supported by beams of equal stiffness. Approximate solutions of two-way continuous plates over rigid supports have been presented by Bittner (7) and Maugh and Pan (8). Engelbreth (9) and Newmark (10) independently developed approximate distribution procedures for determining the total moments across any section for plates continuous over rigid beams. Lechter (11) extended the flexibility method to two-way continuous plates over rigid supports. The basic structure in this approach was a simply supported rectangular plate. Angular functions were defined in terms of influence coefficients for deflection of a simple plate obtained from a set of tables prepared by Tuma, Havner and French (12). Single panel solutions were extended by the method of moment distribution to the analysis of two-way continuous rectangular plates supported by beams with flexural and torsional rigidities by Ang and Newmark (13).

1.2. Scope of Study. This study extends the flexibility method of approach to the solution of rectangular plates continuous in two directions and supported by flexible beams. Torsional stiffness of the beams is not taken into consideration because of the complex nature of the problem.

The essentials of the flexibility approach to continuous rectangular plates were discussed by Tuma in a graduate course in plate structures during fall 1961-1962. A rectangular plate supported by four columns at corners and having all edges free, is selected as a basic structure. A method of obtaining influence coefficients for deflection for the selected basic structure is described in Chapter II. In Chapter III general moment and reaction equations are derived in terms of flexibilities from compatibility conditions at the junction of two adjacent panels, and a matrix formulation for the solution is presented. All the flexibilities are defined in terms of the influence coefficients for deflection in Chapter IV. An example problem is worked out in Chapter V and a summary and conclusions are given in Chapter VI.

CHAPTER II

DEFLECTION INFLUENCE COEFFICIENTS BY FINITE DIFFERENCES

2.1. <u>General Finite Difference Equation</u>. Consider a thin rectangular plate subjected to normal loads and supported by four columns at the four corners (Figure 2.1).

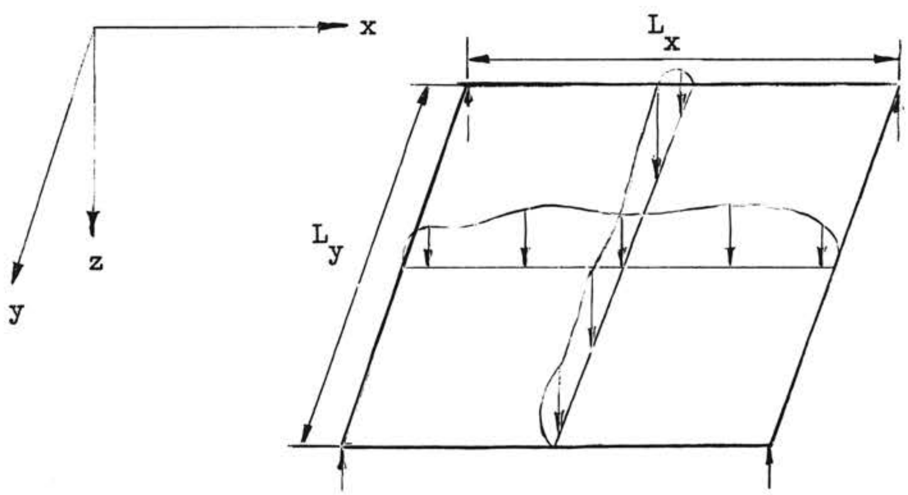


Figure 2.1. Basic Plate Structure

With usual assumptions, the deflections are governed by Lagrange's differential equation:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{D} \qquad (2.1)$$

where,

x, y = Coordinates of a point on the plate surface.

w = Deflection at any point x, y.

p = Intensity of loading at the point.

D = Flexural rigidity of the plate.

Equation 2.1 can be resolved into two equations:

$$\frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} = -p \tag{2.2}$$

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = -\frac{M}{D} \tag{2.3}$$

where,

$$M = -D \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

Consider that the plate structure shown in Figure 2.1 is divided into an arbitrary number of equal size rectangular elements Δx and Δy along x and y-axes. A typical detail of gridwork thus formed is shown in Figure 2.2.

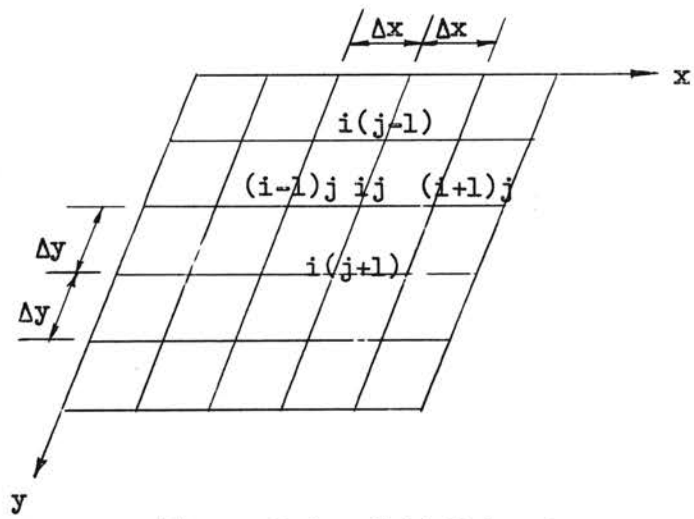


Figure 2.2. Grid Network

Expressing Equations 2.2 and 2.3 in finite difference form at any general point ij,

$$\frac{M_{(i+1)j} - 2M_{ij} + M_{(i-1)j}}{\Delta x^2} + \frac{M_{i(j+1)} - 2M_{ij} + M_{i(j-1)}}{\Delta y^2} = -p_{ij} \qquad (2.4)$$

$$\frac{w_{(i+1)j} - 2w_{ij} + w_{(i-1)j}}{\Delta x^2} + \frac{w_{i(j+1)} - 2w_{ij} + w_{i(j-1)}}{\Delta y^2} = -\frac{M_{ij}}{D} \qquad (2.5)$$

With the following notation

$$t = \frac{\Delta x}{\Delta y}, \quad a = \frac{1}{2(1+t^2)}, \quad b = \frac{t^2}{2(1+t^2)}$$

$$\lambda = \frac{t}{2(1+t^2)} \quad \text{and} \quad \Delta A = \Delta x \cdot \Delta y$$

the above equations reduce to

$$M_{ij} - a\left(M_{(i+1)j} + M_{(i-1)j}\right) - b\left(M_{i(j+1)} + M_{i(j-1)}\right) = p_{ij}\lambda\Delta A \qquad (2.6)$$

$$w_{ij} - a\left(w_{(i+1)j} + w_{(i-1)j}\right) - b\left(w_{i(j+1)} + w_{i(j-1)}\right) = \frac{M_{ij}}{D}\lambda\Delta A \qquad (2.7)$$

If Equation 2.7 is written at points (i+1)j, (i-1)j, i(j+1) and i(j-1) successively, the following equations will result:

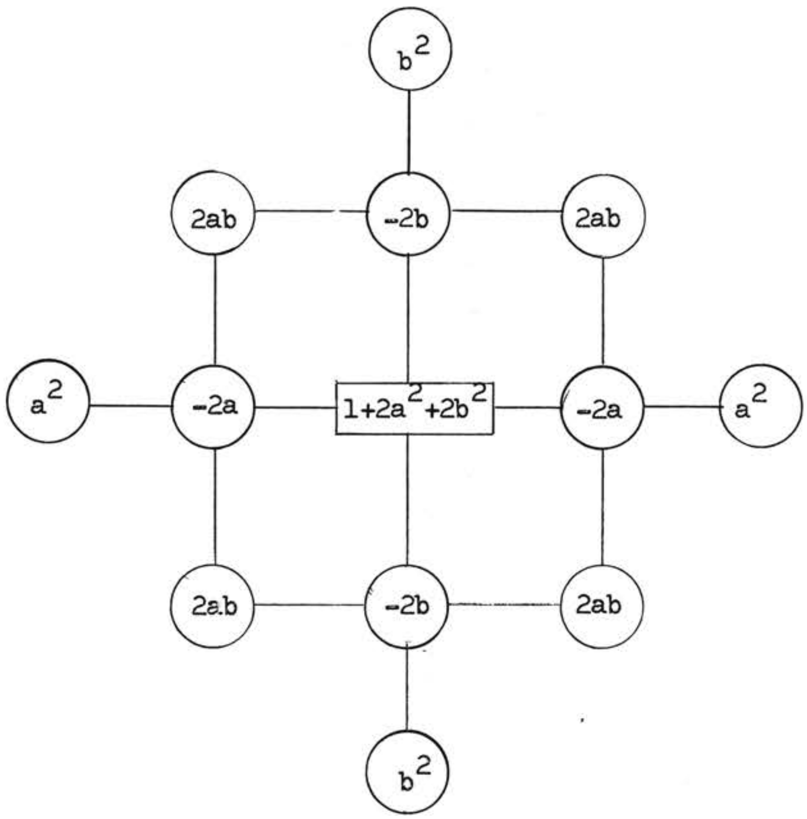
$$\frac{M_{(i+1)j}}{D}\lambda\Delta A = w_{(i+1)j} - a\left(w_{(i+2)j} + w_{ij}\right) - b\left(w_{(i+1)(j+1)} + w_{(i+1)(j-1)}\right)$$

$$\frac{M_{(i-1)j}}{D}\lambda\Delta A = w_{(i-1)j} - a\left(w_{ij} + w_{(i-2)j}\right) - b\left(w_{(i-1)(j+1)} + w_{(i-1)(j-1)}\right)$$

$$\frac{M_{i(j+1)}}{D}\lambda\Delta A = w_{i(j+1)} - a\left(w_{(i+1)(j+1)} + w_{(i-1)(j+1)}\right) - b\left(w_{i(j+2)} + w_{ij}\right)$$

$$\frac{M_{i(j-1)}}{D}\lambda\Delta A = w_{i(j-1)} - a\left(w_{(i+1)(j-1)} + w_{(i-1)(j-1)}\right) - b\left(w_{ij} + w_{i(j+2)}\right) \quad (2.8)$$

Substituting Equations 2.7 and 2.8 in Equation 2.6, Lagrange's equation is obtained in finite difference form which can be represented as,



$$(w) = \frac{p}{D}(\lambda\Delta A)^2 \quad (2.9)$$

This equation is valid for all the points lying in the rectangle formed by second interior lines from the edges.

2.2. Finite Difference Equations for Typical Points Near the Edges. The general finite difference equation will be modified for various points near the edges as follows:

(a) Point on First Interior Line parallel to the y-axis (Figure 2.3a).

The boundary condition is

$$(M_x)_{edge} = -D \left(\frac{\partial^2 w}{\partial x^2} + \upsilon \frac{\partial^2 w}{\partial y^2}\right) = 0$$

or $$(M)_{edge} = -D\,(1-\upsilon)\,\frac{\partial^2 w}{\partial y^2}$$

Applying this condition to the point $(i-1)j$

$$\frac{M_{(i-1)j}}{D} = -\frac{1-\upsilon}{\Delta y^2}\left(w_{(i-1)(j-1)} - 2w_{(i-1)j} + w_{(i-1)(j+1)}\right)$$

Denoting $\frac{1-\upsilon}{(\Delta y)^2}\lambda\Delta A = (1-\upsilon)\lambda t = d$

the equation can be written as

$$\frac{M_{(i-1)j}}{D}\lambda\Delta A = -d\left(w_{(i-1)(j-1)} - 2w_{(i-1)j} + w_{(i-1)(j+1)}\right) \qquad (2.10)$$

Substituting this in Equation 2.6 along with Equations 2.7 and 2.8,

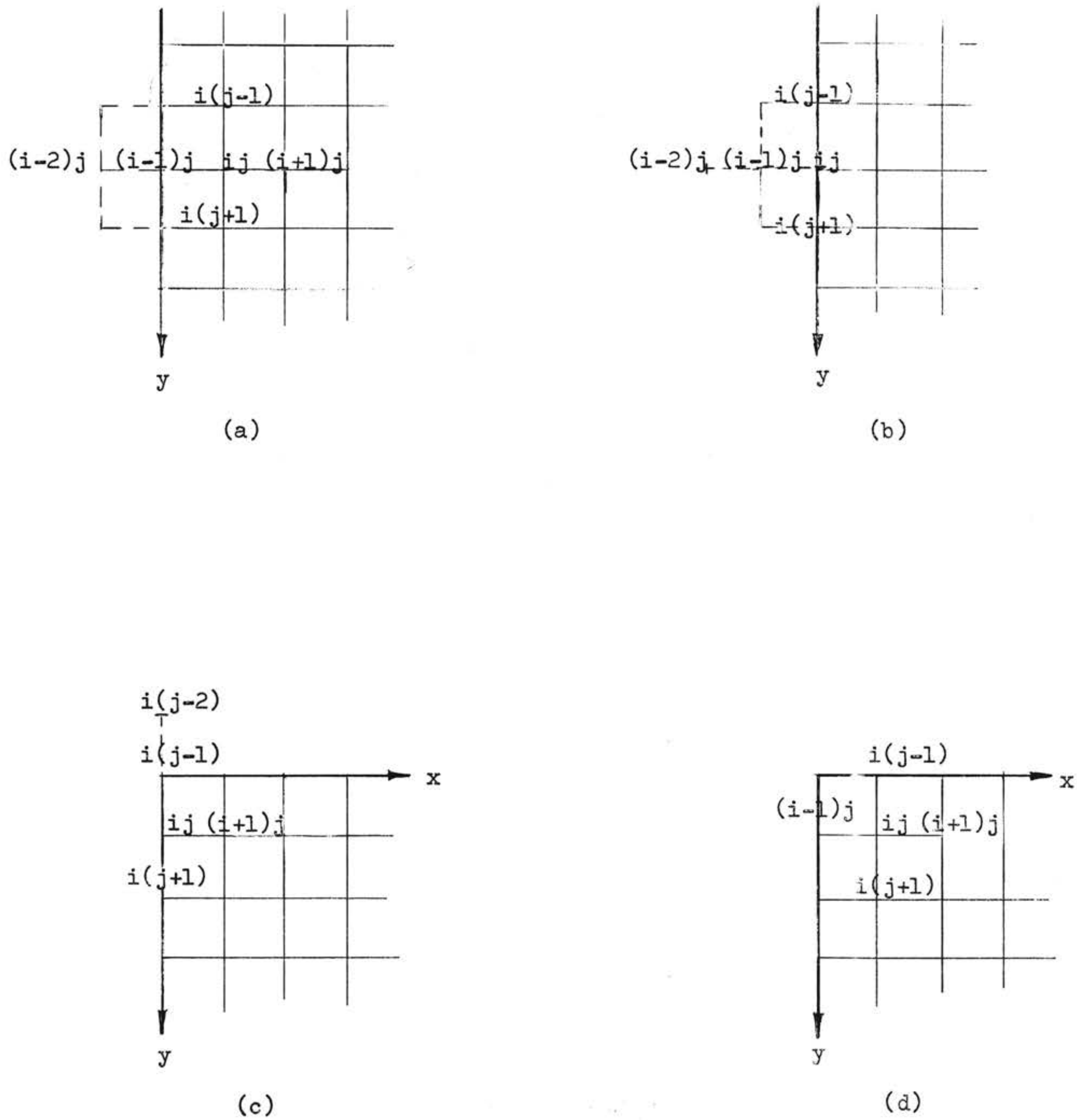
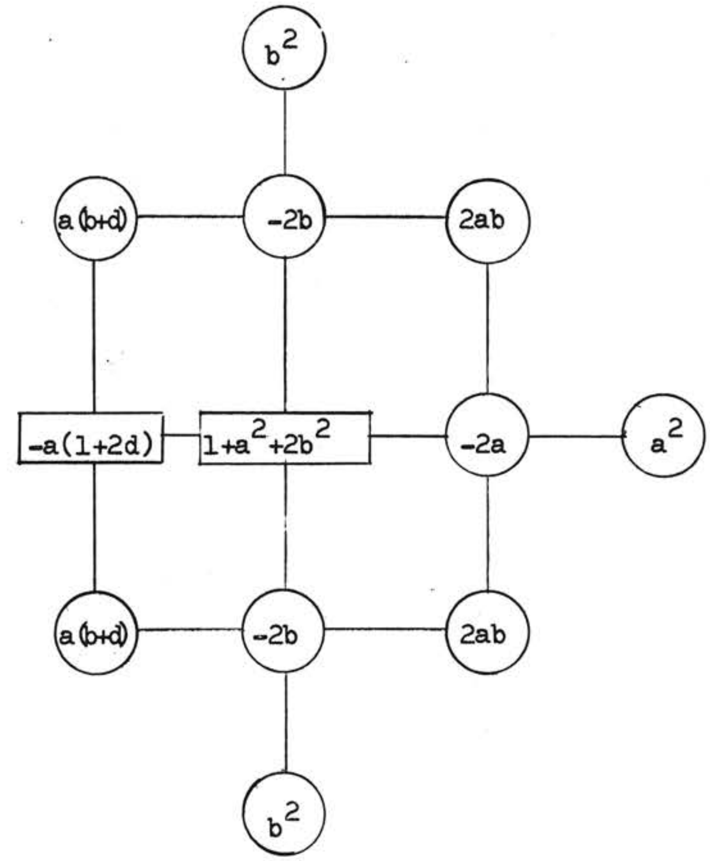
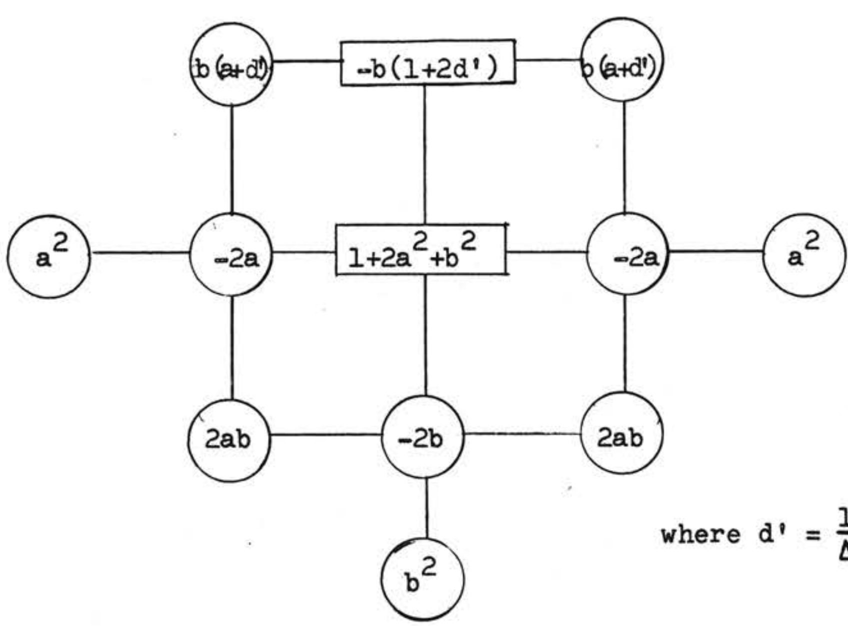


Figure 2.3. Typical Points Near the Edges



$$(w) = \frac{p}{D}(\lambda \Delta A)^2 \qquad (2.11)$$

If the point lies on the first interior point parallel to the x-axis this equation modifies to



$$(w) = \frac{p}{D}(\lambda \Delta A)^2 \qquad (2.12)$$

where $d' = \frac{1-\nu}{\Delta x^2} \lambda \Delta y = \frac{\lambda(1-\nu)}{t}$

(b) Point on the Free Edge parallel to the y-axis (Figure 2.3b).

