

A MATHEMATICAL MODEL OF VIRUS  
TRANSPORT AND SURVIVAL  
IN GROUNDWATER

By

ANNIE D. HOGSETT  
" "

Bachelor of Science in Arts and Science

Oklahoma State University

Stillwater, Oklahoma

1981

Submitted to the Faculty of the  
Graduate College of the  
Oklahoma State University  
in partial fulfillment of  
the requirements for  
the Degree of  
MASTER OF SCIENCE  
July, 1985

Thesis  
1985  
H 716m  
cop. 2



A MATHEMATICAL MODEL OF VIRUS  
TRANSPORT AND SURVIVAL  
IN GROUNDWATER

Thesis Approved:

*Jan Wignar*  
\_\_\_\_\_  
Thesis Adviser

*Mayis Seapan*  
\_\_\_\_\_

*Merrin J. Keen*  
\_\_\_\_\_

*Norman D. Murham*  
\_\_\_\_\_  
Dean of the Graduate College

## PREFACE

This project presents a mathematical model for predicting virus transport and survival capabilities in groundwater. The basis for the model is a virus mass balance which results in a nonlinear partial differential equation. Closed form solutions to the general equation have been developed utilizing both the linear and Freundlich isotherms.

The solutions formulated for each isotherm have been converted into an interactive FORTRAN computer program. The purpose of the program is to provide easily obtainable predictions of safe distances between home septic tank systems and private drinking water supplies.

The model was tested using typical values for soil hydraulic properties and adsorption coefficients. Inactivation rates used in the tests were those previously reported in the literature. The results of the tests were good, and when more specific data is available, the testing should be completed.

I would like to express my sincere thanks to my friends, family, and co-workers for their support and patience both

during my stay at Oklahoma State and during the completion of this project. I would also like to thank Dr. M. Keener for his mathematical justification and Dr. J. Wagner, the major adviser of the project.

## TABLE OF CONTENTS

Chapter	Page
I. INTRODUCTION. . . . .	1
Statement of Problem . . . . .	1
Objectives . . . . .	2
II. LITERATURE REVIEW . . . . .	4
Introduction . . . . .	4
The Compartment Vector Approach. . . . .	4
The Virus Mass Balance Approach. . . . .	5
III. THE MODEL . . . . .	9
Development of the Model . . . . .	9
Limitations . . . . .	9
Assumptions . . . . .	10
Mass Balance. . . . .	11
Expansion of Solid Phase Term . . . . .	14
Expansion of Inactivation Term. . . . .	16
Mathematical Solution of the Model . . . . .	18
Computer Solution of the Model . . . . .	21
IV. RESULTS AND DISCUSSION. . . . .	27
V. CONCLUSION AND RECOMMENDATIONS. . . . .	56
SELECTED BIBLIOGRAPHY. . . . .	58
APPENDIXES . . . . .	60
APPENDIX A - THE COMPUTER PROGRAM . . . . .	61
APPENDIX B - COMPLETE OUTPUT OF A COMPUTER RUN. . . . .	73

## LIST OF TABLES

Table	Page
I. Typical Parameters Used for Model Verification Calculations . . . . .	29
II. Definition Page for a Computer Run Using Large Solid and Liquid Phase Inactivation Rates . . . . .	31
III. Tabular Output Utilizing the Linear Isotherm and Showing the Reduction of Virus Concentration Due to Large Liquid and Solid Phase Inactivation Rates . . . . .	32
IV. Tabular Output Utilizing the Freundlich Isotherm and Showing the Reduction in Virus Concentration Due to Large Liquid and Solid Phase Inactivation Rates. . . . .	35
V. Definition Page for a Computer Run Using Small Solid and Liquid Phase Inactivation Rates . . . . .	40
VI. Tabular Output Utilizing the Linear Isotherm and Showing the Increase in Virus Concentration Due to a Large Source Inactivation Rate . . . . .	41
VII. Tabular Output Utilizing the Freundlich Isotherm and Showing the Increase in Virus Concentration Due to a Large Source Inactivation Rate. . . . .	44
VIII. Definition Page for a Computer Run with Equal Inactivation Rates. . . . .	48
IX. Tabular Output Utilizing the Linear Isotherm and Showing a Constant Virus Concentration Due to Equal Inactivation Rates . . . . .	49

Table

Page

X.	Tabular Output Utilizing the Freundlich Isotherm and Showing a Slight Increase in Virus Concentration Due to Plug Flow . . .	52
----	--	----



LIST OF FIGURES

Figure	Page
1. Virus Flow Through Differential Control Volume for the Virus Mass Balance . . . . .	11
2. Computer Program Flow Chart . . . . .	23
3. A Comparison of Virus Concentrations Resulting from Calculations Utilizing the Linear (—) and Freundlich (---) Isotherms When Small Source Inactivation Rates are Used. . . . .	38
4. A Comparison of Virus Concentrations Resulting from Calculations Utilizing the Linear (—) and Freundlich (---) Isotherms When Large Source Inactivation Rates are Used. . . . .	47
5. A Comparison of Virus Concentrations Resulting from Calculations Utilizing the Linear (—) and Freundlich (---) Isotherms When Inactivation Rates are Equal . . . . .	55

## NOMENCLATURE

a	A unique solution to Equation 34a, used only in Appendix A
A	Two dimensional area of the differential control volume, distance squared
C	Virus concentration, plaque units per volume of fluid
$C_0$	Initial virus concentration, plaque units per volume
$C_s$	Virus concentration adsorbed on grains, plaque units per volume
$C_T$	The total virus concentration, plaque units per volume of porous media
$\hat{C}(x)$	The limit of concentration as time approaches infinity, used only in Appendix A
$\dot{C}(u)$	The differential function of concentration for the parameter u, used only in Appendix A
f(C)	A general function of concentration representing the individual isotherms
g(C)	A general function of concentration representing the partial differential of the individual isotherms with respect to concentration
k	The adsorption coefficient, volume per mass
K	Concentration at sometime t, used only in Appendix A
n	The exponent in the Freundlich Isotherm
p(t)	Some general function of concentration in time at a distance of zero

$Q$	Flux
$r_t$	Virus natural inactivation rate
$t$	time
$\dot{t}(u)$	The differential function of time in a system of equations, used only in Appendix A
$t(u)$	A general function of time in a system of equations, used only in Appendix A
$u$	Some general parameter of concentration, time and distance, used only in Appendix A
$\bar{u}$	The value for the parameter $u$ for which $x(u) \equiv x$ , used only in Appendix A
$u_0$	The value for the parameter $u$ for which $C(u_0) = 0$ , used only in Appendix A
$u_1$	The value for the parameter $u$ for which $x(u_1) = x$ and $t(u_1) = t$ , used only in Appendix A
$V$	Superficial or Darcy Velocity, distance per time
$x$	Distance
$\dot{x}(u)$	The differential function of distance in a system of equations, used only in Appendix A.
$x(u)$	A general function of distance in a system of equations, used only in Appendix A
$\dot{y}(u)$	A differential equation representing part of the transform of system (4a), used only in Appendix A
$y(u)$	A general function of the parameter $u$ representing one dimension of the transform of system (4a), used only in Appendix A
$\dot{z}(u)$	A differential equation of the parameter $u$ , used only in Appendix A
$z(u)$	A general function of the parameter $u$ , used only in Appendix A

$\alpha$	The first order decay rate of the virus concentration, 1/time
$\lambda_f$	The virus inactivation rate in the fluid phase, 1/time
$\lambda_s$	The virus inactivation rate in the solid phase, 1/time
$\omega(x)$	A function of distance, used only in Appendix A
$\mu$	Viscosity
$\rho_\beta$	The bulk density of the soil, mass per volume
$\gamma$	A function of the parameter $u$ , used only in Appendix A
$\theta$	Porosity

## CHAPTER I

### INTRODUCTION

#### Statement of the Problem

The purpose of this project is to develop a mathematical model for determining safe distances between domestic waste water disposal sites and drinking-water supplies. On-site water supply wells and septic tanks are among the most common and economical ways to provide suburban and rural areas with potable water and waste disposal capabilities. As these areas grow in population and geographical size, supplies of unpolluted water become dangerously small. In addition, the cumulative effect of waste percolation into the groundwater has increased to an alarming level.

The United States is dependent upon groundwater for 95 percent of its fresh water supply (1). However, biological contamination of the groundwater by septic tanks and cesspools is a major problem. An estimated 60,000,000 people in North America use septic tanks for their waste water disposal. Most sparsely populated rural areas, as

well as many densely populated areas, depend solely on septic tank systems (2). The reduction of the organic load, fecal bacteria, and virus populations by these on-site treatment systems is very limited. In actuality, the majority of the waste water treatment must be provided by the soil (3).

Over half of the waterborne disease outbreaks in the United States are caused by contaminated groundwater (4). Some 800 billion gallons of raw sewage from septic tanks and cesspools leach into the ground on an annual basis. In addition, 250 billion gallons of raw and treated sewage reach the soil from leaking municipal sewage systems (5). From these few facts, it is obvious that the present septic tank systems are inadequate, and therefore must be compensated for. In other words, if the soil is left to complete the necessary treatment, either the amount of waste deposited must be decreased, or the distance between deposition points and drinking water supplies must be increased.

### Objectives

This paper presents a deterministic mathematical model that predicts the survival and transport capabilities of viruses in groundwater. The purpose is to evaluate the

movement of viruses from a home septic tank system to a private water well supply.

## CHAPTER II

### LITERATURE REVIEW

#### Introduction

In the last ten years, there have been several models of virus transport presented in the literature. Generally, the models can be separated into two types according to their basic format. That is to say that one category of model is based on virus mass balances, while the other is based on what is referred to as a "compartment vector" approach. Because the models within each category are very similar, we will not provide an exhaustive analysis of each model. Instead, an explanation of each model group will be provided, along with representative examples.

#### The Compartment Vector Approach

Two groups (6,7) have published models that use a "compartment vector" approach. In this method, a mathematical statement represents each discrete point in the virus movement through the subsurface. This type of model is often called a lumped parameter system. In this type of model, the



path of the virus is followed as it travels through the environment. Each point in the pathway is described by a separate mathematical statement.

Models using this method have been successful in fitting the available data. However, there are several reasons why the resulting models are undesirable for application in this project. It is hoped that the model resulting from this project will eventually be used to help solve the rural and suburban problems mentioned earlier. Thus, the final model must be easily applicable to any area of the country. In addition, the model should require both minimal computer time and minimal input data. The compartment vector approach results in a series of equations, and the computer time necessary to solve them is quite extensive. In addition, these models usually require large amounts of input data, even as detailed as subsurface temperature, soil pH, and amount of rainfall.

#### The Virus Mass Balance Approach

The second major type of model is based on virus mass balances. Although there are often large differences in the final form of these equations, an understanding of their common basis makes their development more clear.

Vilker (1,8,9) has been one of the most prominent authors in this area. His mass balance consists of the sum of a liquid-phase depletion term ( $\partial C/\partial t$ ), a convective transport term ( $V\partial C/\partial x$ ), and a solid phase accumulation term. Vilker's model fits data from batch systems (10,11) well. However, he incorporated some simplifying assumptions. Elimination of one of these assumptions would increase the similarity of the resulting model to the in vivo situation.

Vilker assumed that natural inactivation was accounted for within the other three terms. On the contrary, the mass balance is more correctly stated as "IN-OUT+GENERATION=ACCUMULATION." In a virus transport model, the "IN" and "OUT" terms are accounted for in the convective transport term. The accumulation term is expressed by the solid phase accumulation term. In addition, there must be a "GENERATION" term. In the case of viruses in the subsurface, the generation term will be negative. The virus concentrations are constantly being depleted by natural inactivation. Depletion of viruses due to natural inactivation is separate from losses due to soil adsorption or exiting viruses by convective transport.

Another criticism of Vilker's model involves the liquid phase virus concentration. As just stated, the convection

term that Vilker included actually handled any concentration changes in the liquid phase. The viruses that leave the liquid phase accumulate in the solid phase. That transfer only needs to be accounted for once. Therefore, the liquid phase depletion term and the solid phase accumulation term express the same physical situation.

Yet, despite any minor differences in approach or format, Vilker's model fit the data well. Moreover, his studies laid the ground work for this and many other studies.

Grosser (4) is another author whose work is formatted much like Vilker's. Grosser also began with the concept of a virus mass balance. His model was an improvement over Vilker's model in terms of natural inactivation; his included a separate term expressing natural die-off. However, the model had the same short-comings in the treatment of liquid phase depletion.

The mass balance on which Grosser based his model consisted of a convective transport term, a term for natural inactivation, a dispersion term, and a term for the changes in concentration with respect to time. As stated earlier, the liquid phase concentration has been accounted for in the convective transport term. The liquid phase depletion term and the solid phase accumulation are redundant.

The final point to be discussed concerning Grosser's model is that of dispersion. As stated previously, Grosser included a dispersion term in his model. It is well known that dispersion of virus concentrations occurs in the subsurface (4). Thus, the inclusion of such a term should increase the accuracy of the model. Certainly, including dispersion improves the model's representation of the actual subsurface condition. However, actual dispersion rates have not been accurately quantified at this time. Thus, the model may be a more realistic representation, but at the same time, less accurate.

Like Vilker's model, Grosser's work was instructive for further studies. His solution using the linear isotherm fitted the data (12) relatively well. In this project, the model will be based on a virus mass balance, with much of the same approach that Vilker and Grosser took. However, instead of stopping with a solution only by the linear isotherm, we also hope to solve the model with the Freundlich Isotherm.

## CHAPTER III

### THE MODEL

#### Development of the Model

##### Limitations

Unfortunately, the availability of actual data concerning the behavior of viruses in the subsurface is extremely limited. Many in vitro studies have been, and are being performed. However, these provide, at best, only an estimate of the in vivo situation.

For example, as mentioned in the preceeding chapter, one factor that affects the survival of viruses in the groundwater is natural inactivation or viral die-off. Some viruses can lay dormant for extended periods of time, but all will eventually die without a host. The rate of natural inactivation can be estimated in the laboratory (13). However, it is also known that virus concentrations will disperse in the subsurface. This dispersion rate cannot at this time, be predicted by in vitro studies.

In summary, data are not available to accurately quantify what is actually happening below the surface.

This deficiency requires that several basic assumptions be incorporated into the model. The following section discusses all these assumptions.

### Assumptions

In this project, we have assumed that: (1) the solutions are dilute; (2) the tracers are non-conservative; (3) mass transfer occurs primarily through convective transport; (4) flow is uniform and one dimensional; and (5) the medium is homogeneous, non-compressible, porous and void of strata.

The assumption that all solutions are dilute infers that the fluid properties are independent of concentration. Thus, the density, viscosity, and other similar characteristics of the waste water solution can be considered equivalent to those of pure water. This assumption is beneficial not only in simplifying the model, but also in minimizing the required input data.

Assuming a non-conservative tracer means that the viruses are affected by the environment. The two major modes of elimination are adsorption on the soil grains and natural inactivation.

In the subsurface, the possible modes of virus transport are convective transport, hydrodynamic dispersion,

and molecular diffusion. As of this time accurate values for dispersion have not been quantified. Thus, including a dispersion term only reduces the validity of the resulting model. Molecular diffusion can be measured, but its influence is negligible in comparison to that of convective transport. As a result, virus transport has been assumed to occur only by convective transport.

### Mass Balance

Once these assumptions are established, a mass balance can be written for a differential control volume of soil. Recall that one dimensional flow has been assumed. Thus, "A" will refer to the two dimensional area through which the virus concentration flows.

