Harvey Mudd College
Mathematics

Senior Thesis
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Research Proposal: Turing Pattern Dynamics for Spatiotemporal Models with Growth and Curvature

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1 Introduction

Many different reaction-diffusion models have been proposed to study pattern formation in biological organisms [3]. Most of this development has occurred after Turing's paper in 1952 [1] where he proposed a system of two simple coupled nonlinear reaction-diffusion equations to model the spatiotemporal movement of the concentration of two different chemical substances which he called morphogens. Even though diffusion usually brings systems to equilibrium, Turing prosed a diffusion-driven instability in the case of a linearly stable spatially uniform state, which leads to the formation of these patterns as a result to the movement of the morphogens.

2 Proposed Research

Turing's diffusion-driven instability has not been fully accepted as a pattern formation mechanism in biology. One criticism has been the sensitivity of these Turing patterns to initial conditions, boundary conditions, and perturbations in parameter values. However, the original Turing paper did not consider a number of naturally occurring phenomena that affect pattern formation, such as the geometry of the domain and its growth, as well as the movement of the concentrations of morphogens that affect intensity of color, position of stripes, movement of the stripes and formation of new ones as a result to growing, etc.

Most work has been done on simple one-dimensional domains [4], and [2] provides a study of pattern formation in general two-dimension domains taking into account geometry and growth of the domains. However, the examples considered in [2] are only restricted to isotropic growth, thus accurately modelling organisms that grow proportionally and whose adult body is a magnification of the body in early stages. However, other organisms' growth does not preserve the proportions between their linear typical dimensions, called allometric growth. One possible area that I plan to further research in my senior thesis is allometric growth, and also seasonal effects on the growth of an organism.

Additionally, I want to perform some theoretical analysis regarding existence of solutions with certain qualitative behavior, explore stability through linear stability analysis, and perform local and global bifurcation analysis for the reaction-diffusion models for the dynamics of one and two-dimensional domains in $\mathbb{R}^3$.

3 Prior Research

I have taken an introductory course in Partial Differential Equations (Math 180) and I am currently taking Partial Differential Equations (Math 182) that will give me the necessary preparation to read and understand previous work in the area of Turing pattern formation.

Last summer, I did research in mathematical biology which was my first exposure to modelling biological phenomena using differential equations. I am also going to participate in a research project this summer with Professor Jon Jacobsen, that will introduce me to the most important work done in this area of Turing pattern formation. Furthermore, I will participate in the Park City Utah summer program whose topic this year is mathematical biology, and attend classes to strengthen my concepts in modelling naturally occurring phenomena using partial differential equations.

References

[1] A. M. Turing, *The chemical basis of morphogenesis*, Philos. Trans. R. Soc. Lond. B. 237 (1952), pp. 37-72.

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[3] J. D. Murray, *Mathematical Biology, second ed.*, Springer-Verlag, 1993.

[4] E. J. Crampin, E.A. Gaffney & P. K. Maini, *Reaction and diffusion growing domains: scenarios for robust pattern formation*, Bull. Math. Biol. 61 (1999), pp. 1093-1120.