A BAYESIAN ANALYSIS OF AUTOREGRESSIVE PROCESSES: TIME AND FREQUENCY DOMAIN

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PREFACE

A Bayesian approach to inferences in the time and frequency domains is developed for multivariate autoregressive models. Examples are presented to demonstrate the use and limitations of the techniques. The approach handles a very wide class of models, including a rich class of nonstationary (and explosive) models. To the best of my knowledge, the AR spectral analysis approach as described in Chapters IV and V is new.

I wish to express extreme indebtedness to all of the people who have contributed to the development of this thesis. My thesis advisor, Professor L. D. Broemeling has been instrumental in the development of this thesis by providing guidance, concern, encouragement, and my introduction to the Bayesian approach to statistical inference.

The other committee members also deserve thanks. Professor Ron W. McNew showed me that a procedure must work in practice. The application of statistical procedures can be quite different than the theory. Professor N. Mukhopadhyay was always willing to discuss the philosophy of various statistical approaches. And, Professor John Rea introduced me to the application of statistics to economic problems.

There are other people that deserve thanks. Professor W. Stewart was instrumental in my deciding to formally study statistics. Professor D. L. Weeks showed me a boundless sense of humor whenever he asked me "But, does it work?" Finally without the pioneers in probability and statistics like A. N. Kolmogorov and Harold Jeffreys, the topic of this thesis would not exist.

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CHAPTER I

INTRODUCTION

Many areas of the physical and social sciences require the use of time series models. One of the popular families of models is the linear system of difference equations with constant coefficients. When randomness is added the model is called the vector autoregressive process (VAP or $AR_k(p)$) where the observable variables are k-dimensional random vectors covering p lags. For example, in economics one might suppose that the current values of Gross National Product and Unemployment are linear combinations of their past values. Also, one might suppose that these two economic variables interact so that if unemployment increases by some percentage then Gross National Product will change according to its past history as well as the past history of unemployment.

Many observed portions of time series (especially economic series) exhibit growth. An example is Gross National Product. Population increase in the absence of resource constraints should provide for growth of national output. (Note that over the last 100 years there have been no resource constraints which prevented real long term growth of national output in the United States.) Classical techniques for modeling nonstationary time series generally require that the observable process be transformed into a process that is covariance stationary. But, it may not be possible to determine a priori which transformation should be used. Further, even an approximate transformation may be difficult to determine. If the wrong type of nonstationarity is removed, one may produce a distorted series which leads to poor predictions. The need to work with a covariance stationary series is motivated by widespread use of asymptotic approximations used in classical inference procedures. Small sample inference procedures which do not use asymptotic approximations and are valid for a wide class of nonstationary models are likely to provide a simpler modeling procedure which is less likely to produce poor predictions.

Since the nature of many processes changes over time, there is a need to detect when these changes occur. Changing parameter values may make asymptotic approximations untenable.

The goal of this paper is to describe a Bayesian approach to making inferences about the change point as well as other parameters in the changing VAP. The approach to be described can be applied to nonstationary VAP's, allows for the use of concentrated (proper) or nonconcentrated (improper) prior information, does not use asymptotic approximations, and does not require iterative computation to produce the final analysis.

Chapter II contains a review of the relevant literature. The topics included are classical and Bayesian linear and changing linear models, classical and Bayesian changing and nonchanging time series models, and Bayesian inferences in the frequency domain for time series models. Chapter III describes Bayesian inferences in the time domain for the changing $AR_k(p)$ process. Chapter IV describes a Bayesian approach to making inferences in the frequency domain for a nonchanging $AR_1(p)$ process. Chapter V describes Bayesian frequency domain inferences for the $AR_k(p)$ operator. Finally, Chapter VI summarizes the main results which have been derived.

CHAPTER II

REVIEW OF THE LITERATURE

Much has been written about autoregressive-moving average (ARMA) time series models in the last 25 years. Perhaps the first modern book to achieve popularity was by Blackman and Tukey (1954) followed by Jenkins and Watts (1968). Kendall (1973) and Chatfield (1975) provided general introductions to time series. Bloomfield (1976) paid special attention to spectral analysis as did Koopmans (1974). Anderson (1971) provided a thorough mathematical treatment of ARMA models for the univariate case. Fuller (1976) described both univariate and multivariate ARMA models and their associated classical estimation techniques in a theoretical manner requiring background in advanced calculus but not measure theory. Advanced books on time series were written by Hannon (1970) and Brillinger (1975).

Perhaps the most important book on time series was written by Box and Jenkins (1976). Their interactive approach to model identification, parameter estimating, forecasting, and diagnostic checking seems to have made time series analysis popular by describing practical analysis procedures. Their book remains the bible of time series analysis.

Ali (1977) derived the exact likelihood function for an ARMA process. He also described a convenient way to numerically evaluate the likelihood function. Brillinger and Tiao (1978) contains a collection of articles discussing the foundations and future of time series analysis.

Finally, Akaike (1978) describes how to use maximum likelihood estimation in the time domain to make inferences in the frequency domain for ARMA models.

Harrison and Stevens (1976) defined the dynamic linear model and showed how a forecaster could use both data and prior knowledge to forecast the future values of a very general class of time series models. Smith (1979) described a generalization of the Bayesian steady forecasting model. Since the Bayesian conditions on past data, the analysis of the linear model is relevant. References are Lindley and Smith (1972), Press (1982), DeGroot (1970), Berger (1981), and Zellner (1971).

Newbold (1973) described a Bayesian approach to estimating Box-Jenkins transfer function noise models using approximations for the moving average part of the model.

Chow (1960) described tests of equality between sets of coefficients in two linear regressions. Farley, Hinick, and McGuire (1975) described several tests for a shift in the slope of a multivariate linear time series model. Hinkley (1969) derived the asymptotic properties of the maximum likelihood estimator for the intersection of two straight lines in the two-phase regression problem. Quandt (1958, 1972) found the maximum likelihood estimator for the shift point and parameters in two-phase regression when the error variances are different in the two phases.

Broemeling (1972) and Smith (1975) described Bayesian approaches to inferences about changing sequences of random variables. Broemeling (1977) continued his previous work by deriving the joint predictive distributions for the changing normal, Bernoulli, and exponential sequences. Bacon and Watts (1971) used a Bayesian approach to the changing straight line problem as did Ferriera (1975). Chin Choy and Broemeling (1980) generalized the previous works of Ferriera (1975) and of Holbert and Broemeling (1977) by using proper prior distributions for the changing linear model. Booth (1982) described a general Bayesian approach to the change point problem using a vague prior and allowing for the possibility of no change. Booth examined changing univariate and multivariate normal sequences, changing linear models, and changing linear time series models. But, the posterior distributions derived by Booth need to be analyzed numerically, requiring the inversion of large matrices due to the moving average components.

Salazar, Broemeling, and Chi (1981) examined the changing linear model with autocorrelated errors. Holbert (1982) provided a nice description of the Bayesian analysis of switching linear models which included an interesting application to a financial problem.

Finally, Salazar (1982) described the Bayesian analysis of changing univariate autoregressive models.

Regarding Bayesian spectral inferences, Shore and Holt (1980) seems to be the only reference. Shore and Holt provide some asymptotic results.

CHAPTER III

BAYESIAN ANALYSIS OF THE CHANGING $AR_k(p)$: TIME DOMAIN INFERENCES

Description of the $AR_k(p)$ Likelihood Function

The model described in this chapter is a generalization of one of the models discussed in Salazar (1982). Specifically, where Salazar (1982) discussed the univariate pth order autoregressive process ($AR_1(p)$), this chapter discusses the k-variate pth order autoregressive process. Let

$x(t)$, $t = 1,2,\ldots,n$ be $k\times 1$ observable random vectors,

A_i, $i = 1,2$ be $kp\times k$ unknown constant matrices,

$z(t-1) = (x'(t-1),x'(t-2),\ldots,x'(t-p))'$, $t = 1,2,\ldots,n$, with $z(0)$ a known initial condition,

$e(t)$, $t = 1,2,\ldots,n \sim$ iid $N_k(0,T)$, where T is an unknown $k\times k$ positive definite precision matrix,

$m \in \{1,2,\ldots,n-1\}$, and primes denote a transpose.

Then the model is

$$\begin{bmatrix} x'(1) \\ x'(2) \\ \vdots \\ x'(m) \\ x'(m+1) \\ x'(m+2) \\ \vdots \\ x(n) \end{bmatrix} = \begin{bmatrix} z'(0) & 0 \\ z'(1) & 0 \\ \vdots & \vdots \\ z'(m-1) & 0 \\ 0 & z'(m) \\ 0 & z'(m+1) \\ \vdots & \vdots \\ 0 & z'(n-1) \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} + \begin{bmatrix} e'(1) \\ e'(2) \\ \vdots \\ e'(m) \\ e'(m+1) \\ e'(m+2) \\ \vdots \\ e'(n) \end{bmatrix}, \qquad (3.1)$$

where the autoregressive coefficients change after x(m) has been observed. That is, A_1 is the set of coefficients before the change, and A_2 contains the coefficients after the change. Expression (3.1) can be written more compactly as

$$\underset{n\times k}{X(m)} = \underset{n\times 2pk}{Z(m)} \underset{2pk\times k}{A} + \underset{n\times k}{E(m)}, \qquad (3.2)$$

$$m = 1,2,\ldots,n-1.$$

Expression (3.2) is a compact matrix notation for a single partial realization of the $AR_k(p)$ process with exactly one change. There are situations (such as when a time series is observed for each of several independent machines) where more than one partial realization can be obtained. Using superscripts to denote random matrices corresponding to r independent partial realizations, each with n observations, one can write

$$\underset{rn\times k}{\begin{bmatrix} X^1(m) \\ X^2(m) \\ \vdots \\ X^r(m) \end{bmatrix}} = \underset{rn\times 2pk}{\begin{bmatrix} Z^1(m) \\ Z^2(m) \\ \vdots \\ Z^r(m) \end{bmatrix}} \underset{2pk\times k}{A} + \underset{rn\times k}{\begin{bmatrix} E^1(m) \\ E^2(m) \\ \vdots \\ E^r(m) \end{bmatrix}}. \qquad (3.3)$$

Expression (3.3) explicitly allows one to combine data for r independent partial realizations. However, the form of (3.2) is the same as (3.3). It is convenient to merely alter the dimension of (3.2) to account for the r independent partial realizations. This produces

$$\underset{rn\times k}{X(m)} = \underset{rn\times 2pk}{Z(m)} \cdot \underset{2pk\times k}{A} + E(m), \tag{3.4}$$

$m = 1,2,\ldots,n-1.$

Under the assumption of normality and with r independent partial realizations, the likelihood function for the model (3.4) is

$$f(X/A,T,m) \propto |T|^{\frac{rn}{2}} \exp -\frac{1}{2}\mathrm{tr}(X(m) - Z(m)A)'(X(m) - Z(m)A)T, \tag{3.5}$$

$A \in R^{2pk\times k}$, $T \in R^{k\times k}$ and p.d., $m \in \{1,2,\ldots,n-1\}$.

Except for the dependence on m (and the hidden dependence on z(0)), the likelihood function (3.5) is of the same form as the usual multivariate regression model. It is also important to point out that the elements of A have not been restricted in any manner. In particular, A can correspond to a nonstationary process. Actually, since z(0) is a known initial condition, the family of models considered above possesses a mean that is explicitly a function of time as well as the change point.

The Prior Distribution

To use our Bayesian approach to this statistical inference problem one must specify a prior distribution for the parameters A, T, and m. For a given value of m, the class of prior distributions considered here is the conjugate Normal-Wishart family of prior distributions. The form is

$$f(A/T,m) \propto |T|^{\frac{2pk}{2}} \exp -\frac{1}{2}\mathrm{tr}(A-B)'P(A-B)T, \tag{3.6}$$

$$f(T/m) \propto |T|^{\frac{a-(k+1)}{2}} \exp -\frac{1}{2}\mathrm{tr}\,\Sigma T, \tag{3.7}$$

$$f(m) \propto 1, \tag{3.8}$$

where $A \in R^{2pk\times k}$, $T \in R^{k\times k}$, and $m \in \{1,2,\ldots,n-1\}$. The hyperparameters are

$$\left.\begin{array}{l} B \in R^{2pk\times k} \\ P \in R^{2pk\times 2pk}, \text{ positive definite,} \\ \Sigma \in R^{k\times k}, \text{ positive definite,} \\ \text{and } a > 0 \end{array}\right\}. \tag{3.9}$$

The joint prior is obtained from

$$f(A,T,m) \propto f(A/T,m)f(T/m)f(m). \tag{3.10}$$

Note that f(A,T/m) is a Normal-Wishart prior distribution. Also, note that f(A,T,m) is a proper distribution which can be concentrated for A and T. But it cannot be concentrated for m. Also note that if $P \to 0$, $a \to -pk$, $\Sigma \to 0$, then f(A,T/m) is Jeffreys' prior conditional upon the change point m. In this case the support of m must be restricted to $m \in \{k,k+1,\ldots,n-k\}$ so that the posterior distributions will be proper. In summary, the prior used herein allows considerable flexibility in representing prior information for A and T. One might use prior information to restrict the support of m. But, the prior chosen in this thesis requires that the support be uniform for possible change points with positive probability.

The Posterior Distributions

After completing the square on the matrix A, the joint posterior distribution for all parameters is given by

$$f(A,T,m/X) \propto f(X/A,T,m)f(A,T,m) \propto |T|^{\frac{rn+2pk+a-(k+1)}{2}} \times$$

$$\exp -\frac{1}{2}\mathrm{tr}\{(A-\hat{A}(m))'C_1(m)(A-\hat{A}(m))+R(m)\}T, \qquad (3.11)$$

where

$$C_1(m) = Z'(m)Z(m) + P, \quad 2pk\times 2pk$$

$$C_2(m) = Z'(m)X(m) + PB, \quad 2pk\times k$$

$$\hat{A}(m) = C_1^{-1}(m)C_2(m), \quad 2pk\times k$$

$$R(m) = X'(m)X(m) + B'PB + \Sigma - C_2'(m)C_1^{-1}(m)C_2(m), \quad k\times k.$$

Conditional upon m, (3.11) is a Normal-Wishart distribution.

In order to obtain the marginal posterior mass function for m, one first integrates with respect to A and then T. As a result,

$$f(T,m/X) \propto |C_1(m)|^{-\frac{k}{2}} |T|^{\frac{rn+a-(k+1)}{2}} \exp -\frac{1}{2}\mathrm{tr}R(m)T,$$

$$f(m/X) \propto |C_1(m)|^{-\frac{k}{2}} |R(m)|^{-\frac{rn+a}{2}}, \quad m = 1,2,\ldots,n-1. \qquad (3.12)$$

The posterior probability mass function for the change point m, given by (3.12), is not a standard distribution. It must be analyzed numerically. It is important to point out that if rn is large one may encounter over or underflow when using a computer to evaluate (3.12). One way to avoid such numerical difficulties is to scale the determinants to be powered up. Another is to realize that one may be interested primarily in some measure of central tendency. If so, then

$$\log f(m/X) \propto \frac{k}{2}\log|C_1^{-1}(m)| + (\frac{rn+a}{2}) \log |R^{-1}(m)|, \tag{3.13}$$

can easily be evaluated with no over or underflow problems. One can obtain the posterior mode for m from (3.13) and then obtain conditional inferences for A and T.

If one is interested in inferences for A and T conditional upon m, then

$$f(A/m,X) \propto |(A-\hat{A}(m))'C_1(m)(A-\hat{A}(m))+R(m)|^{-\frac{v+(k+2pk-1)}{2}}, \tag{3.14}$$

where $v = rn+a-k+1$

$$f(T/m,X) \propto |T|^{\frac{rn+a-(k+1)}{2}} \exp -\frac{1}{2}trR(m)T, \tag{3.15}$$

That is A/m,X is distributed as a matrix-T, and T/m,X is distributed as a Wishart.

Letting $\underset{\sim}{A}_i$ be the i^{th} column of A,

$\underset{\sim}{A} = (\underset{\sim}{A}_1', \underset{\sim}{A}_2', \ldots, \underset{\sim}{A}_k')$, partitioning

$\hat{A}(m)$ similar to A, and $R^{ij}(m) = [R(m)]_{ij}^{-1}$,

$$E[\underset{\sim}{A}/m,X] = \underset{\sim}{\hat{A}}(m), \tag{3.16}$$

$$Var(\underset{\sim}{A}/m,X) = [(v-2)C_1(m)]^{-1} \otimes R(m), \tag{3.17}$$

where $\otimes$ denotes the Kroenecker product. Also,

$$E[T/m,X] = (rn+a)R^{-1}(m), \tag{3.18}$$

$$Cov(T_{ij},T_{k\ell}|m,X) = (rn+a)[R^{ij}(m)R^{j\ell}(m) + R^{i\ell}(m)R^{jk}(m)]. \tag{3.19}$$

Rather than making inferences conditional upon a point estimate for m, one may wish to make inferences on A and T based on their marginal distributions. Using (3.12) and (3.14), the marginal posterior distribution for A is

$$f(A/X) \propto \sum_{m=1}^{n-1} f(m/X)f(Z/m,X)$$

$$\propto \sum_{m=1}^{n-1} f(m/X)T_{2pk\times k}(v,A(m),C_1(m),R(m)), \tag{3.20}$$

where T denotes the matrix T distribution. Then

$$E[A/X] = \sum_{m=1}^{n-1} f(m/X)\hat{A}(m), \tag{3.21}$$

$$\underset{m}{E}\, Var(\underset{\sim}{A}/m,X) = \sum_{m=1}^{n-1} f(m/X)[(v-2)C_1(m)]^{-1}\otimes R(m), \tag{3.22}$$

$$\underset{m}{Var}\, E[\underset{\sim}{A}/m,X] = \sum_{m=1}^{n-1} f(m/X)[E[\underset{\sim}{A}/m,X]E[\underset{\sim}{A}'/m,X] - E[\underset{\sim}{A}/X]E[\underset{\sim}{A}'/X]], \tag{3.23}$$

$$Var(\underset{\sim}{A}/X) = \underset{m}{E}\, Var(\underset{\sim}{A}/m,X) + \underset{m}{Var}\, E[\underset{\sim}{A}/m,X].$$

For the precision T, the marginal posterior distribution is

$$f(T/X) \propto \sum_{m=1}^{n-1} f(m/X)f(T/m,X) \propto \sum_{m=1}^{n-1} f(m/X)W_{k\times k}(rn+a,R^{-1}(m)), \tag{3.25}$$

where W denotes the Wishart distribution. The moments are

$$E[T/X] = \sum_{m=1}^{n-1} f(m/X)(rn+a)R^{-1}(m), \tag{3.26}$$

$$\underset{m}{E}\ \mathrm{Cov}(T_{ij},T_{k\ell}/m,X) = \sum_{m=1}^{n-1} f(m/X)\mathrm{Cov}(T_{ij},T_{k\ell}/m,X), \tag{3.27}$$

$$\underset{m}{\mathrm{Cov}}\ E(\binom{T_{ij}}{T_{k\ell}})(m,X) = \sum_{m=1}^{n-1} f(m/X)[E[\binom{T_{ij}}{T_{k\ell}})(T_{ij},T_{k\ell})/m,X]$$

$$- E[\binom{T_{ij}}{T_{k\ell}})/X]E[T_{ij},T_{k\ell})/X]], \tag{3.28}$$

$$\mathrm{Cov}(T_{ij},T_{k}\ /X) = \underset{m}{E}\ \mathrm{Cov}(T_{ij},T_{k\ell}/m,X) + \underset{m}{\mathrm{Cov}}\ E[\binom{T_{ij}}{T_{k\ell}})/m,X]. \tag{3.29}$$

Expressions (3.12) - (3.29) are exact formulas which do not depend on asymptotic results. However, if rn is "very large" only the conditional formulas may be computable. Recall that authors such as Box and Jenkins (1976) suggest that one may need at least one hundred observations to get reasonably accurate point estimates for the parameters of an autoregressive process. It should be remarked that if the limiting (Jeffreys) version of (3.10) is used, then (3.12) will only exist for $m = k,k+1, \ldots,n-k$ since $R(m)$ will be singular.

No restrictions of any kind were imposed on the values of A in order to obtain expressions (3.12) - (3.29). The formulas are valid whether the $AR_k(p)$ process is (when extended over an infinite time span) stationary or nonstationary. In fact, (3.1) contains a rich family of nonstationary models obtained by differencing the data. When one contemplates the difficulties of transforming a multivariate process into stationarity, one is presented with great practical difficulties. Should one examine the components of the process individually and try to transform each into a stationary process before considering the components as a group? Should one try to view all variables together to discover a relevant transformation? Or, would it be desirable to allow an inference procedure to be applied to a rich class of nonstationary models? The posterior inferences described in this section are valid for a wide range of nonstationary models. It is possible that the Bayesian approach, which allows exact small sample inferences, will simplify the modeling phase of the analysis.

The Predictive Distribution of Future Observations

The predictive distribution for the changing $AR_k(p)$ will now be discussed. The predictive distribution is the one Bayesians may wish to use for making forecasts or extrapolating into the future. Let

$$\underset{1\times k}{x'(n+1)} = \underset{1\times 2pk \times 2pk\times k}{WA} + \underset{1\times k}{e'(n+1)}, \tag{3.30}$$

where W is any 1×2pk matrix. (In particular it might be chosen as (0',z'(n)).) Then

$$f(x'(n+1)/A,T,m,W) \propto |T|^{\frac{1}{2}} \exp -\frac{1}{2}\mathrm{tr}(x'(n+1) - WA)' \times (x'(n+1) - WA)T \,. \tag{3.31}$$

Let

$$\left.\begin{aligned}
D_1(m) &= W'W + C_1(m), \quad 2pk\times 2pk \\
F_1(m) &= 1 - WD_1^{-1}(m)W', \quad 1\times 1 \\
F_2(m) &= WD_1^{-1}(m)C_2(m), \quad 1\times k \\
\hat{x}(n) &= F_1^{-1}(m)F_2(m), \quad 1\times k \\
R_2(m) &= X'(m)X(m) + B'PB + \Sigma - F_2'(m)F_1^{-1}(m)F_2(m) \\
&\quad - C_2'(m)D_1^{-1}(m)C_2(m), \quad k\times k
\end{aligned}\right\}, \tag{3.32}$$

then the predictive distribution of x(n+1) conditional on m is given by

$$f(x(n+1)/m,X) \propto [(x(n+1) - \hat{x}(m))'R_2^{-1}(m)(x(n+1) - \hat{x}(m)) + F_1^{-1}(m)]^{-\frac{rn+a-k+1+k}{2}}. \tag{3.33}$$

That is, $x(n+1)/m,X$ is distributed as a multivariate T, $T_k(rn+a-k+1,$ $E = \hat{x}(m),\ V = \frac{R_2(m)F_1^{-1}(m)}{rn+a-k-1})$. The marginal predictive distribution of $x(n+1)$ is given by the mixture

$$f(x(n+1)/X) \propto \sum_{m=1}^{n-1} f(m/X)f(x(n+1)/m,X). \tag{3.34}$$

So that

$$E[x(n+1)/X] = \sum_{m=1}^{n-1} f(m/X)\hat{x}(m), \tag{3.35}$$

$$\underset{m}{\mathrm{Var}}\, E[x(n+1)/m,X] = \sum_{m=1}^{n-1} f(m/X)\{E[x(n+1)/m,X]E[x'(n+1)/m,X] - E[x(n+1)/X]E[x'(n+1)/X]\}, \tag{3.36}$$

$$\underset{m}{E}\, \mathrm{Var}(x(n+1)/m,X) = \sum_{m=1}^{n-1} f(m/X)\frac{R_2(m)F_1^{-1}(m)}{rn+a-k-1}, \tag{3.37}$$

and finally,

$$Var(x(n+1)/X) = \underset{m}{E}\, Var(x(n+1)/m,X) + \underset{m}{Var}\, E[x(n+1)m,X]. \qquad (3.38)$$

Using (3.35) and (3.38) one can make forecasts for x(n+1). One can also obtain conditional forecasts several steps ahead where one conditions on the previous forecasts. Here, the work of Harrison and Stevens (1976) may be of value. The joint predictive distribution for several steps ahead can be obtained, but unfortunately it is difficult to handle. For those interested in the joint predictive distribution for a forecast containing several steps see Broemeling and Land (1982).

The theoretical development of the posterior and predictive distributions for the changing multivariate autoregressive process is now concluded. The remainder of the chapter will be devoted to a description of some numerical studies which examine the effect of the sample size on the identification of the change point and conditional estimation of the $AR_k(p)$ parameters.

A Numerical Study of the Changing $AR_1(1)$ Model

The primary focus in Salazar (1982) was on the change point posterior mass function. Table I, found in Appendix A, summarizes results not available in Salazar (1982). The models examined were of the form

$$x(t) = A1 \cdot x(t-1) + e(t), \quad t = 1,2,\ldots,CHANGE$$

$$x(t) = A2 \cdot x(t-1) + e(t), \quad t = CHANGE + 1,\ldots,N$$

where

CHANGE the actual value of the change point, always INT(N/2), A1 the AR coefficient before the change,

A2 the AR coefficient after the change,

N sample size (not including the initial value),

X0 the initial starting value,

TAU the precision of e(t).

Conditional posterior information is presented in Table I. The column headings are

OBS an identification number for the different models,

MODEM the posterior mode of m, calculated from (3.13),

EA1GMM the posterior expectation of A1 given that the change point is MODEM, calculated from (3.16),

EA2GMM the posterior expectation of A2 given that the change point is MODEM, calculated for (3.16),

VA1GMM the posterior variance of A1 given that the change point is MODEM, calculated from (3.17),

VA2GMM the posterior variance of A2 given that the change point is MODEM, calculated from (3.17),

ETAUGMM the posterior expectation of TAU given that the change point is MODEM, calculated from (3.18),

VTAUGMM the posterior variance of TAU given that the change is MODEM, calculated from (3.19).

Two hundred pseudorandom N(0,1) deviates were generated. These deviates were used to construct all data sets. The computer experiments were not repeated sampling experiments. In all cases, the limiting (or Jeffreys) form of the prior was used. Two values for A1 were used to generate the models. They were A1 = .5 and .9. The sample size N was varied from N = 10 to N = 200. The AR coefficient A2 was varied from -.9 to .9 in increments of .2 when A1 = .5. When A1 = .9, values of A2 used to generate data were A2 = .9, 1.00, 1.01, 1.02, and 1.03. The different data sets were sorted first by sample size N, then by the "before" coefficient A1, and finally by the "after" coefficient A2. The results are listed in Table I.

To read Table I, consider the first line of the body of the table where OBS = 1. The data analyzed corresponds to a model where N = 10, CHANGE = 5, X0 = 1, TAU = 1, A1 = .5, and A2 = -.9. The posterior mode of the possible change points is MODEM = 8. That is, the true change occurred on the fifth observation and the posterior mode estimates the change as occurring on the eighth observation. Under the assumption that the change point occurred at m = 8, conditional means and variances for the AR parameters were

$$E[A1|m=8,\text{data}] = -.31644,$$

$$E[A2|m=8,\text{data}] = -3.9105,$$

$$\text{Var}(A1|m=8,\text{data}) = .108961,$$

$$\text{Var}(A2|m=8,\text{data}) = 10.9482,$$

$$E[TAU|m=8,\text{data}] = 3.02622,$$

$$\text{Var}(TAU|m=8,\text{data}) = 2.28950.$$

It should be remarked that experience suggests that for small samples or for small changes in the AR coefficient that there is a tendency for the posterior change point mass function to have large values at the end points. It was decided to restrict the support for the change point to $m \in \{2,3,\ldots,N-2\}$.

Table I indicates that it is easier to detect a change when the AR coefficient changes in sign from positive to negative. As the sample size is increased, the change point is more readily detected.

Overall impressions gained from the computer experiments are

i) It is easier to detect a change and estimate parameters for nonstationary models.

ii) From the conditional posterior variances, it appears that 200 or more observations are needed to obtain reliable point estimates of the AR coefficients unless one uses concentrated prior information.

iii) The "after" parameter A2 was more accurately estimated than the "before" parameter A1.

A Numerical Study of the $AR_2(1)$ Model

with No Change

Before conducting a numerical study of the changing $AR_2(1)$, it was decided that insight should first be gained investigating Bayesian inferences for the nonchanging $AR_2(1)$. Data sets were generated from models of the following form

$$(x_1(t),x_2(t)) = (x_1(t-1),x_2(t-1))\begin{bmatrix} A11 & A12 \\ A21 & A22 \end{bmatrix} + (e_1(t),e_2(t))$$

where $x_1(0) = x10 = 1$, $x_2(0) = x20 = 1$, $t = 1,2,\ldots,N$, and $(e_1(t),e_2(t))' \sim$ iid $N_2(0,T)$, $T = I$. For the numerical study, 200 normal random deviates were generated in SAS82 and were used to build models where N and

A22 were varied. All AR coefficients except A22 were held fixed so that All = .5, A21 = 0, A21 = .5 for all data sets.

Tables II, III, and IV, found in Appendix A, provide a listing of the posterior means and variances of the elements of the AR coefficient matrix and the precision matrix. The posterior results were obtained using the vague Jeffreys prior. Since so many means and variances needed to be listed it was decided to use three tables to make the complete listing. Each table contains posterior information for all models examined. But each table lists a different type of posterior information. In presenting the results, the models (or data sets) were first sorted by the sample size N and then by the AR coefficient A22. The column heading OBS is a model identifier that links a particular model across the three tables. For instance, when OBS = 1 in Tables II - IV, the model used to generate the data was

$$(x_1(t),x_2(t)) = (x_1(t-1),x_2(t-1))\begin{bmatrix} .5 & .5 \\ 0 & .95 \end{bmatrix} + (e_1(t),e_2(t))$$

$$t = 1,2,\ldots,N = 10,\ x_1(0) = x10 = 1,\ x_2(0) = x20 = 1.$$

Table II contains the posterior means for the AR coefficients so that

$$E\left[\begin{matrix} A11 & A12 \\ A21 & A22 \end{matrix}\middle|\, data\right] = \begin{bmatrix} EA11 & EA12 \\ EA21 & EA22 \end{bmatrix}$$

$$= \begin{bmatrix} -.26033 & .957197 \\ -.27203 & .830643 \end{bmatrix},\ \text{for OBS} = 1.$$

Table III contains posterior variances for the AR coefficients so that

$$\begin{bmatrix} Var(A11|data) \\ Var(A12|data) \\ Var(A21|data) \\ Var(A22|data) \end{bmatrix} = \begin{bmatrix} VA11 \\ VA12 \\ VA21 \\ VA22 \end{bmatrix} = \begin{bmatrix} .140609 \\ .216120 \\ .0407406 \\ .0626197 \end{bmatrix},$$

when OBS = 1.

Table IV contains the posterior means and variances of the precision matrix T. So that for OBS = 1,

$$E[T|data] = E[\begin{matrix} T11 & T12 \\ T21 & T22 \end{matrix}|data]$$

$$= \begin{bmatrix} ET11 & ET12 \\ ET21 & ET22 \end{bmatrix}$$

$$= \begin{bmatrix} 2.56620 & .307190 \\ .307190 & 1.66958 \end{bmatrix}.$$

Also,

$$\begin{bmatrix} Var(T11|data) \\ Var(T12|data) \\ Var(T22|data) \end{bmatrix} = \begin{bmatrix} VT11 \\ VT12 \\ VT22 \end{bmatrix} = \begin{bmatrix} 1.64634 \\ .547354 \\ .69687 \end{bmatrix}, \quad \text{when OBS = 1.}$$

The posterior expectations of the AR coefficients and variances were computed using nonchanging versions of (3.16) and (3.17), respectively. The posterior expectations and variances of the precision matrix were computed using nonchanging versions of (3.18) and (3.19), respectively.

