CODEE Journal

Volume 12 *Linking Differential Equations to Social Justice and Environmental Concerns*

Article 7

2-13-2019

The Mathematics of Gossip

Jessica Deters
Virginia Polytechnic Institute and State University

Izabel P. Aguiar
Stanford University

Jacquie Feuerborn

The Mathematics of Gossip

Cover Page Footnote (optional)
All authors would like to thank the Colorado School of Mines Department of Applied Mathematics and Statistics for providing an encouraging environment for this research.

The Mathematics of Gossip

Jessica Deters
Virginia Polytechnic Institute and State University

Izabel Aguiar
Stanford University

Jacquie Feuerborn

Keywords: Fake news, gossip, ODEs, humanistic mathematics, problem-based learning, active learning
Manuscript received on September 27, 2018; published on February 13, 2019.

Abstract: How does a lie spread through a community? The purpose of this paper is two-fold: to provide an educational tool for teaching Ordinary Differential Equations (ODEs) and sensitivity analysis through a culturally relevant topic (fake news), and to examine the social justice implications of misinformation. Under the assumption that people are susceptible to, can be infected with, and recover from a lie, we model the spread of false information with the classic Susceptible-Infected-Recovered (SIR) model. We develop a system of ODEs with lie-dependent parameter values to examine the pervasiveness of a lie through a community.

The model presents the opportunity for the education of ODEs in a classroom setting through a creative application. The model brings a socially and culturally relevant topic into the classroom, allowing students who may not relate with purely technical examples to connect with the material. Including diverse perspectives in the discussion and development of mathematics and engineering will enable creative and differing approaches to the worlds' problems.

1 Introduction

The current instruction of ODEs lacks socially-relevant examples geared towards helping students understand the social implications and applications of their work. By including a broader diversity of examples in current questions in technical fields, we will be able to address social and cultural problems in more creative and inclusive ways.

The term "fake news" has become increasingly culturally relevant as technology and social media further increases the ability for individuals and media groups to make unfounded claims. Furthermore, the phenomena of fake news has pervaded society as a fundamentally epistemic challenge. The ability to judge what is truth and not reaches beyond evidence and into philosophical arguments. However, the ability to distinguish truth is critical to making informed decisions that inevitably impact society.

The spread of misinformation throughout society has become one of the most pressing issues of our generation. In order to address such an epistemic challenge, we must prepare our future leaders, future developers, future mathematicians to think with this mindset.

The purpose of this paper is two-fold: to provide an educational tool for teaching ODEs and sensitivity analysis through a culturally relevant topic (fake news), and to examine the social justice implications of misinformation. The intentions behind these purposes are to engage students with a culturally relevant example and to encourage students to think creatively about how to apply mathematical tools to societal issues.

The organization of the paper is as follows. In Section 2 we detail the educational foundation behind problem based learning. In Section 3 we discuss the background of the proposed project, develop the Gossip Model used to understand the spread of a rumor, and discuss the use of *anthropomorphized sensitivity analysis* to examine parameter dependence. Suggestions for classroom implementation for the instruction of the Gossip Model are discussed in Section 4. We conclude in Section 5 with discussion and suggestions for future work.

2 Educational Foundation: Problem Based Learning and Student Interest

Active learning approaches have been shown to boost student motivation and student performance. Motivation comes, in part, from interest [1]. Accordingly, if students are interested in the problem and application, they will be more motivated to learn the concept at hand. We posit that many students are interested in the spread of fake news, and more human-centric applications than currently addressed in mathematics classrooms. As a result, by connecting ODEs to the spread of fake news, students will be more motivated to learn ODEs, and more prepared to creatively translate societal issues to mathematical problems.

We provide two classroom implementation suggestions: a **simple implementation** designed for one class period and an **independent work implementation** designed for multiple class periods.

The two implementations offer various levels of an active, problem-based learning pedagogy, meaning students learn by working to understand or solve a problem [3]. Teachers facilitate student learning by guiding their students through the problem and offering help when needed. Students then work in small groups to solve the problem at hand [3]. It is believed that this experience helps students develop problem-solving skills and acquire new information through self-directed inquiry into the problem.

A more detailed explanation of how to implement this project into a class is provided in Section 4.

