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METHODS OF ADAPTIVE CONTROL

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Paul Rutland Hinrichs

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CHAPTER I

ADAPTIVE CONTROL

Introduction

The term "adaptive" was borrowed by Ashby[1] from the biologists where it describes the ability of an organism to adjust to its environment. The connotation of the word "adaptive" to describe a control system implies that the system is self-compensating in some manner for changes that occur relative to the system. The definition of adaptive systems to be used here will be the same as the one given by Drenick and Shahbender[2] and others and is, in fact, a restriction upon the above noted connotation.

Definition: An adaptive control system is a control system designed for operation in a slowly-changing

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[1] W. R. Ashby, Design for a Brain (New York: John Wiley and Sons, Inc., 1954), p.57.

[2] R. F. Drenick and R. A. Shahbender, "Adaptive Servomechanisms," AIEE Trans. (Applications to Industry), Vol. 76 (November, 1957), pp. 286-292.

environment in such a way that compensation is made for the environmental change.

The phrase <u>slowly-changing environment</u> is used to denote environmental changes which are slow compared to the system's time-constants. That is, at any particular time the system's response can be considered to be time-invariant for the duration of the calculation, and only cases where this approximation or assumption is valid are to be considered.

Zadeh[3] has attmepted a more precise definition of adaptive control, but the definition appears to the author to be more useful when applied to the learning process than to the adaptive process. In general, the two processes can be distinguished by the following characteristics. An adaptive process senses in some manner the state of the plant and uses this information to control some of the system's variables to effect a useful control system. A learning process generally tries different methods of control, and in some usually random manner weights are associated with each trial in accordance with the degree of success of the trial. The trials are usually not weighted too heavily at the beginning, but as the number of trials increases, and the statistics begin to show trends, the weights associated with

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[3] L. A. Zadeh, "On the Definition of Adaptivity," <u>Proc</u>. <u>IEEE</u> (Correspondence Section), Vol. 51, No. 3 (March, 1963), pp. 469-470.

certain choices increase cumulatively with "good" guesses and decrease cumulatively with "bad" guesses.

It is now reasonable to ask, "With the environmental restriction noted in the definition, are there enough problems in this category of adaptive control systems to warrant the amount of attention that they are now receiving in the literature?" (That this list is large and growing rapidly is illustrated by Aseltine, et al.,[4] in a literature survey that was published in 1958.) The answer to the above question is yes, and several examples will be cited to illustrate applications of adaptive systems as defined herein.

Example 1. Interest in adaptive controls has been stimulated because of the real need for it in the aircraft flight stabilization field. As aircraft travel faster and higher, the response of the vehicle varies over a wide range depending on the changing environmental conditions, and it becomes necessary to compensate in some way for the change. A jet pilot, for example, will subconsciously develop the habit of injecting a continual, minute, 1.5 cps test signal to the control stick to get the "feel" of the aircraft as the environment changes, something the pilot of a conventional aircraft does not do.[5] He is thus kept up to date on

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[4] J. A. Aseltine, A. R. Mancini, and C. W. Sarture, "A Survey of Adaptive Control Systems," IRE Trans. on Automatic Control, Vol. AC-3 (December, 1958), pp. 102-108.

[5] N. D. Diamantides, "Informative Feedback in Jet-pilot Control-stick Motion," Trans. AIEE, Vol. 76, Pt. 2 (November, 1957), pp. 243-249.

the vehicle response so that he can apply the correct stick motion for stable maneuvering.

It is just this characteristic of the human being - adapting his behavior to a changing environment - that is being imitated in adaptive servos. (It has even been suggested that _all_ feedback control systems have been designed, consciously or not, to imitate the behavior of a human being in his comparison, actuation, and anticipation functions while controlling a process.)[6]

Example 2. Drenick and Shahbender[7] designed an adaptive radar system for tracking aircraft. They noted that when a target flies a straight path, little bandwidth is needed for tracking. In this situation, the bandwidth is reduced which, in turn, reduces the noise so that tracking accuracy is increased. When a target uses evasive maneuvers, bandwidth (and therefore noise) is increased to hold the target. The spectra of the tracking signal is sensed, and this information is used to make bandwidth adjustments.

Example 3. The mass and moment of inertia of a missile change as the fuel burns. It may be that the rate of change of these variables is so fast that the system must be considered strictly as a time-varying problem and the system

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[6]L. Braun, "On Adaptive Control Systems," _IRE Trans. on Automatic Control_, Vol. AC-4 (November, 1959), pp. 30-42.

[7]R. F. Drenick and R. A. Shahbender, _loc. cit._, pp. 286-292.

pre-programmed to make the necessary corrections or adjustments for these changes. However, if this is not the case, then it may be more feasible to build-in an adaptive adjustment than to build-in a pre-programmed system. This would be especially true if the missile behavior was not completely predictable in the various situations it would encounter.

Example 4. A radio receiver with automatic gain control is a good example of an adaptive system that has been in use for many years. If the average signal level at the detector starts to decrease (increase) a corrective signal is developed that increases (decreases) the gain in the earlier stages of the receiver. A radio receiver's gain or, for that matter, any amplifier's gain may also change with temperature, radiation, power supply variations, etc. The gain of an amplifier is also effectively lowered if it is saturated by an excessively high signal level.

There is also a large class of systems, especially rockets and high speed aircraft, that are designed adaptively due to necessity. This is because the equations of the system are not known with any degree of accuracy until the system is built and operating, and at this time, it is generally imperative to have a working control system.

In referring to a system, reference is made to a complete feedback system considered to be made up of two parts:

1) the controlled element which is an element or collection of elements belonging to the system

and having characteristics which <u>cannot</u> be modified for control purposes;

2) the <u>controller</u> which is an element or collection of elements belonging to a system and having characteristics which <u>can</u> be modified for control purposes.

Of concern here are systems in which the controller is capable of automatic adjustment or modification of its characteristics to obtain the "best" system with changes in inputs and controlled element characteristics. Controllers with this capability are referred to as adaptive controllers, and systems containing adaptive controllers are referred to as adaptive systems.

Classification of Adaptive Control Systems

The number of variables to be sensed and the number to be adjusted depend, of course, on the particular system to be designed. For this reason and the fact that there are about as many ways to implement an adaptive system as there are control system engineers, the number of ways in which adaptive systems might be classified is almost unlimited. The recent interest in adaptive systems has produced many papers on this subject, and many specific systems have been proposed.[8] The proposed systems cover a wide range of

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[8] J. A. Aseltine, A. R. Mancini, and C. W. Sarture, *loc. cit.*, pp. 102-108.

applications including process control, flight control, fire control, and most other areas where automatic control systems have found widespread application. Although superficially some of these systems differ markedly, it is apparent that the same principles of operation, viewed either mathematically or intuitively, are being used in different systems. This has prompted several investigators to suggest classification schemes which emphasize these similarities and provide a framework for further study in the area of adaptive control. Schemes suggested by Aseltine, et al.,[9] Levin,[10] and Dandois[11] are abstracted below and serve to review the adaptive principles presently being considered by control engineers.

Aseltine and his colleagues classify adaptive systems according to the way adaptive behavior is achieved and suggest the following five classes.

1) _Passive Adaptation_: systems which exhibit adaptive behavior without parameter changes.

2) _Input Signal Adaptation_: systems which adjust their parameters in accordance with input signal characteristics.

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[9] J. A. Aseltine, A. R. Mancini, and C. W. Sarture, _loc. cit._, pp. 102-108.

[10] M. J. Levin, "Methods for the Realization of Self-Optimizing Systems," _ISA Paper No. ISA-FCS2-58_ (April, 1958).

[11] M. Dandois, "Self-adaptive Control Systems," Rept. No. FZM-1242, presented at the Specialist Meeting of the Institute of Aeronautical Sciences, Texas Section, Arlington State College, Arlington, Texas (September, 1958).

3) Extremum Adaptation: systems which self-adjust for maximum or minimum of some system variable.

4) System-Variable Adaptation: systems which make self-adjustments based on measurements of system variables.

5) System-Characteristic Adaptation: systems which make self-adjustments based on measurements of transfer characteristics.

Levin in his classification of adaptive systems considers only three categories.

1) Input-Sensing Systems: systems in which the adaptive controller measures input signal characteristics during operation and adjusts the system on the basis of the measurements.

2) Plant-Sensing Systems: systems in which the adaptive controller measures plant parameters and adjusts the system on the basis of the measurements.

3) Performance-Criterion Sensing Systems: systems in which some quantity indicating the quality of system performance is measured and the adaptive controller acts in some way to drive this quantity to a certain value (usually a minimum or maximum).

Dandois, who is primarily interested in flight control systems, also considers three categories of adaptive control.

1) Control through design for operation over wide

variations in environment without parameter adjustments.

2) Control by parameter adjustments as functions of the system's input and output variables.

3) Control by parameter adjustments based on measurements of the system's dynamic characteristics.

These authors present a variety of systems under each category. Comparing these three classifications and noticing their similarity, one can see that most proposed adaptive systems are covered by relatively few operating principles. Levin's classification is probably most suitable for the purposes of this dissertation since his plant-sensing system classification best describes the sense in which the author's system is adaptive. Chapter II will also contain examples of plant-sensing adaptation schemes.

It is also convenient to classify the time-varying parameters into two categories for plant-sensing systems.

1) Parameters that can be directly compensated so that the controller can adjust the parameter to yield the correct transfer function.

2) Parameters that must be compensated for their changes indirectly by a feedback or feedforward arrangement.

Talkin[12] and others have designed systems that depend upon a signal being artificially injected into the system in

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[12] A. I Talkin, "Adaptive Servo Tracking," IRE Trans. on Automatic Control, Vol. AC-6, No. 2 (May, 1961), pp. 167-172.

order to work. In general this is undesirable; however, the authors point out that it may only be necessary to update the system periodically, or it may be possible to filter the unwanted signal from the output, or it may be possible to keep the output level that results from this injected signal to a low enough value that it is not too objectionable. Talkin's[13] system is mainly a transfer function tracking device and can, in theory, track two parameters for each perturbing frequency introduced. Anderson, et al.,[14] inject what they call discrete-interval binary noise and cross-correlate this with the output to recover the impulse response of the system. This will be discussed further in Chapter II.

Another possibly less objectionable technique used in signal adaptive systems is that of perturbating or dithering a parameter. This technique has been used by McGrath and his associates,[15,16] and it has been discussed also by Mosner.[17] The perturbating signal usually appears as

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[13] A. I. Talkin, "Adaptive Servo Tracking," IRE Trans. on Automatic Control, Vol. AC-6, No. 2 (May, 1961), pp. 167-172.

[14] G. W. Anderson, R. N. Buland, and G. R. Cooper, "A Self-adjusting System for Optimum Dynamic Performance," IRE National Convention Rec., Pt. 4 (1958), pp. 102-108.

[15] R. J. McGrath and V. C. Rideout, "A Simulator Study of a Two-Parameter Adaptive System," IRE Trans. on Automatic Control, Vol. AC-6, No. 1 (February, 1961), pp. 35-42.

[16] R. J. McGrath, V. Rajaraman, and V. C. Rideout, "A Parameter-perturbation Adaptive Control System," IRE Trans. on Automatic Control, Vol. AC-6, No. 2 (May, 1961), pp. 154-162.

[17] P. Mosner, "Perturbation Approach to the Response of a Control System," IRE Trans. on Automatic Control (Correspondence Section), Vol. AC-6, No. 3 (Sept., 1961), pp. 361-362.

modulation on the output signal; thus, when there is no input signal, there is no output. For this reason, parameter perturbation might be less objectionable than signal perturbation. Of course, during periods of no input there will be no adaptive control signal; whereas, signal injection does have the advantage of continuously up-dating the control system.

Adaptive control schemes which perturbate the system's parameters, inject artificially an input signal, or generate in any way an unwanted signal at the output when the system is correctly adjusted will be considered undesirable; therefore, the adaptive method to be presented by the author will be an effort to circumvent these schemes.

Historical Background[18]

The intensive and widespread interest in adaptive control has grown not only because of the fascinating potentialities of adaptive systems, but perhaps even more as a consequence of the status of control technology in the late 1950s. During the 1940s the standard analytical and design techniques for feedback control systems appeared in rapid succession in the aftermath of World War II. By 1950, the Nyquist, Bode, and Evans plots were widely accepted approaches to linear control system design, and the literature already contained detailed expositions of the describing-function and phase-plane analysis of nonlinear feedback systems. Thus, 1950

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[18]E. Mishkin and L. Braun, Jr., Adaptive Control Systems (New York: McGraw Hill Book Co., Inc., 1961), pp. 1-3.

marked a leveling-off point in the development of control technology.

The first half of the 1951-1960 decade was characterized by a solidification of the technical position gained during the 1940s: university courses in automatic control became numerous and popular, many texts appeared, and the journals published many specific applications.

Another development entered into the control-system picture around 1956. Computer technology reached the point at which it became feasible to consider the inclusion of reasonably complex analog or digital computers as real-time elements of the control systems. Digital computers, for example, became admissible elements because of the improvement of solid-state devices, the development of smaller storage units, and the continuing decreases in computation time, so that not only was the computer feasible from power-drain and size standpoints, but it also was capable of performing the control computations with reasonable reliability and in time intervals appreciably less than the system response time (so that time multiplexing was possible, with the consequent reduction in computer cost and size per control channel).

Summary

An attempt has been made to define what adaptive control systems are and to illustrate the wide variety of such systems with several examples. The classification of these systems was discussed and it was pointed out that they can be conveniently placed into relatively few categories. A few comments were also included to indicate the classification of the systems to be considered in this dissertation and what some of the objectives of this dissertation will be. A more complete description will follow in the sequel.

For any discussion on adaptive control systems to be complete, some discussion of the more representative contributions should be included. This will be done in Chapter II where the four systems to be discussed are those of M. F. Marx of the General Electric Company; S. S. Osder of the Sperry Gyroscope Company; G. Wm. Anderson, R. N. Buland, and G. R. Cooper of Aeronutronics Systems, Inc.; and H. P. Whitaker of Massachusetts Institute of Technology. All four of these systems belong to the plant-sensing categroy of control systems, the same as the systems to be presented in this dissertation.

Chapter III will describe the proposed method in detail and develop the general theory for its operation. An example using a linear control system will be presented together with an analysis of the adaptive feedback-loop. This will aid in giving insight into the nature of the feedback-loop and yield valuable information concerning the adaptive-loop's stability and expected performance.

Chapter IV will point out some general remarks concerning noise introduced into the feedback control system and its effect upon the adaptive portion of the system.

The example used in Chapter III dealt with a system whose parameters could be directly compensated. Chapter V will present an example of the control of the pitch-rate of an aircraft. Here, the parameters have many degrees of freedom, and they cannot be compensated for directly. The approach to the problem is to embed the aircraft into a feedback control system in such a way as to make the system depend, in the main, upon some parameter that can be compensated directly. An effort is also made to develop the simplest adaptive system in terms of the amount of hardware needed.

Chapter VI will create an extension of the results of Chapter III to a multiple parameter adaptive system, and again, an example will be presented to illustrate the effectiveness of the method.

Chapter VII, the conclusion, will summarize the results and will compare the author's system to those of Chapter II.

CHAPTER II

PRESENT METHODS OF ADAPTIVE CONTROL

This chapter will present a summary of four papers on adaptive control systems. All four illustrate different methods of automatic adjustment of system parameters to maintain specified closed loop pole-zero configurations. Presumably, the ideal pole-zero configurations are based on those of Graham and Lathrop's,[19] McDonald's,[20] analog simulation, or possibly as a solution of the Wiener-Hopf equation.[21] These papers represent, to some extent, the state of the art of adaptive control, but more importantly, they illustrate the variety of methods of attack used in solving these problems. An effort has been made to include only the basic ideas of each author along with critical comments as to the advantages, disadvantages, and limitations of each.

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[19] D. Graham and R. C. Lathrup, "The Synthesis of Optimum Response: Criteria and Standard Forms," _Trans_. _AIEE_, Vol. 72, Pt. 2 (November, 1953), pp. 273-288.

[20] T. McDonald, "A Development of Standard Form Using an Integral Error Criteria" (unpublished Master's dissertation, School of Electrical Engineering, University of Oklahoma, 1963).

[21] Y. W. Lee, _Statistical Theory of Communication_ (New York: John Wiley and Sons, Inc., 1960), p. 369.

Marx's System

Marx's[22] approach is best illustrated by the simple second-order system shown in Fig. 1. The desired transmittance is

$$\frac{E(s)}{R(s)} = \frac{s(s + 2\zeta\omega_n)}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad (2.1)$$

which is obtained when $K_1K_2 = 1$. The function of the controller is to adjust K_2 as K_1 varies to keep $K_1K_2 = 1$. Fig. 2 shows the pole-zero pattern and Fig. 3 shows the energy density spectrum of E(s) for a step input for three different values of the product K_1K_2. The operation of the adaptive loop will now be described. When the gain K_1K_2 is low, corresponding to peak in the energy density spectrum at ω_1, the output from the low-pass filter section exceeds that from the high-pass filter section and the integrator increases K_2 until the peak in the frequency occurs at ω_n; the input to the integrator is then zero and the open-loop gain K_1K_2 is unity. The reverse operation occurs when the open-loop gain is initially too high. The actual depolarizers used by Marx were full-wave rectifiers.

This system was built by Marx and found to operate satisfactorily for step inputs. The system is obviously

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[22] M. F. Marx, "Recent Adaptive Work at the General Electric Company," Proc. Self-Adaptive Flight Control Systems Symp. (January, 1959), pp. 201-215.

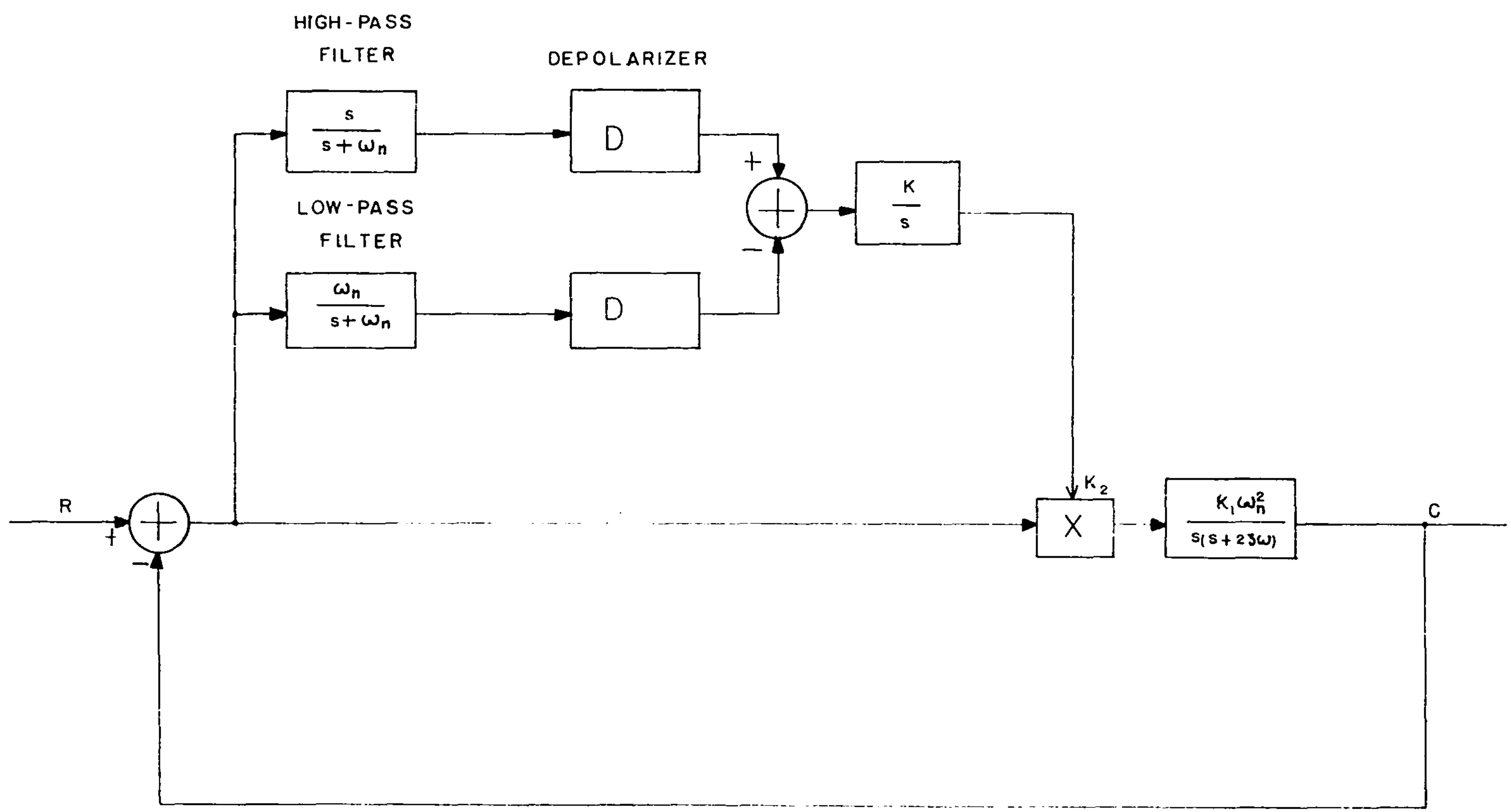

FIG. I. MARX'S FREQUENCY SENSITIVE SERVO. FROM ADAPTIVE CONTROL SYSTEMS, BY MISHKIN & BRAUN, COPYRIGHT 1961. USED BY PERMISSION.

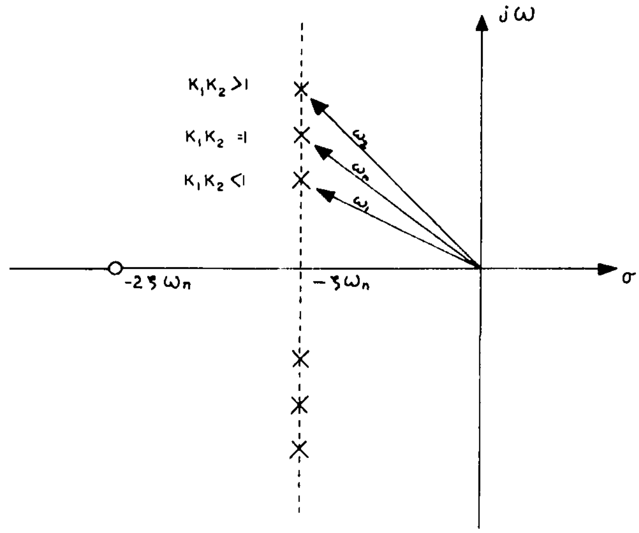

FIG. 2. POLE-ZERO DIAGRAM FOR DIFFERENT VALUES OF THE PRODUCT K_1K_2 WHEN THE INPUT IS A STEP. USED BY PERMISSION.

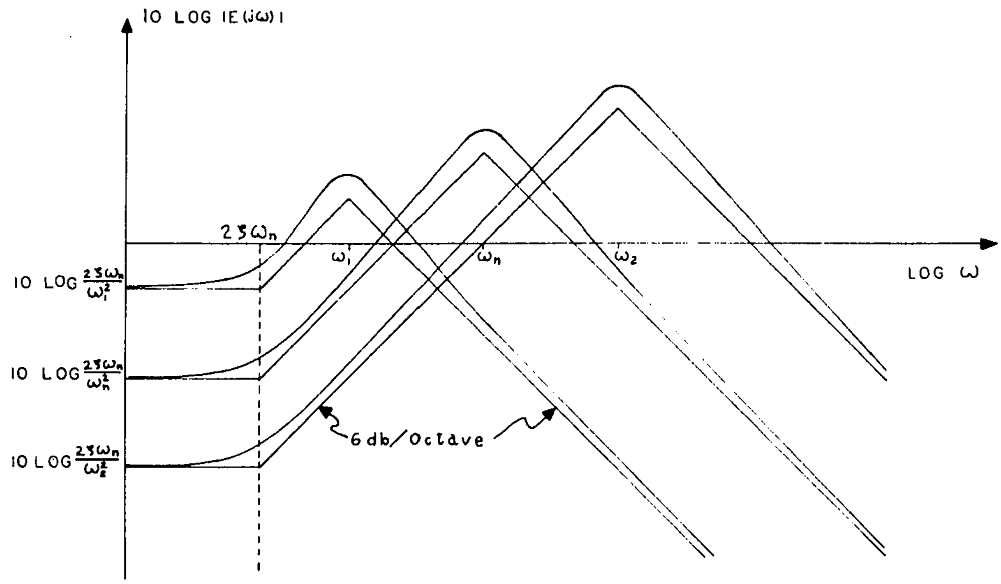

FIG. 3. ENERGY DENSITY FOR DIFFERENT VALUES OF THE PRODUCT K_1K_2 WHEN THE INPUT IS A STEP. USED BY PERMISSION.

limited as to the types of inputs that it can accept and will tend to be correct only if the input is a step. Also, the theory of operation is, in actuality, based on the average energy spectral density since the value of K_2 should be changed only if total high frequency energy is not equal to the total low frequency energy, but K_2 should not respond to the instantaneous outputs of the depolarizer since K_1 by definition is a slowly time-varying gain. Therefore, the only way to overcome this difficulty is to reduce the gain of the integrator circuit. Marx applied his adaptive loop to a higher order system (an aircraft system which will be treated in Chapter III), but he was unable to stablize the system even for step inputs. The following is a direct quotation from Marx's article.[23]

> Extension to the control of the airplane mode for the case on hand has led to difficulty due to the low frequency closed loop pole caused by the integrator. For cases where the required open loop gain is low, this pole results in low frequency components in the error which the frequency servo interprets as resulting from insufficient gain. Consequently, successive commands progressively increase the system gain until the actuator roots become oscillatory.

One answer to this difficulty noted by Marx is more elaborate filtering, for example, by removing these troublesome low frequency components. The system, of course, would continue to have the disadvantage that it is input type limited, and to remove this limitation will require a basic change in the adaptive loop philosophy.

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[23] M. F. Marx, loc. cit., pp. 201-215.

The main contribution of Marx, so far as this author is concerned, is in the suggested use of filters and depolarizers with a system to derive a feedback signal to be used for the readjustment of the system's parameters. These ideas are developed further in the next chapter where parameter control is achieved independent of the parameter or input.