As with Table I, it is important to emphasize that these computer simulations were not repeated sampling experiments.

The results of the computer experiments will now be discussed. The point estimates of the AR coefficients, as given by their posterior means, seemed poor until a sample size of about N = 200 was used. Since the posterior distribution of the AR coefficients (marginally) are t-distributions, a Highest Posterior Density (HPD) region with content of about 95% is given by the posterior mean plus or minus two posterior standard deviations. Such intervals included the true values of the parameters. However, the true values were frequently near the endpoints of the intervals. So, one can gain insight into whether enough data was used by examining the width of such an HPD region. If one requires at least one accurate digit in the posterior mean of the AR coefficients, it appears that 200 or more observations will be required. Since Jeffreys' prior was used, the posterior means for the AR coefficients will be the same as for maximum likelihood.

A similar pattern can be seen in Table IV where the posterior results for the precision matrix is listed. It appears that 100 to 200 observations are needed for precise results to be obtained.

Overall, Tables II - IV suggest that one must be cautious when modeling a data set with fewer than 100 or 200 observations.

A Numerical Study of the Changing $AR_2(1)$ Model

Tables V and VI, found in Appendix A, summarize computer experiments for the bivariate-first order AR process with one change. Using PROC MATRIX of SAS82, 200 bivariate normal deviates with mean vector zero and identity precision matrix numerous data sets were constructed. The models were of the form

$$(x_1(t),x_2(t)) = (x_1(t-1),x_2(t-1))\begin{bmatrix} A11 & A12 \\ A21 & A22 \end{bmatrix} + (e_1(t),e_2(t))$$

for t = 1,2,...,CHANGE,

$$(x_1(t),x_2(t)) = (x_1(t-1),x_2(t-1))\begin{bmatrix} B11 & B12 \\ B21 & B22 \end{bmatrix} + e_1(t),e_2(t))$$

for t = CHANGE + 1,...,N,

where $x_1(0)$ = x10 = 1, $x_2(0)$ = x20 = 1, A11 = B11 = .5, A12 = B12 = .5, A21 = B21 = 0, and A22 and B22 were allowed to vary. The true change occurred at CHANGE which was always in the middle of the series. That is, CHANGE = INT(N/2), N = 25, 50, 100, 200. Jeffreys' prior was used to construct a posterior distribution for the parameters. Table V lists conditional posterior means for the AR coefficients. Table VI lists conditional posterior means for the precision matrix. The tables are constructed so that a row with the same OBS number in both Table V and Table VI corresponds to the same set of generated data. The models were sorted first by sample size N, then by A22, and finally by B22.

Formula (3.13) was used to determine the posterior mode of the possible change points. This calculated mode is called MODEM in Tables V and VI. Formula (3.16) was used to calculate the condition posterior mean for the AR coefficients given that m = MODEM. Formula (3.18) was used to calculate the conditional posterior mean of the precision matrix given that m = MODEM. So, MODEM is an estimate of the true change point where the acutal change point occurred at m = CHANGE = INT(N/2).

In Tables V and VI, when OBS = 1, the true model was

$$A = \begin{bmatrix} .5 & .5 \\ 0 & .5 \end{bmatrix}, \quad B = \begin{bmatrix} .5 & .5 \\ 0 & -.9 \end{bmatrix}, \quad N = 25,$$

$$x_1(0) = x_2(0) = 1, \quad \text{CHANGE} = 12.$$

So, in Table V when OBS = 1, MODEM = 2,

$$E[A|m=MODEM,data] = \begin{bmatrix} EA11GM & EA12GM \\ EA21GM & EA22GM \end{bmatrix} = \begin{bmatrix} -8.2539 & 14.3766 \\ 4.22784 & -6.9027 \end{bmatrix}$$

$$E[B|m=MODEM,data] = \begin{bmatrix} EB11GM & EB12GM \\ EB21GM & EB22GM \end{bmatrix} = \begin{bmatrix} .307530 & .842358 \\ -.14028 & -.74450 \end{bmatrix}.$$

In Table VI when OBS = 1,

$$E[T|m=MODEM,data] = \begin{bmatrix} ET11GM & ET12GM \\ ET21GM & ET22GM \end{bmatrix}$$

$$= \begin{bmatrix} 1.36997 & .123891 \\ .123891 & .70201 \end{bmatrix}.$$

When OBS = 2, a new model was used. It should be emphasized that the computer experiments were not repeated sampling experiments. Data was generated once for each of 108 models and for each data set, conditional posterior calculations were made.

MODEM is listed in both Table V and Table VI. As can be seen, the posterior mode of the change point was a poor estimate of the true change point until N = 100. When N = 100, only changes of the largest magnitude were detected. For N = 200, the change was detected a little bit better. One reason for the poor performance might be that the type of change was subtle. The "before" and "after" models were different only in one element of the AR coefficient matrices. Another reason for the poor performance might be the size of the variance of the innovations. Also note that many of the changes were quite small in magnitude. Further, misidentification of the change resulted in bad point estimates for either the "before" or the "after" model but rarely for both. In addition, it seems that estimates of the precision are better than estimates of the AR coefficients. These results were not unexpected. With N = 200, there were actually only 100 observations a model. If, as described in the previous section of this thesis, 100 or 200 observations seem to be the least one should have, then in the change point problem one should have at least 100 observations before the change point and 100 observations after the change point.

In summary, with changes of "small" magnitude and with "small" samples, a change may be difficult to detect.

CHAPTER IV

BAYESIAN INFERENCES FOR THE $AR_1(p)$ PROCESS: THE FREQUENCY DOMAIN

Time Domain Notation

As remarked in the review of the literature at the end of Chapter II, Shore and Holt (1980) seems to be the only reference for the Bayesian approach to spectral inferences. However, the approach by Shore is not directly tied to AR processes. In recent years attention has turned away from nonparametric approaches to estimating spectral density functions and toward estimating the time domain parameters of an AR process (possibly of high order and possibly using moving average components) and transforming to the frequency domain. References for this approach are Parzen (1978) and Akaike (1978). This chapter is devoted to transforming Bayesian time domain inferences for the $AR_1(p)$ into frequency domain inferences.

The notation used in this chapter is similar to that to Chapter III except that there is no change point. Let

$$x(t) = a_1x(t-1) + a_2x(t-2) + \dots + a_px(t-p) + e(t), \qquad (4.1)$$

where $x(t)$ is a 1×1 observable random variable,

$t = 1,2,\dots,n$, $e(t) \sim \text{iid } N(0,\tau)$ with $\tau > 0$ a precision, and

$x(-p+1),\dots,x(0)$ are known initial values.

The matrix formulation is

$$\begin{bmatrix} x(1) \\ x(2) \\ \vdots \\ x(n) \end{bmatrix} = \begin{bmatrix} x(0) & x(-1) & \cdots & x(-p+1) \\ x(1) & x(0) & \cdots & x(-p+2) \\ \vdots & & & \\ x(n-1) & x(n-2) & \cdots & x(n-p) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} + \begin{bmatrix} e(1) \\ e(2) \\ \vdots \\ e(n) \end{bmatrix}, \text{ or}$$

$$X = Za + E. \tag{4.2}$$

Again, after allowing for r independent partial realizations each with n observations, the model (4.2) becomes

$$\underset{rn\times 1}{X} = \underset{rn\times p}{Z} \cdot \underset{p\times 1}{a} + \underset{rn\times 1}{E}. \tag{4.3}$$

The likelihood function for (4.3) is

$$f(X/a,\tau) \propto \tau^{\frac{rn}{2}} \exp -\frac{\tau}{2}(X - Za)'(X - Za). \tag{4.4}$$

The family of natural conjugate Normal-Gamma prior distributions will be used. So

$$\left.\begin{aligned} f(a/\tau) &\propto \tau^{\frac{p}{2}} \exp -\frac{\tau}{2}(a-b)'P(a-b) \\ f(\tau) &\propto \tau^{\frac{c}{2}-1} \exp -\frac{d}{2}\tau \end{aligned}\right\} \tag{4.5}$$

with hyperparameters $c > 0$, $d > 0$, $b \in R^p$, $P \in R^{p\times p}$, p.d. In order to express the posterior distribution conveniently, let

$$\left.\begin{aligned} B_1 &= Z'Z + P, \\ B_2 &= Z'X + Pb, \\ \hat{a} &= B_1^{-1}B_2 \\ R &= X'X + b'Pb + d - B_2'B_1^{-1}B_2 \end{aligned}\right\}. \tag{4.6}$$

Then, the posterior distribution can be expressed as

$$f(a,\tau/X) \propto \tau^{\frac{rn+c+P}{2}-1} \exp -\frac{\tau}{2}\{(a-\hat{a})'B_1(a-\hat{a}) + R\}. \qquad (4.7)$$

As can be seen from (4.7),

$$\left.\begin{aligned} &a/\tau,X \sim N_p(\hat{a},v(\tau)),\ v(\tau) = \tau^{-1}B_1^{-1} \\ &\tau/X \sim \text{Gamma}\ (\tfrac{rn+c}{2}, \tfrac{R}{2}) \end{aligned}\right\}. \qquad (4.8)$$

The above results are described in such references as Zellner (1971) and establishes the notation to be used in the rest of this chapter.

The Spectral Density of the $AR_1(p)$ Process

In order to define the spectral density function of the $AR_1(p)$ process it is convenient to express (4.1) in terms of the backshift operator B. Then (4.1) becomes

$$a(B)x(t) = e(t),\quad a(B) = 1-a_1 B - a_2B^2 - \ldots - a_pB^p. \qquad (4.9)$$

The spectral density of (4.9) can be found in Fuller (1976) and is given by

$$S(w) = [2\pi\tau a(e^{-iw})a(e^{iw})]^{-1},\quad -\pi \leq w \leq \pi, \qquad (4.10)$$

so long as $\{x_t\}$ is covariance stationary. The $AR_1(p)$ process (defined over an infinite time span) will be stationary only when a(B) is invertible. However, the multiplicative inverse of S(w), denoted by $S^{-1}(w)$, always exists for the $AR_1(p)$ process. This can be seen by considering the process $\{y_t\}$ defined by

$y(t) = a(B)\delta(t)$, where $\delta(t) \sim$ iid $N(0,(2\pi)^2\tau)$ and $(2\pi)^2\tau$ is a variance. Then

$$S_y(w) = 2\pi\tau a(e^{-iw})a(e^{iw}). \tag{4.11}$$

So (4.11) is the same as $S^{-1}(w)$. $S^{-1}(w)$ always exists since it is the spectral density function for a moving average process (see Fuller (1976)).

While expressions (4.10) and (4.11) are convenient for the purposes of interpretation, they are inconvenient for the purposes of inference. $S^{-1}(w)$ will now be expressed as a quadratic form. Let

$$\left.\begin{aligned} a^* &= (-1,a_1,\ldots,a_p)', \\ \hat{a}^* &= (-1,\hat{a}_1,\ldots,\hat{a}_p)', \\ s(w) &= (\sin 0w,\ldots,\sin pw)', \\ c(w) &= (\cos 0w,\ldots,\cos pw)', \\ D(w) &= c(w)c'(w) + s(w)s'(w), \text{ and} \\ B_1^{*-1} &= \begin{pmatrix} 0 & 0 \\ 0 & B_1^{-1} \end{pmatrix} \end{aligned}\right\}. \tag{4.12}$$

Then

$$a^*/\tau, X \sim N_{p+1}(\hat{a}^*, \tau^{-1}B_1^{*-1}). \tag{4.13}$$

$S^{-1}(w)$ can now be expressed as

$$S^{-1}(w) = 2\pi\tau a^{*\prime}D(w)a^*, \quad -\pi \le w \le \pi. \tag{4.14}$$

Expression (4.13) and (4.14) offer a convenient form for obtaining posterior moments of $S^{-1}(w)$. More will be said about this later.

The Exact Posterior Distribution of $S(w)$ for the $AR_1(1)$

Upon inserting p = 1 into (4.9), (4.12) and (4.13) one sees that

$$S(w) = \{2\pi\tau[(a - \cos w)^2 + \sin^2 w]\}^{-1} \quad \text{for the } AR_1(1).$$

Let $k_1(\tau) = 2\pi\tau$ and $k_2(\tau;w) = 2\pi\tau\sin^2 w$. Then

$$S(w) = [k_1(\tau)(a - \cos w)^2 + k_2(\tau;w)]^{-1}. \tag{4.15}$$

In order to obtain the exact posterior distribution of $S(w)$, let $a(w) = a - \cos w$, $\hat{a}(w) = \hat{a} - \cos w$. Then from (4.8) one sees that

$$a(w)/\tau,X \sim N_1(\hat{a}(w),v(\tau)), \quad v(\tau) = \tau^{-1}B_1^{-1}.$$

So

$$f(a(w)/\tau,X) = (2\pi v(\tau))^{-\frac{1}{2}} \exp -\frac{v^{-1}(\tau)}{2}(a(w) - \hat{a}(w))^2.$$

The posterior density for $S(w)$ can be obtained using several additional transformations. Let $z_1(w) = a^2(w)$, $z_2(w) = k_1(\tau)z_1(w) + k_2(\tau;w)$, and $S(w) = z_2^{-1}(w)$. Then, the posterior distribution for $S(w)$ is given by

$$f(S(w)/\tau,X) = \frac{(2\pi v(\tau))^{-\frac{1}{2}}}{2k_1(\tau)} S^{-2}(w)\left(\frac{S^{-1}(w)-k_2(\tau;w)}{k_1(\tau)}\right)^{-\frac{1}{2}}$$

$$\times\{\exp-\frac{v(\tau)}{2}[(\frac{S^{-1}(w) - k_2(\tau;w)}{k_1(\tau)})^{\frac{1}{2}} - \hat{a}(w)]^2$$

$$+ \exp-\frac{v(\tau)}{2}[(\frac{S^{-1}(w) - k_2(\tau;w)}{k_1(\tau)})^{\frac{1}{2}} + \hat{a}(w)]^2\} \quad \text{for } 0 < S(w) < k_2^{-1}(\tau;w). \tag{4.16}$$

And

$$f(S(w)/X) = \int_0^\infty f(S(w)/\tau,X)f(\tau/X)d\tau. \tag{4.17}$$

Expression (4.17) gives the exact posterior distribution for S(w). As can be seen (4.17) is inconvenient to handle. The use of (4.17) would be to obtain exact Highest Posterior Probability regions for S(w). For each frequency w where one would like percentage points for S(w) one would have to use numerical integrations to determine upper and lower bounds for percentage points of S(w). The bounds would have to define the smallest posterior region containing a preselected amount of posterior probability. While one can use (4.17) to obtain an exact analysis of S(w) for the $AR_1(1)$ it may not be computationally feasible to plot an interval. Also, the posterior for S(w) is difficult to handle for processes with p > 1 estimate for S(w) over many frequencies. An alternative approach which is computationally feasible will be described in the next section.

Posterior Moments of $S^{-1}(w)$ for the $AR_1(p)$

The exact posterior mean and variance for $S^{-1}(w)$, as defined in (4.14) will now be derived. In order to avoid numerical integrations or asymptotic approximations for obtaining interval estimates of S(w) and $S^{-1}(w)$, Chebychev's inequality will be employed to define a posterior probability region. While the approximation introduced by Chebychev's inequality is not the most desirable one, it is appropriate for all sample sizes and is computationally efficient.

Using (4.13) and (4.14) one need only use standard results on quadratic forms (see Press (1982)) to obtain the mean and variance of $S^{-1}(w)$. One finds that

$$E[S^{-1}(w)|\tau,X] = 2\pi\tau\{\mathrm{tr}\ D(w)\tau^{-1}B_1^{*-1} + \hat{a}^{*\prime}D(w)\hat{a}^*$$

$$= 2\pi\ \mathrm{tr}\ D(w)B_1^{*-1} + 2\pi\tau\hat{a}^{*\prime}D(w)\hat{a}^*,$$

$$E[S^{-1}(w)|X] = 2\pi\ \mathrm{tr}\ D(w)B_1^{*-1} + 2\pi(\frac{rn+c}{R})\hat{a}^{*\prime}D(w)\hat{a}^*, \tag{4.18}$$

$$\mathop{\mathrm{Var}}_{\tau}(E[S^{-1}(w)|\tau,X]) = (2\pi\hat{a}^{*\prime}D(w)\hat{a}^*)^2\ 2(rn+c)R^{-2}, \tag{4.19}$$

$$\mathrm{Var}(S^{-1}(w)|\tau,X) = (2\pi\tau)^2[2\ \mathrm{tr}\ D(w)\tau^{-1}B_1^{*-1}D(w)B_1^{*-1}$$

$$+ 4\hat{a}^{*\prime}D(w)\tau^{-1}B_1^{*-1}D(w)\hat{a}^*]$$

$$= 2(2\pi)^2\ \mathrm{tr}\ D(w)B_1^{*-1}D(w)B_1^{*-1}$$

$$+ 4(2\pi)^2\tau\hat{a}^{*\prime}D(w)B_1^{*-1}D(w)\hat{a}^*, \tag{4.20}$$

$$\mathop{E}_{\tau}[\mathrm{Var}(S^{-1}(w)|\tau,X)] = 2(2\pi)^2\ \mathrm{tr}\ D(w)B_1^{*-1}D(w)B_1^{*-1}$$

$$+ 4(2\pi)^2(rn+c)R^{-1}\hat{a}^{*\prime}D(w)B_1^{*-1}D(w)\hat{a}^*, \tag{4.21}$$

Finally, the variance of $S^{-1}(w)$ is given by

$$\mathrm{Var}(S^{-1}(w)|X] = \mathop{\mathrm{Var}}_{\tau}[E[S^{-1}(w)|\tau,X]) + \mathop{E}_{\tau}[\mathrm{Var}(S^{-1}(w)|\tau,X)]. \tag{4.22}$$

Using the exact posterior mean and variance of $S^{-1}(w)$ one can now define a posterior probability region for $S^{-1}(w)$ using Chebychev's inequality. Define

$$\left.\begin{aligned} S_L^{-1}(w) &= E[S^{-1}(w)|X] - k\, Var^{\frac{1}{2}}(S^{-1}(w)|X) \\ S_U^{-1}(w) &= E[S^{-1}(w)|X] + k\, Var^{\frac{1}{2}}(S^{-1}(w)|X) \end{aligned}\right\} . \tag{4.23}$$

Then, the interval $(S_L^{-1}(w),\ S_U^{-1}(w))$ is a posterior probability region for $S^{-1}(w)$ with posterior probability of at least $1 - \frac{1}{k^2}$, $k > 1$.

Since the reciprocal transformation is monotonically decreasing, a posterior probability region for $S(w)$ is given by $([S_U^{-1}(w)]^{-1},[S_L^{-1}(w)]^{-1})$. This interval also has posterior probability of at least $1 - \frac{1}{k^2}$.

Expression (4.23) defines posterior intervals for $S(w)$ and $S^{-1}(w)$ which are valid for all sample sizes and are computationally very efficient. These intervals will be conservative and are unlikely to lead anyone astray.

Posterior Moments for the Reciprocal of the Squared Gain

Some researchers may be interested in the squared gain of $\{x(t)\}$, denoted $G^2(w)$, rather than $S(w)$. The exact mean and variance of $G^{-2}(w)$ can be obtained in a manner similar to the approach used in the previous section where $S^{-1}(w)$ was discussed. Using the exact mean and variance of $G^{-2}(w)$ one can define posterior probability regions for $G^{-2}(w)$ and $G^2(w)$ based upon Chebychev's inequality.

From Fuller (1976) one finds that

$$G^{-2}(w) = a^{*\prime}D(w)a^*, \qquad -\pi \leq w \leq \pi. \tag{4.24}$$

The expression (4.24) is almost the same as the expression for $S^{-1}(w)$. The difference is that $G^{-2}(w)$ does not have the factor of $2\pi\tau$. From (4.13) and (4.24) one can use standard results for quadratic forms to see that

$$E[G^{-2}(w)|\tau,X] = \text{tr } D(w)\tau^{-1}B_1^{*-1} + \hat{a}^{*\prime}D(w)\hat{a}^*, \tag{4.25}$$

$$E[G^{-2}(w)|X] = \frac{R}{rn+c-2}\text{tr } D(w)B_1^{*-1} + \hat{a}^*D(w)\hat{a}^*, \tag{4.26}$$

$$\text{Var}(E[G^{-2}(w)|\tau,X]) = 2R^2(rn+c-2)^{-2}(rn+c-4)^{-1}(\text{tr}D(w)B_1^{*-1})^2, \tag{4.27}$$

$$\text{Var}(G^{-2}(w)|\tau,X) = 2\tau^{-2}\text{ tr } D(w)B_1^{*-1}D(w)B_1^{*-1}$$

$$+ 4\tau^{-1}\hat{a}^{*\prime}D(w)B_1^{*-1}D(w)\hat{a}^* \tag{4.28}$$

$$\underset{\tau}{E}[\text{Var}(G^{-2}(w)|\tau,X)] = \frac{2R^2}{(rn+c-2)(rn+c-4)}\text{tr } D(w)B_1^{*-1}D(w)B_1^{*-1}$$

$$+ \frac{4R}{(rn+c-2)}\hat{a}^{*\prime}D(w)B_1^{*-1}D(w)\hat{a}^*, \tag{4.29}$$

and

$$\text{Var}(G^{-2}(w)|X) = \underset{\tau}{\text{Var}}(E[G^{-2}(w)|\tau,X]) + \underset{\tau}{E}[\text{Var}(G^{-2}(w)|\tau,X)]. \tag{4.30}$$

Expressions (4.26) and (4.30) given the exact posterior mean and variance for $G^{-2}(w)$.

To obtain posterior probability regions for $G(w)$ and $G^{-2}(w)$, define

$$G_L^{-2}(w) = E[G^{-2}(w)|X] - k\ \mathrm{Var}^{\frac{1}{2}}(G^{-2}(w)|X)$$
$$G_U^{-2}(w) = E[G^{-2}(w)|X] + k\ \mathrm{Var}^{\frac{1}{2}}(G^{-2}(w)|X) \quad . \qquad (4.31)$$

Then, by Chebychev's inequality, the intervals $(G_L^{-2}(w),\ G_U^{-2}(w))$ and $([G_U^{-2}(w)]^{-1},\ [G_L^{-2}(w)]^{-1})$ define posterior probability regions for $G^{-2}(w)$ and $G(w)$, respectively. These regions have content at least $1 - \frac{1}{k^2}$, $k > 1$. These intervals are valid for all sample sizes and they are easy to compute.

Some Computer Experiments for the $AR_1(1)$:

Frequency Domain Inferences

In order to demonstrate the Bayesian spectral analysis procedure developed in the last section, five computer experiments were conducted. Models of the form

$$x(t) = A \cdot x(t-1) + e(t), \qquad t = 1,2,\ldots,N$$

were used to construct generated data sets to be analyzed. PROC MATRIX of SAS82 was used to generate 200 pseudo-random $N(0,\tau)$, $\tau = 1$ deviates. These deviates were used to construct observable sequences of x(t) values. Data where A = -.5 and .5 with N = 100 and 200 were generated. Also, data with A = 1.02 and N = 100 were generated. For each of the five model-sample length combinations, relevant posterior calculations were carried out in order to perform a spectral analysis of the data. In all cases, Jeffreys' vague prior distribution was used. Also, a starting value of x(0) = .3 was used in all cases.

Tables VII - XI, found in Appendix A, are computer printouts of SAS data sets listing the posterior information. Figures 1 - 10, found in Appendix B, are graphs of S(w), $S^{-1}(w)$ and their associated posterior probability regions with content at least $1 - \frac{1}{k^2}$, k =2. The posterior calculations for S(w) and $S^{-1}(w)$ occurred at 60 equally spaced frequencies between 0 and π.

The column headings in Tables VII - XI are the following:

OBS	identifiers associated with calculations at different frequencies
W	frequency
ESM1	$E[S^{-1}(w)/data]$ at frequency W
VSM1	$Var[S^{-1}(w)/data]$ at frequency W
(SM1L,SM1U)	lower and upper limits, respectively, for $S^{-1}(w)$ with posterior content at least $1 - \frac{1}{k^2}$, k = 2 at W
(SL,SU)	lower and upper limits, respectively, for S(w) with posterior content at least $1 - \frac{1}{k^2}$, k = 2 at W
ISPEC	the actual value of $S^{-1}(w)$ at W
SPE	the actual value of S(w) at W

Table VII may be difficult to interpret. So, Figure 1 superimposes (SM1L,SM1U) with $S^{-1}(w)$. With only 100 observations the AR coefficient had a posterior mean of E[A/data] = .297, a point estimate that seems poor. Figure 1 shows that (SM1L,SM1U) surrounds the actual value of $S^{-1}(w)$, though there is considerable uncertainty. Figure 2 superimposes (SL,SU) with S(w). Again, the interval surrounds the true S(w), but there is considerable uncertainty near w = 0.

Table VIII lists the results when A = .5 and N = 200. As can be seen, $Var(S^{-1}(w)/data)$ was reduced by about a factor of five. Figures 3 and 4 are graphs of the posterior information for $S^{-1}(w)$ and S(w), respectively. As can be seen, the effect of increasing the data length from N = 100 produced a definite improvement. E[A/data] = .446 and the posterior intervals for $S^{-1}(w)$ and S(w) are shorter. Note that for the $AR_1(1)$ Figures 2 and 4 indicate that the "best" information is produced at the higher frequencies.

Table IX lists the posterior information for the model with A = -.5 and N = 100. Figures 5 and 6 plot the posterior information for $S^{-1}(w)$ and S(w), respectively. For N = 100, E[A/data] = -.624 and the posterior interval for $S^{-1}(w)$ seems quite wide particularly near w = 0. However, Figure 6 shows an exceptionally "good" for most frequencies except those very near π.

Table X lists the results for the model with A = -.5 and N = 200. E[A/data] = -.562, which shows the improvement of doubling the sample size. Also, VSM1 was reduced by about one half. Note that for N = 200 $E[S^{-1}(w)/data]$ one to two accurate digits throughout the frequency range. Also note that with N = 200 the procedure did not perform so badly near $w = \pi$. Also note that taking the log of S(w) might make the graph of S(w) easier to interpret. (Log S(w)is frequently of interest to engineers. Such a transformation was not used in the study in order to make a fair comparison of many models and sample sizes.)

Table XI shows the posterior results when A = 1.02 and N = 100. Note that $E[S^{-1}(w)/data]$ and the actual value of S(w) generally agree up to one or two significant digits. As remarked in the previous section S(w) exists only for covariance stationary processes. When A = 1.02, {x(t)} will not be covariance stationary. Figures 9 and 10 are graphs of $S^{-1}(w)$ and S(w), respectively. Figure 9 shows that while $S^{-1}(w)$ is well behaved, that virtually all of the density in S(w) is concentrated near w = 0. Figure 9 shows that in order to produce white noise from the observable {x(t)} process that power (variance) of {x(t)} must be alternated at low frequencies (long periods) and amplified at high frequencies (short periods). This is intuitively reasonable since that long run growth of $x(t) = 1.02 \cdot x(t-1) + e(t)$ is what makes that process nonstationary.

Figure 10 shows that to produce an explosive sequence {x(t)} from a stationary sequence {e(t)} large amounts of variance must be amplified at low frequencies (long periods). Of course, when A = 1.02 S(w) does not actually exist. In real data situations one may not know a prior that an observed partial realization of a process is generated by a stationary process. (Consider a Gross National Product sequence.) It is hoped that the procedure outlined above can be used to help determine whether or not an observed process is stationary or nonstationary.

Bayesian AR Spectral Analysis of the Wolfer Sunspot Data

The last section provided some insight regarding the performance of the Bayesian AR spectral approach (denoted BARS in this section). However, a practical example using real data may provide even more insight. The SAS/ETS User's Guide, 1980 Edition, lists annual sunspot activity covering the years 1749 to 1924. This data set can be found in Table XII, found in Appendix A, of this thesis or on page 7.9 of the SAS/ETS guide. The column headings of Table XII are:

OBS	the SAS data set observation number,
ACTIVITY	the number of sunspots of a particular year,
YEAR	the year containing the sunspot activity,
ADJ	the number of sunspots minus the mean number of sunspots.

A quick glance at the table of data shows that over the years sunspot activity increases and decreases. A graph of the data is not provided. It would need to be continued over several pages and would be difficult to interpret.

Using the notation used in the beginning of this chapter, Table XIII, found in Appendix A, summarizes the posterior information for the time domain parameters under the assumption that an $AR_1(2)$ model is useful. A vague Jeffreys' prior was used to analyze the Wolfer sunspot data. Table XIII shows that E[a/data] = AH = (1.33,-.648)' with VA = Var(a/data).

Table XIV, found in Appendix A, lists the posterior information for the frequency domain parameters. The column headings are the same as in Table VII. The major point to note is that SL is maximum at w = .52333 and SU is maximum at .57567 (i.e. 12.0 and 10.91 years, respectively). In order to produce Table XIV, 60 equally spaced frequencies from 0 to π were examined. One could of course examine other frequencies as is done in Table XV where 60 frequencies equally spaced between 0 and $\pi/2$ were examined.

Since tables are frequently difficult to read, Figures 11 and 12, which graph $S^{-1}(w)$ and $S(w)$, and are found in Appendix B, may be of more interest. In particular, one can see from Figure 12 that $w \doteq .5$ shows a peak of the spectral density. Figures 13 and 14, found in Appendix B, merely show plots expanded from 0 to $\pi/2$.

Table XVI, found in Appendix A, shows the posterior information under the assumption that an $AR_1(3)$ generated the data. The labels are the same as for Table XIII. A marginal highest posterior density region for a_3 includes zero. So, an $AR_1(2)$ seems a simpler description of the process. Table XVII, found in Appendix A, lists the frequency domain posterior information and Figures 15 and 16, found in Appendix B, are graphs of $S^{-1}(w)$ and $S(w)$, respectively, under the assumption that an $AR_1(3)$ generated the data. The spectral graphs for the $AR_1(2)$ and the $AR_1(3)$ are very similar. This represents additional confirmation that a_3 is probably not needed in the model.

In summary, the BARS approach with Jeffreys' prior suggests that there is periodic behavior in sunspot activity. The period length of the dominant cycle is about 10.5 to 11.6 years long. The "interval" approach to making inferences in the frequency domain is not universally available in computer packages. Except for BARS, interval estimates for spectral density estimation require asymptotic justification. It is not obvious that the 176 observations used in the sunspot analysis constitutes a "large" set of data. But, the interval estimates of BARS are appropriate for a sample size of 176.

Summary of BARS for the $AR_1(p)$

Chapter IV derived the exact posterior probability density function for the sprectral density function of the $AR_1(1)$ process. That p.d.f. is difficult to work with. So, the exact posterior mean and variance of $S^{-1}(w)$ was derived. It was shown that the posterior mean and variance of

$S^{-1}(w)$ is very easy to use. Also, approximate posterior probability regions for $S^{-1}(w)$ and $S(w)$ were derived using Chebychev's inequality. The result is a conservative, easy to use approach for making interval estimates of $S^{-1}(w)$ and $S(w)$ which are valid for all sample sizes. The complete Bayesian approach (both time and frequency domain) can be used to determine whether or not a particular sample size has sufficient precision for one's purposes.

