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A Model of the Transmission of Cholera in a Population with Contaminated Water

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Abstract: Cholera is an infectious disease that is a major concern in countries with inadequate access to clean water and proper sanitation. According to the World Health Organization (WHO), "cholera is a disease of inequity—an ancient illness that today sickens and kills only the poorest and most vulnerable people…The map of cholera is essentially the same as a map of poverty." [27] We implement a published model [9] of a SIR model that includes a bacterial reservoir. Bacterial concentration in the water, λ, or the "force of infection," is modeled by the Monod Equation in microbiology. We investigate the sensitivity of the models to some parameters. We use parameter values for cholera in Haiti that are consistent with the ranges in meta-analysis [9] and other sources. We show the results of our numerical approximation of solutions. Our goal is to use this system of nonlinear ordinary differential equations to raise awareness among the mathematics community of the dynamics of cholera. We discuss the enhancement of undergraduate experiences by motivating learning with a real-world context in social justice implications of global health.

1 Overview

Cholera is a well-established waterborne disease that spreads through populations of people who are already overly burdened. Controlling cholera will require investment of monetary and human resources, issues beyond the scope of this paper. Here, we seek to understand and illustrate more about the disease through a mathematical model. We begin with an overview of the disease and of the outbreak in Haiti. We review the basic disease model and its parameters, introduce and analyze a model for the force of infection, and examine the combined system of differential equations. We show the results of our numerical solution with respect to the compartments of the population and to the level of bacteria in the water supply, and we examine the sensitivity of the model to parameter values. We share perspectives about teaching mathematics with and about social justice, and we point to the broader view of experts as well. As we indicate in Section 8, two of the authors are novices or near-novices in teaching mathematics with social justice, and one author is well versed in social justice in general. We describe the context of and support for our work at our institution. We discuss several mathematical communities with resources available to anyone wishing to include projects in teaching differential equations. Some are venues to which new work may be contributed. Greater sharing of materials and pedagogies also constitutes engagement in social justice.

2 Cholera is a Dreadful Disease

The world is dealing with the seventh pandemic of cholera in the past two hundred years, and it has lasted for almost sixty years: "Cholera is a 'genie that escaped from the bottle' and is proving very difficult to put back in." [31] We summarize relevant information about cholera from [27, 26, 2, 12]. Humans throughout the world can contract cholera, an infectious diarrhoeal disease. The majority have few or no symptoms but can still spread the disease. Treatments are usually quite effective when administered promptly. Preventative measures include vaccination, drinking clean water, and washing hands well – all of which assumes that people have easy access to these resources. Consider, however, that one million to over four million people contract cholera each year, with deaths estimated between twenty thousand to nearly one hundred fifty thousand annually.

People mostly become infected by eating food or drinking water that is contaminated with the bacteria *Vibrio cholerae*. The bacteria can brew in someone's system for up to twelve days before she or he develops diarrhea, which can lead to dangerous dehydration. Most deaths occur on the first day of illness; in fact, death can occur within hours without treatment. "[T]he bacteria are present in their faeces for 1-10 days after infection and are shed back into the environment, potentially infecting other people." [26] In a layperson's terms, the poop of people who are sick is full of bacteria, and if bacteria gets into food that others eat, or a water supply from which people drink, others also get sick, and a cycle of infection develops. The dynamics of the disease indicate that it is intimately linked to inadequate access to clean water and to improper treatment of human waste.

Unfortunately, many people across the globe live with inadequate sanitation and clean water. We report some 2015 estimates about the global situation [28]: 71%, or 5.2 billion people, used a safely managed drinking water service; (that is, one located on premises, available when needed and free from contamination); 263 million people spent over 30 minutes per round trip to collect water from an improved source; 844 million people still lack even a basic drinking water service.

For sanitation, the situation is even worse; only 39%, or 2.9 billion people, used a safely managed sanitation service in which excreta are safely disposed of in situ or treated off-site; 2.3 billion people still lacked even a basic sanitation service; 600 million people shared improved facilities with other households; 892 million people worldwide still practiced open defecation; more than 2 billion drink water from sources that are faecally contaminated, and 2.4 billion are without basic sanitation facilities, exposing them to a range of water-related diseases including cholera. Moreover, "Half the world lacks access to essential health services, [and] 100 million [are] still pushed into extreme poverty because of health expenses." [32]

The case of Haiti has raised attention to cholera and demonstrates that cholera can become entrenched very quickly. Haiti had been essentially cholera-free until the unintentional introduction in the fall of 2010 by a United Nations (U.N.) peacekeeping team: "A panel of experts appointed by the U.N. found that the strain of cholera that popped up in Haiti was 'a perfect match' for a strain found in Nepal. Nepalese peacekeepers were staying at the U.N. camp, and poor sanitation sent sewage from the camp into local waterways." [8] How ironic that some who came to help with security made general conditions more dire. The response of the U.N. Secretary-General [24] to the legal determination of the U.N.'s responsibility is a poignant illustration of the magnitude of suffering experienced by those with cholera, as well as the general situation of many countries in which cholera is endemic.

