

A DISCRETE TIME ITERATIVE NEGOTIATION
ALGORITHM FOR PHOSPHORUS
REDUCTIONS IN THE
ILLINOIS RIVER
BASIN

By

JEONG-SOON PARK

Bachelor of Arts

Sungkyunkwan University

Seoul, Korea

1984

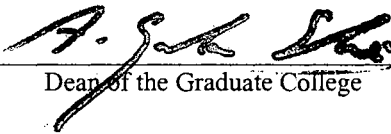
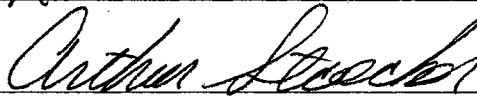
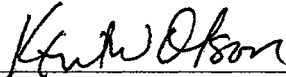
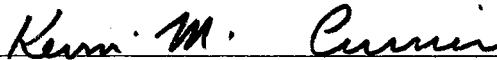
Submitted to the Faculty of the
Graduate College of the
Oklahoma State University
in partial fulfillment of
the requirements for
the Degree of
DOCTOR OF PHILOSOPHY
May, 2005

A DISCRETE TIME ITERATIVE NEGOTIATION
ALGORITHM FOR PHOSPHORUS
REDUCTIONS IN THE
ILLINOIS RIVER
BASIN

Dissertation Approved:



Dissertation Advisor



Dean of the Graduate College

ACKNOWLEDGMENTS

First and foremost thanks are due to my advisor, Dr. Keith D. Willett. This study was motivated by his advice. Without his guidance, this dissertation would not have been completed. I wish to express my sincere thanks to Dr. Kevin M. Currier and Dr. Gerald M. Lage who taught me Microeconomics. The time working with Dr. Kevin M. Currier provided me with insights and ideas over the past three years. I want to also thank Dr. Arthur Louis Stoecker who taught me Environmental Economics. I would like to express my grateful appreciation to all members of my committee. To Mrs. Ruby Ladd, thank you for your immediate help in the past few years. You have always helped me with a warm smile.

Thanks for financial support go to Oklahoma State University and the Department of Economics and Legal Studies in Business. I thank Korea Energy Economics Institute for allowing me to study abroad.

I wish to express my sincerest thanks to my wife, Keum-Bae Lee, and our two children, Seo-Hyun Park and Joo-Hyung Park, for their patience and endless love. I am deeply indebted to my brother Ik-Soon Park, who has cared for my parents on behalf of me since I left.

Finally, I hope my father, who passed away during my absence from home, can hear me. I dedicate this dissertation to my parents.

TABLE OF CONTENTS

Chapter	Page
I. INTRODUCTION.....	1
Motivation.....	1
Objectives	3
II. LITERATURE REVIEW	4
Common Studies.....	4
Transboundary Pollution Studies	5
III. THEORETICAL BACKGROUND	8
Lange-Arrow-Hurwicz Approach.....	8
Malinvaud Approach	10
Dynamic Process with Public Goods	11
Lindahl Process.....	14
Lindahl-Malinvaud Process	16
Malinvaud-Drèze-Poussin Process	17
Modeling Transboundary Pollution	20
Modeling Negotiations	20
Modeling Cost Sharing	23
Cost Functions	24
Damage Functions	24
IV. METHODOLOGY	32
Unidirectional Externalities Approach with Side-payment	34
Enforcement Rule Approach.....	36
Negotiation Approach.....	36
Payoffs	39
V. NUMERICAL APPLICATION	43
Data.....	44
Software	46
Numerical Application.....	47
Transition of Emission.....	48
Ecological Surplus	50
Financial Transfers	50
Surplus Sharing.....	53
Payoffs	54

Chapter	Page
VI. CONCLUSIONS	55
SELECTED BIBLIOGRAPHY	57
APPENDICES	60
APPENDIX A: CONVERGENCE PROPERTY OF SOLUTION.....	61
APPENDIX B: PROPERTY OF SURPLUS	64
APPENDIX C: DATA CONSTRUCTION	65
APPENDIX D: NUMERICAL APPLICATION	77
APPENDIX E: RESULTS BY SCENARIO	79

LIST OF TABLES

Table	Page
5.1 P Emissions and Parameters.....	48
5.2 Transport Coefficients.....	48
5.3 Evolution of P Emission.....	48
5.4 Costs Reduction Without/With Transfers	53
5.5 Net Benefits.....	54
A.1 P Loading in the Arkansas Region.....	66
A.2 P Loading in the Oklahoma Region	67
A.3 Illinois River Phosphorus Mass Balance Inputs	68
A.4 WWTP Cost Functions.....	68
A.5 Abatement Cost and Emissions	69
A.6 Parameters and Initial Values.....	71
A.7 Total Phosphorus Transported to the Lake Tenkiller	71
A.8 Transport Coefficient to Tenkiller.....	73
A.9 Transport Coefficient Construction Procedure.....	74
A.10 Status of Data after Data Construction	75
A.11 Total Costs per Region.....	78
A.12 Evolution of Total Costs per Region (3% Reduction).....	79
A.13 Evolution of Total Costs per Region (3% Reduction).....	80
A.14 Financial Transfers (3% Reduction).....	81
A.15 Evolution of P Emissions (5% Reduction).....	82
A.16 Evolution of Total Costs per Region (5% Reduction).....	83
A.17 Financial Transfers (5% Reduction).....	84
A.18 Evolution of P Emissions (10% Reduction).....	85

Table	Page
A.19 Evolution of Total Costs per Region (10% Reduction).....	86
A.20 Financial Transfers (10% Reduction).....	87

LIST OF FIGURES

Figure	Page
3.1 Framework of Dynamic Process	13
3.2 Lindahl (L) Process.....	15
3.3 Lindahl-Malinvaud (LM) Process	17
3.4 Malinvaud-Drèze-Poussin (MDP) Process	19
3.5 Modeling Negotiations.....	22
5.1 Evolution of P Emissions	49
5.2 Evolution of Total Cost per Region	51
5.3 Evolution of Transfers per Region	52
A.1 Abatement Cost (SR-1)	70
A.2 Abatement Cost (SR-2)	70
A.3 Abatement Cost (SR-3)	70
A.4 Abatement Cost (SR-4)	70
A.5 Direction of Phosphorus Transport	72
A.6 Transport Coefficient Matrix.....	74
A.7 Data Availability and Regional Selection	76
A.8 Abatement Cost Sharing Parameter	77
A.9 Evolution of P Emissions (3% Reduction).....	79
A.10 Evolutions of Total Costs per Region (3% Reduction)	80
A.11 Evolutions of Transfers per Region (3% Reduction)	81
A.12 Evolution of P Emissions (5% Reduction).....	82
A.13 Evolutions of Total Costs per Region (5% Reduction)	83
A.14 Evolutions of Transfers per Region (5% Reduction)	84
A.15 Evolution of P Emissions (10% Reduction).....	85

Figure	Page
A.16 Evolutions of Total Costs per Region (10% Reduction)	86
A.17 Evolutions of Transfers per Region (10% Reduction)	87

CHAPTER I

INTRODUCTION

Motivation

Over the course of the first 25 years of the Water Quality Act, the primary focus was on regulation of point sources with technology-based standards. The recent renewed emphasis on water quality is captured in proposed total maximum daily load (TMDL) rules. A TMDL identifies the amount of a pollutant that is allowed in a water body, allocates allowable pollutant loads among sources, and provides a foundation for achieving water quality levels. The TMDL program is characterized as an ambient regulation where regulation and reporting are more concerned with the in-situ quality of water bodies. The most important characteristic of TMDL rules is the impact on nonpoint source regulation.

James Boyd (2002) identifies a number of important challenges facing the implementation of TMDL rules in a critical review of the proposed TMDL rules. The list of important challenges includes the lack of correspondence between state boundaries and watersheds, which provides the foundation for jurisdictional disputes. This issue is important because the downstream segments could inherit water quality problems from upstream sources located in other states. Furthermore, if the case is that standards may differ across states, the conflicts may easily occur. Less stringent water quality standards upstream coupled with less effective implementation upstream can be the cause of impairments in a downstream state with stricter water quality standards. In this case, the proposed TMDL rules require states to identify a process for resolving disagreements between jurisdictions.

An example of a region where jurisdiction conflicts appear to be evolving is the Illinois River Basin in eastern Oklahoma and northwest Arkansas. Significant growth in poultry production has been accompanied by water quality problems. The Illinois River Basin covers an area approximately equal to 433,160 hectares; roughly 54 percent of the total basin is located in Oklahoma. The river corridor in

Oklahoma is a popular tourist and recreational attraction and was the first river designated as wild and scenic by the state of Oklahoma. Annually, a large number of people float the Illinois River by canoe, raft, or kayak while an even larger number enjoy swimming, fishing, camping, hiking, birding, and hunting opportunities. The Illinois River is a source of drinking water for several municipalities, irrigation water for farms and nurseries, and a habitat for several state and federal threatened and endangered species (Bality et al., 1998). Tourism contributes a great deal to the economic base in the Basin, especially the portion in Oklahoma. A substantial amount is also derived from agriculture, plant nurseries, forestry, and gravel and limestone mining. The agricultural activities include cattle ranching and poultry operations, with the latter showing significant growth in recent years.

The historical evolution of water policy in the Illinois River Basin with respect to Arkansas and Oklahoma is highlighted by controversy in recent years. For example, a controversy in the late 1980s and early 1990s involved the discharge of municipal water into the Illinois River by the city of Fayetteville, Arkansas. Given the river's designation as wild and scenic in Oklahoma, increased wastewater discharges from sources in Arkansas triggered a lawsuit by Oklahoma. This legal action resulted in a U.S. Supreme Court decision in 1992 that resolved the conflict in favor of Arkansas. The lawsuit was based on the proposition that Fayetteville must meet Oklahoma's water quality standards at the state line, but it was concluded that existing evidence did not prove that the city of Fayetteville violated Oklahoma water quality laws. In 1996 a report on Tenkiller Lake, which is downstream on the Illinois River, contained a recommendation for phosphorus reductions in the Illinois River Basin (Oklahoma Water Resources Board/Oklahoma State University, 1996). In 1997, the Arkansas River Compact Commission established a goal to reduce phosphorus in the Basin by 40 percent.

The continued decline in water quality has led to discussion focused on developing and implementing a phosphorus standard. Three different phosphorus standards have been discussed: an EPA-recommended eco-region nutrient criterion standard of 0.010 mg./l.; an Oklahoma Scenic Rivers Commission standard of 0.020 mg./l.; and a target level of 0.0375 mg./l., which is designed to control algal growth. Yet another proposal has suggested a standard of 0.05 mg./l. The state of Oklahoma has decided to adopt the target level of 0.0375 mg./l.

As things currently stand, both Oklahoma and Arkansas may remain far apart on the issue of water quality in the Illinois River Basin. Several questions have been raised: Is it possible for both states to reach an agreement? What is Pareto optimum for both states? What is a best strategy for both states? To find answers to these questions may be a first step to resolve the phosphorus problem in the Illinois River Basin.

Objectives

The purpose of this study is to propose a modeling structure to resolve the jurisdictional disputes on water quality between Arkansas and Oklahoma in the Illinois River Basin. Three options are considered from the viewpoint of theory and policy. The first case is that a side-payment is used. A side-payment is a way for Oklahoma to control the phosphorus emission from Arkansas. But the question of whether this option is reasonable and acceptable to Oklahoma must be examined. The second case is based on the use of an enforcement rule. The latter is a way for Oklahoma to improve water quality without additional abatement cost. The acceptability and reasonableness of this option for Arkansas must be examined. The third option is to use a negotiation scheme. It is an option never examined to resolve the phosphorus problem in the Illinois River Basin. It is important to examine how a negotiating scheme is different from other two options in its properties and payoffs for both regions. It is important to look at whether negotiation can overcome the limits of other options to resolve the phosphorus problem in the Illinois River Basin. The objective of this study is to identify the limits of initial policies and propose negotiation as an effective alternative. There are other important issues such as free rider or the reliability of information, but these issues are beyond the scope of this study and are not discussed.

The organization of the dissertation is as follows. Chapter II presents a selected review of studies concerned with phosphorus problem and transboundary pollution studies. Chapter III explains the theoretical background and main modules to be used for modeling. Chapter IV introduces three options and proposes the modeling structure. Chapter V explains the data development procedure, model application, and the results of the numerical application. Chapter VI summarizes the discussion, and presents implications and conclusions.

CHAPTER II

LITERATURE REVIEW

Common Studies

The overriding concern in the Illinois River Basin for both Arkansas and Oklahoma continues to be land applications of poultry litter as the primary method of disposal. The primary sources of phosphorus in the Illinois River are derived from the land application of poultry litter throughout the basin along with phosphorus discharges from municipal treatment plants in the Basin. A number of studies have looked at the economic and environmental impacts of poultry litter generation and use of fertilizer as well as its disposal.

There are studies by Govindasamy and Cochran (1995, 1998); Govindasamy, Cochran, and Buchanberger (1994); as well as Xu, Prato, and Fulcher (1993). A common feature of the model structures used in these studies is that poultry litter is treated as a factor of production. Thus, the model structures are concerned with the “derived demand” for poultry.

The models of Govindasamy and Cochran (1995, 1998) and Govindasamy, Cochran, and Buchanberger (1994) focus more on poultry litter applications and pay less attention to nutrients, particularly nitrogen and phosphorus. Poultry is introduced into the model structures used in these studies through a balance equation that traces the use of poultry litter. The right-hand side of the equation shows the quantity of poultry litter produced in the watershed and is treated as a parameter. The left-hand side shows the endogenously determined uses of poultry litter. The uses are the amounts of litter applied to crops in the region as well as the amount shipped out of the watershed. No explicit consideration is given to nutrient demands in these studies.

Xu, Prato, and Fulcher (1993) explicitly model the demand for nutrients and also consider the tradeoffs between litter, as a source of crop nutrients, and commercial fertilizers. The supply sources of

litter are treated as exogenous in all of these models. Among other things, it is assumed that the generation of poultry litter is proportional to poultry production. This implies that reductions in poultry litter require a proportional reduction in poultry production. An overriding concern is that the opportunity cost of adjustments to environmental policies may be overstated.

Transboundary Pollution Studies

The typical way of dealing with a river pollution problem is the unidirectional externalities approach. This approach assumes unique upstream polluters and unique downstream victims. Another way of dealing with the problem is a regional reciprocal externalities approach. This approach assumes that there is a common property resource with free access for many agents. This approach makes it possible to examine the case where all agents are polluters and victims at the same time.

The phosphorus problem in the Illinois River Basin can be examined from the viewpoint of an unidirectional externalities approach. But this approach has become incompatible as targeted pollution is shifting from point source to nonpoint source. The sub-regions in the Illinois River Basin could be polluters as well as victims. It means the possibility of the latter to be applied to the Illinois River Basin as an alternative of the former. But such a viewpoint has not received attention in the relevant studies on this area.

In recent years, the modeling of international negotiations on transboundary pollution has been focused. An early contributor is Mäler (1989) who proposed the acid rain game formulation for transboundary air pollution. Kaitala, Pohjola and Tahvonen (1992) applied Mäler's formulation to analyze the problem of transboundary pollution between Finland and the Former Soviet Union.

The significant shortcoming of these studies is that they have a global information requirement. Using global information implies that information about the entire emission abatement cost functions and entire deposition damage cost functions of the relevant regions are known. But it is not easy to estimate the entire costs due to wide changes in emission and deposition in the real world.

Tulkens (1979) presented a model for gradual emission reduction using only local information. Chander and Tulkens (1991) suggested the general framework which is not based on the global information. The local information assumption implies that a player knows only its marginal abatement cost and marginal damage cost at any level of emission. Kaitala, Mäler and Tulkens (1995) applied the acid rain

game model (Mäler, 1989) with local formulation (Tulkens) to the air pollution problem among Finland, Russia and Estonia. The model of Kaitala, Mäler and Tulkens (1995) is an improvement over other approaches, but the problem is an impractical assumption that the time scale is continuous. It means that the negotiation should be continuous.

Germain, Toint and Tulkens (1996) suggested a negotiation model of discrete time approach with local information. They assume that a member of the coalition knows only its marginal abatement cost and damage cost at any level of emission. It is also assumed that the damage function is linear and emission is lower bounded due to the technological limit. The lower bound of emissions becomes lower as abatement technologies are developed and abatement proceeds. The lower bound of emission is updated by new information at each stage of the negotiation. The emission reduction target is also set at each stage of the negotiation. What is different under the negotiation is that the members of the coalition are required to reduce more emission which is deposited in another region. Such a cooperative emission reduction leads the participants to Pareto optimum. Aggregate costs become lower than the costs under a noncooperative Nash equilibrium. The saved cost is called ecological surplus. In their model, ecological surplus is used as a source of financial transfers between participants in the coalition. Ecological surplus sharing is made by a rule which is formulated by Chander and Tulkens (1991, 1992). Germain, Toint and Tulkens (1996) applied the model to negotiations on the acid rains problem in Northern Europe. Germain and Toint (2000) recast this negotiation algorithm using a quadratic damage function with the same data, but the result is similar with that of Germain, Toint and Tulkens (1996).

The negotiation algorithm suggested by Germain, Toint and Tulkens (1996) is capable of being applied to the phosphorus problem in the Illinois River Basin, because it is one of the most reasonable versions of reciprocal externalities approach. The essential module of Germain, Toint and Tulkens (1996) consists of local information, discrete time, and financial transfers. Among them, a factor of local information is based on the formulation suggested by Tulkens (1979). But the origin of the methodology is the theory of resource allocation processes established in 1960. Arrow and Hurwicz (1977), Malinvaud (1970) and Drèze and de la Vallée Poussin (1971) contributed to the foundation of this theory. The financial transfer is based on the formulation presented by Chander and Tulkens (1991, 1992). The other

factors such as damage functions and cost functions as well as the acid rain game are formulated by Mäler.

Chapter III reviews these modules as theoretical background.

CHAPTER III

THEORETICAL BACKGROUND

Arrow and Hurwicz (1960) formalize the planning theory of Lange (1936). Prices are used in an iterative and decentralized resource allocation procedure. Malinvaud (1967) establishes the process that converges to a solution using a sequence of feasible plans with a monotonic property of the utility function. But this procedure was not a gradient process but a discrete-step. On the other hand, the dynamic process with public good is developed by Lindahl (1958). Malinvaud (1970) reformulated the Lindahl process. The gradual method in the environmental economic issue was developed by Malinvaud (1971, 1972), Drèze and Poussin (1971) which is called MDP process. Tulkens (1979) applies the resource allocation process to the economic-ecological system. Based on the MDP process, Tulkens (1992) formulated negotiations on a voluntary provision of public goods. Chander and Tulkens (1991) suggested an ecological surplus sharing rule.

Lange-Arrow-Hurwicz Approach

The purpose of planning in this approach is to find a resource allocation to maximize the objective function. Assume n firms and s commodities in the economy. The planning problem is

$$\max U(y_1, \dots, y_s) \quad \text{s.t.} -y_i + \sum_{j=1}^n g_{ij}(x_j) + e_i \geq 0, \quad i = 1, \dots, s \quad (3.1)$$

$g_{ij}(x_j)$: amount of good i produced by firm j .

$-y_i$: inputs.

$\sum_{j=1}^n g_{ij}(x_j)$: total net output of good i .

e_i : initial endowment of good i .

By the Kuhn-Tucker theorem, necessary and sufficient conditions for (y_1, \dots, y_s) and (x_1, \dots, x_n) are

$$y_i = 0: U_i - \lambda_i \leq 0. \quad y_i > 0: U_i - \lambda_i = 0. \quad (3.2)$$

where $U_i = \partial U(y_1, \dots, y_s) / \partial y_i$.

$$x_i = 0: \lambda_i \sum_{j=1}^n g_{ij}(x) \leq 0 \quad x_i > 0: \lambda_i \sum_{j=1}^n g_{ij}(x) = 0 \quad (3.3)$$

$$\lambda_i = 0: \sum_{j=1}^n g_{ij}(x_j) + e_i - y_i \geq 0 \quad \lambda_i > 0: \sum_{j=1}^n g_{ij}(x_j) + e_i - y_i = 0 \quad (3.4)$$

where $(\lambda_1, \dots, \lambda_s)$ are dual variables associated with the constraints in (3.1).

Now suppose that this economy is separately run by firm managers and a distributor, and λ_i is a price of good i . Managers maximize profits. A distributor maximizes the difference in the values between the objective function and the cost of the final demand vector. The managers' maximization problem is

$$\max \sum_{j=1}^s \lambda_j g_{ij}(x_j) = 0 \quad \text{s.t. } x_j \geq 0, \quad j = 1, \dots, n \quad (3.5)$$

The necessary- sufficient conditions for solution are

$$x_j > 0: \sum_{i=1}^s \lambda_i \frac{\partial g_{ij}}{\partial x_j} = 0 \quad x_j = 0: \sum_{i=1}^s \lambda_i \frac{\partial g_{ij}}{\partial x_j} \leq 0 \quad j = 1, \dots, n \quad (3.6)$$

The distributor's maximization problem is

$$\max U(y_1, \dots, y_s) - \sum_{i=1}^s \lambda_i y_i \quad \text{s.t. } y_i \geq 0 \quad (3.7)$$

The necessary- sufficient conditions for solution are

$$y_i > 0: U_i - \lambda_i = 0 \quad y_i = 0: U_i - \lambda_i \leq 0 \quad i = 1, \dots, s \quad (3.8)$$

(3.6)-(3.8) are identical with (3.2)-(3.3). The solutions obtained from (3.1) and (3.5)-(3.7) are same when the relevant functions are structurally concave. The same consumption and production vectors are chosen. Thus, the solution of (3.1) is achieved if appropriate prices are quoted. The appropriate prices are calculated from an iterative procedure as follows. First of all, the adjustment of the final demand for good i depends on the difference between its marginal contribution to the objective function and its price.

$$\begin{cases} \dot{y}_i = 0 & \text{if } y_i = 0: U_i - \lambda_i \leq 0 \\ \dot{y}_i = a(U_i - \lambda_i) & \text{otherwise} \end{cases} \quad (3.9)$$

Each firm adjusts its operations to raise its profits.

$$\begin{cases} \dot{x}_j = 0 & \text{if } x_i = 0: \sum_{j=1}^s \lambda_j \frac{\partial g_{ij}}{\partial x_j} \leq 0 \\ \dot{x}_j = a \sum_{j=1}^s \lambda_j \frac{\partial g_{ij}}{\partial x_j} & \text{otherwise} \end{cases} \quad (3.10)$$

The price of a good is positively related to the excess demand.

$$\begin{cases} \dot{\lambda}_i = 0 & \text{if } \dot{\lambda}_i = 0: \lambda_i = 0: \sum_{j=1}^n g_{ij}(x_j) + e_i - y_i \geq 0 \\ \dot{\lambda}_i = a \left(\sum_{j=1}^n g_{ij}(x_j) + e_i - y_i \right) & \text{otherwise} \end{cases} \quad (3.11)$$

The information transmission mechanism is as follows. The centre sets prices and communicates them to firms and the distributor. Firms and the distributor calculate and inform the centre of demands and supplies. Thereby, the centre can calculate the price adjustment. These processes make it possible to converge to a solution to the problem (3.1). Thus, an optimal resource allocation can be found without receiving information about the production possibilities. Mathematically, these processes are interpreted as defining the gradient process to find the saddle-point of the Lagrangian corresponding to (3.1).

Malinvaud Approach

The planning problem analyzed by Malinvaud (1967) is as follows.

$$\max U(\mathbf{y}) \quad \text{s.t.} \quad 0 \leq \mathbf{y} \leq \mathbf{x}_j + \mathbf{e}, \quad \mathbf{x}_j \in X_j, \quad j=1, \dots, n. \quad (3.12)$$

\mathbf{y} : vector of final demand allocated.

\mathbf{e} : vector of the endowments.

$\mathbf{x}_j \in X_j$: constraints.

\mathbf{x}_j : firm j 's production program.

X_j : set of all program feasible for j .

At t^{th} iteration, the constraint $\mathbf{x}_j \in X'_j$ is used as an approximation to $\mathbf{x}_j \in X_j$. The production possibility sets are calculated by firms based on the prices announced by the central authority. Thus, the approximation to X_j is defined by

$$X'_j = \left[x \mid x = \sum_{i=1}^I \lambda_i \mathbf{x}_j^i, \lambda_i \geq 0, \sum_{i=1}^I \lambda_i = 1 \right].$$

The planning problem is rewritten as follows.

$$\max U(\mathbf{y}) \quad \text{s.t.} \quad 0 \leq \mathbf{y} \leq \mathbf{x}'_j + \mathbf{e}, \quad \mathbf{x}'_j \in X_j'^{t-1}, \quad j=1, \dots, n. \quad (3.13)$$

$$X_j^{t-1} = \left[x \mid x = \sum_{i=1}^{t-1} \lambda_i x_j^i, \lambda_i \geq 0, \sum_{i=1}^t \lambda_i = 1 \right]$$

x_j^i : firm j 's responses at earlier iteration.

The central authority solves this maximization problem and announces a set of prices (dual variables λ_i) corresponding to the solution. Firm j calculates profit-maximizing production program x_j and inform the centre of it. The constraint is $x_j \in X_j^{t-1}$ at t^{th} iteration of planning process defined by

$$X_j^{t-1} = \left[x \mid x = \sum_{i=1}^{t-1} \lambda_i x_j^i, \lambda_i \geq 0, \sum_{i=1}^t \lambda_i = 1 \right].$$

The centre establishes new approximations X_j^t to the X_j . The constraint X_j^{t-1} is replaced by X_j^t , and then the procedure is repeated. The procedure (3.13) converges to a solution of the problem (3.12).¹

Dynamic Process with Public Goods²

The process is a theoretical framework to explain how the economies to reach an equilibrium. The main factors of this framework are states of economy, solution concepts and processes. States of economy are defined by consumption, production, budget constraint, and institutional and behavioral assumptions. The typical example is as follows.

The State of economy is described as a vector $x \equiv (y_1, \dots, y_n; z_1, \dots, z_n; z, w)$. It assumes that one public good z and one private good y ($y_i \geq 0, z_i \geq 0$), and endowment ω of private good. The consumers set is $N = \{i \mid i = 1, \dots, n\}$. The utility function $u_i(y_i, z_i)$ is a C^2 class quasi-concave function.

The feasibility condition for economy is

$$\sum_{i \in N} y_i + w = \sum_{i \in N} \omega_i \tag{3.14}$$

The consumption of the private good is different, but that of public good consumption is the same.

$$z_i = z \quad \forall i \in N, \quad w = g(z). \tag{3.15}$$

The production is assumed as follows.

¹ Heal, G. (1986).

² Henry Tulkens, "Dynamic Processes for Public Goods," Journal of Public Economics, 9 (1978): 163-201.

$$w = g(z) \quad \text{where} \quad \frac{dg}{dz} > 0, \quad \text{private good input } w \geq 0 \quad \text{and public good output } z \geq 0. \quad (3.16)$$

A unit tax θ ($\theta > 0$), which is quantity of private good per unit of public good, is considered.³ θ (θ_i) is called unit tax (individualized unit tax).

