A LINEAR PROGRAMMING APPROACH

TO RESOURCE ALLOCATION IN

A PUBLIC SCHOOL SYSTEM

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TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

Few business enterprises face resource allocation problems any more complicated or restrictive than the allocation problems faced by public school systems. Often the resource allocation problems peculiar to public schools arise because (1) the upper bound on a public school system's revenue inflow is relatively fixed, (2) educational programs sometimes have a low priority relative to other public programs, and (3) the taxpaying function of the patrons of a system places them in a customer role whether or not they consume the product of the system. Generally patrons expect public school system output beyond that achievable within the fiscal provision allocated by the voters. Thus the administration of the public school system takes place in a political environment in which the expectations of some patrons must be sacrificed in order to meet those of others.

At the same time, some striking similarities exist between the administration of public schools and business enterprises, e.g., labor and materials are often scarce and investments in productive facilities are long-lived and require large cash outlays. Observing these similarities, one might expect both to employ similar management accounting and management science techniques. Yet several writers have observed that public schools have not generally used the management tools which business enterprises have used successfully. One explanation frequently advanced is the difficulty of measuring the goals and objectives of public schools.[1] Dyer (1973, p. 30) recognizes the validity of this explanation and then points out the need to attempt to invalidate it:

> It is a task that never has been, and probably never can be, fully accomplished. Yet it is one that must be constantly attempted if school systems and the public that supports them are to take seriously the idea that the schools are indeed accountable for meeting the developmental needs of students.

Three comparatively new developments affecting education administration have made the employment of some of the advanced tools of management accounting and management science feasible for modern school administrators: (1) application of Planning, Programming, Budgeting Systems (PPBS), (2) accumulation of cost information by program, called "accountability," and (3) the advent of low cost computer services.

PPBS involves explicit statements of (1) program goals, (2) intermediate objectives for achieving those goals, and (3) the means by which progress toward the goals might be measured. Within the limitations imposed by some basic assumptions, such as quantifiable criteria, the PPBS procedure in an educational context seeks to answer the following questions:

1. What results would we like for our educational system to produce by a specified date?

2. Which intermediate objectives are most likely to help us achieve our goals?

3. How will we know when we have accomplished our objectives and eventually reached our goals?

An essential element in the second development, accountability,

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[1] In this study and in general usage by educators the terms "goals" and "objectives" are not synonymous. These terms are defined on page 8.

requires organizing a public school district's accounting system so that costs can be identified with specific programs. In the past the accounting process has identified expenditures by object using line-item budgeting, e.g., instructional supplies, salaries, gasoline, etc. When an object classification is employed, one cannot readily identify expenditures with specific programs without conducting a special investigation. Thus the cost of various academic and extracurricular programs cannot be determined easily. In contrast, when expenditures are classified by program, the cost of each can be ascertained by summing the charges to the appropriate account. Costs are recorded under each program by object and, therefore, enter the books in a two dimensional classification--by object and by program. An important benefit is that administrators can study the behavior of costs by object for each program. The behavior of some costs is likely to be obvious. For example, in most instructional programs facilities are fixed costs and instructional supplies consumed by students are variable. For other costs, administrators will probably be unable to approximate the behavior until they have gathered enough data to use techniques such as regression analysis. As a starting point for the latter group of costs, administrators may rely on rough estimates.

The availability of comparatively inexpensive computers and numerous time-sharing installations is a third development enhancing use of advanced management techniques by school administrators. The combination of available data and ready access to computers makes it feasible for almost all school districts, regardless of size, to employ advanced management techniques.

Purposes of the Study

The purposes of this study are to:

1. Develop a general model incorporating linear programming (LP) in the planning procedures of a public school system.

2. Alter the general model as needed to make it responsive to the environment in which public schools must operate.

3. Identify the limitations of the LP approach to planning public school resource allocations.

Given the three new developments in education management, PPBS, accountability, and low-cost computer services, administrators should have sufficient tools to employ LP for resource allocation and planning and control. It is recognized that data sufficient to introduce LP may exist within other record-keeping systems, but the natural output data of PPBS and accountability exist in a form which can be easily incorporated by an LP model.

The reader will recall that one of the stages in the development of PPBS requires participants to describe how they plan to measure progress toward achievement of objectives. These measurements provide some of the quantitative data necessary for employment of a linear programming approach. While some of the measurement procedures have weaknesses, there are several reasons why such shortcomings do not preclude their use. First, the measurements are the best available. Second, they represent the first step to more accurate procedures. Third, resource allocations can improve as available information improves, because the LP algorithm enables administrators to insert other measurements easily. Finally, most administrators are well aware of the shortcomings of such measurements and make allowances for resulting weaknesses as they make decisions.

Organization and Methodology of the Study

Many disciplines such as economics and industrial engineering have used the LP algorithm extensively in solving resource allocation problems. The first phase of this research analyzes the literature related to education system models (especially linear programming) in order to benefit from the research in these fields. The findings are summarized in Chapter II. The second phase quantifies as completely as possible the environmental constraints which have a bearing on resource allocation in the public schools, such as state laws, manpower, finances, etc. This segment of the research involves further library study as well as interviews with professional educators, and it culminates with the construction of the general public school resource allocation model (Chapter III).

The third and fourth phases of the study (Chapters IV and V) obtain data from, and adapt the general model to, the specific goals and conditions of the Bartlesville School System. Bartlesville was selected to participate in this study because it is a progressive, well-managed school system with a national reputation for educational excellence (Community Education Council, 1972, p. 3). Further, since Bartlesville was one of the first participants in the PPBS/accountability pilot study in Oklahoma, it has been able to accumulate more data relevant to this study than other Oklahoma school systems.

To adapt the general model the researcher asked the superintendent and assistant superintendent to assign a weighting factor to the goals of the Bartlesville School System. These goals were defined by the

Community Education Council (CEC) and school officials in a system-wide study in 1972 when PPBS was in its early stages in Bartlesville. The weighting factors provided by the superintendents are the coefficients of variables in the objective function. Thus the model maximizes the goal weights of programs undertaken by the school system. Some advantages of the goal weighting approach are examined in later sections of this chapter and Chapter II (see pages 8, 20, and 31).

In addition to the objective function, a number of linear equations were written reflecting the environmental constraints of the system. These equations are called "operating constraints." Quantitative data relevant to these constraints were obtained from Bartlesville's records. When the objective function and constraints had been defined, the model was solved using the IBM MPSX360 program (a linear programming algorithm). The writer discussed this solution with the Bartlesville administrators who identified modifications of the model required to achieve specified administrative or political demands. For example, the model omitted certain academic courses from the "optimal" solution, but the administrators felt patrons would insist that a minimum level of such courses be offered. Therefore, constraints were added forcing these minimum levels into the solution.

The distinction between operating constraints and constraints inserted to achieve specific administrative or political demands is that the former make the model operable and the latter make the model responsive to the environment in which a particular school system must function. Insertion of the latter type of constraint allows administrators to observe what sacrifices must be made in order to meet specific patron demands. Such constraints enter into policy-setting decisions and thus are described as "policy constraints." The policy constraints of the Bartlesville School System are explained in Chapter V.

Both Chapters IV and V contain descriptions of the fifth phase of this research--interpreting the solution.

Chapter VI discusses limitations of the LP approach, describes ways to avert or mitigate these limitations, and explains some of the expected consequences of using the model if the limitations cannot be averted.

Chapter VII summarizes the study and highlights the conclusions drawn. In addition, this chapter contains recommendations for further research.

Terminology

The following terms are defined relative to their use in the study and, to the extent possible, as they are used by educators or accountants.

Weighting factor: The weighting factor is a number assigned to a specific goal which reflects the goal's relative importance to the person who assigned the weight. One advantage of the weighting factor is that it provides a common unit for making comparisons of various activities.[2] These factors are reported in Appendix A.

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[2] To illustrate the comparison of activities, suppose the optimal solution indicated no drivers training courses should be offered. Further, suppose patrons insist at least two units of drivers training be offered. In response to the patrons' insistence, a minimum constraint for two units of drivers training could be inserted in the LP model. When a minimum amount of drivers training is forced into the solution, a determinable amount of some other program will be forced out. Patrons can be shown that in order to have minimum drivers training they must be willing to sacrifice some amount of another program.

Goal: Goals are general and enduring statements of purpose that express the system's fundamental intentions, and provide guidelines for planning the future development of the school system (McFarland, 1966, p. 5).

Objective: Objectives are results to be achieved or points to be reached in pursuit of a goal (*The American Heritage Dictionary*, 1970, p. 905).

Resource allocation: Resource allocation is the deliberate distribution of all the factors of production available to an educational system with the objective of producing a given product, such as, graduates with "employable skills."

Optimal solution: The optimal solution is that allocation of resources which results in the maximum number of goal weights being produced by the school system within the constraints imposed by the environment, the state, administrators, and patrons.

Limitations of the Study

One impediment to research in the not-for-profit sector is the difficulty of expressing variables in a common unit. To mitigate the effects of this problem, goal weights are employed; to the extent that this common unit fails to capture all the qualities of the "ideal" common unit, the study is limited. However, since the weighting factors enable one to observe the units of one program which must be given up in order to obtain specified units of another program, it appears that more objective comparisons of the myriad of educational programs are possible with weights than without them.

The objective of the study is to develop a technique for making resource allocation decisions in a public school system. Although testing the generality of the technique seems desirable, numerous school systems would probably have to participate in the research for such tests to be persuasive. It is likely that the magnitude of the testing project would justify a separate research effort in itself. Therefore, this study is restricted to the development of the technique in a carefully selected school system. It can be viewed as the first step in a series of research projects which hopefully will yield benefits in the management of educational enterprises.

School administrators assigned the weighting factors in this study, and, consequently, their opinion has an important impact on the selection of the system's programs. This condition does not appear to limit the present study. In fact, it resembles typical school operations, i.e., administrators usually assign priorities to programs in practice (at least implicitly). If the use of administrator-determined weights appears to inhibit future use of the model or create unexpected problems, other techniques for determining the weighting factors can be employed.

Significance of the Study

Through applications in numerous disciplines the linear programming algorithm has proven to be a powerful tool for examining numerous and diverse criteria bearing on a decision. The development of an LP model for public school districts could have far-reaching benefits in the process of identifying optimal allocations of education-related resources. At least three characteristics of the LP model support this view: (1) comprehensive studies are feasible,[3] (2) the interrelationships of variables included in the study are apparent in both the setup of the problem and its solution, and (3) the cost of obtaining solutions is minute by comparison to the cost of running "live" experiments. Using the linear programming algorithm, school administrators can examine the expected results of decisions or events in advance and then take steps to change controllable variables so that the expected results more nearly approximate those desired. It appears, therefore, that obtaining many of the benefits from the implementation of PPBS requires a linear programming approach.

The linear programming algorithm is quite flexible. By specifying the policy constraints, experimenters can modify a general model to meet the requirements of school districts whose goals and resources are widely divergent from the "typical" school district. Therefore, the approach could be employed by virtually every school district regardless of size, patronage, or other characteristics. Once a general LP model has been perfected, it could be maintained by the State Board of Education and made available to any school district upon request at nominal cost. Thus school administrators might determine the sensitivity of proposed policies to controllable and uncontrollable variables.

Finally, the computer solutions to linear programming problems can be used to explain to patrons, state officials, and federal agency

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[3]The OSU computer configuration can handle up to 5,000 constraints efficiently; with multiple steps and diminished efficiency the system can handle up to 13,729 constraints (Mathematical Programming Systems - Extended (MPSX), and Generalized Upper Bounding (GUB) Program (SH20-0968-1), Revised, 1973, pp. 330-331).

representatives the rationale behind decisions reached by the local board of education or superintendent. Where controversial trade-offs are involved, the interested parties can vote for the activity which they prefer.

In conclusion, it appears that the linear programming approach to allocation problems in the not-for-profit sector of the economy, and more specifically in the administration of public school systems, can result in experimentation and decisions founded on objective criteria, sound logic, and computerized LP modeling rather than intuition and political pressure or the high cost of irreversible "live" experiments.

CHAPTER II

A REVIEW OF RELEVANT STUDIES

Educational enterprises and their related operations have been described by numerous models. Generally these are mathematical representations of colleges, universities, vocational-technical institutions, and public school districts which attempt to accomplish one of the following three objectives:

1. Describe the operations which convert inputs (students, dollars, supplies, teachers' services, etc.) into educational outputs (students with desired skills, morals, and character traits).
2. Highlight adjustments required of the administrators resulting from changes in assumptions or inputs to the model.
3. Support arguments for or against changes in the current methods of distributing state and federal aid.

In a few cases, litigants have used data generated by the models to strengthen their arguments concerning the equity of extant fund distribution procedures (Schoettle, 1972, p. 459).

This chapter examines selected studies by describing (1) common characteristics of the models employed, (2) the applications of the models, the data used, and the purposes of the studies, and (3) some operations not performed by these studies. Special attention is directed to those studies which have employed the linear programming algorithm. Upon this background, Chapter III builds the model for allocating resources in a public school district.

Characteristics of Models

To provide a point of departure for the subsequent discussion this section first describes general features of the LP technique. Then it focuses on four characteristics of the linear programming procedure which have a special impact on this research. Finally, it examines characteristics of other (non-LP) models which can be adapted to, or provide support for, techniques used in this research.

General Features of the Linear Programming Technique

Linear programming is a mathematical technique in which some criterion of effectiveness (known as the objective function) is optimized (maximized or minimized) subject to all operating restrictions (known as constraints). It can be characterized by:

1. One linear equation expressing the interrelationships among the variables in the system and their parameters in the effectiveness criterion.
2. A set of linear equations or inequalities expressing constraints imposed on the system by the environment or the voluntary actions of the planner.

Generally, the mathematical format is as follows:

Optimize (maximize or minimize) $X_0 = C_1X_1 + C_2X_2 + \ldots + C_nX_n$

Subject to:

$$A_{11}X_1 + A_{12}X_2 + \ldots + A_{1n}X_n \leq b_1$$

$$A_{21}X_1 + A_{22}X_2 + \ldots + A_{2n}X_n \leq b_2$$

...

...

...

$$A_{m1}X_1 + A_{m2}X_2 + \ldots + A_{mn}X_n \leq b_m$$

with all $X_i \geq 0$, where:

X_1 represents the activity level of the variables in the system,

A_{ij} represents the parameters of the constraints,

C_{ij} represents the parameters of the objective function, and

b_i represents the capacity limitation of each constraint.

Characteristics of LP Models

The four categories convenient for examining characteristics common to linear programming studies are (1) the objective function, (2) the constraint set, (3) the ability of the planner to intervene in the process, and (4) the impact that the process has on other segments of the economy.

The Objective Function. In most of the education-related LP models the objective function has maximized social welfare surrogated by lifetime personal income. For example, in a study of the Illinois State University curriculum, Koch (1973, p. 495) approximated the value of the university's output by "the present discounted value of the change in the lifetime income streams of the students who obtained education at the university." In a similar study related to the entire educational system of Northern Nigeria, Bowles (1967, p. 191) attempted to maximize the increment in discounted lifetime earnings attributable to additional years of education for all forms of formal education (excluding on-the-job training and self study).

McNamara (1971) approached the social welfare problem with a supply-demand function which optimized funds allocated to, and students enrolled in, programs located in a specific labor market area. The effect of his study was to minimize the unmet demand for manpower in critical occupational categories in a given area. For future research, he suggested a study which considers different lifetime earnings of graduates employed in various skills, and another study which maximizes a student's employment opportunities, i.e., mobility (McNamara, 1971, pp. 338, 361).

Cognizant of the frequent attacks in economic literature on measurements of social welfare, each of the above authors qualified his findings to the extent that his measure of welfare was invalid. Later in this study these qualifications supply a partial justification for the goal weight approach (see pages 19 and 20).

The Constraint Set. The following subdivisions are typical of the constraint set of an educational system's LP model:

1. A budget or financial resources constraint.
2. A physical facility constraint.
3. An available students constraint.
4. A teacher and support personnel constraint.

Applications of these constraints in the literature reviewed are similar to the applications in this study.

Generally, a modeler can adjust the budget constraint for changes in local, state, and federal funds easily. Capitalizing on this feature, Bruno (1969, pp. 488, 492, and 495) wrote constraints forcing a certain percentage of local and state participation in school district finance. In the Bowles (1967, pp. 190, 192) study the personnel constraint (4 above) could be expanded by importing teachers and the student constraint (3 above) could be modified by exporting students for their education.

Like finances and personnel, physical facilities (buildings and equipment) can be adjustable. Although the planning horizon of most models is usually short enough to preclude new school construction, a lease alternative makes it unnecessary to consider physical facilities constant. In contrast to the short range assumptions of other studies, Bowles (1967, p. 192) programmed a time frame sufficient to react to students who finished their education and re-entered the model as teachers. Few resources remain fixed over such a lengthy period, including physical facilities. Going one step further, Bowles assumed changing educational technology. While such intertemporal considerations complicate any model, most algorithms, especially linear programming, can be designed to cope with resource variations and re-entry. Thus, it is not necessary to assume that any resources are fixed.

Except in model designs which allow for students who are not promoted to succeeding grades, the quantity of students in each grade normally changes very slowly. On the other hand, the quality of students is rarely constant, yet few studies report any attempt to consider the students' abilities or aspirations. Most studies presume that this weakness is alleviated by the price mechanism of the job market which operates to insure congruity of available jobs and qualified students training to fill those jobs; but most studies concede a possibility for unmeasurable error if non-economic forces are ignored.

Other resources which nearly every study considers are teachers and support personnel. Given the teacher re-entry and importation possibilities of the Bowles study, one need not consider the supply of teachers restricted. Due to teacher shortages, this treatment may not have been realistic several years ago, but it is probably safe today.

The model in Chapter III groups the constraints in a somewhat different fashion. The four constraint subdivisions mentioned above are one group--the environment group. Another group captures as completely as possible the school laws. Together these are the operating constraints to which Chapter I refers (see page 6).

The Ability of the Planner to Intervene in the Process. One of the useful features of a linear programming algorithm is the planners' override capability effected by injecting new constraints, relaxing binding constraints, or altering system parameters. In the Koch study, one can observe a pragmatic application of the intervention feature. Due in part to an objective function based on lifetime earnings and a comparatively low pay scale for elementary school teachers, the first optimal solution generated by Koch's model completely eradicated the elementary education department at Illinois State University. For political reasons, few educational systems will be able to make such drastic changes in one year (if at all). Therefore, Koch mitigated the first solution by placing a lower bound on any department's staff equal to 75 percent of the previous year's staff (1973, p. 497).