Osder's System

Osder's system[24] is best illustrated with a third order plant such as the one shown in Fig. 4. Except for the order of the system, the design problem is similar to the previous one used to illustrate Marx's system. Again, K_1 is assumed to vary so that it will be the job of the performance computer to keep the product $K_1K_2 = K_a$ (a constant). The root-locus plot of the system is shown in Fig. 5. Over the range of values that K_1K_2 is allowed to have (shown in heavy lines on the root-locus) in this system, ω_n is approximately held constant, and in Marx's system, the time constant was kept fixed. Like Marx's system, which was especially designed to take advantage of a system with a particular root-locus, Osder's system was also oriented to take advantage of the constant ω_n.

The response of the system is determined primarily by the complex roots of the system, and it is the function of

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[24] S. S. Osder, "Sperry Adaptive Flight Control System," Proc. Self-Adaptive Flight Control Systems Symp. (January, 1959), pp. 81-122.

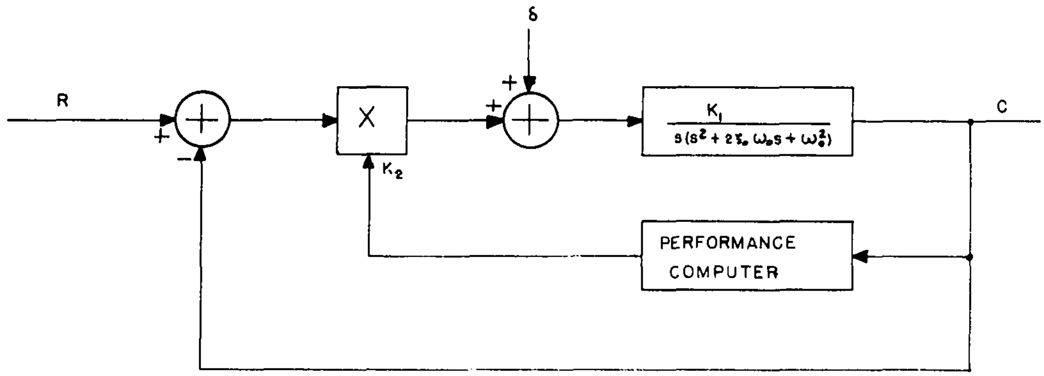

FIG. 4. OSDER'S SYSTEM. FROM ADAPTIVE CONTROL SYSTEMS, BY MISHKIN & BRAUN. COPYRIGHT 1961. McGRAW-HILL BOOK CO., INC. USED BY PERMISSION.

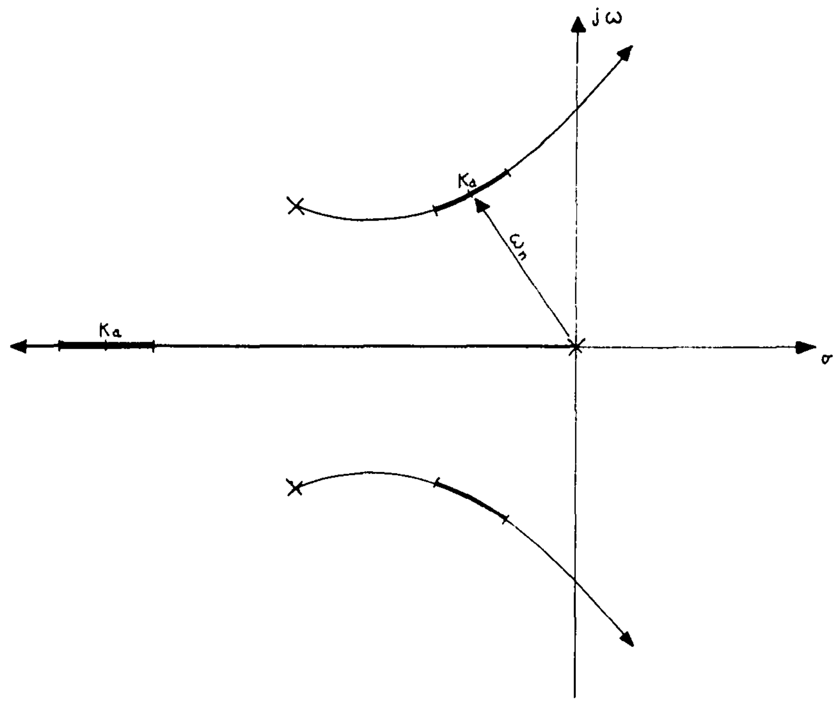

FIG. 5. ROOT-LOCUS DIAGRAM OF OSDER'S SYSTEM.

the performance computer to determine their location. This is done by applying an impulse function as noted and counting number of sign reversals in a given period of time at the output. The impulse response is approximately

$$\frac{C}{\delta} = \frac{K_t}{s^2 + 2\zeta\omega_n s + \omega_n^2} . \qquad (2.2)$$

The time response when δ is a unit impulse is approximately

$$c(t) \approx \frac{K_t}{\omega_n(1-\zeta^2)^{\frac{1}{2}}} \epsilon^{-\zeta\omega_n t} \sin(\omega_n t\sqrt{1-\zeta^2}) . \qquad (2.3)$$

From Equation (2.3) it is clear that the impulse response changes sign when

$$t = \frac{m\pi}{\omega_n(1-\zeta^2)^{\frac{1}{2}}} \quad m = 1,2,3,\ldots \qquad (2.4)$$

Since ω_n is essentially constant for the allowable range of values of the product K_1K_2, the values of t for which c = 0 are directly related to ζ. The number of reversals in sign of the impulse response during the fixed sampling interval is, therefore, a measure of the system's damping ratio. When the desired damping ratio is denoted by ζ_d, the number of sign reversals in the sampling interval for $\zeta < \zeta_d$ is larger than $\zeta = \zeta_d$, and the number of sign reversals for $\zeta_d < \zeta$ is less than for $\zeta = \zeta_d$. The performance computer counts the number of sign reversals in a fixed period of time and, based on this count, adjusts the gain K_2.

When no input signal $r(t)$ is applied, the operation described is valid. When there is an applied command signal, the validity of the foregoing relation between the system damping ratio and the number of changes in sign at the output is questionable. This problem is circumvented by applying the test impulses when no command is present or when the transients due to an applied command signal have subsided. The system has been simulated for an F-100C aircraft, and satisfactory performance was obtained.

The deficiencies of this system are obvious. First, it is not easily generalized since it depends upon a root-locus with certain desirable characteristics. Second, it depends upon the application of a signal which perturbs the output to some degree. Third, the updating must be done during a period when there are no input signals present and all previous transients have subsided. In many systems, these last two deficiencies would be too objectionable to be considered as a practical solution to the problem.

The System of Anderson, Buland, and Cooper[25]

The system which is discussed in this section utilizes the principles of statistical design theory to determine the

[25]G. W. Anderson, R. N. Buland, and G. R. Cooper, *loc. cit.*, pp. 102-108.

impulse response of the plant. Again, a plant which can be represented as a second-order system is to be controlled in such a way that the damping ratio ζ is maintained constant. A figure-of-merit computer is used to establish the damping ratio based on the impulse response measurement. The output from the figure-of-merit computer is used to adjust the system open-loop gain to the correct value of the damping ratio.

The impulse response of the system is to be approximated by

$$G(s) = \frac{\omega_n^{\ 2}}{s^2 + 2\zeta\omega_n s + \omega_n^{\ 2}} \tag{2.5}$$

or

$$g(t) = \frac{\omega n}{(1-\zeta^2)^{\frac{1}{2}}} \epsilon^{-\zeta\omega_n t} \sin(\omega_n t\sqrt{1-\zeta^2}). \tag{2.6}$$

The impulse response given in Equation (2.6) is represented in Fig. 6. The figure of merit used for the system is constructed from the positive and negative areas A_+ and A_-, respectively, of the impulse response. The figure of merit is defined as

$$M = A_+ - kA_- \tag{2.7}$$

where k is chosen to make M = 0 when the desired system damping ratio is attained.

Anderson, et al., show that

$$M = \frac{1-k\ \exp(-\zeta\pi\sqrt{1-\zeta^2})}{1-\exp(-\zeta\pi\sqrt{1-\zeta^2})} \tag{2.8}$$

When $k = 6.1$, M versus ζ is shown in Fig. 7. In this case, when $\zeta > \frac{1}{2}$ the figure of merit is positive and when $\zeta < \frac{1}{2}$ the figure of merit is negative. Furthermore, the figure of merit can be made to null at any value of ζ by proper choice of k. The output from the figure-of-merit computer is, therefore, an effective error signal on which to base automatic changes of compensation.

A periodic signal was injected into the system so that the crosscorrelation function could be obtained by integrating only over this period. In the system built, twelve crosscorrelators were used to find the values for the impulse response at the times τ_i indicated in Fig. 8. The figure of merit was approximated by

$$M = \sum_{m=1}^{12} A_m g(\tau_m) \tag{2.9}$$

where
$$A_m = \begin{cases} a_m & g(\tau_m) > 0 \\ ka_m & g(\tau_m) < 0 \end{cases}$$

and
$$a_m = \tfrac{1}{2}(\tau_{m+1} - \tau_{m-1}). \tag{2.9}$$

The over-all configuration is shown in Fig. 9.

This technique of attack to the solution to adaptive systems should be a powerful one. However, for simple systems, like the one here, the complexity of the auxiliary equipment to perform such a relatively simple task is overwhelming. Also, errors were introduced as a result of nonideal integrators, the use of a relatively short time interval

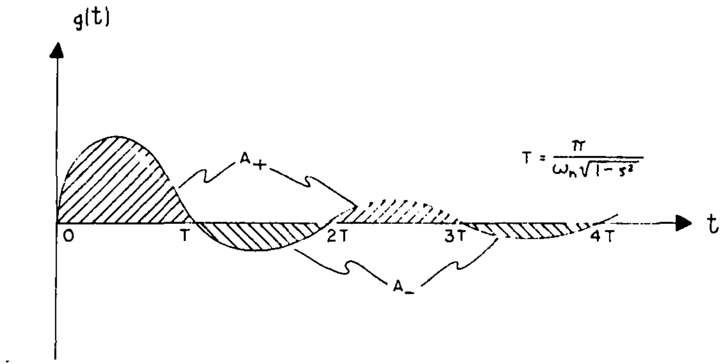

FIG. 6. IMPULSE RESPONSE OF A SECOND-ORDER SYSTEM. FROM ADAPTIVE CONTROL SYSTEMS, BY MISHKIN & BRAUN. COPYRIGHT 1961. McGRAW-HILL BOOK CO., INC. USED BY PERMISSION.

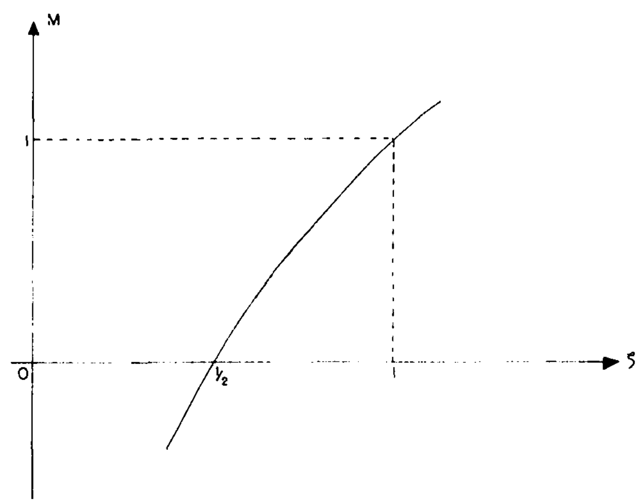

FIG. 7. FIGURE OF MERIT VERSUS DAMPING RATIO FOR K = 6.1. USED BY PERMISSION.

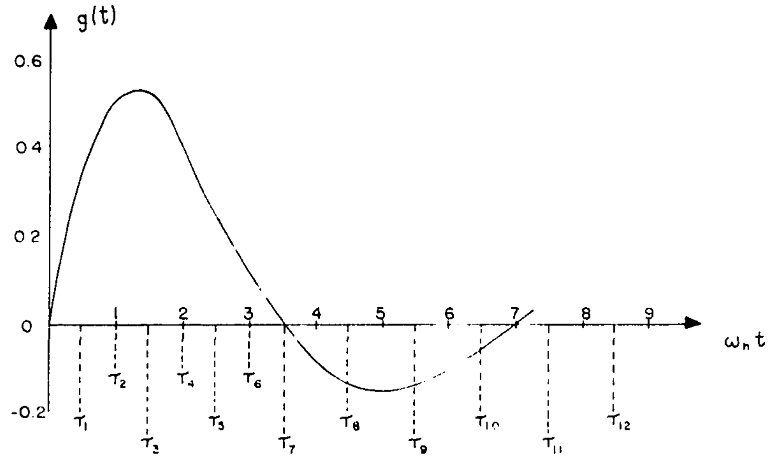

FIG. 8. IMPULSE RESPONSE OF SECOND-ORDER SYSTEM WITH ζ=0.5. USED BY PERMISSION.

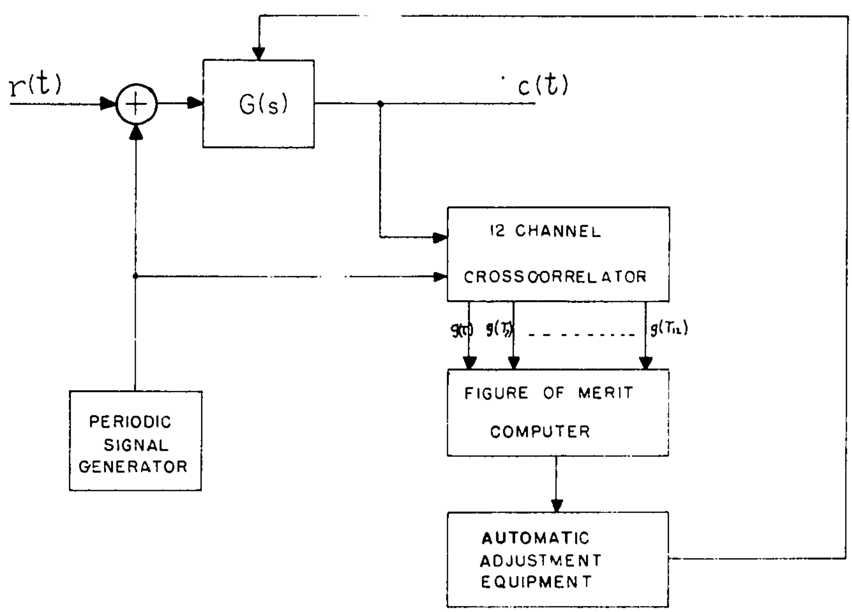

FIG. 9. SYSTEM OF ANDERSON, ET AL. USED BY PERMISSION.

to approximate the impulse response, and Equation (2.9). To adapt this technique to more general systems, figures of merit that can be readily calculated must be determined.

Whitaker's Method

The philosophy described in the previous sections requires the identification of some significant characteristic to obtain their adaptive capabilities. In Whitaker's system,[26] the desired adaptivity is achieved through automatic adjustment of the system's gains to minimize or null error quantities generated from the error between the output of a model and the system's output.

The basic principles of operation are illustrated in the system of Fig. 10. When no signal is applied to the system, the switches S_1 and S_2 are in position b. With the initiation of an input signal, S_1 and S_2 are both switched to position a for 1 and 10 seconds, respectively, and they are then returned to position b. The switching operation of S_1 and S_2 occurs each time the input signal changes level. S_3 is switched in accordance with the sign of the input signal so that the error signal will not be sensitive to the input polarity.

Each time the level changes, the gains K_{12} and K_{22} are altered by an amount which is proportional to the

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[26] H. P. Whitaker, "MIT Presentation," Proc. Self-Adaptive Flight Control Systems Symp. (January, 1959), pp. 58-80.

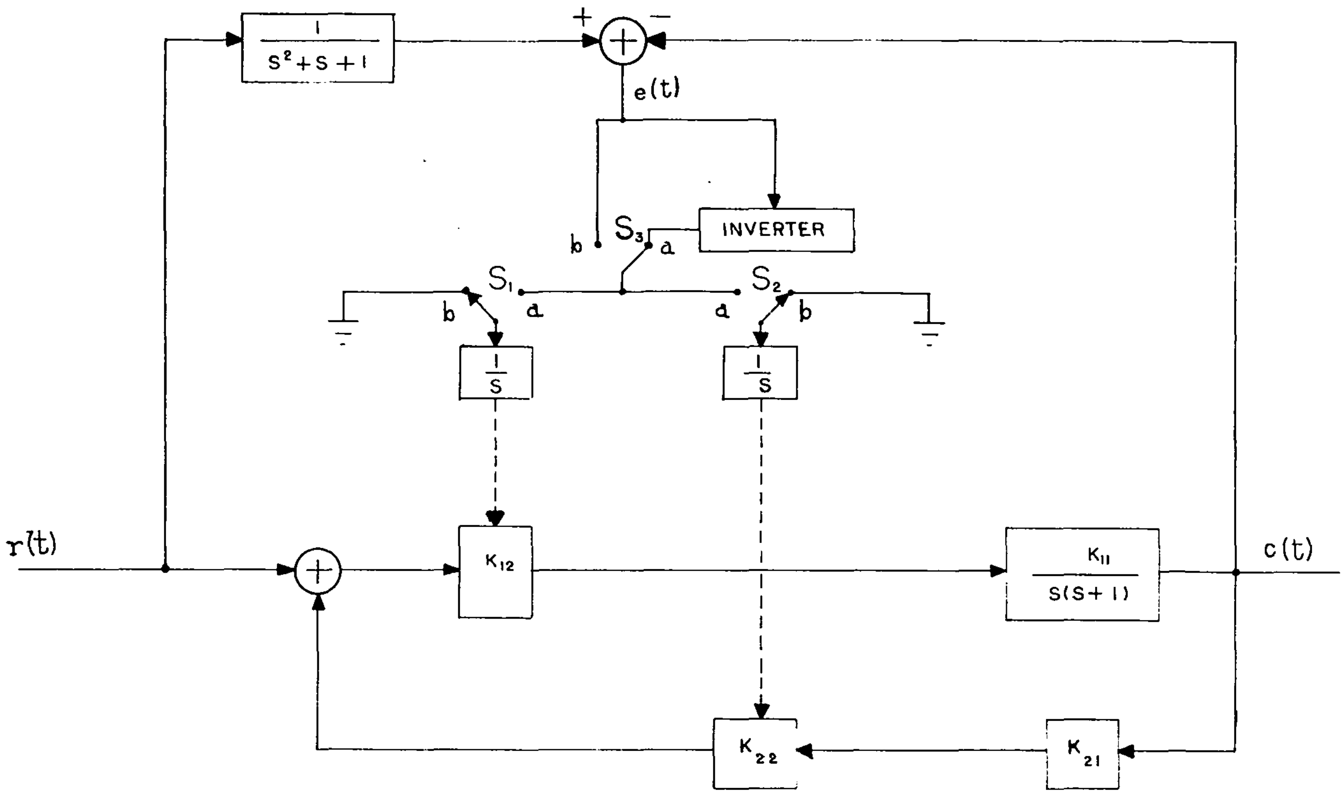

FIG. 10. MODEL-REFERENCE ADAPTIVE SYSTEM. FROM ADAPTIVE CONTROL SYSTEMS, BY MISHKIN & BRAUN. COPYRIGHT 1961. McGRAW-HILL BOOK CO., INC. USED BY PERMISSION.

respective error quantities until K_{12} and K_{22} are such that the error criteria

$$\int_{t_o}^{1-t_o} e(t)dt = \int_{t_o}^{10-t_o} e(t)dt = 0 \tag{2.10}$$

are satisfied. From this point on, the model and system responses are about the same so long as K_{11} and K_{21} do not change.

The error quantities are chosen so that they are more sensitive to the system parameter which they control than to the remaining parameters. Bongiorno[27] has illustrated the effectiveness of this error criteria for the situation when the input is a positive step and shows that it does, indeed, have the desired properties.

Care must be taken when specifying the desired error criteria in order to insure convergence and to be sure that the convergence is single-valued. Analog computer studies and flight tests programs have indicated satisfactory performance of these model-reference systems.[28]

Summary

These papers serve to illustrate several facts concerning the state of the art of adaptive systems. One, no

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[27] E. Mishkin and L. Braun, Jr., _loc. cit_, pp. 339-342.

[28] H. P. Whitaker, _loc. cit._, pp. 58-80.

adaptive scheme, to date, is generally applicable to all control systems. Also, there are about as many ways to add an adaptive-loop to a system as there are groups who have considered the problem. The methods discussed were all oriented towards adjusting the gain, and it is not clear in each case what would be necessary if it was desirable to control some other parameter.

The designers of these systems have shown considerable ingenuity in both their methods of control and in their ability to take advantage of a particular system's characteristics. The system to be presented in the following chapter is applicable to other parameters as well as the gain. It should be pointed out, however, that the gain is a natural parameter to adjust since it is usually accessible and variations in other parameters can often be compensated for directly by changing the gain.

Most of the advantages and disadvantages were pointed out in each section. A comparison of these systems with the one presented in this dissertation is made in Chapter VII.

CHAPTER III

ERROR COMPARISON USED IN PARAMETER CONTROL

This chapter presents a method of control which, although simple in both concept and mechanization, has not been heretofore disclosed in the literature. The system is to be system adaptive as opposed to being signal adaptive, i.e., some plant parameter is slowly time-varying, and for each state of this parameter, there is a desired transmittance to be achieved. An example is given in this chapter where the plant gain is the time-varying parameter so that the adaptive loop varies the system's open-loop gain such that direct compensation is achieved. Since it is not always possible to directly compensate for the time-varying parameter (it will not always be accessible), Chapters V and VI will be concerned, in part, with methods to circumvent this event. When the parameter is accessible and direct compensation is feasible, then the adaptive portion is implemented in exactly the same way as in the example.

The present method will have certain advantages over those presented in Chapter II; however, a general comparison of the methods will be reserved for Chapter VII. The adaptive systems of the previous chapter were oriented towards the adjustment of a particular parameter, the gain, and were generally "tuned" to certain type of inputs for correct operation. The method to be presented represents an approach to overcoming both of these diffuculties. The case where a single slowly time-varying parameter is to be adjusted will be considered in this chapter together with an example, while Chapter VI will extend the results to multiple time-varying parameter plants.

Let $c(r,K,t)$ be the output of a control system where K is the slowly time-varying parameter, and it is assumed that K can be compensated for directly. Let $c(r,K',t)$ and $c(r,K'',t)$ be the outputs from two auxiliary plants that differ from the main system or plant only in the value of the parameter K. Two error signals are then developed as

$$e_1(t) = c(r,K,t) - c(r,K',t) \tag{3.1}$$

and

$$e_2(t) = c(r,K,t) - c(r,K'',t)\,. \tag{3.2}$$

These error signals will ordinarily be sensitive to the sign of the input so they are applied to depolarizers whose outputs are nonnegative and have the following properties:

1) $d\left[e\right] = 0$ if and only if $e = 0$,
2) $d\left[e_1\right] = d\left[e_2\right]$ if and only if $\left|e_1\right| = \left|e_2\right|$,
3) $d\left[e_1\right] < d\left[e_2\right]$ if and only if $\left|e_1\right| < \left|e_2\right|$.

A signal $u(r,K,K',K'',t)$ is formed by taking the difference of the two depolarized outputs. This gives

$$u(r,K,K',K'',t) = d\left[c(r,K,t) - c(r,K',t)\right] - d\left[c(r,K,t) - c(r,K'',t)\right] = d\left[e_1\right] - d\left[e_2\right] \tag{3.3}$$

and

$$\overline{\Delta u} = \left\{\overline{\frac{\partial d}{\partial e_1}}\,\overline{\frac{\partial e_1}{\partial c}}\left(\overline{\frac{\partial c(r,K,t)}{\partial r}} - \overline{\frac{\partial c(r,K',t)}{\partial r}}\right) - \overline{\frac{\partial d}{\partial e_2}}\,\overline{\frac{\partial e_2}{\partial c}}\left(\overline{\frac{\partial c(r,K,t)}{\partial r}} - \overline{\frac{\partial (r,K'',t)}{\partial r}}\right)\right\}\Delta r + \left(\overline{\frac{\partial d}{\partial e_1}}\,\overline{\frac{\partial e_1}{\partial c}}\;\overline{\frac{\partial c(r,K,t)}{\partial K}} - \overline{\frac{\partial d}{\partial e_2}}\,\overline{\frac{\partial e_2}{\partial c}}\,\overline{\frac{\partial c(r,K,t)}{\partial K}}\right)\Delta K + \cdots \tag{3.4}$$

where the bar indicates that an average has been taken over the last T seconds which exceeds the time duration for the system's transients to subside. This averaging is to decrease the system's adaptive loop sensitivity to the instantaneous time t, and the actual techniques used will be discussed later. Equation (3.4) illustrates the motivation for this connection scheme. If K' and K" are restricted to a small region about K, then

$$\overline{\frac{\partial c(r,K,t)}{\partial r}} - \overline{\frac{\partial c(r,K',t)}{\partial r}} \text{ and } \overline{\frac{\partial c(r,K,t)}{\partial r}} - \overline{\frac{\partial c(r,K'',t)}{\partial r}}$$

are relatively small. Within this same small region, K' and K" are chosen such that $\overline{\frac{\partial d}{\partial e_1}}$ is a monotonically increasing function of K as K K' and $\overline{\frac{\partial d}{\partial e_2}}$ is a montonically decreasing function as K K" for K in the interval (K',K"). This will make the term

$$\overline{\frac{\partial d}{\partial e_1}}\;\overline{\frac{\partial e_1}{\partial c}}\;\overline{\frac{\partial c(r,K,t)}{\partial K}} - \overline{\frac{\partial d}{\partial e_2}}\;\overline{\frac{\partial e_2}{\partial c}}\;\overline{\frac{\partial c(r,K,t)}{\partial K}} =$$

$$\left(\overline{\frac{\partial d}{\partial e_1}} - \overline{\frac{\partial d}{\partial e_2}}\right)\overline{\frac{\partial e_1}{\partial c}}\;\overline{\frac{\partial c}{\partial K}} \tag{3.5}$$

a monotonic function of K in the interval (K',K"). Also, the terms comprising the coefficients of ΔK tend to be additive while the coefficients of Δr tend to become small. It appears that this connection scheme has the desired properties, viz., being sensitive to changes in K and less sensitive to changes in the input r, at least if K is constrained to small enough interval (K',K"). The size of the region that can actually be used is determined by the range of the interval (K',K") over which the above results generally hold true. In many cases, the range of K can exceed the interval (K',K") since the monotonic properties noted will often hold true outside this interval. Figure 11 illustrates the general adaptive scheme for control of a single parameter.