When point ij falls on the edge, points (i-2)j, (i-1)j (i-1)(j-1) and (i-1)(j+1) lie outside the plate. The deflections at these points are expressed in terms of the deflections at the points on the plate by making use of the boundary conditions:

The first boundary condition is

$$(M_x)_{edge} = 0 \text{ or } (M)_{edge} = -D(1-\upsilon)\frac{\partial^2 w}{\partial y^2} .$$

In finite difference form this becomes,

$$\frac{M_{ij}\lambda\Delta A}{D} = w_{ij} - a\left(w_{(i+1)j} + w_{(i-1)j}\right) - b\left(w_{i(j+1)} + w_{i(j-1)}\right)$$

$$= -d\left(w_{i(j-1)} - 2w_{ij} + w_{i(j+1)}\right)$$

from which

$$w_{(i-1)j} = w_{ij}\,\frac{(1-2d)}{a} + \frac{(d-b)}{a}\left(w_{i(j+1)} + w_{i(j-1)}\right) - w_{(i+1)j} \tag{2.13}$$

Similarly, expressing the same boundary condition at points i(j+1) and i(j-1):

$$w_{(i-1)(j+1)} = w_{i(j+1)}\,\frac{(1-2d)}{a} + \frac{(d-b)}{a}\left(w_{ij} + w_{i(j+2)}\right) - w_{(i+1)(j+1)} \tag{2.14}$$

$$w_{(i-1)(j-1)} = w_{i(j-1)}\,\frac{1-2d}{a} + \frac{(d-b)}{a}\left(w_{ij} + w_{i(j-2)}\right) - w_{(i+1)(j-1)} \tag{2.15}$$

The second boundary condition is

$$R_x = -D\left(\frac{\partial^3 w}{\partial x^3} + (2-\upsilon)\frac{\partial^3 w}{\partial x \partial y^2}\right) = 0.$$

Expressing this in finite difference form

$$\frac{1}{2\Delta x^2}\left(w_{(i+2)j} - 2w_{(i+1)j} + 2w_{(i-1)j} - w_{(i-2)j}\right) + \frac{2-\nu}{2\Delta x\Delta y^2}\left(w_{(i+1)(j+1)}\right.$$

$$- w_{(i-1)(j+1)} - 2w_{(i+1)j} + 2w_{(i-1)j} - w_{(i-1)(j-1)} +$$

$$\left. + w_{(i+1)(j-1)}\right) = 0 \ .$$

Solving for $w_{(i-2)j}$,

$$w_{(i-2)j} = w_{(i+2)j} - 2w_{i+1)j} + 2w_{(i-1)j} + c\left(w_{(i+1)(j+1)} -\right.$$

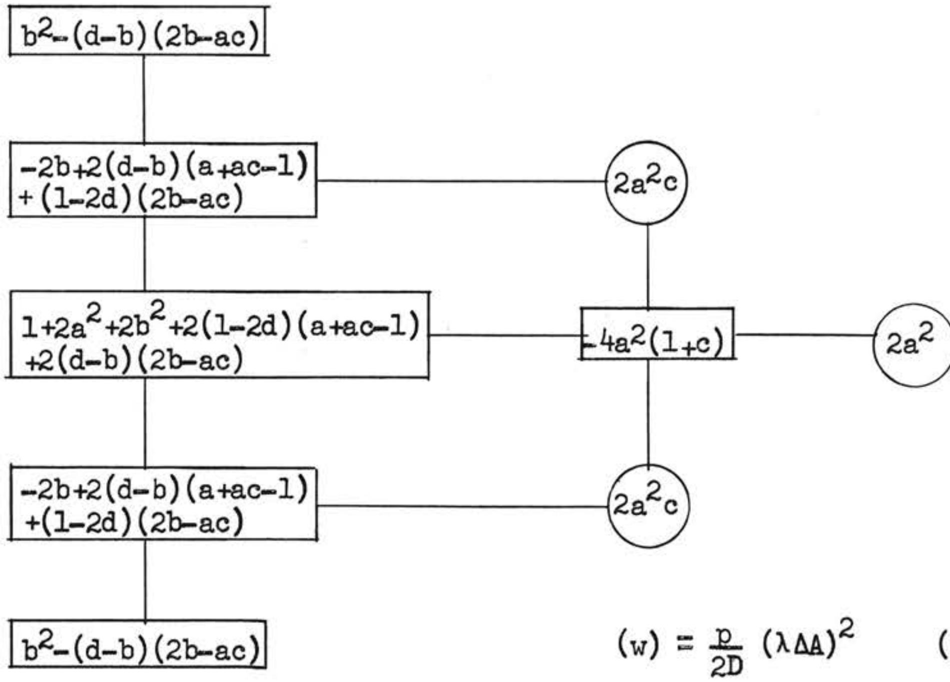
$$- w_{(i-1)(j+1)} - 2w_{(i+1)j} + 2w_{(i-1)j} - w_{(i-1)(j-1)} +$$

$$\left. + w_{(i+1)(j-1)}\right) \qquad (2.16)$$

where $c = (2-\nu)t^2$

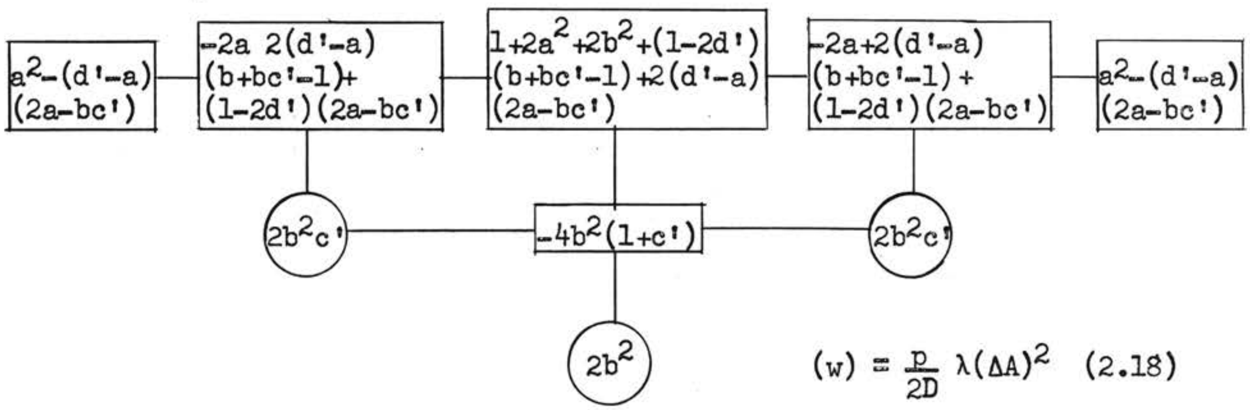
The unknown values $w_{(i-1)j}$, $w_{(i-1)(j+1)}$ and $w_{(i-1)(j-1)}$ can be eliminated from the above equation by using Equations 2.13, 2.14 and 2.15.

Substituting Equations 2.13 to 2.16 in the general equation (2.9), the operator equation obtained is



$$(w) = \frac{p}{2D} (\lambda \Delta A)^2 \qquad (2.17)$$

If the point lies on the free edge parallel to the x-axis, the equation modifies to



$$(w) = \frac{p}{2D} \lambda (\Delta A)^2 \qquad (2.18)$$

where $d' = \frac{(1-\nu)}{t}$ and $c' = \frac{(2-\nu)}{t^2}$

(c) Point adjacent to a support on the Free Edge parallel to the y-axis (Figure 2.3c).

The boundary conditions are:

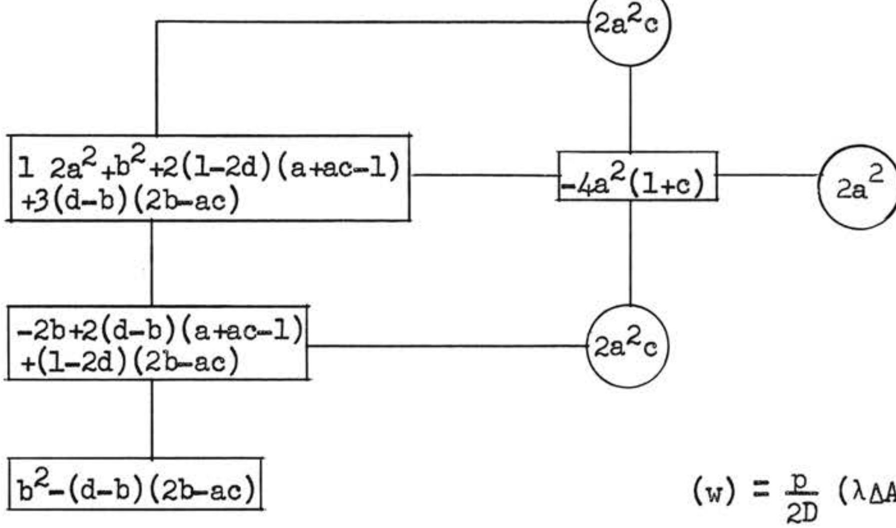
(i) $w_{i(j-1)} = 0 \qquad (2.19)$

(ii) $(M_x)_{i(j-1)} = (M_y)_{i(j-1)} = 0$

From which it follows that

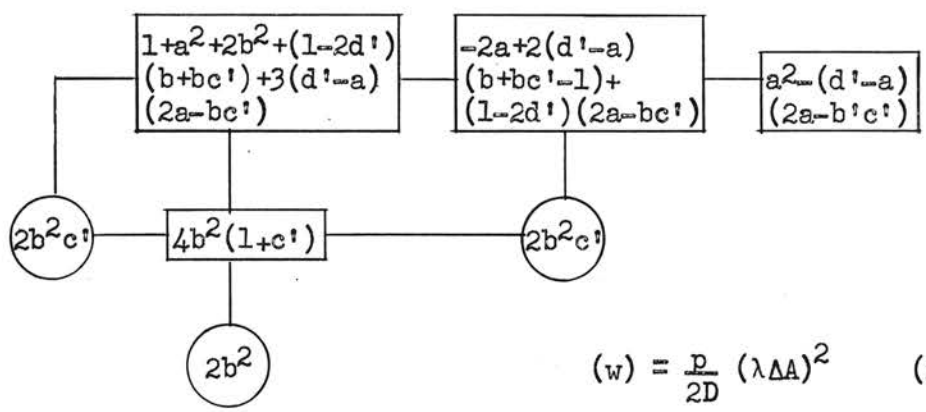
$$w_{(i-2)j} = -w_{ij} \tag{2.20}$$

Substituting in Equation 2.17,



$$(w) = \frac{p}{2D} (\lambda \Delta A)^2 \tag{2.21}$$

If the point is adjacent to a support on the free edge parallel to the x-axis, the above equation takes the form,



$$(w) = \frac{p}{2D} (\lambda \Delta A)^2 \tag{2.22}$$

(d) First Interior Corner Point (Figure 2.3d).

The boundary conditions are:

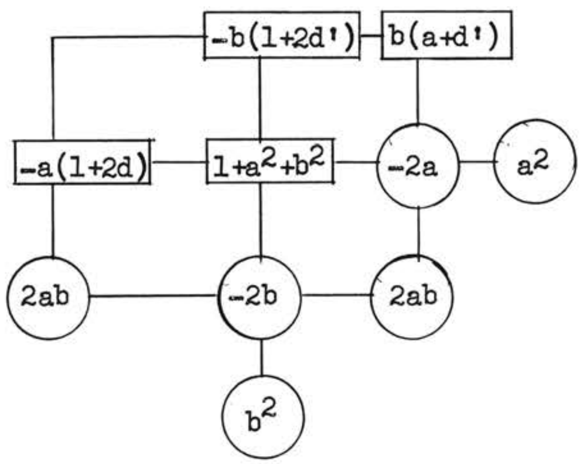
(i) $w_{(i-1)(j-1)} = 0$

(ii) $(M_x)_{(i-1)j} = (M_y)_{i(j-1)} = 0$

Applying this condition at points (i-1)j and i(j-1),

$$\left.\begin{aligned} \frac{M_{(i-1)j}}{D}\lambda\Delta A &= -d\left(w_{(i-1)(j+1)} - 2w_{(i-1)j} + w_{(i-1)(j-1)}\right) \\ \frac{M_{i(j-1)}}{D}\lambda\Delta A &= -d'\left(w_{(i-1)(j-1)} - 2w_{i(j-1)} + w_{(i+1)(j-1)}\right) \end{aligned}\right\} \quad (2.23)$$

Substituting Equations 2.23 along with Equations 2.7 and 2.8 in 2.6,



$$(w) = \frac{p}{D}\,(\lambda\Delta A)^2 \qquad (2.24)$$

2.3. <u>Deflection Influence Coefficients</u>. The deflection at a point ij due to a unit load at point kl is defined as "Deflection Influence Coefficient" and is denoted by η_{ij}^{kl}. The deflections at all network points of the basic plate structure shown in Figure 2.1, due to a unit load at any point kl, can be determined by writing the equations derived in previous sections at various points and solving them simultaneously. A matrix formulation and computer solution is very convenient in such cases. Using the abbreviated notation, the matrix formulation takes the form

$$[A]\ [w] = [p] \tag{2.25}$$

where

$[A]$ = Coefficient matrix

$[w]$ = Deflection matrix

$[p]$ = Load matrix

From Equation 2.25

$$[w] = [A]^{-1}\ [p] \tag{2.26}$$

Thus the deflection influence coefficients due to unit load at kl can readily be obtained by inverting the coefficient matrix A by means of a computer and post multiplying it by load matrix p which has unity as the element corresponding to kl, the remaining elements being zero.

If η_{ij}^{kl} is the deflection influence coefficient as defined above, the deflection at ij due to a unit load at kl becomes

$$w_{ij}^{kl} = \frac{\Delta x \Delta y}{D}\ \eta_{ij}^{kl}. \tag{2.27}$$

The deflection influence coefficients due to unit load at other points can be obtained by changing the elements of matrix p successively and pre-multiplying by $[A]^{-1}$.

If the computer being utilized has internal storage capable of directly inverting a matrix of order 'n' and if the order of matrix A exceeds this, a method which utilizes the geometric symmetry of the basic structure and the principle of superposition can be applied.

An unsymmetrically loaded symmetrical plate can be represented as the summation of four symmetrically and antisymmetrically loaded plates as shown in Figure 2.4.

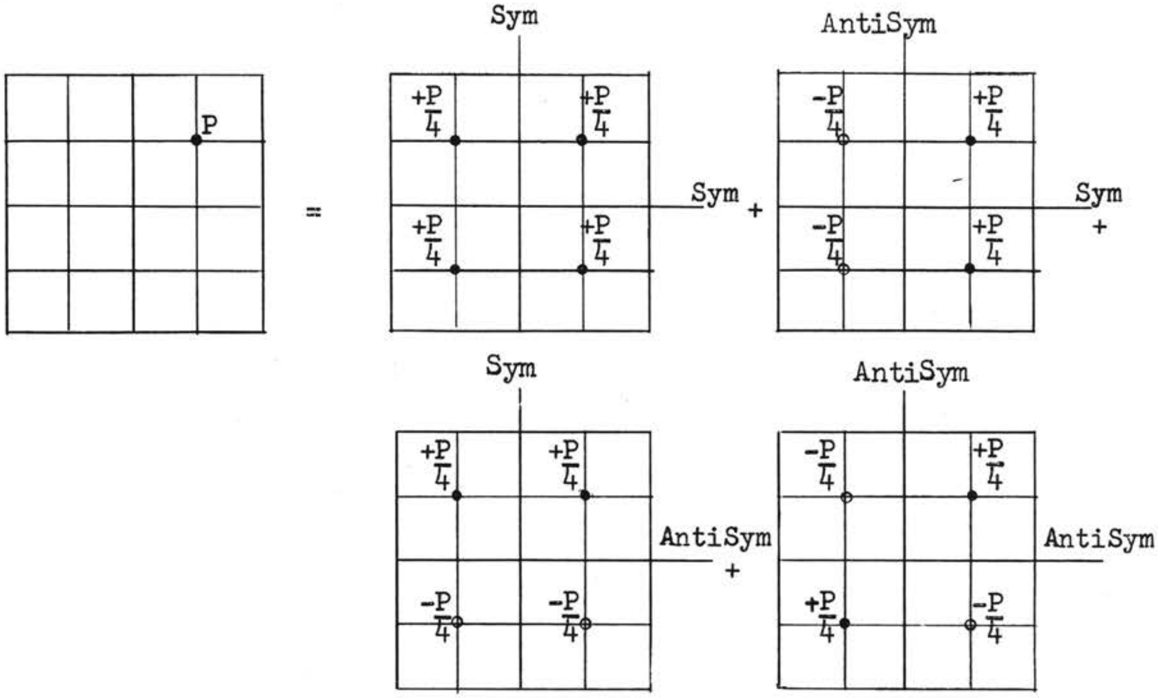


Figure 2.4. Resolution of Antisymmetrically Loaded Symmetrical Plate.

It can be easily seen that the deflections in the upper right quadrant are sufficient to determine the deflections throughout the entire plate in each of the four plates on the right side. When once the deflections throughout these plates are obtained, the deflections due to unsymmetrical load P on the plate on the left side can be obtained by the method of superposition. Thus, the problem reduces to inversion of matrices of much smaller order than A.

CHAPTER III

GENERAL MOMENT AND REACTION EQUATIONS

3.1. Derivation of Moment and Reaction Equations. A continuous rectangular plate subjected to loads normal to the middle plane of the plate is considered (Figure 3.1). The flexural rigidity in any panel is constant. The supporting beams are flexible and their torsional rigidity is neglected.

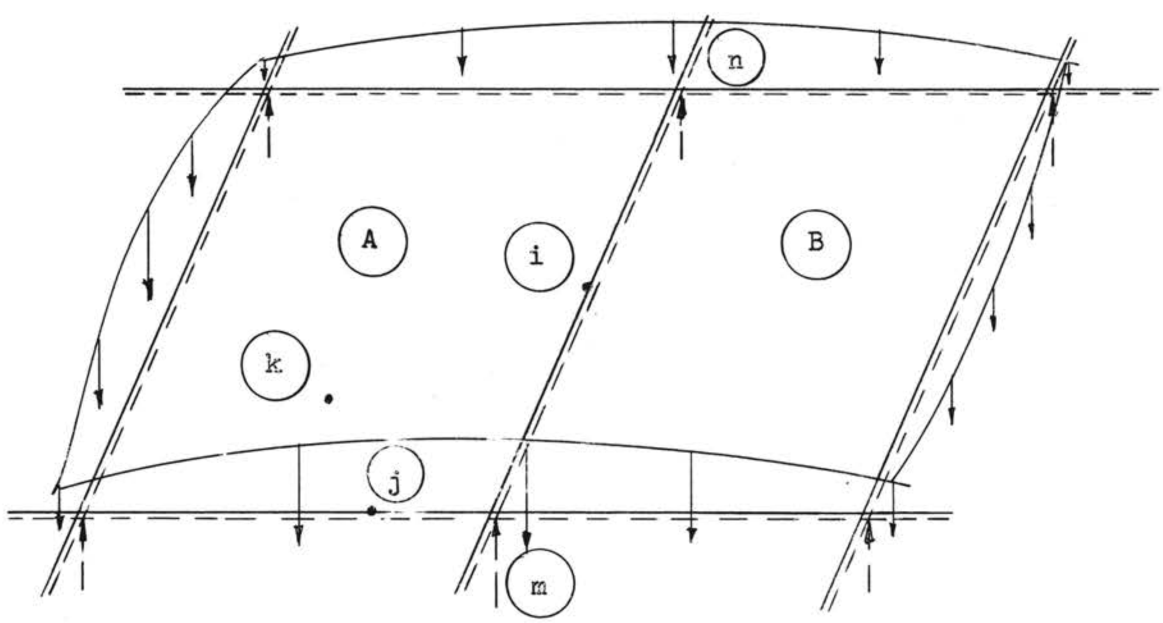


Figure 3.1. General Structure

There are three unknowns at any point along the common boundary of any two adjacent panels A and B. These are the moment and the two reactions between the panels and the beam. These can be obtained from the compatibility conditions:

(i) The sum of the normal slopes of adjacent panels at any point along a supporting beam is zero.

