ANALYSIS OF COUPLED CIRCUITS

BY SERIES EXPANSION

By

EDMUND W. SCHEDLER JR.

Bachelor of Science

Oklahoma Agricultural and Mechanical College

1949

Submitted to the Faculty of the Graduate School of

the Oklahoma Agricultural and Mechanical College

in Partial Fulfillment of the Requirements

for the Degree of

MASTER OF SCIENCE

1952

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THESIS AND ABSTRACT APPROVED:

Thesis Adviser

Faculty Representative

Dean of the Graduate School

PREFACE

An attempt to plot the response of a band-pass amplifier often confronts the radio engineer. In this thesis a method is worked out in which the response of the circuit to frequencies very near the resonant frequency may be readily calculated. It will be assumed that the reader is familiar with alternating current theory and its associated mathematics.

Excellent work has been done in analyzing double tuned circuits by modifying the coupling circuit in one manner or another.[1] Usually in this type of analysis some of the circuit reactances are assumed to remain constant with variations in frequency. Although this assumption generally contributes negligible errors in design, occasionally an engineer needs to work a problem to a specified degree of accuracy. This thesis was written in order to eliminate the tedious work of an exact solution and yet allow an engineer some knowledge of the degree of accuracy of the solution of his problem.

[1] Lawrence Baker Arguimbau, Vacuum-Tube Circuits, pp. 210-13.

ACKNOWLEDGEMENT

The author wishes to express his sincere thanks to Professor H. T. Fristoe and Dr. A. L. Betts for their encouragement and aid in preparing this thesis. The Algebraic Expansion of Taylor's Series, a timely suggestion for shortening the work, was brought forth by Professor Betts.

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CHAPTER I

INTRODUCTION

The typical band-pass amplifier may be analyzed by considering the circuit to the left of terminals A and B to be an equivalent generator with an internal impedance. The impedance looking to the left

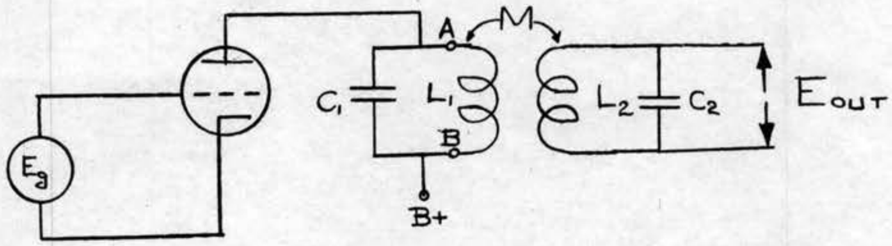

Figure 1 Circuit for a band-pass tuned, radio-frequency amplifier

of terminals A and B is R_p in parallel with X_{c1}.[1]

$$Z_g = \frac{R_P X_{C1}^2 - jR_P^2 X_{C1}}{R_P^2 + X_{C1}^2} \tag{1}$$

The reactive term is $\frac{-jX_{C1}\ R_P^2}{R_P^2 + X_{C1}^2}$. With a pentode tube the value of R_p is generally much larger than X_{c1}. The expression for Z_g then simplifies to

$$Z_g = \frac{X_{C1}^2}{R_P} - j X_{C1}. \tag{2}$$

The equivalent circuit then becomes

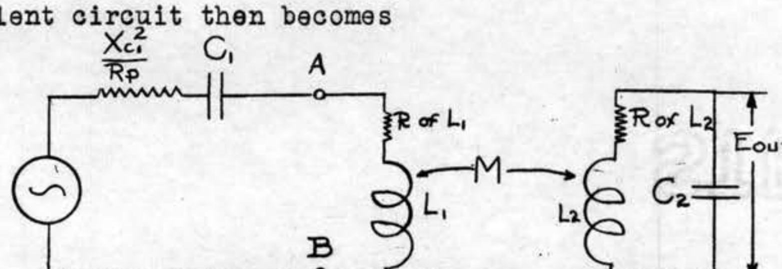

Figure 2 Equivalent circuit

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[1] W.L. Everitt, E.E., Ph. D., Communication Engineering, p. 496.

The loading of the generator introduces the additional resistance in series with the primary circuit.

The following symbols will be used to simplify the circuit to a greater extent.