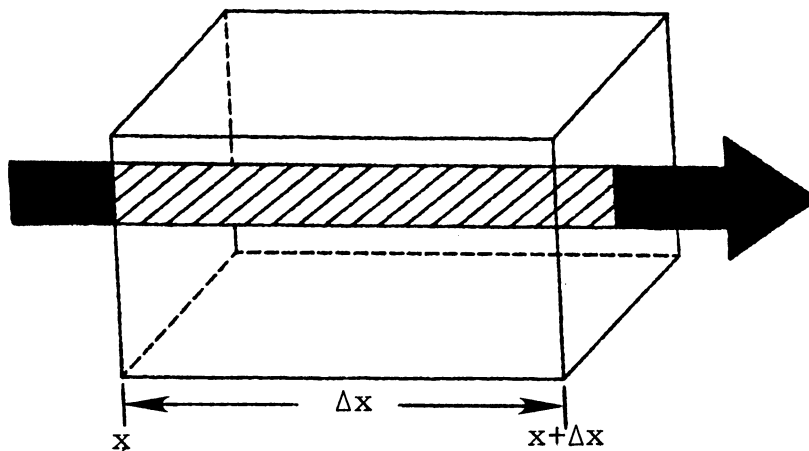


Figure 1. Virus Flow Through Differential Control Volume for the Virus Mass Balance

Considering the dimensions of the control volume in conjunction with Darcy's Law,

$$\frac{Q}{A} = \frac{-k(\Delta P)}{\mu \Delta x} = v \quad (1)$$

Therefore,

$$(V)(A) = q = \left( \frac{\text{Unit Volume of Liquid}}{\text{Time}} \right) \quad (2)$$

and multiplying by the concentration, we have

$$(C)(V)(A) = \left( \frac{\text{Mass of Viruses}}{\text{Unit Volume, Liquid}} \right) \times \left( \frac{\text{Unit Volume, Liquid}}{\text{Time}} \right) \quad (3)$$

Equation (3) reduces to

$$(C)(V)(A) = \left( \frac{\text{Mass of Viruses}}{\text{Time}} \right) \quad (4)$$

Now, recall the basic mass balance equation, "IN-OUT + GENERATION = ACCUMULATION." As stated, the equation requires an expression for each term; accumulation, generation, and entering and exiting virus concentrations. The assumption of one dimensional flow specifies that there is no infiltration of viruses except from the source. In addition, the viruses cannot reproduce without a host, but instead are continuously dying off. Thus, the generation term is negative. The accumulation term is due to the



adsorption of the viruses onto the soil grains. Now, elaborating on each term of the mass balance equation,

$$\text{IN} - \text{OUT} + \text{GENERATION} = \text{ACCUMULATION} \quad (5)$$

$$\begin{array}{l} \text{VIRUSES ENTERING} - \text{VIRUSES EXITING} + \text{RATE OF VIRUSES} \\ \text{IN LIQUID} \qquad \qquad \text{IN LIQUID} \qquad \qquad \text{PRODUCED} \end{array} \quad (6)$$

$$= \begin{array}{l} \text{VIRUSES IN ROCK} - \text{VIRUSES IN ROCK} \\ \text{AT TIME } t+\Delta t \qquad \qquad \text{AT TIME } t \end{array}$$

Rewriting the equation in terms of the symbols defined in the nomenclature yields

$$\begin{aligned} & \left( CVA \Big|_x \Delta t \right) - \left( CVA \Big|_{x+\Delta x} \Delta t \right) + r_t A \Delta x \Delta t \\ & = \left( C_T A \Big|_{t+\Delta t} \Delta x \right) - \left( C_T A \Big|_t \Delta x \right). \end{aligned} \quad (7)$$

Now dividing by the area of the control volume, A, and rearranging, the equation becomes

$$\left( CV \Big|_{x+\Delta x} - CV \Big|_x \right) \Delta t + \left( C_T \Big|_{t+\Delta t} - C_T \Big|_t \right) \Delta x = r_t \quad (8)$$

Further, dividing by  $\Delta x \Delta t$  and taking the limit as  $\Delta x$  and  $\Delta t$  approach zero, Equation (8) becomes,

$$v \frac{\partial C}{\partial x} + \frac{\partial C_T}{\partial t} = r_t \quad (9)$$

Expansion of Solid Phase Term

The term expressing the total mass of viruses per unit volume of porous medium,  $C_T$ , can be expanded. Recall from the nomenclature,  $C_S$ , the virus concentration attached to the soil grains, the soil bulk density,  $\rho_\beta$ , and the soil porosity,  $\theta$ , then,

$$\begin{aligned} \frac{\text{Mass of Viruses}}{\text{Unit Volume of Porous Media}} &= \frac{\text{Volume of Voids}}{\text{Volume of Porous Media}} \\ &\times \frac{\text{Mass of Viruses}}{\text{Volume of Voids}} \\ &+ \frac{\text{Mass of Solids}}{\text{Volume of Porous Media}} \\ &\times \frac{\text{Mass of Viruses}}{\text{Mass of Solids}} \end{aligned} \quad (10)$$

This can also be expressed mathematically as,

$$C_T = \theta C + \rho_\beta C_S \quad (11)$$

Furthermore, recall that the medium was assumed to be homogeneous, nondeformable, and void of strata. Another way to state this assumption is that the soil has constant physical properties. By assuming constant physical properties, Equation (11) can be partially differentiated as,

$$\frac{\partial C_T}{\partial t} = \theta \frac{\partial C}{\partial t} + \rho_B \frac{\partial C_S}{\partial t} .$$

Note that  $C_S$ , previously defined as the virus concentration attached to the soil grains, is some function of the source concentration or  $C_S = f(C)$ . Also by the chain rule,

$$\frac{\partial C_S}{\partial t} = \frac{\partial C_S}{\partial C} \times \frac{\partial C}{\partial t} . \quad (13)$$

Now Equation (12) has two unknowns,  $C_S$  and  $C$ . Thus, a second equation is required. There are three possible adsorption isotherms that can replace the  $C_S$  term. Two of these isotherms have been used in this project; the linear isotherm,

$$C_S = kC ; \quad (14)$$

and the Freundlich isotherm,

$$C_S = kC^n . \quad (15)$$

To simplify the form of Equation (13), let the derivative of  $C_S$  with respect to concentration be some general function of concentration  $g(C)$ . Then the partial differential of the isotherms with respect to time, can be expressed as

$$\frac{\partial C_s}{\partial t} = g(C) \frac{\partial C}{\partial t} \quad (16)$$

Substituting Equation (12) and (16) into Equation (5), and rearranging yields,

$$\left[ \theta + \rho_\beta g(C) \right] \frac{\partial C}{\partial t} + v \frac{\partial C}{\partial x} = r_t \quad (17)$$

Equation (17) must be solved with the derivation of both the linear and Freundlich isotherms (with respect to concentration) substituted in for  $g(C)$ .

#### Expansion of Inactivation Term

At this point consider the natural inactivation (or viral die-off) term alone. This paper will consider three possible inactivation rates that the virus can experience. The source inactivation rate refers to the inactivation rate that the virus experiences at the point where it is discharged into the system. The data of Gerba (13) and that of Burge (10) show that source inactivation follows first order decay.

On the other hand, the virus inactivation rates within the subsurface are not necessarily the same. The depletion rate within the liquid phase should be allowed to vary

independently of both the depletion rate within the solid phase (adsorbed onto the soil grains) and the source inactivation rate. All three values might be equal in a particular situation. Still, the model should have the capability of allowing the rates to vary independently of one another.

A realistic representation of the subsurface requires that natural inactivation be separated as

$$\begin{aligned}
 \frac{\text{Rate of Mass Degraded}}{\text{Volume of Porous Media}} &= \frac{\text{Rate of Mass Degraded}}{\text{Unit Volume of Fluid}} \\
 &\times \frac{\text{Volume of Fluid}}{\text{Volume of Porous Media}} \\
 &+ \frac{\text{Rate of Mass Degraded}}{\text{Unit Mass of Solids}} \\
 &\times \frac{\text{Mass of Solids}}{\text{Volume of Porous Media}} . \quad (18)
 \end{aligned}$$

The same equation can be expressed mathematically as

$$r_t = \frac{\partial C_T}{\partial t} = \theta \frac{\partial C}{\partial t} + \rho \beta \frac{\partial C_S}{\partial t} . \quad (19)$$

To simplify the form of the natural elimination terms, the concentration of the dying viruses in the liquid phase may be redefined as

$$\frac{\partial C}{\partial t} = \lambda_f C \quad . \quad (20)$$

In the same way, the inactivation rate in the solid phase is redefined as

$$\frac{\partial C_s}{\partial t} = \lambda_s C_s \quad . \quad (21)$$

Equation (20) and (21) can be substituted into Equation (19) to yield

$$r_t = \theta \lambda_f C + \rho_\beta \lambda_s C_s \quad . \quad (22)$$

Again, recall that  $C$  is some general function of liquid phase concentration,  $C = f(C)$ . Thus substitution of Equation (22) into Equation (17) gives the final equation,

$$\left[ \theta + \rho_\beta g(C) \right] \frac{\partial C}{\partial t} + v \frac{\partial C}{\partial x} = \theta \lambda_f C + \rho_\beta \lambda_s f(C) \quad . \quad (23)$$

The adsorption isotherms will be substituted into Equation (23) in place of  $f(C)$ .

#### Mathematical Solution of the Model

Equation (23) is a nonlinear partial differential equation. The objective is to solve this equation for the concentration in terms of distance and time. These values

of concentration will be computed only where the function is differentiable in each isotherm stated for  $g(C)$ .

Also, the concentration will be investigated for two boundary conditions. At some distance  $x$ , at time zero, the concentration will be zero (for all  $x$  greater than zero). However, at a distance of zero, at some time  $t$ , the concentration will have a definite value computed as a function of time only. Temporarily define that computed value as some function of time  $p(t)$  (for all  $t$  greater than or equal to zero). The function  $p(t)$  is a positive, continuously differentiable function on the interval  $[0, \infty)$ , for which its derivative with respect to time is less than or equal to zero.

Furthermore, consider the function  $p(t)$  in two forms: (1)  $p(t) = k$ ; and (2)  $p(t) = ke^{-\alpha t}$ . Condition (1) expresses constant source virus concentration; whereas, condition (2) is representative of first order decay of the source. The term  $\beta$  is defined as the rate constant for the source inactivation. Also, note that if  $\alpha$  is set to zero, condition (2) reduces to condition (1). Therefore, Equation (23) must be solved for both conditions of  $p(t)$  and for both isotherms.

There are several possible methods of solution for Equation (23). The method of characteristic equations was

chosen. The general idea behind this method is first to reduce the original equation to a system of ordinary differential equations. Then, transform the system of equations into another variable system, and solve the new system that represents the original equation. The resulting equation is a closed form solution. Such a solution was obtained for both the linear and the Freundlich isotherms. The final forms of the equations are as follows:

incorporating the linear isotherm,

$$C(x,t) = C_0 e^{-\alpha t + [\alpha(\theta + k\rho_\beta) + (\theta\lambda_f + k\rho_\beta\lambda_s)]\frac{x}{V}}$$

$$\text{for } 0 \leq \frac{x}{V} \leq \frac{t}{\theta + k\rho_\beta} \quad ; \quad (24)$$

and using the Freundlich isotherm,

$$C(x,t) = C_0 e^{-a\left(\alpha + \frac{\lambda_s}{n}\right) + \frac{\theta}{V}[\lambda_f x + \frac{\lambda_s}{n}\left(\frac{V}{\theta}t - x\right)]} \quad (25)$$

$$\text{where } t = a + \frac{\theta x}{V} - \frac{n}{(n-1)\lambda_s}$$

$$x \ln \left[ 1 + \frac{k\rho_\beta\lambda_s}{\theta\lambda} C_0^{(n-1)} e^{-\alpha a(n-1)} (1 - e^{-(n-1)\theta\lambda_f x}) \right]$$

$$\text{and for } 0 \leq x \leq \frac{Vt}{\theta} \quad . \quad (26)$$



Realize that the terms  $tV/(\theta+k\rho_\beta)$ , in the linear isotherm and  $Vt/\theta$ , in the Freundlich Isotherm, refer to the "front". That is, the front is the line in the two dimensional control volume where on one side of the line there is a virus concentration of some value  $C(x,t)$  and on the other side of that line the virus concentration is equal to zero. In other words, the front is the leading edge of the virus concentration.

Thus, Equations (24) and (25) are only concerned with values of distance  $x$ , less than the distance at the front. The virus concentration at a distance greater than the front is zero.

A complete derivation of these equations is presented in Appendix A.

#### Computer Solution of the Model

An interactive computer program has been written to translate the mathematical statements and solution algorithms into "machine" instructions. The program is written in FORTRAN 77 and is compatible with most FORTRAN compilers.

The flow chart of the program is shown in Figure 2. As illustrated, the initial portion of the program allows

the user to enter all of the information necessary to run the program. The user must enter: (1) a title, date or time; (2) the units preferred by the user for distance, time, density, and concentration; (3) the soil hydraulic properties; (4) the virus characteristics; and (5) the time and distance to be tested.

After this information has been entered, the user will be prompted to choose an isotherm and enter any additional information necessary for solution by that isotherm. (For example, the exponent  $n$  is needed for the Freundlich isotherm.) Choosing the isotherm causes a program control variable to be set. This will be used later to determine which subroutine to call.

The values of time and distance are then used to calculate the number of points of observation, and the number of rows and columns in the output table. At this point, the appropriate subroutine is called, based on the chosen isotherm.

The subroutine begins to calculate values of the virus concentration. The calculations will first be performed for the initial time and distance. This procedure will be repeated at increments set by the user, until both time and distance are equal to their final values.

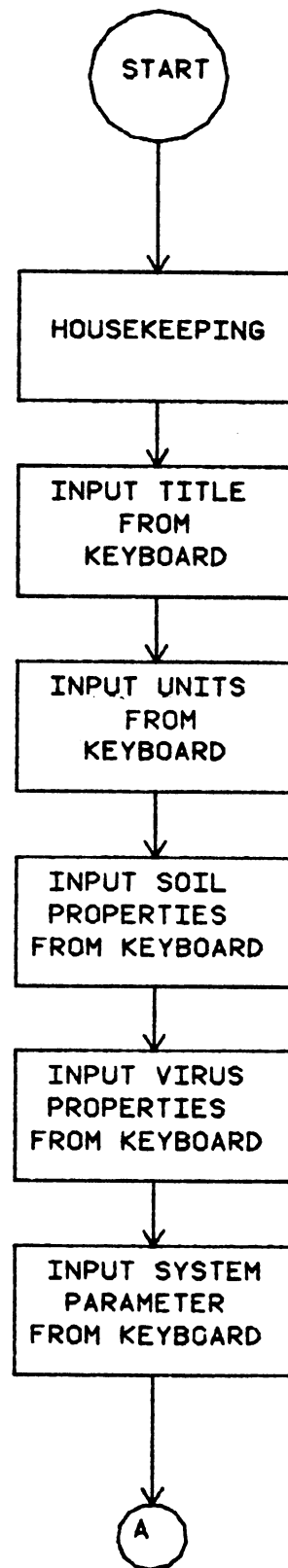


Figure 2. Computer Program Flow Chart

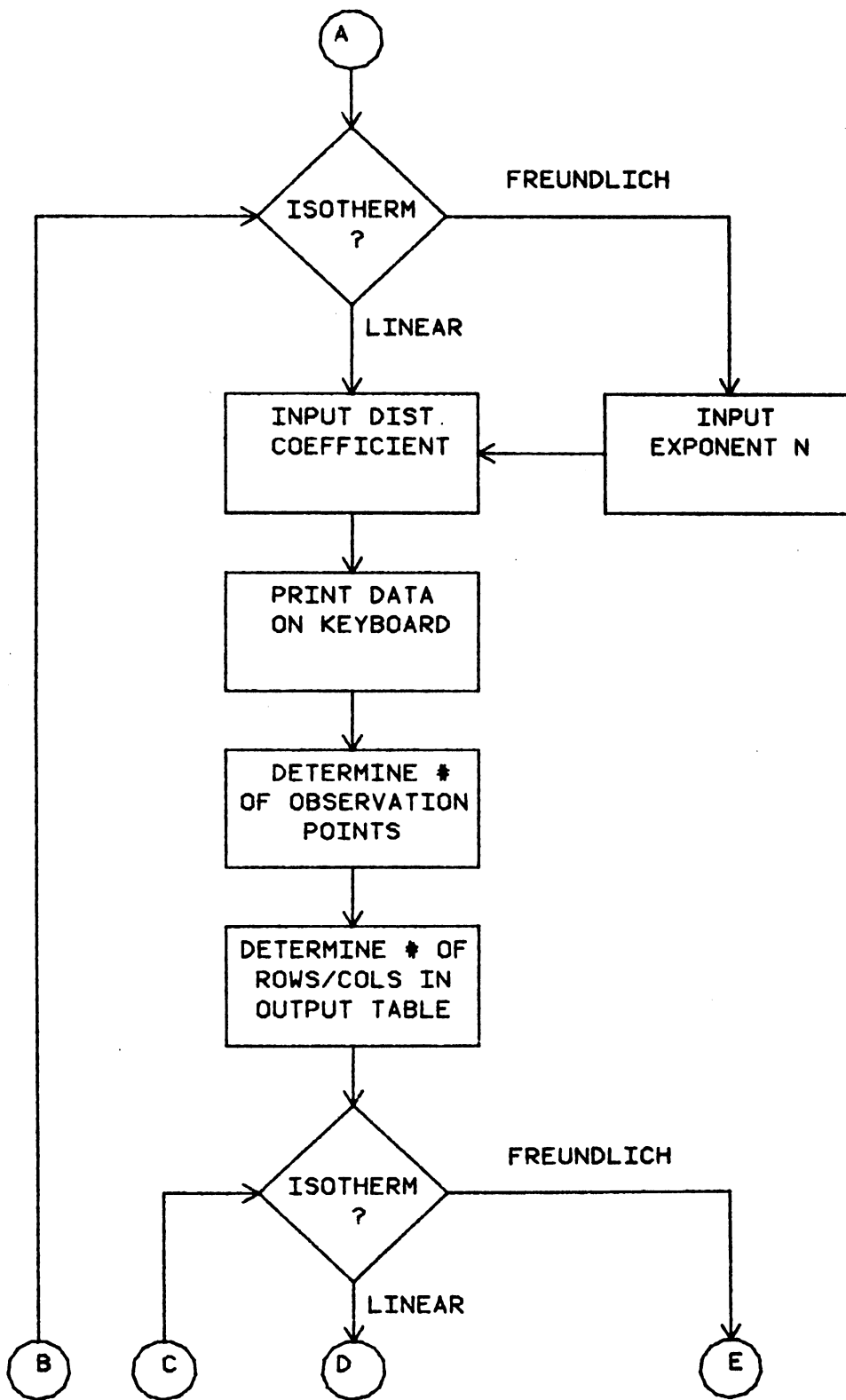


Figure 2. (Continued)