Institutional and behavioral assumptions are as follows. Each agent i maximizes its own utility based on the formula $\pi_i(\cdot) \equiv \frac{\partial u_i / \partial z_i}{\partial u_i / \partial y_i}$ ⁴ $\forall i \in N$ and budget set $\{(y_i, z_i) \mid y_i + \theta z_i \leq \omega_i; y_i \geq 0, z_i \geq 0\}$, $\pi_i(y_i, z_i) = \theta$; the demands for private and public good are functions of θ and ω_i . That is, $y_i = \xi(\theta, \omega_i)$ and $z_i = \zeta(\theta, \omega_i)$. Lump sum transfer T_i of private good is incorporated. It is given to i if $T_i < 0$ (taken from i if $T_i > 0$). Then, the budget set is rewritten as $\{(y_i, z_i) \mid y_i + \theta z_i + T_i \leq \omega_i\}$, and the demand functions are rewritten as $y_i = \xi_i(\theta, T_i)$ and $z_i = \zeta_i(\theta, T_i)$.

³ It is called unit tax by Tulkens. It has different names: price of the public good (Milleron, 1972), pseudo-tax-price (Samuelson, 1966), tax-price (Buchanan, 1968), contribution rate (Malinvaud, 1971), etc.

⁴ It is the marginal rate of substitution of agent i between z_i and y_i . It can be interpreted as a price of z_i in y_i or the marginal willingness to pay (WTP) for improvement in environmental quality.

<p>Assumptions One public good z, one private good y. $y_i \geq 0, z_i \geq 0$. Endowment of private good ω. Consumers set: $N = \{i i = 1, \dots, n\}$. Utility function: $u_i(y_i, z_i)$. C^2 class quasi-concave function. Production function: $w = g(z)$. $\frac{dg}{dz} > 0$. Private good input $w \geq 0$. Public good output $z \geq 0$. Quantity of private good per unit of public good θ.</p>
<p>Utility maximization $\pi_i(\cdot) \equiv \frac{\partial u_i / \partial z_i}{\partial u_i / \partial y_i}, \{(y_i, z_i) y_i + \theta z_i \leq \omega_i; y_i \geq 0, z_i \geq 0\}, \pi_i(y_i, z_i) = \theta$.</p>
<p>Demands $z_i(t) = \zeta(\theta_i(t), \omega_i), y_i(t) = \xi(\theta_i(t), \omega_i)$.</p>
<p>Lump sum transfer of private good T_i, Budget set $\{(y_i, z_i) y_i + \theta z_i + T_i \leq \omega_i\}$, Rewritten demand functions $y_i = \xi_i(\theta_i, T_i), z_i = \zeta_i(\theta_i, T_i)$.</p>
<p>Solution depends on the defined concept of solution.</p>
<p>Process depends on the solution concept defined.</p>

Figure 3.1 Framework of Dynamic Process

Solution concept is a problem of interpretation about economic rationality depending on its definition. There are relevant issues such as existence, uniqueness and relations of solutions. The process is to explain how the economy moves from a given initial state to equilibrium which satisfies a defined solution concept. These discussions can be summarized as Figure 3.1.

There are three basic processes: Lindahl (L) process, Lindahl-Malinvaud (LM) process and Malinvaud-Drèze-de la Vallée Poussin (MDP) process. But the processes are typically classified into LM and MDP (Tulkens, 1978).⁵ Among them, the first dynamic process with public good is the Lindahl (L) process.

⁵ There are many solution concepts for economies with public goods as many classified equilibria as follows: 1) Noncooperative equilibrium (Buchanan, 1968; Malinvaud, 1969-72); 2) Bowen equilibrium (Bowen, 1944; Bergstrom and Goodman, 1973); 3) Fourgeaud equilibrium (Fourgeaud, 1969); 4) Pareto optimum (Samuelson, 1954), Pseudo equilibrium (Samuelson, 1966), Public competitive equilibrium (Foley, 1967); 5) Individually rational Pareto optimum (Drèze and de la Vallée Poussin, 1971); 6) Core allocation (Foley, 1970); 7) Lindahl equilibrium (Lindahl, 1919; Milleron, 1972); 8) Pseudo-Shapley value (Champsaur, 1975); 9) Pseudo-nucleolus (Champsaur, 1975); 10) Public equilibrium (Ruys, 1974); 11) Groves-Ledyard equilibrium (Groves and Ledyard, 1977) etc. In the case of 2, 4, 5, 7

Lindahl Process⁶

This process assumes one public good, one private good; endowments ω_1 and ω_2 ; two consumers ($i = 1, 2$), and producer. At starting point ($t = 0$), unit taxes vector is (θ_1, θ_2) such that $\theta_1 + \theta_2 = \gamma$. Demands for public good $z_i(0) = \zeta(\theta_i(0), \omega_i)$ and private good $y_i(0) = \xi(\theta_i(0), \omega_i)$ are obtained from utility maximization. Supply of public good $z(0)$ is passive because of an assumption that the producer responds passively. If condition (3.14), (3.15) and (3.16) are satisfied, then the process is terminated. If not the case, the process proceeds. When the change of unit tax is denoted by $\dot{\theta} \equiv \frac{d\theta_i}{dt}$, it is defined as follows.

$$\dot{\theta}_i(t) = a \left[\zeta_i(\theta_i(t), \omega_i) - \zeta_j(\theta_j(t), \omega_j) \right] \quad i, j = 1, 2 \text{ and } i \neq j, \quad a \in (0, \infty), \quad t \geq 0. \quad (3.17)$$

This system gives a solution of unit taxes $\theta_1(t)$, $\theta_2(t)$. Then, unit tax affects the demand $\zeta_i(\theta_i(t), \omega_i)$, $\xi_i(\theta_i(t), \omega_i)$. Consumer decreases (increases) its demand if θ is higher (lower). The demand affects the unit tax in turn. The unit tax is higher (lower) if the individual demand increases (decreases). This adjustment is repeated until the state of economy satisfies the feasibility condition. At equilibrium, the consumption amount of the private good is different between agents with the same price, but the amount of the public good is same between agents with different price.

Lindahl process is meaningful in the sense that it is the first dynamic process with public good. But it does not offer the convergence to an optimum. The utility change of the agent i is written as

$$\dot{u}_i = \frac{\partial u_i}{\partial y_i} \dot{y}_i + \frac{\partial u_i}{\partial z_i} \dot{z}_i = \frac{\partial u_i}{\partial y_i} (\dot{y}_i + \pi_i \dot{z}_i) = - \frac{\partial u_i}{\partial y_i} \dot{\theta}_i z_i \cdot \dot{u}_i < 0 \text{ if } \dot{\theta}_i > 0. \quad (3.18)$$

Consider that the public good supply should equal the smallest demand,

$$x^L \equiv (y_i^L, y_j^L; z_i^L, z_j^L; z^L, w^L) \quad (3.19)$$

and 10, the process is defined. The difference between them is in the interpretation of economic rationality (Tulkens, 1978).

⁶ Erik Lindahl. Just taxation—a positive solution. In Richard Musgrave and Alan Peacock, editors, *Classics in the Theory of Public Finance*, pages 98–123. Macmillan, London, 1958. Malinvaud (1971).

⁷ $\dot{u}_i = \frac{\partial u_i}{\partial y_i} \left(-(\theta_i \dot{z}_i + \dot{\theta}_i z_i) + \pi_i \dot{z}_i \right)$
 $= - \frac{\partial u_i}{\partial y_i} \theta_i \dot{z}_i - \frac{\partial u_i}{\partial y_i} \dot{\theta}_i z_i + \frac{\partial u_i}{\partial y_i} \pi_i \dot{z}_i = - \frac{\partial u_i}{\partial y_i} \dot{\theta}_i z_i + \frac{\partial u_i}{\partial y_i} (\pi_i \dot{z}_i - \theta_i \dot{z}_i) = - \frac{\partial u_i}{\partial y_i} \dot{\theta}_i z_i$
 $\because (\pi_i \dot{z}_i - \theta_i \dot{z}_i) = 0$ by individual utility maximization hypothesis $\pi_i = \theta_i$.

$$z^L = \min_{k \in \{i,j\}} \{ \zeta_k(\theta_k, \omega_k) \}$$

$$z_k^L = z^L$$

$$y_k^L = \omega_k - \theta_k z^L \quad \forall k \in \{i, j\}$$

$$w^L = \gamma z^L.$$

The utility function is $u_i(y_i^L, z^L)$. The utility change is $\dot{u}_i = \frac{\partial u_i}{\partial y_i} [(\pi_i - \theta_i) z^L - \dot{\theta}_i z^L]$. $\dot{u}_i > 0$ if $\dot{\theta}_i > 0$ when the demand is not the smallest $\pi_i \neq \theta_i$ (especially, $\pi_i(y_i^L, z^L)$ is sufficiently greater than θ_i). This result contradicts $\dot{u}_i < 0$ if $\dot{\theta}_i > 0$ above. In other words, both agents are better off ($\dot{u}_i > 0$) as θ_i increases if z is not minimum. Both agents are worse off ($\dot{u}_i < 0$) as θ_i increases if z is a minimum. This type of evolution of utility change may be impossible at the stationary state of an economy. As a result, the Lindahl solution will be the initial vector $(\theta_1(0), \theta_2(0))$.⁸

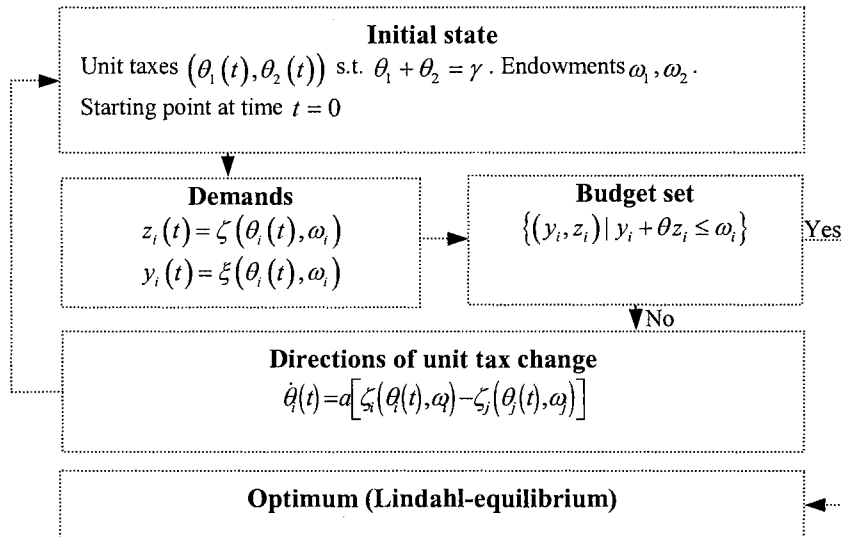


Figure 3.2 Lindahl (L) Process

⁸ Malinvaud (1971), p.102.

Lindahl-Malinvaud Process

Malinvaud (1970, 1971) reformulated Lindahl's idea to resolve the absence of convergence to an optimum and the asymmetric treatment of the agents. The Lindahl-Malinvaud process assumes more than two consumers and one producer. It introduces lump sum transfers T_i where $\sum_{i \in N} T_i = 0$.

At the starting point, unit taxes vector is $(\theta_1(0), \dots, \theta_i(0), \dots, \theta_n(0))$ s.t. $\sum_{i \in N} \theta_i(0) = \gamma$. Lump sum transfers vector is $(T_1(0), \dots, T_i(0), \dots, T_n(0))$ s.t. $\sum_{i \in N} T_i(0) = 0$. Demands are $z_i(0) = \zeta(\theta_i(0), T_i(0))$ and $y_i(0) = \xi(\theta_i(0), T_i(0)) \forall i \in N$. Supply $\bar{z}(0)$ is an average of individual demand z_i (the passive producer assumption is still valid). If condition (3.14), (3.15) and (3.16) are satisfied, it leads to pseudo-equilibrium. If not the case, then the process in a change of unit taxes and lump sum transfers proceed.

$$\dot{\theta}_i = a \left[\zeta_i(\theta_i(t), T_i(t)) - \frac{1}{n} \sum_{j \in N} \zeta_j(\theta_j(t), T_j(t)) \right] \forall i \in N. \quad (3.20)$$

$$\dot{T}_i = -\frac{1}{n} \sum_{j \in N} \zeta_j(\theta_j(t), T_j(t)) \dot{\theta}_j \quad \forall i \in N. \quad (3.21)$$

This system gives the values of unit taxes vector $(\theta_1(t), \dots, \theta_i(t), \dots, \theta_n(t))$ and lump sum transfers vector $(T_1(t), \dots, T_i(t), \dots, T_n(t))$ as solutions. Unit taxes and lump sum transfers vectors affect the demand $\zeta_i(\theta_i(t), T_i)$, $\xi_i(\theta_i(t), T_i)$ in turn. That is, consumers adjust demand for z_i and y_i . Producer adjusts supply to the change in the average demand \bar{z} . A change in unit tax increases (decreases) if demand is greater (smaller) than average demand. A change in lump sum transfers compensate agent for the raise (or decline) in his individual unit tax.

By the side condition $\dot{y}_i + \theta_i \dot{z}_i + \dot{\theta}_i z_i + \dot{T}_i = 0$ and the utility maximization hypothesis $\theta_i = \pi_i$,

$$\dot{u}_i = \frac{\partial u_i}{\partial y_i} \dot{y}_i + \frac{\partial u_i}{\partial z_i} \dot{z}_i = -\frac{\partial u_i}{\partial y_i} (\dot{\theta}_i z_i + \dot{T}_i).$$

$$\dot{u}_i = -\frac{\partial u_i}{\partial y_i} \left[-a(z_i - \bar{z})\bar{z} + a(z_i - \bar{z})z_i \right] = -\frac{\partial u_i}{\partial y_i} a(z_i - \bar{z})^2 \leq 0. \quad (3.22)$$

This result implies the property of non-decreasing monotonicity. In the LM process all agents are treated in symmetric way which is different from the case of the Lindahl process.

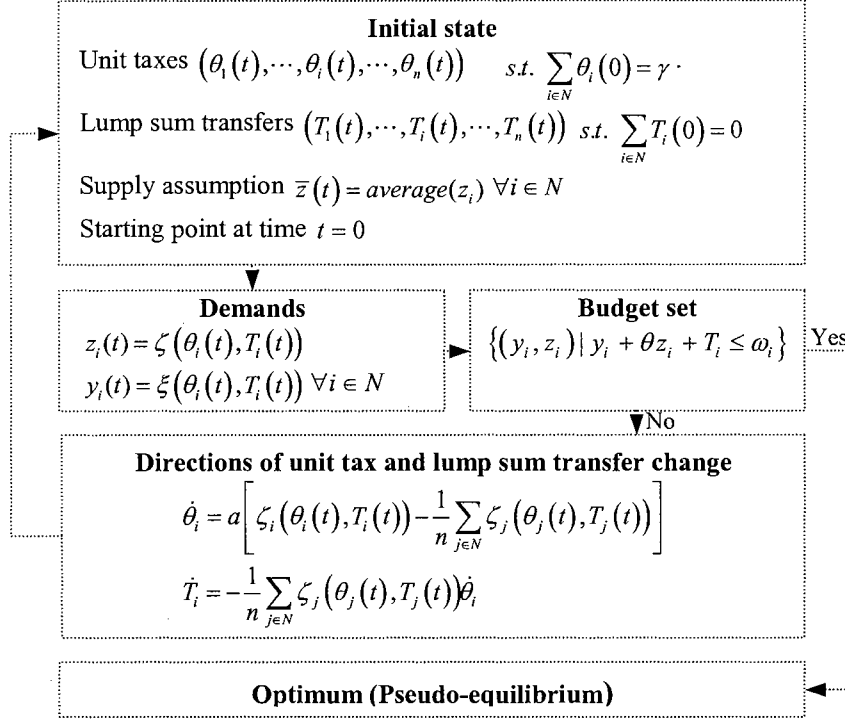


Figure 3.3 Lindahl-Malinvaud (LM) Process

Malinvaud-Drèze-Poussin Process

The Malinvaud-Drèze-Poussin process is different from the Lindahl process and the Lindahl-Malinvaud process. First, it does not use unit taxes, utility maximization hypothesis and demand functions. Lump sum transfers are generalized to all commodities. Linearity assumption of the production function is weakened by convexity assumption.

At the starting point, the state of economy is defined as

⁹ From the directions of unit tax and lump sum transfer change $\dot{\theta}_i = a(z_i - \bar{z})$, $\dot{T}_i = -\bar{z}\dot{\theta}_i$ and $\dot{\theta}_i z_i = a(z_i - \bar{z})z_i$,

$$\dot{u}_i = -\frac{\partial u_i}{\partial y_i} (\dot{\theta}_i z_i + \dot{T}_i) = -\frac{\partial u_i}{\partial y_i} [a(z_i - \bar{z})z_i - \bar{z}\dot{\theta}_i] = -\frac{\partial u_i}{\partial y_i} [a(z_i - \bar{z})z_i - \bar{z}a(z_i - \bar{z})].$$

$$\therefore \dot{u}_i = -\frac{\partial u_i}{\partial y_i} a(z_i - \bar{z})^2 \leq 0.$$

$$x(0) = [y_1(0), \dots, y_n(0); z_1(0), \dots, z_n(0); w(0), z(0)].$$

Consumer's marginal rate of substitution is $\pi_i(0) = \pi_i(y_i(0), z_i(0))$.

Producer's marginal cost is $\gamma(0) = \frac{dg(z(0))}{dz(0)}$.

If $\sum_{i \in N} \pi_i(0) = \gamma(0)$, the state $x(0)$ is an optimum. That is, there is no change.

If $\sum_{i \in N} \pi_i(0) \neq \gamma(0)$, the process proceeds as follows.

$$\dot{z}(t) = \dot{z}_i(t) = a \left(\sum_{j \in N} \pi_j(t) - \gamma(t) \right) \quad \forall i \in N \quad (3.23)$$

$$\dot{w}(t) = \gamma(t) \dot{z}(t) \quad (3.24)$$

$$\dot{y}_i(t) = -\pi_i(t) \dot{z}(t) + \delta_i a \left(\sum_{j \in N} \pi_j(t) - \gamma(t) \right)^2 \quad \forall i \in N \quad (3.25)$$

where the set of distributive weights $\delta_i \geq 0$, $\sum_{i \in N} \delta_i = 1$; adjustment coefficient $a \in (0, +\infty)$, $t \geq 0$.

The first equation (3.23) is a formalization of the idea that the change of public good is determined by the discrepancy between the sum of expressed willingness to pay for the public good and the marginal cost. The second equation (3.24) is the consequential adjustment to the private good input to produce the public good. The third equation (3.25) implies the agent's allocation of the private good. Thus, the direction of change in the private good \dot{y}_i depends on the expressed willingness to pay for the public good $(-\pi_i(t) \dot{z}(t))$, the distributive weight δ_i , and the squared error in the first order conditions. The squared error term plays a role of a device to guarantee that the feasibility condition holds in the process.¹⁰

This system gives the solution of $x(t)$. This state affects the consumer's marginal rate of substitution $\pi_i(t)$ and the producer's marginal cost $\gamma(t)$. The changes $\dot{z}(t)$, $\dot{z}_i(t)$, $\dot{w}(t)$ and $\dot{y}_i(t)$ are determined by $\pi_i(t)$ and $\gamma(t)$ in turn.

¹⁰ If the squared error term is not included in the equation, $\dot{y}_i = -\pi_i \dot{z}$ $i \in N$. The feasibility condition is

$\sum_{i \in N} \dot{y}_i + \gamma \dot{z} = -\sum_{i \in N} \pi_i a \left(\sum_{j \in N} \pi_j - \gamma \right) + \gamma a \left(\sum_{j \in N} \pi_j - \gamma \right) = -a \left(\sum_{j \in N} \pi_j - \gamma \right)^2 < 0$. That is, the process will violate the feasibility conditions if the squared error term is omitted.

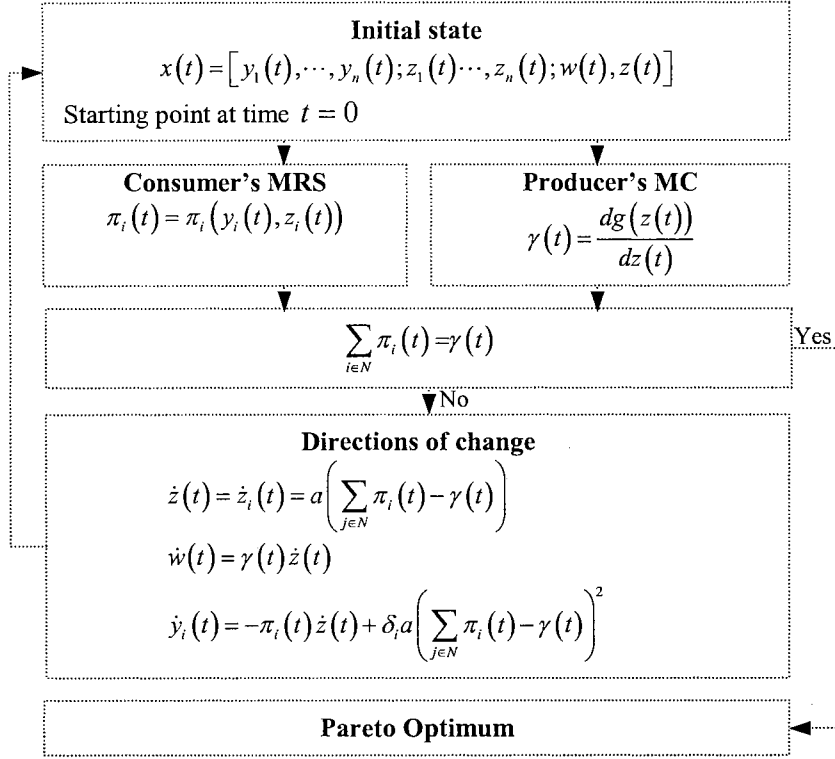


Figure 3.4 Malinvaud-Drèze-Poussin (MDP) Process

The utility change of the agent is

$$\dot{u}_i = \frac{\partial u_i}{\partial y_i} \dot{y}_i + \frac{\partial u_i}{\partial z_i} \dot{z}_i = \frac{\partial u_i}{\partial y_i} (\dot{y}_i + \pi_i \dot{z}_i) = \frac{\partial u_i}{\partial y_i} \delta_i a \left(\sum_{j \in N} \pi_j - \gamma \right)^2 \quad \text{and} \quad \dot{u}_i = \frac{\partial u_i}{\partial y_i} \delta_i a \left(\sum_{j \in N} \pi_j - \gamma \right)^2 \geq 0.$$

This result shows that the MDP process has a non-decreasing monotonicity property when $\dot{u}_i > 0$. This property implies that the utilities of all agents change in the positive direction and that there is an incentive for each participant in the process. The process converges to the Pareto optimum with individual rationality with respect to the initial state (allocation) when $\dot{u}_i = 0$.

¹¹ $\dot{y}_i + \pi_i \dot{z}_i = -\pi_i \dot{z}_i + \delta_i a \left(\sum_{j \in N} \pi_j - \gamma \right)^2 + \pi_i \dot{z}_i = \delta_i a \left(\sum_{j \in N} \pi_j - \gamma \right)^2$.

Modeling Transboundary Pollution¹²

The assumptions and the framework are as follows.

One private good $x_i \geq 0$ and one public good (public bad) $z \leq 0$;

Set of countries $N = \{i | i = 1, \dots, n\}$, a common environmental resource is shared among them;

Utility function $u_i(x_i, z)$, $\frac{\partial u_i}{\partial x_i} > 0$, $\frac{\partial u_i}{\partial z} > 0$;

Country i 's willingness to pay for an improvement in environmental quality $\pi_i = \frac{\partial u_i / \partial z}{\partial u_i / \partial x_i}$;

Production function $f_i(y_i, p_i) = 0$, $\frac{\partial f_i}{\partial y_i} > 0$, $\frac{\partial f_i}{\partial p_i} \leq 0$, private good output y_i is accompanied by

pollutant discharge p_i ;

Marginal cost in y_i of its discharge abatement $\gamma_i = -\frac{\partial f_i / \partial p_i}{\partial f_i / \partial y_i} \geq 0$;

Transferable amount of private good T_i (taken from i if $T_i < 0$, given to i if $T_i > 0$);

Feasibility condition $x_i = y_i + T_i$ for country i and $\sum x_i = \sum y_i + \sum T_i$ for all countries;

Transfer function $z = F(p_1, \dots, p_i, \dots, p_n)$, $z = \sum p_i$.

$\pi_i = \gamma_i$ at a non-cooperative equilibrium; $\pi_N \left(\equiv \sum_j^n \pi_j \right) = \gamma_i$ at a Pareto optimum.

The non-cooperative equilibrium is not a desired optimum. Only the Pareto optima are considered.

Modeling Negotiations

Two methods are considered for the economic-ecological system to move from a noncooperative equilibrium to a preferred optimum. That is, global (one shot) method and gradual (sequential) method. In the one shot method, the optimum is directly computed from the system's model. It is implemented by

¹² Parkash Chander and Henry Tulkens, "Theoretical foundations of negotiations and cost sharing in transfrontier pollution problems," *European Economic Review*, 36 (1992): 388-398. Henry Tulkens, "An Economic Model of International Negotiations Relating to Transfrontier Pollution," (Chapter 16, pp.199-212), in: K. Krippendorff, ed., *Communication and Control in Society* (Gordon and Breach, New York).

appropriate and/or incentive schemes¹³. It requires complete information on the preference, production (or cost) and transfer functions. In the sequential method, computations are simpler and repeated over time. It is based on local properties of relevant functions. The changes are designed in such a way that repetition entails a sequence of states that converges to the desired optimum. The gradual method in environmental economic issue is developed from the planning or tâtonnement approach for public goods (Malinvaud, 1970-1971; Drèze and Poussin, 1975). Tulkens (1979) applied the resource allocation process to the economic-ecological system. Planning or tâtonnement approach is a branch of the theory of resource allocation processes (Arrow and Hurwicz) as explained at the beginning of this chapter. It describes the agents' action without a planner or an auctioneer. For this reason, this model is used in modeling negotiations on a voluntary provision of public goods (Tulkens, 1992). The statement of the process is as follows.

$$\dot{p}_i = -(\pi_N - \gamma_i), \quad i = 1, \dots, n \quad (3.26)$$

$$\dot{z} = -\sum \dot{p}_i \quad (3.27)$$

$$\dot{y}_i = \gamma_i \dot{p}_i \quad (3.28)$$

$$\dot{x}_i = \dot{y}_i + \dot{T}_i \quad (3.29)$$

$$\dot{T}_i = -\gamma_i \dot{p}_i - \pi_i \dot{z} + \delta_i \sum_{j=1}^n (\pi_N - \gamma_j)^2, \quad 0 \leq \delta_i \leq 0, \quad \forall i, \quad \sum_i \delta_i = 1 \quad (3.30)$$

Equation (3.26) is the change of discharge in country i . At a Pareto optimum, $\dot{p}_i = 0$, otherwise $\dot{p}_i \neq 0$.

