SUPPLY CHAIN COORDINATION UNDER ADVANCE-PURCHASE DISCOUNT CONTRACT WITH SALES EFFORT AND TRANSSHIPMENT

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Abstract: In today's business environment, a competition is no longer about competing between firms, but between supply chains. Improving supply chain's performance has become necessary for companies to survive. Supply chain coordination ensures a maximum performance of a supply chain. This dissertation studies impacts of an advance-purchase contract and supply chain coordination in two different supply chains.

We first consider the supply chain with the manufacturer and retailer who can exert sales effort to stipulate demand. We develop the contract that combines the advance-purchase contract and the target rebate contract to coordinate the retailer's ordering and effort decisions. We analytically show that supply chain coordination is achievable, but profit splitting may not be fully flexible depending on market conditions.

We second consider the supply chain with the manufacturer and two retailers who can transship products to satisfy unmet demand as a result of an inventory shortage. We establish a new mechanism that integrates the advance-purchase contract to coordinate the supply chain. The coordination mechanism follows in two steps: it first aligns the objective of the retailer group with the objective of the supply chain, and second aligns the individual objective of each individual retailer with the joint objective of the retailer group. We analytically show that supply chain coordination and arbitrary profit split is achievable.

The coordinating contracts lead to Pareto improving situations. The numerical analyses show the performance improvement of the supply chain from the inclusion of the advance-purchase contract. We also conduct the sensitivity analyses to see the impacts of the contract terms on the retailers' optimal decisions, and the impacts of market conditions on the contracts. The potential future research directions for both studies are also discussed.

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CHAPTER 1

INTRODUCTION

Consider retailers in two different regions who sell a product to end customers within their own region in a single selling-season. Due to the stochastic nature of customer demand, the demand and the supply of each retailer may not be perfectly matched, and therefore there are chances that the retailers might have unsatisfied demand or remaining inventory at the end of the selling-season. Companies, such as, Bosch and Toyota, employ the distributor integration strategy, aka transshipment, to alleviate any impact from such circumstances. Toyota dealerships have implemented a real-time inventory information system to monitor inventory among them. When a particular model and color of a vehicle desired by a customer is not available, a dealer searches for that specificity via the inventory system. If that is found, the "dealer trade" occurs, for which two dealerships, the "finding" and "found" dealerships, swap their vehicles, or the former pays the latter the manufacturer's wholesale price for obtaining the car. If that is not found, the customer is considered lost. Another transshipment application on a German paper wholesaler is discussed in [1].

Transshipment is a form of virtual pooling, which is one of the inventory sharing strategies. It is particularly useful in an industry which is required to maintain a high service level where inventory cost is expensive, or involves a high-end or necessary product. Effective transshipment is enabled with good information flow and reliable transportation links. It mimics the effect of risk pooling, which reduces variability of aggregated demand and results in a lower safety-stock level or higher customer service level or both. In general, there are two major types of transshipment: lateral transshipment and preventive transshipment. Lateral transshipment is employed as a reactive action where after satisfying own demand, a location with insufficient inventory makes a request for an emergency supply to another location with excess inventory. In contrast, preventive transshipment re-balances inventory at locations before demand observation. Apparently, for a single selling-season supply chain, lateral transshipment is more desirable unless forecast information changes after an inventory decision has been made. The pioneer studies on lateral transshipment in a single-echelon supply chain are Robinson [2] and Rudi et al. [3], which the latter focuses on supply chain coordination.

Sales effort can be used as a performance-improvement strategy from both marketing and operation perspectives. For example, a retailer can offer a promotion or a discount (see, e.g., [4]), expand shelf-space, invest in effective advertising, enhance demand forecasting (see, e.g., [5, 6]), provide better after-sales service or better educate customers about a product (see, e.g., [7]). These activities have become commonly used strategies in practice. For instance, retailers such as Wal-Mart and Sprouts mail out their weekly-sales ads to households, and car dealerships often offer a 0% interest or a deep discount from MSRP on their new cars. Even though these actions can enhance supply chain profit, a proper mechanism is needed to ensure optimal behaviors of the players in these circumstances.

Supply chain coordination is to make decisions from the supply chain's standpoint, rather than from the individual company's standpoint. Professor Hau Lee from Stanford University states, "The battle for market supremacy will not be between enterprises but between supply chains." If the supply chain wins (loses), every player in the supply chain wins (loses). Double marginalization causes supply chain inefficiency, which exists when an objective of supply chain's players is not aligned with the objective of the supply chain. A properly designed supply contract uses pricing and incentive mechanisms to resolve the double marginalization and to achieve supply chain coordination. Amongst coordinating contracts, a revenue-sharing (see, e.g., [8]), a buy-back (see, e.g., [9, 10]), and a rebate (see, e.g., [11, 12] contracts are the most studied and utilized due to their simplicity and applicability. Practical applications also appear in industries. For example, a revenue-sharing contract is used in the video rental industry, a buy-back contract is used in the book or magazine industry, and a rebate contract is used in the automotive industry.

In contrast, an advance-purchase discount contract, hereafter referred to as an advance-purchase contract for short, provides a retailer two ordering opportunities associated with two different wholesale prices: an advance (or first) wholesale price and a regular (or second) wholesale price. An incentive is offered by pricing an advance wholesale price lower than a regular wholesale price. It was first studied by Cachon [13] and later applied by Berndt and Hurvitz [14] in the healthcare industry.

This dissertation studies impacts of the advance-purchase contract and supply chain coordination in two different supply chains. The first study considers a supply chain that includes a manufacturer and a retailer, where the manufacturer offers a target rebate contract to the retailer who can exert a sales effort, which is costly and noncontractable. Taylor [11] shows that with only a target rebate contract, supply chain coordination is not achievable. He then combines a buy-back contract with a target rebate contract to coordinate the supply chain. He et al. [15] utilize a rebate contract and a penalty mechanism to coordinate the similar supply chain. However, there are associated practical drawbacks. First, a buy-back contract creates additional undesirable processes, which may incur extra costs, such as cost of handling, administering or transportation. Second, lost sale is difficult to monitor, especially in today's e-commerce environment. For example, in the retail industry when an online retailer is out of stock for a particular product, an "Out-of-Stock" notice is displayed on the web page. In that circumstance, the customer may choose to be noticed via an email when the product becomes available; however, not every customer opts into such strategy. To address these problems, we replace a buy-back contract or a lost-sale penalty with an advance-purchase contract.

The game between the manufacturer and retailer is modeled as a Stackelberg game, where the manufacturer (the leader) designs and offers a target rebate plus an advance-purchase contract to the retailer (the follower). The retailer sees the contract terms and make their ordering and effort decisions. We follow Taylor's approach (see [11]) to derive a coordinating second wholesale price and a coordinating unit rebate, then use a rebate target level and a first wholesale price to attain arbitrary profit split between the manufacturer and the retailer.

Next, we study the second supply chain with a manufacturer and two independent retailers with transshipment. In addition to a double marginalization effect created between the manufacturer and the retailers, transshipment causes another double marginalization effect between the retailers because the retailer intends to make as much profit as possible from providing transshipment by setting a high transshipment price. To coordinate the supply chain, both effects need be resolved. Although Rudi et al. [3] establish coordinating transshipment prices to eliminate the double marginalization effect between the retailers, there is still a need to resolve the other double marginalization effect while sustaining the outcome of Rudi et al. [3]. We develop the two-stage coordinating approach, in which first the supply chain profit is split between the manufacturer and the retailers through an advance-purchase contract, and then the retailers split their profit through coordinating transshipment prices.

The game between the manufacturer and retailers is a Stackelberg game, where the manufacturer (the leader) designs and offers an advance-purchase plus transshipment contract to the retailers (the follower). The retailers see the contract terms and make their ordering decisions. We show that the supply chain profit can be expressed as a linear combination of the manufacturer's and retailers' profits, where the coefficients are a function of an arbitrary profit split parameter (see [16] for more details). To accomplish this, we introduce a transshipment premium to motivate the manufacturer to facilitate transshipment. We then follow Rudi et al. [3] to derive coordinating transshipment prices, and use them for splitting profit between the two retailers.

The organization of this dissertation study is as follows. The next chapter reviews existing related literatures. The problem statement, research objectives and contributions of each study in this dissertation are described in Chapter 3. Chapter 4 presents a study of the supply chain under a target rebate contract with sales effort. The supply chain with transshipment is studied in Chapter 5. Chapter 6 summarizes the studies in this dissertation and discusses future research directions.

CHAPTER 2

LITERATURE REVIEW

Self-interest and decentralized decision making do not naturally lead to 100% supply chain efficiency so optimal supply chain performance is not guaranteed when every party in a supply chain optimizes its own individual performance. To improve the performance of the global supply chain, supply chain coordination helps align individual performance of every player with the global supply chain objective. Thomas and Griffin [17] emphasize the importance of supply chain coordination in the field of supply chain management. They review literatures pertaining to operational planning, which includes the coordination between buyer-vendor, production-distribution, inventory-distribution, and decision making and planning at the strategic level.

From our perspective, we can categorize supply chain coordination for a multi-echelon supply chain into two broad categories. One is the vertical coordination, while the other one is the horizontal coordination. Although both help coordinate the supply chain, their mechanism and interpretation are different. Generally, coordination across echelons can be efficiently achieved via risk sharing approaches through supply contracts, e.g., a buy-back contract, a revenue sharing contract, an options contract, an advance-purchase contract, etc. Whereas, coordination within an echelon can be accomplished via risk pooling strategies, e.g., inventory pooling, virtual pooling, product pooling, transshipment, etc.

In the following, we provide the three relevant streams of the literature review: (1) coordination via risk sharing, (2) coordination via risk pooling, and (3) a supply chain with sales effort.

2.1 Coordination via Risk Sharing: Supply Contracts

The study of vertical supply chain coordination was motivated by Cachon and Terwiesch [18], "Even if every firm in a supply chain chooses actions to maximize its own expected profit, the total profit earned in the supply chain may be less than the entire supply chain's maximum profit". Although there always exists a conflicting incentive for every player in the supply chain, there also exists a common objective of every player in the supply chain, which is each firm tries to maximize its profit. Because of the conflict of incentive, total supply chain performance by locally maximizing is never guaranteed to achieve the supply chain performance by globally maximizing, considering the global benefit of the supply chain.

Without a supply contract in place, Cachon and Zipkin [19] show that under the competition, firms are trying to lower its inventories more than they would do under the cooperation, resulting in less system efficiency. Supply contracts are typically agreed between two or more players across two different echelons, which allows risks to be shared between them, and generally provides a downstream player(s) an incentive to place a larger order quantity, which helps eliminate the effect of double marginalization. Thereby, the total supply chain profit increases, and is able to achieve the optimal supply chain profit. Cachon [20] summarizes necessary characteristics for good supply contracts. A variety of supply contracts have been studied by a number of researchers. Pasternack [9], Taylor [11] and Zhang et al. [21] focus on a buy-back contract. Cachon and Lariviere [8], Pasternack [22], Wang et al. [23], Hu et al. [24] and Zhang et al. [21] study a revenue-sharing contract and its application in airline industry. Taylor [11], Lee et al. [25] and Dahai and Liu [12] examine a rebate contract.

In this work, we consider a supply chain in a single selling-season similar to Cachon [13] and Dong and Zhu [26], and study a specific supply contract which allows a retailer to place two orders: an early order before demand observation, and a late order immediately after demand observation.

In the next section, we review the literatures related to the advance-purchase contract and related applications.

2.1.1 Advance-Purchase Contract

An advance-purchase contract provides two ordering opportunities to a retailer to place an order to a manufacturer at different times. Specifically, all the units in the first order, which is placed in advance, get a discount from the manufacturer, and all the units in the second order, which is placed at the beginning of the selling-season, are charged at a regular wholesale price. There are two different types of an advance-purchase contracts in regard to the manufacturer's production decision. For the first type, the manufacturer has two production opportunities, corresponding to two ordering opportunities of the retailers. So, the manufacturer can start production after he receives each order from the retailer. Whereas the second type gives the manufacturer only one production opportunity, which occurs in advance of the selling-season to satisfy both first and second orders of the retailer due to either a long lead-time or limited production capacity. This way, the manufacturer has to speculate on the second order of the retailer and build his inventory to satisfy that.

The studies of an advance-purchase contract have been limited due to the complexity of the model. In the following, we present those that are relevant to our work. Cachon [13] studies inventory management and production management under a newsvendor setting by applying a push-pull strategy and an advance-purchase contract. He shows that the advance-purchase contract could lead to global supply chain efficiency, and introduces the Pareto contract, which sometimes requires arbitrary and frequent changes of wholesale prices. Later on, Dong and Zhu [26] state that in reality it is impossible for a firm to keep drastically changing wholesale prices because the prices of negotiations and renegotiations involve a lot of sensitive issues among firms. They, therefore, study multiple pricing scenarios of wholesale prices to determine which scenario gives the most complete set of a Pareto improvement.

Özer and Wei [27] study an advance-purchase contract in the capacity commitment problem when the demand information is asymmetric, under which an upstream firm may not trust provided information of a downstream firm. The authors show the existence of the optimal order quantity for the first ordering opportunity of a retailer, production quantity of a manufacturer, and optimal prices. In addition, channel coordination can be achieved via a combined mechanism of an advance-purchase contract and a payback agreement.

Cho and Tang [28] analyze the two procurement opportunities in a supply chain with a manufacturer and a retailer under uncertain production yield in vaccine industry. For an practical application in healthcare industry, Berndt and Hurvitz [14] study a problem in vaccine production under an advance-purchase agreement.

Although the literatures have studied advance-purchase contracts from both theoretical and practical standpoints, they have never utilized an advance-purchase contract as a mechanism to coordinate the supply chains that we consider in this dissertation. Further, our advance-purchase contract is different from those. We relax an assumption of a long production lead-time to provide a manufacturer a capability to satisfy a second order of a retailer without having to produce in advance.

2.2 Coordination via Risk Pooling: Transshipment

Typically, transshipment can be categorized into two different transshipment policies, namely preventive transshipment and lateral transshipment.

Preventive Transshipment is the transshipment that happens between two consecutive subperiods when the demand information is partially observed. Rong [29] studies preventive transshipment problems by developing a model for a decentralized supply chain, which consists of two independent retailers who submit their order quantity to a supplier before the actual demand is observed. The author shows that there exists a Nash equilibrium for an order quantity of both retailers, and the transfer payments alone cannot coordinate the supply chain.

Another advantage of preventive transshipment is it allows firms to re-balance their inventory between two consecutive time periods. Gullu et al. [30] study the multi-period framework, which comprises of a single supplier and two independent retailers with supplier's and retailers' lead-times to determine their optimal individual order-up-to levels in a periodic review system. Before products are shipped, the retailers are allowed to transfer stocks at the supplier to improve expected costs. The authors derive the existence of a unique Nash equilibrium of the retailers' order-up-to levels, which lead to close-to-optimal performance.

Lateral transshipment is a special case of preventive transshipment where transshipment occurs at the end of time period when the demand information is fully known. It is typically employed as an emergency supply to "immediately" fulfill unmet demand as a result of an inventory shortage. Reyes [31] uses Game Theory to solve a small transshipment problem in a fully centralized supply chain under none cooperative, partially cooperative and fully cooperative environments.

For a single-echelon, centralized supply chain, Axsater [32] evaluates and derives the decision rule for lateral transshipment in a single-echelon, centralized inventory system, consisting of multiple warehouses facing Poisson demand. The instantaneous transshipment is allowed between the warehouses with transshipment cost. The author derives decision rule, analogous to a savings algorithm, and use it as a heuristic approach, which guarantees cost savings and performs considerably well, especially in complex supply chains. Yang and Qin [33] develop a model for one manufacturing company, consisting of two capacitated plants, which are located in two different regions, where the demand in each of the regions is stochastic, and can be satisfied by a remote plant via "Virtual Lateral Transshipment". The authors show the modifiedbase-stock property is the optimal production policy, and the demand-by-demand property as the optimal transshipment policy.

For a single-echelon, decentralized supply chain, Zhao et al. [34] study the multi-period inventory sharing and rationing game in a network with two independent retailers, each facing two demand classes: (1) retailer's own customers with high priority, and (2) the other retailer's customers who request inventory sharing with low priority. Each retailer determines the base-stock level and the rationing level. The authors derive the Nash equilibrium, and show (1) the inventory sharing games are supermodular under particular conditions, and therefore there exists a pure-strategy Nash equilibrium, (2) there exists a dominant strategy equilibrium for the retailers' rationing level in the inventory rationing game when the base-stock level of both retailers are fixed, and (3) the inventory sharing and inventory rationing game is not supermodular over the entire strategy space, but remains supermodular over most of the entire strategy space and hence the Nash equilibrium exists.

Rudi et al. [3] establish coordinating transshipment for a supply chain with two independent retailers facing stochastic demand. Hu et al. [35] extend Rudi et al. [3] to derive the necessary and sufficient conditions for the existence of coordinating linear transfer prices under the lateral transshipment policy in a supply chain with two production locations, facing stochastic demand and uncertain production capacity. They found the coordinating linear transshipment prices only exist for symmetric facilities, which complements the results of Rudi et al. [3], which has no production capacity.

For the multiple-echelon, decentralized supply chain, Özen et al. [36] analyze the problem in a supply chain with a warehouse and multiple retailers facing the stochastic demand. A retailer is allowed to transship his/her excess inventory to another. The authors study two allocation games: Forced Allocation and Relaxed Allocation games, where a warehouse is a cross-dock facility and a DC, respectively.

The authors demonstrate that in both games, there always exists an equilibrium at which an optimal allocation and order quantity of a retailer maximizes total expected profit of the retailers.

Anupindi et al. [37] study the inventory ordering and allocation decisions in a decentralized distribution system, which consists of warehouses and retailers who face stochastic demand. The retailers secure inventories prior to the demand observation, and fulfill their demand after the demand observation. Subsequently, if there are an excess inventory and unmet demand in the system, an owner of that inventory may share his/her inventory to satisfy the unmet demand to gain residual profit through an inventory exchange. Using the two-step solution approach by which in the first step the retailers agree on the allocation of the residuals, and in the second step they individually make the decision on their inventory level, the authors establish the core allocation in the residuals allocation game, and the conditions for the existence of the pure strategy Nash equilibrium in the inventory game.

Sošić [38] extends Anupindi et al. [37] to consider two different types of the retailers: myopic and farsighted retailers, who receive products from a supplier. The retailers decide whether or not to participate in the transshipment game. The model has two stages in which the ordering decision to the supplier, and the transshipment decision take place in the first and second stages, respectively. A myopic retailer views only an immediate payoff, whereas a farsighted retailer considers the reaction of other retailers. The author shows that without an extra mechanism the grand coalition for myopic retailers is not stable, and is stable for the farsighted symmetric retailers. Also, the author provides the condition, under which the grand coalition for asymmetric retailers is farsightedly stable.

There are more existing studies of transshipment, which can be further categorized based on characteristics of a supply chain. For centralized supply chains, Herer et al. [39] and Hu et al. [40] study single-period, single-echelon supply chains, Chen et al. [41] and Rosales et al. [42] consider single-period, multi-echelon supply chains, Robinson [2], Hu et al. [43] and van Wijk et al. [44] focus on multi-period, single-echelon supply chains, and Özdemir et al. [45], Paterson et al. [46], Yang et al. [47], Özdemir et al. [48] and Gong and Yücesan [49] analyze multi-period, multi-echelon supply chains.

For decentralized supply chains, Huang and Sošić [50] study a single-period, single-echelon supply chain, Wee and Dada [51] focus on a single-period, multi-echelon supply chain, Huang and Sošić [52], Granot and Sošić [53] and Van der Heide and Roodbergen [54] analyze multi-period, single-echelon supply chains, and Çömez et al. [55] and Satır et al. [56] consider multi-period, multi-echelon supply chains.