R_{11} = resistance around the primary mesh

L_{11} = inductance of the primary mesh with the secondary open circuited

C_{11} = capacitance of the primary mesh

R_{22} = resistance of the secondary mesh

L_{22} = inductance of the secondary mesh

C_{22} = capacitance of the secondary including distributed capacity and input capacitance of the next tube

E = primary applied voltage which is $g_m E_g X_{C1}$. This analysis will assume that E remains constant throughout a small band of frequencies. Another chapter will cover the case of E as a function of frequency.

The circuit of Figure 2 simplifies into Figure 3.

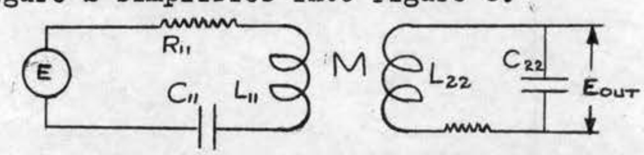

Figure 3 Equivalent circuit of mesh impedances.

The impedance of mesh one with all other meshes open-circuited is Z_{11}. The impedance of mesh two with all other meshes open-circuited is Z_{22}. The mutual impedance between mesh one and mesh two is Z_{12}.[2] From Figure 3

$$Z_{11} = R_{11} + jWL_{11} + \frac{1}{jWC} \tag{3}$$

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[2] Ibid., pp.217-23.

$$Z_{22} = R_{22} + jWL_{22} + \frac{1}{jWC_{22}} \quad (4)$$

$$Z_{12} = -jWM. \quad (5)$$

The E.M.F. equations for each mesh may be written

$$E = I_1 Z_{11} + I_2 Z_{12} \quad (6)$$

$$O = I Z_{12} + I Z_{22} \quad (7)$$

The current I_2 may be found by solving the simultaneous equations

$$I_2 = \frac{\begin{vmatrix} Z_{11} & E \\ Z_{12} & O \end{vmatrix}}{\begin{vmatrix} Z_{11} & Z_{12} \\ Z_{12} & Z_{22} \end{vmatrix}} \quad (8)$$

$$I_2 = \frac{-E(-jWM)}{(R_{11} + jWL_{11} + \frac{1}{jWC_{11}})(R_{22} + jWL_{22} + \frac{1}{jWC_{22}}) + W^2M^2}. \quad (9)$$

The output voltage is $I_2 X_{c_{22}}$.

$$E_{OUT} = \frac{E\frac{M}{C_{22}}}{(R_{11} + jWL_{11} + \frac{1}{jWC})(R_{22} + jWL_{22} + \frac{1}{jWC}) + W^2M^2} \quad (10)$$

Examining (10) with principal interest on the denominator, some facts are apparent. If both meshes are individually resonant at the same frequency, the denominator will be resistive at resonance. The reactive term in the denominator at frequencies slightly different from resonance will be the difference between two relatively large numbers. This makes it necessary to compute the reactance of each term to a high degree of accuracy before (10) would be of much value in plotting a response. With this in mind it would be well to seek another method of plotting the response without having to resort to extensive multiplication to evaluate the reactive terms.

CHAPTER II

TAYLOR'S SERIES SOLUTION

Taylor's theorem as applied to the expansion of a single variable is[1]

$$f(X) = f(X_o + h) = f(X_o) + h\frac{d f(X)}{d X}\bigg|_{X=X_o} + \frac{h^2}{\underline{|2}}\,\frac{d^2 f(X)}{d X^2}\bigg|_{X=X_o} + \frac{h^3}{\underline{|3}}\,\frac{d^3 f(X)}{d X^3}\bigg|_{X=X_o} + \text{-----------} \qquad (11)$$

In an effort to provide an accurate equation for the response of a band-pass amplifier this theorem will be applied. Let E_{out} be some function of omega.

$$E_{OUT} = f(W) = \frac{E\frac{M}{C_{22}}}{\left[R_{11} + (WL_{11} - \frac{1}{WC_{11}})\right]\left[R_{22} + (WL_{22} - \frac{1}{WC_{22}})\right] + W^2M^2} \qquad (12)$$

and let W_o occur when

$$W_o L_{11} = \frac{1}{W_o C_{11}} \qquad (13)$$

$$W_o L_{22} = \frac{1}{W_o C_{22}} \qquad (14)$$

The first two terms of the expansion then become

$$E_{OUT} = \frac{\frac{EM}{C_{22}}}{R_{11}\ R_{22} + W_o^2 M^2} - h\,\frac{EM}{C_{22}}\left[\frac{2W_o M^2 + j(2R_{11}L_{22} + 2R_{22}L_{11})}{R_{11}\ R_{22} + W_o^2 M^2}\right] \qquad (15)$$

These two terms are simple enough and would be easy to apply.