From any given set of constraints and an objective function, at least one mathematically optimal solution will be produced (assuming a solution is bounded and feasible). In other words, the LP algorithm maximizes the goal weights in the objective function subject to whatever constraints the model includes. Whether this solution is in fact optimal depends in part on the modeler's skill in capturing the school system's environment with his objective function and constraint set. Since it is unlikely that the optimal solution produced will be identical to the existing system or a desired system, the planner may find it desirable to employ one of the above override features to produce the desired result.

An additional benefit of intervention is that it facilitates experiments with specific environmental or policy changes. From an optimal solution reference point, the planner can observe changes in the objective function emanating from experimental alterations of the model.

A further benefit of intervention is that it enables the experimenter to develop a study involving several time periods. The user first develops a model of the system and solves this model using the first year's data. With the new parameters developed in this first solution and the estimated data for year two, the experimenter solves the model again. To illustrate the change in parameters, assume a school system starts with ninety classrooms. The first year's solution is, therefore, constrained to ninety rooms. If a first year variable indicates seven more rooms could and should be built, the experimenter can relax the classroom constraint by seven rooms in the second year.

Although a model could adjust for several years without successive modeling, it is conceivable that computer capacity restrictions would make successive modeling more expedient. For example, a model which has 4,000 general equations for the first year cannot represent more than three years if each equation is repeated for each additional year, because the MPSX program is limited to 13,729 equations.

When a new constraint is injected into the model or a binding constraint is relaxed, the planner has altered the solution through a "policy constraint" as described in Chapter I. Policy constraints appear in Chapter V where the general model (Chapter III) as applied to Bartlesville (Chapter IV) is altered in accordance with the administrators' intervention.

Impact on Other Segments of the Economy. Apparently most researchers have assumed that an optimal solution in the educational segment of the economy will not suboptimize other elements in the overall social welfare function. Again Bowles' study is an exception--he made no such assumption. Instead, he actually studied the intersegmental effects of several levels of education and concluded that education, especially primary, "has an extremely strong claim on economic resources" (1967, p. 191). In addition, he discovered a high level of productivity for new educational technologies and imported teachers (Bowles, 1967, p. 191).

In a related vein, most researchers have employed earnings to measure social welfare (as noted earlier) thereby exposing their study to criticism for restricting considerations to market-based criteria. Nonmarket criteria probably have an impact on educational and noneducational segments of the economy. Ignoring this impact could result in solutions which are not optimal. However, until better noneconomic measurements are available, the importance of this omission is difficult to assess.

One of the assumptions of this study is that local administrators are in the best position to sense the impact the school has on other segments of the economy. An administrator's longevity may be construed as an indicator of his skill in interpreting that impact with respect to his community. This condition provides additional support to the use of goal weights determined by the administrator rather than "neutral" economic data such as increased lifetime earnings attributable to education.

To conclude the discussion of LP model characteristics, one property mentioned in connection with planner intervention deserves restatement for emphasis and contrast with other modeling techniques. Any feasible LP model will yield a mathematically optimal solution for the given objective function and constraints. Other modeling procedures will produce a solution, but the question remains: "Is this the best solution?" Apparently when using linear programming the programmer must have primary concern for constructing an LP model which successfully captures the environment. When he has done that, the algorithm will assure him the best solution.

Characteristics of Other Models

This research dichotomizes studies pertaining to educational system modeling into linear programming studies and "other" studies in order to stress those features which cause linear programming to be the preferred approach. Nevertheless, several nonLP projects have contributed directly or indirectly to the current investigation. Some important contributions are described below.

A Cost Effectiveness Model. Recognizing the fact that "educators seem to know very little about the processes that take the inputs of education and link them to educational productivity," Temkin (1969, p. 58) built a cost effectiveness model of public school systems which suggests a need for four basic information files:

1. A set of valued overall objectives which serve as a standardizing parameter set against which evaluations are made.
2. A fundamental structure relating system activities to the overall objectives.
3. A set of performance criteria and a performance outcome for each criteria which determines the extent to which the activity produced what it was designed to produce.
4. A set of activity expenditures.

Temkin's information files tie directly to the model developed in this study. His first file helps provide data for the goal weights, the second determines the constraint set and objective function equations, the third supplies data for the objective function, and the fourth determines the parameters for each variable in each constraint.

Temkin (1969, p. 17) prefaced the development of his model by stressing the need for "a systematic method for evaluation of ongoing educational systems so that future period allocations can be made with full awareness of the appropriate decision inputs."

Computer Simulations. As if in direct response to Temkin's observation, Nielsen and LoCascio (1972) developed a computer-assisted planning model for school districts which computes staff, facility, and financial requirements for the district based on projected enrollment and desired programs. Their model classifies variables as:

1. Environmental variables which are largely beyond the planner's control but still have an impact upon the system.
2. State variables which reflect administrative policies and the stock of resources at a given time.
3. Decision variables which can be controlled by administrators to achieve their objectives.

As described earlier, a similar classification applies to constraints in the Chapter III model--environmental constraints, legal constraints, and policy constraints. Unlike most of the other models studied, Nielsen and LoCascio's model has been implemented by several school systems and continuously improved by the staff of Peat, Marwick, Mitchell and Company.[1]

State Aid Formulas. Most states employ simple finance distribution models related to average daily attendance (ADA). One objective of these formulas is to equitably distribute state aid. However, numerous law suits have successfully challenged their "equity" by showing that residents of economically handicapped areas pay a higher percent of their gross income for education than residents of affluent areas, even though the per capita education expenditure in depressed areas is lower than in affluent areas. Actually the funds received for average daily attendance are generally uniform. Different property

[1]Mr. LoCascio supplied this information in a telephone conversation with the writer dated January 6, 1975.

tax valuation bases cause the disparity between neighborhoods. Nevertheless, many patrons believe ADA funds are the best vehicle to close the gap, i.e., ADA funds should be unequal as needed to make the per capita expenditures on education in a state equal in all neighborhoods. After the Supreme Court of California found that the property tax based financing of California's schools denied equal protection of the laws guaranteed by the United States Constitution in the *Serrano v Priest* case, courts in Minnesota, Texas, New Jersey, Wyoming, and Arizona reached the same conclusion. By January, 1972, similar suits were in litigation in eleven other states (Schoettle, 1972, p. 455).

Partially to compensate for the inequities spawned by the simplistic state aid formulas, lawmakers attached additional simple formulas. Thus, after espousing the objectives of assuring "full educational opportunities for every child in Oklahoma..." and "equal educational opportunity," Oklahoma lawmakers designed a state support formula which involves more than thirteen separate computations for each school district (*School Laws of Oklahoma*, 1974, pp. 138-144).

In response to the inequities in public school finance and anticipating additional Supreme Court rulings, Schoettle suggested another formula to improve financial aid distribution to school districts which he asserts mixes local and state financing with equalization payments and places all commercial and industrial property taxes in the hands of the state for fairer allocation. Schoettle (1972, p. 466) cites three advantages of this model. First, it results in a uniform tax rate for all commercial and industrial property; second, it leaves control of the school district to district residents; and third, it facilitates ratio comparisons among school districts.

The search for equitable aid distribution systems is likely to create substantial differences in the allocation formulas used from year to year and from state to state. The model proposed in Chapter III can adapt to any of the computational schemes suggested to date.

An Ability/Effort Model. Hines (1972) proposed a model similar to Schoettle's in that it allowed transfer of funds from wealthy to poor school areas. Using regression Hines developed an "ability model" (based on adjusted per capita income) and an "effort model" (based on adjusted per capita educational expenditures). Then considering (1) regional spillins and spillouts of educated people due to a mobile labor force, (2) return on educational expenditures, and (3) effort to ability ratios, Hines (1970, p. 80) decided that, in the interest of equity, "underachiever" regions should transfer funds to "overachiever" regions. In general, "underachiever" regions had high per capita incomes (ability) and spent a lower percentage of their incomes on education (effort) than "overachiever" regions. For political reasons it is improbable that the Hines model will ever be implemented on an interregional basis, but, given recent court sentiments, intrastate implementation is more plausible. The model developed in Chapter III will be capable of dealing with this procedure also.

Hines' model has another impact on this study. He discovered that, due to a mobile population, education benefits spilled out of some regions and into others distorting returns on investment in education in most regions and raising the question of who should finance educational investment (1972, p. 37). This finding further supports the election of this study to use a nonincome based objective function, i.e., goal weights.

The Accountability Model. Accountability, a comparatively new educational concept, merits discussion here because it generates information vital to this study. As described in Chapter I, planning, programming, budgeting systems (PPBS) specify goals and the plans for realizing the goals. When someone sets a goal, he must account for his efforts to achieve it. In its broadest context the concept of accountability encompasses all acts of accounting for one's efforts in a public school system, although the term "accountability" is often narrowly construed as "financial management."

While accountability applies to financial and nonfinancial data, probably the most significant forward step is related to financial information. Prior to accountability financial records were collected only on the basis of the object of expenditures. Presently the Oklahoma Pilot Study Schools use a 17 digit code which identifies each expenditure by:

1. The fiscal year of the expenditure (1 digit)
2. The fund from which the expenditure will be drawn (1 digit)
3. The source of the fund (2 digits)
4. The function of the expenditure (3 digits)
5. The object of the expenditure (3 digits)
6. The organization making the expenditure (2 digits)
7. The subject matter promoted by the expenditure (2 digits)
8. The school site benefiting from the expenditure (3 digits).

Probably full use of such a complete expenditure classification requires a computer, but the Durham-Middlefield, Connecticut, School District Number 13 operates a somewhat less exhaustive system without a computer (_Regional District No. 13_, 1974-75).

The preceding model characteristics have contributed significantly to the development of the model to be described in Chapter III. The application, data sources, and purposes of these and other models have further aided in the development of this study and will be discussed in the next section.

Applications, Data Sources, and Purposes of Education System Models

The preceding section examined general characteristics of mathematical models citing several which employed procedures relevant to this research. This section discusses educational applications of models, data sources found in other studies, and the purposes (objectives) of the other models. The primary focus of this section is on the usefulness of mathematical models in educational settings because it appears that few administrators have found them helpful, state aid formulas and accountability procedures excepted. One might hope that greater social benefits would emerge from research efforts in education administration than can be inferred from administrative acceptance.

The applications of models will be the nucleus of organization in this discussion, but each subsection will also consider data sources and purposes of the model under examination. In the context of this section the term "application" means the administrative unit to which mathematical models have been applied, namely, a university, a group of counties served by area vocational-technical schools, and a state.

Application of Models to a University

While the Koch model was applied specifically to Illinois State University to maximize the aggregate incremental expected lifetime earnings of graduates, it could be applied to any public university for the purpose of distributing limited resources to academic departments with unlimited demands for resources. Most of Koch's data came from the university's records and National Science Foundation studies (Koch, 1973), but numerous other sources of similar data exist, e.g., Department of Labor statistics and Illinois Employment Commission records. Since the model is general and data are readily available, repeated application of the model at ISU and elsewhere is possible at fairly modest marginal cost.

Koch encountered one of the most serious obstacles to implementation of the model--personnel problems in departments experiencing a reduction in staff with adoption of the optimal solution (see page 17 for a description of this problem).

A problem which appears to be almost as acute as staff reduction is, ironically, the problem of staff increases resulting from rapid jumps in enrollment in some high-demand disciplines. While constraints in the model limit the number of new faculty to the number of people with proper credentials, such constraints are probably not capable of detecting those people who "fit in." Typically the search for acceptable high quality professional talent in rapidly expanding disciplines requires considerable time and thus inhibits rapid response to staff level increases suggested by the model.

In a similar manner student aspirations and talents present an obstacle. Frequently students lack either the ability or the motivation to succeed in some of the high-demand subject areas. Aptitude and personality test scores can supply data to make the model responsive to the student supply problem (Smith, et al.,1974, p. 4), and statistics concerning position openings and people seeking positions supplied by various professional organizations (e.g., American Accounting Association and Oklahoma Educators Association) can make the model responsive to the professional supply problem.

Applications of Models to Area Vocational-Technical Schools

A large portion of mathematical modeling in education exists in the vocational-technical sector of education.[2] One such model which has already been mentioned, the McNamara study (1971), applied to four occupational programs in a five county area around Philadelphia. The State (of Pennsylvania) Department of Labor and Industry furnished much of the data used (McNamara, 1971, p. 341), but vital data came from other studies, estimates, and previous phases of McNamara's study. Although McNamara stresses the general applicability of his model, it appears that reliance on estimates and data from other studies may impede adoption of the model on a wide scale.

One of the purposes of another area vocational-technical school study was:

> To establish data collection procedures for the variables of the Linear Programming Model including source, method of capture, and system entry (Smith, et al, p. 2).

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[2] A possible explanation for this is the extensive data bank available from various agencies such as the Office of Education, the State Departments of Vocational-Technical Education, and OTIS (Occupational Training Information System).

Numerous sources of data were incorporated, such as OTIS, General Aptitude Test Battery (GATB) scores, and state and federal agency statistics. One especially interesting source of data was the Operations and Procedures Manual of the Oklahoma State Department of Vocational-Technical Education because a similar "rule book," the Administrators Handbook, is the major source of the constraint equations in Chapter III. One of the unique qualities of a model built around such a book is that it assures compliance with state and federal regulations (assuming the regulations are included in the book).

Applications of Models to State Level Administrative Units

Most legislative education models have statewide application. Ordinarily the data used is generated internally or compiled from required district reports. Typically the model's purpose is to distribute financial support to the districts which comply with state regulations (i.e., become accredited), and to give a measure of equality of educational opportunity to all residents of the state.

Dissatisfied with the equity of existing state models, Schoettle (1972) and Bruno (1969), in studies mentioned earlier, designed models to be applied statewide for more equitable resource allocations. The data used in both models was taken from state files. Compared with existing allocation schemes both models, especially Bruno's, appear to distribute resources far more equitably. Since courts have ruled that many state school financing programs deny equal protection of the law, and since data is readily available for either model, the residents of many states are likely to see the adoption of a model similar to the ones proposed by Schoettle or Bruno.

The only apparent impact the adoption of a Bruno or Schoettle model will have on the model in this study is a minor alteration of the budget constraint. For instance, one alteration might be a change in the budget constraint coefficient for one or more revenue variables. The most extensive alteration would probably require no more than the addition of a special budget constraint to reflect new allocation procedures plus a few new variables.

Techniques Not Employed in Previous Studies

The primary purpose of the model described in Chapter III is to assist public school administrators in planning and administering available resources optimally in relation to explicit goals. To accomplish this purpose, the model employs two previously unused techniques in combination with selected practices described earlier in this chapter.[3] While other studies do not use these two techniques, they lend support to such use. The techniques and supporting studies are discussed below.

The Objective Function Is Based on Noneconomic Goals Set by Administrators and Patrons

The objective function (usually a maximand) employed in other

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[3]The selected practices are:

1. Allowing the planner to intervene in the problem solution,
2. Classifying constraints into environmental, legal, and policy constraint categories (primarily for conceptual rather than computational reasons),
3. Drawing data from existing files to the extent practical,
4. Using published handbooks to identify legal constraints.

studies is usually some economic-based goal. To illustrate, one study maximized the amount of money returned to the economy by reduced welfare payments and increased taxes resulting from vocational-technical education (Smith, et al.,1974, p. 3). As standard procedure, authors qualify the conclusions they draw to the extent of the inappropriateness of their maximand (see Bowles, 1967, p. 195). Reacting to a wide range of conditions, practicing administrators select both economic and noneconomic goals to which they assign priorities for achievement. Whether or not these goals and priorities are explicit depends, in part, on the degree of implementation of PPBS by the system. Frequently the goals are explicit but the priorities are not, even though a prioritizing mechanism exists, e.g., questionnaires, administrators' statements, etc. Given prioritized goals, an educational model need not be confined to economic surrogates of social welfare. Recognizing the probable variability in goal priorities and the nonneutral impact of administrators' opinions, the conclusions of this study should be qualified to the extent the maximand is inappropriate.[4] Nevertheless, the use of weighted goals which are not necessarily economic appears to be more appropriate for elementary and secondary school models than economically surrogated goals because relatively greater effort goes into preparing students to coexist with the rest of society than preparing them for specific occupations. In contrast, college and vocational-technical schools place greater emphasis on occupational preparation supporting the use of economic goals.

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[4]Further consideration of the limitations of goal weights appears in Chapter III (see page 38).

Further support for the use of goals which are not necessarily surrogated by economic measures emerges from the Hines study. Given the spillouts of educational benefits identified by Hines, if a school system in any one of several states tried to maximize the return on the district's educational investment, it would offer no college preparatory programs. Few people would argue that such a policy was in the best interest of the district.

A final argument in support of using prioritized goals is that the administrators are in a better position to interpret their patrons' consensus than remote employers who set wages without regard for patrons' feelings. The following typical situation illustrates the validity of this argument: If (1) substantial noneconomic benefits accrue to the community through a strong music program, (2) musicians' salaries are low, and (3) the administrator's decision model maximizes expected lifetime earnings, the music program's worth to the community will be seriously understated and resources devoted to music will be less than they would be in a prioritized goal solution.

The Model Considers the Interrelationships of the Environment and State and Local Laws

Since the Chapter III model is primarily a management tool for local school administrators, the ideal model will embody all the constraints operating on the school system, including the laws of the relevant state and community. Basically school laws can enter the model in two ways, by definition and by equation, as discussed below.

Entry by Definition. *The School Laws of Oklahoma* (1974) prescribe minimum requirements for a number of resources and activities in order for the school district to be accredited. If the minimum legal requirements are accepted as the definition of a resource or activity (variable), then one portion of the law is built into the model. This study assumes that all schools in a district want to be accredited because district finances depend on accreditation and because alumnae of accredited schools gain admission to the next echelon of schools far more readily than the alumnae of nonaccredited schools. Thus all variables are assumed to meet the minimum legal requirements. Consider the following example of entry by definition: In order for a school to be accredited, all the teachers in that school must be certified. Consequently, a teacher is defined as something more than a person who conducts classes; a teacher is one who holds a current certificate to teach the subject to which he/she will be assigned. To further illustrate, a junior high school is not the sum of the students in grades 7, 8, and 9. A junior high school must also include a separate structure (wing or building) and separate faculty, labs, athletic programs, and library (School Laws of Oklahoma, 1974).