(ii) The displacement of each of the panels at any point along the beam must be equal to the displacement of the beam at that point.

If $(\theta_i)_A$ and $(\theta_i)_B$ are rotations at i of panels A and B, the first compatibility condition requires that (See Figure 3.2)

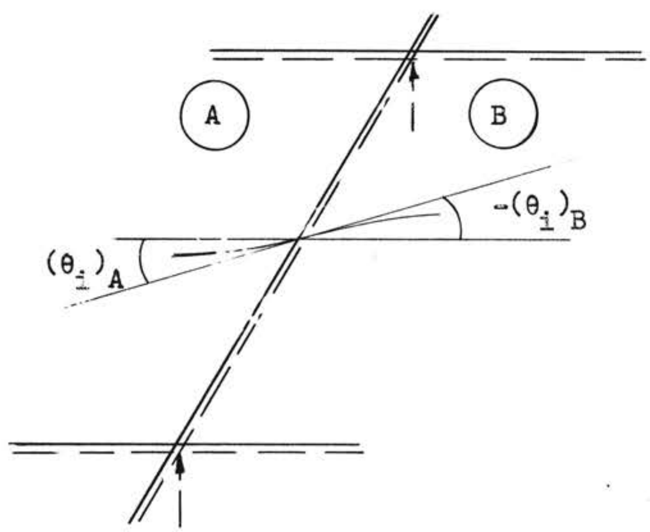


Figure 3.2. Slope Compatibility of Adjacent Panels

$$(\theta_i)_A + (\theta_i)_B = 0 \quad . \tag{3.1}$$

If $(\Delta_i)_A$, $(\Delta_i)_B$ and $(\Delta_i)^{Beam}$ are the displacements at i of panels A, B and the supporting beam, respectively, the second compatibility condition requires that (Figure 3.3)

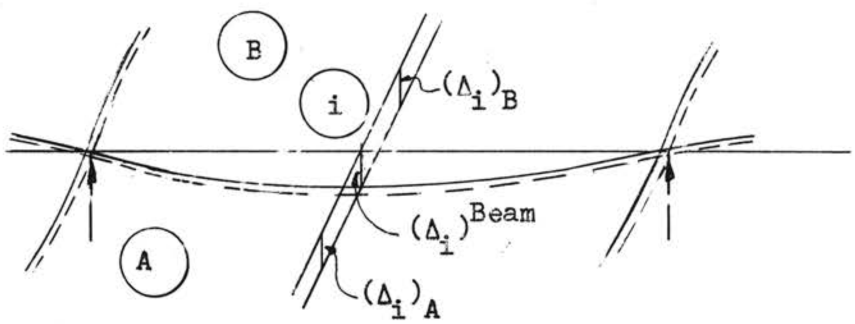


Figure 3.3. Displacement Compatibility

$$(\Delta_i)_A = (\Delta_i)^{Beam} \tag{3.2}$$

$$(\Delta_i)_B = (\Delta_i)^{Beam} \quad . \tag{3.3}$$

The algebraic expressions for the slopes and displacements are:

$$(\theta_i)_A = \sum_A \tau_{ik}P_k + (F_i)_A M_i + \sum_A G_{ij}M_j + (-T_i)'R_i' + \sum_A (-Q_{ij})'R_j' \tag{3.4}$$

$$(\theta_i)_B = \sum_B \tau_{ik}P_k + (F_i)_B M_i + \sum_B G_{ij}M_j + (-T_i)''R_i'' + \sum_B (-Q_{ij})''R_j'' \tag{3.5}$$

$$(\Delta_i)_A = \sum_A \delta_{ik}'P_k + (T_i)'M_i + \sum_A Q_{ij}'M_j + (-D_i)'R_i' + \sum_A (-H_{ij})'R_j' \tag{3.6}$$

$$(\Delta_i)_B = \sum_B \delta_{ik}''P_k + (T_i)''M_i + \sum_B Q_{ij}''M_j + (-D_i)''R_i'' + \sum_B (-H_{ij})''R_j'' \tag{3.7}$$

$$(\Delta_i)^{Beam} = (\delta_i)^B + R_i D_i^B + \sum_{Beam} H_{ij}^B R_j + Q_{im}^B M_m + Q_{in}^B M_n \qquad (3.8)$$

Where:

j is any point, other than i, on the boundary of the panels.

k is a typical interior point of the panels.

The angular load function τ_{ik} is the edge slope at i due to a unit load at k, considering the plate as supported by columns only at the corners (hereafterwards referred to as "basic structure").

The displacement load function δ'_{ik}, (δ''_{ik}) is the edge displacement at i of the basic structure A, (B) due to a unit shear at k.

The angular flexibility $(F_i)_{A,B}$ is the edge slope at i of the basic structure A or B respectively due to a unit moment at i.

The angular-displacement flexibility T'_i, (T''_i) is the edge displacement at i of the basic structure A, (B) due to a unit moment at i. By virtue of Maxwell-Betti Reciprocal Theorem, T_i can also be defined as the edge slope at i due to a unit shear at i.

The angular carry-over G_{ij} is the edge slope at i of the basic structure due to a unit moment at j.

The angular-displacement carry-over Q'_{ij}, (Q''_{ij}) is the edge deflection at i of the basic structure A (B) due to a unit moment at j. From Maxwell-Betti Theorem Q_{ij} can also be defined as the slope at i due to a unit shear at j.

The displacement flexibility D'_i (D''_i) is the edge displacement at i of the basic structure A (B) due to a unit shear at i.

The displacement carry-over H'_{ij}, (H''_{ij}) is the edge slope at i of the basic structure A (B) due to a unit shear at j.

The displacement load function δ_i^B is the displacement at i due to the weight of the beam.

The displacement flexibility D_i^B is the displacement at i of the beam mn due to a unit shear 'i', considering the beam to be simply supported.

The displacement-angular carry-over $(Q_i)^B_{m,n}$ is the displacement at i of the beam mn due to a unit moment at m or n respectively, considering the beam to be simply supported.

The displacement carry-over H^B_{ij} is the displacement at i of the beam mn due to a unit shear at any other point j on the beam.

P_k is any load applied at k.

M_i and M_j are the bending moments at i and j respectively.

R_i' and R_i'' are the reactions at i of the panels A and B respectively, assumed to be acting upwards (Figure 3.4).

R_j' and R_j'' are the reactions at j of the panels A and B respectively, assumed to be acting upwards.

M_m^B and M_n^B are the moments at ends of the beam mn.

R_i and R_j are the reactions on the beam mn at i and j assumed to be acting downwards (Figure 3.4).

A graphical illustration of the various angular and displacement functions is given in Chapter IV.

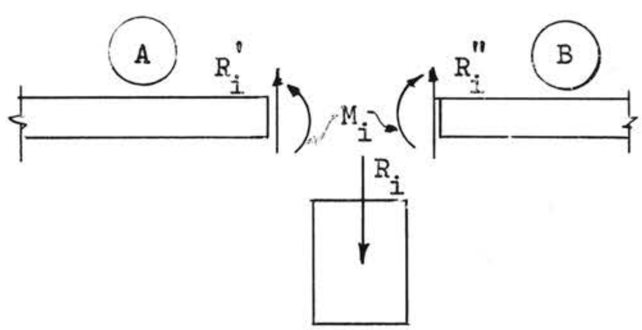


Figure 3.4. Reactions Between Plates and Supporting Beam

From Figure 3.4 it can be easily seen that

$$R_i = R_i' + R_i'' + q \tag{3.9}$$

where

q is the weight of beam of length Δx or Δy.

Similarly,

$$R_j = R_j' + R_j'' + q \quad . \tag{3.10}$$

Substituting Equations 3.4 and 3.5 into 3.1, Equations 3.6, 3.7, 3.8, 3.9 and 3.10 into 3.2 and 3.3 the general moment and reaction equations are obtained:

$$\sum_{A,B} P_k \tau_{ik} + M_i \sum_{A,B} G_i + \sum_{A,B} M_j G_{ij} + R_i'(-T_i)_A + \sum_A (-Q_{ij}) R_j' + R_i''(-T_i)_B + + \sum_B (-Q_{ij}) R_j'' = 0 \tag{3.11}$$

$$(\sum_A \delta_{ik} P_k + \delta_i^B) + M_i (T_i)_A + \sum_A Q_{ij} M_j - R_i' \left\{ (D_i)_A + D_i^B \right\} - R_i'' D_i^B - \sum_A H_{ij} R_j' - - \sum_{Beam} H_{ij}^B R_j' - \sum_{Beam} R_j'' H_{ij}^B - M_m Q_{im}^B - M_n Q_{in}^B = 0 \tag{3.12}$$

$$(\sum_B \delta_{ik} P_k + \delta_i^B) + M_i (T_i)_B + \sum_B Q_{ij} M_j - R_i' \left\{ (D_i)_B + D_i^B \right\} - R_i'' D_i^B - \sum_B H_{ij} R_j' - - \sum_{Beam} H_{ij}^B R_j' - \sum_{Beam} R_j'' H_{ij}^B - M_m Q_{im}^B - M_n Q_{in}^B = 0 \tag{3.13}$$

The moments M_m and M_n can be determined from the continuity condition of the supporting beam. Two isolated spans of a continuous supporting beam loaded by reactions from the plate are shown in Figure 3.5. Using the flexibility method of analysis of continuous beams (14), the three moment equation is

$$\sum_{lm,mn} \tau_m^B + M_m (F_{ml}^B + F_{mn}^B) + M_l G_{ml}^B + M_n G_{mn}^B + \sum_{lm,mn} R_i Q_{mi}^B = 0 \; . \tag{3.14}$$

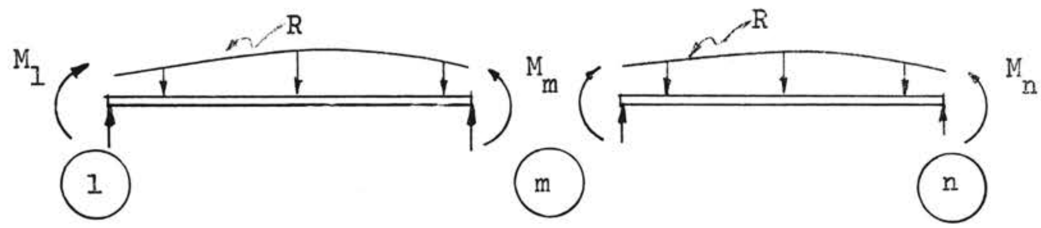


Figure 3.5. Continuous Supporting Beam

where

Angular load function τ_m^B is the slope at m of the beam mn due to its weight.

Angular flexibility F_{mn}^B (or F_{ml}^B) is the slope of the beam mn (or lm) at m due to a unit moment at m.

Angular carry-over G_{mn} (or G_{ml}) is the slope of m at the beam mn (or lm) due to a unit moment at n (or l).

The displacement-angular carry-over Q_{mi} is the end slope at m due to unit shear i.

As many such equations as the number of continuous supports can be written for each supporting continuous beam. Equations thus obtained, when combined with those obtained by writing Equations 3.11, 3.12 and 3.13 at various points along the boundaries of continuous plate panels, yield the complete solution of the problem.

A digital computer solution suggests itself because of the large number of unknowns involved. A matrix from the above general moment and reaction equations can be formulated as follows:

$$
\begin{bmatrix} \Sigma\tau_1 P_k \\ \Sigma\tau_2 P_k \\ \cdot \\ \Sigma\tau_s P_k \\ \Sigma\delta_1 P_k \\ \Sigma\delta_2 P_k \\ \cdot \\ \Sigma\delta_s P_k \\ \Sigma\tau^B_m \\ \Sigma\tau^B_n \\ \cdot \\ \Sigma\tau^B_r \end{bmatrix}
+
\begin{bmatrix}
\Sigma F_1 & G_{12} & \cdot & \cdot & G_{1s} & T_1 & Q_{12} & \cdot & \cdot & Q_{1s} & 0 & \cdot & \cdot & 0 \\
G_{21} & \Sigma F_1 & \cdot & \cdot & G_{2s} & Q_{21} & T_2 & \cdot & \cdot & Q_{2s} & 0 & \cdot & \cdot & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
G_{s1} & G_{s2} & \cdot & \cdot & \Sigma F_s & Q_{s1} & Q_{s2} & \cdot & \cdot & T_s & 0 & \cdot & \cdot & \cdot \\
T_1 & Q_{12} & \cdot & \cdot & Q_{1s} & \Sigma D_1 & H_{12} & \cdot & \cdot & H_{1s} & Q^B_{m1} & Q^B_{n1} & 0 & 0 \\
Q_{21} & T_2 & \cdot & \cdot & Q_{2s} & H_{21} & \Sigma D_2 & \cdot & \cdot & H_{2s} & Q^B_{2m} & Q^B_{2n} & 0 & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
Q_{s1} & Q_{s2} & \cdot & \cdot & T_s & H_{s1} & H_{s2} & \cdot & \cdot & \Sigma D_s & \cdot & \cdot & \cdot & Q_{sr} \\
0 & 0 & \cdot & \cdot & 0 & Q^B_{m1} & Q^B_{m2} & \cdot & \cdot & Q^B_{ms} & \Sigma F^B_m & G_{mn} & 0 & 0 \\
0 & 0 & \cdot & \cdot & 0 & Q^B_{n1} & Q^B_{n2} & \cdot & \cdot & Q^B_{ns} & G_{nm} & \Sigma F_n & 0 & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0 & 0 & \cdot & \cdot & 0 & Q^B_{r1} & Q^B_{r2} & \cdot & \cdot & Q^B_{rs} & \cdot & \cdot & \cdot & \Sigma F_r
\end{bmatrix}
\begin{bmatrix} M_1 \\ M_2 \\ \cdot \\ M_s \\ -R_1 \\ -R_2 \\ \cdot \\ -R_s \\ -M^B_m \\ -M^B_n \\ \cdot \\ -M^B_r \end{bmatrix}
= 0 \qquad (3.15)
$$

The General Moment and Reaction Matrix Equation where the subscript s corresponds to the total number of boundary points. Mm, Mn, etc. are the moments in the beam over the supports.

The solution can be obtained by inverting the coefficient matrix, as already explained in Chapter II.

In case the unknown moments and reactions are too many to be handled by the computer, they can be reduced by adopting the following matrix reduction.

Using abbreviated notation, the general moment and reaction matrix can be written as,

$$\begin{bmatrix} [G] & [Q] & \\ [Q] & [H] & [Q^B] \\ & [Q^B] & [G^B] \end{bmatrix} \begin{bmatrix} [M] \\ [-R] \\ [-M^B] \end{bmatrix} = - \begin{bmatrix} [\tau] \\ [\delta] \\ [\tau^B] \end{bmatrix} \qquad (3.16)$$

where

$[G]$ and $[G^B]$ are the submatrices of the angular functions of the plate and beam, respectively.

$[Q]$ and $[Q^B]$ are the submatrices of the angular-displacement carry-overs of the plate and beam, respectively.

$[H]$ is the submatrix of the displacement functions of the plate and beam.

$[M]$ is the submatrix of the moments in the plate over the continuous supports.

$[R]$ is the submatrix of the reactions between the plate and supporting beams.

$[M^B]$ is the submatrix of the moments in the beam over the supports.

$[\tau]$ and $[\tau^B]$ are the submatrices of the angular load functions of the plate and beam, respectively.

$[\delta]$ is the submatrix of the displacement load functions of the plate and beam.

Resolving Equation 3.15 into three equations,

$$[G]\,[M] - [Q]\,[R] = -[\tau] \qquad (3.17)$$

$$[Q]\,[M] - [H]\,[R] - [Q^B]\,[M^B] = -[\delta] \qquad (3.18)$$

$$-[Q^B]\,[R] - [G^B]\,[M^B] = -[\tau^B] \qquad (3.19)$$

Solving for $[M]$

$$[M] = -[G^B]^{-1}[\tau] + [G]^{-1}[Q] \cdot$$
$$\Big[\big[[Q]^{-1}[Q^B]\big]^{-1}\Big([Q]^{-1}[H] - [G]^{-1}[Q]\Big) - [G^B]^{-1}[Q^B]\Big]^{-1} \cdot$$
$$\Big[\big[[Q]^{-1}[Q^B]\big]^{-1}\Big([Q]^{-1}[\delta] - [G]^{-1}[\tau]\Big) - [G^B]^{-1}[\tau^B]\Big]$$

$$(3.20)$$

Thus by successive inversions and other algebraic operations of coefficient submatrices, the moments can be evaluated. The values of $[R]$ and $[M^B]$ are obtained as

$$[R] = [Q]^{-1}[\tau] + [Q]^{-1}[G]\,[M] \qquad (3.21)$$

$$[M^B] = [G^B]^{-1}[\tau^B] - [G^B]^{-1}[Q^B]\,[R] \qquad (3.22)$$

Thus the remaining unknowns $[R]$ and $[M^B]$ can be determined successively from $[M]$.

CHAPTER IV

ANGULAR AND DISPLACEMENT FUNCTIONS

4.1. __Angular and Displacement Load Functions.__ Consider a rectangular plate supported by columns at the corners to be acted upon by a load P = 1 at point k (Figure 4.1). From the definitions given in the previous chapter, the slope of the deflection curve at i due to $P_k = 1$ is the angular load function τ_{ik} and the displacement at i due to $P_k = 1$ is the displacement load function δ_{ik}.

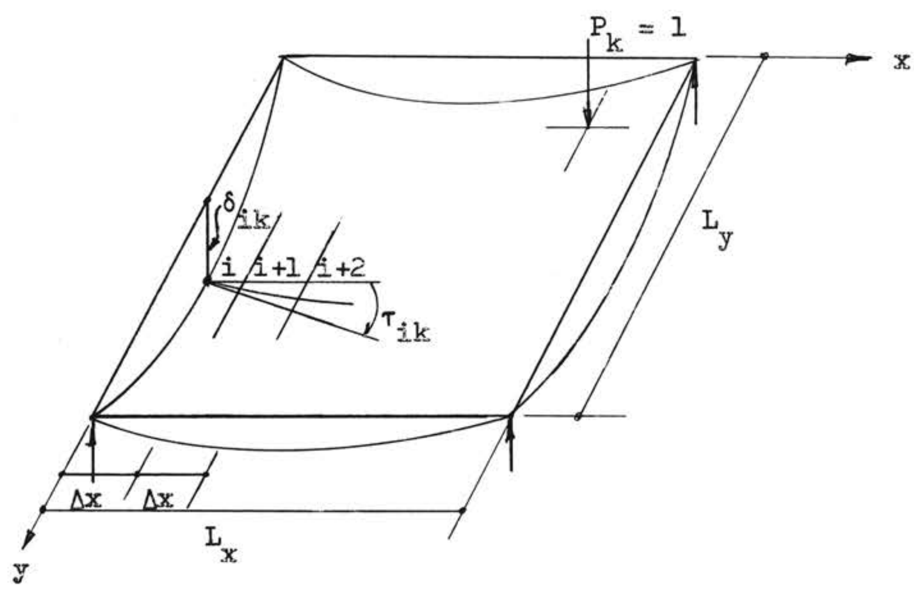


Figure 4.1. Angular Load and Displacement Functions

If the plate is divided into an arbitrary number of equally sized rectangular elements with sides Δx and Δy in the directions x and y respectively, the slope of deflection curve at 'i' can be approximated as:

$$\theta_i = \frac{w_{i+1} - w_i}{\Delta x} \tag{4.1}$$

where

w_{i+1} and w_i are displacements at points i+1 and i.

