IDENTIFYING CONCEPTS AND PROCESSES IN MATHEMATICS NEEDED FOR THE ADEQUATE PREPARATION OF ELEMENTARY TEACHERS

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To

My wife, Wilma,

whose aid, encouragement, and

inspiration have been invaluable.

PREFACE

The status of the elementary teacher with reference to his preparation and ability to teach arithmetic, in the opinions of many writers in the field, reached a low ebb after World War II. Through the efforts of many people and many agencies, conditions have improved and are still improving.

Identifying the concepts and processes of mathematics needed by an elementary teacher in teaching elementary arithmetic seemed to be necessary in order that the concepts and processes might be emphasized in the teachers' training. It was hoped that this study would aid in the training of the teachers of elementary arithmetic.

Profound gratitude is expressed to Dr. James H. Zant, Professor of Mathematics, whose inspiration, guidance, and patience have made this study possible.

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The writer is also indebted to the experts and elementary teachers who contributed to this investigation by giving their time and opinions in answering the questionnaire.

R. C.

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CHAPTER I

THE PROBLEM

The purpose of this study is to identify the concepts and processes of mathematics needed by an elementary teacher to teach arithmetic adequately in grades one to six.

Importance

The preparation of an elementary teacher is multiphasic in its specific demands and general in that it covers most of the areas of a liberal education. A liberal education is a necessity for every teacher.

> For the teacher, however, being well educated is a necessity. Without it, the teacher cannot interpret any field of knowledge in its proper relationship to the whole of society. And without it, the teacher will not be respected by a society which is itself becoming increasingly well educated.[1]

Most elementary teachers are assigned a grade to teach and are expected to teach one group of students all subjects throughout the school year. The teacher must know well the subject matter of the particular grade in which he is to teach, and he should know what has preceded as well as that which is to follow. Modern psychology has shown that the child should be developed into a well integrated individual. Then the teacher must know the past, work in the present, and plan for the future

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[1] W. Earl Armstrong, "The Teacher Education Curriculum," The Journal of Teacher Education, VIII (September, 1957), 235-236.

for each and every child in his class.

It must be remembered that the mathematical concepts, skills, and quantitative understandings that the students acquire can be no better than those possessed by their teachers. Hence, teachers must strive constantly to increase their own understandings as well as to search for more "know how" for effective teaching.[2]

Instruction in mathematics should place emphasis on the development of a vocabulary of mathematical terms as well as on understandings. Most of the vocabulary difficulty in mathematics is caused by a lack of understanding of technical terms, such as, exponent, congruent, equation, numerator, denominator and percentage.[3]

The problem of what constitutes adequate training for the elementary school teacher is broad and has many ramifications. It is impractical to attack this problem in its entirety. Hence, the problem must be limited. To teach arithmetic adequately, the elementary teacher must be familiar with certain concepts and processes of mathematics.

If you should wish to qualify for such a position (elementary teacher of arithmetic), the main requirement would be that you "understand" arithmetic. You cannot teach what you do not know. Here is the arithmetic that you would have to teach in the first six grades:

(1) Basic concepts, processes and vocabulary of arithmetic; (2) our decimal system of numeration, including the concept of decimal fractions; (3) computation, whole numbers and common and decimal fractions; (4) principal units of measurement for everyday use; (5) solution of problems involving computation and units of measurement; (6) identification of geometric figures; (7) use of simple graphs; (8) estimation and checking of answers to problems.[4]

Additional examples are given in Chapter II, pages 13 to 16, concerning the importance of mathematical concepts and processes to the elementary teacher.

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[2] Improvement of the Teaching of Mathematics, Oklahoma State Department of Education, 1957, p. 64.

[3] Ibid., p. 68.

[4] "Guidance Report of the Commission on Post War Plans," The Mathematics Teacher, XL (November, 1947), 324.

Featherston and Hull[5] state that research does not

> . . . throw much light on the problem of which specific concepts should be included in teacher education courses. It sheds no light at all on the most desirable ways of teaching these concepts. In short, it seems from the 1955-56 research that the important specific understandings in mathematics that should be included in the teacher education program have not been determined.

Thus, since the concepts and processes of elementary arithmetic must be used in teaching for understanding and meaning, they must first be identified. The teacher must know and understand the concepts and processes which he is to teach or else he will be poor and inefficient. Hence, the identification of these concepts and processes is most important for the teacher.

Need for the Study

Many articles have appeared giving the shortcomings of the teaching of arithmetic. A test of over one thousand ninth grade pupils in three eastern states showed extreme weaknesses in all but the most simple examples of computation, problem solving, understandings and judgments.[6] Almost identical failings were shown in a test given to one thousand freshmen in college.

> The evidence points clearly that we are not achieving functional competence in arithmetic at the elementary school level, at the junior high level, and at the senior high school level.[7]

Many pupils not only fail to learn the processes and concepts of arithmetic, but stop taking mathematics as soon as possible. Some of

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[5] E. Glenn Featherston and J. Dan Hull, Analysis of Research in the Teaching of Mathematics, 1955 and 1956, U. S. Department of Health, Education and Welfare, Office of Education, p. 20.

[6] Ben A. Sueltz and John W. Beredick, "The Need for Extending Arithmetical Learnings," Mathematics Teacher, XLIII (February, 1950), 71.

[7] Ibid., 72.

the main reasons for the drop-outs point directly to the teachers.[8] Glennon[9] reports the following in a doctoral study at Harvard University in 1948: A test consisting of 80 items on basic understandings was given to three groups of students and teachers of elementary arithmetic. The group consisted of 144 freshmen at the time of entrance into college, 172 seniors just before graduation, and 160 in-service teachers at the end of a school year. All had indicated that they wanted to become elementary teachers or were already elementary teachers.

Some of the items on the test and apparent degree of difficulty follow. An easy item was: Changing the order of addends in an addition example does not change the value of the answer. An item of medium difficulty was: Dividing the dividend and divisor by ten does not change the value of the answer (quotient). One of the most difficult items in the test was: A digit in the units' place represents a value one-tenth as large as the same digit in the tens' place.

The average number of items correctly answered by the freshmen was 35.45 and the per cent of total (80) was 44.31. The average number of items correctly answered by the seniors was 34.19 and the per cent of the total was 42.73. The teachers worked correctly an average of 43.81 items or 54.77 per cent of the total.

These findings seem to suggest several aspects of needed redirection in the program of in-service development of teachers of arithmetic. Curriculum revision of the professional courses must be concerned with emphasizing the subject matter as well as with the principles of teaching the subject matter.[10]

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[8] Henry S. Dyer, Robert Kalin, and Frederic M. Lord, Problems in Mathematical Education (Princeton, 1956), p. 3.

[9] Vincent J. Glennon, "A Study of the Growth and Mastery of Certain Basic Mathematical Understandings of Seven Educational Levels," Mathematics Teacher, XLII (December, 1949), 389-396.

[10] Ibid., 395.

Research shows that there is a lack of understanding of meanings in elementary arithmetic by the teachers. Research further shows that many teachers are weak in mathematical processes such as computation, problem solving, and judgments. Also, a great number of elementary teachers exhibit an unfavorable attitude toward arithmetic.[11] Thus, there is a need for the identification of the concepts and processes of mathematics.

Basic Assumptions

The basic assumptions for this study are as follows:

1. Meeting various classroom situations which require making decisions concerning the use of text material requires an understanding of the concepts and processes of mathematics on the part of the elementary teacher.

2. To teach arithmetic adequately for pupil learning with meaning and understanding, the teacher must himself understand the underlying mathematical concepts and processes.

3. Elementary teacher needs, involving concepts and processes of arithmetic, are of prime importance in their training.

Hypothesis

The consensus among the experts who best know the field of arithmetic and its teaching regarding the concepts and processes needed by an elementary teacher for adequacy in the classroom can be identified and stated as catagories which can be used as criteria for planning a program of preparation of elementary teachers for giving effective

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[11] W. H. Dutton, "Attitudes of Prospective Teachers Toward Arithmetic," Elementary School Journal, LII (October, 1951), 84-90.

instruction in arithmetic.

Definitions of Terms

Certain terms need to be defined with reference to their use in this paper:

Concepts are the elements of knowledge. Concepts as used here are identified by mathematical terms which are used by the teacher or textbook to develop the child in his mathematical understanding, thinking, and reasoning. If a term has quantitative or spatial significance, it is included as a concept.

Process is an operation, a course of procedure, a series of actions, motions, or operations definitely conducive to an end.

> By process is meant the way in which the learner operates in order to attain certain learning products. Process refers to the way in which one learns. But it, also, has a larger significance because process, a way of learning, itself becomes established and a particular way of learning with its consequent meaning for a way of attacking new problems is often as important to the individual as the particular product.[12]

Plan of Study

Preliminary identification of concepts and processes was made on a frequency of occurrence basis from a number of selected elementary arithmetic textbooks. The concepts and processes were used to formulate a questionnaire which was sent to a selected group of college teachers in mathematics and mathematics education. The questionnaire was also sent to a much larger group of selected elementary teachers. The final selection of concepts and processes was based upon the importance attached to

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[12] G. Lester Anderson and Arthur I. Gates, "The General Nature of Learning," Learning and Instruction. Forty-ninth Yearbook of the National Society for the Study of Education, Part I (University of Chicago, 1950), pp. 27-28.

each item by both the experts and the elementary teachers as shown on the questionnaire.

Percentages were given for each item according to the three check spaces on the questionnaire. An average was figured between the teachers and experts. This average was used to determine the importance of the concepts and processes. An average rating of sixty per cent or higher in any check space was used to determine its importance. If none of the three check spaces received a sixty per cent or higher rating, then a combination of two space ratings was used to determine the importance of the concept or process. A sixty per cent or higher rating in any check space indicates a good majority. Also, a low rating by one group would necessitate an extremely high rating by the other group to have a rating of sixty per cent.

CHAPTER II

PARTIAL REVIEW OF RELATED LITERATURE

Introduction

The objectives of this chapter were: (1) to show the changes which have taken place in the teaching of mathematics over the past half century, (2) to emphasize especially the latest theories of learning as related to mathematics, (3) to relate these to the present study, and (4) to indicate the plight of the elementary teacher.