Entry by Equation. In addition to defining the minimum requirements for certain variables, the School Laws of Oklahoma (1974) mandate certain relationships among the variables of a school district. After expressing these legal relationships mathematically, the modeler can incorporate another portion of the law into the model. As an illustration of this procedure, consider the following equation which computes one part of a school's revenue:

$$\$260\ (EADA) + \$312\ (SADA) + 75\%\ (ADH)(PCAT) + \ldots + IA = DR,$$

Where:

EADA = Elementary average daily attendance,

SADA = Secondary average daily attendance

ADH = Average daily haul,

PCAT = Per capita allowance for transportation,

IA = Incentive aid,

DR = District revenue.

As far as possible, school laws will be captured in the model by either definition or equation. A few laws, however, are difficult to commit to mathematical expression. In general these are expressions of philosophy, purpose, or ideals such as the following: "There should be a concern for democratic, moral, and intellectual values and special attention to the needs of society...(Annual Bulletin, 1974, p. 30). Through a loose interpretation of the "entry by definition" concept, such laws can be brought into the model, but their impact on the objective function (and thus school management) is likely to be undetectable. Therefore, the underlying philosophies, purposes, and ideals, are presumed to exist throughout the school system regardless of the variable mix prescribed by the objective function and are built into administrators' goal weight assignments.

Summary

This chapter has examined the literature related to school management models to identify characteristics and techniques which will be helpful in development of the model in Chapter III. While the emphasis has been on linear programming models, the applications, data sources, and purposes of several nonlinear programming studies have supplied essential techniques or support to this study. With this background, the stage is set for the development of the public school system resource allocation model.

CHAPTER III

THE GENERAL SCHOOL RESOURCE ALLOCATION MODEL

All states have rules prescribing certain operating policies or activity levels for their accredited public schools. These rules, together with numerous natural restrictions, construct a general framework within which each school system must function. For example, the number of children in a given school district (a natural restriction) divided by the maximum student to teacher ratio allowed by law (a state rule) determines the minimum number of teachers a school district must employ. Each school district adds its own rules to those imposed by nature and state laws in order to attain the school system's goals as defined by the administrators and patrons of the system.

This chapter has three sections. The first section, Sources, examines the sources of the three types of equations listed below in the public school resource allocation linear programming (LP) model:

1. The objective function (goals of the school system)
2. Operating constraints (legal and natural restrictions within which the school system must operate)
3. Policy constraints (requirements imposed on the system by the system's administrators, patrons, or nonadministrative personnel).

These three equation types are then combined for the LP model in the second section (Description of the Model).

The model can be processed with an IBM MPSX360 computer program providing extensive management information. The third section of Chapter III (Information in the Solution) explains the information provided in the computer program output related to resources (constraints) and variables (defined on page 50).

Sources

While most school systems do not construct formal mathematical models of their operations, they implicitly combine the goals of the district, operating constraints, and policy constraints for their year-to-year functions. This section describes many of the sources from which goals, operating constraints, and policy constraints are obtained.

Goals

Basically five sources help administrators define the goals (the objective function) for public schools: Patron and Staff Surveys, Independent Studies, PPBS Studies, Administrative Statements, and Economic Studies.

Patron and Staff Surveys. Many public school systems have circulated opinion questionnaires among selected samples of their patrons and/or staff. Where properly designed, these surveys provide a fairly representative expression of the beliefs of the groups polled and the relative importance attached to each belief by the group. In effect, then, these surveys have identified the goals and their related weighting factors (as defined on page 7) for the objective function.

Independent Studies. Sometimes groups which are independent of the school administration will conduct studies to identify the goals of the school system and activities which will lead to the realization of those goals. Such studies are an excellent objective function source. The Community Education Council (CEC) of Bartlesville is a case in point. The CEC defined twenty-four separate goals for the Bartlesville schools which the Bartlesville Board of Education adopted without alteration. Since the CEC goals are the source of the objective function of this study, they are discussed further on pages 44 and 45.

PPBS Studies. During the Johnson Administration many federal government divisions instigated planning, programming, budgeting systems to improve their effectiveness. This management technique has gained popularity with public school administrators recently, and, consequently, many school systems have altered their data-gathering framework so that it will be more compatible with the PPBS technique.[1]

One of the first steps in adopting a PPBS system is stating the enterprise's goals. These goal statements are another good source from which an objective function can be derived.

Administrative Statements. Probably in public schools, as in other enterprises, the goals of the organization reflect the goals of the most dominant members of the organization. In the absence of explicitly written goals, administrators are likely to perceive different goals (or priorities) from those perceived by patrons, teachers, or students

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[1]For example, Regional District 13 of Durham, Connecticut, installed a noncomputerized accounting system in conjunction with an overall PPBS format (Regional District 13 Board of Education Budget, 1974-75). Selected schools in Oklahoma have participated in the Pilot Study to improve an accounting system to be used with PPBS (Pilot Project, 1973).

(Pingleton, 1962). In spite of the apparent possibility of goal incongruencies, administrators' expressions of the enterprise's goals may be a reliable source for building the objective function since ongoing management of the system is in the administrators' hands. When written goals exist as a result of, say, a PPBS plan, the administrators' weighting of the goals may give an indication of the relative role each goal plays in the objective function.

Economic Studies. As stated in Chapter II, most LP models of educational systems have used an objective function expressed in strictly economic terms, such as maximization of expected lifetime earnings. Economic studies, usually conducted by government agencies, may be an excellent objective function source.

Operating Constraints

Operating constraints impose boundaries within which an administrator must operate and which are beyond his immediate control. The operating constraints arising from legal proclamations are generally not difficult to identify although they may be quite difficult to quantify. Operating constraints arising from states of nature are generally obvious in both existence and quantity. Five sources of operating constraints are discussed below.

State and Federal Laws. Although school laws are generally spread throughout the statutes, most state boards of education have perused the law books and grouped those laws relevant to school administration in a single book.[2] Since most of the laws can be

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[2] In Oklahoma, for example, the book is School Laws of Oklahoma. The Oklahoma State Board of Education revises this book annually.

obtained from sources which are easier to interpret and quantify than the statutes themselves, such as, administrators' handbooks, the school lawbook is not the best source for identifying operating constraints.

Except for special grant contracts, most federal laws determine a minimum level of operations for state laws and thus are redundant with the state laws. Federal grants generally carry their own operating constraints in the contract.

Local Laws. Local laws are the authoritative pronouncements of the city, county, or other political subdivisions in which a school district operates. The relationship between local laws and state laws is similar to the relationship between state laws and federal laws, i.e., state laws determine a minimum level of operations for local laws. Thus when a local law is introduced into the model, a state law pertaining to the same topic usually becomes redundant.

Local laws should be distinguished from local board of education policies because, in this study, the local board is considered a part of the administrative team which sets the policies for the school system (included as a part of the policy constraints). Local laws set conditions which cannot be altered by the board since they emanate from an authority over which the board has no control. For example, millage levels are set by the local electorate in annual elections. Administrators can try to persuade voters to vote for a given millage level, but, other than persuasive efforts prior to the election, the board of education has no control over millage levels. Once the voters have acted, however, the board may have fairly broad discretion over the spending of the funds. Their discretionary policies governing expenditures of the funds are policy constraints (to be discussed later), not operating constraints.

Administrators' Handbooks. As mentioned earlier, the volume and legal terminology of the laws make law books an impractical reference source for school administrators. Therefore, many state departments of education have published handbooks to guide administrators in routine school management. These books are probably the best source of legal operating constraints.

Independent Associations. Most school districts voluntarily join one or more association, such as the Oklahoma Secondary Schools Activities Association, which specify certain operating constraints. Schools are not required to join such associations, but as a matter of practicality they do because the interscholastic competitive events in which nonmember schools can participate are highly limited.

The constitution, by-laws, and handbooks of independent associations are probably the second best source from which operating constraints can be gleaned.

Natural Restriction. The operating constraints described to this point are set by authoritative bodies beyond the control of school administrators. Natural conditions establish some constraints as well. In many cases the natural limitations are wholly contained in the legal constraints (recall the teacher-pupil ratio illustration in the introduction). The statistical reports of a school system, legal constraints containing natural limitations, and the modeler's observations are the best sources from which to derive natural limitations. The following are examples of natural constraints:

1. The sum of all the students enrolled in high school times the maximum number of credits a student can take each semester

cannot exceed the total number of teachers times the maximum student-to-teacher ratio.

2. The sum of unearmarked revenues collected by a school district less the sum of general fund expenditures cannot be negative (assuming debt financing of general fund expenditures is illegal).

3. The sum of the teachers employed by a school district cannot exceed the sum of certified personnel residing in the district (assuming commuting from other districts is impractical).

Policy Constraints

The distinction between policy constraints and operating constraints may appear to be blurred in some cases. For instance, a school system is not required to have interscholastic football activities, but local pressures may be such that the administration believes its only practical course of action is to participate in football competition. With respect to the linear programming algorithm, the distinction between policy and operating constraints is meaningless, but the writer believes it is important because the solution of the model without policy constraints compared to the solution with policy constraints enables a manager to examine every trade off as optional programs (variables not forced into the system by laws or nature) consume remaining resources (constraints). Furthermore, some seemingly obligatory policy constraints may not bind administrators as rigidly in the short run as operating constraints. Consider the following illustration:

> ABC school system receives tax revenues early in the year, and, therefore, has cash reserves to invest. If an investment opportunity pays a high rate of return but commits the cash for a period slightly longer than the present fund reserves, say, five days, ABC will need to borrow general operating funds for five days or forego the investment opportunity. If state laws prohibit borrowing for the general fund (an operating constraint) and local procedures require prompt payment of obligations (a policy constraint) the system will be unable to benefit from the high return opportunity. However, if the rewards of the investment opportunity are great enough and the period of cash shortage short enough, ABC may make arrangements with their creditors to delay payment briefly. The policy constraint could be altered slightly by administrators whereas the operating constraint could not.

Policy constraints may be written or implied. In either case, they generally come from the four sources discussed below.

Administrators. Administrators usually have standard operating procedures which free them to concentrate on exceptional events. Frequently these procedures appear in operating manuals which are obvious sources for policy constraints. A not so obvious source is administrators' unwritten policies. For example, a superintendent may decide never to appropriate more than 95 percent of the revenues he anticipates, even though he could legally appropriate more. Constraints of this type can only be discovered through observation or interviews with the administrator. Administrators can "experiment" with an LP model by imposing this type of constraint on the system and observing the results. Experimentation will be discussed more completely in Chapter VII (see page 117).

Patrons. Patrons sometimes impose limitations on the system. For instance, when the parents of musically talented students insist on the same per capita expenditure on music as on athletics, they are creating a constraint. Whether this kind of restriction is a policy constraint or an operating constraint depends (1) upon whether the patrons are more correctly viewed as part of the management team or part of the environment with which administrators must cope, and (2) upon the consequences of resisting the patron's demands. Probably such a determination must be made on an event-by-event basis.

Personnel. Generally the constraints that nonadministrative personnel impose on the school system, such as teachers' union contracts, are operating constraints. Occasionally the limitations might be considered policy constraints because (1) they are not required by laws or natural conditions, (2) they arise voluntarily from nonadversary type proceedings, (3) they utilize resources of the system and thus alter the model (however slightly), (4) they are in the best interests of the school system and not necessarily the best interest of the personnel, and (5) they could be prevented by the administrators. When teachers agree to cooperate in a student teacher program, for instance, they are committing the resources of the school system in a manner which meets the five criteria listed above. Therefore, one may view their actions as policy constraints.

Description of the Model

This section describes the three elements of the public school resources allocation model: the objective function, constraints, and variables.

The Objective Function

In 1972 a group of Bartlesville citizens, the Community Education Council (CEC), identified twenty-four goals (Appendix A) which "the system (Bartlesville) might adopt to promote learner development to more effectively serve the needs of the community" (Community Education Council, 1971-72). These goals provide the basic structure of the objective function, i.e., each variable helps achieve one goal.

At the researcher's request, Bartlesville's superintendent and assistant superintendent independently assigned a weighting factor between 0 and 100 to each CEC goal. The superintendent's weighting factor was added to the assistant superintendent's weighting factor. This sum (hereafter called "goal weight") serves as the objective function coefficient for all the variables perceived by the researcher to aid in accomplishing a particular goal. The objective function in this research maximizes the goal weight of the Bartlesville Independent School District No. 30 and can be formally stated as follows:

$$\text{Max } GW_o = GW_1 \left(\sum_{i=1}^{a} X_i \right) + GW_2 \left(\sum_{i=1}^{b} X_i \right) \cdots GW_{24} \left(\sum_{i=1}^{x} X_i \right)$$

where: GW_1, $GW_2 \ldots GW_{24}$ are the goal weights for goals 1 through 24 (see Appendix A),

a, b, ..., x are the number of variables which affect goals 1, 2, ..., 24 respectively,

X_i is the activity level of the ith variable, and

GW_o is the aggregate of the product of all goal weights, GW_1, through GW_{24}, times the activity levels of the variables.

Constraints

The two basic kinds of constraints, operating and policy, are defined on page 6. Since policy constraints are discussed in Chapter V, this subsection describes only operating constraints. After identifying the sources from which operating constraints were drawn, this subsection presents three examples of operating constraints--a budget constraint, a curriculum constraint, and a natural constraint.

Operating constraints were taken from four publications and the researcher's observations of natural restrictions (to be discussed below). The first publication, Annual Bulletin for Elementary and Secondary Schools (Administrators' Handbook, 1974), hereafter called the Annual Bulletin, provided most of the constraints because the rules in this book were the easiest to write in equation form. Most of these deal with curricula, student activities, and teachers' work loads. The second publication, The Constitution and By-Laws of the Oklahoma Secondary Schools Activities Association, (1974), provided most of the interscholastic competitive activity constraints, e.g., football playoff, marching band, and debate contest requirements.

The transportation constraints and financial management and accounting constraints were taken from State Board of Education Regulations for Administration and Handbook on Budgeting and Business Management, hereafter called the State Board Regulations. Most of the constraints drawn from the Annual Bulletin and State Board Regulations could have been taken from the School Laws of Oklahoma, 1974, because the first two are interpretations and summaries of the latter. However, as pointed out on pages 40 and 41, it is much easier to read and write equations for rules in handbook-type publications than in statutory compendia. Consequently, only a few constraints were drawn from the School Laws of Oklahoma.

Through reading and discussions with public school administrators the researcher identified as many natural constraints as possible. Many of the natural constraints are embodied in one or more of the constraints drawn from the preceding sources.

The following are examples of operating constraints in the general model:

The Budget Constraint. Programs of the school system are limited by the amount of available financial resources, i.e., the budget constraint. The State Board Regulations (1974, p. 34) provide that:

> All independent school districts shall be in good financial condition and shall give the State Board of Education sufficient evidence of being able to administer the fiscal affairs of the district in a proper manner.

This constraint is written (mathematically) to assure a nonnegative financial position as follows:

$$(\sum \text{Revenues} - \sum \text{Committed Costs}) - \sum \text{Discretionary Costs} \geq 0$$

For this study revenues are any financial resource inflow to the general fund, including borrowed resources. (Funds other than the general fund are beyond the scope of this study.) Committed costs are defined as costs which the school district must incur as long as it remains in operation, such as, the superintendent's salary. (An independent school district is required to have a superintendent.) Subtracting these committed costs from revenues leaves the financial resources available to initiate discretionary programs (the remainder in the brackets above).

Discretionary costs are the financial resource requirements of programs which a school system initiates in pursuit of its own goals and not to satisfy legal stipulations. The budget constraint allows whatever combination of discretionary programs maximizes the goal weights without exceeding available resources.

<u>A Curriculum Constraint</u>. The Annual Bulletin requires each high school to offer a minimum of 36 units of course work with at least 4 units of mathematics and science; 5 units of language arts and social studies; 2 units of foreign language, fine arts, and physical education; and 12 units of applied vocations. Therefore:

$$\sum_{i=10th}^{12th} \text{Math}_i + \sum_{i=10th}^{12th} \text{Science}_i + \sum_{i=10th}^{12th} \text{Language Arts}_i +$$

$$\sum_{i=10th}^{12th} \text{Social Studies}_i + \sum_{i=10th}^{12th} \text{Foreign Language}_i +$$

$$\sum_{i=10th}^{12th} \text{Fine Arts}_i + \sum_{i=10th}^{12th} \text{Physical Education}_i +$$

$$\sum_{i=10th}^{12th} \text{Applied Vocations} - 36\,(\text{the number of high schools}) \geqq 0,$$

and

$$\sum_{i=10th}^{12th} \text{Math}_i - 4(\text{the number of high schools}) \geqq 0,$$

$$\sum_{i=10th}^{12th} \text{Science}_i - 4(\text{the number of high schools}) \geqq 0,$$

$$\sum_{i=10th}^{12th} \text{Language Arts}_i - 5(\text{the number of high schools}) \geqq 0,$$

$$\sum_{i=10th}^{12th} \text{Social Studies}_i - 5(\text{the number of high schools}) \geqq 0,$$

$$\sum_{i=10th}^{12th} \text{Foreign Language}_i - 2(\text{the number of high schools}) \geqq 0,$$

$$\sum_{i=10th}^{12th} \text{Fine Arts}_i - 2(\text{the number of high schools}) \geqq 0,$$

$$\sum_{i=10th}^{12th} \text{Physical Education}_i - 2(\text{the number of high schools}) \geqq 0,$$

$$\sum_{i=10th}^{12th} \text{Applied Vocations}_i - 12(\text{the number of high schools}) \geq 0,$$

where:

(1) the number of high schools means the number of high schools in the district,

(2) i is the grade level at which a course is offered, e.g., 10th, 11th, and 12th grade math, 10th, 11th, 12th grade science; etc. and,

(3) the rest of the symbols (e.g., Math, Physical Education, and Applied Vocations) designate the subject being taught.

The reason for writing these constraints with zero righthand sides is explained in Appendix B, Modeling Techniques Facilitating Implementation.

A Natural Constraint. All teachers in Oklahoma public schools must hold a valid certificate for their particular teaching area. Therefore, the number of courses a system can offer in a given subject is limited by the number of teachers certified to teach that subject. The following is an example of a natural restriction for courses in Russian:

$$\sum_{i=10th}^{12th} \text{Russian language courses}_i - 5 \sum \text{The number of certified Russian teachers} \leq 0$$

A legal restriction is embodied in this natural restriction. In Oklahoma teachers may not teach more than five courses per day and a unit of credit requires one hour each day for two semesters. Consequently, if only one person in a school district is certified to teach Russian, the school system cannot offer more than five units of Russian each year.