> The Haitian people have faced enormous hardships and obstacles over the years: endemic poverty; political instability; and, of course, the devastating earthquake of 2010. The cholera epidemic that soon followed added a deeper layer of tragedy and suffering. Most recently this was compounded by the horrendous hurricane that put the country under new serious strains. [From 2010 through 2016], cholera has afflicted nearly 800,000 people and claimed the lives of more than 9,000 Haitians.

According to [4], 57.7% of Haiitians had access to a drinking water source in 2015, and 27.6% had access to improved sanitation, or just 19.2% of the rural population. The report of the appointed panel of experts, [6], describes the conditions in Haiti in 2010 that can help us understand multiple factors contributing to this tragic outbreak. This includes what had formerly been fairly safe "human activity along [the Meye Tributary System of the Artibonite River], with women washing, people bathing, people collecting water for drinking, and children playing," [6], as well as regular exposure of agricultural workers to irrigation water from the river. The report describes leaky pipes that allowed cross contamination at the U.N. camp, even though there were no reported cases of cholera at the camp, as well as open pits of human waste that could wash elsewhere in rains. Other conditions also contributed to the "explosive cholera outbreak:" lack of immunity, "optimal environmental conditions for rapid proliferation" of the bacteria in the river, movement of infected people to other areas, an extra-potent strain of bacteria, and poor sanitation conditions in treatment centers and across Haiti. Although the U.N. has subsequently been held responsible, "[t]he Independent Panel conclude[d] that the Haiti cholera outbreak was caused by the confluence of circumstances ... and was not the fault of, or deliberate action of, a group or individual." [6]

3 Basic SIR Model and Meaning of Parameters

We remind the reader of a basic SIR model of the spread of a disease. See [21] for an easily accessible, straightforward, general introduction of a simpler model than we cover here, or [10] for an SIR model of Avian flu in birds. We provide some indications of the need for an improved model in the case of cholera.

We consider N people who are either "susceptible", "infectious", or "removed" from getting the disease. The variables S, I, R are often transformed into fractions of N for mathematical convenience. We consider the following general model with notation and parameter values from [9], acknowledging the issue of non-integer numbers of people for unscaled S, I, R variables.

$$\frac{dS}{dt} = -\lambda\, S + \mu_b\, N - \mu_d\, S \tag{3.1a}$$

$$\frac{dI}{dt} = \lambda\, S - \gamma\, I - (\mu_c + \mu_d)\, I \tag{3.1b}$$

$$\frac{dR}{dt} = \gamma\, I - \mu_d\, R \tag{3.1c}$$

$$N = S\; +\; I\; +\; R \tag{3.1d}$$

Parameters μ_b, μ_d, and μ_c represent the rates at which people are born, die from general causes, and die from cholera, respectively. A common form of the SIR model uses a net death rate by combining $\mu_c + \mu_d$, but separate parameters are needed here because cholera can greatly increase the death rate. In the case of Haiti, an intensive study of four sites indicated that, during the first seven months of the epidemic, "the crude mortality rate increased by an estimated 2.9–fold (2.1 – 4.0–fold across sites) compared with baseline data." [12]

The parameter γ represents the rate at which people who are infectious recover, with $1/\gamma$ representing the length of time that an infected person is ill. Generally, in the basic model, these are all constants for a particular disease and population.

The term $\lambda\, S$ in Equations (3.1 a, b) is often modeled as $\alpha\, S\, I$ where α has a constant value, as in [21] and [10]. In this system, then, λ is often considered as a function only of the number of infected people.

For cholera, the disease dynamics indicate that λ include the interplay between people and the bacterial reservoir, capturing the "force of infection" (FOI), which is well explained in [9]. Both [9] and [1] present a meta-analysis of multiple research sources for models of cholera, particularly related to the outbreak in Haiti. The main article in [9], primarily for healthcare practitioners, also has a supplement that states the differential equations, describes the parameters, and provides ranges of parameter values with units. Five models of cholera of increasing complexity are included in [1], as well as ranges of parameter values. We use both sources for our parameter values in the expanded model.

We first share how we have led undergraduate students through an understanding of the new FOI model as related simply to bacteria – not to the SIR components. We then describe the expanded SIR model expanded to include bacterial growth and interaction with humans, and we show the results of our numerical solution of the SIRB system. We discuss the enhancement of undergraduate experiences by motivating learning with a real-world context with social justice implications of global health.