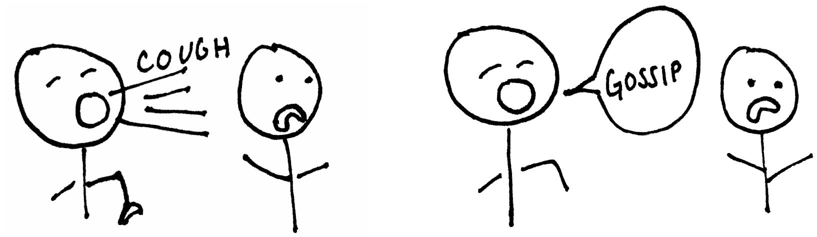

Figure 1: Modeling the spread of gossip with the modeling tools developed to understand infectious diseases.

3 The Gossip Model

In this section we discuss the background, assumptions, and development of the Gossip Model, a system of ODEs used to understand the spread of a lie. The Gossip Model proposed here is that which we use to prompt students to question the societal implications of the spread of a lie, as well as the role of mathematics in addressing such an issue.

3.1 Background and related work

The SIR model for modeling the population biology of infectious diseases was originally developed by Kermack and McKendrick in 1927 [2]. The SIR model describes the dynamics of three populations – those susceptible to (S), infected with (I), and recovered from (R) the infectious disease, and is common in the instruction of ODEs. The system of ODEs describe the following population dynamics: S becomes infected at a transmission rate β when interacting with the I population. I recover (and join the R population) at a rate γ.

When discussing the traditional SIR model, the language used (something *spreading*, people being *infected* and *recovering*) hints toward a more socially-driven application. This shared vocabulary, the idea of the *transmission* of something through interaction (see Figure 1), leads to the next sections wherein we develop and describe the Gossip Model.

3.2 Assumptions of the model

We begin to develop our model by stating our assumptions. Foremost, we assume that the gossip that is spreading is *false*. Similar to the SIR model, we assume that there are three possible populations: those susceptible to the gossip (S), infected with the gossip (I), and recovered from the gossip (R). The S population consists of people who have not yet heard the gossip. The I population consists of people who have heard the gossip *and believe it is true.* The R population consists of people who have heard the gossip *and know it is false.* Furthermore, gossip is only spread through direct interaction with the infected (it's not airborne), and there are a fixed amount of people in the system, $N = S + I + R$.

3.3 The Model

We define four lie-dependent parameters, ρ, β, γ, and α. We assume that, upon hearing the gossip for the first time, $(1 - \rho)$ proportion of the population will immediately know

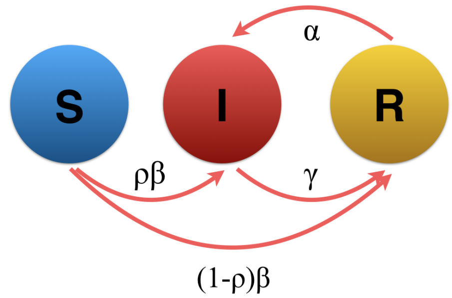

Figure 2: The SIR model described by (3.1). Parameters β and γ describe the rates at which the lie is believed and rejected as false, respectively, ρ describes the percentage of the population immune to the lie, and α describes the rate at which the recovered population become re-infected with the lie.

it is false: that is, they will move directly from S to the R population. This proportion of people describes those in the population who may have existing knowledge counteracting the gossip, or who are perhaps naturally skeptical people.

The rate of transmission, β, describes how "contagious" the gossip is. It is the per-capita rate per unit time that the interaction between populations will lead to infection. This parameter is dependent on the environment in which the gossip is being spread (e.g., maybe gossip spreads faster in a middle school than in an office), the nature of the gossip (e.g., perhaps people will talk more about relationships than ice cream flavors), and the origin of the gossip (e.g., the reliability of the original source).

The rate of recovery, γ describes how easy the gossip is to counteract with other evidence/hearsay. Again, this parameter is dependent on the environment and the nature of the gossip.

The fourth parameter, α, describes how convincing the believers of the gossip (those in the I population) are. This rate acts through the interaction between the I and R populations, and results in the R population becoming re-infected with the gossip.

The system of ODEs for the SIR model described above and shown in Figure 2, is written as,

$$
\begin{aligned}
\frac{\partial S}{\partial t} &= -\beta \cdot S \cdot I \\
\frac{\partial I}{\partial t} &= \rho \cdot \beta \cdot S \cdot I - \gamma \cdot I + \alpha \cdot R \\
\frac{\partial R}{\partial t} &= \gamma \cdot I - \alpha \cdot R + (1-\rho) \cdot \beta \cdot S \cdot I.
\end{aligned}
\tag{3.1}
$$

3.4 An Example

To demonstrate an example of how the model works, consider the gossip, *"Jo made out with Jaimie last night,"* and assume that this statement is a false rumor started by one jealous friend. The following scenes describe the movement between populations upon hearing the rumor. The scene is also demonstrated in the schematic shown in Figure 3.