Variables

Variables are activities which consume one or more resources and which contribute to the accomplishment of the system's goals. The Russian course example above has two variables--Russian Courses and Certified Russian Teachers--as long as an unlimited quantity of either courses or teachers is available. However, when the amount of a resource consumed by a variable is fixed at a given level the variable changes to a constraint. For example, if the number of certified Russian teachers is fixed (constant) at 2, the variable, certified Russian teachers, becomes a limited resource constraining the number of Russian courses that can be offered. The Russian course constraint can then be rewritten as follows:

$$\sum_{i=10th}^{12th} \text{Russian Courses} - 10 \doteq 0, \text{ or}$$

$$\sum_{i=10th}^{12th} \text{Russian Courses} \doteq 10.$$

As pointed out in Appendix B, when a variable is constrained by a bound (upper, lower, or equality), both modeling efficiencies <u>and</u> interpretation problems arise.

Information in the Solution

The two parts of this section, Constraints and Variables, examine the information which the MPSX360 output provides for school administrators.

Constraints

Extensive management information related to constraints (available resources) appears in the computer output. This part discusses the information in the following divisions: Activity Levels, Cost of Changes, Relevant Range for Cost of Changes, Limiting Processes, and Summary

Activity Levels. Constraints prescribe a maximum amount or a minimum amount of a given resource which may be consumed by the variables in the model. Often not all of the resource is consumed. The amount of a given resource used by variables in the optimal solution (the resource's activity) appears in the printout. For example, if ten first grade teachers reside in (i.e., are available to) a school district, but only seven first grade classes are offered (i.e., only seven first grade teachers are used) in the optimal solution, the activity would be seven. The amount of unused resources (Slack Activity) also appears in the printout. In the illustration just given, it would be three (ten teachers available minus seven teachers used).

Cost of Changes. When the resource constraining the activity of variables in the optimal solution is consumed at an "intermediate" point between its upper and lower physical limits, the constraint is referred to as a "basis" constraint. For instance, in the first grade teacher example just used the constraining resources had an activity level (seven) between its upper limit (ten) and its lower limit (zero).

Unless an alternate optimal solution exists, any change (increase or decrease) in the activity level of a resource in a basis constraint causes a reduction in the aggregate goal weights. If this were not true, the solution could not be optimal. This reduction is the goal weight loss of changing a basis resource, usually called Unit Cost. Two Unit Costs are reported for each basis resource; one is the reduction in the aggregate goal weights related to decreases in the activity level of the resource, the other is the reduction in the aggregate goal weights related to increases in the resource's activity level.

To illustrate the Unit Cost for a basis constraint, consider the first grade teacher example again. The school system had ten first grade teachers available, but the optimal solution employed only seven. If the Unit Cost values were 52/24, the administrator knows that for every teacher terminated (from seven downward to the Lower Limit) the objective function value declines by 52 goal weights; and for every teacher added, the objective function value declines by 24 goal weights.

When a resource has been completely consumed or has been used at its lowest allowable level in the optimal solution, i.e., when the resource is at its Upper Limit or Lower Limit respectively, the constraint is binding rather than basis. Hereafter, resources in binding constraints will be referred to as "stopped resources." If it were possible to obtain one more unit (for a resource stopped at its upper limit) or eliminate one more unit (for a resource stopped at its lower limit), the system's aggregate goal weight would increase. The amount of this increase is called the dual activity or "shadow price" and is the stopped resource's Unit Cost in a negative sense. In other words, for stopped resources, the shadow price is the cost of not changing. For example, if a school system had eleven second grade teachers, all of whom were used in the optimal solution, and if the dual activity reported for second grade teachers were 75, the aggregate goal weight of the system would increase by 75 units for each additional available teacher. The cost of _not_ adding an additional teacher is 75 goal weights.

Note that the shadow price is in units of goal weights, not dollars. The administrator is comparing the impact that one additional second grade teacher will have on goal realization. The dollars and all other resources involved in the decision are automatically processed in the budget and other constraint equations.

Relevant Range for Cost of Changes. The cost of changing (and cost of _not_ changing) described above is valid only over a given range for each resource. The computer program calculates the lower end and upper end of that range (called Lower Activity/Upper Activity). Beyond these ends, the cost of changing (Dual Activity or Unit Cost) changes. To illustrate, assume the Lower Activity/Upper Activity for the _second_ grade teachers mentioned above is 8/13 (see Cost of Changes). For each teacher above 8 who is added to the system up to 13, the aggregate goal weight of the system will increase by 75. Both basis resources and stopped resources have relevant ranges. Note that the Lower Activity/Upper Activity values do not necessarily correspond to the lower limits or upper limits placed on the resource by nature or the experimenter (eleven teachers in this example). The upper limit of eleven was imposed by exogenous influences such as natural constraints, state laws, or policy constraints. Lower Activities/Upper Activities are imposed by endogenous influences within the model itself as explained in _Limiting Processes_.

Limiting Processes. Beyond the Lower Activity/Upper Activity levels just discussed, the Dual Activity (or Unit Cost) changes. The new Dual Activity cannot be determined because when a resource reaches the Lower Activity or Upper Activity another resource or variable in the model reaches one of its limits (becomes "stopped"). The resource or variable which reaches its limit is called a Limiting Process: it is the endogenous influence referred to above.

To illustrate with the second grade teachers again, suppose there are only thirteen second grade rooms available. If the school system is able to obtain more than thirteen second grade _teachers_ (the Upper Activity), it will be constrained by second grade _rooms_ instead of second grade teachers. Thus, second grade rooms is the Limiting Process at the teachers' Upper Activity.

Summary. The following list summarizes the information provided to administrators in the output related to resources:

1. The amount of the resource used and unused (Activity).
2. The sacrifices that must be made (in aggregate goal weights) to alter a basis resource either upward or downward (Unit Cost).
3. The costs of not changing the exogenous limits on stopped resources (shadow price or Dual Activity).
4. The activity ranges for which the costs of changing or not changing are valid (Lower Activity/Upper Activity).
5. The resources or variables in the model which are affected when the ranges in 4 are exceeded (Limiting Processes).

Variables

Some of the information pertaining to variables closely parallels information pertaining to resources (constraints). To avoid confusing variables in the illustrations used below with resources in the preceding subsection's illustrations the following distinction is made: A resource is a stock of a given category of assets, e.g., second grade teachers; a variable is a process which may utilize some or all of those assets, e.g., second grade teachers employed in Sunset Elementary School.

The division titles for this subsection are: Activity Levels, Goal Penalties, Relevant Range for Goal Penalties, Sensitivity, Limiting Processes, and Summary.

Activity Levels. Activity is the amount of a given resource which is assigned to a variable. Thus, if two of the seven first grade teachers in the preceding subsection's example were employed by Sunset Elementary School, the activity for the Sunset first grade teachers would be two.

Goal Penalties. A reduction in the aggregate goal weight (a goal penalty) results from forcing a one unit change in the activity level of a variable. This goal penalty is usually called the "Unit Cost" of a variable. For example, suppose the activity level of Sunset first grade teachers is two and the Unit Cost is 44. If three first grade teachers are employed at Sunset the aggregate goal weight will decline by 44 units.

Relevant Range for Goal Penalties. A variable's Unit Cost is relevant over a specific range called the Lower Activity/Upper Activity. The interpretation of Lower Activity/Upper Activity is exactly the same for variables as it was for constraints (resources) above.

Sensitivity. A variable's goal weight in the objective function is called its Input Cost. The Input Costs in this study were determined by the superintendent and assistant superintendent (see page 45). Since this figure is fed into the computer by the experimenter, it is not new information. However, the Input Cost is one of the most significant factors in determining the activity level of a variable in the optimal solution. An administrator might like to know how sensitive his model is to changes in a given variable's Input Cost. This information, called Upper Cost/Lower Cost, is supplied by the computer printout. For instance, if the Input Cost for Sunset second grade teachers is 95 (i.e., the goal weight is 95) and the Upper Cost/Lower Cost is 105/2, the Sunset second grade teacher goal weight could fall from 95 to 2 before its activity in the optimal solution would change; or the goal weight could increase slightly to 105 before an activity change occurred. In other words, these data give the administrator a feeling for the sensitivity of variables to the goal weights assigned to them.

Limiting Process. The Limiting Process for variables bears the same interpretation it had for resources.

Summary. The solution to a linear programming problem tells an administrator the following things about the variables with which he is working:

1. The optimal activity levels for the variables (Activity)
2. The goal weights lost (goal penalties) by forcing a change in the variables' activity levels (Unit Cost)
3. The range of activity levels over which the goal penalties above are relevant (Lower Activity/Upper Activity)
4. The sensitivity of those activity levels to changes in the variables' goal weight (Lower Cost/Upper Cost)
5. The resources or variables in the model which are affected when the ranges in 3 above are exceeded (Limiting Processes).

Summary

Chapter III identifies the sources of the three types of equations in the general school resource allocation model (the objective function, operating constraints, and policy constraints). From these sources the researcher extracted the equations which constitute the model used in this research and described in the second section of Chapter III. Using an IBM MPSX360 program to solve the model, administrators can obtain volumes of information useful for making resource allocation decisions. This information is described and explained in the final section of this chapter. Chapter IV describes the first application of the model in Oklahoma Independent School District Number 30.

CHAPTER IV

APPLYING THE GENERAL MODEL

Chapter IV describes the application of the general model developed in Chapter III to Oklahoma Independent School District No. 30 (Bartlesville, Oklahoma) before policy constraints were introduced by Bartlesville administrators. Without policy constraints, the general model is optimal only in the context of meeting minimum natural and legal requirements and will be called the minimum model hereafter. Therefore, the results reported in Chapter IV should be viewed as a point of departure to which administrators add policy constraints to obtain a solution which is optimal in the context of meeting their school system's goals. As noted on page 42, the purpose of identifying this point of departure is to facilitate examination of the trade-offs in resource utilization when optimal programs are initiated.

Chapter IV has two sections. The first section, Sources of Data, identifies the data sources used in this solution; the second section, Solution, discusses the computer solution for this phase of the study.

Sources of Data

Data emanate from numerous reports prepared for external agencies and internal management and from information prepared specifically for this study.

Reports Prepared for External Agencies and Internal Management

Four report categories were utilized:

1. Reports to State Agencies
2. Reports to Internal Management
3. Reports to the School Board
4. Reports to (or from) Others

Generally these reports are statistical tabulations of either historical data or carefully supported predictions. Since a report's underlying documentation may be examined by state auditors, reported information is seldom based on purely subjective interpretations of a given school system's conditions.

Reports to State Agencies

All states require independent school districts to report statistics regarding their financial activities, curricula, attendance, transportation, etc., to one or more central agencies for approval and/or accumulation of data. This research uses three reports required in Oklahoma--the Estimate of Needs, the Application for Accrediting, and the Annual Statistical Report.

Estimate of Needs. Each independent school district prepares a School District 19X1-19X2 Estimate of Needs and Financial Statement for the Fiscal Year 19X0-19X1 under the auspices of an independent Certified Public Accountant. The Estimate of Needs is (1) filed with the County Clerk for approval by the Excise Board, (2) filed with the State Auditor, and (3) published (in part) in legal journals in the school district's home county. It includes estimates of revenues and expenses for all funds for the ensuing year and statements of revenues and expenses for the ending year. The coefficients for revenues in the current study's budget constraint came from this report.

Application for Accrediting. Each school submits an Application for Accrediting to the Accreditation Section, State Board of Education, between October 1 and October 15 every year. If a school fails to comply with this provision it may lose both state financial aid and accreditation. The following data were drawn from the Application for Accrediting of the eighteen schools in Independent District Number 30.

1. The number of students enrolled (by grade and by school)
2. The number of staff personnel employed (by school)
3. The number of faculty members employed (by grade, school, and subject)
4. The number of administrators employed
5. Professional improvement data
6. Required course data
7. Curricula data
8. Counselor-pupil ratios
9. Library expenditures
10. Physical facility data.

These data were used (1) as capacity limitations in operating constraints, (2) as bounds on bounded variables, and (3) as decision variable coefficients in various constraints.

Annual Statistical Report. The district superintendent files an Annual Statistical Report to State Department of Education for the Year Ending June 30, 19XX with the Finance Division, State Department of Education, at the end of each year. Some of the information in this report duplicates information available in the Application for Accrediting. Other information appearing in the Annual Statistical Report includes:

1. Student-days of absence and attendance
2. Transportation data
3. Summer school data
4. The number of high school graduates
5. The number of rooms used, abandoned, and added
6. The number of teaching days and professional days
7. Nonteaching staff information.

The Estimate of Needs, Application for Accrediting, and Annual Statistical Report provide most of the data for the operating constraints. In general, these data reveal facts which enable authorities to ascertain compliance with minimum requirements. Data compiled to aid district administrators in effectively employing their resources appear under the next heading.

Reports to Internal Management

School administrators receive internal reports which help them determine how to spend resources to meet patrons' expectations. These documents supply information not available in reports to state agencies because they contain (1) additional detail, (2) new information, and (3) predictions not found in reports to state agencies.

Additional Detail. The data in records maintained by schools usually contain more information than is reported to government agencies. For instance, school districts maintain files on their teachers which are the basis for state reports but which generally carry more information than required by the state. In state reports an administrator must affirm that each teacher is certified for the subject he or she is teaching. Most teachers are certified in several subjects and could be assigned to those subjects if necessary. Thus state reports show that the district complies with the laws, but they do not show the alternate teacher arrangements available to the school. Administrators need this information to plan their course offerings and recruiting efforts. Examples of other pertinent personnel data not reported externally (but used in this study) are pay scales and fringe benefits above the state minimum, and salary allocations for teaching and supervising activities.

Property inventories further illustrate the added detail available in internal reports. School districts are required to keep an inventory of their equipment and buildings. A district reports the number of classrooms and buses available to the State Board of Education, but administrators need more information for their planning. For example, what are the capacity and condition of buses? Is the maintenance staff adequate? What are the plans for retirement, replacement, and expansion of transportation facilities? Classroom facilities? The answers to such questions can be found in internal reports.

New Information. Bartlesville's internal management reports provide the following information not found (in any form) in state reports:

1. Pupil intelligence and aptitude test scores
2. Data pertaining to interscholastic competitive events
3. Data pertaining to extracurricular activities
4. Operating cost breakdowns by school, subject, object, etc.

The volume of new internal management information available in Bartlesville is probably greater than for most medium-size systems because Bartlesville leases its own computer and has effectively utilized the computer in developing its information system.

Predictions. Bartlesville has implemented a PPBS program built around the twenty-four goals listed in Appendix A. Each teacher has developed activity statements listing projects designed to achieve these goals. This study uses data on projected activity levels from these statements. Other examples of predicted conditions which were used are estimates of the number of pupils in each grade in the ensuing year and estimates of teachers available in the Bartlesville area.

Reports to the School Board

Generally this study considers the local school board as part of the management team because it helps determine system policies. However, the data source section of this research separates the school board from internal management because the board receives several special reports.

Most reports prepared for the school board contain primarily financial data. This research uses data from the following three reports:

1. General Fund Estimated Appropriations
2. Financial Statements and Reports
3. Appropriation and Encumbered Ledger.

General Fund Estimated Appropriations. The General Fund Estimated Appropriations report shows:

1. Estimated revenues from local, county, state, and federal sources

2. General fund appropriations in three broad categories--instruction, support services, and designated accounts, and
3. Estimated building fund income and appropriations.

Most of the data used in this study come from the estimated revenues section because a more detailed breakdown of the general fund and building fund appropriations is available in the Appropriation and Encumbered Ledger (discussed below).

Financial Statements and Reports. The Financial Statements and Reports summarize actual receipts and disbursements for the general fund, the building fund, the cafeteria fund, and activity funds, and the number of meals served at each school. The current study uses data from all these financial statements and reports except the building fund which is beyond the scope of this study.

Appropriation and Encumbered Ledger. The Bartlesville data processing center provides an expenditure summary, the Appropriation and Encumbered Ledger, from which most of the coefficients for expenditures in the budget constraint are drawn. This ledger classifies general fund expenditures in the following ways:

1. Expenditures by Site (classifying expenditures by location, e.g., Central Junior High, Sooner High, etc.)
2. Subject Standard Budget Summary (classifying expenditures by academic subjects, e.g., art, mathematics, general elementary education, etc.)
3. General Fund Standard Budget Summary (classifying expenditures by function, e.g., instruction, legal services, school counseling, etc.).

The total cost of each major subject category in high school and junior high school (from the Subject Standard Budget Summary) is divided by the number of sections offered (from the Application for Accrediting) to determine the average cost per section of a given subject. The resulting quotient serves as the subject's budget constraint coefficient. The writer recognizes the weaknesses of using an average cost where, ideally, a marginal cost should be used. This and other limitations of the current research are discussed in Chapter VI.

The General Fund Standard Budget Summary provides budget constraint coefficients for such variables as legal and accounting services, media area direction, speech pathology, and public relations. Most of the expenditures in these categories are fees and salaries of professional people and do not vary materially from year to year. Therefore, these expenditures and activities were treated as committed costs (see page 47).

Reports To or From Others

School districts transmit or receive miscellaneous reports from parties other than the state, internal management, or the school board. Only one report of this type is used in this study, the 1972 Community Education Council goals study. Since the goal study is discussed on page 44, no further discussion is given here.

Information Prepared Specifically for This Study

Sometimes data useful to an LP model are not contained (in usable form) in existing reports. A researcher must generate his own data under these circumstances by (1) conducting special studies,

(2) rearranging existing data, or (3) estimating data.

Conducting Special Studies

When data vital to an LP approach do not exist, they must be created before the study can progress. For instance, the Community Education Council study did not assign weighting factors to their twenty-four goals. Therefore (as described on page 45), the superintendent and assistant superintendent provided the goal weights for this study by assigning a number from 0 to 100 to each goal. Other data created for this research pertained to activity programs, e.g., the number of students involved in activities and the teachers sponsoring activities.

Rearranging Existing Data

Some data vital to an LP study exist in the reports to the state, internal management, etc., but their form must be altered to permit effective use. For instance, the average cost per section of the major subject categories required dividing the total cost of each major subject category by the number of sections offered (see pages 64 and 65). No other data rearrangements were considered necessary for this study.

Estimating Data

Frequently historical data exist in a form useful for the LP procedure, but due to changed conditions the data may not produce reliable results. For example, adequate records on school bus fuel consumption are readily available. However, since a price increase for petroleum products appeared imminent, an estimated price of 55 cents per gallon was used.