4 Force of Infection Model and Effect of Parameter Values

In the model from microbiology described by [9], λ is the *force of infection (FOI)*, the per capita rate of infection experienced by susceptible individuals. It is reasonable to assume that the chance of any one person becoming infected by *V. cholerae* rises with the concentration of the bacteria in the water. Let B represent this changing level of contamination of the water supply. The force of infection, *lambda*, increases with B but has a maximal level, β, the "contact rate" of the susceptible population with contaminated water. The concentration of bacteria for which the infection rate is half of its maximum is κ, which is a shape parameter in the model. Measured in bacterial cells per mL water, this is the concentration of *V. cholerae* that yields 50% chance of infection [9, Table S5]; this is also referred to as the half-saturation constant of vibrios [1].

$$\lambda = \beta \frac{B}{B + \kappa} \tag{4.1}$$

We use the following parameter values within the ranges provided by [9]: a contact rate of the susceptible population with contaminated water 0.07 per day and a level of 10^5 cells of *V. cholerae* per mL water at which there is a 50% chance of infection. We use these parameter values for Figure 1, with an exaggerated scale for B to see the mathematical meaning of the definition of κ and how κ relates to β.

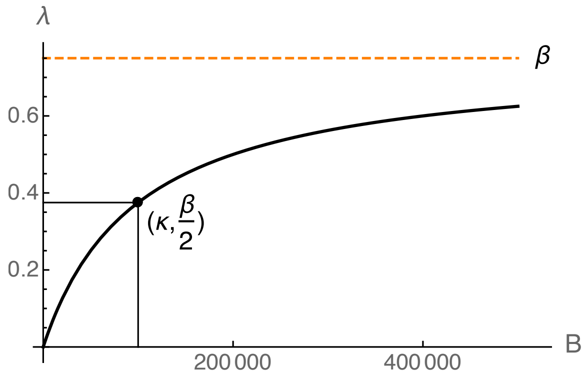

Figure 1: Role of Parameters in the Force of Infection Model

Equation (4.1) has Michaelis-Menten dynamics, a term that may be familiar to our readers; it is the Monod equation in microbiology. Translating the epidemiological statements, we can verify that

$$\lambda\Big|_{B=\kappa} = \frac{\beta}{2} \tag{4.2}$$

and λ has limiting value β as the bacteria level, B, increases.

The FOI is a function of B with parameters κ and β; this is four dimensional if we consider all variable quantities. We will examine this in two and three dimensions. The different asymptotic behaviors of the FOI functions for multiple values of β, all with $\kappa = 10^5$ cells/mL, are apparent in Figure 2. We see the FOI functions for multiple values of κ, all with $\beta = 0.07$/day, in Figure 3; the curves have the same asymptote. Our 2D graphs are traces of the 3D graph, which include the points defined by Equation (4.2). We can see that cholera worsens for larger values of β and smaller values of κ.

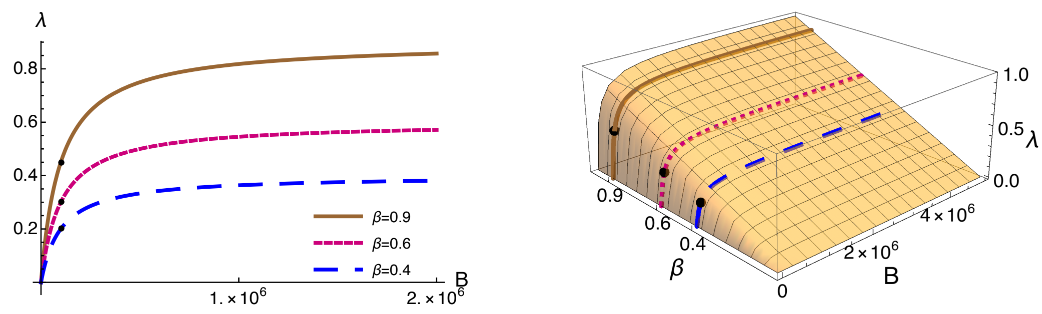

Figure 2: Effect on FOI of Varying β with constant κ

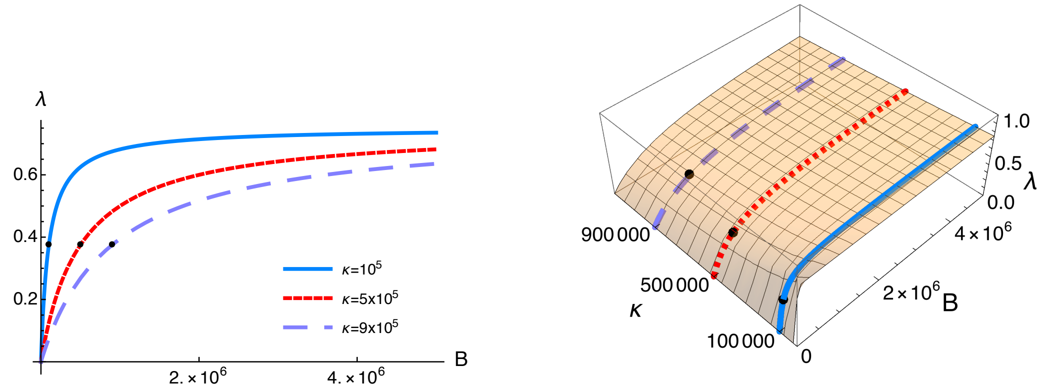

Figure 3: Effect on FOI of Varying κ with constant β

Note that in Figures 1, 2, and 3, the bacteria level is thought of as increasing. In our system of differential equations, the bacteria level can rise and fall, leading the FOI to rise and fall. We remind students that these graphs do not show time.