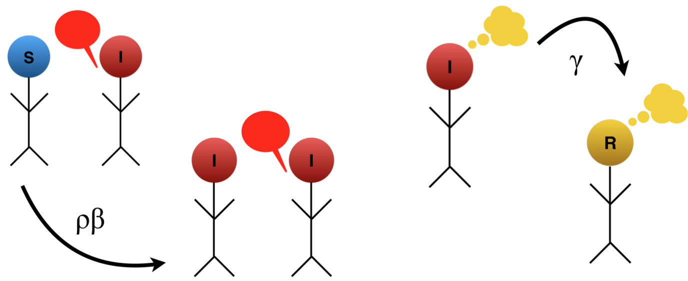

Figure 3: Schematic showing the rates at which the gossip spreads and dies. Leftmost shows the rate $\rho\beta$ at which people who haven't heard the gossip hear and believe it. Middle shows the rate γ at which those who believe the gossip stop believing it. Rightmost shows the rate α at which particularly convincing believers can re-convince nonbelievers of the gossip.

Scene: Susceptible interacting with Infected and becoming Recovered according to $(1-\rho)\beta$
I: Did you hear that Jo made out with Jaimie last night?
S (becoming R): Um no, I was with them both all night.
I: Believe what you want.

Scene: Susceptible interacting with Infected and becoming Infected at $\rho\beta$
I: Did you hear that Jo made out with Jaimie last night?
S (becoming I): OMG I knew they were acting weird.
I: I know right.

Scene: Infected recovering at rate γ
I (becoming R): *(thinking aloud)* It really doesn't make sense for Jo to make out with Jaime... Jo has been home sick with the flu all week.

(Scene: Recovered interacting with Infected and becoming re-Infected at rate α*)*
I: Did you hear that Jo made out with Jaimie last night?
R: You know that's a dumb rumor right?
I: No it's not, I saw them with my own eyes.
R (becoming I): Wow really? I guess it makes sense...

3.5 Anthropomorphized Sensitivity Analysis

As discussed in Section 3.3, parameters ρ, β, γ, and α depend on many characteristics of the gossip scenario. To highlight these differences and how they impact the dynamics of the system, we conduct *Anthropomorphized Sensitivity Analysis*. This approach allows students to observe and practice the purposes of conducting sensitivity analysis, understand the contextual meaning of the parameters, and creatively and humanistically translate social characteristics to mathematics.

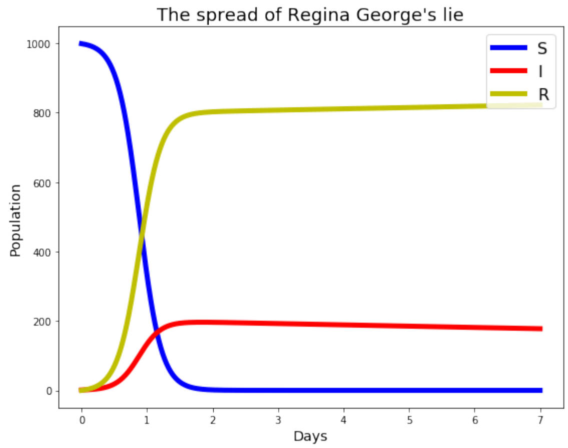

Figure 4: The dynamics of a petty rumor spread by Regina George. We see that by the end of the second day after the gossip started, everybody in the school has heard the rumor. However, by the end of the week, more than 80% of the school knows that the rumor is false.

3.5.1 Regina George

Regina George is one of the main characters in the film *Mean Girls*. Regina George is well-known as a meanspirited, but extremely popular, member of her high school. We assume that Regina George and her gossip have a high priority status at the high school, and thus that the transmission rate, $\beta = 0.03$, is extremely high: each infected person spreads the gossip to 30 other people, per day. Likewise, we assume that the nature of Regina George's gossip is petty, and thus relatively easy to disprove or counteract. This characteristic of the rumor lends to a high rate at which people recover and reject the rumor as false ($\gamma = 0.02$, 20 people per day). We also assume that Regina George is well-known to spread gossip, and thus is relatively unreliable. This unreliability is reflected in that 80% of people ($(1 - \rho) = 0.8$) who hear the gossip automatically discount it as false – they won't be tricked by yet another Regina George rumor.