The data for this study were gathered from the sources noted in the preceding discussions, entered on the data cards prescribed by the MPSX360 program, and processed. The next section describes the output of the model before policy constraints were introduced.

Solution

The MPSX360 program displays the optimal solution to an LP problem in two major sections entitled "Rows" and "Columns." The data in the "Rows" section reveal the model's conformity to constraints and sensitivity to the changes in the constraints; the data in the "Columns" section reveal the variables to which resources are assigned, the quantity of resources assigned, and the sensitivity of the model to variations in the assigned quantities. The two parts of this section parallel the MPSX360 program display: Constraint Analysis and Variable Analysis.

Five general considerations deserve mention before the above parts are discussed:

1. The term "Unit Cost" was discussed on pages 52, 53, and 55. Those discussions yield the following definition: Unit Cost is the aggregate goal weight change related to (1) changes in a variable's activity level, or (2) changes in the capacity limitations of constraints. A Unit Cost can be a positive cost and thus reduce the aggregate goal weight of the model; or it can be a negative cost and thus increase the aggregate goal weight of the model. Since the term "Unit Cost" is used in the MPSX360 printout, it has been used up to

this point in this research. However, continued use of this terminology could be a source of confusion throughout the remainder of this study because a negative quantity increases goal weights and a positive quantity decreases goal weights. To avoid this confusion, a goal weight increase or decrease will be described as a goal weight gain or loss, respectively, throughout the rest of this study.

To further clarify the terms "positive cost" and "negative cost," consider the following:

(1) A basis variable's "Unit Cost" is always positive because any change in the variable's activity level decreases the aggregate goal weight of the solution (a necessary condition for optimality explained on page 51).

(2) A "stopped" resource's "Unit Cost" is negative when the limits on the resource are relaxed, because the new capacity limitation allows more goal weight producing activity. For example, if the minimum number of students a school district must transport daily is lowered from 1,200 to 1,000 (assuming the model suggests busing only the minimum number of students, i.e., is "stopped" at the lower limit), resources will be released for application in other school system activities whose benefit/cost ratio (defined on page 70) exceeds that of busing. Therefore, instead of costing the system goal weights, relaxing the transportation constraint will earn the system goal weights. Hence, the term "negative cost."

(3) A "stopped" resource's "Unit Cost" is positive when the limits on the resource are tightened, because the new capacity limitation diverts resources away from high benefit/cost-ratio activities to low benefit/cost-ratio activities. In the transportation example above, suppose the minimum number of students the school district must transport daily is increased from 1,200 to 1,300. In this case the additional resources consumed by busing activities will be drawn from activities with higher benefit/cost-ratios, reducing the solution's aggregate goal weight.

2. The sum of the goal weights in the optimal solution of the minimum model is 7,921,388. The amounts of the gains or losses in the rest of this chapter will be added to or subtracted from the current aggregate of 7,921,388.

3. An analysis of each variable and constraint individually is not practical because of the size of the model. Furthermore, the

activity and sensitivity of variables and constraints comprising broad categories do not differ greatly, one from another. Therefore, the solution is described herein in terms of broad categories unless extraordinary conditions warrant more specific discussion.

4. Frequently a variable is constrained to a range of values above zero. For instance, a school system must offer drug abuse classes in all schools. Thus, Bartlesville must offer a minimum of 18 drug abuse classes. Constraint equations could be written for the range limitations, but placing a lower and an upper bound on the variable is a more economical method of accomplishing the same result (see Appendix B).

Bounding techniques have been used frequently in this study. For reasons explained in Appendix B the effects of bounding are reported in the Variable Analysis sections as well as the Constraint Analysis section.

5. Some efforts by a school system make smaller demands on the system's resources than others. For example, the tuition for summer school courses offsets some of the program's costs. Thus, offering a course in the summer has a lower net financial resource drain than offering the same course during the regular term. Another example arises from group activity courses such as physical education and instrumental music which have larger class sizes than academic courses such as English and math. Group activity courses place lower demands on teacher and physical plant resources than academic courses. Each of these activities generates the amount of goal weights that was assigned to the activity by the administrators (see page 66). The ratio of the goal weights generated to the resource demands of an

activity is the primary determinant of the activity's entry in the optimal solution. Use of the term "benefit/cost-ratio" in subsequent sections of this research refers to the goal-weights-generated-to-resources-demanded ratio for the variable under consideration.

Constraint Analysis

Several terms defined elsewhere in this study will be used in the Constraint Analysis. For the reader's convenience, the definitions are repeated here.

1. The amount of a resource used in the optimal solution is its Activity Level. The unused amount of the resource is its Slack.

2. The net increase (decrease) in aggregate goal weights which would result from a change in the consumption of a resource is its goal weight gain (loss).

3. When a resource is being consumed at an externally imposed limit, it is stopped. When a resource is being consumed between externally imposed limits, it is basis.

4. The activity range over which goal weight gains or losses are valid is the relevant range, sometimes called Lower Activity/Upper Activity.

5. The resource or variable in the model which reaches an upper or lower limit when the relevant range is exceeded is the limiting process, also called the endogenous influence.

All the constraints in the model can be associated with one of the following ten constraint categories: Financial Resources, Required Physical Education Courses, Pupil/Teacher Ratios, Student Teacher Programs, Junior High School Curriculum, High School Curriculum, Summer School Programs, Interscholastic Competitive Activities, Teacher Utilization, and Transportation.

Financial Resources

From the budget equation on page 47 it is obvious that the model will produce a nonnegative financial result. Under some circumstances deficit spending is legal for general operations,[1] but the model in this research was designed for a debt-free program.

In this study, all the financial resources are consumed. For each additional available dollar, the model will generate a goal weight gain of .85 weights over the next $168,826. The upper Limiting Process for the budget is the number of students in the adult grade school achievement course, i.e., the $168,826 will go into adult grade school achievement programs. One cannot determine what would happen if more than $168,826 were available, because the upper Limiting Process reaches a limit at that level (see page 54 for a complete discussion of this process).

Required Physical Education

The School Laws of Oklahoma require all students to take one physical education class each year. The model offers just enough physical education courses to meet this constraint at the lower level with a fairly high goal weight gain (1441 units) for each section that can be eliminated. Total physical education credits allowed by law

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[1]General operations are activities charged to the general fund as opposed to activities charged to the general fixed assets group of accounts, general bonded debt and interest fund, debt service fund, etc.

toward graduation from high school (two per student) are the limiting factor for such courses in high school. Adult education courses are the limiting factor for physical education in junior high and elementary schools. The model suggests meeting the legal requirements for physical education by getting students involved in interscholastic sports (see Interscholastic Competitive Activities on page 74).

Pupil/Teacher Ratio

The elementary pupil/teacher ratio prescribed by the model is "basis" at 29 to 1, less than the legal maximum of 35 to 1. The loss attached to deviating from this ratio is moderate, 191 goal weights. Thus, if Bartlesville administrators hire enough teachers to move the ratio to 28:1, the aggregate goal weights will decline by 191 units.

Kindergarten and secondary school pupil/teacher ratios have reached their upper legal limits of 25 to 1 and 32 to 1, respectively. Forcing any lower ratio on the system results in an infinite goal weight loss, i.e., an infeasible solution, because the model has employed all the secondary teachers available in Bartlesville.

Student Teacher Program

The student teacher program is constrained by the number of cooperating supervising teachers. In the minimum model all of these teachers are engaged in student teacher supervision with a gain of 168 goal weights for each additional supervising teacher added to the project.

Junior High Curriculum

The law requires a junior high school to offer three courses in English and math, and one course in United States history and laboratory science. These constraints are met at the minimum level. Nonrequired subjects such as social science and industrial arts are not in the minimum optimal solution because their benefit to cost ratio is low and because no minimum number of credits in junior high is prescribed by law. The goal weight loss from additional required curriculum courses is low (about 55 goal weights), but it is fairly high (between 1,576 and infinity) for industrial arts and social science classes. The adult grade school achievement program is the limiting process for required courses which, in turn, limit nonrequired subjects.

High School Curriculum

Oklahoma school law is more explicit about the high school curriculum than the junior high school curriculum. It requires 18 credits per student in high school with minimum offerings of English, math, science, foreigh languages, physical education, social studies, and applied vocational courses. The foreign language, fine arts, and applied vocational course constraints are met at the minimum legal level with goal weight losses for additional sections of about 1,500 units. The remaining high school curriculum constraints are basis with an infinite goal weight loss for reduction in offerings and about a 15,000 goal weight loss for an increase in offerings. The limiting processes for required course constraints are the courses in the constraints. Adult grade school achievement activities limit nonrequired courses.

Summer School Programs

According to the model, all resources committed to summer school programs should be for office occupations courses. However, those courses are quite sensitive to input cost changes, and any of the other vocational-technical subjects can substitute for office occupations at a slight goal weight loss. Therefore, it seems acceptable to broaden the interpretation of the model by saying that all resources committed to summer school programs should be for vocational-technical courses.

Interscholastic Competitive Activities

The model prescribes minimum activity levels (usually zero) for resources committed to secondary school interscholastic competitive activities. In the light of the Required Physical Education section on page 71, this suggests that graduation credits earned in interscholastic competition must be in self-supporting sports. The goal weight loss from forcing nonself-supporting activities into the solution is prohibitive because resources must be drawn from high benefit/cost programs.

Teacher Utilization

The minimum model employs every available teacher. The goal weight gain for each additional teacher is 188. The teacher/pupil ratio is the limiting factor at the lower limit for teachers employed.

Transportation

The model calls for transporting the minimum number of students with a high goal weight loss for each additional student transported.

Increased transportation activities would draw vital resources away from required high school courses, the upper limiting factor.

Variable Analysis

The five definitions at the beginning of the Constraint Analysis discussion apply here except that a variable has no slack activity. One additional definition applies to variables: the responsiveness of a variable's activity level to changes in its goal weight (Input Cost) is the variable's Input Cost Sensitivity.

All the variables in the model can be associated with one of the following six categories: Extracurricular Activities, Athletic Programs, Curricula, Ancillary Services, Revenues, and Personnel.

Extracurricular Activities

Extracurricular activities and elementary school excursions are basis in the minimum model solution. The benefit/cost ratio for these programs is high in comparison to the same ratio for athletic and academic programs. Consequently, many resources, especially financial, are committed to extracurricular activities and excursions in preference to athletic, academic, special education, and ancillary service[2] programs. The goal weight loss for activity level changes in extracurricular activities and excursions is high and both variables are insensitive to declines in the goal weights assigned to them by administrators (Input Cost).

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[2]An ancillary service program is one which helps accomplish the primary education mission of the school system, but is not indispensible, e.g., guidance and testing, cafeteria, transportation, and audio visual programs.

Athletic Programs

The activity levels for athletic program variables are at the lowest level allowed by the lower bound constraints. Small to substantial gains (between 28 and 1,600 goal weights) arise if these limits can be relaxed. The limiting factor for every athletic program is the adult grade school achievement program. Athletics are fairly insensitive to changes in their Input Cost; the narrowest range of variation in Input Cost before activity levels change is from infinity to 170 goal weights.

Curricula

Elementary School Courses. The activity levels for elementary school courses are at the lowest limit allowed by their lower bounds. For each of these variables, the limiting factor is the adult grade school achievement program. About 1,350 goal weights can be gained by relaxing the lower limits by one unit.

Required Secondary School Courses. Required courses (i.e., English, math, foreign languages, natural sciences, and Oklahoma and United States history) are basis in junior high and stopped at their lower limit in high school. This result might appear to conflict with the results in the constraint section just preceding where junior high curricula constraints are stopped and high school curricula constraints are basis (see page 73). To resolve this conflict, consider the following: The law requires a minimum total number of courses to be offered in junior high and in high school; this requirement produces the curriculum constraints. The law further requires a minimum number of courses in certain subjects; this requirement produces lower bounds (constraints) for variable activity levels. In junior high the sum of the lower bounds on variables is less than the minimum total number of courses in the curriculum constraint. Hence, variables (courses) are basis and constraints (total curricula) are stopped at the lower limit. The opposite condition holds in high school. The sum of required courses (lower bounds) exceeds the minimum total course requirement (constraints). Thus, high school constraints are basis and variables are at their lower limit.

Summer School Courses. Based on conversations with Bartlesville administrators, the researcher estimated that 800 students would be available for summer school (but not necessarily enrolled). Since a student can take only one course in summer school each year, the 800 students became the limiting factor for all summer school courses. As described on page 74 all these students were assigned to vocational-technical programs. Consequently the model offers no academic courses in summer school, and forcing them into the solution results in moderately high losses (around 300 goal weights).

Tuition payments of summer school students increase the benefit/cost ratios of academic summer school courses. However, relative to other optional programs summer school courses have low ratios and are, therefore, insensitive to Input Cost changes.

Correspondence Courses. Correspondence courses are not offered. However, their goal weight loss is quite low (under 50 goal weights) and they are very sensitive to increases in their Input Cost because their benefit/cost ratio is fairly high in relation to other optional programs.

Vocational Courses. According to Oklahoma laws, school districts must offer a minimum of twelve vocational credits. All twelve credits are allotted to vocational agriculture, health occupations, technical education, and trade and industrial courses in the minimum model. If more sections of these courses are forced into the solution, the goal weight loss is high (about 1,700). In addition, these courses are very sensitive to downward changes in their Input Cost.

Other vocational courses, home economics, business, office occupations, distributive education, and journalism, enter the solution at their lower limit, usually zero, and are comparatively insensitive to Input Cost changes. Home economics, business, and office occupations cause fairly low goal weight losses when their activity is increased (46 to 188 goal weights), whereas distributive education and journalism courses cause high goal weight losses when their activity is increased (1,500 to 2,100 goal weights).

Finally, with respect to vocational programs, the model solution indicates that Bartlesville should rely heavily upon the area vocational-technical school to provide vocational courses by suggesting that 1,943 credits should be earned at the area vocational-technical school and transferred to Bartlesville high schools. This policy is virtually insensitive to changes in the Input Costs, because the benefit/cost ratio for area vocational-technical school courses is greater than that ratio for in-house courses.

Special Education Programs. Special programs for the handicapped children are at their lower limits in the minimum model. The loss from increasing these programs is high, over 7,500 goal weights. Special programs for gifted children, on the other hand, are at their upper limits with a slight goal weight gain resulting from program increases. Support activities such as special transportation of exceptional children, work-study cooperative programs, etc., also carry slight goal weight gains from increased activity.

Ancillary Activities

In every case, ancillary activities are offered at their lower limit with moderate to extremely high goal weight losses from increased activity. The losses for increasing the guidance and testing programs by one unit exceeded 125,000 goal weights; and for increasing the learning resources center (library and audio-visual facilities) losses exceeded 60,000 goal weights. All ancillary activities are insensitive to Input Cost changes because their benefit/cost ratios are approaching zero.

Revenues

Revenues, of course, are at upper limits, and they carry tremendous goal weight gains if the limits can be relaxed. For example, if ad valorem taxes could be doubled, the Bartlesville School System could increase their aggregate goal weights by 1,684,717.

Personnel

The model affords modest goal weight gains for each additional student. Teachers are the limiting factor for the number of students to be added to the system by way of the student/teacher ratio constraint.

Teachers' aides and nonteaching professional personnel, such as school nurses, accountants, doctors and speech pathologists, all enter the model at minimum levels with (1) virtually no sensitivity to Input Cost changes and (2) high goal weight losses attendant to increased activity levels.

Summary

Chapter IV has described the initial application of the general model developed in Chapter III by, first, outlining the numerous sources of data which were used in the LP model, and second, detailing the wealth of information produced by solving the model using the IBM MPSX360 program. The model applied here (the minimum model) does not include policy constraints. Therefore, several worthwhile programs (e.g., academic courses in summer school) are not included in the solution. Chapter V describes the second application of the model (the optimum model), in which school administrators insert optional programs, i.e., those not required by state laws or nature.

CHAPTER V

POLICY CONSTRAINTS OF THE BARTLESVILLE SCHOOL SYSTEM

Chapter V describes one of the most important phases of this research, the administrator's imposition of optional programs on the model. The model developed in Chapter III and employed in Chapter IV merely insures that the school system's operations comply with school and natural laws. Using policy constraints administrators can add any programs to Chapter III's minimum model as long as the additions do not cause the system to violate any of the legal or natural constraints. These additional programs presumably help the school system achieve its long-range goals; hence the Chapter V model is optimal in the context of the school system's defined goals and hereafter will be called the optimum model.

Chapter V has two sections. The first, Model Alterations, describes (1) optional restructuring of the minimum model, and (2) policy constraints imposed by the Bartlesville administrators. The second, Solution, examines the _differences_ in the solution of the minimum model and the optimum model, i.e., the impact of the administrators' alterations.

Model Alterations

The _School Laws of Oklahoma_ provide numerous options in the operating and financial management of a school system. For example, a school system can either buy or lease school buses. If a researcher designs the model around one option, but administrators follow another, it may be necessary to restructure the model slightly to reflect the operations of the system being modeled.

The first part of this section, Restructuring, discusses a budgeting option assumed in the minimum model which was changed to reflect procedures followed by the Bartlesville system. The second part, Policy Constraints, describes policy constraints suggested by Bartlesville administrators. The third part, Qualifying Policy Constraints, considers a limitation to the optimum model reported in this research.

Restructuring

Title 62 of the Oklahoma Statutes, § 335, states that:

> When money is due any county, city, town, or school district in this State from sale, lease, or rental of any public property, or royalty, or for compensation for service of public employees or other purpose, it shall be paid over to the lawful treasurer thereof.
>
> The governing board shall have authority to direct by written resolution duly entered in the minutes of its meeting at the time such money is received or prior thereto that such money shall be credited to the fund account from which such property was derived or from which payment has been or will be made for such services rendered or other purposes.
>
> If there be no resolution by the governing board directing the disposition of the money received as contemplated herein it shall be the duty of the treasurer to credit such money so received to the general fund.

In accordance with this provision a school district may credit the proceeds of athletic events and other activities to the general fund or to a special fund. The minimum model in Chapter III reflects the general fund option. However, Bartlesville credits a special fund restricted to student activity programs. Accordingly, the minimum model was restructured to give effect to the option followed by Bartlesville. The effects of this modification are examined in the Solution section on page 85.