If $\eta_{(i+1)k}$ and η_{ik} are the influence coefficients for the displacements at i+1 and i respectively due to unit load at k,

$$w_{(i+1)k} = \frac{1}{D} \Delta x \Delta y \, \eta_{(i+1)k} \tag{4.2}$$

$$w_{ik} = \frac{1}{D} \Delta x \Delta y \, \eta_{ik} \tag{4.3}$$

where

D is the flexural rigidity of the plate.

From Equations 4.2 and 4.3

$$\tau_{ik} = \frac{\Delta x \Delta y}{D} \; \frac{1}{\Delta x}\left(\eta_{(i+1)k} - \eta_{ik}\right)$$

or

$$\tau_{ik} = \frac{\Delta y}{D}\left(\eta_{(i+1)k} - \eta_{ik}\right) \tag{4.4}$$

and

$$\delta_{ik} = \frac{\Delta x \Delta y}{D} \eta_{ik}. \tag{4.5}$$

If i is on edge parallel to the x-axis,

$$\tau_{ik} = \frac{\Delta x}{D}\left(\eta_{(i+1)k} - \eta_{ik}\right). \tag{4.6}$$

4.2. <u>Angular and Angular-Displacement Flexibilities</u>. Consider a rectangular plate supported by columns at the corners, to be acted upon by a unit moment at point i (Figure 4.2). From the definitions given in the previous chapter, the rotation and displacement at i due to $M_i = 1$ are angular and angular-displacement flexibilities F_i and T_i respectively.

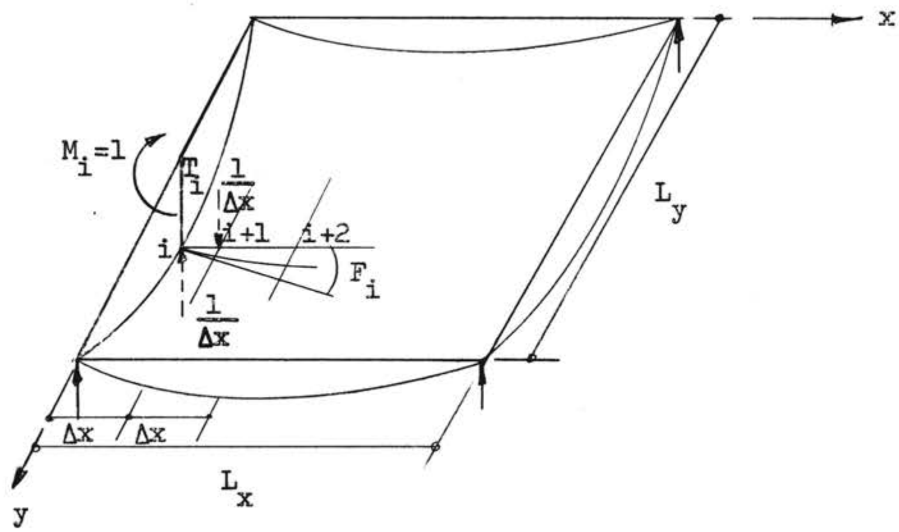


Figure 4.2. Angular and Angular-Displacement Flexibilities

The moment can be replaced by a couple with forces $\frac{1}{\Delta x}$ at i and i+1 as shown with dotted lines in Figure 3.2. From the definition of the influence coefficients for the displacement,

$$F_i = \frac{w_{i+1} - w_i}{\Delta x} = \frac{\Delta x \Delta y}{D} \frac{1}{(\Delta x)^2} \left\{ \left(\eta_{(i+1)(i+1)} - \eta_{(i+1)i} \right) - \left(\eta_{i(i+1)} - \eta_{ii} \right) \right\}.$$

But from Maxwell's Reciprocal Theorem, $\eta_{(i+1)i} = \eta_{i(i+1)}$. Therefore, the above equation can be written as:

$$F_i = \frac{1}{D} \frac{\Delta y}{\Delta x} \left\{ \eta_{(i+1)(i+1)} - 2\eta_{i(i+1)} + \eta_{ii} \right\} \tag{4.7}$$

From the discussion above, it follows that

$$T_i = \frac{\Delta x \Delta y}{D} \frac{1}{\Delta x} \left(\eta_{i(i+1)} - \eta_{ii} \right)$$

or

$$T_i = \frac{\Delta y}{D} \left(\eta_{i(i+1)} - \eta_{ii} \right). \tag{4.8}$$

If i is on edge parallel to x-axis,

$$F_i = \frac{1}{D} \frac{\Delta x}{\Delta y} \left\{ \eta_{(i+1)(i+1)} - 2\eta_{i(i+1)} + \eta_{ii} \right\} \tag{4.9}$$

$$T_i = \frac{\Delta x}{D} \left(\eta_{i(i+1)} - \eta_{ii} \right) \tag{4.10}$$

4.3. Angular and Angular-Displacement Carry-Overs. Consider a column supported rectangular plate to be acted upon by a unit moment at j (Figure 4.3). From the definitions, the rotation and displacement at i due to $M_j = 1$ are the angular and angular-displacement carry-overs G_{ij} and Q_{ij} respectively.

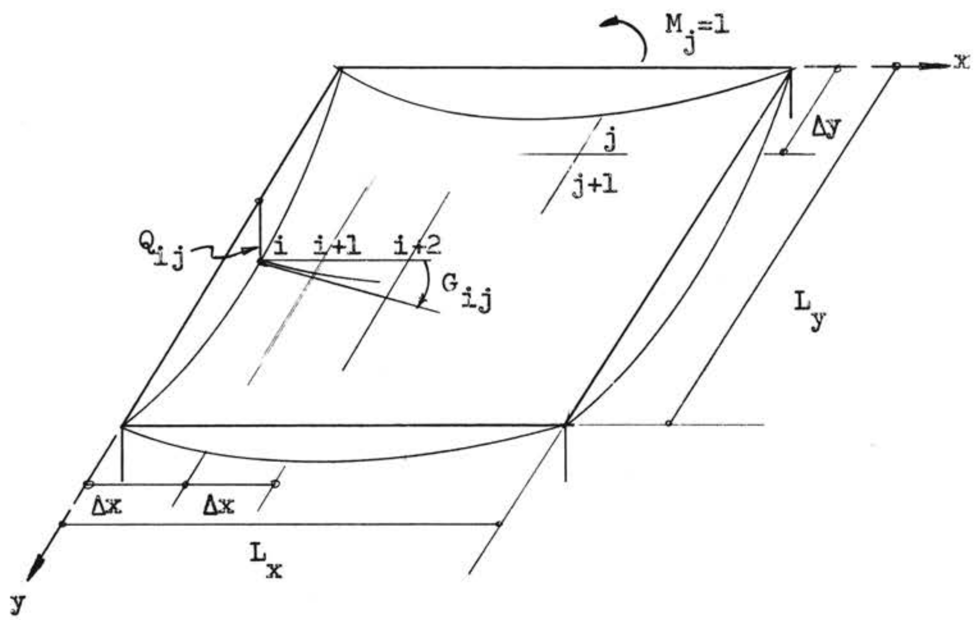


Figure 4.3. Angular and Angular-Displacement Carry-Overs

The moment can be replaced by a couple with forces $\frac{1}{\Delta y}$ at j and j+1. From the definition,

$$G_{ij} = \frac{w_{i+1} - w_i}{\Delta x} = \frac{\Delta x \Delta y}{D} \frac{1}{\Delta x} \frac{1}{\Delta y} \left\{ \left(\eta_{(i+1)(j+1)} - \eta_{(i+1)j} \right) - \left(\eta_{i(j+1)} - \eta_{ij} \right) \right\} .$$

But,

$$\eta_{(i+1)j} = \eta_{i(j+1)} \quad .$$

Substituting in the above equation,

$$G_{ij} = \frac{1}{D} \left\{ \eta_{(i+1)(j+1)} - 2\eta_{i(j+1)} + \eta_{ij} \right\} . \tag{4.11}$$

Similarly from the definition,

$$Q_{ij} = \frac{\Delta x}{D} \left(\eta_{i(j+1)} - \eta_{ij} \right) \quad . \tag{4.12}$$

If i is on edge parallel to x-axis and j on edge parallel to y-axis,

$$Q_{ij} = \frac{\Delta y}{D} \left(\eta_{i(j+1)} - \eta_{ij} \right) \quad . \tag{4.13}$$

If i and j are on parallel edges, (normal to x-direction)

$$G_{ij} = \frac{1}{D} \frac{\Delta x}{\Delta y} \left\{ \eta_{(i+1)(j+1)} - 2\eta_{i(j+1)} + \eta_{ij} \right\} \tag{4.14}$$

(normal to y-direction)

$$G_{ij} = \frac{1}{D} \frac{\Delta y}{\Delta x} \left\{ \eta_{(i+1)(j+1)} - 2\eta_{i(j+1)} + \eta_{ij} \right\} \tag{4.15}$$

4.4. Displacement Flexibility and Carry-Over. Consider a column supported rectangular plate to be acted upon by a unit shear at i. By definition, the displacement at i due to a unit shear at i is the displacement flexibility D_i (Figure 4.4).

$$D_i = w_i = \frac{\Delta x \Delta y}{D} \eta_{ii} \tag{4.16}$$

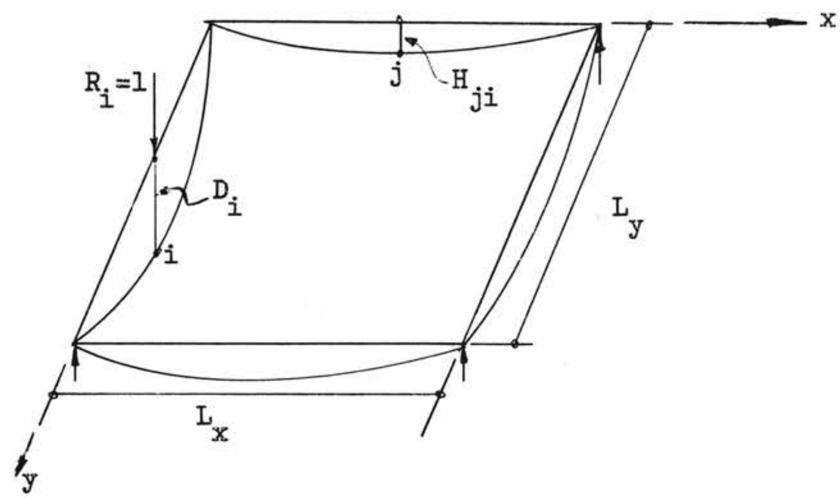


Figure 4.4. Displacement Flexibility and Carry-Over

The displacement at j due to a unit shear at i is the displacement carry-over H_{ji}.

$$H_{ji} = w_j = \frac{\Delta x \Delta y}{D} \eta_{ji}$$

By virtue of the Maxwell-Betti Reciprocal Theorem $H_{ji} = H_{ij}$, the displacement at i due to a unit load at j. Therefore, it follows that

$$H_{ij} = \frac{\Delta x \Delta y}{D} \eta_{ij} = \frac{\Delta x \Delta y}{D} \eta_{ij} \quad . \tag{4.17}$$

4.5. Beam Flexibilities.

(a) Angular and displacement load functions:

Consider a simply supported beam to be acted upon by its own weight (Figure 4.5). By definition, the slope of the beam at m and displacement at i are angular and displacement load functions τ_m^B and δ_i^B respectively.

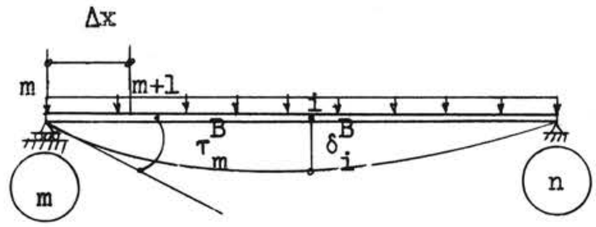


Figure 4.5. Angular and Displacement Load Functions of Beam

If η_{ij}^B is the influence coefficient for the deflection at i due to a unit load at j, then

$$w_{ij} = \frac{\Delta x^3}{EI} \eta_{ij}^B$$

where

EI is the flexural rigidity of the beam.

The angular load function can now be expressed as

$$\tau_m^B = \frac{w_{m+1}}{\Delta x} = \frac{\Delta x^3}{EI} q \sum_{j=1}^{p} \eta_{(m+1)j}^B \qquad (4.18)$$

where

p is the number of strips in the beam and q is the weight of each strip.

The displacement load function becomes

$$\delta_i^B = \frac{\Delta x^3}{EI} q \sum_{j=1}^{p} \eta_{ij}^B \quad . \qquad (4.19)$$

(b) Displacement flexibility and displacement and displacement-angular carry-over:

Consider a simply supported beam mn to be acted upon by a unit shear at i (Figure 4.6). By definition the displacements at i and j due to unit shear at i are displacement flexibility D_i^B and displacement carry-over H_{ji} respectively.

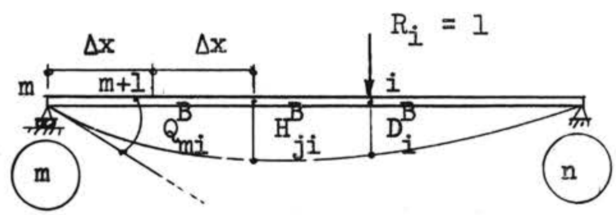


Figure 4.6. Displacement Flexibility and Carry-Overs of Beam

These can be expressed as

$$D_i^B = w_{ii} = \frac{\Delta x^3}{EI} \eta_{ii}^B \tag{4.20}$$

and

$$H_{ji} = w_{ji} = \frac{\Delta x^3}{EI} \eta_{ji}^B \ . \tag{4.21}$$

The rotation at m due to unit shear at i is displacement-angular carry-over Q_{mi}^B.

$$Q_{mi}^B = \frac{w_{m+1}}{\Delta x} = \frac{\Delta x^2}{EI} \eta_{(m+1)i}^B \tag{4.22}$$

(c) Angular flexibilities and angular and angular-displacement carry-overs:

Consider a simply supported beam mn to be acted upon by a unit moment at m (Figure 4.7). The slope at m due to a unit moment at m is the angular flexibility of the beam F^B_{mm}.

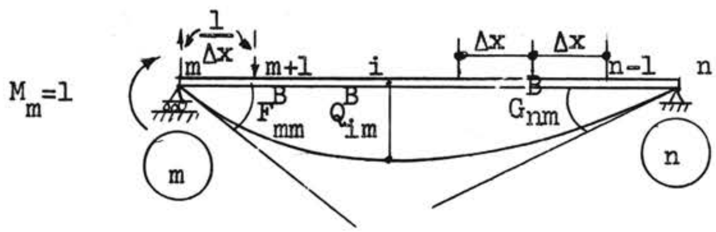


Figure 4.7. Angular Flexibilities and Carry-Over Functions of Beam

Replacing the unit moment by a statically equivalent couple of forces $\frac{1}{\Delta x}$ a distance Δx apart as shown in Figure 4.6,

$$F^B_{mm} = \frac{w_{(m+1)(m+1)}}{\Delta x} = \frac{\Delta x^3}{\Delta x} \frac{1}{EI} \eta^B_{(m+1)(m+1)}$$

or

$$F^B_{mm} = \frac{\Delta x^2}{EI} \eta^B_{(m+1)(m+1)}. \tag{4.23}$$

The slope at n due to a unit moment at m is the angular carry-over G^B_{nm} (Figure 4.7),

$$G^B_{nm} = \frac{w_{(n-1)(m+1)}}{\Delta x} = \frac{\Delta x^3}{\Delta x} \frac{1}{EI} \eta^B_{(n-1)(m+1)}$$

or

$$G^B_{nm} = \frac{\Delta x^2}{EI} \eta^B_{(n-1)(m+1)} \tag{4.24}$$

In view of the reciprocal relations for angular carry-overs and deflection influence coefficients, it can be written

$$G^B_{nm} = \frac{\Delta x^2}{EI} \eta_{(m+1)(n-1)} \; . \tag{4.25}$$

The deflection at any point i on the beam due to a unit moment at m is the angular displacement carry-over Q^B_{im}.

$$Q^B_{im} = w_{i(m+1)} = \frac{\Delta x^3}{EI} \eta^B_{i(m+1)} \tag{4.26}$$

CHAPTER V

NUMERICAL EXAMPLE

The plate structure shown in Figure 5.1 is analyzed for a uniformly distributed load of 100 pounds per square foot. All the panels are four inches thick. Edge beams are provided around the plate structure. The dimensions of edge beams are:

width = 9 inches

Depth = 1 foot 6 inches.

Poisson's ratio is taken as zero. The structure is supported by rigid columns.

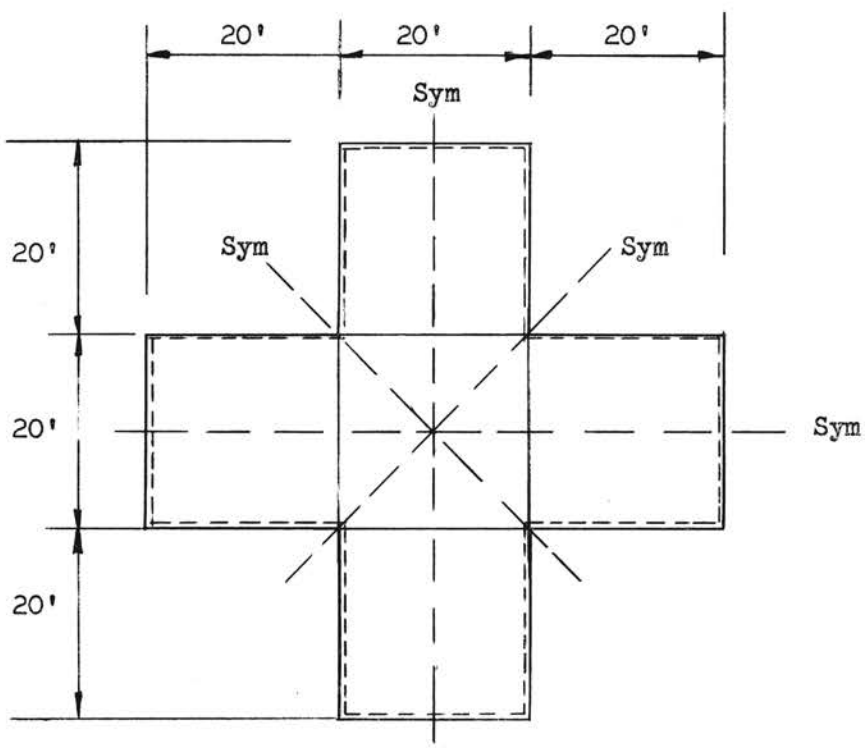


Figure 5.1. Continuous Plate Structure

A square panel supported by four columns at the corners is taken as a basic unit and is covered by a sixty-four unit finite difference network as shown in Figure 5.2.