There have been a few studies of supply chain coordination with transshipment. In a single-echelon supply chain, Rudi et al. [3] construct coordinating transshipment prices for two independent retailers. Hezarkhani and Kubiak [57] study the coordination of a single-echelon supply chain with two companies and develop an "implicit pricing mechanism" to determine transshipment prices as a function of the production quantity of the two companies. The authors show that given an optimal production quantity, there could be multiple values for transshipment prices that coordinate the supply chain, and the set of such transshipment prices is never empty.

In a multi-echelon supply chain, Dong et al. [58] propose the "transshipment" contract for a decentralized supply chain with a soft drink manufacturer and two bottlers: a national and regional bottlers. The optimal order quantity of the regional bottler is exogenously given, equivalent to that of a newsvendor. The authors consider one-way transshipment from the national bottler to the regional bottler. The national bottler offers the regional bottler the transshipment contract, which consists of the total payment including profit without transshipment and fixed bonus for transshipment, and inventory sharing ratios for transshipment. The manufacturer then offers an "incentive" contract to the national bottler in both ex-ante and expost perspectives. The authors show that the optimal order quantity is decreasing in transshipment cost, and when the transshipment cost is high, the transshipment opportunity tends to be limited. In addition, there exists a situation, under which transshipment is not a worthwhile collaboration option. Although the authors successfully coordinate their supply chain, their transshipment mechanism is limited. In addition, unlike our study, the authors do not use a pricing mechanism to coordinate a supply chain.

Li et al. [59] study supply chain coordination with transshipment in a single-supplier, multiple-symmetric-retailers supply chain in two time periods, where both demand and supply opportunities exist in both periods. At the beginning of the second period, the retailers can request for transshipment, buy more inventory from the supplier, and/or sell inventory back to the supplier to adjust their inventories. The authors identify two misalignments: horizontal incentive conflict, which is caused by the difference in marginal values of inventory, and (vertical) double marginalization, which is caused by the difference in profit margins. The contract uses a lump sum to guarantee full participation of the retailers, and the transshipment and buy-back prices are dependent on the system on-hand beginning inventory in the second period. The authors assume a retailer is not allowed to keep inventory at the end of the first period. Hence, this model can be decomposed into two newsvendor problems, where in each time period each individual retailer order quantity is at the newsvendor critical fractile. The coordination mechanism in the first period is purely a wholesale price contract. Whereas, the coordination mechanism in the second period is more interesting, by which retailers with existing inventory becomes suppliers, providing transshipment to retailers who need more inventory to begin with in the second time period. Although the coordination is achieved in this work, the supply chain setting is drastically different from ours. Because the model can be decomposed into two newsvendor problems, the existence and impacts of transshipment are significantly reduced.

Even though a lot of research has studied transshipment problems, and a few of them have successfully coordinated a supply chain, supply chain coordination in a supply chain studied in this dissertation has never been achieved.

2.3 Supply Chain with Sales Effort

Research on sales effort has appeared in marketing and management science literatures in many different appearances. For example, the manufacturer can improve product quality or extend a shelf-life of a product (see, e.g., [60, 61, 62]). Whereas, the retailer can offer a promotion or discount (see, e.g., [4]), expand shelf-space, invest in effective advertising, enhance demand forecasting (see, e.g., [5, 6]), provide better after-sale service or better educate customers about a product (see, e.g., [7]).

Taylor [11] studies a supply chain with a supplier and a retailer who can exert a sales effort to multiplicatively stipulate the demand. The author shows that supply chain coordination is not achievable under a target rebate-only contract, and then establishes a coordinating contract by utilizing a target rebate contract and a buy-back contract. For the same supply chain setting, Krishnan et al. [4] study a promotional effort, which can be exerted after observing demand, and the sales effort cost is both effort and observed demand dependent. They show that a buy-back-only contract cannot coordinate the supply chain, hence proposing three coordinating contracts: an effort sharing contract, a markdown allowance contract and a constrained buy-buy contract. In addition, He et al. [15] consider the similar supply chain, where a demand distribution is conditional on effort and a retail price. The manufacturer specifies contract terms, and the retailer makes a decision on order quantity, an effort level and a retail price. The authors study a revenue-sharing contract, a buy-back contract, and a sales rebate and penalty (SRP) contract, and show that only an integration of a buy-back contract, and a SRP contract can coordinate the supply chain.

A supply chain with sales effort also appears in different applications. Ferguson et al. [7] study a false failure return problem in a supply chain with a manufacturer and a retailer. To coordinate the supply chain, the authors use a target rebate contract, under which the retailer receives a rebate for every unit below the return target level, to encourage the retailer to exert more effort to reduce the number of the false failure return. Dahai and Liu [12] analyze the effect of free riding in the supply chain with a manufacturer, an online retailer, and a traditional retailer who exerts sales effort. The authors establish a coordinating contract, which allows the manufacturer to offer a selective rebate contract when the traditional retailer guarantees price match. Taylor and Xiao [5] study the effectiveness of a rebate contract and a buy-back contract in a supply chain with a manufacturer and a retailer who can exert forecasting effort with a costly cost of exertion. They show that the rebate contract is more effective in encouraging forecasting effort than the return contract. However, when demand is dependent of sales effort or a retail price, the return is much more effective. Shin and Tunca [6] study supply chain coordination of retailers with demand forecast investments, and analyze the impacts of supply contracts on forecast investments in both observable and unobservable cases.

Mukhopadhyay et al. [63] study a supply chain with mixed channels in a symmetric and asymmetric information cases, where customer demand is price and effort dependent. For multiple time periods, Chu and Desai [62] study the supply chain, in which both the manufacturer and retailer can exert efforts to induce customer demand. Heese and Swaminathan [64] study the inventory and sales effort problem with unobservable lost sales in multiple time periods using Bayesian process to update the future demand.

For deterministic demand Ma et al. [65] investigate a supply chain, where a manufacturer can invest in a quality improvement, and a retailer can exert a marketing effort to amplify customer demand. To coordinate a supply chain, the authors use a two-part tariff contract, and allow investment costs of both efforts to be shared. Li and Liu [66] study supply chain coordination for a supply chain with a supplier and a retailer facing deterministic, price-effort-dependent demand, and show that a two-part tariff contract can achieve supply chain coordination.

In marketing, Lariviere and Padmanabhan [60] study the problem of slotting allowance in new product introduction by allowing a retailer to exert merchandising effort and decide a retail price to influence customer demand. Desiraju [67] examines the relative value of the uniform and the brand-by-brand methods of setting slotting fees and determines the preferred method of setting slotting allowances.

Even if supply chain coordination in a supply chain with sales effort has been extensively studied, existing coordination mechanisms are evolved from a buy-back contract or lost-sale penalty or a combination of both. These two mechanisms have the practical disadvantages (previously discussed in Chapter 1); thus, there is a need to develop a different coordinating mechanism.

2.4 Summary of the Literature Review

Taylor [11], Krishnan et al. [4] and He et al. [15] serve as a motivation and foundation of our work in Chapter 4. The reviews of them can be found in Section 2.3. As opposed to a buy-back contract or lost sale penalty, an advance-purchase contract provides practical advantages. For example, some products when returned may require lengthly and/or costly processes or become worthless, and therefore executing a return may not be worthwhile and desirable by both players. In addition, a lost sale is difficult to detect, especially with today's advanced e-commerce technology, where a product may be out of stock, and a lost sale is never reported. To make both schemes work seamlessly, the supply chain may result in overinvesting; thus, reducing supply chain efficiency.

We summarize the most relevant literatures to the study in Chapter 4 in Table 2.1.

Table 2.1: Summary of Relevant Literatures to the study in Chapter 4

Papers	**Topics**				
	Rebate Contract	Target Rebate Contract	Buy-Back Contract	Penalty Mechanism	Adv.-Pur. Contract
Taylor [11]		✓	✓	✓	
Krishnan et al. [4]			✓		
He et al. [15]	✓		✓	✓	
This dissertation		✓			✓

Although these papers successfully establish coordinating contracts in a supply chain with sales effort, our coordinating contract is different from those by utilizing an advance-purchase contract in conjunction with a target rebate contract.

In addition, Rudi et al. [3], Dong and Rudi [68], Zhao and Atkins [69] and Shao et al. [70] serve as a motivation and foundation of our work in Chapter 5. Dong and Rudi [68] study the supply chain with a manufacturer and n retailers in a single time period. The authors analyze the impacts of transshipment from both manufacturer's and retailers' perspectives when a wholesale price is exogenously given and endogenously determined by the manufacturer. For the exogenous wholesale price, the retailers always benefit from employing transshipment through the effect of risk pooling. For the endogenous wholesale price, the authors show that the retailers are much less sensitive to the wholesale price and the number of participating retailers. Hence, the manufacturer benefits from setting the higher wholesale price, although this makes the retailers worse off. Zhang [71] generalizes Dong and Rudi [68] to general demand distributions, and further examines their key results by demonstrating that the transshipment problem is equivalent to a newsvendor problem with adjusted demand. Then, the impact of transshipment can be examined by analyzing the relationship between the adjusted demand and the demand of a newsvendor without transshipment. The author summarizes that while keeping the mean of demand constant, transshipment reduces the demand variability, resulting in the retailers' optimal order quantity being closer to the demand mean.

Zhao and Atkins [69] study the substitution game and transshipment game in a supply chain with two retailers, who face stochastic demand, and compete in their retail prices to influence the demand. The retailers make a decision on their retial price and order quantity. When customers are not able to find the product they need at a particular retailer, they either switch to another retailer who carries a substitutable product (in the substitution game), or wait at the same retailer until transshipment from the other retailer arrives (in the transshipment game). The authors show that under the transshipment game, the retail price and safety stock increase in the transshipment price. Compared to a substitution game, a transshipment game is more beneficial when transshipment is expensive and a competition level is low. Also, if a transshipment price is endogenously set by the retailers, there is an optimal transshipment price that maximizes the retailer's profit, not the supply chain's profit.

Shao et al. [70] examine transshipment incentives in a supply chain with a manufacturer, and independent retailers or a chain store, i.e., centralized retailers. The authors find that the retailer's order quantity under transshipment is increasing in the transshipment price. In addition, when the manufacturer controls the transshipment prices, he/she always sets them as high as possible, and thus the manufacturer's profit increases, and the retailers' profit can be lower with transshipment. When retailers jointly make transshipment price decision, the retailers benefits from transshipment, and the manufacturer can be harmful. Additionally, the results and findings hold in a case of two asymmetric retailers.

We summarize the most relevant literatures to the study in Chapter 5 in Table 2.2.

Table 2.2: Summary of Relevant Literatures to the study in Chapter 5

Papers	**Topics**		
	Advance-Purchase Contract	Transshipment Mechanism	Transshipment Coordination[1]
Rudi et al. [3]		✓	✓
Cachon [13]	✓		
Dong and Rudi [68]		✓	
Dong and Zhu [26]	✓		
Zhao and Atkins [69]		✓	
Shao et al. [70]		✓	
This dissertation	✓	✓	✓

Although there have been a number of studies on transshipment problems, none of them has developed a coordinating contract that achieves supply chain coordination in a single-period, multi-echelon supply chain. To the best of our knowledge, this dissertation is the first study, successfully developing a coordinating contract for such a supply chain.

[1]The coordination is established in a supply chain setting different from this work.

CHAPTER 3

PROBLEM STATEMENT, RESEARCH OBJECTIVES AND CONTRIBUTIONS

This dissertation studies impacts of an advance-purchase contract and supply chain coordination in a supply chain with sales effort (Chapter 4), and in a supply chain with transshipment (Chapter 5). In the following, we describe the research objectives and contributions, specific to each study.

3.1 Supply Chain with Sales Effort

3.1.1 Problem Statement

There are two major research problems in this study. The first is to study the advance-purchase contract on the supply chain with sales effort. With today's advanced manufacturing technology, expediting production or acquiring on-demand capacity to fulfill an "emergency" order is very efficient and effective. Our advance-purchase contract takes advantage of that by allowing the manufacturer to satisfy the retailer's second order with a negligible lead-time. In addition, an advance-purchase contract allows the manufacturer to observe the demand partially in advance, thus providing better understanding of the demand and an adequate amount of time for production. Therefore, it is worthwhile to consider this contract when developing a mechanism to coordinate the supply chain.

Literature focuses on different mechanisms to coordinate retailer's decisions in this supply chain. A number of existing coordinating mechanisms utilize a buy-back contract or a penalty mechanism. However, there are associated practical drawbacks.

Thus, the second research problem is to replace a buy-back contract or lost-sale penalty with the advance-purchase contract, and then combine that with a target rebate contract to coordinate the supply chain.

3.1.2 Research Objectives

Taylor [11] shows that the target rebate contract alone cannot coordinate the supply chain with sales effort, but a target rebate contract and a buy-back contract combined can. In practice, neither the manufacturer nor the retailer would want to spend an extra effort on additional activities for completing a return process. Such activities include, but are not limited to, administering, handling, transporting, and recycling unsold products. In this research, we address this issue by establishing a new contractual mechanism, which replaces a buy-back contract in [11] with an advance-purchase contract. In the following, we describe the objectives of this research study.

Objective 1: Examine the interactions between a target rebate contract and an advance-purchase contract.

Under a target rebate contract and an advance-purchase contract, there are two opposite pulling directions on retailer's optimal order quantity. The target rebate contract increases the retailer's optimal order size, whereas the advance-purchase contract lowers the optimal order quantity of the supply chain. Hence, it is unclear how these two contracts interact via the setting of their contract terms.

Objective 2: Determine whether or not the integration of a target rebate contract and an advance-purchase contract can achieve supply chain coordination with arbitrary profit split.

By combining the advance-purchase contract with the target rebate contract, we analytically and numerically demonstrate that supply chain coordination with arbitrary profit split is achievable. This is necessary for a contract to result in a Pareto improvement. A Pareto improvement is the scenario in which no player is worse off, and one player is strictly better off.

Objective 3: Quantity the improvements and compare the performance the proposed contract to that in a decentralized supply chain and a newsvendor supply chain.

To quantify the improvements, we would like to both analytically and numerically compare the performance of our proposed contract to that of the target rebate-only contract, and that of no contract. Besides, we also would like to analytically and numerically compare the values of the decision variables, i.e., the retailer's optimal order quantities and optimal effort level, and the optimal profits under different scenarios.

3.1.3 Contributions

Although a lot of research focuses on a problem in a supply chain with sales effort and successfully establishes a coordinating contract, they utilize a buy-back contract or lost-sale penalty, which associates with practical drawbacks as described earlier. In contrast, this study utilizes a target rebate contract and an advance-purchase contract to coordinate the order quantity and sales effort. Not only does this research contribute to the existing literature, but also it provides managerial insights on how to adopt these two contracts to improve supply chain's performance. The followings are the unique characteristics of our model framework studied in this research.

1. None of the previous studies on a supply chain with sales effort allows a retailer to place an order at a manufacturer twice. This work incorporates this initiative by allowing the retailer to place the second order to satisfy unmet demand as a result of an inventory shortage.

2. This study provides the coordinating mechanism that allows arbitrary profit

split via the combination of the target rebate contract and the advance-purchase contract.

3. Managerial insights will help explain practical influences and impacts of the contract terms, and strategic behaviors of the players in this setting.

3.2 Supply Chain with Transshipment

3.2.1 Problem Statement

Transshipment is widely used in industries in which great customer satisfaction is required. In the retail industry, "lateral" transshipment allows two retailers to transship or resell products between them for an "emergency" supply. This allows the retailers to improve customer satisfaction and/or to lower safety-stock quantity. Transshipment has two opposite impacts on retailers' optimal inventory. On one hand, it lowers the optimal inventory because it additionally provides a retailer a late option to satisfy demand. On the other hand, it increases the optimal inventory as a retailer sees an opportunity in a resale. Of course, these impacts are influenced by who has control on the transshipment price. Literature shows when the wholesale price is endogenously determined by a manufacturer, he can be harmful if retailers have control on the transshipment price. In contrast, a manufacturer benefits from transshipment, but a retailer is harmful by transshipment when the manufacturer has control on the transshipment price. The outcomes are unclear. Thus, there is a need for a remedy, which addresses this ambiguity and results in a Pareto improvement.

In addition, for the same reason mentioned in the problem statement of the first study, it is worthwhile to adopt the advance-purchase contract to coordinate this supply chain. Thus, the research problem of this study is to develop a coordinating mechanism that embeds the advance-purchase contract into the framework of the supply chain with transshipment, and results in a Pareto improvement.

3.2.2 Research Objectives

It has been shown that the transshipment mechanism alone does not coordinate the supply chain. In addition, their does not exist a contractual mechanism that coordinates the supply chain with transshipment. This research incorporates an advance-purchase contract to achieve supply chain coordination. Specifically, we would like to investigate how the manufacturer optimally makes pricing decisions on the transshipment prices and the wholesale prices. In the following, we elaborate the objectives to be accomplished in this research study.

Objective 1: Define the interactions between transshipment and an advance-purchase contract.

There are two misalignments in a supply chain with transshipment. One is the horizontal misalignment, where a retailer makes profit from selling products to another retailer through transshipment. The other is the vertical misalignment, where a manufacturer gains profit from satisfying retailers' orders. The advance-purchase contract helps re-align a vertical incentive, thus alleviating the impacts of the vertical misalignment, but does not help on the horizontal misalignment. When embedded with transshipment, how would such a contract help re-align both the vertical and the horizontal incentives, and what are the associated impacts?

Objective 2: Determine whether or not an advance-purchase contract and transshipment can achieve supply chain coordination with arbitrary profit split.

With these two individual mechanisms combined, we analytically and numerically show that supply chain coordination with arbitrary profit split is attainable. This is necessary for a contract to result in a Pareto improvement.

Objective 3: Quantity the improvements and compare the performance the proposed contract to that in a decentralized supply chain and a newsvendor supply chain.

To quantify the improvements, we would like to both analytically and numerically compare the performance of our proposed contract to that of the transshipment-only agreement, and that of no contract. Besides, we also would like to analytically and numerically compare the retailer's optimal order quantities and the optimal profits under different scenarios.

3.2.3 Contributions

To achieve the objectives above, we develop the framework that represents the transshipment game in the multi-echelon supply chain, and utilize Game Theory to formulate the model and analyze the game. This research study provides contributions to the existing domain knowledge from both theoretical and practical standpoints. The followings are the unique characteristics of our model framework in this research.

1. Because a retailer would charge as high as possible for providing transshipment if the retailer has control on the transshipment price, this research establishes a new contract, which allows the manufacturer to set a priori and include the transshipment prices as part of the contract.

2. None of the previous studies allows retailers to have the second order placed to a manufacturer when allowing transshipment. This work supports that idea by having two wholesale prices, the lower one for the first order and the higher one for the second order.

3. Besides setting the transshipment prices, the manufacturer also specifies the contract terms in the advance-purchase contract, i.e., the wholesale prices, as well as the transshipment premium and the profit split parameter.

4. Insights gained from this study will help explain realistic strategic behaviors of players within the newsvendor setting and transshipment.

CHAPTER 4

Order Quantity and Sales Effort Coordination with Advance-Purchase and Target Rebate Contract

In this chapter, we study a supply chain with a manufacturer and a retailer who faces a stochastic, effort-dependent demand for a single product in a single selling season. We design a contract with a target rebate and advance-purchase agreement to coordinate both ordering and effort decisions. The contract offers the retailer two ordering opportunities in conjunction with a target rebate to encourage the retailer to place a larger first order and exert more sales effort. With two ordering opportunities, the retailer places her first order to obtain her inventory, and after the demand is observed, places her second order to fulfill the unmet demand.

4.1 Model Formulation

4.1.1 Notations

The sequence of events is as follows. In Stage 1, the manufacturer determines the advance-purchase and target rebate contract parameters. In Stage 2, the retailer decides on her sales effort level e, and places her first order to the manufacturer at a wholesale price w_1. Subsequently, the manufacturer produces products with marginal cost c_1. In Stage 3, the retailer receives her inventory, and then observes and fulfills her demand at a retail price r. Also, the manufacturer provides a rebate u for every unit the retailer sells above the target level T from her inventory. In Stage 4, if there is unmet demand, the retailer places her second order to the manufacturer at a wholesale price w_2. Subsequently, the manufacturer immediately produces products with marginal cost c_2 to satisfy the retailer's order. Then the retailer receives her order and satisfies her remaining demand at a retail price r.