Policy Constraints

As pointed out in Chapter IV, many worthwhile programs (especially academic courses) are left out of the minimum model's solution either because their benefit/cost ratio was lower than the same ratio for other optional programs[1] or because state laws did not force the program into the solution. To develop the optimum model, administrators impose policy constraints prescribing minimum (or maximum) activity levels for programs which they believe are desirable (or excessive). The following four policy constraints were injected into the model in this research: Foreign Languages, Music and Art, and Industrial Arts courses in junior high school; and Vocational-Technical courses in high school.

Foreign Languages. State law does not require school districts to offer foreign language credits in junior high school, but Spanish and French courses are suggested in the *Annual Bulletin* (1974, p. 39). Bartlesville offers French, Spanish, and Latin to ninth graders in both junior high schools (*Application for Accrediting*, 1974-75, p. 4). Since no students enrolled in French in 1974-1975 the lower limit policy constraints for foreign language courses was set at two courses.

[1]For a description of the benefit/cost ratio's influence on a variable's entry into the solution, see page 69.

Music and Art. State law distinguishes music and art theory from music and art laboratory classes. In this research music and art theory are presumed to be integrated with the physical science classes. Thus the music and art policy constraints injected into the model are for laboratory sections, i.e., band, orchestra, chorus, and art. The constraints require three music laboratory courses and one art laboratory course per grade.

Industrial Arts. The Annual Bulletin (1974, p. 40) suggests (but does not require) that a school system offer several industrial arts courses in junior high school. Bartlesville offers industrial arts activities in a staggered pattern that enables a policy constraint of two courses per year to fulfill their requirements. Thus a lower bound of two was placed on the industrial arts variable in the optimum model.

Vocational-Technical. The law requires school districts to offer twelve vocational-technical courses in high school. Agriculture, health occupations, technical education, and trade and industrial classes satisfy the twelve course requirement in the minimum model (see page 78). To distribute the subjects covered more evenly and to offer several other useful programs, administrators imposed a policy constraint requiring the offering of two credits in each of the following courses: vocational agriculture, business, distributive education, health occupations, home economics, office occupations, technical education, and trade and industrial occupations.

Qualifying Policy Constraints

The Bartlesville administrators were most generous with their time and other resources in this study. Nevertheless, given additional time to understand the MPSX360 printout in greater depth, they probably would have injected numerous other policy constraints into the model. Accordingly, the term "optimum model" should be interpreted in relation to this qualification.

Solution

The first part of this section examines the effects of the budget restructuring for student activity funds. The second and third parts, Constraint Analysis and Variable Analysis, discuss the differences in the solutions of the minimum and optimum models (defined on pages 58 and 81 respectively) with respect to activity levels, goal weight gains and losses, relevant ranges, and limiting factors for both constraints and variables, and Input Cost sensitivity for variables.

Effects of Restructuring

As a result of the athletic and student activity budget being separated from the general fund budget (see page 82), the model exhibited the following three characteristics:

(1) An Additional Budget Constraint

(2) Increased Sensitivity

(3) New Limiting Factors.

An Additional Budget Constraint. The following constraint was inserted into the model establishing a separate student activity budget:

Σ Receipts from Student Activities + Σ Contribution from the General Fund (if any) - Σ Student Activity Expenses $\geq$ 0.

All school systems account for the proceeds and expenditures of students' entrepreneurial efforts, such as car washes, in an Activity Fund. The money in the Activity Fund is raised primarily as a result of student planning and effort. These funds are beyond the scope of this study because the school administration's authority in regard to these funds is fiduciary and not managerial.

In contrast, the money contemplated in the new budget constraint equation results from activities organized primarily by the school system such as ticket sales at athletic contests. Although some fiduciary overtones exist in accounting for the receipts and disbursements of such activities, management control aspects predominate. The new budget equation restricts receipts and disbursements to student activities; otherwise it functions in the same manner as the general budget equation given on page 47.

Increased Sensitivity. In general the model became more responsive[2] to changes in the quantity of available resources and the activity levels of variables when the student activity budget was separated from the general budget. The following observations support this statement:

(1) Many goal weight gains or losses for constraints and variables increased more than 2,000 percent in the restructured model solution. For example, the loss for an additional high school football game was 1,558 goal weights before splitoff and 28,257 afterward.

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[2] A model is more responsive to changes in a given variable than another model when (1) identical activity level changes cause greater goal weight gains or losses, (2) the ranges over which the goal weight gains or losses are valid decrease, or (3) the upper or lower Input Costs are closer to the goal weights assigned by the administrators.

(2) The relevant ranges usually decreased from 10 to 75 percent. For instance, the range over which the number of high school math sections could vary before the goal weight losses (or gains) from one more (or less) section would change was 82 sections before separation and 52 sections afterward, i.e., the relevant range decreased by 30 sections (37 percent).

(3) Variables included in the student activity budget were less sensitive to Input Cost changes after the budgets were separated, but most other variables showed little change in Input Cost sensitivity.

The following analysis explains the restructured model's increased responsiveness: Some funds which initially could be applied to projects in the order of their descending benefit to cost ratios were restricted by the restructuring to pay for programs with lower ratios. Hence use of the remaining unrestricted funds became even more critical.

New Limiting Factors. Before the student activity budget was separated from the general budget, the number of students in activities was the limiting factor for only one constraint, and the adult grade school achievement program was the limiting factor for sixteen constraints, including the budget. After separation, the former was the limiting factor for thirteen constraints and the latter for twelve.

The budget affects more variables than any other constraint and its limiting factor dominates the rest of the model. Thus one would expect earmarking financial resources for special programs to produce a dominant limiting factor for the special budget. Restructuring caused the number of students in activities to dominate the activity segment of the model while the adult program continued to dominate the rest.

In addition to the new budget equation several policy constraints were added to the model (see pages 83 to 85). The effects of those policy constraints on the other constraints and the variables are described in the remainder of this chapter.

Constraint Analysis

As a result of the policy constraints inserted by the Bartlesville administrators, the optimum model's solution exhibited many values which differed from the minimum model's solution. Most of these were slight (less than 5 percent) and probably would not influence an administrator's decision process. However, in this researcher's opinion, the six differences discussed below are large enough to alter an administrator's behavior.

The Adult Grade School Achievement Program. The demand that policy constraints place on resources consumed by both the adult program and policy constraint programs (especially financial resources) cuts the adult program approximately in half. As noted on pages 71 and 87, the limiting factor for the minimum model budget constraint is the number of students in the adult grade school achievement program. Every time a program requiring financial resources is forced into the solution by a policy constraint, it draws funds away from the adult program, thereby reducing the number of adult students accommodated. For example, a junior high school band section costs $3,335. For each additional band section forced into the solution, the funds available for the adult program diminish by $3,335.

Introduction of policy constraints has another impact on the adult program. The goal weight loss for increasing the adult program quadrupled while the relevant range was reduced by 97 percent. In other words, increases in the adult program drew resources from programs with much higher benefit/cost ratios, and fewer nonrequired programs were available from which the adult program could draw funds. Both of these changes were comparatively large, and they emphasize the impact of the policy constraints' claim on resources originally allotted to the adult program.

The Number of Students in Activities. Due to restructuring, the number of students in activities was cut from 13,398 to 7,199.[3] The minimum model's student activity program drew financial resources from the general fund commensurate with its benefit-to-cost ratio. As a result of restructuring the student activity program became self supporting, i.e., it could not draw funds from the general fund. Consequently, the student activity program and the number of students in activities were sharply curtailed.

In both the minimum and optimum models the number of students in activities is "basis." Deviating from the optimum model level (7,199) in either direction results in fairly high goal weight losses.

Junior High School Subjects. The minimum model contained variables for junior high school courses in foreign languages, music and art, and industrial arts; but, due to their low benefit/cost ratios, these courses entered the solution at a zero activity level. For reasons explained in Appendix B, the researcher used lower bound constraints to force the activity of these variables to the desired level as follows:

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[3] A student can be in more than one activity. Consequently, even though Bartlesville has only 6,699 students, it can have more than 6,699 students in activities.

Foreign Language Courses $\geq$ 2

Music Laboratory Courses $\geq$ 3

etc.

Further discussion of the impact of the lower bound constraints appears in the section entitled Junior High School Courses on page 92.

Total Credits. The law requires every student to earn at least 18 credits in high school, including 3 1/2 credits earned in the ninth grade.[4] After the policy constraints were imposed on the model, the total number of credits earned increased slightly reflecting the enrollment in new courses. The goal weight gains and losses for the total credit constraint remained the same while the relevant range declined from 54,446 credits to 53,156 credits, reflecting the diversion of resources from extracurricular activities to the new courses.

Applied Vocational Courses. Among the subjects forced into the high school curriculum were four additional vocational-technical courses. (The minimum model offered twelve courses.) Since the total high school vocational-technical course constraint calls for a lower limit of twelve classes, forcing in four more causes the constraint to operate at a level between its lower limit of 12 and its undefined upper limit, i.e., to become "basis." The vo-tech course constraint exhibits no change in its goal weight gains and losses or in its limiting factor (the adult program) as a result of the added courses, but its relevant range decreases slightly. This means that any further increases in the vocational-technical sections offered will continue to draw resources

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[4]A credit is two semesters' work in a given subject. Thus, Oklahoma history (a one semester course) is half a credit, whereas United States history (a two semester course) is a full credit.

from the adult program, but fewer resources remain to be drawn

Teachers Sponsoring Extracurricular Activities. Each student activity must be sponsored by at least one teacher. Therefore, the number of student activities is constrained by the number of sponsoring teachers. The drop in the number of students in activities (see p. 89) is accompanied by a reduction of teachers sponsoring activities to about one-third the original level. At the same time the loss related to further reductions jumps from 31 goal weights to infinity and the relevant range declines by over 70 percent. The infinite goal loss means that any further reduction in the use of resources in this constraint will violate the constraint and cause an infeasibility condition. Therefore, the number of teachers sponsoring extracurricular activities is at its lowest feasible level.

Variable Analysis

This section discusses important differences between the minimum and optimum models' activity levels, goal weight gains and losses, relevant ranges, limiting factors, and Input Cost sensitivities for four specific variables--Student Activities, Junior High School Courses, Vocational-Technical Courses, and Adult Grade School Achievement Courses. The differences in other variables are slight and are not likely to affect the administrator's decision. On page 69 this study noted that bounding techniques cause an overlap in topics covered in Chapter IV's Constraint Analysis section and the Variable Analysis section. Again bounding techniques have caused some variables to be reported as constraints and variables simultaneously, thus accounting for the overlap in topics covered in this section and the one immediately preceding (Junior High School Courses, for instance).

Student Activities. The optimum model solution calls for fewer students in every extracurricular activity except athletics (which are already at their lower limit in the minimum model). The budget restructuring described on page 82 limits all student activities through their new budget which, in turn, is limited by the *number* of students in activities.[5]

Two important changes in athletic activity values deserve notice. First, the goal weight losses from forcing more athletic events into the optimum model solution are from 20 to 90 times as great as goal weight losses in the minimum model. Second, although athletic events are much less sensitive to Input Cost changes in the optimum model solution than the minimum model solution, they basically are insensitive in either model.[6] These value changes reflect the impact of restructuring and policy constraints on student activity programs.

Junior High School Courses. The foreign language, industrial arts, and music and art subjects forced into the solution by policy constraints carry high goal weight losses over a medium relevant range if more sections are injected into the solution. Their upper limiting factor is the adult program. The new courses are insensitive to Input Cost

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[5] Student activities are variables which are constrained by the number of students in activities. State law limits the number of activities in which students can participate to two per day (*Annual Bulletin*, 1974, pp. 34, 50, and 73).

[6] The goal weight assigned to athletic programs by Bartlesville administrators is 142. Goal weights for athletics would have to increase to about 500 in the minimum model and about 5,000 in the optimum model before the solution of the two models would change. One could say the optimum model is one-tenth as sensitive as the minimum model, but actually both models are insensitive.

changes, requiring a goal weight increase from the current 192 up to 1,580 before current activity levels change.

These courses entered the minimum model at zero activity level due to their low benefit/cost ratio. Even when variables have a zero activity level, they have goal weight gain or loss values. The minimum model solution goal weight gain or loss values for the junior high school subjects did not change in the optimum model, but their relevant ranges were much shorter. The explanation for these conditions is that these variables continued to draw resources from the same limiting factor, but fewer resources remained.

Vocational-Technical Courses. Ordinarily goal weight loss increases are accompanied by relevant range decreases, but forcing four new vo-tech courses into the high school curriculum caused the goal weight losses related to additional courses to increase from 50 percent to over 4,000 percent while the relevant range increased by about 150 percent. The explanation for this unusual behavior is as follows:

In the minimum model solution vocational agriculture, health occupations, technical education, and trade and industrial courses consumed all resources which were allocated to vocational-technical programs to fill the twelve credit minimum constraint (see page 78). If any one of these subjects had twelve sections, it would drive all the other vocational-technical subjects to their lower limit of zero. Thus twelve units was the upper end of the relevant range for any vocational-technical course, and, due to the nonnegativity constraint, zero was the lower end. In short, the limiting factor in the minimum model for any vocational-technical course was any other vo-tech course.

On the other hand, in the optimum model the sum of the lower limits for all vo-tech courses equals sixteen, four more than the state requirement of twelve. Therefore, increasing any vo-tech subject draws resources from some nonvocational-technical subject (specifically, the adult program). Since it takes more sections of vocational-technical courses to draw all the resources from the adult program in the optimum model, the relevant range has increased. Moreover, the benefit to cost ratio for the adult program is greater than for the vo-tech courses. Hence, the goal weight losses from forcing another vo-tech course into the solution are greater in the optimum model than in the minimum model.

Basically, then, the reason for the unusual behavior of an increasing goal weight loss accompanied by an increasing relevant range is a limiting factor change in which the new limiting factor has a higher benefit/cost ratio than the old and a longer scale of common resources.

Number of Students in the Adult Grade School Achievement Courses. Since the adult program is the limiting factor for many variables (see page 88 and the section immediately preceding, for example) and thus has broad exposure to the activity levels of those variables, one would expect it to be quite sensitive to policy constraints. These expectations are confirmed. First, the number of students in the optimum model solution is about half the number in the minimum model solution, and second, the relevant range has been cut by 99 percent.

The adult program is basis in both solutions, and, therefore, a goal weight loss attaches to any change (increase or decrease) in its activity level. The loss related to allowing fewer students into the program is 88 and 18 goal weights in the mimimum and optimum models respectively, and for allowing more students the loss is 47 and 166 in the minimum and optimum models respectively. Relative to goal weight losses for activity changes in other programs these losses are small implying that this program might be the one which administrators could consider manipulating to meet other goals.

The output of the school resource allocation model contains a wealth of information similar to the data discussed in this section. The user must take care not to apply unjustified interpretations to this information. Accordingly, Chapter VI identifies some limitations of the LP approach so that users can avoid interpretation errors.

Summary

Once the general model described in Chapter III has been developed, administrators can alter the model in pursuit of the overall goals of the school system. The first section of Chapter V explained two classes of model alterations made by the Bartlesville administrators--restructuring and policy constraints. The last section of Chapter V described the effects the above alterations had on the solution of the model.

CHAPTER VI

LIMITATIONS OF THE LINEAR PROGRAMMING APPROACH

Many writers have recognized the inadequacies of surrogating a real life condition with a mathematical model. For example, Hadley (1968, p. 2), states:

> An important thing to realize is that it is essentially never true that the nature of the real world can be described with complete accuracy in a model. Certain approximations must always be made. The nature of these approximations can vary widely with the circumstances. Whether or not a given approximation can be considered valid depends on the accuracy needed in the results. One of the most difficult tasks in constructing models is deciding what are realistic and allowable approximations to make.

The model developed and applied in this research has required several approximations. The three sections of this chapter--Modeling Limitations, Data Limitations, and Implementation Limitations--recognize the limitations of approximating actual school operations with a linear programming model and discuss steps that can be taken to mitigate or avert them.

Modeling Limitations

The conclusions one draws from the output of any model can be no stronger than the model itself. Ideally any public school resource allocation algorithm will insure decisions leading to goal achievement; but, like other modeling techniques, linear programming fails to reach this ideal. Failures can generally be traced to one of the following modeling errors: Inappropriate Formulation, Inadequate Formulation, Improper Interpretation of Results.

Inappropriate Formulation

The LP algorithm assumes (1) constrained activities, (2) linear relationships, and (3) a static environment. When the system being modeled fails to meet any of these assumptions, the possibility of error arises. The severity of the error is generally a function of the inappropriateness of the assumption.

Constrained Activities. A school resource allocation problem is not likely to fail the constrained activities assumption. In fact, many activities are restricted on both the lower and upper level. For example, a school system must employ at least one teacher for every 160 students in high school (a lower limit). It may employ more teachers until it exhausts its least abundant resource, e.g., classrooms, certified personnel, or funds (an upper limit).

Linear Relationships. Few real world relationships in public schools are precisely linear, but the linearity assumption is often tolerable over a given range. For instance, the marginal cost of any given subject may closely approximate a linear function of the number of students enrolled over a range from one to six sections. However, since the law forbids a teacher meeting more than six classes each day (Annual Bulletin, 1974, p. 71) a new teacher must be added for each additional block of six sections, making that portion of the cost of the subject a stepped function, at six-section intervals.

The LP algorithm often produces useful information even though some relationships are nonlinear because the error over a specified range is immaterial. Furthermore, statistical techniques such as regression can estimate the incremental change (i.e., marginal cost) and compute the probability that the estimate falls within a predesignated interval around the true incremental change. When an administrator uses such estimates, he knows the level of risk he is taking and can adjust his decisions accordingly.

If the errors associated with nonlinearity appear to invalidate conclusions drawn from an LP model, a modeler may find one of many existing nonlinear programming techniques acceptable. Taha (1971, Chapter 17) describes algorithms for separable, stochastic, quadratic, and geometric programming. However, the assumptions upon which these algorithms are based are often more restrictive than linear programming assumptions, thus limiting their utility.

The model developed in this research is inhibited if one or more of the constraints fails the linearity assumption and such failure cannot be mitigated by (1) allowing for relatively inconsequential errors, or (2) employing nonlinear programming algorithms. Based on experiments with the Chapter III and Chapter V models the researcher believes administrators will not find linearity failures which render the LP algorithm useless.

Static Environment. Once constructed, an LP model assumes invariant relationships. For example, the model may prescribe one principal and one vice principal for every secondary school without allowing for unexpected vacancies in these jobs. As another example the model may not allow for one-time bargain purchases of supplies or services consumed by the school. The static environment assumption is too restrictive to be met in the dynamic circumstances of a public school system. However, violations of the assumption are probably not that critical. In the first place, most violations are likely to be inconsequential in the long run. Considering the principal or vice principal vacancy example, few schools (let alone an entire school system) would have to completely restructure operations due to an unexpected vacancy. In the second place, when violations of the static environment assumption are substantial (as a result of a natural disaster, for instance) the model can generally be redesigned to respond to new conditions. In fact, quick, accurate, and significant decisions are most urgent in emergency conditions, therefore, an LP model may be more helpful in emergencies than under normal conditions.