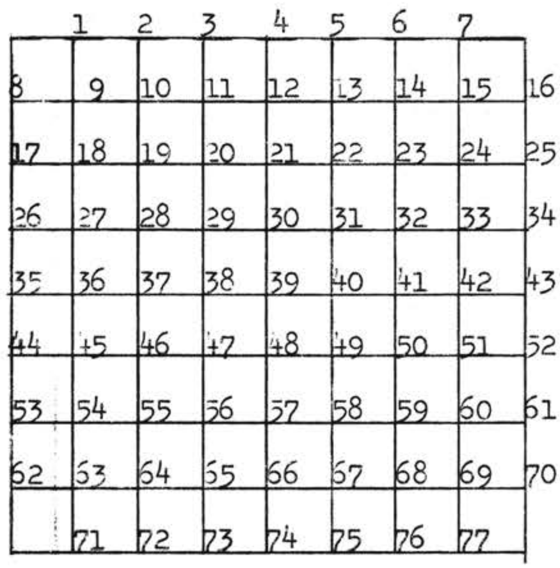


Figure 5.2. Basic Panel

Deflection influence coefficients, obtained by taking advantage of symmetry of the basic unit as outlined in Chapter II, are presented in Table 5.1.

Table 5.1 Deflection Influence Coefficients

Influence Coefficient η at Point	Unit Load at Point 1	2	3	4	5	6	7	8	9	10	11	12	13
1	.818840	1.101826	1.49206	1.055982	.868135	.615076	.391441	.131003	.663020	.927819	.999178	.940357	.792011
2	1.101826	1.961205	2.147943	2.007521	1.663367	1.183180	.615076	.195153	1.002760	1.618569	1.833221	1.762699	1.499814
3	1.149206	2.147943	2.815238	2.750041	2.318917	1.663367	.868135	.213914	1.068691	1.871057	2.350175	2.365027	2.05222
4	1.055982	2.007521	2.750041	3.125266	2.750041	2.007521	1.055982	.203556	1.051139	1.810111	2.376587	2.618289	2.376587
5	.868135	1.663367	2.318917	2.750041	2.815238	2.147943	1.139206	.172605	.883560	1.533804	2.055221	2.365027	2.350175
6	.615076	1.183180	1.663367	2.007521	2.147943	1.961205	1.101826	.126412	.636362	1.108239	1.499813	1.722699	1.833221
7	.391144	.615076	.868135	1.055982	1.149206	1.101826	.818840	.067678	.334430	.582546	.792011	.940357	.999178
8	.131003	.195153	.213914	.203445	.172605	.126312	.067678	.818830	.663020	.551876	.363918	.388526	.315935
9	.662030	1.002760	1.106869	1.051139	.883560	.636562	.334430	.630200	1.104867	1.262710	1.267955	1.165904	.984327
10	.927819	1.618569	1.871057	1.810111	1.533804	1.108239	.582536	.551876	1.262708	1.818915	1.962523	1.863254	1.595132
11	.999178	1.833221	2.350175	2.376587	2.055221	1.499813	.792011	.464918	1.267955	1.962523	2.413650	2.390138	2.087027
12	.940357	1.762609	2.365027	2.618289	2.365027	1.672699	.930357	.388526	1.165903	1.863254	2.390138	2.632154	2.391038
13	.792011	1.499814	2.052220	2.376587	2.350175	1.833221	.999178	.315935	.983327	1.595143	2.087027	2.391038	2.41365
14	.582536	1.108239	1.533804	1.810111	1.871057	1.618569	.927819	.233828	.735083	.935501	1.595132	1.863254	1.962523
15	.334430	.636453	.883561	1.051139	1.106869	1.002760	.663020	.170677	.367542	.754709	.984328	1.165904	1.267955
16	.067678	.126312	.172606	.203445	.213914	.195153	.131033	.096332	.170677	.338812	.315935	.388526	.464918
17	.195153	.306025	.353302	.344491	.396834	.219299	.118134	1.101826	1.002760	.896275	.785286	.671985	.554307
18	.551876	.896275	1.036629	1.014912	.871539	.647637	.338911	.927819	1.272708	1.414766	1.416810	1.307486	1.115386
19	.722105	1.345127	1.611466	1.606598	1.391509	1.021458	.532847	.772105	1.362500	1.904929	1.955286	1.876441	1.630215
20	.853028	1.535539	1.964851	2.030532	1.793276	1.331360	.711361	.639929	1.337860	1.926246	2.285383	2.296453	2.042557
21	.822818	1.523011	2.010011	2.192061	2.010011	1.523011	.822818	.5272343	1.223857	1.839372	2.287260	2.449981	2.287260
22	.711261	1.331360	1.794277	2.030532	1.964851	1.535539	.853028	.428293	1.045258	1.603461	2.042557	2.296453	2.285383
23	.542848	1.021458	1.391125	1.606508	1.611366	1.345127	.772105	.338661	.821467	1.264948	1.630215	1.876331	1.955286
24	.338812	.637637	.871530	1.014912	1.016329	.896275	.551878	.255100	.569573	.862895	1.115387	1.307468	1.416810
25	.118134	.219239	.296835	.344491	.353302	.209625	.951533	.195044	.305260	.432104	.554307	.671985	.785286
26	.213914	.353302	.315672	.414105	.360201	.269926	.136165	1.159206	1.106869	1.036379	.938977	.821018	.687496
27	.464918	.785286	.938977	.942639	.824465	.611330	.327834	.999177	1.262955	1.416890	1.433636	1.339524	1.158243
28	.639279	1.122256	1.475360	1.503085	1.236890	.920510	.493838	.853028	1.337860	1.698395	1.847538	1.792143	1.586591
29	.718268	1.294700	1.644581	1.722112	1.545564	1.162190	.628137	.718269	1.312140	1.801937	2.093644	2.124575	1.921918
30	.707991	1.298955	1.696551	1.839473	1.696551	1.298955	.707991	.598081	1.208882	1.738489	2.108921	2.247828	2.108921
31	.626260	1.162100	1.545565	1.922112	1.644581	1.294700	.718268	.491145	1.042800	1.544082	1.921918	2.123575	2.093644
32	.493839	.920519	1.236892	1.493085	1.375360	1.122254	.639279	.395104	.836619	1.256299	1.586592	1.297143	1.847438
33	.326846	.611330	.824466	.942639	.938977	.785286	.464918	.307336	.621444	.912089	1.518244	1.339524	1.433636
34	.146165	.409269	.362008	.414105	.415672	.335302	.213914	.224719	.386250	.541705	.687496	.821018	.938977
35	.203445	.344491	.414105	.519371	.370911	.278685	.151551	1.05598	1.051139	1.041912	.942639	.839616	.712467
36	.388527	.671985	.821009	.839616	.745002	.553280	.301499	.940357	1.065903	1.307468	1.339524	1.268290	1.110901
37	.527234	.931875	1.159422	1.200988	1.074086	.807089	.436568	.822818	1.223857	1.525833	1.667114	1.630906	1.468631
38	.598081	1.076948	1.370938	1.447844	1.313061	.996574	.541222	.707991	1.208882	1.671761	1.861650	1.899533	1.740855
39	.5996120	1.095400	1.421352	1.536308	1.321352	1.093590	.5996120	.5996120	1.126585	1.576842	1.883451	1.993864	1.883451
40	.540123	.996575	1.313962	1.447844	1.370938	1.076948	.598091	.599181	.990282	1.422013	1.220856	1.899544	1.861650
41	.436567	.807898	1.074062	1.200988	1.159422	.931875	.527243	.406614	.815187	1.181797	1.468632	1.640226	1.667114
42	.301499	.558280	.745003	.839616	.821018	.671985	.388527	.320814	.616335	.886918	.110902	1.268290	1.339524
43	.151552	.278686	.370912	.419371	.414105	.355591	.204544	.239535	.302274	.566765	.712468	.839616	.942639
44	.172605	.296834	.362008	.370911	.330872	.230008	.136818	.868135	.835600	.871539	.824465	.745002	.638912
45	.315935	.554307	.687396	.712468	.638912	.482267	.261298	.792011	.984327	1.115386	1.158243	1.110901	.983899
46	.428293	.762433	.958679	1.004149	.906867	.687291	.373197	.711261	1.045258	1.301881	1.432085	1.225304	1.289282
47	.491145	.805558	1.130772	1.201210	1.096810	.837452	.456586	.626296	1.942800	1.321082	1.598638	1.639538	1.515013
48	.499181	.909212	1.177506	1.271047	1.117750	.909121	.499181	.541123	.990282	1.370653	1.625296	1.716055	1.625296
49	.456586	.837452	1.096810	1.201210	1.130772	.885528	.391145	.356586	.880720	1.249286	1.515014	1.639539	1.598638
50	.373197	.687291	.906867	1.004149	.958679	.762433	.428293	.374560	.731489	1.028074	1.289282	1.425304	1.432985
51	.261798	.482467	.638912	.712467	.687496	.554307	.315935	.295543	.556622	.793051	.983899	1.110901	1.158243
52	.136818	.250008	.330872	.370911	.362008	.296834	.172605	.218966	.369958	.512060	.638912	.745002	.824465
53	.126312	.219299	.269926	.278685	.250008	.198589	.103609	.615076	.636462	.637637	.611330	.558280	.482467
54	.243828	.388628	.541704	.566765	.512060	.388628	.211425	.582536	.745086	.862895	.912075	.886917	.793051
55	.338661	.562027	.768960	.811813	.738070	.562027	.305957	.542848	.821467	1.301500	1.160439	1.166989	1.064136
56	.395104	.714229	.915145	.976290	.895222	.685793	.374559	.493849	.846619	1.135641	1.313745	1.355492	1.258702
57	.406614	.740453	.958482	1.034251	.958482	.740453	.406614	.436569	.815187	1.134579	1.347354	1.422087	1.347354
58	.374559	.685793	.895222	.976290	.915145	.714229	.395104	.373197	.731489	1.040417	1.258702	1.355492	1.313745
59	.305957	.562027	.738070	.811813	.768960	.562027	.338661	.305957	.605609	.868711	1.064136	1.166989	1.160439
60	.211425	.388628	.512060	.566765	.541704	.388628	.243828	.236715	.451433	.692464	.793051	.886917	.912075
61	.103609	.198589	.250008	.278685	.269926	.219299	.126312	.166899	.281573	.338628	.482467	.558280	.611330
62	.067678	.118134	.146165	.151551	.136818	.103609	.056661	.319144	.334430	.338812	.327846	.301499	.261798
63	.170677	.281573	.386254	.407264	.369958	.281573	.153240	.334430	.467542	.569574	.621443	.616335	.556622
64	.255100	.434703	.587121	.623631	.569510	.434703	.236715	.338812	.569574	.735138	.864058	.881922	.809470
65	.307336	.541344	.717382	.767920	.705893	.541344	.295543	.327846	.621444	.864058	1.017020	1.058612	.985235
66	.320814	.585259	.758360	.818549	.758360	.585259	.320814	.301499	.616335	.881923	1.059117	1.120551	1.059117
67	.295543	.541344	.705893	.767920	.717382	.541344	.307336	.261798	.556622	.809470	.985235	1.058612	1.017020
68	.236715	.434703	.569510	.623631	.587121	.434703	.255100	.211425	.451433	.658950	.809470	.881922	.864058
69	.153240	.281573	.369958	.407264	.386254	.281573	.170677	.153240	.311120	.451433	.556622	.616335	.621443
70	.056661	.103609	.136518	.151551	.146165	.118134	.067678	.090584	.153240	.211425	.261798	.301499	.327846
71	.096332	.175044	.224719	.239535	.218966	.166899	.090584	.234949	.170677	.255100	.307336	.328814	.295543
72	.175044	.318727	.410375	.438779	.402185	.307163	.166899	.118134	.305260	.450041	.557735	.585258	.541344
73	.224719	.410375	.530669	.570114	.524847	.402185	.218966	.146165	.386250	.587121	.717381	.758359	.705893
74	.239535	.438779	.570114	.615936	.570114	.438779	.239535	.151552	.407274	.623631	.767920	.818549	.767920
75	.218966	.402185	.524847	.570114	.530669	.410375	.224719	.136818	.369958	.569510	.705893	.758359	.717381
76	.166899	.307163	.402185	.438779	.410375	.318727	.175044	.103609	.281573	.434703	.541344	.585258	.557736
77	.090584	.166899	.218966	.239535	.224719	.175044	.096332	.056661	.153240	.236715	.295543	.328814	.307336

Table 5.1. (Continued)

Influence Coefficient η at Point

Point	14	15	16	17	18	19	20	21	22	23	24	25	26
	Unit Load at Point												
1	.582536	.334430	.067678	.195153	.551876	.772105	.853028	.822818	.711261	.542848	.338812	.118134	.213914
2	1.108239	.636463	.126312	.309625	.896275	1.345127	1.545549	1.523011	1.331360	1.021458	.637637	.219299	.353302
3	1.533804	.883561	.172606	.353302	1.036379	1.611466	1.964851	2.010011	1.794277	1.391175	.871540	.296835	.415672
4	1.810111	1.051139	.203445	.344491	1.014912	1.606508	2.030532	2.194061	2.030532	1.606508	1.014912	.344491	.414105
5	1.871057	1.106869	.213914	.296834	.871539	1.391509	1.794276	2.010011	1.964851	1.611466	1.036379	.353302	.362008
6	1.618569	1.002760	.195153	.219299	.637637	1.021458	1.331360	1.523011	1.545549	1.345127	.896275	.309625	.269926
7	.927819	.663020	.131003	.118134	.338811	.542847	.711261	.822818	.853028	.772105	.551876	.195153	.146165
8	.243828	.170677	.096332	1.101826	.927819	.772105	.639927	.527234	.428293	.338661	.255100	.175044	1.149206
9	.745084	.467542	.170677	1.202760	1.262708	1.362500	1.337860	1.223857	1.045258	.821467	.569574	.305260	1.106869
10	.845501	.754509	.338812	.896275	1.414766	1.804929	1.926246	1.839372	1.603461	1.264948	.862895	.432104	1.036379
11	1.595132	.984328	.315935	.785786	1.4168100	1.955286	2.285383	2.287260	2.042557	1.630215	1.115387	.554307	.938977
12	1.863254	1.165903	.388526	.671985	1.307468	1.876441	2.296453	2.449981	2.296453	1.876441	1.307468	.671985	.821018
13	1.962523	1.267955	.464918	.554307	1.115386	1.630215	2.042557	2.287260	2.285383	1.955286	1.416810	.785786	.687496
14	1.818915	1.26271	.551876	.432127	.862895	1.264947	1.603461	1.839372	1.962460	1.804929	1.414766	.896275	.541704
15	1.26271	1.104867	.663020	.305260	.569574	.821467	1.045258	1.223857	1.337860	1.362500	1.262708	1.002760	.386254
16	.551876	.663020	.818840	.175044	.255100	.338661	.428293	.527234	.639979	.772105	.927819	1.101826	.224719
17	.432127	.305260	.175044	1.961205	1.618564	1.345127	1.122256	.931875	.762433	.606722	.460041	.318727	2.147943
18	.862895	.569574	.255100	1.618564	1.818915	1.804929	1.68395	1.525833	1.301881	1.301500	.755138	.460041	1.871057
19	1.264947	.821467	.338661	1.345127	1.804929	2.164009	2.201980	2.064191	1.801144	1.448832	1.040308	.606722	1.611466
20	1.603461	1.045258	.428293	1.122256	1.68395	2.201980	2.528865	2.476645	2.210812	1.801144	1.301881	.762434	1.375360
21	1.839372	1.223857	.5272343	.931875	1.525833	2.064191	2.476645	2.675431	2.476645	2.064191	1.525833	.931875	1.159422
22	1.962460	1.337860	.639979	.762433	1.301881	1.801144	2.210812	2.476645	2.528865	2.201980	1.68395	1.122256	.958679
23	1.804929	1.362500	.772105	.606722	1.301500	1.448832	1.801144	2.064191	2.201980	2.164009	1.804929	1.345127	.768960
24	1.414766	1.262708	.927819	.460041	.755138	1.040308	1.301881	1.525833	1.68395	1.804929	1.818915	1.618564	.587121
25	.896275	1.002760	1.101826	.318727	.460041	.606722	.762434	.931875	1.122256	1.345127	1.618564	1.961205	.410375
26	.541704	.386254	.224719	2.147943	1.871057	1.611466	1.375360	1.159422	.958679	.768960	.587121	.410375	2.815238
27	.912075	.621444	.307336	1.833221	1.962523	1.955286	1.847438	1.667114	1.432985	1.160440	.864058	.557736	2.350175
28	1.256798	.846619	.395104	1.545549	1.504929	2.201980	2.250133	2.124535	1.872850	1.532138	1.135641	.714229	1.964851
29	1.544081	1.047775	.491145	1.294700	1.801936	2.23279	2.489526	2.465811	2.232887	1.856509	1.391083	.885529	1.644581
30	1.738489	1.208882	.598081	1.076948	1.617767	2.102553	2.459930	2.608952	2.459930	2.102553	1.617767	1.076948	1.370938
31	1.801937	1.312140	.718268	.885528	1.391082	1.856509	2.232887	2.465611	2.489526	2.23279	1.801936	1.294700	1.130772
32	1.698395	1.337860	.853028	.714229	1.135641	1.532139	1.872861	2.124636	2.250133	2.201980	1.804929	1.545549	.915145
33	1.416809	1.267955	.999177	.557736	.864058	1.160439	1.432986	1.667114	1.847438	1.955286	1.962523	1.833221	.717322
34	1.036379	1.106869	1.149206	.410375	.587121	.768960	.958680	1.159422	1.375360	1.611466	1.871057	2.147943	.520669
35	.566765	.407264	.239535	2.007521	1.810111	1.606508	1.403085	1.200988	1.004149	.811813	.623631	.438779	2.750041
36	.886917	.616335	.320814	1.762699	1.863254	1.876441	1.797132	1.640906	1.425304	1.166989	.881922	.585258	2.365027
37	1.181797	.815187	.406614	1.523011	1.839372	2.064191	2.124637	2.030086	1.811420	1.502027	1.134579	.740453	2.010011
38	1.422012	.990282	.499181	1.298955	1.738489	2.102553	2.311397	2.308666	2.119197	1.790016	1.370653	.909121	1.696551
39	1.576842	1.126585	.5996120	1.094500	1.576840	2.000681	2.301220	2.415249	2.301220	2.000681	1.576840	1.094500	1.421352
40	1.617761	1.208882	.707991	.909121	1.370653	1.790016	2.119198	2.308666	2.311397	2.102553	1.738489	1.298955	1.177506
41	1.525833	1.223857	.822818	.740453	1.134579	1.502027	1.811421	2.030086	2.124637	2.064191	1.839372	1.623011	.958482
42	1.307468	1.165903	.940357	.585259	.881923	1.166990	1.425305	1.640906	1.797132	1.876441	1.863254	1.762699	.758360
43	1.014912	1.051139	1.05598	.438779	.623631	.811813	1.004150	1.200988	1.403085	1.606508	1.810111	2.007521	.570114
44	.512060	.369958	.218966	1.663367	1.533804	1.391509	1.236890	1.074086	.906867	.738070	.569510	.402185	2.318917
45	.793051	.556622	.295543	1.499814	1.595132	1.630215	1.586591	1.468631	1.289282	1.064136	.809470	.541344	2.055222
46	1.048074	.731489	.374560	1.331360	1.603461	1.801144	1.872860	1.811420	1.633819	1.366756	1.040417	.685793	1.794277
47	1.249386	.880720	.456586	1.162109	1.544082	1.856509	2.037395	2.049070	1.898732	1.618204	1.249385	.837452	1.545565
48	1.370653	.990282	.541123	.996575	1.422013	1.790016	2.043648	2.135691	2.043648	1.790016	1.422013	.996575	1.313062
49	1.391082	1.047800	.626769	.837452	1.249385	1.618204	1.898732	2.049070	2.037395	1.856509	1.544082	1.162109	1.096810
50	1.301881	1.045258	.711261	.683793	1.040417	1.366756	1.633819	1.811420	1.872860	1.801144	1.603461	1.331360	.895222
51	1.115386	.984327	.792011	.541344	.809470	1.064136	1.289282	1.468631	1.586591	1.630215	1.595132	1.499814	.705893
52	.871539	.883560	.868135	.402185	.569510	.738070	.906867	1.074086	1.236890	1.391509	1.533804	1.663367	.524847
53	.388628	.281573	.166899	1.183180	1.108239	1.021458	.920510	.807893	.687291	.562027	.434703	.307163	1.663367
54	.692464	.451433	.236715	1.108239	.845501	1.264947	1.256798	1.181797	1.048074	.868711	.658950	.434703	1.533804
55	.868711	.605609	.305957	1.021458	1.264948	1.448832	1.532138	1.502027	1.366756	1.146760	.868711	.562027	1.391175
56	1.040417	.731489	.373197	.920510	1.256799	1.532139	1.695954	1.719198	1.602096	1.366756	1.048074	.687291	1.236892
57	1.134579	.815187	.436569	.807893	1.181800	1.502027	1.719198	1.796507	1.719198	1.502027	1.181800	.807893	1.074662
58	1.135641	.846619	.493849	.687291	1.048074	1.366756	1.602096	1.719198	1.695954	1.532139	1.256799	.920510	.906867
59	1.301600	.821467	.542848	.562027	.868711	1.146760	1.366756	1.502027	1.532138	1.448832	1.264948	1.021458	.738070
60	.862895	.745086	.582536	.434703	.658950	.868711	1.048074	1.181797	1.256798	1.264947	.845501	1.108239	.569510
61	.637637	.636462	.615076	.307163	.434703	.562027	.687291	.807893	.920510	1.021458	1.108239	1.183180	.402185
62	.211425	.153240	.090584	.615076	.582536	.542847	.493850	.436568	.373197	.305957	.236715	.166899	.868135
63	.451433	.311120	.153240	.636463	.745086	.821467	.846619	.815187	.731489	.605609	.451433	.281573	.883561
64	.658950	.451433	.211425	.637637	.862895	1.040308	1.135641	1.134579	1.040417	.868711	.692464	.388628	.871540
65	.809470	.556622	.261798	.611330	.912089	1.160439	1.313744	1.347354	1.258702	1.064136	.793051	.482467	.824466
66	.881923	.616335	.301499	.558280	.886918	1.166990	1.355492	1.422087	1.355492	1.166990	.886918	.558280	.745003
67	.864058	.621444	.327546	.482467	.793051	1.064136	1.258702	1.347354	1.313744	1.160439	.912089	.611330	.638912
68	.755138	.549574	.338812	.388628	.692464	.868711	1.040417	1.13458	1.135641	1.040308	.862895	.637637	.512060
69	.569574	.467542	.334430	.281573	.451433	.605609	.731489	.815187	.846619	.821467	.745086	.636463	.369958
70	.338812	.334430	.319144	.166899	.236715	.305957	.373197	.436568	.493850	.542847	.582536	.615076	.218966
71	.236715	.153240	.056661	.126312	.243828	.338661	.395104	.406614	.374559	.305957	.211425	.103609	.172606
72	.434703	.281573	.103609	.219299	.432104	.606722	.714229	.740453	.685793	.562027	.388628	.198589	.256835
73	.569510	.369958	.136818	.409269	.541705	.768960	.915145	.958483	.895222	.738070	.512060	.250008	.362008
74	.623631	.407274	.151552	.273686	.566765	.811813	.976289	1.034251	.976289	.811813	.566765	.278686	.370912
75	.587121	.386250	.146165	.250008	.512060	.738070	.895222	.958482	.915145	.768960	.541205	.409269	.330872
76	.460041	.305260	.118134	.198589	.388628	.562027	.685793	.740453	.714229	.606722	.432104	.219299	.250008
77	.255100	.170677	.234949	.103609	.211425	.305957	.374559	.406614	.395104	.338661	.243828	.126312	.136816