The averaging device is necessary in order to keep the adaptive action from responding to instantaneous changes in the error signals. For example, take the case when K has the correct value and no corrective signal should be sent to

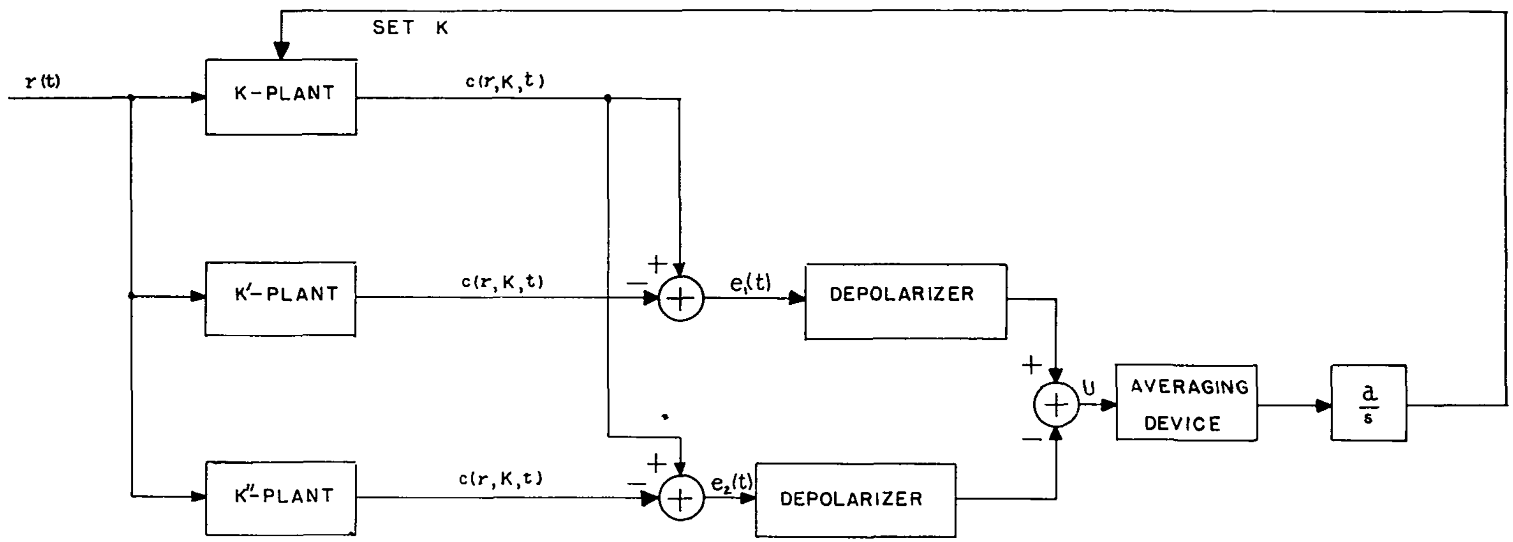

FIG. 11. ERROR COMPARISON ADAPTIVE CONTROL SYSTEM.

the integrator. The difference between the depolarizer outputs will not cancel to give an output that is identically zero; however, for this scheme to work, we know that the average input to the integrator must be zero if we are not going to change the output of the integrator. This average, then, is to be a function of time, and it should emphasize the last T seconds. If the average p(t) were going to be calculated on a digital or computer then the function

$$p_1(t) = \frac{1}{T}\int_{t-T}^{t}\left(d\left[c(\tau,K) - c(\tau K')\right] - d\left[c(\tau,K) - c(\tau,K'')\right]\right) d\tau \tag{3.6}$$

might be a reasonable average to work with. However, since linear systems were involved already, the function

$$p_2(t) = \int_{0}^{t} \epsilon^{\frac{-(t-\tau)}{T}} \left(d\left[c(\tau,K)-c(\tau,K')\right] - d\left[c(\tau,K)-c(\tau,K'')\right]\right) d\tau \tag{3.7}$$

was used due to the simplicity of its physical implementation. It should be realized that neither $p_1(t)$ nor $p_2(t)$ will, in general, be identically zero even when K is set to the correct value, and, for this reason, the output of the integrator that follows the averaging device will cause K to vary about its correct value. These variations can be minimized by reducing a in Fig. 11 at the expense of decreasing the parameter-loop gain which, of course, reduces the amount of correction that can be applied to K for a given input. The variations can also be minimized by averaging over a longer time interval, but this will also reduce the effectiveness of the adaptive loop. In practice, some compromise must be reached between the adaptive loop gain and allowable deviation of the parameter K from its correct value. Note that K will only receive correction during periods when input transients exist since the system is linear up to the depolarizing element, all plant functions have the same final value, and $d\left[0\right] = 0$.

The depolarizer $d_1\left[e\right] = e^2$ and $d_2\left[e\right] = \left|e\right|$ will be the ones actually used. $d_1\left[e\right]$ is used since it can be easily implemented on the analog computer and lends itself to analysis. $d_2\left[e\right]$ is used since it can be implemented with rather elementary hardware. Some general statements about $d_1\left[e\right]$ and $d_2\left[e\right]$ can now be made. Suppose r(t) is applied to the input of the system of Figure 11 giving rise to error e(t). Then, since the system is linear up to the depolarizer, if Br(t) is applied to the input where B is a constant, the error will be Be(t). $d_1\left[Be(t)\right] = B^2e^2(t)$, and because both the averaging device and the integrator are linear, the parameter's open loop output at the integrator varies as the square of the input for a given r(t). Similarly, when $d_2\left[e\right]$ is used, the parameter's open loop output at the integrator varies as the absolute value of the input for a given r(t). If the lag time for the avererage device were large enough so that the value of K was not effectively changed for the duration of the input and plant transients, then the above statements would also be true for the closed loop parameter gain.

The first experimental system that was built is shown in Fig. 12 where the operations of squaring and multiplying are performed in the time domain. Physically, the plant represents any system that can adequately be described with poles normalized to 0.0 and -1.0 and with slowly time-varying gain K_2. The object of the closed loop system is to keep the overall gain product $K_1K_2 = K$ constant in the same manner as Marx[29]. Although a multiplier was actually used, the gain K_1 could just as easily have been set with a servo driven potentiometer; by picking a servo motor with suitable characteristics, both the integrator and the averaging network could possibly be eliminated.

An attempt at analysis of the system of Fig. 12 will now be made for the class of input functions r(t) of the form Be^{-At}, which includes both the step and impulse functions. The Laplace transforms of the two errors are

$$L\left[e_1\right] = R(s)\left[F(s,K)-F(s,K')\right] \quad \text{and} \tag{3.8}$$

$$L\left[e_2\right] = R(s)\left[F(s,K)-F(s,K'')\right] \; . \tag{3.9}$$

Realizing that multiplication in the time domain corresponds

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[29] M.F. Marx, loc. cit., pp. 201-215

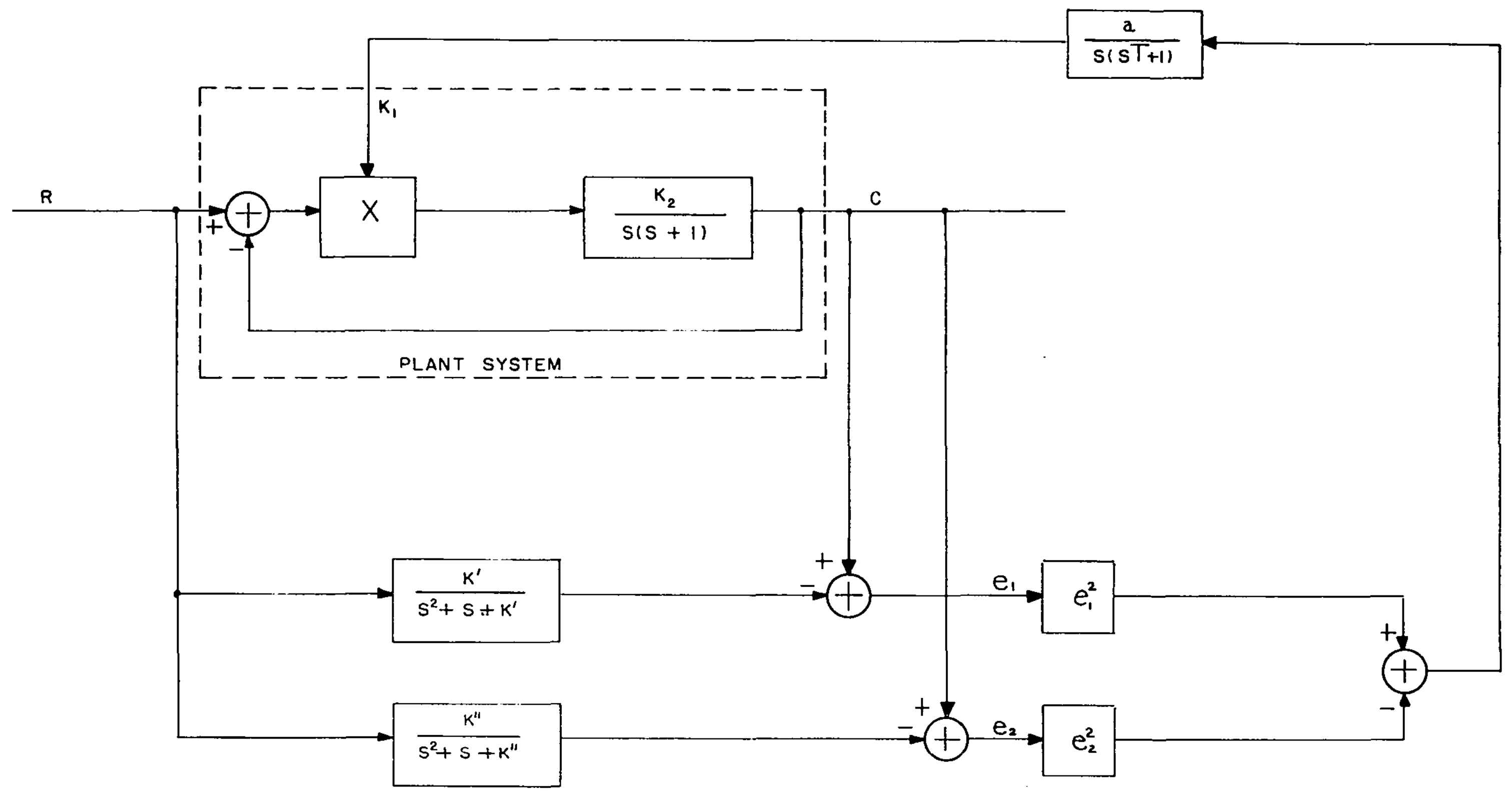

FIG. 12. EXPERIMENTAL SECOND-ORDER PLANT WITH VARIABLE GAIN K_2.

to complex convolution in the transform domain,[30] it follows that

$$L\left[e_1{}^2-e_1{}^2\right] \equiv \frac{1}{2\pi j}\int_{Br} R(\lambda)R(s-\lambda)\Big\{F(\lambda,K')F(s-\lambda,K')$$

$$-F(\lambda,K'')F(s-\lambda,K'')-2F(s-\lambda,K)\left[F(\lambda,K')-F(\lambda,K'')\right]\Big\}d\lambda \quad (3.10)$$

where advantage has been taken of the fact that

$$\int_{Br} F_1(\lambda)F_2(s-\lambda)\,d\lambda = \int_{Br} F_1(s-\lambda)F_2(\lambda)\,d\lambda \quad (3.11)$$

as long as the integration can be performed over a suitable contour Br. The open loop change in K, ΔK, for a given input function B^{-At} is just the final change in the value of the integrator's output. This can be calculated as

$$\Delta K = \lim_{s\to 0} \frac{sa}{s(sT+1)} \; L\left[e_1{}^2 - e_2{}^2\right]$$

$$= \frac{aB^2}{2\pi j}\int_{Br} \frac{1}{A^2-\lambda^2}\Big\{F(\lambda,K')F(-\lambda,K')-F(\lambda,K'')F(-\lambda,K'')$$

$$-2F(-\lambda,K)\left[F(\lambda,K')-F(\lambda,K'')\right]\Big\}\;d\lambda. \quad (3.12)$$

Since the poles of $F(\lambda,K)$ are in the left half-plane, the poles of $F(-\lambda,K)$ will live in the right half-plane so that

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[30] M. F. Gardner and J. L. Barnes, Transients in Linear Systems, Vol. 1 (New York: John Wiley and Sons, Inc., 1957), pp. 275-277.

a suitable contour of integration is along the j-axis of the λ-plane.

A typical term of Equation (3.12) is of the form

$$\frac{aB^2K_1K_2}{2\pi j}\int_{Br}\frac{d\lambda}{(A^2-\lambda^2)\ (\lambda^2+\lambda+K_1)\ (\lambda^2-\lambda+K_2)}\ .$$

Closing the contour, Br, about the left half-plane, the residues are as follows:

$$\mathrm{Res}\left[\lambda = -A\right] = \frac{aB^2K_1K_2}{2\pi j}\ \frac{1}{2A(A^2-A+K_1)\,(A^2+A+K_2)}$$

$$\mathrm{Res}\left[\lambda = \frac{-1-\sqrt{1-4K_1}}{2}\right] = \frac{aB^2K_1K_2}{2\pi j\,(1-4K)^{\frac{1}{2}}\left[1+K_2-K_1+(1-4K_1)^{\frac{1}{2}}\right]}$$

$$\cdot\frac{1}{A^2+K_1-\frac{1}{2}+\frac{1}{2}(1-4K_1)^{\frac{1}{2}}}$$

$$\mathrm{Res}\left[\lambda = \frac{-1+\quad 1-4K_1}{2}\right] = \frac{-aB^2K_1K_2}{2\pi j\,(1-4K_1)^{\frac{1}{2}}\left[1+K_2-K_1+(1-4K_1)^{\frac{1}{2}}\right]}$$

$$\cdot\frac{1}{A^2+K_1-\frac{1}{2}-\frac{1}{2}(1-4K_1)^{\frac{1}{2}}}$$

$\mathrm{Res}\left[\lambda = -A\right]$ is to be read as the residue corresponding to the root $\lambda = -A$, etc. From these residues, it follows that

$$\frac{aB^2}{2\pi j}\int_{Br}\frac{1}{A^2-\lambda^2}\;F(\lambda,K_1)F(-\lambda,K_2)\,d\lambda =$$

$$\frac{aB^2K_1K_2\left[4A^3+\ 8A^2+A(2K_1+2K_2+4)+(K_1-K_2)^2+2K_1+2K_2\right]}{2A(A^2+A+K_1)\ \ (A^2+A+K_2)\left[(K_1-K_2)^2+2K_1+2K_2\right]}. \tag{3.13}$$

As a check on this result, it should be noted that it is symmetrical in K_1 and K_2 which must follow from the property of Equation (3.11).

From Equation (3.12),

$$\Delta K = aB^2\left(\frac{K'\left[A^3+2A^2+A(K'+1)+K'\right]}{2A(A^2+A+K')^2} - \frac{K''\left[A^3+2A^2+A(K''+1)+K''\right]}{2A(A^2+A+K'')^2}\right.$$

$$-\frac{KK'\left[4A^3+8A^2+A(2K+2K'+4)+(K-K')^2+2K+2K'\right]}{A(A^2+A+K)\ (A^2+A+K')\left[(K-K')^2+2K+2K'\right]}$$

$$\left.+\frac{KK''\left[4A^3+8A^2+A(2K+2K''+4)+(K-K'')^2+2K+2K''\right]}{A(A^2+A+K)\ (A^2+A+K'')\left[(K-K'')^2+2K+2K''\right]}\right). \tag{3.14}$$

Thus ΔK can be evaluated for a whole class as A varies in the interval $(0,\infty)$. As A becomes small, the input approaches a step function, and if B = A and $A\to\infty$, the input approaches an impulse. In Appendix A the function

$$\int_0^\infty e^2\ (K_1,K_2,t)\,dt$$

is tabulated for a range of values of K_1 and K_2 while A was allowed to vary through the interval $10^{-3} \le A \le 10^3$. Actually, the calculations were made for a wider range of A than indicated; however, the results remained the same as A was made less than 10^{-3} and the result decreased exactly as A^{-2} as A was made larger than 10^3.

The tables in Appendix A have several uses, e.g., the value of K such that

$$I = \int_0^\infty \left[e_1^2(t) - e_2^2(t)\right] dt = 0 \tag{3.15}$$

can be found. For this value of K, there would be no error developed to change K when the system is excited with this particular input. From physical reasoning, it can be argued that this value of K is in the interval K"<K<K'. Also,

$$\int_0^\infty e_1^2(K',K',t)\,dt - \int_0^\infty e_2^2(K',K'',t)\,dt = \int_0^\infty e_1^2(K'',K',t)\,dt - \int_0^\infty e_2^2(K'',K'',t)\,dt \tag{3.16}$$

so that the error is of the same magnitude when K = K' as it is when K = K". The last two comments are true in general and do not depend upon the particular plant. For the system to be of much practical value, the value of K that satisfies Equation (3.15) should be reasonably well-behaved so that the final pole-zero configuration is not overly dependent upon type of input applied. Fig. 13 is a graph of a root of Equation (3.15) versus A which shows that the root remains remarkably constant. In fact, for all values of K' and K" that were checked, the value of K turned out to be approximately

$$K \approx \frac{K'+K''}{2} \tag{3.17}$$

over the entire range of A which indicates that the desired desensitivity to the input r has apparently been achieved over this range.

The tables in Appendix A can also be used to determine I for the range of K, and this has been done in Table 1 for K' = 1.4 and K" = 0.7. Table 1 shows that for a given A the open loop error is a reasonably linear function of K over the entire range of values recorded. For step inputs ($A \rightarrow 0$), and, in this case, Table 1 shows that I is approximately

$$I \approx 1-K \tag{3.18}$$

for K' = 1.4 and K" = 0.7. More generally,

$$I \approx G(A)\ \left(\frac{K'+K''}{2} - K\right). \tag{3.19}$$

The open loop adaptive gain would equal the closed loop adaptive gain if there was a sufficient time delay in the adaptive loop so that system transients have effectively subsided before K is reset to a new value. Physically, this time lag is created by the two integrations involved. In this case, the change in loop gain K can be represented by

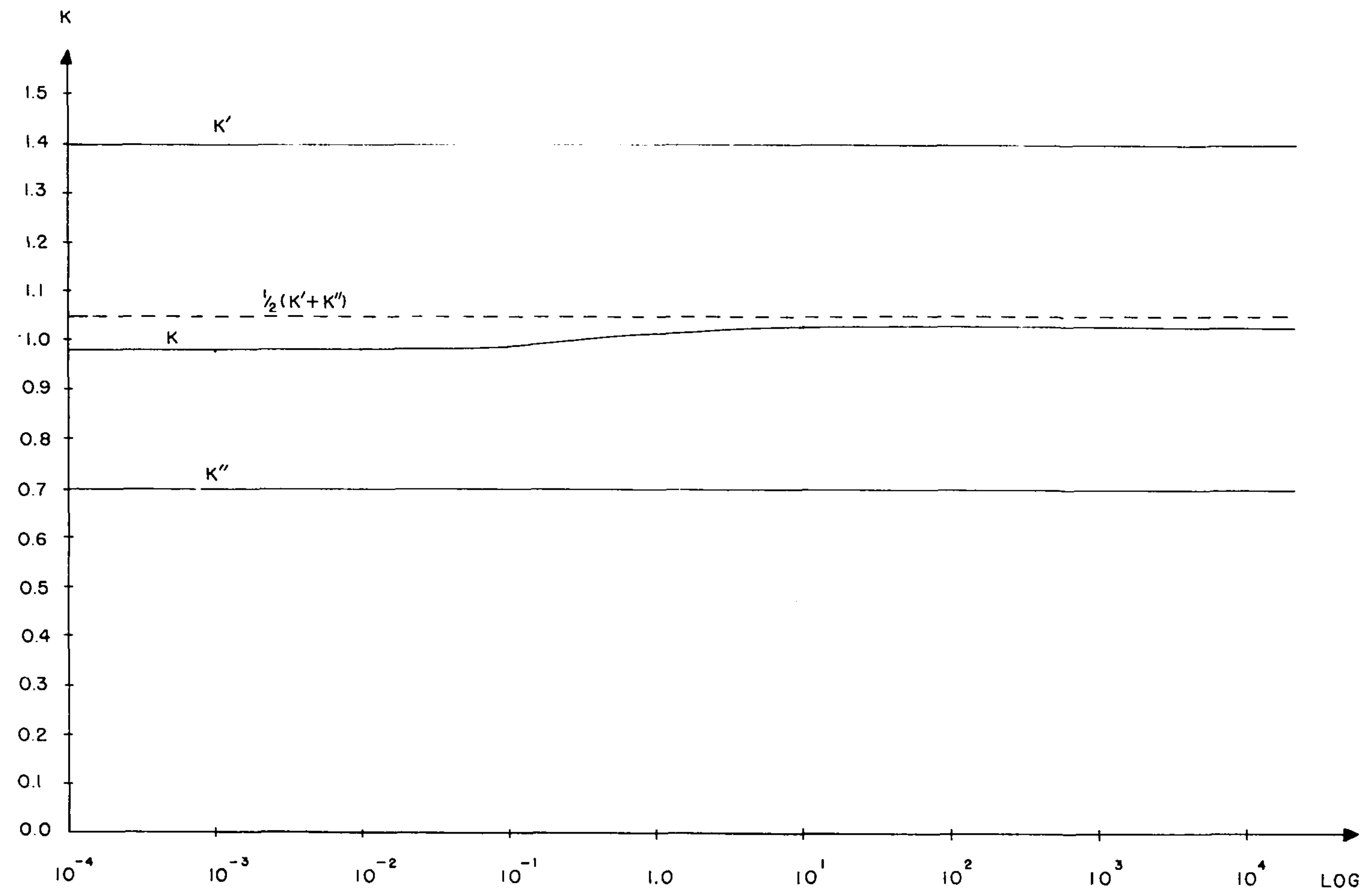

FIG. 13. VALUE OF K FOR ZERO ERROR.

TABLE 1

INTEGRAL ERROR EVALUATION

K' = 1.4 K" = 0.7 $I = \int_0^\infty (e_1^2 - e_2^2)\,dt$

$I = 0.xxxx(10^n) = 0.xxxx(n)$

K	$A = 10^{-3}$	$A = 10^{-2}$	$A = 10^{-1}$
0.6	0.4471(0)	0.4432(0)	0.4033(0)
0.7	.3228(0)	.3205(0)	.2956(0)
0.8	.2036(0)	.2025(0)	.1894(0)
0.9	.0918(0)	.0916(0)	.0881(0)
1.0	-.0109(0)	-.0078(0)	-.0061(0)
1.1	-.1039(0)	-.1029(0)	-.0919(0)
1.2	-.1868(0)	-.1852(0)	-.1689(0)
1.3	-.2597(0)	-.2577(0)	-.2366(0)
1.4	-.3228(0)	-.3205(0)	-.2956(0)
1.5	-.3769(0)	-.3743(0)	-.3461(0)

TABLE 1--Continued

A = 1.0	A = 10	A = 10^2	A = 10^3
0.1341(0)	0.2657(-2)	0.2715(-4)	0.2696(-6)
.1045(0)	.2110(-2)	.2141(-4)	.2142(-6)
.0724(0)	.1507(-2)	.1503(-4)	.1503(-6)
.0392(0)	.0871(-2)	.0886(-4)	.0886(-6)
.0065(0)	.0127(-2)	.0233(-4)	.0233(-6)
-.0249(0)	-.0407(-2)	-.0408(-4)	-.0408(-6)
-.0541(0)	-.1014(-2)	-.1028(-4)	-.1028(-6)
-.0808(0)	-.1585(-2)	-.1608(-4)	-.1608(-6)
-.1045(0)	-.2110(-2)	-.2142(-4)	-.2142(-6)
-.1250(0)	-.2584(-2)	-.2624(-4)	-.2625(-6)

$$\Delta K_1 = K_{1,n} - K_{1,n-1} = aB^2G(A)\left(\frac{K'+K''}{2} - K_{1,n-1}K_2\right) \tag{3.20}$$

which is a difference equation with solution

$$K = \frac{K'+K''}{2} + \left(K_o - \frac{K+K''}{2}\right)(1-aB^2K_2G(A))^n \tag{3.21}$$

where K_o is the initial value of K, $(K_{1,n-1}K_2)$, and n is the number of times that an input of the form B^{-At} has been applied allowing the transients to subside after each input. For stability,

$$\left|1-aB^2K_2G(A)\right| \leq 1 \tag{3.22}$$

or

$$0 \leq aB^2K_2G(A) \leq 2, \tag{3.23}$$

but since $aB^2K_2G(A)$ is non-negative, only the upper limit is of concern. For a step, G(A) is approximately one so that for stability, the magnitude B of an input step must be restricted:

$$B \leq \sqrt{\frac{2}{aK_2}}\ . \tag{3.24}$$

This equation illustrates the fact that maximum allowable range of B depends upon K_2 as well as a. Thus, the choice of a will depend upon the largest value that K_2 will be allowed to assume. This fact is more apparent when it is recognized that for a given input $K = K_2(K_1+\Delta K_1)$, and, if K_2 is large, then K_1 is small so that the change in K can be appreciable.

Fig. 14 is a graph of K versus n with $K_o = 0.5$ and $\frac{K'+K''}{2} = 1.0$ as $aB^2K_2G(A)$ takes on different values.

The system was built, and it performed pretty much as predicted. It was observed that the reset adaptive loop error was proportional to the square of the input amplitude and, of course, Equation (3.23) had to be satisfied. However, Fig. 14 indicates that it would also be benificial if the average value of $aB^2K_2G(0) = 1$, thus minimizing the number of inputs necessary to reset K to its final value. Actually, for the system built, the time delay did not sufficiently approximate an ideal delay so that K was always disturbed about its correct value even after reaching steady-state. A setting of the average value of $aB^2K_2G(0)$ to one-fourth proved to be effective with very little disturbance of K from its steady-state value.