It seemed advisable to discuss the theories of learning which have affected the changes in the teaching of arithmetic. Thorndike's psychology led to the "drill" theory of learning in mathematics. The field theories of learning which stem from the Gestalt psychology led to the "meaning" theory of learning in mathematics. Meanings and understandings are the essence of the "meaning" theory. The meanings and understandings of the concepts and processes of arithmetic are important in the training of elementary teachers. Lists of mathematical concepts and processes have been made, but few attempts have been made to determine the importance of them. The purpose of the present study has been the identification of the concepts and processes of arithmetic needed by elementary teachers.

The demand for elementary teachers since World War II has been so great that many unqualified teachers have been certified to teach. The requirements which were of necessity relaxed have been gradually brought back to normal and have been raised generally throughout the

country.

Learning Theories in Mathematics

Too little is known about how children learn in mathematics. Psychology has made great progress in many fields, but little in mathematics.[1] Some discussion of two general theories of learning is given in order to describe the development and application of learning theories in mathematics.

Learning theories fall into two major families: stimulus-response theories and cognitive theories, but not all theories belong to these two families.[2]

Stimulus-response Theories. The stimulus-response theories of Edward L. Thorndike and his followers have dominated learning for over half a century. The laws of learning: (1) effect, (2) readiness, and (3) exercise which Thorndike first promulgated had a great influence upon all education. He later made fundamental revisions in the laws of exercise and effect. However, the law of exercise had made its impact upon the teaching of mathematics in the form of the "drill" theory.

Transfer of learning, also, played a part in the early mathematics program. It was given as a reason for studying mathematics. Discipline of the mind and transfer were practically synonymous. The study of mathematics, Latin, and Greek afforded the necessary discipline to carry over into any profession.

Thorndike disagreed with this idea of transfer of learning. He explained transfer by what he called identical elements in the different

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[1] Henry S. Dyer, Robert Kalin, and Frederic M. Lord, Problems in Mathematical Education (Princeton, 1956), p. 4.

[2] Ernest R. Hilgard, Theories of Learning (2nd ed., New York, 1956), p. 8.

situations. Similar elements would carry over from one learning situation to another, thus helping to effect a solution to a new situation.

Glennon and Hunnicutt[3] reported that numerous studies which were made earlier in the century have shown the effect of meaningfulness of the material being learned on the facility with which it is learned and on the permanance of learning. The studies reported by McLellan and Dewey[4] in 1895 and Thorndike[5] in 1922, writing specifically on the psychology of arithmetic, stressed the importance of teaching for meanings and understandings.[6] Their writings, however, were often misinterpreted and in general did not bring about any significant change in methodology from drill teaching to meaningful teaching.

> McLellan and Dewey issued The Psychology of Numbers in 1895 ... . It taught us . . . the importance of the whole. The authors said in substance, let us begin with wholes, because they give significance to parts. Let us not believe that we should begin with parts and that the pupil can in some way put them together to make meaningful wholes.[7]

Thorndike seemingly tried to change to emphasis on meanings, but the drill method was too well established.

Field Theories. The cognitive or field theories of learning stem

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[3] Vincent J. Glennon and C. W. Hunnicutt. What Does Research Say About Arithmetic? Association for Supervision and Curriculum Development, A Department of the National Education Association (Washington, 1953), p. 12.

[4] James A. McLellan and John Dewey, The Psychology of Numbers (New York, 1916).

[5] Edward L. Thorndike, The Psychology of Arithmetic (New York, 1922), p. 19.

[6] Edward L. Thorndike, New Methods in Teaching Arithmetic (New York, 1921), pp. 58-59.

[7] B. R. Buckingham, "Significance, Meaning, Insight - These Three," Mathematics Teacher, XXXI (January, 1938), 26.

from the Gestalt psychology.

Field theory claims that learning is not so much a matter of establishing connections between stimuli and responses as it is of finding patterns in the stimuli perceived, "that is, seeing some sort of organization and meaning in the field of experience."[8]

Structuring and understandings, or insights, are basic to this type of learning. Thinking is reorganizing understandings, experiences, and facts to effect a solution to a problem.

McConnell[9] states:

On the contrary, it is part and parcel of a theory of learning which stresses organization rather than discreteness, understanding rather than memorization, and exercise of the higher mental processes rather than dependence upon lower-order habits.

The field psychologist thinks in terms of the organization and systematic arrangement of the whole rather than in terms of elements set out in unrelated, disconnected form. Wholes are organized structures of parts rather than a mere collection of parts and are more than the collection of all the parts.[10]

Practice definitely has a place only after understandings have been developed. The function of practice is to increase efficiency of performance in operations which are already clearly understood. The structure of the number system and the systematic character of number relations should be enhanced through the drill program.[11]

Furthermore, present learning theory stresses the importance of meanings throughout the whole range of number operations in addition to

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[8] Dyer, Kalin and Lord, p. 7.

[9] T. R. McConnell, "Recent Trends in Learning Theory," Arithmetic in General Education. Sixteenth Yearbook of the National Council of Teachers of Mathematics (Columbia University, 1941), p. 276.

[10] G. T. Buswell, "The Psychology of Learning in Relation to the Teaching of Arithmetic," The Teaching of Arithmetic. Fiftieth Yearbook of the National Society for the Study of Education, Part II (University of Chicago, 1951), p. 146.

[11] Ibid., p. 147.

continued insistence on competence in computation.[12]

The "meaning" theory of teaching arithmetic which grew out of the "field" theories of learning was first promulgated by Brownell[13] in 1935.

> The "meaning" theory conceives of arithmetic as a closely knit system of understandable ideas, principles, and processes . . . . The true test (of learning) is an intelligent grasp upon number relations and the ability to deal with arithmetical situations with proper comprehension of their mathematical as well as their practical significance.[14]

Definitions of terms, concepts, and processes do not constitute meanings or understandings. There is no meaning in stating from memory that $5 \nearrow 7 = 12$. There is meaning when we say that 12 is 10 and 2 and that $5 \nearrow 7 = (5 \nearrow 5) \nearrow 2$ or $10 \nearrow 2$. There is meaning in every mathematical concept and process. There is meaning in each step of every process.

> Meanings are the paths to all desirable outcomes in arithmetical instruction, and unless meanings are comprehended the outcomes are never reached . . . . In arithmetic, meaning is the import of relationships inherent in number study, the sense which the relationships are intended to express. Relationships constitute the meanings.[15]

> Meanings and ideas are dynamic facts of experience. Moreover, they shed light back upon the experiences from which they grew and thus give larger meanings to these experiences.[16]

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[12] Ibid., p. 149.

[13] William A. Brownell, "Psychological Consideration in the Learning and the Teaching of Arithmetic," Teaching of Arithmetic. Tenth Yearbook of the National Council of Teachers of Mathematics (Columbia University, 1935), p. 19.

[14] Ibid.

[15] C. Newton Stokes, Teaching the Meanings of Arithmetic (New York, 1951), p. 4.

[16] Harry Grove Wheat, The Psychology of Teaching of Arithmetic (Boston, 1937), p. 149.

An experience can have meaning only in terms of previous understandings and insights. Meaning is always based upon experience, and experience is meaningful only in terms of what the learner already understands. . . . Insight is more than understanding. It is a mental state in which the learner is fully aware of the conditions and relations which constitute a given behavior pattern and from which other useful patterns may evolve or take form.[17]

Importance of Meanings in Arithmetic

Mathematical terms have always been interspersed in all reading materials from the daily newspaper to the most technical books of any area of knowledge. Very little conversation takes place which does not include many mathematical terms.

Horn[18] reports:

Every investigator has shown the incidence of arithmetical terms to be very large - how large depends upon how broadly "arithmetical terms" are defined. If indefinite and marginal terms are included, such as more, heavy and high, the incidence shown in an analysis of recently published geography texts runs as high as one word in seven. This is not surprising when one realizes that, of the first 1069 words in the list compiled by Thorndike and Lorge,[19] more than one in ten are reasonably specific arithmetical, geometrical, or statistical terms, and if indefinite mathematical terms are included, the proportion is about one in four.

Pressey[20] reports a study made to determine the "absolutely essential" words, the "important" but not essential words, and the "unimportant" words in 19 different subjects. She had each text checked by two different people and used as many texts and people as necessary to raise the

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[17] Stokes, pp. 8-9.

[18] Ernest Horn, "Arithmetic in the Elementary School Curriculum," The Teaching of Arithmetic. Fiftieth Yearbook of the National Society for the Study of Education, Part II (University of Chicago, 1951), p. 10.

[19] Edward L. Thorndike and Irving Lorge, The Teacher's Word Book of 30,000 Words (New York, 1944).

[20] Luella Cole Pressey, "The Determination of the Technical Vocabulary of the School Subjects," School and Society, XX (July 19, 1924), 91-96.

reliability to a high level. Then, the list of arithmetical words was checked by 103 summer school teachers. They rated 117 of these mathematical words "absolutely essential" and 26 words "important" under common mathematical words. They also rated 83 words "absolutely essential," 274 "important" and 49 "unimportant" but included in the texts under arithmetic.

> A very considerable proportion of the words rated as absolutely essential by teachers of mathematics and arithmetic are also deemed essential by teachers of other subjects. Among the words considered essential in art, for example, are area, balance, breadth, circle, cube, depth, dimension, distance, horizontal, length, measure, parallel, perpendicular, rectangle, square, triangle, and unit.[21]

Thus, a pupil may have a good vocabulary and be a good reader at any state of development, yet fail because of the technical words of some subject. The incidence of mathematical words in all reading material makes the learning of meanings most important.

Brownell[22] gives importance to meanings by listing their advantages or values:

> (1) Arithmetic can function in intelligent living only when it is understood. In practical living we must be intelligent in quantitative situations . . . . To the degree that situations differ from the completely familiar, we must be able to think . . . . and one does not think effectively with mechanical skills alone. Thinking is possible only to him who possesses rich meanings.
>
> (2) Meanings facilitate learning. Through meanings we secure insights and note relationships which, without meanings, we should not likely hit upon. The insights in turn enable us to foresee connections and to tie together various aspects of the learning task which without understanding, would have to be mastered separately one at a time.
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> (3) Meanings increase the chances of transfer. It is because meanings do transfer that they facilitate learning . . . . The effects of

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[21] Horn, p. 10.