Table 5.1. (Continued)

Influence Coefficient η at Point / Unit Load at Point

Point	27	28	29	30	31	32	33	34	35	36	37	38	39
1	.464918	.639779	.718268	.707991	.626769	.493849	.327846	.146165	.203445	.388527	.5272347	.598081	.5996120
2	.785786	1.122256	1.294700	1.298955	1.162109	.920510	.611330	.409269	.344491	.671985	.931875	1.075948	1.094500
3	.938977	1.375360	1.644581	1.696551	1.545565	1.236892	.824466	.362008	.414105	.821018	1.159422	1.370938	1.421352
4	.942639	1.403085	1.722112	1.839473	1.722112	1.403085	.942639	.414105	.419371	.839616	1.200988	1.447844	1.536308
5	.824465	1.236890	1.545564	1.696551	1.644581	1.375360	.938977	.415672	.370911	.745002	1.074086	1.313061	1.421352
6	.611330	.920510	1.162109	1.298955	1.294700	1.122256	.785286	.353302	.278685	.553280	.8020893	.996574	1.094500
7	.327845	.493848	.628137	.707991	.718268	.639779	.464918	.213914	.151551	.301499	.436568	.541122	.599612
8	.999178	.853028	.718268	.598081	.491145	.395104	.307336	.224719	1.055982	.940357	.822818	.707991	.5996120
9	1.267955	1.337860	1.312142	1.208862	1.047800	.846619	.621444	.386250	1.051139	1.165903	1.223857	1.208882	1.126585
10	1.416809	1.698395	1.801937	1.738489	1.544082	1.256799	.912089	.541705	1.014912	1.307468	1.525833	1.617761	1.576842
11	1.433636	1.847438	2.093644	2.108921	1.921918	1.586592	1.158244	.687496	.942639	1.339524	1.667114	1.861650	1.383451
12	1.339524	1.797132	2.123575	2.247828	2.123575	1.797132	1.339524	.821018	.839616	1.268290	1.640906	1.899533	1.993864
13	1.158243	1.586591	1.921918	2.108921	2.093644	1.847438	1.433636	.938977	.713467	1.110901	1.468631	1.740855	1.883451
14	.912075	1.256798	1.544081	1.738489	1.801937	1.698395	1.416809	1.036379	.566765	.886917	1.181797	1.422012	1.376842
15	.621443	.846619	1.047775	1.208882	1.312140	1.337860	1.267955	1.106869	.407264	.616335	.815187	.9902817	1.126585
16	.307336	.395104	.491145	.598081	.718268	.853028	.999177	1.149206	.239535	.320814	.406614	.499181	.5996120
17	1.833271	1.545549	1.294700	1.076948	.885528	.714229	.557736	.410375	2.007521	1.762699	1.523011	1.298955	1.094500
18	1.962523	1.804929	1.801936	1.617767	1.391082	1.135641	.864058	.587121	1.810111	1.863254	1.839372	1.738489	1.576840
19	1.955286	2.201980	2.23279	2.102553	1.856509	1.532139	1.160439	.763960	1.606508	1.876441	2.064191	2.102553	2.000681
20	1.847438	2.250133	2.498526	2.459930	2.232887	1.872861	1.432986	.958680	1.403085	1.797132	2.124637	2.311397	2.301220
21	1.667114	2.124636	2.465611	2.608952	2.465611	2.124636	1.667114	1.159422	1.200988	1.64096	2.030086	2.308666	2.415249
22	1.432985	1.872860	2.232887	2.459930	2.489526	2.250133	1.847438	1.375360	1.004149	1.425304	1.811420	2.119197	2.301220
23	1.160440	1.532138	1.856509	2.102553	2.23279	2.201980	1.955286	1.611466	.811813	1.166989	1.502027	1.790016	2.000631
24	.864058	1.135641	1.391083	1.617767	1.801936	1.804929	1.962523	1.871057	.623631	.88192	1.134579	1.37065	1.576840
25	.557736	.714229	.885529	1.076948	1.294700	1.545549	1.833221	2.147943	.438279	.585258	.740453	.909121	1.094500
26	2.350175	1.964851	1.644581	1.370938	1.130772	.951145	.717381	.530670	2.750041	2.365027	2.010011	1.696551	1.421352
27	2.413646	2.285383	2.09364	1.861650	1.598638	1.313745	1.017020	.717381	2.376587	2.391038	2.287260	2.108921	1.883448
28	2.285383	2.528865	2.489526	2.311397	2.037395	1.695954	1.313744	.915145	2.030532	2.296453	2.476645	2.459930	2.301218
29	2.09364	2.489526	2.744074	2.663980	2.407198	2.037395	1.598639	1.130772	1.722112	2.123575	2.465611	2.662980	2.614448
30	1.861650	2.311397	2.662980	2.839561	2.662980	2.311397	1.861650	1.370938	1.447844	1.899533	2.308660	2.616344	2.747652
31	1.598638	2.037395	2.407198	2.662980	2.744074	2.489526	2.09364	1.644581	1.201210	1.639538	2.049070	2.389927	2.614480
32	1.313745	1.695954	2.037395	2.311397	2.489526	2.528865	2.285383	1.964851	.976290	1.355492	1.719198	2.043648	2.301218
33	1.017020	1.313744	1.598639	1.861650	2.09364	2.285383	2.413646	2.350175	.767920	1.058612	1.347354	1.625796	1.883448
34	.717381	.915145	1.130772	1.370938	1.644581	1.964851	2.350175	2.815238	.570114	.758359	.958483	1.177506	1.421352
35	2.376587	2.030532	1.722112	1.447844	1.201210	.976290	.767920	.570114	3.125266	2.618289	2.194061	1.839473	1.536305
36	2.391038	2.296453	2.123575	1.899533	1.639538	1.355492	1.058612	.758359	2.618289	2.637154	2.449982	2.24782	1.993864
37	2.287260	2.476645	2.465611	2.308660	2.049070	1.719198	1.347354	.958483	2.194061	2.449982	2.675431	2.608952	2.415249
38	2.108921	2.459930	2.662980	2.616344	2.389927	2.043648	1.625796	1.177505	1.839473	2.247828	2.608952	2.839561	2.747652
39	1.883448	2.301218	2.614448	2.747652	2.614448	2.301218	1.883448	1.421352	1.536305	1.993864	2.415249	2.747652	2.917413
40	1.625796	2.043648	2.389927	2.616344	2.662980	2.459930	2.108921	1.696551	1.271005	1.716053	2.135691	2.49358	2.747652
41	1.347354	1.719198	2.049070	2.308660	2.465611	2.476645	2.287260	2.010011	1.034251	1.432087	1.795507	2.135691	2.415249
42	1.059117	1.35549	1.639538	1.899533	2.123575	2.296453	2.391038	2.365027	.818549	1.120551	1.422087	1.716056	1.993864
43	.767920	.976289	1.201211	1.447844	1.722112	2.030532	2.376587	2.750041	.615936	.818549	1.034251	1.271047	1.536305
44	2.055221	1.353615	1.545564	1.313061	1.096810	.895222	.705893	.524847	2.750041	2.365027	2.010011	1.696551	1.421352
45	2.087027	2.042557	1.921918	1.740855	1.515013	1.258702	.985235	.705893	2.376587	2.391038	2.287260	2.108921	1.883448
46	2.042557	2.210812	2.232887	2.119197	1.898732	1.602096	1.287020	.895222	2.030532	2.296453	2.476645	2.459930	2.301218
47	1.921918	2.232886	2.407198	2.389927	2.205626	1.898732	1.515013	1.096810	1.722112	2.123575	2.465611	2.662980	2.614448
48	1.740856	2.119198	2.389927	2.493580	2.389927	2.119198	1.740856	1.313062	1.447844	1.899533	2.308666	2.616344	2.747652
49	1.515013	1.898732	2.205626	2.389927	2.407198	2.232586	1.921918	1.545565	1.201210	1.639538	2.049070	2.389927	2.014448
50	1.287020	1.602096	1.898732	2.119197	2.232887	2.210812	2.042557	1.794277	.976290	1.355492	1.719198	2.043648	2.301218
51	.985235	1.258702	1.515013	1.740855	1.921918	2.042557	2.087027	2.055222	.767420	1.058612	1.347354	1.625796	1.883448
52	.705893	.895222	1.096810	1.313061	1.545564	1.353615	2.055221	2.318917	.570114	.758359	.958483	1.177506	1.421352
53	1.499813	1.351359	1.162109	.996574	.837452	.685793	.541344	.403185	2.007521	1.762699	1.523011	1.298955	1.094500
54	1.595132	1.603461	1.544081	1.422012	1.249385	1.040417	.809470	.569510	1.810111	1.863254	1.839378	1.738489	1.576840
55	1.630215	1.801144	1.856509	1.790017	1.618204	1.366796	1.064136	.738070	1.606508	1.876441	2.064191	2.102553	2.000681
56	1.586592	1.872861	2.037395	2.04365	1.898732	1.633819	1.289282	.906867	1.403085	1.797132	2.124636	2.311397	2.301218
57	1.463632	1.811421	2.049070	2.135691	2.049070	1.811421	1.468632	1.074662	1.200988	1.640906	2.030086	2.308666	2.415249
58	1.289282	1.633819	1.898732	2.043648	2.037395	1.872861	1.586592	1.236892	1.004149	1.425304	1.811420	2.119197	2.301218
59	1.064136	1.366796	1.618204	1.790016	1.856509	1.801144	1.630215	1.391175	.811813	1.166989	1.502027	1.790016	2.000680
60	.809470	1.040417	1.249385	1.422012	1.544081	1.603461	1.595132	1.495804	.623631	.881922	1.134579	1.370653	1.576840
61	.541344	.685793	.837452	.996574	1.162109	1.351359	1.499813	1.663367	.438279	.585258	.740453	.909121	1.094500
62	.293011	.371361	.628137	.541122	.495587	.374556	.295543	.213966	1.055082	.940357	.822818	.707991	.599612
63	.984328	1.045258	1.047775	.990282	.880720	.731489	.556622	.369958	1.051139	1.165904	1.223857	1.208282	1.126585
64	1.115387	1.301831	1.391083	1.370653	1.249386	1.045074	.793051	.513060	1.014912	1.307468	1.525833	1.617767	1.576842
65	1.158244	1.432986	1.593639	1.625796	1.515013	1.289282	.983899	.638912	.942639	1.339524	1.667114	1.861650	1.883451
66	1.110902	1.425305	1.639538	1.716056	1.639538	1.425305	1.110902	.745003	.839616	1.268290	1.640906	1.899533	1.993864
67	.983899	1.289282	1.515013	1.625796	1.593639	1.432986	1.158244	.824466	.713467	1.110901	1.468631	1.740855	1.883451
68	.793051	1.048074	1.249386	1.370653	1.391083	1.301881	1.115387	.871540	.566765	.886917	1.181797	1.422012	1.576842
69	.556622	.731489	.880720	.990282	1.047775	1.045258	.984328	.853561	.407264	.616335	.815187	.990282	1.126585
70	.295543	.374556	.455587	.541122	.628137	.711361	.793011	.868135	.239535	.320814	.406614	.499181	.599612
71	.315935	.423292	.491145	.499181	.455586	.373197	.261728	.136818	.203445	.388527	.522334	.598081	.5996120
72	.594307	.762434	.885529	.909121	.852452	.687591	.482467	.250008	.344491	.671585	.931875	1.075948	1.094500
73	.832497	.958680	1.130772	1.177505	1.096810	.906867	.638912	.330672	.414105	.821018	1.159422	1.370938	1.421352
74	.712468	1.004150	1.201211	1.271097	1.201211	1.004150	.712468	.370912	.419371	.839616	1.200988	1.447844	1.536305
75	.638912	.906867	1.096810	1.177505	1.130772	.958680	.832497	.362008	.370911	.745002	1.074086	1.313061	1.421352
76	.434467	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.276685	.553280	.802089	.996574	1.094500
77	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.301499	.436568	.541122	.5996120

Table 5.1. (Continued)