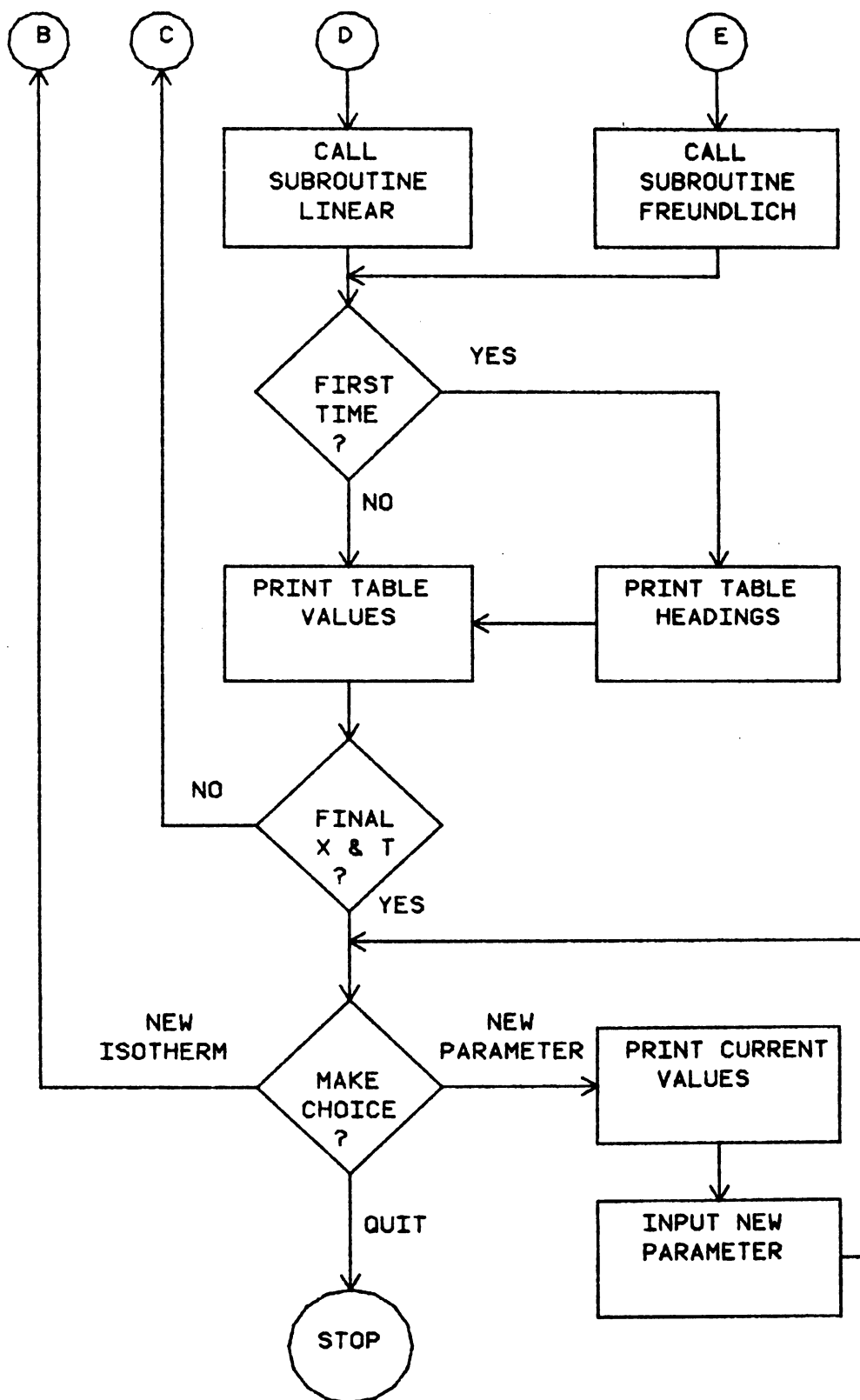


Figure 2. (Continued)

After the calculations have been completed and the values printed, the user is given a choice in the form of a menu:

Would you like to choose another isotherm,  
change your data or stop

- 1 - linear isotherm
- 2 - Freundlich isotherm
- 3 - Change Input Data
- 4 - Stop.

Options 1, 2, and 4 are obvious and will not be explained further. If Option 3 is chosen, the computer will print the current values of all user input data. The user may then change any of the information (including the title), make no change, or stop. Each time a value is changed, an updated list will be printed and the user has the same options to change the data. If "make no change" is chosen, the previous menu will be displayed, allowing the user to choose an isotherm, change data, or quit.

Several important design features were adhered to in this program. The program is compartmentalized to facilitate additions or omissions. The user is always provided with an easily understood menu. Also, all input from the user is validated by the program, and any invalid entry causes an error message to be printed. The prompt is then repeated until a valid entry is made.

## CHAPTER IV

### RESULTS AND DISCUSSION

The preceding chapters presented the development of the model, and in so doing, showed the main differences between this model and its predecessors described in the literature review. One of the differences is in the expansion of the natural inactivation term. As explained previously, natural inactivation has been separated into solid and liquid phase depletion terms to present what was thought to be a more realistic picture of the subsurface.

Dr. Charles P. Gerba, of Arizona State University, has been measuring virus inactivation rates in the laboratory. As of this time, he has been unable to determine the inactivation rate of the solid phase. Thus, the benefit derived from including these terms separately cannot yet be ascertained.

An important aspect of designing any mathematical model is testing it against documented data. The complete testing of this model, and thus the completion of this report cannot wait indefinitely for the remaining data to

be gathered. Furthermore, Dr. Gerba's preliminary studies have indicated that the source inactivation rate is virtually equal to the liquid phase inactivation rate. Also, because the viruses adsorbed onto the soil grains are no longer within the transport medium, he now considers the solid phase inactivation rate of little importance (13). However, regardless of the new developments in our understanding of virus behavior in the groundwater, the aim of this project remains the same. The testing of the computer program must be complete, even if the values tested depict an unrealistic situation. Therefore, to test the logic of the computer program, the solid and liquid phase inactivation rates have been given values that are: (1) equal to, (2) greater than, and (3) less than the value of the source inactivation rate.

Average values for soil hydraulic properties, and adsorption coefficients were used. Typical input values are shown on Table I. The values for the inactivation rates that were used initially were reported by Gerba (14). The values were then varied to check sensitivity.

The output from several computer runs are included in the following pages. To interpret this data, at least enough to check the workability of the model, recall the definitions of the inactivation rates. The source inactivation is the first order decay rate of the viruses.



That is, the source inactivation rate is the die-off rate at the point where the virus concentration is discharged into the system. That rate is represented by the equation

$$C(0,t) = C_0 e^{-\alpha t} \quad (24)$$

where  $\alpha$  is referred to as the rate constant of the source inactivation. (Note that the equation will reduce to a constant value if  $\alpha$  is equal to zero.)

TABLE I  
TYPICAL PARAMETERS USED FOR MODEL  
VERIFICATION CALCULATIONS

Parameters	Values
Superficial Velocity, $V$	0.200 cm/min
Porosity, $\theta$	0.300
Column Bulk Density, $\rho_\beta$	1.500 gm/cc
Adsorption Coefficient, $k$	0.010 cc/gm
Source Inactivation, $\alpha$	-0.001 1/min
Liquid Phase Inactivation, $\lambda_f$	-0.001 1/min
Solid Phase Inactivation, $\lambda_s$	-0.001 1/min
Initial Concentration, $C_0$	1000.000 plu/cc

From the development of the mass balance, recall the definition of the  $\lambda$  terms. The rate of change in virus concentration in the fluid phase is referred to as  $\lambda_f$ . In the same way,  $\lambda_s$  is defined as the rate of change of virus concentration in the solid phase. The solid phase refers to the viruses that are adsorbed onto the soil grains. Both  $\lambda_f$  and  $\lambda_s$  refer to subsurface conditions.

To interpret the output data, begin by referring to Table II. The program will print a page like this before each table of output values, to define that computer run. In other words, Table II prints all the input data given in that computer run to differentiate it from past or future runs. Note that Table II shows the inactivation rates in the solid and liquid phases greater than the value of the source inactivation rate. This situation can be thought of as the viruses experiencing higher inactivation rates within the subsurface than they would otherwise.

Thus, refer to Table III. The virus concentration values decrease for a single value of time as distance increases. This reduction is due to the higher inactivation rates within the subsurface. The linear isotherm was used to generate the values printed in Table III.

TABLE II

DEFINITION PAGE FOR A COMPUTER RUN USING LARGE SOLID  
AND LIQUID PHASE INACTIVATION RATES

---

VIRUS TRANSPORT  
PAGE 1

TRIAL 1 12/14/85

SOIL HYDRAULIC PROPERTIES	
POROSITY	0.30000
BULK DENSITY	1.50000 GM/CC
SUPERFICIAL VELOCITY	0.02000 CM/MN
DISTRIBUTION COEFFICIENT	0.010000 CC/GM
VIRUS INACTIVATION RATES	
SUSPENDED IN FLUID	-0.001000 1/MN
ADSORBED ON SOIL	-0.001000 1/MN
SOURCE VALUES	
INACTIVATION RATE	-0.000100 1/MN
INITIAL CONCENTRATION	1000.00 PLU/CC

POINTS OF OBSERVATION IN DISTANCE (CM) AND TIME (MN)

INITIAL X =	0.0	DELTA X =	10.0	FINAL X =	140.0
INITIAL T =	0.0	DELTA T =	300.0	FINAL T =	6000.0

---

If the Freundlich isotherm is used, the concentration values will be qualitatively the same as with the linear isotherm. Table IV shows the results of a computer run using the same input values that were presented for Table III. In this example, the exponent,  $n$ , for the Freundlich isotherm was set at  $n = 1.10$ . The difference in the resulting concentration values are shown in Figure 3.

TABLE III

TABULAR OUTPUT UTILIZING THE LINEAR ISOTHERM AND SHOWING  
THE REDUCTION OF VIRUS CONCENTRATION DUE TO LARGE  
LIQUID AND SOLID PHASE INACTIVATION RATES

VIRUS TRANSPORT

TRIAL 2 12/14/84

PAGE 2

## LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		0.00	10.00	20.00	30.00	40.00
	0.0	1000.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	970.4453	842.1897	0.0000	0.0000	0.0000
I	600.0	941.7644	817.2993	709.2837	615.5435	0.0000
M	900.0	913.9312	793.1443	688.3213	597.3513	518.4045
E	1200.0	886.9204	769.7034	667.9783	579.6970	503.0833
	1500.0	860.7078	746.9553	648.2366	562.5645	488.2148
M	1800.0	835.2700	724.8794	629.0781	545.9380	473.7859
N	2100.0	810.5842	703.4561	610.4861	529.8032	459.7834
	2400.0	786.6277	682.6655	592.4434	514.1450	446.1948
	2700.0	763.3794	662.4900	574.9341	498.9497	433.0078
	3000.0	740.8181	642.9104	557.9424	484.2036	420.2104
	3300.0	718.9236	623.9094	541.4526	469.8933	407.7913
	3600.0	697.6763	605.4702	525.4502	456.0059	395.7393
	3900.0	677.0569	587.5757	509.9209	442.5288	384.0435
	4200.0	657.0466	570.2102	494.8503	429.4500	372.6931
	4500.0	637.6279	553.3579	480.2253	416.7578	361.6785
	4800.0	618.7832	537.0039	466.0325	404.4409	350.9895
	5100.0	600.4954	521.1331	452.2593	392.4878	340.6162
	5400.0	582.7480	505.7312	438.8928	380.8879	330.5491
	5700.0	565.5254	490.7844	425.9216	369.6311	320.7800
	6000.0	548.8115	476.2795	413.3337	358.7068	311.2996

TABLE III (Continued)

VIRUS TRANSPORT TRIAL 1 12/14/84 PAGE 3

## LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		50.00	60.00	70.00	80.00	90.00
	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	0.0000	0.0000	0.0000	0.0000	0.0000
I	600.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	900.0	449.8914	0.0000	0.0000	0.0000	0.0000
E	1200.0	436.5950	378.8938	328.8186	0.0000	0.0000
	1500.0	423.6917	367.6958	319.1008	276.9277	240.3287
M	1800.0	411.1697	356.8289	309.6697	268.7432	233.2259
N	2100.0	399.0178	346.2832	300.5176	260.8008	226.3331
	2400.0	387.2249	336.0486	291.6360	253.0929	219.6438
	2700.0	375.7808	326.1169	283.0171	245.6129	213.1524
	3000.0	364.6748	316.4790	274.6523	238.3540	206.8528
	3300.0	353.8970	307.1255	266.5352	231.3095	200.7393
	3600.0	343.4380	298.0483	258.6580	224.4733	194.8066
	3900.0	333.2878	289.2400	251.0134	217.8392	189.0492
	4200.0	323.4375	280.6917	243.5949	211.4011	183.4621
	4500.0	313.8787	272.3958	236.3956	205.1531	178.0398
	4800.0	304.6021	264.3452	229.4091	199.0900	172.7780
	5100.0	295.5996	256.5327	222.6289	193.2060	167.6717
	5400.0	286.8633	248.9512	216.0493	187.4958	162.7161
	5700.0	278.3853	241.5935	209.6642	181.9545	157.9072
	6000.0	270.1580	234.4534	203.4675	176.5770	153.2403

TABLE III (Continued)

VIRUS TRANSPORT TRIAL 1 12/14/84 PAGE 4

## LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		100.00	110.00	120.00	130.00	140.00
	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	0.0000	0.0000	0.0000	0.0000	0.0000
I	600.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	900.0	0.0000	0.0000	0.0000	0.0000	0.0000
E	1200.0	0.0000	0.0000	0.0000	0.0000	0.0000
	1500.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	1800.0	202.4025	175.6526	0.0000	0.0000	0.0000
N	2100.0	196.4207	170.4613	147.9328	128.3818	0.0000
	2400.0	190.6154	165.4233	143.5606	124.5874	108.1218
	2700.0	184.9819	160.5343	139.3177	120.9053	104.9263
	3000.0	179.5149	155.7898	135.2003	117.3321	101.8253
	3300.0	174.2094	151.1854	131.2045	113.8643	98.8159
	3600.0	169.0607	146.7173	127.3268	110.4991	95.8955
	3900.0	164.0643	142.3812	123.5638	107.2335	93.0613
	4200.0	159.2155	138.1732	119.9120	104.0642	90.3110
	4500.0	154.5098	134.0895	116.3679	100.9886	87.6418
	4800.0	149.9434	130.1266	112.9288	98.0040	85.0517
	5100.0	145.5120	126.2808	109.5912	95.1075	82.5380
	5400.0	141.2113	122.5486	106.3523	92.2966	80.0986
	5700.0	137.0379	118.9267	103.2091	89.5688	77.7313
	6000.0	132.9879	115.4119	100.1589	86.9217	75.4340

TABLE IV

TABULAR OUTPUT UTILIZING THE FREUNDLICH ISOTHERM AND  
SHOWING THE REDUCTION IN VIRUS CONCENTRATION DUE TO  
LARGE LIQUID AND SOLID PHASE INACTIVATION RATES

VIRUS TRANSPORT

TRIAL 1 12/14/85

PAGE 6

## LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		0.00	10.00	20.00	30.00	40.00
	0.0	1000.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	970.4453	836.7832	0.0000	0.0000	0.0000
I	600.0	941.7644	812.0750	700.3884	604.1912	0.0000
M	900.0	913.9312	788.0879	679.7178	586.3694	505.9485
E	1200.0	886.9204	764.8105	659.6521	569.0740	491.0332
	1500.0	860.7078	742.2253	640.1841	552.2883	476.5571
M	1800.0	835.2700	720.3025	621.2900	535.9973	462.5085
N	2100.0	810.5842	699.0264	602.9485	520.1826	448.8730
	2400.0	786.6277	678.3840	585.1533	504.8389	435.6401
	2700.0	763.3794	658.3462	567.8792	489.9478	422.7971
	3000.0	740.8181	638.8997	551.1191	475.4954	410.3330
	3300.0	718.9236	620.0334	534.8489	461.4697	398.2358
	3600.0	697.6763	601.7187	519.0637	447.8579	386.4954
	3900.0	677.0569	583.9470	503.7468	434.6455	375.1028
	4200.0	657.0466	566.6987	488.8752	421.8247	364.0444
	4500.0	637.6279	549.9629	474.4456	409.3823	353.3123
	4800.0	618.7832	533.7178	460.4395	397.3062	342.8962
	5100.0	600.4954	517.9536	446.8511	385.5835	332.7874
	5400.0	582.7480	502.6584	433.6584	374.2100	322.9763
	5700.0	565.5254	487.8101	420.8604	363.1719	313.4548
	6000.0	548.8115	473.4023	408.4351	352.4565	304.2141