Equation (3.27) implies that the ambient quality change \dot{z} , which is assumed to have a simple additive form, is determined by the pollutant discharges. It represents the ecological feasibility. Equation (3.28) is a change in abatement cost. It represents the technological feasibility. Equation (3.29) is an adjustment of domestic private good production. It represents the consumption feasibility. Equation (3.30) is the net transfer. It consists of three terms. The first term is an amount received by country i . The second term is

the amount to be paid by country i . Third term $\left(\sum_{j \in N} \pi_N - \gamma_j \right)^2$ is the ecological surplus. δ_i is country i 's

fraction of ecological surplus which is constant parameter.

¹³ Mäler (1990)

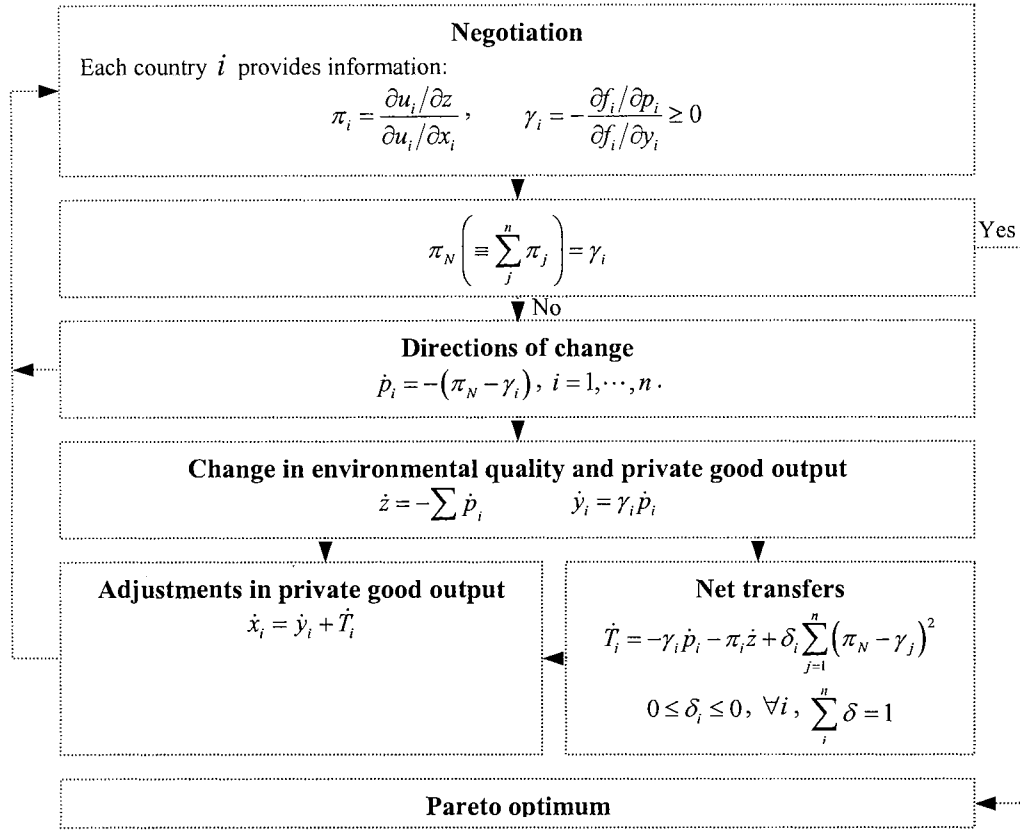


Figure 3.5 Modeling Negotiations

In the negotiation, each country provides information on its willingness $\pi_i(t)$ to pay for a marginal improvement in the environmental quality $z(t)$ and the marginal cost $\gamma_i(t)$ of discharge reduction $p_i(t)$. A discharge change \dot{p}_i is calculated by given information. The changes \dot{z} of environmental quality and \dot{y}_i of abatement cost in the private good are determined by the sum of discharge changes \dot{p}_i . With \dot{p}_i , \dot{z} and given δ_i , the net transfers \dot{T}_i are calculated. The transfers are given to i if $\dot{T}_i > 0$ and taken from i if $\dot{T}_i < 0$. If the country i does not join the coalition for the negotiation, there are no net transfers ($\dot{T}_i = 0$) and no change in the utility ($\dot{u}_i = 0$). If the country i joins the coalition, it is possible that $\dot{T}_i > 0$ and $\dot{u}_i > 0$ because of the possibility that $\pi_N \left(\equiv \sum_j \pi_j \right) > \gamma_i$ if $\pi_i = \gamma_i$. This shows that there is an incentive for country i to join the coalition for the negotiation. In addition, a Pareto optimum can be reached through negotiations.

Modeling Cost Sharing

This is a problem of an ecological surplus sharing rule. Chander and Tulkens (1991) defined the distribution profile $\delta = (\delta_1, \dots, \delta_i, \dots, \delta_n)$ as follows.

$$\delta_i = \pi_i(t) / \pi_N(t), \quad i = 1, \dots, n. \quad (3.31)$$

Rewrite (3.30) using (3.31),¹⁴

$$\dot{T}_i = -\gamma_i \dot{p}_i + \frac{\pi_i(t)}{\pi_N(t)} \sum_{j=1}^n \gamma_j \dot{p}_j, \quad i = 1, \dots, n. \quad (3.32)$$

Rewrite (3.29) using (3.32) and (3.28),

$$\dot{x}_i = \frac{\pi_i(t)}{\pi_N(t)} \sum_{j=1}^n \gamma_j \dot{p}_j, \quad i = 1, \dots, n. \quad (3.33)$$

Equation (3.33) shows that the consumption cost (foregone consumption) change \dot{x} is proportional to the preferences intensity π_i of country i relative to the total preferences intensity π_N for environmental quality. The idea of cost sharing rule is that the ecological surplus is distributed depending to the contribution of each country to the improvement of environmental quality.

Another relevant issue is the correctness of information which is provided by the members of the coalition. Three cases are discussed in literatures: First, incorrect information on its preferences π_i and/or

¹⁴ From (3.30), where $0 \leq \delta_i \leq 1$, $\sum_{i=1}^n \delta_i = 1$,

$$\begin{aligned} \dot{T}_i &= -\gamma_i \dot{p}_i - \pi_i \dot{z} + \delta_i \sum_{j=1}^n (\pi_N - \gamma_j)^2 \\ &= -\gamma_i \dot{p}_i - \pi_i (-\sum \dot{p}_i) + \delta_i \sum_{j=1}^n (\pi_N - \gamma_j)^2 \quad \text{using (3.27)} \\ &= -\gamma_i \dot{p}_i - \pi_i \sum (\pi_N - \gamma_j) + \delta_i \sum_{j=1}^n (\pi_N - \gamma_j)^2 \\ &= -\gamma_i \dot{p}_i - \pi_i \sum (\pi_N - \gamma_j) + \frac{\pi_i(t)}{\pi_N(t)} \sum_{j=1}^n (\pi_N - \gamma_j)^2 \quad \because \delta_i = \frac{\pi_i(t)}{\pi_N(t)} \text{ in (3.31).} \\ &= -\gamma_i \dot{p}_i + \frac{\pi_i(t)}{\pi_N(t)} \sum_{j=1}^n [-(\pi_N^2 - \pi_N \gamma_j) + (\pi_N^2 - 2\pi_N \gamma_j + \gamma_j^2)]^2 \\ &= -\gamma_i \dot{p}_i + \frac{\pi_i(t)}{\pi_N(t)} \sum_{j=1}^n [-(\pi_N - \gamma_j) \gamma_j] \\ &= -\gamma_i \dot{p}_i + \frac{\pi_i(t)}{\pi_N(t)} \sum_{j=1}^n \gamma_j \dot{p}_j \quad \text{by (3.26).} \end{aligned}$$

cost γ_i ; Second, no participation; Third, the direction of change of the pollutant discharges, \dot{p}_i , determined by its own interest. Local games were introduced to analyze these behaviors in the 70s.¹⁵ This issue is also important, but it is not discussed in the dissertation.

Cost Functions

Consider the abatement cost function $C_i(E_i)$ in terms of emission E_i where C_i denotes the total abatement cost and E_i denotes the emission from the region i . The abatement cost function is assumed to be a C^2 class convex function. Using the quadratic approximation technique, the abatement cost function can be evaluated at \bar{E}_i as follows.

$$C_i(E_i) = C_i(\bar{E}_i) + C_i'(\bar{E}_i)(E_i - \bar{E}_i) + \frac{1}{2}C_i''(\bar{E}_i)(E_i - \bar{E}_i)^2 \cdot C_i' < 0, C_i'' > 0.$$

$$C_i(E_i) = \gamma_i + \alpha_i(\bar{E}_i - E_i) + \beta_i(\bar{E}_i - E_i)^2 \text{ where } \gamma_i = C_i(\bar{E}_i), \alpha_i = -C_i'(\bar{E}_i), \text{ and } \beta_i = \frac{1}{2}C_i''(\bar{E}_i) \text{ and } \gamma_i, \alpha_i, \beta_i > 0.$$

γ_i is total abatement cost. α_i is the marginal abatement cost. β_i is a parameter to be estimated. This form of abatement cost function is assumed in the negotiation model because of its appropriate properties.

Damage Functions¹⁶

To derive damage function, assume that the environment has well-defined characteristics, then

$$Y \in R^m.$$

Y : Ambient concentration.

R : Consumers' preferences.

Assume that the physical changes in environmental quality due to changes in human activities, then environmental interaction function F can be defined as follows.

¹⁵ Roberts, J., "Incentives in Planning Procedures for the Provision of Public Goods," *Review of Economic Studies*, 46, no. 2, 1979: 283-292; Chander, P. and H. Tulkens, 1991, "Strategically Stable Cost Sharing in an Economic-Ecological Negotiation Process," Paper presented at the Second Congress of the European Association of Environment and Resource Economists, Stockholm, June: CORE discussion paper no. 9135; Chander, P., 1987, "Cost-Sharing Local Games in Dynamic Processes for Public Goods," in: B. Dutta and D. Ray, eds, *Theoretical Issues in Development Economics* (Oxford University Press, India); Chander and Tulkens, "Theoretical Foundations of Negotiations and Cost Sharing in Transfrontier Pollution Problems," *European Economic Review*, 36, 1992: p.397

¹⁶ Karl-Göran Mäler, "Damage Functions and Their Estimation: A Theoretical Survey," Environmental Damage Costs, OECD, (1974): 223-252; Andreu Mas-Colell, Michael D. Whinston and Jerry R. Green, "Microeconomic Theory," Oxford University Press, 1995; Geoffrey A. Jehle and Philip J. Reny, "Advanced Microeconomic Theory," 1998.

$$Y = F(z) \quad (3.34)$$

z : Activities.

F : Environmental interaction function (physical damage function).

Assume that environmental quality is a public good. Thus, vector Y has a property of a public good. It means that no one can change the quality only for himself.

Utility maximization problem (UMP):

$$u(\mathbf{c}, Y) \quad (3.35)$$

A utility function u represents the consumers' preferences for private consumption and for environmental quality. It is assumed to be a C^2 class function defined on R^{n+m} . The n -vector \mathbf{c} is net demands for private marketable goods and services. It stands for consumption if positive and supply if negative.

Budget constraint is

$$\mathbf{p}^T \mathbf{c} \leq w \quad (3.36)$$

\mathbf{p} : price vectors.

w : wealth or lump sum income.

$\mathbf{p}^T \mathbf{c}$: inner product of \mathbf{p} and \mathbf{c} .

First order necessary condition is

$$u_i - \alpha p_i = 0 \quad (i = 1, \dots, n) \quad (3.37)$$

$$u_i = \frac{\partial u}{\partial c_i}$$

α : Lagrange multiplier.

The solutions to the UMP is the Marshallian demand functions c .

$$\mathbf{c} = \mathbf{c}(\mathbf{p}, Y, w). \quad (3.38)$$

The indirect utility function v is obtained by substituting (3.38) into (3.35).

$$v(\mathbf{p}, Y, w) = u(\mathbf{c}(\mathbf{p}, Y, w), Y) \quad (3.39)$$

Expenditure minimization problem (EMP):

$$\text{Min } \mathbf{p}^T \mathbf{c} \quad \text{s.t. } u(\mathbf{c}, Y) \geq \bar{u}.$$

This is the dual problem of the expenditure minimization at a given utility level \bar{u} .

First order necessary condition is

$$\beta u_i - p_i = 0 \quad (i = 1, \dots, n) \quad (3.40)$$

β : Lagrange multiplier.

If the solution to the EMP is c^* , the Hicksian (or compensated) demand function is

$$\mathbf{h} = \mathbf{c}^*(\mathbf{p}, Y, \bar{u}). \quad (3.41)$$

The expenditure function is

$$e(\mathbf{p}, Y, \bar{u}) = \mathbf{p}^\top \mathbf{h}(\mathbf{p}, Y, \bar{u}) \quad (3.42)$$

By Shephard's lemma,

$$\frac{\partial e(\mathbf{p}, Y, \bar{u})}{\partial p_i} = h_i(\mathbf{p}, Y, \bar{u}).$$

Taking derivative with respect to p_j ,

$$\begin{aligned} \frac{\partial}{\partial p_j} \left(\frac{\partial e(\mathbf{p}, Y, \bar{u})}{\partial p_i} \right) &= \frac{\partial h_i(\mathbf{p}, Y, \bar{u})}{\partial p_j} = \frac{\partial c_i(p_i, Y, e(p_i, Y, \bar{u}))}{\partial p_j} \\ \frac{\partial^2 e}{\partial p_j \partial p_i} &= \frac{\partial c_i(p_i, Y, e(p_i))}{\partial p_j} + \frac{\partial c_i(p_i, Y, e(p_i))}{\partial w} \frac{\partial e(p_i)}{\partial p_j}. \end{aligned}$$

Slutsky equation is

$$\frac{\partial^2 e}{\partial p_j \partial p_i} - \frac{\partial c_i(p_i, Y, e(p_i))}{\partial p_j} - \frac{\partial c_i(p_i, Y, e(p_i))}{\partial w} \frac{\partial e(p_i)}{\partial p_j} = 0 \quad (3.43)$$

or
$$\frac{\partial^2 e}{\partial p_j \partial p_i} - \frac{\partial c_i(p_i, Y, e(p_i))}{\partial p_j} - \frac{\partial c_i(p_i, Y, e(p_i))}{\partial w} c_i(p, Y, w) = 0$$

Thus, the expenditure function has properties as follows.

Shephard's lemma:
$$\frac{\partial e(\mathbf{p}, Y, \bar{u})}{\partial p_i} = h_i(\mathbf{p}, Y, \bar{u}).$$

Relationship between Marshallian and Hicksian demand:

$$c(\mathbf{p}, e(\mathbf{p}, Y, \bar{u})) = \mathbf{h}(\mathbf{p}, Y, \bar{u}) \cdot \quad \mathbf{h}(\mathbf{p}, v(\mathbf{p}, w), Y) = c(\mathbf{p}, w, Y).$$

e : concave function in \mathbf{p} .

Slutsky equation:
$$\frac{\partial^2 e}{\partial p_j \partial p_i} - \frac{\partial c_i(p_i, Y, e(p_i))}{\partial p_j} - \frac{\partial c_i(p_i, Y, e(p_i))}{\partial w} \frac{\partial e(p_i)}{\partial p_j} = 0.$$

$$\text{Symmetric substitution: } \frac{\partial h_i}{\partial p_j} = \frac{\partial h_j}{\partial p_i}.$$

If u is quasi concave in (p, Y) , then e is convex in Y .

Damage functions for one individual:

The damage function is defined as what must be compensated to the individual as the environment deteriorates. Thus, the damage function for one individual is the compensating variation as

$$CV(p, Y, \bar{u}) = e(p, Y, \bar{u}) - I \quad (3.44)$$

\bar{u} : predetermined level of utility.

I : Consumer's income level.

Assume three goods (labor supply, medical care and private good). Consider two situations of Y' and Y'' . Y' (Y'') is a case that the individual is healthy (sick). The damage function is defined as the compensating variation as

$$CV = e(p, Y'', \bar{u}) - I = \sum_{i=1}^n p_i (c_i^*(p, Y'', \bar{u}) - c_i(p, Y', I)).$$

If the individual is sick, the sickness will prevent him from supplying labor c_1 .

$$c_1^*(p, Y'', \bar{u}) = c_1(p, Y'', I) = 0. \quad (3.45)$$

If the individual is healthy, he will not need to spend money on medical care c_2 .

$$c_2(p, Y', I) = 0. \quad (3.46)$$

Assume that the income effect on the demand for medical care c_2 is ignored.

$$c_2^*(p, Y'', \bar{u}) = c_2(p, Y'', I). \quad (3.47)$$

From the three equations above, the damage function will be

$$\begin{aligned} CV &= e(p, Y'', \bar{u}) - I = \sum_{i=1}^n p_i (c_i^*(p, Y'', \bar{u}) - c_i(p, Y', I)) \\ &= p_1 (c_1^*(p, Y'', \bar{u}) - c_1(p, Y', I)) + p_2 (c_2^*(p, Y'', \bar{u}) - c_2(p, Y', I)) + p_3 (c_3^*(p, Y'', \bar{u}) - c_3(p, Y', I)) \text{ by (3.45)} \\ &= p_2 (c_2^*(p, Y'', \bar{u}) - c_2(p, Y', I)) = p_2 c_2(p, Y'', \bar{u}) \text{ by (3.46) and (3.47)}. \\ \therefore CV &= -p_1 c_1(p, Y', I) + p_2 c_2(p, Y'', \bar{u}) + p_3 c_3^*(p, Y'', \bar{u}) - p_3 c_3(p, Y', I). \end{aligned}$$

This implies that the compensating variation consists of the loss in wage income $(-p_1c_1(p, Y', I))$, an increase in expenditure for medical care $(p_2c_2(p, Y'', \bar{u}))$, and an increase in expenditure for private good consumption $(p_3c_3^*(p, Y'', \bar{u}) - p_3c_3(p, Y', I))$ to compensate the individual for the sickness. Thus, estimating individual damage functions is a problem of estimating the individual expenditure functions. The problem is that the computed expenditure function from the individual demand is not a function of environmental qualities but a function of prices.

Assume that the utility function is $u(c, r, Y)$, $r \in R^k$ where r is a quality vector of goods and services. The quality r may be lower (higher) by the pollution (maintenance). Suppose that $x \in R^n$ is a vector of goods and services used as inputs for maintenance. Thus, the quality vector is a function of x and Y as $r = g(x, Y)$. The rewritten demand function is $\hat{c} = (c, x)$. The rewritten utility function is $\hat{u}(\hat{c}, Y) = u(c, g(x, Y), Y)$. The rewritten utility function is different from the previous one, because it includes maintenance.

Now consider the production side. The pollution affects the production possibilities. This may lead to the decrease (ΔI) of the profits of the firm. This change may appear on the individual income and damage functions. The demand price for environmental quality is defined as

$$\delta_j = -\frac{\partial e}{\partial Y} \quad (3.48)$$

Social damage functions:

Assumptions are as follows. Consumer set is $\{s|1, \dots, S\}$. All markets are cleared. All prices reflect social costs from which environmental costs are subtracted. Social welfare function represents the preferences of the decision maker. The decision maker uses the damage function to make it possible to judge whether a change in the human activity vector z is desirable or not.

Suppose that revenue R and the damage D are yielded by the change in z . The desirable change occurs only if $R > D$. The human activity may affect different persons differently. Considering the difference in the preferences of individuals, it should be possible to use the income redistribution as a tool for decision maker to change the situation for individual. It is called lump sum transfers. Two cases are considered. The one is a case that lump sum transfers are feasible. The other is a case that lump sum

transfers are not feasible. However, the basis for the decision maker to change the situation is only Pareto criterion.

When the lump sum transfers are feasible Pareto criterion can be explained as follows. First, the aggregated expenditure function is defined as the sum of the individual expenditure functions.

$$e(p, Y, \bar{u}^1, \dots, \bar{u}^s) = \sum_{s=1}^S e^s(p, Y, \bar{u}^s) \quad (3.49)$$

The total lump sum income (wealth) I is

$$I = \sum_{s=1}^S I^s \quad (3.50)$$

The social damage function is defined as

$$CV = e(p, Y, \bar{u}^1, \dots, \bar{u}^s) - I = \sum_{s=1}^S CV^s \quad (3.51)$$

CV^s : compensating variation for individual s .

If the revenue R , which is yield by the human activity z , is greater than the compensating variation CV , the decision maker can transfer revenue so that every individual can have necessary or more amount to compensate himself. But if $R > D$ does not hold, the lump sum transfers are not feasible from the viewpoint of Pareto criterion.

When the lump sum transfers are feasible the social welfare function is as follows. Assume that preferences on the distribution of real income, environmental quality, and the idea of consumer sovereignty represent the preferences of the decision maker. These preferences are assumed to be represented by an indirect social welfare utility function.

$$V(p, Y, I^1, \dots, I^s) \quad (3.52)$$

By Roy's identity when $\frac{\partial V}{\partial I^s} > 0$ ($s=1, \dots, S$) is assumed,

$$\frac{\partial V}{\partial p^i} = - \sum_{s=1}^S c_i^s \frac{\partial V}{\partial I^s}, \quad i=1, \dots, n. \quad (3.53)$$

Equation (3.53) can be rewritten with equation (3.48) as

$$\frac{\partial V}{\partial Y^i} = \sum_{s=1}^S \delta_j \frac{\partial V}{\partial I^s}, \quad j=1, \dots, m. \quad (3.54)$$

This equation says that the welfare change, which is due to a change in environmental quality, equals the sum of the individual preferences on the environmental quality. This is a condition for the existence of a welfare function $W(c_1, \dots, c_s)$ such that

$$V(p, Y, I^1, \dots, I^S) = W(v^1(p, Y, I^1), \dots, v^S(p, Y, I^S)) \quad (3.55)$$

$v^s(p, Y, I^s)$: indirect utility function of individual s .

The social expenditure function can be defined in the similar way as the individual expenditure.

Expenditure minimization problem is

$$\text{Min } p^T \sum_{s=1}^S c^s \quad \text{s.t. } W(u^1(c^1, Y), \dots, u^S(c^S, Y)) \geq \bar{U}$$

\bar{U} : give social welfare level.

Its solution is the social compensated demand functions c^s .

$$c^s = c^{s*}(p, Y, \bar{U}), \quad s = 1, \dots, S. \quad (3.56)$$

The social expenditure function is

$$E(p, Y, \bar{U}) = p^T c^{s*}(p, Y, \bar{U}). \quad (3.57)$$

To examine the relationship between the social expenditure function and the individual expenditure function, assume that

$$\bar{u}^s = u^s(c^{s*}(p, Y, \bar{U}), Y), \quad s = 1, \dots, S. \quad (3.58)$$

This individual utility function depends on the consumptions which are adjusted by the lump sum transfers.

The social expenditure function is the sum of the individual expenditure functions as

$$E(p, Y, \bar{U}) = \sum_{s=1}^S e^s(p, Y, \bar{u}^s) \quad (3.59)$$

The social damage function is written as

$$CV = E(p, Y, \bar{U}) - I \quad (3.60)$$

Estimation of social damage functions:

The environmental quality as a public good does not have a market in which the individual preferences can be revealed. Thus, the demand function and the expenditure function cannot be derived based on the revealed preference as in the case of the private goods.

The typical ways to estimate the damage function are evaluation of direct costs, asking people about willingness to pay, voting, a study of individual responses to environmental deterioration, and a study of market responses and so forth. The evaluation of direct costs is to calculate expenditures on medical care and the wage income loss. This method is not popular from the theoretical viewpoint. The way to ask people about their willingness to pay is used often to estimate damage function. The problem in this method is that there is the possibility of distorted answer when people have incentives to do so. Voting is used to get information on the opinion of people about environmental quality, but it is not an appropriate method to get information on the damage function. A study of individual responses to environmental deterioration is used when the relationship between the environmental quality and the private good is a perfect substitute. A study of market responses can be used when the individual responses to the change in the environmental quality cause the change in the market price.

CHAPTER IV

METHODOLOGY

The purpose of this chapter is to suggest the model structure to resolve disputes on the phosphorus problem in the Illinois River Basin. The main issue is how to reflect the characteristics of the problem in the model theoretically and practically. The phosphorus problem in the Illinois River Basin can be viewed by three different approaches as follows.

First, the unidirectional externalities approach with side-payment may be considered. There is no practical evidence that this approach has been considered. But theoretically this is one of the typical ways to deal with a river pollution problem. Especially, side-payment is a device for the downstream region to control the pollution discharged from the upstream region. The basic assumption of this approach is that the upstream region is unique polluter and the downstream region is unique victim. Thus, the polluter pays principle¹⁷ is an important issue of this approach. There are some issues to be examined as TMDL is introduced. The first one is whether the assumption of this approach is appropriate. This is because the sub-regions could be victims as well as polluters if the pollution is not unidirectional. The second one is whether its solution captures benefits and costs correctly. This is because unrevealed benefits can be accrued in the upstream region with a same reason. If the results of the analysis support these ideas, we will be able to understand the reason why this approach has not been considered.

¹⁷ The polluter pays principle (PPP) was adopted by the OECD in 1972 as a guideline for domestic environmental policies. The reasons for adopting PPP are as follows. Any subsidized pollution abatement can be biased incentives and cause distortions in the domestic economy. Second, considering the long run technical development, companies can be rewarded by developing better and less expensive abatement technologies. Third, the application of PPP would not create unintended distortions in foreign trade. That is, the exporter would have to absorb the total social cost for the product (Mäler, 1990).

Second, the enforcement rule approach may be considered. The US Supreme Court decision¹⁸ in 1992 is practical evidence that this approach was considered. This approach is same as the first approach without side-payment. They are based on the same theoretical assumption of unidirectional externality. There are some questions to be investigated with a same reason as above. The first one is whether its assumption is appropriate as in the first approach. The second one is whether its solution captures benefits and costs correctly. This is also same as in the first approach. Third one is whether the guideline of enforced abatement can be acceptable to AR. If the results of analysis support these ideas, we will be able to understand the reason why this approach could not be effective.

Third, the negotiation approach may be considered. But this approach has never been examined to resolve the problem in the Illinois River Basin. This approach is based on the assumption of reciprocal externality. It means that all players can be victims as well as polluters at the same time. This assumption is compatible with a change in situation and policy due to the introduction of TMDL. The previous literatures¹⁹ proved that this approach satisfies the individual rationality and group rationality. Also, it is told that this approach provides all players with incentives to participate in the coalition for the negotiation when financial transfers are used. One concern in applying this approach to the Illinois River Basin is the availability of required data. But it is an issue related to numerical application which will be discussed in the next chapter.