We run this simulation for 7 days and analyze the situation that has arisen from Regina George's rumor (see Figure 4 for analysis and interpretation).

3.5.2 Dr. Neverheardofher

Dr. Neverheardofher is the top expert in her field. Her field, however, is extremely specific, esoteric, and small (perhaps she studies the ligaments of the Argentine ant). Consider the possibility that Dr. Neverheardofher wants to test her scientific community, and, to do so, introduces a lie in her most recent publication (perhaps that the female Argentine ant is double-jointed). The transmission rate at 1 person per ten days ($\beta = 0.0001$) of this esoteric lie is considerably less impressive than Regina George's (people don't tend to chat about the ligaments of female Argentine ants).

Furthermore, the rate at which people recover from this lie ($\gamma = 0.00001$, one person every one hundred days) is incredibly small – the esoteric and harmless lie does not necessarily inspire independent counter-arguments – and Dr. Neverheardofher is *the*

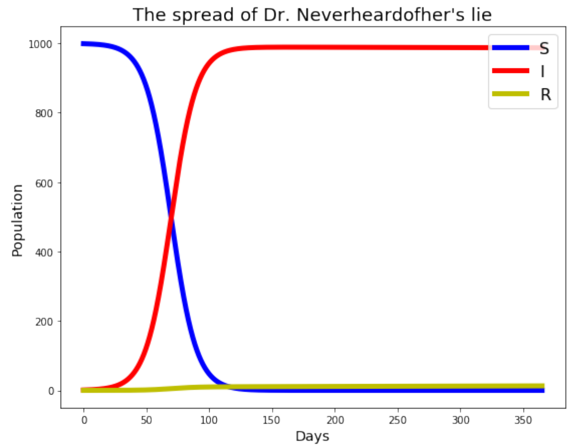

Figure 5: The spread of an esoteric lie started by Dr. Neverheardofher. We see that although it takes considerably longer for her community to be saturated with the lie (about 100 days), by the end of the year, the majority of the community still believes her lie. We do, however, see that the population of the infected is very slowly decreasing.

expert in her field: there is no other evidence counteracting her claim. Finally, because Dr. Neverheardofher is a well-known expert, has her PhD, and has a reputation of producing top-notch science, we assume that only 1% of people who hear the lie ($(1-\rho) = 0.01$) reject it as false. This one percent might represent people who are naturally cynical, people who have some sort of direct experience with the nature of the lie, or have other reasons not to believe it.

We run the scenario for a year to observe the dynamics (see Figure 5 for analysis).

3.5.3 The Conwoman

The Conwoman is charismatic. Her lie, her vision, her movement, is one of which she is so convincing that all of her followers are able to re-convince non-believers of the lie. This charisma is reflected through the parameter, $\alpha = 0.009$. This parameter represents the ability for the Recovered population to become re-Infected upon interacting with the Infected population (see the rightmost picture in Figure 3).

We assume that the Conwoman and her followers' rate of infection ($\beta = 0.003$, three people per day) is higher than that of Dr. Neverheardofher, but lower than that of Regina George–she's popular and influential, but not on the scale as Regina George. We also assume that the Conwoman's lie is easier to recover from ($\gamma = 0.001$, one person per day) than Dr. Neverheardofher's, although not as easy as Regina George's. Furthermore, we assume the Conwoman is a fairly prominent figure with no prior reputation of lying. Thus we assume that only 30% of people ($(1-\rho) = 0.3$) automatically reject the rumor as false. See Figure 6 for the dynamics and analysis.

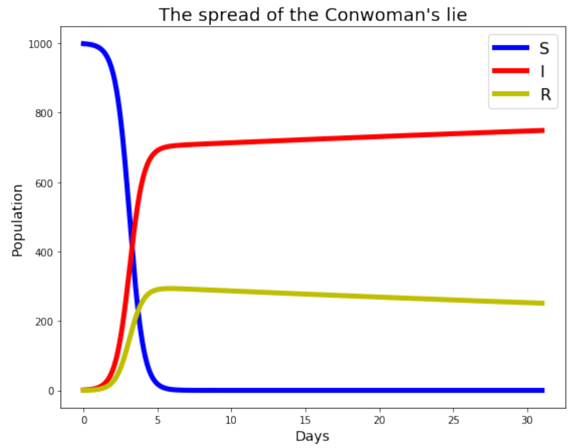

Figure 6: The spread of a lie started by the Conwomanand perpetuated by her charismatic followers. We see that by the end of the month, nearly 80% of the population believes her lie. Furthermore, contrary to the decline in infection we saw in the other two cases, we see that the number of people in the Infected population is increasing.