[22] William A. Brownell, "When is Arithmetic Meaningful?" Journal of Educational Research, XXXVIII (March, 1945), 494-497.

meanings are cumulative; their contributions to learning increase in amount as they enable the learner to gain new insights, to discover short cuts, and to apply in new ways what has been learned.

(4) Meaningful arithmetic is better retained and is more easily rehabilitated than is mechanically learned arithmetic. Meanings strengthen skills by supplying a structure to support them. When the skills no longer function, the structure remains, and on this basis the skills can be renewed.

Many examples can be given showing the importance of meanings. A second grade class which had worked on meanings of position in counting and a few simple addition computations, all less than ten, suddenly had the problem 19 + 9 + 9 to work. Finally, a boy came up with this solution: Nineteen is 1 ten and 9, take 1 from the second 9 and add 1 to the first 9 in 19 to make 2 tens, then take 1 from the 8 and add to the last 9 to make 3 tens and 7 which is 37. The entire class thoroughly understood and worked several other examples.[23]

Meanings in Arithmetic Teachers Should Develop

Many lists of mathematical terms, concepts, processes, and phrases have been made with little or no agreement as to which are essential. Probably the most complete list, together with definitions, is to be found throughout Buckingham's book, Elementary Arithmetic, Its Meaning and Practice.[24] Also, the better arithmetic textbooks would contain good lists of meanings.

Omitting such topics as measurement, Brownell suggests four categories of meanings:[25]

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[23] Ibid., 495-496.

[24] B. R. Buckingham, Elementary Arithmetic, Its Meaning and Practice (Boston, 1947).

[25] William A. Brownell, "The Place of Meaning in the Teaching of Arithmetic," Elementary School Journal, XLVII (January, 1957), 257-258.

1. One group consists of whole numbers, common fractions, decimal fractions, per cent, ratio and proportion, denominate numbers, and the technical terms of arithmetic -- addend, divisor, common denominator, etc.

2. A second group of arithmetical meanings include understandings of the fundamental operations. Children must know what happens with each operation (to the numbers) and when to use each operation.

3. A third group of meanings is composed of the more important principles, relationships, and generalizations of arithmetic. This includes such principles as the order of adding or multiplying does not matter, and both numerator and denominator may be multiplied or divided by the same number without change of value.

4. A fourth group of meanings relates to the understanding of our decimal number system and its use in rationalizing our computational procedures and algorisms. This includes place values and uses (applications) in "borrowing" and "carrying."

Of course, the teacher must have adequate training to teach these meanings. This means more than bare definitions of terms. It means analysis and synthesis so that the student will discover and generalize the meanings.

Status of Arithmetic Teachers

The great shortage of elementary teachers since World War II has brought about undesirable results. Many unqualified teachers were hired, and certification standards were lowered or ignored. Furthermore, most of the teachers colleges have no mathematical requirements for elementary teachers. Thus, there has been a wide range in the abilities of the teachers.

The minimum requirements for certification of elementary teachers by states in 1955 showed that one state required less than one year of college training, four states required one but less than two years, 12 required two but less than three years, two required three but less than four years, and 29 required four years of college training. The 1955 report

showed improvement over the last reports in 1949 and 1953.[26] However, many unqualified persons are teaching each year on temporary certificates.

The requirements in mathematics for the teachers are extremely low. Many elementary teachers are teaching with one or two years of high school mathematics, and the arithmetic they had in elementary school.

Grossnickle[27] gives some requirements in mathematics for certification of elementary teachers. Three states had blanket requirements (a choice of fields which included mathematics), 35 states had no requirements, and 10 states had specific requirements which averaged 3.4 semester hours of mathematics.

Layton[28] states that the average requirements over the whole nation for the lowest certificate was mathematics content .52 semester hours, and for methods in mathematics .16 semester hours. The means for the highest certificates differ very little from these.

More than three-fourths of the teachers colleges require no mathematics of any kind for admission, and two-thirds of them require no courses in background mathematics for elementary teachers. Also, in more than half of the colleges offering curricula which prepare teachers for the elementary grades, a background course in mathematics is missing.[29]

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[26] W. Earl Armstrong and T. M. Stinnett, A Manual on Certification Requirements for School Personnel in the United States, National Education Association (Washington, 1955), pp. 1-3.

[27] Foster E. Grossnickle, "The Training of Teachers of Arithmetic," The Teaching of Arithmetic. Fiftieth Yearbook of the National Society for the Study of Education, Part II (University of Chicago, 1951), p. 205.

[28] W. I. Layton, "The Certification of Teachers of Mathematics," The Mathematics Teacher, XLII (December, 1949), 378.

[29] Grossnickle, pp. 208, 210.

Although the modal length of the training program for elementary teachers during the last twenty-five years increased from two years to four years, the amount of training required in mathematics decreased during that interval.[30]

The picture is not nearly so bad as painted, however. In the twenty-five years since Brownell started expounding the "meaning" theory, a great change has taken place. Nevertheless, Mueller and Moser[31] warn that:

> Reports from consultants working with the in-service training of teachers indicate that the biggest single barrier to a more effective implementation of meaningful arithmetic is the inadequacy in the mathematical background of the teachers themselves. Teachers cannot do a creditable job teaching that which they neither practice nor understand.

They further give three reasons for improved mathematics teaching: (1) Teachers have accepted the meaning approach and are energetic and enthusiastic about improving their teaching. (2) The emerging point of view is that arithmetic must be taught as a structured system of related ideas, principles and processes with imbedded social applications. (3) This movement for better instruction in mathematics rests on the broadest possible base for the greatest number of teachers and will begin with the concepts essential for providing a solid foundation for later mathematical learning.

Summary

Teaching of arithmetic has run the gamut from the strict disciplinary ideas of the nineteenth century to the other extreme of

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[30] Ibid., p. 229.

[31] Francis Mueller and Harold Moser, "Background Mathematics for Teachers of Arithmetic," Emerging Practices in Mathematics Education. Twenty-second Yearbook of the National Council of Teachers of Mathematics (Washington, D. C., 1954), p. 181.

advocating no mathematics in the elementary school in the twentieth century.[32] The change has been deliberate and gradual, and today neither extreme exists.

Arithmetic was taught for discipline of the mind and body. This attitude was dominant well into this century. Drill, memory work, and forced learning of abstract arithmetic were the essentials of the old masters. Discipline in mathematics was closely associated with punishment. Anything which was hard to do and took a lot of time contributed to discipline. An idle mind was the devil's workshop. Hence, if a student were busy with drill work, memory work, or outlandish problems, he never gave the teacher any trouble.

Soon after the turn of the century teaching of arithmetic began to take on new meaning. Experiments were performed. Psychology made great progress in analyzing the learning process and establishing connections between interests and learning. The "meaning" theory of teaching arithmetic began to displace the "drill" theory. New texts based upon meanings, understanding, interests, and needs were written. These books have utilized the latest knowledge and theories of learning. They are well organized and well written, but not too up-to-date. Publishing companies are reluctant to revise and to include the new and modern concepts.

The teachers themselves were the chief deterrent to the change from the "drill" theory to the "meaning" theory of teaching arithmetic. They were taught and trained in the "drill" method and have been very slow to change. The "drill" method still predominates with many teachers.

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[32] E. R. Breslich, "Importance of Mathematics in General Education," Mathematics Teacher, XLIV (January, 1951), 4; Glennon and Hunnicutt, 17.

CHAPTER III

THE PROCEDURE FOR THE STUDY

Introduction

The aims of this chapter were to describe and analyze the procedures used to obtain the data for this investigation.

The identification of the concepts and processes of elementary arithmetic necessitated: (1) a preliminary selection of mathematical concepts and processes from selected elementary arithmetic textbooks, (2) the making of a questionnaire to help determine the importance of the concepts and processes, (3) the checking of the questionnaire by a group of experts and by a group of elementary teachers.

The procedures of selecting the concepts and processes, of making the questionnaire, of selecting both the experts and teachers have been described in this chapter. Also, certain personal data such as the amount of college credit in mathematics and mathematics methods, units of high school mathematics, number of years teaching experience which were collected from the teachers have been analyzed in this chapter. A comparison was made between the percentages of answered questionnaires in this investigation, and published reports of returns of questionnaires in general.

Selection of Concepts and Processes

The elementary arithmetic textbooks have been completely designed

and written within the past twenty years, and have included the latest in psychology of learning and the "meaning" theory of teaching arithmetic. These textbooks which are well written and well organized contain a complete set of concepts and processes.

The following items were considered in the selection of the arithmetic books from which the concepts and processes were chosen:

1. Those books authored by outstanding educators.
2. The extent of the use of the books.
3. The major publishing companies of elementary books.
4. Recency of publication.
5. The Oklahoma state adoptions in arithmetic.

Six complete sets of elementary arithmetic books were selected and analyzed to obtain the major concepts and processes. Also, a few other books were checked for possible concepts missed or not appearing in three or more of the six sets. The criterion of selection was that a concept or process must appear in three or more of the books to be considered important.

The analysis of the arithmetic books (See Appendix C p. 98) consisted of a page by page scanning for mathematical concepts and processes which were recorded by code for particular text and grade in a notebook. For example, the Rowe-Peterson arithmetic series was listed I. Hence, I, 1 would indicate Rowe-Peterson Book I (Grade 1) and under I, 6 would be recorded all new concepts and processes appearing in Rowe-Peterson Book 6 (Grade 6). An attempt was made to make the lists as complete as possible and not repeat concepts and processes from later books in a series. The following sets of books were used:

Brueckner, Leo J. et al., Arithmetic We Use (Grades One to Six)

Clark, John R. et al., Growth in Arithmetic (Grades One to Six)

Mallory, Virgil S. et al., Using Arithmetic (Grades One to Six)

Morton, Robert Lee, et al., Making Sure of Arithmetic (Grades One to Six)

Studebaker, J. A. et al., Study Arithmetics (Grades One to Six)

Wheat, H. G. et al., Rowe-Peterson Arithmetic (Books One to Six)

Two other books were used:

Bartoo, G. C. et al., Adventures with Numbers (Grade I)

Stern, Catherine. Discovering Arithmetic (Grade I)

Then, two alphabetical listings were made on large cardboard sheets. The first group consisted of concepts and processes for the first three grades, and the second group consisted of concepts and processes for the next three grades. The lists were then checked for frequency in the six sets of books and the extra ones.