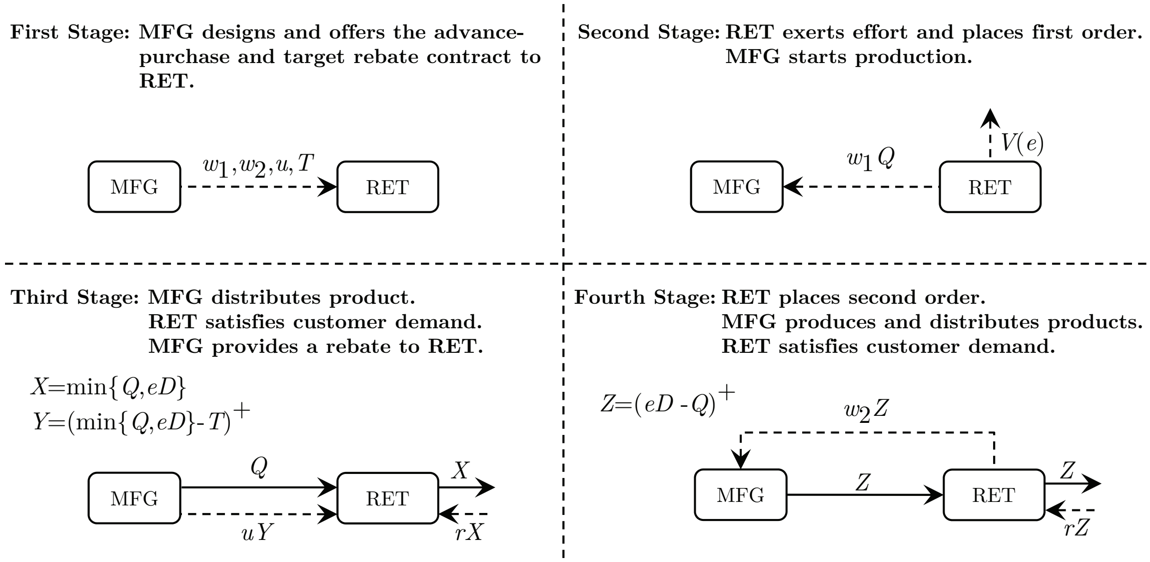

Figure 4.1: Operations and Financial Transactions in the Supply chain

Figure 4.1 summarizes the sequence of operations and financial transactions in the supply chain, where the solid lines are product flows and the dotted lines are financial flows.

We assume that r is exogenous. This assumption is justified when a retailer is a price taker as a result of a competitive market. Also, we assume $w_1 > c_1$ and $w_2 > c_2$ to ensure positive margins for the manufacturer. In addition, we assume $c_2 > c_1$ because the second production may require an expedition or acquisition of additional capacity, and therefore incur extra cost. For simplicity, we assume remaining inventory at the end of the selling season has zero value. Notice that under the advance-purchase contract all demand will always be satisfied at the end of the selling season.

Let D denote retailer's stochastic demand, which has a probability density function $\phi(\cdot)$, $\phi(D) > 0$ for all $D \geq 0$, and a cumulative distribution function $\Phi(\cdot)$, where continuity and differentiability are satisfied. The retailer can exert sales effort

e, $e \geq 1$, at a cost of $V(e)$ to influence demand in the multiplicative form, i.e., eD. We assume $V(e)$ is convex and increasing to model an increasing marginal cost of sales effort. This type of effort-demand model is also used in Taylor [11] and Krishnan et al. [4].

The summary of the notations used in this chapter is presented in Table 4.1.

Table 4.1: Notation Summary

Notation	**Description**
D	Retailer's stochastic demand with PDF $\phi(\cdot)$ and CDF $\Phi(\cdot)$
r	Unit retail price
c_i	Unit production cost for manufacturer's i production opportunity, $i = \{1, 2\}$
w_i	Unit wholesale price for retailer's i ordering opportunity, $i = \{1, 2\}$
u	Unit rebate
T	Target level of a rebate
Q^C	Order quantity of a centralized supply chain
e^C	Effort level of a centralized supply chain
Π^C	Expected profit of a centralized supply chain
Q^A	Order quantity under an advance-purchase contract
e^A	Effort level under an advance-purchase contract
Π_i^A	Expected profit of i under an advance-purchase contract, $i = \{M, R\}$, denoting the manufacturer and retailer, respectively
Q^B	Order quantity under an advance-purchase and target rebate contract
e^B	Effort level under an advance-purchase and target rebate contract
Π_i^B	Expected profit of i under an advance-purchase and target rebate contract, $i = \{M, R\}$
Q^*	Optimal order quantity under an advance-purchase and target rebate contract
e^*	Optimal effort level under an advance-purchase and target rebate contract

4.1.2 Centralized Supply Chain

The centralized supply chain serves as the benchmark for comparisons of supply chain's performance and retailer's decisions under coordination in the contract. In the centralized supply chain, both the manufacturer and the retailer belong to the same company. Two production opportunities are available. The first opportunity is available in advance with marginal cost c_1, and the second opportunity becomes available after the demand is realized with marginal cost $c_2 > c_1$. Let $(x)^+$ denote $\max\{x, 0\}$. The expected profit of the supply chain is

$$\Pi^C = -c_1 Q + \mathbb{E}\left\{r \min\{Q, eD\} + (r - c_2)(eD - Q)^+\right\} - V(e), \tag{4.1}$$

where the first term is the production cost of the first order, the second term is the revenue of the first order, the third term is the profit of the second order, and the last term is cost of sales effort. Let $\Gamma(Q) = \int_0^Q D d\Phi(D)$. The optimal effort level is e^C which satisfies $(\partial/\partial e) V(e) = c_2 \Gamma\left(Q^C/e\right) + (r - c_2)\mathbb{E}\{D\}$, and the optimal order quantity is $Q^C = e\Phi^{-1}((c_2 - c_1)/c_2)$. Note that $\Phi^{-1}((c_2 - c_1)/c_2)$ is the optimal order quantity with the minimum sales effort. The derivation for such scenario can be found in Appendix A.

4.1.3 Decentralized Retailer under Advance-Purchase Contract

In the decentralized supply chain, the manufacturer and retailer are independent. Under the advance-purchase contract, the retailer has two opportunities to place her orders: the first order is placed in advance at a wholesale price w_1, and the second order is placed during the selling season at a wholesale price w_2. Corresponding to the two ordering opportunities of the retailer, the manufacturer has two production opportunities. Under the advance-purchase contract, the manufacturer's expected profit is

$$\Pi_M^A = (w_1 - c_1) Q + (w_2 - c_2)\mathbb{E}\left\{(eD - Q)^+\right\}, \tag{4.2}$$

where the manufacturer makes profit from satisfying retailer's first and second ordering opportunities, represented by the first and second terms, respectively.

The retailer's expected profit is

$$\Pi_R^A = -w_1 Q + \mathbb{E}\left\{r \min\{Q, eD\} + (r - w_2)(eD - Q)^+\right\} - V(e), \tag{4.3}$$

where the retailer pays a unit cost w_1 for her first order quantity Q, and sells them at the retail price r, and places her second order quantity if necessary. Additionally, the retailer pays the cost $V(e)$ to exert a sales effort e to amplify her demand. By using integrals, the retailer's expected profit can be expressed as follows:

$$\Pi_R^A = (r - w_1) Q - r \int_0^{Q/e} (Q - eD)\, d\Phi(D) + (r - w_2) \int_{Q/e}^{\infty} (eD - Q)\, d\Phi(D) - V(e). \tag{4.4}$$

Thus, the first order derivative with respect to Q is

$$(\partial/\partial Q)\, \Pi_R^A = r - w_1 - r\Phi(Q/e) - (r - w_2)\left[1 - \Phi(Q/e)\right], \tag{4.5}$$

$$= w_2 - w_1 - w_2 \Phi(Q/e). \tag{4.6}$$

The optimal effort level is e^A which satisfies $(\partial/\partial e)\, V(e) = w_2 \Gamma(Q^A/e) + (r - w_2)\mathbb{E}\{D\}$. The optimal order quantity is $Q^A = e\Phi^{-1}\left((w_2 - w_1)/w_2\right)$, where $\Phi^{-1}\left((w_2 - w_1)/w_2\right)$ is the retailer's optimal order quantity with the minimum sales effort. The derivation for such scenario can be found in Appendix A.

4.1.4 Decentralized Retailer under Advance-Purchase and Target Rebate Contract

In this section, we develop a new contract, called an advance-purchase and target rebate contract, which is the integration of the advance-purchase contract and the target rebate contract. This contract works similarly as the advance-purchase contract, except the manufacturer uses a rebate mechanism to provide an incentive to the retailer to increase her effort. Specifically, the retailer earns a rebate u for every unit she sells above the target level T.

Under the advance-purchase and target rebate contract, the retailer's expected profit is

$$\Pi_R^B = -w_1 Q + \mathbb{E}\left\{ r\min\{Q, eD\} + u\left(\min\{Q, eD\} - T\right)^+ + \left(r - w_2\right)\left(eD - Q\right)^+ \right\} - V\left(e\right). \tag{4.7}$$

To understand Eq. (4.7), the retailer pays $w_1 Q$ to obtain her inventory, and sells products at the retail price r, receives the rebate u from the manufacturer for every unit from her inventory sold above the target level T, and makes profit of $r - w_2$ for each unit in her second order quantity, respectively. In addition, the retailer is able to amplify her demand by exerting sales effort e at a cost of exertion $V(e)$ to maximize her expected profit.

The expected profit of the manufacturer is

$$\Pi_M^B = \left(w_1 - c_1\right) Q + \mathbb{E}\left\{ \left(w_2 - c_2\right)\left(eD - Q\right)^+ - u\left(\min\{Q, eD\} - T\right)^+ \right\}, \tag{4.8}$$

where he makes profit from satisfying retailer's first and second orders, and offering the retailer an incentive through a target rebate. In the following, we follow the two-step procedure to determine the characteristics of the retailer's optimal policies. In this procedure, we first determine the retailer's optimal ordering policy for any given sales effort level, and later use the result to determine the retailer's optimal policy on a sales effort. To facilitate the analysis, the retailer expected profit, shown in Eq. (4.7), is alternatively expressed as

$$\Pi_R^B = \begin{cases} \left(r - w_1\right) Q - r\int_0^{Q/e}\left(Q - eD\right) d\Phi\left(D\right) \\ \qquad\qquad + \left(r - w_2\right)\int_{Q/e}^{\infty}\left(eD - Q\right) d\Phi\left(D\right) - V(e), \quad \text{if } Q \le T; \\ \left(r - w_1\right) Q - r\int_0^{Q/e}\left(Q - eD\right) d\Phi\left(D\right) \\ +u\left(\int_{T/e}^{Q/e}\left(eD - T\right) d\Phi\left(D\right) + \left(Q - T\right)\left[1 - \Phi\left(Q/e\right)\right]\right) \\ \qquad\qquad + \left(r - w_2\right)\int_{Q/e}^{\infty}\left(eD - Q\right) d\Phi\left(D\right) - V(e), \quad \text{if } Q > T. \end{cases} \tag{4.9}$$

The retailer's expected profit in Eq. (4.9) is isolated into two conditions, "without rebate" and "with rebate", where a target rebate term exists in the latter, but does not exist in the former. Next, we begin to characterize retailer's optimal order quantity Q^* for any given e. The first and second order derivatives of the retailers's expected profit with respect to retailer's order quantity are

$$(\partial/\partial Q)\,\Pi_R^B = \begin{cases} r - w_1 - r\Phi\left(Q/e\right) - \left(r - w_2\right)\left[1 - \Phi\left(Q/e\right)\right], & \text{if } Q \leq T; \\ r - w_1 - r\Phi\left(Q/e\right) + u\left[1 - \Phi\left(Q/e\right)\right] & \\ \qquad - \left(r - w_2\right)\left[1 - \Phi\left(Q/e\right)\right], & \text{if } Q > T, \end{cases} \tag{4.10}$$

$$= \begin{cases} w_2 - w_1 - w_2\Phi\left(Q/e\right), & \text{if } Q \leq T; \\ w_2 + u - w_1 - \left(w_2 + u\right)\Phi\left(Q/e\right), & \text{if } Q > T, \end{cases} \tag{4.11}$$

and

$$\left(\partial^2/\partial Q^2\right)\Pi_R^B = \begin{cases} -w_2\phi\left(Q/e\right)1/e, & \text{if } Q \leq T; \\ -\left(w_2 + u\right)\phi\left(Q/e\right)1/e, & \text{if } Q > T. \end{cases} \tag{4.12}$$

Thus, Π_R^B is concave on $[0, T)$ and (T, ∞). Although π_R^B is continuous,

$$\lim_{Q \to T^-} (\partial/\partial Q)\,\Pi_R^B < \lim_{Q \to T^+} (\partial/\partial Q)\,\Pi_R^B. \tag{4.13}$$

Define $Q^B \equiv e\Phi^{-1}\left(\left(w_2 + u - w_1\right)/\left(w_2 + u\right)\right)$. Note that $Q^B > Q^A$. Also, define $f_1(T) \equiv \Pi_R^B\left(Q^A, e|T\right) - \Pi_R^B\left(Q^B, e|T\right)$ on $T \in \left[Q^A, Q^B\right]$, and define τ_1, which is the target level that makes the retailer indifferent between "going for" and "not going for" the rebate, i.e., $f_1(\tau_1) = 0$.

Lemma 4.1 *For any given e, the optimal order quantity for the retailer under an advance-purchase and target rebate contract, Q^*, is given by the following: If $\tau_1 < T$, then $Q^* = Q^A$; if $\tau_1 > T$, then $Q^* = Q^B$; if $\tau_1 = T$, then the retailer is indifferent between ordering Q^A and Q^B.*

Proof. It is straightforward to show that if $T \leq Q^A$, then Q^B maximizes $\Pi_R^B(\cdot, e|T)$, and if $T \geq Q^B$, then Q^A maximizes $\Pi_R^B(\cdot, e|T)$. If $Q^A \leq T \leq Q^B$, then $\Pi_R^B\left(Q^A, e|T\right) = ew_2\Gamma\left(Q^A/e\right) + e(r - w_2)\mathbb{E}\{D\} - V(e)$ and $\Pi_R^B\left(Q^B, e|T\right) = e(w_2 + u)\Gamma\left(Q^B/e\right) - u\left[e\Gamma(T/e) + T(1 - \Phi(T/e))\right] + e(r - w_2)\mathbb{E}\{D\} - V(e)$. Because $f_1\left(Q^A\right) < 0 < f_1\left(Q^B\right)$ and $f_1(\cdot)$ is continuous and increasing, there exists a single-valued inverse function f_1^{-1} and a unique τ_1; further, $\tau_1 \in \left(Q^A, Q^B\right)$. If $Q^A < T < Q^B$, then $\lim_{Q\to T^-} (\partial/\partial Q)\Pi_R^B(Q, e|T) < 0 < \lim_{Q\to T^+} (\partial/\partial Q)\Pi_R^B(Q, e|T)$. Because Q^A maximizes $\Pi_R^B(\cdot, e|T)$ on $[0, T)$ and Q^B maximizes $\Pi_R^B(\cdot, e|T)$ on $(T, \infty]$, $Q^* = \arg\max_{Q\in\{Q^A, Q^B\}} \Pi_R^B(Q, e|T)$. If $T < \tau_1$, then $f_1(T) < 0$ and $\Pi_R^B\left(Q^B, e|T\right) > \Pi_R^B\left(Q^A, e|T\right)$. If $T > \tau_1$, then $f_1(T) > 0$. ■

To understand the intuition behind Lemma 4.1, let's consider two extreme cases of T. When T is extremely low, e.g., equals to 0, the retailer gets a rebate of u for every unit that she sold. The retailer optimal order quantity can be derived with the margin of $r + u - w$. When T is extremely high, the retailer has to exert a considerable amount of sales effort to have a shot at getting the rebate. However, the cost of sales effort exceeds the expected revenue from the rebate. As a result, the retailer behaves as if the rebate does not exist. When T is intermediate, the retailer considers her marginal cost and revenue to decide whether or not she wants to go for the rebate. There is an optimal order quantity associated with each case. As T increases, the incentive from the rebate decreases, and when $T = \tau_1$, she is indifferent between placing the large and small orders.

Corollary 4.1 shows this threshold is the multiplication of an effort level e and a similar threshold under the no-sales-effort scenario. Let τ_0 be analogous to τ_1 when the retailer exerts minimum sales effort. The derivation and proof of existence and uniqueness for τ_0 can be found in Appendix A.

Corollary 4.1 $\tau_1 = e\tau_0$.

Proof. Because $Q^A = e\Phi^{-1}\left(\left(w_2 - w_1\right)/w_2\right)$ and $Q^B = e\Phi^{-1}\left(\left(w_2 + u - w_1\right)/\left(w_2 + u\right)\right)$, which are the multiplication of e and the retailer's optimal order quantity when sales effort is minimum in each respective case, $\tau_1 = e\tau_0$. ■

Lemma 4.1 and Corollary 4.1 suggest that the retailer's expected profit under the advance-purchase and target rebate contract can be expressed as a function of a single decision variable, a sales effort level e. Next, we characterize the optimal level of the sales effort. Let $Q^*(e)$ denote the retailer's optimal quantity given sales effort level e. Then,

$$\Pi_R^B(Q^*(e), e|T) = \begin{cases} w_2 e\Gamma\left(Q^A/e\right) + \left(r - w_2\right) e\mathbb{E}\left\{D\right\} - V(e), & \text{if } e \leq T/\tau_0; \\ \left(w_2 + u\right) e\Gamma\left(Q^B/e\right) - u\left(e\Gamma\left(T/e\right) + T\left[1 - \Phi\left(T/e\right)\right]\right) & \\ \qquad + \left(r - w_2\right) e\mathbb{E}\left\{D\right\} - V(e), & \text{if } e > T/\tau_0. \end{cases} \tag{4.14}$$

Thus, the first and second order derivatives with respect to e are

$$(\partial/\partial e)\,\Pi_R^B(Q^*(e), e|T) = \begin{cases} w_2\Gamma\left(Q^A/e\right) + \left(r - w_2\right)\mathbb{E}\left\{D\right\} - (\partial/\partial e)\,V(e), & \text{if } e \leq T/\tau_0; \\ \left(w_2 + u\right)\Gamma\left(Q^B/e\right) - u\Gamma\left(T/e\right) & \\ \qquad + \left(r - w_2\right)\mathbb{E}\left\{D\right\} - (\partial/\partial e)\,V(e), & \text{if } e > T/\tau_0, \end{cases} \tag{4.15}$$

and

$$\left(\partial^2/\partial e^2\right)\Pi_R^B(Q^*(e), e|T) = \begin{cases} -\left(\partial^2/\partial e^2\right)V(e), & \text{if } e \leq T/\tau_0; \\ e^{-3}uT^2\phi\left(T/e\right) - \left(\partial^2/\partial e^2\right)V(e), & \text{if } e > T/\tau_0. \end{cases} \tag{4.16}$$

Although $\Pi_R^B(Q^*(e), e|T)$ is continuous,

$$\lim_{e\to(T/\tau_0)^-}(\partial/\partial e)\,\Pi_R^B(Q^*(e), e|T) < \lim_{e\to(T/\tau_0)^+}(\partial/\partial e)\,\Pi_R^B(Q^*(e), e|T). \tag{4.17}$$

Therefore, T/τ_0 cannot be an optimal sales effort level, and $\Pi_R^B(Q^*(e), e|T)$ is not differentiable at T/τ_0.