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[1]Austin V. Eastman, M.S., FUNDAMENTALS OF VACUUM TUBES, p. 499.

However, three terms and possibly more are necessary. The second term should be examined <u>before</u> it is evaluated at $W = W_0$. The first derivative is

$$\frac{EM}{C_{22}}\left[\frac{2WM^2 + \frac{2}{W^3 C_{11} C_{22}} - 2WL_{11}L_{22} + j\left(R_{11}L_{22} + R_{22}L_{11} + \frac{R_{11}}{W^2 C_{22}} + \frac{R_{22}}{W^2 C_{11}}\right)}{\left[\left(R_{11} + j\left[WL_{11} - \frac{1}{WC_{11}}\right]\right)\left(R_{22} + j\left[WL_{22} - \frac{1}{WC_{22}}\right]\right) + W^2M^2\right]^2}\right] \tag{16}$$

The third term of the expansion requires the differentiation of the second derivative before the second derivative is evaluated at $W = W_0$. Since the second derivative itself is a very complicated equation, the third derivative would be extremely laborious to evaluate. Therefore, it would be better to use some other approach to solve the problem.

CHAPTER III

ALGEBRAIC EXPANSION OF TAYLOR'S SERIES

The shape of equation (10) is known to be like Figure 4, depending, of course, upon the degree of coupling of the circuit.

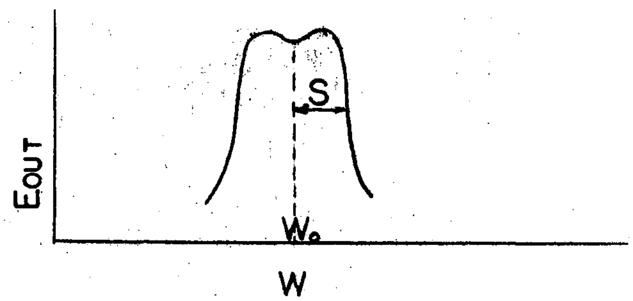

Figure 4 Response of a slightly over coupled band-pass amplifier.

Equation (10) can be rewritten for convenience.

$$E_{OUT} = \frac{\frac{E\,M}{C_{22}}}{\left[R_{11}+j\left(WL_{11}-\frac{1}{WC_{11}}\right)\right]\left[R_{22}+j\left(WL_{22}-\frac{1}{W\,C_{22}}\right)\right]+W^2M^2} \qquad (10)$$

The angular velocity W can be expressed as an W_0 + S.

$$W = W_0 + S \qquad (17)$$

The value of S may be either a positive or negative angular velocity.

In actual practice the primary and secondary meshes are seldom identical. However, if identical primary and secondary circuits are assumed, the the process of computing the response will be much simpler. Assuming identical circuits, the output voltage can be written as

$$E_{OUT} = \frac{\frac{E\,M}{C_{22}}}{\left[R_{11}+j\left(WL_{11}-\frac{1}{WC_{11}}\right)\right]^2+W^2M^2} \qquad (18)$$

$$E_{OUT} = \frac{\frac{EM}{C}}{R^2 + \frac{2L}{C} - \frac{(W^4L^2C^2 - W^4M^2C^2 + 1)}{W^2C^2} + j2R\frac{(W^2LC - 1)}{WC}} \quad (19)$$

Multiplying both the numerator and denominator by $W^2\ C^2$ gives

$$E_{OUT} = \frac{E\ M\ W^2 C}{W^4(M^2C^2 - L^2C^2) + j2RW^3LC^2 + W^2(R^2C^2 + 2LC) - j2RWC - 1} \quad (20)$$

The powers of W may be computed using equation (17).