One question that was too difficult to answer analytically was: What happens for input types that were not among the class of functions considered, and, in particular, what is the effect of applying another input before transients from a previous input have subsided? The system remained stable and continued to operate successfully under both conditions provided the total peak excursion during approximately one plant time-constant did not exceed the value of B determined by Equation (3.23). As previously mentioned, when the value $aB^2K_2G(0)$ was equal to one-fourth, K was not effectively disturbed from its steady-state values. For

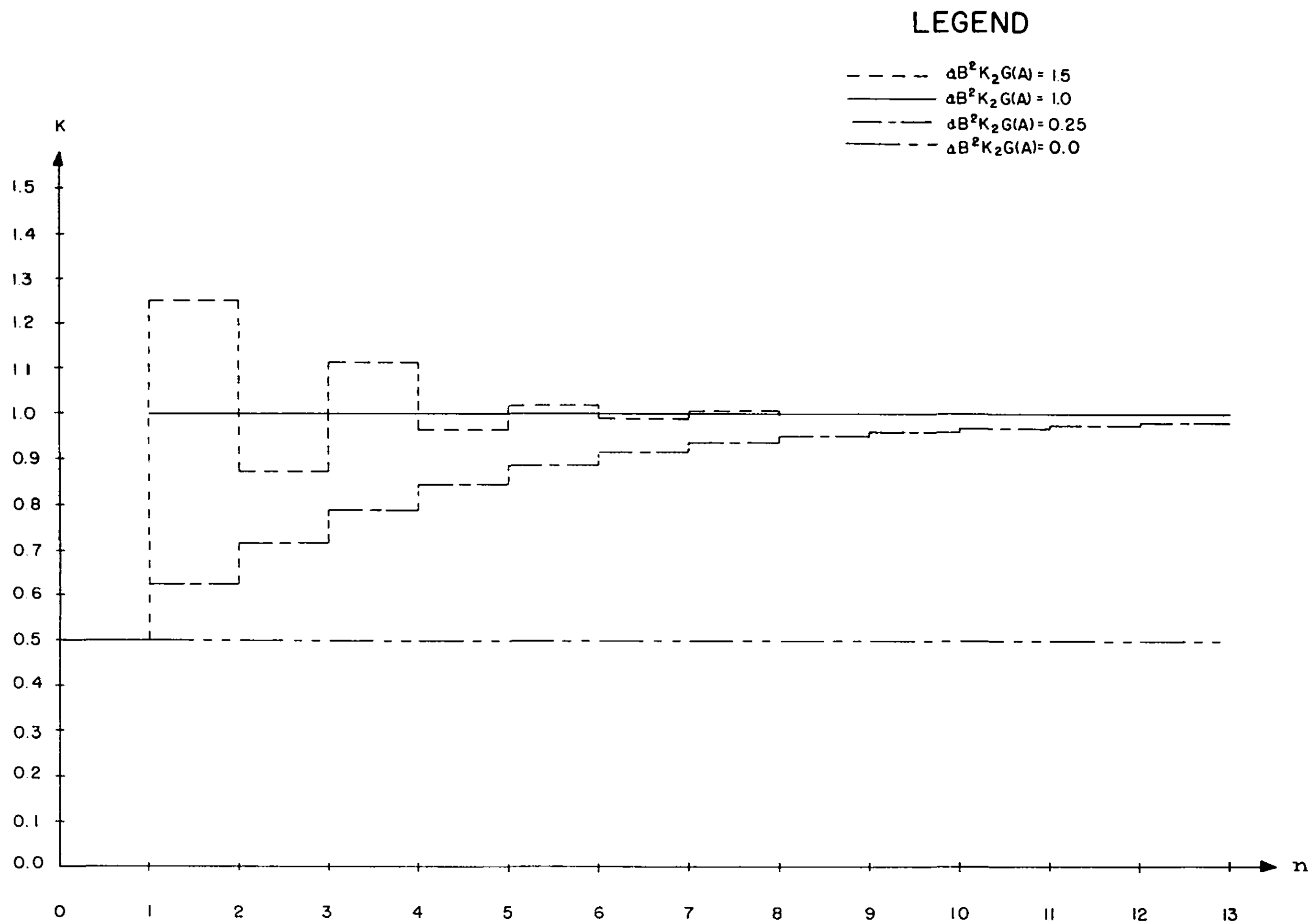

FIG. 14. K VERSUS THE NUMBER OF INPUTS (n) FOR $aB^2G(A)K_2$ = CONSTANT.

step inputs with this value of B, at least eight could occur with the same polarity without any danger of making the adaptive loop unstable. Also, there is no apparent harm in driving the system with an input so large that the adaptive loop is unstable so long as it does not happen repeatedly. This large input will cause the system to overshoot to a value that is farther from its steady-state value than before it occurred, but if this is followed by a series of smaller inputs, the system will stabilize, and K will approach its correct value. Whether or not it would be desirable to set a to a value this large would have to be determined for a particular system.

The most apparent limitation of the system lies in the fact that a perfect time delay is not realizable. If it were, the value of K could be held stationary until the transients from an input signal had subsided. The integrated error signal (if one existed) could then be used to reset K. This would allow one to choose the maximum gain without regard to any consequences other than stability. However, the system as it stands works quite well with the average of $aB^2K_2G(A) = \frac{1}{4}$, and there is much to be said for the economics of such a simple system.

The actual graphs of input and output signals, as well as the values of K_1 and K_2, are not displayed here since they did not differ pictorially from those of the system to be discussed next.

In an effort to explore the possibilities of building a more economical system, full-wave rectifiers were substituted for the squaring devices used as the depolarizing elements. This turned out to be a considerable improvement. The gain a could be increased about 50 per cent while maintaining the same margin of stability. The ratio of the maximum input that did not appreciably disturb K from its steady-state value to the maximum allowable input for stability was also increased. It was further observed that the adaptive loop error voltage varied directly with the amplitude of the input variable.

Fig. 15 is a record of the operation of this system. At the top is the parameter K_2 which has been disturbed at various intervals to illustrate the adaptive nature of the system. The degree of disturbance was considerably larger than that which would be encountered in what has been defined as a slowly time-varying system. However, the parameter K_2 could drift during periods of no input so that the product K_1K_2 could deviate considerably form its correct value. The next strip down shows K_2, and this should be compared with Fig. 14. The next strip down shows the input which was all square waves with frequency low enough to allow the system to reach steady-state for each half cycle, and the bottom strip shows the output.

In Figs. 15, 16, and 17, $K' = 1.4$ and $K'' = 0.7$ so that $K_{ave} = 1.05$. Steady-state values of K, K_1, and K_2 are

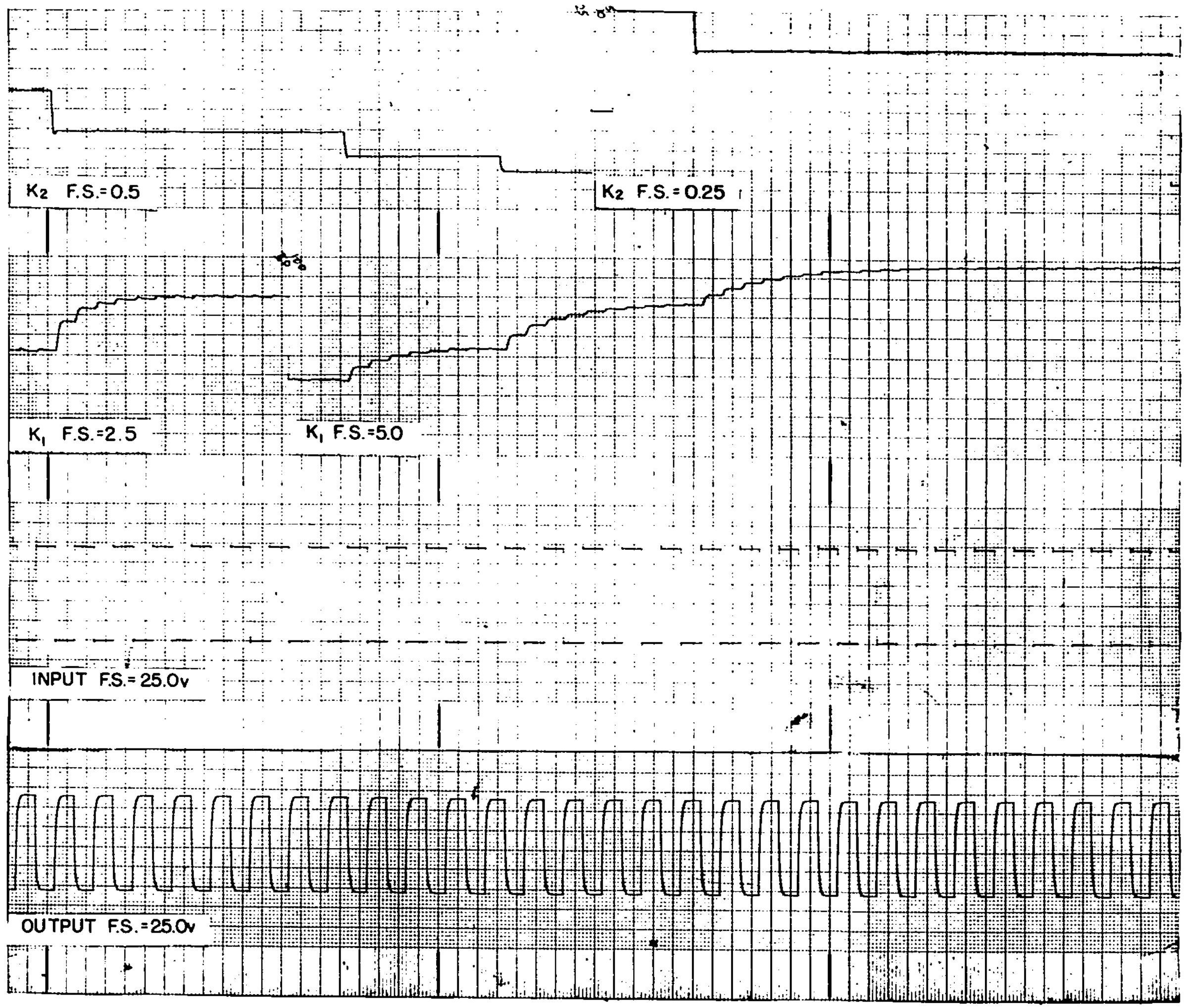

FIG. 15. PARAMETERS OF ADAPTIVE CONTROL SYSTEM. TIME: 20 SECONDS = 1.0 MAJOR DIVISION.

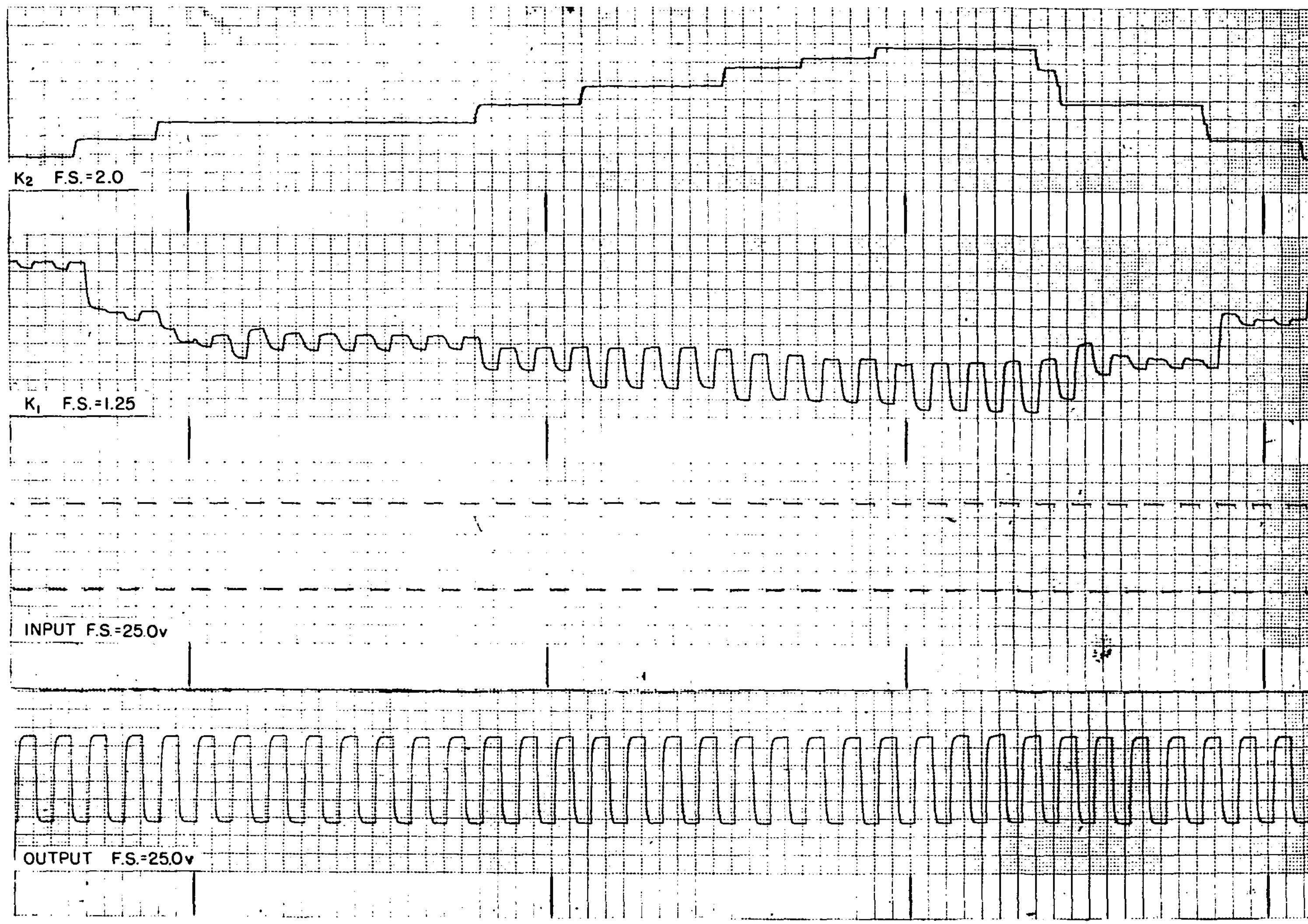

FIG. 16. PARAMETERS OF ADAPTIVE CONTROL SYSTEM. TIME: 20 SECONDS = 1.0 MAJOR DIVISION.

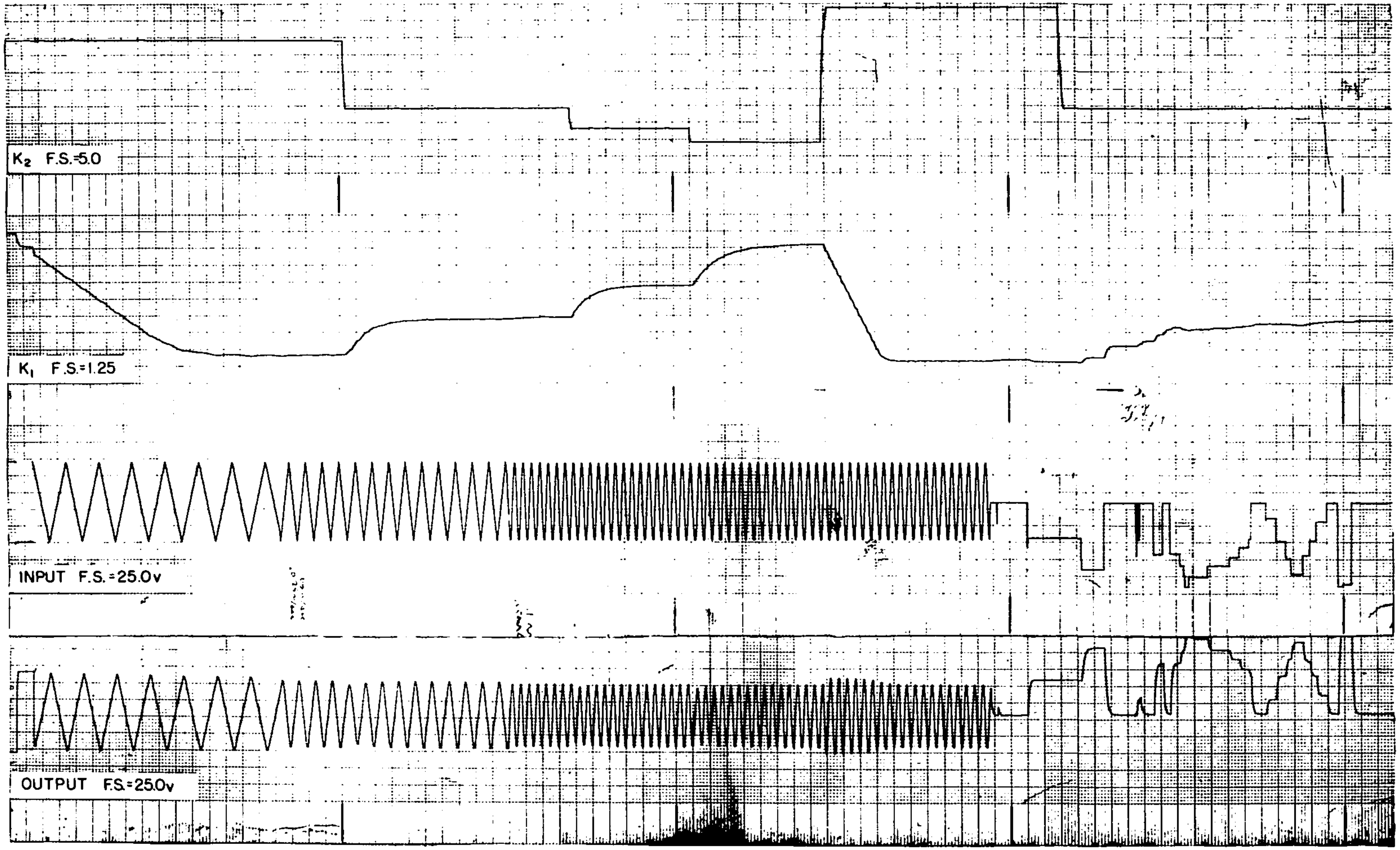

FIG. 17. PARAMETERS OF ADAPTIVE CONTROL SYSTEM. TIME: 20 SECONDS = 1.0 MAJOR DIVISION.

shown in Table 2.

TABLE 2

VALUES FOR THE ADAPTIVE PARAMETERS

Type Input	K_1	K_2	K_1K_2
Square	1.30	0.720	0.936
Square	2.00	0.475	0.950
Square	2.70	0.350	0.945
Square	4.85	0.200	0.970
Triangular	0.85	1.000	0.850
Triangular	1.90	0.475	0.904
Sine	1.95	0.475	0.925
Sine	2.90	0.325	0.944
Sine	4.20	0.225	0.945

Fig. 16 is similar to Fig. 15 except that the value of $aB^2K_2G(0)$ is much larger which was due, in this case, to a large value of K_2. This illustrates that a system can be stable and still not be too useful since the product K_1K_2 is disturbed too far from its steady-state value. A more nearly perfect time delay would be helpful in reducing these effects.

Fig. 17 depicts the adaptive process for a variety of input signals.

Before continuing, a review of the system from the standpoint of practicality will be made. First, it is noted that no restrictions were ever found necessary on the types of input that could be applied to the system, even with inputs that never allowed the system to reach a steady state, as long as the total peak changes during a system time constant remained below some upper bound. However, it was found that the parameter's loop gain had to be set the lowest for step-type inputs in order that the parameter loop be stable.

In terms of economics, it has already been noted that a servo driven potentiometer could be used to set K_1, although it should be noted that it would have to be driven with a suitable power amplifier. The auxiliary plants can usually be synthesized economically from passive networks, and using two difference amplifiers, the final system might take the form as shown in Fig. 18. As can be seen, this is not a complicated system considering its capabilities. If the values for parameters were not feasible, then active networks would be necessary. For these simple transfer functions, the cost would still not be excessive.

The system discussed has been kept simple purposely to allow an analysis of the system. It could be considered to be realistic for a more complicated system provided that the system could still be adequately described by a pair of conjugate poles and, further, that the system have two more poles than zeros. In this case the root-locus would have the general form shown in Fig. 19 with two of the complex roots approaching asymptotes of 90° and 270°. These systems

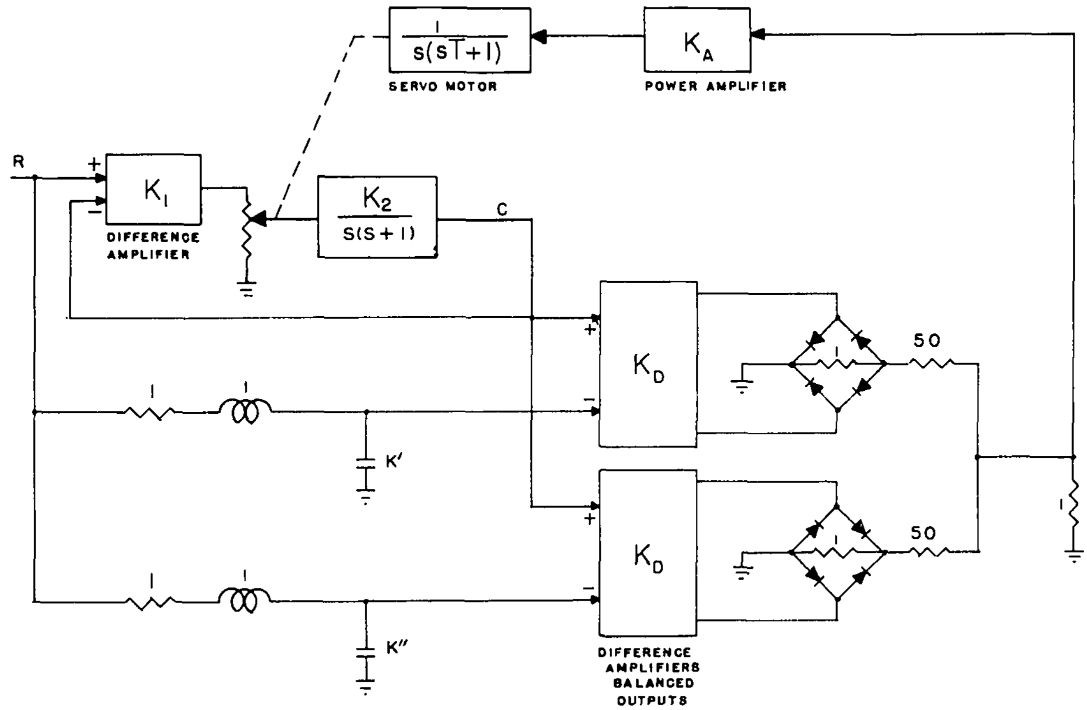

FIG. 18. SECOND-ORDER SYSTEM COMPONENTS.

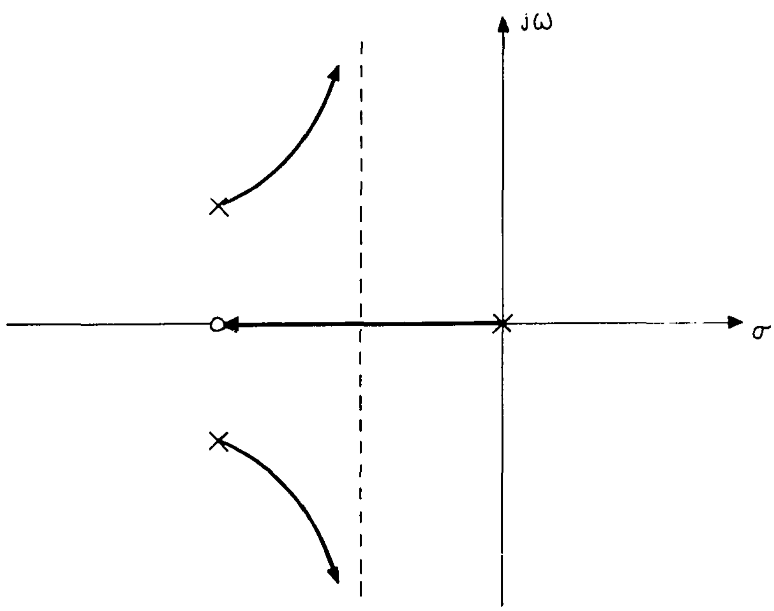

FIG. 19. POLE-ZERO PLOT OF SECOND-ORDER SYSTEM.

also have the particularly nice feature that they are usually stable for all values of positive K (although the adaptive loop might not be) if all of the poles and zeros are originally in the left-half plane as well as the break-point for the asymptotes.

This example was in terms of a linear feedback system, but the general theory of operation is not so restricted. The linear system could be analyzed thoroughly so that experimental and actual results could be compared.

The most serious limitation with this method of control apparently will be the size of the range of the time-varying parameter over which the monotonic properties hold as well as the property of being insensitive to input types. It may be that only a class of functions need to be considered as possible inputs, and over this class the range of the parameter can be extended. This is not the same as saying that only members of this class can be applied to the input; it only means that members of this class are the most probable, and an occasional function from outside the class applied to the input will not upset the operation as a whole.

CHAPTER IV

NOISE CONSIDERATIONS

For the case when the depolarizer output $d[e] = e^2$, the noise problem can be handled with some facility; however, when $d[e] = |e|$, only a few remarks can be made in general. The over-all configuration has been redrawn in Fig. 20 with noise sources introduced as well as the coefficients α and β. The auxiliary plants are assumed to be noiseless as compared with the main plant. This assumption is made simply because it is realistic, not because these noise sources would be any more difficult to handle than the ones shown.

The possibilities of applying different percentages of the depolarizer outputs, e.g., $\alpha d[e_1] - \beta\, d[e_2]$, should not be overlooked since this is a way of changing the steady-state value of K without changing the parameters K' and K". This would be the case if the auxiliary plant functions were constructed for a particular value of steady-state K, and then it was desired to operate with some other value of K. Another case might be when the particular values of K' and K" are not conveniently achievable. Of course, it is still necessary to maintain K in the range $K'' < K < K'$.