These three approaches are considered as option-1, option-2 and option-3 respectively. The organization of this chapter is as follows. First part describes each option. Second part investigates option-3 to see whether the negotiation approach provides incentives to all participants (sub-regions of AR and OK). Third part examines payoffs obtained from three options. Through these procedures, the priority of options to AR and OK may be revealed. But any implications for the sub-regions may not be available. This is because specific aspects of transition between optima can be captured by numerical application. This

¹⁸ Tomas S. Soerens, Edward H. Fite, OSRC, Janie Hipp, "Water Quality in the Illinois River: Conflict and Cooperation between Oklahoma and Arkansas." Diffuse Pollution Conference Dublin 2003

¹⁹ M. Germain and PH.L. Toint (2000), "An Iterative Process for International Negotiations on Acid Rain in Northern Europe Using a General Convex Formulation," *Environmental and Resource Economics*, 15: 199-216; Kaitala, Mäler, and Tulkens (1995), "The Acid Rain Game as a Resource Allocation Process with an Application to the International Cooperation Among Finland, Russia and Estonia," *Scandinavian Journal of Economics*, 97 (2): 325-343.

chapter suggests the framework of the model and formulas which will be used for numerical application in Chapter V.

Unidirectional Externalities Approach with Side-payment (option-1)

Let's begin with the unidirectional externalities approach without side-payment. Region-1 and region-2 are unique upstream polluter and unique downstream victim respectively. Suppose that region-1 reduces the pollution. The payoffs are

$$\text{Payoff for region-1: } NB_1 = -C(R). \quad (4.1)$$

$$\text{Payoff for region-2: } NB_2 = B(R). \quad (4.2)$$

$C(R)$: cost function for region-1.

$B(R)$: benefit function for region-2.

If region-1 has the initial right to pollute the river, there is no incentive for region-1 to reduce its pollution. There is no way for region-2 to control pollutions discharged from region-1. Both sides cannot reach an agreement in this framework. However, the Coase theorem says that upstream and downstream regions can reach an agreement that the upstream region restricts its pollution to E^* when some assumptions are satisfied.²⁰

Now consider unidirectional externalities approach with side-payment (option-1). Assume that region-1 agrees to the pollution reduction with a side-payment S paid by region-2. Specifically, region-1 will reduce its pollution by R_1 if a side-payment S_1 is paid by region-2. Region-2 will pay S_2 if region-1 reduces the pollution with R_2 . The payoffs for region-1 and region-2 are written as

$$\text{Payoff for region-1: } NB_1 = S - C(R). \quad (4.3)$$

$$\text{Payoff for region-2: } NB_2 = B(R) - S. \quad (4.4)$$

²⁰ The assumptions are as follows: "(1) Both countries know the abatement cost function and the damage cost function in each country, (2) there are no transaction costs, (3) the original distribution of property rights is well defined, (4) the pollution of the river can be seen in isolation from other international relations, (5) the change in the distribution of the rights between the countries will not change the abatement-cost or the damage-cost functions." Mäler (1990), "International Environmental Problems." *Oxford Review of Economic Policy*, Vol. 6, No. 1, p.84.

Let (R_1, S_1) denote the offer from region-1. Let (R_2, S_2) denote the offer from region-2. The condition for an agreement is as follows. Region-1 will accept (R_2, S_2) if the payoff obtained from (R_2, S_2) is equal to or greater than the payoff from its own offer. Region-2 will accept (R_1, S_1) if the payoff obtained from (R_1, S_1) is equal to or greater than the payoff from its own offer. These conditions are written as

$$\text{Region-1 accepts } (R_2, S_2) \text{ if } S_2 - C(R_2) \geq S_1 - C(R_1). \quad (4.5)$$

$$\text{Region-2 accepts } (R_1, S_1) \text{ if } B(R_1) - S_1 \geq B(R_2) - S_2. \quad (4.6)$$

Best reply is a net benefit maximization problem subject to agreement condition. Net benefit maximization problem for region-1 is

$$\text{Max } [S - C(R)] \quad \text{s.t. } B(R) - S \geq B(R_2) - S_2. \quad (4.7)$$

If the solution to this problem is $R_1 = R^*$,

$$\text{Side-payment: } S_1 = B(R^*) - B(R_2) + S_2 \quad (4.8)$$

$$\text{Best reply: } (S_1, R^*) \text{ if } [S - C(R)] \geq 0; (0, 0) \text{ otherwise.} \quad (4.9)$$

Net benefit maximization problem for region-2 is

$$\text{Max } [B(R) - S] \quad \text{s.t. } S - C(R) \geq S_1 - C(R_1). \quad (4.10)$$

If the solution to this problem is $R_2 = R^*$,

$$\text{Side-payment: } S_2 = C(R^*) - C(R_1) + S_1 \quad (4.11)$$

$$\text{Best reply: } (S_2, R^*) \text{ if } [B(R^*) - S] \geq 0; (0, 0) \text{ otherwise.} \quad (4.12)$$

The results are as follows.

$$\text{Nash non-cooperative equilibrium set: } \{(R^*, S) : B(R^*) \geq S \geq C(R^*)\}. \quad (4.13)$$

$$\text{Equilibrium payoffs set: } \{(NB_1, NB_2) : NB_1 + NB_2 = B(R^*) - C(R^*)\}. \quad (4.14)$$

With the assumptions of identical bargaining power and cooperation, the results are written as

$$\text{Total gains: } B(R^*) - C(R^*). \quad (4.15)$$

$$\text{Payoffs: } NB_1 = NB_2 = \frac{1}{2}[B(R^*) - C(R^*)]. \quad (4.16)$$

$$\text{Side-payment: } S = \frac{1}{2}[B(R^*) + C(R^*)]. \quad (4.17)$$

Enforcement Rule Approach (option-2)

Suppose that region-1 is required to reduce its pollution by R . The payoffs are written as

$$\text{Payoff for region-1: } NB_1 = -C(R) < 0. \quad (4.1)$$

$$\text{Payoff for region-2: } NB_2 = B(R) > 0. \quad (4.2)$$

This is same as unidirectional externalities approach without side-payment.

Negotiation Approach (option-3)

The situation assumed by this approach is as follows. Consider sub-regions which are indexed by i ($i = 1, \dots, n$). These sub-regions belong to region-1 or region-2 indexed by z ($z = 1, 2$). These n sub-regions join the coalition for the negotiation to reduce pollution. Each sub-region knows its current level of emission and cost function. Each member must reduce emission every year by τ (ratio to the emission level of the previous year). The negotiation is held once a year until Pareto optimum is reached. The model structure is as follows.

$$\text{Region set: } Z = \{z | z = 1, 2\}.$$

$$\text{Sub-region set: } I = \{i | i = 1, \dots, n\}. I \subset Z.$$

$$\text{Aggregate total cost function: } J_i(E) = C_i(E_i) + D_i(Q_i(E)) \quad (4.18)$$

$$E : \text{emission } (n \times 1). E > 0 \quad (4.19)$$

Total abatement cost: $C_i(E_i)$. $C'_i < 0$, $C''_i > 0$. (Assume C^2 class convex function).

$$\text{Deposition function: } Q = AE. \quad (4.20)$$

$$A = (a_{ij}): \text{transport coefficient matrix } (n \times n). i, j = 1, \dots, n.$$

$$a_{ij} \geq 0. i \text{ is receiving region. } j \text{ is emitting region.}$$

$$\text{Damage function: } D(Q) = \pi Q. D'_i \geq 0, D''_i \geq 0. \text{ (Assume linear function).} \quad (4.21)$$

$$Q_i = \sum_{j=1}^n a_{ij} E_j : \text{deposition } (n \times 1).$$

π : value of damage per unit of deposition $(1 \times n)$.

Let's begin with a case that global information is available. Under noncooperation, sub-region i 's aggregate cost minimization problem is

$$\text{Min}_{E_i} J_i(E_i) = C_i(E_i) + D_i(Q_i(E)) \quad \text{s.t. } Q = AE, \quad E_i > 0, \quad i, j = 1, \dots, n. \quad (4.22)$$

The optimality condition to solve this problem is

$$C'_i(E_i) + a_{ii} D'_i(Q_i(E)) = 0. \quad (4.23)$$

Under cooperation, sub-region i 's aggregate cost minimization problem is

$$\text{Min}_{E_1, E_2, \dots, E_n} J = \sum_{i=1}^n (C_i(E_i) + D_i(Q_i(E))) \quad \text{s.t. } Q = AE, \quad E_i > 0. \quad (4.24)$$

The optimality condition to solve this problem is

$$C'_i(E_i) + \sum_{j=1}^n a_{ij} D'_j(Q_j(E)) = 0 \quad i, j = 1, \dots, n. \quad (4.25)$$

There is a difference in optimality conditions between noncooperation and cooperation. Additional term under cooperation is $\sum_{j=1}^n a_{ij} D'_j(Q_j(E))$, $j \neq i$. This implies a sub-region is required to reduce greater emission under cooperation. This is an important property to bring more benefits under cooperation.

Now suppose that only local information is available. Assume that each sub-region i knows its current emission level, marginal abatement cost and marginal damage cost. It is more realistic situation, because total aggregate cost function J may not be known to sub-regions in usual cases. With discrete time scale, the constraints are rewritten as

$$Q_t = AE_t, \quad E_t > F_{t,i}. \quad t : \text{stage of the negotiation.} \quad (4.26)$$

Assume that emission is lower-bounded due to technological limit. Information about emission level and its lower bound is updated at each stage of the negotiation. These assumptions are written as

$$E_{i,t} > F_{i,t} \geq 0. \quad (4.27)$$

$$F_{i,t+1} = \min[F_{i,t}, \tau E_{i,t+1}] \quad (0 \leq \tau_{i,t+1} \leq \tau, 0 < \tau \leq 1). \quad (4.28)$$

The emission converges to the Pareto optimal level if level set of cost is upper-bounded as²¹

$$L_0 = \{E : J(E) \leq J(\tilde{E}) \text{ and } E_i \text{ feasible } (i=1, \dots, n)\}; \quad \lim_{\|\tilde{E}\| \rightarrow \infty} \sum_{i=1}^n D_i(Q_i(E)) = \infty. \quad (4.29)$$

The initial level of emission \tilde{E} is assumed as a noncooperative Nash equilibrium. As emission level changes from an initial level to Pareto optimum, aggregate cost also changes. Difference in aggregate cost during the transition of emission level from a noncooperative Nash equilibrium to Pareto optimum is

$$J_i(E^*) - J_i(\tilde{E}) = C_i(E^*) - C_i(\tilde{E}) + D_i(E^*) - D_i(\tilde{E}). \quad (4.30)$$

\tilde{E} : initial level of emission (noncooperative Nash equilibrium).

E^* : Pareto optimum.

If $J_i(E^*) < J_i(\tilde{E})$, sub-region i benefits from the negotiation. The difference $J_i(\tilde{E}) - J_i(E^*)$ is called ecological surplus. The ecological surplus is not known until the end of the negotiation procedures. To make ecological surplus available at the current stage of the negotiation, it should be redefined. Total surplus between two consecutive negotiation stages is written as

$$\Delta J_t = J_t - J_{t+1} = \sum_{i=1}^n \Delta J_{i,t} = \sum_{i=1}^n [\Delta C_{i,t} + \Delta D_{i,t}]. \quad (4.31)$$

$$\Delta J_t \leq \sum_{i=1}^n \Delta C_{i,t} + \sum_{i=1}^n \pi_i \sum_{j=1}^n a_{ij} \Delta E_{j,t} = \sum_{j=1}^n \left[\Delta C_{j,t} + \Delta E_{j,t} \sum_{i=1}^n a_{ij} \pi_j \right] \equiv \sum_{j=1}^n \Delta G_{j,t}. \quad (4.32)$$

$\Delta G_{j,t}$, which is assumed as $\Delta G_{j,t} \leq 0$ ²², is total surplus obtained between two stages of the negotiation.

Ecological surplus consists of saved abatement cost and reduced damage. This is a source of the financial transfers.

Now consider the financial transfer $T_{i,t}$. $T_{i,t}$ is received by i if it is negative. $T_{i,t}$ is paid by i if it is positive. The financial transfer is defined as

²¹ see Appendix A.

²² see Appendix B

$$\Delta T_{i,t} = -\Delta C_{i,t} - \Delta D_{i,t} + \sum_j \delta_{ij} \Delta G_{j,t} \quad (0 \leq \delta_{ij} \leq 1, \sum_{j=1}^n \delta_{ij} = 1, \forall j). \quad (4.33)$$

The budget of transfers is balanced at each stage (feasibility condition).

$$\sum_{i=1}^n \Delta T_{i,t} = 0, \quad \forall t. \quad (4.34)$$

Aggregate cost with financial transfer for sub-region i is

$$J_i^T = C_i(E_i) + D_i(Q_i(E)) + T_i \quad (4.35)$$

Aggregate cost decreases as the negotiation proceeds as follows.

$$\Delta J_i^T = \Delta C_i + \Delta D_i + \Delta T_i = \sum_{j=1}^n \delta_{ij} \Delta G_{j,t} \leq 0 \quad (4.36)$$

δ_{ij} : surplus sharing rule.

$$\delta_{ij} = \frac{\partial D_i / \partial E_j}{\sum_{k=1}^n \partial D_k / \partial E_j} = \frac{a_{ij} \pi_i}{\sum_{k=1}^n a_{kj} \pi_k}, \quad k = 1, \dots, n. \quad (4.37)$$

From these results, $J_i(E^*) - J_i(\tilde{E})$, $T_i(E^*)$, and $J_i^T(E^*) - J_i^T(\tilde{E})$ can be calculated for sub-region i and region z ($z = 1, 2; i = 1, \dots, n$).

Payoffs

Now the task is to formulate net benefits obtained from each option. Let's begin with option-1.

Suppose that pollution abatement is $R^* = \tilde{E} - E^*$. The payoff from option-1 is as follows.

$$NB_1 = S - C(R^*)$$

$$NB_2 = B(R^*) - S$$

$$S = \frac{1}{2}[B(R^*) + C(R^*)]$$

Option-1 considers only benefit (of region-2) obtained from reduced damage, side-payment and abatement cost (accrued to region-1). Even if region-1's benefit obtained from the reduced damage existed, it is omitted. Such a result is due to the assumption of this approach. Suppose there is the omitted benefit of region-1 denoted by $B_1(R^*)$. $B_1(R^*)$ was not conceived when option-1 was applied. Thus, $B_1(R^*)$ is the extra benefit to region-1. On the other hand, $B_1(R^*)$ was not conceived when side-payment S was paid by

region-2. Thus, $B_1(R^*)$ is the extra cost to region-2. To resolve this problem, the payoffs can be reevaluated

as

$$\begin{aligned} NB'_1 &= NB_1 + B_1(R^*) = (S - C(R^*)) + (D_1(\tilde{E}) - D_1(E^*)) \\ &= S - C(R^*) + \sum_{i \in \{z=1\}} D_i(\tilde{E}) - \sum_{i \in \{z=1\}} D_i(E^*) \end{aligned} \quad (4.38)$$

$$\begin{aligned} NB'_2 &= NB_2 - B_1(R^*) = (B_2(R^*) - S) - (D_1(\tilde{E}) - D_1(E^*)) \\ &= B_2(R^*) - S - \sum_{i \in \{z=1\}} D_i(\tilde{E}) + \sum_{i \in \{z=1\}} D_i(E^*) \end{aligned} \quad (4.39)$$

$$B_1(R^*) = (D_1(\tilde{E}) - D_1(E^*))$$

$$D_1(\tilde{E}) = \sum_{i \in \{z=1\}} D_i(\tilde{E})$$

$$D_1(E^*) = \sum_{i \in \{z=1\}} D_i(E^*)$$

Option-2 has same problem. The payoffs from option-2 are as follows.

$$NB_1 = -C(R^*).$$

$$NB_2 = B_2(R^*).$$

With the same reason, the payoffs from option-2 are reevaluated as

$$NB''_1 = -C(R^*) + (D_1(\tilde{E}) - D_1(E^*)) \quad (4.40)$$

$$NB''_2 = B_2(R^*) \quad (4.41)$$

Now consider payoffs obtained from option-3. The payoff for region z is as follows.

$$\begin{aligned} NB_z &= \sum_{i \in \{z\}} \left[(D_i(\tilde{E}) - D_i(E^*)) - (C_i(E_i^*) - C_i(\tilde{E}_i)) \right] \\ &= \sum_{i \in \{z\}} \left[(C_i(\tilde{E}_i) + D_i(\tilde{E})) - (C_i(E_i^*) + D_i(E^*)) \right] \\ &= \sum_{i \in z}^n (J_i(\tilde{E}) - J_i(E^*)). \quad z = 1, 2. \end{aligned} \quad (4.42)$$

The financial transfer for sub-region i is defined as

$$T_{i,t} = \sum_{t=1}^s \Delta T_{i,t} = \sum_{t=1}^s \left(-\Delta C_{i,t} - \Delta D_{i,t} + \sum_j \delta_{ij} \Delta G_{j,t} \right) \quad (4.43)$$

s : required number of negotiation stages until the Pareto optimum is reached.

The financial transfer for region z is defined as

$$T_z = \sum_{t=1}^s \sum_{i \in z} \Delta T_{i,t} = \sum_{t=1}^s \sum_{i \in \{z\}} \left(-\Delta C_{i,t} - \Delta D_{i,t} + \sum_j \delta_{ij} \Delta G_{j,t} \right), \quad z = 1, 2. \quad i, j = 1, \dots, n. \quad (4.44)$$

The net benefits including financial transfers are written as

$$NB_z^T = \sum_{i \in \{z\}} \left(J_i(\tilde{E}) - J_i(E^*) + T_i \right), \quad z = 1, 2. \quad (4.45)$$

The payoffs for region z are summarized as follows.

Option-1

Region-1:

$$\begin{aligned} NB_1' &= (S - C(R^*)) + (D_1(\tilde{E}) - D_1(E^*)) \\ &= S - C(R^*) + \sum_{i \in \{z=1\}} D_i(\tilde{E}) - \sum_{i \in \{z=1\}} D_i(E^*) \end{aligned} \quad (4.46)$$

Region-2:

$$\begin{aligned} NB_2' &= (B(R^*) - S) - (D_1(\tilde{E}) - D_1(E^*)) \\ &= B_2(R^*) - S - \sum_{i \in \{z=1\}} D_i(\tilde{E}) + \sum_{i \in \{z=1\}} D_i(E^*) \end{aligned} \quad (4.47)$$

$$D_1(\tilde{E}) = \sum_{i \in \{z=1\}} D_i(\tilde{E}), \quad D_1(E^*) = \sum_{i \in \{z=1\}} D_i(E^*).$$

Option-2

Region-1:

$$\begin{aligned} NB_1'' &= -C(R^*) + (D_1(\tilde{E}) - D_1(E^*)) \\ &= -C(R^*) + \sum_{i \in \{z=1\}} D_i(\tilde{E}) - \sum_{i \in \{z=1\}} D_i(E^*) \end{aligned} \quad (4.48)$$

Region-2:

$$NB_2'' = B_2(R^*) \quad (4.49)$$

Option-3 (without financial transfers)

Region-1:

$$NB_1^* = \sum_{i \in \{z=1\}}^n (J_i(\tilde{E}) - J_i(E^*)) \quad (4.50)$$

Region-2:

$$NB_2^* = \sum_{i \in \{z=2\}}^n (J_i(\tilde{E}) - J_i(E^*)) \quad (4.51)$$

Option-3 (with financial transfers)

Region-1:

$$NB_1^T = \sum_{i \in \{z=1\}}^n (J_i(\tilde{E}) - J_i(E^*) + T_i) \quad (4.52)$$

Region-2:

$$NB_2^T = \sum_{i \in \{z=2\}}^n (J_i(\tilde{E}) - J_i(E^*) + T_i) \quad (4.53)$$

The preliminary conclusion is as follows. Region-1 prefers option-1 and region-2 prefers option-2. After reevaluating payoffs from option-1, it shows that the net benefit for region-1 is underestimated and the net benefit for region-2 is overestimated. After reevaluating payoffs from option-2, it shows that the net benefit for region-1 is underestimated.

The payoff from option-3 is still unobvious. This is because the comparison is possible only with specific information on the evolutions of emissions and costs. In addition, specific information on the distribution of ecological surplus (or financial transfer) has a key to determine the incentives for the sub-regions (participants). Thus, the more interesting conclusions can be obtained from the results of numerical application in Chapter V.

CHAPTER V

NUMERICAL APPLICATION

The purpose of this chapter is to provide specific results using model structure and formula developed in Chapter IV. This chapter also gives explanations about the status of data and data construction. The reason is that the data problem was a big hurdle to the topic of this dissertation. The data problem implies that most required data are not available. The only way to overcome this barrier was to construct required data from other available data. The purpose of numerical application in this chapter is not to use the result as it is to the real world. It is a device to see how the model is working and how different results are obtained from different options. What the result of numerical application represents is only tendency revealed by each option. In this context, there is no problem in numerical application using constructed data. Although data do not have practical purpose, it may be desirable to construct data based on the feasible logic and assumptions. The specific process to construct data is explained in Appendix C. Program writing for numerical application was tried using Gauss, Gams, and VBA. Among them, VBA was selected based on accessibility and compatibility. Information obtained from program writing procedure is added to this chapter.

The regional selection for numerical application is limited by the data status. The selected sub-regions cover Rogers (AR), Springdale (AR), Siloam Spring (AR) and Tahlequah (OK). These four sub-regions are not sufficient to represent the entire feature of the Illinois River Basin. But these sub-regions are sufficient to show all possible results of the model. Numerical application is focused on showing the results and comparing options.

This chapter consists of two parts. First part describes the status of data, software and data construction procedures. Second part explains the results of numerical application. Numerical application is organized by five sub-parts. First one is transition of emissions. This one shows how emission level evolves

from a noncooperative Nash equilibrium to Pareto optimum. It also presents the number of the negotiation stages required to reach Pareto optimum. Second one is ecological surplus. This is to show the feature of saved aggregate costs (in sub-regions of AR and OK) during the transition of emission level between optima. Third one is financial transfers. This is to illustrate how to share ecological surplus between members (sub-regions of AR and OK) of a coalition for the negotiation. Fourth one is surplus sharing. This is to demonstrate how surplus sharing leads all participants (sub-regions of AR and OK) to net gains and how it provides participants with incentives to join a coalition for the negotiation. Fifth one is payoffs. This is to show net benefits accrued to AR and OK which are obtained from each option. In addition, it explains why traditional approach should be ineffective and why the negotiation approach can be a candidate to resolve the disputes in the Illinois River Basin.

Data

The required data include phosphorus emissions, cost functions, transport coefficients, and marginal damage. Initial plan was to cover a wide range of sub-regions including the fifteen watersheds of the Upper Illinois River Basin. However, it should be narrowed by the data availability.

First, transport coefficient data are not available. It should be constructed from other available data. The useful relevant data was provided by Gade (Dissertation, 1998). From this data, the transport coefficients matrix was constructed for thirteen sub-regions, which include Prairie Grove (AR), Rogers (AR), Fayetteville (AR), Springdale (AR), Lincoln (AR), Gentry (AR), Siloam Springs (AR), Watts (OK), Westville (OK), Midwestern Nursery (OK), Tahlequah (OK), Cherokee Nation (OK), and Tenkiller (OK). The procedure to construct transport coefficient data is explained as in Appendix C.

Second, phosphorus emission data are available, but only some of them. Annual data is rarely found. Most useful data are offered by Water Quality Monitoring Report.²³ This source covers eight sub-regions such as Ark04 (Flint Creek), Ark05 (Sager Creek), Ark06 (Illinois River), Ark07 (Baron Fork), Rogers (AR), Springdale (AR), Illinois River near Watts (OK), Illinois River Near Tahlequah (OK), Flint Creek Near Kansas (OK), and Baron Fork at Eldon (OK) for 1980-2002. Other data sources such as USGS,

²³ Arkansas-Oklahoma Compact, "Water Quality Monitoring Report -- Illinois River Basin," CY2002.

STORET, EPA, DEQ (AR, OK) are available. But these data sources do not provide annual data. It is required to calculate annual data using raw data (daily/monthly data; stream flow; phosphorus load etc.). The problem in using raw data is missing of data for long period of time. Another data source may be simulated data. Storm et al. (1996) provides simulated annual data. It covers the fifteen watersheds of the Upper Illinois River Basin for 1960-1980's. There is another useful source of data. Nelson and White (2002) offer phosphorus annual data for Rogers (AR) and Springdale (AR) for 1997-2001. From these data sources, data for ten sub-regions, which include six sub-regions in AR and 4 sub-regions in OK, are collected.

Third, the abatement cost data are not available. Some valuable data could be found from Willett and Mitchell (2001). Rogers (AR), Fayetteville (AR), Springdale (AR), Siloam Springs (AR), and Tahlequah (OK) are covered by this data source. The data provides cost data as a functional form of $C_i(X_i) = \omega_0 + \omega_1 X - \omega_2 X^2 + \omega_3 X^3$ in which X denotes a percentage of phosphorus reduction and ω denotes coefficients. The function $C_i(X_i)$ has been converted into a function of emissions $C_i(E_i)$ through a procedure using quadratic approximation technique and econometric method. First step is to define the cost function as

$$C_i(E_i) = \gamma_i + \alpha_i (\bar{E}_i - E_i) + \beta_i (\bar{E}_i - E_i)^2$$

Using the definition of X ,

$$X = 1 - \frac{\bar{E}_i}{E_p}$$

The function $C_i(X_i)$ can be rewritten as

$$C_i(\bar{E}_i) = \omega_0 + \omega_1 \left(1 - \frac{\bar{E}_i}{E_p}\right) - \omega_2 \left(1 - \frac{\bar{E}_i}{E_p}\right)^2 + \omega_3 \left(1 - \frac{\bar{E}_i}{E_p}\right)^3$$

$$C_i'(\bar{E}_i) = -\frac{\omega_1}{E_p} + 2\omega_2 \left(1 - \frac{\bar{E}_i}{E_p}\right) \frac{1}{E_p} - 3\omega_3 \left(1 - \frac{\bar{E}_i}{E_p}\right)^2 \frac{1}{E_p}$$

If the cost function $C_i(E_i)$ is evaluated at \bar{E}_i by quadratic approximation technique, it will be written as

$$C_i(E_i) = C_i(\bar{E}_i) + C_i'(\bar{E}_i)(E_i - \bar{E}_i) + (0.5)C_i''(\bar{E}_i)(E_i - \bar{E}_i)^2$$

The function $C_i(E_i)$ can be rewritten as

$$C_i(E_i) = \gamma_i + \alpha_i(\bar{E}_i - E_i) + \beta_i(\bar{E}_i - E_i)^2$$

$$\gamma_i = C_i(\bar{E}_i)$$

$$\alpha_i = -C'_i(\bar{E}_i)$$

$$\beta_i = (0.5)C''_i(\bar{E}_i) \quad \gamma_i, \alpha_i, \beta_i > 0$$

The parameter γ_i is total abatement cost at $E_i = \bar{E}_i$. α_i is marginal abatement cost. β_i is a coefficient of squared error at the level of emission \bar{E}_i . Second step is to find the value of γ_i , α_i and β_i . The values of γ_i and α_i are obtained using econometric method. The value of β_i can be estimated using these values. From these procedures, the obtained cost function data covers three sub-regions of AR and one sub-region of OK.