$$W^2 = W_o^2 + 2W_oS + S^2 \quad (21)$$

$$W^3 = W_o^3 + 3W_o^2S + 3W_oS^2 + S^3 \quad (22)$$

$$W^4 = W^4 + 4W^3S + 6W^2S^2 + 4WS^3 + S^4 \quad (23)$$

Substitute for the $W^4(M^2C^2 - L^2C^2)$ term in the denominator $(W_o^4 + 4W_o^3S + 6W_o^2S^2 + 4W_oS^3 + S^4)\ (M^2C^2 - L^2C^2)$. This will give $W_o^4\ (M^2C^2 - L^2C^2) + 4W_o^3S(M^2C^2 - L^2C^2) + 6W_o^2S^2(M^2C^2 - L^2C^2) + 4W_oS^3(M^2C^2 - L^2C^2) + S^4(M^2C^2 - L^2C^2)$. Substitute for the $j2RW^3LC^2$ term $j2RLC^2W_o^3 + j6RLC^2W_o^2\ S + j6RLC^2W_oS^2 + j2RLC^2S^3$. Substitute for the $W^2(R^2C^2 + 2LC)$ term $W_o^2(R^2C^2 + 2LC) + 2W_oS(R^2C^2 + 2LC) + S^2(R^2C^2 + 2LC)$. Substitute for the $-j2RWC$ term $-j2RCW_o - j2RCS$.

The next process is to collect coefficients of each power of S. The coefficient of each power of S is listed on the next page.

Let G equal the coefficients of S^0.

$$G = W_0^4(M^2C^2 - L^2C^2) + j2RLC^2W_0^3 + W_0^2\ (R^2C^2 + 2LC) - j2RCW_0 - 1 \tag{24}$$

Let F equal the coefficients of S.

$$F = 4W_0^3(M^2C^2 - L^2C^2) + j6RLC^2W_0^2 + 2W_0(R^2C^2 + 2LC) - j2RC \tag{25}$$

Let E equal the coefficients of S^2.

$$E = 6W_0^2(M^2C^2 - L^2C^2) + j6RLC^2W_0 + R^2C^2 + 2LC \tag{26}$$

Let H equal the coefficients of S^3.

$$H = 4W_0(M^2C^2 - L^2C^2) + j2RLC^2 \tag{27}$$

Let D equal the coefficients of S^4.

$$D = M^2C^2 - L^2C^2 \tag{28}$$

The numerator of (20) can be expressed as

$$W^2EMC = (W_0^2 + 2W_0S + S^2)EMC = W_0^2EMC + 2W_0SEMC + S^2\ EMC \tag{29}$$

Again, the coefficients of each power of S must be separated.

$$C = W_0^2\ EMC \tag{30}$$

$$B = 2W_0\ EMC \tag{31}$$

$$A = EMC \tag{32}$$

The substitutions (24) through (32) reduce the output voltage across the condenser to

$$E_{out} = \frac{C + BS + AS^2}{G + FS + ES^2 + HS^3 + DS^4} \tag{33}$$

By long division the expression for the output voltage becomes

$$E_{out} = C/G + S\frac{(B - CF/G)}{G} + S^2\frac{(A - CE/G - FB/G + CF^2/G^2)}{G} \tag{34}$$

$$+ S^3\frac{(2CEF^2/G^2 - BE/G - CH/G - AF/G + BF^2/G^2 - CF^3/G^3)}{G} +$$

$$+ S^4\left(2CFH/G^3 - BH/G^2 - DC/G^2 - AE/G^2 - 3CEF^2/G^4 - F^3B/G^4 - CF^4/G^5 + CE^2/G^3 + 2FBE/G^3 + AF^2/G^3 \right) \quad (34)$$

It might be well to pause at this point and examine the meaning of (34). The first term corresponds to the value of E_{out} at the resonant frequency. If S is zero then the output will be C/G. This can be shown to be logically true by observing Figure 4. The coefficient of S is the evaluation of the first derivative at $W = W_0$. The coefficient of S^2 is the evaluation of the second derivative at $W = W_0$. The coefficients of S^3 and S^4 are the evaluations of the third and fourth derivatives respectively at $W = W_0$. The S in (34) corresponds to the _h_ in the Taylors expansion or (11).