If a concept was checked under three or more of the book columns, it was considered important enough to go into the questionnaire. The concepts were again arranged into related groups and put into a questionnaire.

The Questionnaire

The guiding principles in the construction of the questionnaire were:

1. Give clear and distinct instructions and explanations at the beginning of the questionnaire.

2. Minimize respondents' work by the use of simple checks. Thus, a three-item-rating scale was used: (1) essential, (2) desirable, (3) unimportant.

3. Group related concepts and processes in order to facilitate checking.

4. Refine the questionnaire through conferences and interviews with members of the investigator's committee and colleagues.

The questionnaire (Appendix A) was divided into two parts: One part included the concepts and processes selected by the above procedure for grades one, two, and three; and the other part embodied the concepts and processes of grades four, five, and six. The assumption was that a teacher in grades one, two, or three would not feel competent to check the importance of the concepts and processes of grades four, five, and six, and that a teacher of grades four, five, or six, would not wish to check the questionnaire for the first three grades. There seems to be a natural grouping in our school systems of the first three grades into one subgroup, and the next three grades into a second subgroup, and the seventh and eighth, or seventh, eighth, and ninth grades into a third subgroup. The universality of the first two subgroups and the variability of the third subgroup prompted the limitation of this study to grades one to six. The rating scale was reduced from five possible checks to three in order to facilitate the task of the respondent. The directions for marking and explanations were clearly stated. Suggestions for refinement of the questionnaire were made by members of the investigator's committee and by his colleagues.

The first page of the questionnaire follows:

Directions for Scoring

The following concepts and processes of elementary arithmetic were selected on a frequency of occurrence basis from six major sets of elementary arithmetic books. These concepts and processes may vary in importance in the mathematical development of the child.

Concepts are the elements of knowledge. Concepts as used here are simply mathematical terms which are used by the teacher or textbook to develop the child in his mathematical understanding, thinking and reasoning. Processes are the operations such as adding, subtracting, multiplying, dividing, measuring, etc., which the child performs with numbers.

Please check in the space at the right according to the importance which you consider the concept or process to have in the mathematical development of understanding, reasoning, and thinking of the child. The ratings are (1) essential, (2) desirable, (3) unimportant.

CONCEPTS

PART I: FIRST, SECOND AND THIRD GRADES

1. Essential
2. Desirable
3. Unimportant

		1	2	3
	ADDITION			
1	add			
2	and			
3	column			
4	plus (+)			
5	how many			
6	sum			
7	total			
8	altogether			
9	together			
	SUBTRACTION			
10	count change			
11	cross (out)			
12	difference			
13	how many left			
14	left (over)			
15	minus (-)			
16	remainder			
17	take away			
	MEASURE			
	Time:			
18	clock (o'clock)			
19	short hand (hr)			
20	long hand (min)			
21	hour			
22	half-hour			
23	half-past			
24	days (names)			
25	hours			
26	minutes			
27	seconds			
28	calendar			
29	week			
30	months (names)			
31	months (length			
32	year			
33	Weight:			
34	ounce			
35	pound (lb)			
36	Length:			
37	inch			
38	foot (feet)			
39	foot ruler			
40	yard			

The questionnaire was then sent to a group of experts in the area of mathematics education, and to a selected group of elementary arithmetic teachers.

Selection of the Experts

It was assumed that college teachers working in the field of mathematics education were qualified to pass judgment upon the relative importance of the concepts and processes in the quantitative development of the child.

The selection of the experts was based upon the following achievements:

1. Authors of elementary arithmetic books.
2. Authors of other books and articles related to the teaching of arithmetic.
3. Those working particularly in the area of mathematics education.
4. Elementary supervisors, especially in teachers colleges and large city school systems.

Table I shows the response of the experts to the questionnaire. Thirty-two questionnaires were sent to college personnel. One questionnaire was returned since the recipient was deceased. The responses were as follows: 68 per cent checked and returned the questionnaire; 6.5 per cent had completely retired; 13 per cent did not respond; 3 per cent reported lack of time; 3 per cent were in executive positions and referred the questionnaire to the education department with no further answer; 6.5 per cent refused to check it. Two experts refused to check the questionnaire. One returned it unchecked, stating there was a dichotomy between the directions on the questionnaire and the letter of trans-

mittal, but stated that if he checked the questionnaire in line with the purpose of this study he would have to check all items essential. The other expert who did not check the questionnaire stated he saw little value in this type of study.

The 68 per cent returns on the eight-page questionnaire compared favorably with Shannon's[1] 67 per cent on questionnaire of five or more pages.

TABLE I

PERCENTAGE OF QUESTIONNAIRE RETURNS FOR THE EXPERTS

	Answered	Unanswered	Not Returned
Experts	67.7	4.2	28.1

Selection of Elementary Teachers

The elementary teachers' opinions concerning the items on the questionnaire were needed in the identification of the concepts and processes. Also, those teachers who actually teach these concepts and processes to the children should know which ones are important. The following criteria were used:

1. Only school systems which were members of the North Central Association of Colleges and Secondary Schools were selected.

2. Only independent school districts were chosen.

3. Only elementary schools of 12 teachers or more were chosen.

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[1] J. R. Shannon, "Percentages of Returns on Questionnaires in Reputable Educational Research," Journal of Educational Research, XLII (October, 1948), 140.

In some cases where one elementary principal had two or more buildings under his supervision, the minimum of 12 teachers possibly consisted of all the elementary teachers in the system.

These criteria were based upon the assumptions that the requirements for teachers were higher in these schools, that the teachers were better qualified and did a higher type of teaching because of supervision, in-service meetings, and conferences.

A letter (Appendix B) was written to the elementary principal asking for the name, address, and grade of teaching of five or more of his better teachers from grades one to six. He was asked to select the teachers on the basis of the following criteria:

1. Do they seem to be better teachers in comparison to others in their field?

2. Do pupils seem to rate higher in arithmetic after being taught by these teachers in comparison with other teachers?

3. Have the teachers been successful teachers over a period of years?

The principals were quite aware of the mathematics situation in the schools and the need for improving the teaching of mathematics. They felt they had many good teachers and were interested in the new ones having the best preparation possible. They further felt that the experienced and better teachers could give much aid to the young and new teacher. Consequently, there was almost one hundred per cent response by the principals. Also, most of them gave more names than the minimum of five which was requested. More than four hundred names of elementary arithmetic teachers were received. Three hundred sixty questionnaires were sent, and two hundred forty-five were checked and returned.

Sixty-eight per cent of the elementary teachers checked and returned the questionnaire. Six per cent returned the questionnaires unanswered and 26 per cent did not return the questionnaires. This average was low compared to Shannon's[2] 69.9 per cent for 4-5 page questionnaires, but it compares favorably to Trow's[3] opinion. He stated that usually a third to a half of the persons circulated did not answer.

The per cent of answered questionnaires (Table II) from elementary

TABLE II

QUESTIONNAIRE RETURNS FOR ELEMENTARY TEACHERS

Grade	Answered	Unanswered	Not Returned
First	70.0	6.0	24.0
Second	68.0	4.0	28.0
Third	74.1	3.7	22.2
Fourth	60.7	11.5	27.8
Fifth	61.9	9.5	28.6
Sixth	69.6	2.9	27.5
Average	68.0	6.1	25.9

teachers ranged from a low of 60.7 per cent for the fourth grade teachers to a high of 74.1 per cent for the third grade teachers. Fifth grade teachers responded only slightly better with 61.9 per cent checking the questionnaires. The total returned questionnaires (answered and unanswered) showed fourth grade 72.2 per cent, fifth grade 71.4 per cent and sixth grade 72.5 per cent. The reasons for the low returns for the fourth and

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[2] Ibid., 140.

[3] William Clark Trow. Scientific Method in Education (Boston, 1925), p. 101.

fifth grades are unknown, but the close and fairly high total returns of questionnaires would indicate something was amiss. It might indicate a lack of knowledge of concepts and processes of the other two grades in the subgroup. Also, the high rating of the third grade teachers might indicate a more thorough knowledge of the concepts and processes of that subgroup. The sixth grade teachers showed a relatively high percentage 69.9) of answered questionnaires which would indicate a good knowledge of the concepts and processes of the fourth, fifth and sixth grades.

The main objective in giving the data in Tables I and II was to compare the responses to the questionnaire with the responses to questionnaires in general. No other treatment of this data seemed pertinent to this problem except the comparisons which were just given.

There is evidence (Table III) that the principals followed the criteria in the selection of the teachers. Five people of the 245 who

TABLE III

AMOUNT OF MATHEMATICS AND TEACHING EXPERIENCE OF ELEMENTARY TEACHERS

GRADE	High School Mathematics (units)	College Mathematics (sem. hrs.)	Mathematics Methods (sem. hrs.)	Teaching Experience (years)
First	2.45	4.14	1.55	19.07
Second	2.88	3.73	1.89	19.64
Third	2.87	3.84	1.87	18.60
Fourth	2.92	4.89	1.92	17.31
Fifth	2.97	6.13	2.83	16.09
Sixth	3.23	7.37	2.47	20.78
Total Average	2.91	5.11	2.10	18.66

answered questionnaires gave two years' teaching experience, two showed

three years' teaching experience and five showed four years' teaching experience. All others had more than four years of teaching experience. The average number of years of teaching experience ranged from a low of 16.09 years among the fifth grade teachers to a high of 20.78 years among the sixth grade teachers. The average number of years of teaching for all teachers who answered the questionnaires was 18.66 years.