The parameters β and κ are on very different scales, so comparing how sensitive the FOI is to each is not clear from Figure 2 and Figure 3. We employ the method in [14]: for a function f with parameter p, the *sensitivity function* is defined as $S(f;p) = \frac{df}{dp} \ / \ \frac{p}{f}$. This is based on $\triangle f/f$ representing the average proportion of change in f, which would be divided by $\triangle p/p$ representing the average proportion of change in p. The sensitivity function is like a ratio of relative rates of change. We evaluate $S(f;p)$ at baseline values of the independent variables and parameters. $B = 1000, \beta = 0.07, \kappa = 10^5$. The result can be interpreted as an approximate rate of change of the function value with respect to a one percent increase in the parameter value.

The sensitivity of FOI to the half-saturation constant is $S(\lambda;\kappa) = -\kappa/(B+\kappa)$, which is approximately -0.96 for our baseline values. The sensitivity of the force of infection to the contact rate of the susceptible population with contaminated water is $S(\lambda;\beta) \equiv 1$.

The FOI is barely more sensitive to the rate at which susceptible people contact the bacteria in the reservoir than to the half-saturation constant of vibrios. FOI decreases by about 1% for a 1% increase κ near its baseline value, and FOI increases by 1% as β increases 1%. FOI is a bit more stable with changes in β than it is to changes in κ. Even if we vary the bacterial concentration from $B = 100$ to $B = 9000$, $S(\lambda;\kappa)$ only varies between -0.92 and 1.

5 Disease Model with Bacterial Reservoir

5.1 SIRB Model

Now we consider the combined system of Equations (3.1) with Equation (4.1), with the additional differential equation modeling bacteria in the water reservoir. Infected persons contribute to the vibrio concentration at rate ξ, and the bacteria die at rate δ.

$$\frac{dS}{dt} = -\beta\,\frac{B}{B+\kappa}\,S + \mu_b\,(S\;+\;I\;+\;R) - \mu_d\,S \tag{5.1a}$$
$$\frac{dI}{dt} = \beta\,\frac{B}{B+\kappa}\,S - \gamma\,I - (\mu_c + \mu_d)\,I \tag{5.1b}$$
$$\frac{dR}{dt} = \gamma\,I - \mu_d\,R \tag{5.1c}$$
$$\frac{dB}{dt} = \xi\,I - \delta\,B \tag{5.1d}$$

We can see from our SIRB model in Equations (5.1) that cholera worsens for larger values of β, ξ, and μ_c, as well as with smaller values of γ, δ, and κ.

5.2 Our Simulation of a Moderately Severe Outbreak

We return to the parameter set that was used in Figure 1: $\beta = 0.07$/day and $\kappa = 10^5$ cells/mL. These model a fairly severe cholera situation, being on the worse end of the ranges in [9]. The duration of cholera infection for a person is $1/\gamma$ days, which is a little over two weeks for this parameter set. The natural lifespan of cholera in a water reservoir is $1/\delta$ days, which is a little over seven weeks for this parameter set.

We include additional parameter values to enable a numerical solution. We model a subpopulation of size $N = 15,000$, which is similar to the sizes of subpopulations of study sites within Cap-Haïtien and North Department considered in [12]. Haiti is divided into ten Departments, which are geographic subdivisions like states or provinces. [4, 1] For the daily birth rate, we convert the 2017 estimate from [4] of 23 births per thousand people annually to a daily rate of $\mu_b = 23/1000/365$. We base our death rates on [1]: $\mu_d = 9/1000/365$ for deaths unrelated to cholera, and $\mu_c = 25/1000/365$ for deaths related to cholera. We base our other parameter values on [9]: $\xi = 10.0$ cells per mL water per day shed into the water source by infectious individuals, and $\kappa = 10^5$ cells per mL water as the half-saturation rate. We modeled through 100 days, which is about 14 weeks as in [29, Figure 2].

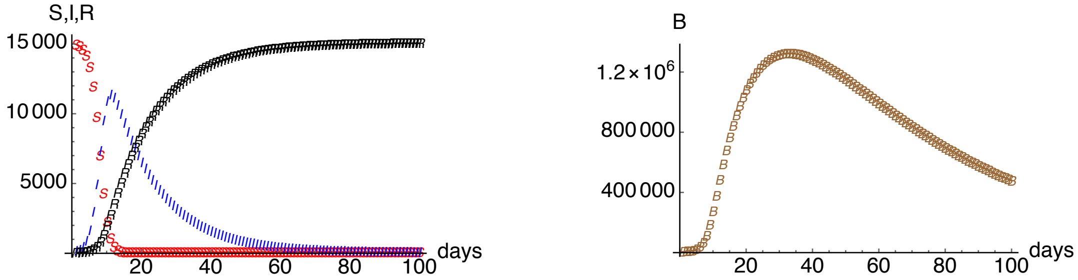

Figure 4: Simulation of Moderately Severe Outbreak: left SIR, right B.