CHAPTER IV

SAMPLE PROBLEM

An example of the series solution and how it compares with an exact solution would be very instructive. The primary and secondary

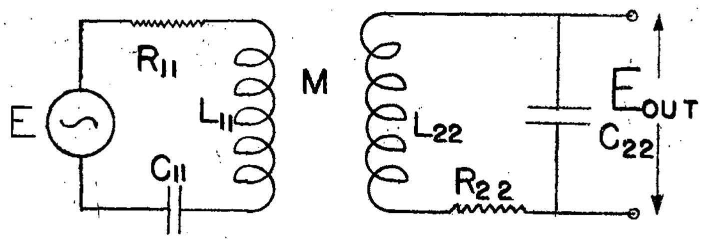

Figure 5 Tuned circuits with identical primary and secondary

circuits are to be identical. The circuit constants used are values that one might expect from an ordinary intermediate frequency transformer.

$R = R_{11} = R_{22} = 20$ ohms

$L = L_{11} = L_{22} = 1$ millihenry

$C = C_{11} = C_{22} = 125$ micromicrofarads

K = coefficient of coupling = 0.008

$E = 1$ volt

$M = 8$ microhenry

$X_0 = 1/W_0C = W_0L = 2{,}830$ ohms

$f_0 = W_0/2\pi = 450{,}158$ cycles per second

f_0 = resonant frequency of the primary and secondary tuned individually

Equations 24 to 32 must be evaluated.

$C = 8 \times 10^{-3}$

$B = 5660 \times 10^{-12}$

$A = 10^{-15}$

$$D = -15.625 \times 10^{-27}$$

$$E = 500 \times 10^{-15}$$

$$F = -j10^{-8}$$

$$G = 114 \times 10^{-6}$$

$$H = -176.8 \times 10^{-21}$$

$$C/G = 70.2 \qquad (35)$$

$$\frac{B - CF/G}{G} = -j\ 0.616 \times 10^{-2} \qquad (36)$$

$$\frac{A - CE/G - FB/G + CF^2/G^2}{G} = -229.5 \times 10^{-9} \qquad (37)$$

$$\frac{2CEF^2 - BE/G - CH/G - AF/G + BF^2/G^2 - CF^3/G^3}{G} = j47.1 \times 10^{-12} \qquad (38)$$

$$2CFH/G^3 - BH/G^2 - DC/G^2 - AE/G^2 - 3CEF^2/G^4 - F^3B/G^4 - CF^4/G^5$$
$$2FBE/G^3 + CE^2/G^3 + AF^2/G^3 = 9.881 \times 10^{-15} \qquad (39)$$

The values above are approximate because they are, for the most part, slide rule accuracy. The expansion to five terms is

$$E_{out} = 70.2 + S\,(-j\ 0.616 \times 10^{-2}) + S^2\,(-229.5 \times 10^{-9}) + S^3\,(+j\ 47.1 \times 10^{-12}) + S^4\,(9.881 \times 10^{-15}). \qquad (40)$$

The values of S at which the peak output voltage occurs may be found by differentiating the equation of the output voltage written with four terms of the series.

$$E_{out} = 70.2 + S\,(-j\ 0.616 \times 10^{-2}) + S^2\,(-229.5 \times 10^{-9}) + S^3\,(+j\ 47.1 \times 10^{-12}) \qquad (41)$$

$$\frac{d\,E_{out}}{ds} = 0 = -j\ 0.616 \times 10^{-2} + 2S\,(-229.5 \times 10^{-9}) + 3S^2\,(+j47.1 \times 10^{-12}) \qquad (42)$$

Solving equation 42 for S gives

$$\text{Real part of } S = \pm \frac{j\ 18.09 \times 10^{-7}}{j\ 6 \times 47.1 \times 10^{-12}} = 6400 \text{ radians/second.}$$

The upper and lower peak frequencies from resonance would be

$$6400/2\pi = 1018 \text{ cycles.}$$

It might be asked why the fifth term was omitted. If the fifth term was considered the derivative would have involved solving a cubic equation. The solution of a cubic equation besides being more tedious, usually involves successive approximations. Using the first four terms is a satisfactory approximation by itself.

Using (40) and allowing S to vary in steps of 628 radians the response was computed. For comparison the response was computed using the exact solution of (18). Both responses are shown on page 14. The series response was computed with only a slide rule. The exact response was computed on a computing machine with the accuracy maintained high enough so that the final result was correct to four significant figures.

For frequencies very near the resonant frequency, the series solution is very accurate. The work done for the exact solution was much longer and required extensive trigonometric tables. Notice that the series solution becomes quite inaccurate above about 800 cycles. However, the value of frequency that produces a peak output as determined by the exact solution appears to be around 900 cycles. This agrees remarkably well with the value 1018 cycles as determined by the series solution.