TABLE IV (Continued)

VIRUS TRANSPORT TRIAL 1 12/14/84 PAGE 7

## LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		50.00	60.00	70.00	80.00	90.00
	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	0.0000	0.0000	0.0000	0.0000	0.0000
I	600.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	900.0	436.6460	0.0000	0.0000	0.0000	0.0000
E	1200.0	423.7810	365.8137	315.8364	0.0000	0.0000
	1500.0	411.2947	355.0410	306.5403	264.7185	228.6445
M	1800.0	399.1765	344.5886	297.5208	256.9314	221.9222
N	2100.0	387.4180	334.4414	288.7642	249.3756	215.3996
	2400.0	376.0029	324.5923	280.2673	242.0401	209.0685
	2700.0	364.9243	315.0364	272.0186	234.9222	202.9220
	3000.0	354.1719	305.7590	264.0146	228.0117	196.9577
	3300.0	343.7366	296.7554	256.2441	221.3063	191.1672
	3600.0	333.6116	288.0164	248.7026	214.7964	185.5484
	3900.0	323.7825	279.5378	241.3855	208.4806	180.0938
	4200.0	314.2424	271.3062	234.2811	202.3477	174.7988
	4500.0	304.9834	263.3164	227.3857	196.3954	169.6612
	4800.0	295.9973	255.5625	220.6951	190.6198	164.6733
	5100.0	287.2751	248.0384	214.1994	185.0126	159.8331
	5400.0	278.8113	240.7342	207.8951	179.5703	155.1341
	5700.0	270.5957	233.6445	201.7765	174.2891	150.5731
	6000.0	262.6226	226.7644	195.8389	169.1620	146.1461



TABLE IV (Continued)

VIRUS TRANSPORT TRIAL 1 12/14/84 PAGE 8

## LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		100.00	110.00	120.00	130.00	140.00
	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	0.0000	0.0000	0.0000	0.0000	0.0000
I	600.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	900.0	0.0000	0.0000	0.0000	0.0000	0.0000
E	1200.0	0.0000	0.0000	0.0000	0.0000	0.0000
	1500.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	1800.0	191.7197	165.6563	0.0000	0.0000	0.0000
N	2100.0	186.0862	160.7925	138.9610	120.1154	0.0000
	2400.0	180.6198	156.0718	134.8834	116.5918	100.7985
	2700.0	175.3140	151.4884	130.9244	113.1723	97.8430
	3000.0	170.1624	147.0407	127.0825	109.8522	94.9751
	3300.0	165.1640	142.7223	123.3522	106.6306	92.1904
	3600.0	160.3107	138.5320	119.7327	103.5023	89.4872
	3900.0	155.6020	134.4641	116.2187	100.4662	86.8638
	4200.0	151.0298	130.5151	112.8074	97.5190	84.3176
	4500.0	146.5918	126.6830	109.4972	94.6589	81.8454
	4800.0	142.2855	122.9628	106.2831	91.8819	79.4457
	5100.0	138.1046	119.3516	103.1635	89.1864	77.1162
	5400.0	134.0466	115.8474	100.1354	86.5701	74.8551
	6000.0	126.2856	109.1429	94.3434	81.5652	70.5294

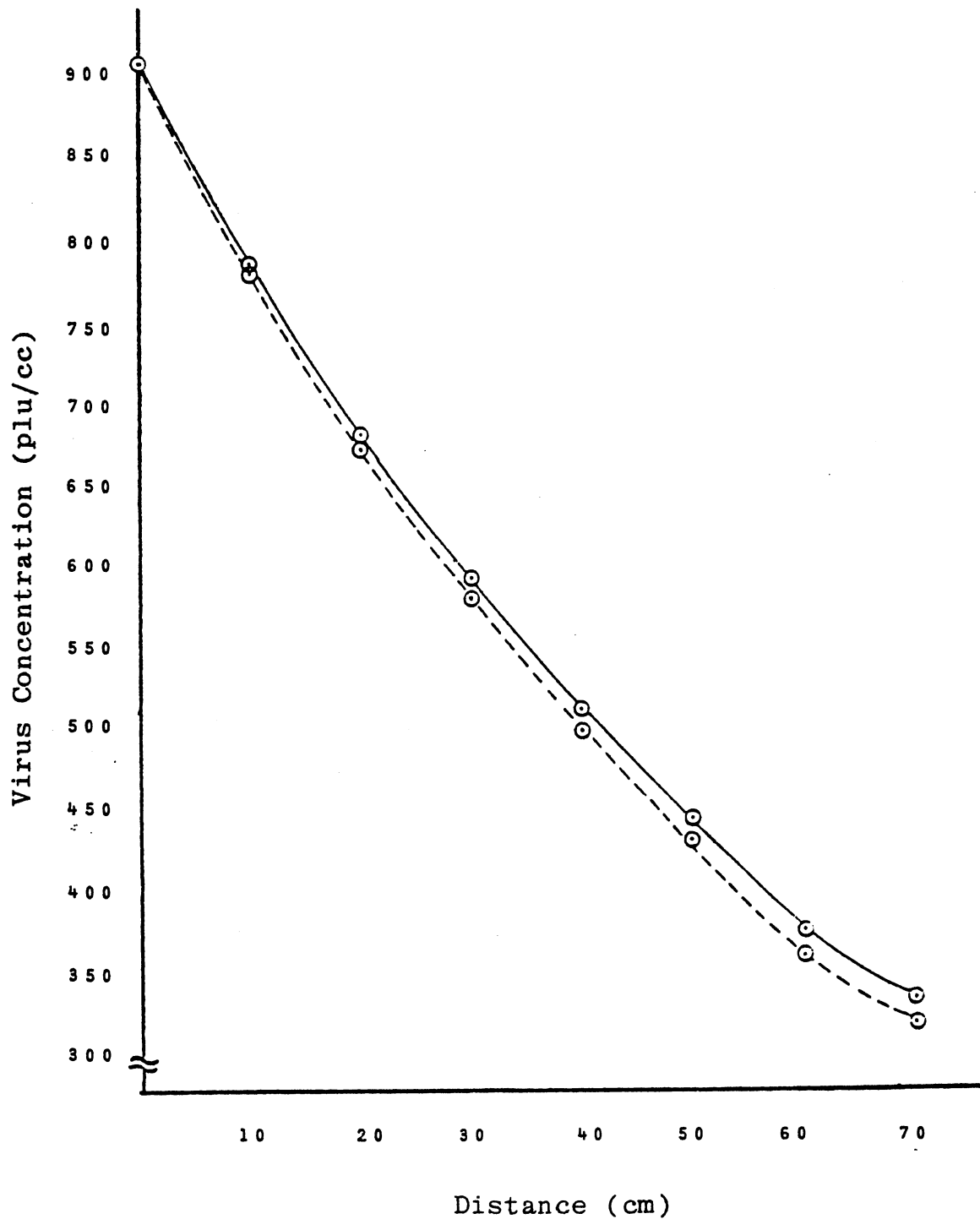


Figure 3. A Comparison of Virus Concentrations Resulting from Calculations Utilizing the Linear (—) and Freundlich (---) Isotherms When Small Source Inactivation Rates are Used

Now observe the converse situation. Table V is the definition table for a run in which the solid and liquid inactivation rates are less than the source inactivation rate. In other words, as the viruses enter the subsurface they are entering an environment characterized by a reduced viral depletion rate. Thus, at a single value of time the concentration will be the greatest at its farthest point of travel. Reference to Table VI will show that this is indeed what the program has found. Table VII shows the results of the same computer run except this time the Freundlich isotherm was used. Again there is only a slight quantitative difference between the results of the model when using the two isotherms. Figure 4 shows this quantitative difference.

Finally, if all three inactivation rates are set at equal values, the linear isotherm shows no change in concentration for a single value of time as distance increases (refer to Table VIII and IX). This constant value is due to the equilibrium established behind the front. As always, the concentration before the front is equal to zero.

For the same set of input values, Table X gives the resulting virus concentration values when the Freundlich isotherm is used. As shown numerically in the table, and

again graphically in Figure 5, the concentration increases very slightly as distance increases for a single value of time. This increase is due to the previous assumption of plug flow. As the viruses move through the subsurface the heaviest concentration of viruses will be at the front. However, there will be viruses left behind attached to the soil grains. In addition, these viruses will continue to die-off further reducing the concentration.

TABLE V

DEFINITION PAGE FOR A COMPUTER RUN USING SMALL  
SOLID AND LIQUID PHASE INACTIVATION RATES

---

VIRUS TRANSPORT  
PAGE 9

TRIAL 1 12/14/85

SOIL HYDRAULIC PROPERTIES

POROSITY	0.30000
BULK DENSITY	1.50000 GM/CC
SUPERFICIAL VELOCITY	0.02000 CM/MN
DISTRIBUTION COEFFICEINT	0.010000 CC/GM

VIRUS INACTIVATION RATES

SUSPENDED IN FLUID	-0.000100 1/MN
ADSORBED ON SOIL	-0.000100 1/MN

SOURCE VALUES

INACTIVATION RATE	-0.001000 1/MN
INITIAL CONCENTRATION	1000.00 PLU/CC

POINTS OF OBSERVATION IN DISTANCE (CM) AND TIME (MN)

INITIAL X =	0.0	DELTA X =	10.0	FINAL X =	140.0
INITIAL T =	0.0	DELTA T =	300.0	FINAL T =	6000.0

---

TABLE VI

TABULAR OUTPUT UTILIZING THE LINEAR ISOTHERM AND SHOWING  
THE INCREASE IN VIRUS CONCENTRATION DUE TO A LARGE  
SOURCE INACTIVATION RATE

VIRUS TRANSPORT TRIAL 1 12/14/84 PAGE 10

LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		0.00	10.00	20.00	30.00	40.00
	0.0	1000.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	740.8181	853.6362	0.0000	0.0000	0.0000
I	600.0	548.8115	632.3892	728.6946	839.6665	0.0000
M	900.0	406.5696	468.4854	539.8303	622.0400	716.7698
E	1200.0	301.1941	347.0627	399.9160	460.8186	530.9961
	1500.0	223.1303	257.1106	296.2654	341.3833	393.3718
M	1800.0	165.2990	190.4722	219.4788	252.9030	291.4170
N	2100.0	122.4565	141.1053	162.5939	187.3551	215.8870
	2400.0	90.7180	104.5333	120.4525	138.7961	159.9330
	2700.0	67.2055	77.4402	89.2334	102.8226	118.4813
	3000.0	49.7871	57.3691	66.1058	76.1729	87.7731
	3300.0	36.8832	42.5001	48.9723	56.4303	65.0240
	3600.0	27.3237	31.4848	36.2796	41.8046	48.1709
	3900.0	20.2419	23.3245	26.8766	30.9696	35.6859
	4200.0	14.9956	17.2793	19.9107	22.9428	26.4368
	4500.0	11.1090	12.8008	14.7502	16.9965	19.5848
	4800.0	8.2298	9.4831	10.9272	12.5913	14.5088
	5100.0	6.0968	7.0252	8.0951	9.3279	10.7484
	5400.0	4.5166	5.2044	5.9970	6.9102	7.9626
	5700.0	3.3460	3.8555	4.4427	5.1192	5.8988
	6000.0	2.4788	2.8562	3.2912	3.7924	4.3700

TABLE VI (Continued)

VIRUS TRANSPORT TRIAL 1 12/14/84 PAGE 11

## LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		50.00	60.00	70.00	80.00	90.00
	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	0.0000	0.0000	0.0000	0.0000	0.0000
I	600.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	900.0	825.9253	0.0000	0.0000	0.0000	0.0000
E	1200.0	611.8606	705.0398	812.4092	0.0000	0.0000
	1500.0	453.2778	522.3066	601.8481	693.5024	799.1145
M	1800.0	335.7966	386.9343	445.8599	513.7593	591.9985
N	2100.0	248.7642	286.6479	330.3013	380.6021	438.5632
	2400.0	184.2890	212.3540	244.6932	281.9568	324.8955
	2700.0	136.5247	157.3157	181.2731	208.8788	240.6886
	3000.0	101.1400	116.5424	134.2906	154.7413	178.3066
	3300.0	74.9264	86.3367	99.4848	114.6352	132.0928
	3600.0	55.5068	63.9598	73.7002	84.9238	97.8567
	3900.0	41.1204	47.3826	54.5984	62.9131	72.4940
	4200.0	30.4628	35.1019	40.4475	46.6072	53.7049
	4500.0	22.5674	26.0041	29.9643	34.5275	39.7856
	4800.0	16.7183	19.2643	22.1981	25.5786	29.4739
	5100.0	12.3852	14.2714	16.4447	18.9491	21.8348
	5400.0	9.1752	10.5725	12.1826	14.0378	16.1756
	5700.0	6.7972	7.8323	9.0251	10.3995	11.9832
	6000.0	5.0355	5.8023	6.6859	7.7041	8.8774

TABLE VI (Continued)

VIRUS TRANSPORT TRIAL 1 12/14/84 PAGE 12

## LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		100.00	110.00	120.00	130.00	140.00
	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	0.0000	0.0000	0.0000	0.0000	0.0000
I	600.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	900.0	0.0000	0.0000	0.0000	0.0000	0.0000
E	1200.0	0.0000	0.0000	0.0000	0.0000	0.0000
	1500.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	1800.0	682.1526	786.0371	0.0000	0.0000	0.0000
N	2100.0	505.3511	582.3103	670.9897	773.1733	0.0000
	2400.0	374.3730	431.3860	497.0813	572.7808	660.0083
	2700.0	277.3423	319.5786	368.2468	424.3264	488.9460
	3000.0	205.4605	236.7499	272.8042	314.3489	362.2205
	3300.0	152.2089	175.3886	202.0983	232.8755	268.3394
	3600.0	112.7590	129.9310	149.7181	172.5183	198.7908
	3900.0	83.5339	96.2552	110.9139	127.8047	147.2678
	4200.0	61.8835	71.3077	82.1671	94.6801	109.0987
	4500.0	45.8444	52.8260	60.8708	70.1407	80.8223
	4800.0	33.9624	39.1345	45.0942	51.9615	59.8746
	5100.0	25.1599	28.9915	33.4066	38.4940	44.3562
	5400.0	18.6389	21.4774	24.7482	28.5171	32.8599
	5700.0	13.8081	15.9109	18.3339	21.1260	24.3432
	6000.0	10.2293	11.7871	13.5821	15.6505	18.0339

TABLE VII

TABULAR OUTPUT UTILIZING THE FREUNDLICH ISOTHERM AND  
SHOWING THE INCREASE IN VIRUS CONCENTRATION DUE  
TO A LARGE SOURCE INACTIVATION RATE

VIRUS TRANSPORT TRIAL 1 12/14/84 PAGE 14

LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		0.00	10.00	20.00	30.00	40.00
	0.0	1000.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	740.8181	860.4373	0.0000	0.0000	0.0000
I	600.0	548.8115	637.3062	740.0684	859.3137	0.0000
M	900.0	406.5696	471.9934	547.9956	636.2332	738.6033
E	1200.0	301.1941	349.5947	405.8108	471.0203	546.7046
	1500.0	223.1303	258.9368	300.4893	348.7087	404.6636
M	1800.0	165.2990	191.7888	222.5234	258.1829	299.5269
N	2100.0	122.4565	142.0536	164.7867	191.1375	221.7242
	2400.0	90.7180	105.2160	122.0190	141.5172	164.1320
	2700.0	67.2055	77.9236	90.3595	104.7786	121.4878
	3000.0	49.7871	57.7162	66.9146	77.5775	89.9318
	3300.0	36.8832	42.7491	49.5522	57.4380	66.5724
	3600.0	27.3237	31.6633	36.6952	42.5268	49.2804
	3900.0	20.2419	23.4545	27.1741	31.4866	36.4800
	4200.0	14.9956	17.3722	20.1253	23.3125	27.0070
	4500.0	11.1090	12.8672	14.9036	17.2621	19.9921
	4800.0	8.2298	9.5304	11.0366	12.7808	14.8006
	5100.0	6.0968	7.0590	8.1730	9.4628	10.9562
	5400.0	4.5166	5.2284	6.0530	7.0069	8.1112
	5700.0	3.3460	3.8726	4.4824	5.1884	6.0049
	6000.0	2.4788	2.8686	3.3197	3.8414	4.4456



TABLE VII (Continued)