Fourth, marginal damage data are not available. The way to get this data was based on the assumption that 1997 is a year characterized as a noncooperative Nash equilibrium. The reason is that the state of AR and OK agreed to the reduction of total phosphorus loading in 1997 as explained in Chapter I (Introduction). Under the Nash equilibrium assumption, the marginal damage π is observable from the relationship that $-C'_i(E_i) = \pi_i D'_i(Q_i)$ (Chapter IV, Negotiation). The marginal damage data for Rogers (AR), Springdale (AR), and Siloam Springs (AR), and Tahlequah (OK) became available from such a process.

The data availability made by data construction is summarized as follows: Transport coefficient data for thirteen sub-regions; the phosphorus emission data for ten sub-regions; the abatement cost and marginal damage data for four sub-regions. Finally the data sets for four sub-regions of AR and one sub-region of OK could be used for numerical application. This range of sub-regions includes Rogers (AR), Springdale (AR), Siloam Springs (AR), and Tahlequah (OK) which are indexed by subscript i ($i = 1, 2, 3, 4$).

Software

The required function of software is nonlinear optimization. There are many qualified software such as Gauss, Gams, FORTRAN, Mathematica, AIMMS, Matlab, SAS, VBA and so forth. Gauss has

packages which use Newton/Quasi-Newton method. Gams has nonlinear programming (NLP) solvers such as MINOS, CONOPT and SNOPT. Among them, SNOPT uses nonlinear function and gradient values. SNOPT is said to be effective when the problem has a nonlinear objective function and large numbers of sparse linear constraints. MINOS is known as more efficient than SNOPT if the objective and its gradients are cheap to evaluate. FORTRAN has a package of LANCELOT which uses Trust-Region method. The LANCELOT package is based on FORTRAN77. Other software (SAS, AIMMS and Mathematica) also have Trust-Region method packages. Those solvers or packages are said to use mathematical techniques such as descent methods, conjugate gradient methods, Newton-type methods and so forth. This study uses VBA. The strength of this software is flexibility and compatibility with excel software.

Numerical Application

The assumed situation is as follows. Three sub-regions ($i=1,2,3$) of AR and one sub-region ($i=4$) of OK agree to form a coalition for the negotiation to reduce phosphorus emissions in the Illinois River Basin. Each sub-region ($SR-i, i=1,2,3,4$) reduces phosphorus emission by 5% to the level of previous year. Each member ($SR-i$) knows its marginal abatement cost and marginal damage cost at any level of phosphorus emission. 1997 is chosen as an initial state and characterized as a noncooperative Nash equilibrium. The reason is same as explained in Chapter I (Introduction) and Chapter V (Data). The negotiation is terminated at the stage which all sub-regions reach Pareto optimum.

The total cost function of $SR-i$ is

$$J_i(E) = C_i(E_i) + D_i(Q_i(E))$$

$$C_i(E_i) = \gamma_i + \alpha_i(\bar{E}_i - E_i) + \beta_i(\bar{E}_i - E_i)^2$$

$$D(Q) = \pi Q$$

$$Q = AE$$

The initial values of parameters and emissions are as follows.

Table 5.1 P Emissions and Parameters

i	\bar{E}_i (ton/Yr)	γ (million dollars)	α (dollar/Kg)	β (dollar/ton ²)	π (dollar/Kg)
$i=1$	26.8	3.9	47.3	8.4	52.12
$i=2$	55.0	6.9	47.3	7.9	52.65
$i=3$	94.5	2.0	4.4	0.2	5.86
$i=4$	49.8	2.0	2.3	0.0	74.62

Table 5.2 Transport Coefficients

	$i=1$	$i=2$	$i=3$	$i=4$
$j=1$	0.908	0.093	0.000	0.000
$j=2$	0.083	0.898	0.000	0.000
$j=3$	0.000	0.000	0.753	0.000
$j=4$	0.072	0.080	0.194	0.031

i : Emitting sub-region. j : Receiving sub-region.

Transition of Emission

The first task is to examine the Pareto optimal level of emission of $SR-i$. If $SR-i$ reaches the Pareto optimum, it does not need to reduce emission any more. The evolution of emissions during the transition from a noncooperative Nash equilibrium to Pareto optimum is as follows.

Table 5.3 Evolution of P Emission (ton/Yr)

	$SR-1$	$SR-2$	$SR-3$	$SR-4$
0	27	55	95	50
1	26	54	90	50
2	26	54	85	50
3	26	54	81	50
4	26	54	77	50
5	26	54	73	50
6	26	54	69	50
7	26	54	66	50
8	26	54	63	50
9	26	54	60	50
10	26	54	57	50
11	26	54	54	50
12	26	54	51	50
13	26	54	49	50
14	26	54	48	50
15	26	54	48	50
16	26	54	48	50
17	26	54	48	50
18	26	54	48	50
19	26	54	48	50
20	26	54	48	50

Figure 5.1 Evolution of P Emissions
5% Reduction

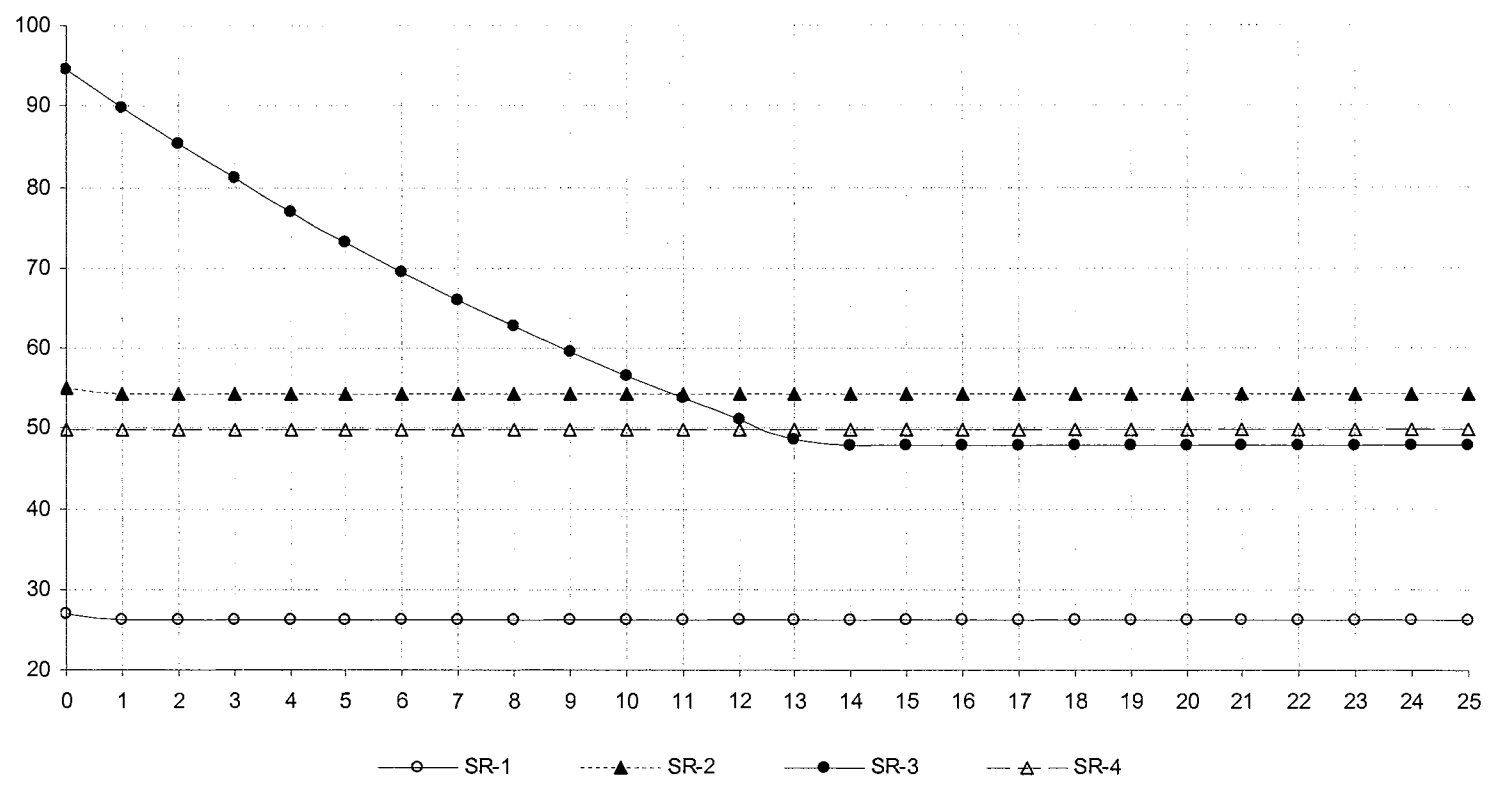


Table 5.3 and Figure 5.1 show the evolution of phosphorus emissions. $SR-1$ and $SR-2$ reach the optimum at the first stage. $SR-3$ reaches the optimum at the fourteenth stage. $SR-4$ does not need to reduce its phosphorus emission. The negotiation is terminated at the fourteenth stage.

$SR-3$ is required to reduce greater emission than other sub-regions. The reason is that its effect of emission on other region is greater. $SR-4$ is not required to reduce its emission additionally. This is because it does not affect other region. $SR-1$ and $SR-2$ are required to reduce their emissions, but they are small since their effects on other region is small.

Ecological Surplus

Figure 5.2 shows the evolution of aggregate cost during the transition from a noncooperative Nash equilibrium to Pareto optimum. The cost of $SR-3$ increases remarkably. The cost $SR-4$ decreases considerably. The costs of $SR-1$ decreases slightly. The cost of $SR-2$ increases slightly.

Cost reductions consist of saved abatement costs and reduced damage costs. In $SR-4$, the reduced damage cost is greater than other regions. In $SR-3$ case, increased abatement cost dominates the reduced damage cost. In $SR-1$ and $SR-2$, the evolution of costs are also obtained from changes in damage costs and abatement costs. The saved aggregate cost is ecological surplus as defined in Chapter IV. Thus, ecological surplus occurs in $SR-1$ and $SR-4$.

Financial Transfers

Figure 5.3 shows the evolution of transfers. The financial transfer is received if it is negative and paid if it is positive as explained in Chapter IV (Negotiation approach). $SR-4$ is a biggest payer and $SR-3$ is a biggest recipient of the financial transfer. $SR-1$ and $SR-2$ are also recipients of the financial transfer. This result implies that the ecological surplus can be shared between sub-regions using financial transfers.

Figure 5.2 Evolution of Total Costs per Region
(5% Reduction)

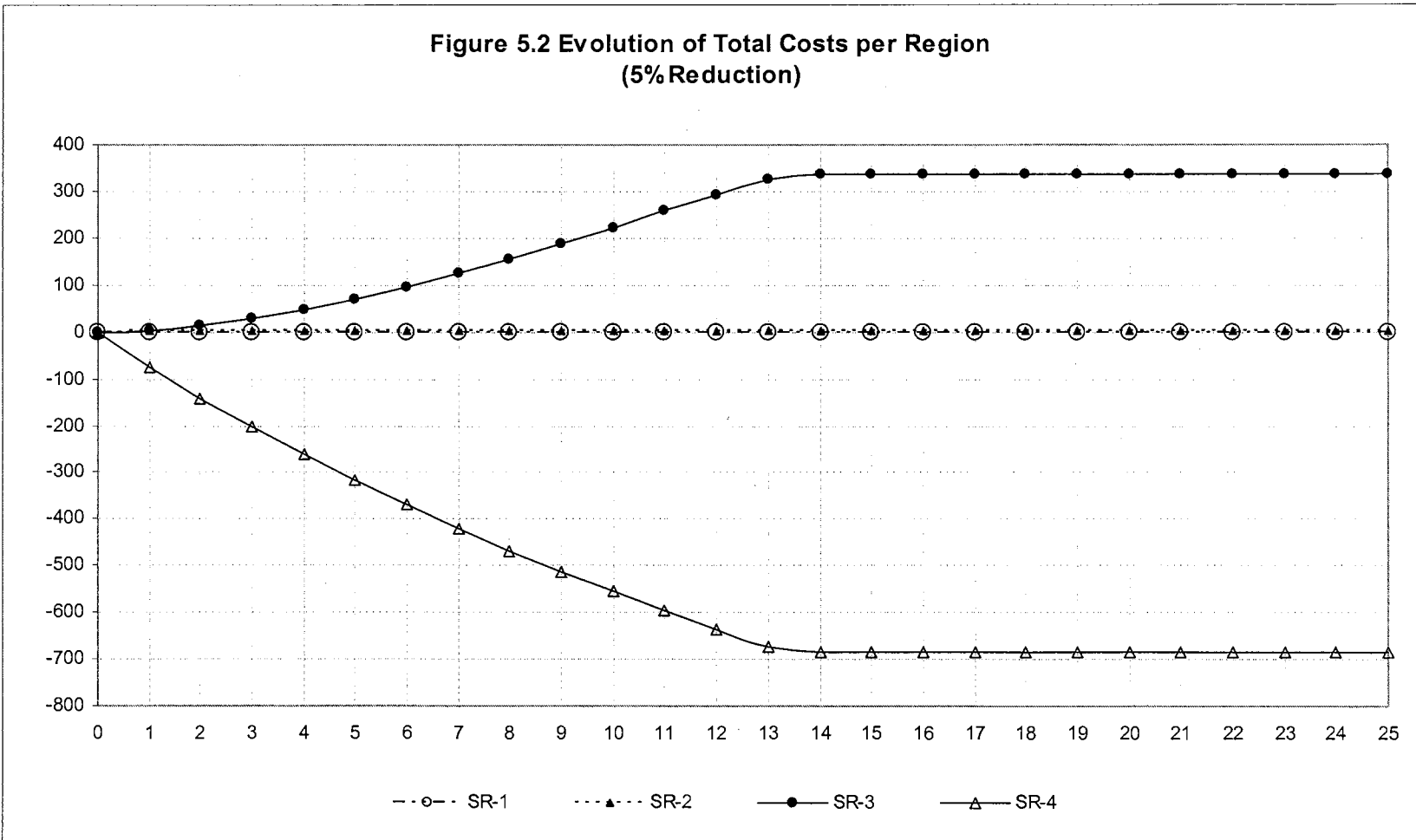
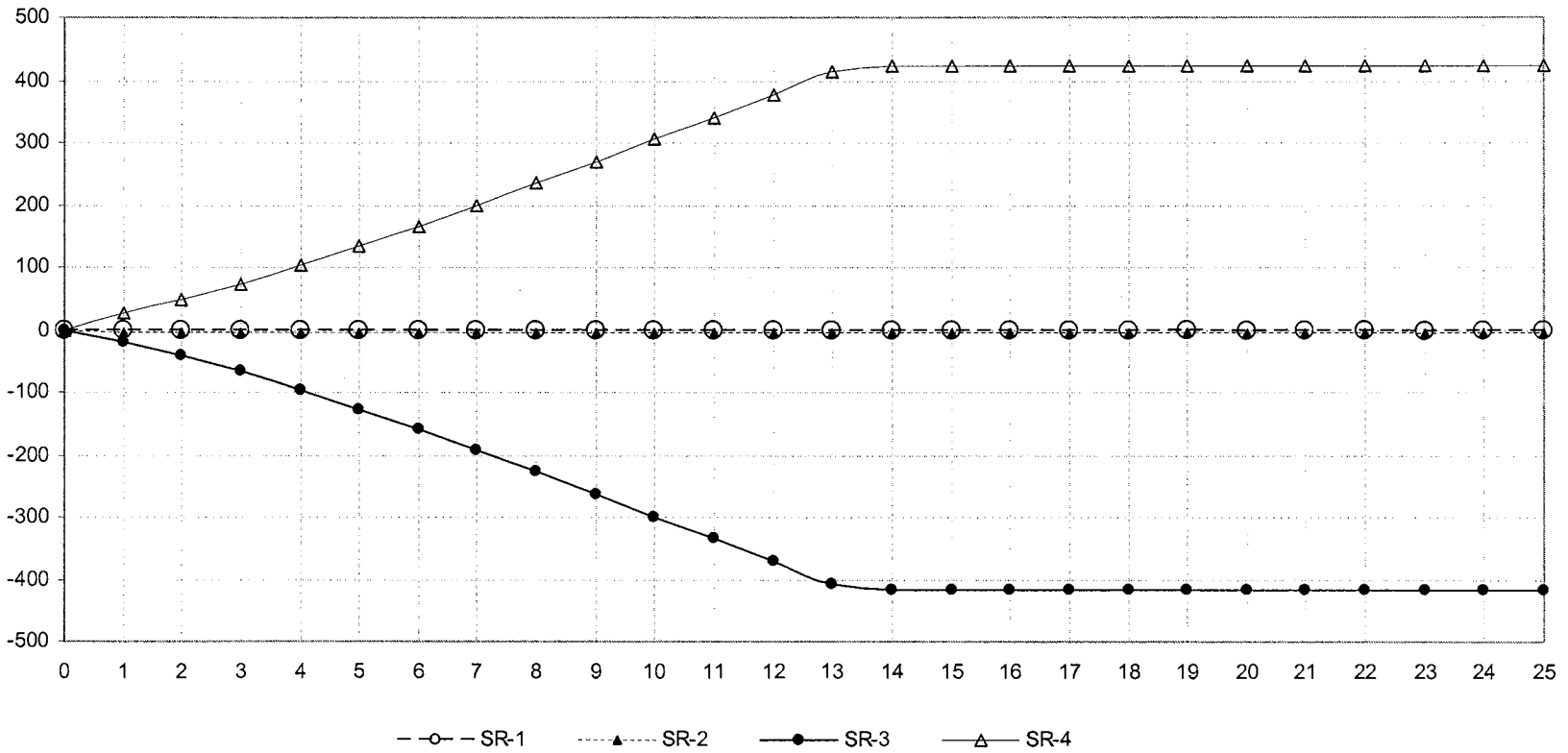


Figure 5.3 Evolution of Transfers per Region
(5% Reduction)



Surplus Sharing

Table 5.4 shows cost reductions when financial transfers are used or not. As shown in (1), aggregate costs increase in *SR-2* and *SR-3*. Aggregate costs decrease in *SR-1* and *SR-4*. These results are obtained from changes in abatement cost and damage cost as explained in Figure 5.2.

Financial transfers are as (2) in the Table 5.4. The allocated amount of the financial transfer is determined by surplus sharing rule δ_{ij} as explained in Chapter IV (Negotiation approach). The total amount of financial transfers is zero. This implies that the budget of transfers is balanced at each stage (feasibility condition), i.e. $\sum_{i=1}^n \Delta T_{i,t} = 0, \forall t$.

Aggregate costs with financial transfers are as shown in (3) in Table 5.4. Comparing with (1), aggregate cost decreases in all sub-regions. This is a result of surplus sharing using financial transfers. Based on this result, we can say that the negotiation approach with financial transfers leads all players to aggregate cost reduction. In other words, the negotiation approach provides all members with incentives to participate in a coalition for the negotiation to reduce emissions when financial transfers are used.

Table 5.4 Costs Reduction Without/With Transfers (m.\$)

	(1)	(2)	(3)	(4)
	$(J_i(E^*) - J_i(\tilde{E}))$	$T_i(E^*)$	$J_i^T(E^*) - J_i^T(\tilde{E})$	$\frac{(J_i^T(E^*) - J_i^T(\tilde{E}))}{J_i^T(\tilde{E})}$
SR-1	-0.5	-2.1	-2.6	-0.17%
SR-2	1.2	-4.4	-3.2	-0.12%
SR-3	338.5	-406.4	-67.9	-534.87%
SR-4	-684.2	413.0	-271.3	-11.44%
Total	-345.0	0.0	-345.0	-5.20%

Payoffs

This section examines the payoffs of jurisdictional regions indexed by z ($z=1,2$) in Chapter IV. The payoff obtained from each option is as shown in Table 5.5. The payoff from option-1 is NB in which AR and OK share benefits equally. But this result is distorted as explained in Chapter IV (Payoffs). The reevaluated payoff is NB' which show net gain to AR and net loss to OK. This result says that option-1 is acceptable to AR, option-1 is not acceptable to OK. Thus, both sides may not reach an agreement.

The payoff from option-2 is NB which shows net loss to AR and net gain to OK. This result is also distorted by missing extra benefit of AR. The reevaluated payoff is NB'' . Net loss to AR becomes lower than before reevaluation. Option-2 is not acceptable to AR, but option-2 is acceptable to OK. Thus, both sides may not reach an agreement.

The payoffs from option-3 are NB^* and NB^T . NB^* is payoffs from option-3 when financial transfers are not used. The result shows net loss to AR and net gain to OK. Thus, both sides may not reach an agreement when option-3 is used without financial transfers.

NB^T is payoffs from option-3 when financial transfers are used. The result shows net gain to AR and OK. Both sides may be able to reach an agreement, because incentives for both sides exist. Based on this result, the negotiation approach with financial transfers can be best strategy to AR and OK.

Table 5.5 Net Benefits

		AR	OK	Notes
Option-1	NB	36.5	36.5	Eq.(4.16)
	NB'	308.2	-235.5	Eq.(4.46), (4.47)
Option-2	NB	-647.7	684.2	Eq.(4.1), (4.2)
	NB''	-375.7	684.2	Eq.(4.48), (4.49)
Option-3	NB^*	-339.2	684.2	Eq.(4.50), (4.51)
	NB^T	73.8	271.3	Eq.(4.52),(4.53)

NB : Net benefits based on unidirectional externalities approach.

NB' , NB'' : Net benefits adjusted to reflect the extra benefits.

NB^* : Net benefits obtained during the transition between equilibria.

NB^T : Net benefits when the financial transfers are used.

CHAPTER VI

CONCLUSIONS

Discussions are summarized as follows. The phosphorus problem in the Illinois River basin has been regarded as a river pollution problem. Initial efforts to resolve the problem have been made based on the assumption of unidirectional externalities approach. The traditional approaches become incompatible with the property of the problem in the Illinois River Basin as the target of abatement has shifted from point source to nonpoint source pollutions. Such a situation is requiring new approach. This is a motivation of the dissertation. To investigate the problem, three options are considered. First option is the unidirectional externalities approach with side-payment. Second option is the enforcement rule approach. Third option is the negotiation approach. These three options are analyzed and compared by designated model and numerical application. Required data should be constructed, because they are not readymade. Mathematical logics, plausible assumptions and other available data were used in constructing data. The regional selection for numerical application was limited by the status of data. Finally, three sub-regions of AR and one sub-region of OK could be used. These four sub-regions data successfully show all possible results. Numerical application supports that the negotiation approach is a best strategy to AR and OK.

Implications for data are as follows. First, it is imminent to improve the data collection system. Current system is not friendly supporting economic research. Data for phosphorus emissions, phosphorus abatement costs, transport coefficients and damage costs are required to apply the negotiation to the Illinois River Basin. Second, the data for all relevant watersheds of the Illinois River Basin are required. Third, the data need to be developed so as to capture not only point source, but also nonpoint source discharge of phosphorus. Fourth, standardized abatement cost data should be developed to cover various sources of phosphorus. The improvement of data system will be a first priority to resolve the phosphorus problem in the Illinois River Basin (Appendix C; Chapter V).

Implications for approaches are as follows. First, traditional approaches are ineffective. These approaches cannot lead AR and OK to an agreement. Second, the negotiation approach provides all participants (sub-regions) with incentives to join a coalition for the negotiation. Third, the negotiation is unique approach to guarantee an agreement between AR and OK. Fourth, the negotiation approach is most persuasive resolution. It offers specific outlook such as required periods for negotiation, expected payoffs and the features of surplus sharing (Appendix E).

New findings are as follows. First, it identified that ineffectiveness of traditional approaches is due to an impractical assumption. Second, it demonstrated the possibility that the negotiation approach could be applied successfully to the phosphorus problem in the Illinois River Basin. Third, it found the fact that the lack of data is a barrier to apply the negotiation to the Illinois River Basin. Fourth, it found that the multilateral negotiation between all relevant sub-regions is more desirable than the bilateral negotiation between AR and OK. As far as the dispute is viewed as a problem between AR and OK, the problem may not be resolved. The reason is that such a viewpoint is to adopt unidirectional externalities assumption. Fifth, in resolving the problem in the Illinois River Basin, each approach is distinguished as follows. Under the enforcement rule, the negotiation is not possible. Under the unidirectional externalities approach with side-payment, negotiation is possible, but an agreement is not possible. Under the negotiation approach, both negotiations and an agreement are possible. Thus, the negotiation approach is the most practical way to resolve the jurisdictional disputes on water quality between Arkansas and Oklahoma in the Illinois River Basin.

The limit of this study is as follows. First, the constructed data are biased to point source. It was inevitable because of the lack of data. But it should be improved in the future. Second, there are accompanied issues such as information reliability and free riders. Although they are important issues, they could not be included. Third, a wide range of sub-regions could not be covered. Even though this is also due to the lack of data, it should be improved to produce more practical results.

SELECTED BIBLIOGRAPHY

- Boyd, J. (2000). "The New Face of the Clean Water Act: A Critical Review of the EPA's Proposed TMDL Rules." Resources for the Future Discussion Paper 00-12.
- Chander, P. and H. Tulkens (1992). "Theoretical Foundations of Negotiations and Cost Sharing in Transfrontier Pollution Problems," *European Economic Review*, 36(2/3): 288-299.
- Doye, D.M., J.G. Berry and P.E. Norris (1993). "Broiler Production: Considerations for Potential New Growers," OSU Extension Fact Sheet No. 202.
- Frandsen, P.E., K. Jonasson, H.B. Nielsen, and O. Tingleff (2004). "Unconstrained Optimization," 3.Edition, *Informatics and Mathematical Modelling*, Technical University of Denmark.
- Gade, D.R. (1998). "An Investigation of The Sources and Transport of Nonpoint Source Nutrients in the Illinois River Basin in Oklahoma and Arkansas," Dissertation.
- Germain, M. and P.Toint (2000). "An Iterative Process for International Negotiations on Acid Rain in Northern Europe Using a General Convex Formulation," *Environmental and Resource Economics*, 15: 199-216.
- Germain, M., P. Toint and H. Tulkens (1996). "International Negotiations on Acid Rains in Northern Europe: A Discrete Time Iterative Process", in Anastasios Xepapadeas (ed.), *Economic Policy for the Environment and Natural Resources*, Department of Economics, University Crete. 217-236.
- Gonzalez-Alcorta, M., J.H. Dorfman, and G.M. Pesti (1994). "Maximizing Profit in Broiler Production as Prices Change: A Simple Approximation with Practical Value," *Agribusiness*, 10: 389-399.
- Govindasamy, R. and M.J. Cochran (1995). "Implications of Alternative Environmental Policies on Phosphorus Loading from Poultry Litter," *Agricultural Economics*, 13: 137-148.
- Govindasamy, R. and M.J. Cochran (1998). "Implications of Policy Regulations on Land Applications of Poultry Litter," *Agricultural and Resource Economic Review*, 27: 85-94.
- Govindasamy, R., M.J. Cochran, and E. Buchanberger (1994). "Economic Implications of Phosphorus Loading from Poultry Litter," *Water Resource Bulletin*, 30: 901-910.
- Heal, G. (1986). "Planning," In Kenneth J. Arrow and Michael D. Intriligator (ed.). *Handbook of Mathematical Economics*, Volume III: 1483-1510.
- Jehle, G.A. and P.J. Reny (1998). "Advanced Microeconomic Theory."
- Kaitala, V., K. Mäler, and H. Tulkens (1995). "The Acid Rain Game as a Resource Allocation Process with an Application to the International Cooperation Among Finland, Russia and Estonia," *Scandinavian Journal of Economics*, Vol.97, No. 2: 325-343.