Frequency	Frequency off Resonance	E_{out} By Series	E_{out} Exact By (18)
448,158	− 1200 Cycles	$64.8 \angle 24.05^\circ$	$70.30 \angle 40.50$
449,138	− 1020 Cycles	$66.5 \angle 24.05^\circ$	$70.74 \angle 34.55$
449,158	− 1000 Cycles	$67.0 \angle 23.80^\circ$	$70.76 \angle 33.80$
449,258	− 900 Cycles	$68.2 \angle 22.6^\circ$	$70.84 \angle 30.10$
449,358	− 800 Cycles	$69.8 \angle 21.10^\circ$	$70.83 \angle 26.45$
449,458	− 700 Cycles	$69.8 \angle 19.35^\circ$	$70.77 \angle 22.95$
449,558	− 600 Cycles	$70.0 \angle 17.18^\circ$	$70.68 \angle 19.45$
449,658	− 500 Cycles	$70.2 \angle 14.75^\circ$	$70.57 \angle 16.10$
449,758	− 400 Cycles	$70.3 \angle 12.10^\circ$	$70.45 \angle 12.78$
449,858	− 300 Cycles	$70.2 \angle 9.26^\circ$	$70.35 \angle 9.54$
449,958	− 200 Cycles	$70.2 \angle 6.25^\circ$	$70.27 \angle 6.33$
450,058	− 100 Cycles	$70.2 \angle 3.165^\circ$	$70.20 \angle 3.17$
450,158	± 0 Cycles	$70.2 \angle 0^\circ$	$70.17 \angle 0$
450,258	+ 100 Cycles	$70.2 \angle -3.165^\circ$	$70.17 \angle -3.15$
450,358	+ 200 Cycles	$70.2 \angle -6.25^\circ$	$70.19 \angle -6.32$
450,458	+ 300 Cycles	$70.2 \angle -9.26^\circ$	$70.24 \angle -9.51$
450,558	+ 400 Cycles	$70.3 \angle -12.1^\circ$	$70.31 \angle -12.75$
450,658	+ 500 Cycles	$70.2 \angle -14.75^\circ$	$70.39 \angle -16.05$
450,758	+ 600 Cycles	$70.0 \angle -17.18^\circ$	$70.50 \angle -19.36$
450,858	+ 700 Cycles	$69.8 \angle -19.35^\circ$	$70.54 \angle -22.80$
450,958	+ 800 Cycles	$69.8 \angle -21.1^\circ$	$70.57 \angle -26.15$
451,058	+ 900 Cycles	$68.2 \angle -22.6^\circ$	$70.56 \angle -29.90$
451,158	+ 1000 Cycles	$67.0 \angle -23.8^\circ$	$70.40 \angle -33.60$
451,178	+ 1020 Cycles	$66.5 \angle -24.05^\circ$	$70.44 \angle -34.25$
451,358	+ 1200 Cycles	$64.8 \angle 24.05^\circ$	$70.03 \angle -41.2$

CHAPTER V

THE SOLUTION WITH APPLIED VOLTAGE VARYING WITH FREQUENCY

Equation 33 was developed assuming that the applied voltage does not change with frequency. Figure 2 shows that the assumption is not true. From Figure 2 the E.M.F. equations may be written as

$$g_m E_g 1/WC_1 = I_1 Z_{11} + I_2 Z_{12} \tag{43}$$

$$0 = I_1 Z_{12} + I_2 Z_{12}. \tag{44}$$

The current I_2 may be found by solving (43) and (44) by determinants.

$$I_2 = \frac{-Z_{12} g_m E_g/WC_1}{Z_{11} Z_{12} - Z_{12}^2} \tag{45}$$

$$I_2 = \frac{\dfrac{j G_M E_G M}{C_1}}{W^2 M^2 + \left[R_{L_1} + \dfrac{1}{(WC_1)^2 R_P} + j\left(WL_1 - \dfrac{1}{WC_1}\right)\right]\left[R_{L_2} + j\left(WL_2 - \dfrac{1}{WC_2}\right)\right]} \tag{46}$$