Figure 4 shows the results of our numerical solution. The graph of "Infected" is slightly skewed to the right; the SIRB model indicates that people are infected more quickly than in an SIR model. The rapidity of infection in our model is reasonable, given the following about the outbreak in Haiti: "The first cases were confirmed on October 22nd, 2010. As of April 17th, 2011, 285,931 cases of cholera had been reported, with 154,041 (54.0%) patients hospitalized, and 4,870 (1.7%) deaths." [6] Furthermore, a single hospital had under a total of 70 hospitalizations for cholera and no deaths from October 8 through October 19 in 2010, and then had 404 hospitalizations and 44 deaths *on the next day!* [6]

The graphs of "Recovered" and "Susceptible" indicate a speedier recovery than we might expect in a subpopulation of Haiti. That is reasonable because ours is still a simple model. The simulation of "Bacteria" concentration in the water reservoir seems consistent with the values and description in [16] of Haitian cholera bacteria levels.

5.3 Effect of Parameter Values: Less Severe Outbreak

The SIRB model can yield widely different results, which can be useful in modeling a different situation. We changed some parameter values, still within the ranges determined by [9]: $\beta = 0.75 \longrightarrow 0.6$, $\gamma = 0.07 \longrightarrow 0.12$, $\delta = 0.02 \longrightarrow 0.15$, $\xi = 10 \longrightarrow 5$.

Here, the contact rate, β of the susceptible population with contaminated water is lower; a person is sick with cholera for $1/\gamma = 8.3$ days instead of over two weeks; cholera lives in the water reservoir for $1/\delta = 6.7$ days instead of over seven weeks; and infected people shed bacteria into the water source at half the rate. We see in Figure 5 that this second parameter set models a much less severe outbreak. Although the graph of those who recover looks quite similar, the the number of susceptible people decreases more slowly, and the peak number of people infected is much lower. The peak bacterial level in the water is dramatically lower.

Consider the following parameter set with values all still within ranges given by [9]: the contact rate with contaminated water, duration of infectiousness, and lifespan of bacteria in the water are all significantly smaller. $\beta \longrightarrow 10^{-4}$, $\gamma \longrightarrow 0.20$, $\delta \longrightarrow 0.20$, $\xi \longrightarrow 5$. Not one person in our simulation of a small population contracted cholera.

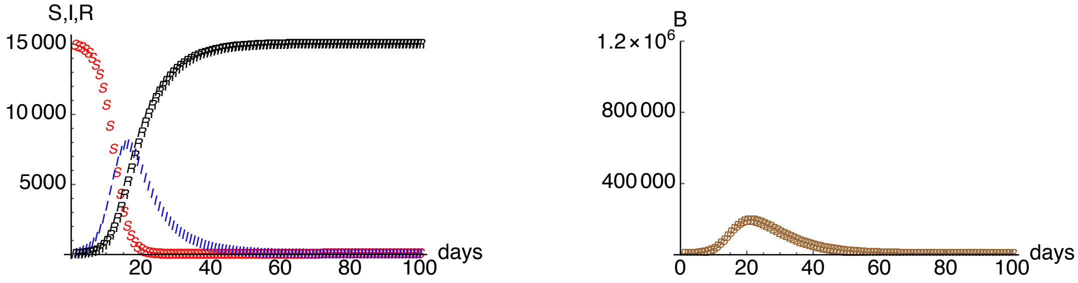

Figure 5: Simulation of Less Severe Outbreak: left SIR, right B.

Our choice of the basic SIRB model for a short-term analysis is affirmed by the assessment of multiple models by [1]: "We find that within the majority of the individual Haitian departments, for the timeframe of about 15 months, the simple SIRB model enjoys the most support."

5.4 Simulation Methods

The simulations are produced with Euler's Method. We chose this for multiple reasons: simplicity; intuition for students; and sufficient accuracy. Figure 6 shows the results of another numerical solution, using *Mathematica's* sophisticated numerical differential equation solver, NDSolve. Figure 4 is comparable, indicating the adequacy of Euler's Method for this system.