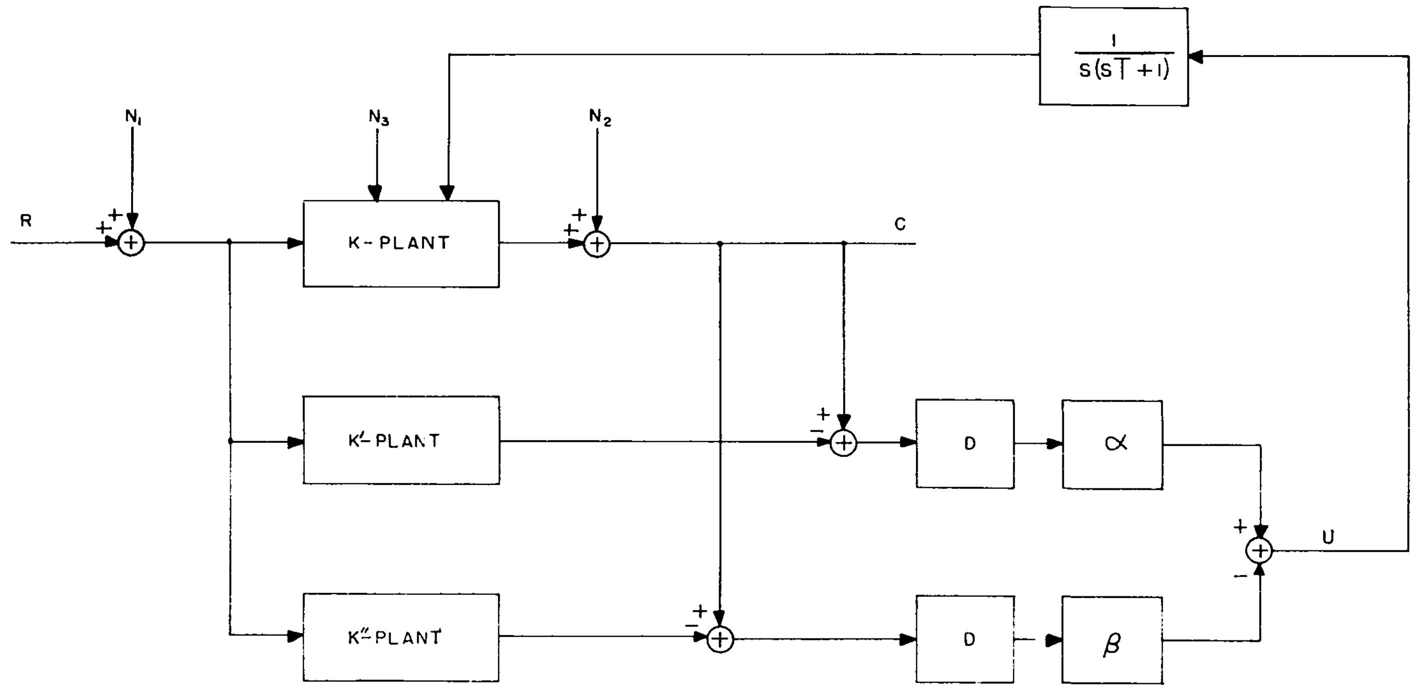

FIG. 20. ERROR FEEDBACK SYSTEM WITH NOISE SOURCES.

The coefficients α and β can have desirable settings when a noisy system is considered as in Fig. 20 where the effect of noise entering the system at different points is to be examined. The effect of the noise can be examined most easily if each noise source is considered separately as though it were the only source of noise in the system.

First it is noted that the noise source $n_1(t)$ at the input is welcome so far as the adaptive portion of the system is concerned. In fact, if there is no particular harm done for a particular system, then it would be desirable to inject a small amount of noise into the input. The noise could be injected only periodically, just often enough to make sure that the system is properly updated. This, then, would remove one of the disadvantages of the system, viz., that the system's parameters can develop lags during periods when there is no input. All of the systems which are continuously updated depend upon some sort of signal injection[31,32] which causes the system's output to be disturbed. This output disturbance is analyzed and appropriate corrections to the system are made. Therefore, the present system has no disadvantages at all when compared to these other methods because if signal injection is allowable, it will be used to advantage; if it is not allowable, none of these other systems could even be considered.

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[31] S. S. Osder, loc. cit., pp. 81-122.

[32] G. W. Anderson, R. N. Buland, and G. R. Cooper, loc. cit., pp. 102-108.

The noise source $n_2(t)$ represents noise introduced at the output. It might physically result from a radar antenna's being buffeted by wind or an aircraft's heading being varied as it encounters air currents. If the outputs of the plants labeled K, K', and K" are designated as s(t), s'(t), and s"(t) respectively, then u(t), as shown in Fig. 20, is

$$u(t) = \alpha d\left[s(t)-s'(t)+n_2(t)\right] - \beta d\left[s(t)-s''(t)+n_2(t)\right] . \tag{4.1}$$

In the absence of an input signal, it would be desirable for the noise not to alter the value of K. This will only be the case when $\alpha = \beta$. When signals are present, about the most that can be hoped for is that the noise will not alter the average value of u(t). Consider the case when $d\left[e\right] = e^2$. Then

$$\begin{aligned} u(t) = \alpha\Big(&s^2(t)+s'^2(t)+n_2^2(t)-2s(t)s'(t)+2s(t)n_2(t) \\ &-2s'(t)n_2(t)\Big) - \beta\Big(s^2(t)+s''^2(t)+n_2^2(t) \\ &-2s(t)s''(t)+2s(t)n_2(t)-2s''(t)n_2(t)\Big) \end{aligned} \tag{4.2}$$

and

$$\begin{aligned} \overline{u(t)} = \alpha\Big[&\overline{s^2(t)}+\overline{s'^2(t)}+\overline{n_2^2(t)}-2\overline{s(t)s'(t)}+2\overline{s(t)n_2(t)} \\ &-2\overline{s'(t)n_2(t)}\Big] - \beta\Big[\overline{s^2(t)}+\overline{s''^2(t)}+\overline{n_2^2(t)}-2\overline{s(t)s''(t)} \\ &+2\overline{s(t)n_2(t)}-\overline{s''(t)n_2(t)}\Big] \end{aligned} \tag{4.3}$$

where the bar is used to indicate the ensemble average value. If the signals are independent from the noise so that the average of these products can be set equal to the product of their averages, and if the noise $n_2(t)$ has zero for its mean value, then Equation (4.3) reduces to

$$\overline{u(t)} = \alpha\left[\overline{s^2(t)}+\overline{s'^2(t)}+\overline{n_2{}^2(t)}-2\overline{s(t)s'(t)}\right] -\beta\left[\overline{s^2(t)}+\overline{s''^2(t)}+\overline{n_2{}^2(t)}-2\overline{s(t)s''(t)}\right] . \tag{4.4}$$

Again, if $\alpha = \beta$, then

$$\overline{u(t)} = \alpha\left[\overline{s'^2(t)}-\overline{s''^2(t)}-2\overline{s(t)s'(t)}+2\overline{s(t)s''(t)}\right] \tag{4.5}$$

so that with the assumptions made, u(t) has its correct average value even in the presence of additive noise.

For the system discussed in Chapter III, it was shown analytically that $\overline{u(t)}$ was relatively insensitive to an entire class of input signals to the system. It was also found experimentally that $\overline{u(t)}$ was just as insensitive to signals not in this class. The adaptive feedback loop is used to force the time average of u(t) to zero. Assume now that the time average of u(t) is zero, so that the parameter K is not varying. Assume further that when this occurs the signals and noise in the previous equations are stationary and ergodic. The ensemble averages can then be replaced with their time averages. If noise is present and it is desirable to have the system converge to some other value than it does when $\alpha = \beta$, Equation (4.4) must be used. The averages indicated must all be calculated, and the subsequent values for which $\overline{u(t)} = 0$ must be determined. These averages are not known, in general, and must be estimated using a priori assumptions. For many systems these averages are not even stationary; thus, it will generally be expedient to set $\alpha = \beta$ when there is output noise present.

Even when $\alpha = \beta$ the noise terms can cancel only when long term averages are considered. The averaging that actually takes place in the adaptive portion must be as short as it is deemed practical since longer averaging times reduce the effectiveness of the adaptive loop. For this reason, it might be desirable to reduce the effects of the noise introduced at the output as much as possible before developing the difference signals. One way of doing this is to construct a filter that minimizes the mean-squared error caused by the noise's having been added to the signal. In this case, it is desirable to minimize

$$\overline{e^2(t)} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \left[\int_{-\infty}^{\infty} h(\tau)\; s(t-\tau)-n_2(t-\tau)\; d\tau - s(t) \right]^2 dt \tag{4.6}$$

which can be shown[33] to reduce

$$\overline{e^2(t)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau)h(\sigma)\varphi_{ii}(\tau-\sigma)\,d\sigma d\tau - 2\int_{-\infty}^{\infty} h(\tau)\varphi_{id}(\tau)\,d\tau - \varphi_{dd}(0) \tag{4.7}$$

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[33] Y. W. Lee, loc. cit., p. 369.

where

$h(t)$ = the unit-impulse response of the filter

$\varphi_{ii}(t)$ = the autocorrelation function of the filter input (the signal $s(t)$ plus the noise $n_2(t)$)

$\varphi_{id}(t)$ = the input-desired-output crosscorrelation function

$\varphi_{dd}(0)$ = the mean square value of the desired filter output, viz., the signal $s(t)$.

The minimization of Equation (4.7) requires the solution of the well-known Wiener-Hopf equation

$$\varphi_{id}(\tau) = \int_{-\infty}^{\infty} h(\sigma)\varphi_{ii}(\tau-\sigma)d\sigma \text{ for } \tau \geq 0. \tag{4.8}$$

When the above autocorrelation and crosscorrelation functions are known, $h(\sigma)$ can be solved for using the methods illustrated by Lee.[34] A major difficulty generally exists when using this method of attack, viz., the filters are realizable only in the sense that their output will not precede their input and that their Laplace transforms exist. However, they do not, in general, turn out to be simple ratios of polynomials in s that can be realized by using the simple processes of differentiation and integration. Often, the desired filters must be approximated, thus giving rise to another source of error.

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[34]Y. W. Lee, loc. cit., p. 369.

Additive noise sources introduced within the plant, $n_3(t)$, will give rise to a noise term added to the signal at the output if the plant is linear. This noise when referred to the output can then be treated the same as an additive output noise so that the remarks concerning the output noise are also true for these noise sources when they are referred to the output.

In the case of a nonlinear plant, little can be said in general about the effects of noise introduced within the plant; however, when $\alpha = \beta$ and there is no signal present, the noise will subtract out from the combined depolarizer outputs.

CHAPTER V

APPLICATION OF ADAPTIVE CONTROL TO AN AIRCRAFT

The control of high performance aircraft has probably done more to stimulate interest in adaptive control than any other control application. Preprogramming to compensate for the changing parameters such as speed, altitude, fuel consumption, etc., cannot be predetermined as precisely as in the case of a missile. This is due, in part, to the vast number of situations which can occur with an aircraft, such situations being created by the aircraft's mission, weather, etc.

Improvements in aircraft control are necessary for a pilot to maintain good control during all the many situations he may encounter. There are many things to consider in an aircraft's control system, and one of the important considerations is the time-lag problem. Barron and Pennington[35] have listed the typical sequence of events shown:

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[35] R. L. Barron and A. J. Pennington, "Dodco, Inc. Research in Optimum Adaptive Flight Control," Proc. Self-Adaptive Flight Control Systems Symp. (January, 1959), pp. 216-251.

1. The pilot reacts to a combination of stimuli.
2. A control-stick motion or force applied by the pilot is converted to a command signal by the controller.
3. Hydraulic fluid flows in the servo actuator.
4. The control surface undergoes acceleration.
5. Once sufficient time has elapsed, a significant change in control surface displacement will occur, and thus the aerodynamic flow pattern about the elevator begins to change.
6. After another delay, the new aerodynamic circulation field is obtained, and a new resultant force is produced on the control surface.
7. This control force will, in general, alter the angular accelerations, $\ddot{\theta}$ (see Fig. 21).
8. The angular acceleration integrates to a value θ, which occurs, initially, primarily in the form of an α increment.
9. The new value of α changes (in the course of time) the circulation about the wings and the resulting lift.
10. The change in lift produces a change in $\dot{\gamma}$.
11. $\dot{\gamma}$ integrates to a new flight path inclination γ.

If the aircraft does not have a proper control system, these lags can produce a dangerous dead time that can result in loss of control.

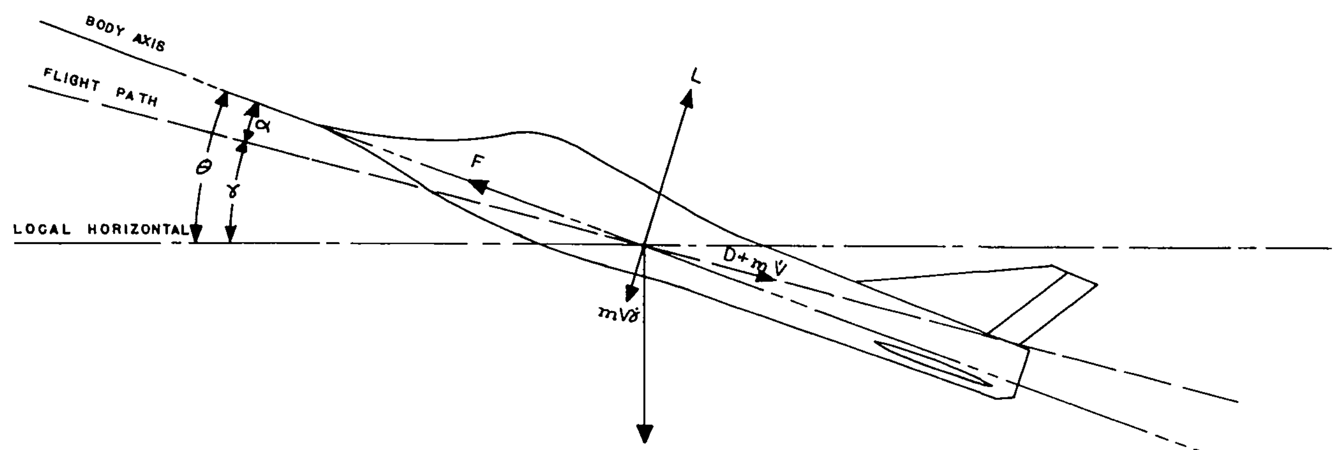

FIG. 21. MOTION OF AN AIRCRAFT WITHIN THE VERTICAL PLANE.

There are still other advantages in improving an aircraft's control system. For example, better control results in better fuel consumption; thus, the range is extended. It also results in improved navigation accuracy and better control while bombing, firing missiles, etc. There is a considerable effort now being made in France, England, and the United States to develop a completely "hands-off" automatic landing facility for aircraft so that planes can land safely in zero-ceiling weather, and because the landing of today's larger jets requires a high degree of pilot skill. These automatic landing devices will require the best response possible from the aircraft's control system. During a landing, the plane is subjected to large changes in both speed and altitude so that the aircraft's dynamics vary considerably, which points out the necessity of making the aircraft's control system adaptive. Pilots even use additional control surface at slow speeds to maintain control, and the TFX airplane being built by General Dynamics will use a completely different wing configuration for take-off and landing than it will for cruising.

Control of the aircraft's pitch-rate $\dot{\theta}$ is usually the the most difficult, so it will be the one dealt with here. There are changes with the aircraft's attitude, speed, altitude, fuel consumption, fuel consumption rate, etc. In order to emphasize the control system and not the aircraft as such, a subsonic low-altitude aircraft, viz., a B-25, will be used as an example. In a super-sonic high-altitude aircraft, many additional factors would have to be dealt with, e.g., if the altitudes are in excess of 150,000 feet, the effect of the variation of gravity potential with altitude must be included.

If the aircraft motion is restricted to a vertical plane (see Fig. 21), and if we ignore such things as structural elasticity, then the longitudinal equations of force and moment balance are:

along the flight path

$$F \cos \alpha - D - m\dot{V} - mg \sin \gamma = 0 \qquad (5.1)$$

normal to the flight path

$$F \sin \alpha + L - mV\dot{\gamma} - mg \cos \gamma = 0 \qquad (5.2)$$

moments about the mass center

$$M_S + M_D + M_C - \frac{d(J\dot{\theta})}{dt} = 0 \qquad (5.3)$$

where

F = effective engine thrust

α = angle of attack (between body axis and velocity vector)

D = drag = $qS(C_D + kC_L{}^2)$

m = mass

γ = inclination of the velocity vector

L = lift = $qSa \sin \alpha$

M_S = static stability moment = $qS_C \alpha C_\alpha$

M_D = damping moment = $qS_C(\dot{\alpha}C_{\dot{\alpha}} + \dot{\theta}C_{\dot{\theta}})$

M_C = control moment = $qS_C(\delta C_\delta + C_i \int \delta dt)$

J = polar moment of inertia in pitch

θ = angle between body axis and local horizontal = $\alpha + \gamma$

δ = elevator deflection

S = wing area

S_C = control surface area

V = velocity along flight path

q, k, C's are parameters that can be considered as slowly time-varying.

This is about the simplest form that can be used to represent the longitudinal dynamics of an aircraft. Equation (5.3) can be written in the form

$$J\ddot{\theta} = qS_C[\alpha C_\alpha + \dot{\alpha}C_{\dot{\alpha}} + \dot{\theta}(C_{\dot{\theta}} - \frac{\dot{J}}{qS_C}) + \delta C_\delta + C_i\int\delta dt] \qquad (5.4)$$

Note the J term whose effects may become significant during periods of high fuel flow-rate, such as occurs with afterburner use.

Redefining the coefficients and considering them to be slowly time-varying, Equation (5.4) can be transformed so that the aircraft's transmittance is

$$\frac{\dot{\theta}}{\delta} = \frac{K_C(s + \omega_{CO})}{s^2 + s\zeta_C\omega_C s + \omega_C{}^2} \, . \qquad (5.5)$$

It is now desirable to build an adaptive control system such that the effects of the slowly time-varying parameters in Equation (5.5) can be minimized. The usual approach[36,37]

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[36] M. F. Marx, loc. cit., pp. 201-215.

[37] S. S. Osder, loc. cit., pp. 81-122.

is to introduce additional high-frequency poles (actuator poles) along with a suitable feedback arrangement so that aircraft's response depends mainly upon the high-frequency poles introduced. This same approach will be used here. Consider the control system in Fig. 22 which also defines the variables. Using data compatible with a B-25 aircraft,[38] the root-locus will vary as in Fig. 23. If the poles can be maintained automatically within the boxes shown, suitable operation of the aircraft can be maintained.

The system discussed in Chapter III actually had only one degree of freedom, K; whereas, the aircraft has many degrees of freedom. In fact, the aircraft zero varies from -1 to -5 radians per second, and the aircraft's poles, from $-2 \pm j1$ to $-2 \pm j10$ radians per second. With each position of the aircraft's poles and zero, a different gain setting is necessary to keep the system's poles within the boxes shown in Fig. 23. A general method of handling multidimensional problems where many degrees of freedom of the error are involved will appear in a later chapter. However, the system at hand has the important characteristic that its response has been made to depend largely upon a pair of dominant roots, and if an error can be developed that is a measure in some sense of the location of these dominant roots, then perhaps it could be used to develop a feedback signal to alter the system's open-loop gain.

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[38]M. F. Marx, loc. cit., pp. 201-215.

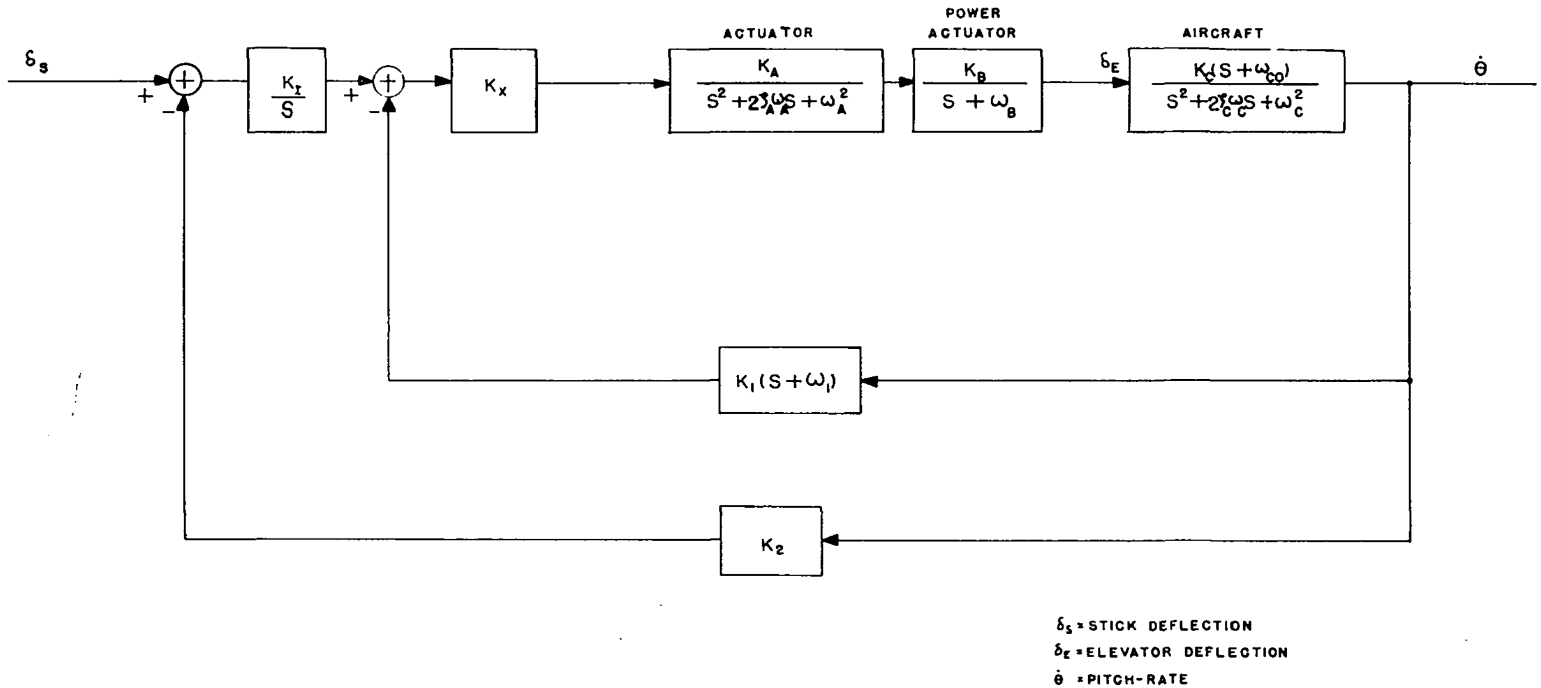

FIG. 22. A PITCH-RATE CONTROL SYSTEM FOR A B-25 AIRCRAFT.

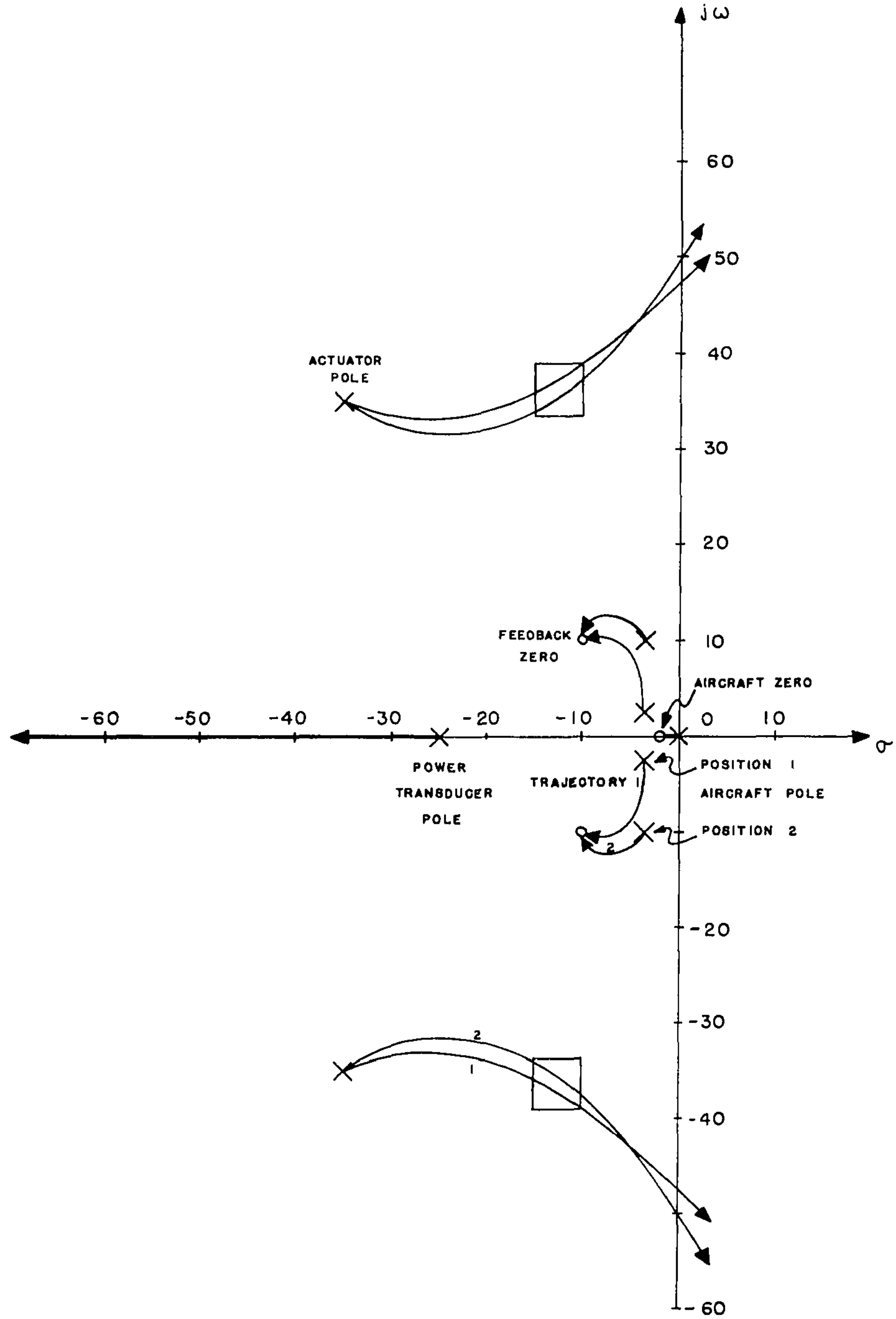

FIG. 23. POLE-ZERO PLOT OF AIRCRAFT CONTROL SYSTEM.

For academic purposes, an effort was made to find the simplest plant functions that would maintain adequate control of the aircraft. If the poles near the origin were all assumed to be cancelled by the zeros near the origin, and if the pole introduced by the power actuator were neglected, then only the roots introduced by the actuator would be of concern. This situation is illustrated in Fig. 24 where only the approximate locus of these two roots has been shown. It is approximate since the exact trajectory depends upon the exact location of the aircraft's poles and zeros. It will also be assumed that no events will occur that will cause the poles to migrate outside the area darkened on the locus. With these assumptions, an attempt to use second-order plants to develop the error signal was used, and it was reasonably successful. The plant's poles were placed at each end of the darkened trajectory approximately equidistant from the boxes. The exact pole locations for one plant were $-10 \pm j38$ radians per second while the pole locations for the second plant were $-15 \pm j33$ radians per second.