- Kaitala, V., K. Mäler and H. Tulkens (1995). "The Acid Rain Game as a Resource Allocation Process with an Application to the International Cooperation among Finland, Russia and Estonia," *Scandinavian Journal of Economics*, 97(2): 325-343.
- Kaitala, V., M. Pohjola and O. Tahvonen (1992). "An Economic Analysis of Transboundary Air Pollution Between Finland and the Former Soviet Union," *Scandinavian Journal of Economics*, 94(3): 409-424.
- Kaitala, V., M. Pohjola, and O. Tahvonen (1992). "An Economic Analysis of Transboundary Air Pollution between Finland and the Former Soviet Union," *Scandinavian Journal of Economics*, Vol.94, No. 3: 409-424.
- Kennedy, J.O.S. (1986). "Rules for Optimal Fertilizer Carryover: An Alternative Explanation," *Review of Marketing and Agricultural Economics*, 53: 3-10.
- Knoeber, C.R. and W.N. Thurman (1995). "Don't Count Your Chickens . . . Risk and Risk Sharing in the Broiler Industry," *American Journal of Agricultural Economics*, 77: 486-496.
- Lindahl, E. (1958). "Just Taxation-A Positive Solution. In Richard Musgrave and Alan Peacock, editors, *Classics in the Theory of Public Finance*, pages 98-123. Macmillan, London.
- Mäler, K. (1974). "Damage Functions and Their Estimation: A Theoretical Survey, Environmental Damage Costs", OECD, 223-252.
- Mäler, K. (1974). "Environmental Economics: A Theoretical Inquiry," Published for Resources for the Future, Inc., The Johns Hopkins University Press.
- Mäler, K. (1989). "The Acid Rain Game" in: *Valuation Methods and Policy Making in Environmental Economics*, H. Folmer and E. van Ierland (eds.), Elsevier, 231-252.
- Mäler, K. (1989). "The Acid Rain Game" in: *Valuation Methods and Policy Making in Environmental Economics*, H. Folmer and E. van Ierland (eds.), Elsevier, 231-252.
- Mäler, K. (1990). "International Environmental Problems," *Oxford Review of Economic Policy*, Vol.6, No.1.
- Mas-Colell, A., M.D. Whinston, and J.R. Green (1995). *Microeconomic Theory*, Oxford University Press.
- Melack, J.M. (1995). "Transport and Transformations of P, Fluvial and Lacustrine Ecosystems," SCOPE54, PHOSPHORUS IN THE GLOBAL ENVIRONMENT, Edited by Holm Tiessen, Scientific Committee on Problems of the Environment (SCOPE), 245-254.
- Missfeldt, F. (1999). "Game-Theoretic Modeling of Transboundary Pollution," *Journal of Economic Surveys*, Vol. 13, No. 3.
- Mitchell, D.M. (2001). "An Examination of Non-Regulatory Methods for Controlling Nonpoint Source Pollution," Dissertation.
- Mizgalewicz, P.J. (1996). "Modeling Agrichemical Transport in Midwest Rivers Using Geographic Information Systems," Dissertation.
- Nelson, M. and K. White (2002). "Illinois River Phosphorus Sampling Results and Mass Balance Computation," University of Arkansas.
- Pesti, G.M., R.A. Arreas, and B.R. Miller (1985). "Use of the Quadratic Growth Response to Dietary and Energy Concentrations in Least-Cost Feed Formulation," *Poultry Science*, 65: 1041-1051.

- Siebert, H. (1995). *Economics of the Environment: Theory and Practice*. Berlin: Springer, 4th edition.
- Smith, D.A. "Phosphorus in Oklahoma's Scenic Rivers, Justification for 0.037 mg/L Justification for 0.037 mg/L," Oklahoma Water Resources Board.
- Soerens, T., Fite, E., and Hipp, J. (2003). "Water Quality In The Illinois River: Conflict And Cooperation Between Oklahoma and Arkansas," Proceedings of the 7th International Conference - Diffuse Pollution and Basin Management, Dublin, Ireland.
- Soerens, T.S., E.H. Fite, OSRC, and J.Hipp (2003). "Water Quality in the Illinois River: Conflict and Cooperation between Oklahoma and Arkansas," Diffuse Pollution Conference Dublin 2003.
- Storm, D.E. et al. (1996). "Basin-wide Pollution Inventory for the Illinois River Comprehensive Basin Management Program," Final Report.
- Storm, D.E., A.L. Stoecker et al. (2003). "Evaluating Cost Effective Technologies to Reduce Phosphorus Loading to Surface Waters for the Ozark Region," Oklahoma State University.
- Tahvonen, O., V. Kaitala, and M. Pohjola (1993). "A Finish-Soviet Acid Rain Game: Noncooperative Equilibria, Cost Efficiency, and Sulfur Agreement," *Journal of Environmental Economics and Management*, 24: 87-100.
- Tulkens, H. (1978). "Dynamic Processes for Public Goods," *Journal of Public Economics*, 9: 163-201.
- Tulkens, H. (1979). "An Economic Model of International Negotiation Relating to Transfrontier Pollution," C.O.R.E., Universite Catholique de Louvain, Belgium.
- Tulkens, H. (1979). "An Economic Model of International Negotiations Relating to Transfrontier Pollution," (Chapter 16, pp.199-212), in: K. Krippendorff, ed., *Communication and Control in Society*, Gordon and Breach, New York.
- Vieux, B. and F.G. Moreda (2003). "Nutrient Loading Assessment in the Illinois River Basin Using a Synthetic Approach," *Journal of the American Water Resources Association*, 39: 757-769.
- Water Quality Monitoring Report for the Illinois River Basin--Arkansas-Oklahoma Compact, 2003.
- Willett, K.D. and D.M.Mitchell (2001). "WWTP Cost Functions," working paper.
- Xu, F.T., T. Prato, and C. Fulchur (1993). "Broiler Litter Application to Land in an Agricultural Watershed: A GIS Approach," *Water Science and Technology*, 28: 111-118.

APPENDICES

APPENDIX A: CONVERGENCE PROPERTY OF SOLUTION

Germain, Toint and Tulkens (1996) proved the convergence property of solution as follows.

$$(A.1) \quad Q = Q(E) = AE.$$

$$(A.2) \quad J_i(E) = C_i(E_i) + D_i(Q_i(E))$$

$$(A.3) \quad \min_E J(E) = \sum_{i=1}^n J_i(E).$$

$$(A.4) \quad E_i \geq 0.$$

$$(A.5) \quad \frac{dE_i(t)}{dt} = -K \left[C'_i(E_i(t)) + \sum_{j=1}^n a_{ji} D'_j(Q_j(E(t))) \right]$$

$$(A.6) \quad Q_i = Q(E_i) = AE_i$$

$$(A.7) \quad F_{i,t} \leq F_i$$

$$(A.8) \quad F_{i,t+1} = \min [F_{i,t}, \tau_{i,t+1} E_{i,t+1}]$$

$$(A.9) \quad 0 \leq \tau_{i,t+1} \leq \tau$$

$$(A.10) \quad L_0 = \left\{ E \mid J(E) \leq J(\bar{E}) \text{ and } E_i \text{ feasible } (i = 1, \dots, n) \right\}$$

$$(A.11) \quad \lim_{\|E\| \rightarrow \infty} \sum_{i=1}^n D_i(Q_i(E)) = \infty$$

Theorem:

- (i) Under the conditions stated above, the sequence $\{E_t\}$ converges to a solution of optimization problem in (A.3)-(A.1)-(A.4).
- (ii) Furthermore, this convergence occurs in a number of stages if all limiting values of the emissions are strictly positive.

Proof:

- (i) The sequence $\{E_t\}$ converges to a solution of (A.3)-(A.1)-(A.4)

Consider the case where the sequence $\{E_t\}$ is infinite.

The boundedness of the level set L_0 and its definition in (A.10) ensure that it is compact.

Using (A.9), extract a subsequence $\{t_j\}_{j=1}^\infty \subseteq \{t\}_{t=1}^\infty$ such that $\{E_{t_j}\}_{j=1}^\infty$ and $\{\tau_{i,t_j}\}_{j=1}^\infty$ ($i=1, \dots, n$) are convergent,

with

$$\lim_{j \rightarrow \infty} E_{t_j} = \bar{E} \text{ and } \lim_{j \rightarrow \infty} \tau_{i,t_j} = \bar{\tau}_i \text{ (} i=1, \dots, n \text{)}. \quad (\text{A.12})$$

If $I \subseteq \{1, \dots, n\}$ is defined as the set of indices such that $E_{i,t} = F_{i,t-1}$ for infinitely many t_j , this set must contain least one element, otherwise the procedure will stop and the convergence will be finite.

Deduce that, for each $i \in I$,

$$\bar{E}_i = \lim_{j \rightarrow \infty} E_{i,t_j} = \lim_{j \rightarrow \infty} F_{i,t_j-1} \leq \lim_{j \rightarrow \infty} F_{i,t_j-1} \leq \lim_{j \rightarrow \infty} \tau_{i,t_j-1} E_{i,t_j-1} = \bar{\tau}_i \bar{E}_i. \quad (\text{A.13})$$

where (A.8) is used. But $\bar{\tau}_i \leq \tau_{i,t+1} < 1$ by (A.9), and thus (A.4) and (A.13) imply that

$$\bar{E}_i = 0 \quad \forall i \in I \quad (\text{A.14})$$

If the first order optimality conditions for the local problems (A.3)-(A.1)-(A.4) at each stage, the result is obtained as

$$\frac{\partial J(E_{t_j})}{\partial E_i} \begin{cases} \geq 0 & \text{if } E_{i,t_j} = F_{i,t_j-1} \\ = 0 & \text{if } E_{i,t_j} > F_{i,t_j-1} \end{cases} \quad (\text{A.15})$$

In the limit, what is deduced from (A.15) and the definition of I is

$$\frac{\partial J(\bar{E})}{\partial E_i} = \nabla J(\bar{E})_i \begin{cases} \geq 0 & i \in I \\ = 0 & i \notin I \end{cases} \quad (\text{A.16})$$

Now let E^* denote a solution of global problem (A.3)-(A.1)-(A.4), which obviously ensures that

$$J(\bar{E}) \geq J(E^*).$$

If $J(\bar{E}) > J(E^*)$, then

$$\begin{aligned} 0 > J(E^*) - J(\bar{E}) &\geq \nabla J(\bar{E})^T (E^* - \bar{E}) \\ &= \sum_{i \in I} \nabla J(\bar{E})_i (E_i^* - \bar{E}_i) \\ &= \sum_{i \in I} \nabla J(\bar{E})_i E_i^* \geq 0 \end{aligned} \quad (\text{A.17})$$

where the convexity of $J(\cdot)$, (A.16) and (A.14) are successfully used. But (A.17) is impossible.

Thus, $J(E^*) = J(\bar{E})$ is obtained as desired.

If the sequence $\{E_t\}$ is finite, $E_{i,t+1} = E_{i,t} = \bar{E}_i$ for some t and for all i .

$$\therefore 0 \leq F_{i,t} \leq \tau \bar{E}_i < \bar{E}_i \quad (\text{A.18})$$

And the solution of the local program (A.3)-(A.1)-(A.7) is not constrained by any of the bounds (A.7):

therefore, it is also solution of the global program.

- (ii) The convergence occurs in a number of stages if all limiting values of the emissions are strictly positive.

Assume that the sequence $\{E_t\}$ is infinite. As in the first part of the proof, it is obtained that (A.14) is satisfied for some non-empty set I , which is impossible if all limiting values of the emissions are strictly positive. Thus, this latter condition implies that the sequence $\{E_t\}$ is finite and the theorem is proved.

QED

APPENDIX B: PROPERTY OF SURPLUS

Germain et al. (1996) proved the property of surplus as follows.

Theorem: At each stage t of the negotiation process and for all regions, it must be that $\Delta G_{j,t} \leq 0$.

Proof:

For stage t , Consider the functions

$$G_j(E_j) = C_j(E_j) + E_j \sum_{i=1}^n a_{ij} \pi_i \quad (\text{B.1})$$

This definition implies that

$$\Delta G_{j,t} = G_j(E_{j,t+1}) - G_j(E_{j,t}) \quad (\text{B.2})$$

The convexity of G_j in E_j yields that

$$\begin{aligned} G_j(E_{j,t}) - G_j(E_{j,t+1}) &\geq G'_j(E_{j,t+1}) [E_{j,t} - E_{j,t+1}] \\ G_j(E_{j,t+1}) - G_j(E_{j,t}) &\leq G'_j(E_{j,t+1}) [E_{j,t+1} - E_{j,t}] \end{aligned} \quad (\text{B.3})$$

Taking derivative of (B.1),

$$G'_j(E_j) = C'_j(E_j) + \sum_{i=1}^n a_{ij} \pi_i \quad (\text{B.4})$$

$$\Delta G_{j,t} \leq G'_j(E_{j,t+1}) [E_{j,t+1} - E_{j,t}] \quad \text{from (B.3)}$$

$$\text{(i) If } E_{j,t+1} \leq E_{j,t}, \quad G'_j(E_{j,t+1}) = C'_{i,t}(E_{j,t+1}) + \sum_{i=1}^n a_{ij} \pi_i \geq 0 \quad (\text{B.5})$$

$$\Delta G_{j,t} = G'_j(E_{j,t+1}) [E_{j,t+1} - E_{j,t}] < 0$$

$$\text{(ii) If } E_{j,t} < E_{j,t+1}, \quad G'_j(E_{j,t+1}) = C'_{i,t}(E_{j,t+1}) + \sum_{i=1}^n a_{ij} \pi_i = 0 \quad (\text{B.6})$$

$$\therefore \Delta G_{j,t} = 0$$

QED

APPENDIX C: DATA CONSTRUCTION

Phosphorus Emission Data Sources:

Water Quality Monitoring Report – Illinois River Basin (CY2002, Arkansas-Oklahoma Compact) provides P emission data for the period of 1980-2002. The data cover ARK04A, ARK05, ARK07, Watts, Tahlequah, Kansas and Eldon. Data from Water-Resources Investigations Report (USGS) cover phosphorus concentrations, loads, and yields in the Illinois River Basin. The data cover five regions including Illinois River near Watts, Flint Creek near Kansas, Illinois River at Chewey, Illinois River near Tahlequah, and Baron Fork at Eldon. Other data sources are EPA, USGS, STORET and DEQ (OK, AR) etc. These sources provide similar data. The concentration of phosphorus data is provided. The data are collected once a month. To get annual data, it should be calculated from stream water and phosphorus data. But these data are missing for long period of time in many cases. Marc Nelson and Kati White (2002) provide Illinois River Phosphorus Mass Balance inputs data. Storm et al. (1996) provides basin-wide pollution inventory data for the Illinois River Basin. These data are simulated by independent and continuous simulation techniques. The data cover fifteen watersheds in the Upper Illinois River Basin for the period of 1962-1986.

Transport coefficients, Cost and Other Data Sources:

Gade (Dissertation, 1998) examines the transport of nonpoint source nutrients in the Illinois River Basin in Oklahoma and Arkansas. This study clarifies the main sources of phosphorus discharge and transport mechanism. Storm et al. (2003) is dealing with evaluating cost effective technologies to reduce phosphorus for the Ozark region using SWAT model. Mizgalewicz (Dissertation, 1996) sets up the model of agrichemical transport in Midwest River using GIS technique. This study describes the mathematical logic and how to operate the SWAT model in detail. It provides specific results about cost effectiveness and optimal policy for phosphorus abatement.

Table A.1 P Loading in the Arkansas Region

Year	(1) Ark04 - Flint Creek			(2) Ark05 - Sager Creek			(3) Ark06 - Illinois River			(4) Ark07 - Baron Fork		
	Flow (cfs)	Pt (mg/L)	Pt (kg/yr)	Flow (cfs)	Pt (mg/L)	Pt (kg/yr)	Flow (cfs)	Pt (mg/L)	Pt (kg/yr)	Flow (cfs)	Pt (mg/L)	Pt (kg/yr)
1980	11.90	0.13	1382	4.50	2.43	9766	169.00	0.47	70939	9.50	0.10	878
1981	19.80	0.15	2635	6.50	2.13	12336	197.00	0.42	73895	18.40	0.14	2218
1982	29.90	0.17	4566	9.00	2.03	16277	591.00	0.37	195294	37.40	0.48	16167
1983	19.00	0.07	1239	6.30	1.96	11050	352.00	0.39	121347	27.20	0.13	3037
1984	53.50	0.11	5351	15.40	0.95	13066	706.00	0.44	278693	51.80	0.18	8466
1985	91.30	0.06	5137	24.80	1.74	38450	947.00	0.29	244426	79.40	0.21	14962
1986	78.40	0.07	4691	21.10	0.83	15716	879.00	0.31	239436	64.00	0.15	8402
1987	58.30	0.05	2551	16.70	0.95	14136	815.00	0.29	213996	63.20	0.13	7563
1988	41.80	0.03	1157	12.60	1.15	12986	531.00	0.25	119982	31.80	0.10	2755
1989	38.00	0.05	1697	11.70	1.23	12821	558.00	0.29	145020	50.20	0.12	5559
1990	71.30	0.06	3821	20.20	0.86	15515	1127.00	0.20	205331	102.00	0.11	9929
1991	51.60	0.05	2489	15.50	0.91	12653	724.00	0.22	142253	49.40	0.09	3794
1992	56.10	0.05	2355	16.50	1.28	18921	760.00	0.22	150684	47.90	0.13	5433
1993	88.20	0.05	3545	24.60	0.64	13995	1163.00	0.18	188000	104.00	0.08	7709
1994	53.00	0.05	2414	15.70	0.72	10110	674.00	0.19	114370	37.00	0.08	2677
1995	61.30	0.08	4106	17.80	0.70	11080	783.00	0.24	165733	54.20	0.16	7842
1996	33.50	0.05	1496	11.00	0.92	9028	667.00	0.23	134032	64.40	0.08	4831
1997	37.30	0.07	2448	17.80	1.03	16354	497.00	0.21	94504	35.90	0.07	2151
1998	42.90	0.06	2142	18.10	0.86	13876	668.00	0.25	146960	61.10	0.11	5822
1999	63.50	0.05	2578	24.50	0.98	21429	737.00	0.21	135413	45.80	0.10	4176
2000	55.60	0.04	1893	30.70	0.82	22469	597.00	0.23	122831	52.60	0.13	6230
2001	39.40	0.05	1636	21.20	0.80	15201	598.00	0.29	156581	41.40	0.07	2387
2002	44.60	0.05	1850	21.80	1.19	23231	619.00	0.28	156009	38.00	0.10	3536
Avg.	50.00	0.07	3047	19.00	1.18	19816	668.00	0.28	167762	51.00	0.13	6012

Source: Arkansas-Oklahoma Compact, "Water Quality Monitoring Report Illinois River Basin," CY2002.

Table A.2 P Loading in the Oklahoma Region

Year	(5) Illinois River near Watts			(6) Illinois River Near Tahlequah			(7) Flint Creek Near Kansas			(8) Baron Fork at Eldon		
	Flow (cfs)	Pt (mg/L)	Pt (kg/yr)	Flow (cfs)	Pt (mg/L)	Pt (kg/yr)	Flow (cfs)	Pt (mg/L)	Pt (kg/yr)	Flow (cfs)	Pt (mg/L)	Pt (kg/yr)
1980	173.0	0.42	65279	249.0	-	-	32.00	0.19	5454	77	-	-
1981	260.0	0.19	44119	384.0	-	-	57.00	0.18	9077	201	-	-
1982	591.0	-	-	812.0	-	-	69.00	0.19	11537	296	-	-
1983	352.0	-	-	537.0	-	-	49.00	0.28	12415	184	-	-
1984	706.0	-	-	1157.0	-	-	143.00	0.24	30532	364	-	-
1985	947.0	-	-	1651.0	-	-	237.00	0.22	47591	593	-	-
1986	879.0	-	-	1452.0	-	-	183.00	0.22	36430	536	-	-
1987	815.0	-	-	1218.0	-	-	141.00	0.16	19840	491	-	-
1988	531.0	-	-	820.0	-	-	97.00	0.27	22946	269	-	-
1989	558.0	0.21	104653	808.0	-	-	90.00	0.56	44981	320	-	-
1990	1127.0	0.18	182432	1695.0	0.10	147579	-	0.11	-	666	-	-
1991	724.0	0.16	104534	1094.0	0.08	76796	-	0.12	-	451	0.06	24,145
1992	760.0	0.16	109571	1207.0	0.08	86205	-	0.12	-	440	0.095	37,315
1993	1163.0	0.28	287317	1751.0	0.10	154647	182.00	0.16	25359	700	0.108	67,234
1994	674.0	0.17	101127	1071.0	0.08	80223	136.00	0.13	15418	328	0.037	10,878
1995	783.0	0.14	100233	1123.0	0.08	80229	140.00	0.19	23207	422	0.263	98,819
1996	693.0	0.19	116542	938.0	0.09	71207	76.00	0.15	10294	432	0.025	9,645
1997	573.0	0.16	83415	812.0	0.07	49797	94.80	0.12	9871	332	0.023	6,671
1998	713.0	0.14	87876	1044.0	0.08	75524	96.50	0.13	10945	409	0.033	12,054
1999	793.0	0.18	130314	1143.0	0.24	239891	137.00	0.21	25817	361	0.042	13,541
2000	648.0	0.34	197346	1083.0	0.19	185708	133.00	0.16	19243	376	0.164	55,072
2001	649.0	0.39	223154	1033.0	0.19	172521	101.00	0.17	15154	343	0.076	23,281
2002	619.0	0.42	229424	851.0	0.34	256129	82.00	0.21	15452	262	0.203	47,500
Avg.	684.0	0.23	142368	1041.0	0.13	121852	114.00	0.19	19774	385	0.094	32,305

Source: Arkansas-Oklahoma Compact, "Water Quality Monitoring Report Illinois River Basin," CY2002.

Table A.3 Illinois River Phosphorus Mass Balance inputs (pound/Yr)

	1997	1998	1999	2000	2001
Prairie Grove	349	377	1,237	18,633	503
Rogers	59,168	76,820	68,632	71,458	76,525
Springdale	121,185	109,455	124,115	134,270	192,361

Source: Marc Nelson and Kati White (2002), "Illinois River Phosphorus Sampling Results and Mass Balance Computation," University of Arkansas.

Abatement Cost Functions

Cost data are available only for Tahlequah (OK), Rogers (AR), Fayetteville (AR), Springdale (AR), and Siloam Springs (AR). The available data have a form of $C_i(X_i) = \omega_i^0 + \omega_i^1 X - \omega_i^2 X^2 + \omega_i^3 X^3$ where X denotes a percentage of phosphorus reduction and ω denotes coefficients.

Table A.4 WWTP Cost Functions

(1) Tahlequah (OK)	$TC = 1,678,837 + 19,099.9 X - 521.59 X^2 + 4.7015 X^3$
(2) Rogers (AR)	$TC = 2,353,899 + 25,633.2 X - 699.2 X^2 + 4.7015 X^3$
(3) Fayetteville (AR)	$TC = 3,894,306 + 42,942.6 X - 1,171.8 X^2 + 10.566 X^3$
(4) Springdale (AR)	$TC = 3,630,080 + 38,081.6 X - 1,037.7 X^2 + 9.3627 X^3$
(5) Siloam Springs (AR)	$TC = 1,154,362 + 10,885 X - 294.19 X^2 + 2.6763 X^3$

* WWTP: Wastewater Treatment Plant.

Source: K.D. Willett and D..M. Mitchell (2001), Working paper.

To use an abatement cost function as a function of emissions, it is necessary to convert the function $C_i(X_i)$ into $C_i(E_i)$. Suppose that the function is defined as $C_i(E_i) = \gamma_i + \alpha_i (\bar{E}_i - E_i) + \beta_i (\bar{E}_i - E_i)^2$ where \bar{E}_i is the emission level of initial state and E_i is current level of emission. The problem is to find the value of γ_i , α_i , and β_i .

By the definition of X ,

$$X = 1 - \frac{\bar{E}_i}{E_p}$$

The cost function is rewritten as

$$C_i(\tilde{E}_i) = \varpi_0 + \varpi_1\left(1 - \frac{\tilde{E}_i}{\tilde{E}_p}\right) - \varpi_2\left(1 - \frac{\tilde{E}_i}{\tilde{E}_p}\right)^2 + \varpi_3\left(1 - \frac{\tilde{E}_i}{\tilde{E}_p}\right)^3.$$

$$C_i'(\tilde{E}_i) = -\frac{\varpi_1}{\tilde{E}_p} + 2\varpi_2\left(1 - \frac{\tilde{E}_i}{\tilde{E}_p}\right)\frac{1}{\tilde{E}_p} - 3\varpi_3\left(1 - \frac{\tilde{E}_i}{\tilde{E}_p}\right)^2\frac{1}{\tilde{E}_p}.$$

Using quadratic approximation technique, the cost function $C_i(E_i)$ is evaluated at \tilde{E}_i as

$$C_i(E_i) = C_i(\tilde{E}_i) + C_i'(\tilde{E}_i)(E_i - \tilde{E}_i) + (0.5)C_i''(\tilde{E}_i)(E_i - \tilde{E}_i)^2.$$

$C_i(E_i)$ is rewritten as

$$C_i(E_i) = \gamma_i + \alpha_i(\tilde{E}_i - E_i) + \beta_i(\tilde{E}_i - E_i)^2$$

$$\gamma_i = C_i(\tilde{E}_i),$$

$$\alpha_i = -C_i'(\tilde{E}_i),$$

$$\beta_i = (0.5)C_i''(\tilde{E}_i). \quad \gamma_i, \alpha_i, \beta_i > 0.$$

The parameter γ_i is an abatement cost, and α_i is marginal abatement cost, and β_i is a coefficient of variance at the level of emission \tilde{E}_i . If the current level of emission E_i is equal to \tilde{E}_i , the abatement cost will be $\gamma_i = C_i(\tilde{E}_i)$.