Two teachers with one year of experience returned the questionnaire unanswered and gave incompetence as the reason for not checking. One teacher with two years' experience returned the unanswered questionnaire with the same explanation. Two teachers returned the unanswered questionnaire and gave lack of time, two others gave illness, and two others returned the unanswered questionnaire with no reason. One teacher gave substitute teaching as a reason for not answering the questionnaire. The most frequent reason given for not answering the questionnaire was lack of time. A total of nine people, or less than 3 per cent, gave lack of time as the reason for not answering it. The next most frequent reason given was incompetency. Seven people, or 2 per cent, gave unqualified or incompetent as the reason for not checking the questionnaire. Other reasons were: too many school duties, lost or misplaced questionnaires, illness, and extension work. Most of these reasons were given on a card included in a follow-up letter concerning the questionnaire.

The elementary teachers showed a low of 2.45 units of high school mathematics in the first grade to a high of 3.23 units in the sixth grade. The order of high school units in mathematics from highest to lowest was sixth (3.23), fifth (2.97), fourth (2.92), second (2.88), third (2.87), and first (2.45). The average of 2.91 units in high school compared favorably with the mathematics requirements to enter most colleges,

especially teachers colleges.

The same order exists for the number of semester hours of college mathematics as for units of high school mathematics. The sixth grade teachers had a high of 7.37 semester hours in mathematics; fifth grade teachers showed 6.13 semester hours; fourth grade teachers showed 4.89 semester hours; first grade teachers had 4.14 semester hours; third grade teachers had 3.84 semester hours; and second grade teachers were low with 3.73 semester hours. Fifty or 20 per cent of the elementary teachers had no college mathematics. The average of 4.11 semester hours is extremely high compared with the amount required for certification of elementary teachers.

The number of semester hours in mathematics methods ranged from a low of 1.55 semester hours for first grade teachers to a high of 2.83 semester hours for fifth grade teachers. The order again from highest to lowest was fifth grade, sixth grade, fourth grade, second grade, third grade, and first grade. The average number of semester hours of methods in mathematics for the 245 elementary teachers was 2.10 semester hours which was much higher than the amount required for certification.

SUMMARY

The concepts and processes of elementary arithmetic were selected on a frequency of occurrence basis from six sets of elementary books, grades 1 to 6. These concepts and processes were arranged in related groups and put into a two-division questionnaire. The entire questionnaire was sent to a group of experts in the field of mathematics and mathematics education. The first section of the questionnaire including the concepts and processes of the first, second, and third grades

was sent to a selected group of first, second, and third grade teachers. The second part of the questionnaire, including the concepts and processes of the fourth, fifth, and sixth grades, was sent to a selected group of fourth, fifth, and sixth grade teachers.

The data concerning the teachers indicated they were well-trained, experienced, and capable teachers. No teacher of less than two years' experience answered the questionnaire. The average was 18.66 years' experience. Their training showed an average of 2.9 high school units in mathematics, 5.1 semester hours in college mathematics, and 2.1 semester hours of mathematics methods. The rank in amount of training both in high school and college from highest showed sixth, fifth, fourth, third, second, and first.

The per cent of returns of the questionnaires was good in comparison to published reports on similar types of surveys. There was some indication from the per cent of returns from the elementary teachers that possibly the fourth grade teachers felt most incompetent to check the concepts and processes of the fifth and sixth grades. Also, there was a slight indication that the fifth grade teachers felt incompetent to check fourth and sixth grade concepts and processes. There was some evidence that third and sixth grade teachers were most confident in checking the concepts and processes in their respective groups.

CHAPTER IV

ANALYSIS OF DATA, CONCEPTS

The objective of this chapter was to analyze the data in terms of the categories as mentioned in the hypothesis. Also, the concepts have been rated as set forth at the end of Chapter I. An average rating of sixty per cent or higher in any check space indicated the importance of the concept. If none of the three check spaces received a sixty per cent or higher rating, then a combination of two space ratings has been used to indicate whether the concept was, or was not, important.

Concepts are the elements of knowledge. Concepts are identified by mathematical terms or words which are used by the teacher or textbook to develop the child in his mathematical understanding, thinking and reasoning.

The Categories of Concepts

The categories were based upon the findings of an unpublished doctoral dissertation by J. J. Stipanowich,[1] and six sets of arithmetic books listed in Chapter III. Stipanowich lists 33 topics in arithmetic which were recommended by 75 per cent of the educators who answered his questionnaire as being needed in a basic mathematics course for

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[1] J. J. Stipanowich, "The Development and Appraisal of a Course in Basic Mathematics for Prospective Elementary School Teachers," (unpublished Ed. D. dissertation, Northwestern University, 1956), pp. 85-87.

elementary teachers. Twenty-six of these topics were recommended by 90 per cent of the educators. The major topic headings were:

Growth in Numbers

Hindu-Arabic Numerals

Numbers in Our System

Denominate Numbers and Measurement

The Fundamental Operations Using Integers

Checking the Results

The Fundamental Operations using Common Fractions

The Fundamental Operations using Decimal Fractions

Aids to Problem Solving in Arithmetic

The 33 topics listed were all subheadings under these major topics. Also, the major topic headings correspond somewhat to the chapter headings in the arithmetic books which were used in the survey of this study. Hence, the categories were based upon these two related sources of topics.

The questionnaire corresponds closely to the following categories involving concepts which were considered important to the elementary teacher. The first eleven of the categories are as follows:

I. The concepts of order.
   (a) Numbers (or integers) in order.
   (b) Numbers in rank.

II. The concepts involving synthesis.
   (a) Addition.
   (b) Multiplication.

III. The concepts involving analysis.
   (a) Subtraction.
   (b) Division.

IV. The concepts of comparison.

(a) Certain antonyms or marginal mathematical terms.

V. The concepts of measure.

(a) Area.

(b) Capacity.

(c) Volume.

(d) Counting.

(e) Length.

(f) Values (or money).

(g) Parts.

(h) Time.

(i) Weight.

(j) Other related concepts.

(k) Various systems of measurement.

1. English.

2. Metric.

VI. The concepts involved in a thorough understanding of the number system.

(a) Other number systems.

(b) Structure of the number system.

(c) Natural numbers.

(d) Common fractions.

(e) Decimal fractions.

(f) Percentage.

VII. The concepts of a family budget.

VIII. Certain concepts of business.

IX. The concepts of graphs.

X. Certain concepts pertaining to verbal problems.

XI. Other mathematical concepts.

Treatment of Data

A table which corresponds to each category has been constructed and lists the data, after treatment, from the questionnaire. All tables in this chapter express in per cent the opinions of the teachers concerning the concepts of elementary arithmetic according to the three checks on the questionnaire. Each concept was checked by the elementary teachers in one of three spaces as follows: (1) essential, (2) desirable, (3) unimportant. These check points under each classification were totaled and per cents, based on the total answered questionnaires, were calculated. Similar data was given for the experts. An average was figured between the per cents of the teachers and experts. All analyses were based upon the average rankings. The conclusions for the category as a whole were based upon the ratings of a majority of the concepts in each category. Some of the subheadings were also rated by this same method.

Concepts of Order. Table IV lists the concepts of order and rank. The ordering principle in mathematics states that any integer has an antecedent which is smaller than the given integer, and a successor which is larger. Concepts of order are the natural numbers. Rank means the position held as first, second, third, etc. The integers, both symbols and words from one to ten, were rated "essential" by 99 per cent of both the experts and elementary teachers. The integers from eleven to twenty were rated "essential" by the experts and teachers in per cents ranging from 88 to 92. The integers by tens to 100 were rated "essential" by 82 per cent or more of both the experts and elementary teachers.

The concepts of rank were rated "essential" from a high of 91 per

TABLE IV

THE IMPORTANCE OF MATHEMATICAL CONCEPTS OF ORDER AND RANK FOR ELEMENTARY TEACHERS

Percentage of (1) essential, (2) desirable, (3) unimportant, (4) blank.

Concepts	Teachers				Experts				Average			
	1	2	3	4	1	2	3	4	1	2	3	4
Numbers												
count	84	12	3	1	95	0	0	5	89.5	6.0	1.5	3.0
each	73	23	3	1	86	9	0	5	79.5	16.0	1.5	3.0
1 one	98	2	0	0	100	0	0	0	99.0	1.0	0.0	0.0
2 two	98	2	0	0	100	0	0	0	99.0	1.0	0.0	0.0
3 three	98	2	0	0	100	0	0	0	99.0	1.0	0.0	0.0
4 four	98	2	0	0	100	0	0	0	99.0	1.0	0.0	0.0
5 five	98	2	0	0	100	0	0	0	99.0	1.0	0.0	0.0
6 six	98	2	0	0	100	0	0	0	99.0	1.0	0.0	0.0
7 seven	98	2	0	0	100	0	0	0	99.0	1.0	0.0	0.0
8 eight	98	2	0	0	100	0	0	0	99.0	1.0	0.0	0.0
9 nine	98	2	0	0	100	0	0	0	99.0	1.0	0.0	0.0
10 ten	98	2	0	0	100	0	0	0	99.0	1.0	0.0	0.0
11 eleven	84	14	1	1	100	0	0	0	92.0	7.0	0.5	0.5
12 twelve	83	14	1	2	100	0	0	0	91.5	7.0	0.5	1.0
13 thirteen	78	18	2	2	100	0	0	0	89.0	9.0	1.0	1.0
14 fourteen	78	18	2	2	100	0	0	0	89.0	9.0	1.0	1.0
15 fifteen	77	19	2	2	100	0	0	0	88.5	9.5	1.0	1.0
16 sixteen	77	21	1	1	100	0	0	0	88.5	10.5	0.5	0.5
17 seventeen	76	21	2	1	100	0	0	0	88.0	10.5	1.0	0.5
18 eighteen	76	21	2	1	100	0	0	0	88.0	10.5	1.0	0.5
19 nineteen	76	20	2	2	100	0	0	0	88.0	10.0	1.0	1.0
20 twenty	78	20	2	0	100	0	0	0	89.0	10.0	1.0	0.0
30 thirty	72	22	3	3	100	0	0	0	86.0	11.0	1.5	1.5
40 forty	72	22	3	3	95	5	0	0	83.5	13.5	1.5	1.5
50 fifty	72	22	3	3	95	5	0	0	83.5	13.5	1.5	1.5
60 sixty	70	24	3	3	95	5	0	0	82.5	14.5	1.5	1.5
70 seventy	70	24	3	3	95	5	0	0	82.5	14.5	1.5	1.5
80 eighty	70	23	3	4	95	5	0	0	82.5	14.0	1.5	2.0
90 ninety	69	24	3	4	95	5	0	0	82.0	14.5	1.5	2.0
100 one hundred	74	19	3	4	95	5	0	0	84.5	12.0	1.5	2.0
Ordinal numbers												
first	87	13	0	0	95	5	0	0	91.0	9.0	0.0	0.0
second	87	13	0	0	95	5	0	0	91.0	9.0	0.0	0.0
third	87	13	0	0	95	5	0	0	91.0	9.0	0.0	0.0
fourth	80	20	0	0	95	5	0	0	87.5	12.5	0.0	0.0
fifth	76	24	0	0	95	5	0	0	85.5	14.5	0.0	0.0
sixth	66	27	2	5	86	14	0	0	76.0	20.5	1.0	2.5
seventh	64	29	2	5	86	14	0	0	75.0	21.5	1.0	2.5
eighth	64	29	2	5	81	14	5	0	72.5	21.5	3.5	2.5
ninth	64	29	2	5	81	14	5	0	72.5	21.5	3.5	2.5
tenth	63	30	1	6	81	14	5	0	72.0	22.0	3.0	3.0