One final approach should be mentioned. One could obtain conditional point estimates for the frequency domain functions. If one used Jeffreys' prior and inserted the marginal posterior modes for the AR coefficients and the precision, one would obtain the same result as with maximum likelihood. If one possessed an "unambiguously large" sample size, the asymptotic properties of maximum likelihood estimators could be brought into play. One may consult Akaike (1978) for the maximum likelihood approach. However, the Bayesian approach offers three advantages:

i) one can use prior information, and

ii) one need not rely on asymptotic approximations;

iii) one can use the Bayesian approach to estimate nonstationarities from a very rich class.

It is hoped that further examination of the Bayesian autoregressive approach to frequency domain inferences will provide more confirmation of its usefulness.

CHAPTER V

BAYESIAN INFERENCES FOR THE $AR_k(p)$: FREQUENCY DOMAIN

Notation

A natural extension of the results of Chapter VI would be to include multivariate processes. As in the previous chapter, the notation for the posterior distributions of the time domain parameters will be developed. The time domain parameters will then be transformed to the frequency domain. As indicated in Chapter III, Press (1982) is a good reference for multivariate regression.

Let

$$x'(t) = x'(t-1)A_1 + \ldots + x'(t-p)A_p + e'(t), \quad t = 1,2,\ldots,n \tag{5.1}$$

where x_t is a k×1 observable random vector with $x(-p+1),\ldots,x(0)$ known initial values, A_j are k×k unknown constant matrices, $A_j \in R^{k\times k}$, $e_t \sim$ iid $N_k(0,T^{-1})$ where T is a k×k unknown positive definite precision matrix. As with Cahpter III, let (5.1) denote one of r independent realizations. Then the complete matrix formulation is

$$\underset{rn\ k}{X} = \underset{rn\times pk}{Z} + \underset{pk\times k}{A} + \underset{rn\times k}{E} . \tag{5.2}$$

The likelihood function for (5.2) is

$$f(X/A,T) \propto |T|^{\frac{rn}{2}} \exp -\frac{1}{2}\mathrm{tr}(X - ZA)'(X - ZA)T.$$

The natural conjugate prior is reasonably rich and will be parameterized as

$$\left.\begin{aligned} f(A/T) &\propto |T|^{\frac{pk}{2}} \exp -\frac{1}{2}\mathrm{tr}(A - C)'P(A - C)T \\ f(T) &\propto |T|^{\frac{d-(k+1)}{2}} \exp -\frac{1}{2}\mathrm{tr}\,\Sigma T \end{aligned}\right\}. \tag{5.3}$$

Finally, letting

$$\left.\begin{aligned} D_1 &= Z'Z + P, \\ D_2 &= Z'X + PC, \\ \hat{A} &= D_1^{-1}D_2, \\ R &= X'X + C'PC + \Sigma - D_2'D_1^{-1}D_2 \end{aligned}\right\}, \tag{5.4}$$

the posterior distribution for all parameters can be expressed as

$$\left.\begin{aligned} f(A/T,X) &\propto \exp -\frac{1}{2}\mathrm{tr}(A - \hat{A})'D_1(A - \hat{A})T \\ f(T/X) &\propto |T|^{\frac{rn+d-(k+1)}{2}} \exp -\frac{1}{2}\mathrm{tr}\,RT \end{aligned}\right\}. \tag{5.5}$$

The following partitions are useful:

$$\left.\begin{aligned} A &= (\underset{\sim}{A}_1,\ldots,\underset{\sim}{A}_k),\ \underset{\sim}{A} = (\underset{\sim}{A}_1',\ldots,\underset{\sim}{A}_k')' \\ \hat{A} &= (\hat{\underset{\sim}{A}}_1,\ldots,\hat{\underset{\sim}{A}}_k),\ \hat{\underset{\sim}{A}} = (\hat{\underset{\sim}{A}}_1',\ldots,\hat{\underset{\sim}{A}}_k')' \end{aligned}\right\}. \tag{5.6}$$

Then, from (5.5) one sees that

$$\underset{\sim}{A}/T,X \sim N_{k^2p}(\hat{\underset{\sim}{A}}, T^{-1} \otimes D_1^{-1})$$

$$T/X \sim W_{k\,k}(rn+d, R^{-1}) \quad . \qquad (5.7)$$

Further notation needed is

$$\left.\begin{aligned} T^{\ell m} &= [T^{-1}]_{\ell m} \\ T^{-1}(\ell,m) &= \begin{vmatrix} T^{\ell\ell} & T^{\ell m} \\ T^{m\ell} & T^{mm} \end{vmatrix} \\ \mathrm{Var}\left(\binom{A_\ell}{A_m}/T,X\right) &= T^{-1}(\ell,m) \otimes D_1^{-1} \end{aligned}\right\} . \qquad (5.8)$$

Now

$$\left.\begin{aligned} E[T^{-1}/X] &= R/rn+d-2k-2, \qquad rn+d-2k > 2. \\ \mathrm{Var}(T^{\ell m}/X) &= \frac{[R_{\ell\ell}R_{mm} + \frac{rn+d-2k}{rn+d-2k-2}R_{\ell m}^2]}{(rn+d-2k-1)(rn+d-2k-2)(rn+d-2k-4)} \\ \mathrm{Cov}(T^{ij},T^{k\ell}/X) &= \frac{[\frac{2}{rn+d-2k-2}R_{ij}R_{k\ell} + R_{ij}R_{j\ell} + R_{i\ell}R_{kj}]}{(rn+d-2k-1)(rn+d-2k-2)(rn+d-2k-4)} \\ E[T^{ij}T^{k\ell}/X] &= \mathrm{Cov}(T^{ij},T^{k\ell}/X) + E[T^{ij}/X]E[T^{k\ell}/X] \end{aligned}\right\} . \qquad (5.9)$$

Expressions (5.5) - (5.9) can be found in Press (1982).

Inferences in the Frequency Domain

It is now convenient to reexpress (5.1) in terms of the backshift operator B. Let

$$A(B) = I - A_1B - A_2B^2 - \dots - A_pB^p. \qquad (5.9)$$

Then (5.1) is

$$x_t'A(B) = e_t'. \tag{5.10}$$

Rather than making inferences about the spectral density of $\{x_t\}$, inferences will be made regarding the operator (or filter) $A(B)$ where the coefficient matrices must be estimated. The properties of the filter $A(B)$ can be determined by examining its effects upon a stochastic input of known type, namely white noise with unit variance (almost). Consider the auxiliary process $\{y_t\}$ where

$$y_t' = \partial_t'A(B), \qquad \partial_t \sim N_k(0,2\pi I). \tag{5.11}$$

Since $\{y_t\}$ is a moving average process it is guaranteed to be covariance stationary and its spectral density function exists (see Fuller, 1976). Letting $S_y(w)$ denote the spectral density of $\{y_t\}$ one can consult Fuller (1976) to find that

$$S_y(w) = A'(e^{-iw})A(e^{iw}), \qquad -\pi \leq w \leq \pi, \tag{5.12}$$

where $i^2 = -1$.

Now, expression (5.12) is inconvenient to handle. It will be expanded, manipulations made, and new notation established which will allow inferences to be made for $S_y(w)$.

$$\begin{aligned} S_y(w) &= [I - A_1e^{-iw} - A_2e^{-2iw} - \dots - A_pe^{-piw}]' \\ &\quad \times [I - A_1e^{iw} - A_2e^{2iw} - \dots - A_pe^{piw}], \\ &= [A_0e^{0iw} + A_1e^{-iw} + \dots + A_pe^{-piw}]' \\ &\quad \times [A_0e^{0iw} + A_1e^{iw} + \dots + A_pe^{piw}], \text{ where } A_0 = -I, \\ &= \sum_{r=0}^{p} A_r'e^{-riw} \sum_{s=0}^{p} A_se^{+siw}, \end{aligned}$$

$$= \sum_{r=0}^{p} \sum_{s=0}^{p} A_r'A_s e^{-i(r-s)w},$$

$$= Co(w) - iQu(w),$$

where
$$Co(w) = \sum_{r=0}^{p} \sum_{s=0}^{p} A_r' \cos(r-s)wA_s$$

$$Qu(w) = \sum_{r=0}^{p} \sum_{s=0}^{q} A_r' \sin(r-s)wA_s \qquad . \qquad (5.13)$$

The expressions in (5.13) give the cospectral and quadrature spectral densities, respectively. The summation notation can be eliminated by letting

$$A^* = \begin{bmatrix} -I \\ A_1 \\ \vdots \\ A_p \end{bmatrix}, \quad c(w) = \begin{pmatrix} I \cos 0w \\ \cdot \\ \cdot \\ \cdot \\ I \cos pw \end{pmatrix}, \quad s(w) = \begin{pmatrix} I \sin 0w \\ \cdot \\ \cdot \\ \cdot \\ I \sin pw \end{pmatrix} \quad . \qquad (5.14)$$

Then (5.13) can be expressed as

$$Co(w) = A^{*\prime}[c(w)c'(w) + s(w)s'(w)]A^*$$

$$Qu(w) = A^{*\prime}[s(w)c'(w) - c(w)s'(w)]A^*.$$

To express individual components, let

$$A^*(\ell,m) = \binom{\tilde{A}^*_\ell}{\tilde{A}^*_m},$$

$$T^{\ell m} = [T^{-1}]_{\ell m}$$

$$T^{-1}(\ell,m) = \begin{pmatrix} T^{\ell\ell} & T^{\ell m} \\ T^{m\ell} & T^{mm} \end{pmatrix}$$

$$\mathrm{Var}(A^*(\ell,m)/T,X) = V(\ell,m/T,X) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & T^{\ell\ell}D_1^{-1} & 0 & T^{\ell m}D_1^{-1} \\ 0 & 0 & 0 & 0 \\ 0 & T^{m\ell}D_1^{-1} & 0 & T^{mm}D_1^{-1} \end{pmatrix}$$

$$= T^{-1}(\ell,m) \otimes D_1^{*-1},$$

$$G_c(w) = c(w)c'(w) + s(w)s'(w)$$

$$G_c^*(w) = \begin{pmatrix} 0 & \frac{1}{2}G_c(w) \\ \frac{1}{2}G_c(w) & 0 \end{pmatrix}$$

$$G_Q(w) = s(w)c'(w) - c(w)s'(w)$$

$$G_Q^*(w) = \begin{bmatrix} 0 & \frac{1}{2}G_Q(w) \\ \frac{1}{2}G_Q(w) & 0 \end{bmatrix} .$$

Then, one can finally rewrite (5.13) as

$$\begin{aligned} Co(\ell,m;w) &= A^{*\prime}(\ell,m)G_c^*(w)A^*(\ell,m) \\ Qu(\ell,m;w) &= A^{*\prime}(\ell,m)G_Q^*(w)A^*(\ell,m) \end{aligned} \quad . \tag{5.14}$$

Expression (5.14) is in terms of quadratic forms and is the desired simplification of (5.12). Using standard results on quadratic forms, one finds that

$$E[Co(\ell,m;w)/T,X] = \mathrm{tr}(G_c^*(w)T^{-1}(\ell,m) \otimes D_1^{*-1}) + \hat{A}^{*\prime}(\ell,m)G_c^*(w)\hat{A}^*(\ell,m), \tag{5.15}$$

$$E[Co(\ell,m;w)/X] = tr(G_c^*(w)E[T^{-1}(\ell,m)/X] \otimes D_1^{*-1})$$

$$+ \hat{A}^{*\prime}(\ell,m)G_c^*(w)\hat{A}^*(\ell,m), \tag{5.16}$$

$$Var(E[Co(\ell,m;w)/T,X]) = Var(tr(G_c^*(w)T^{-1}(\ell,m) \otimes D_1^{*-1}))$$

$$= \frac{1}{4}tr^2(G_c(w)\begin{pmatrix}0 & 0\\ 0 & D_1^{-1}\end{pmatrix})[2Var(T^{m\ell}/X) + 2Cov(T^{m\ell},T^{\ell m}/X)], \tag{5.17}$$

$$Var(Co(\ell,m;w)/T,X) = 2tr(G_c^*(w)T^{-1}(\ell,m) \otimes D_1^{*-1}G_c^*(w)T^{-1}(\ell,m) \otimes D_1^{*-1}$$

$$+ 4\hat{A}^{*\prime}(\ell,m)G_c{}^*(w)T^{-1}(\ell,m) \otimes D_1^{*-1}G_c^*(w)\hat{A}^*(\ell,m), \tag{5.18}$$

$$E[Var(Co(\ell,m;w)/T,X)] = E[T^{m\ell}T^{m\ell} + T^{mm}T^{\ell\ell}/X]tr(G_c(w)\begin{pmatrix}0 & 0\\ 0 & D_1^{-1}\end{pmatrix}\times$$

$$G_c(w)\begin{pmatrix}0 & 0\\ 0 & D_1^{-1}\end{pmatrix}) + 4A^{*\prime}(\ell,m)G_c^*(w)\frac{R(\ell,m)}{rn+d-2k-2} \otimes D_1^{*-1}G_c^*(w)\hat{A}^*(\ell,m), \tag{5.19}$$

$$Var(Co(\ell,m;w)/X) = Var(E[Co(\ell,m;w)/T,X])$$

$$+ E[Var(Co(\ell,m;w)/T,X)]. \tag{5.20}$$

Expressions (5.16) and (5.20) give the exact posterior mean and variance of $Co(\ell,m;w)$. (Note that (5.9) is needed to evaluate (5.16) and (5.19).) With the posterior means and variances for the cospectral density function one can construct a posterior probability region for each element of the cospectral density at any $w \in [-\pi,\pi]$.

The posterior mean and variance for $Qu(\ell,m;w)$, $\ell \neq m$ can be obtained by replacing $G_c^*(w)$ found in (5.16) and (5.20) with $G_Q^*(w)$. Therefore small sample Bayesian inference for the cospectral and quadratic spectral and quadrature spectral density functions of the auxiliary process $\{y_t\}$

have been described. An extension to polar coordinates is desirable but has not yet been accomplished.

One might also note that one could insert the posterior mode for the AR parameters into

$$S_x(w) = [2\pi A(e^{iw})TA'(e^{-iw})]^{-1}$$

to produce a conditional point estimate of $S_x(w)$, where $S_x(w)$ is the spectral density function of the $AR_k(p)$ process. If one were to use Jeffreys' prior, this conditional point estimate will be the same as with maximum likelihood. If one desired to use asymptotic approximations, one may exploit the wide body of classical literature (see Akaike (1978)). As a result, a complete asymptotic Bayesian spectral analysis approach exists and is essentially the same as the maximum likelihood approach. However, this chapter described additional small sample results that are possible with the Bayesian approach.

CHAPTER VI

SUMMARY

Chapter III described new inference procedures for the $AR_k(p)$ process with one change. The procedures represent an extension to the work of Salazar (1982). The Bayesian approach to changing time series models has two major advantages over the classical approaches:

i) One can use prior information in the modeling process which is not in the form of raw measurements but can be represented with a probability distribution.

ii) Since asymptotic approximations are not used, one can apply the Bayesian approach to a rich class of nonstationary models. Since the class of nonstationary models used includes those one obtains by differencing the data, one step of the modeling process can be eliminated.

Chapter IV described a new complete Bayesian approach to making inferences for the spectral density function of a process that is well represented as an $AR_1(p)$ process. While approximations based on Chebychev's inequality were used, the approach described promises to work quite well in practice. Nevertheless, it would be desirable to obtain sharper bounds on interval estimates in the frequency domain which do not use Chebychev's inequality or asymptotic approximations.

Chapter V described a new Bayesian approach for making inferences for the $AR_k(p)$ operator. Extensions to the approach are desirable. In particular, extending the results of Chapter V to include:

i) inferences for the actual spectral density function of the $AR_k(p)$ process, and

ii) inferences for the gain and phase angle.

While asymptotic results are available to provide these extensions, it is conjectured that small sample results can be obtained.

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APPENDIXES

APPENDIX A

TABLES

TABLE I

POSTERIOR INFORMATION FOR CHANGING $AR_1(1)$, $x(t) = A1 \cdot x(t-1)+e(t)$, $t = 1,2,\ldots,CHANGE$, $x(t) = A2 \cdot x(t-1)+e(t)$, $t = CHANGE+1,\ldots,N$

N=10 CHANGE=5 XO=1 TAU=1 A1=0.5

OBS	A2	MODEM	EA1GMM	EA2GMM	VA1GMM	VA2GMM	ETAUGMM	VTAUGMM
1	-0.9	8	-0.31644	-3.9105	0.108961	10.9482	3.02622	2.28950
2	-0.7	8	-0.30067	-3.6301	0.111445	10.7599	2.89084	2.08924
3	-0.5	8	-0.28056	-2.8254	0.115366	7.4957	2.73088	1.86443
4	-0.3	8	-0.25653	-2.0337	0.121748	5.3938	2.53603	1.60786
5	-0.1	8	-0.22937	-1.3974	0.129329	4.5362	2.33877	1.36746
6	0.1	8	-0.19707	-0.9021	0.136288	4.6418	2.16036	1.16679
7	0.3	7	-0.14994	-0.7193	0.140566	6.3399	2.00903	1.00905
8	0.5	8	-0.08322	-1.3839	0.141207	18.2550	1.86913	0.87341
9	0.7	8	0.02531	-1.3858	0.140694	14.1082	1.69495	0.71821
10	0.9	5	-0.37871	0.7831	0.161151	0.1575	2.36337	1.39638

N=10 CHANGE=5 XO=1 TAU=1 A1=0.9

OBS	A2	MODEM	EA1GMM	EA2GMM	VA1GMM	VA2GMM	ETAUGMM	VTAUGMM
11	0.90	8	0.13079	-0.044940	0.134567	3.03693	2.32269	1.34873
12	1.00	5	-0.13765	0.734641	0.159856	0.23161	2.83294	2.00639
13	1.01	5	-0.13765	0.745933	0.159666	0.22382	2.83632	2.01118
14	1.02	5	-0.13765	0.757392	0.159477	0.21618	2.83967	2.01593
15	1.03	5	-0.13765	0.769010	0.159291	0.20869	2.84299	2.02065

TABLE I (Continued)

---------------------------- N=20 CHANGE=10 XO=1 TAU=1 A1=0.5 ----------------------------

OBS	A2	MODEM	EA1GMM	EA2GMM	VA1GMM	VA2GMM	ETAUGMM	VTAUGMM
16	-0.9	2	0.03338	-0.47940	0.936513	0.055196	1.15427	0.148037
17	-0.7	15	-0.14174	-0.73543	0.092441	0.105535	1.19909	0.159758
18	-0.5	15	-0.05148	-0.69318	0.094910	0.104141	1.21186	0.163179
19	-0.3	15	0.02698	-0.62194	0.092525	0.111415	1.23098	0.168367
20	-0.1	15	0.10127	-0.53703	0.089338	0.123342	1.24160	0.171286
21	0.1	15	0.16690	-0.43924	0.086930	0.137094	1.23895	0.170554
22	0.3	2	0.03338	0.02230	0.949365	0.062663	1.13864	0.144056
23	0.5	5	-0.37871	0.23455	0.258467	0.069173	1.24329	0.171754
24	0.7	5	-0.37871	0.33787	0.245482	0.066682	1.30906	0.190404
25	0.9	5	-0.37871	0.51574	0.241096	0.059232	1.33287	0.197394

---------------------------- N=20 CHANGE=10 XO=1 TAU=1 A1=0.9 ----------------------------

OBS	A2	MODEM	EA1GMM	EA2GMM	VA1GMM	VA2GMM	ETAUGMM	VTAUGMM
26	0.90	5	-0.13765	0.622805	0.289107	0.0498010	1.32167	0.194089
27	1.00	5	-0.13765	0.812850	0.300575	0.0356631	1.27124	0.179561
28	1.01	5	-0.13765	0.834182	0.302077	0.0339555	1.26492	0.177780
29	1.02	5	-0.13765	0.855616	0.303598	0.0322242	1.25858	0.176003
30	1.03	5	-0.13765	0.877057	0.305127	0.0304802	1.25228	0.174244

TABLE I (Continued)

N=30 CHANGE=15 XO=1 TAU=1 A1=0.5

OBS	A2	MODEM	EA1GMM	EA2GMM	VA1GMM	VA2GMM	ETAUGMM	VTAUGMM
31	-0.9	15	0.24856	-0.7263	0.113392	0.038702	0.85551	0.052279
32	-0.7	28	-0.22389	-1.2706	0.038152	0.174540	0.83314	0.049581
33	-0.5	28	-0.16328	-1.0744	0.040148	0.154346	0.83503	0.049806
34	-0.3	15	0.24856	-0.5010	0.112932	0.045464	0.85900	0.052706
35	-0.1	15	0.24856	-0.4280	0.107509	0.047917	0.90233	0.058157
36	0.1	15	0.24856	-0.3598	0.100802	0.051258	0.96237	0.066154
37	0.3	27	0.03059	-0.6746	0.041978	0.200040	1.03664	0.076758
38	0.5	27	0.09061	-0.6868	0.040832	0.225908	1.13724	0.092380
39	0.7	27	0.15373	-0.7075	0.039453	0.256753	1.23581	0.109087
40	0.9	27	0.23486	-0.7320	0.037357	0.272480	1.33470	0.127244

N=30 CHANGE=15 XO=1 TAU=1 A1=0.9

OBS	A2	MODEM	EA1GMM	EA2GMM	VA1GMM	VA2GMM	ETAUGMM	VTAUGMM
41	0.90	27	0.54099	-0.59703	0.028883	0.249075	1.23491	0.108928
42	1.00	5	-0.13765	0.66500	0.352507	0.023600	1.03764	0.076906
43	1.01	5	-0.13765	0.68955	0.355486	0.022443	1.02894	0.075622
44	1.02	5	-0.13765	0.71486	0.358689	0.021247	1.01975	0.074278
45	1.03	5	-0.13765	0.74078	0.362094	0.020020	1.01016	0.072888

TABLE I (Continued)

----------------------------- N=50 CHANGE=25 XO=1 TAU=1 A1=0.5 -----------------------------

OBS	A2	MODEM	EA1GMM	EA2GMM	VA1GMM	VA2GMM	ETAUGMM	VTAUGMM
46	-0.9	25	0.121358	-0.77190	0.049501	0.0179806	1.06566	0.0473178
47	-0.7	25	0.121358	-0.66768	0.050433	0.0232156	1.04595	0.0455837
48	-0.5	25	0.121358	-0.56397	0.050313	0.0263169	1.04846	0.0458027
49	-0.3	25	0.121358	-0.45899	0.049561	0.0295432	1.06436	0.0472028
50	-0.1	15	0.248564	-0.27310	0.086995	0.0255630	1.08048	0.0486431
51	0.1	2	0.033379	-0.06508	0.934096	0.0216666	1.07339	0.0480073
52	0.3	2	0.033379	0.03835	0.908936	0.0217313	1.10311	0.0507019
53	0.5	35	-0.035167	0.49085	0.030427	0.0530555	1.18559	0.0585675
54	0.7	35	0.004499	0.67387	0.030606	0.0387488	1.22738	0.0627689
55	0.9	38	0.048376	0.82977	0.030237	0.0285528	1.26376	0.0665455

----------------------------- N=50 CHANGE=25 XO=1 TAU=1 A1=0.9 -----------------------------

OBS	A2	MODEM	EA1GMM	EA2GMM	VA1GMM	VA2GMM	ETAUGMM	VTAUGMM
56	0.90	5	-0.13765	0.634806	0.304921	0.0146650	1.16232	0.0562907
57	1.00	35	0.42667	0.900926	0.024678	0.0263713	1.22059	0.0620762
58	1.01	35	0.42944	0.911876	0.024640	0.0257515	1.22185	0.0622047
59	1.02	35	0.43252	0.924670	0.024602	0.0249460	1.22253	0.0622746
60	1.03	35	0.43598	0.939726	0.024566	0.0238953	1.22253	0.0622737

TABLE I (Continued)

N=100 CHANGE=50 XO=1 TAU=1 A1=0.5

OBS	A2	MODEM	EA1GMM	EA2GMM	VA1GMM	VA2GMM	ETAUGMM	VTAUGMM
61	-0.9	53	0.160966	-0.95917	0.0214229	0.001562	1.03042	0.0216687
62	-0.7	56	0.148094	-0.85667	0.0200664	0.005520	1.05428	0.0226840
63	-0.5	55	0.163862	-0.71798	0.0199994	0.010016	1.06046	0.0229503
64	-0.3	54	0.176311	-0.55203	0.0199741	0.014329	1.05608	0.0227613
65	-0.1	54	0.190761	-0.36787	0.0198970	0.017800	1.04809	0.0224180
66	0.1	97	0.011629	0.79796	0.0103611	0.842435	1.00649	0.0206741
67	0.3	97	0.137469	0.89929	0.0101525	0.905863	1.01559	0.0210494
68	0.5	35	-0.035167	0.41224	0.0342236	0.012297	1.03117	0.0217004
69	0.7	35	-0.035167	0.64226	0.0345697	0.008699	1.02085	0.0212681
70	0.9	38	-0.023436	0.89706	0.0345130	0.003086	0.99592	0.0202418

N=200 CHANGE=100 XO=1 TAU=1 A1=0.5

OBS	A2	MODEM	EA1GMM	EA2GMM	VA1GMM	VA2GMM	ETAUGMM	VTAUGMM
71	-0.9	100	0.293974	-0.91874	0.0096586	0.00163	0.955776	0.00922734
72	-0.7	100	0.293974	-0.71289	0.0096675	0.00492	0.954893	0.00921030
73	-0.5	100	0.293974	-0.49156	0.0096687	0.00749	0.954774	0.00920802
74	-0.3	100	0.293974	-0.27071	0.0096645	0.00914	0.955185	0.00921595
75	-0.1	108	0.283476	-0.07050	0.0090241	0.01058	0.956193	0.00923540
76	0.1	198	0.219622	1.13264	0.0048560	1.13099	0.954720	0.00920696
77	0.3	198	0.321983	1.34927	0.0045682	1.00188	0.960578	0.00932030
78	0.5	35	-0.035167	0.50367	0.0359365	0.00447	0.971798	0.00953931
79	0.7	82	0.194154	0.73276	0.0122110	0.00402	0.973425	0.00957127
80	0.9	82	0.194154	0.91208	0.0123149	0.00152	0.965211	0.00941043

TABLE II

POSTERIOR INFORMATION FOR $AR_2(1)$--AR COEFFICIENTS

$$x'(t) = x'(t-1)A + e'(t)$$

------------------------ N=10 X10=1 X20=1 A11=0.5 A12=0.5 A21=0 ------------------------

OBS	A22	EA11	EA12	EA21	EA22
1	0.95	-0.26033	0.957197	-0.27203	0.830643
2	0.99	-0.26342	0.936492	-0.26797	0.841957
3	1.00	-0.26416	0.930925	-0.26757	0.844107
4	1.01	-0.26493	0.925197	-0.26749	0.845928
5	1.02	-0.26574	0.919305	-0.26778	0.847383
6	1.03	-0.26662	0.913246	-0.26847	0.848434

------------------------ N=20 X10=1 X20=1 A11=0.5 A12=0.5 A21=0 ------------------------

OBS	A22	EA11	EA12	EA21	EA22
7	0.95	0.124325	0.196191	0.0484178	0.770848
8	0.99	0.124011	0.181838	0.0470651	0.777294
9	1.00	0.123967	0.177573	0.0468245	0.777318
10	1.01	0.123956	0.173068	0.0466884	0.776707
11	1.02	0.123997	0.168370	0.0467122	0.775498
12	1.03	0.124113	0.163553	0.0469707	0.773759

------------------------ N=25 X10=1 X20=1 A11=0.5 A12=0.5 A21=0 ------------------------

OBS	A22	EA11	EA12	EA21	EA22
13	-0.9	0.136531	0.579730	-0.06735	-0.86209
14	-0.7	0.126780	0.583515	-0.12539	-0.59002
15	-0.5	0.108991	0.596909	-0.15953	-0.38857
16	-0.3	0.087708	0.603256	-0.18472	-0.21774
17	-0.1	0.066221	0.599996	-0.20178	-0.05709
18	0.1	0.049376	0.588387	-0.20574	0.10033
19	0.3	0.043452	0.570489	-0.19078	0.25645
20	0.5	0.052792	0.550271	-0.15573	0.41369
21	0.7	0.072409	0.539670	-0.11083	0.58996

TABLE II (Continued)

------------------------ N=25 X10=1 X20=1 A11=0.5 A12=0.5 A21=0 ------------------------

OBS	A22	EA11	EA12	EA21	EA22
22	0.9	0.0872872	0.5464	-0.077471	0.804792

------------------------ N=30 X10=1 X20=1 A11=0.5 A12=0.5 A21=0 ------------------------

OBS	A22	EA11	EA12	EA21	EA22
23	0.95	-0.049083	0.511781	-0.056306	0.829347
24	0.99	-0.046645	0.512788	-0.049386	0.872759
25	1.00	-0.046135	0.513120	-0.047895	0.883929
26	1.01	-0.045647	0.513501	-0.046465	0.895240
27	1.02	-0.045168	0.513950	-0.045068	0.906717
28	1.03	-0.044681	0.514495	-0.043672	0.918402

------------------------ N=50 X10=1 X20=1 A11=0.5 A12=0.5 A21=0 ------------------------

OBS	A22	EA11	EA12	EA21	EA22
29	-0.90	0.179652	0.521293	-0.07871	-0.89215
30	-0.70	0.178950	0.509316	-0.14910	-0.60494
31	-0.50	0.174554	0.512367	-0.18504	-0.38458
32	-0.30	0.166893	0.518990	-0.20724	-0.20769
33	-0.10	0.155439	0.523916	-0.20919	-0.04005
34	0.10	0.145095	0.525355	-0.18994	0.12913
35	0.30	0.140616	0.523598	-0.15298	0.30000
36	0.50	0.143600	0.519919	-0.10242	0.47090
37	0.70	0.151836	0.517508	-0.04317	0.64422
38	0.90	0.155840	0.516499	-0.00966	0.80329
39	0.95	0.155505	0.513591	-0.01186	0.83638
40	0.99	0.156163	0.515491	-0.00554	0.87376
41	1.00	0.156514	0.518210	-0.00072	0.88885
42	1.01	0.156629	0.522126	0.00554	0.90778

TABLE II (Continued)

------------------------ N=50 X10=1 X20=1 A11=0.5 A12=0.5 A21=0 ------------------------

OBS	A22	EA11	EA12	EA21	EA22
43	1.02	0.156079	0.526919	0.0126268	0.930735
44	1.03	0.154361	0.531830	0.0194459	0.956853

----------------------- N=100 X10=1 X20=1 A11=0.5 A12=0.5 A21=0 ------------------------

OBS	A22	EA11	EA12	EA21	EA22
45	-0.9	0.295061	0.537298	0.01403	-0.93926
46	-0.7	0.294088	0.541896	-0.00328	-0.74483
47	-0.5	0.297914	0.542496	-0.05567	-0.53049
48	-0.3	0.300196	0.541755	-0.09662	-0.32200
49	-0.1	0.298367	0.541045	-0.11483	-0.11846
50	0.1	0.294559	0.540441	-0.11397	0.08012
51	0.3	0.290998	0.539603	-0.10232	0.27472
52	0.5	0.288150	0.538282	-0.08738	0.46914
53	0.7	0.285099	0.536585	-0.07206	0.66952
54	0.9	0.258625	0.532669	-0.08540	0.88147

----------------------- N=200 X10=1 X20=1 A11=0.5 A12=0.5 A21=0 ------------------------

OBS	A22	EA11	EA12	EA21	EA22
55	-0.9	0.445308	0.522985	0.038200	-0.90763
56	-0.7	0.441598	0.522222	0.023949	-0.69033
57	-0.5	0.447258	0.520239	-0.021177	-0.48001
58	-0.3	0.452720	0.519901	-0.052023	-0.27994
59	-0.1	0.454961	0.520939	-0.063369	-0.08665
60	0.1	0.454634	0.522944	-0.060213	0.10163
61	0.3	0.452922	0.525500	-0.048601	0.28750
62	0.5	0.450243	0.528252	-0.031141	0.47457
63	0.7	0.446112	0.530766	-0.008346	0.66887