To obtain further analytical results, we assume the demand follows the uniform distribution between 0 and 1, i.e., $D \sim$ Uniform(0,1), and the cost of sales effort, $V(e) = ae^2/2$, where $a > 0$. In the following, we show there exists a unique threshold Υ such that if the target level exceeds that threshold, then the retailer behaves as if the rebate does not exist, i.e., the optimal sales effort level and the optimal order quantity fall equal to those in the without-rebate case. If the target level is below the threshold, then the optimal effort level and the optimal order quantity are greater than those in the without-rebate case. If the target level equals the threshold, then the retailer is indifferent between the high and low effort-quantity pairs (see Corollary 4.2).

The analysis of retailer's optimal sales effort level proceeds in two steps. First, we show in Lemma 4.2 that the objective function, $\Pi_R^B(Q^*(\cdot), \cdot|T)$, is concave on $[0, T/\tau_0)$ and either convex and then concave or simply concave on $(T/\tau_0, \infty)$. Second, we use that result to specify the optimal effort level in Lemma 4.3.

Lemma 4.2 *If* $T < u\tau_0^3/a$, *then* $\Pi_R^B(Q^*(e), e|T)$ *is concave in* $[0, T/\tau_0)$ *and* $([uT^2/a]^{1/3}, \infty)$ *and convex in* $(T/\tau_0, [uT^2/a]^{1/3})$; *if* $T \geq u\tau_0^3/a$, *then* $\Pi_R^B(Q^*(e), e|T)$ *is concave in* $[0, T/\tau_0)$ *and* $(T/\tau_0, \infty)$.

Proof. For $e \in [0, T/\tau_0)$, $(\partial^2/\partial e^2)\, \Pi_R^B(Q^*(e), e|T) = -a < 0$. For $e \in (T/\tau_0, \infty)$, $(\partial^2/\partial e^2)\, \Pi_R^B(Q^*(e), e|T) = uT^2e^{-3} - a$. If $T \geq u\tau_0^3/a$, then $uT^2e^{-3} - a < u\tau_0^3/T - a \leq 0$. Suppose $T < u\tau_0^3/a$. Note $uT^2e^{-3} - a > 0$ if and only if $e < (uT^2/a)^{1/3}$. ■

A consequence of Lemma 4.2 is that $\lim_{e\to\infty} \Pi_R^B(Q^*(e), e|T) = -\infty$, and $\Pi_R^B(Q^*(e), e|T)$ has one maximizer in $[0, T/\tau_0]$ and at most one maximizer in $(T/\tau_0, \infty)$. Denote the maximizer on $[0, T/\tau_0]$ by $\tilde{e}$. Let e^B be the maximizer on $(T/\tau_0, \infty)$, if it exists; let $e^B = T/\tau_0$ if no such maximizer exists. Note $\tilde{e} = \min\{e^A, T/\tau_0\}$. Define $j(T) \equiv \Pi_R^B(Q^*(\tilde{e}), \tilde{e}|T) - \Pi_R^B(Q^*(e^B), e^B|T)$ on $[\tau_0 e^A, \infty)$ to represent the difference in the retailer's profit when exerting the optimal sales effort level of the supply chains with and without a rebate. In addition, define Υ to satisfy $j(\Upsilon) = 0$, i.e., the target level threshold that makes the retailer indifferent between exerting the optimal effort level of the supply chains with and without a rebate.

Lemma 4.3 *Suppose $D \sim$ Uniform(0,1). There exists a unique Υ such that if $T < \Upsilon$, then $e^* = e^B$ and further $e^B > T/\tau_0$; if $T > \Upsilon$, then $e^* = e^A$ and further $e^A < T/\tau_0$; if $T = \Upsilon$, then the optimal sales effort level is indifferent between e^A and e^B and further $e^A < T/\tau_0 < e^B$.*

Proof. It can be verified that $\Pi_R^B(Q^*(e^B), e^B|\cdot)$ is decreasing, and $\Pi_R^B(Q^*(\tilde{e}), \tilde{e}|\cdot)$ is weakly increasing. Therefore, $j(\cdot)$ is increasing. If $T \leq \tau_0 e^A$, then $0 \leq \lim_{e\to(T/\tau_0)^-} (\partial/\partial e)\Pi_R^B(Q^*(e), e|T) < \lim_{e\to(T/\tau_0)^+}(\partial/\partial e)\Pi_R^B(Q^*(e), e|T)$. Because $\Pi_R^B(Q^*(\cdot), \cdot|T)$ is concave on $[0, T/\tau_0)$ and $\lim_{e\to(T/\tau_0)^-} (\partial/\partial e)\Pi_R^B(Q^*(e), e|T) \geq 0$, $(\partial/\partial e)\Pi_R^B(Q^*(e), e|T)|_{e\in[0,T/\tau_0)} > 0$. Because $\lim_{e\to(T/\tau_0)^+}(\partial/\partial e)\Pi_R^B(Q^*(e), e|T) > 0$, $\Pi_R^B(Q^*(\cdot), \cdot|T)$ has one stationary point on $(T/\tau_0, \infty)$, and the second-order condition is satisfied at that point. Therefore, $\Pi_R^B(Q^*(e^B), e^B|\cdot) > \Pi_R^B(Q^*(\tilde{e}), \tilde{e}|\cdot)$ and $j(\tau_0 e^A) < 0$. Clearly, $\lim_{T\to\infty} \Pi_R^B(Q^*(\tilde{e}), \tilde{e}|T) = \lim_{T\to\infty} \Pi_R^B(Q^*(e^A), e^A|T) = \Pi_R^A(Q^A, e^A) < \infty$. It can be verified that $j(\cdot)$ is continuous, and $\lim_{T\to\infty} \Pi_R^B(Q^*(e^B), e^B|T) = -\infty$; thus, $\lim_{T\to\infty} j(T) = +\infty$. Because $j(\cdot)$ is continuous and increasing, $j(\tau_0 e^A) < 0$, and $\lim_{T\to\infty} j(T) = +\infty$, there exists a single-valued inverse function j^{-1} and a unique Υ; further $\Upsilon \in (\tau_0 e^A, \infty)$. Recall $e = T/\tau_0$ cannot be the optimal effort level. Thus, if $T < \Upsilon$, then $\Pi_R^B(Q^*(e^B), e^B|T) > \Pi_R^B(Q^*(\tilde{e}), \tilde{e}|T)$ and $e^* = e^B > T/\tau_0$; if $T > \Upsilon$, then $\Pi_R^B(Q^*(e^B), e^B|T) < \Pi_R^B(Q^*(\tilde{e}), \tilde{e}|T)$ and $e^* = e^A < T/\tau_0$; if $T = \Upsilon$, then $\Pi_R^B(Q^*(e^B), e^B|T) = \Pi_R^B(Q^*(\tilde{e}), \tilde{e}|T)$ and $e^* = e^B$ or e^A. ∎

The results from Lemmas 4.1 and 4.3 directly yield the result in Corollary 4.2, which specifies the retailer's optimal order quantity and sales effort.

Corollary 4.2 *Suppose $D \sim$ Uniform(0,1). If $T < \Upsilon$, then $e^* = e^B$ and $Q^* = Q^B$; if $T > \Upsilon$, then $e^* = e^A$ and $Q^* = Q^A$; if $T = \Upsilon$, then the retailer is indifferent between*

(e^A, Q^A) and (e^B, Q^B).

Corollary 4.2 shows that there exists the threshold for T, above which the retailer's optimal decisions in the "without-rebate" case are optimal, and under which the retailer's optimal decisions in the "with-rebate" case are optimal. The intuition is the retailer behaves as if the rebate does not exist when the target level is high, and takes an incentive of a rebate into consideration when the target level is low.

4.2 Supply Chain Coordination

In this section, we design the contract to coordinate the ordering and sales effort decisions. Under the advance-purchase and target rebate contract $(w_1, u(T), w_2(T), T)$, let $\bar{L}(T)$ be the retailer's expected profit when she exerts effort e^C where $e^C > T/\tau_0$, and let $\underline{L}(T, w_1)$ be the retailer's expected profit when she exerts effort e^A where $e^A < T/\tau_0$.

Theorem 4.1 *For any $\kappa \in (0, \Pi^C(Q^C, e^C))$, under the advance-purchase and target rebate contract $(w_1,\ u(T),\ w_2(T),\ T)$, the supply chain coordination can be achieved when $u = u(T^*)$; $w_2 = w_2(T^*)$; T^*, and w_1^* are set such that $\bar{L}(T^*, w_1^*) = \underline{L}(T^*, w_1^*) + \epsilon = \kappa$, and $w_1^* \leq w_2$, where ϵ is sufficient small, and*

$$u(T) = \frac{(c_1 - 2c_2)(c_1 - w_1)\Delta^2}{c_2\left[\Delta^2 - \zeta(T)\right]}, \tag{4.18}$$

$$w_2(T) = \frac{w_1\left[((c_1 - c_2)\Delta)^2 - c_2^2\zeta(T)\right] - c_1^2(c_1 - 2c_2)\Delta^2}{c_1c_2\left[\Delta^2 - \zeta(T)\right]}, \tag{4.19}$$

where $\Delta = (c_1^2 - 2c_1c_2 + c_2r)$, and $\zeta(T) = 4a^2c_2^2T^2$.

Proof. Define $y(T) = \underline{L}(T) - \bar{L}(T)$, and $y(T^*) = 0$. It can be verified that $(\partial/\partial w_1)$ $\underline{L}(T, w_1) < 0$, $\underline{L}(T, c_1) = \Pi^C$, and $\underline{L}(T, \cdot)$ is continuous. Hence, for a given T there exists a single-valued inverse function with respect to w_1, $\underline{L}^{-1}$, and a unique $w_1^*(T)$ such that $\underline{L}(T, w_1^*(T)) = \kappa - \epsilon$.

Define $T_1 = e^A\tau_0$ and $T_2 = e^C\tau_0$. Because $e^A < e^C$, $T_1 < T_2$. For $T < T_2$, we have $e^C > T/\tau_0$, and hence $(\partial/\partial e)\Pi_R^B(Q^*(e), e|T)|_{e=e^C} = 0$ because $u^* = u(T)$ and $w_2^* = w_2(T)$. For $T > T_1$, we have $e^A < T/\tau_0$, and $\Pi_R^B(Q^*(\tilde{e}), \tilde{e}|T) = \underline{L}(T)$. Because $\lim_{e\to e^{A+}}(\partial/\partial e)\Pi_R^B(Q^*(e), e|T_1) > 0$, $\Pi_R^B(Q^*(\cdot), \cdot|T)$ is convex and then concave or simply concave, and $e^C > T_1/\tau_0(T_1)$, $\Pi_R^B(Q^*(e^B), e^B|T_1) = \bar{L}(T_1)$, and $\bar{L}(T_1) > \underline{L}(T_1)$; hence $y(T_1) < 0$. Because $\lim_{e\to e^{C-}}(\partial/\partial e)\Pi_R^B(Q^*(e), e|T_2) < \lim_{e\to e^{C+}}(\partial/\partial e)\Pi_R^B(Q^*(e), e|T_2) = 0$, thus $\Pi_R^B(Q^*(\tilde{e}), \tilde{e}|T_2) = \underline{L}(T_2) > \bar{L}(T_2)$; hence $y(T_2) > 0$. Because $y(T_1) < 0 < y(T_2)$ and $y(\cdot)$ is continuous, there exists T^* and $e^A\tau_0(T^*) < T^* < e^C\tau_0(T^*)$. Because $T^* < e^C\tau_0$, e^C is a stationary point of $\Pi_R^B(Q^*(\cdot), \cdot|T^*)$ on $(T^*/\tau_0(T^*), \infty)$. Because $\Pi_R^B(Q^*(\cdot), \cdot|T^*)$ may be convex and then concave, there may be a second stationary point on $(T^*/\tau_0(T^*), \infty)$, denoted by e_0 if existing. This is true only if $(\partial/\partial e)\Pi_R^B(Q^*(e), e|T^*)|_{e=(T^*/\tau_0(T^*))^+} < 0$. Then, $\Pi_R^B(Q^*(e_0), e_0|T^*) < \Pi_R^B(Q^*(\tilde{e}), \tilde{e}|T^*) = \Pi_R^B(Q^*(e^A), e^A|T^*)$. Because $\Pi_R^B(Q^*(e^C), e^C|T^*) = \bar{L}(T^*) > \underline{L}(T^*) = \Pi_R^B(Q^*(e^A), e^A|T^*) > \Pi_R^B(Q^*(e_0), e_0|T^*)$, $e^C = \arg\max \Pi_R^B(Q^*(e), e|T^*)$.

Because $e^C > T^*/\tau_0(T^*)$, $Q^* = Q^B$ (by Lemma 4.1). Because $u^* = u(T^*)$ and $w_2^* = w_2(T^*)$, $Q^B = Q^C$, and thus $Q^* = Q^C$. ■

The supply chain achieves coordination when the retailer places the optimal order quantity and exerts the optimal sales effort. Under the advance-purchase and target rebate contract, her optimal decision depends on a target level T: when T is sufficiently high, she makes the decision as if the target rebate does not exist, i.e., ordering Q^A and exerting e^A, and when T is sufficiently low, she makes the decision considering an incentive provided under the target rebate, i.e., ordering Q^B and exerting e^B. The manufacturer ensures supply chain coordination and a Pareto improvement by setting $w_1 = w_1^*$ and $T = T^*$ because $u = u(T^*)$ in Eq. (4.18) and $w_2 = w_2(T^*)$ in Eq. (4.19), $Q^B = Q^C$ and $e^B = e^C$.

4.3 Sensitivity Analysis

Proposition 4.1 shows the impact of w_1 and T on a coordinating $u(T)$ and $w_2(T)$.

Proposition 4.1 *(a) $u(T)$ is increasing in T; and $u(T)$ is increasing in w_1 if $\Delta^2/\zeta(T) > 1$, and is decreasing in w_1 if $\Delta^2/\zeta(T) < 1$. (b) $w_2(T)$ is decreasing in T; and $w_2(T)$ is increasing in w_1 if $\Delta^2/\zeta(T) < 1$, and is decreasing in w_1 if $1 < \Delta^2/\zeta(T) < c_2^2/(c_2 - c_1)^2$.*

Proof. $\partial u/\partial w_1 = (2c_2 - c_1)^2\Delta^2/c_2(\Delta^2 - \zeta(T))$. If $\Delta^2/\zeta(T) > 1$, then $\partial u/\partial w_1 > 0$. Otherwise, $\partial u/\partial w_1 < 0$. $\partial u/\partial T = 8a^2c_2T(2c_2 - c_1)(w_1 - c_1)\Delta^2/(\Delta^2 - \zeta(T))^2 > 0$. $\partial w_2/\partial w_1 = (c_2 - c_1)^2\Delta^2 - c_2^2\zeta(T)/c_1c_2(\Delta^2 - \zeta(T))$. If $\Delta^2/\zeta(T) < 1$, then $(c_2 - c_1)^2\Delta^2 - c_2^2\zeta(T) < 0$ and $c_1c_2(\Delta^2 - \zeta(T)) < 0$. Hence, $\partial w_2/\partial w_1 > 0$. If $\Delta^2/\zeta(T) > c_2^2/(c_2 - c_1)^2$, then $\partial w_2/\partial w_1 > 0$. If $1 < \Delta^2/\zeta(T) < c_2^2/(c_2 - c_1)^2$, then $\partial w_2/\partial w_1 < 0$. $\partial w_2/\partial T = -8a^2c_2T(2c_2 - c_1)(w_1 - c_1)\Delta^2/(\Delta^2 - \zeta(T))^2 < 0$. ■

$\Delta^2/\zeta(T)$ can be considered as the value of T relative to the values of the market parameters. The results in Proposition 4.1 can be described as follows. For the impacts of T, to maintain the incentive level as T increases, u increases and w_2 decreases. The impacts of w_1 on u and w_2 are dependent of T. First, for a sufficiently high T such that $\Delta^2/\zeta(T) < 1$, the retailer makes decisions as if the target rebate contract does not exist. Thus, u is decreasing in w_1, and w_2 is increasing in w_1, where the former is to show the reduced impact of the rebate contract, and the latter is to offer the incentive to the retailer. Second, for a sufficiently low T, the retailer considers the incentives of both the target rebate contract and the advance-purchase contract when making her decisions. When $1 < \Delta^2/\zeta(T)$, u is increasing and w_2 is decreasing in w_1, to maintain the incentive level. Further, when T is relatively small such that $c_2^2/(c_2 - c_1)^2 < \Delta^2/\zeta(T)$, u is also small, and as a result the incentive of the target rebate is insufficient. In this case, w_2 is increasing in w_1 to supplement the incentive.

When $u(T)$ and $w_2(T)$ deviate from those given in Eqs. (4.18) and (4.19), respectively, the advance-purchase contract and target rebate does not achieve supply chain coordination, and their effects on the retailer's optimal decisions, i.e., e^* and Q^*, are summarized in Proposition 4.2. We include only the case when $T \neq \Upsilon$ because when $T = \Upsilon$ the optimal decision can take multiple values.

Proposition 4.2 *Suppose $D \sim$ Uniform(0,1), and $T \neq \Upsilon$. (a) e^* is constant in u if $T > \Upsilon$, and is increasing in u if $T < \Upsilon$. Q^* is constant in u if $T > \Upsilon$, and is increasing in u if $T < \Upsilon$. (b) e^* is decreasing in w_2. When $T > \Upsilon$, Q^* is increasing in w_2 if $2w_1^2 + rw_2 > 3w_1w_2$, and is decreasing in w_2 if $2w_1^2 + rw_2 < 3w_1w_2$. When $T < \Upsilon$, Q^* is increasing in w_2 if $e^B w1/(w_2+u)^2 + (w_2+u-w_1)/(w_2+u)\partial e^B/\partial w_2 > 0$, and is decreasing if $e^B w1/(w_2+u)^2 + (w_2+u-w_1)/(w_2+u)\partial e^B/\partial w_2 < 0$.*

Proof. If $T > \Upsilon$, then $e^* = e^A$ and $Q^* = Q^A$, which are independent of u. Hence, $\partial e^A/\partial u = 0$ and $\partial Q^A/\partial u = 0$. $\partial e^A/\partial w_2 = -w_1^2/(2aw_2^2) < 0$. $\partial Q^A/\partial w_2 = w_1(2w_1^2 + rw_2 - 3w_1w_2)/(2aw_2^3)$. If $T < \Upsilon$, then $e^* = e^B$. By Implicit Function Theorem, $\partial e^B/\partial u = -[(\partial^2/\partial e \partial u)A]/[(\partial^2/\partial^2 e)A]|_{e=e^B}$. Because $(\partial^2/\partial e\partial u)A > 0$, $\partial e^B/\partial u > 0$. Similarly, $\partial e^B/\partial w_2 = -[(\partial^2/\partial e\partial w_2)A]/[(\partial^2/\partial^2 e)A]|_{e=e^B}$. Because $(\partial^2/\partial e\partial w_2)A < 0$, $\partial e^B/\partial w_2 < 0$. If $T < \Upsilon$, then $Q^* = Q^B$. $\partial[(w_2+u-w_1)/(w_2+u)]/\partial u = \partial[(w_2+u-w_1)/(w_2+u)]/\partial w_2 = w_1/(w_2+u)^2 > 0$. Hence, $\partial Q^B/\partial u = e^B(w_1/(w_2+u)^2) + (w_2+u-w_1)/(w_2+u)\partial e^B/\partial u > 0$, and $\partial Q^B/\partial w_2 = e^B w_1/(w_2+u)^2 + (w_2+u-w_1)/(w_2+u)\partial e^B/\partial w_2$. ■

Intuitively, when the manufacturer provides larger u, the retailer exerts more sales effort and places a larger order. Then, when the manufacturer increases w_2, the retailer lowers her sales effort due to a decrease in profit margin in her second ordering opportunity. Also, when the target level is sufficiently high, the retailer relatively compares the retailer price to w_1 and w_2 to decide whether or not to decrease or increase her order quantity. When the target level is sufficiently low, the retailer consider the total effect to decide her order quantity. Two effects are pulling in two opposite directions. Note that e is decreasing in w_2, but Q^B is increasing in w_2.