cent for the concepts first, second, and third to a low of 72 per cent for the concept tenth. Others were fourth, fifth, sixth, seventh, eighth and ninth. All concepts of order were rated "essential."

Synthesis and Analysis. Table V shows the concepts pertaining to synthesis in elementary arithmetic. Snythesis in arithmetic is the act of putting groups together. The following concepts, related to addition, were rated "essential": add, and, plus (+), how many, altogether, together. Three concepts -- column, sum, and total -- were rated "desirable." The last three terms appeared only on the first half of the questionnaire and might have received a higher rating by the fourth, fifth, and sixth grade teachers. The concept total received a 62 per cent "essential"

TABLE V

THE IMPORTANCE OF MATHEMATICAL CONCEPTS OF SNYTHESIS FOR ELEMENTARY TEACHERS

Percentage of (1) essential, (2) desirable, (3) unimportant, (4) blank.

Concepts	Teachers				Experts				Average			
	1	2	3	4	1	2	3	4	1	2	3	4
Addition												
add	93	3	0	4	100	0	0	0	96.5	1.5	0.0	2.0
and	78	14	1	7	81	14	5	0	79.5	14.0	3.0	3.5
column	58	23	15	4	48	33	19	0	53.0	28.0	17.0	2.0
plus (+)	76	18	3	3	76	19	5	0	76.0	18.5	4.0	1.5
how many	92	6	2	0	90	10	0	0	91.0	8.0	1.0	0.0
sum	58	28	13	1	57	33	5	5	57.5	30.5	9.0	3.0
total	44	38	15	3	62	33	5	0	53.0	35.5	10.0	1.5
altogether	67	29	2	2	76	14	10	0	71.5	21.5	6.0	1.0
together	54	32	8	6	67	24	9	0	60.5	28.0	8.5	3.0
Multiplication												
carrying	98	2	0	0	90	5	5	0	94.0	3.5	2.5	0.0
multiplier	90	9	0	1	72	24	0	0	83.0	16.5	0.0	0.5
multiply	47	17	27	9	90	5	5	0	68.5	11.0	16.0	4.5
product	90	10	0	0	81	19	0	0	85.5	14.5	0.0	0.0
partial product	67	23	9	1	52	48	0	0	59.5	35.5	4.5	0.5
tables	91	7	1	1	52	29	14	5	71.5	18.0	7.5	3.0
twice	75	12	7	6	66	24	5	5	70.5	18.0	6.0	5.5

rating by the experts, but only 44 per cent "essential" by the teachers.

The following concepts, related to multiplication, were rated "essential": carrying, multiplier, multiply, product, tables, and twice. One concept, partial product, was rated "desirable."

Twelve concepts under addition and multiplication were rated "essential," and four concepts were rated "desirable." Thus, the category was rated "essential" because the majority of the items were so rated.

Table VI shows the concepts pertaining to analysis in elementary arithmetic. The following concepts, related to subtraction, were rated "essential": difference, how many left, minus (-), take away. The following group of concepts were rated "desirable": count change, left (over), remainder, exceeds, minuend, subtrahend, and needs. One concept, cross (out), was rated "unimportant."

The following concepts, related to division, were rated "essential": fractions, borrowing, dividend, quotient, trial quotient. Cancellation was rated "desirable" and caret (^) was rated "unimportant." The experts rated the concept cancellation low in the (1) "essential" column, and about average, or 48 per cent, in the (3) "unimportant" check space. Opinions expressed by the experts were to the effect that the operation of reducing fractions was a division process and violated the meaning of the word cancellation. Fifty-five per cent of the elementary teachers believed cancellation was an "essential" concept in elementary arithmetic, and 24 per cent of the elementary teachers believed it was a "desirable" concept.

The category was rated "desirable" because there was not a distinct majority for either check space. There were 9 concepts rated "essential," 8 concepts rated "desirable," and 2 concepts rated "unimportant." Subtraction was rated "desirable" because a majority of the items were so

rated. Division was rated "essential" because a majority of the items were rated "essential."

TABLE VI

THE IMPORTANCE OF THE CONCEPTS OF ANALYSIS FOR THE ELEMENTARY TEACHER

Percentage of (1) essential, (2) desirable, (3) unimportant, (4) blank.

Concepts	Teachers				Experts				Average			
	1	2	3	4	1	2	3	4	1	2	3	4
Subtraction												
count change	57	33	6	4	57	33	5	5	57.0	33.0	5.0	4.5
cross (out)	31	42	22	5	19	10	57	14	25.0	26.0	39.5	9.5
difference	67	22	9	2	71	24	0	5	69.0	23.0	4.5	3.5
how many left	88	11	0	1	95	5	0	0	91.5	8.0	0.0	0.5
left (over)	47	35	12	6	71	29	0	0	59.0	32.0	6.0	3.0
minus (-)	72	22	4	2	71	19	10	0	71.5	20.5	7.0	1.0
remainder	42	31	23	4	57	38	5	0	49.5	34.5	14.0	2.0
take away	87	6	5	2	81	14	5	0	84.0	10.0	5.0	1.0
exceeds	58	29	8	5	57	24	14	5	57.5	26.5	11.0	5.0
minuend	72	21	2	5	33	34	33	0	52.5	27.5	17.5	2.5
subtrahend	72	21	4	3	33	34	33	0	52.5	27.5	18.5	1.5
need(s)	41	33	18	8	66	19	5	10	53.5	26.0	11.5	9.0
Division												
fractions	86	11	2	1	100	0	0	0	93.0	5.5	1.0	0.5
borrowing	88	7	4	1	67	5	19	9	77.5	6.0	11.5	5.0
caret (^)	29	35	30	6	29	33	38	0	29.0	34.0	34.0	3.0
cancellation	55	24	16	5	19	33	48	0	37.0	28.5	32.0	2.5
dividend	88	10	1	1	67	33	0	0	77.5	21.5	0.5	0.5
division	89	10	0	1	81	19	0	0	85.0	14.5	0.0	0.5
quotient	90	9	0	1	81	19	0	0	85.5	14.0	0.0	0.5
trial quotient	70	23	4	3	67	28	5	0	68.5	25.5	4.5	1.5

Comparison. Certain concepts of comparison are considered important by the teachers, experts, and textbooks in developing a sense of relationships such as a sense of space and space relationships, a sense of direction and direction relationships, a sense of size and size relationships. These are not wholly mathematical concepts but are marginal concepts of a mathematical nature. Table VII lists the concepts of comparison.

TABLE VII

THE IMPORTANCE OF MATHEMATICAL CONCEPTS OF COMPARISON FOR ELEMENTARY TEACHERS

Percentage of (1) essential, (2) desirable, (3) unimportant, (4) blank.

Concepts	Teachers				Experts				Average			
	1	2	3	4	1	2	3	4	1	2	3	4
Comparisons												
above-below	77	18	2	3	86	14	0	0	81.5	16.0	1.0	1.5
big-little	92	7	1	0	95	5	0	0	93.5	6.0	0.5	0.0
bottom-top	87	12	1	0	86	14	0	0	86.5	13.0	0.5	0.0
fast-slow	82	16	1	1	95	5	0	0	88.5	10.5	0.5	0.5
few-many	82	13	3	2	95	5	0	0	88.5	9.0	1.5	1.0
fewer-more	70	24	3	3	90	10	0	0	80.0	17.0	1.5	1.5
first-last	92	7	1	0	95	5	0	0	93.5	6.0	0.5	0.0
heavier-lighter	46	37	14	3	76	24	0	0	61.0	30.5	7.0	1.5
heavy-light	48	40	8	4	90	10	0	0	69.0	25.0	4.0	2.0
in-out	73	23	2	2	71	19	10	0	72.0	21.0	6.0	1.0
large-small	89	9	1	1	95	5	0	0	92.0	7.0	0.5	0.5
larger-smaller	69	28	2	1	90	10	0	0	79.5	19.0	1.0	0.5
largest-smallest	68	27	3	2	86	14	0	0	77.0	20.5	1.5	1.0
left-right	93	7	0	0	95	5	0	0	94.0	6.0	0.0	0.0
long-short	87	12	0	1	95	5	0	0	91.0	8.5	0.0	0.5
longer-shorter	66	29	2	3	90	10	0	0	78.0	19.5	1.0	1.5
longest-shortest	65	28	4	3	86	14	0	0	75.5	21.0	2.0	1.5
more-less	90	7	2	1	90	10	0	0	90.0	8.5	1.0	0.5
old-young	63	27	7	3	86	9	5	0	74.5	18.0	6.0	1.5
older-younger	48	42	8	2	71	24	5	0	59.5	33.0	6.5	1.0
oldest-youngest	44	42	10	4	67	28	5	0	55.5	35.0	7.5	2.0
tall-short	79	18	2	1	95	5	0	0	87.0	11.5	1.0	0.5
taller-shorter	58	35	3	4	86	14	0	0	72.0	24.5	1.5	2.0
tallest-shortest	58	32	7	3	81	19	0	0	69.5	25.5	3.5	1.5
wide-narrow	56	32	10	2	76	14	10	0	66.0	23.0	10.0	1.0
high-low	81	18	1	0	86	14	0	0	83.5	16.0	0.5	0.0
higher-lower	54	38	5	3	86	14	0	0	70.0	26.0	2.5	1.5
highest-lowest	54	34	7	5	81	19	0	0	67.5	26.5	3.5	2.5
before-after	63	30	6	1	76	14	0	10	69.5	22.0	3.0	5.5
buy-sell	64	30	3	3	76	19	0	5	70.0	24.5	1.5	4.0
full-empty	61	34	4	1	76	19	0	5	68.5	26.5	2.0	3.0
east-west	62	30	7	1	76	19	0	5	69.0	24.5	3.5	3.0
north-south	63	29	7	1	76	19	0	5	69.5	24.0	3.5	3.0
increase-decrease	69	22	7	2	71	24	0	5	70.0	23.0	3.5	3.5
more-less	78	16	2	4	81	14	0	5	79.5	15.0	1.0	4.5
nearest-farthest	65	28	4	3	71	24	0	5	68.0	26.0	2.0	4.0
part-whole	79	16	3	2	86	9	0	5	82.5	12.5	1.5	3.5
share-keep	48	37	9	6	76	19	0	5	62.0	28.0	4.5	5.5
sum-difference	84	10	3	3	90	5	0	5	87.0	7.5	1.5	4.0
upward-downward	57	34	5	4	86	9	0	5	71.5	21.5	2.5	4.5