TABLE II (Continued)

---------- N=200 X10=1 X20=1 A11=0.5 A12=0.5 A21=0 ----------					
OBS	A22	EA11	EA12	EA21	EA22
64	0.9	0.444099	0.527481	-0.000028648	0.885672

TABLE III

POSTERIOR INFORMATION FOR $AR_2(1)$--VARIANCE OF AR COEFFICIENTS, $x'(t) = x'(t-1)A + e'(t)$

------------------------ N=10 X10=1 X20=1 A11=0.5 A12=0.5 A21=0 ------------------------

OBS	A22	VA11	VA12	VA21	VA22
1	0.95	0.140609	0.216120	0.0407406	0.0626197
2	0.99	0.144090	0.212955	0.0419355	0.0619778
3	1.00	0.144884	0.211890	0.0423590	0.0619492
4	1.01	0.145633	0.210706	0.0428426	0.0619858
5	1.02	0.146330	0.209399	0.0433919	0.0620939
6	1.03	0.146967	0.207963	0.0440133	0.0622802

------------------------ N=20 X10=1 X20=1 A11=0.5 A12=0.5 A21=0 ------------------------

OBS	A22	VA11	VA12	VA21	VA22
7	0.95	0.0672113	0.0772663	0.0324300	0.0372816
8	0.99	0.0674959	0.0758984	0.0332806	0.0374236
9	1.00	0.0675897	0.0754792	0.0336359	0.0375622
10	1.01	0.0676879	0.0750217	0.0340479	0.0377369
11	1.02	0.0677862	0.0745226	0.0345141	0.0379440
12	1.03	0.0678787	0.0739793	0.0350307	0.0381790

------------------------ N=25 X10=1 X20=1 A11=0.5 A12=0.5 A21=0 ------------------------

OBS	A22	VA11	VA12	VA21	VA22
13	-0.9	0.0479913	0.0836037	0.0080013	0.0139388
14	-0.7	0.0465631	0.0815278	0.0152466	0.0266955
15	-0.5	0.0460270	0.0820558	0.0193256	0.0344532
16	-0.3	0.0462061	0.0839841	0.0223555	0.0406334
17	-0.1	0.0470506	0.0865466	0.0246938	0.0454226
18	0.1	0.0484339	0.0891848	0.0260035	0.0478821
19	0.3	0.0500423	0.0910135	0.0259075	0.0471188
20	0.5	0.0512487	0.0908707	0.0240547	0.0426522
21	0.7	0.0512980	0.0884956	0.0197018	0.0339881

TABLE III (Continued)

------------------------ N=25 X10=1 X20=1 A11=0.5 A12=0.5 A21=0 ------------------------

OBS	A22	VA11	VA12	VA21	VA22
22	0.9	0.0502493	0.0857928	0.0126596	0.0216142

------------------------ N=30 X10=1 X20=1 A11=0.5 A12=0.5 A21=0 ------------------------

OBS	A22	VA11	VA12	VA21	VA22
23	0.95	0.0400380	0.0504420	0.0104325	0.0131434
24	0.99	0.0398176	0.0497779	0.0085077	0.0106359
25	1.00	0.0397651	0.0496194	0.0080615	0.0100593
26	1.01	0.0397137	0.0494652	0.0076312	0.0095050
27	1.02	0.0396633	0.0493159	0.0072158	0.0089718
28	1.03	0.0396136	0.0491726	0.0068132	0.0084572

------------------------ N=50 X10=1 X20=1 A11=0.5 A12=0.5 A21=0 ------------------------

OBS	A22	VA11	VA12	VA21	VA22
29	-0.90	0.0208153	0.0239276	0.0039416	0.0045309
30	-0.70	0.0204836	0.0234328	0.0105721	0.0120942
31	-0.50	0.0201813	0.0232484	0.0134228	0.0154627
32	-0.30	0.0199114	0.0233254	0.0144190	0.0168913
33	-0.10	0.0198837	0.0234374	0.0147857	0.0174282
34	0.10	0.0201569	0.0235818	0.0148140	0.0173311
35	0.30	0.0206031	0.0237231	0.0143101	0.0164771
36	0.50	0.0210214	0.0237471	0.0129086	0.0145823
37	0.70	0.0212268	0.0235360	0.0101980	0.0113074
38	0.90	0.0212213	0.0229385	0.0068204	0.0073724
39	0.95	0.0212284	0.0226156	0.0062000	0.0066052
40	0.99	0.0212145	0.0223920	0.0053912	0.0056904
41	1.00	0.0212027	0.0223943	0.0050051	0.0052864
42	1.01	0.0211887	0.0224509	0.0044952	0.0047630

TABLE III (Continued)

------------------------ N=50 X10=1 X20=1 A11=0.5 A12=0.5 A21=0 ------------------------

OBS	A22	VA11	VA12	VA21	VA22
43	1.02	0.0211769	0.0225788	0.00385980	0.00411532
44	1.03	0.0211740	0.0227855	0.00313484	0.00337342

------------------------ N=100 X10=1 X20=1 A11=0.5 A12=0.5 A21=0 ------------------------

OBS	A22	VA11	VA12	VA21	VA22
45	-0.9	0.00953362	0.0112362	0.00085709	0.00101015
46	-0.7	0.00955513	0.0113534	0.00327364	0.00388975
47	-0.5	0.00951723	0.0114234	0.00515356	0.00618577
48	-0.3	0.00942869	0.0114340	0.00621852	0.00754111
49	-0.1	0.00936720	0.0114174	0.00669103	0.00815549
50	0.1	0.00936123	0.0114049	0.00672025	0.00818736
51	0.3	0.00939365	0.0114070	0.00633861	0.00769719
52	0.5	0.00943813	0.0114228	0.00549125	0.00664597
53	0.7	0.00948670	0.0114624	0.00407043	0.00491815
54	0.9	0.00959665	0.0118542	0.00216900	0.00267924

------------------------ N=200 X10=1 X20=1 A11=0.5 A12=0.5 A21=0 ------------------------

OBS	A22	VA11	VA12	VA21	VA22
55	-0.9	0.00409452	0.00405361	0.00065713	0.00065057
56	-0.7	0.00415871	0.00407722	0.00219511	0.00215210
57	-0.5	0.00421078	0.00412393	0.00327349	0.00320597
58	-0.3	0.00423118	0.00415635	0.00383889	0.00377100
59	-0.1	0.00423724	0.00416993	0.00400831	0.00394463
60	0.1	0.00423924	0.00417148	0.00386809	0.00380626
61	0.3	0.00424049	0.00416629	0.00346639	0.00340574
62	0.5	0.00424320	0.00415775	0.00281925	0.00276247
63	0.7	0.00425126	0.00415298	0.00190315	0.00185916

TABLE III (Continued)

------------------------ N=200	X10=1	X20=1	A11=0.5	A12=0.5	A21=0 ------------------------
OBS	A22	VA11	VA12	VA21	VA22
64	0.9	0.00419999	0.00410656	0.000675762	0.000660728

TABLE IV

POSTERIOR INFORMATION FOR $AR_2(1)$--PRECISION MATRIX, $x'(t) = x'(t-1)A + e'(t)$

N=10 X10=1 X20=1 A11=0.5 A12=0.5 A21=0

OBS	A22	ET11	ET12	ET22	VT11	VT12	VT22
1	0.95	2.56620	0.307190	1.66958	1.64634	0.547354	0.696871
2	0.99	2.54999	0.368245	1.72538	1.62561	0.566911	0.744230
3	1.00	2.54840	0.385260	1.74252	1.62359	0.573633	0.759094
4	1.01	2.54816	0.403235	1.76120	1.62327	0.581302	0.775459
5	1.02	2.54946	0.422341	1.78159	1.62494	0.590059	0.793518
6	1.03	2.55255	0.442778	1.80388	1.62888	0.600069	0.813498

N=20 X10=1 X20=1 A11=0.5 A12=0.5 A21=0

OBS	A22	ET11	ET12	ET22	VT11	VT12	VT22
7	0.95	1.51485	0.678337	1.31772	0.254976	0.136461	0.192932
8	0.99	1.52007	0.692598	1.35179	0.256736	0.140806	0.203038
9	1.00	1.52195	0.697373	1.36287	0.257371	0.142253	0.206379
10	1.01	1.52407	0.702660	1.37509	0.258089	0.143859	0.210096
11	1.02	1.52642	0.708424	1.38844	0.258884	0.145622	0.214196
12	1.03	1.52893	0.714581	1.40285	0.259737	0.147527	0.218666

N=25 X10=1 X20=1 A11=0.5 A12=0.5 A21=0

OBS	A22	ET11	ET12	ET22	VT11	VT12	VT22
13	-0.9	1.31118	0.259479	0.752658	0.149494	0.0458347	0.0492604
14	-0.7	1.32879	0.245605	0.758912	0.153537	0.0464675	0.0500824
15	-0.5	1.34753	0.247823	0.755860	0.157899	0.0469547	0.0496803
16	-0.3	1.36751	0.258479	0.752375	0.162617	0.0476389	0.0492233
17	-0.1	1.38473	0.272628	0.752800	0.166736	0.0485543	0.0492789
18	0.1	1.39356	0.286968	0.756804	0.168869	0.0494347	0.0498045
19	0.3	1.38848	0.298836	0.763431	0.167640	0.0499699	0.0506806
20	0.5	1.36927	0.306101	0.772232	0.163035	0.0500475	0.0518558
21	0.7	1.34531	0.306199	0.779831	0.157378	0.0496900	0.0528814

TABLE IV (Continued)

N=25 X10=1 X20=1 A11=0.5 A12=0.5 A21=0

OBS	A22	ET11	ET12	ET22	VT11	VT12	VT22
22	0.9	1.33396	0.303312	0.781307	0.154734	0.0493143	0.0530818

N=30 X10=1 X20=1 A11=0.5 A12=0.5 A21=0

OBS	A22	ET11	ET12	ET22	VT11	VT12	VT22
23	0.95	1.11049	0.213453	0.881446	0.0880854	0.0365858	0.0554962
24	0.99	1.11062	0.216050	0.888394	0.0881061	0.0369053	0.0563746
25	1.00	1.11079	0.216791	0.890188	0.0881324	0.0369932	0.0566025
26	1.01	1.11098	0.217537	0.891962	0.0881625	0.0370812	0.0568283
27	1.02	1.11117	0.218260	0.893680	0.0881928	0.0371667	0.0570475
28	1.03	1.11133	0.218921	0.895293	0.0882185	0.0372462	0.0572535

N=50 X10=1 X20=1 A11=0.5 A12=0.5 A21=0

OBS	A22	ET11	ET12	ET22	VT11	VT12	VT22
29	-0.90	1.16581	0.146898	1.01418	0.0566297	0.0250816	0.0428563
30	-0.70	1.17240	0.123214	1.02484	0.0572713	0.0253479	0.0437625
31	-0.50	1.18243	0.119157	1.02643	0.0582554	0.0255807	0.0438982
32	-0.30	1.19461	0.125763	1.01976	0.0594626	0.0257092	0.0433299
33	-0.10	1.19684	0.135958	1.01537	0.0596847	0.0257026	0.0429574
34	0.10	1.18643	0.144795	1.01412	0.0586507	0.0255031	0.0428515
35	0.30	1.16848	0.150731	1.01481	0.0568897	0.0251772	0.0429098
36	0.50	1.14908	0.153638	1.01719	0.0550161	0.0248425	0.0431116
37	0.70	1.13313	0.152427	1.02195	0.0534994	0.0246092	0.0435163
38	0.90	1.12862	0.152872	1.04413	0.0530746	0.0250375	0.0454254
39	0.95	1.12976	0.157188	1.06047	0.0531818	0.0254746	0.0468578
40	0.99	1.12884	0.156141	1.06948	0.0530951	0.0256595	0.0476579
41	1.00	1.12795	0.153439	1.06793	0.0530117	0.0255859	0.0475200
42	1.01	1.12709	0.149503	1.06372	0.0529300	0.0254428	0.0471459

TABLE IV (Continued)

N=50 X10=1 X20=1 A11=0.5 A12=0.5 A21=0

OBS	A22	ET11	ET12	ET22	VT11	VT12	VT22
43	1.02	1.12675	0.144743	1.05679	0.0528981	0.0252433	0.0465332
44	1.03	1.12758	0.140016	1.04783	0.0529765	0.0250233	0.0457481

N=100 X10=1 X20=1 A11=0.5 A12=0.5 A21=0

OBS	A22	ET11	ET12	ET22	VT11	VT12	VT22
45	-0.9	1.00959	0.131764	0.856613	0.0208015	0.00900194	0.0149752
46	-0.7	1.00889	0.136428	0.849090	0.0207727	0.00893115	0.0147133
47	-0.5	1.01614	0.139413	0.846582	0.0210724	0.00897637	0.0146265
48	-0.3	1.02594	0.140714	0.846006	0.0214806	0.00905869	0.0146067
49	-0.1	1.03096	0.141131	0.845831	0.0216913	0.00910137	0.0146006
50	0.1	1.03061	0.141275	0.845934	0.0216767	0.00909987	0.0146042
51	0.3	1.02778	0.141579	0.846373	0.0215577	0.00908091	0.0146193
52	0.5	1.02521	0.142063	0.847079	0.0214500	0.00906749	0.0146437
53	0.7	1.02412	0.142419	0.847595	0.0214045	0.00906451	0.0146616
54	0.9	1.04741	0.147156	0.847937	0.0223890	0.00928357	0.0146734

N=200 X10=1 X20=1 A11=0.5 A12=0.5 A21=0

OBS	A22	ET11	ET12	ET22	VT11	VT12	VT22
55	-0.9	0.954435	0.0781307	0.964068	0.00920148	0.00467800	0.00938815
56	-0.7	0.945332	0.0800604	0.964226	0.00902679	0.00463598	0.00939123
57	-0.5	0.944536	0.0788653	0.964429	0.00901160	0.00463211	0.00939519
58	-0.3	0.947188	0.0783765	0.964242	0.00906228	0.00464374	0.00939154
59	-0.1	0.948682	0.0787782	0.963996	0.00909089	0.00465016	0.00938675
60	0.1	0.948518	0.0796642	0.963927	0.00908775	0.00464974	0.00938540
61	0.3	0.947438	0.0805837	0.964312	0.00906706	0.00464707	0.00939290
62	0.5	0.945888	0.0810538	0.965327	0.00903741	0.00464475	0.00941269
63	0.7	0.944279	0.0803216	0.966623	0.00900669	0.00464249	0.00943799

TABLE IV (Continued)

N=200	X10=1	X20=1	A11=0.5	A12=0.5	A21=0		
OBS	A22	ET11	ET12	ET22	VT11	VT12	VT22
64	0.9	0.943981	0.079537	0.965459	0.00900101	0.00463485	0.00941527

TABLE V

POSTERIOR INFORMATION FOR CHANGING $AR_2(1)$--AR COEFFICIENTS $x'(t) = x'(t-1)A + e'(t)$, $t = 1, 2, \ldots,$ CHANGE, $x'(t) = x'(t-1)B + e'(t)$, $t =$ CHANGE $+ 1, \ldots, N$

------------------- N=25 X10=1 X20=2 A11=0.5 A12=0.5 A21=0 CHANGE=12 -------------------

OBS	A22	B22	MODEM	EA11GM	EA12GM	EA21GM	EA22GM	EB11GM	EB12GM	EB21GM	EB22GM
1	0.50	-0.90	2	-8.2539	14.3766	4.22784	-6.9027	0.307530	0.842358	-0.14028	-0.74450
2	0.50	-0.70	2	-8.2539	14.3766	4.22784	-6.9027	0.322935	0.853405	-0.20610	-0.62721
3	0.50	-0.50	2	-8.2539	14.3766	4.22784	-6.9027	0.322774	0.858297	-0.23964	-0.48764
4	0.50	-0.30	2	-8.2539	14.3766	4.22784	-6.9027	0.311905	0.858279	-0.24983	-0.34918
5	0.50	-0.10	2	-8.2539	14.3766	4.22784	-6.9027	0.299206	0.852563	-0.24516	-0.21378
6	0.50	0.10	2	-8.2539	14.3766	4.22784	-6.9027	0.289832	0.843459	-0.23281	-0.08271
7	0.50	0.30	2	-8.2539	14.3766	4.22784	-6.9027	0.284873	0.834045	-0.21964	0.04347
8	0.50	0.50	2	-8.2539	14.3766	4.22784	-6.9027	0.282760	0.825540	-0.21143	0.16641
9	0.50	0.70	2	-8.2539	14.3766	4.22784	-6.9027	0.280564	0.814873	-0.21178	0.28540
10	0.50	0.90	2	-8.2539	14.3766	4.22784	-6.9027	0.271075	0.785510	-0.22644	0.37815
11	0.70	1.00	2	-2.2960	3.9447	1.24888	-1.4867	0.286656	0.826442	-0.14212	0.49583
12	0.70	1.01	2	-2.2960	3.9447	1.24888	-1.4867	0.286645	0.826148	-0.14210	0.49735
13	0.70	1.02	2	-2.2960	3.9447	1.24888	-1.4867	0.286653	0.825946	-0.14202	0.49884
14	0.70	1.03	2	-2.2960	3.9447	1.24888	-1.4867	0.286685	0.825855	-0.14189	0.50036
15	0.80	1.00	2	-1.6453	2.8053	0.92352	-0.8170	0.288447	0.882434	-0.05665	0.61780
16	0.80	1.01	2	-1.6453	2.8053	0.92352	-0.8170	0.288362	0.884365	-0.05467	0.62663
17	0.80	1.02	2	-1.6453	2.8053	0.92352	-0.8170	0.288251	0.886351	-0.05255	0.63657
18	0.80	1.03	2	-1.6453	2.8053	0.92352	-0.8170	0.288107	0.888349	-0.05029	0.64777
19	0.90	1.00	2	-1.2635	2.1369	0.73265	-0.3828	0.261500	0.907055	0.01567	0.79830
20	0.90	1.01	2	-1.2635	2.1369	0.73265	-0.3828	0.260271	0.905463	0.01685	0.81401
21	0.90	1.02	2	-1.2635	2.1369	0.73265	-0.3828	0.259043	0.903131	0.01787	0.83102
22	0.90	1.03	2	-1.2635	2.1369	0.73265	-0.3828	0.257834	0.899959	0.01870	0.84930
23	0.95	0.99	2	-1.1263	1.8966	0.66403	-0.2127	0.243863	0.903380	0.03082	0.86585
24	0.95	1.00	2	-1.1263	1.8966	0.66403	-0.2127	0.242578	0.899981	0.03089	0.88018
25	0.95	1.01	2	-1.1263	1.8966	0.66403	-0.2127	0.241393	0.895735	0.03074	0.89545
26	0.95	1.02	2	-1.1263	1.8966	0.66403	-0.2127	0.240335	0.890578	0.03036	0.91159
27	0.95	1.03	2	-1.1263	1.8966	0.66403	-0.2127	0.239431	0.884468	0.02974	0.92848

TABLE V (Continued)

------------------- N=50	X10=1	X20=2	A11=0.5	A12=0.5	A21=0	CHANGE=25 -------------------						
OBS	A22	B22	MODEM	EA11GM	EA12GM	EA21GM	EA22GM	EB11GM	EB12GM	EB21GM	EB22GM	
28	0.50	-0.90	48	0.4952	0.7404	0.08251	-0.6653	22.9603	15.0913	-3.2167	-3.0237	
29	0.50	-0.70	47	0.4799	0.6899	0.04034	-0.3448	14.0791	17.0780	-4.5912	-6.3570	
30	0.50	-0.50	48	0.4804	0.6511	-0.02407	-0.1522	7.0443	4.5833	-2.8600	-2.3882	
31	0.50	-0.30	2	-8.2539	14.3766	4.22784	-6.9027	0.4861	0.6269	-0.0853	-0.0122	
32	0.50	-0.10	2	-8.2539	14.3766	4.22784	-6.9027	0.4903	0.6259	-0.1234	0.1274	
33	0.50	0.10	2	-8.2539	14.3766	4.22784	-6.9027	0.4910	0.6295	-0.1433	0.2617	
34	0.50	0.30	2	-8.2539	14.3766	4.22784	-6.9027	0.4876	0.6332	-0.1465	0.4049	
35	0.50	0.50	2	-8.2539	14.3766	4.22784	-6.9027	0.4799	0.6354	-0.1333	0.5677	
36	0.50	0.70	2	-8.2539	14.3766	4.22784	-6.9027	0.4679	0.6376	-0.1033	0.7484	
37	0.50	0.90	2	-8.2539	14.3766	4.22784	-6.9027	0.4415	0.6400	-0.0786	0.9232	
38	0.70	1.00	2	-2.2960	3.9447	1.24888	-1.4867	0.3720	0.6211	-0.1059	0.9972	
39	0.70	1.01	2	-2.2960	3.9447	1.24888	-1.4867	0.3518	0.6137	-0.1147	1.0011	
40	0.70	1.02	2	-2.2960	3.9447	1.24888	-1.4867	0.3279	0.6041	-0.1240	1.0043	
41	0.70	1.03	2	-2.2960	3.9447	1.24888	-1.4867	0.3014	0.5921	-0.1325	1.0071	
42	0.80	1.00	2	-1.6453	2.8053	0.92352	-0.8170	0.3947	0.6176	-0.0936	0.9961	
43	0.80	1.01	2	-1.6453	2.8053	0.92352	-0.8170	0.3786	0.6103	-0.1018	0.9998	
44	0.80	1.02	2	-1.6453	2.8053	0.92352	-0.8170	0.3589	0.6009	-0.1112	1.0027	
45	0.80	1.03	2	-1.6453	2.8053	0.92352	-0.8170	0.3354	0.5889	-0.1212	1.0047	
46	0.90	1.00	2	-1.2635	2.1369	0.73265	-0.3828	0.4333	0.6121	-0.0668	0.9887	
47	0.90	1.01	2	-1.2635	2.1369	0.73265	-0.3828	0.4253	0.6049	-0.0721	0.9922	
48	0.90	1.02	2	-1.2635	2.1369	0.73265	-0.3828	0.4151	0.5961	-0.0786	0.9948	
49	0.90	1.03	2	-1.2635	2.1369	0.73265	-0.3828	0.4025	0.5851	-0.0863	0.9963	
50	0.95	0.99	2	-1.1263	1.8966	0.66403	-0.2127	0.4666	0.6206	-0.0345	0.9770	
51	0.95	1.00	2	-1.1263	1.8966	0.66403	-0.2127	0.4654	0.6155	-0.0352	0.9814	
52	0.95	1.01	2	-1.1263	1.8966	0.66403	-0.2127	0.4641	0.6099	-0.0361	0.9855	
53	0.95	1.02	2	-1.1263	1.8966	0.66403	-0.2127	0.4626	0.6038	-0.0373	0.9891	
54	0.95	1.03	2	-1.1263	1.8966	0.66403	-0.2127	0.4609	0.5971	-0.0386	0.9923	

TABLE V (Continued)

------------------ N=100 X10=1 X20=2 A11=0.5 A12=0.5 A21=0 CHANGE=50 ------------------

OBS	A22	B22	MODEM	EA11GM	EA12GM	EA21GM	EA22GM	EB11GM	EB12GM	EB21GM	EB22GM
55	0.50	-0.90	47	0.4835	0.6148	-0.12313	0.5686	0.53440	0.43825	-0.00813	-0.83017
56	0.50	-0.70	47	0.4835	0.6148	-0.12313	0.5686	0.53918	0.44446	-0.02897	-0.67015
57	0.50	-0.50	47	0.4835	0.6148	-0.12313	0.5686	0.54170	0.45251	-0.03787	-0.51034
58	0.50	-0.30	47	0.4835	0.6148	-0.12313	0.5686	0.54116	0.45263	-0.03335	-0.31396
59	0.50	-0.10	2	-8.2539	14.3766	4.22784	-6.9027	0.52731	0.44978	-0.08617	0.34039
60	0.50	0.10	2	-8.2539	14.3766	4.22784	-6.9027	0.52805	0.46497	-0.08140	0.41143
61	0.50	0.30	2	-8.2539	14.3766	4.22784	-6.9027	0.52915	0.48459	-0.07782	0.48099
62	0.50	0.50	98	0.5318	0.5135	-0.07694	0.5464	8.29503	3.82617	9.45247	5.74299
63	0.50	0.70	2	-8.2539	14.3766	4.22784	-6.9027	0.52863	0.52406	-0.06921	0.68826
64	0.50	0.90	2	-8.2539	14.3766	4.22784	-6.9027	0.51979	0.52296	-0.03663	0.89005
65	0.70	1.00	26	0.3351	0.7921	-0.15044	0.3608	0.54460	0.48354	-0.00822	1.02586
66	0.70	1.01	26	0.3351	0.7921	-0.15044	0.3608	0.54445	0.48412	-0.00678	1.03406
67	0.70	1.02	26	0.3351	0.7921	-0.15044	0.3608	0.54431	0.48477	-0.00565	1.04228
68	0.70	1.03	26	0.3351	0.7921	-0.15044	0.3608	0.54419	0.48555	-0.00486	1.05059
69	0.80	1.00	2	-1.6453	2.8053	0.92352	-0.8170	0.51291	0.53798	-0.01513	1.01680
70	0.80	1.01	2	-1.6453	2.8053	0.92352	-0.8170	0.51274	0.53897	-0.01389	1.02549
71	0.80	1.02	2	-1.6453	2.8053	0.92352	-0.8170	0.51246	0.54004	-0.01331	1.03390
72	0.80	1.03	2	-1.6453	2.8053	0.92352	-0.8170	0.51193	0.54118	-0.01377	1.04171
73	0.90	1.00	2	-1.2635	2.1369	0.73265	-0.3828	0.50714	0.53774	-0.02436	1.01667
74	0.90	1.01	2	-1.2635	2.1369	0.73265	-0.3828	0.50546	0.53849	-0.02578	1.02224
75	0.90	1.02	2	-1.2635	2.1369	0.73265	-0.3828	0.50206	0.53792	-0.02978	1.02502
76	0.90	1.03	2	-1.2635	2.1369	0.73265	-0.3828	0.49470	0.53267	-0.03835	1.02041
77	0.95	0.99	2	-1.1263	1.8966	0.66403	-0.2127	0.50444	0.53221	-0.02740	1.00956
78	0.95	1.00	2	-1.1263	1.8966	0.66403	-0.2127	0.50234	0.53240	-0.02878	1.01412
79	0.95	1.01	2	-1.1263	1.8966	0.66403	-0.2127	0.49857	0.53095	-0.03205	1.01604
80	0.95	1.02	2	-1.1263	1.8966	0.66403	-0.2127	0.49177	0.52490	-0.03770	1.01220
81	0.95	1.03	2	-1.1263	1.8966	0.66403	-0.2127	0.48257	0.50914	-0.04197	1.00075

TABLE V (Continued)

------------------ N=200 X10=1 X20=2 A11=0.5 A12=0.5 A21=0 CHANGE=100 ------------------

OBS	A22	B22	MODEM	EA11GM	EA12GM	EA21GM	EA22GM	EB11GM	EB12GM	EB21GM	EB22GM
82	0.50	-0.90	102	0.5298	0.5004	-0.07693	0.5525	0.47501	0.387865	0.11708	-0.87756
83	0.50	-0.70	101	0.5301	0.5000	-0.07705	0.5536	0.45260	0.384238	0.10403	-0.75242
84	0.50	-0.50	101	0.5301	0.4998	-0.07705	0.5545	0.45010	0.392673	0.04497	-0.59595
85	0.50	-0.30	101	0.5301	0.4997	-0.07705	0.5554	0.45531	0.397459	-0.00628	-0.41044
86	0.50	-0.10	104	0.5176	0.5005	-0.07670	0.5543	0.47064	0.394321	-0.03180	-0.20898
87	0.50	0.10	198	0.4913	0.4305	-0.06062	0.3987	5.66143	0.906874	7.48320	-0.66013
88	0.50	0.30	2	-8.2539	14.3766	4.22784	-6.9027	0.49383	0.441708	-0.05210	0.46733
89	0.50	0.50	2	-8.2539	14.3766	4.22784	-6.9027	0.49096	0.442423	-0.03401	0.54544
90	0.50	0.70	2	-8.2539	14.3766	4.22784	-6.9027	0.48693	0.439938	-0.01305	0.66156
91	0.50	0.90	26	0.3343	0.7837	-0.21223	0.1227	0.49116	0.377908	0.01029	0.89271
92	0.70	1.00	27	0.4328	0.7848	-0.17013	0.3623	0.48134	0.394549	0.00452	0.99900
93	0.70	1.01	27	0.4328	0.7848	-0.17013	0.3623	0.48214	0.393729	0.00295	1.00919
94	0.70	1.02	27	0.4328	0.7848	-0.17013	0.3623	0.48343	0.392883	0.00162	1.01938
95	0.70	1.03	91	0.5155	0.5636	-0.08665	0.7494	0.46278	0.340427	0.00087	1.02995
96	0.80	1.00	2	-1.6453	2.8053	0.92352	-0.8170	0.47772	0.444883	0.00406	0.99796
97	0.80	1.01	2	-1.6453	2.8053	0.92352	-0.8170	0.47806	0.444260	0.00274	1.00859
98	0.80	1.02	2	-1.6453	2.8053	0.92352	-0.8170	0.47908	0.443575	0.00153	1.01909
99	0.80	1.03	2	-1.6453	2.8053	0.92352	-0.8170	0.48053	0.442909	0.00067	1.02944
100	0.90	1.00	2	-1.2635	2.1369	0.73265	-0.3828	0.47728	0.455865	0.00384	0.99945
101	0.90	1.01	2	-1.2635	2.1369	0.73265	-0.3828	0.47722	0.456780	0.00260	1.00936
102	0.90	1.02	2	-1.2635	2.1369	0.73265	-0.3828	0.47825	0.457474	0.00140	1.01947
103	0.90	1.03	2	-1.2635	2.1369	0.73265	-0.3828	0.47998	0.457994	0.00058	1.02963
104	0.95	0.99	2	-1.1263	1.8966	0.66403	-0.2127	0.47818	0.453104	0.00429	0.99231
105	0.95	1.00	2	-1.1263	1.8966	0.66403	-0.2127	0.47659	0.455613	0.00366	1.00068
106	0.95	1.01	2	-1.1263	1.8966	0.66403	-0.2127	0.47629	0.457958	0.00244	1.00995
107	0.95	1.02	2	-1.1263	1.8966	0.66403	-0.2127	0.47748	0.459954	0.00125	1.01974
108	0.95	1.03	2	-1.1263	1.8966	0.66403	-0.2127	0.47953	0.461545	0.00049	1.02976

TABLE VI

POSTERIOR INFORMATION FOR CHANGING $AR_2(1)$--PRECISION MATRIX $x'(t) = x'(t-1)A + e'(t)$, $t = 1,2,\ldots,CHANGE$, $x'(t) = x'(t-1)B + e'(t)$, $t = CHANGE+1,\ldots,N$