VIRUS TRANSPORT TRIAL 1 12/14/84 PAGE 15

## LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		50.00	60.00	70.00	80.00	90.00
	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	0.0000	0.0000	0.0000	0.0000	0.0000
I	600.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	900.0	857.4556	0.0000	0.0000	0.0000	0.0000
E	1200.0	634.5532	736.4448	854.7769	0.0000	0.0000
	1500.0	469.5942	544.8962	632.2698	733.6523	851.2900
M	1800.0	347.5220	403.1665	467.7280	542.6233	629.5105
N	2100.0	257.1802	298.3069	346.0066	401.3347	465.4651
	2400.0	190.3424	220.7376	255.9876	296.8350	344.2344
	2700.0	140.8614	163.3242	189.3687	219.5669	254.5540
	3000.0	104.2532	120.8551	140.1006	162.4108	188.2560
	3300.0	77.1592	89.4294	103.6508	120.1336	139.2239
	3600.0	57.1064	66.1752	76.6841	88.8616	102.9630
	3900.0	42.2692	48.9724	56.7386	65.7300	76.1534
	4200.0	31.2840	36.2382	41.9770	48.6244	56.3245
	4500.0	23.1559	26.8178	31.0588	35.9704	41.6587
	4800.0	17.1380	19.8463	22.9804	26.6095	30.8116
	5100.0	12.6852	14.6872	17.0033	19.6846	22.7910
	5400.0	9.3894	10.8691	12.5820	14.5633	16.8567
	5700.0	6.9499	8.0436	9.3094	10.7744	12.4687
	6000.0	5.1447	5.9532	6.8887	7.9712	9.2239

TABLE VII (Continued)

VIURS TRANSPORT TRIAL 1 12/14/84 PAGE 16

## LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		100.00	110.00	120.00	130.00	140.00
	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	0.0000	0.0000	0.0000	0.0000	0.0000
I	600.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	900.0	0.0000	0.0000	0.0000	0.0000	0.0000
E	1200.0	0.0000	0.0000	0.0000	0.0000	0.0000
	1500.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	1800.0	730.2397	847.0862	0.0000	0.0000	0.0000
N	2100.0	539.8940	626.1633	726.2170	842.1768	0.0000
	2400.0	399.1638	462.8579	536.7151	622.2974	721.5942
	2700.0	295.1172	342.1433	396.6626	459.8691	533.0957
	3000.0	218.2122	252.9356	293.1841	339.8044	393.8752
	3300.0	161.3481	186.9870	216.7004	251.1111	291.0129
	3600.0	119.3019	138.2331	160.1690	185.5852	215.0135
	3900.0	88.2212	102.2013	118.3961	137.1579	158.8771
	4200.0	65.2376	75.5612	87.5183	101.3669	117.4080
	4500.0	48.2419	55.8653	64.6994	74.9233	86.7545
	4800.0	35.6772	41.3072	47.8256	55.3779	64.1165
	5100.0	26.3825	30.5429	35.3593	40.9313	47.3812
	5400.0	19.5112	22.5837	26.1400	30.2562	35.0174
	5700.0	14.4308	16.7001	19.3263	22.3653	25.8798
	6000.0	10.6723	12.3494	14.2886	16.5324	19.1285

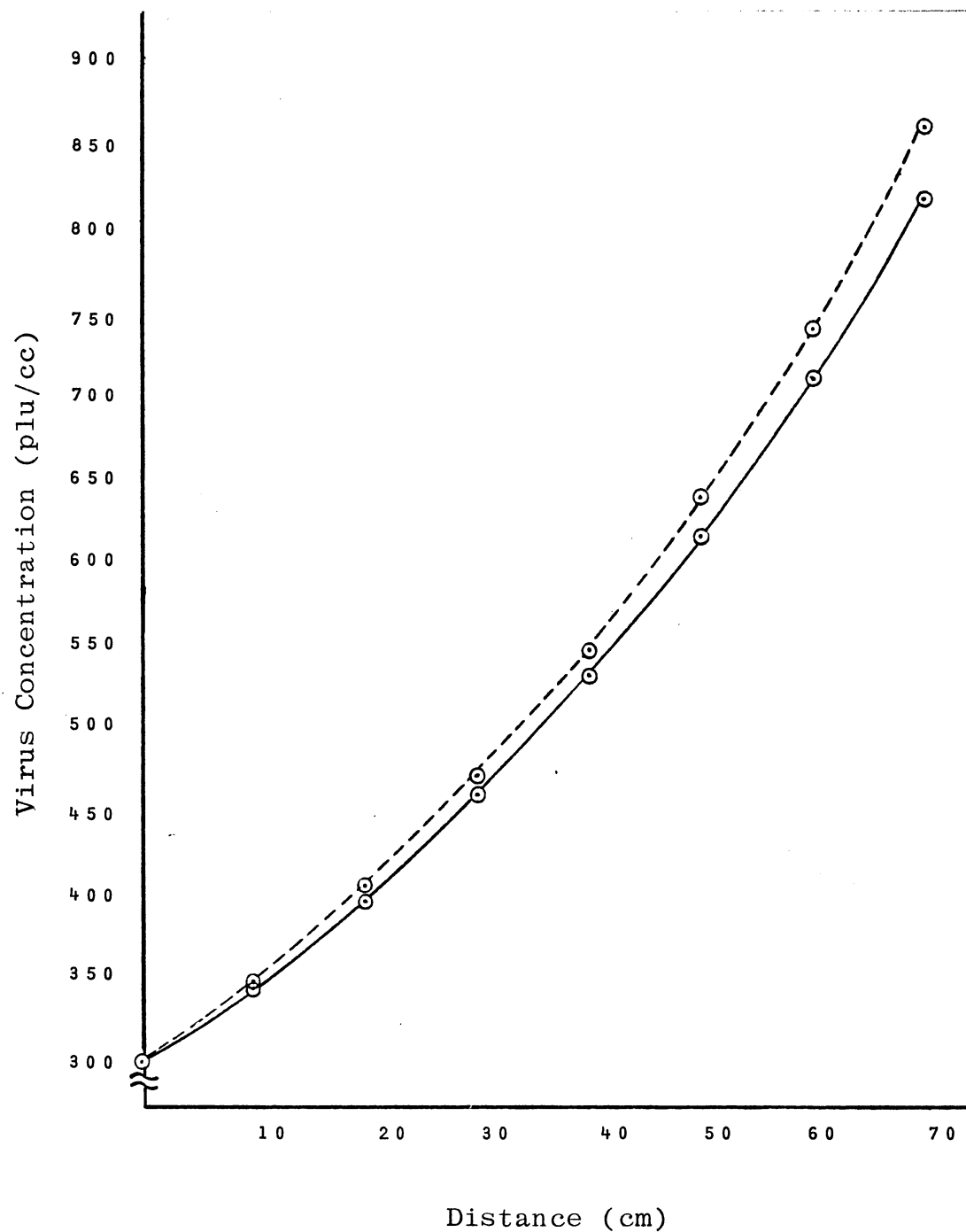


Figure 4. A Comparison of Virus Concentrations Resulting from Calculations Utilizing the Linear (—) and Freundlich (---) Isotherms When Large Source Inactivation Rates are Used

## TABLE VIII

DEFINITION PAGE FOR A COMPUTER RUN  
WITH EQUAL INACTIVATION RATES

---

VIRUS TRANSPORT  
PAGE 17

TRIAL 1 12/14/85

SOIL HYDRAULIC PROPERTIES

POROSITY	0.30000
BULK DENSITY	1.50000 GM/CC
SUPERFICIAL VELOCITY	0.02000 CM/MN
DISTRIBUTION COEFFICEINT	0.010000 CC/GM

VIRUS INACTIVATION RATES

SUSPENDED IN FLUID	-0.001000 1/MN
ADSORBED ON SOIL	-0.001000 1/MN

SOURCE VALUES

INACTIVATION RATE	-0.001000 1/MN
INITIAL CONCENTRATION	1000.00 PLU/CC

POINTS OF OBSERVATION IN DISTANCE (CM) AND TIME (MN)

INITIAL X =	0.0	DELTA X =	10.0	FINAL X =	140.0
INITIAL T =	0.0	DELTA T =	300.0	FINAL T =	6000.0

---

TABLE IX

TABULAR OUTPUT UTILIZING THE LINEAR ISOTHERM AND  
SHOWING A CONSTANT VIRUS CONCENTRATION  
DUE TO EQUAL INACTIVATION RATES

VIRUS TRANSPORT TRIAL 1 12/14/84 PAGE 18

LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		0.00	10.00	20.00	30.00	40.00
	0.0	1000.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	740.8181	740.8181	0.0000	0.0000	0.0000
I	600.0	548.8115	548.8115	548.8115	548.8115	0.0000
M	900.0	406.5696	406.5696	406.5696	406.5696	406.5696
E	1200.0	301.1941	301.1941	301.1941	301.1941	301.1941
	1500.0	223.1303	223.1303	223.1303	223.1303	223.1303
M	1800.0	165.2990	165.2990	165.2990	165.2990	165.2990
N	2100.0	122.4565	122.4565	122.4565	122.4565	122.4565
	2400.0	90.7180	90.7180	90.7180	90.7180	90.7180
	2700.0	67.2055	67.2055	67.2055	67.2055	67.2055
	3000.0	49.7871	49.7871	49.7871	49.7871	49.7871
	3300.0	36.8832	36.8832	36.8832	36.8832	36.8832
	3600.0	27.3237	27.3237	27.3237	27.3237	27.3237
	3900.0	20.2419	20.2419	20.2419	20.2419	20.2419
	4200.0	14.9956	14.9956	14.9956	14.9956	14.9956
	4500.0	11.1090	11.1090	11.1090	11.1090	11.1090
	4800.0	8.2298	8.2298	8.2298	8.2298	8.2298
	5100.0	6.0968	6.0968	6.0968	6.0968	6.0968
	5400.0	4.5166	4.5166	4.5166	4.5166	4.5166
	5700.0	3.3460	3.3460	3.3460	3.3460	3.3460
	6000.0	2.4788	2.4788	2.4788	2.4788	2.4788

TABLE IX (Continued)

VIRUS TRANSPORT TRIAL 1 12/14/84 PAGE 19

## LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		50.00	60.00	70.00	80.00	90.00
	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	0.0000	0.0000	0.0000	0.0000	0.0000
I	600.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	900.0	406.5696	0.0000	0.0000	0.0000	0.0000
E	1200.0	301.1941	301.1941	301.1941	0.0000	0.0000
	1500.0	223.1303	223.1303	223.1303	223.1303	223.1303
M	1800.0	165.2990	165.2990	165.2990	165.2990	165.2990
N	2100.0	122.4565	122.4565	122.4565	122.4565	122.4565
	2400.0	90.7180	90.7180	90.7180	90.7180	90.7180
	2700.0	67.2055	67.2055	67.2055	67.2055	67.2055
	3000.0	49.7871	49.7871	49.7871	49.7871	49.7871
	3300.0	36.8832	36.8832	36.8832	36.8832	36.8832
	3600.0	27.3237	27.3237	27.3237	27.3237	27.3237
	3900.0	20.2419	20.2419	20.2419	20.2419	20.2419
	4200.0	14.9956	14.9956	14.9956	14.9956	14.9956
	4500.0	11.1090	11.1090	11.1090	11.1090	11.1090
	4800.0	8.2298	8.2298	8.2298	8.2298	8.2298
	5100.0	6.0968	6.0968	6.0968	6.0968	6.0968
	5400.0	4.5166	4.5166	4.5166	4.5166	4.5166
	5700.0	3.3460	3.3460	3.3460	3.3460	3.3460
	6000.0	2.4788	2.4788	2.4788	2.4788	2.4788

TABLE IX (Continued)

VIRUS TRANSPORT TRIAL 1 12/14/84 PAGE 20

## LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		100.00	110.00	120.00	130.00	140.00
	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	0.0000	0.0000	0.0000	0.0000	0.0000
I	600.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	900.0	0.0000	0.0000	0.0000	0.0000	0.0000
E	1200.0	0.0000	0.0000	0.0000	0.0000	0.0000
	1500.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	1800.0	165.2990	165.2990	0.0000	0.0000	0.0000
N	2100.0	122.4565	122.4565	122.4565	122.4565	0.0000
	2400.0	90.7180	90.7180	90.7180	90.7180	90.7180
	2700.0	67.2055	67.2055	67.2055	67.2055	67.2055
	3000.0	49.7871	49.7871	49.7871	49.7871	49.7871
	3300.0	36.8832	36.8832	36.8832	36.8832	36.8832
	3600.0	27.3237	27.3237	27.3237	27.3237	27.3237
	3900.0	20.2419	20.2419	20.2419	20.2419	20.2419
	4200.0	14.9956	14.9956	14.9956	14.9956	14.9956
	4500.0	11.1090	11.1090	11.1090	11.1090	11.1090
	4800.0	8.2298	8.2298	8.2298	8.2298	8.2298
	5100.0	6.0968	6.0968	6.0968	6.0968	6.0968
	5400.0	4.5166	4.5166	4.5166	4.5166	4.5166
	5700.0	3.3460	3.3460	3.3460	3.3460	3.3460
	6000.0	2.4788	2.4788	2.4788	2.4788	2.4788

TABLE X

TABULAR OUTPUT UTILIZING THE FREUNDLICH ISOTHERM AND  
SHOWING A SLIGHT INCREASE IN VIRUS CONCENTRATION  
DUE TO PLUG FLOW

VIRUS TRANSPORT TRIAL 1 12/14/84 PAGE 22

LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		0.00	10.00	20.00	30.00	40.00
	0.0	1000.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	740.8181	741.9023	0.0000	0.0000	0.0000
I	600.0	548.8115	549.6028	550.3821	551.1497	0.0000
M	900.0	406.5696	407.1470	407.7158	408.2759	408.8274
E	1200.0	301.1941	301.6162	302.0312	302.4402	302.8425
	1500.0	223.1303	223.4381	223.7413	224.0398	224.3333
M	1800.0	165.2990	165.5239	165.7449	165.9628	166.1772
N	2100.0	122.4565	122.6207	122.7820	122.9408	123.0973
	2400.0	90.7180	90.8379	90.9556	91.0715	91.1857
	2700.0	67.2055	67.2930	67.3790	67.4635	67.5469
	3000.0	49.7871	49.8509	49.9137	49.9753	50.0362
	3300.0	36.8832	36.9298	36.9755	37.0206	37.0650
	3600.0	27.3237	27.3577	27.3911	27.4240	27.4564
	3900.0	20.2419	20.2668	20.2912	20.3152	20.3388
	4200.0	14.9956	15.0138	15.0315	15.0491	15.0663
	4500.0	11.1090	11.1223	11.1353	11.1480	11.1606
	4800.0	8.2298	8.2394	8.2489	8.2582	8.2674
	5100.0	6.0968	6.1038	6.1107	6.1175	6.1243
	5400.0	4.5166	4.5217	4.5268	4.5318	4.5366
	5700.0	3.3460	3.3497	3.3534	3.3570	3.3606
	6000.0	2.4788	2.4815	2.4842	2.4868	2.4894



TABLE X (Continued)

VIRUS TRANSPORT TRIAL 1 12/14/84 PAGE 23

## LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		50.00	60.00	70.00	80.00	90.00
	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	0.0000	0.0000	0.0000	0.0000	0.0000
I	600.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	900.0	409.3718	0.0000	0.0000	0.0000	0.0000
E	1200.0	303.2397	303.6306	304.0154	0.0000	0.0000
	1500.0	224.6230	224.9083	225.1892	225.4660	225.7388
M	1800.0	166.3887	166.5964	166.8015	167.0036	167.2027
N	2100.0	123.2517	123.4036	123.5530	123.7005	123.8459
	2400.0	91.2983	91.4092	91.5183	91.6259	91.7320
	2700.0	67.6291	67.7100	67.7896	67.8682	67.9456
	3000.0	50.0962	50.1553	50.2134	50.2707	50.3272
	3300.0	37.1088	37.1519	37.1943	37.2361	37.2773
	3600.0	27.4884	27.5198	27.5508	27.5813	27.6114
	3900.0	20.3621	20.3851	20.4077	20.4300	20.4519
	4200.0	15.0833	15.1001	15.1165	15.1328	15.1488
	4500.0	11.1730	11.1853	11.1973	11.2091	11.2208
	4800.0	8.2765	8.2854	8.2942	8.3028	8.3114
	5100.0	6.1309	6.1374	6.1438	6.1501	6.1563
	5400.0	4.5415	4.5462	4.5509	4.5555	4.5601
	5700.0	3.3641	3.3676	3.3710	3.3744	3.3777
	6000.0	2.4920	2.4945	2.4970	2.4995	2.5019

TABLE X (Continued)

VIRUS TRANSPORT TRIAL 1 12/14/84 PAGE 24

## LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		100.00	110.00	120.00	130.00	140.00
	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	0.0000	0.0000	0.0000	0.0000	0.0000
I	600.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	900.0	0.0000	0.0000	0.0000	0.0000	0.0000
E	1200.0	0.0000	0.0000	0.0000	0.0000	0.0000
	1500.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	1800.0	167.3988	167.5919	0.0000	0.0000	0.0000
N	2100.0	123.9889	124.1298	124.2687	124.4057	0.0000
	2400.0	91.8364	91.9391	92.0406	92.1405	92.2388
	2700.0	68.0218	68.0969	68.1708	68.2437	68.3154
	3000.0	50.3828	50.4376	50.4915	50.5447	50.5970
	3300.0	37.3179	37.3579	37.3973	37.4361	37.4743
	3600.0	27.6410	27.6702	27.6989	27.7273	27.7551
	3900.0	20.4735	20.4948	20.5158	20.5365	20.5568
	4200.0	15.1646	15.1802	15.1955	15.2105	15.2254
	4500.0	11.2324	11.2437	11.2549	11.2659	11.2767
	4800.0	8.3198	8.3280	8.3362	8.3442	8.3521
	5100.0	6.1624	6.1685	6.1744	6.1803	6.1861
	5400.0	4.5645	4.5689	4.5733	4.5776	4.5818
	5700.0	3.3810	3.3842	3.3873	3.3905	3.3935
	6000.0	2.5043	2.5066	2.5090	2.5112	2.5135