Figs. 25 and 26 show some of the adaptive characteristics of system. When the aircraft's poles or zero was moved, the aircraft's gain was also altered and this is recorded along the top strip of both figures. The next strip down shows the system's open-loop gain, while the third and fourth strips show the system's input and output respectively.

The system operated quite well with step inputs as shown in Fig. 25, and it also operated correctly with random

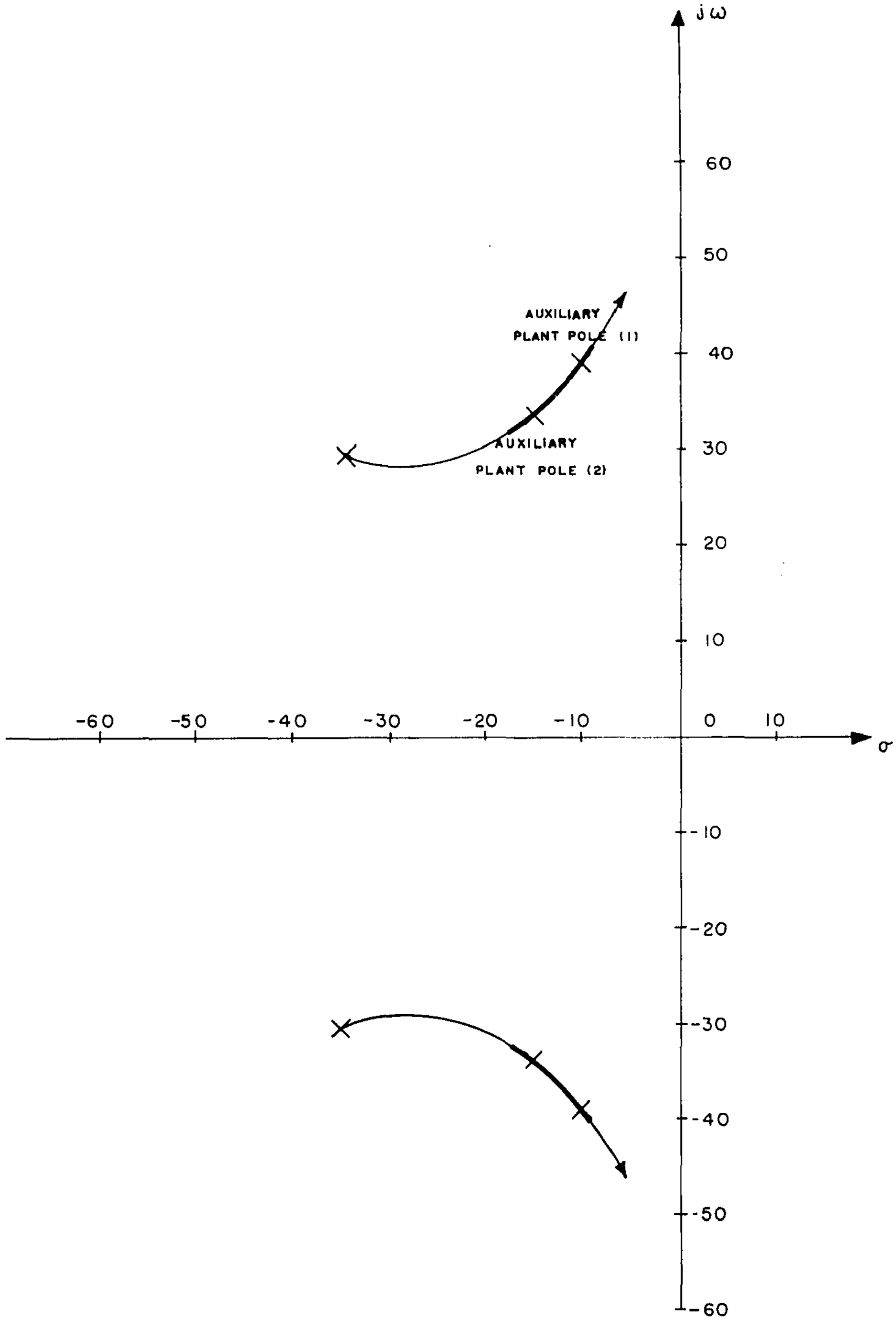

FIG. 24. POSITION OF SECOND-ORDER AUXILIARY PLANT POLES.

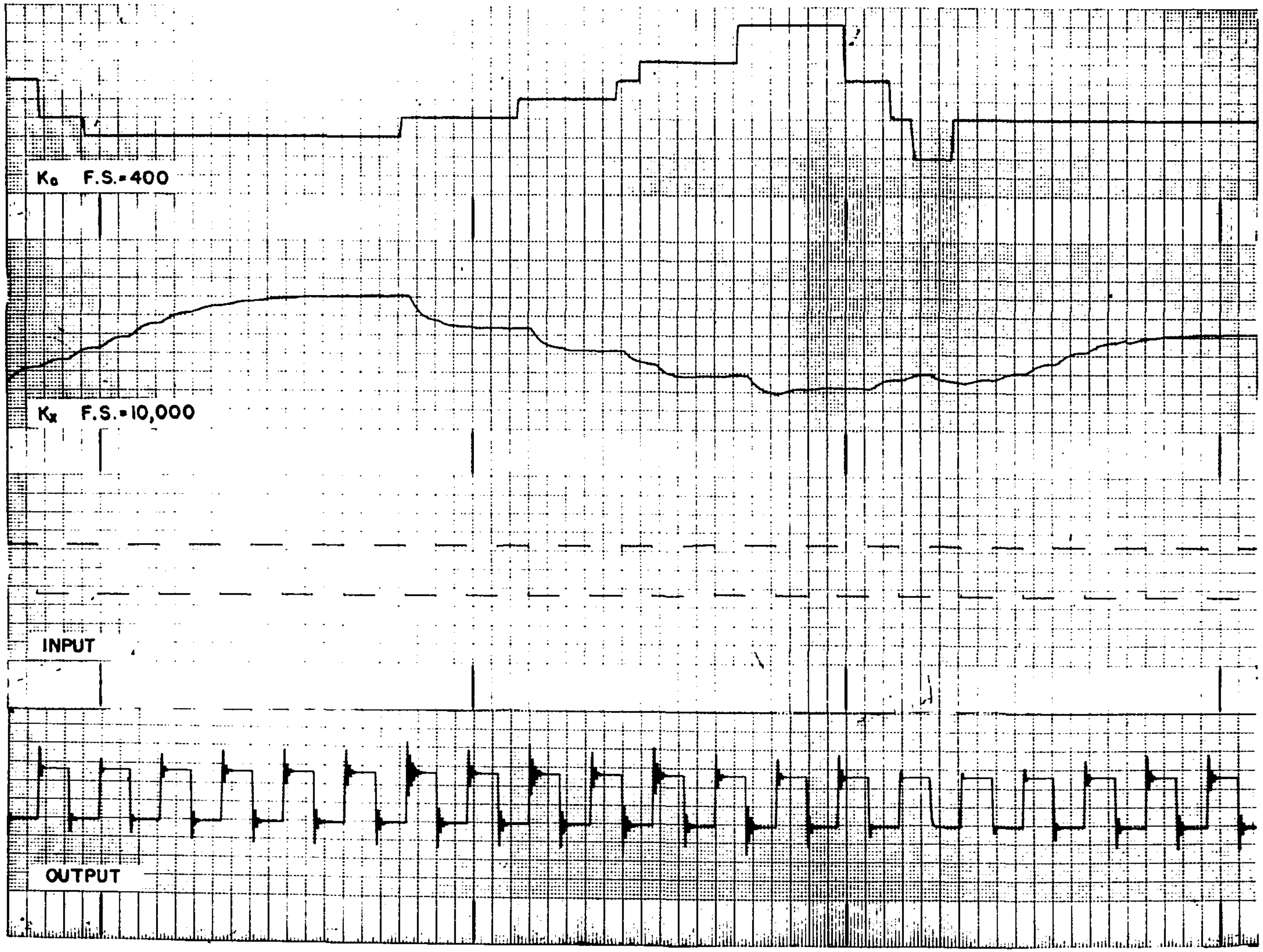

FIG. 25. AIRCRAFT ADAPTIVE PITCH-RATE CONTROL SYSTEM. TIME: 1 SEC./MAJOR DIVISION.

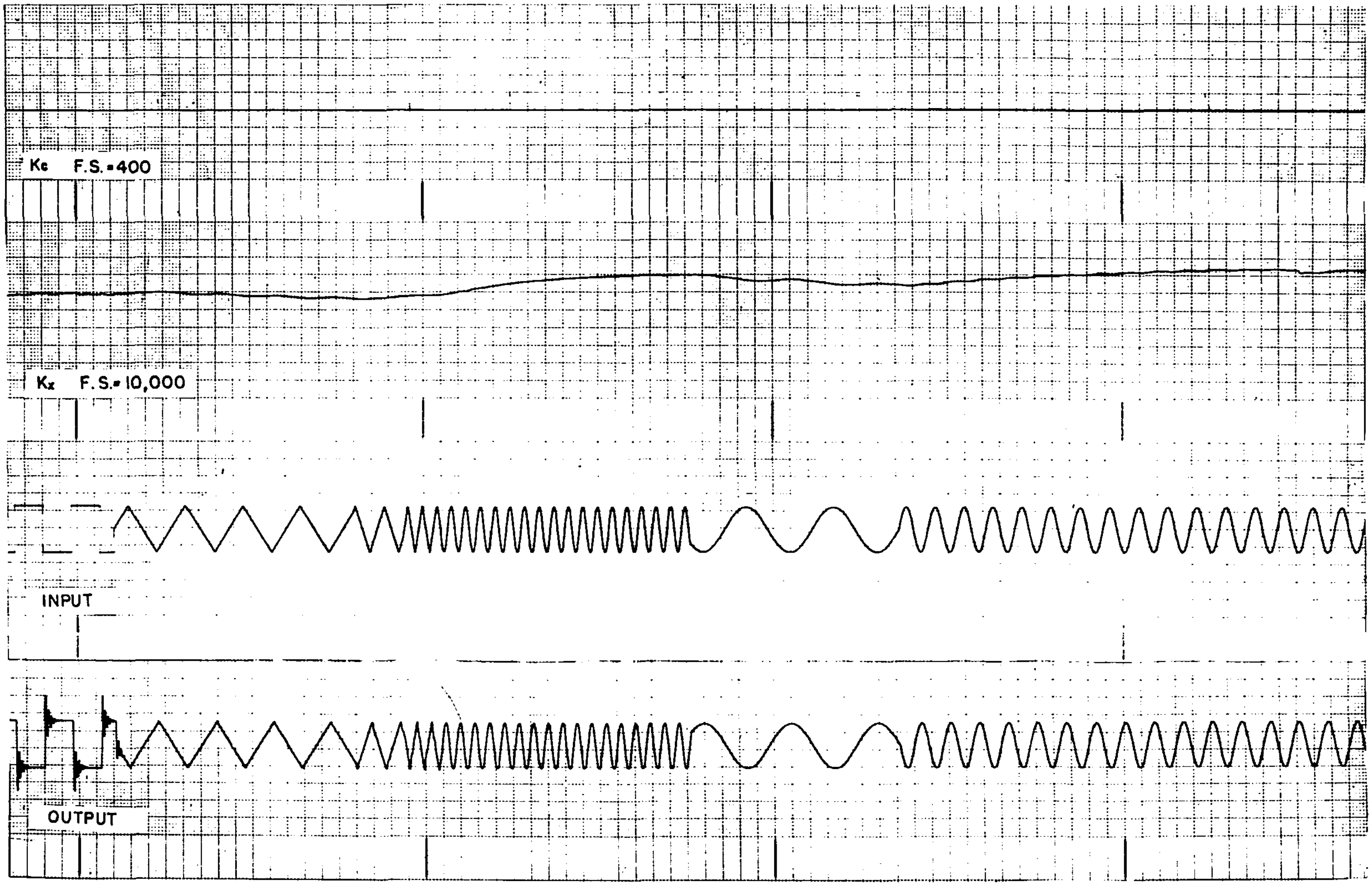

FIG. 26. AIRCRAFT ADAPTIVE PITCH-RATE CONTROL SYSTEM. TIME: 1 SEC./MAJOR DIVISION.

type inputs. In fact, the only time incorrect operation was observed was when low-frequency ramp-type inputs (triangular waves) and high-frequency sinuscids were injected for extended periods of time. The low-frequency triangular waveforms tended to decrease the gain while the high-frequency sinusoids tended to increase the gain. This can be observed somewhat in Fig. 26. The high-frequency sinusoids were about one cycle per second, and the low-frequency triangular waves used to cause a significant drop in gain were about one-third cycle per second. Neither type of input could be considered realistic; however, it would be desirable to reduce these effects as much as possible. A 17 per cent increase and a 16 per cent decrease in gain were observed in the worst case; however, operation was maintained within the limits darkened on the locus although these extreme conditions represented operation outside the boxes of Fig. 23.

The most logical explanation for this adverse operation noted is that the auxilary plants were oversimplified. For example, take the case where a high-frequency signal is injected into the system and allowed to reach steady-state. For frequencies near forty radians per second, the plant that has this frequency as its natural frequency will have a large output. When the aircraft's actuator roots are in this vicinity, a large output will also emanate from the aircraft. If the system were properly represented by Fig. 24, then the error signal developed as both actuator poles and auxiliary plant poles approached the same vicinity should cancel. However, if either the phase or amplitude of these signals differ considerably a large error signal can still be developed giving rise to an improper error signal.

Two other phenomena were noticed. The aircraft control system was not sensitive to the location of the aircraft zero. Varying the zero from -1 to -5 radians per second had virtually no effect upon the closed loop system. The method actually used to vary the aircraft's poles also varied the aircraft's gain. When the gain was readjusted so that the actuator roots were in the correct position, the loop gain was approximately the same as the aircraft's poles were varied (an open-loop gain of 93,400 was required to keep the actuator poles centered in the box when the aircraft's poles were in position one of Fig. 23, while a gain of 86,300 was needed when the aircraft's poles were in position two). This showed that the system was most sensitive to the aircraft gain as opposed to the aircraft roots. The second phenomenon was that the system generally migrated nearer to the auxiliary plant whose roots were closer to the j-axis than to a point, say, half way between the auxiliary plants. This was probably due to the fact that the errors could not be made zero as in Chapter III when a plant root was placed directly over an auxiliary plant root, since the auxiliary plant is now only an approximation to the main plant system. The trajectories also vary.

The next step in auxiliary plant sophistication was to introduce a pole corresponding to the one introduced by the power actuator, as this could be one of the larger sources of error introduced. The locations of these roots are shown in Fig. 27. Again, the roots tended to migrate too close to the j-axis so that the auxiliary plants' roots were located further back on the trajectory shown in Fig. 23. The location of the roots for these auxiliary plants were as follows: plant one, -52.5, $-12 \pm j36$; plant two, -57.5, $-18 \pm j30$. However, the frequency dependence of the adaptive portion of the control system was eliminated, and good control followed. This is best seen in Fig. 28.

Figs. 29 and 30 show the control system's output together with those of the auxiliary plants. Here, it is more apparent that the actuator roots tend towards the j-axis. These figures also show that the system is adaptive well beyond the range of the location of the auxiliary plant root locations.

It appears from these simulator studies that this adaptive method could be used for the control of aircraft. The system using third-order auxiliary plants proved to be successful over the range of inputs applied. Although direct compensation of the aircraft's parameters was not feasible, an indirect approach was used. The system was made to depend upon the location of the actuator roots, and these roots could be located in approximately the right position by adjusting the open-loop gain of the control system.

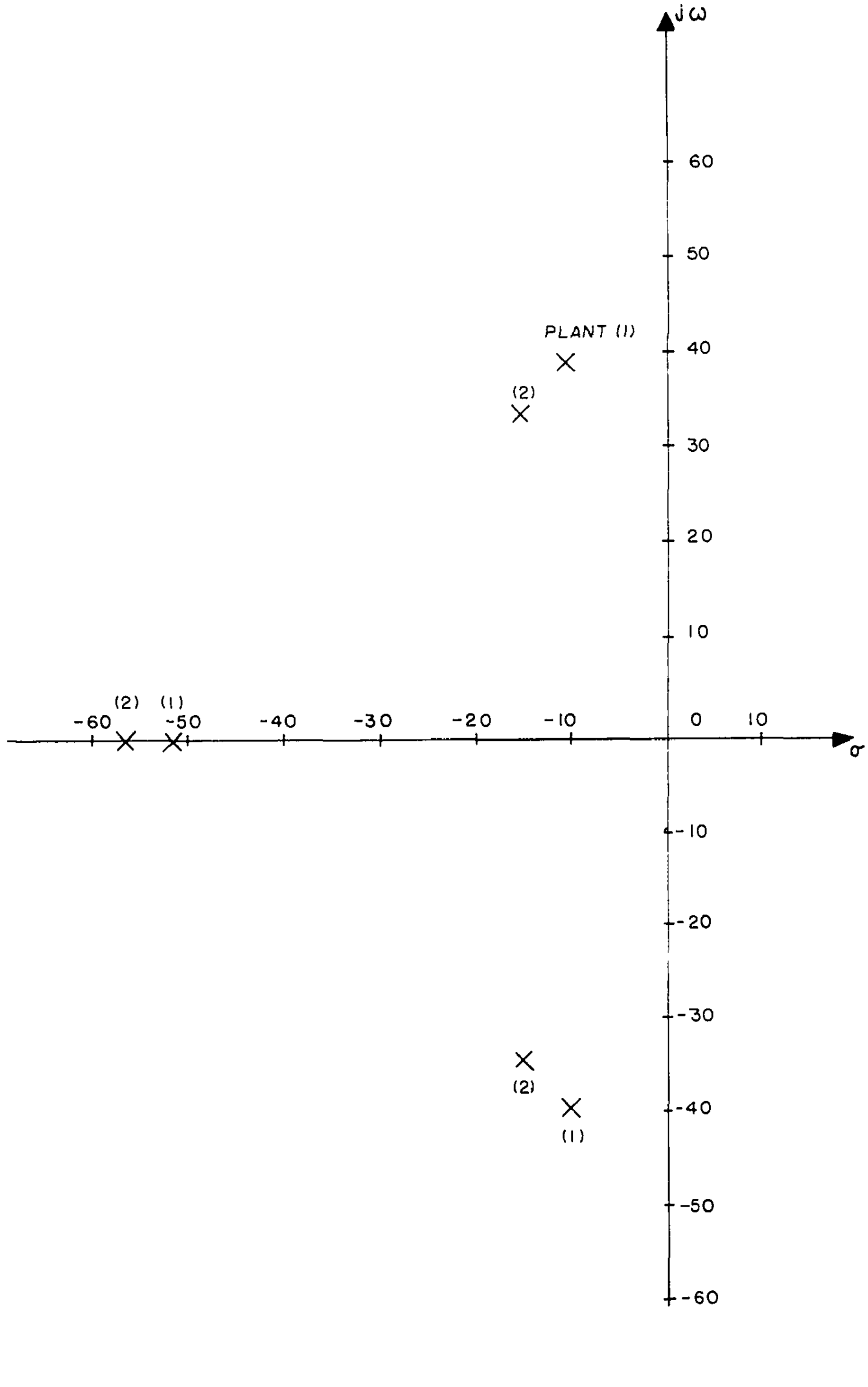

FIG. 27. POSITION OF THIRD-ORDER AUXILIARY PLANT POLES.

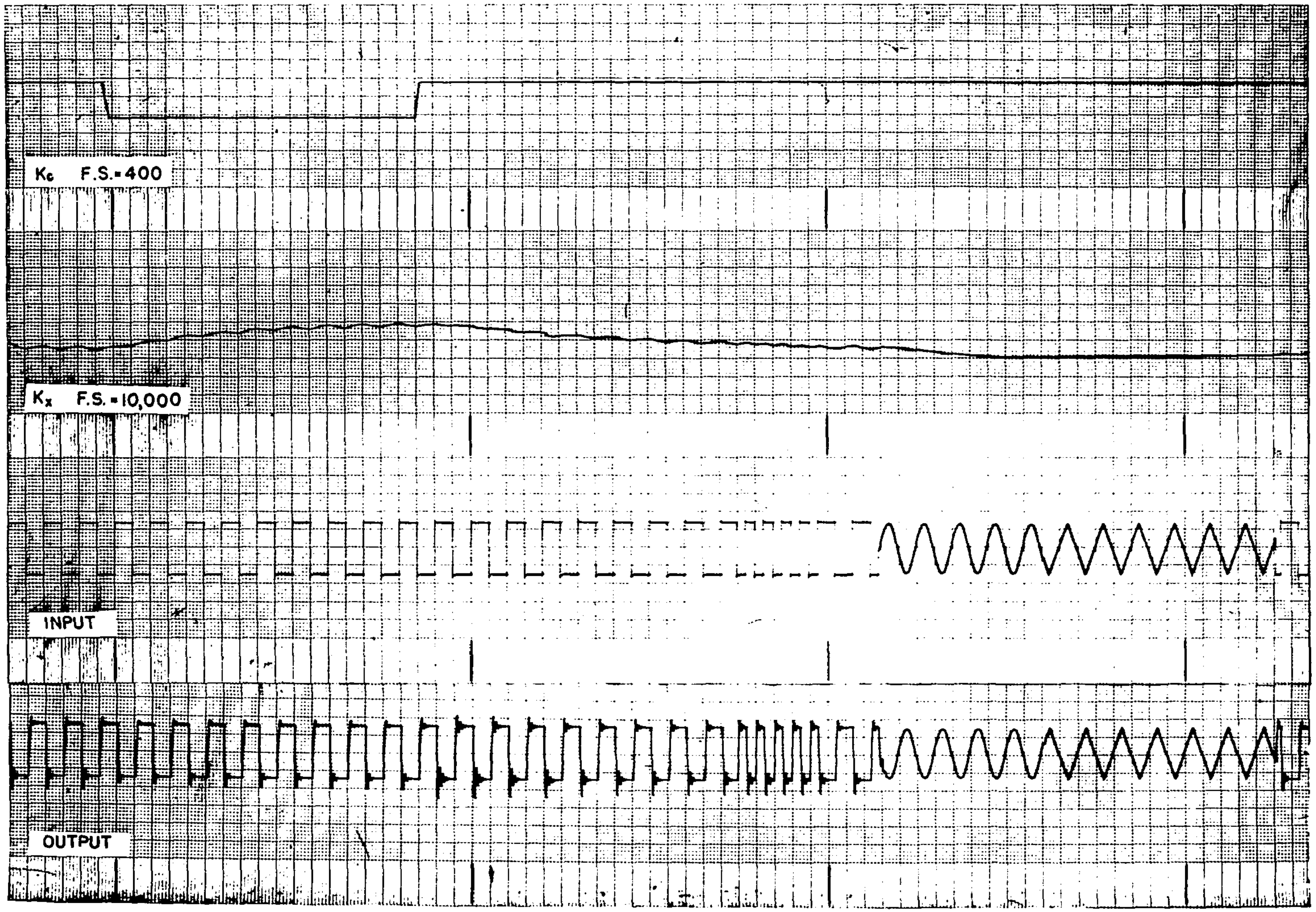

FIG. 28. AIRCRAFT CONTROL SYSTEM WITH THIRD-ORDER AUXILIARY PLANTS. 1 SEC./MAJOR DIVISION.

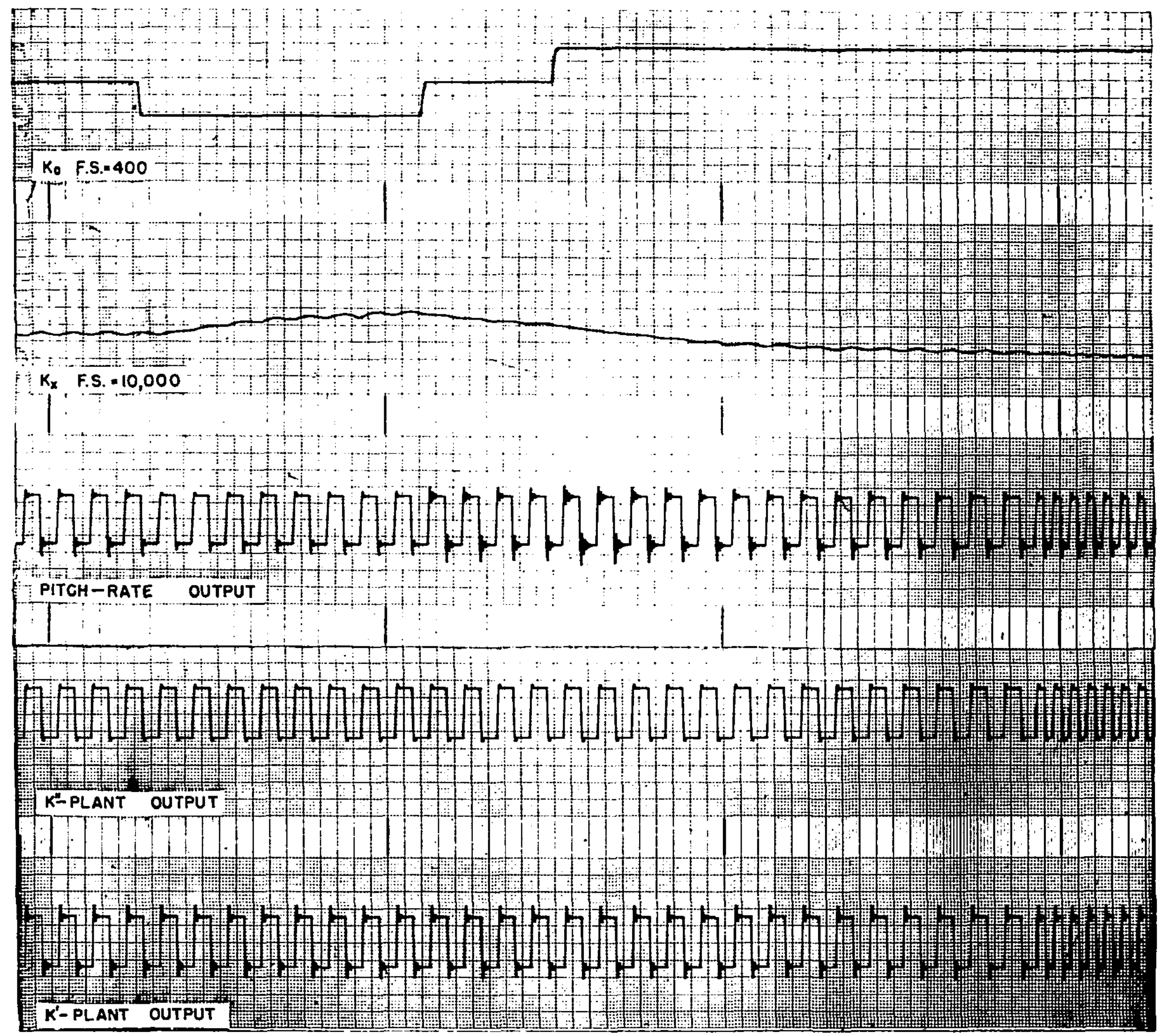

FIG. 29. AIRCRAFT CONTROL SYSTEM AND PLANT OUTPUTS. 1 SEC./MAJOR DIVISION.