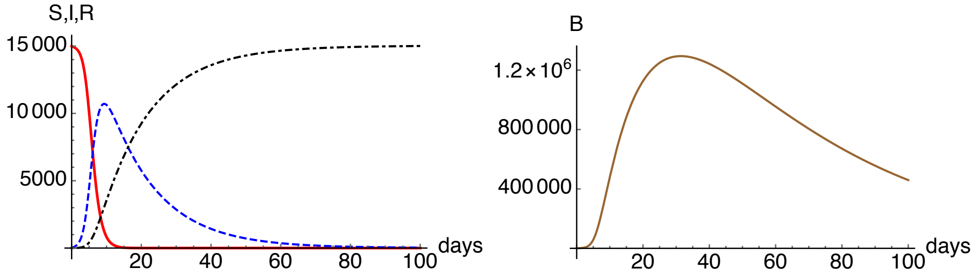

Figure 6: Solution using NDSolve

6 Relating to Social Justice

6.1 Data Collection and Choices of Model and Parameters

We used a fairly simple model with published data and information about parameter values. These choices are appropriate for teaching undergraduates about mathematical modeling in general, or about differential equations. Estimating parameters can be within the realm of undergraduate mathematics, if information is carefully prepared. We can indicate the use of sophisticated statistical methods used in parameter estimation, such as the maximum likelihood methods in [1]. Whatever the level of complexity that instructors choose to engage students in modeling, it is appropriate for students – and instructors – to understand the variation in, power of, and limitations in, modeling.

"Models are simple, but not simplistic representations of the real world. They are used to capture the 'essence' of a complex phenomenon." [9] Models can illustrate the effect of vaccinations, as in [9] with cholera, or [10] with Avian flu. Models might help in setting public policy about the frequency and extent of vaccinations. Waning immunity is included in [9]. These models are more complex than our simple SIRB model. Other models consider subpopulations, which may be defined by a geographical region, age structure, sex, or health condition, some of which are issues of social justice. Five models with various subpopulation structures according to level of susceptibility, etc. are considered in [1], along with other models that take into account movement between the Departments of Haiti. Models can be spatiotemporal, as in [16], to account for the seasonal effects of flooding. Accurate models for Haiti over a longer time period than ours would require considering that: "Rainfall possibly increased surface water prone to infection (possibly through open defecation) and dissemination through runoff. The mountainous eastern areas ... are prone to both, as sanitation conditions are extremely poor and the topography increases the risk of transmission through runoff." [3] Departments in Haiti vary in population and poverty levels. [1, Table 2] "Considering its high population density and poor sanitation, there is a high risk for continued cholera transmission in [the vast Artibonite estuary]." [3] (Multiple maps are also included in [3].) Notice how intimately linked the modeling process is to inequities. The mathematical model chosen can lead to more or less appreciation for the severity of the social justice issue because models vary in the degree of "explanation" for the spread of the outbreak and the severity on the populations.

So, which model do we use? "There is tremendous disagreement throughout both kinds of research, biological and mathematical, about what parameter values are realistic and what modelling assumptions are important." [1] Both [9] and [1] include tables with a wide range of possible parameter values for SIR and variant models. Model misspecification and parameter uncertainty are explicitly discussed in [9].

Models and data influence public policy. Public policy influences the type, amount, and quality of data collected. An epidemiological model and its parameters can be based somewhat on theoretical information about a disease and its transmission, but the ability to validate our choices for the model and the parameters requires reliable data. Evidence of issues with data collection in the case of Haiti is offered in [3].

> [M]issing data at the epidemic's beginning led to a difference of over 20,000 patients compared with the number of patients reported by [Haiti's Ministry of Health]. ... In addition, approximately 2% of registered patients had no geographic information, or had geographic information that could not be traced. ... [D]ata are [based on health facilities], and patients who did not seek care were not included, ... [which] would be particularly important in the remote mountains regions where access to care is the most limited.

Haiti had 0.7 hospital beds per 1,000 people in 2013 [4], hardly enough to manage the tremendous number of cholera cases, let alone to ensure high quality healthcare or to handle accurate data collection. Moreover, "decentralization of healthcare structures in Haiti was and remains difficult in very remote areas such as some villages in North Department that require a *10-hour walk* to get to the nearest healthcare facility." [12] "[A]ssessment of cholera-related deaths ... estimated that *87% of deaths were not recorded* in the hospital records." [12] (Emphases added.)

The difficulty in data accuracy and in modeling is well described by [1]:

> [I]n addition to the expected surveillance errors given the setting, there were surely additional errors in representing the data numerically. We believe that these errors are typical, and that the data will have noise in any large-scale cholera study. As a result, if there were a 'true' model that could describe the surveillance data in such an epidemiological setting, then that 'true' model would not be a 'true' representation of the actual disease dynamics. This is simply unavoidable, and we assume that the model which best describes the noisy data would be the best model to describe the actual data if it were possible to discover that data.

Good modeling demands good data. That, in and of itself, assumes that someone has the resources – and the authority (though not necessarily the "right") – to collect data on individuals, as well as to report at least aggregate information to public agencies.