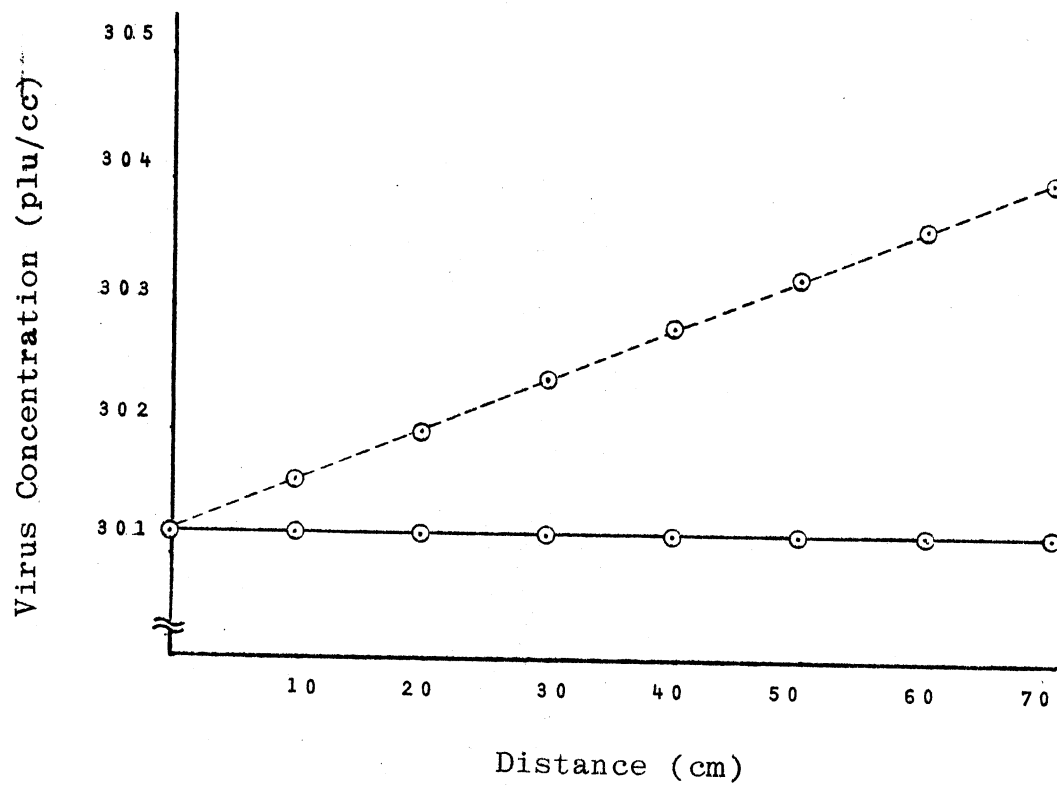


Figure 5. A Comparison of Virus Concentrations Resulting from Calculations Utilizing the Linear (—) and Freundlich (---) Isotherms When Inactivation Rates are Equal

## CHAPTER V

### CONCLUSIONS AND RECOMMENDATIONS

This paper presents a mathematical model for predicting the survival and transport capabilities of viruses in groundwater. The model is intended to be used as a tool in determining safe distances between home septic tank systems and private water well supplies.

The mathematical statements and solution algorithms of the model have been converted into a FORTRAN computer program. The program was designed to require a minimal amount of input data. This allows a person with a basic understanding of soil hydraulology to use the program with little training.

This paper is not intended to completely verify the results of the model. It is only one part of a joint study by Oklahoma State University and Arizona State University. The other major part of this joint effort is to establish data on virus inactivation rates. Until this data is known, the full potential of the model cannot be realized. However, results of the computer program

qualitatively agree with the known behavior of viruses in groundwater (15).

There are several additional areas that should be investigated.

(1) Solutions to the model are currently available using the linear and the Freundlich isotherms. An additional solution using the Langmuir Isotherm would allow further comparison to laboratory data.

(2) As mentioned above, the data on solid phase inactivation rates are not yet available. When the data become available, testing of the model with that data should be performed.

(3) Dispersion is not included in the model. When data are available to support dispersion calculations, they should be added to the model.

## SELECTED BIBLIOGRAPHY

1. Vilker, L., L. H. Frommham, R. Kamdar, and S. Sundaram, "Application of Ion Exchange/Adsorption Models to Virus Transport in Percolating Beds." American Institute of Chemical Engineers Symposium Series on Water, 74, No. 178, 84-92 (1978).
2. Viraraghavan, T. "Effects of Septic Tank Systems on Environmental Quality." J. Environmental Management, 15, 63-70 (1982).
3. Hagedorn, C., E. McCoy and T. Rahe, "The Potential for Ground Water Contamination from Septic Effluents." J. Environmental Quality, 10, No. 1, 1-8 (1981).
4. Grosser, P., "A One-Dimensional Mathematical Model of Virus Transport." Second International Conference on Ground-Water Quality Research, 3 (1984).
5. Keeley, J., "Meeting the Groundwater Contamination Problem." Water Sewage Works, 124 88 (1977).
6. Sheridan, R., M. Abernathy, J. Yeager, "A Computer Model to Determine Sludge Pathogen Transport Through Environmental Pathways." U.S. Environmental Protection Agency, U.S. Government Printing Office, Washington, D. C. (1981).
7. Dawson, J., K. Hain, B. McClure, R. Sheridan, and J. Yeager, "Sewage Sludge Pathogen Transport Model Project." U.S. Environmental Protection Agency, U.S. Government Printing Office, Washington, D.C. (1981).
8. Vilker, V., and W. Burge, "Adsorption Mass Transfer Model for Virus Transport in Soils." Water Research 14, 783-790 (1979).
9. Vilker, V. "An Adsorption Model for Prediction of Virus Breakthrough From Fixed Beds." State of Knowledge in Land Treatment of Wastewater, 2, 381-388 (1978).

10. Burge, W., N. Enkiri, "Virus Adsorption by Five Soils." J. Environmental Quality, 7, 73-76 (1978a).
11. Burge, W., N. Enkiri, "Adsorption Kinetics of Bacteriophage  $\phi$ x-174 on Soil." J. Environmental Quality, 7, 536-541 (1978b).
12. Yeager, J., and R. O'Brien, "Enteroviruses and Bacteriophage Inactivation in Subsurface Waters and Translocation in Soil." New Mexico Water Resources Institute (1977).
13. Gerba, C., Personal Communication, 1984-1985.
14. Keswick, B., and C. Gerba, "Survival in Enteric Viruses and Indicator Bacteria in Groundwater." J. Environmental Science and Health, 17, 903-912 (1982).
15. Wagner, J., Personal Communication, 1983-1985.

APPENDIXES



APPENDIX A

THE COMPUTER PROGRAM

```

C*****
C
C
C          VIRUS TRANSPORT & SURVIVAL
C          IN GROUND WATER
C
C
C*****
C
C
C
C MANUAL
C VERSION # 1
C
C
C THIS PROGRAM MODELS THE TRANSPORT AND SURVIVAL
C   CAPABILITIES OF VIRUSES IN GROUNDWATER.
C
C THE ASSUMPTIONS ARE AS FOLLOWS:
C   -ONE DIMENSIONAL, CONSTANT, & UNIFORM FLOW
C   -HOMOGENEOUS, NON-DEFORMABLE MEDIUM
C   -MASS TRANSPORT PRIMARILY BY CONVECTION
C   -NON CONSERVATIVE TRACER
C   -FLUID PROPERTIES INDEPENDENT OF CONCENTRATION
C
C
C   DIMENSION TITLE(30),UNIT(5),ARRAY(2),ARAY(2)
C   DIMENSION CON(7),COL(7),IC(15),IT(2)
C   COMMON /PARAM/ P,BD,V,LF,LS,CO,ALPHA
C   REAL K,N,LF,LS
C
C READ DEVICE: IN=5          WRITE DEVICE: LP=6
C
C   IN=5
C   LP=6
C
C
C INITIALIZE PROGRAM FLOW CONTROL VARIABLES
C   IEDIT=1
C   MAXCOL=5
C   MAXROW=40
C   NPAGE=1
C
C
C PRINT THE TITLE BOX
C
C   WRITE(LP,2)
C 2 FORMAT(5X,'*****'
C   .,/,5X,'**',46X,'**',/,5X,'**',7X,'THIS PROGRAM MODELS THE MOVEME
C   .NT',7X,'**',/,5X,'**',6X,'OF VIRUSES THROUGH THE GROUNDWATER',6X,
C   . '**',/,5X,'**',46X,'**')
C   WRITE(LP,3)
C 3 FORMAT(5X,'*****'
C   .///)

```

```

C
C ENTER THE TITLE
C
9   WRITE(LP,10)
10  FORMAT(//,5X,'ENTER TITLE',/,5X, '?')
    READ(IN,12) (TITLE(I),I=1,30)
12  FORMAT(30A2)
    GOTO (15,530), IEDIT
C
C DEFINE UNITS
C
15  WRITE(LP,16)
16  FORMAT(//, ' *** ENTER THE UNITS OF THE SYSTEM ***')
C
C ENTER THE UNITS OF LENGTH
17  WRITE(LP,20)
20  FORMAT(//,5X,'ENTER UNITS FOR LENGTH',/,5X,
    .'(2 CHARACTERS)',/,5X, '?')
    READ(IN,25) IL
25  FORMAT(A2)
C
C ENTER THE UNITS OF TIME
26  WRITE(LP,27)
27  FORMAT(/,5X,'ENTER UNITS FOR TIME',/,5X,
    .'(2 CHARACTERS)',/,5X, '?')
    READ(IN,28) (IT(II),II=1,2)
28  FORMAT(2A1)
C
C ENTER THE UNITS OF CONCENTRATION
34  WRITE(LP,35)
35  FORMAT(/,5X,'ENTER CONCENTRATION UNITS',/,5X,
    .'(6 CHARACTERS)',/,5X, '?')
    READ(IN,36) IM1,IM2,IM3
36  FORMAT(3A2)
C
C ENTER THE UNITS OF DENSITY
37  WRITE(LP,38)
38  FORMAT(/,5X,'ENTER THE UNITS FOR DENSITY',/,5X,
    .'(5 CHARACTERS)',/,5X, '?')
    READ(IN,39) (UNIT(K),K=1,5)
39  FORMAT(5A1)
C
C
    DO 42 JJ=1,2
      ARRAY(JJ)=UNIT(JJ)
42  CONTINUE
    JJ=JJ+1
C
C
    DO 46 I=1,2
      ARAY(I)=UNIT(JJ)
      JJ=JJ+1
46  CONTINUE
C

```

```

C ENTER DATA FOR FIRST PROBLEM
C
C
C SOIL/HYDRAULIC PROPERTIES
C
47 WRITE(LP,49)
49 FORMAT(//,' *** ENTER THE SOIL/HYDRAULIC PROPERTIES ***')
C
C POROSITY
53 WRITE(LP,54)
54 FORMAT(/,5X,'ENTER FORMATION POROSITY',/,5X, '?')
   READ(IN,55) P
55 FORMAT(F10.5)
60 IF(P.GT.0.0.AND.P.LT.1.0) GOTO 70
   WRITE(LP,65)
65 FORMAT(5X,'POROSITY MUST BE GREATER THAN ZERO AND LESS THAN '
   .ONE:',/,6X,' --REENTER  ?',/)
   READ(IN,*) P
   GOTO 60
70 GOTO (75,530),IEDIT
C
C BULK DENSITY
75 WRITE(LP,80) (UNIT(K),K=1,5)
80 FORMAT(/,5X,'ENTER BULK DENSITY OF THE FORMATION IN ',5A1,
   ./,5X, '?')
   READ(IN,85) BD
85 FORMAT(F10.5)
86 IF(BD.GT.0.0) GOTO 90
   WRITE(LP,87)
87 FORMAT(5X,'BULK DENSITY MUST BE GREATER THAN ZERO:',/,
   .6X,' --REENTER  ?')
   READ(IN,55) BD
   GOTO 86
90 GOTO (95,530),IEDIT
C
C DARCY VELOCITY
95 WRITE(LP,100) IL,IT(1),IT(2)
100 FORMAT(/,5X,'ENTER DARCY (SUPERFICIAL) VELOCITY, ',A2,'/',
   .a1,a1,/,5X, '?')
   READ(IN,55) V
110 IF(V.GT.0.0) GOTO 120
   WRITE(LP,115)
115 FORMAT(5X,'VELOCITY MUST BE GREATER THAN ZERO:',/,
   .6X,' --REENTER  ?')
   READ(IN,55) V
   GOTO 110
120 GOTO (121,530),IEDIT
C
C
C VIRUS CHARACTERISTICS
C
121 WRITE(LP,122)
122 FORMAT(//,' *** ENTER THE VIRUS CHARACTERISTICS ***')

```

```

C SOURCE INACTIVATION RATE
123  WRITE(LP,124) IT(1),IT(2)
124  FORMAT(/,5X,'ENTER THE SOURCE INACTIVATION RATE (ALPHA) IN 1/'
    .,A1,A1,/,5X, '?')
125  READ(IN,142,ERR=123) ALPHA
126  IF(ALPHA.GT.0.0) GOTO 127
    GOTO 129
127  WRITE(LP,128)
128  FORMAT(5X,'ALPHA MUST BE LESS THAN ZERO:',/,6X,' --REENTER  ?')
    GOTO 125
129  GOTO (130,530),IEDIT
C
C VIRUS INACTIVATION RATE, LAMBDA(F)
130  WRITE(IN,140) IT(1),IT(2)
140  FORMAT(/,5X,'ENTER VIRUS INACTIVATION RATE',/,5X,
    . 'SUSPENDED IN THE FLUID (LAMBDA(F)) IN 1/',A1,A1,/,5X, '?')
141  READ(IN,142,ERR=130) LF
142  FORMAT(F10.6)
144  IF(LF.GT.0.0) GOTO 146
    GOTO 150
146  WRITE(LP,148)
148  FORMAT(5X,'LAMBDA(F) MUST BE LESS THAN ZERO:',/,6X,
    . ' --REENTER?')
    GOTO 141
150  GOTO (156,530),IEDIT
C
C VIRUS INACTIVATION RATE, LAMBDA(S)
156  WRITE(IN,158) IT(1),IT(2)
158  FORMAT(/,5X,'ENTER VIRUS INACTIVATION RATE',/,5X,
    . 'ADSORBED ON THE SOIL (LAMBDA(S)) IN 1/',A1,A1,/,5X, '?')
160  READ(IN,162,ERR=156) LS
162  FORMAT(F10.6)
    IF(LS.GT.0.0) GOTO 166
    GOTO 174
166  WRITE(LP,168)
168  FORMAT(5X,'LAMBDA(S) MUST BE LESS THAN ZERO:',/,6X,
    . ' --REENTER?')
    GOTO 160
174  GOTO (176,530),IEDIT
C
C GENERAL PARAMETERS OF THE SYSTEM
C
176  WRITE(LP,178)
178  FORMAT(//, ' *** ENTER THE GENERAL PARAMETERS OF THE SYSTEM ***')
C
C INITIAL CONCENTRATION
180  WRITE(LP,182)
182  FORMAT(/,5X,'ENTER THE INITIAL CONCENTRATION',/,5X, '?')
184  READ(IN,186) C0
186  FORMAT(F15.5)
    IF(C0.GT.0.0) GOTO 192
188  WRITE(LP,190)
190  FORMAT(5X,'THE INITIAL CONCENTRATION MUST BE GREATER '
    . 'THAN ZERO:',/,6X,' --REENTER  ?')
    GOTO 184

```

```

192 GOTO (194,530),IEDIT
C
C TIME OF INTEREST
194 WRITE(LP,196)IT(1),IT(2)
196 FORMAT(/,5X,'ENTER THE TIME OF INTEREST.',/,10X,'STEADY '
.'STATE HAS NOT BEEN ASSUMED.',/,10X,'THEREFORE ENTER INITIAL, '
.'FINAL AND DELTA TIMES',/,10X,'IN ',A1,A1,' .')
198 CONTINUE
READ(IN,*,ERR=194) TI,TF,DELT
DELT=ABS(DELT)
200 IF(TI.GE.0.0.AND.DELT.LE.1.0E-06) GOTO 212
IF(TI.GE.0.0) GOTO 206
202 WRITE(LP,204)
204 FORMAT(/,5X,'THE INITIAL TIME MUST BE > OR = ZERO:',/,6X, T
.'REENTER ?')
READ(IN,85)TI
GOTO 200
206 IF(TF.GE.0.0) GOTO 214
208 WRITE(LP,210)
210 FORMAT(/,5X,'THE FINAL TIME MUST BE > OR = ZERO:',/,6X, R
.' --REENTER ?')
READ(IN,*,ERR=208) TF
GOTO 206
212 TF=TI
214 GOTO (216,530),IEDIT
C
C THE DISTANCE OF INTEREST
216 WRITE(LP,218) IL
218 FORMAT(/,5X,'ENTER THE DISTANCE OF INTEREST',/,10X,'ENTER ' T
.'THE INITIAL, FINAL AND DELTA DISTANCES',/,10X,'IN ',A2,' .')
READ(IN,*,ERR=216) XI,XF,DELX
DELX=ABS(DELX)
220 IF(XI.GE.0.0.AND.DELX.LE.1.0E-06) GOTO 232
IF(XI.GE.0.0) GOTO 226
222 WRITE(LP,224)
224 FORMAT(/,5X,'THE INITIAL DISTANCE MUST BE > OR = ZERO:',/,6X, R
.' --REENTER ?')
READ(IN,85,ERR=222) XI
GOTO 220
226 IF(XF.GE.0.0) GOTO 234
228 WRITE(IN,230)
230 FORMAT(/,5X,'THE FINAL DISTANCE MUST BE > OR = ZERO:',/,6X,7 E
.' --REENTER ?')
READ(IN,85,ERR=228) XF
GOTO 226
232 XF=XI
234 GOTO (240,530),IEDIT
C
C CHOICE OF THE ISOTHERM
C
240 WRITE(LP,242)
242 FORMAT(/,5X,'WHICH ISOTHERM DO YOU WANT TO USE?',/,13X,
.'ENTER 1--LINEAR',/,20X,'2--FREUNDLICH'
./,20X,'?')