The denominator of the expression for I_2 should be simplified and can be expressed as

$$R_{L_1}R_{L_2} + \frac{R_{L_2}}{(WC_1)^2 R_P} + jR_{L_2}(WL_1 - \frac{1}{WC_1}) + j(R_{L_1} + \frac{1}{(WC_1)^2 R_P})(WL_2 - \frac{1}{WC_2})$$

$$-(W^2 L_1 L_2 - \frac{L_1}{C_2} - \frac{L_2}{C_1} + \frac{1}{W^2 C_1 C_2}) + W^2 M^2$$

Multiplying the numerator and denominator by W^3 yields for the numerator

$$\frac{j G_M \; E_G \; W^3 M}{C_1}$$

and for the denominator

$$W^3(R_{L_1}R_{L_2} + \frac{L_1}{C_2} + \frac{L_2}{C_1}) + W^2(\frac{j L_2}{C_1^2 R_P} - \frac{j R_{L_1}}{C_2} - \frac{j R_{L_2}}{C_1}) + W(\frac{R_{L_2}}{C_1^2 R_P} - \frac{1}{C_1 C_2})$$

$$W^4(jR_{L_2} L_1 + jR_{L_1} L_2) + W^5 (M^2 - L_1 L_2)$$

Since the output voltage is $I_2 X_{C_2}$ or $I_2 1/jWC_2$ the numerator above becomes

$$\frac{G_M \; E_G \; W^2 M}{C_1 \; C_2}$$

and the denominator remains the same.

The complete equation for the output voltage is (47). Notice that the principal difference between (47) and (20) is that (47) has a fifth power equation in the denominator whereas (20) has only a fourth power equation in the denominator.

$$E_{OUT} = \frac{G_M \; E_G \; \dfrac{W^2 M}{C_1 C_2}}{W^5(M^2 - L_1 L_2) + W^4(jR_{L_2}L_1 + jR_L L_2) + W^3\left(R_{L_1}R_{L_2} + \dfrac{L_1}{C_2} + \dfrac{L_2}{C_1}\right) + W^2\left(\dfrac{jL_2}{C_1^2 R_p} + \dfrac{jR_{L_1}}{C_2} + \dfrac{jR_{L_2}}{C_1}\right) + W\left(\dfrac{R_{L2}}{C_1^2 R_p} - \dfrac{1}{C_1 C_2}\right) - \dfrac{j}{C_1^2 C_2 R_p}} \tag{47}$$

For convenience (17), (21), (22), and (23) will be repeated.

$$W = W_o + S \tag{17}$$

$$W^2 = W_o^2 + 2W_oS + S^2 \tag{21}$$

$$W^3 = W_o^3 + 3W_o^2S + 3W_oS^2 + S^3 \tag{22}$$

$$W^4 = W_o^4 + 4W_o^3S + 6W_o^2S^2 + 4W_oS^3 + S^4 \tag{23}$$

A fifth power term in W must be evaluated.

$$W^5 = W_o^5 + 5W_o^4S + 10W_o^3S^2 + 10W_o^2 S^3 + 5W_oS^4 + S^5 \tag{48}$$

Equations (17), (21), (22), (23), and (48) must be substituted into (47). After the substitution is carried out, the coefficients of each power of S must be determined. Let the constants A, B, C, D, E, and F be be equal to the coefficients of S^o, S, S^2, S^3, S^4, and S^5 in the denominator respectively.

$$A = W_o^5 (M^2 - L_1L_2) + W_o^4 (jR_{L_2} L_1 + jR_{L_1} L_2) + W_o^3 (R_{L_1} R_{L_2} + L_1/C_2)$$

$$+ W_o^3 (L_2/C_1) + W_o^2 (j L_2/C_1^2 R_p - j R_{L_1}/C_2 - j R_{L_2}/C_1) \quad (49)$$

$$+ W_o (R_{L_2}/C_1^2 R_p - 1/C_1C_2) - j/C_1^2 C_2 R_p$$

$$B = 5W_o^4 (M^2 - L_1L_2) + 4W_o^3 (jR_{L_2} L_1 + jR_{L_1} L_2) + 3W_o^2 (L_1/C_2) + \quad (50)$$

$$3W_o^2 (R_{L_1} R_{L_2} + L_2/C_1) + 2W_o (jL_2/C_1^2 R_p - jR_{L_1}/C_2 - jR_{L_2}/C_1)$$