------------------ N=25 X10=1 X20=2 A11=0.5 A12=0.5 A21=0 CHANGE=12 ------------------

OBS	A22	B22	MODEM	ET11GM	ET21GM	ET22GM
1	0.50	-0.90	2	1.36997	0.123891	0.70201
2	0.50	-0.70	2	1.45841	0.154797	0.76545
3	0.50	-0.50	2	1.50106	0.183986	0.84641
4	0.50	-0.30	2	1.50098	0.206849	0.92073
5	0.50	-0.10	2	1.47475	0.217100	0.97588
6	0.50	0.10	2	1.44020	0.214336	1.00355
7	0.50	0.30	2	1.41050	0.203274	1.00241
8	0.50	0.50	2	1.39230	0.189835	0.97542
9	0.50	0.70	2	1.38827	0.180540	0.92984
10	0.50	0.90	2	1.40829	0.194626	0.89643
11	0.70	1.00	2	1.31636	0.162833	0.95786
12	0.70	1.01	2	1.31654	0.164224	0.95953
13	0.70	1.02	2	1.31667	0.165552	0.96119
14	0.70	1.03	2	1.31673	0.166796	0.96281
15	0.80	1.00	2	1.25804	0.093295	0.92537
16	0.80	1.01	2	1.25715	0.091701	0.92029
17	0.80	1.02	2	1.25620	0.089813	0.91415
18	0.80	1.03	2	1.25520	0.087624	0.90683
19	0.90	1.00	2	1.24130	0.033667	0.86720
20	0.90	1.01	2	1.24157	0.033193	0.85741
21	0.90	1.02	2	1.24187	0.032821	0.84654
22	0.90	1.03	2	1.24219	0.032583	0.83466
23	0.95	0.99	2	1.24774	0.018008	0.85675
24	0.95	1.00	2	1.24831	0.018978	0.84845
25	0.95	1.01	2	1.24884	0.020160	0.83916
26	0.95	1.02	2	1.24932	0.021563	0.82898
27	0.95	1.03	2	1.24974	0.023188	0.81804

TABLE VI (Continued)

------------------ N=50 X10=1 X20=2 A11=0.5 A12=0.5 A21=0 CHANGE=25 ------------------

OBS	A22	B22	MODEM	ET11GM	ET21GM	ET22GM
28	0.50	-0.90	48	1.35515	0.371711	0.654821
29	0.50	-0.70	47	1.30671	0.393386	0.818687
30	0.50	-0.50	48	1.22243	0.313331	0.853465
31	0.50	-0.30	2	1.17693	0.240210	0.886206
32	0.50	-0.10	2	1.17834	0.214092	0.914951
33	0.50	0.10	2	1.18197	0.190151	0.913737
34	0.50	0.30	2	1.18214	0.163573	0.881530
35	0.50	0.50	2	1.17561	0.130972	0.821209
36	0.50	0.70	2	1.16136	0.094414	0.740668
37	0.50	0.90	2	1.15935	0.064682	0.649472
38	0.70	1.00	2	1.22202	0.107590	0.696418
39	0.70	1.01	2	1.24254	0.112112	0.692975
40	0.70	1.02	2	1.26784	0.118376	0.690114
41	0.70	1.03	2	1.29714	0.126459	0.687946
42	0.80	1.00	2	1.20214	0.127215	0.737363
43	0.80	1.01	2	1.21805	0.131553	0.734033
44	0.80	1.02	2	1.23840	0.137659	0.731377
45	0.80	1.03	2	1.26365	0.145898	0.729596
46	0.90	1.00	2	1.16876	0.145190	0.777931
47	0.90	1.01	2	1.17663	0.148633	0.774797
48	0.90	1.02	2	1.18682	0.153345	0.772332
49	0.90	1.03	2	1.20001	0.159736	0.770788
50	0.95	0.99	2	1.13825	0.148049	0.803969
51	0.95	1.00	2	1.13940	0.149350	0.800816
52	0.95	1.01	2	1.14077	0.150869	0.797887
53	0.95	1.02	2	1.14236	0.152658	0.795259
54	0.95	1.03	2	1.14420	0.154765	0.793016

TABLE VI (Continued)

------------------ N=100 X10=1 X20=2 A11=0.5 A12=0.5 A21=0 CHANGE=50 ------------------

OBS	A22	B22	MODEM	ET11GM	ET21GM	ET22GM
55	0.50	-0.90	47	0.959102	0.049345	0.908409
56	0.50	-0.70	47	0.959551	0.047048	0.907378
57	0.50	-0.50	47	0.959660	0.045842	0.909872
58	0.50	-0.30	47	0.959064	0.043455	0.910529
59	0.50	-0.10	2	0.959566	0.054626	0.774558
60	0.50	0.10	2	0.957692	0.050762	0.841617
61	0.50	0.30	2	0.956436	0.042142	0.888380
62	0.50	0.50	98	0.956006	0.039750	0.896091
63	0.50	0.70	2	0.957501	0.013094	0.887109
64	0.50	0.90	2	0.952781	-0.013206	0.805780
65	0.70	1.00	26	0.959168	0.028664	0.977953
66	0.70	1.01	26	0.958800	0.028167	0.974967
67	0.70	1.02	26	0.958514	0.027612	0.971450
68	0.70	1.03	26	0.958313	0.026955	0.967301
69	0.80	1.00	2	0.948546	0.002207	0.870431
70	0.80	1.01	2	0.948190	0.001027	0.864411
71	0.80	1.02	2	0.948132	-0.000093	0.857731
72	0.80	1.03	2	0.948570	-0.001059	0.850273
73	0.90	1.00	2	0.955260	0.016211	0.890828
74	0.90	1.01	2	0.956679	0.016283	0.884142
75	0.90	1.02	2	0.960045	0.017937	0.877080
76	0.90	1.03	2	0.967592	0.024508	0.872724
77	0.95	0.99	2	0.958199	0.024647	0.901448
78	0.95	1.00	2	0.959894	0.025534	0.895406
79	0.95	1.01	2	0.963329	0.028150	0.889087
80	0.95	1.02	2	0.969737	0.035213	0.885499
81	0.95	1.03	2	0.978134	0.050692	0.894102

TABLE VI (Continued)

N=200 X10=1 X20=2 A11=0.5 A12=0.5 A21=0 CHANGE=100

OBS	A22	B22	MODEM	ET11GM	ET21GM	ET22GM
82	0.50	-0.90	102	0.939132	0.026913	0.99617
83	0.50	-0.70	101	0.917801	0.013882	0.99771
84	0.50	-0.50	101	0.908154	0.017115	1.00160
85	0.50	-0.30	101	0.907024	0.020795	1.00156
86	0.50	-0.10	104	0.906777	0.020351	1.00053
87	0.50	0.10	198	0.922361	0.026640	0.89891
88	0.50	0.30	2	0.906377	0.021108	0.96156
89	0.50	0.50	2	0.904158	0.017989	0.99628
90	0.50	0.70	2	0.902384	0.006770	0.98369
91	0.50	0.90	26	0.910891	0.018609	0.98496
92	0.70	1.00	27	0.908124	0.013225	0.98797
93	0.70	1.01	27	0.907827	0.012423	0.98184
94	0.70	1.02	27	0.907342	0.011664	0.97547
95	0.70	1.03	91	0.912891	0.029469	0.99674
96	0.80	1.00	2	0.903747	-0.006810	0.95065
97	0.80	1.01	2	0.903679	-0.008005	0.94327
98	0.80	1.02	2	0.903318	-0.009099	0.93552
99	0.80	1.03	2	0.902800	-0.010112	0.92744
100	0.90	1.00	2	0.903983	0.001757	0.98777
101	0.90	1.01	2	0.904044	-0.000330	0.98184
102	0.90	1.02	2	0.903615	-0.002265	0.97503
103	0.90	1.03	2	0.902924	-0.004025	0.96737
104	0.95	0.99	2	0.903834	0.012892	1.00158
105	0.95	1.00	2	0.904503	0.010126	0.99918
106	0.95	1.01	2	0.904602	0.007311	0.99559
107	0.95	1.02	2	0.904012	0.004665	0.99065
108	0.95	1.03	2	0.903104	0.002296	0.98431

TABLE VII

POSTERIOR INFORMATION FOR $AR_1(1)$, A = .5, N = 100, FREQUENCY DOMAIN

A=0.5 EA=0.2972529 N=100

OBS	W	ESM1	VSM1	SM1L	ISPEC	SM1U	SL	SPE	SU
1	0.05233	3.14960	0.90936	1.24239	1.57860	5.0568	0.197753	0.633474	0.804898
2	0.10467	3.16485	0.90299	1.26433	1.60437	5.0654	0.197419	0.623299	0.790932
3	0.15700	3.19022	0.89253	1.30075	1.64724	5.0797	0.196862	0.607076	0.768789
4	0.20933	3.22564	0.87818	1.35141	1.70709	5.0999	0.196084	0.585791	0.739968
5	0.26167	3.27101	0.86026	1.41600	1.78377	5.1260	0.195083	0.560610	0.706214
6	0.31400	3.32620	0.83915	1.49410	1.87706	5.1583	0.193862	0.532749	0.669299
7	0.36633	3.39108	0.81531	1.58519	1.98670	5.1970	0.192420	0.503348	0.630840
8	0.41867	3.46546	0.78927	1.68864	2.11239	5.2423	0.190757	0.473397	0.592194
9	0.47100	3.54913	0.76163	1.80370	2.25380	5.2946	0.188873	0.443695	0.554415
10	0.52333	3.64186	0.73302	1.92953	2.41053	5.3542	0.186769	0.414847	0.518262
11	0.57567	3.74342	0.70416	2.06513	2.58215	5.4217	0.184444	0.387274	0.484232
12	0.62800	3.85350	0.67578	2.20938	2.76820	5.4976	0.181897	0.361246	0.452615
13	0.68033	3.97182	0.64865	2.36105	2.96816	5.5826	0.179128	0.336909	0.423541
14	0.73267	4.09805	0.62355	2.51875	3.18149	5.6774	0.176138	0.314318	0.397022
15	0.78500	4.23185	0.60128	2.68100	3.40760	5.7827	0.172930	0.293462	0.372995
16	0.83733	4.37284	0.58264	2.84621	3.64588	5.8995	0.169507	0.274282	0.351344
17	0.88967	4.52064	0.56843	3.01275	3.89567	6.0285	0.165878	0.256696	0.331922
18	0.94200	4.67485	0.55940	3.17898	4.15628	6.1707	0.162056	0.240600	0.314566
19	0.99433	4.83504	0.55631	3.34332	4.42701	6.3268	0.158059	0.225886	0.299104
20	1.04667	5.00078	0.55984	3.50433	4.70711	6.4972	0.153912	0.212444	0.285361
21	1.09900	5.17161	0.57064	3.66079	4.99582	6.6824	0.149646	0.200167	0.273165
22	1.15133	5.34707	0.58932	3.81173	5.29234	6.8824	0.145298	0.188952	0.262348
23	1.20367	5.52667	0.61638	3.95647	5.59587	7.0969	0.140907	0.178703	0.252751
24	1.25600	5.70992	0.65229	4.09464	5.90557	7.3252	0.136515	0.169332	0.244222
25	1.30833	5.89632	0.69740	4.22612	6.22059	7.5665	0.132161	0.160756	0.236624
26	1.36067	6.08537	0.75199	4.35101	6.54008	7.8197	0.127882	0.152903	0.229831
27	1.41300	6.27653	0.81626	4.46958	6.86315	8.0835	0.123709	0.145706	0.223735
28	1.46533	6.46929	0.89030	4.58218	7.18892	8.3564	0.119669	0.139103	0.218237
29	1.51767	6.66313	0.97409	4.68921	7.51650	8.6370	0.115780	0.133041	0.213256
30	1.57000	6.85750	1.06752	4.79109	7.84500	8.9239	0.112058	0.127470	0.208721
31	1.62233	7.05189	1.17037	4.88821	8.17351	9.2156	0.108512	0.122346	0.204574
32	1.67467	7.24575	1.28233	4.98095	8.50113	9.5105	0.105146	0.117631	0.200765
33	1.72700	7.43855	1.40297	5.06961	8.82697	9.8075	0.101963	0.113289	0.197254

TABLE VII (Continued)

A=0.5 EA=0.2972529 N=100

OBS	W	ESM1	VSM1	SM1L	ISPEC	SM1U	SL	SPE	SU
34	1.77933	7.6298	1.53178	5.15447	9.1501	10.1051	0.0989602	0.109288	0.194006
35	1.83167	7.8189	1.66813	5.23576	9.4697	10.4020	0.0961353	0.105599	0.190994
36	1.88400	8.0054	1.81133	5.31367	9.7849	10.6971	0.0934834	0.102198	0.188194
37	1.93633	8.1887	1.96057	5.38833	10.0948	10.9891	0.0909990	0.099061	0.185586
38	1.98867	8.3684	2.11498	5.45985	10.3985	11.2770	0.0886757	0.096168	0.183155
39	2.04100	8.5440	2.27363	5.52833	10.6953	11.5597	0.0865071	0.093499	0.180887
40	2.09333	8.7150	2.43550	5.59380	10.9842	11.8362	0.0844863	0.091040	0.178769
41	2.14567	8.8809	2.59952	5.65631	11.2646	12.1055	0.0826069	0.088774	0.176794
42	2.19800	9.0413	2.76460	5.71587	11.5356	12.3667	0.0808623	0.086688	0.174951
43	2.25033	9.1957	2.92959	5.77247	11.7966	12.6189	0.0792462	0.084770	0.173236
44	2.30267	9.3437	3.09333	5.82612	12.0467	12.8613	0.0777529	0.083010	0.171641
45	2.35500	9.4849	3.25464	5.87677	12.2853	13.0930	0.0763766	0.081398	0.170162
46	2.40733	9.6189	3.41233	5.92441	12.5118	13.3134	0.0751122	0.079924	0.168793
47	2.45967	9.7454	3.56525	5.96900	12.7255	13.5218	0.0739549	0.078582	0.167532
48	2.51200	9.8639	3.71224	6.01051	12.9259	13.7174	0.0729003	0.077364	0.166375
49	2.56433	9.9743	3.85217	6.04889	13.1124	13.8997	0.0719441	0.076264	0.165320
50	2.61667	10.0761	3.98399	6.08411	13.2845	14.0681	0.0710829	0.075276	0.164363
51	2.66900	10.1691	4.10667	6.11612	13.4417	14.2221	0.0703131	0.074396	0.163502
52	2.72133	10.2531	4.21925	6.14490	13.5835	14.3612	0.0696320	0.073619	0.162737
53	2.77367	10.3277	4.32085	6.17039	13.7097	14.4851	0.0690367	0.072941	0.162064
54	2.82600	10.3929	4.41067	6.19257	13.8198	14.5932	0.0685250	0.072360	0.161484
55	2.87833	10.4484	4.48799	6.21141	13.9136	14.6854	0.0680950	0.071872	0.160994
56	2.93067	10.4941	4.55219	6.22688	13.9908	14.7612	0.0677450	0.071475	0.160594
57	2.98300	10.5298	4.60278	6.23896	14.0512	14.8206	0.0674737	0.071168	0.160283
58	3.03533	10.5555	4.63933	6.24763	14.0946	14.8633	0.0672799	0.070949	0.160061
59	3.08767	10.5710	4.66156	6.25288	14.1209	14.8891	0.0671631	0.070817	0.159926
60	3.14000	10.5764	4.66928	6.25470	14.1300	14.8981	0.0671226	0.070771	0.159880

TABLE VIII

POSTERIOR INFORMATION FOR $AR_1(1)$, A = .5, N = 200, FREQUENCY DOMAIN

---------------------------------- A=0.5 EA=0.4457908 N=200 ----------------------------------

OBS	W	ESM1	VSM1	SM1L	ISPEC	SM1U	SL	SPE	SU
1	0.05233	1.84817	0.208587	0.93475	1.57860	2.7616	0.362109	0.633474	1.06981
2	0.10467	1.86981	0.206811	0.96028	1.60437	2.7793	0.359797	0.623299	1.04136
3	0.15700	1.90582	0.203922	1.00266	1.64724	2.8090	0.356002	0.607076	0.99734
4	0.20933	1.95608	0.200020	1.06161	1.70709	2.8506	0.350809	0.585791	0.94196
5	0.26167	2.02048	0.195246	1.13674	1.78377	2.9042	0.344328	0.560610	0.87971
6	0.31400	2.09882	0.189780	1.22755	1.87706	2.9701	0.336690	0.532749	0.81463
7	0.36633	2.19090	0.183834	1.33338	1.98670	3.0484	0.328039	0.503348	0.74997
8	0.41867	2.29646	0.177655	1.45348	2.11239	3.1394	0.318528	0.473397	0.68801
9	0.47100	2.41521	0.171516	1.58692	2.25380	3.2435	0.308309	0.443695	0.63015
10	0.52333	2.54684	0.165717	1.73267	2.41053	3.3610	0.297530	0.414847	0.57714
11	0.57567	2.69097	0.160580	1.88952	2.58215	3.4924	0.286335	0.387274	0.52924
12	0.62800	2.84721	0.156444	2.05615	2.76820	3.6383	0.274856	0.361246	0.48634
13	0.68033	3.01514	0.153659	2.23116	2.96816	3.7991	0.263218	0.336909	0.44820
14	0.73267	3.19430	0.152586	2.41306	3.18149	3.9755	0.251538	0.314318	0.41441
15	0.78500	3.38419	0.153587	2.60039	3.40760	4.1680	0.239923	0.293462	0.38456
16	0.83733	3.58430	0.157024	2.79178	3.64588	4.3768	0.228476	0.274282	0.35819
17	0.88967	3.79408	0.163250	2.98599	3.89567	4.6022	0.217289	0.256696	0.33490
18	0.94200	4.01294	0.172610	3.18202	4.15628	4.8439	0.206446	0.240600	0.31427
19	0.99433	4.24031	0.185429	3.37908	4.42701	5.1015	0.196019	0.225886	0.29594
20	1.04667	4.47554	0.202012	3.57663	4.70711	5.3745	0.186065	0.212444	0.27959
21	1.09900	4.71800	0.222639	3.77431	4.99582	5.6617	0.176626	0.200167	0.26495
22	1.15133	4.96703	0.247559	3.97192	5.29234	5.9621	0.167725	0.188952	0.25177
23	1.20367	5.22193	0.276988	4.16934	5.59587	6.2745	0.159375	0.178703	0.23985
24	1.25600	5.48202	0.311103	4.36649	5.90557	6.5976	0.151571	0.169332	0.22902
25	1.30833	5.74658	0.350040	4.56330	6.22059	6.9299	0.144303	0.160756	0.21914
26	1.36067	6.01489	0.393892	4.75967	6.54008	7.2701	0.137550	0.152903	0.21010
27	1.41300	6.28621	0.442704	4.95549	6.86315	7.6169	0.131287	0.145706	0.20180
28	1.46533	6.55980	0.496473	5.15058	7.18892	7.9690	0.125486	0.139103	0.19415
29	1.51767	6.83490	0.555148	5.34474	7.51650	8.3251	0.120119	0.133041	0.18710
30	1.57000	7.11078	0.618625	5.53773	7.84500	8.6838	0.115157	0.127470	0.18058
31	1.62233	7.38667	0.686750	5.72926	8.17351	9.0441	0.110570	0.122346	0.17454
32	1.67467	7.66181	0.759317	5.91903	8.50113	9.4046	0.106331	0.117631	0.16895
33	1.72700	7.93546	0.836073	6.10672	8.82697	9.7642	0.102415	0.113289	0.16375

TABLE VIII (Continued)

A=0.5 EA=0.4457908 N=200

OBS	W	ESM1	VSM1	SM1L	ISPEC	SM1U	SL	SPE	SU
34	1.77933	8.2069	0.91671	6.29195	9.1501	10.1218	0.0987971	0.109288	0.158933
35	1.83167	8.4753	1.00089	6.47438	9.4697	10.4762	0.0954549	0.105599	0.154455
36	1.88400	8.7400	1.08820	6.65362	9.7849	10.8263	0.0923678	0.102198	0.150294
37	1.93633	9.0002	1.17821	6.82928	10.0948	11.1711	0.0895168	0.099061	0.146428
38	1.98867	9.2553	1.27045	7.00098	10.3985	11.5095	0.0868844	0.096168	0.142837
39	2.04100	9.5045	1.36440	7.16832	10.6953	11.8406	0.0844550	0.093499	0.139503
40	2.09333	9.7471	1.45953	7.33093	10.9842	12.1634	0.0822141	0.091040	0.136408
41	2.14567	9.9826	1.55527	7.48840	11.2646	12.4768	0.0801487	0.088774	0.133540
42	2.19800	10.2102	1.65103	7.64037	11.5356	12.7801	0.0782469	0.086688	0.130884
43	2.25033	10.4294	1.74622	7.78647	11.7966	13.0722	0.0764980	0.084770	0.128428
44	2.30267	10.6394	1.84021	7.92633	12.0467	13.3525	0.0748923	0.083010	0.126162
45	2.35500	10.8398	1.93240	8.05961	12.2853	13.6200	0.0734212	0.081398	0.124075
46	2.40733	11.0300	2.02216	8.18598	12.5118	13.8741	0.0720768	0.079924	0.122160
47	2.45967	11.2095	2.10889	8.30513	12.7255	14.1139	0.0708520	0.078582	0.120408
48	2.51200	11.3778	2.19198	8.41674	12.9259	14.3389	0.0697405	0.077364	0.118811
49	2.56433	11.5344	2.27087	8.52053	13.1124	14.5483	0.0687366	0.076264	0.117364
50	2.61667	11.6789	2.34499	8.61625	13.2845	14.7416	0.0678353	0.075276	0.116060
51	2.66900	11.8109	2.41381	8.70364	13.4417	14.9182	0.0670321	0.074396	0.114894
52	2.72133	11.9301	2.47684	8.78248	13.5835	15.0777	0.0663232	0.073619	0.113863
53	2.77367	12.0360	2.53363	8.85257	13.7097	15.2195	0.0657051	0.072941	0.112962
54	2.82600	12.1285	2.58375	8.91373	13.8198	15.3434	0.0651748	0.072360	0.112186
55	2.87833	12.2073	2.62685	8.96579	13.9136	15.4488	0.0647299	0.071872	0.111535
56	2.93067	12.2721	2.66260	9.00863	13.9908	15.5356	0.0643682	0.071475	0.111005
57	2.98300	12.3228	2.69074	9.04213	14.0512	15.6035	0.0640881	0.071168	0.110593
58	3.03533	12.3593	2.71107	9.06620	14.0946	15.6523	0.0638883	0.070949	0.110300
59	3.08767	12.3813	2.72342	9.08079	14.1209	15.6819	0.0637678	0.070817	0.110123
60	3.14000	12.3890	2.72771	9.08585	14.1300	15.6922	0.0637261	0.070771	0.110061

TABLE IX

POSTERIOR INFORMATION FOR $AR_1(1)$, A = -.5, N = 100, FREQUENCY DOMAIN

A=-0.5 EA=-0.624342 N=100

OBS	W	ESM1	VSM1	SM1L	ISPEC	SM1U	SL	SPE	SU
1	0.05233	16.2094	7.72676	10.6500	14.1214	21.7688	0.045937	0.070814	0.093897
2	0.10467	16.1779	7.69392	10.6304	14.0956	21.7255	0.046029	0.070944	0.094070
3	0.15700	16.1257	7.63945	10.5978	14.0528	21.6536	0.046182	0.071160	0.094360
4	0.20933	16.0527	7.56372	10.5522	13.9929	21.5531	0.046397	0.071465	0.094767
5	0.26167	15.9592	7.46727	10.4939	13.9162	21.4244	0.046676	0.071859	0.095293
6	0.31400	15.8454	7.35078	10.4229	13.8229	21.2679	0.047019	0.072343	0.095942
7	0.36633	15.7117	7.21506	10.3395	13.7133	21.0839	0.047430	0.072922	0.096716
8	0.41867	15.5584	7.06104	10.2439	13.5876	20.8729	0.047909	0.073596	0.097619
9	0.47100	15.3860	6.88980	10.1363	13.4462	20.6357	0.048460	0.074370	0.098655
10	0.52333	15.1949	6.70250	10.0170	13.2895	20.3727	0.049085	0.075248	0.099830
11	0.57567	14.9856	6.50042	9.8864	13.1178	20.0847	0.049789	0.076232	0.101149
12	0.62800	14.7587	6.28492	9.7447	12.9318	19.7726	0.050575	0.077329	0.102620
13	0.68033	14.5148	6.05744	9.5925	12.7318	19.4372	0.051448	0.078543	0.104249
14	0.73267	14.2547	5.81946	9.4300	12.5185	19.0794	0.052413	0.079882	0.106045
15	0.78500	13.9789	5.57255	9.2577	12.2924	18.7002	0.053475	0.081351	0.108018
16	0.83733	13.6884	5.31828	9.0761	12.0541	18.3007	0.054643	0.082959	0.110180
17	0.88967	13.3838	5.05825	8.8857	11.8043	17.8819	0.055923	0.084715	0.112541
18	0.94200	13.0659	4.79407	8.6869	11.5437	17.4450	0.057323	0.086627	0.115116
19	0.99433	12.7358	4.52734	8.4803	11.2730	16.9913	0.058854	0.088708	0.117920
20	1.04667	12.3942	4.25963	8.2664	10.9929	16.5220	0.060525	0.090968	0.120971
21	1.09900	12.0422	3.99250	8.0459	10.7042	16.0384	0.062350	0.093421	0.124287
22	1.15133	11.6806	3.72742	7.8192	10.4077	15.5419	0.064342	0.096083	0.127890
23	1.20367	11.3104	3.46583	7.5871	10.1041	15.0338	0.066517	0.098969	0.131803
24	1.25600	10.9327	3.20909	7.3500	9.7944	14.5155	0.068892	0.102099	0.136055
25	1.30833	10.5486	2.95848	7.1085	9.4794	13.9886	0.071487	0.105492	0.140676
26	1.36067	10.1590	2.71516	6.8634	9.1599	13.4545	0.074324	0.109171	0.145700
27	1.41300	9.7650	2.48023	6.6153	8.8369	12.9147	0.077431	0.113162	0.151166
28	1.46533	9.3677	2.25465	6.3646	8.5111	12.3708	0.080835	0.117494	0.157118
29	1.51767	8.9682	2.03928	6.1122	8.1835	11.8243	0.084572	0.122197	0.163608
30	1.57000	8.5677	1.83484	5.8585	7.8550	11.2768	0.088678	0.127307	0.170691
31	1.62233	8.1670	1.64195	5.6043	7.5265	10.7298	0.093198	0.132864	0.178435
32	1.67467	7.7675	1.46109	5.3500	7.1989	10.1850	0.098183	0.138911	0.186916
33	1.72700	7.3702	1.29261	5.0963	6.8730	9.6440	0.103691	0.145496	0.196221

TABLE IX (Continued)

A=-0.5 EA=-0.624342 N=100

OBS	W	ESM1	VSM1	SM1L	ISPEC	SM1U	SL	SPE	SU
34	1.77933	6.97607	1.13674	4.84371	6.54986	9.10844	0.109788	0.152675	0.20645
35	1.83167	6.58632	0.99359	4.59274	6.23025	8.57990	0.116551	0.160507	0.21774
36	1.88400	6.20198	0.86314	4.34387	5.91508	8.06008	0.124068	0.169059	0.23021
37	1.93633	5.82409	0.74525	4.09754	5.60521	7.55065	0.132439	0.178405	0.24405
38	1.98867	5.45371	0.63967	3.85412	5.30148	7.05330	0.141778	0.188626	0.25946
39	2.04100	5.09183	0.54607	3.61391	5.00473	6.56976	0.152213	0.199811	0.27671
40	2.09333	4.73946	0.46397	3.37714	4.71578	6.10177	0.163887	0.212054	0.29611
41	2.14567	4.39755	0.39287	3.14396	4.43540	5.65113	0.176956	0.225459	0.31807
42	2.19800	4.06704	0.33213	2.91442	4.16438	5.21966	0.191583	0.240132	0.34312
43	2.25033	3.74883	0.28109	2.68847	3.90344	4.80920	0.207935	0.256184	0.37196
44	2.30267	3.44381	0.23901	2.46605	3.65331	4.42158	0.226164	0.273724	0.40551
45	2.35500	3.15280	0.20510	2.24704	3.41468	4.05856	0.246393	0.292853	0.44503
46	2.40733	2.87660	0.17856	2.03147	3.18818	3.72172	0.268693	0.313658	0.49225
47	2.45967	2.61596	0.15855	1.81960	2.97446	3.41233	0.293055	0.336196	0.54957
48	2.51200	2.37161	0.14423	1.61204	2.77408	3.13117	0.319369	0.360480	0.62033
49	2.56433	2.14420	0.13477	1.40997	2.58760	2.87844	0.347411	0.386458	0.70923
50	2.61667	1.93437	0.12935	1.21507	2.41553	2.65367	0.376837	0.413987	0.82300
51	2.66900	1.74268	0.12716	1.02949	2.25834	2.45587	0.407187	0.442802	0.97135
52	2.72133	1.56967	0.12746	0.85564	2.11647	2.28369	0.437887	0.472486	1.16872
53	2.77367	1.41579	0.12953	0.69598	1.99029	2.13561	0.468250	0.502440	1.43683
54	2.82600	1.28149	0.13274	0.55282	1.88015	2.01016	0.497473	0.531872	1.80891
55	2.87833	1.16712	0.13649	0.42822	1.78637	1.90602	0.524654	0.559796	2.33526
56	2.93067	1.07299	0.14029	0.32389	1.70918	1.82210	0.548819	0.585076	3.08747
57	2.98300	0.99937	0.14370	0.24121	1.64881	1.75753	0.568979	0.606498	4.14572
58	3.03533	0.94646	0.14640	0.18122	1.60542	1.71169	0.584216	0.622890	5.51800
59	3.08767	0.91440	0.14813	0.14465	1.57913	1.68414	0.593774	0.633261	6.91308
60	3.14000	0.90328	0.14874	0.13193	1.57001	1.67462	0.597149	0.636939	7.57994