```

```

244 READ(IN,246) ITYPE
246 FORMAT(I1)
      GOTO (250,270), ITYPE
      WRITE(LP,248)
248 FORMAT(5X, 'YOU MUST CHOOSE 1 OR 2 --REENTER',/, '      ?')
      GOTO 244

C
C
C THE LINEAR ISOTHERM
C THE DISTRIBUTION COEFFICEINT
250 WRITE(LP,252) (ARAY(KK),KK=1,2), (ARRAY(JJ),JJ=1,2)
252 FORMAT(/,5X, 'ENTER THE ADSORPTION DISTRIBUTION COEFFICEINT'
      .,/,10X, 'IN',1X,2A1, '/',2A1, ' .')
      READ(IN,254,ERR=250) K
254 FORMAT(F10.6)
256 IF(K.GT.0.0) GOTO 260
      WRITE(LP,258)
258 FORMAT(5X, 'THE DISTRIBUTION COEFFICEINT MUST BE GREATER THAN ' :
      . 'ZERO:',/,6X, ' --REENTER  ?')
      READ(IN,254) K
      GOTO 256
260 GOTO (446,530), IEDIT
C
C THE FREUNDLICH ISOTHERM
270 WRITE(LP,272)
272 FORMAT(5X, 'ENTER THE EXPONENT, N',/,5X, '?')
      READ(IN,85,ERR=270) N
C
C THE DISTRIBUTION COEFFICEINT
274 WRITE(LP,276) (ARAY(KK),KK=1,2), (ARRAY(JJ),JJ=1,2)
276 FORMAT(/,5X, 'ENTER THE ADSORPTION DISTRIBUTION COEFFICEINT'
      .,/,10X, 'IN',1X,2A1, '/',2A1, ' .')
      READ(IN,278,ERR=274) K
278 FORMAT(F10.6)
280 IF(K.GT.0.0) GOTO 284
      WRITE(LP,282)
282 FORMAT(5X, 'THE DISTRIBUTION COEFFICEINT MUST BE GREATER THAN ' :
      . 'ZERO:',/,6X, ' --REENTER  ?')
      READ(IN,278) K
      GOTO 280
284 GOTO (446,530), IEDIT
C
C LIST THE CURRENT VALUES TO DEFINE THE PROBLEM
C
446 WRITE(LP,447) NPAGE, (TITLE(I), I=1,30)
447 FORMAT(1H1,/,5X, 'VIRUS TRANSPORT',/,5X, 'PAGE', I3,///,5X,30A2,///)
      NPAGE=NPAGE+1
      WRITE(LP,449) P,BD, (UNIT(KK),KK=1,5), V, IL, IT(1), IT(2), K,
      . (ARAY(KK),KK=1,2), (ARRAY(JJ),JJ=1,2)
449 FORMAT(5X, 'SOIL HYDRAULIC PROPERTIES',/,10X,
      . 'POROSITY', F10.5,/,10X,
      . 'BULK DENSITY', F10.5,1X,5A1,/,10X,
      . 'SUPERFICIAL VELOCITY', F10.5,1X,A2, '/',A1,A1,/
      .,10X, 'DISTRIBUTION COEFFICEINT', F10.6,1X,2A1, '/',2A1)
      WRITE(LP,451) LF, IT(1), IT(2), LS, IT(1), IT(2)

```

```

451  FORMAT(/,5X,'VIRUS INACTIVATION RATES',/,10X,
      . 'SUSPENDED IN FLUID           ',F10.6,1X,'1/',A1,A1,/,10X,
      . 'ADSORBED ON SOIL            ',F10.6,1X,'1/',A1,A1)
      WRITE(LP,453) ALPHA,IT(1),IT(2),CO,IM1,IM2,IM3
453  FORMAT(/,5X,'SOURCE VALUES',/,10X,
      . 'INACTIVATION RATE           ',F10.6,1X,'1/',A1,A1,/,10X,
      . 'INITIAL CONCENTRATION        ',F10.2,1X,3A2)
C
C LIST THE TIME AND SPACE PARAMETERS
      WRITE(LP,455) IL,IT(1),IT(2),XI,DELX,XF,TI,DELT,TF
455  FORMAT(//,5X,'POINTS OF OBSERVATION IN DISTANCE ('A2,')', AND TI
      . IME ('A1,A1,')',/,10X,'INITIAL X =',F7.1,2X,'DELTA X =',F7.1
      . ,2X,'FINAL X =',F7.1,/,10X,'INITIAL T =',F7.1,2X,'DELTA T =',
      . F7.1,2X,'FINAL T =',F7.1)
C
C
C THIS ROUTINE CALLS THE SUBROUTINES TO SOLVE THE PROBLEM WITH
C   EACH ISOTHERM AND PRINTS THE RESULTS IN TABULAR FORM OF X/T.
C
C
C DETERMINE THE NUMBER OF POINTS OF OBSERVATION IN TIME AND SPACE
456  CONTINUE
      NPX=1
      XDEL=DELX
      IF(DELX.LT.1.0E-03) GOTO 458
      NPX=ABS(XF-XI)/DELX+1
      IF(XI.GT.XF)XDEL=-DELX
458  CONTINUE
      NPT=1
      TDEL=DELT
      IF(DELT.LT.1.0E-03) GOTO 460
      NPT=ABS(TF-TI)/DELT+1
      IF(TI.GT.TF) TDEL=-DELT
460  CONTINUE
C
      NCFLG=1
      LPRT=1
462  NROW1=1
      NROW2=MAXROW
464  IF(NROW2.GT.NPT) NROW2=NPT
C
C
      DO 502 NROW=NROW1,NROW2
          GOTO (466,468),NCFLG
466  NCOL1=1
          NCOL2=MAXCOL
468  IF(NCOL2.GT.NPX) NCOL2 = NPX
          NCOL=MAXCOL
          IF(NCOL2.EQ.NPX) NCOL=NCOL2-NCOL1+1
C
          T=TI+FLOAT(NROW-1)*TDEL
          IF(NROW.EQ.NPT)T=TF
C
C

```



```

C      THIS SECTION CALLS THE SUBROUTINES
C
      DO 480 I=NCOL1,NCOL2
          J=I-NCOL1+1
          X=XI+FLOAT(I-1)*XDEL
          IF(I.EQ.NPX) X=XF
          GOTO (470,472),ITYPE
470      CALL LINEAR(C,X,T,K)
          GOTO 476
472      CALL FREUND(C,X,T,K,N)
476      CON(J)=C
          COL(J)=X
480      CONTINUE
C
      PRINT CONCENTRATION DISTRIBUTION
C
          GOTO (490,498),LPRT
490      WRITE(LP,447) NPAGE,(TITLE(I),I=1,30)
          NPAGE=NPAGE+1
          WRITE(LP,492) IL,(COL(J),J=1,NCOL)
492      FORMAT(11X,'LIQUID PHASE CONCENTRATION DISTRIBUTION',/1X,
          '-----',
          '/,5X,'|',/,5X,'|',15X,'DISTANCE ',A2,/,5X,'|',/,1X,
          '-----',
          '/,5X,'|',7X,'|',/,5X,'|',7X,'|',5F9.2,/,5X,'|',7X,'|',/,5X,'|',
          '-----')
498      IF(NROW.EQ.1) GOTO 600
          IF(NROW.EQ.2) GOTO 605
          IF(NROW.EQ.3) GOTO 610
          IF(NROW.EQ.4) GOTO 615
          IF(NROW.EQ.5) GOTO 620
          IF(NROW.EQ.6) GOTO 600
          IF(NROW.EQ.7) GOTO 625
          IF(NROW.EQ.8) GOTO 630
          IF(NROW.GE.9) GOTO 600
600      WRITE(LP,601) T,(CON(L),L=1,NCOL)
601      FORMAT(5X,'|',7X,'|',/,5X,'|',F7.1,'|',5F9.4)
          GOTO 500
605      WRITE(LP,606) T,(CON(L),L=1,NCOL)
606      FORMAT(5X,'|',7X,'|',/,2X,'T',2X,'|',F7.1,'|',5F9.4)
          GOTO 500
610      WRITE(LP,611) T,(CON(L),L=1,NCOL)
611      FORMAT(5X,'|',7X,'|',/,2X,'I',2X,'|',F7.1,'|',5F9.4)
          GOTO 500
615      WRITE(LP,616) T,(CON(L),L=1,NCOL)
616      FORMAT(5X,'|',7X,'|',/,2X,'M',2X,'|',F7.1,'|',5F9.4)
          GOTO 500
620      WRITE(LP,621) T,(CON(L),L=1,NCOL)
621      FORMAT(5X,'|',7X,'|',/,2X,'E',2X,'|',F7.1,'|',5F9.4)
          GOTO 500
625      WRITE(LP,626) IT(1),T,(CON(L),L=1,NCOL)
626      FORMAT(5X,'|',7X,'|',/,2X,A1,2X,'|',F7.1,'|',5F9.4)
          GOTO 500
630      WRITE(LP,631) IT(2),T,(CON(L),L=1,NCOL)
631      FORMAT(5X,'|',7X,'|',/,2X,A1,2X,'|',F7.1,'|',5F9.4)

```

```

500     LPRT=2
502     CONTINUE
        IF(NROW2.EQ.NPT) GOTO 504
        NROW1=NROW1+MAXROW
        NROW2=NROW2+MAXROW
        LPRT=1
        NCFLG=2
        GOTO 464
504     IF(NCOL2.EQ.NPX) GOTO 506
        NCOL1=NCOL1+MAXCOL
        NCOL2=NCOL2+MAXCOL
        LPRT=1
        NCFLG=2
        GOTO 462
506     CONTINUE
C
C MENU THE NEXT POSSIBLE OPERATION OR CHANGES TO DATA
C
C NEXT OPERATION OPTIONS
508     WRITE(LP,510)
510     FORMAT(//,5X,'WOULD YOU LIKE TO CONTINUE WITH AN ISOTHERM','/,10X,
        . 'OR WOULD YOU LIKE TO STOP?',/,13X,'ENTER 1--LINEAR',/,20X,
        . '2--FREUNDLICH',/,20X,'3--CHANGE INPUT',/,20X,
        . '4--STOP',/,20X, '?')
512     READ(IN,246)I
        ITYPE=I
        IEDIT = 1
        GOTO (250,270,530,700),ITYPE
        WRITE(LP,514)
514     FORMAT(5X,'YOU MUST CHOOSE A 1, 2, 3 OR 4 --REENTER',/,5X, '?')
        GOTO 512
C
C CHANGE INPUT OPTIONS
530     WRITE(LP,535) (TITLE(I),I=1,30),P,BD,(UNIT(KK),KK=1,5),V,IL,IT(1)
        .,IT(2)
535     FORMAT(/,5X,'THESE ARE THE VALUES NOW ENTERED',/,20X,'1--TITLE = '
        .,30A2,/,20X,'2--POROSITY = ',F10.5,/,20X,'3--BULK DENSITY = ',F10.
        .5,1X,5A1,/,20X,'4--VELOCITY = ',F10.5,1X,A2,'/',A1,A1)
C
        WRITE(LP,540) LF,IT(1),IT(2),LS,IT(1),IT(2),K,(ARRAY(KK),KK=1,2),
        .(ARRAY(JJ),JJ=1,2)
540     FORMAT(20X,'5--LAMBDA(F) = ',F10.6,1X,'1/',A1,A1,/
        .,20X,'6--LAMBDA(S)= ',F10.6,1X,'1/',A1,A2,/,20X,
        . '7--DISTRIBUTION COEFFICEINT = ',F9.6,1X,2A1,'/',2A1)
C
        WRITE(LP,545) C0,IM1,IM2,IM3,ALPHA,IT(1),IT(2)
545     FORMAT(20X,'8--INITIAL CONCENTRATION = ',F10.4,1X,3A2,/,20X,
        . '9--ALPHA = ',F10.6,1X,'1/',A1,A1,/,19X,
        . '10--MAKE NO CHANGE',/,19X,'11--STOP')
        IEDIT=2
C
C

```

```

      read(in,*) ij
      GOTO (9,53,75,95,130,156,556,180,123,560,700),IJ
      WRITE(LP,555)
555  FORMAT(/,5X,'YOU MUST ENTER A 1, 2, 3, 4, 5, 6, 7, 8, 9, OR A 10'
      .,/,10X,'--PLEASE REENTER')
      GOTO 530
556  WRITE(IN,557)
557  FORMAT(5X,'WHICH ISOTHERM ARE YOU USING?',/,13X,
      . 'ENTER 1--LINEAR',/,20X,'2--FREUNDLICH'
      .,/,20X,'?')
      READ(IN,246,ERR=556) J
      GOTO(250,270),J
560  EDIT=1
      GOTO 508
C
700  STOP
      END
C
C
C
C
C
C
C
C
C SUBROUTINES TO SOLVE WITH EACH RESPECTIVE ISOTHERM
C
C
C
C SUBROUTINE TO SOLVE WITH THE LINEAR ISOTHERM
      SUBROUTINE LINEAR(C,X,T,K)
      REAL K,LF,LS
      COMMON /PARAM/ P,BD,V,LF,LS,CO,ALPHA
C
      FRONT=V*T/(P+K*BD)
      ALFA=ABS(ALPHA)
      IF(X.LE.FRONT) GOTO 10
      C=0.0
      RETURN
C
10  EX=ALFA*(P+K*BD)+(P*LF+K*BD*LS)
      EXPO=-ALFA*T+(EX*X/V)
      IF(EXPO.LT.-1.0E32) EXPO=-1.0E32
      IF(EXPO.GT.1.0E32) EXPO= 1.0E32
      C=CO*EXP(EXPO)
      RETURN
      END
C
C
C

```

```

C
C
C SUBROUTINE TO SOLVE WITH THE FREUNDLICH ISOTHERM
  SUBROUTINE FREUND(C,X,T,K,N)
    REAL K,N,LS,LF
    COMMON /PARAM/ P,BD,V,LF,LS,CO,ALPHA
C
  ALFA=ABS(ALPHA)
4  IF(X.GT.0.001) GOTO 5
  C=CO*EXP(-ALFA*T)
  RETURN
C
5  FRONT=(V/P)*T
  IF(X.LE.FRONT) GOTO 10
  C=0.0
  RETURN
C
10 A=T/2.0
  T1=1.0-EXP((N-1.0)*P*LF*X/V)
  T2=-1.0*ALFA*A*(N-1.0)
  T3=((K*BD*LS)/(P*LF))*CO**(N-1.0)
  T4=N/((N-1.0)*LS)
  T5=P*X/V
  IF(ALFA.LT.1.0E-6) GOTO 40
C
20 FUNC=A-T+T5-T4*ALOG(1.0+T3*EXP(T2)*T1)
  IF(ABS(FUNC).LT.0.01) GOTO 30
  F1=-1.0*ALFA*(N-1.0)
  FSLOPE=1.0-T4*((T3*T1*EXP(T2)*F1)/(1.0+T3*T1*EXP(T2)))
  A=A-(FUNC/FSLOPE)
  GOTO20
C
30 EX=(LS/N)*((V/P)*T-X)
  EXPO=P/V*(LF*X+EX)
  EXPON=-A*(ALFA+LS/N)+EXPO
  IF(EXPON.LT.-1.0E32) EXPON=-1.0E32
  IF(EXPON.GT. 1.0E32) EXPON= 1.0E32
  C=CO*EXP(EXPON)
  RETURN
C
40 F2=P*LF*X/V
  F3=CO*EXP(F2)
  F4=1.0/(N-1.0)
  C=F3/((1.0+T3*T1)**F4)
  RETURN
END

```

APPENDIX B  
COMPLETE OUTPUT OF A  
COMPUTER RUN

```
*****
**
**          THIS PROGRAM MODELS THE MOVEMENT          **
**        OF VIRUSES THROUGH THE GROUNDWATER          **
**
**
*****
```

ENTER TITLE ?  
trial 1 12/10/84

\*\*\* ENTER THE UNITS OF THE SYSTEM \*\*\*

ENTER UNITS FOR LENGTH  
(2 CHARACTERS) ?  
cm

ENTER UNITS FOR TIME  
(2 CHARACTERS) ?  
mn

ENTER CONCENTRATION UNITS  
(6 CHARACTERS) ?  
plu/cc

ENTER THE UNITS FOR DENSITY  
(5 CHARACTERS) ?  
gm/cc

\*\*\* ENTER THE SOIL/HYDRAULIC PROPERTIES \*\*\*

ENTER FORMATION POROSITY ?  
.4

ENTER BULK DENSITY OF THE FORMATION IN GM/CC ?  
1.25

ENTER DARCY (SUPERFICIAL) VELOCITY, CM/MN ?  
.03

\*\*\* ENTER THE VIRUS CHARACTERISTICS \*\*\*

ENTER THE SOURCE INACTIVATION RATE (ALPHA) IN 1/MN ?  
-.0007

ENTER VIRUS INACTIVATION RATE  
SUSPENDED IN THE FLUID (LAMBDA(F)) IN 1/MN ?  
-.0007

ENTER VIRUS INACTIVATION RATE  
ADSORBED ON THE SOIL (LAMBDA(S)) IN 1/MN ?  
-.0004

\*\*\* ENTER THE GENERAL PARAMETERS OF THE SYSTEM \*\*\*

ENTER THE INITIAL CONCENTRATION ?  
1000.