4.4 Numerical Analysis

In this section, we numerically compare the performance of the coordinating contract to that in the newsvendor and the target rebate-only contract cases. Let's assume retailer's demand follows the uniform distribution between 0 and 200, and consider the following example to demonstrate numerical results.

Table 4.2: Parameters for the Numerical Example

		Coordination	Newsvendor	Target Rebate-Only
Exogenous parameters:	r	40	40	40
	c_1	10	-	-
	c_2	20	-	-
	c	-	10	10
	a	20	20	20
Decision variables:	w_1	21	-	-
	w_2	24.11	-	-
	u	17.89	-	-
	T	6965.50	-	-
	w^b	-	16	-
	w^d	-	-	16.65
	u^d	-	-	15
	T^d	-	-	10000

The model formulation of the supply chain under the target rebate-only contract can be found in Appendix A. For the centralization, we use the same values of market parameters under coordination. Note that "-" means the parameters or variables are not applicable. Also, the contract parameters under coordination satisfy the coordinating conditions in Theorem 4.1.

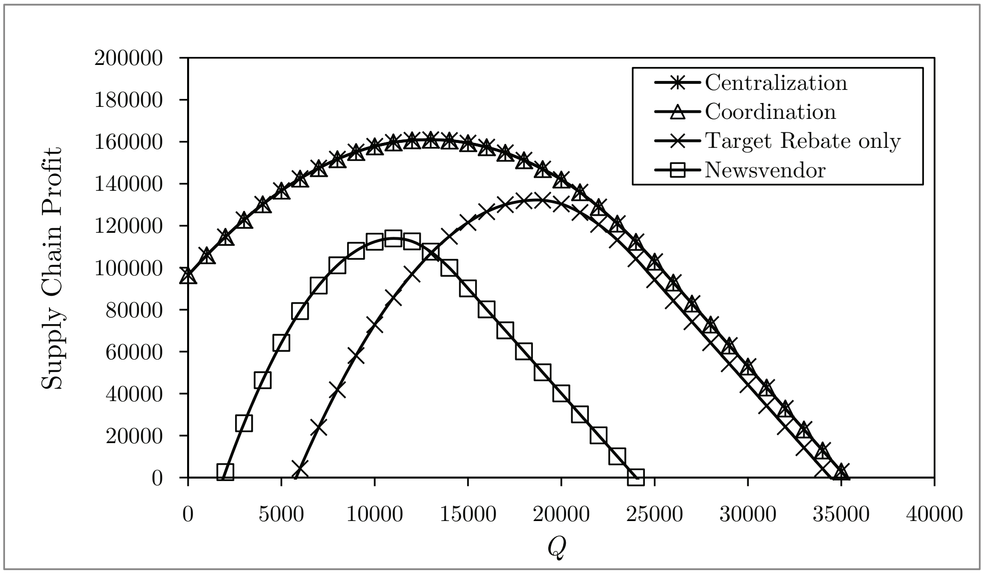

Figure 4.2: Profit Comparison

Figure 4.2 compares the total supply chain profits under four scenarios, i.e., centralization, coordination, target rebate-only contract and newsvendor, and reveals the following insights. Firstly, the total profit curves under coordination and under centralization overlap, and hence the retailer's optimal order quantity under both scenarios are identical. Secondly, the supply chain profit under coordination is always higher than those under target rebate-only contract and newsvendor. This describes the improvement from including the advance-purchase contract. Lastly, the optimal order quantity of the supply chain under coordination is lower than that under the target rebate-only contract and newsvendor. The intuition is that the second ordering opportunity inserts a profit margin when fulfilling the demand from the second order, and as a result lowers an incentive to satisfy the demand from inventory.

Figure 5.5 shows the supply chain profit, the manufacturer's profit and the retailer's profit under coordination.

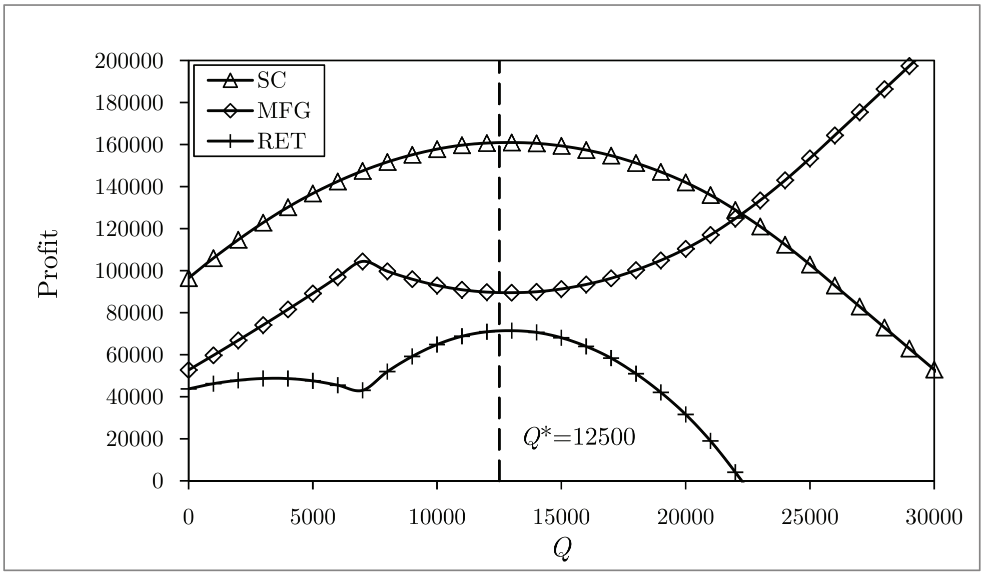

Figure 4.3: Profits under Coordination

Figure 5.5 shows the coordinating contract achieves supply chain coordination, i.e., the optimal order quantity of the retailer under coordination is identical to the optimal order quantity of the supply chain.

Figures 4.4 and 4.5 respectively show the retailer's and manufacturer's profits under coordination, the target rebate-only contract and newsvendor.

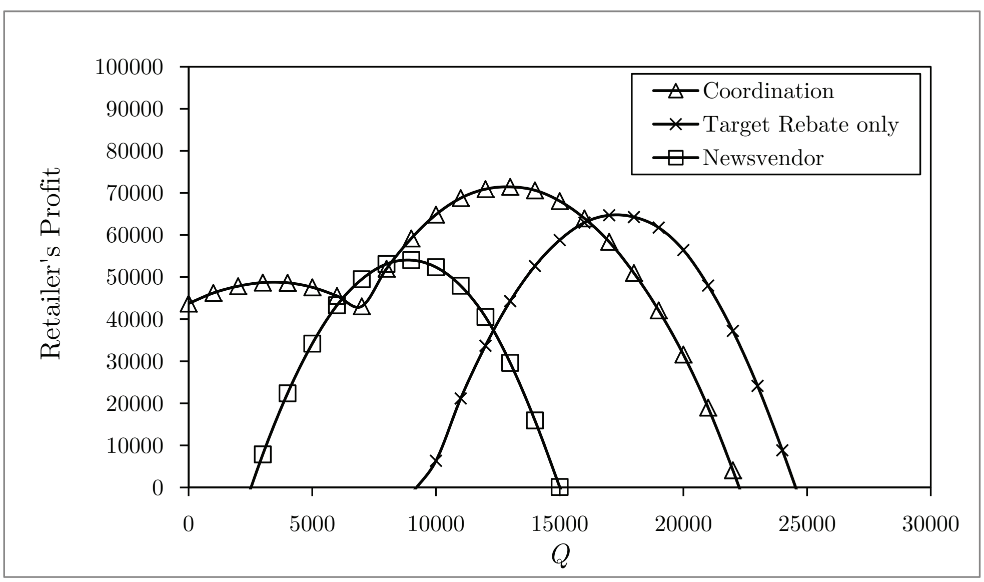

Figure 4.4: Retailer's Profit Comparison

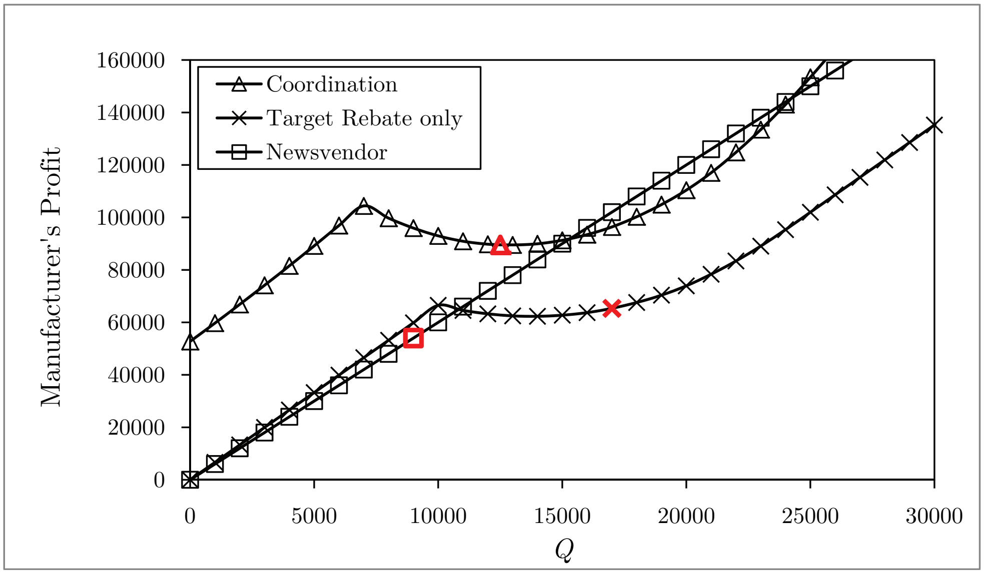

Figure 4.5: Manufacturer's Profit Comparison

In Figure 4.4, the retailer decides the optimal effort level and the optimal order quantity in each scenario, which accordingly determines the corresponding manufacturer's profit, highlighted in Figure 4.5. These two figures show the coordinating contract results in the Pareto improvement.

We now present coordinating contracts and exhibit arbitrary profit splitting in Table 4.3. Let $\underline{\Pi}^C = \Pi^C(Q^C, e^C)$, and $\underline{\Pi}_R^B = \Pi_R^B(Q^*, e^*)$. For the centralized supply chain, $\underline{\Pi}^C$, Q^C and e^C are 160,968.72, 12,500 and 125, respectively.

Table 4.3: Coordinating Contracts and Arbitrarily Profit Splitting

$\underline{\Pi}_R^B$	w_1	u	T	w_2
$0.28\underline{\Pi}^C$	27.40	27.40	5436.33	27.40
$0.30\underline{\Pi}^C$	26.25	25.70	5667.86	26.80
$0.50\underline{\Pi}^C$	19.31	15.34	7496.33	23.28
$0.70\underline{\Pi}^C$	14.75	8.29	9362.92	21.21
$0.90\underline{\Pi}^C$	11.40	2.52	10210.33	20.28
$0.99\underline{\Pi}^C$	10.15	0.30	12376.81	20.00

In Table 4.3, the resulting profits of the retailer under the advance-purchase discount contract $\underline{\Pi}_R^B$ are carried out by the percentage of the total profit of the centralized supply chain, and for each of those profits, the associated coordinating contract is presented.

Table 4.3 suggests that to allocate more profit to the retailer, the manufacturer lowers w_1 and w_2, while the retailer compromises on a less attractive rebate, a lower u and a higher T. In contrast, the manufacturer increases w_1 and w_2 to receive a higher profit share, and makes the rebate more attractive by raising u and lower T to retain the coordination. This result resembles that in Taylor [11] although we replace his buy-back contract with the advance-purchase contract.

Further, it can also be observed in Table 4.3 that to offer the retailer a significantly small amount of profit, the assumption, $w_1 < w_2$, may be violated. Hence, arbitrary profit split is not fully flexible in this parameter setting.[1] The intuition behind this outcome is that when w_1 becomes sufficiently large the retailer dramatically reduces her order size, and heavily relies on her second order opportunity. In order to induce the optimal order quantity, the manufacturer provides compensation through a rebate by increasing u and lowering T.

4.5 Summary

In this chapter, we study a decentralized supply chain with a manufacturer and a retailer who faces a stochastic effort-dependent demand. The sales effort incurs exertion cost and is non-contractible because the cost for sales effort is difficult to verify by the manufacturer, and thus is hard to be shared proportionally by the manufacturer.

To coordinate the supply chain, we study an advance-purchase and target rebate

[1] We also perform the analysis with a different set of market parameters, and are able to achieve fully-flexible arbitrary profit split.

contract to establish a new contract, called an advance-purchase and target rebate contract. Under this contract, the retailer has two opportunities to place order at two different wholesale prices to satisfy the demand. The first ordering opportunity is available prior to demand observation, and the second ordering opportunity becomes available after demand observation during the selling season. Correspondingly, the manufacturer also has two production opportunities with two different marginal costs.

As opposed to other coordinating approaches, such as a buy-back contract or lost sale penalty, the advance-purchase contract is preferable in some industries because of its practical advantages. For example, some products when returned may require lengthly and/or costly processes or become worthless, and therefore executing a buy-back may not be worthwhile and desirable by both players. In addition, a lost sale is difficult to tract, especially with today's advanced e-commerce technology, where a product may be out of stock, and a lost sale is never reported.

We show that supply chain coordination with arbitrary profit split and Pareto improvements are achievable. In this approach, the manufacturer ensures the retailer's optimal decisions by properly specifying the rebate, target level and wholesale prices to assure a win-win situation. Numerical results show that the inclusion of the advance-purchase contract greatly improves supply chain profit even though it lowers the supply chain's and retailer's optimal order quantities. In addition, the contract has sufficient flexibility for arbitrary profit split.

There are multiple possible directions for future research. One of which is to eliminate such possibility to assure fully-flexible arbitrary profit split. The second direction is to establish a new contract which allows the players to share the cost of sales effort. Also, it would be interesting to see results when demand is both sales-effort and retail-price dependent, or a demand-effort model takes a different form.

CHAPTER 5

Supply Chain Coordination with Transshipment and Advance-Purchase Contract

In this chapter, we analyze impacts of the advance-purchase contract on a supply chain with transshipment. The supply chain consists of a manufacturer and two retailers who face stochastic demand for a single product in a single selling-season. The manufacturer facilitates transshipment between the two retailers. This problem is particularly of our interest because it is unclear how transshipment would influence manufacturer's and retailers' decisions, and whether or not supply chain coordination is achievable. In general, the impact of transshipment is two-folded since a retailer might place a smaller order as she hopes to rely on transshipment from another retailer to satisfy her unmet demand. Alternatively, she might place a larger order as she hopes to make profit from accepting a transshipment request from the other retailer. With an advance-purchase contract, this problem has become more complicated. We study how such contract influences the manufacturer's and retailers' decisions and performance of the supply chain, and design a contract to coordinate an ordering decision and allow an arbitrary profit split.

5.1 Model Formulation

5.1.1 Notation

We study a supply chain with a manufacturer and two retailers selling the same product in two regions. Throughout this chapter, we use the subscripts i and j to refer to these two retailers, and suppress the terms, $i = \{1, 2\}$ and $i, j = \{1, 2\}, i \neq j$, for short when referencing to retailer i, and retailer i and j, respectively. The manufacturer offers the advance-purchase contract to the retailers, and allows transshipment between the retailers to satisfy demand if he/she faces a stock-out situation. Under the advance-purchase contract, the manufacturer provides two ordering opportunities to the retailers: one before the selling season, and the other one during the selling season. The manufacturer has two production opportunities corresponding to the two ordering opportunities. Retailer i then sells products at a retail price r_i to satisfy customers' demand in region i, D_i. The demand in the two regions are i.i.d with a probability density function $\phi(\cdot)$, $\phi(D_i) > 0$ for all $D_i \geq 0$, and a cumulative distribution function $\Phi(\cdot)$. The retail prices are assumed to be exogenous. This assumption is justified when a retailer is a price taker as a result of a competitive market. For simplicity, we also assume the remaining inventory at the end of the selling season has zero value.

The manufacturer and the retailers follow the following sequence of events. In Stage 1, retailer i places her first order to the manufacturer at a wholesale price w_1 to obtain products. Each unit in this order is produced at marginal cost c_1. As soon as the retailer receives her inventory in Stage 2, she immediately satisfies her demand. In Stage 3, if she needs more supply, then she submits a transshipment request to retailer j. If the transshipment request is accepted, then retailer i pays a transshipment price c_{ji} to retailer j for every transshipped unit. To facilitate this transshipment, the manufacturer charges retailer j a unit transportation cost τ_{ji} and retailer i a unit transshipment premium ρ_i. This is justified because the manufacturer provides transportation for transshipment, and charges the premium for cost of rebranding products. In Stage 4, if retailer i still requires more products, then she places her second order to the manufacturer at a unit wholesale price w_2 to obtain products. Each unit in this order is produced at marginal cost c_2.

We assume $c_2 > c_1$ because the second production may require an expedition and therefore incur extra cost. We also assume $w_1 \geq c_1$ and $w_2 \geq c_2$ to ensure positive profit margins of the manufacturer. To guarantee that transshipment is preferred to the second order, we assume $c_{ji} + \rho_i < w_2$. Notice that under the advance-purchase contract all demand will always be satisfied at the end of the selling season.

We summarize the sequence of operations and financial transactions in the supply chain in Figure 5.1, where the solid lines are product flows and the dotted lines are financial flows.

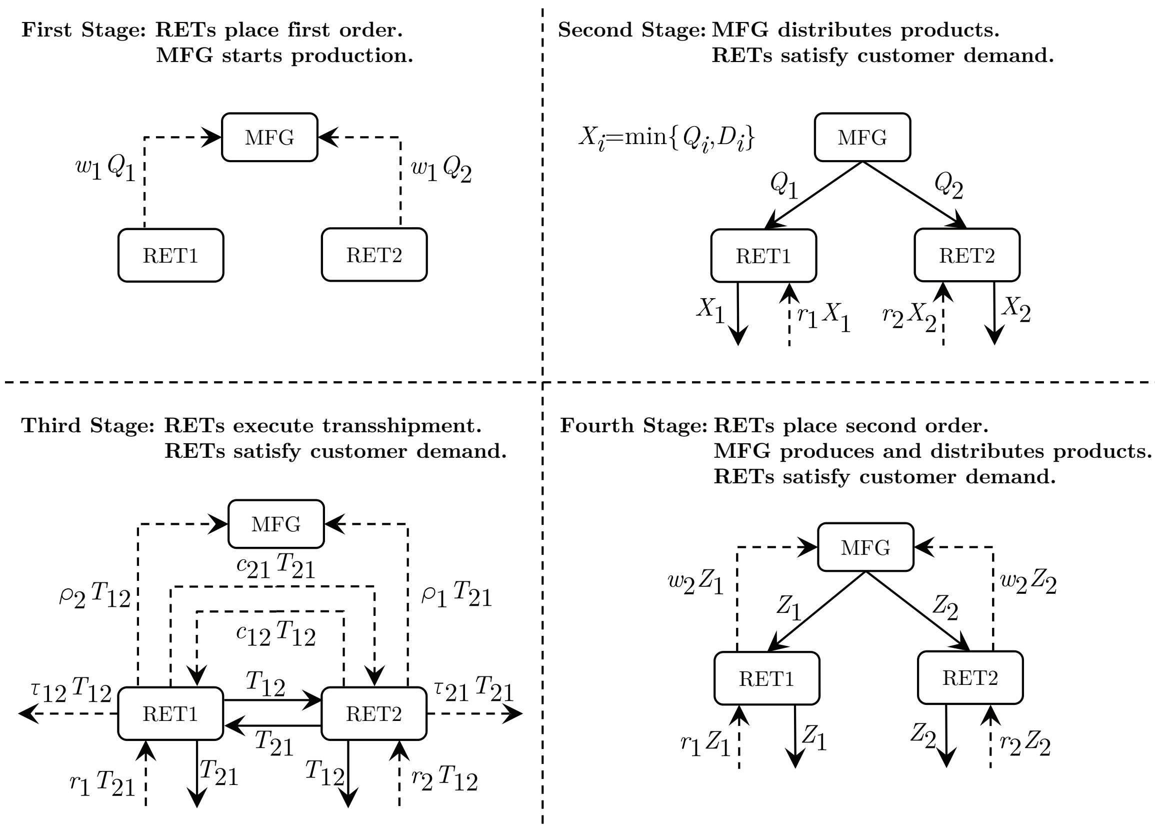

Figure 5.1: Operations and Financial Transactions in the Supply chain

The summary of the notation used in this chapter is presented in Table 5.1.