Apparently the real life conditions of a public school system do not conform to two of the three assumptions of the LP algorithm. However, these failures do not seem to be so detrimental as to render the model useless.

Inadequate Formulation

The researcher believes the most insidious problems of the linear programming approach to public school resource allocation arise from omissions of essential relationships from the model. Since omissions are not readily detectible, users cannot allow for output errors spawned by them. Even worse, an incomplete model may produce misleading results. The following incident which occured in the later stages of this study emphasizes the importance of a complete model:

The *School Laws of Oklahoma* (1974, pp. 187 and 197) state that issuing warrants in excess of income and revenue provided or accumulated for the year is unlawful. It follows that a school system is constrained by the amount of revenues received in a given year or carried over from prior years, and borrowing funds for general operations is an unlawful amelioration of that constraint. Consequently, prudent administrators will exclude anticipated revenue from their budget for the ensuing year when the probability of receiving it is low. When such revenue (sometimes called soft money) is received, it either forms a padding against unexpected expenditures or enables the board of education to initiate optional programs.

Data used in this research indicated that Bartlesville would have a $4,976,758.22 budget for 1974-75. During the year Bartlesville received $270,049.94 in soft money, a fact initially unknown to the researcher. The optimum model solution using the lower budget indicated that undertaking all of Bartlesville's optional programs would be infeasible--a misleading indication. When the soft money oversight was discovered and the model corrected, the model produced the results reported in Chapters IV and V.

Avoiding inadequate model formulation requires the modeler to communicate regularly with administrators and to attend to minute details. As long as the model has a bounded, feasible solution, the LP algorithm contains no internal checks to assure complete formulation. Therefore, the planning and development stages for the model are crucial.

Improper Interpretation of Results

The preceding section described what the researcher considers the most crucial problem in applying LP to public school resource allocation situations. Improper interpretation of the results of the model is probably the second most crucial problem. This section examines the interpretation problem in two parts--(1) The Simplistic Nature of the LP Algorithm, and (2) Meaningless Precision in the Output.

The Simplistic Nature of the LP Algorithm

The LP algorithm produces results (1) containing fractional values and (2) optimizing a single function. The simplistic nature of linear programming has been criticized because often fractional quantities have no meaning in reality (e.g., half a student) and most enterprises have more than a single objective. The two algorithms discussed below (Integer Linear Programming and Goal Programming) attempt to overcome this criticism.

Integer Linear Programming. Some LP problems restrict all or some of the optimum solution variables to integer values. Generally the optimal *integer* solution differs from the optimal *simplex*[1] solution which has been rounded to the nearest integer (Taha, 1971, p. 304). Therefore, when integer values are required the user should employ an integer LP algorithm. Interested readers should refer to Taha (1971, Chapter 10) for more detail on integer programming procedures.

Integer programming algorithms have two characteristics which limit their use. First, designing integer LP models is more complicated. Second, computer solutions of integer models are usually

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[1]The simplex algorithm was used in Chapters IV and V. It is the most common mathematical programming algorithm.

several times more expensive than solutions of a comparable simplex LP model because they require more computer core space and central processing unit (CPU) time.

The researcher believes integer programming is unnecessary in public school resource allocation problems because fractional quantities of all the variables considered in this study have a reasonable "real world" interpretation. For instance, half a math section means a math section with half the allowable number of students. Half a teacher is a part-time teacher. Furthermore, administrators should recognize that the model does not make decisions; hopefully it aids them in making decisions. Hence, restricting the model to integer values seems to be an unnecessary impediment. Nevertheless, if future research indicates integer programming is needed, the model contained in this research can be adjusted accordingly.

Goal Programming. Since most enterprises, including public school systems, have multiple goals, one would expect a model which optimizes more than one objective function to be more useful than a single function model. According to Killough and Sanders (1973, p. 278):

> Goal programming can be effectively utilized where the firm has multiple, incompatible, and incommensurable goals. Goal programming does not impose on management a requirement that their goals be compressed into a unidimensional decision criterion...
>
> The most desirable feature of goal programming is the opportunity it gives to the planning team to review critically its hierarchy of goals after an initial solution has been obtained from the planning model. Both the priority structure for goals and constraints can be modified to attain the most desirable set of objectives.

The researcher plans future studies applying goal programming to public school resource allocation problems. The major hinderance to its immediate application appears to be the lack of a generally available computer program.

The researcher felt the use of goal programming in this study was not mandatory because prior studies have indicated that the solutions of goal programming and linear programming models are practically identical (Bailey, et al, 1974). Therefore, the cost of developing a computer program and refining the model might exceed the benefits generated. The future research mentioned in the preceding paragraph will be directed, in part, to resolving the cost/benefit question related to goal programming and other mathematical programming techniques for school resource allocation problems.

Meaningless Precision in the Output

The MPSX360 program carries computations out to five decimal places. For example, the minimum model activity levels for ninth grade United States and Oklahoma history classes are 82.08736 and 82.04986 sections respectively. Such precision is generally meaningless due, in part, to the data quality limitations described in the next section, and an administrator can probably round both activity levels to 82 without jeopardizing the utility of the remainder of the solution. While this statement may seem to conflict with the integer programming discussion on page 101 no conflict really exists. To begin with, fractional sections of any subject have a "real world" meaning up to 1/32 of a section if classes are limited to 32 students. To illustrate, 82 and X/32nds of a section means there are 83 sections with X students in one of the sections. Finally, as a practical matter, rounding the output of the model probably introduces less error than any of the data quality limitations discussed below.

The suggestion of rounding raises another important question: How far can a researcher round a number? Considering the Oklahoma history course example, could an administrator round to the nearest ten sections (80)? the nearest hundred sections (100)? or the nearest thousand sections (0)? The answers to these might be: Probably. Maybe. No!, respectively.

In this researcher's opinion the rounding question must be resolved by affected administrators on a case-by-case basis. As the model and data are refined, the need for rounding should diminish. In the meantime, users should take care not to attribute more significance to the output of the model than is justified.

Assuming all the challenges of modeling discussed in this section can be overcome, use of linear programming by a public school system is still restricted by available data. This limitation is considered in the next section.

Data Limitations

Data limitations take two basic forms--Availability and Quality.

Data Availability

From the data source discussion on pages 58 to 67 one might conclude that data availability poses few problems for school resource allocation studies. This conclusion is probably valid for school districts using computerized record systems which, among other things, allocate costs to programs, courses, sites, etc. (often called the

Pilot Project[2] in Oklahoma).

Besides compiling volumes of data for external agencies and internal management, Pilot Project data processing can generally develop special data at a reasonable cost. In contrast, many school districts have not yet adopted accounting systems which enable them to relate costs to programs, courses, and sites. In order to use LP these districts would have to prepare estimates. If properly done, these estimates could make the LP approach too costly. This fact underscores one of the important arguments favoring the Pilot Project--complete data collection procedures enable administrators to develop information vital for effective management.

Data Quality

Often the quality of available data limits its utility. For instance the researcher considered historical transportation cost records less reliable than estimated transportation costs incorporating 55 cent per gallon gasoline prices (see page 66). Therefore, estimated values were used in place of historical values.

The researcher believes the most serious data quality limitation in this study emerges from the use of average course costs. The estimated cost per section in a given subject was determined by dividing the total amount spent on that subject by the number of sections offered as described on pages 64 and 66. This average cost per section

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[2]Twelve Oklahoma school districts participated in initial efforts to construct, test, and implement a computerized accounting system in conjunction with "accountability" programs. The system emerging from their efforts--the Pilot Project--is basically a chart of account codes used to identify expenditures by function, object, programs, etc. (see page 25).

became the course's coefficient in the budget constraint where, ideally, a marginal cost should have been used. Since insufficient data existed to estimate marginal costs, the average cost appeared to be the next best alternative.

To illustrate the effect that using average costs may have had on the model, consider the following hypothetical drivers education situation: Bartlesville spent $33,007 in 1974 to offer 28 drivers education classes at an average cost of $1,179 per class. If two-thirds of that cost were fixed (such as insurance, licenses, taxes, etc.) the average variable cost per class would be $393 (1,179 x 1/3). Therefore, the budget constraint coefficient for drivers education should be $393 rather than $1,179 as used in this study, and three times as many sections could be added before the budget constraint was violated because the incremental cost per section would be one-third as great as the incremental cost used. If average cost is not a reasonable approximation of marginal cost for most courses in the model, the budget coefficients for the subjects included in the study could produce erroneous results such as an infeasible solution. In fact, before the researcher learned that $270,049.96 in soft money had been left out of the model (see page 100), he believed one important cause of the model's infeasibility was the use of average course costs.

Data availability and quality limitations can often be averted. When administrators discover that certain data are needed and can be accumulated at a reasonable cost, they can design a recording system to fulfill their needs. In addition, administrators can use statistical techniques when such procedures yield equally reliable information at lower cost, e.g., administrators can use regression analysis to obtain better marginal cost data. The important thing is to recognize weaknesses in data and make the appropriate allowances.

When the model is appropriate and reliable data is available, a final limitation exists--resources required to implement the system.

Implementation Limitations

A school system must acquire two resources to implement the linear programming approach--equipment and skilled personnel.

Equipment Limitations

LP routines are available for most computers with sufficient core size to make the routine useful. In general, then, software is not an obstacle to linear programming. Hardware is. Probably only very large school districts can afford a computer large enough to process reasonably complex LP problems.

This limitation really is not as forbidding as it may seem because a school district can buy computer time from numerous commercial services. In addition, if the demand for linear programming and/or other mathematical models justifies such services, the state board of education or a university might develop a consulting agency to help school systems design, process, and interpret education administration models. Chapter VII gives further consideration to the consulting service suggestion.

Personnel Limitations

Canned LP programs are simple to use. A person with limited knowledge of math, computer operations, keypunching, and the linear programming algorithm can master the mechanics of the MPSX360 program after minimal training. However, highly skilled personnel are required at two stages in linear programming studies--model design and output interpretation. To avoid misusing the LP algorithm a school system must rely on a researcher who recognizes the limitations of both the model and the data used, and interprets the model's solution in relation to those limitations.

If the equipment limitations discussed above are overcome by establishing an education administration consulting agency, the personnel limitation will probably be overcome at the same time. (Presumably the agency will be staffed by competent personnel.) This approach appears to be the most efficient attack on both equipment and personnel limitations.

School districts have at least one alternative. They can release a selected teacher from some of his or her instructional responsibilities to concentrate on education administration modeling. Probably the best qualified person for such an assignment would be one with a strong quantitative background, e.g., a math teacher. Advantages of this approach are likely to emerge from (1) the modeler's intimate knowledge of the system being studied, and (2) the potential training for higher administration positions. These two advantages notwithstanding, this researcher believes the central agency alternative would be the more efficient because numerous systems, not just one, could benefit from the modeler's training.

Summary

A mathematical model can be used effectively or misused. When decisions are based on the output of a model which has been improperly designed or applied, a correct decision is a fortuitous event. Chapter VI has identified numerous limitations related to the linear programming approach to school resource allocation problems and steps that administrators can take to avert or mitigate these limitations. The purpose of Chapter VI is to help those who may wish to apply linear programming to school administration problems to avoid modeling, data, and implementation pitfalls.

In spite of the potential errors from misuse of LP, the researcher believes the approach offers a methodical, reliable, efficient, and inexpensive technique for examining resource allocation problems. It can be used for developing policies, experimenting with resource allocation options, and supporting arguments for or against proposed legislation. The Conclusions section of Chapter VII expounds on and supports these beliefs.

CHAPTER VII

SUMMARY, CONCLUSIONS, AND SUGGESTED RESEARCH

Chapter VII summarizes the research reported in Chapters II through VI, reports the conclusions drawn from this study, and suggests promising areas for future research in public school resource allocation.

Summary

The objectives of this research have been to:

1. Develop a general model incorporating linear programming in the planning procedures of a public school system,
2. Alter the general model as needed to make it responsive to the environment in which public schools must operate,
3. Identify the limitations of the LP approach to planning public school resource allocations.

To achieve these objectives, this study has progressed through five important phases: Researching the Literature, Constructing the Model, Obtaining Data, Applying the Model, Interpreting the Solution.

Researching the Literature

The literature review revealed numerous models of public school systems. Some models have employed the linear programming algorithm and provide a background from which this research drew general procedures. Others have employed nonlinear programming techniques which have been incorporated in this study's model. Finally, one, the accountability model, has provided data for this research.

Constructing the Model

The current study constructed a linear programming resource allocation model for a public school system. The model has four types of elements: An Objective Function, Operating Constraints, Policy Constraints, and Variables.

The Objective Function. The linear programming algorithm maximizes or minimizes any given relationship (called the objective function) among the variables of the model. Often the objective function is expressed in economic terms such as "maximize profits" or "minimize costs." Such terms do not appear to express the mission of a public school system adequately because some goals of a public school system do not lend themselves to economic measurement. For example, a frequently mentioned goal of public school systems is to prepare young men and women to assume responsible citizenship roles upon graduation from high school.

School administrators and patrons perceive certain goals of their school system as having greater importance than others. Furthermore, certain activities contribute to the realization of each goal. If administrators and/or patrons assign a weight to each of the system's goals according to its importance to them, a researcher can design a linear programming model which allocates the school system's limited resources to various activities in a manner that will maximize the assigned goal weights.

Independent School District Number 30 in Bartlesville, Oklahoma, cooperated in this research. District 30 identified 24 goals in 1972. These goals plus a weight assigned to each by the superintendent and assistant superintendent served as the objective function for this study.

Operating Constraints. State and natural laws restrict a school system's use of limited resources. For example, when a state law requires a minimum number of physical education courses, it encumbers whatever quantity of the system's resources (teachers, dollars, etc.) are necessary to comply with the law. These resources cannot be allocated to other activities. Such restrictions are called operating constraints in this research.

To construct the operating constraints, the researcher examined the School Laws of Oklahoma and interviewed school administrators. These sources produced 146 distinct operating constraints.

Policy Constraints. Operating constraints do not define a complete public school system; they prescribe minimum or maximum levels for specific activities. School administrators are free to require higher minimums or lower maximums and to initiate additional activities in order to complete their overall program. The restrictions that administrators voluntarily impose on their school system in order to complete it are called policy constraints in this research. Policy constraints are not allowed to violate operating constraints.

To develop policy constraints for this research the model was first solved with operating constraints only (i.e., without policy constraints). Then the Bartlesville administrators were asked to alter the model in whatever manner they saw fit (as long as their actions did not violate operating constraints). The administrators alterations resulted in 41 policy constraints.

Variables. Each activity or program which consumes at least one of a school system's scarce resources and contributes to the realization of a goal is a variable. Thus, for example, a seventh grade math class is a variable because it consumes (a) a teacher's time, (b) a classroom, and (c) some amount of the district's financial resources; and it contributes to the goal of providing students with quantitative skills.

The model developed in this study contains 387 variables which were identified in the literature search, interviews with administrators, and the data collection process.

Obtaining Data

The law requires school districts to submit numerous reports to federal, state, and local governmental agencies. In addition to these external reports, administrators request data compilations for internal management decisions. Consequently, an enormous volume of data exists and these data facilitate linear programming studies of public school systems. Some of the available data can be used without alteration, some require alteration, and some should be used only until more reliable data can be generated.

The only data generated especially for this study were the goal weights in the objective function. All other data were taken directly from existing reports.

Applying the Model

Two versions of the model were applied to the Bartlesville system. The first included only operating constraints. It was called the "minimum model" because it merely required compliance with state and natural laws. The second contained both operating and policy constraints and was called the "optimum model" because it prescribed the optimum resource allocation scheme in view of the system's own goals.

The reason for applying the minimum model was to identify the starting point to which administrators added programs in pursuit of the system's goals. By comparing the minimum and optimum model solutions, administrators could observe the "sacrifices" required to achieve their goals.

Interpreting the Solution

The IBM MPSX360 program was used to solve the model in this study. The printout of the MPSX360 program supplies at least five statistics for each variable:

1. The variable's activity level
2. The cost of forcing a change in that activity level
3. The range of activity levels over which the costs in 2 above are valid
4. The internal limiting factors
5. The variable's sensitivity to changes in its goal weight.

It also supplies four statistics for each resource (constraint):

1. The amount of the resource being consumed and not being consumed

2. The gains and losses associated with increases or decreases in each available resource
3. The range of activity levels over which the costs in 2 above are valid
4. The internal limiting factors.

Various configurations of the MPSX360 program can produce other useful data.

While a decision maker can obtain an impressive volume of helpful information from the LP algorithm, he _must_ know enough about the algorithm, the model, and the data to recognize and neutralize interpretation problems. Perhaps the best way to avoid such problems is by utilizing a consulting service such as mentioned in the next section.

Conclusions

To help organize this section the conclusions are grouped into three general categories: Usefulness of LP in School Resource Allocation, Problems and Limitations of LP, and Implementation Suggestions for LP. The research reported herein was conducted at a single school system--Independent School District Number 30 in Bartlesville, Oklahoma. Therefore, the conclusions are subject to affirmation through additional research conducted in other school systems with differing sizes, resources, and goals.

Usefulness of LP in School Resource Allocation

The current research indicates that a linear programming resource allocation model based on a goal weight objective function can be used by a public school system for three basic administrative activities:

Examining New Programs, Developing Policy, and Influencing the Activities of Authoritative Groups. A brief comment on information economics precedes the discussion of these three LP uses.

Information Economics. Accumulating and processing data requires an expenditure of resources. Presumably the reason for engaging in such activities is to produce benefits which exceed the resource expenditures.

From this study it appears the benefits will be greater than the costs when the linear programming school resource allocation model is used to examine new programs, develop policies, and influence the activities of authoritative groups. The researcher believes the costs will be modest for two reasons. First, most of the work done in this study will not have to be repeated for other Oklahoma schools because the operating constraint portion of the model is common to all Oklahoma independent school districts. Second, the hardware and software necessary to implement LP procedures have been refined to the extent that operating costs are modest. All of the computer runs of the model used in this research cost less than $15 to process.