Concepts of comparison rated "essential" were: above-below, big-little, bottom-top, fast-slow, few-many, fewer-more, first-last, heavier-lighter, heavy-light, in-out, large-small, larger-smaller, largest-smallest, left-right, long-short, longer-shorter, longest-shortest, more-less, old-young, tall-short, taller-shorter, tallest-shortest, wide-narrow, high-low, higher-lower, highest-lowest, before-after, buy-sell, full-empty, east-west, north-south, increase-decrease, more-less, nearest-farthest, part-whole, share-keep, sum-difference, and upward-downward. Only two pairs of concepts of comparison were rated "desirable." These were older-younger, and oldest-youngest.

This category was rated "essential" because 38 pairs of concepts were rated "essential" and 2 pairs were rated "desirable."

Measure. The idea of measure is most important to all mankind in this scientific age. Certainly, concepts of measure vary in their importance to man according to their use. Table VIII lists the concepts of measure and their importance as rated by the teachers and the experts.

The concepts of capacity (volume) rated "essential" were teaspoon, tablespoon, cup, pint, quart, gallon, peck, and bushel. Those rated "desirable" were half pint and barrel. The concept gill was rated "unimportant."

No concepts strictly of volume were rated "essential." Cubic units such as cubic inches, cubic feet, cubic yards were rated "desirable." Board feet and cubic centimeters were rated "unimportant."

The concepts of counting which were rated "essential" were units, pairs, dozen, and zero. Score and gross were rated "desirable." Quire and ream were considered "unimportant."

TABLE VIII

THE IMPORTANCE OF THE CONCEPTS OF MEASURE FOR THE ELEMENTARY TEACHER

Percentage of (1) essential, (2) desirable, (3) unimportant, (4) blank.

Concepts	Teachers				Experts				Average			
	1	2	3	4	1	2	3	4	1	2	3	4
Area												
surface	76	13	6	5	95	0	0	5	85.5	6.5	3.0	5.0
square units (in., ft., yds.)	71	15	7	7	95	5	0	0	83.0	10.0	3.5	3.5
acre	59	26	12	3	67	33	0	0	63.0	29.5	6.0	1.5
section (sq. mi.)	42	36	17	5	33	43	19	5	37.5	39.5	18.0	5.0
Capacity (volume)												
teaspoon	76	22	1	1	71	29	0	0	73.5	25.5	0.5	0.5
tablespoon	76	22	1	1	71	24	0	5	73.5	23.0	0.5	3.0
cup	80	19	1	0	71	24	0	5	75.5	21.5	0.5	2.5
gill	17	39	40	4	19	57	19	5	18.0	48.0	29.5	4.5
pint	93	7	0	0	95	5	0	0	94.0	6.0	0.0	0.0
half pint (cup)	35	43	15	7	57	38	5	0	46.0	40.5	10.0	3.5
quart	93	7	0	0	95	5	0	0	94.0	6.0	0.0	0.0
gallon	93	7	0	0	95	5	0	0	94.0	6.0	0.0	0.0
peck	84	11	5	0	81	19	0	0	82.5	15.0	2.5	0.0
bushel	83	12	5	0	86	14	0	0	84.5	13.0	2.5	0.0
barrel	39	42	19	0	24	15	19	5	31.5	47.0	19.0	2.5
Volume												
cubic units (in., ft., yds.)	30	24	30	16	67	5	14	14	48.5	14.5	22.0	15.0
board feet	23	31	41	5	28	39	28	5	25.5	35.0	34.5	5.0
cubic centimeters	6	21	70	3	29	33	33	5	17.5	27.0	51.5	4.0
Counting												
units	94	5	0	1	95	5	0	0	94.5	5.0	0.0	0.5
pairs	90	7	3	0	100	0	0	0	95.0	3.5	1.5	0.0
dozen	91	8	1	0	90	5	0	5	90.5	6.5	0.5	2.5
score	55	34	9	2	43	28	24	5	49.0	31.0	16.5	3.5
gross	33	39	25	3	33	43	19	5	33.0	41.0	22.0	4.0
quire	12	37	48	3	5	42	48	5	8.5	39.5	48.0	4.0
ream	15	43	39	3	14	43	38	5	14.5	43.0	38.5	4.0
zero	89	8	2	1	95	0	0	5	92.0	4.0	1.0	3.0
Length												
inch	67	25	6	2	95	5	0	0	81.0	15.0	3.0	1.0
foot (feet)	74	17	6	3	95	5	0	0	84.5	11.0	3.0	1.5
foot ruler	69	24	5	2	86	14	0	0	77.5	19.0	2.5	1.0
yard	58	31	9	2	86	14	0	0	72.0	22.5	4.5	1.0
yardstick	43	40	8	9	52	38	5	5	42.5	39.0	6.5	7.0
linear	74	20	5	1	95	5	0	0	84.5	12.5	2.5	0.5
rod	49	34	16	1	29	62	9	0	39.0	48.0	12.5	0.5

TABLE VIII (Continued)

Percentage of (1) essential, (2) desirable, (3) unimportant, (4) blank.

Concepts	Teachers				Experts				Average			
	1	2	3	4	1	2	3	4	1	2	3	4
Length, contd.												
mile	82	15	3	0	100	0	0	0	91.0	7.5	1.5	0.0
hand	4	34	58	4	9	48	38	5	6.5	41.0	48.0	4.5
span	2	37	57	4	5	52	38	5	3.5	44.5	47.5	4.5
pace	2	34	59	5	5	52	38	5	3.5	43.0	48.5	5.0
fathom	4	44	48	4	5	43	47	5	4.5	43.5	47.5	4.5
nautical mile	6	40	51	3	14	48	38	0	10.0	44.0	44.5	1.5
knot	8	44	44	4	19	48	28	5	13.5	46.0	36.0	4.5
Money												
change	57	36	5	2	81	9	0	10	69.0	22.5	2.5	6.0
coins	64	27	6	3	90	10	0	0	77.0	18.5	3.0	1.5
cent	92	7	1	0	95	5	0	0	93.5	6.0	0.5	0.0
penny	92	7	0	1	90	5	5	0	91.0	6.0	2.5	0.5
nickel	95	5	0	0	95	5	0	0	95.0	5.0	0.0	0.0
dime	95	5	0	0	95	5	0	0	95.0	5.0	0.0	0.0
quarter	87	12	1	0	90	10	0	0	88.5	11.0	0.5	0.0
half-dollar	82	17	1	0	81	19	0	0	81.5	18.0	0.5	0.0
dollar, silver	56	34	9	1	62	33	5	0	59.0	33.5	7.0	0.5
dollar, bill	74	18	6	2	90	10	0	0	82.0	14.0	3.0	1.0
Parts												
halves	80	16	2	2	90	10	0	0	85.0	13.0	1.0	1.0
thirds	53	36	7	4	71	29	0	0	62.0	32.5	3.5	2.0
fourths	58	29	8	5	81	19	0	0	69.5	24.0	4.0	2.5
sixths	12	31	45	12	43	38	19	0	27.5	34.5	32.0	6.0
Time												
clock (o'clock)	83	12	2	3	90	10	0	0	86.5	11.0	1.0	1.5
short hand (hr.)	87	9	2	2	81	19	0	0	84.0	14.0	1.0	1.0
long hand (min.)	87	10	2	1	76	24	0	0	81.5	17.0	1.0	0.5
hour	91	7	1	1	90	10	0	0	90.5	8.5	0.5	0.5
half-hour	77	18	2	3	90	10	0	0	83.5	14.0	1.0	1.5
half-past	65	27	5	3	57	33	5	5	61.0	30.0	5.0	4.0
days (names)	82	17	1	0	90	10	0	0	86.0	13.5	0.5	0.0
hours	75	24	1	0	90	10	0	0	82.5	17.0	0.5	0.0
minutes	64	26	7	3	81	19	0	0	72.5	22.5	3.5	1.5
seconds	33	31	31	5	52	29	19	0	42.5	30.0	25.0	2.5
calendar	72	23	0	0	90	10	0	0	83.5	16.5	0.0	0.0
week	82	17	0	1	86	14	0	0	84.0	15.5	0.0	0.5
months (names)	67	32	1	0	90	10	0	0	78.5	21.0	0.5	0.0
months (length)	12	52	1	0	62	33	5	0	37.0	42.5	18.5	2.0
year	98	2	0	0	95	5	0	0	96.5	3.5	0.0	0.0
leap year	79	19	2	0	86	9	5	0	82.5	14.0	3.5	0.0
decade	60	32	7	1	71	19	10	0	65.5	25.5	8.5	0.5
century	70	25	5	0	86	9	5	0	78.0	17.0	5.0	0.0

TABLE VIII (Continued)

Percentage of (1) essential (2) desirable, (3) unimportant, (4) blank.