Table 5.1: Notation Summary

Notation	**Description**
c_1	Unit manufacturing cost in manufacturer's first production
c_2	Unit manufacturing cost in manufacturer's second production
r_i	Unit retail price of retailer i
τ_{ij}	Unit transportation cost paid by retailer i for transshipment to retailer j
Q_i	First order quantity of retailer i
T_{ij}	Expected transshipment quantity sent from retailer i to retailer j
Z_i	Expected unmet demand at retailer i, after satisfying demand from transshipment
π^c	Expected profit of a centralized supply chain
$\pi^d_{R_i}$	Expected profit of retailer i under decentralization
π^d_R	Expected total profit of retailers under decentralization, $\sum_{i=1,2} \pi^d_{R_i}$
π^d_M	Expected profit of a manufacturer under decentralization
w_1	Unit wholesale price in retailers' first ordering opportunity
w_2	Unit wholesale price in retailers' second ordering opportunity
c_{ij}	Unit transshipment price paid by retailer j for transshipment from retailer i
ρ	Unit transshipment premium paid by a retailer to a manufacturer
λ	Profit split parameter

5.1.2 Centralized Supply Chain

The centralized supply chain serves as the benchmark to compare supply chain's performance and retailers' optimal decision under coordination when deriving a coordinating contract. Under centralization, the entire supply chain belongs to a single firm. Let $x^+ = \max\{x, 0\}$. Define $T_{ij} = \min\left\{(Q_i - D_i)^+,\ (D_j - Q_j)^+\right\}$ to represent transshipment quantity from retailer i to retailer j, and $Z_i = D_i - (\min\{D_i, Q_i\} + T_{ji})$ to represent the remaining demand of retailer i after transshipment. The supply chain's expected profit is given by

$$\pi^c = \mathbb{E}\left\{\sum_{i=1,2} r_i \min\{D_i, Q_i\} + \sum_{\substack{i,j=1,2\\ i\neq j}} (r_j - \tau_{ij})\, T_{ij} + \sum_{i=1,2} (r_i - c_2)\, Z_i\right\} - c_1 \sum_{i=1,2} Q_i. \tag{5.1}$$

The first, second and third terms in Eq. (5.1) are revenues from satisfying demand from inventory, from transshipment and from the second order, respectively. The last term is the procurement cost. Plugging in the definition Z_i into Eq. (5.1) and rearrangement yield

$$\pi^c = \mathbb{E}\left\{\sum_{i=1,2} c_2 \min\{D_i, Q_i\} + \sum_{\substack{i,j=1,2\\ i\neq j}} (c_2 - \tau_{ij})\, T_{ij} + \sum_{i=1,2} (r_i - c_2)\, D_i\right\} - c_1 \sum_{i=1,2} Q_i. \tag{5.2}$$

The optimal solution of Eq. (5.2) satisfies the following condition,

$$F(Q_i) = \frac{c_2 - c_1}{c_2} + \frac{c_2 - \tau_{ij}}{c_2}\frac{\partial \mathbb{E}T_{ij}}{\partial Q_i} + \frac{c_2 - \tau_{ji}}{c_2}\frac{\partial \mathbb{E}T_{ji}}{\partial Q_i} = 0, \quad \text{for } i, j = 1, 2, i \neq j. \tag{5.3}$$

5.1.3 Decentralized Supply Chain

When retailers are independent, each party in the supply chain attempts to maximize his or her own profit. Under an advance-purchase contract, the manufacturer designs a contract by specifying contract parameters, i.e., $w_1, w_2, c_{ij}, c_{ji}, \rho$, to coordinate the supply chain. His expected profit is

$$\pi_M^d = (w_1 - c_1) \sum_{i=1,2} Q_i + \mathbb{E} \left\{ \rho \sum_{\substack{i,j=1,2 \\ i \neq j}} T_{ij} + (w_2 - c_2) \sum_{i=1,2} Z_i \right\}. \tag{5.4}$$

The three terms are the profits from satisfying retailers' first order, from receiving the transshipment premium, and from satisfying retailers' second order, respectively.

Given a contract, retailer i, $i = 1, 2$, then decides an order quantity Q_i to maximize her expected profit

$$\pi_{R_i}^d = \mathbb{E} \left\{ r_i \min \{D_i, Q_i\} + (r_i - c_{ji} - \rho) T_{ji} + (c_{ij} - \tau_{ij}) T_{ij} + (r_i - w_2) Z_i \right\} - w_1 Q_i. \tag{5.5}$$

The first four terms in Eq. (5.5) are the revenues by satisfying demand from her inventory, by satisfying demand from receiving transshipment, by providing transshipment, and by satisfying demand from her second order, respectively. The last term is the procurement cost. Note that without an advance-purchase contract, retailer's expected profit is identical to $\pi_{R_i}^d$ in Eq. (5.5) with $Z_i = 0$, representing the absence of the second order.

The optimal order quantity, Q_i, of Eq. (5.5) satisfies the following conditions,

$$F(Q_i) = \frac{w_2 - w_1}{w_2} + \frac{c_{ij} - \tau_{ij}}{w_2} \frac{\partial \mathbb{E} T_{ij}}{\partial Q_i} + \frac{w_2 - c_{ji} - \rho}{w_2} \frac{\partial \mathbb{E} T_{ji}}{\partial Q_i} = 0, \quad \text{for } i, j = 1, 2, i \neq j. \tag{5.6}$$

The summation of the expected profit of the retailers is

$$\begin{aligned} \pi_R^d &= \sum_{i=1,2} \pi_{R_i}^d \\ &= \mathbb{E} \left\{ \sum_{i=1,2} w_2 \min \{D_i, Q_i\} + \sum_{\substack{i,j=1,2 \\ i \neq j}} (w_2 - \tau_{ji} - \rho) T_{ji} \right. \\ &\qquad \left. + \sum_{i=1,2} (r_i - w_2) D_i \right\} - w_1 \sum_{i=1,2} Q_i. \end{aligned} \tag{5.7}$$

In the next section, we develop a mechanism and a contract that achieve supply chain coordination with arbitrary profit split, and assures a win-win situation.

5.2 Supply Chain Coordination

Our objective in this section is to design a contract to coordinate a supply chain with transshipment, and leads to a Pareto improvement. The coordinating mechanism follows in two steps. In the first step, the manufacturer uses the two wholesale prices and the transshipment premium to split total expected profit of the supply chain into two portions: one for himself and the other for the two retailers as elaborated in Theorem 5.1. Then the retailers use transshipment prices to further split the second portion as described in Theorem 5.2.

Let $\Delta = \sum_{i=1,2} r_i D_i$ denote expected total supply chain revenue.

Theorem 5.1 *Let λ be a parameter set by the manufacturer for an arbitrary profit split. Supply chain coordination can be achieved by setting w_1, w_2 and ρ_i to satisfy the following conditions:*

$$w_1 = \lambda c_1, \tag{5.8}$$

$$w_2 = \lambda c_2, \tag{5.9}$$

$$\rho_i = (\lambda - 1)\,\tau_{ij}, \tag{5.10}$$

where $1 \leq \lambda \leq \Delta/(\Delta - \pi^c)$.

Proof. Applying coordination conditions in Eqs. (5.8), (5.9) and (5.10) to Eq. (5.7), we obtain

$$\pi_R^d = \lambda \pi^c + (1 - \lambda)\,\Delta, \tag{5.11}$$

$$\pi_M^d = (1 - \lambda)\,\pi^c - (1 - \lambda)\,\Delta. \tag{5.12}$$

Eq. (5.11) shows that retailer's optimal decision is also supply chain's optimal decision. Furthermore, it is easy to verify that $\pi_M^d \geq 0$ if and only if $\lambda \geq 1$, and $\pi_R^d \geq 0$ if and only if $\lambda \leq \Delta/(\Delta - \pi^c)$. Therefore, the valid range of λ for supply chain coordination is

$$1 \leq \lambda \leq \frac{\Delta}{\Delta - \pi^c}. \tag{5.13}$$

■

Rudi et al. [3] show Nash equilibrium, the solution to a traditional transshipment game, is unique. Likewise, a similar proof can be constructed to show a uniqueness of Nash equilibrium, Q_i given by Eq. (5.6), for our coordination game.

Let's denote

$$\alpha_i(Q_i) = \Pr(D_i < Q_i), \tag{5.14}$$

$$\beta_i(Q_i, Q_j) = \Pr(Q_i + Q_j - D_j < D_i < Q_i), \tag{5.15}$$

$$\gamma_i(Q_i, Q_j) = \Pr(Q_i < D_i < Q_i + Q_j - D_j), \tag{5.16}$$

graphically shown in Figure 5.2

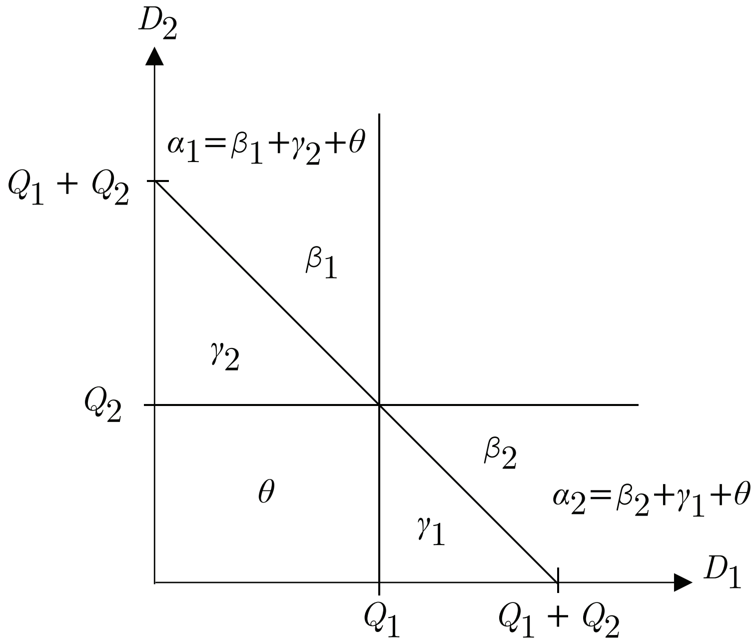

Figure 5.2: Graphical Illustration of Probabilities

We use the same probability notations defined in Rudi et al. [3] to derive optimal transshipment prices. These probabilities are associated with the events for a given Q_i and Q_j. α_i is for the scenario that retailer i has excess inventory. β_i is for the scenario that retailer i has excess inventory, but the total retailers' inventory is less than the total retailers' demand. In this case, retailer j receives transshipment from retailer i, but will not be able to fully satisfy her unmet demand. γ_i is for the scenario that retailer i is stock out, and the total retailers' inventory is more than the total retailers' demand. In this case, retailer i receives transshipment from retailer j, and will be able to completely satisfy her unmet demand. In Theorem 5.2, we derive the coordinating transshipment prices.

Theorem 5.2 *Given the contract parameters specified in Theorem 5.1, supply chain coordination can be achieved by setting transshipment prices*

$$c_{ij} = \frac{(w_2 - \rho)\,\beta_i\beta_j + (\tau_{ji} - w_2 + \rho)\,\beta_j\gamma_i - \tau_{ij}\gamma_i\gamma_j}{\beta_i\beta_j - \gamma_i\gamma_j}. \tag{5.17}$$

Proof. From Eq. (5.7), setting $\partial \pi_R^d / \partial Q_i = 0$ yields

$$\alpha_i + \frac{(w_2 - \tau_{ji} - \rho)}{w_2}\gamma_i - \frac{(w_2 - \tau_{ij} - \rho)}{w_2}\beta_i = \frac{w_2 - w_1}{w_2}, \quad \text{for } i, j = 1, 2, i \neq j. \quad (5.18)$$

From Eq. (5.5), setting $\partial \pi_{R_i}^d / \partial Q_i = 0$ yields

$$\alpha_i + \frac{(w_2 - c_{ji} - \rho)}{w_2}\gamma_i - \frac{(c_{ij} - \tau_{ij})}{w_2}\beta_i = \frac{w_2 - w_1}{w_2}, \quad \text{for } i, j = 1, 2, i \neq j. \quad (5.19)$$

By equating the left-hand sides of Eqs. (5.18) and (5.19) with the following transshipment prices, retailer i's optimal order quantity is also optimal for π_R^d.

$$c_{ij} = \frac{(w_2 - \rho)\,\beta_i\beta_j + (\tau_{ji} - w_2 + \rho)\,\beta_j\gamma_i - \tau_{ij}\gamma_i\gamma_j}{\beta_i\beta_j - \gamma_i\gamma_j}. \quad (5.20)$$

■

The parameters $(w_1^*, w_2^*, \rho^*, c_{ij}^*, c_{ji}^*)$ of a coordinating contract are defined by Eqs. (5.8), (5.9), (5.10) and (5.17). In the following, we study the properties of the proposed coordinating contract.

Proposition 5.1 *Given a contract $(w_1^*, w_2^*, \rho^*, c_{ij}^*, c_{ji}^*)$, retailers' optimal order quantity Q_i is independent of λ. In addition, retailers' optimal ordering decision is also optimal to the supply chain.*

Proof. Eq. (5.11) shows π_R^d is linearly proportional to π^c. Hence, retailers' optimal order quantity Q_i is independent of λ. Substituting contract parameters in Eqs. (5.8), (5.9), (5.10) and (5.17) into Eq. (5.6), and arranging yield Eq. (5.3). ■

Proposition 5.1 shows the supply chain profit can be split arbitrarily, which is necessary to achieve Pareto improvements. If the manufacturer properly sets the contract parameters and allows the retailers to make their ordering decision freely, Proposition 5.1 assures that retailers' optimal ordering decision is also optimal for the whole supply chain. By Propositions 5.1, a coordinating contract achieves an arbitrary profit split and Pareto improvements.

Proposition 5.2 characterizes the role of λ in the arbitrary profit split.

Proposition 5.2 *There exists $\underline{\lambda}$, below which a manufacturer would prefer a decentralized supply chain under transshipment over a coordinated supply chain. Likewise, there exists $\bar{\lambda}$, above which a retailer would prefer a decentralized supply chain under transshipment over a coordinated supply chain. In addition, supply chain coordination and a win-win situation are attained when $\lambda \in [\underline{\lambda}, \bar{\lambda}]$.*

Proof. This is to show that when λ is below $\underline{\lambda}$, $\pi_M^d < \pi_M^t$, and when λ is above $\bar{\lambda}$, $\pi_{R_i}^d < \pi_{R_i}^t$, where π_M^t, and $\pi_{R_i}^t$ respectively denote manufacturer's and retailer's profit in a supply chain under transshipment but no advance-purchase contract. In addition, when λ is between $\underline{\lambda}$ and $\bar{\lambda}$, $\pi_M^d \geq \pi_M^t$ and $\pi_{R_i}^d \geq \pi_{R_i}^t$, where the equality is achieved when $\lambda = \underline{\lambda}$ or $\lambda = \bar{\lambda}$, respectively. From Eqs. (5.11) and (5.12), it is easy to verify that π_M^d is linearly increasing in λ, and $\pi_{R_i}^d$ is linearly decreasing in λ for $\lambda \in [1, \Delta/(\Delta - \pi^c)]$. So, there exists $\underline{\lambda}$, below which $\pi_M^d < \pi_M^t$, which means the manufacturer prefers a transshipment agreement to the coordinating contract. Likewise, because $\pi_{R_i}^d$ is linearly decreasing in λ, is continuous on $\lambda \in [1, \Delta/(\Delta - \pi^c)]$, $\pi_{R_i}^d = \pi^c$ when $\lambda = 1$, and $\pi_{R_i}^d = 0$ when $\lambda = \Delta/(\Delta - \pi^c)$, there exists $\bar{\lambda}$, above which $\pi_{R_i}^d < \pi_{R_i}^t$, which implies the retailers prefer a transshipment agreement to the coordinating contract. In addition, $\underline{\lambda} < \bar{\lambda}$ because at least one player always prefers the coordinating contract. Moreover, when $\underline{\lambda} \leq \lambda \leq \bar{\lambda}$, $\pi_M^d \geq \pi_M^t$ and $\pi_{R_i}^d \geq \pi_{R_i}^t$ because $\sum_{i \in R} \pi_{R_i}^d + \pi_M^d > \sum_{i \in R} \pi_{R_i}^t + \pi_M^t$. Particularly, $\bar{\lambda} = \arg\max_{\lambda \in [1, \Delta/(\Delta - \pi^c)]} \pi_M^d(\lambda)$, and $\pi_{R_i}^d = \pi_{R_i}^t$. ■

The intuition of Proposition 5.2 is that because manufacturer's profit is linearly increasing in λ and retailer's profit is linearly decreasing in λ, the manufacturer can select λ that leaves the retailers not worse off and himself better off, compared to the performances in the decentralized case.

5.3 Sensitivity Analysis

In this section, we examine the impacts of the contract parameters and the market parameters on the order quantities and the profits. Note that when analyzing the impact of a parameter of interest, the other parameters are fixed.

Proposition 5.3 *Q_i increases when w_1 decreases, when w_2 increases, when c_{ij} and c_{ji} increase, or when ρ increases.*

Proof. Proof can be derived directly from Nash equilibrium conditions given in Eq. (5.6). ■

These results are intuitive. When w_1 decreases, retailer i has more incentive to order a larger quantity due to a larger profit margin. When c_{ij} or ρ is higher, the retailer has to pay more to obtain products through transshipment. As a result, she would avoid requesting transshipment by placing a larger first order. In addition, when c_{ij} is higher, facilitating transshipment becomes more profitable; hence, the retailer places a larger order. When w_2 is higher, the retailer has to pay more to obtain products in her second order.

Proposition 5.4

- *Q_i is decreasing in c_1.*
- *Q_i is non-decreasing in c_2 if $\lambda c_1+(c_2\partial c_{ij}/\partial c_2-(c_{ij}-\tau_{ij}))\partial\mathbb{E}T_{ij}/\partial Q_i \geq (c_2\partial c_{ji}/\partial c_2-c_{ji}+(\lambda-1)\tau_{ij})\partial\mathbb{E}T_{ji}/\partial Q_i$; decreasing in c_2 if $\lambda c_1+(c_2\partial c_{ij}/\partial c_2-(c_{ij}-\tau_{ij}))\partial\mathbb{E}T_{ij}/\partial Q_i < (c_2\partial c_{ji}/\partial c_2 - c_{ji} + (\lambda-1)\tau_{ij})\partial\mathbb{E}T_{ji}/\partial Q_i$.*
- *Q_i is non-decreasing in τ_{ij} if $\partial\mathbb{E}T_{ij}/\partial Q_i(\partial c_{ij}/\partial\tau_{ij}-1) \geq \partial\mathbb{E}T_{ji}/\partial Q_i(\partial c_{ji}/\partial\tau_{ij}+(\lambda-1))$; decreasing in τ_{ij} if $\partial\mathbb{E}T_{ij}/\partial Q_i(\partial c_{ij}/\partial\tau_{ij}-1) < \partial\mathbb{E}T_{ji}/\partial Q_i(\partial c_{ji}/\partial\tau_{ij}+(\lambda-1))$.*
- *Q_i is independent of r_i.*

Proof. It can be shown that at equilibrium, $\frac{\partial \mathbb{E}T_{ij}}{\partial Q_i} \geq 0$, and $\frac{\partial \mathbb{E}T_{ji}}{\partial Q_i} \leq 0$.