TABLE X

POSTERIOR INFORMATION FOR $AR_1(1)$, A = -.5, N = 200, FREQUENCY DOMAIN

---------------------------------- A=-0.5 EA=-0.56217 N=200 ----------------------------------

OBS	W	ESM1	VSM1	SM1L	ISPEC	SM1U	SL	SPE	SU
1	0.05233	14.4757	3.28422	10.8512	14.1214	18.1002	0.055248	0.070814	0.092156
2	0.10467	14.4483	3.27007	10.8317	14.0956	18.0650	0.055356	0.070944	0.092322
3	0.15700	14.4028	3.24659	10.7992	14.0528	18.0065	0.055536	0.071160	0.092600
4	0.20933	14.3393	3.21396	10.7538	13.9929	17.9248	0.055789	0.071465	0.092990
5	0.26167	14.2580	3.17241	10.6957	13.9162	17.8202	0.056116	0.071859	0.093496
6	0.31400	14.1590	3.12224	10.6250	13.8229	17.6929	0.056520	0.072343	0.094118
7	0.36633	14.0426	3.06379	10.5419	13.7133	17.5434	0.057002	0.072922	0.094860
8	0.41867	13.9092	2.99748	10.4466	13.5876	17.3719	0.057564	0.073596	0.095725
9	0.47100	13.7592	2.92378	10.3394	13.4462	17.1790	0.058211	0.074370	0.096718
10	0.52333	13.5929	2.84320	10.2205	13.2895	16.9652	0.058944	0.075248	0.097842
11	0.57567	13.4107	2.75629	10.0903	13.1178	16.7312	0.059769	0.076232	0.099105
12	0.62800	13.2133	2.66364	9.9492	12.9318	16.4775	0.060689	0.077329	0.100511
13	0.68033	13.0011	2.56590	9.7974	12.7318	16.2048	0.061710	0.078543	0.102067
14	0.73267	12.7748	2.46370	9.6355	12.5185	15.9140	0.062838	0.079882	0.103783
15	0.78500	12.5348	2.35773	9.4638	12.2924	15.6058	0.064079	0.081351	0.105665
16	0.83733	12.2820	2.24867	9.2829	12.0541	15.2811	0.065440	0.082959	0.107725
17	0.88967	12.0169	2.13722	9.0931	11.8043	14.9408	0.066931	0.084715	0.109974
18	0.94200	11.7403	2.02409	8.8949	11.5437	14.5858	0.068560	0.086627	0.112423
19	0.99433	11.4531	1.90997	8.6890	11.2730	14.2171	0.070338	0.088708	0.115088
20	1.04667	11.1558	1.79554	8.4759	10.9929	13.8358	0.072276	0.090968	0.117982
21	1.09900	10.8495	1.68148	8.2560	10.7042	13.4429	0.074389	0.093421	0.121124
22	1.15133	10.5348	1.56843	8.0301	10.4077	13.0395	0.076690	0.096083	0.124532
23	1.20367	10.2127	1.45701	7.7986	10.1041	12.6269	0.079196	0.098969	0.128228
24	1.25600	9.8841	1.34782	7.5622	9.7944	12.2060	0.081927	0.102099	0.132237
25	1.30833	9.5498	1.24140	7.3214	9.4794	11.7782	0.084903	0.105492	0.136585
26	1.36067	9.2108	1.13827	7.0770	9.1599	11.3446	0.088148	0.109171	0.141303
27	1.41300	8.8679	1.03888	6.8294	8.8369	10.9065	0.091689	0.113162	0.146425
28	1.46533	8.5222	0.94366	6.5794	8.5111	10.4651	0.095556	0.117494	0.151990
29	1.51767	8.1746	0.85297	6.3275	8.1835	10.0218	0.099783	0.122197	0.158040
30	1.57000	7.8260	0.76712	6.0743	7.8550	9.5778	0.104409	0.127307	0.164627
31	1.62233	7.4774	0.68637	5.8205	7.5265	9.1344	0.109476	0.132864	0.171807
32	1.67467	7.1298	0.61091	5.5666	7.1989	8.6930	0.115035	0.138911	0.179644
33	1.72700	6.7840	0.54090	5.3131	6.8730	8.2549	0.121140	0.145496	0.188214

TABLE X (Continued)

A=-0.5	EA=-0.56217	N=200							
OBS	W	ESM1	VSM1	SM1L	ISPEC	SM1U	SL	SPE	SU
34	1.77933	6.44109	0.476418	5.06062	6.54986	7.82155	0.127852	0.152675	0.19760
35	1.83167	6.10193	0.417503	4.80964	6.23025	7.39422	0.135241	0.160507	0.20792
36	1.88400	5.76749	0.364134	4.56062	5.91508	6.97436	0.143382	0.169059	0.21927
37	1.93633	5.43866	0.316239	4.31396	5.60521	6.56337	0.152361	0.178405	0.23181
38	1.98867	5.11636	0.273699	4.07004	5.30148	6.16269	0.162267	0.188626	0.24570
39	2.04100	4.80147	0.236348	3.82915	5.00473	5.77378	0.173197	0.199811	0.26115
40	2.09333	4.49484	0.203977	3.59156	4.71578	5.39811	0.185250	0.212054	0.27843
41	2.14567	4.19732	0.176340	3.35746	4.43540	5.03717	0.198524	0.225459	0.29784
42	2.19800	3.90972	0.153157	3.12701	4.16438	4.69242	0.213110	0.240132	0.31979
43	2.25033	3.63282	0.134118	2.90038	3.90344	4.36527	0.229081	0.256184	0.34478
44	2.30267	3.36740	0.118888	2.67780	3.65331	4.05700	0.246487	0.273724	0.37344
45	2.35500	3.11417	0.107115	2.45960	3.41468	3.76874	0.265341	0.292853	0.40657
46	2.40733	2.87383	0.098430	2.24635	3.18818	3.50130	0.285608	0.313658	0.44517
47	2.45967	2.64703	0.092459	2.03889	2.97446	3.25517	0.307204	0.336196	0.49046
48	2.51200	2.43440	0.088821	1.83834	2.77408	3.03045	0.329984	0.360480	0.54397
49	2.56433	2.23651	0.087141	1.64612	2.58760	2.82691	0.353743	0.386458	0.60749
50	2.61667	2.05392	0.087049	1.46384	2.41553	2.64400	0.378214	0.413987	0.68313
51	2.66900	1.88712	0.088188	1.29319	2.25834	2.48105	0.403055	0.442802	0.77328
52	2.72133	1.73656	0.090219	1.13584	2.11647	2.33729	0.427845	0.472486	0.88041
53	2.77367	1.60267	0.092824	0.99333	1.99029	2.21201	0.452077	0.502440	1.00672
54	2.82600	1.48580	0.095714	0.86705	1.88015	2.10455	0.475160	0.531872	1.15334
55	2.87833	1.38628	0.098626	0.75818	1.78637	2.01437	0.496433	0.559796	1.31895
56	2.93067	1.30437	0.101335	0.66771	1.70918	1.94103	0.515189	0.585076	1.49766
57	2.98300	1.24031	0.103650	0.59642	1.64881	1.88420	0.530728	0.606498	1.67668
58	3.03533	1.19427	0.105420	0.54490	1.60542	1.84363	0.542407	0.622890	1.83521
59	3.08767	1.16637	0.106536	0.51357	1.57913	1.81916	0.549703	0.633261	1.94715
60	3.14000	1.15669	0.106931	0.50268	1.57001	1.81069	0.552275	0.636939	1.98933

TABLE XI

POSTERIOR INFORMATION FOR $AR_1(1)$, A = 1.02, N = 100, FREQUENCY DOMAIN

A=1.02 EA=1.004444 N=100

OBS	W	ESM1	VSM1	SM1L	ISPEC	SM1U	SL	SPE	SU
1	0.05233	0.0214	0.00006	0.0066	0.0201	0.0362	27.5955	49.8716	152.587
2	0.10467	0.0709	0.00015	0.0468	0.0726	0.0949	10.5323	13.7699	21.384
3	0.15700	0.1531	0.00052	0.1074	0.1601	0.1988	5.0290	6.2469	9.309
4	0.20933	0.2680	0.00152	0.1899	0.2822	0.3461	2.8893	3.5438	5.265
5	0.26167	0.4152	0.00362	0.2949	0.4386	0.5355	1.8675	2.2800	3.391
6	0.31400	0.5942	0.00739	0.4223	0.6289	0.7661	1.3052	1.5901	2.368
7	0.36633	0.8046	0.01354	0.5719	0.8526	1.0374	0.9639	1.1729	1.749
8	0.41867	1.0459	0.02289	0.7433	1.1090	1.3485	0.7416	0.9017	1.345
9	0.47100	1.3173	0.03632	0.9361	1.3975	1.6984	0.5888	0.7156	1.068
10	0.52333	1.6181	0.05482	1.1498	1.7172	2.0863	0.4793	0.5823	0.870
11	0.57567	1.9475	0.07943	1.3838	2.0673	2.5111	0.3982	0.4837	0.723
12	0.62800	2.3045	0.11126	1.6374	2.4468	2.9716	0.3365	0.4087	0.611
13	0.68033	2.6883	0.15144	1.9100	2.8548	3.4666	0.2885	0.3503	0.524
14	0.73267	3.0977	0.20112	2.2008	3.2900	3.9947	0.2503	0.3040	0.454
15	0.78500	3.5317	0.26146	2.5091	3.7512	4.5544	0.2196	0.2666	0.399
16	0.83733	3.9890	0.33360	2.8339	4.2373	5.1442	0.1944	0.2360	0.353
17	0.88967	4.4684	0.41865	3.1744	4.7469	5.7625	0.1735	0.2107	0.315
18	0.94200	4.9686	0.51768	3.5296	5.2785	6.4076	0.1561	0.1894	0.283
19	0.99433	5.4882	0.63167	3.8987	5.8308	7.0778	0.1413	0.1715	0.256
20	1.04667	6.0258	0.76154	4.2805	6.4022	7.7711	0.1287	0.1562	0.234
21	1.09900	6.5799	0.90809	4.6740	6.9912	8.4858	0.1178	0.1430	0.214
22	1.15133	7.1490	1.07203	5.0782	7.5961	9.2198	0.1085	0.1316	0.197
23	1.20367	7.7315	1.25392	5.4920	8.2153	9.9711	0.1003	0.1217	0.182
24	1.25600	8.3259	1.45420	5.9141	8.8471	10.7377	0.0931	0.1130	0.169
25	1.30833	8.9305	1.67314	6.3435	9.4897	11.5175	0.0868	0.1054	0.158
26	1.36067	9.5437	1.91085	6.7790	10.1415	12.3084	0.0812	0.0986	0.148
27	1.41300	10.1638	2.16728	7.2194	10.8005	13.1081	0.0763	0.0926	0.139
28	1.46533	10.7890	2.44220	7.6635	11.4651	13.9145	0.0719	0.0872	0.130
29	1.51767	11.4177	2.73520	8.1100	12.1334	14.7254	0.0679	0.0824	0.123
30	1.57000	12.0482	3.04567	8.5578	12.8035	15.5385	0.0644	0.0781	0.117
31	1.62233	12.6787	3.37285	9.0056	13.4737	16.3517	0.0612	0.0742	0.111
32	1.67467	13.3075	3.71577	9.4522	14.1420	17.1627	0.0583	0.0707	0.106
33	1.72700	13.9328	4.07328	9.8964	14.8067	17.9693	0.0557	0.0675	0.101

TABLE XI (Continued)

A=1.02 EA=1.004444 N=100

OBS	W	ESM1	VSM1	SM1L	ISPEC	SM1U	SL	SPE	SU
34	1.77933	14.5531	4.4441	10.3369	15.4660	18.7693	0.0532786	0.0646580	0.0967411
35	1.83167	15.1665	4.8267	10.7725	16.1180	19.5604	0.0511237	0.0620425	0.0928287
36	1.88400	15.7714	5.2194	11.2022	16.7609	20.3406	0.0491628	0.0596625	0.0892686
37	1.93633	16.3661	5.6206	11.6245	17.3931	21.1076	0.0473762	0.0574941	0.0860249
38	1.98867	16.9490	6.0281	12.0386	18.0127	21.8595	0.0457468	0.0555164	0.0830664
39	2.04100	17.5185	6.4401	12.4431	18.6181	22.5940	0.0442595	0.0537113	0.0803661
40	2.09333	18.0731	6.8544	12.8369	19.2075	23.3093	0.0429013	0.0520629	0.0779002
41	2.14567	18.6112	7.2687	13.2191	19.7795	24.0034	0.0416609	0.0505574	0.0756479
42	2.19800	19.1314	7.6808	13.5886	20.3324	24.6742	0.0405281	0.0491826	0.0735913
43	2.25033	19.6322	8.0882	13.9443	20.8647	25.3202	0.0394942	0.0479279	0.0717141
44	2.30267	20.1123	8.4886	14.2852	21.3750	25.9393	0.0385515	0.0467837	0.0700025
45	2.35500	20.5703	8.8797	14.6105	21.8618	26.5300	0.0376931	0.0457419	0.0684439
46	2.40733	21.0050	9.2590	14.9192	22.3238	27.0907	0.0369131	0.0447952	0.0670276
47	2.45967	21.4152	9.6242	15.2106	22.7598	27.6197	0.0362060	0.0439371	0.0657438
48	2.51200	21.7997	9.9730	15.4837	23.1686	28.1157	0.0355673	0.0431619	0.0645840
49	2.56433	22.1576	10.3032	15.7379	23.5490	28.5774	0.0349927	0.0424646	0.0635409
50	2.61667	22.4879	10.6127	15.9725	23.9000	29.0033	0.0344788	0.0418410	0.0626078
51	2.66900	22.7896	10.8994	16.1867	24.2207	29.3924	0.0340224	0.0412870	0.0617790
52	2.72133	23.0619	11.1614	16.3801	24.5101	29.7436	0.0336207	0.0407995	0.0610496
53	2.77367	23.3040	11.3971	16.5521	24.7675	30.0559	0.0332713	0.0403754	0.0604153
54	2.82600	23.5154	11.6048	16.7022	24.9922	30.3286	0.0329722	0.0400125	0.0598722
55	2.87833	23.6954	11.7832	16.8301	25.1835	30.5607	0.0327217	0.0397085	0.0594174
56	2.93067	23.8435	11.9310	16.9353	25.3410	30.7518	0.0325184	0.0394618	0.0590483
57	2.98300	23.9594	12.0472	17.0176	25.4641	30.9012	0.0323612	0.0392709	0.0587628
58	3.03533	24.0427	12.1311	17.0767	25.5527	31.0086	0.0322491	0.0391349	0.0585592
59	3.08767	24.0931	12.1821	17.1126	25.6063	31.0737	0.0321815	0.0390529	0.0584366
60	3.14000	24.1106	12.1998	17.1250	25.6249	31.0963	0.0321582	0.0390245	0.0583942

TABLE XII

WOLFER SUNSPOT DATA, TAKEN FROM
SAS/ETS MANUAL P. 7.9

OBS	YEAR	ACTIVITY	ADJ
1	1749	809	362.98
2	1750	834	387.98
3	1751	477	30.98
4	1752	478	31.98
5	1753	307	-139.02
6	1754	122	-324.02
7	1755	96	-350.02
8	1756	102	-344.02
9	1757	324	-122.02
10	1758	476	29.98
11	1759	540	93.98
12	1760	629	182.98
13	1761	859	412.98
14	1762	612	165.98
15	1763	451	4.98
16	1764	364	-82.02
17	1765	209	-237.02
18	1766	114	-332.02
19	1767	378	-68.02
20	1768	698	251.98
21	1769	1061	614.98
22	1770	1008	561.98
23	1771	816	369.98
24	1772	665	218.98
25	1773	348	-98.02
26	1774	306	-140.02
27	1775	70	-376.02
28	1776	198	-248.02
29	1777	925	478.98
30	1778	1544	1097.98
31	1779	1259	812.98
32	1780	848	401.98
33	1781	681	234.98
34	1782	385	-61.02
35	1783	228	-218.02

TABLE XII (Continued)

OBS	YEAR	ACTIVITY	ADJ
36	1784	102	-344.02
37	1785	241	-205.02
38	1786	829	382.98
39	1787	1320	873.98
40	1788	1309	862.98
41	1789	1181	734.98
42	1790	899	452.98
43	1791	666	219.98
44	1792	600	153.98
45	1793	469	22.98
46	1794	410	-36.02
47	1795	213	-233.02
48	1796	160	-286.02
49	1797	64	-382.02
50	1798	41	-405.02
51	1799	68	-378.02
52	1800	145	-301.02
53	1801	340	-106.02
54	1802	450	3.98
55	1803	431	-15.02
56	1804	475	28.98
57	1805	422	-24.02
58	1806	281	-165.02
59	1807	101	-345.02
60	1808	81	-365.02
61	1809	25	-421.02
62	1810	0	-446.02
63	1811	14	-432.02
64	1812	50	-396.02
65	1813	122	-324.02
66	1814	139	-307.02
67	1815	354	-92.02
68	1816	458	11.98
69	1817	411	-35.02
70	1818	304	-142.02

TABLE XII (Continued)

OBS	YEAR	ACTIVITY	ADJ
71	1819	239	-207.02
72	1820	157	-289.02
73	1821	66	-380.02
74	1822	40	-406.02
75	1823	18	-428.02
76	1824	85	-361.02
77	1825	166	-280.02
78	1826	363	-83.02
79	1827	497	50.98
80	1828	625	178.98
81	1829	670	223.98
82	1830	710	263.98
83	1831	478	31.98
84	1832	275	-171.02
85	1833	85	-361.02
86	1834	132	-314.02
87	1835	569	122.98
88	1836	1215	768.98
89	1837	1383	936.98
90	1838	1032	585.98
91	1839	858	411.98
92	1840	632	185.98
93	1841	386	-60.02
94	1842	242	-204.02
95	1843	107	-339.02
96	1844	150	-296.02
97	1845	401	-45.02
98	1846	615	168.98
99	1847	985	538.98
100	1848	1243	796.98
101	1849	959	512.98
102	1850	665	218.98
103	1851	645	198.98
104	1852	542	95.98
105	1853	390	-56.02

TABLE XII (Continued)

OBS	YEAR	ACTIVITY	ADJ
106	1854	206	-240.02
107	1855	67	-379.02
108	1856	43	-403.02
109	1857	228	-218.02
110	1858	548	101.98
111	1859	938	491.98
112	1860	957	510.98
113	1861	772	325.98
114	1862	591	144.98
115	1863	440	-6.02
116	1864	470	23.98
117	1865	305	-141.02
118	1866	163	-283.02
119	1867	73	-373.02
120	1868	373	-73.02
121	1869	739	292.98
122	1870	1391	944.98
123	1871	1112	665.98
124	1872	1017	570.98
125	1873	663	216.98
126	1874	447	0.98
127	1875	171	-275.02
128	1876	113	-333.02
129	1877	123	-323.02
130	1878	34	-412.02
131	1879	60	-386.02
132	1880	323	-123.02
133	1881	543	96.98
134	1882	597	150.98
135	1883	637	190.98
136	1884	635	188.98
137	1885	522	75.98
138	1886	254	-192.02
139	1887	131	-315.02
140	1888	68	-378.02

TABLE XII (Continued)

OBS	YEAR	ACTIVITY	ADJ
141	1889	63	-383.02
142	1890	71	-375.02
143	1891	356	-90.02
144	1892	730	283.98
145	1893	849	402.98
146	1894	780	333.98
147	1895	640	193.98
148	1896	418	-28.02
149	1897	262	-184.02
150	1898	267	-179.02
151	1899	121	-325.02
152	1900	95	-351.02
153	1901	27	-419.02
154	1902	50	-396.02
155	1903	244	-202.02
156	1904	420	-26.02
157	1905	635	188.98
158	1906	538	91.98
159	1907	320	-126.02
160	1908	485	38.98
161	1909	439	-7.02
162	1910	186	-260.02
163	1911	57	-389.02
164	1912	36	-410.02
165	1913	14	-432.02
166	1914	96	-350.02
167	1915	474	27.98
168	1916	571	124.98
169	1917	1034	587.98
170	1918	806	359.98
171	1919	636	189.98
172	1920	376	-70.02
173	1921	261	-185.02
174	1922	142	-304.02
175	1923	58	-388.02

TABLE XII (Continued)

OBS	YEAR	ACTIVITY	ADJ
176	1924	167	-279.02

TABLE XIII

POSTERIOR INFORMATION FOR WOLFER SUNSPOTS, $AR_1(2)$ = -TIME DOMAIN

AH=MARGINAL POSTERIOR MEAN FOR AR COEFFICIENTS
VA=MARGINAL POSTERIOR VARIANCE FOR AR COEFFICIENTS

AH	COL1
ROW1	1.33244
ROW2	-0.647869

VA	COL1	COL2
ROW1	0.00340928	-0.0027624
ROW2	-0.0027624	0.00341235

TABLE XIV

POSTERIOR INFORMATION FOR WOLFER SUNSPOTS
$AR_1(2)$--FREQUENCY DOMAIN, $w \in [0,3.14]$

N=174

OBS	ROW	W	ESM1	VSM1	SM1L	SM1U	SL	SU
1	ROW1	0.05233	0.000025810	4.13127E-11	0.000012955	0.000038665	25863.5	77193
2	ROW2	0.10467	0.000024993	3.89869E-11	0.000012505	0.000037481	26680.0	79965
3	ROW3	0.15700	0.000023686	3.53722E-11	0.000011791	0.000035580	28105.3	84813
4	ROW4	0.20933	0.000021964	3.08333E-11	0.000010859	0.000033070	30239.1	92092
5	ROW5	0.26167	0.000019937	2.58272E-11	0.000009773	0.000030101	33221.1	102321
6	ROW6	0.31400	0.000017741	2.08584E-11	0.000008607	0.000026875	37209.1	116188
7	ROW7	0.36633	0.000015538	1.64291E-11	0.000007431	0.000023645	42293.1	134564
8	ROW8	0.41867	0.000013516	1.29938E-11	0.000006306	0.000020725	48251.0	158574
9	ROW9	0.47100	0.000011883	1.09236E-11	0.000005273	0.000018493	54074.4	189656
10	ROW10	0.52333	0.000010868	1.04877E-11	0.000004391	0.000017345	57653.7	227735
11	ROW11	0.57567	0.000010715	1.18585E-11	0.000003828	0.000017602	56810.1	261233
12	ROW12	0.62800	0.000011682	1.51460E-11	0.000003898	0.000019465	51374.0	256545
13	ROW13	0.68033	0.000014033	2.04648E-11	0.000004985	0.000023081	43326.4	200586
14	ROW14	0.73267	0.000018041	2.80369E-11	0.000007451	0.000028631	34926.8	134203
15	ROW15	0.78500	0.000023980	3.83296E-11	0.000011598	0.000036362	27501.1	86223
16	ROW16	0.83733	0.000032120	5.22261E-11	0.000017667	0.000046574	21471.3	56603
17	ROW17	0.88967	0.000042728	7.12242E-11	0.000025849	0.000059606	16776.7	38687
18	ROW18	0.94200	0.000056057	9.76555E-11	0.000036293	0.000075821	13188.9	27553
19	ROW19	0.99433	0.000072351	1.34916E-10	0.000049120	0.000095582	10462.3	20358
20	ROW20	1.04667	0.000091833	1.87695E-10	0.000064433	0.000119234	8386.9	15520
21	ROW21	1.09900	0.000114707	2.62194E-10	0.000082323	0.000147092	6798.5	12147
22	ROW22	1.15133	0.000141152	3.66313E-10	0.000102874	0.000179431	5573.2	9721
23	ROW23	1.20367	0.000171320	5.09801E-10	0.000126162	0.000216478	4619.4	7926
24	ROW24	1.25600	0.000205331	7.04344E-10	0.000152252	0.000258410	3869.8	6568
25	ROW25	1.30833	0.000243275	9.63591E-10	0.000181192	0.000305359	3274.8	5519
26	ROW26	1.36067	0.000285204	1.30310E-09	0.000213007	0.000357401	2798.0	4695
27	ROW27	1.41300	0.000331135	1.74018E-09	0.000247704	0.000414566	2412.2	4037
28	ROW28	1.46533	0.000381045	2.29367E-09	0.000285261	0.000476830	2097.2	3506
29	ROW29	1.51767	0.000434874	2.98360E-09	0.000325629	0.000544118	1837.8	3071
30	ROW30	1.57000	0.000492518	3.83073E-09	0.000368732	0.000616304	1622.6	2712
31	ROW31	1.62233	0.000553836	4.85608E-09	0.000414465	0.000693207	1442.6	2413
32	ROW32	1.67467	0.000618645	6.08029E-09	0.000462692	0.000774597	1291.0	2161
33	ROW33	1.72700	0.000686723	7.52297E-09	0.000513253	0.000860193	1162.5	1948

TABLE XIV (Continued)

N=174

OBS	ROW	W	ESM1	VSM1	SM1L	SM1U	SL	SU
34	ROW34	1.77933	0.00075781	9.20202E-09	0.00056596	0.00094966	1053.00	1766.92
35	ROW35	1.83167	0.00083161	1.11328E-08	0.00062058	0.00104263	959.11	1611.39
36	ROW36	1.88400	0.00090778	1.33275E-08	0.00067689	0.00113867	878.21	1477.33
37	ROW37	1.93633	0.00098598	1.57944E-08	0.00073462	0.00123733	808.19	1361.24
38	ROW38	1.98867	0.00106579	1.85372E-08	0.00079348	0.00133809	747.34	1260.27
39	ROW39	2.04100	0.00114679	2.15541E-08	0.00085317	0.00144042	694.24	1172.10
40	ROW40	2.09333	0.00122855	2.48381E-08	0.00091335	0.00154375	647.77	1094.87
41	ROW41	2.14567	0.00131060	2.83757E-08	0.00097370	0.00164750	606.98	1027.01
42	ROW42	2.19800	0.00139245	3.21473E-08	0.00103386	0.00175104	571.09	967.25
43	ROW43	2.25033	0.00147361	3.61271E-08	0.00109347	0.00185376	539.44	914.52
44	ROW44	2.30267	0.00155359	4.02827E-08	0.00115218	0.00195500	511.51	867.92
45	ROW45	2.35500	0.00163188	4.45760E-08	0.00120962	0.00205414	486.82	826.71
46	ROW46	2.40733	0.00170798	4.89633E-08	0.00126542	0.00215053	465.00	790.25
47	ROW47	2.45967	0.00178138	5.33962E-08	0.00131923	0.00224354	445.72	758.02
48	ROW48	2.51200	0.00185162	5.78220E-08	0.00137069	0.00233254	428.72	729.56
49	ROW49	2.56433	0.00191821	6.21851E-08	0.00141947	0.00241695	413.75	704.49
50	ROW50	2.61667	0.00198070	6.64278E-08	0.00146523	0.00249617	400.61	682.49
51	ROW51	2.66900	0.00203867	7.04917E-08	0.00150766	0.00256967	389.15	663.28
52	ROW52	2.72133	0.00209171	7.43187E-08	0.00154648	0.00263694	379.23	646.63
53	ROW53	2.77367	0.00213946	7.78522E-08	0.00158142	0.00269750	370.71	632.34
54	ROW54	2.82600	0.00218158	8.10389E-08	0.00161224	0.00275093	363.51	620.26
55	ROW55	2.87833	0.00221778	8.38295E-08	0.00163871	0.00279685	357.55	610.24
56	ROW56	2.93067	0.00224779	8.61801E-08	0.00166066	0.00283492	352.74	602.17
57	ROW57	2.98300	0.00227141	8.80530E-08	0.00167793	0.00286488	349.05	595.97
58	ROW58	3.03533	0.00228846	8.94180E-08	0.00169040	0.00288651	346.44	591.58
59	ROW59	3.08767	0.00229882	9.02528E-08	0.00169798	0.00289966	344.87	588.94
60	ROW60	3.14000	0.00230242	9.05438E-08	0.00170061	0.00290423	344.33	588.03

TABLE XV

POSTERIOR INFORMATION FOR WOLFER SUNSPOTS,
$AR_1(2)$--FREQUENCY DOMAIN, $w \in [0,1.57]$

N=174

OBS	ROW	W	ESM1	VSM1	SM1L	SM1U	SL	SU
1	ROW1	0.026167	0.0000260173	4.19131E-11	0.0000130692	0.0000389653	25663.8	76516
2	ROW2	0.052333	0.0000258095	4.13127E-11	0.0000129545	0.0000386645	25863.5	77193
3	ROW3	0.078500	0.0000254666	4.03289E-11	0.0000127655	0.0000381676	26200.3	78336
4	ROW4	0.104667	0.0000249933	3.89869E-11	0.0000125054	0.0000374812	26680.0	79965
5	ROW5	0.130833	0.0000243967	3.73205E-11	0.0000121786	0.0000366148	27311.4	82111
6	ROW6	0.157000	0.0000236855	3.53722E-11	0.0000117906	0.0000355804	28105.3	84813
7	ROW7	0.183167	0.0000228705	3.31914E-11	0.0000113481	0.0000343929	29075.8	88120
8	ROW8	0.209333	0.0000219642	3.08333E-11	0.0000108587	0.0000330698	30239.1	92092
9	ROW9	0.235500	0.0000209811	2.83576E-11	0.0000103307	0.0000316315	31614.1	96799
10	ROW10	0.261667	0.0000199373	2.58272E-11	0.0000097732	0.0000301014	33221.1	102321
11	ROW11	0.287833	0.0000188507	2.33061E-11	0.0000091954	0.0000285060	35080.3	108750
12	ROW12	0.314000	0.0000177409	2.08584E-11	0.0000086067	0.0000268751	37209.1	116188
13	ROW13	0.340167	0.0000166291	1.85463E-11	0.0000080160	0.0000252422	39616.2	124750
14	ROW14	0.366333	0.0000155380	1.64291E-11	0.0000074314	0.0000236445	42293.1	134564
15	ROW15	0.392500	0.0000144916	1.45617E-11	0.0000068597	0.0000221236	45200.6	145780
16	ROW16	0.418667	0.0000135156	1.29938E-11	0.0000063062	0.0000207250	48251.0	158574
17	ROW17	0.444833	0.0000126367	1.17688E-11	0.0000057755	0.0000194978	51287.9	173144
18	ROW18	0.471000	0.0000118829	1.09236E-11	0.0000052727	0.0000184930	54074.4	189656
19	ROW19	0.497167	0.0000112833	1.04886E-11	0.0000048061	0.0000177605	56304.8	208070
20	ROW20	0.523333	0.0000108680	1.04877E-11	0.0000043911	0.0000173450	57653.7	227735
21	ROW21	0.549500	0.0000106681	1.09396E-11	0.0000040531	0.0000172831	57860.1	246727
22	ROW22	0.575667	0.0000107152	1.18585E-11	0.0000038280	0.0000176025	56810.1	261233
23	ROW23	0.601833	0.0000110420	1.32567E-11	0.0000037601	0.0000183240	54573.3	265950
24	ROW24	0.628000	0.0000116815	1.51460E-11	0.0000038980	0.0000194651	51374.0	256545
25	ROW25	0.654167	0.0000126672	1.75416E-11	0.0000042907	0.0000210438	47520.0	233063
26	ROW26	0.680333	0.0000140330	2.04648E-11	0.0000049854	0.0000230806	43326.4	200586
27	ROW27	0.706500	0.0000158130	2.39475E-11	0.0000060257	0.0000256002	39062.2	165955
28	ROW28	0.732667	0.0000180414	2.80369E-11	0.0000074514	0.0000286313	34926.8	134203
29	ROW29	0.758833	0.0000207523	3.27999E-11	0.0000092981	0.0000322066	31049.5	107549
30	ROW30	0.785000	0.0000239800	3.83296E-11	0.0000115978	0.0000363622	27501.1	86223
31	ROW31	0.811167	0.0000277582	4.47507E-11	0.0000143790	0.0000411374	24308.8	69546
32	ROW32	0.837333	0.0000321203	5.22261E-11	0.0000176668	0.0000465738	21471.3	56603
33	ROW33	0.863500	0.0000370993	6.09640E-11	0.0000214834	0.0000527152	18969.9	46547

TABLE XV (Continued)