$$C = 10W_o^3 (M^2-L_1L_2) + 6W_o^2 (jR_{L_2} L_1 + jR_{L_1} L_2) + 3W_o (L_1/C_2) \quad (51)$$

$$+ 3W_o (L_2/C_1 + R_{L_1} R_{L_2}) + j(L_2/C_1^2 R_p - R_{L_1}/C_2 - R_{L_2}/C_1)$$

$$D = 10W_o^2 (M^2-L_1L_2) + 4W_o (jR_{L_2} L_1 + jR_{L_1} L_2) + L_1/C_2 + \quad (52)$$

$$L_2/C_1 + R_{L_1} R_{L_2}$$

$$E = 5W_o (M^2-L_1L_2) + j(R_{L_2} L_1 + R_{L_1} L_2) \quad (53)$$

$$F = M^2 - L_1 L_2 \quad (54)$$

The numerator of (47) can be expressed as

$$\frac{g_m E_g M}{C_1 C_2} (W_o^2 + 2W_oS + S^2) = \frac{g_m E_g M W_o^2}{C_1 C_2} + \frac{g_m E_g M 2 W_o S}{C_1 C_2}$$

$$+ \frac{g_m E_g M S^2}{C_1 C_2} .$$

This can be written as $G + H S + I S^2$ where

$$G = \frac{g_m E_g M W_o^2}{C_1 C_2}$$

$$H = \frac{g_m E_g M 2 W_o}{C_1 C_2} \tag{57}$$

$$I = \frac{g_m E_g M}{C_1 C_2} . \tag{58}$$

Using (49) to (57) the output voltage equation simplifies to

$$E_{out} = \frac{G + H S + I S^2}{A + B S + C S^2 + D S^3 + E S^4 + F S^5} . \tag{59}$$

If the division indicated in (59) is carried out the quotient is a Taylor's expansion of the output voltage.

$$E_{out} = G/A + S/A (H - GB/A) + S^2/A (I - GC/A - BH/A + B^2G/A^2)$$
$$+ S^3/A (2CGB/A^2 + B^2H/A^2 - CH/A - GD/A - BI/A - B^3G/A^3) \tag{60}$$

In (60) the $\underline{S}$ terms correspond to the $\underline{h}$ terms in (11). The coefficients of each S term in (60) is nothing more than the evaluation of each derivative at the frequency selected to expand around.

CHAPTER VI

CONCLUSIONS

A comparison of the calculated response of equation 18 and equation 33 shows that the series is extremely accurate for frequencies near resonance. However, certain approximations were made in evaluating the derivatives at W_0. If the smaller component of a derivative was less than 1/10 the larger component, then the smaller component was neglected. Actually, this need not have been done, but it shortened the computations considerably. Increased accuracy over a wider range with less terms of the series used could probably be attained by using both components of each derivative.

The series converges very rapidly near the resonant frequency. With frequencies far from resonance the terms required for convergence becomes large. The error in the series at any frequency can be made as small as desired by taking a sufficient number of terms.

At frequencies several thousand cycles from resonance the exact method for evaluating response (by equation 18) should be perfectly satisfactory. The difference in the reactive terms in the denominator should be large enough to be readily determined by a slide rule.

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VITA

EDMUND W. SCHEDLER JR.
candidate for the degree of
Master of Science

Thesis: ANALYSIS OF COUPLED CIRCUITS BY SERIES EXPANSION

Major: Electrical Engineering

Biographical and Other Items:

Born: October 13, 1926 at Manila, Philippines
Undergraduate Study: Oklahoma Agricultural and Mechanical College, 1944-1949

Graduate Study: Oklahoma Agricultural and Mechanical College, 1950-1952

Experiences: Navy, 1945-46; Continental Oil Company, 1949-1950; Employed by the School of Electrical Engineering in 1950

Member of Eta Kappa Nu, Sigma Tau, Phi Kappa Phi, and a student member of The Institute of Radio Engineers.

Date of Final Examination: May 13, 1952

THESIS TITLE: Analysis of Coupled Circuits by Series Expansion

AUTHOR: Edmund W. Schedler Jr.

THESIS ADVISER: Dr. A. L. Betts

The content and form have been checked and approved by the author and thesis adviser. Changes or corrections in the thesis are not made by the Graduate School office or by any committee. The copies are sent to the bindery just as they are approved by the author and faculty adviser.

TYPIST: Edmund W. Schedler Jr.