In contrast to the modest costs, the benefits are likely to be extensive. First, the model produces a unique optimal solution for any given set of conditions. Second, it produces information on the costs of changing the model, relevant ranges for values in the solution, internal limiting factors, and input cost sensitivities. Third, it enables administrators to manipulate the school system in a synthetic environment before disrupting real operations in any way. It appears, then, that the benefit to cost ratio will be favorable.

Examining New Programs. When school administrators must make a decision concerning new programs, they can use the model developed in this research to "experiment" with optional program arrangements. To illustrate, suppose a patron tenders an airplane to a school district to encourage their offering a private aviation program. Although the district will not have a large capital outlay for the airplane, it may incur substantial annual cash disbursements to keep the aircraft safely maintained and licensed by the Federal Aviation Administration. Therefore, the school system may not be able to accept the patron's offer. By inserting the private aviation program variables and constraints into the linear programming model, administrators can "experiment" with the plan to answer two questions. First, will the proposed program violate any operating constraint,[1] i.e., will the solution be infeasible? If so, the district must either reject the program or apply for special permission to proceed from the State Department of Education. Second, assuming the solution is feasible, will the school district be willing to sacrifice the optional program(s) which the private aviation course will supplant? Since available resources are limited, initiating a new program requires curtailing an old one. The model's solution presents both the activity level of the proposed program and the activity levels of all other programs. Consequently, an administrator can identify the projects which the model suggests curtailing.

An administrator can add and subtract programs one at a time or in combinations to observe their effects on the total system. Probably

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[1]Recall that an operating constraint is a nonoptional constraint imposed by authorities beyond the control of the school district.

only a few "experiments" will be required because the model solution presents several statistics in addition to the activity levels of the programs (e.g., relevant range, input cost sensitivity, and limiting factors). After examining the impact of new programs on the school system, administrators can narrow the range of alternatives. Thus the examination stage moves toward the second application of the model--developing policy.

Developing Policy. Administrators can use the linear programming approach to help develop policies for their school district. The distinction between examining new programs and developing policy is not easily drawn. The mechanics (inserting variables and constraints) are the same, but the purpose is different. Examining new programs answers "What if...?" type questions while policy development answers, "Which do you prefer...?" type questions.

To illustrate, using the private aviation example of the preceding section, administrators might ask themselves "What happens to the rest of our curriculum if we initiate a flying course?" Suppose the solution indicates that either a computer science course or a swimming team would have to be eliminated if the private aviation program were initiated. Now the question becomes, "Which program best enables us to achieve our overall goals?" Faced with a choice among the three programs, administrators may select the program themselves, or they may present alternatives to the school board, or they may ask patrons to express a preference through PTA groups, surveys, etc. Examples of policies administrators might adopt for accepting or rejecting projects are: select the programs which (1) benefit the greatest number of students, or (2) are most likely to result in statewide prestige for the district, or (3) might lead to the greatest number of impressive scores on national college entrance examinations.

This researcher believes the linear programming resource allocation model can help develop policies in numerous situations, such as:

1. Teachers' union contract negotiations
2. Physical plant expansion considerations
3. Student activity considerations
4. Community service programs.

Influencing the Activities of Authoritative Groups. The third application for the resource allocation model is demonstrating the impact of proposed programs to patrons, legislators, or state administrators. The following incident illustrates this application: In 1975 the Oklahoma State Legislature passed a bill which (1) required minimum salaries for public school teachers to be raised from approximately \$7,000 per year to \$7,700 per year, and (2) provided that the state would supply the \$700 to implement the raise (House Bill No. 1410, p. 8). However, the legislature failed to appropriate funds for the increased social security and fringe benefit costs. This oversight caused financial problems for many school districts, forcing some to discontinue prior years' projects to meet the added payroll costs.

The model used in this study may well have detected this oversight, demonstrating to legislators the need to provide funds beyond the base salary increase. In fact, it detected a similar error in an earlier phase of this study. The reader will recall that the optimum model reported an infeasibility because \$270,049.94 in "soft money" had been omitted from the model. Apparently the model is relatively sensitive to financial changes; it may have sounded an early warning of the legislators' oversight.

Problems and Limitations of LP

Chapter VI identified several limitations of the linear programming approach. Although none of the limitations seemed to present insurmountable barriers to the use of linear programming for school resource allocation, two are important enough in this researcher's opinion to warrant further discussion. Data Considerations and Interpretations of Linear Programming Results.

Data Considerations. Chapter IV described volumes of internal and external data available for use in the linear programming model. However, Chapter VI recognized some problems related to these data and suggested a search for other methods of accumulating statistics for use in the model.

Much of the data used in the current study are factual, and they are always subject to audit by state authorities. However, the solution of the model using estimated values may be more reliable than the solution based on factual records, particularly when the probability of changes is high, such as the increase in the price of gasoline, or when estimates have stronger theoretical support, such as estimated marginal costs versus average costs (see pages 105 and 106).

It follows from the above arguments that whether the data reflect historical facts or well conceived estimates is irrelevant to their utility in the model. The important criterion in selecting data is whether it leads to sound decisions as defined by the school system using the LP approach.

Interpretation of Linear Programming Results. The following axiom may apply to any mathematical modeling system: the conclusions drawn from a given model should not exceed the model's capabilities for providing good decision information. Administrators must know enough about the linear programming approach to recognize weaknesses in both the model and the data and to adjust for those weaknesses appropriately. Furthermore, they must insure that they do not default their decision making function to a computer model which generates data but does not make decisions.

Implementation Suggestions for LP

Due to the resource requirements of an LP system, e.g., equipment, skilled personnel, etc., the researcher believes the most efficient way to implement linear programming for public school systems is to establish a central agency which has access to adequate computer equipment and is able to hire sufficiently skilled personnel to make the system useful. The system could be available to public school districts within one or more states.

Among other things, such a facility will reduce the cost of model development because common elements have already been developed. As the number of uses increases, the model's efficiency will be enhanced and the committed cost of developing and executing linear programming systems will be spread over a broader base. In addition to spreading the cost, a centralized system can (1) develop a data bank and (2) upgrade the model continuously (with the help of practicing school administrators).

Either the State Department of Education or a major university are feasible locations for the centralized system. The State Department of Education offers the following advantages:

1. Information processing would be easier because reports filed by schools throughout the state are readily accessible.
2. Administrators at the state level may have greater expertise in school laws than any other group.
3. The State Department of Education probably works more closely with the legislature in matters of finance and school law than any other group. They could use the output of the model to influence education legislation.

Advantages of a major university are:

1. Linear programming software is more likely to be available at a university than at the State Department of Education.
2. The use of computer equipment by other groups for such activities as instruction, research, and institutional information processing reduces the incremental cost for all users, including school administrators.
3. A university based agency could draw on the expertise of many disciplines in addition to education administration. For example, a university team could be comprised of personnel from the computer center, industrial engineering, accounting, administrative sciences, the legal department, and many others.

Whether the State Department of Education or a major university is the selected location for the central modeling group, both should cooperate in the development and improvement of linear programming techniques for public school resource allocation.

Suggestions for Future Research

Throughout this dissertation problems which might be attacked fruitfully in future research have been identified (see pages 98 and 107 for example). To organize this section, potential research areas have been classified in two groups:

1. Further research in mathematical programming
2. Research in supportive areas.

Further Research in Mathematical Programming

Further research in mathematical programming could take two directions, one extending the current study and another examining other mathematical programming techniques.

Extending the Current Study. The introduction to the preceding section recognized the need to corroborate the conclusions of this study by additional research in other school systems. Such studies should proceed in two phases. First, additional studies in Oklahoma should confirm the conclusions in this research and improve the model. If these first studies concur with the current research on the utility of the LP approach, the second phase should develop models for other states. This sequence is suggested because (1) the work in the first phase can be reduced by building on the current study and (2) negative conclusions in the first phase would probably obviate the second phase.

Other Mathematical Programming Techniques. The limitations discussion in Chapter VI described the advantages and shortcomings of several other mathematical programming systems, such as goal programming, integer linear programming, etc. Goal programming appears to be the most promising mathematical algorithm for public school administration research because it allows administrators to optimize a system containing several objective functions (goals). Dynamic programming also appears to be useful for public school systems. It enables administrators to observe the impact of given policies over several years. Finally, parametric programming can be useful, but its utility depends upon the feasibility of developing parameters through the supportive research described below. Therefore goal, dynamic, and parametric programming for public schools research should be undertaken.

Most other mathematical programming systems are quite restrictive because they require specific conditions not likely to be found in most public school situations. For example, quadratic programming assumes a quadratic relationship among the variables in a constraint. The applicability of linear programming may not extend over a very broad range in a public school system, but the relevant range for quadratic programming is probably even narrower.

Supportive Research

Additional research is warranted in three areas relating directly to the linear programming model: Accounting Improvements, Data Improvements, and Information Content Studies.

Accounting Improvements. One who studies governmental operations is likely to find object accounting (accounting for items purchased) rather than program accounting (assigning costs to programs)[2]. The former will limit the experimenter's ability to apply a linear programming technique to the system because he will be unable to relate resource expenditures with the programs benefited. The current

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[2]The Pilot Study discussed on page 25 is based on program accounting.

research requires program accounting because it provides more definitive information for decision making. Therefore, this researcher urges further applications and refinement of program accounting.

Data Improvements. Research should be directed toward generating better data for LP models, especially identifying a program's marginal costs. Regression analysis of program costs will probably produce better marginal cost estimates than the crude averages used in this study. Linear regression, for example, produces the equation

$$y = \alpha + \beta x$$

where

α = the fixed portion of the program's cost

β = the marginal cost of the program

x = the number of programs

and y = the total cost of x programs.

In the budget equation given on page 47, α would be a committed cost and β a discretionary cost.

Information Content Studies. A body of research in the fields of accounting and finance--usually called "efficient capital markets" (ECM)--attempts to determine if accounting reports contain new information by observing the behavior of security prices in a public market such as the New York Stock Exchange. ECM proponents argue that those accounting reports which contain new information will cause investors to reassess the investment quality of affected securities and the security prices will reflect investors' aggregate revaluation.

A parallel can be drawn between investors' behavior regarding publicly traded securities and the behavior of school district patrons with respect to bond issues and annual millage elections. More specifically, if two school systems report their operations under different systems (e.g., program accounting vs. object accounting) and either system provides superior information, one might argue that the patrons' behavior resulting from the superior information will be reflected in their propensity to vote for or against bond issues or millage levels.

Many school administrators have resisted implementing program accounting procedures. ECM studies may supply evidence on the wisdom or folly of administrators' resistance to program accounting. If ECM research implies that patrons understand the costs of public school programs better when program accounting is used, administrators are more likely to adopt program accounting. As a result, mathematical models will be useful for more school systems in the future.

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APPENDIX A

GOALS AND GOAL WEIGHTS OF THE

BARTLESVILLE SCHOOL SYSTEM

The attached goals of the Bartlesville School System were compiled by the Community Education Council in 1972. These goals and their related weights provided the basic framework for the objective function in the linear programming model developed in this study.

Goal Weight	Goal
142	Motivate nonparticipating students to become active in at least one activity.
153	Determine and emphasize those activities which further the educational, cultural, physical and social development of each student.
142	Optimize the number of extracurricular activities toward maximizing student interest and participation.
180	Increase communications between individual schools and their patrons.
192	Strengthen the teaching of communication skills, beginning at the elementary level and continuing through high school. Improvements are not only recommended in the basic abilities to read, write, and speak but also in foreign language exposure and in mathematical fundamentals.
180	Provide better incentives for students to learn more about the world in which they live through an applied understanding of the physical and life sciences.
183	Design social science and humanities courses to be more relevant and meaningful so that students may be better prepared for rapid societal changes and increasing challenges in an increasingly complex society.

Goal Weight	Goal
180	Provide adequate physical education at all levels of schooling.
170	Create more interest in and recognition for music and art as a part of the liberal education.
170	Provide better vocational and occupational preparation for career-minded students who need marketable job skills upon completion of high school.
180	Recognize slow learners and provide better educational opportunities for them.
168	Provide more effective and more complete professional counseling for the proper guidance and motivation of students at all levels.
175	Increase the educational values of study periods and library facilities.
160	Coordinate and select texts to insure comprehensive coverage of essential subject matter throughout all grades.
150	Design and plan, grade by grade, a coordinated approach to teaching course content in critical subjects to insure mastery at each level.
192	Continue to seek optimum utilization of available funds with priority given to those expenditures motivating maximum learner development.
165	Provide appropriate facilities in proper locations to meet the changing learner, community, and administrative needs.
145	Establish a cooperative means for use of existing facilities for community purposes.

Goal Weight	Goal
175	Develop a strong sense of responsibility for buildings and equipment on the part of administrators, faculty, and students.
147	Include and emphasize the maintenance and functional points of view in all plans for future construction and or remodeling.
135	Optimize bus services.
162	Establish clear and definite lines of communication for maintenance and repairs.
188	Increase the effectiveness of the school system by developing adequate organizational structure to conduct, measure, and improve the educational process.
185	Establish a meaningful, planned, program of professional development.

APPENDIX B

MODELING TECHNIQUES FACILITATING IMPLEMENTATION

An objective of this research is to develop a model which school administrators can use to examine resource allocation problems. Accordingly, the model incorporates features which facilitate its implementation. One of these features, Bounding, produces two benefits--cost reduction and work reduction. At the same time, it produces an effect which requires clarification--Double Reporting.

Bounding

In some cases state law prescribes a maximum or minimum activity level for a given variable or class of variables. For instance, the law requires at least one drug abuse program for every school. A modeler can write a constraint for this particular law as follows (assuming 18 schools in the district):

$$\sum_{i=1}^{18} \text{Drug Abuse Program}_i \geq 18.$$

where i is the i^{th} school.

Every time a constraint equation is added to the model, the cost of the computer solution increases because (1) the computer must reserve more core, and (2) the solution may require more iterations.

An alternative to the above constraint equation is to set a lower bound (LO) on each drug abuse program variable as follows:

$$\text{Drug Abuse Program}_1 \geq 1$$

$$\text{Drug Abuse Program}_2 \geq 1$$

...

...

...

$$\text{Drug Abuse Program}_{18} \geq 1$$

where the subscripts 1, 2, ..., 18 represent schools 1, 2, ..., 18.

In linear programming all variables are constrained to a nonnegative value because negative quantities of variables, such as a minus one drug abuse program, make no sense in real life. Therefore, the lower bound for variables is at least zero. The bounding option simply moves the zero lower bound up to the desired nonzero level. Thus the bounding option can accomplish the same end result as constraint equations which establish a lower activity level for a single variable or class of variables without consuming extra core space or generating more iterations.

Occasionally a modeler may want to impose an upper bound on a given variable's activity level. For instance, the Oklahoma Secondary Schools Activities Association limits the number of football games in which a member school can participate to less than 14 games per year, excluding playoffs (1974, p. 40). The MPSX360 program enables a user to designate an upper bound (UP) for a given variable just as he can designate a lower bound. The same cost-saving advantage accrues.

Sometimes a user may want to fix the value of a variable at one specific (i.e., constant) level. He can do this with the MPSX360 program by setting a lower bound equal to an upper bound (FIX).

In addition to actuating the cost-saving benefit of the LO and UP options, the FIX option sometimes creates another benefit--work reduction. Often capacity limitations constrain more than one relationship among variables. For example, the number of students in high school constrains the financial resources received by the school, the number of teachers to be employed, the number of courses to be offered, and ten other relationships (thirteen in all). When several constraint equations have a common capacity limitation, the user can reduce the amount of work required for "experimenting" with the model in the following way:

The constraints in a linear programming model typically are written

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i$$

where a_{ij} is the coefficient in the i^{th} constraint for the j^{th} variable

x_j is the decision variable for the j^{th} variable, and

b_i is the capacity limitation of the i^{th} constraint.

Mathematically the constraints could be written:

$$\sum_{j=1}^{n} a_{ij} x_j - b_i \leq 0.$$

In the latter formulation the capacity limitation b_i enters the computer program as a "variable," but when the FIX option is applied to b_i, it is actually a constant.

An administrator may be curious about the effects a change in the number of students in a particular high school would have on the rest of the system. If the constraints have been written in the usual way (i.e., the first formulation above), one must change the b_i value for thirteen separate constraints. On the other hand, if the constraints have been written with b_i on the left side of the inequality sign (the second formulation above) and if the FIX bounding option has been used on the b_i value, one needs only to change the FIX value. Thus an experimenter's work is cut to a fraction (1/13th in this case) of what it would have been using the typical formulation. At the same time the opportunity for human error is greatly reduced.

When a large number of b_is are common to many constraints, the work and error reduction benefits of the latter formulation can be significant indeed. However, this approach creates the interpretation problem discussed in the next section.

Double Reporting

Capacity limitations, b_is, are called RHSs because they are typically written on the Right Hand Side of the inequality sign in a constraint. On the other hand, variables and their coefficients are typically written on the left hand side of the inequality sign. In the second formulation above the computer sees the b_i as a variable rather than an RHS because it has been moved to the left of the inequality sign. Some terminology conflicts result. The b_i is not really a variable; it is a capacity limitation with a constant value. Nevertheless, b_i is reported in the "variables" section of the computer solution with a variable's sensitivity analysis.

An important distinction between b_is and decision variables is that the b_is have a zero coefficient in the objective function in this study. Therefore, they neither add to, nor subtract from, the aggregate goal weights of the model. Decision variables have a nonzero coefficient in the objective function, specifically their assigned goal weight.

A b_i's activity level, goal weight gain or loss, relevant range, and limiting factors are the same whether it is modeled as an RHS or a "fixed" variable. One value is displayed for the b_i when it is modeled as a variable that is not displayed when it is modeled as an RHS--an input cost sensitivity. This value is meaningless and can be ignored because as a fixed variable the b_i will never have a nonzero coefficient in the objective function.

When the b_i is modeled as a variable, the constraint to which it relates is still reported in the "Constraint Analysis" section of the computer printout. Hence, the double reporting problem. Double reporting is not a major problem, because, although the data in the "Constraint Analysis" section require a few minor interpretation modifications (beyond the scope of this paper), the information contained therein is identical to (1) the information that would have been presented had the b_i been shown as an RHS, and (2) the information shown for the b_i in the "Variable Analysis" section.

Double reporting and bounding account for the discussion of lower bound constraints, in the "Variable Analysis" section of Chapter IV. This apparent contrariety notwithstanding, it seems double reporting and bounding pose no substantive problems. Since both formulations produce the same information, the one which produces the information most conveniently should be chosen. In the current research the fixed variable information is unquestionably the most convenient.

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