N=174

OBS	ROW	W	ESM1	VSM1	SM1L	SM1U	SL	SU
34	ROW34	0.88967	0.000042728	7.12242E-11	0.000025849	0.000059606	16776.7	38686.7
35	ROW35	0.91583	0.000049037	8.33261E-11	0.000030780	0.000067293	14860.3	32488.6
36	ROW36	0.94200	0.000056057	9.76555E-11	0.000036293	0.000075821	13188.9	27553.5
37	ROW37	0.96817	0.000063819	1.14672E-10	0.000042402	0.000085236	11732.1	23583.7
38	ROW38	0.99433	0.000072351	1.34916E-10	0.000049120	0.000095582	10462.3	20358.2
39	ROW39	1.02050	0.000081680	1.59016E-10	0.000056460	0.000106901	9354.5	17711.6
40	ROW40	1.04667	0.000091833	1.87695E-10	0.000064433	0.000119234	8386.9	15520.0
41	ROW41	1.07283	0.000102835	2.21777E-10	0.000073050	0.000132619	7540.4	13689.2
42	ROW42	1.09900	0.000114707	2.62194E-10	0.000082323	0.000147092	6798.5	12147.3
43	ROW43	1.12517	0.000127473	3.09986E-10	0.000092261	0.000162686	6146.8	10838.9
44	ROW44	1.15133	0.000141152	3.66313E-10	0.000102874	0.000179431	5573.2	9720.6
45	ROW45	1.17750	0.000155763	4.32451E-10	0.000114172	0.000197354	5067.0	8758.7
46	ROW46	1.20367	0.000171320	5.09801E-10	0.000126162	0.000216478	4619.4	7926.3
47	ROW47	1.22983	0.000187839	5.99883E-10	0.000138854	0.000236824	4222.5	7201.8
48	ROW48	1.25600	0.000205331	7.04344E-10	0.000152252	0.000258410	3869.8	6568.0
49	ROW49	1.28217	0.000223808	8.24951E-10	0.000166364	0.000281251	3555.5	6010.9
50	ROW50	1.30833	0.000243275	9.63591E-10	0.000181192	0.000305359	3274.8	5519.0
51	ROW51	1.33450	0.000263740	1.12227E-09	0.000196739	0.000330740	3023.5	5082.9
52	ROW52	1.36067	0.000285204	1.30310E-09	0.000213007	0.000357401	2798.0	4694.7
53	ROW53	1.38683	0.000307670	1.50830E-09	0.000229996	0.000385343	2595.1	4347.9
54	ROW54	1.41300	0.000331135	1.74018E-09	0.000247704	0.000414566	2412.2	4037.1
55	ROW55	1.43917	0.000355596	2.00115E-09	0.000266127	0.000445064	2246.9	3757.6
56	ROW56	1.46533	0.000381045	2.29367E-09	0.000285261	0.000476830	2097.2	3505.6
57	ROW57	1.49150	0.000407475	2.62030E-09	0.000305098	0.000509853	1961.3	3277.6
58	ROW58	1.51767	0.000434874	2.98360E-09	0.000325629	0.000544118	1837.8	3071.0
59	ROW59	1.54383	0.000463227	3.38620E-09	0.000346845	0.000579609	1725.3	2883.1
60	ROW60	1.57000	0.000492518	3.83073E-09	0.000368732	0.000616304	1622.6	2712.0

TABLE XVI

POSTERIOR INFORMATION FOR WOLFER SUNSPOTS, $AR_1(3)$--TIME DOMAIN

AH=MARGINAL POSTERIOR MEAN FOR AR COEFFICIENTS
VA=MARGINAL POSTERIOR VARIANCE FOR AR COEFFICIENTS

AH	COL1
ROW1	1.28498
ROW2	-0.541705
ROW3	-0.0783066

VA	COL1	COL2	COL3
ROW1	0.00582783	-0.00776999	0.00377601
ROW2	-0.00776999	0.0137452	-0.00777074
ROW3	0.00377601	-0.00777074	0.00582597

TABLE XVII

POSTERIOR INFORMATION FOR WOLFER SUNSPOTS,
$AR_1(3)$--FREQUENCY DOMAIN

N=173

OBS	ROW	W	ESM1	VSM1	SM1L	SM1U	SL	SU
1	ROW1	0.05233	0.000029449	6.54442E-11	0.000013269	0.00004563	21916.4	75364
2	ROW2	0.10467	0.000028338	6.01912E-11	0.000012822	0.00004385	22802.5	77993
3	ROW3	0.15700	0.000026562	5.22954E-11	0.000012099	0.00004103	24375.2	82651
4	ROW4	0.20933	0.000024231	4.28795E-11	0.000011134	0.00003733	26790.1	89813
5	ROW5	0.26167	0.000021495	3.32144E-11	0.000009969	0.00003302	30283.4	100315
6	ROW6	0.31400	0.000018544	2.44901E-11	0.000008647	0.00002844	35159.6	115651
7	ROW7	0.36633	0.000015602	1.76094E-11	0.000007209	0.00002399	41676.0	138712
8	ROW8	0.41867	0.000012921	1.30521E-11	0.000005696	0.00002015	49635.5	175568
9	ROW9	0.47100	0.000010781	1.08492E-11	0.000004193	0.00001737	57576.0	238488
10	ROW10	0.52333	0.000009477	1.06904E-11	0.000002938	0.00001602	62434.9	340340
11	ROW11	0.57567	0.000009323	1.21693E-11	0.000002346	0.00001630	61351.9	426331
12	ROW12	0.62800	0.000010634	1.51479E-11	0.000002850	0.00001842	54294.3	350876
13	ROW13	0.68033	0.000013731	2.01984E-11	0.000004743	0.00002272	44014.1	210838
14	ROW14	0.73267	0.000018929	2.90637E-11	0.000008147	0.00002971	33657.5	122749
15	ROW15	0.78500	0.000026529	4.50664E-11	0.000013103	0.00003996	25028.0	76321
16	ROW16	0.83733	0.000036817	7.33909E-11	0.000019683	0.00005395	18535.4	50804
17	ROW17	0.88967	0.000050056	1.21176E-10	0.000028040	0.00007207	13875.0	35663
18	ROW18	0.94200	0.000066480	1.97369E-10	0.000038382	0.00009458	10573.3	26054
19	ROW19	0.99433	0.000086291	3.12322E-10	0.000050946	0.00012164	8221.2	19629
20	ROW20	1.04667	0.000109655	4.77143E-10	0.000065967	0.00015334	6521.4	15159
21	ROW21	1.09900	0.000136695	7.02861E-10	0.000083672	0.00018972	5271.0	11951
22	ROW22	1.15133	0.000167497	9.99480E-10	0.000104267	0.00023073	4334.2	9591
23	ROW23	1.20367	0.000202097	1.37505E-09	0.000127933	0.00027626	3619.8	7817
24	ROW24	1.25600	0.000240488	1.83489E-09	0.000154817	0.00032616	3066.0	6459
25	ROW25	1.30833	0.000282620	2.38110E-09	0.000185027	0.00038021	2630.1	5405
26	ROW26	1.36067	0.000328393	3.01249E-09	0.000218621	0.00043817	2282.2	4574
27	ROW27	1.41300	0.000377668	3.72503E-09	0.000255602	0.00049973	2001.1	3912
28	ROW28	1.46533	0.000430261	4.51295E-09	0.000295904	0.00056462	1771.1	3379
29	ROW29	1.51767	0.000485951	5.37028E-09	0.000339387	0.00063252	1581.0	2946
30	ROW30	1.57000	0.000544481	6.29312E-09	0.000385822	0.00070314	1422.2	2592
31	ROW31	1.62233	0.000605561	7.28209E-09	0.000434890	0.00077623	1288.3	2299
32	ROW32	1.67467	0.000668873	8.34512E-09	0.000486170	0.00085158	1174.3	2057
33	ROW33	1.72700	0.000734078	9.50011E-09	0.000539141	0.00092901	1076.4	1855

TABLE XVII (Continued)

N=173

OBS	ROW	W	ESM1	VSM1	SM1L	SM1U	SL	SU
34	ROW34	1.77933	0.00080081	1.07772E-08	0.00059319	0.00100844	991.629	1685.81
35	ROW35	1.83167	0.00086871	1.22207E-08	0.00064762	0.00108980	917.596	1544.13
36	ROW36	1.88400	0.00093738	1.38893E-08	0.00070167	0.00117309	852.453	1425.16
37	ROW37	1.93633	0.00100644	1.58561E-08	0.00075460	0.00125828	794.736	1325.21
38	ROW38	1.98867	0.00107549	1.82068E-08	0.00080563	0.00134536	743.296	1241.27
39	ROW39	2.04100	0.00114417	2.10362E-08	0.00085409	0.00143424	697.232	1170.84
40	ROW40	2.09333	0.00121208	2.44435E-08	0.00089939	0.00152476	655.839	1111.87
41	ROW41	2.14567	0.00127886	2.85267E-08	0.00094106	0.00161666	618.560	1062.63
42	ROW42	2.19800	0.00134417	3.33758E-08	0.00097879	0.00170955	584.949	1021.67
43	ROW43	2.25033	0.00140767	3.90648E-08	0.00101237	0.00180297	554.642	987.78
44	ROW44	2.30267	0.00146905	4.56446E-08	0.00104176	0.00189634	527.332	959.92
45	ROW45	2.35500	0.00152801	5.31357E-08	0.00106699	0.00198904	502.756	937.22
46	ROW46	2.40733	0.00158429	6.15217E-08	0.00108822	0.00208036	480.685	918.93
47	ROW47	2.45967	0.00163764	7.07449E-08	0.00110568	0.00216960	460.915	904.42
48	ROW48	2.51200	0.00168783	8.07034E-08	0.00111967	0.00225600	443.263	893.12
49	ROW49	2.56433	0.00173467	9.12505E-08	0.00113051	0.00233882	427.566	884.55
50	ROW50	2.61667	0.00177797	1.02197E-07	0.00113860	0.00241733	413.679	878.27
51	ROW51	2.66900	0.00181758	1.13315E-07	0.00114433	0.00249083	401.473	873.87
52	ROW52	2.72133	0.00185336	1.24348E-07	0.00114810	0.00255862	390.835	871.00
53	ROW53	2.77367	0.00188520	1.35016E-07	0.00115031	0.00262009	381.666	869.33
54	ROW54	2.82600	0.00191299	1.45030E-07	0.00115133	0.00267465	373.881	868.56
55	ROW55	2.87833	0.00193666	1.54104E-07	0.00115154	0.00272178	367.407	868.40
56	ROW56	2.93067	0.00195613	1.61966E-07	0.00115123	0.00276103	362.183	868.63
57	ROW57	2.98300	0.00197136	1.68373E-07	0.00115070	0.00279203	358.163	869.04
58	ROW58	3.03533	0.00198231	1.73122E-07	0.00115015	0.00281446	355.307	869.45
59	ROW59	3.08767	0.00198894	1.76059E-07	0.00114975	0.00282813	353.591	869.76
60	ROW60	3.14000	0.00199124	1.77089E-07	0.00114960	0.00283288	352.998	869.87