ENTER THE TIME OF INTEREST.  
STEADY STATE HAS NOT BEEN ASSUMED,  
THEREFORE ENTER INITIAL, FINAL AND DELTA TIMES  
IN MN ?  
0.0,5400.,300.

ENTER THE DISTANCE OF INTEREST  
ENTER THE INITIAL, FINAL AND DELTA DISTANCES  
IN CM ?  
0.0,140.,10.

WHICH ISOTHERM DO YOU WANT TO USE ?  
ENTER 1--LINEAR  
2--FREUNDLICH  
1

ENTER THE ADSORPTION DISTRIBUTION COEFFICIENT  
IN CC/GM .  
.01

VIRUS TRANSPORT  
PAGE 1

TRIAL 1 12/10/84

SOIL HYDRAULIC PROPERTIES	
POROSITY	0.40000
BULK DENSITY	1.25000 GM/CC
SUPERFICIAL VELOCITY	0.03000 CM/MN
DISTRIBUTION COEFFICIENT	0.010000 CC/GM

VIRUS INACTIVATION RATES	
SUSPENDED IN FLUID	-0.000700 1/MN
ADSORBED ON SOIL	-0.000400 1/MN

SOURCE VALUES	
INACTIVATION RATE	-0.000700 1/MN
INITIAL CONCENTRATION	1000.00 PLU/CC

POINTS OF OBSERVATION IN DISTANCE (CM) AND TIME (MN)

INITIAL X =	0.0	DELTA X =	10.0	FINAL X =	140.0
INITIAL T =	0.0	DELTA T =	300.0	FINAL T =	5400.0

VIRUS TRANSPORT  
PAGE 2

TRIAL 1 12/10/84

LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		0.00	10.00	20.00	30.00	40.00
	0.0	1000.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	810.5842	811.5981	812.6133	0.0000	0.0000
I	600.0	657.0466	657.8687	658.6914	659.5154	660.3403
M	900.0	532.5918	533.2578	533.9250	534.5928	535.2615
E	1200.0	431.7104	432.2505	432.7913	433.3325	433.8745
	1500.0	349.9380	350.3757	350.8140	351.2529	351.6924
M	1800.0	283.6541	284.0090	284.3643	284.7200	285.0762
N	2100.0	229.9256	230.2133	230.5013	230.7897	231.0784
	2400.0	186.3741	186.6072	186.8407	187.0745	187.3085
	2700.0	151.0719	151.2609	151.4501	151.6396	151.8293
	3000.0	122.4565	122.6097	122.7631	122.9167	123.0704
	3300.0	99.2613	99.3855	99.5098	99.6343	99.7590
	3600.0	80.4596	80.5603	80.6611	80.7620	80.8631
	3900.0	65.2193	65.3009	65.3826	65.4644	65.5463
	4200.0	52.8657	52.9319	52.9981	53.0644	53.1308
	4500.0	42.8521	42.9057	42.9594	43.0132	43.0670
	4800.0	34.7353	34.7787	34.8222	34.8658	34.9094
	5100.0	28.1559	28.1911	28.2263	28.2617	28.2970
	5400.0	22.8227	22.8512	22.8798	22.9084	22.9371



VIRUS TRANSPORT  
PAGE 3

TRIAL 1 12/10/84

LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		50.00	60.00	70.00	80.00	90.00
	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	0.0000	0.0000	0.0000	0.0000	0.0000
I	600.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	900.0	535.9312	536.6013	0.0000	0.0000	0.0000
E	1200.0	434.4172	434.9607	435.5046	436.0496	0.0000
	1500.0	352.1321	352.5725	353.0137	353.4553	353.8975
M	1800.0	285.4326	285.7898	286.1472	286.5051	286.8638
N	2100.0	231.3673	231.6567	231.9465	232.2367	232.5273
	2400.0	187.5427	187.7773	188.0122	188.2474	188.4829
	2700.0	152.0191	152.2093	152.3997	152.5904	152.7812
	3000.0	123.2243	123.3784	123.5328	123.6873	123.8421
	3300.0	99.8837	100.0086	100.1337	100.2590	100.3845
	3600.0	80.9641	81.0654	81.1668	81.2684	81.3700
	3900.0	65.6282	65.7104	65.7926	65.8749	65.9572
	4200.0	53.1972	53.2638	53.3304	53.3971	53.4639
	4500.0	43.1208	43.1748	43.2288	43.2829	43.3370
	4800.0	34.9530	34.9968	35.0406	35.0844	35.1283
	5100.0	28.3324	28.3678	28.4033	28.4389	28.4744
	5400.0	22.9658	22.9945	23.0233	23.0521	23.0809

VIRUS TRANSPORT  
PAGE 4

TRIAL 1 12/10/84

LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		100.00	110.00	120.00	130.00	140.00
	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	0.0000	0.0000	0.0000	0.0000	0.0000
I	600.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	900.0	0.0000	0.0000	0.0000	0.0000	0.0000
E	1200.0	0.0000	0.0000	0.0000	0.0000	0.0000
	1500.0	354.3398	0.0000	0.0000	0.0000	0.0000
M	1800.0	287.2222	287.5815	287.9414	288.3015	0.0000
N	2100.0	232.8179	233.1092	233.4008	233.6928	233.9852
	2400.0	188.7186	188.9547	189.1910	189.4277	189.6647
	2700.0	152.9723	153.1637	153.3552	153.5471	153.7392
	3000.0	123.9969	124.1520	124.3073	124.4628	124.6186
	3300.0	100.5099	100.6357	100.7616	100.8876	101.0138
	3600.0	81.4717	81.5737	81.6758	81.7779	81.8802
	3900.0	66.0397	66.1223	66.2051	66.2879	66.3708
	4200.0	53.5307	53.5977	53.6648	53.7319	53.7991
	4500.0	43.3912	43.4455	43.4998	43.5542	43.6087
	4800.0	35.1722	35.2162	35.2603	35.3044	35.3485
	5100.0	28.5100	28.5457	28.5814	28.6172	28.6530
	5400.0	23.1098	23.1387	23.1676	23.1966	23.2256

WOULD YOU LIKE TO CONTINUE WITH AN ISOTHERM,  
OR WOULD YOU LIKE TO STOP ?

ENTER 1--LINEAR  
2--FREUNDLICH  
3--CHANGE INPUT  
4--STOP

3

THESE ARE THE VALUES NOW ENTERED

1--TITLE = TRIAL 1 12/10/84  
2--POROSITY = 0.40000  
3--BULK DENSITY = 1.25000 GM/CC  
4--VELOCITY = 0.03000 CM/MN  
5--LAMBDA(F) = -0.000700 1/MN  
6--LAMBDA(S) = -0.000400 1/MN  
7--DISTRIBUTION COEFFICEINT = 0.010000 CC/GM  
8--INITIAL CONCENTRATION = 1000.0000 PLU/CC  
9--ALPHA = -0.000700 1/MN  
10--MAKE NO CHANGE  
11--STOP

3

ENTER BULK DENSITY OF THE FORMATION IN GM/CC ?

1.55

THESE ARE THE VALUES NOW ENTERED

1--TITLE = TRIAL 1 12/10/84  
2--POROSITY = 0.40000  
3--BULK DENSITY = 1.55000 GM/CC  
4--VELOCITY = 0.03000 CM/MN  
5--LAMBDA(F) = -0.000700 1/MN  
6--LAMBDA(S) = -0.000400 1/MN  
7--DISTRIBUTION COEFFICEINT = 6.000000 CC/GM  
8--INITIAL CONCENTRATION = 1000.0000 PLU/CC  
9--ALPHA = -0.000700 1/MN  
10--MAKE NO CHANGE  
11--STOP

10

WOULD YOU LIKE TO CONTINUE WITH AN ISOTHERM,  
OR WOULD YOU LIKE TO STOP?

ENTER 1--LINEAR  
2--FREUNDLICH  
3--CHANGE INPUT  
4--STOP

2

ENTER THE EXPONENT, N ?

1.11

ENTER THE ADSORPTION DISTRIBUTION COEFFICEINT  
IN CC/GM .

.01

VIRUS TRANSPORT  
PAGE 5

TRIAL 1 12/10/84

SOIL HYDRAULIC PROPERTIES

POROSITY	0.40000
BULK DENSITY	1.55000 GM/CC
SUPERFICIAL VELOCITY	0.03000 CM/MN
DISTRIBUTION COEFFICEINT	0.010000 CC/GM

VIRUS INACTIVATION RATES

SUSPENDED IN FLUID	-0.000700 1/MN
ADSORBED ON SOIL	-0.000400 1/MN

SOURCE VALUES

INACTIVATION RATE	-0.000700 1/MN
INITIAL CONCENTRATION	1000.00 PLU/CC

POINTS OF OBSERVATION IN DISTANCE (CM) AND TIME (MN)

INITIAL X =	0.0	DELTA X =	10.0	FINAL X =	140.0
INITIAL T =	0.0	DELTA T =	300.0	FINAL T =	5400.0

VIRUS TRANSPORT  
PAGE 6

TRIAL 1 12/10/84

LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		0.00	10.00	20.00	30.00	40.00
	0.0	1000.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	810.5842	813.9075	817.2117	0.0000	0.0000
I	600.0	657.0466	659.7092	662.3540	664.9863	667.5947
M	900.0	532.5918	534.7239	536.8457	538.9529	541.0452
E	1200.0	431.7104	433.4209	435.1196	436.8062	438.4841
	1500.0	349.9380	351.3071	352.6697	354.0222	355.3650
M	1800.0	283.6541	284.7524	285.8428	286.9253	288.0017
N	2100.0	229.9256	230.8046	231.6790	232.5470	233.4099
	2400.0	186.3741	187.0791	187.7787	188.4745	189.1649
	2700.0	151.0719	151.6361	152.1980	152.7545	153.3090
	3000.0	122.4565	122.9087	123.3583	123.8053	124.2483
	3300.0	99.2613	99.6239	99.9841	100.3417	100.6964
	3600.0	80.4596	80.7503	81.0389	81.3253	81.6097
	3900.0	65.2193	65.4522	65.6836	65.9131	66.1407
	4200.0	52.8657	53.0524	53.2377	53.4216	53.6043
	4500.0	42.8521	43.0017	43.1502	43.2976	43.4437
	4800.0	34.7353	34.8551	34.9740	35.0921	35.2094
	5100.0	28.1559	28.2518	28.3473	28.4418	28.5357
	5400.0	22.8227	22.8996	22.9760	23.0517	23.1270

VIRUS TRANSPORT  
PAGE 7

TRIAL 1 12/10/84

LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		50.00	60.00	70.00	80.00	90.00
	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	0.0000	0.0000	0.0000	0.0000	0.0000
I	600.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	900.0	543.1218	545.1853	0.0000	0.0000	0.0000
E	1200.0	440.1460	441.7986	443.4407	445.0725	0.0000
	1500.0	356.6975	358.0222	359.3369	360.6428	361.9399
M	1800.0	289.0701	290.1296	291.1831	292.2312	293.2686
N	2100.0	234.2643	235.1157	235.9580	236.7971	237.6277
	2400.0	189.8509	190.5316	191.2081	191.8786	192.5449
	2700.0	153.8571	154.4025	154.9445	155.4823	156.0159
	3000.0	124.6879	125.1258	125.5587	125.9900	126.4177
	3300.0	101.0495	101.3992	101.7470	102.0923	102.4346
	3600.0	81.8923	82.1726	82.4510	82.7275	83.0021
	3900.0	66.3671	66.5920	66.8150	67.0363	67.2561
	4200.0	53.7856	53.9653	54.1441	54.3213	54.4977
	4500.0	43.5889	43.7330	43.8763	44.0184	44.1595
	4800.0	35.3256	35.4412	35.5558	35.6696	35.7825
	5100.0	28.6290	28.7213	28.8133	28.9044	28.9950
	5400.0	23.2017	23.2758	23.3495	23.4225	23.4949

VIRUS TRANSPORT  
PAGE 8

TRIAL 1 12/10/84

LIQUID PHASE CONCENTRATION DISTRIBUTION

		DISTANCE CM				
		100.00	110.00	120.00	130.00	140.00
	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
T	300.0	0.0000	0.0000	0.0000	0.0000	0.0000
I	600.0	0.0000	0.0000	0.0000	0.0000	0.0000
M	900.0	0.0000	0.0000	0.0000	0.0000	0.0000
E	1200.0	0.0000	0.0000	0.0000	0.0000	0.0000
	1500.0	363.2263	364.5051	0.0000	0.0000	0.0000
M	1800.0	294.3010	295.3254	296.3403	297.3486	0.0000
N	2100.0	238.4550	239.2751	240.0897	240.8973	241.6990
	2400.0	193.2077	193.8644	194.5165	195.1627	195.8061
	2700.0	156.5459	157.0718	157.5938	158.1123	158.6272
	3000.0	126.8423	127.2627	127.6818	128.0967	128.5086
	3300.0	102.7745	103.1122	103.4470	103.7800	104.1095
	3600.0	83.2742	83.5443	83.8134	84.0792	84.3437
	3900.0	67.4744	67.6905	67.9057	68.1193	68.3308
	4200.0	54.6723	54.8457	55.0177	55.1889	55.3583
	4500.0	44.2996	44.4383	44.5762	44.7131	44.8488
	4800.0	35.8948	36.0060	36.1165	36.2260	36.3348
	5100.0	29.0849	29.1738	29.2624	29.3501	29.4374
	5400.0	23.5669	23.6382	23.7091	23.7794	23.8493

WOULD YOU LIKE TO CONTINUE WITH AN ISOTHERM,  
OR WOULD YOU LIKE TO STOP? d10

ENTER 1--LINEAR

2--FREUNDLICH

3--CHANGE INPUT

4--STOP

?



VITA<sup>1</sup>

Annie D. Hogsett

Candidate for the Degree of

Master of Science

Thesis: A MATHEMATICAL MODEL OF VIRUS TRANSPORT & SURVIVAL  
IN GROUNDWATER

Major Field: Chemical Engineering

Biographical:

Personal Data: Born in Dallas, Texas, January 26,  
1959.

Education: Graduated from St. Stephen's Episcopal  
School, Austin, Texas in May, 1977; received  
Bachelor of Science Degree in Chemistry from  
Oklahoma State University in May, 1981; com-  
pleted requirements for a Master of Science  
degree at Oklahoma State University in July,  
1985.

Professional Experience: Research Assistant in Depart-  
ment of Chemistry at Oklahoma State University,  
January, 1979 to May, 1980; Research Technician,  
M.D. Anderson Tumor Institute, Houston, Texas,  
May to September, 1980; Teaching Assistant in  
the Department of Chemistry at Oklahoma State  
University, January, 1981 to May, 1981; Medical  
Technologist and Supervisor of Special Chemistry  
Department, Lufkin Memorial Hospital, Lufkin,  
Texas, August 1981 to May 1982; Process Engineer,  
Bendix Corporation, Kansas City, Missouri, June,  
1981 to present.