Then, $\frac{\partial F(Q_i)}{\partial c_1} = -\lambda/w_2 < 0$.

$$\frac{\partial F(Q_i)}{\partial c_2} = c_1/c_2^2 + \frac{\left(c_2\frac{\partial c_{ij}}{\partial c_2} - (c_{ij} - \tau_{ij})\right)}{(\lambda c_2)^2}\lambda\frac{\partial \mathbb{E}T_{ij}}{\partial Q_i} - \frac{\left(c_2\frac{\partial c_{ji}}{\partial c_2} - (c_{ji} - (\lambda-1)\tau_{ij})\right)}{(\lambda c_2)^2}\lambda\frac{\partial \mathbb{E}T_{ji}}{\partial Q_i}.$$

Hence,

$\frac{\partial F(Q_i)}{\partial c_2} \geq 0$ if $\lambda c_1 + \left(c_2\frac{\partial c_{ij}}{\partial c_2} - (c_{ij} - \tau_{ij})\right)\frac{\partial \mathbb{E}T_{ij}}{\partial Q_i} \geq \left(c_2\frac{\partial c_{ji}}{\partial c_2} - (c_{ji} - (\lambda-1)\tau_{ij})\right)\frac{\partial \mathbb{E}T_{ji}}{\partial Q_i}$,

and $\frac{\partial F(Q_i)}{\partial c_2} < 0$ otherwise.

$$\frac{\partial F(Q_i)}{\partial \tau_{ij}} = \frac{1}{\lambda c_2}\frac{\partial \mathbb{E}T_{ij}}{\partial Q_i}\left(\frac{\partial c_{ij}}{\partial \tau_{ij}} - 1\right) - \frac{1}{\lambda c_2}\frac{\partial \mathbb{E}T_{ji}}{\partial Q_i}\left(\frac{\partial c_{ji}}{\partial \tau_{ij}} + (\lambda - 1)\right),$$

where $\frac{\partial c_{ij}}{\partial \tau_{ij}} = [(\lambda-1)\beta_j(\gamma_i - \beta_i) - \gamma_i\gamma_j]/(\beta_i\beta_j - \gamma_i\gamma_j)$, and $\frac{\partial c_{ji}}{\partial \tau_{ij}} = \beta_i\gamma_j/(\beta_i\beta_j - \gamma_i\gamma_j)$.

Therefore,

$\frac{\partial F(Q_i)}{\partial \tau_{ij}} \geq 0$ if $\frac{\partial \mathbb{E}T_{ij}}{\partial Q_i}\left(\frac{\partial c_{ij}}{\partial \tau_{ij}} - 1\right) \geq \frac{\partial \mathbb{E}T_{ji}}{\partial Q_i}\left(\frac{\partial c_{ji}}{\partial \tau_{ij}} + (\lambda-1)\right)$, and $\frac{\partial F(Q_i)}{\partial \tau_{ij}} < 0$ otherwise.

$F(Q_i)$ is independent of r_i. ■

Intuitively, when c_1 increases, the manufacturer will increase w_1 to offset, and therefore a retailer orders less in the first order. For the effect of c_2, this result initially appears counter-intuitive if we forget c_{ij} and c_{ji} depend on c_2. When c_2 is sufficiently high and increases, a retailer places a larger first order to avoid paying high w_2, which is a function of c_2. Alternatively, when c_2 is relatively low and increases, a retailer may lower her first order quantity due to two effects: (1) a retailer is less sensitive to an increase of c_2 because she can still rely on transshipment to satisfy her demand, and (2) a retailer experiences providing transshipment becoming less profitable and requesting transshipment becoming less expensive

It seems counter-intuitive to see a retailer orders more when τ_{ij} increases for the same reason. Specifically, if c_{ij} and c_{ji} are significantly increasing in τ_{ij}, then a retailer raises her order quantity because transshipment is more profitable. On the other hand, if transshipment prices c_{ij} and c_{ji} are not increasing in transshipment cost τ_{ij}, transshipment becomes less attractive; hence, a retailer reduces her order size. Finally, the retailer's optimal order quantity is independent of r_i.

Proposition 5.5

- *π^c is increasing in r_i, decreasing in c_1, non-increasing in c_2, and non-increasing in τ_{ij}.*
- *π^d_M is independent of r_i, increasing in c_1, non-decreasing in c_2, and non-decreasing in τ_{ij}.*
- *$\pi^d_{R_i}$ is increasing in r_i, decreasing in c_1, non-increasing in c_2, non-decreasing in τ_{ij} if $\mathbb{E}\{(\partial c_{ij}/\partial\tau_{ij} - 1)T_{ij}\} \geq \mathbb{E}\{(\partial c_{ji}/\partial\tau_{ij} + (\lambda-1))T_{ji}\}$, and decreasing in τ_{ij} if $\mathbb{E}\{(\partial c_{ij}/\partial\tau_{ij} - 1)T_{ij}\} < \mathbb{E}\{(\partial c_{ji}/\partial\tau_{ij} + (\lambda-1))T_{ji}\}$.*

Proof. For π^c, consider Eq. (5.1) and substitute in $Z_i = D_i - (\min\{D_i, Q_i\} + T_{ji})$. $\partial\pi^c/\partial r_i = \mathbb{E}\{D_i\} > 0$, $\partial\pi^c/\partial c_1 = -\sum_i Q_i < 0$, $\partial\pi^c/\partial c_2 = \mathbb{E}\{\sum_{i=1,2}\min\{D_i, Q_i\} + \sum_{i,j=1,2,i\neq j} T_{ij} - \sum_{i=1,2} D_i\} \leq 0$, and $\partial\pi^c/\partial\tau_{ij} = -\mathbb{E}\{T_{ij}\} \leq 0$.
For π^d_M, consider Eq. (5.4) and substitute in Z_i. π^d_M is independent of r_i, $\partial\pi^d_M/\partial c_1 = (\lambda-1)\sum_{i=1,2} Q_i > 0$, $\partial\pi^d_M/\partial c_2 = \mathbb{E}\{-(\lambda-1)\sum_{i,j=1,2,i\neq j} T_{ij} - (\lambda-1)\min\{D_i, Q_i\} + (\lambda-1)\sum_{i=1,2} D_i\} \geq 0$, and $\partial\pi^d_M/\partial\tau_{ij} = \mathbb{E}\{(\lambda-1)\sum_{i,j=1,2,i\neq j} T_{ij}\} \geq 0$.
For $\pi^d_{R_i}$, consider Eq. (5.5) and substitute in Z_i. $\partial\pi^d_{R_i}/\partial r_i = \mathbb{E}\{D_i\} > 0$, $\partial\pi^d_{R_i}/\partial c_1 = -\lambda Q_i < 0$. In addition, $\partial\pi^d_{R_i}/\partial c_2 = \mathbb{E}\{\lambda\min\{D_i, Q_i\} + (\lambda - \partial c_{ji}/\partial c_2)T_{ji} + \partial c_{ij}/\partial c_2 T_{ij} - \lambda D_i\}$. Lastly, $\partial\pi^d_{R_i}/\partial\tau_{ij} = \mathbb{E}\{-(\partial c_{ji}/\partial\tau_{ij} + (\lambda-1))T_{ji} + (\partial c_{ij}/\partial\tau_{ij} - 1)T_{ij}\}$. Hence, $\pi^d_{R_i}$ is non-decreasing in τ_{ij} if $\mathbb{E}\{(\partial c_{ij}/\partial\tau_{ij} - 1)T_{ij}\} \geq \mathbb{E}\{(\partial c_{ji}/\partial\tau_{ij} + (\lambda-1))T_{ji}\}$, and is decreasing in τ_{ij} otherwise. ■

It is straightforward to see π^c increases (decreases) when the profit margin increases (decreases). π^d_M increases in w_1, w_2, and ρ because the manufacturer will collect more revenue. $\pi^d_{R_i}$ decreases in w_1 and w_2 because the retailer incurs higher cost. The impact of the transshipment cost on the retailer's expected profit depends on τ_{ij} on c_{ij} and c_{ji}. Specifically, if c_{ij} sufficiently increases when τ_{ij} increases, then retailer i is able to collect more profit from providing transshipment. On the other hand, if c_{ij} decreases when τ_{ij} increases, transshipment becomes less profitable, and retailer's profit is therefore decreases.

The impacts of the market parameters on retailers' order quantity and the profits are summarized in Table 5.2, where $-$, $\uparrow$, $\downarrow$, represent constant, non-decreasing, non-increasing, respectively. In addition, $\updownarrow$ represents either non-increasing or non-decreasing.

Table 5.2: Summary of Impacts of Market Parameters

	Q_i	π^c	π^d_M	$\pi^d_{R_i}$
r_i	-	$\uparrow$	-	$\uparrow$
c_1	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$
c_2	$\updownarrow$	$\downarrow$	$\uparrow$	$\downarrow$
τ_{ij}	$\updownarrow$	$\downarrow$	$\uparrow$	$\updownarrow$

5.4 Numerical Analysis

In this section, we numerically compare the performance of the coordinating contract to that in the newsvendor and the transshipment-only scenarios. Let's assume retailers' demand is uniformly distributed between 0 to 200, and consider the following example to demonstrate numerical results.

Table 5.3: Parameters for the Numerical Example

		Coordination	Newsvendor	Transshipment-Only
Exogenous parameters:	r_i, r_j	50	50	50
	c_1	10	-	-
	c_2	23	-	-
	τ_{ij}, τ_{ji}	2	-	2
	c	-	10	10
Decision variables:	λ	1.95	-	-
	w_1	19.5	-	-
	w_2	44.85	-	-
	ρ	1.9	-	-
	c_{ij}, c_{ji}	19.78	-	-
	w^b	-	20	-
	w^d	-	-	20
	c^d_{ij}, c^d_{ji}	-	-	45

In Table 5.3, we use the same values of market parameters for the centralized case as under coordination. Note that "-" means the parameter or variable is not applicable. Also, the contract parameters under coordination satisfy the coordinating conditions given in Eqs (5.8), (5.9), (5.10) and (5.17).

The impact of the transshipment prices on the retailers' optimal order quantity is shown in Figure 5.3.

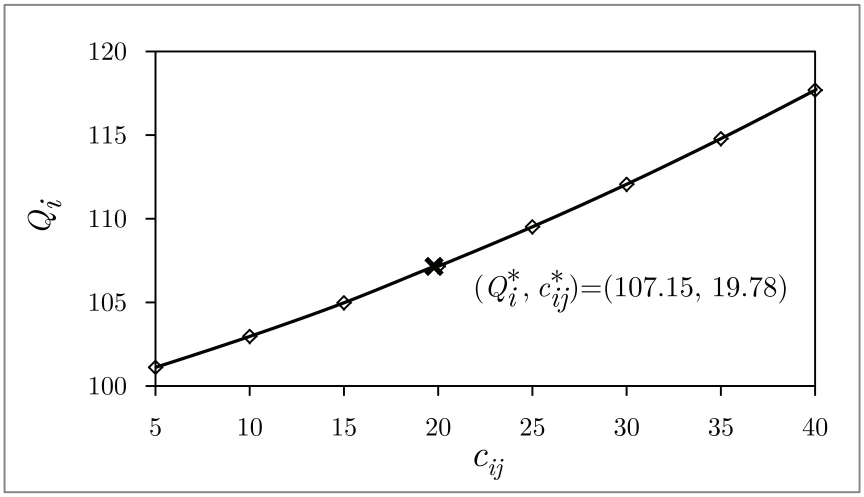

Figure 5.3: Transshipment Prices and Optimal Order Quantity

Figure 5.3 shows that retailers' optimal order quantity is increasing in transshipment prices. In this example, the coordinating transshipment prices c_{ij}^* and c_{ji}^* are 19.78, which yields the retailers' optimal order quantity of 107.15, equal to the optimal order quantity in the centralized case.

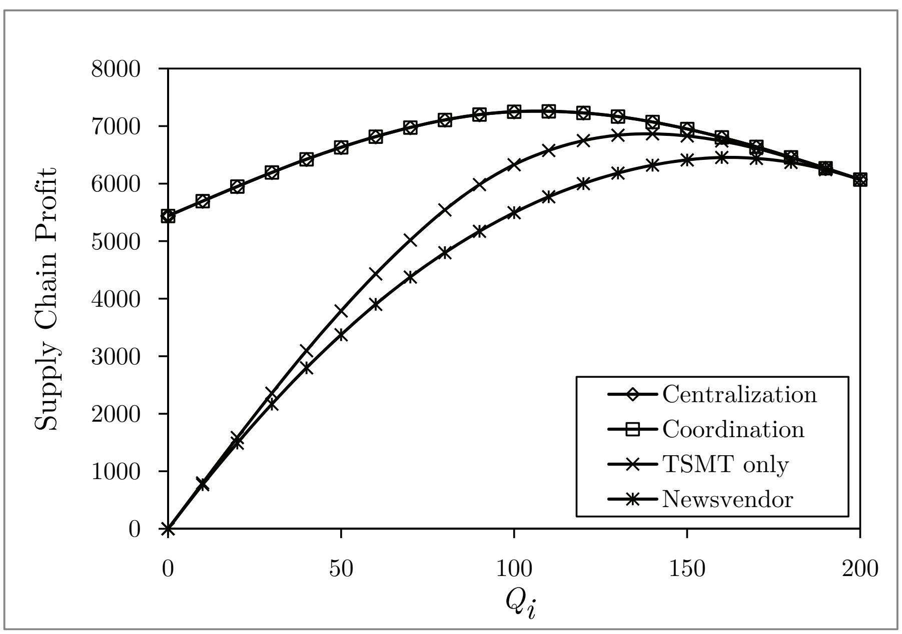

Figure 5.4: Profit Comparison

Figure 5.4 compares the total supply chain profits under four scenarios, i.e., centralization, coordination, transshipment-only and newsvendor, and reveals the following insights. Firstly, the total profit curves under coordination and under centralization overlap, and hence the optimal order quantities under both scenarios are identical. Secondly, the supply chain profit under coordination is at least as high as those under transshipment-only and newsvendor. This shows that the second ordering opportunity helps improve the total supply chain profit. Lastly, the optimal order quantity under coordination is lower than that under transshipment-only and newsvendor. The intuition is that the second ordering opportunity serves as an ample resource to fully satisfy the demand even when the demand is high; thus lowering the first order quantity.

Next, Figure 5.5 presents the supply chain profit, the manufacturer's profit and the retailer's profit under coordination. Because the two retailers are symmetric, we deliberately include only retailer i's profit.

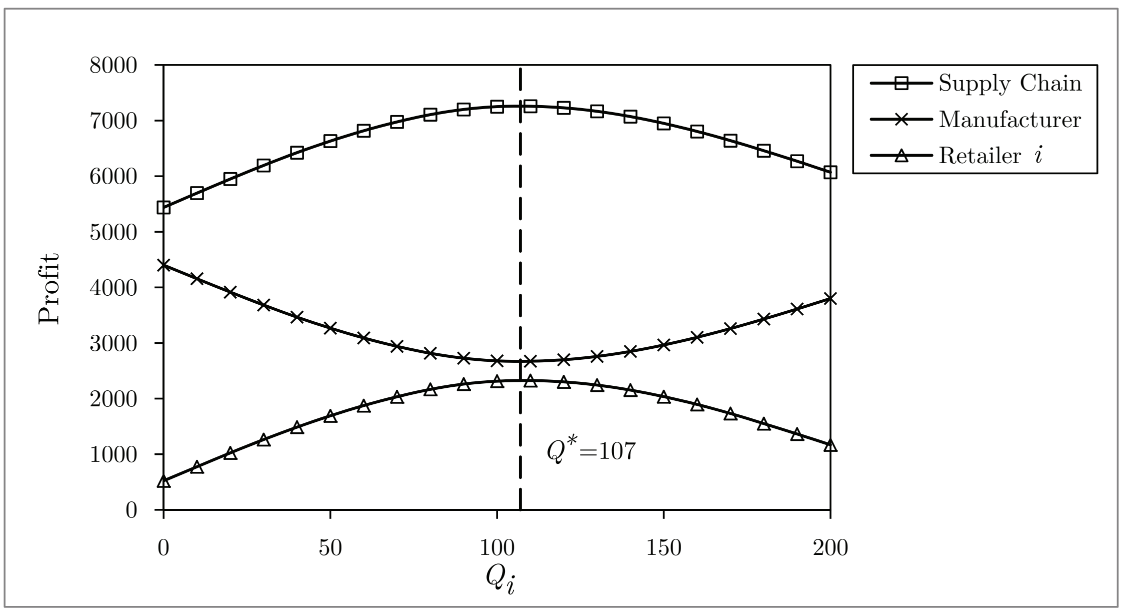

Figure 5.5: Profits under Coordination

Figure 5.5 suggests two insights. First, the coordinating contract achieves supply chain coordination, i.e., the optimal order quantity of the retailer is identical to the optimal order quantity of the supply chain. Second, the manufacturer's profit is convex in the retailer's order quantity. It is interesting to see the manufacturer's profit is literally equal to the total supply chain cost, and the retailers' total profit is equal to the supply chain profit minus the total supply chain cost. Hence, the manufacturer's profit is minimum when the total cost is minimum, which occurs when the retailers' profits are maximum. In this supply chain, there are two cost components, i.e., production cost and transshipment cost. When the retailers' order quantities are lower than the optimal order quantity, the supply chain incurs high production cost due to the larger amount of the retailers' second order. When the retailers' order quantities are higher than the optimal order quantity, the supply chain incurs high transshipment cost due to more transshipment executed. Therefore, the manufacturer's profit function is convex. In reality, once the manufacturer and retailers agree on λ, the manufacturer does not have an incentive to deviate from the contract terms, which are determined by λ. In addition, even if the manufacturer wants to deviate from the coordinating contract to improve his profit, there always exists another λ that provides the manufacturer the desired profit and the retailers higher profit than that of the uncoordinating contract.

Figures 5.6 and 5.7 respectively show the retailer's and manufacturer's profits under coordination, transshipment-only and newsvendor.