APPENDIX B

FIGURES

FIGURE 1 AR1==SM1 3

A=0.5 EA=0.2972529 N=100

PLOT OF SM1L*W SYMBOL USED IS L
PLOT OF ISPEC*W SYMBOL USED IS A
PLOT OF SM1U*W SYMBOL USED IS U

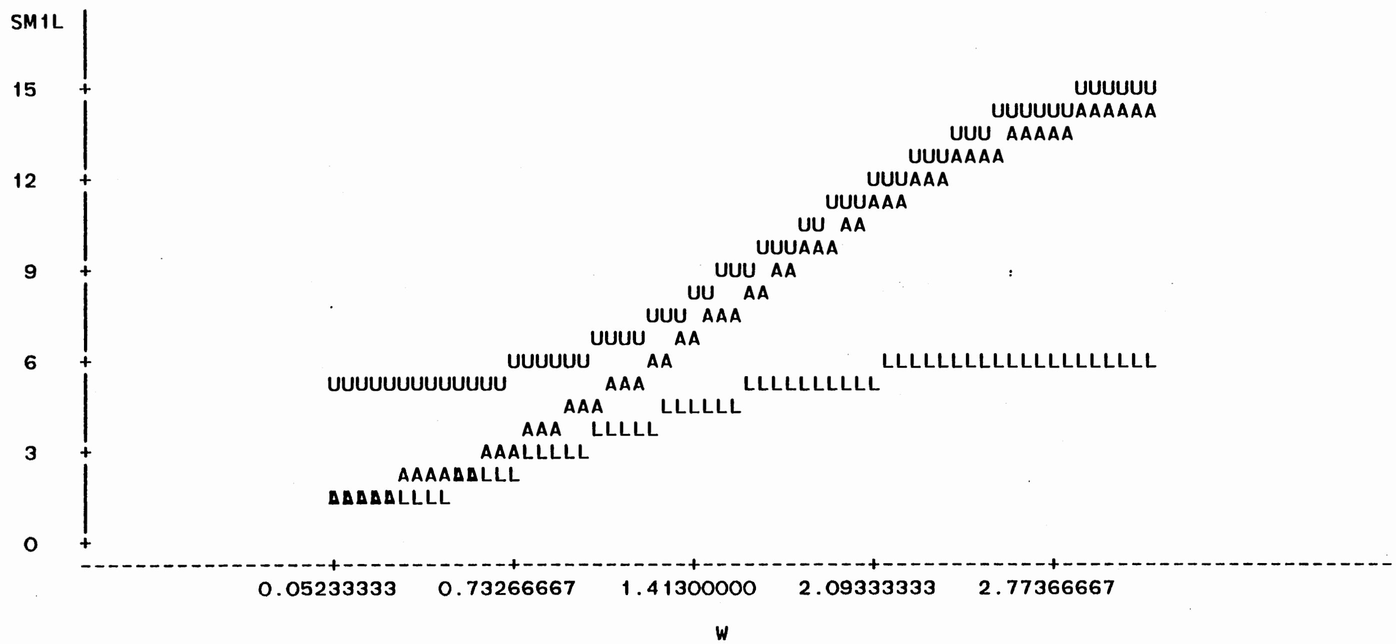

Figure 1. $S^{-1}(w)$, $AR_1(1)$, A = .5, N = 100

FIGURE 2 AR1==S 4
A=0.5 EA=0.2972529 N=100

PLOT OF SL*W SYMBOL USED IS L
PLOT OF SPE*W SYMBOL USED IS A
PLOT OF SU*W SYMBOL USED IS U

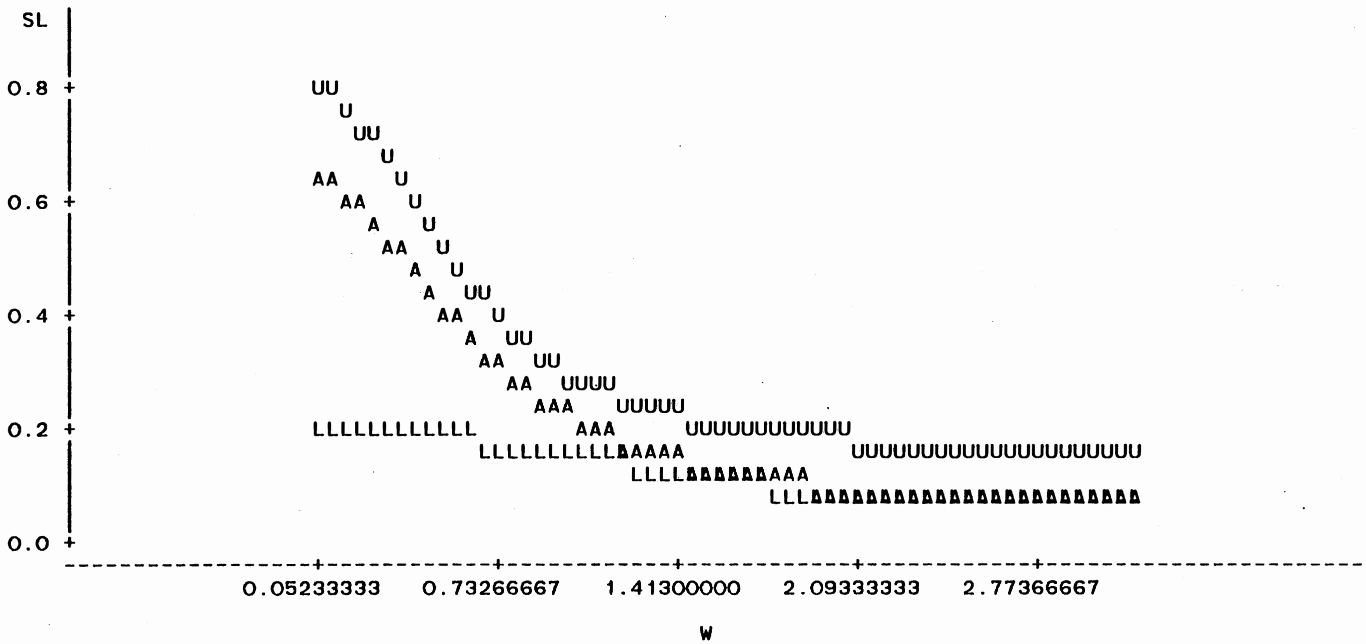

Figure 2. S(w), $AR_1(1)$, A = .5, N = 100

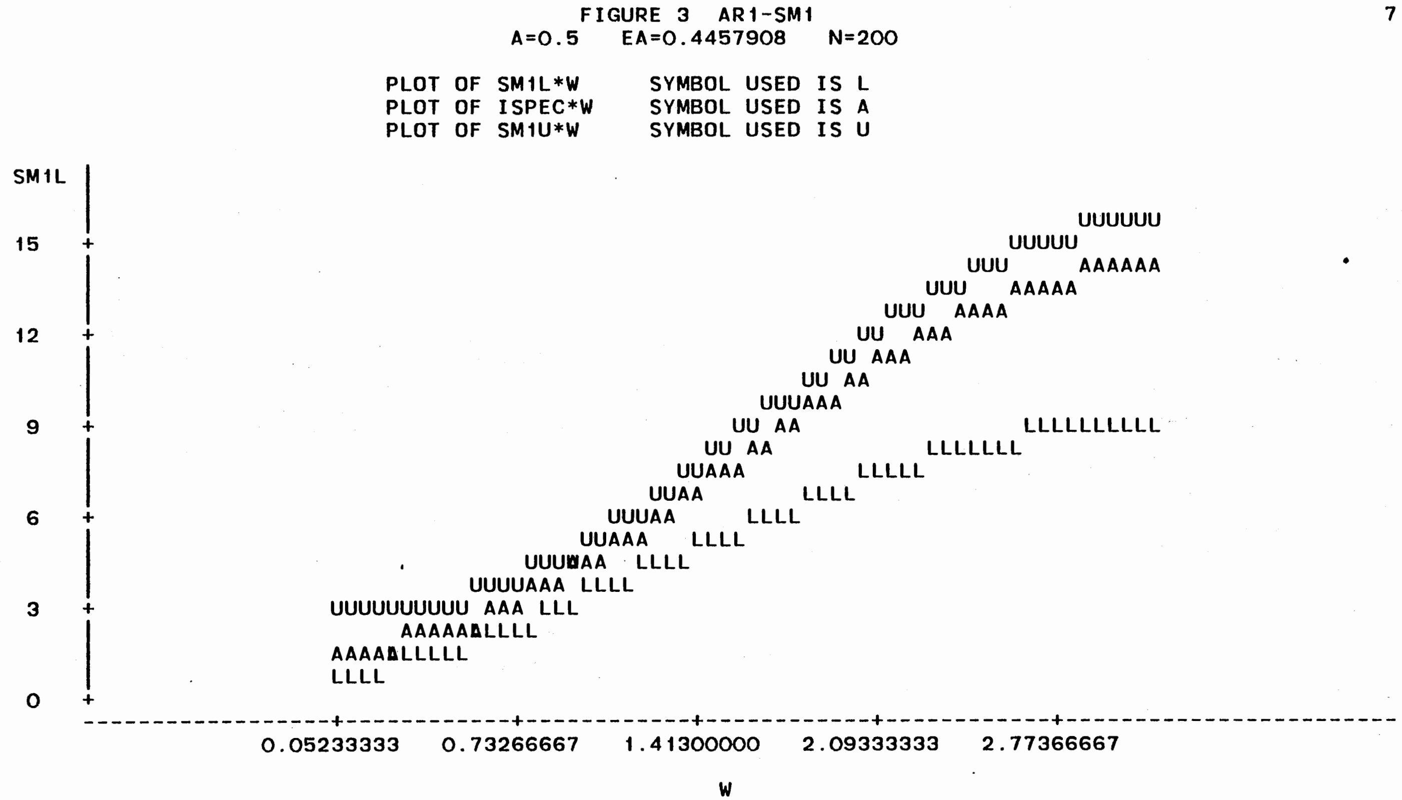

Figure 3. $S^{-1}(w)$, $AR_1(1)$, A = .5, N = 200

FIGURE 4 AR1--S 8

A=0.5 EA=0.4457908 N=200

PLOT OF SL*W SYMBOL USED IS L
PLOT OF SPE*W SYMBOL USED IS A
PLOT OF SU*W SYMBOL USED IS U

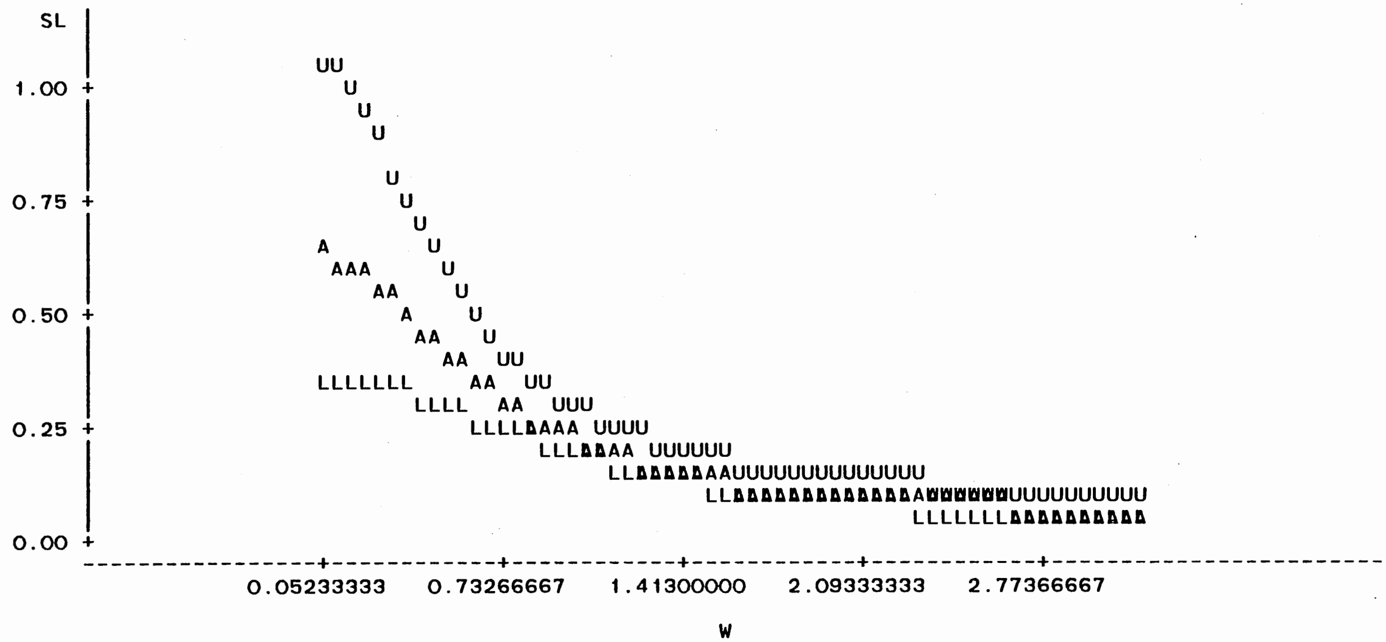

Figure 4. S(w), $AR_1(1)$, A = .5, N = 200

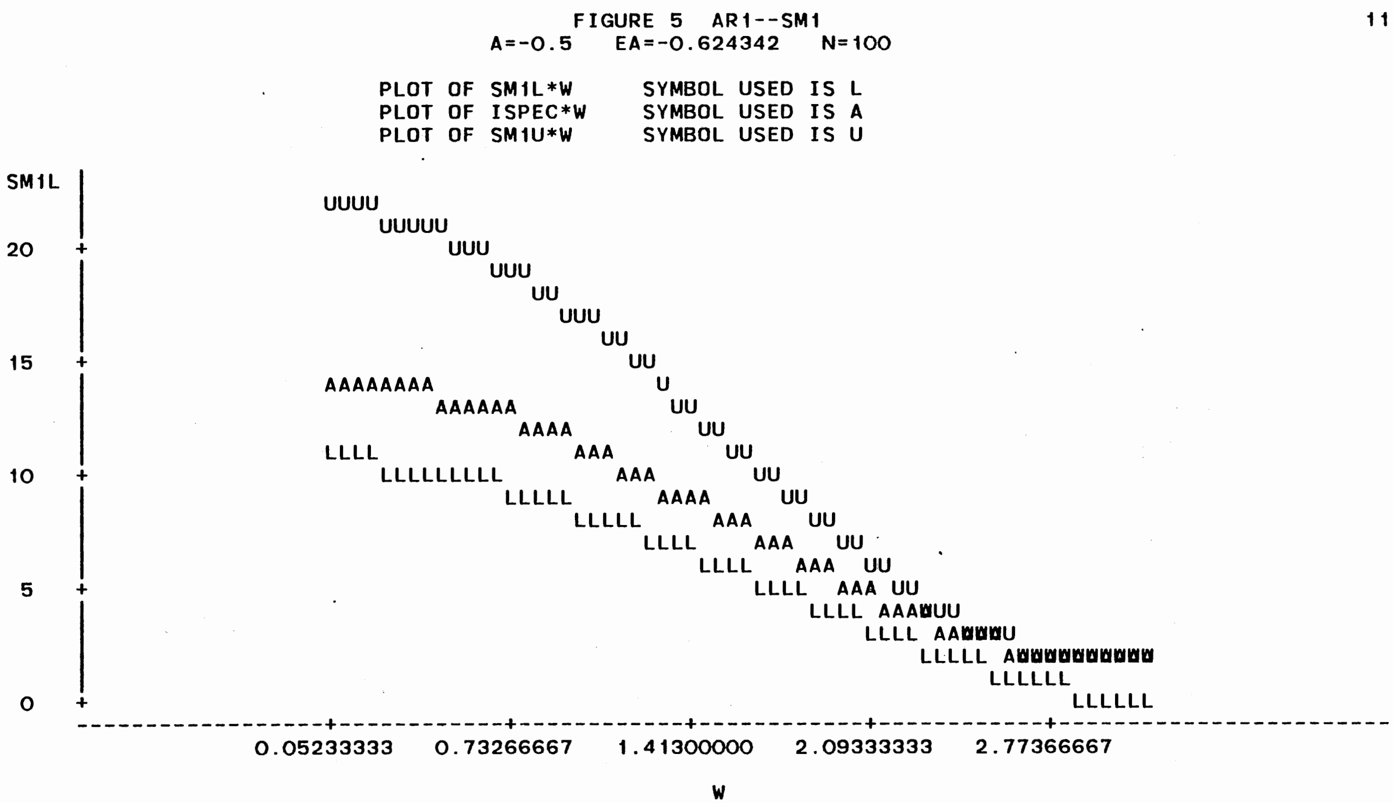

Figure 5. $S^{-1}(w)$, $AR_1(1)$, A = -.5, N = 100

FIGURE 6 AR1--S 12

A=-0.5 EA=-0.624342 N=100

PLOT OF SL*W SYMBOL USED IS L
PLOT OF SPE*W SYMBOL USED IS A
PLOT OF SU*W SYMBOL USED IS U

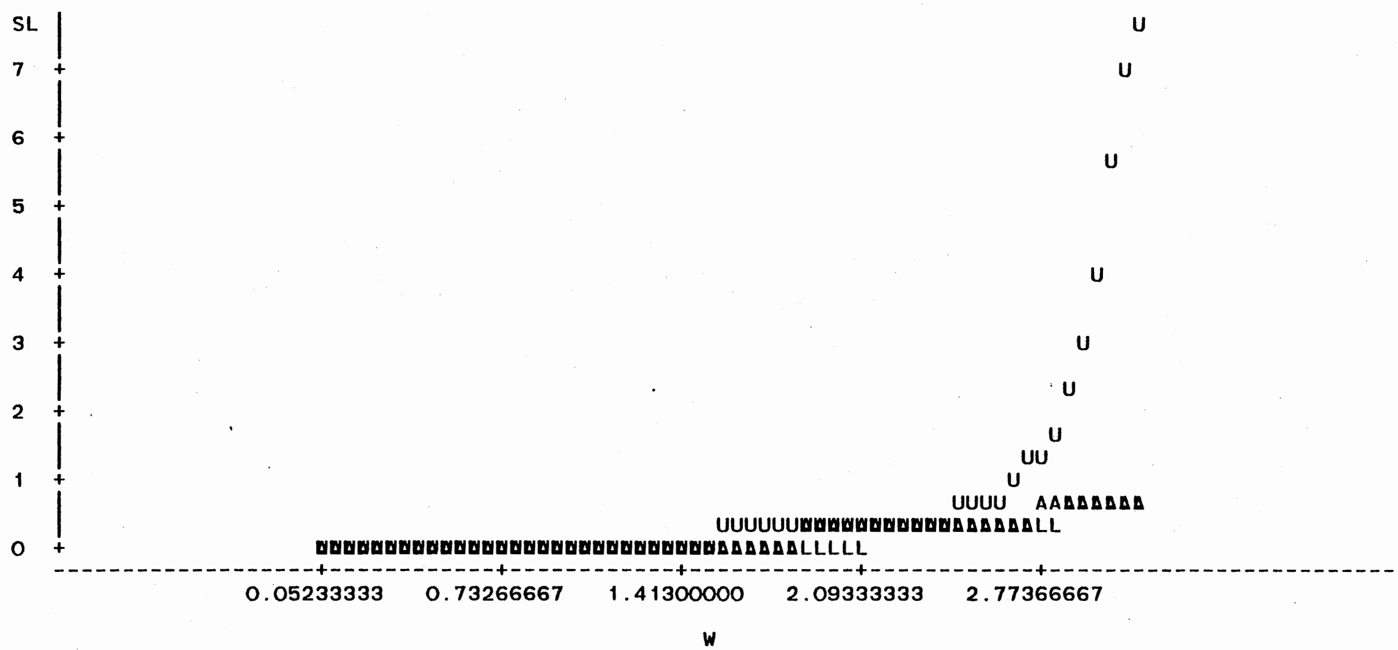

Figure 6. S(w), $AR_1(1)$, A = -.5, N = 100

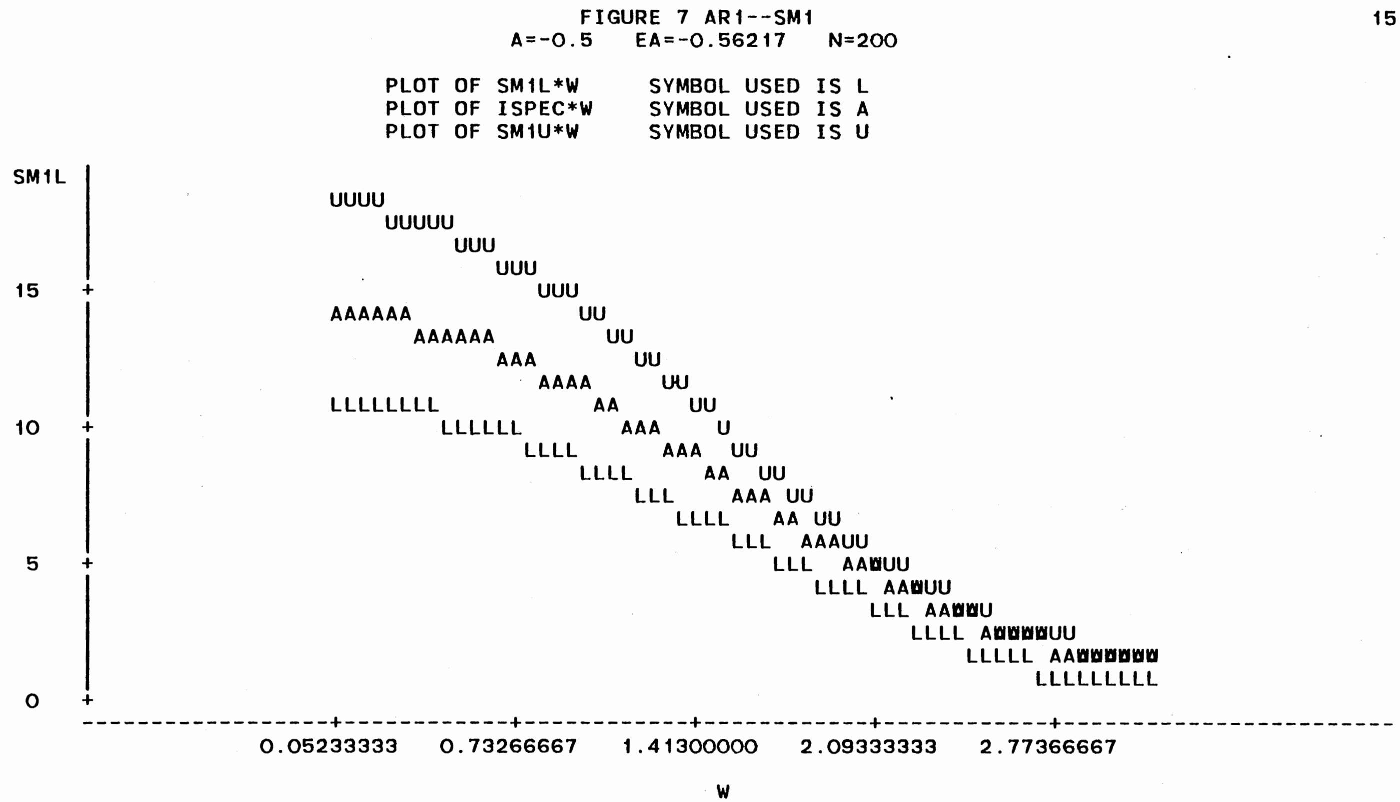

Figure 7. $S^{-1}(w)$, $AR_1(1)$, A = -.5, N = 200

FIGURE 8 AR1-S 16
A=-0.5 EA=-0.56217 N=200

PLOT OF SL*W SYMBOL USED IS L
PLOT OF SPE*W SYMBOL USED IS A
PLOT OF SU*W SYMBOL USED IS U

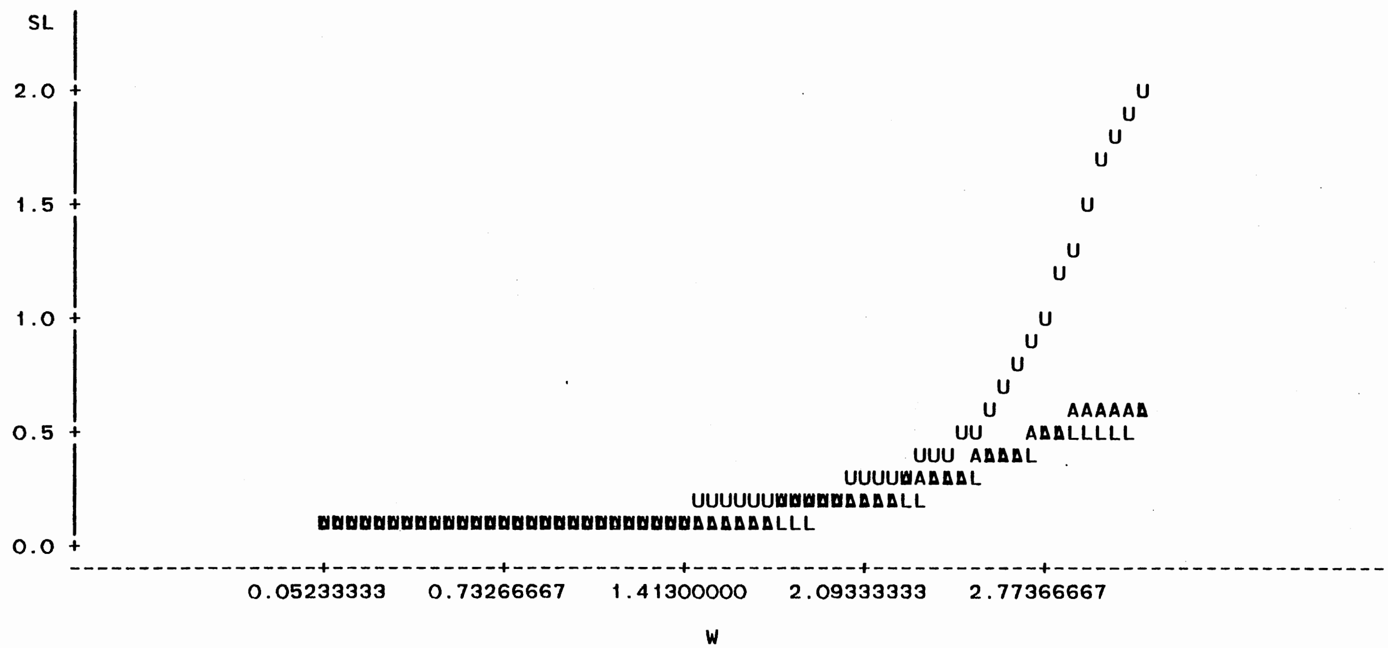

Figure 8. S(w), $AR_1(1)$, A = -.5, N = 200

FIGURE 9 AR1--SM1 19
A=1.02 EA=1.004444 N=100

PLOT OF SM1L*W SYMBOL USED IS L
PLOT OF ISPEC*W SYMBOL USED IS A
PLOT OF SM1U*W SYMBOL USED IS U

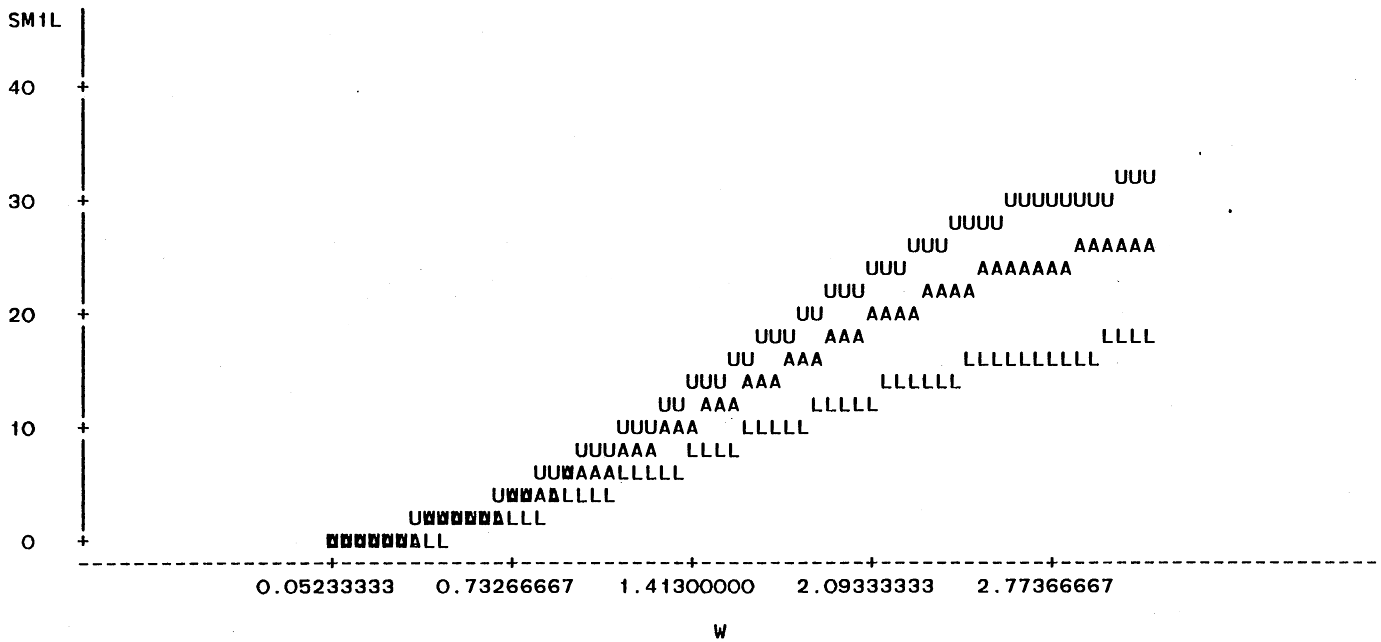

Figure 9. $S^{-1}(w)$, $AR_1(1)$, A = 1.02, N = 100

FIGURE 10 AR1--S 20

A=1.02 EA=1.004444 N=100

PLOT OF SL*W SYMBOL USED IS L
PLOT OF SPE*W SYMBOL USED IS A
PLOT OF SU*W SYMBOL USED IS U

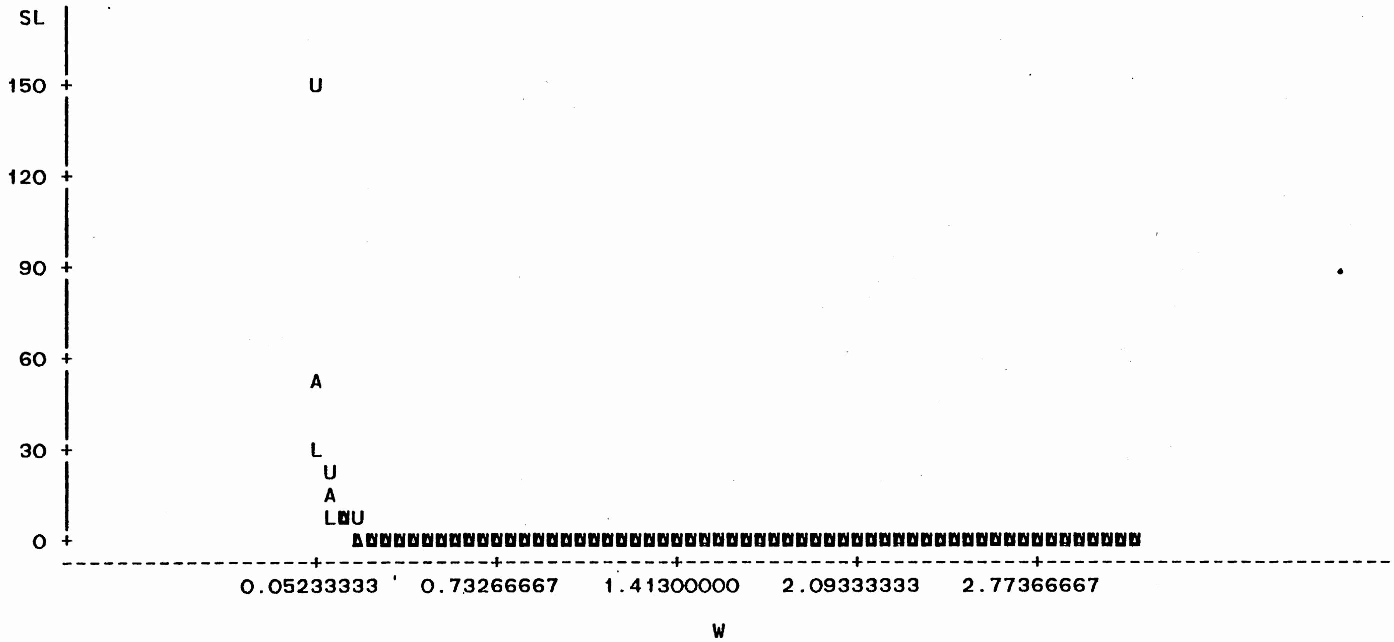

Figure 10. S(w), $AR_1(1)$, A = 1.02, N = 100

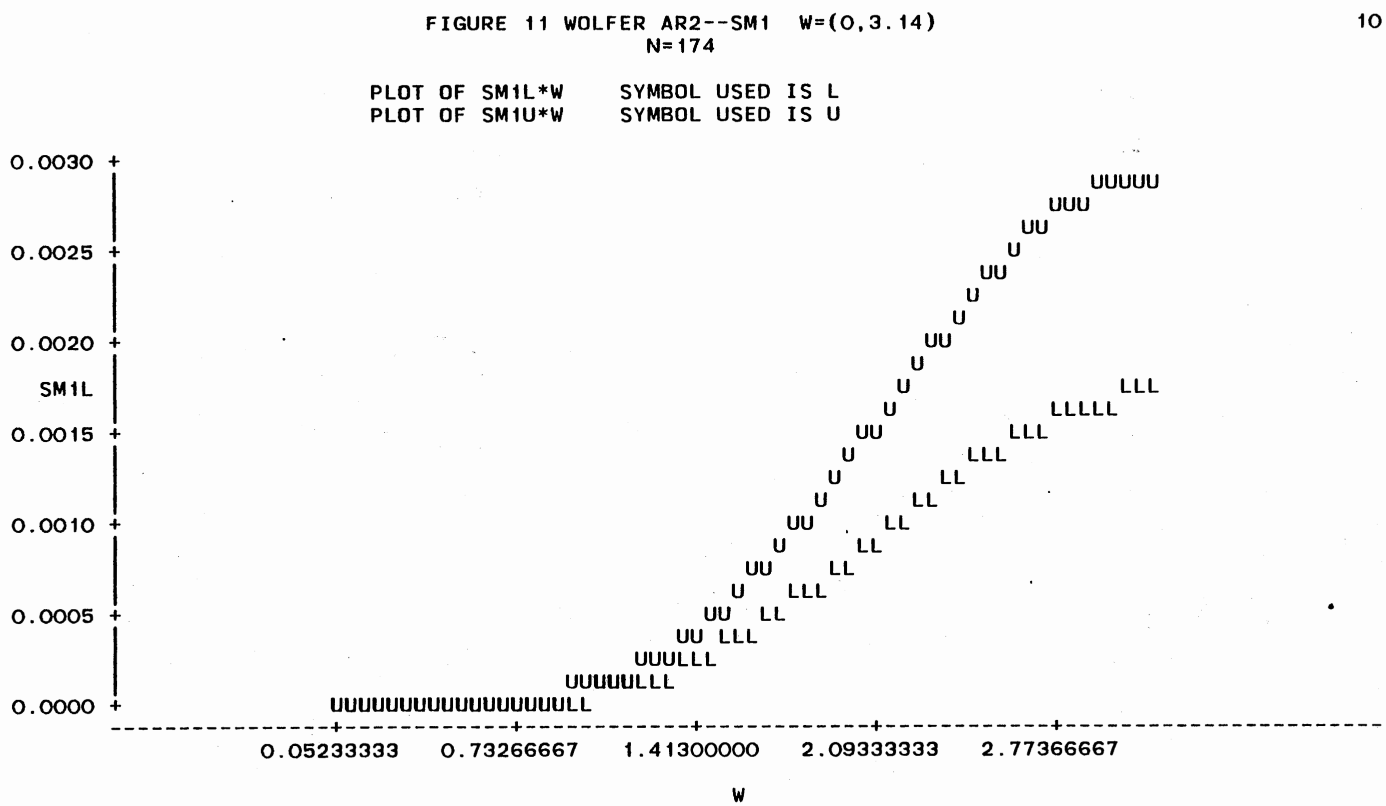

Figure 11. Wolfer, $S^{-1}(w)$, $AR_1(2)$, $w \in [0,3.14]$

FIGURE 12 WOLFER AR2 S W=(0,3.14) 11
N=174

PLOT OF SL*W   SYMBOL USED IS L
PLOT OF SU*W   SYMBOL USED IS U

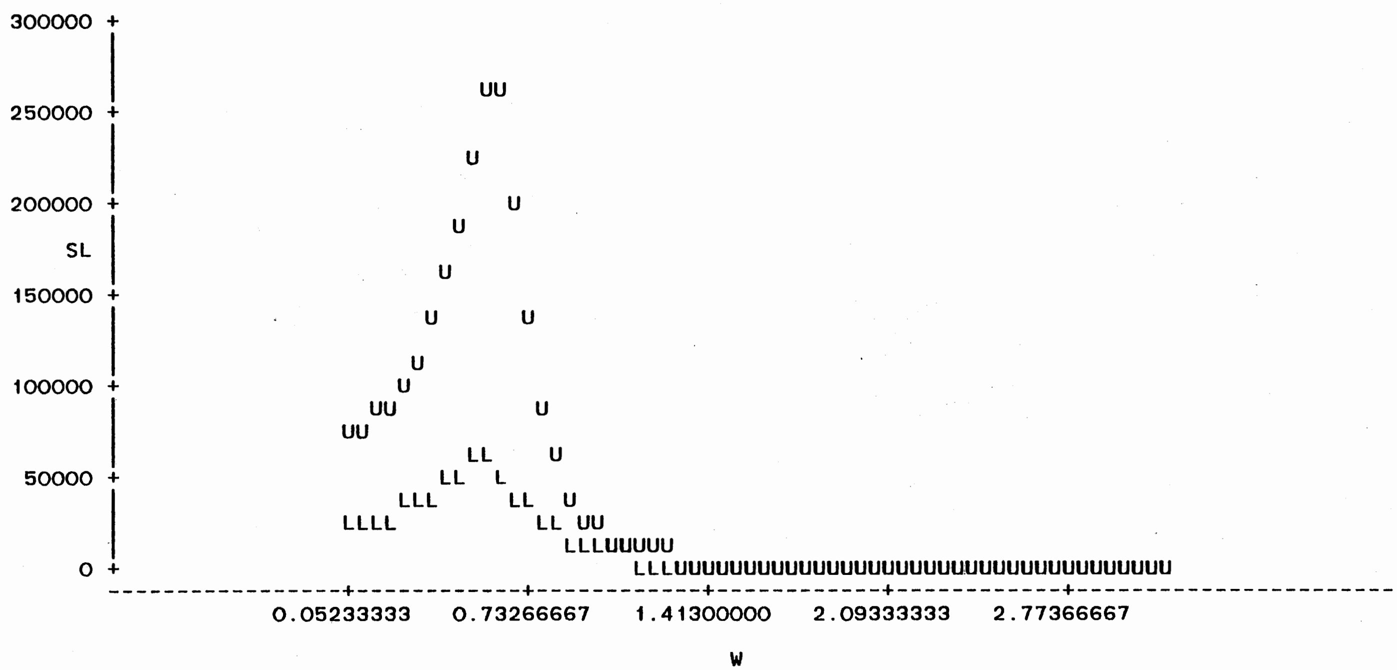

Figure 12. Wolfer, S(w), $AR_1(2)$, $w \in [0,3.14]$

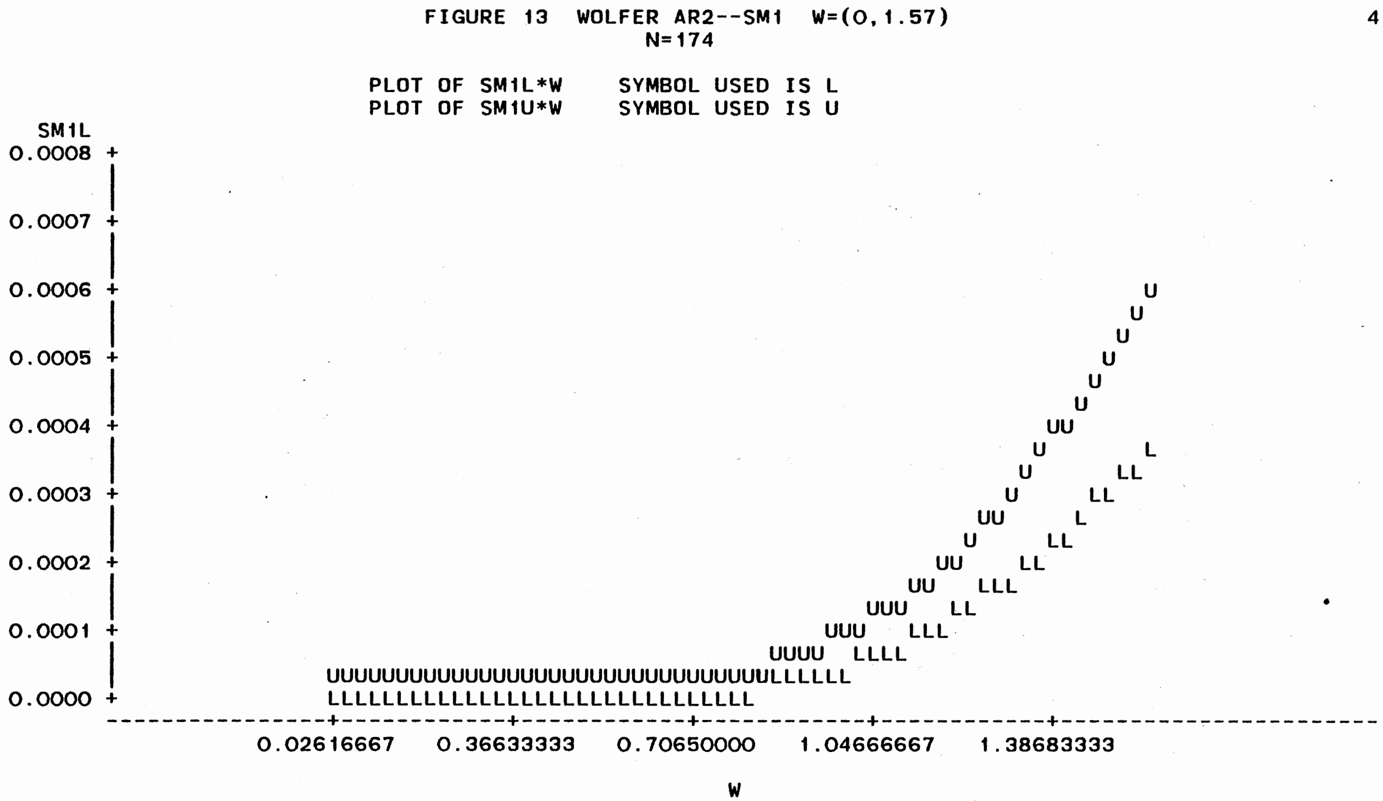

Figure 13. Wolfer, $S^{-1}(w)$, $AR_1(2)$, $w \in [0,1.57]$

FIGURE 14 WOLFER AR2--S W=(0,1.57) 5
N=174

PLOT OF SL*W SYMBOL USED IS L
PLOT OF SU*W SYMBOL USED IS U

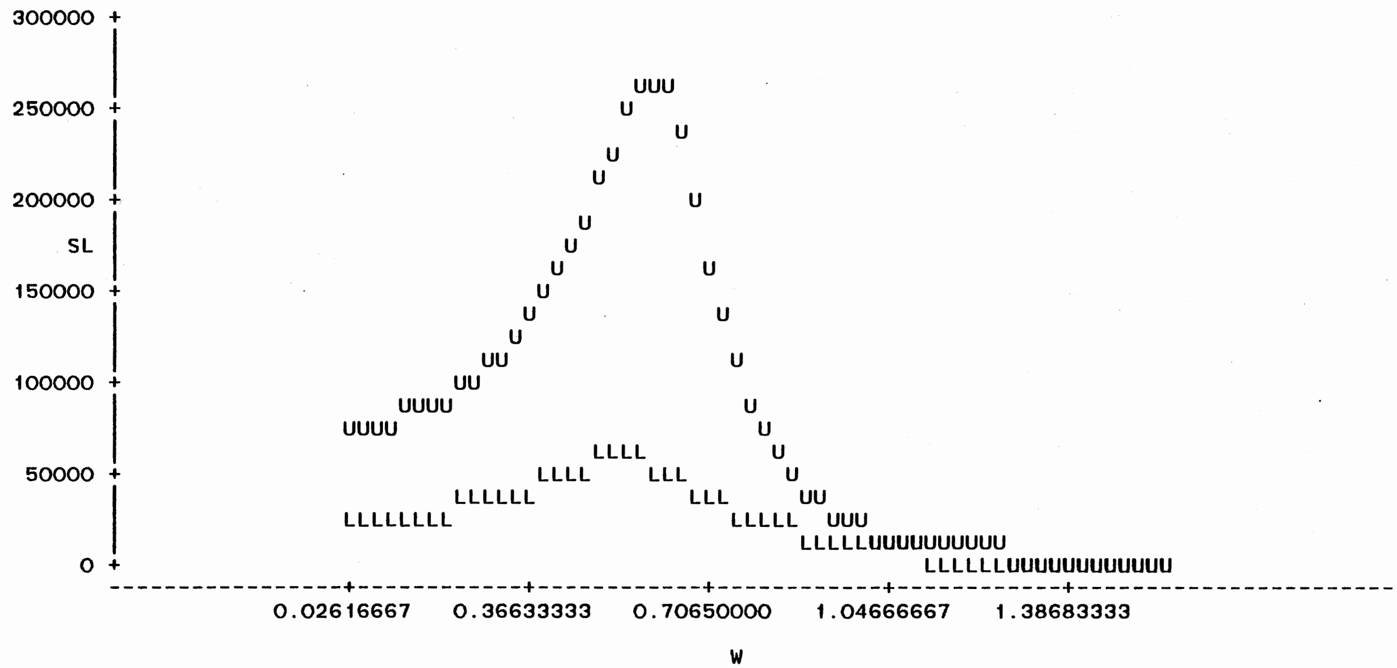

Figure 14. Wolfer, $S(w)$, $AR_1(2)$, $w \in [0,1.57]$

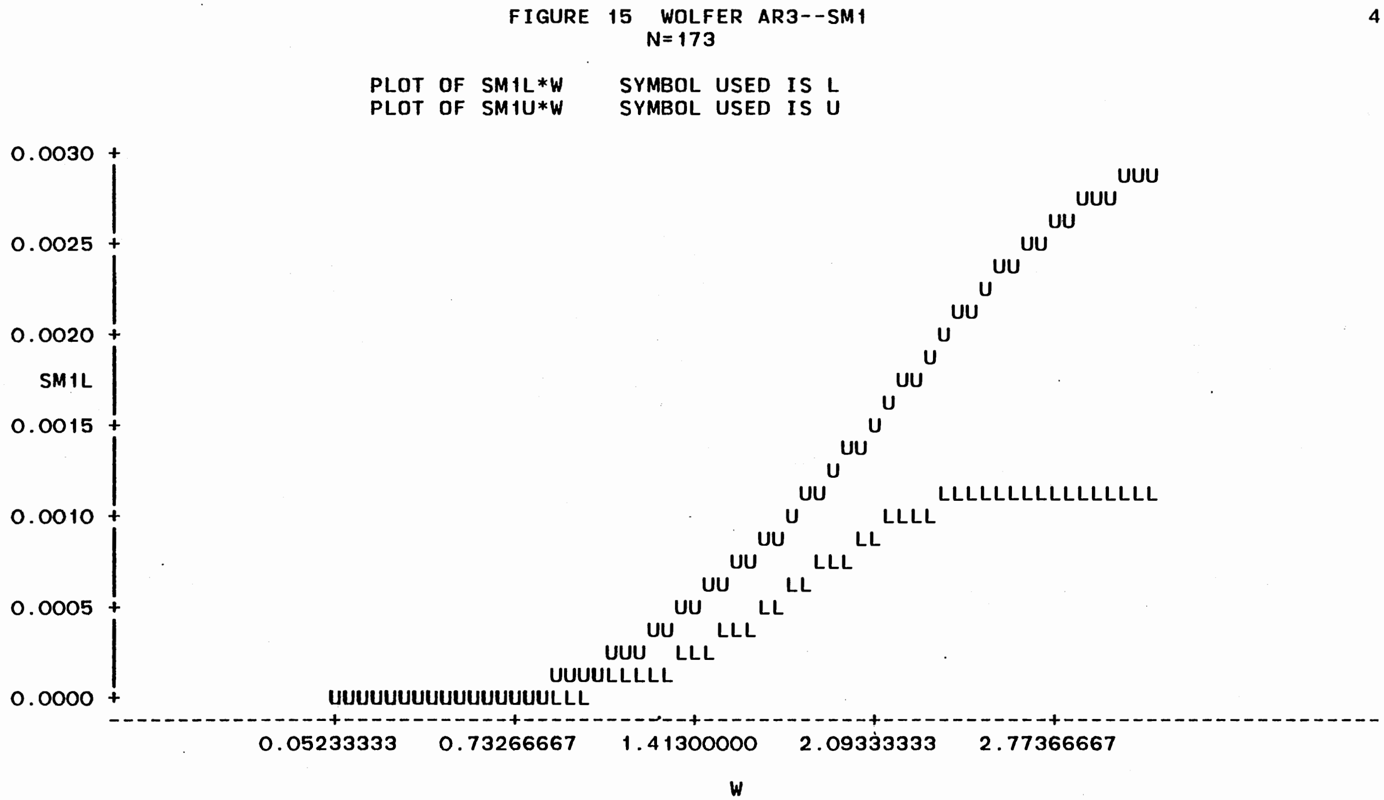

Figure 15. Wolfer, $S^{-1}(w)$, $AR_1(3)$

16 WOLFER AR3--S 5
N=173

PLOT OF SL*W SYMBOL USED IS L
PLOT OF SU*W SYMBOL USED IS U

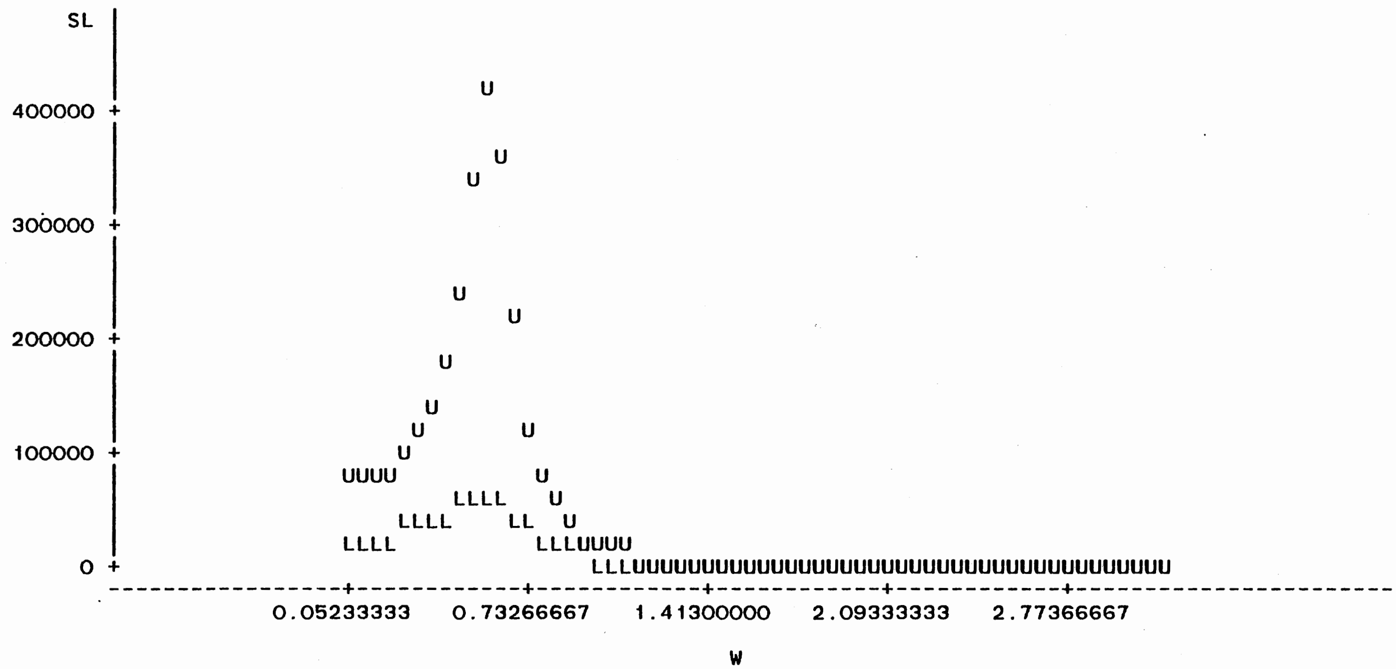

Figure 16. Wolfer, S(w), $AR_1(3)$

VITA 2

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