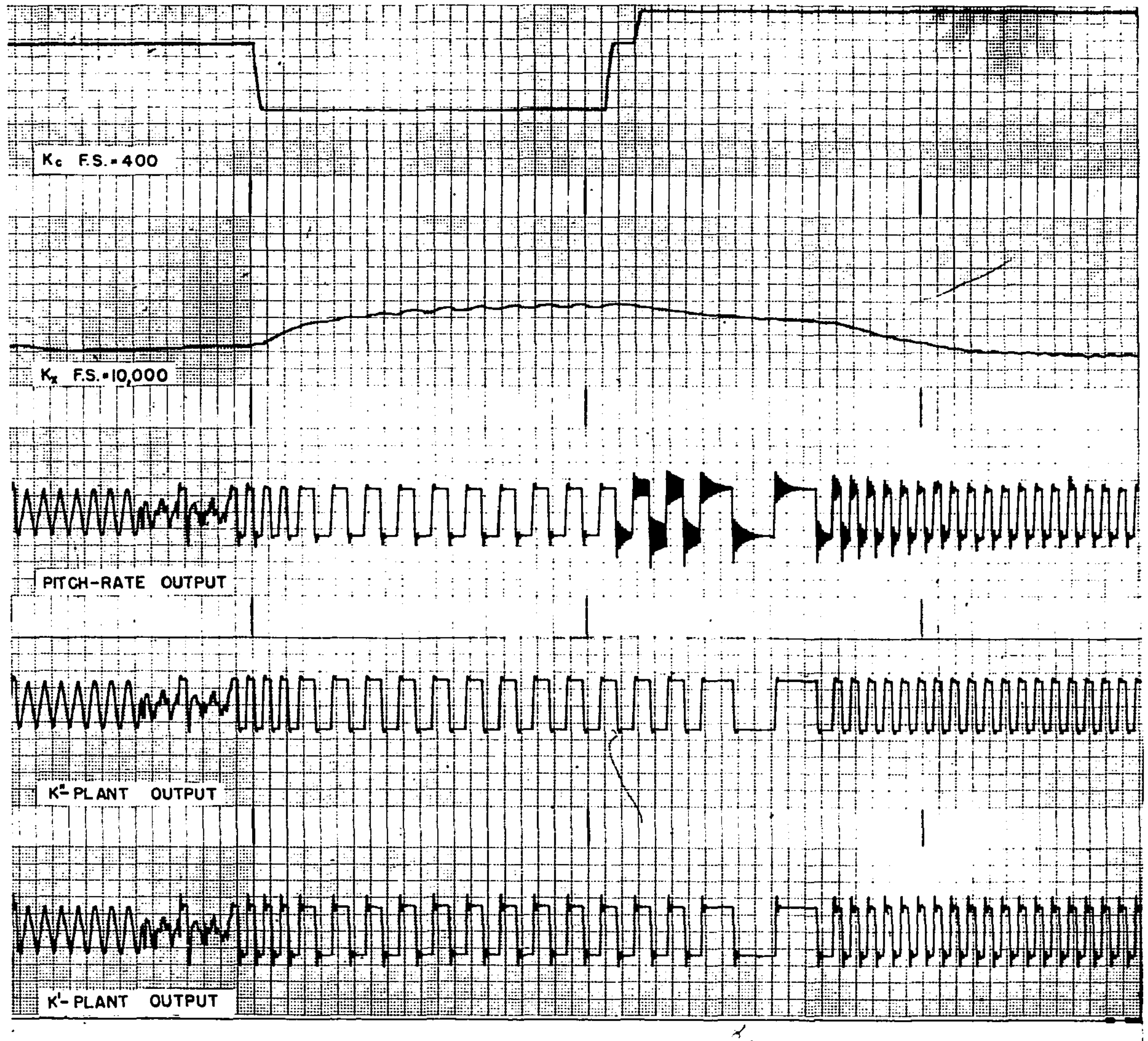

FIG. 30. AIRCRAFT CONTROL SYSTEM AND PLANT OUTPUTS. 1 SEC./MAJOR DIVISION.

CHAPTER VI

SYSTEMS WITH MORE THAN ONE

TIME-VARYING PARAMETER

The example used in Chapter III had one time-varying parameter, the gain, and this parameter was subject to direct compensation. That is, when the plant gain changed, the open-loop gain could be varied to achieve direct compensation. If direct compensation were not possible then the general scheme developed in Chapter III would not be applicable. The aircraft discussed in Chapter V was an example where direct compensation was not practical, and in order to apply the same techniques already developed, additional parameters (the actuator poles) were intorduced in such a way that the system was most sensitive to these parameters. The aircraft's control system then appeared to have a single degree of freedom, the gain, because the poles were introduced so that they would have this feature. Whenever schemes of this sort can be applied, there can be considerable simplification in the control system. This chapter is concerned with the case when direct compensation is not practical and will also include systems with more than one degree of freedom. It is to be expected that when direct compensation is not possible, the complexity of the control system will increase.

It was noted in Chapter III that for correct feedback control within the adaptive-loop, the error signal developed had to be a monotonic function of the variable to be controlled. For the technique which follows to work, the error signals must continue to be monotonic over their entire range of operation. Where more than one degree of freedom is allowable, multiple error signals will be introduced, and these error signals must all be monotonic over the range of the variable that they are monitoring and for all possible values of the other time-varying parameters. This is the only way of guaranteeing the existence of unique solutions, and it may turn out to be a restriction on the system. It may also be possible, when this restriction appears, to redefine the error variables in such a way that they will be monotonic over the region of interest. Also, it is less likely that this restriction will be encountered when direct compensation is used since feedback causes the transfer function of the system to remain somewhat stationary about the time-varying parameter.

Let $u(r,t,K_1,K_2,\ldots,K_n)$ be some function of the errors developed from the outputs of the depolarizers where r is the input signal and K_1, $K_2,\ldots$, K_n are the slowly time-varying parameters. Again, the u's will be developed from difference error signals in an effort to reduce the sensitivity to the type of input signal r and averaged so as to decrease their sensitivity with respect to t. When this is true and the K's are constrained to a relatively small region, the approximation

$$\Delta\bar{u} \approx \frac{\partial \bar{u}}{\partial K_1} \Delta K_1 + \frac{\partial \bar{u}}{\partial K_2} \Delta K_2 + \cdots + \frac{\partial \bar{u}}{\partial K_n} \Delta K_n \tag{6.1}$$

becomes more nearly valid providing these partial derivatives exist and are continuous. From Equation (6.1), it can be seen that $\bar{u}$ will be montonic in the K's about some n-dimensional region. The actual size of this region defines the allowable region of operation, and when direct compensation is used, the region of operation would ordinarily be considerably smaller than otherwise. This is, of course, due to the fact that the error signals developed tend to cause the variables to be reset to their correct value, so that although large changes in a parameter can occur, if the parameters only experience these changes in small steps and if sufficient error signals are developed, the system remains reasonably stationary in these parameters.

When direct compensation is not feasible, the error signals can only be used to indicate the state of the plant and to apply appropriate external compensation. The over-all system continues to operate in an allowable manner, but the plant's parameters are not reset in any way so that the region of operation of the plant is not, in general, constrained to a small range of operation. The monotonicity of the errors developed must be checked for the particular plant, and if they do not have this required property, then some other method of control must be used.

The method of developing the monotonic functions u_i will now be described. The average value of u_i is to be monotonic in K_i independent of the other K_j's. Let K'_i and K''_i be such that

$$\overline{u_i(K''_i)} < \overline{u_i(K'_i)} \, . \tag{6.2}$$

K'_i, K''_i may represent the bounds that the variable K_i can achieve although, generally, this will be somewhat conservative since the function $\overline{u}_i$ will in some cases remain monotonic in K_i in a region larger than that defined by K'_i and K''_i. 2^n auxiliary plants will be necessary to develop the functions u_1, u_2,...u_n, and these will be all of the possible combinations of plants using the extremum values of the K's. For large n, the number of plants needed increases exponentially so that certainly this must be regarded as a limitation. In many cases, however, n will not exceed three, so that the amount of auxiliary equipment necessary is not exorbitant when compared to the task at hand. 2^n error signals are developed in the same style as before, using the difference signals between the main plant and the 2^n auxiliary plants (see Fig. 31). These error signals are depolarized, and difference signals are again formed as follows. To form the function u_i whose average value is to be monotonic in K_i,

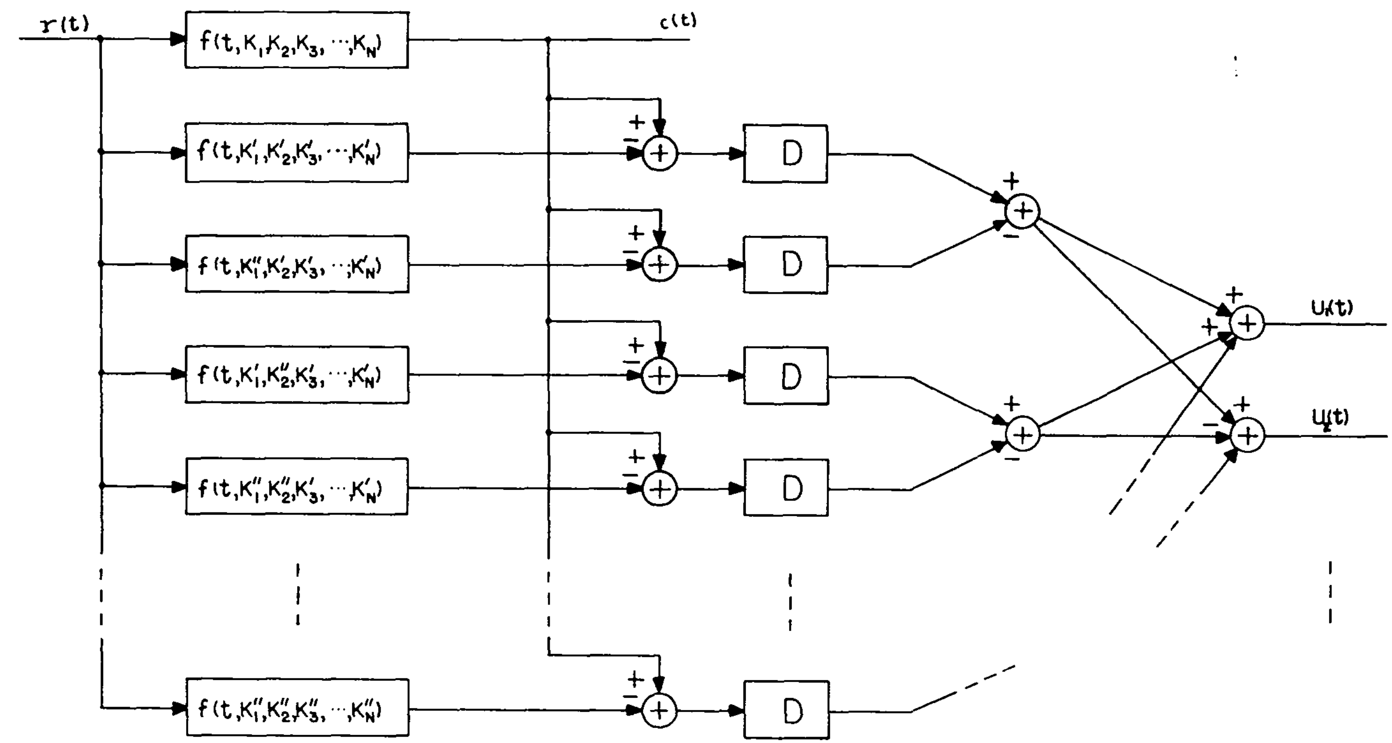

FIG. 31. DEVELOPING THE MONOTONIC U-FUNCTIONS.

connect all of the depolarized signals that result from an auxiliary plant with the parameter K_i' to a summing network; the parameter K_i' is chosen such that the output of each depolarizer involving a plant with K_i' as a parameter will be a monotonically increasing function. Similarly, the depolarizer outputs from auxiliary plants corresponding to the parameter K_i'' will be monotonically decreasing so that the difference between these two signals is a monotonically increasing function of K_i. Note that one-half of the auxiliary plants involving the parameter K_i' involve the parameter K_j', $i \neq j$, and one-half involve the parameter K_j'', so that half of them are monotonically increasing in K_j and half are decreasing in K_j. This connection scheme is an effort to increase the sensitivity of the function u_i to the plant parameter K_i and to decrease the function's sensitivity to other parameters K_j. The relative success or failure of this endeavor will depend upon the particular plant involved.

When direct compensation of the plant's parameters is to be used, the u_i's can be averaged and will directly form the feedback adaptive error signals, since they are monotonic and can be appropriately signed to give corrective action to the K_i's.

When direct compensation cannot be used, the u-functions can still be used to determine the state of the main plant. In this case, the average value of the u-functions need to be normalized to the average value of the depolarized input function $\overline{d[r(t)]}$. This could be done most easily by dividing the u-functions by $d\left[r(t)\right]$, providing $d\left[r(t)\right] \neq 0$. These normalized u-functions can be used to continuously monitor the state of plant's variables; however, it might be more economical to divide the normalized u-functions into discrete intervals and to determine an appropriate plant compensation for each interval. Comparators and logic circuitry would be needed to automate the choice of feedback to be used and, thus, make the system adaptive.

An example using two independent time-varying parameters will now be given to illustrate the effectiveness of the proposed method. Direct compensation will be assumed to be feasible, and a second order plant, similar to the plant used in Chapter III, will be used. This plant will have the transfer function

$$T(s) = \frac{K_{11}}{s^2+K_{21}s} \tag{6.3}$$

where K_{11} and K_{21} are independent, slowly time-varying parameters which can be compensated for directly. The plant is to be imbedded in a unity feedback control system together with the appropriate compensation so that the over-all transfer function is

$$\frac{C(s)}{R(s)} = \frac{K_{11}K_{12}}{s^2+K_{21}K_{22}s+K_{11}K_{12}} = \frac{K_1}{s^2+K_2s+K_1}. \tag{6.4}$$

In this case the limiting values of K_1, (K_1', K_1''), and K_2, (K_2', K_2''), will define a square region of operation in the s-plane as shown in Fig. 32, and the over-all appearance of the system is shown in Fig. 33. The depolarizers used were full-wave rectifiers.

The values of K_1' and K_2' were set to 0.5, while K_1'' and K_2'' were set to 1.0. It was expected that K_1 and K_2 would both approach the average values of 0.75 with the damping ratio $\zeta \approx 0.4$. Fig. 34 shows the output of the control system at the top. The other graphs show the outputs of the four auxiliary plants. The input was a square-wave. The system's parameters were being varied, and the subsequent adaptation was taking place which accounts for the variations in the output.

Fig. 35 shows various modes of adaptation for a square-wave input, while Figs. 36 and 37 depict the adaptive process for different inputs. As in the previous examples, the computer multipliers drifted continually and introduced error signals even when there was no input to the system. Input signals had to be applied to the system at a rate fast enough so that these drift errors could be adapted out. Fig. 37 shows a case where there was no input, and the system was allowed to drift until it almost became unstable. Upon application of an input signal, it can be seen that the system adapted itself back to normal rather quickly. This also illustrates that the system has a rather large range of

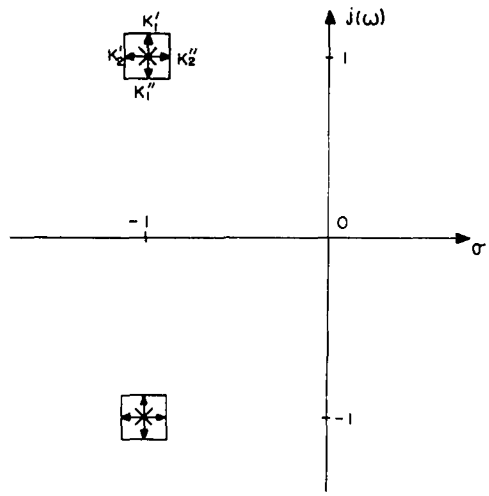

FIG. 32. REGION OF OPERATION FOR A SECOND-ORDER PLANT.

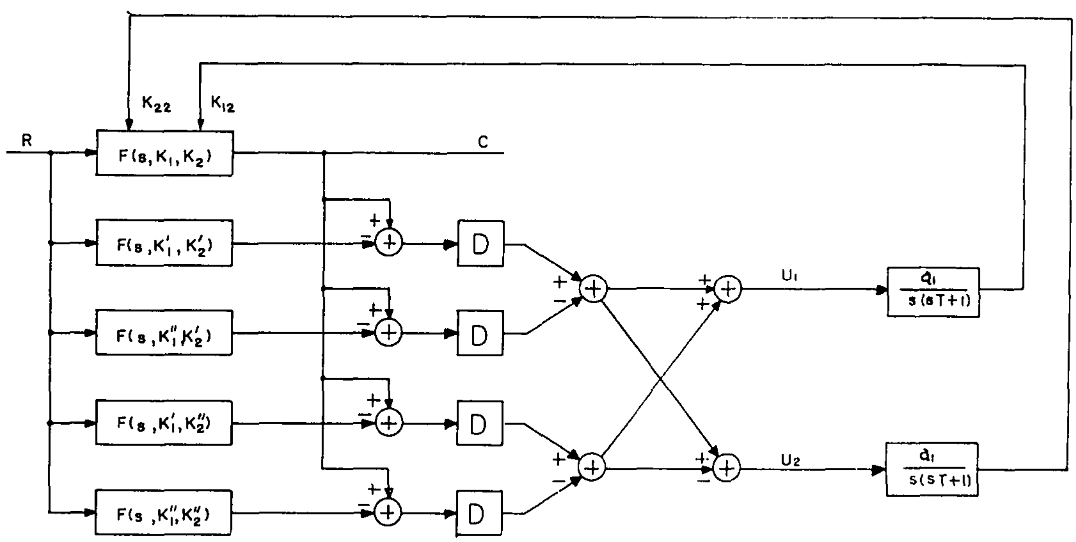

FIG. 33. TWO-PARAMETER ADAPTIVE SYSTEM.

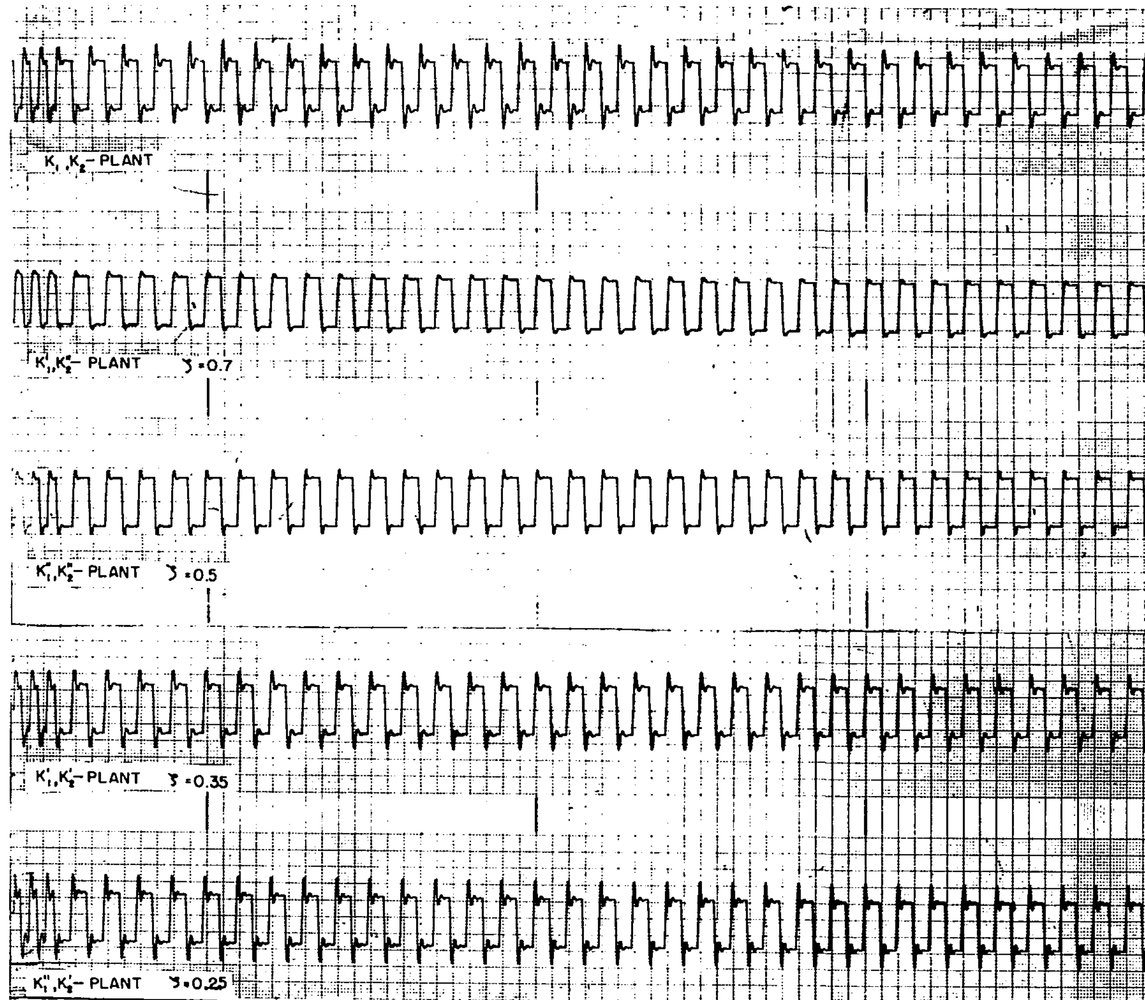

FIG. 34. PLANT OUTPUTS FOR TWO-PARAMETER ADAPTIVE SYSTEM. FULL SCALE = 25 VOLTS. TIME: 20 SECONDS = 1 MAJOR DIVISION.

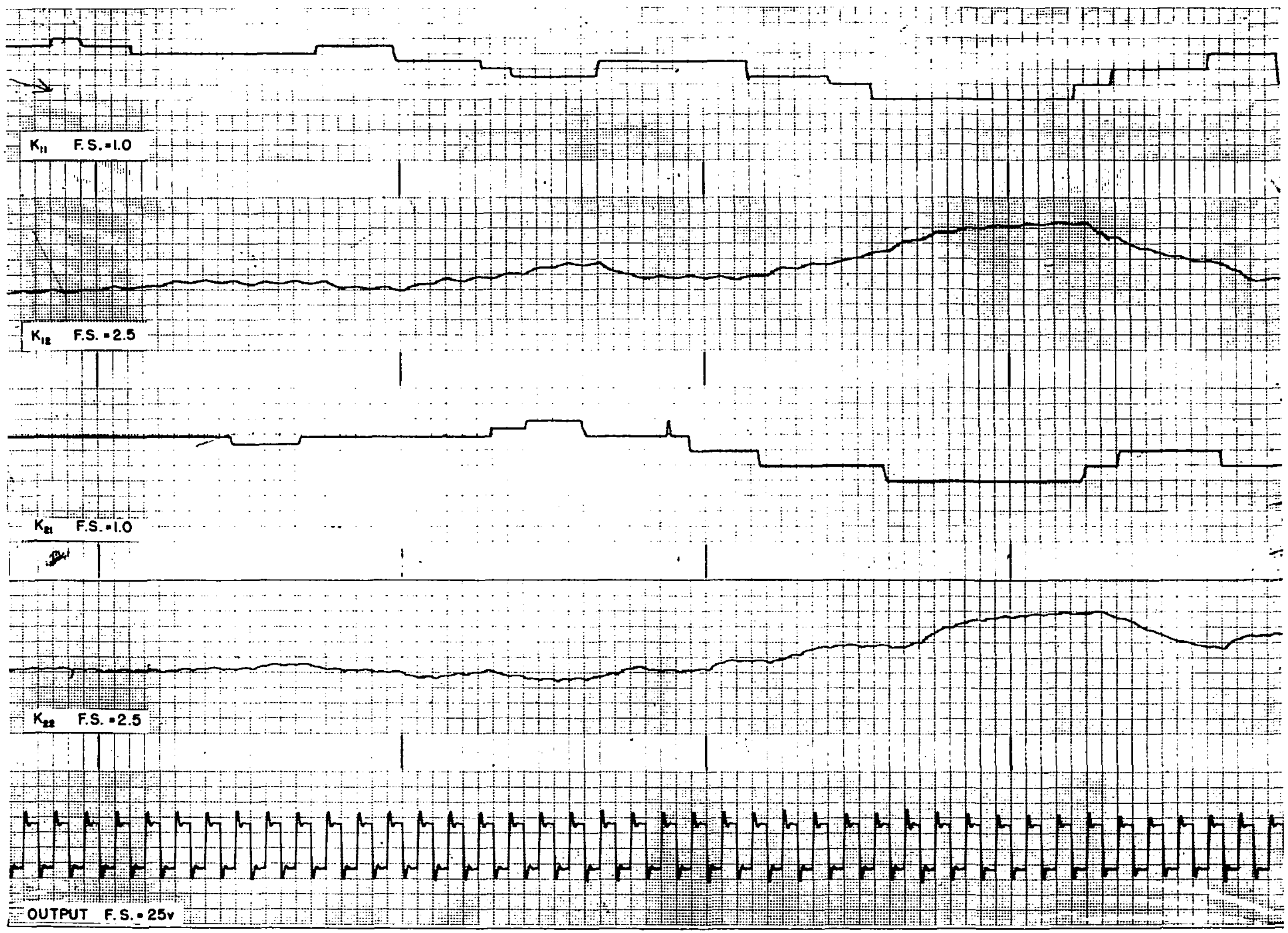

FIG. 35. PARAMETERS OF TWO-PARAMETER ADAPTIVE SYSTEM. TIME: 20 SECONDS = 1 MAJOR DIVISION.