Influence Coefficient η at Point	Unit Load at Point 40	41	42	43	44	45	46	47	48	49	50	51	52
1	.541123	.436569	.301499	.151552	.172606	.315935	.428293	.491145	.499181	.456586	.373197	.261798	.136818
2	.996575	.807893	.558280	.278686	.296834	.554307	.762433	.885528	.909121	.837452	.687291	.482467	.250008
3	1.313062	1.074062	.745003	.370912	.352008	.687496	.958679	1.130772	1.177506	1.096810	.906867	.638912	.330872
4	1.447844	1.200988	.839616	.419371	.370911	.712467	1.004149	1.201210	1.271047	1.201210	1.004149	.712467	.370911
5	1.370938	1.159422	.821018	.414105	.330872	.638912	.906867	1.096810	1.177506	1.130772	.958679	.687496	.362008
6	1.076948	.931875	.671985	.344491	.250008	.482467	.687291	.837452	.909121	.885528	.762433	.554307	.296834
7	.598081	.527234	.388527	.203445	.136818	.261798	.373197	.456586	.499181	.491145	.428293	.315935	.172605
8	.499181	.406614	.320814	.239535	.868135	.792011	.711261	.626769	.541123	.456586	.374560	.295543	.218966
9	.990282	.815187	.616335	.407274	.883560	.984327	1.045258	1.047800	.990282	.880720	.731489	.556622	.369958
10	1.422013	1.181797	.886918	.566765	.871539	1.115386	1.301881	1.391082	1.370653	1.249386	1.048074	.793051	.512060
11	1.740856	1.468632	1.110902	.712468	.829465	1.158243	1.432985	1.598638	1.625796	1.515013	1.289282	.983899	.638912
12	1.899533	1.640906	1.268290	.839616	.745002	1.110901	1.425304	1.639538	1.716055	1.639538	1.425304	1.110901	.745002
13	1.861650	1.667114	1.339524	.942639	.638912	.983899	1.280282	1.515013	1.625796	1.598638	1.432986	1.158243	.824465
14	1.617761	1.525833	1.307468	1.014912	.512060	.793051	1.048074	1.249386	1.370653	1.391082	1.301881	1.115386	.871539
15	1.208882	1.223857	1.165903	1.051139	.369958	.556622	.731489	.880720	.990282	1.047800	1.045258	.984327	.883560
16	.707991	.822818	.940357	1.05598	.218966	.295543	.374560	.456586	.591123	.626769	.711261	.792011	.868135
17	.909121	.740453	.686269	.438780	1.663367	1.490814	1.331360	1.162109	.996575	.837452	.685793	.541344	.402185
18	1.370653	1.134579	.881923	.623631	1.533804	1.595132	1.603461	1.544082	1.422013	1.299385	1.040417	.809470	.569510
19	1.790016	1.602027	1.166990	.811813	1.391509	1.630215	1.801144	1.856509	1.790016	1.618204	1.366756	1.064136	.738070
20	2.119198	1.811421	1.425305	1.004150	1.236890	1.586591	1.872860	2.037395	2.043648	1.898732	1.633819	1.289282	.906867
21	2.308666	2.030086	1.640906	1.200988	1.074086	1.468631	1.811920	2.049070	2.135691	2.049070	1.811420	1.968631	1.074086
22	2.311397	2.124637	1.797132	1.403086	.906867	1.289282	1.663819	1.898732	2.043648	2.037395	1.872860	1.586591	1.236890
23	2.102553	2.065191	1.876441	1.606508	.738070	1.064136	1.366756	1.618204	1.790016	1.856509	1.801144	1.630215	1.391509
24	1.738489	1.839372	1.863254	1.810111	.569510	.809470	1.040417	1.249385	1.422013	1.544082	1.603461	1.595132	1.533804
25	1.298955	1.523011	1.762699	2.007521	.402185	.541344	.685793	.837452	.996575	1.162109	1.331360	1.499814	1.663367
26	1.177501	.958482	.758360	.570114	2.318917	2.055222	1.794277	1.545565	1.313462	1.096810	.895222	.705893	.524847
27	1.625796	1.347354	1.059117	.767920	2.055221	2.087027	2.042557	1.921918	1.740856	1.515013	1.287020	.985235	.705893
28	2.043658	1.719198	1.355492	.976289	1.357615	2.042557	2.210812	2.232886	2.119198	1.898732	1.602096	1.258702	.895222
29	2.389927	2.049070	1.639538	1.201211	1.545564	1.921918	2.232887	2.407198	2.389927	2.205626	1.898732	1.575013	1.096810
30	2.616344	2.308660	1.899533	1.447844	1.313061	1.740866	2.119197	2.389927	2.493580	2.389927	2.119197	1.740855	1.313061
31	2.662980	2.465611	2.123575	1.722112	1.096810	1.515013	1.898732	2.205626	2.389927	2.407198	2.232887	1.421918	1.545564
32	2.459930	2.476645	2.296453	2.030532	.895222	1.258702	1.602096	1.898732	2.119198	2.232886	2.210812	2.042557	1.353615
33	2.108921	2.287260	2.391038	2.376587	.705893	.985235	1.287020	1.515013	1.740856	1.921918	2.042557	2.087027	2.055221
34	1.696551	2.010011	2.365027	2.750041	.524847	.705893	.895222	1.096810	1.313062	1.545565	1.794277	2.055222	2.318917
35	1.271005	1.034251	.818549	.615936	2.750041	2.376587	2.030532	1.722112	1.447844	1.201210	.976290	.767920	.570114
36	1.716055	1.422087	1.120551	.818549	2.365027	2.391038	2.296453	2.123575	1.899533	1.639538	1.355492	1.058612	.758359
37	2.135691	1.796507	1.422087	1.034251	2.010011	2.287260	2.476645	2.465611	2.308666	2.049070	1.719198	1.347356	.958483
38	2.493550	2.135691	1.716056	1.271047	1.696551	2.108921	2.459930	2.662980	2.616344	2.389927	2.043648	1.625796	1.177506
39	2.747652	2.415249	1.993864	1.536305	1.421362	1.883448	2.301218	2.614448	2.747652	2.614448	2.301218	1.883448	1.421352
40	2.839561	2.608952	2.24782	1.839473	1.177506	1.625796	2.043648	2.389927	2.616344	2.662980	2.459930	2.108921	1.696551
41	2.608952	2.675431	2.449982	2.194061	.958483	1.347354	1.719198	2.049070	2.308666	2.465611	2.476645	2.287260	2.010011
42	2.24782	2.449982	2.637154	2.618289	.758359	1.058612	1.355492	1.639538	1.899533	2.123575	2.296453	2.391038	2.365027
43	1.839473	2.194061	2.618289	3.125266	.570114	.767920	.976290	1.201210	1.447844	1.722112	2.030532	2.376587	2.750041
44	1.177506	.958483	.758359	.570114	2.815238	2.350175	1.464851	1.644581	1.370938	1.130772	.915145	.717381	.530669
45	1.625796	1.347354	1.058612	.767920	2.350175	2.413646	2.285383	2.093640	1.861650	1.598639	1.313744	1.017020	.717382
46	2.043648	1.719198	1.355492	.976290	1.964851	2.285383	2.528865	2.489526	2.311397	2.037345	1.695954	1.313745	.915145
47	2.389927	2.049070	1.635538	1.201210	1.644581	2.093650	2.489526	2.744074	2.662980	2.407198	2.037395	1.598638	1.130772
48	2.616344	2.308666	1.899533	1.447844	1.370938	1.861650	2.311397	2.662980	2.839561	2.662980	2.311397	1.861650	1.370938
49	2.662980	2.465611	2.123575	1.922112	1.130772	1.598639	2.037395	2.407198	2.662980	2.744074	2.489527	2.09364	1.644581
50	2.459930	2.476645	2.296453	2.030532	.915145	1.313744	1.695954	2.037395	2.311397	2.489526	2.528865	2.285383	1.964851
51	2.108921	2.287260	2.391038	2.376587	.717381	1.017020	1.313745	1.598638	1.861650	.093640	2.285383	2.413646	2.350175
52	1.696551	2.010011	2.365027	2.750041	.530670	.717381	.951145	1.130772	1.370938	1.644581	1.964851	2.350175	2.815238
53	.909121	.740453	.585255	.438779	2.147943	1.833221	1.545549	1.294700	1.076948	.885529	.714229	.557736	.410375
54	1.370653	1.134579	.881922	.623631	1.871057	1.962523	1.804929	1.801936	1.617767	1.391083	1.135691	.864058	.587121
55	1.790016	1.502027	1.166989	.811813	1.611466	1.955286	2.201980	2.232790	2.102553	1.856509	1.532158	1.160440	.768960
56	2.119197	1.811420	1.425304	1.004149	1.375360	1.847438	2.250133	2.489526	2.459930	2.232887	1.872860	1.432985	.958679
57	2.308666	2.030086	1.640906	1.200988	1.159422	1.667114	2.124636	2.465611	2.608952	2.465611	2.124636	1.667114	1.159422
58	2.311397	2.124636	1.797132	1.403085	.958680	1.432986	1.872861	2.232887	2.459930	2.489526	2.250133	1.847438	1.375360
59	2.102553	2.064191	1.876441	1.606508	.768960	1.160439	1.532139	1.856509	2.102553	2.232790	2.201980	1.955286	1.611466
60	1.738489	1.839378	1.863254	1.810111	.587121	.864058	1.135641	1.391082	1.617767	1.801936	1.804929	1.962523	1.871057
61	1.298955	1.523011	1.762699	2.007521	.410375	.557736	.714229	.885528	1.076948	1.294700	1.545549	1.833221	2.147943
62	.499181	.406614	.320814	.239535	1.149206	.999177	.853028	.718268	.598081	.491145	.395104	.307336	.224719
63	.990282	.815187	.616335	.407264	1.106869	1.267955	1.337860	1.312140	1.208882	1.047775	.896619	.621443	.386254
64	1.422012	1.181797	.886917	.566765	1.036379	1.616809	1.698395	1.801937	1.738489	1.544081	1.259798	.912075	.541704
65	1.740855	1.468631	1.110901	.712467	.938977	1.633636	1.847438	2.093644	2.108921	1.921918	1.586591	1.158243	.687496
66	1.899533	1.640906	1.268290	.839616	.821018	1.339524	1.797132	2.123575	2.247828	2.123575	1.797132	1.339524	.821018
67	1.861650	1.667114	1.339524	.942639	.687496	1.158244	1.586592	1.921918	2.108921	2.093644	1.847438	1.433636	.438977
68	1.617767	1.525833	1.307468	1.014912	.541705	.912089	1.256799	1.549082	1.738489	1.801937	1.698395	1.416809	1.036379
69	1.208882	1.223847	1.165904	1.051139	.386250	.621444	.846619	1.047800	1.20882	1.312142	1.337860	1.267955	1.106869
70	.707991	.822818	.940357	1.055982	.224719	.307336	.395104	.491145	.598081	.908268	.853028	.999178	1.149206
71	.541122	.436568	.301499	.151551	.213914	.464918	.639779	.718268	.707991	.628137	.493848	.327895	.146165
72	.996574	.807089	.558280	.278685	.353302	.785786	1.122256	1.296700	1.298955	1.162109	.920570	.611330	.269926
73	1.313061	1.074086	.745002	.370911	.415672	.938977	1.375360	1.644581	1.696551	1.595564	1.236890	.824465	.362008
74	1.447844	1.200988	.839616	.419371	.414105	.992639	1.403085	1.722112	1.839473	1.722112	1.903085	.942639	.914105
75	1.370938	1.159422	.821018	.414105	.362008	.824466	1.236892	1.565565	1.696551	1.644581	1.375360	.938977	.415672
76	1.076948	.931875	.671985	.344491	.409269	.611330	.920510	1.162109	1.298955	1.296700	1.122250	.785786	.353302
77	.598081	.527234	.385527	.203445	.146165	.327846	.493849	.626769	.707991	.718268	.639779	.464918	.213914

Table 5.1. (Continued)

Influence Coefficient η at Point

	Unit Load at Point												
	53	54	55	56	57	58	59	60	61	62	63	64	65
1	.126312	.243828	.338661	.395144	.406614	.374559	.305957	.211425	.103609	.067678	.170677	.255100	.307336
2	.219299	.388628	.562027	.714229	.740453	.689793	.562027	.388628	.198589	.118134	.281573	.434703	.541344
3	.269926	.541704	.768960	.915145	.958482	.895222	.738470	.512060	.250008	.146165	.386254	.587121	.717382
4	.278685	.566765	.811813	.976290	1.034251	.976290	.811813	.566760	.278685	.151551	.407264	.623631	.767920
5	.250008	.512060	.738070	.895222	.958482	.915145	.768070	.541704	.269926	.136818	.369958	.569510	.705893
6	.198589	.388628	.562427	.689793	.740453	.714229	.606722	.432104	.219299	.103609	.281573	.434703	.541344
7	.103609	.211425	.305957	.374559	.406614	.395104	.338661	.243828	.126312	.056651	.153240	.236315	.295543
8	.615076	.582536	.542848	.493849	.436509	.373197	.305957	.236715	.166899	.319144	.334430	.338812	.327840
9	.636462	.745086	.821467	.846619	.815187	.731489	.605609	.451933	.281573	.334430	.467542	.569573	.621443
10	.637637	.862895	1.301500	1.135641	1.134579	1.040417	.862711	.692464	.388628	.338812	.569574	.755138	.864058
11	.611330	.912475	1.160439	1.313745	1.347354	1.258702	1.064136	.793051	.482467	.327846	.621444	.864058	1.017020
12	.558280	.886917	1.166989	1.355492	1.422037	1.355402	1.166989	.886917	.558280	.301499	.616335	.881923	1.059117
13	.482467	.793051	1.064136	1.258702	1.347354	1.313745	1.160439	.912075	.611330	.261981	.556522	.803470	.985235
14	.388628	.692464	.862711	1.040417	1.134579	1.135641	1.301500	.862895	.637637	.211425	.451433	.658950	.809470
15	.281573	.451433	.605609	.731489	.815187	.846619	.821467	.745086	.636463	.153240	.311120	.451433	.556622
16	.166899	.236715	.305957	.373197	.436509	.493899	.542848	.582536	.615076	.090584	.153240	.211425	.261798
17	1.183180	1.108239	1.021458	.920510	.805893	.687291	.562027	.434703	.307163	.615076	.636462	.637637	.611330
18	1.108239	.845501	1.264948	1.256799	1.181800	1.048074	.868711	.658950	.434703	.582536	.745086	.862895	.912075
19	1.021958	1.264947	1.440832	1.532139	1.502027	1.366750	1.146760	.868711	.562021	.542848	.821467	1.301500	1.160038
20	.920570	1.258798	1.532138	1.695954	1.719198	1.602096	1.366756	1.048074	.687291	.493843	.846619	1.135641	1.313745
21	.807893	1.181797	1.502027	1.719198	1.795507	1.719198	1.502027	1.181797	.807893	.436568	.815187	1.134579	1.347354
22	.687291	1.048074	1.366756	1.602096	1.719198	1.695956	1.532138	1.258798	.920510	.373197	.731489	1.040417	1.258702
23	.562027	.868711	1.146760	1.366756	1.502027	1.532139	1.440832	1.264947	1.021458	.305957	.605609	.868711	1.064136
24	.434703	.658950	.868711	1.048074	1.181800	1.256799	1.264948	.845501	1.108239	.236715	.451433	.692464	.793051
25	.307163	.434703	.562027	.687291	.807493	.920510	1.021458	1.108239	1.183180	.166899	.281573	.388628	.482467
26	1.663367	1.553804	1.391175	1.236892	1.074662	.906867	.738470	.569510	.402185	.862135	.853560	.871530	.824465
27	1.459313	1.595132	1.630215	1.586592	1.468632	1.289282	1.064136	.809470	.541344	.792011	.984327	1.115388	1.158243
28	1.331359	1.603461	1.801144	1.872861	1.811421	1.633819	1.366256	1.040417	.689793	.711361	1.045258	1.301881	1.432985
29	1.162109	1.544081	1.856509	2.037395	2.099070	1.898732	1.612304	1.249385	.837452	.625769	1.047800	1.371082	1.598638
30	.996574	1.422012	1.790016	2.09365	2.135691	2.043649	1.790016	1.422012	.996575	.541123	.990282	1.370653	1.625796
31	.837452	1.249385	1.618204	1.898732	2.040070	2.037395	1.856509	1.544081	1.162109	.456586	.880720	1.249386	1.515013
32	.689793	1.040417	1.366756	1.633819	1.811421	1.872861	1.801144	1.603461	1.331360	.374560	.731489	1.048074	1.289282
33	.541344	.809470	1.064136	1.289282	1.468632	1.586592	1.630215	1.595132	1.459814	.295543	.556622	.793051	.983899
34	.402185	.569510	.738070	.906897	1.074652	1.236892	1.391175	1.553804	1.663364	.218966	.359958	.512060	.638912
35	2.007521	1.810111	1.606508	1.403085	1.200988	1.004150	.811813	.623631	.438779	1.059980	1.051139	1.014912	.942630
36	1.762599	1.863254	1.873441	1.797132	1.640906	1.425304	1.166989	.881922	.585259	.940357	1.165903	1.307468	1.338524
37	1.523011	1.839378	2.064191	2.124636	2.030086	1.811420	1.502037	1.134379	.740453	.822818	1.223857	1.520033	1.667114
38	1.298955	1.738489	2.102553	2.311397	2.308666	2.119137	1.790016	1.370653	.909121	.707491	1.208822	1.617261	1.861650
39	1.094500	1.576840	2.00068	2.301218	2.415249	2.301218	2.000680	1.576840	1.094500	.599612	1.126585	1.576[illegible]42	1.883451
40	.909121	1.370653	1.790016	2.119197	2.308666	2.311397	2.102553	1.738489	1.298955	.499181	.990282	1.422012	1.749855
41	.740453	1.134379	1.502427	1.811420	2.030086	2.124636	2.064191	1.839372	1.523011	.406614	.815187	1.181797	1.459631
42	.585258	.881922	1.186989	1.425304	1.640906	1.797132	1.873441	1.863254	1.462599	.320814	.616335	.886917	1.110901
43	.438779	.623631	.811813	1.004149	1.200988	1.403085	1.606508	1.810111	2.007521	.239535	.407264	.566765	.712467
44	2.149943	1.871057	1.611466	1.375360	1.159422	.959680	.769960	.587121	.410325	1.149206	1.106869	1.036379	.938977
45	1.833221	1.962523	1.959286	1.847438	1.667114	1.432986	1.160439	.864058	.557236	.999177	1.267955	1.416809	1.433636
46	1.545549	1.864[illegible]29	2.201980	2.250133	2.124636	1.873861	1.533139	1.135641	.714229	.853028	1.337860	1.698395	1.847438
47	1.294700	1.701936	2.232790	2.489523	2.465611	2.232887	1.856509	1.391082	.885928	.718068	1.312140	1.801937	2.093644
48	1.076548	1.617267	2.102553	2.459930	2.508952	2.459930	2.102553	1.617267	1.076548	.595081	1.208822	1.738489	2.108921
49	.889529	1.391083	1.856509	2.232887	2.465611	2.489520	2.232700	1.801936	1.294700	.491145	1.047775	1.544481	1.993018
50	.714229	1.135641	1.533138	1.873866	2.124636	2.250133	2.201980	1.864929	1.545549	.395104	.846619	[illegible]	1.586591
51	.557236	.864058	1.160440	1.432985	1.667114	1.847438	1.959286	1.962523	1.833221	.307336	.621444	.913077	1.158243
52	.410375	.587121	.769960	.959680	1.159422	1.375360	1.611466	1.871057	2.149943	.224719	.386254	.541704	.687496
53	1.961305	1.619564	1.345127	1.122256	.931875	.762434	.606722	.460041	.318727	1.101826	1.002760	.896275	.789786
54	1.618564	1.818915	1.804929	1.683950	1.525833	1.301831	1.040308	.755138	.460041	.927819	1.262708	1.414766	1.416810
55	1.345127	1.804929	2.164009	2.201980	2.064191	1.801144	1.448832	1.301500	.606722	.772105	1.362500	1.804929	1.959286
56	1.122256	1.68395	2.201980	2.525865	2.475645	2.201912	1.801144	1.301831	.762433	.639979	1.3377[illegible]50	1.962460	2.285383
57	.931875	1.525833	2.064191	2.475645	2.675431	2.476645	2.064191	1.525833	.931875	.527234	1.223857	1.839372	2.287260
58	.762434	1.301831	1.801149	2.210812	2.475645	2.525865	2.201980	1.683950	1.122236	.428293	1.045258	1.603461	2.042597
59	.606722	1.040308	1.448832	1.801144	2.064191	2.201980	2.164009	1.804929	1.345127	.338661	.821467	1.264947	1.630215
60	.460041	.755138	1.301500	1.301831	1.525833	1.683950	1.804929	1.818915	1.618564	.255100	.569574	.862895	1.115386
61	.318727	.460041	.606722	.762433	.931875	1.122256	1.345127	1.618504	1.951305	.175044	.295250	.433127	.554307
62	1.101826	.927819	.772105	.639979	.527334	.428293	.338661	.255100	.175044	.818840	.663020	.551876	.464918
63	1.062760	1.262708	1.362500	1.337860	1.223857	1.045258	.821467	.569574	.295250	.663020	1.104867	1.262710	1.337955
64	.896275	1.414766	1.804929	1.962460	1.839372	1.603461	1.264947	.862895	.432127	.551876	1.362710	1.818915	1.962523
65	.785786	1.415810	1.959286	2.285383	2.287260	2.042537	1.630215	1.115386	.554307	.464918	1.267955	1.962523	2.413650
66	.671985	1.307468	1.876441	2.246493	2.449051	2.246493	1.876441	1.307468	.671935	.388526	1.165903	1.863254	2.391038
67	.554307	1.115387	1.630215	2.042597	2.287260	2.285383	1.959286	1.416810	.785736	.315935	.984328	1.598132	2.087027
68	.432104	.862895	1.264948	1.603461	1.839372	1.962466	1.804929	1.414766	.896275	.338812	.754509	.845501	1.595132
69	.305960	.569574	.821467	1.045258	1.223857	1.337850	1.362500	1.262708	1.002760	.17[illegible]677	.467542	.745084	.954327
70	.175044	.255100	.338661	.428293	.527234	.630927	.772105	.927819	1.101835	.096532	.170677	.243528	.315935
71	.195153	.551876	.772105	.853028	.852819	.711361	.545547	.33881	.119134	.131083	.663020	.922310	.999178
72	.305625	.896275	1.345127	1.545549	1.523011	1.331360	1.021458	.636[illegible]27	.219299	.195153	1.002760	1.618569	1.833221
73	.353302	1.036379	1.611465	1.964351	2.010011	1.794276	1.319509	.871539	.296834	.213914	1.106869	1.871097	2.350175
74	.344491	1.014712	1.606508	2.030532	2.174071	2.030532	1.606508	1.014312	.344491	.202445	1.051139	1.810111	2.426582
75	.296835	.871504	1.319175	1.794277	2.101011	1.964851	1.611466	1.036379	.353302	.172606	.883581	1.533804	2.052220
76	.219299	.637637	1.021458	1.331360	1.523011	1.545549	1.345127	.896275	.305625	.126312	.636463	1.308339	1.40989
77	.119134	.338812	.543848	.711361	.852818	.853028	.772105	.551876	.195153	.234549	.344491	[illegible]	[illegible]