6.2 Teaching and Learning of Differential Equations, and Mathematical Communities, as Contributions to Equity

Our model of a recent and real cholera epidemic is inherently interdisciplinary, incorporating differential equations, mathematical modeling, epidemiology, social justice, and more. The process of mathematical modeling necessitates honing in on a small part of a larger issue, ignoring detail to grapple with a tractable problem. Mathematicians frequently re-scale variables and eliminate parameters for mathematical convenience. Note that we explicitly included N, the total population, in Equation (3.1d), but in Equations (5.1) we eliminated N in the equations, although the initial population is indicated in Figures 4 and 5. How much more impactful might it be to show the cumulative number of cholera cases, or the number of deaths?

First among the nationally recognized Cognitive Recommendations for all mathematics courses calls for all undergraduate mathematics programs to "promote students' progress in learning to identify and model essential features of a complex situation, modify models as necessary for tractability, and draw useful conclusions." [13, Ref A] However, formerly for some undergraduates, this was a sterile situation, with focus on the mathematical process with problems that are totally devoid of meaning beyond the mathematics.

Many have been working to restore meaning to the math. We mention some related to differential equations. (See 8 for a description of author interactions with these communities.) SIMIODE supports a resurgence of a *modeling-first* approach to the study of differential equations because "[m]odeling engages and motivates students; it makes them curious," [23], which makes students engaged. Peer-reviewed classroom materials for the study of differential equations are offered freely by SIMIODE, IODE [17], and CODEE

[5, Ref B]. CODEE has a long history of working to make differential equations more understandable and visual. In fact, CODEE was originally founded by Bob Borrelli and Courtney Coleman as a Consortium for ODE Experiments, emphasizing the newfound ability to vary parameters and see the results. The introduction of computer graphics in the late 1980s suddenly gave a lot more power to the modeling approach to differential equations. IODE materials focus on conceptual aspects of differential equations. SIMIODE has teacher and student versions of modeling scenarios to facilitate adaptation to individual teaching styles and situations. SIMIODE also points to a number of free e-texts. Both CODEE and SIMIODE accept submissions for peer-review. The National Science Foundation (NSF) has supported each of these communities with grants: SIMIODE 2018-2021, IODE 1999-2003 and 2014-2017, and CODEE 1992-1997 and 2007-2011. All have organized sessions at professional meetings. All promote teaching with a focus on modeling, which is a form of inquiry-oriented learning. SIMIODE has offered multiple professional development opportunities for faculty to learn to teach with a modeling-first approach, as well as to author new materials for publication at the SIMIODE site.

"Mathematical modeling can be used to motivate curricular requirements and can highlight the importance and relevance of mathematics in answering important questions. It can also help students gain transferable skills, such as habits of mind that are pervasive across subject matter." [20] In this way, when taught in context, modeling may be a way to level the playing field for students. The free availability of high quality resources levels the playing field for instructors. Mathematical modeling in differential equations, and the mathematical communities that support its use, are part of a more general social justice context.

6.3 Broader Ideas of Teaching with/for Social Justice

Many different audiences study differential equations, and the course must offer materials and pedagogy to meet their needs. The differential equations course is undergoing great change because of advances in teaching with technology, which has increased the diversity of our students. Differential equations is taught at community colleges as well as four-year institutions, [13, Ref D], and at least one preporatory high school [7] in the United States. The relatively new contest for students, the SIMIODE Challenge Using Differential Equations Modeling (SCUDEM) [23, Ref C], has seen multiple teams of high school students, including one from West Lake High School in Austin, Texas that participated in SCUDEM III at Austin Community College, November 2018. The West Lake students had completed through multivariate calculus, and one was learning differential equations through the OpenCourseWare at the Massachusetts Institute of Technology. [15]

Our readers may consist primarily of those teaching mathematics at the undergraduate level, yet there is much to learn from work at all levels on math and social justice, as well as from the broader context of social justice education. Teachers at all levels can be catalysts for social justice by choosing to exemplify "critical mathematics", designing lessons to examine the world and expose students to inequities. One small but important step is to use person-first language, such as referring to people "who are infectious" rather than the quicker "infecteds."

Consider four possible components of mathematics for social justice [11]:

1. High quality mathematics instruction for all students.
2. Re-centered curriculum around the experiences of students from marginalized communities, while exploring issues of social justice through mathematics.
3. Mathematics as a critical tool for understanding social life; people's positions in society; and issues of power, agency, and oppression.
4. The use of mathematics to radically reconfigure society so that it might be more just.

We argue that the mathematical communities that we describe in Sections 6.2 and 8 contribute to the first of these. The second and third components are fulfilled, in part, by our system of differential equations model of cholera model *when* meaningful examinations of parameter values and model results are informed by the context of the real-world situation, along with a discussion of the limitations of the model.