Table A.5 Abatement Cost and Emissions

E_1	$C_1(E_1)$	E_2	$C_2(E_2)$	E_3	$C_3(E_3)$	E_4	$C_4(E_4)$
32413	2353899	60905	3630080	122831	1154362	185708	1678837
31856	2395949	60311	3666215	119998	1177933	172117	1792526
31298	2434055	59718	3700431	117166	1198572	158526	1861400
30741	2468410	59124	3732779	114333	1216475	144935	1896514
30183	2499205	58531	3763313	111500	1231840	131344	1908928
29626	2526634	57937	3792084	108668	1244863	117753	1909698
29069	2550889	57344	3819143	105835	1255742	104161	1909883
28511	2572163	56750	3844544	103002	1264673	90570	1920539
27954	2590647	56157	3868337	100169	1271854	76979	1952725
27396	2606534	55563	3890575	97337	1277481	63388	2017497
26839	2620018	54970	3911310	94504	1281751	49797	2125914

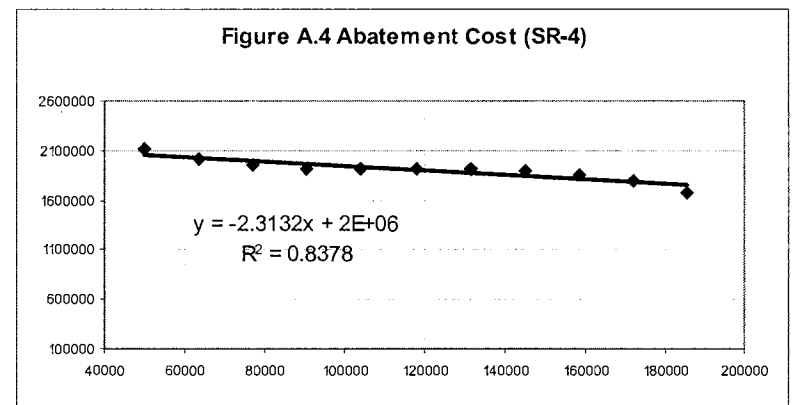
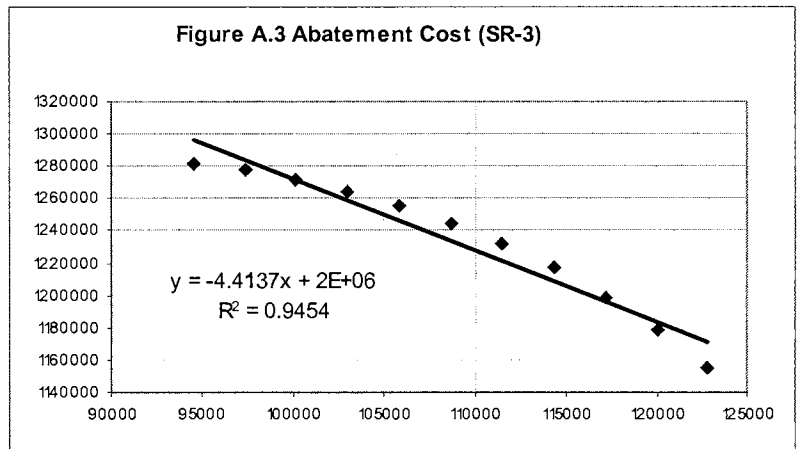
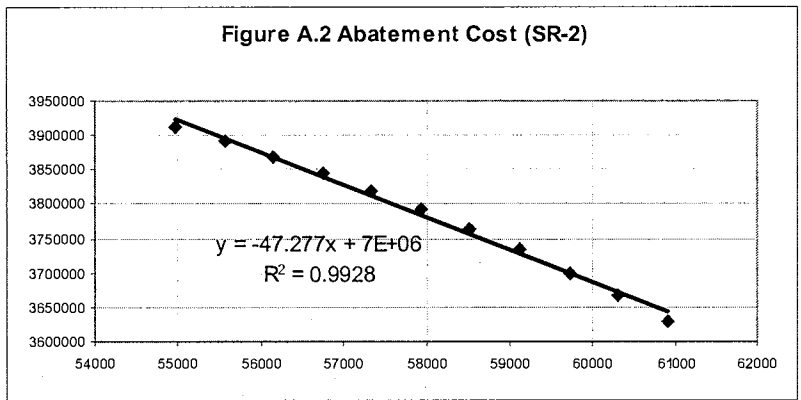
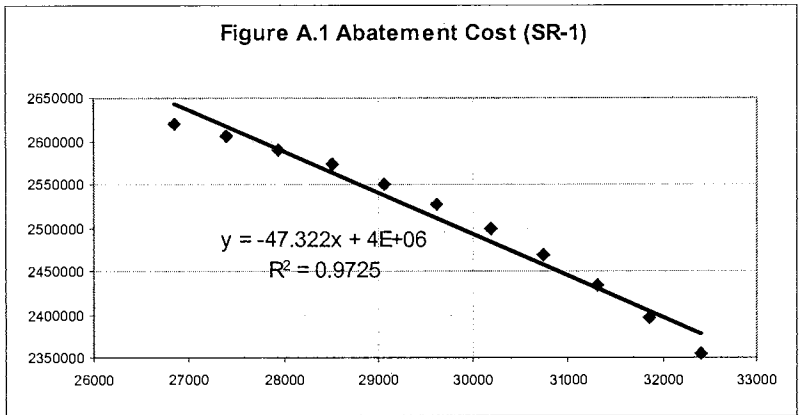


Table A.6 Parameters and Initial Values

i	\tilde{E}_i (ton/Yr)	γ (million\$)	α (\$/Kg)	β (dollar/ton ²)	π (\$/ton)
1	27	4	47.3	8.44	52.1
2	55	7	47.3	7.87	52.6
3	95	2	4.4	0.15	5.9
4	50	2	2.3	0.02	74.6

Transport Coefficients Matrix²⁴

Table A.7 Total Phosphorus Transported to the Lake Tenkiller (1991-1993)

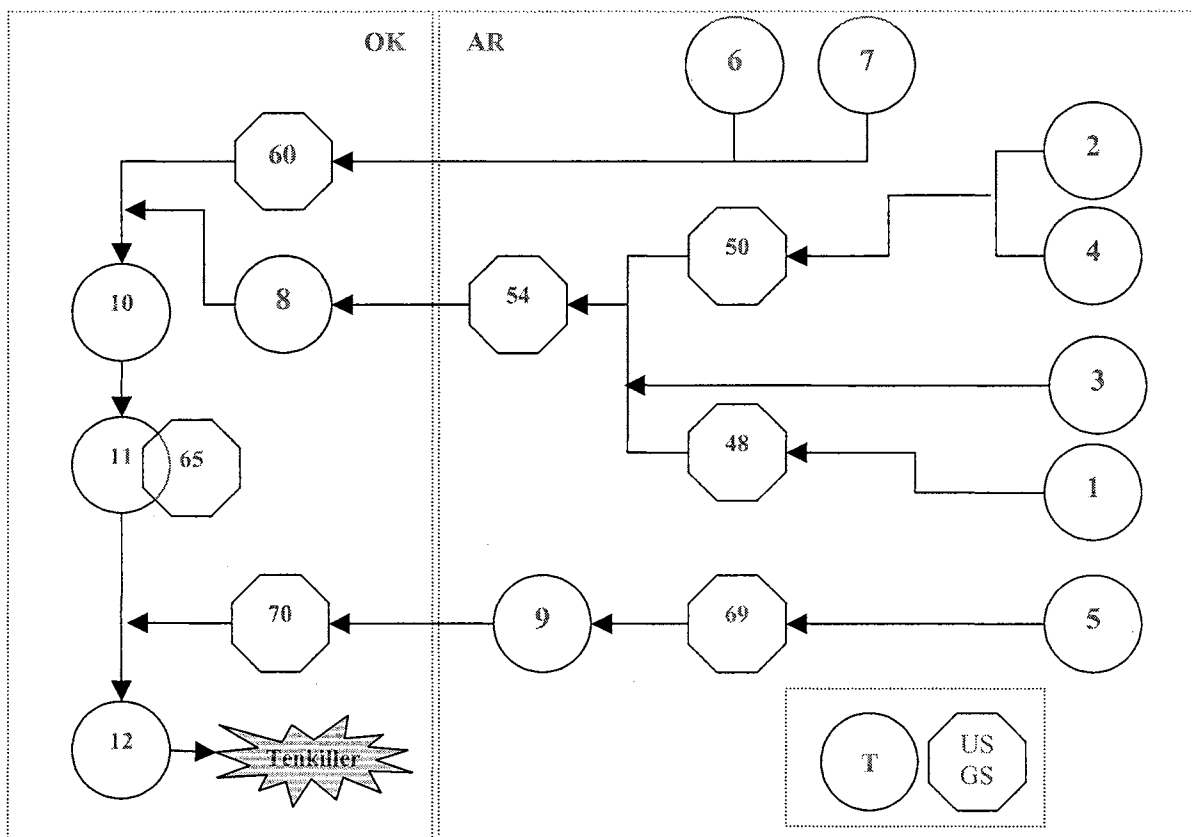
Discharger	Estimated Load at Source (kg/yr)	Distance to Horseshoe Bend (km)	Estimated Corrected Load at Horseshoe Bend			Estimated Annual Total Load (kg/yr)
			Low Flow (kg/yr)	Medium Flow (kg/yr)	High Flow	
Prairie Grove	1,200	161	19	70	23	70
Rogers	21,600	159	355	1,290	417	1,290
Fayetteville	4,500	156	80	284	90	284
Springdale	43,150	153	820	2,860	893	2,860
Lincoln	1,200	130	38	115	31	115
Gentry	1,700	109	85	232	56	232
Siloam Springs	10,000	100	623	1,610	362	1,610
Watts	500	100	31	81	18	81
Westville	2,900	45	615	1,240	187	1,240
Midwestern	600	23	211	391	49	391
Tahlequah	4,700	10	2,200	3,910	441	3,910
Cherokee Nation	530	8	257	455	51	455
Total	92,600		5,300	12,500	2,620	12,500

Sources: David R Gade (Dissertation, 1998)

Transport coefficient matrix for the Illinois River Basin is not available. The relevant data is found from Gade (Dissertation, 1998). These data cover Prairie Grove, Rogers, Fayetteville, Springdale, Lincoln, Gentry, Siloam Springs, Watts, Westville, Midwestern Nursery, Tahlequah, Cherokee Nation, and Tenkiller. This study provides the estimates of total phosphorus transported to the Lake Tenkiller.

²⁴ "Rivers and streams are the major routes of transfer of phosphorus to ocean and many lakes. The main source of phosphorus includes the mining phosphate and its agricultural, industrial and domestic uses. Other activities such as clearing of forests, extensive cultivation and urban waste disposal and drainage systems have enhanced the transport of P from terrestrial to aquatic environments." JOHN M. MELACK, "Transport and transformations of P, Fluvial and Lacustrine Ecosystems," SCOPE54, *PHOSPHORUS IN THE GLOBAL ENVIRONMENT*, Edited by Holm Tiessen, Scientific Committee on Problems of the Environment (SCOPE), 1995: 245-254.

Figure A.5 Direction of Phosphorus Transport



Sources: David R Gade (Thesis, 1998), Arkansas-Oklahoma Compact, "Water Quality Monitoring Report Illinois River Basin," CY2002.

The conceptual diagram flow chart describes the direction and the relationship between the upstream and the downstream points in the Illinois River Basin stream (Gade, 1998).

Using these data, the estimated transport coefficients from each region to the ending point (Tenkiller Lake) are as follows.

i	1	2	3	4	5	6	7	8	9	10	11	12
β	0.058	0.060	0.063	0.066	0.096	0.136	0.161	0.162	0.428	0.652	0.832	0.858

The transport coefficients matrix can be derived based some assumptions. Consider the transport coefficient between the upstream and the next first downstream region. Let $\beta_n, \beta_{n-1}, \dots, \beta_1$ denote the transport coefficient from the upstream region to the ending downstream region. The subscription n denotes the n -th upstream point from the ending point. Let $\alpha_n, \alpha_{n-1}, \dots, \alpha_1$ denote transport coefficients from the upstream region to the next first downstream region.

First, assume that the downstream regions do not affect the upstream regions. From this assumption, the zero values are obtained as transport coefficients from the downstream points to the upstream point.

Second, assume that the transport coefficient β consists of α as $\beta_n = \alpha_n \cdot \alpha_{n-1} \cdots \alpha_2 \cdot \alpha_1$. The values of β is given. Thus, $\beta_1 = \bar{\beta}_1$. It is obvious that $\alpha_1 = \bar{\beta}_1$. With the assumption that $\beta_n = \alpha_n \cdot \alpha_{n-1} \cdots \alpha_2 \cdot \alpha_1$,

$$\alpha_1 = \bar{\beta}_1, \alpha_2 = \frac{\bar{\beta}_2}{\bar{\alpha}_1}, \alpha_3 = \frac{\bar{\beta}_3}{\bar{\alpha}_1 \cdot \bar{\alpha}_2}, \dots, \alpha_n = \frac{\bar{\beta}_n}{\bar{\alpha}_1 \cdots \bar{\alpha}_{n-1}}.$$

From these procedures, the obtained transport coefficient matrix is as shown in Table A.9. The remaining transport coefficients are classified by two groups. The first group is G-group which consists of the coefficients from n to n . The second group is B-group which consists of the coefficients between the bordering regions. By the definition, the coefficient values of G group are obtained from the formula $(1 - \alpha_n)$. Assume that the bordering region j is affected from the bordering region i by the formula that $a_{ji} = (1 - \alpha_i)(\alpha_{ji})$. From these procedures, (13×13) transport coefficient matrix is obtained as shown in Figure A.6.

Table A.9 Transport Coefficient Construction Procedure

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	G1	0.000	B3.1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.000	G2	0.000	B4.2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	B1.3	0.000	G3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	B2.4	0.000	G4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	G5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	G6	B7.6	0.000	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	B6.7	G7	0.000	0.000	0.000	0.000	0.000	0.000
8	0.360	0.369	0.390	0.409	0.000	0.000	0.000	G8	0.000	0.000	0.000	0.000	0.000
9	0.000	0.000	0.000	0.000	0.224	0.000	0.000	0.000	G9	0.000	0.000	0.000	0.000
10	0.090	0.092	0.097	0.102	0.000	0.209	0.247	0.249	0.000	G10	0.000	0.000	0.000
11	0.070	0.072	0.076	0.080	0.176	0.164	0.194	0.195	0.000	0.783	G11	0.000	0.000
12	0.068	0.070	0.074	0.077	0.170	0.159	0.188	0.189	0.498	0.759	0.969	G12	0.000
13	0.058	0.060	0.063	0.066	0.096	0.136	0.161	0.162	0.428	0.652	0.832	0.858	1.000

Figure A.6 Transport Coefficient Matrix

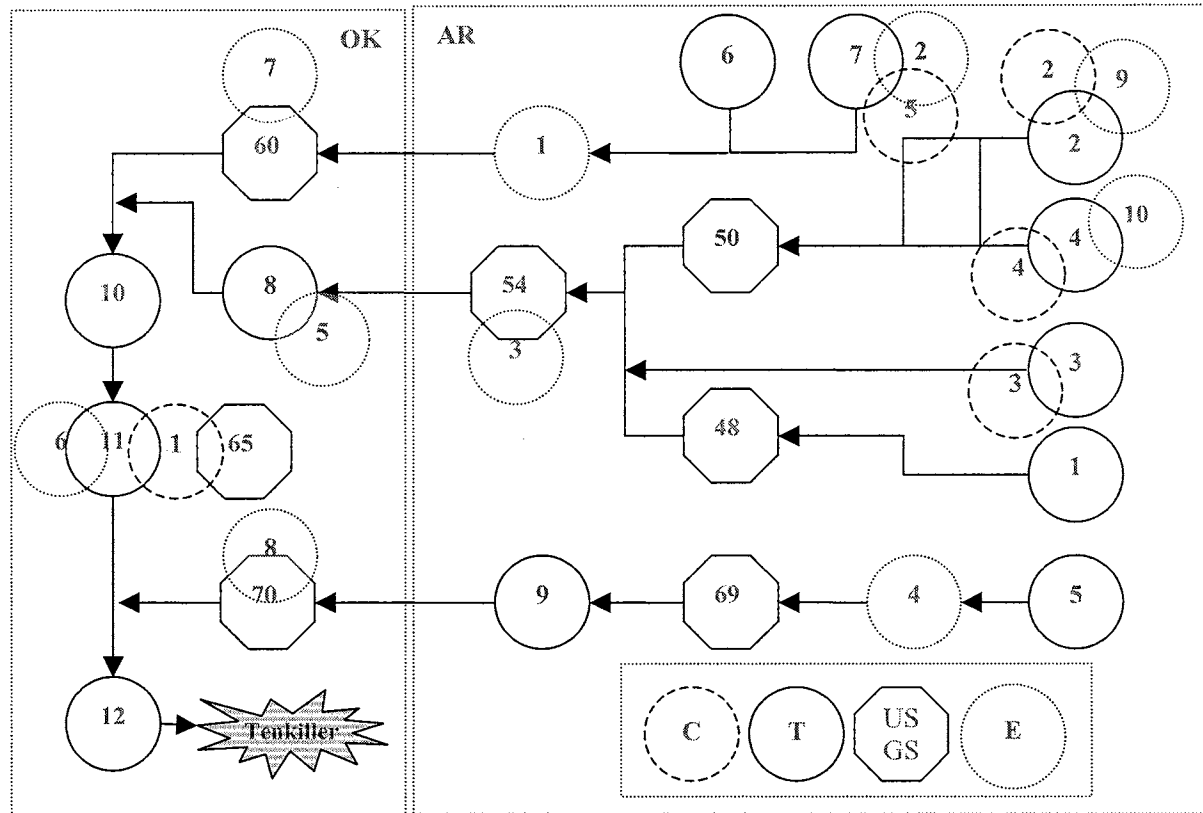
0.910	0.000	0.907	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.908	0.000	0.093	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.907	0.000	0.903	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.083	0.000	0.898	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.776	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.791	0.195	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.157	0.753	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.360	0.369	0.390	0.409	0.000	0.000	0.000	0.751	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.224	0.000	0.000	0.000	0.502	0.000	0.000	0.000	0.000	0.000
0.090	0.092	0.097	0.102	0.000	0.209	0.247	0.249	0.000	0.241	0.000	0.000	0.000	0.000
0.070	0.072	0.076	0.080	0.176	0.164	0.194	0.195	0.000	0.783	0.031	0.000	0.000	0.000
0.068	0.070	0.074	0.077	0.170	0.159	0.188	0.189	0.498	0.759	0.969	0.142	0.000	0.000
0.058	0.060	0.063	0.066	0.096	0.136	0.161	0.162	0.428	0.652	0.832	0.858	1.000	0.000

The availability of data after constructing data is as shown in Table A.10.

Transport coefficients (T)		Emissions (E)		WWTP Cost (C)	
1	Prairie Grove (AR)	1	ARK04A (AR)	1	TAHLEQUAH (OK)
2	ROGERS (AR)	2	ARK05 (AR)	2	ROGERS (AR)
3	Fayetteville (AR)	3	ARK06 (AR)	3	Fayetteville (AR)
4	SPRINGDALE (AR)	4	ARK07 (AR)	4	SPRINGDALE (AR)
5	Lincoln (AR)	5	Watts (OK)	5	Siloam Springs (AR)
6	GENTRY (AR)	6	TAHLEQUAH (OK)		
7	Siloam Springs (AR)	7	Kansas (OK)		
8	Watts (OK)	8	Eldon (OK)		
9	WESTVILLE (OK)	9	Rogers (AR)		
10	Midwestern Nursery (OK)	10	Springdale (AR)		
11	TAHLEQUAH (OK)				
12	Cherokee Nation (OK)				

The final step is to select sub-regions for numerical application. The selected sub-regions should include more than two sub-regions which are bordered each other. If this requirement is not satisfied by available data sets, the numerical application cannot generate all possible results. When the requirement is considered, the availability of data is as shown in Figure A.7. There are four complete data sets which include two bordered sub-regions. Finally, three sub-regions of AR and one region of OK are selected for numerical application. The data sets are used for Chapter V and Appendix D,E.

Figure A.7 Data Availability and Regional Selection



Notes: C = cost data; T = transport coefficient data; E = phosphorus emission data.
 Sources: David R Gade (Thesis, 1998), Arkansas-Oklahoma Compact, "Water Quality Monitoring Report Illinois River Basin," CY2002.

APPENDIX D: NUMERICAL APPLICATION

Table 5.1 Parameters and Initial Levels

i	\tilde{E}_i (ton/Yr)	γ (million\$)	α (\$/Kg)	β (dollar/ton ²)	π (\$/ton)
1	27	4	47.3	8.44	52.1
2	55	7	47.3	7.87	52.6
3	95	2	4.4	0.15	5.9
4	50	2	2.3	0.02	74.6

Table 5.2 Transport Coefficients

Emitting Region (i) \backslash Receiving Region (j)	1	2	3	4
	1	0.908	0.093	0.000
2	0.083	0.898	0.000	0.000
3	0.000	0.000	0.753	0.000
4	0.072	0.080	0.194	0.031

Figure A.8 Abatement Cost Sharing Parameter

$$\delta_{ij} = \frac{a_{ij}\pi_i}{\sum_{k=1}^n a_{kj}\pi_k} = \begin{pmatrix} 0.829 & 0.083 & 0.000 & 0.000 \\ 0.077 & 0.814 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.234 & 0.000 \\ 0.094 & 0.103 & 0.766 & 1.000 \end{pmatrix}$$

Table 5.4 Costs Reduction Without/With Transfers (m.\$)

	(1)	(2)	(3)	(4)
	$(J_i(E^*) - J_i(\tilde{E}))$	$T_i(E^*)$	$J_i^T(E^*) - J_i^T(\tilde{E})$	$\frac{(J_i^T(E^*) - J_i^T(\tilde{E}))}{J_i^T(\tilde{E})}$
SR-1	-0.5	-2.1	-2.6	-0.17%
SR-2	1.2	-4.4	-3.2	-0.12%
SR-3	338.5	-406.4	-67.9	-534.87%
SR-4	-684.2	413.0	-271.3	-11.44%
Total	-345.0	0.0	-345.0	-5.20%

Table A.11 Total Costs ($J_i(E^*) - J_i(\bar{E})$) per Region (m.\$)

AR			OK
SR-1	SR-2	SR-3	SR-4
-0.5	1.2	338.5	-684.2
339.2			-684.2

Relevant Equations

$$C_i(E_i) = \gamma_i + \alpha_i(\bar{E}_i - E_i) + \beta_i(\bar{E}_i - E_i)^2 \quad i = 1, \dots, 4.$$

$$Q = AE,$$

$$A = \begin{pmatrix} 0.908 & 0.093 & 0.000 & 0.000 \\ 0.083 & 0.898 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.753 & 0.000 \\ 0.072 & 0.080 & 0.194 & 0.031 \end{pmatrix}, \quad \mathbf{E} = \begin{pmatrix} 27 \\ 55 \\ 95 \\ 50 \end{pmatrix}$$

$$Q_i(E_i) = \sum_{j=1}^4 a_{ij} E_j \quad j = 1, \dots, 4.$$

$$D = \pi Q$$

$$D_i(E_i) = \pi_i Q_i = \pi_i \sum_{j=1}^4 a_{ij} E_j$$

Noncooperative Nash Equilibrium Condition

$$J_i(E_i, E_j) = C_i(E_i) + D_i(Q_i)$$

$$-C'_i(E_i) = a_{ii} D'_i(Q_i)$$

$$\pi_i = \frac{\alpha_i}{a_{ii}}$$

Cooperative Equilibrium Condition

$$J(E_i, E_j) = J_i(E_i, E_j) + J_j(E_i, E_j)$$

$$-C'_i(E_i) = a_{ii} D'_i(Q_i) + a_{ij} D'_j(Q_j) \quad i, j = 1, \dots, 4. \quad i \neq j.$$

APPENDIX E: RESULTS BY SCENARIO

**Table A.12 Evolution of P Emissions (ton/Yr)
3% Reduction**

	SR-1	SR-2	SR-3	SR-4
0	26.84	54.97	94.50	49.80
1	26.26	54.28	91.67	49.80
2	26.26	54.28	88.92	49.80
3	26.26	54.28	86.25	49.80
4	26.26	54.28	83.66	49.80
5	26.26	54.28	81.15	49.80
6	26.26	54.28	78.72	49.80
7	26.26	54.28	76.36	49.80
8	26.26	54.28	74.07	49.80
9	26.26	54.28	71.84	49.80
10	26.26	54.28	69.69	49.80
11	26.26	54.28	67.60	49.80
12	26.26	54.28	65.57	49.80
13	26.26	54.28	63.60	49.80
14	26.26	54.28	61.70	49.80
15	26.26	54.28	59.84	49.80
16	26.26	54.28	58.05	49.80
17	26.26	54.28	56.31	49.80
18	26.26	54.28	54.62	49.80
19	26.26	54.28	52.98	49.80
20	26.26	54.28	51.39	49.80
21	26.26	54.28	49.85	49.80
22	26.26	54.28	48.35	49.80
23	26.26	54.28	47.73	49.80
24	26.26	54.28	47.73	49.80
25	26.26	54.28	47.73	49.80

Figure A.9

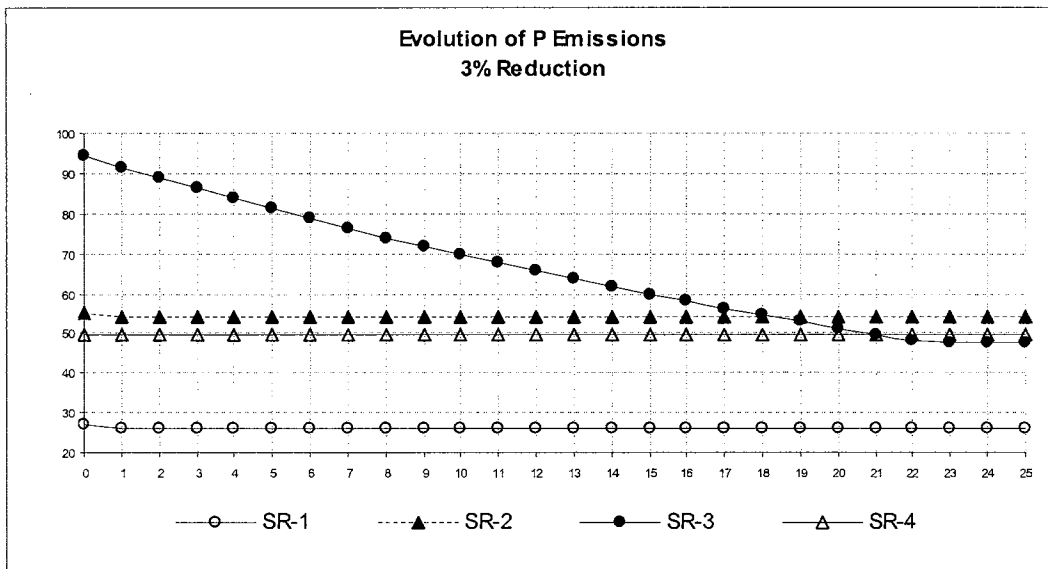


Table A.13 Evolution of Total Costs per Region (m\$)
3% Reduction

	SR-1	SR-2	SR-3	SR-4
0	0.0	0.0	0.0	0.0
1	-0.5	1.2	1.2	-48.2
2	-0.5	1.2	4.8	-88.0
3	-0.5	1.2	10.5	-126.7
4	-0.5	1.2	18.2	-164.1
5	-0.5	1.2	27.6	-200.5
6	-0.5	1.2	38.6	-235.7
7	-0.5	1.2	51.0	-269.9
8	-0.5	1.2	64.6	-303.0
9	-0.5	1.2	79.5	-335.2
10	-0.5	1.2	95.3	-366.4
11	-0.5	1.2	112.0	-396.7
12	-0.5	1.2	129.6	-426.0
13	-0.5	1.2	147.8	-454.5
14	-0.5	1.2	166.6	-482.1
15	-0.5	1.2	185.9	-508.9
16	-0.5	1.2	205.7	-534.9
17	-0.5	1.2	225.8	-560.1
18	-0.5	1.2	246.2	-584.6
19	-0.5	1.2	266.8	-608.3
20	-0.5	1.2	287.7	-631.3
21	-0.5	1.2	308.6	-653.6
22	-0.5	1.2	329.6	-675.3
23	-0.5	1.2	338.5	-684.2
24	-0.5	1.2	338.5	-684.2
25	-0.5	1.2	338.5	-684.2