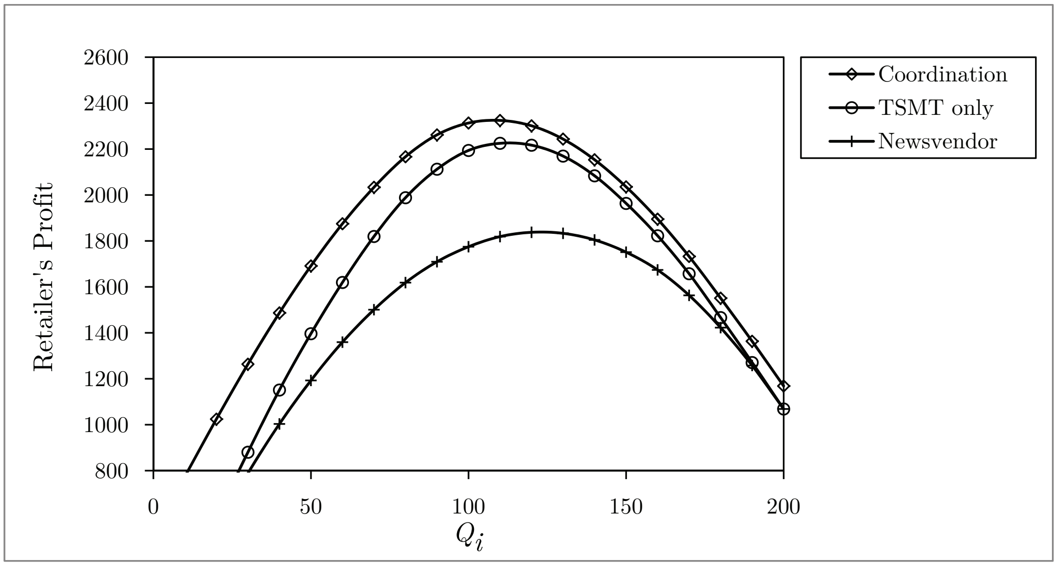

Figure 5.6: Retailer's Profit Comparison

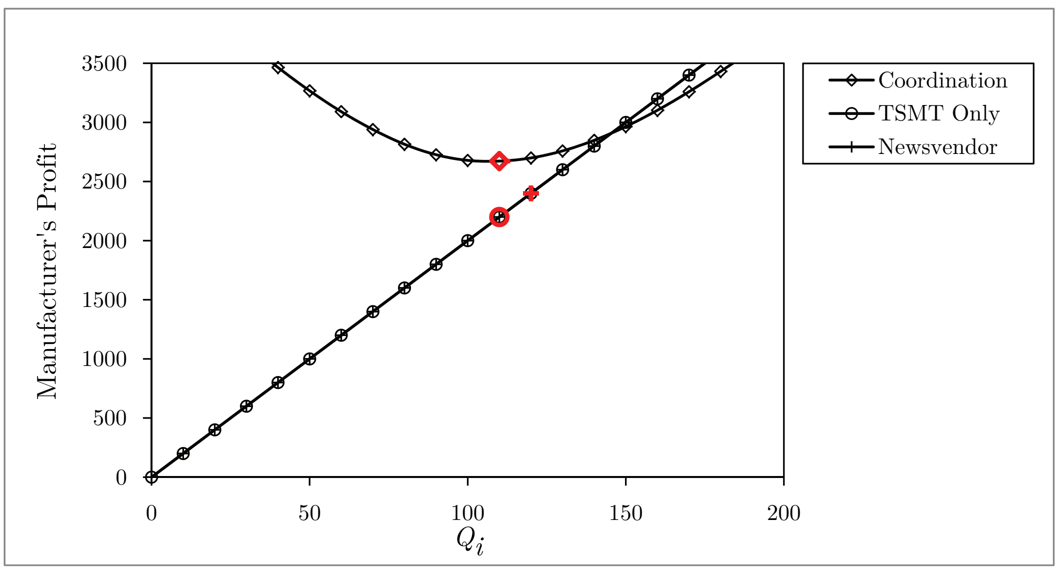

Figure 5.7: Manufacturer's Profit Comparison

In Figure 5.6, the retailer decides the optimal order quantity in each scenario, which accordingly determines the corresponding manufacturer's profit, highlighted in Figure 5.7. It can be observed in Figures 5.7 and 5.6 that the manufacturer's and retailer's profits under coordination are highest compared to those under transshipment-only and newsvendor, which results in the Pareto improvement.

Next, we show the effects of λ on the manufacturer's and retailers' profits under coordination in Figure 5.8. Then, Figure 5.9 displays the resulting profits and the win-win situation after both the manufacturer and the retailers observe the contract terms.

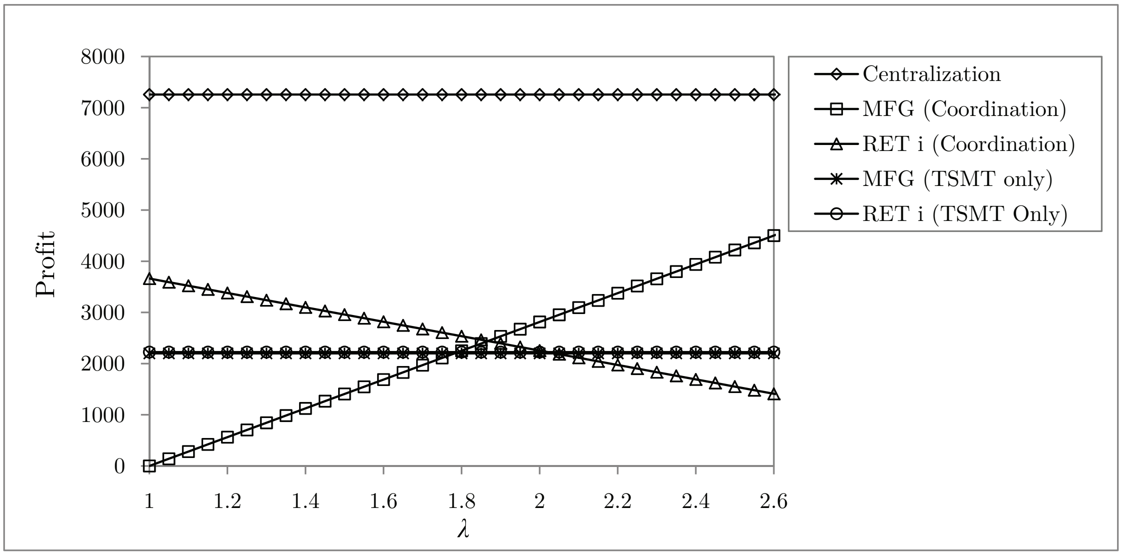

Figure 5.8: Arbitrarily Profit Split

In Figure 5.8, the manufacturer's and the retailer's profits under centralization and under transshipment-only are constant because they are independent of λ. Whereas, the manufacturer's and retailers' profits under coordination are linearly increasing and decreasing in λ, respectively. We can observe that there exists $\underline{\lambda}$, below which the manufacturer profit under coordination is below that under transshipment-only. Similarly, there exists $\bar{\lambda}$, above which the retailer's profit under coordination is below that under transshipment-only.

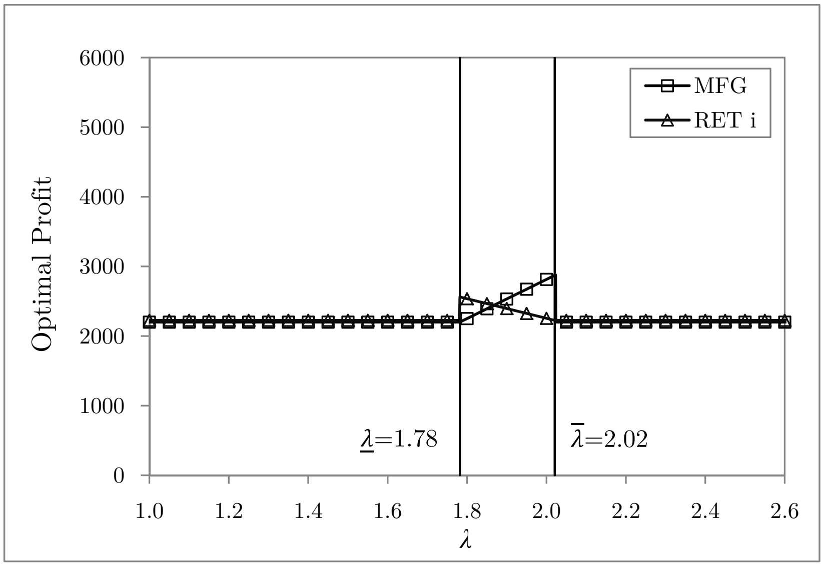

Figure 5.9: Resulting Profits

Figure 5.9 can be directly obtained from Figure 5.8. In Figure 5.9, when $\lambda \leq \underline{\lambda}$, the manufacturer prefers the transshipment agreement to the coordinating contract; when $\lambda \geq \bar{\lambda}$, the retailer prefers the transshipment agreement to the coordinating contract; when $\underline{\lambda} \leq \lambda \leq \bar{\lambda}$, both the manufacturer and the retailer prefer the coordinating contract to the transshipment agreement; thus, both accept the coordinating contract and receive their coordinated profit.

5.5 Summary

This chapter presents the analysis of an advance-purchase contract in a manufacturer-retailers supply chain that allows transshipment between the two retailers. In practice, transshipment is utilized in many industries to reduce the supply-demand mismatch. However, the manufacturer tries to maximize his profit by selling products to the retailers, and the retailers tend to request a high transshipment price to maximize their profit; thus, two double-marginalizations exist.

The proposed advance-purchase contract helps align retailers' and supply chain's objectives; hence, eliminating the double marginalizations. The coordination mechanism includes an advance-purchase contract in a supply chain that consists of a manufacturer and two retailers with transshipment. The manufacturer decides the wholesale prices, transshipment prices and transshipment premium. This contract is developed in two steps, in which it first aligns the joint objective of the retailer group with the objective of the supply chain, and second aligns the objective of each individual retailer with the joint objective of the retailer group. We show that this contract achieves supply chain coordination with the arbitrary profit split, and also ensures a Pareto improvement.

There are three possible directions for future research. First, although our coordinating approach can coordinate a supply chain even when two retailers are asymmetric, we have not shown an extensive analysis when retailers are asymmetric. Second, a supply chain in reality could have more than two retailers. Therefore, a study of such supply chain would be straightforward to pursue. Third, there could be a situation where a manufacturer has limited production capacity when the manufacturer does not allow a job preemption, or when expediting job is not cost-effective. Therefore, he is unable to immediately start his second production to satisfy the second order(s) of retailers in a timely manner. In which case, the manufacturer would only have a single production opportunity, and have to make his production decision in advance taking into consideration the possibility of the second order(s) of the retailers.

CHAPTER 6

CONCLUSIONS

This research is motivated by practical examples in, for instance, the automotive industry, and the advantages of the advance-purchase contract over the existing mechanisms previously developed in the literature. Existing literature uses a buy-back contract and/or lost-sale penalty to coordinate a supply chain with sales effort. However, there are practical drawbacks associated with them. For a buy-back, there are undesirable processes to complete from manufacturer's and retailer's perspectives that would incur extra cost and require additional time and effort to administer. Likewise, it is difficult to accurately track lost sales. Typically, a company addresses this issue by investing in a pricey monitoring system, such as VMI, EDI or PoS, yet are still unable to capture an exact amount of lost sales. In contrast, the advance-purchase contract does not require such processes or system; thus, minimizing an amount of effort, time and investment while improving supply chain's performance.

In this dissertation, we study impacts of the advance-purchase contract and supply chain coordination in two different supply chains. We first consider the supply chain with the manufacturer and the retailer, who can exert sales effort to stipulate customer demand. We then consider the supply chain with the manufacturer and the two retailers, who after observing and fulfilling their own demand can transship products between them to satisfy their unmet demand as a result of an inventory shortage. We apply the advance-purchase contract to both studies to allow the retailers to have two ordering opportunities. The first (advance) order receives a discount, and the second (regular) order is used as an "emergency" action to satisfy unmet demand in the final stage. We use Game Theory to formulate the models and analyze the games. For both supply chains, we develop the contracts, which are specified by the manufacturer, to coordinate the retailers' decisions and allow arbitrary profit split. To the best of our knowledge, this dissertation is the first study that utilizes the advance-purchase contract, and successfully coordinates these two supply chains.

Our coordination approaches work as follow. In the first study, it allows the manufacturer to ensure the retailers optimal decisions by properly specifying the unit rebate and the second wholesale price, and to assure a Pareto improvement by adjusting the first wholesale price and the rebate target level. In the second study, our coordination approach is developed in two steps. It first aligns the objective of the retailer group with the objective of the supply chain, and second aligns the objective of each individual retailer with the objective of the retailer group. In addition, the manufacturer specifies the contract terms, transshipment premium and arbitrary profit split to assure a Pareto improvement.

We analytically and numerically show that supply chain coordination with arbitrary profit split is achievable, and the coordinating contracts lead to Pareto improving situations in both studies. In addition, adding the second ordering opportunity increases the supply chains' profits, and at the same time lowers the optimal order quantity of the supply chains. The reason is the second order allows the retailers to meet all demand at the end of the selling season, while reducing the profit margin of the first order.

In this dissertation, we assume the manufacturer has infinite capacity and can satisfy retailers' order in a timely manner. A possible extension is to consider finite capacity for the manufacturer or a long production lead-time. In which case, the manufacturer must also decide production quantity, which later becomes a constraint in satisfying the second retailers' order. For the first study, possible future research includes the development of a contract which assures fully-flexible arbitrary profit split or allows the players to share the cost of sales effort. The study with different forms of the demand-effort model is also worthwhile to pursue. For the second study, the development of a coordinating mechanism, which includes transshipment in a supply chain with a manufacturer and n ($n > 2$) retailers, is an interesting research direction.

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APPENDIX A

ADDITIONAL DERIVATIONS FOR CHAPTER 4

A.1 Centralized Supply Chain with Minimum Sales Effort

We derive an optimal order quantity of the supply chain. The expected profit of the supply chain is

$$\pi^C = -c_1 Q + \mathbb{E}\left\{r \min\{Q, D\} + (r - c_2)(D - Q)^+\right\}. \tag{A.1}$$

The optimal order quantity satisfies $\Phi^{-1}\left((c_2 - c_1)/c_2\right)$.

A.2 Decentralized Supply Chain under Advance-Purchase Contract with Minimum Sales Effort

We derive an optimal ordering decision of an independent retailer who exerts minimum sales effort. Under this supply chain setting, the retailer's expected profit is

$$\pi_R^A = -w_1 Q + \mathbb{E}\left\{r \min\{Q, D\} + (r - w_2)(D - Q)^+\right\}, \tag{A.2}$$

where the retailer pays $w_1 Q$ for her first order, and sells them at the retail price r, and makes profit of $r - w_2$ for each unit in her second order quantity. Similarly, the retailer's expected profit can be expressed by using integrals as follows:

$$\pi_R^A = (r - w_1) Q - r \int_0^Q (Q - D)\, d\Phi(D) + (r - w_2) \int_Q^\infty (D - Q)\, d\Phi(D). \tag{A.3}$$

Thus,

$$(\partial/\partial Q)\, \pi_R^A = r - w_1 - r\Phi(Q) - (r - w_2)\left[1 - \Phi(Q)\right], \tag{A.4}$$

$$= w_2 - w_1 - w_2 \Phi(Q). \tag{A.5}$$

The optimal order quantity satisfies $\Phi^{-1}\left((w_2 - w_1)/w_2\right)$.

A.3 Decentralized Supply Chain under Advance-Purchase and Target Rebate Contract with Minimum Sales Effort

We first consider an independent retailer, who exerts minimum sales effort. Under this setting, retailer's expected profit is

$$\pi_R^B(Q|T) = -w_1 Q + \mathbb{E}\left\{r\min\{Q,D\} + u\left(\min\{Q,D\}-T\right)^+ + (r-w_2)(D-Q)^+\right\}. \tag{A.6}$$

Compared with Eq. (4.3), this is a special case when $e = 1$. In addition, the retailer receives the rebate u from the manufacturer for every unit she sells from her inventory above the target level T, as represented by the third term in Eq. (A.6). Alternatively, the retailer's expected profit can be expressed based on the condition on her order size and a target level as follows:

$$\pi_R^B = \begin{cases} (r-w_1)Q - r\int_0^Q (Q-D)\,d\Phi(D) \\ \qquad\qquad + (r-w_2)\int_Q^\infty (D-Q)\,d\Phi(D), \quad \text{if } Q \le T; \\ (r-w_1)Q - r\int_0^Q (Q-D)\,d\Phi(D) \\ +u\left(\int_T^Q (D-T)\,d\Phi(D) + (Q-T)\left[1-\Phi(Q)\right]\right) \\ \qquad\qquad + (r-w_2)\int_Q^\infty (D-Q)\,d\Phi(D), \quad \text{if } Q > T. \end{cases} \tag{A.7}$$

Thus, the first and second order derivatives with respect to Q are

$$(\partial/\partial Q)\,\pi_R^B = \begin{cases} r - w_1 - r\Phi(Q) - (r-w_2)\left[1-\Phi(Q)\right], \quad \text{if } Q \le T; \\ r - w_1 - r\Phi(Q) + u\left[1-\Phi(Q)\right] \\ \qquad\qquad - (r-w_2)\left[1-\Phi(Q)\right], \quad \text{if } Q > T, \end{cases} \tag{A.8}$$

$$= \begin{cases} w_2 - w_1 - w_2\Phi(Q), & \text{if } Q \le T; \\ w_2 + u - w_1 - (w_2+u)\Phi(Q), & \text{if } Q > T, \end{cases} \tag{A.9}$$

and

$$\left(\partial^2/\partial Q^2\right)\pi_R^B = \begin{cases} -w_2\phi\left(Q\right), & \text{if } Q \leq T; \\ \\ -\left(w_2+u\right)\phi\left(Q\right), & \text{if } Q > T. \end{cases} \tag{A.10}$$

Define $q^A \equiv \Phi^{-1}\left(\left(w_2 - w_1\right)/w_2\right)$, and $q^B \equiv \Phi^{-1}\left(\left(w_2 + u - w_1\right)/\left(w_2 + u\right)\right)$. Also define $f\left(T\right) \equiv \pi_R^B\left(q^A|T\right) - \pi_R^B\left(q^B|T\right)$ on $T \in \left[q^A, q^B\right]$, and define τ_0 such that $f\left(\tau_0\right) = 0$, i.e., the target level threshold under the minimum-effort scenario that makes the retailer indifferent between ordering the optimal quantity in the with-rebate and without-rebate cases.

Lemma A.1 *(a) τ_0 exists, is unique, and satisfies $\tau_0 \in \left(q^A, q^B\right)$. (b) The optimal order quantity for the retailer under the advance-purchase and target rebate contract, q^*, is given by the following: If $T < \tau_0$, then $q^* = q^B$; if $T > \tau_0$, then $q^* = q^A$; if $T = \tau_0$, then the retailer is indifferent between ordering q^A and q^B.*

Proof. It is straightforward to show that if $T \leq q^A$, then q^B maximizes π_R^B, and if $T \geq q^B$, then q^A maximizes π_R^B. If $q^A \leq T \leq q^B$, then $\pi_R^B\left(q^A|T\right) = w_2\Gamma\left(q^A\right) + \left(r - w_2\right)\mathbb{E}\left\{D\right\}$, and $\pi_R^B\left(q^B|T\right) = \left(w_2+u\right)\Gamma\left(q^B\right) - u[\Gamma\left(T\right) + T\left(1-\phi\left(T\right)\right)] + (r - w_2)\mathbb{E}\left\{D\right\}$. Because $f_0\left(q^A\right) < 0 < f_0\left(q^B\right)$ and $f_0\left(\cdot\right)$ is continuous and increasing, there exists a single-valued inverse function f_0^{-1} and a unique τ_0; further, $\tau_0 \in \left(q^A, q^B\right)$. If $q^A < T < q^B$, then $\lim_{Q\to T^-}\left(\partial/\partial Q\right)\pi_R^B\left(Q|T\right) < 0 < \lim_{Q\to T^+}\left(\partial/\partial Q\right)$ $\pi_R^B\left(Q|T\right)$. Because q^A maximizes $\pi_R^B\left(Q|T\right)$ on $[0, T)$ and q^B maximizes $\pi_R^B\left(Q|T\right)$ on $\left(T, \infty\right)$, $Q_0^* = \arg\max_{Q\in\{q^A,q^B\}}\pi_R^B\left(Q|T\right)$. If $T < \tau_0$, then $f_0\left(T\right) < 0$ and $\pi_R^B\left(q^B|T\right) > \pi_R^B\left(q^A|T\right)$. If $T > \tau_0$, then $f_0\left(T\right) > 0$. ■

A.4 Model Formulation for Supply Chain under Target Rebate-Only Contract with Sales Effort

Under the target rebate-only contract, the retailer's expected profit is

$$\Pi_R^T = -wQ + \mathbb{E}\left\{r\min\left\{Q, eD\right\} + u\left(\min\left\{Q, eD\right\} - T\right)^+\right\} - V\left(e\right). \tag{A.11}$$

The retailer incurs procurement cost and sales effort cost as shown by the first and last terms in Eq. (A.11), and makes profit by fulfilling customer's demand and receiving the rebate from the manufacturer as shown by the second and third terms in Eq. (A.11). The expected manufacturer's profit is

$$\Pi_M^B = (w - c)\, Q - \mathbb{E}\left\{u\left(\min\{Q, eD\} - T\right)^+\right\}. \tag{A.12}$$

The manufacturer makes profit by satisfying the retailer's order, and incurs the cost of providing the rebate to the retailer as respectively given out in Eq. (A.12). For the thorough analysis, we refer readers to Taylor [11] or Chiu et al. [139].

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