We make no claims about our adherence to the last component - yet. Movement toward that goal would mean that we go beyond teaching the mathematical models and processes, beyond simply identifying the factors in Haiti that contribute to cholera outbreaks, beyond modeling how quickly cholera spreads or the numbers of individuals who sicken and die. We would have to allow time to examine what factors - systemic, racial, sociopolitical, etc. - have contributed to the poverty, poor sanitation, poor access to health care, that result in outbreaks and contribute to the high number of deaths. We would also need to examine the under-reporting that is based on limited staff and access to healthcare. For many mathematicians, this seems out of our training and areas of expertise. As we describe in Section 8, we have been making strides in that direction.

We do believe we have exhibited the following: "Mathematics becomes a tool used to examine social environments, increase awareness of social injustice, and serves as a valued language that can be used to further an agenda of social change towards a more just society." [11, P 25]

7 Final Thoughts

The development and analysis of our cholera model are informed by, and highlight inequities in, access: "[H]ouseholds in cholera-affected countries are largely below the global mean with regard to access to basic water and sanitation services." [27] The outbreak of cholera in Haiti in 2010 was the world's largest at the time. However, "in 2017, cholera continues to impact communities already made vulnerable by tragedies such as conflicts and famines. Yemen currently faces the world's largest cholera outbreak, with over 600,000 suspected cases and more than 2,000 deaths reported" between April 2017 and October 2017." [27] Researchers from many disciplines have been involved in identifying and understanding the dynamics of the disease and various situations under which cholera and other devastating diseases thrive. The models that researchers and instructors investigate and create, and the instruction methods that we employ, can prepare the current and future generations to meet the continuing challenges that face our communities and our world.

8 Author Interactions and Acknowledgements

We point to multiple situations that enabled the authors' collaborations. This is to acknowledge others, as well as to encourage our readers to take advantage of opportunities.

Shelton and mathematical colleague John Ross wove social justice assignments into an Introductory Statistics course and produced the article [19], which includes thoughts, supported by a partial literature review about our journey as novices to teaching with social justice, as well as a link to our modules. This began at the 2016 Associated Colleges of the South (ACS) workshop, *Mathematics for Social Justice*. Southwestern University is a member of the ACS consortium. We gratefully acknowledge the organizers (C. Buell, Z. Teymuroglu, J. Wares, C. Yerger) and participants of this conference. They formed a community of mathematicians who care deeply about the issues of social justice within and beyond mathematics/statistics classroom.

Southwestern University has a well-established environment that fosters attention to social justice. The Core Principle: "Fostering a liberal arts community whose values and actions encourage contributions toward the well-being of humanity." The Core Values include "Respecting the worth and dignity of persons," and "Encouraging activism in the pursuit of justice and the common good." [22, Ref A] Southwestern University requires each student to take a course with a significant social justice component to "connect their learning to issues of diversity and inequality." [22, Ref B] Although no math course currently carries the social justice credit, Ross and Shelton made strides in that direction with the statistics modules.

Adrian has extensive experience incorporating social justice into her courses in Education, as well as designing entire courses with such a focus. Shelton began learning from Adrian in 1997 while being one of those to teach a course that all first year students took and that Adrian designed: "Disability, Society, and Ethical Issues." This was when Shelton first learned "person first" language. Southwestern has a long-standing initiative of intentionally making connections, known as *Paideia*^{TM}. [22, Ref C] Adrian has been a leader in *Paideia* since 2012 and served as *Paideia* Director (2014-2018). In this role, Adrian has continued to inform Shelton about social justice, including for the previously mentioned article about introductory statistics.

Two grants to Southwestern University aided Shelton and Groves. The Howard Hughes Medical Institute funds (2012-2016) formed the Inquiry Initiative in the Natural Sciences, supporting the eight-week 2015 Summer Collaborative Opportunities (SCOPE) Faculty-Student Research for Groves, an undergraduate at the time, and Shelton, the faculty supervisor. The W. M. Keck Foundation, Undergraduate Education Program, funds (2015-2018) fostered the integration of molecular biology across the Natural Sciences, supporting Shelton in creating a course module of the force of infection.

Shelton has worked with the SIMIODE [23] community since a 2015 faculty development workshop. Shelton is a co-Principal Investigator in the 2018-2021 NSF grant, was a 2018 DEMARC Fellow (Differential Equations Model and Resource Creator) to author modeling scenarios in differential equations, and co-led a 2018 workshop for MINDE Fellows (Model INstructors in Differential Equations) to guide faculty in practicing a modeling-first approach. These two workshops are the primary purpose of the NSF grant, in order "to promote the use of modeling in motivating and teaching differential equations in high schools and undergraduate institutions." [23, Ref B] Shelton worked with the IODE [17] community as a 2015 and 2016 TIMES Fellow (Teaching Inquiry-oriented Materials: Establishing Supports) using Inquiry-Oriented Differential Equations (IODE).

The authors are grateful for the environment at Southwestern University, to the work of a "Math for Social Justice" Community, to funding opportunities, and to multiples communities in differential equations, mathematical modeling, and general mathematics. We are grateful to the CODEE journal for encouraging this collaboration, which has raised our own awareness of social justice issues in mathematics.

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