Figure A.10

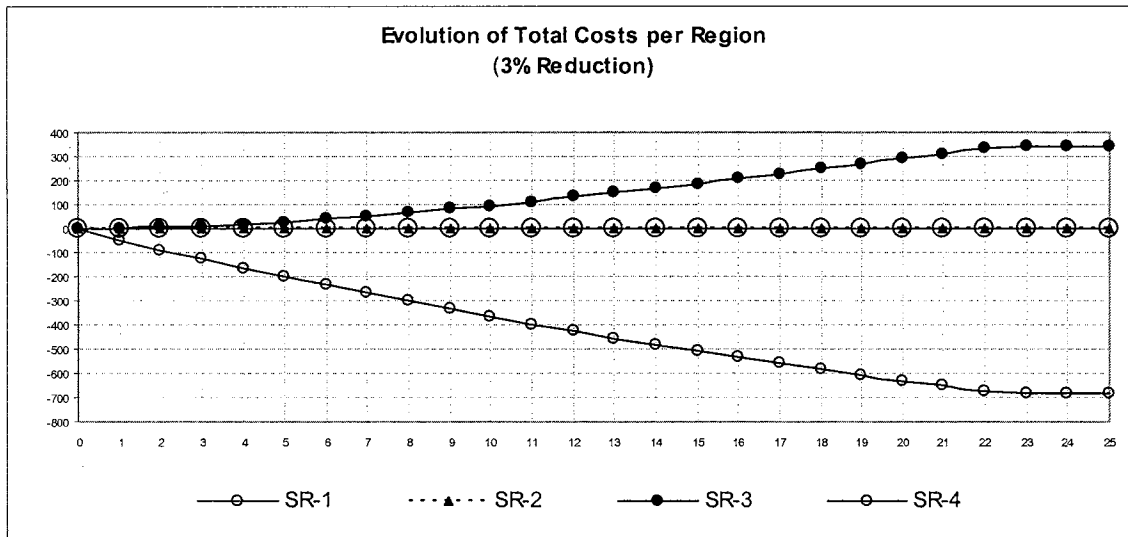
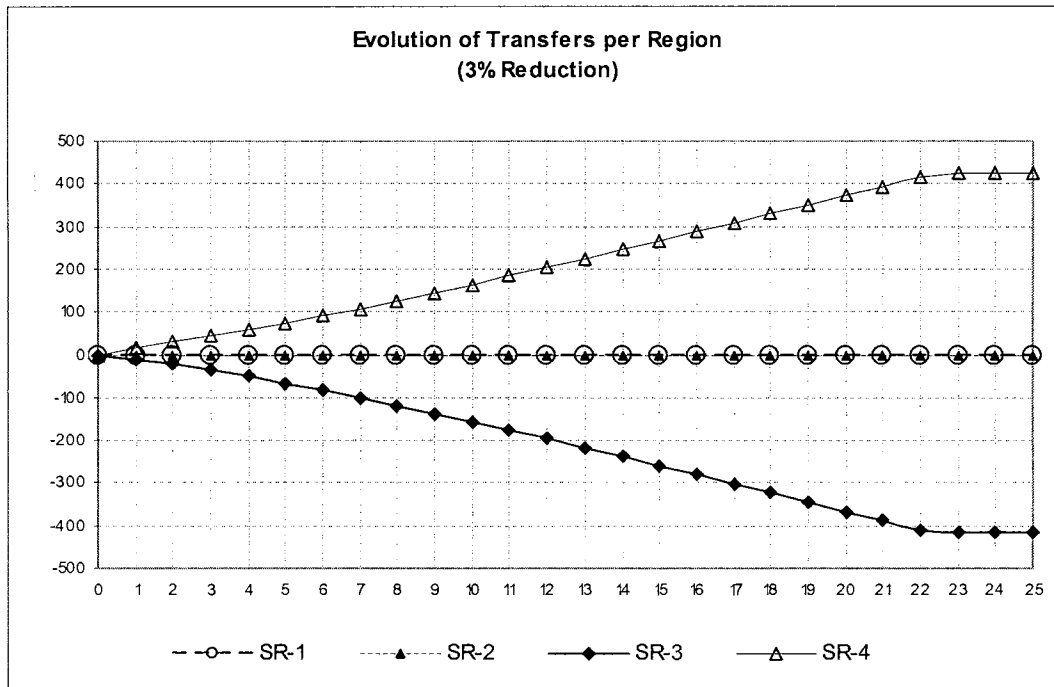


Table A.14 Financial Transfers (m\$)
3% Reduction

	SR-1	SR-2	SR-3	SR-4
0	0	0	0	0
1	-2	-4	-11	17
2	-2	-4	-23	29
3	-2	-4	-36	43
4	-2	-4	-51	57
5	-2	-4	-66	73
6	-2	-4	-83	89
7	-2	-4	-100	107
8	-2	-4	-119	125
9	-2	-4	-138	144
10	-2	-4	-157	164
11	-2	-4	-177	183
12	-2	-4	-197	204
13	-2	-4	-218	224
14	-2	-4	-239	245
15	-2	-4	-260	266
16	-2	-4	-281	287
17	-2	-4	-302	309
18	-2	-4	-324	330
19	-2	-4	-345	351
20	-2	-4	-366	373
21	-2	-4	-388	394
22	-2	-4	-409	415
23	-2	-4	-418	424
24	-2	-4	-418	424
25	-2	-4	-418	424

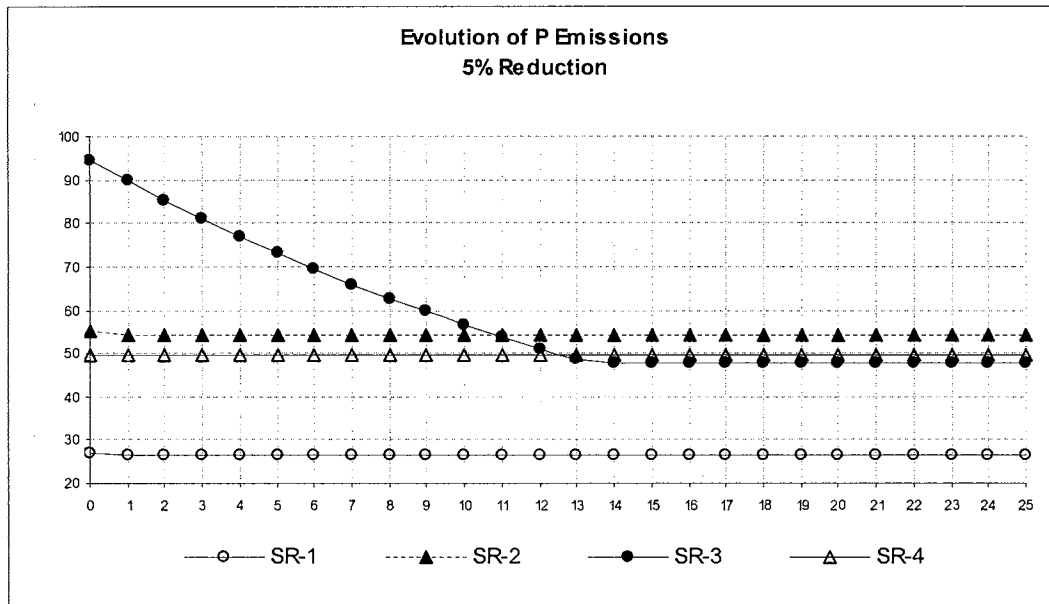
Figure A.11



**Table A.15 Evolution of P Emissions
5% Reduction**

	SR-1	SR-2	SR-3	SR-4
0	26.84	54.97	94.50	49.80
1	26.26	54.28	89.78	49.80
2	26.26	54.28	85.29	49.80
3	26.26	54.28	81.03	49.80
4	26.26	54.28	76.97	49.80
5	26.26	54.28	73.13	49.80
6	26.26	54.28	69.47	49.80
7	26.26	54.28	66.00	49.80
8	26.26	54.28	62.70	49.80
9	26.26	54.28	59.56	49.80
10	26.26	54.28	56.58	49.80
11	26.26	54.28	53.75	49.80
12	26.26	54.28	51.07	49.80
13	26.26	54.28	48.51	49.80
14	26.26	54.28	47.73	49.80
15	26.26	54.28	47.73	49.80
16	26.26	54.28	47.73	49.80
17	26.26	54.28	47.73	49.80
18	26.26	54.28	47.73	49.80
19	26.26	54.28	47.73	49.80
20	26.26	54.28	47.73	49.80
21	26.26	54.28	47.73	49.80
22	26.26	54.28	47.73	49.80
23	26.26	54.28	47.73	49.80
24	26.26	54.28	47.73	49.80
25	26.26	54.28	47.73	49.80

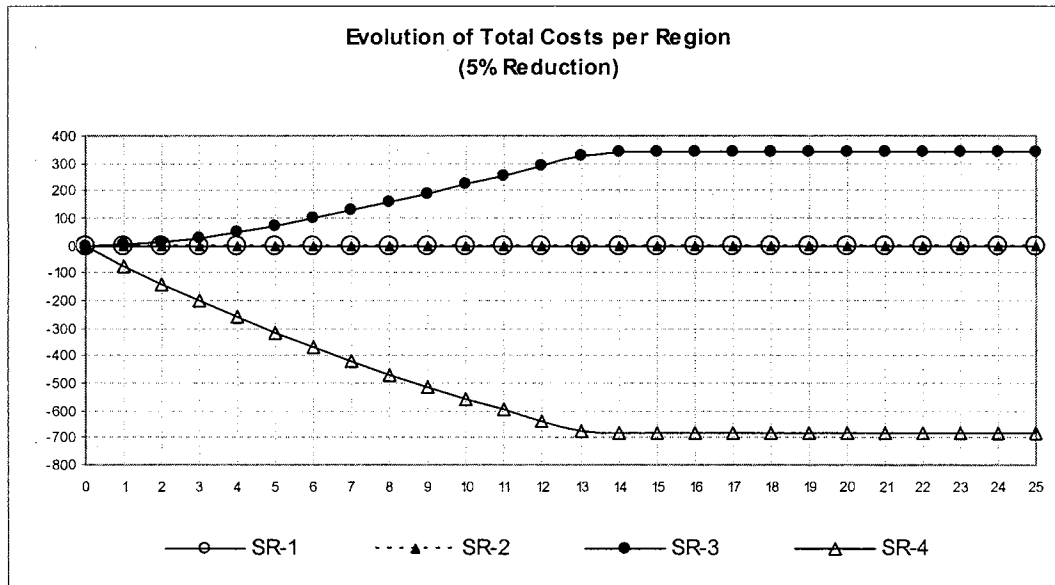
Figure A.12



**Table A.16 Evolution of Total Costs per Region
5% Reduction**

	SR-1	SR-2	SR-3	SR-4
0	0.0	0.0	0.0	0.0
1	-0.5	1.2	3.5	-75.6
2	-0.5	1.2	13.1	-140.6
3	-0.5	1.2	28.1	-202.3
4	-0.5	1.2	47.6	-261.0
5	-0.5	1.2	70.7	-316.7
6	-0.5	1.2	97.0	-369.6
7	-0.5	1.2	125.8	-419.9
8	-0.5	1.2	156.6	-467.7
9	-0.5	1.2	189.0	-513.0
10	-0.5	1.2	222.5	-556.1
11	-0.5	1.2	257.0	-597.1
12	-0.5	1.2	292.0	-636.0
13	-0.5	1.2	327.3	-673.0
14	-0.5	1.2	338.5	-684.2
15	-0.5	1.2	338.5	-684.2
16	-0.5	1.2	338.5	-684.2
17	-0.5	1.2	338.5	-684.2
18	-0.5	1.2	338.5	-684.2
19	-0.5	1.2	338.5	-684.2
20	-0.5	1.2	338.5	-684.2
21	-0.5	1.2	338.5	-684.2
22	-0.5	1.2	338.5	-684.2
23	-0.5	1.2	338.5	-684.2
24	-0.5	1.2	338.5	-684.2
25	-0.5	1.2	338.5	-684.2

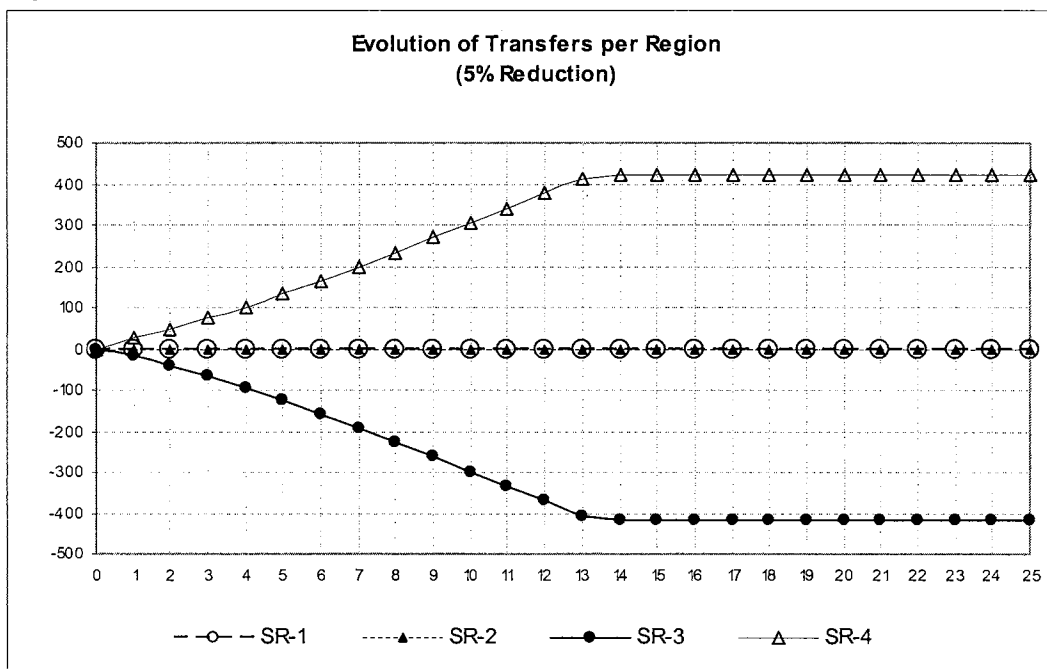
Figure A.13



**Table A.17 Financial Transfers
5% Reduction**

	SR-1	SR-2	SR-3	SR-4
0	0	0	0	0
1	-2	-4	-19	25
2	-2	-4	-41	48
3	-2	-4	-67	74
4	-2	-4	-96	102
5	-2	-4	-127	133
6	-2	-4	-159	166
7	-2	-4	-193	199
8	-2	-4	-228	234
9	-2	-4	-263	270
10	-2	-4	-299	305
11	-2	-4	-335	341
12	-2	-4	-371	377
13	-2	-4	-406	413
14	-2	-4	-418	424
15	-2	-4	-418	424
16	-2	-4	-418	424
17	-2	-4	-418	424
18	-2	-4	-418	424
19	-2	-4	-418	424
20	-2	-4	-418	424
21	-2	-4	-418	424
22	-2	-4	-418	424
23	-2	-4	-418	424
24	-2	-4	-418	424
25	-2	-4	-418	424

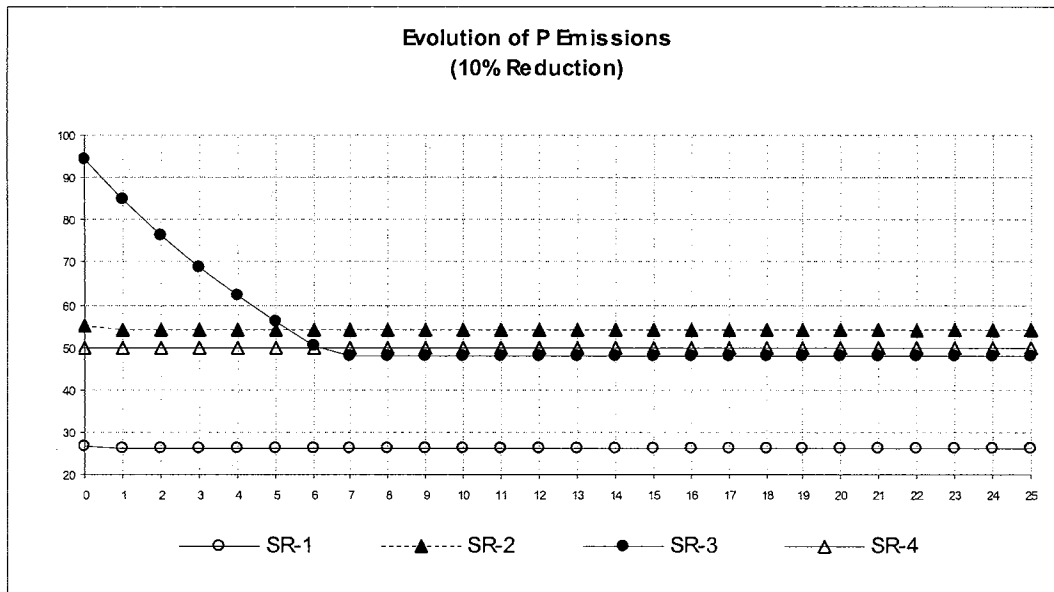
Figure A.14



**Table A.18 Evolution of P Emissions
10% Reduction**

	SR-1	SR-2	SR-3	SR-4
0	26.84	54.97	94.50	49.80
1	26.26	54.28	85.05	49.80
2	26.26	54.28	76.55	49.80
3	26.26	54.28	68.89	49.80
4	26.26	54.28	62.00	49.80
5	26.26	54.28	55.80	49.80
6	26.26	54.28	50.22	49.80
7	26.26	54.28	47.73	49.80
8	26.26	54.28	47.73	49.80
9	26.26	54.28	47.73	49.80
10	26.26	54.28	47.73	49.80
11	26.26	54.28	47.73	49.80
12	26.26	54.28	47.73	49.80
13	26.26	54.28	47.73	49.80
14	26.26	54.28	47.73	49.80
15	26.26	54.28	47.73	49.80
16	26.26	54.28	47.73	49.80
17	26.26	54.28	47.73	49.80
18	26.26	54.28	47.73	49.80
19	26.26	54.28	47.73	49.80
20	26.26	54.28	47.73	49.80
21	26.26	54.28	47.73	49.80
22	26.26	54.28	47.73	49.80
23	26.26	54.28	47.73	49.80
24	26.26	54.28	47.73	49.80
25	26.26	54.28	47.73	49.80

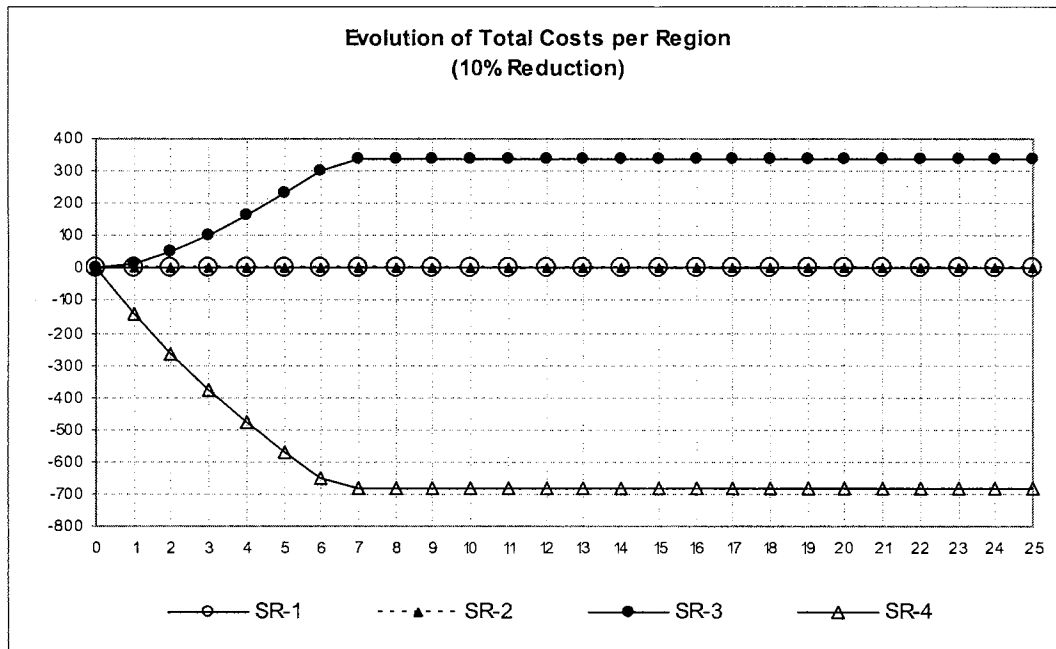
Figure A.15



**Table A.19 Evolution of Total Costs per Region
10% Reduction**

	SR-1	SR-2	SR-3	SR-4
0	0.00	0.00	0.00	0.00
1	-0.50	1.18	13.82	-144.00
2	-0.50	1.18	49.90	-267.13
3	-0.50	1.18	101.51	-377.94
4	-0.50	1.18	163.46	-477.67
5	-0.50	1.18	231.78	-567.43
6	-0.50	1.18	303.45	-648.21
7	-0.50	1.18	338.53	-684.25
8	-0.50	1.18	338.53	-684.25
9	-0.50	1.18	338.53	-684.25
10	-0.50	1.18	338.53	-684.25
11	-0.50	1.18	338.53	-684.25
12	-0.50	1.18	338.53	-684.25
13	-0.50	1.18	338.53	-684.25
14	-0.50	1.18	338.53	-684.25
15	-0.50	1.18	338.53	-684.25
16	-0.50	1.18	338.53	-684.25
17	-0.50	1.18	338.53	-684.25
18	-0.50	1.18	338.53	-684.25
19	-0.50	1.18	338.53	-684.25
20	-0.50	1.18	338.53	-684.25
21	-0.50	1.18	338.53	-684.25
22	-0.50	1.18	338.53	-684.25
23	-0.50	1.18	338.53	-684.25
24	-0.50	1.18	338.53	-684.25
25	-0.50	1.18	338.53	-684.25

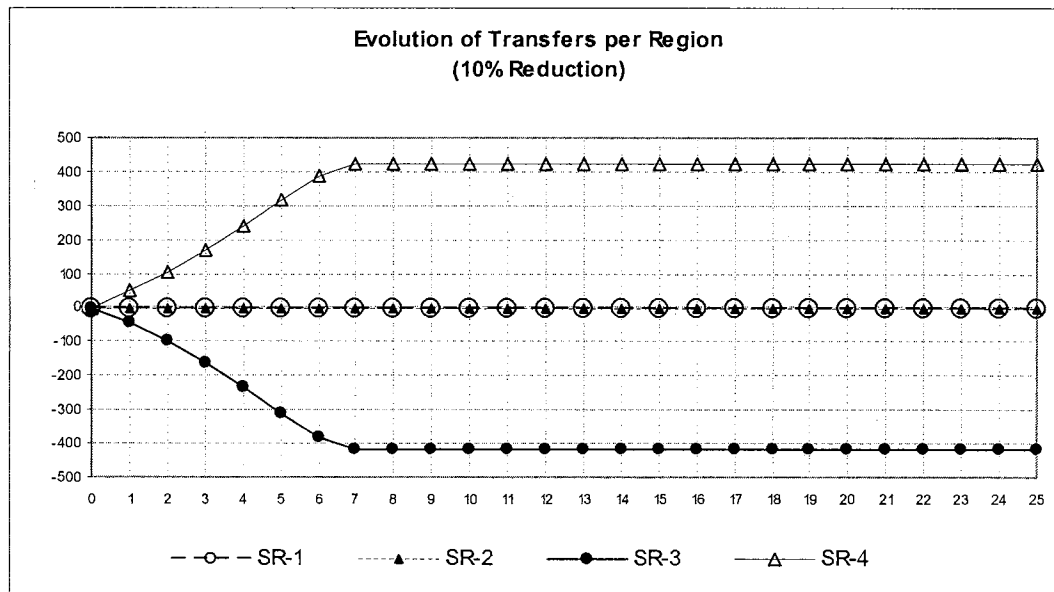
Figure A.16



**Table A.20 Financial Transfers
10% Reduction**

	SR-1	SR-2	SR-3	SR-4
0	0	0	0	0
1	-2	-4	-43	49
2	-2	-4	-99	106
3	-2	-4	-164	171
4	-2	-4	-235	242
5	-2	-4	-309	315
6	-2	-4	-382	389
7	-2	-4	-418	424
8	-2	-4	-418	424
9	-2	-4	-418	424
10	-2	-4	-418	424
11	-2	-4	-418	424
12	-2	-4	-418	424
13	-2	-4	-418	424
14	-2	-4	-418	424
15	-2	-4	-418	424
16	-2	-4	-418	424
17	-2	-4	-418	424
18	-2	-4	-418	424
19	-2	-4	-418	424
20	-2	-4	-418	424
21	-2	-4	-418	424
22	-2	-4	-418	424
23	-2	-4	-418	424
24	-2	-4	-418	424
25	-2	-4	-418	424

Figure A.17



VITA



Jeong-Soon Park

Candidate for the Degree of

DOCTOR OF PHILOSOPHY

Dissertation: A DISCRETE TIME ITERATIVE NEGOTIATION ALGORITHM FOR PHOSPHORUS
REDUCTIONS IN THE ILLINOIS RIVER BASIN

Major Field: Economics

Biographical:

Education: Received Bachelor of Art degree in Economics from Sungkyunkwan University, Seoul, Korea in February 1984. Completed the requirements for the Doctor of Philosophy degree with a major in Economics at Oklahoma State University, Stillwater, Oklahoma in May 2005.

Experience: Employed by Korea Energy Economics Institute as a researcher, 1987 to present; employed by Oklahoma State University, Department of Economics and Legal Studies as a teaching assistant, 2000 to 2004.