Table 5. [illegible]

Influence Coefficient η at Point

	Unit Load at Point												
	66	67	68	69	70	71	72	73	74	75	76	77	Total
1	.320814	.295543	.236715	.153240	.056661	.096332	.175044	.224719	.239535	.218966	.166899	.090584	33.07685
2	.585259	.541344	.434703	.218573	.103609	.175044	.318727	.410375	.438779	.402185	.307163	.166899	59.628669
3	.758360	.705893	.569510	.369958	.136818	.224719	.410375	.530669	.520114	.524847	.402185	.218966	76.220692
4	.818549	.767920	.623631	.407264	.151552	.239535	.438779	.570114	.615936	.570114	.438779	.239535	82.577280
5	.758360	.717382	.587121	.386254	.146165	.218966	.402185	.524847	.570114	.530669	.410375	.224719	76.720692
6	.585259	.557736	.460041	.305260	.118134	.166899	.307163	.402185	.438779	.410375	.318727	.175044	59.628669
7	.328814	.307336	.255100	.170677	.235949	.090584	.116899	.218966	.239535	.224719	.175044	.096332	33.076785
8	.301499	.261798	.211425	.153240	.090584	.067678	.118134	.146165	.151551	.136818	.103609	.056661	33.076785
9	.616335	.556622	.451433	.311120	.153240	.170667	.281573	.286254	.407264	.369958	.281573	.153240	59.763738
10	.885922	.809470	.658950	.451433	.211425	.255100	.434703	.587121	.623631	.569510	.434703	.236715	81.960544
11	1.058612	.985235	.809470	.55662	.261798	.307336	.541344	.717382	.767920	.705893	.454344	.295543	96.372109
12	1.120551	1.059112	.881923	.616335	.301499	.320814	.585259	.758360	.818549	.758360	.485259	.320814	101.380288
13	1.058612	1.017020	.864058	.621444	.327846	.295543	.541344	.705893	.767920	.717382	.541344	.307336	96.372109
14	.881922	.864058	.755138	.569574	.338812	.236715	.434703	.569510	.623631	.587121	.434703	.255100	81.960544
15	.616335	.621443	.569574	.467542	.334430	.153240	.281573	.402364	[illegible]	.386259	.281573	.170677	59.763738
16	.301499	.327846	.338812	.334430	.319144	.056661	.103609	.136818	.151551	.146165	.118134	.067678	33.076885
17	.558280	.482467	.388628	.281573	.166899	.136312	.219299	.269926	.278685	.250008	.198589	.103609	59.628669
18	.886917	.793051	.692464	.451433	.236715	.243828	.388628	.541704	.566765	.512060	.388628	.211425	81.960544
19	1.166987	1.064136	.868711	.605609	.305957	.338661	.562027	.768960	.811313	.738070	.562027	.305957	100.617045
20	1.355492	1.258702	1.040417	.731489	.373197	.395104	.714229	.915145	.976290	.895222	.685793	.375559	112.983999
21	1.022087	1.347354	1.134579	.815187	.436569	.406614	.740453	.958482	1.034251	.958482	.740453	.406614	117.698002
22	1.355492	1.313745	1.135641	.846619	.493849	.374559	.685793	.895222	.976290	.915145	.714229	.395104	112.983999
23	1.166989	1.160439	1.301500	.821467	.542848	.305957	.562027	.738070	.811813	.768960	.562427	.338661	100.617045
24	.886917	.912075	.862895	.745086	.582236	.211425	.388628	.512060	.566765	.541704	.388628	.243828	81.960544
25	.558280	.611330	.637637	.636462	.619076	.103609	.198589	.250008	.278685	.269426	.219299	.136312	59.628669
26	.745002	.638912	.512060	.369958	.218966	.172605	.296834	.362008	.370911	.330872	.250008	.136818	76.720692
27	1.110901	.983899	.793051	.556622	.295543	.315935	.554307	.687496	.712467	.638912	.482467	.261798	96.372109
28	1.425304	1.289282	1.048074	.731489	.374560	.428293	.762433	.958679	1.004149	.906867	.687291	.373197	112.983999
29	1.639558	1.515013	1.249386	.880720	.456486	.991145	.885528	1.130772	1.201210	1.096810	.837452	.456586	124.070109
30	1.716055	1.625796	1.370653	.990282	.591123	.499181	.909121	1.177506	1.271147	1.177506	.909121	.499181	127.947061
31	1.639538	1.598638	1.391082	1.047800	.626769	.456586	.837452	1.096810	1.201210	1.130772	.885528	.491145	124.070109
32	1.425305	1.432985	1.301881	1.045258	.711261	.373197	.687291	.906861	1.004149	.958679	.762433	.428293	112.98399
33	1.110901	1.158243	1.115386	.984327	.792011	.261798	.482467	.638912	.712467	.687496	.554307	.315935	81.960544
34	.745002	.824465	.871539	.883560	.868135	.136818	.250008	.330872	.370911	.362008	.290834	.172605	76.720692
35	.839616	.712468	.566765	.407274	.239535	.203445	.3444919	.414105	.419371	.370912	.278686	.159552	82.577280
36	1.268290	1.110902	.886918	.616335	.320814	.388527	.671985	.821018	.839616	.745003	.558280	.301499	101.380288
37	1.640906	1.468632	1.181797	.815187	.406614	.527234	.931875	1.159422	1.200988	1.074062	.807893	.436567	117.698002
38	1.899533	1.740856	1.422013	.990282	.499181	.598081	1.076948	1.370938	1.447844	1.313062	.996575	.541123	127.947061
39	1.993864	1.883451	1.576842	1.126585	.599612	.599612	1.094500	1.421352	1.556308	1.421352	1.094500	.599612	131.672367
40	1.898533	1.861650	1.617761	1.208882	.707591	.541122	.996570	1.313061	1.447896	1.370938	1.076948	.598081	127.947061
41	1.640906	1.662114	1.525833	1.223857	.822818	.436568	.807089	1.074086	1.200988	1.159422	.931875	.527234	117.698002
42	1.268290	1.339524	1.307963	1.165903	.940357	.301499	.358280	.745002	.839616	.821018	.671985	.388527	101.380283
43	.839676	.942639	1.014912	1.051139	1.055980	.151551	.278684	.370911	.619371	.616105	.344491	.203445	82.577280
44	.821018	.687496	.541705	.386250	.229719	.213914	.353302	.915672	.419105	.362008	.409269	.146165	76.720692
45	1.339524	1.158244	.912039	.621444	.307336	.464918	.785786	.938977	.942639	.824466	.611330	.327846	96.872109
46	1.797132	1.586592	1.256799	.846619	.395104	.639779	1.122256	1.375360	1.403085	1.236892	.920510	.493849	112.983999
47	2.123575	1.921918	1.594082	1.047800	.491145	.718268	1.294700	1.644581	1.722112	1.545565	1.162109	.626769	124.070109
48	2.247828	2.108921	1.738489	1.208882	.598081	.707991	1.298955	1.696551	1.839473	1.696551	1.298955	.707991	127.947061
49	2.123575	2.093644	1.931938	1.312140	.718268	..268137	1.162109	1.545564	1.722112	1.644581	1.294700	.718268	124.070109
50	1.797132	1.847438	1.698395	1.337860	.853028	.493848	.920510	1.236890	1.403085	1.375360	1.122256	.639779	112.98399
51	1.339524	1.433636	1.416809	1.257955	.999177	.327345	.611330	.824465	.942639	.938977	.785786	.464918	96.372109
52	.821018	.938977	1.036379	1.106869	1.149206	.146165	.269936	.360201	.414105	.415672	.353302	.213914	76.720692
53	.671985	.554307	.932104	.305260	.175049	.195153	.309625	.353302	.344491	.296835	.219299	.118134	59.628669
54	1.307968	1.115387	.862895	.569574	.255100	.551876	.896275	1.036379	1.014912	.871540	.637637	.338812	81.960544
55	1.876441	1.630215	1.264948	.821467	.338661	.772105	1.345127	1.611466	1.606508	1.391175	1.021458	.542848	100.617045
56	2.296453	2.042557	1.603461	1.045258	.428293	.853028	1.545544	1.964851	2.030532	1.794277	1.331360	.711261	112.983999
57	2.449981	2.287260	1.839372	1.223557	.527234	.822818	1.523011	2.010011	2.194051	2.010011	1.523011	.822818	117.698002
58	2.296453	2.285384	1.926246	1.337860	.639979	.711261	1.331360	1.794276	2.030532	1.964851	1.545549	.853028	112.983999
59	1.876441	1.955386	1.804929	1.362500	.772105	.542847	1.021458	1.391509	1.606508	1.611466	1.345127	.772105	100.617045
60	1.307468	1.416810	1.414766	1.262707	.927819	.338811	.637637	.871539	1.014912	1.036379	.896275	.551876	81.960544
61	.671985	.785786	.896275	1.002760	1.101826	.118134	.219299	.296534	.344491	.353542	.309625	.195753	59.628669
62	.388526	.315935	.338812	.170677	.096332	.13003	.195153	.213014	.203445	.172606	.12632	.334949	33.076785
63	1.165903	.984328	.754509	.467542	.170677	.663020	1.602760	1.106869	1.051139	.863561	.636463	.334430	59.763738
64	1.1863254	1.595132	.845501	.745384	.243828	.927819	1.618569	1.871057	1.81011	1.533804	1.108239	.582536	81.960544
65	2.391038	2.087027	1.595132	.984327	.315935	.999178	1.833221	2.350175	2.376537	2.052220	1.499814	.792011	96.372109
66	2.637154	2.391038	1.863254	1.165903	.388526	.940357	1.762699	2.365027	2.618289	2.365027	1.762699	.942357	101.372109
67	2.391038	2.413650	1.962523	1.267955	.464918	.792011	1.499813	2.055221	2.378587	2.350175	1.833221	.999178	96.372109
68	1.863254	1.962523	1.818915	1.262708	.551876	.582536	1.108239	1.533804	1.810111	1.871057	1.618569	.927819	81.960544
69	1.165903	1.267955	1.262710	1.104869	.663020	.334430	.636462	.883560	1.051139	1.105869	1.002760	.663420	59.763738
70	.388526	.464918	.551876	.663020	.818840	.067678	.136312	.172605	.203445	.213914	.195153	.131003	33.076785
71	.940357	.792011	.582536	.334430	.067678	.818840	1.101826	1.149206	1.055982	.868135	.615076	.319144	33.076785
72	1.762699	1.499813	1.108239	.636462	.136312	1.101826	1.961205	2.147943	2.007521	1.663367	1.183180	.615076	59.628669
73	2.365027	2.055221	1.533804	.883560	.172605	1.149206	2.197993	2.815258	2.750041	2.318917	1.663367	.868135	76.720692
74	2.618289	2.376587	1.810111	1.051139	.203445	1.055982	2.007521	2.750041	3.125266	2.750041	2.007521	1.055982	82.577280
75	2.365027	2.350175	1.871057	1.106869	.213914	.868135	1.663367	2.318917	2.750041	2.815258	2.147943	1.149206	76.720692
76	1.762699	1.833221	1.618569	1.002760	.195153	.615076	1.183180	1.663367	2.007521	2.147943	1.961305	1.101826	59.628669
77	.940357	.999178	.927819	.663020	.131003	.319144	.615076	.868135	1.055982	1.149206	1.101826	.818840	33.07685

Deflection influence coefficients for a simply supported beam are calculated and presented in Table 5.2.

Table 5.2

η_{ij} Deflection Influence Coefficients for Beam							
i \ j	1	2	3	4	5	6	7
1	2.1869	3.4997	4.0625	3.9900	3.4375	2.500	1.3128
2	3.4997	6.2500	7.4990	7.500	6.4900	4.7470	2.4990
3	4.0625	7.4990	9.6950	9.9940	8.8900	6.4980	3.4364
4	3.9900	7.500	9.9940	11.00	9.9940	7.5000	3.9900
5	3.4375	6.4980	8.8900	9.9940	9.6950	7.4990	4.0625
6	2.500	4.7470	6.4980	7.5000	7.4990	6.2500	3.4997
7	1.3128	2.4990	3.4375	3.9900	4.0625	3.4977	2.1869

The unknowns in the example are moments at points 12, 13, 14, 15 and 16, and reactions at points 1 to 11 (Figure 5.3).

All the angular and displacement functions are obtained by using the relations derived in Chapter IV and utilizing the deflection influence coefficients given in Tables 5.1 and 5.2. A matrix is formulated for the solution of unknowns and is presented on page 47.

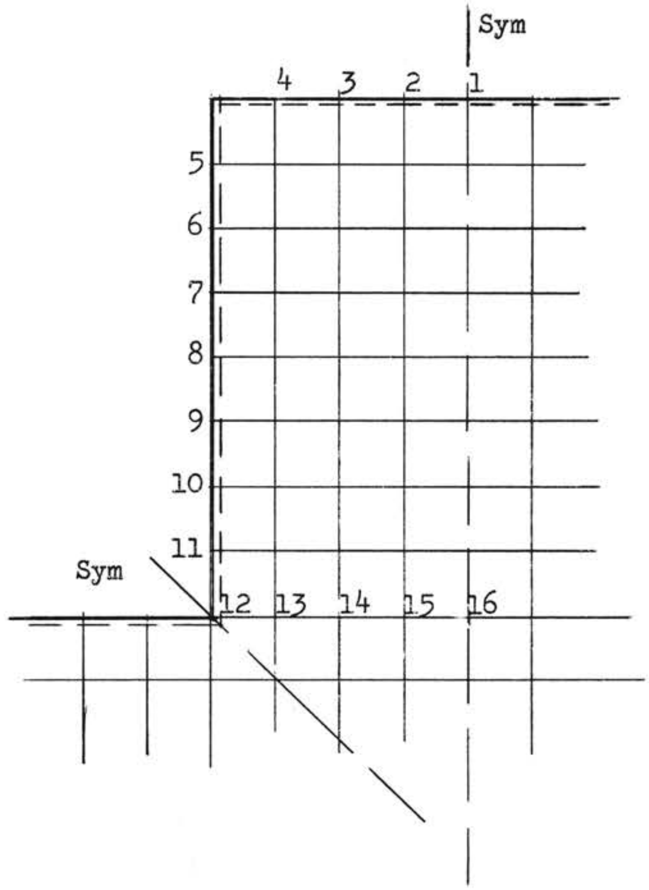


Figure 5.3. Reduced Plate Structure Due to Symmetry

1.803695	.790928	.751447	.634037	.300382	2.281720	3.198602	3.41022	3.24020	2.705230	1.94625	1.034221	.722121	.545236	.763123	.377028	M_{12}	108.8334
.796928	1.66254	.85947	.733182	.346977	1.42736	1.18902	1.12250	1.095089	.956108	.733020	.499021	.341262	.55023	.776250	.416205	M_{13}	83.2576
.751447	.85947	1.62008	.885564	.41506	1.07812	1.88312	1.75812	1.53821	1.29712	.898290	.848260	.439272	.60421	.853123	.462091	M_{14}	71.1907
.634037	.733182	.885564	1.588068	.516741	.981002	1.7351	2.11210	1.86223	1.58682	1.33002	1.18202	.59202	.781209	.915206	.492062	M_{15}	61.8972
.600764	.693954	.830812	1.033482	1.167931	.92012	1.636078	2.02093	2.1023	1.7052	1.44023	1.2700	.616200	.825062	.962036	.505267	M_{16}	58.6670
2.281720	1.42736	1.07812	.981002	.46006	6.202275	8.74232	9.46421	8.88562	7.54011	5.43112	2.841192	.78629	1.42902	1.92518	1.29001	$-R_{11}$	129.5030
3.198602	1.18902	1.88312	1.73510	.818039	8.74232	15.63753	17.6198	22.3501	14.3570	10.37282	5.43127	1.37122	2.62088	3.41202	2.15002	$-R_{10}$	230.07029
3.41022	1.12250	1.75812	2.1121	1.01046	9.46421	17.6198	23.0452	22.9801	19.75292	14.47022	7.49282	1.76208	3.24902	4.3307	2.59697	$-R_9$	300.2082
3.24020	1.095089	1.53821	1.86223	1.0511	8.885623	22.3501	22.9801	25.8329	22.98012	17.3643	8.920170	2.2187	3.89602	4.99672	2.62081	$-R_8$	322.0072
2.70523	.956108	1.29712	1.58682	.8526	7.54011	14.3570	19.75292	22.98012	23.0452	17.6209	9.46421	2.24542	3.8972	4.8572	2.59002	$-R_7$	300.2082
1.94625	.733020	.898290	1.33002	.72011	5.43112	10.37282	14.47022	17.3643	17.6209	15.63722	7.95022	1.95452	3.3372	4.0628	2.13628	$-R_6$	230.07029
1.034221	.499021	.84826	1.18202	.6350	2.8411920	5.43127	7.49282	8.92017	9.46421	7.95022	6.20275	1.24290	2.0072	2.41985	1.27622	$-R_5$	129.5030
.722121	.341262	.439272	.59202	.30810	.786289	1.37122	1.76208	2.2189	2.24542	1.95452	1.2429	5.60277	12.1078	13.49202	7.4925	$-R_4$	129.5030
.545236	.550230	.60421	.781209	.41253	1.42902	2.62088	3.24902	3.89602	3.8972	3.3372	2.0072	12.1078	13.6402	25.4602	14.6129	$-R_3$	230.07029
.763123	.776250	.853123	.915206	.481016	1.92519	3.41202	4.3307	4.99672	4.8572	4.06282	2.41985	13.49202	25.4602	27.9952	19.4026	$-R_2$	300.2082
.377028	.416205	.462091	.492062	.25263	1.29001	2.15002	2.59697	2.62081	2.59002	2.13628	1.27622	7.4925	14.6129	19.402	19.5322	$-R_1$	322.0072

+ … = 0

The solution of this matrix yields the final results which are:

M_{12} = - 58.3272 kip ft.

M_{13} = - 41.6920 kip ft.

M_{14} = - 5.5107 kip ft.

M_{15} = + 10.4092 kip ft.

M_{16} = + 3.7120 kip ft.

R_1 = + 1.7717 kips

R_2 = + 1.0526 kips

R_3 = + 0.8602 kips

R_4 = + 0.6072 kips

R_5 = + 0.7824 kips

R_6 = + 0.9526 kips

R_7 = + 1.0269 kips

R_8 = + 1.0401 kips

R_9 = + 1.5652 kips

R_{10} = - .9652 kips

R_{11} = - 3.5941 kips

CHAPTER VI

SUMMARY AND CONCLUSIONS

6.1. <u>Summary</u>. The application of flexibility methods to two-way continuous rectangular plates supported by flexible beams is presented. The continuous structure is isolated into appropriate basic structures and the support moments and edge shears are selected as redundants. A method of obtaining deflection influence coefficients for the basic structure by a finite difference approximation is given. Angular, displacement and load functions of the basic structure are introduced and are expressed in terms of the deflection influence coefficients. Deformation equations in terms of these functions and the redundants are obtained utilizing the conditions of compatibility of deformations over a continuous support and between plate and the supporting beam. The theory is illustrated by a numerical example.

6.2. <u>Findings and Conclusions</u>. The flexibility method of approach to two-way continuous rectangular plates is direct, can be used for any type of loading, and affords significant reduction in the number of unknowns. The type of basic structure chosen makes possible the application of the flexibility method to a wide range of problems.

The availability of deflection influence coefficient tables for various length-width ratios of the basic structure is a prerequisite to this method. Evaluation of such tables can be accomplished readily by the procedure indicated in this study, with a sufficiently high degree of accuracy obtainable, in most cases, using the same size network.

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