3.6 Classroom Jupyter Notebook

A jupyter notebook with the code for the above plots, question prompts, and problem discussion can be found on GitHub at
`https://github.com/izabelaguiar/The-Mathematics-of-Gossip`.

4 Classroom Implementation

There are several methods by which the instruction of this model can be implemented in a classroom, depending on the preexisting course content and available time. For classes that spend only a few days on SIR models, it may be best to begin with a general explanation of SIR models and then spend another day looking specifically at this application with the Simple Classroom Implementation. Alternatively, this can be used as an introduction to SIR models, if the course does not plan on going deeply into the topic.

For courses that are planning on spending more time looking into different forms of the SIR model, it can be beneficial to use this example as a method of introducing independent student learning, in the form of an assignment or even a presentation. For this depth of learning, the Independent Work Implementation allows students to gain a deeper understanding of the model, create new sensitivity analysis characters or scenarios, and to extend the model beyond current assumptions.

4.1 Simple Classroom Implementation

For a simple classroom implementation that can be done with just one class period, the problem can be explained and worked through by the professor in front of the class. The professor can begin by explaining the background, walking through the formulas and demonstrating an example or two. Demonstrating the parameter sensitivity is best done with the provided Jupyter notebook, which the students should have access to as well. This explanation and demonstration will give students an understanding of the system of ODEs, of the model, and how scenarios translate to parameter values.

Then in small groups, the students can choose their own scenarios to represent through parameter values in the model. The choices should reflect the context of their chosen scenario. This will help students understand the meaning behind the parameters and analyze the sensitivity of the model when changing these parameters.

4.2 Independent Work Implementation

Alternatively, if the goal is to have students gain a deeper understanding of the model and its complexities, the initial model can be taught in a series of class periods. First, the instructor can review the background, formulas, and examples the same way as the Simple Classroom Implementation. Then, the students can be placed in groups or work independently to figure out how to add the alpha term – the movement of individuals from the recovered population back to the infected population – to the equations.

This method will work best if students have already been introduced to various forms of the SIR model in previous classes. This will allow them to connect the ideas from those classes with this Gossip Model and help them to understand the complexity of modeling human interactions. For deeper levels of independent work, the students can introduce other terms into the model. Additional terms could include representing: population growth or decrease, the existence of supporting rumours, and any other factors that students devise. Overall, this teaching method will allow students to gain a deeper understanding of the model as they work to adjust and add to it.

5 Conclusions and Future Work

The model developed in this paper has potential to be altered to include more complicated scenarios. There are many possible topics for students to expand upon this model. First, it would be interesting to consider the ability for lies to spread without direct interaction with the infected, accounting for the impact of social media. Second, the incorporation of a separate, counteracting lie/rumor would represent a more complicated scenario. Furthermore, this topic could be studied through a network-science framework to understand how real data could help analyze the spread of a lie.

In this paper we have developed an educational tool to aid in the instruction of ODEs through a socially-relevant application: the spread of lies. We have provided examples for the model application, suggestions for the instruction of the model, and a supplementary Jupyter notebook to aid in the instruction. This educational tool allows students to study the use of ODEs in modeling, to understand the importance of sensitivity analysis and parameter-dependence, and to develop the skills to translate social issues to mathematics. The model is centered around social dynamics and human environments with the hope to make mathematics more accessible and exciting to a diverse group of students, prepared and excited to use mathematics to address the worlds' most pressing challenges.

References

[1] Brett D. Jones. Motivating students to engage in learning: The MUSIC model of academic motivation. *International Journal of Teaching and Learning in Higher Education*, 21:272–285, 2009.

[2] W. O. Kermack and A. G. McKendrick. A contribution to the mathematical theory of epidemics. 115(772):700–721, 1927. doi: 10.1098/rspa.1927.0118.

[3] Karl Smith, Sheri Sheppard, David Johnson, and Roger Johnson. Pedagogies of engagement: Classroom-based practices. *Journal of Engineering Education*, 94, 2005.