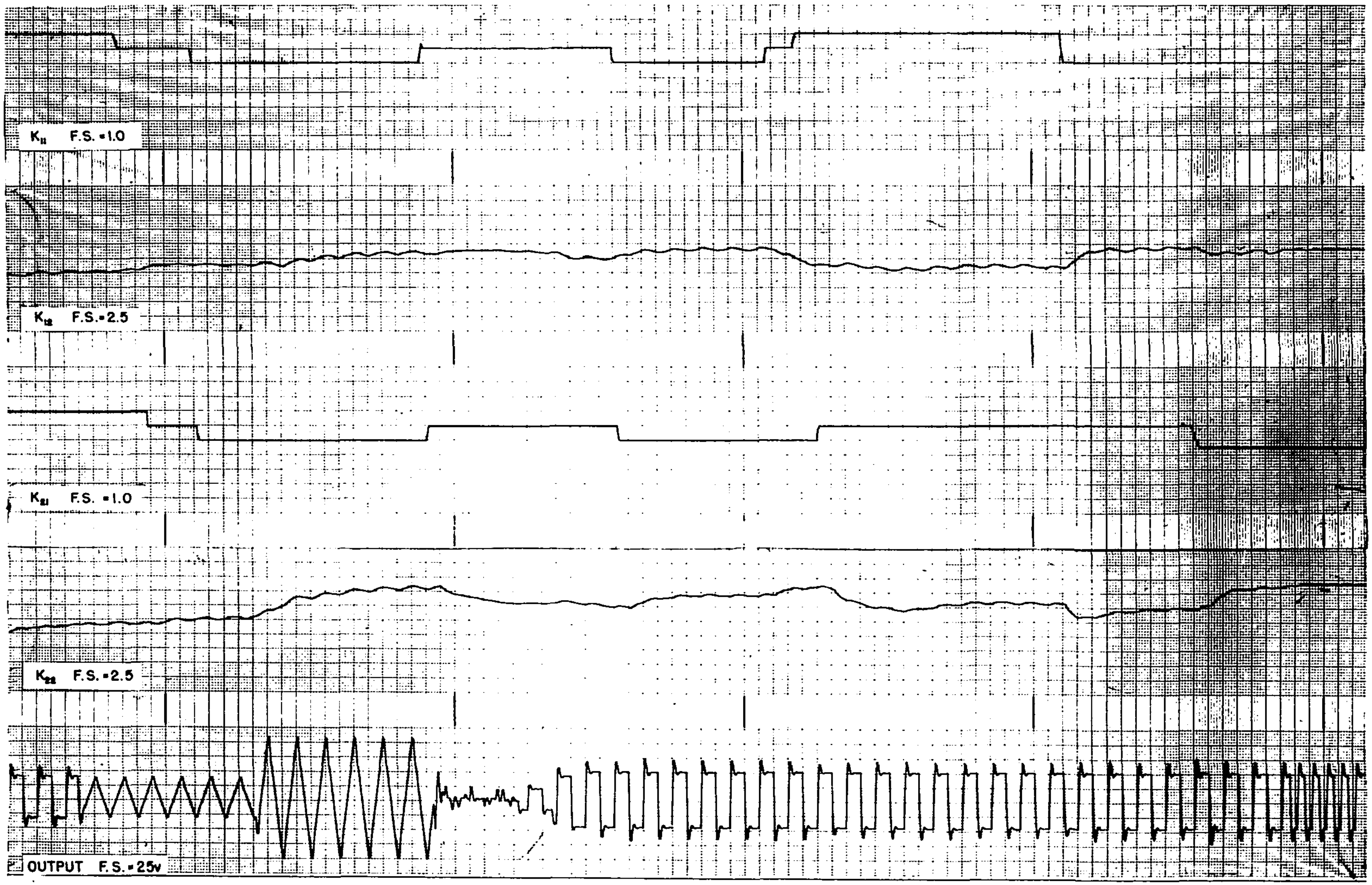

FIG. 36. PARAMETERS OF TWO-PARAMETER ADAPTIVE SYSTEM. TIME: 20 SECONDS = 1 MAJOR DIVISION.

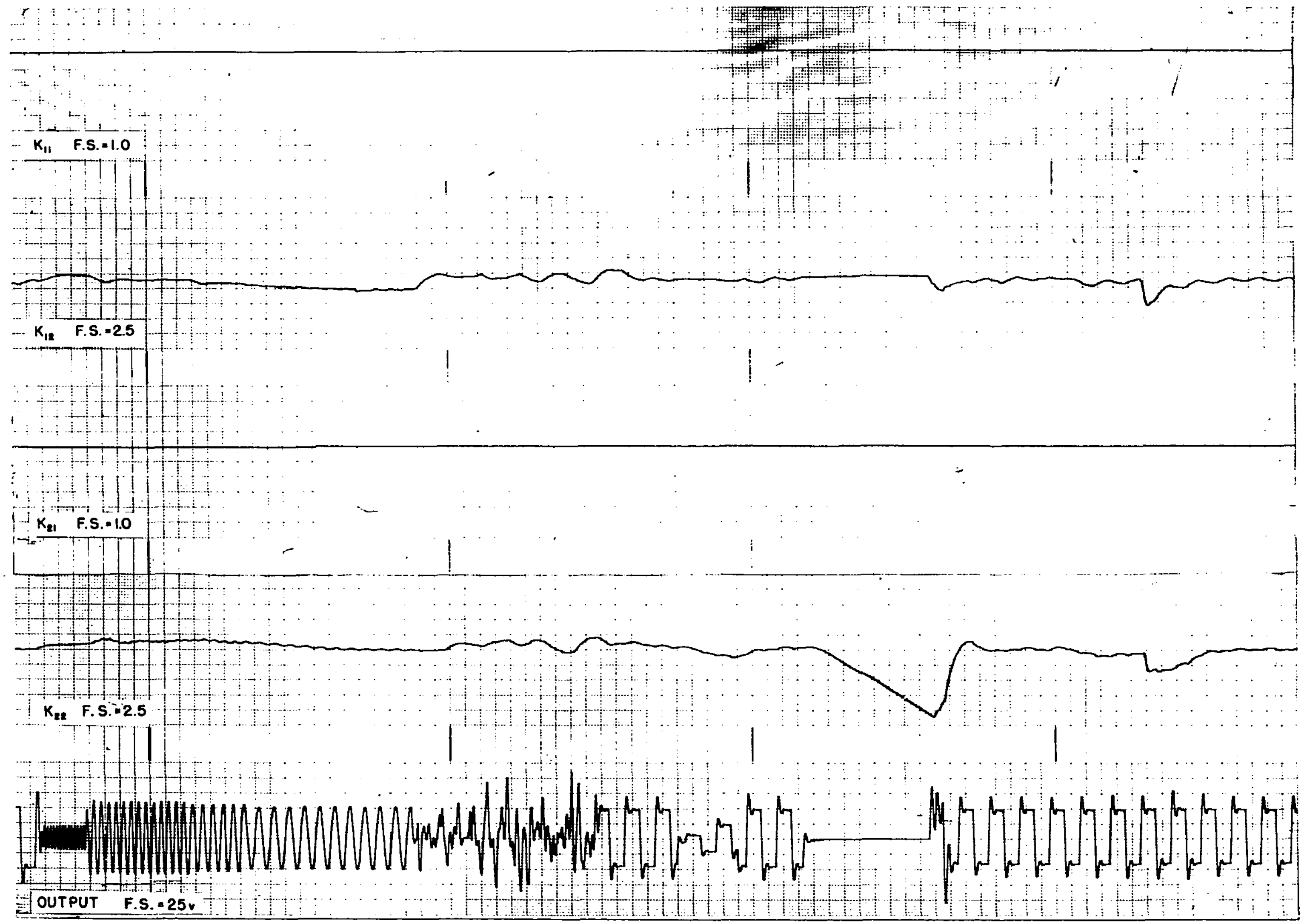

FIG. 37. PARAMETERS OF TWO-PARAMETER ADAPTIVE SYSTEM. TIME: 20 SECONDS = 1 MAJOR DIVISION.

adaptability. Later, on this same record, noise caused both K_{12} and K_{22} to take on erroneous values, and they both re-adjusted themselves in due time. Use of servo-moters would alleviate the drift problem encountered with the electronic multimpliers.

CHAPTER VII

CONCLUSION

An adaptive control method has been developed which has several advantages over schemes presently appearing in the literature. This particular method has been oriented towards plant-sensing adaptive control systems wherein a known control system configuration is considered to be optimum, and it is the function of the adaptive portion to drive the system to this optimum configuration. Presumably these optimum configurations are the "standard forms" of Graham and Lathrop,[39] or come about using the methods proposed by McDonald.[40] The examples of Chapter II were purposely chosen from the literature because they are also classified as plant-sensing systems; therefore, direct comparisons are meaningful.

Marx's system had the distinct advantage of simplicity. His method of control was basically a frequency sensitive servo; thus, the effectiveness of the servo depended upon the input characteristics as well as the control system's transfer function. The system, as described, depended upon a particular

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[39] D. Graham and R. C. Lathrop, loc. cit., pp. 273-288.

[40] T. McDonald, loc. cit.

feedback control system characteristic, viz., the time-varying parameter caused the pole to move parallel to the j-axis of the s-plane. There was no clear way to extend the results to systems in general; in fact, Marx stated that he tried to use the method for a pitch-rate aircraft control system without success.

The next system presented in the sequel was Osder's, and his system also depended upon a particular characteristic of the control system, viz., the time-varying parameter only varied the damping ratio over the variation of interest so that the natural frequency ω_n remained reasonably constant. A performance computer was necessary to count the number of zero crossings during a preset time interval after an impulse function had been injected into the system. The performance counter determined the damping ratio from this information and adjusted the system accordingly. No straight-forward extension to systems in general was apparent, and a moderate amount of hardware might be necessary for the performance computer and the subsequent parameter control. There is a problem also in that the theory only indicates that the system will work correctly when no input signal has been applied. In fact, it can readily be shown that the system will not work when steady-state sinusoids are applied to the input. This problem is circumvented by applying the test impulses only when no command signal is present. Other than this restriction on the test impulses, the system can be almost continuously updated with the corresponding disadvantage of perturbing the output.

The system of Anderson, et al., uses an enormous amount of equipment to perform the relatively simple task of keeping the damping ratio constant of a second-order system. Besides the elaborate and complex equipment, the system also has the disadvantage that signal injection is used to recover the system impulse response. Since it was not practical to perform the crosscorrelations continuously as a function of τ, approximations were necessary and the impulse response was truncated rather quickly. Nonideal integration led to other errors, and these are discussed in the <u>Proceedings of the Self-adaptive Flight Control Systems Symposium</u>.[41] The system has the distinct advantage of being able to update the system almost continuously and independently of the input signal, since this signal is not correlated with the signal injected to recover the impulse response. No straightforward extension to more general systems was proposed; however, it should not be too difficult to relate the impulse response to other system parameters. In most cases, the impulse response would have to be recovered in a more complete style than the example was. Due to the method used in obtaining the impulse response, the method is certainly restricted to linear systems.

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[41] G. W. Anderson, R. N. Buland, and G. R. Cooper, "The Aeronutronic Self-Optimizing Automatic Control System," <u>Proc. of the Self-adaptive Flight Control Systems Symp.</u> (January, 1959), pp. 349-406.

Whitaker's method, like the others, is applicable to the plant-sensing category, but the philosophy used appears to be more general. Or it may be that it just appears to be more general because Whitaker is less specific. Presumably, given a system, one is still left the task of finding the integral error criterion that is most sensitive to the parameter involved. This may or may not be a simple thing to find. He has applied his method to a second-order system with two time-varying parameters with some success.[42] There is also no reason to believe that the method is only applicable to second-order systems, or for that matter, if appropriate error functions can be found, it could even be applicable to nonlinear systems as well. As far as the author knows, there has been no effort upon Whitaker's part to find a general method for determining the proper error functions to use, although some approach similar to the ones used in Chapters III and VI might be applicable.

In all of these systems, one important aspect of the performance of adaptive systems has been omitted as has been noted by Bongiorno,[43] and this is a dynamic analysis of the stability of these systems. That a stability problem exists is obvious: the adaptive circuitry constitutes a feedback path (usually nonlinear), and introduces the question of stability. Unfortunately, the dynamic characteristics of

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[42]H. P. Whitaker, loc. cit., pp. 58-80.

[43]E. Mishkin and L. Braun, Jr., loc. cit., pp. 323-343.

adaptive systems are, in general, described by nonlinear time-varying differential equations of sufficient complexity as to preclude the possibility of a complete dynamic analysis. However, for the proposed system of Chapter III, it was possible to make a few simplifying assumptions which allowed a fairly reasonable analysis of the adaptive loop. This analysis pointed out the way in which the variables enter into the stability problem and added considerable insight into the operation of the adaptive loop. One of the more important results of the analysis was the fact that the stability was a function of the input signal amplitude, with instability occurring when the amplitude was increased beyond a threshold level. Thus, it was illustrated that the adaptive feedback-loop gain must be adjusted in accordance with the maximum expected value of the input signal.

Some of the more important features of the proposed system are the natural way in which the control of any system parameter can be effected and the direct extension to multiple parameter control when these parameters yield to direct compensation. The system does not depend upon any extraneous signal application in order to work, but consequently the system can develop lags over periods when no input has been applied; to date, these two features appear to be mutually inclusive. All of the systems discussed that depend upon an input signal to develop an error signal would also be continuously updated if extraneous signal injection were used.

On the basis of computer studies, the authors of aircraft control systems using test input signals claim that the level of test disturbances can be made low enough not to bother the pilot.[44,45] For any system, this would certainly have to be determined and could restrict the use of such methods. Some systems that are basically linear do not respond linearly to small inputs due to such phenomena as stiction friction.

The results of these comparisons appear in Table 3, where an effort has been made to emphasize the most outstanding characteristics of each system. Whitaker's method appears fairly good on the chart, but this is mainly due to his unspecific approach.

All of the systems can be implemented in a more straight-forward manner when direct compensation of the time-varying parameter is permissible. This feature is not always inherent in the plant, but sometimes this property can be built into the system such as it was with the aircraft example of Chapter V. When direct compensation is not permissible, the complexity of the adaptive portion is increased considerably. Some discussion of how the present method could be extended to such parameters was included in Chapter VI; however, this problem is a difficult one, and perhaps a better solution will be discovered eventually.

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[44] G. W. Anderson, R. N. Buland, and G. R. Cooper, loc. cit., pp. 349-406.

[45] S. S. Osder, loc. cit., pp. 81-122.

TABLE 3

COMPARISON OF THE ADAPTIVE SYSTEMS

	Proposed	Whitaker	Anderson	Osder	Marx
Applicable to any parameter that can be compensated for directly	X				
Does not depend on input signal type	X	X		X	
Does not depend upon an extraneous injected input signal	X	X			X
Does not depend upon some particular control system characteristic	X	X			
Can be straight-forwardly generalized to multiple parameter control	X	X			
System continuously updated			X	X	
Estimate of total hardware needed to control one parameter: (1) least hardware needed; (5) most hardware needed	2	3	5	4	1
Does not depend upon a linear control system	X	X			

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APPENDIX A

SQUARED ERROR EVALUATION FOR SECOND-ORDER SYSTEM

For the second-order systems described by the differential equations

$$\frac{d^2c(t)}{dt^2} + \frac{dc(t)}{dt} + K_1c(t) = K_1r(t) \tag{A.1}$$

and

$$\frac{d^2c(t)}{dt^2} + \frac{dc(t)}{dt} + K_2c(t) = K_2r(t) \tag{A.2}$$

it was necessary to evaluate the integral

$$\int_0^\infty \left[c(r,K_1,t)-c(r,K_2,t)\right]^2 dt = \int_0^\infty e^2(K_1,K_2,t)dt \tag{A.3}$$

where $r(t)$ has the form $\epsilon^{-A(t)}$. Equation (A.3) is tabulated in Table 4 for a range of values of A, K_1, and K_2.

TABLE 4

INTEGRAL EVALUATION FOR SECOND-ORDER SYSTEM

$K_1 = 1.4$ $$\int_{o}^{\infty} e^2 dt = 0.xxxx(10^{-4}) = 0.xxxx(-4)$$

K_2	$A = 10^{-3}$	$A = 10^{-2}$	$A = 10^{-1}$
0.4	0.9534(0)	0.9391(0)	0.8071(0)
0.5	.6643(0)	.6569(0)	.5836(0)
0.6	.4659(0)	.4618(0)	.4194(0)
0.7	.3228(0)	.3205(0)	.2956(0)
0.8	.2173(0)	.2160(0)	.2015(0)
0.9	.1394(0)	.1387(0)	.1305(0)
1.0	.0829(0)	.0852(0)	.0782(0)
1.1	.0435(0)	.0433(0)	.0413(0)
1.2	.0181(0)	.0180(0)	.0172(0)
1.3	.0041(0)	.0041(0)	.0041(0)
1.4	.0000(0)	.0000(0)	.0000(0)
1.5	.0037(0)	.0037(0)	.0036(0)

TABLE 4--Continued

A = 1.0	A = 10	A = 10^2	A = 10^3
0.2174(0)	0.4074(-2)	0.4130(-4)	0.4130(-6)
.1757(0)	.3378(-2)	.3426(-4)	.3426(-6)
.1379(0)	.2719(-2)	.2758(-4)	.2759(-6)
.1045(0)	.2110(-2)	.2141(-4)	.2142(-6)
.0756(0)	.1564(-2)	.1588(-4)	.1588(-6)
.0515(0)	.1091(-2)	.1108(-4)	.1108(-6)
.0323(0)	.0698(-2)	.0710(-4)	.0710(-6)
.0177(0)	.0391(-2)	.0400(-4)	.0398(-6)
.0076(0)	.0173(-2)	.0176(-4)	.0176(-6)
.0018(0)	.0043(-2)	.0043(-4)	.0043(-6)
.0000(0)	.0000(-2)	.0000(-4)	.0000(-6)
.0017(0)	.0041(-2)	.0042(-4)	.0042(-6)

TABLE 4--Continued

$K_1 = 1.2$

K_2	$A = 10^{-3}$	$A = 10^{-2}$	$A = 10^{-1}$
0.4	0.7903(0)	0.7779(0)	0.6631(0)
0.5	.5137(0)	.5075(0)	.4474(0)
0.6	.3304(0)	.3273(0)	.2951(0)
0.7	.2049(0)	.2032(0)	.1861(0)
0.8	.1185(0)	.1177(0)	.1091(0)
0.9	.0607(0)	.0604(0)	.0565(0)
1.0	.0247(0)	.0246(0)	.0232(0)
1.1	.0056(0)	.0056(0)	.0054(0)
1.2	.0000(0)	.0000(0)	.0000(0)
1.3	.0048(0)	.0048(0)	.0046(0)
1.4	.0181(0)	.0180(0)	.0172(0)
1.5	.0377(0)	.0376(0)	.0361(0)

TABLE 4--Continued

A = 1.0	A = 10	A = 10^2	A = 10^3
0.1667(0)	0.2963(-2)	0.3000(-4)	0.3000(-6)
.1260(0)	.2301(-2)	.2330(-4)	.2330(-6)
.0909(0)	.1705(-2)	.1727(-4)	.1727(-6)
.0617(0)	.1187(-2)	.1204(-4)	.1204(-6)
.0385(0)	.0758(-2)	.0769(-4)	.0769(-6)
.0210(0)	.0424(-2)	.0430(-4)	.0430(-6)
.0090(0)	.0186(-2)	.0189(-4)	.0189(-6)
.0022(0)	.0046(-2)	.0047(-4)	.0047(-6)
.0000(0)	.0000(-2)	.0000(-4)	.0000(-6)
.0020(0)	.0044(-2)	.0045(-4)	.0045(-6)
.0076(0)	.0173(-2)	.0176(-4)	.0176(-6)
.0164(0)	.0378(-2)	.0385(-4)	.0385(-6)

TABLE 4--Continued

$K_1 = 1.0$

K_2	$A = 10^{-3}$	$A = 10^{-2}$	$A = 10^{-1}$
0.5	0.3457(0)	0.3412(0)	0.2974(0)
0.6	.1903(0)	.1882(0)	.1679(0)
0.7	.0938(0)	.0930(0)	.0843(0)
0.8	.0370(0)	.0376(0)	.0337(0)
0.9	.0083(0)	.0082(0)	.0076(0)
1.0	.0000(0)	.0000(0)	.0000(0)
1.1	.0067(0)	.0068(0)	.0063(0)
1.2	.0247(0)	.0246(0)	.0232(0)
1.3	.0508(0)	.0507(0)	.0478(0)
1.4	.0829(0)	.0825(0)	.0782(0)
1.5	.1190(0)	.1185(0)	.1125(0)

TABLE 4--Continued

A = 1.0	A = 10	A = 10^2	A = 10^3
0.7692(-1)	0.1331(-2)	0.1346(-4)	0.1346(-6)
.4762(-1)	.0847(-2)	.0857(-4)	.0857(-6)
.2579(-1)	.0471(-2)	.0477(-4)	.0477(-6)
.1100(-1)	.0206(-2)	.0209(-4)	.0209(-6)
.0262(-1)	.0051(-2)	.0051(-4)	.0015(-6)
.0000(-1)	.0000(-2)	.0000(-4)	.0000(-6)
.0238(-1)	.0048(-2)	.0048(-4)	.0049(-6)
.0901(-1)	.0186(-2)	.0189(-4)	.0189(-6)
.1920(-1)	.0406(-2)	.0413(-4)	.0413(-6)
.3226(-1)	.0698(-2)	.0710(-4)	.0710(-6)
.4762(-1)	.1053(-2)	.1071(-4)	.1071(-6)

TABLE 4--Continued

$K_1 = 0.7$

K_2	$A = 10^{-3}$	$A = 10^{-2}$	$A = 10^{-1}$
0.4	0.2563(0)	0.2512(0)	0.2051(0)
0.5	.0911(0)	.0897(0)	.0758(0)
0.6	.0188(0)	.0186(0)	.0161(0)
0.7	.0000(0)	.0000(0)	.0000(0)
0.8	.0137(0)	.0135(0)	.0121(0)
0.9	.0476(0)	.0471(0)	.0424(0)
1.0	.0938(0)	.0930(0)	.0843(0)
1.1	.1474(0)	.1462(0)	.1332(0)
1.2	.2049(0)	.2032(0)	.1861(0)
1.3	.2638(0)	.2618(0)	.2407(0)
1.4	.3228(0)	.3205(0)	.2956(0)
1.5	.3806(0)	.3780(0)	.3497(0)

TABLE 4--Continued

A = 1.0	A = 10	A = 10^2	A = 10^3
0.3930(-1)	0.6040(-3)	0.6091(-5)	0.6092(-7)
.1639(-1)	.2599(-3)	.2623(-5)	.2623(-7)
.0383(-1)	.0626(-3)	.0632(-5)	.0632(-7)
.0000(-1)	.0000(-3)	.0000(-5)	.0000(-7)
.0322(-1)	.0575(-3)	.0581(-5)	.0581(-7)
.1235(-1)	.2197(-3)	.2222(-5)	.2222(-7)
.2579(-1)	.4713(-3)	.4770(-5)	.4771(-7)
.4255(-1)	.7981(-3)	.8084(-5)	.8085(-7)
.6173(-1)	1.1874(-3)	1.2035(-5)	1.2037(-7)
.8257(-1)	1.6279(-3)	1.6511(-5)	1.6514(-7)
1.0448(-1)	2.1098(-3)	2.1415(-5)	2.1418(-7)
1.2670(-1)	2.6250(-3)	2.6662(-5)	2.6667(-7)

TABLE 4--Continued

$K_1 = 0.5$

K_2	$A = 10^{-3}$	$A = 10^{-2}$	$A = 10^{-1}$
0.4	0.0455(0)	0.0443(0)	0.0347(0)
0.5	.0000(0)	.0000(0)	.0000(0)
0.6	.0279(0)	.0274(0)	.0227(0)
0.7	.0911(0)	.0897(0)	.0758(0)
0.8	.1712(0)	.1687(0)	.1444(0)
0.9	.2578(0)	.2543(0)	.2200(0)
1.0	.3457(0)	.3412(0)	.2947(0)
1.1	.4315(0)	.4261(0)	.3738(0)
1.2	.5137(0)	.5076(0)	.4474(0)
1.3	.5914(0)	.5846(0)	.5175(0)
1.4	.6644(0)	.6569(0)	.5836(0)
1.5	.7324(0)	.7244(0)	.6456(0)

TABLE 4--Continued

A = 1.0	A = 10	A = 10^2	A = 10^3
0.0552(-1)	0.0796(-3)	0.0801(-5)	0.0801(-7)
.0000(-1)	.0000(-3)	.0000(-5)	.0001(-7)
.0452(-1)	.0696(-3)	.0701(-5)	.0701(-7)
.1639(-1)	.2599(-3)	.2623(-5)	.2623(-7)
.3346(-1)	.5467(-3)	.5520(-5)	.5520(-7)
.5405(-1)	.9093(-3)	.9188(-5)	.9189(-7)
.7692(-1)	1.3311(-3)	1.3460(-5)	1.3462(-7)
1.0112(-1)	1.7986(-3)	1.8200(-5)	1.8202(-7)
1.2596(-1)	2.3009(-3)	2.3300(-5)	2.3303(-7)
1.5094(-1)	2.8296(-3)	2.8675(-5)	2.8679(-7)
1.7571(-1)	3.3779(-3)	3.4257(-5)	3.4262(-7)
2.0000(-1)	3.9405(-3)	3.9994(-5)	3.9999(-7)

APPENDIX B

NOMENCLATURE

a = constant

A = constant

B = constant

$c(t)$ = output variable

$C(s)$ = Laplace transform of $c(t)$

d = depolarization operator

D = drag

$e(t)$ = error variable

$E(s)$ = Laplace transform of $e(t)$

F = thrust

$f(t)$ = function of time

$F(s)$ = Laplace transform of $f(t)$

$g(t)$ = system impulse response

$G(s)$ = Laplace transform of $g(t)$

$h(t)$ = system impulse response

I = integral

j = square root of -1

J = polar moment

k = constant

K_i = slowly time-varying parameter or a constant

L = lift

m = mass

M = figure of merit

M_i = moment

n(t) = noise signal

p(t) = average of depolarized signals

q = slowly time-varying parameter

r(t) = input signal

R(s) = Laplace transorm of r(t)

s = Laplace transform variable

S_i = switch or area

t = time

T_i = time constant

u(t) = error function

V = velocity

α = constant or angle

β = constant

γ = angle

δ = deflection or impulse function

ϵ = base of natural logarithms

ζ = system damping ratio

θ = angle

λ = dummy integration variable

σ = real part of s

τ = correlation variable

φ_{ij} = correlation function

ω = imaginary part of s