Concepts	Teachers				Experts				Average			
	1	2	3	4	1	2	3	4	1	2	3	4
morning (A.M.)	98	2	0	0	95	5	0	0	96.5	3.5	0.0	0.0
noon	98	2	0	0	95	5	0	0	96.5	3.5	0.0	0.0
afternoon (P.M.)	98	2	0	0	95	5	0	0	96.5	3.5	0.0	0.0
midnight	96	4	0	0	95	5	0	0	95.5	4.5	0.0	0.0
daylight savings	49	34	15	2	71	29	0	0	60.0	31.5	7.5	1.0
table	60	27	12	1	71	29	0	0	65.5	28.0	6.0	0.5
zones, standard	61	29	9	1	62	38	0	0	61.5	33.5	4.5	0.5
Weight												
ounce	23	34	35	8	62	29	9	0	42.5	31.5	22.0	4.0
pound (lb.)	49	35	11	5	90	5	0	5	69.5	20.0	5.5	5.0
hundredweight	32	38	27	3	43	19	33	5	37.5	28.5	30.0	4.0
ton	83	12	5	0	71	24	5	0	77.0	18.0	5.0	0.0
long ton	24	41	30	5	24	43	33	0	24.0	42.0	31.5	2.5
grain	19	28	50	3	24	29	42	5	21.5	28.5	46.0	4.0
carat	2	24	69	5	0	43	52	5	1.0	33.5	60.5	5.0
Other terms												
abacus	21	32	39	8	43	43	9	5	32.0	37.5	24.0	6.5
average	83	15	2	0	90	5	0	5	86.5	10.0	1.0	2.5
census	35	48	14	3	52	33	10	5	43.5	40.5	12.0	4.0
dimensions	71	20	7	2	76	19	0	5	73.5	19.5	3.5	3.5
altitude	62	27	9	2	66	24	5	5	64.0	25.5	7.0	3.5
depth	62	30	7	1	86	5	4	5	74.0	17.5	5.5	3.0
distances	80	16	2	2	90	5	0	5	85.0	10.5	1.0	3.5
height	83	15	1	1	90	5	0	5	86.5	10.0	0.5	3.0
thickness	70	27	2	1	86	9	0	5	78.0	18.0	1.0	3.0
width	81	15	2	2	90	5	0	5	85.5	10.0	1.0	3.5
Measurement												
English system	70	14	12	4	86	9	5	0	78.0	11.5	8.5	2.0
Metric system	16	34	43	7	38	43	19	0	27.0	38.5	31.0	3.5
millimeter	7	38	52	3	24	52	24	0	15.5	45.0	38.0	1.5
centimeter	4	32	58	6	28	48	24	0	15.5	45.0	38.0	1.5
meter	6	34	57	3	33	48	19	0	19.5	41.0	38.0	1.5
kilometer	6	33	58	3	24	52	24	0	15.0	42.5	41.0	1.5
milliliter	5	21	71	3	5	43	52	0	5.0	32.0	61.5	1.5
liter	5	21	74	0	19	48	33	0	12.0	34.5	53.5	0.0
kiloliter	4	21	72	3	5	47	48	0	4.5	34.0	60.0	1.5
milligram	4	31	62	3	14	29	48	9	9.0	30.0	55.0	6.0
gram	43	9	53	4	24	48	28	0	14.0	43.5	40.5	2.0
kilogram	3	20	73	4	9	57	29	5	6.0	38.5	51.0	4.5

The following concepts of length were rated "essential": inch, foot or feet, foot ruler, yard, linear, and mile. Concepts rated "desirable" were yardstick and rod. Other concepts of length little used and rated "unimportant" were hand, span, pace, fathom, nautical mile, and knot.

All concepts of value (money) except silver dollar were rated "essential." The "essential" concepts were change, coins, cent, penny, nickel, dime, quarter, half-dollar, and dollar bill.

The measure of parts as halves, thirds, and fourths were rated "essential." Sixths was rated "desirable."

The concepts of time rated "essential" were clock or o'clock, short (hour) hand, long (minute) hand, hour, half-hour, half-past, days (names), hours, minutes, calendar, week, months (names), year, leap year, decade, century, morning (A.M.), noon, afternoon (P.M., midnight, daylight savings, table, and standard zones. The concept length of months was rated "desirable."

The two concepts of weight which were rated "essential" were pound (lb.) and ton. Those concepts of weight which rated "desirable" were ounce, and hundredweight. Long ton, grain, and carat rated "unimportant."

Some other concepts of measure which were rated "essential" were average, dimensions, altitude, depth, distances, height, thickness, and width. The two concepts abacus and census were rated "desirable."

The English system of measurement was rated "essential" by 78 per cent of the teachers and experts. The metric system and all metric measure concepts were rated "unimportant."

The category on measure was rated "essential" because a large majority of the items were so rated. The subheadings which were rated

"essential" were area, capacity, counting, values (money), parts, time, and other related concepts. Those which were rated "desirable" were length, weight, and various systems of measurement. One subheading, volume, was rated "unimportant." The English system of measurement was rated "essential," but the metric system was rated "unimportant."

The Number System. The Hindu-Arabic number system has probably been one of man's greatest achievements in symbols and logic. A thorough understanding of the number system and all concepts pertaining to it is vital to an elementary teacher. Table IX lists the concepts pertaining to the number system, and the importance which the teachers and experts attach to them.

The concepts of numbers which were rated "essential" were Arabic, even, odd, whole, position, digits, group(s), zero, units, ones' place, tens, tens' place, hundreds, hundreds' place, thousands, millions, and billions. Roman numbers were rated "desirable." Three concepts under group(s) which were rated "unimportant" were oneness, twoness, and fiveness.

The following concepts of fractions were rated "essential": part (of whole), denominator, equal parts, common, numerator, halves, thirds, fourths, fifths, sixths, sevenths, eighths, equivalent, improper, proper, invert, lowest terms, mixed numbers, like (similar), and unlike. Two concepts which were rated "desirable" were ratio, and recipe (mixture).

The concepts pertaining to decimals which were rated "essential" were decimal point, tenths, hundredths, per cent, thousandths, and mixed decimals. One concept, cents point, was rated "desirable."

The category was rated "essential" because 43 concepts were rated "essential," 4 concepts were rated "desirable," and 3 concepts were rated "unimportant."

TABLE IX

THE IMPORTANCE OF THE CONCEPTS OF THE NUMBER SYSTEM FOR THE ELEMENTARY TEACHER

Percentage of (1) essential, (2) desirable, (3) unimportant, (4) blank.

Concepts	Teachers				Experts				Average			
	1	2	3	4	1	2	3	4	1	2	3	4
Numbers												
Arabic	86	11	2	1	76	10	0	14	81.0	10.5	1.0	7.5
even	89	10	1	0	90	5	0	5	89.5	7.5	0.5	2.5
odd	89	10	1	0	90	5	0	5	89.5	7.5	0.5	2.5
whole	93	6	0	1	90	0	0	10	91.5	3.0	0.5	5.0
position	89	10	1	0	95	5	0	0	92.0	7.5	0.5	0.0
digits	84	12	3	1	76	10	9	5	80.0	11.0	6.0	3.0
group(s)	68	23	8	1	95	0	0	5	81.5	11.5	4.0	3.0
oneness	16	14	57	13	52	5	38	5	34.0	9.5	47.5	9.0
twoness	14	16	57	13	48	9	38	5	31.0	12.5	47.5	9.0
fiveness	13	15	59	13	48	9	38	5	30.5	12.0	48.5	9.0
zero	80	14	3	3	95	0	0	5	87.5	7.0	1.5	4.0
units	81	17	1	1	86	5	5	4	83.5	11.0	3.0	2.5
ones' place	72	14	7	7	100	0	0	0	86.0	7.0	3.5	3.5
tens	98	2	0	0	100	0	0	0	99.0	1.0	0.0	0.0
tens' place	72	14	7	7	100	0	0	0	86.0	7.0	3.5	3.5
hundreds	98	2	0	0	100	0	0	0	99.0	1.0	0.0	0.0
hundreds' place	67	15	11	7	90	5	0	5	78.5	10.0	5.5	6.0
thousands	97	3	0	0	100	0	0	0	98.5	1.5	0.0	0.0
millions	76	20	4	0	100	0	0	0	88.0	10.0	2.0	0.0
billions	50	31	16	3	76	24	0	0	63.0	27.5	8.0	1.5
Roman	48	49	3	0	52	43	0	5	50.0	46.0	1.5	2.5
Fractions												
part (of whole)	98	1	0	1	100	0	0	0	99.0	0.5	0.0	0.5
denominator	89	6	4	1	95	5	0	0	92.0	5.5	2.0	0.5
equal parts	94	5	1	0	100	0	0	0	97.0	2.5	0.5	0.0
common	86	7	7	0	95	5	0	0	90.5	6.0	3.5	0.0
numerator	91	5	4	0	95	5	0	0	93.0	5.0	2.0	0.0
halves	95	5	0	0	100	0	0	0	97.5	2.5	0.0	0.0
thirds	94	6	0	0	100	0	0	0	97.0	3.0	0.0	0.0
fourths	94	6	0	0	100	0	0	0	97.0	3.0	0.0	0.0
fifths	89	10	1	0	95	0	5	0	92.0	5.0	3.0	0.0
sixths	85	12	3	0	95	5	0	0	90.0	8.5	1.5	0.0
sevenths	75	20	3	2	57	24	19	0	66.0	22.0	11.0	1.0
eights	80	15	3	2	90	10	0	0	85.0	12.5	1.5	1.0
equivalent	70	19	10	1	100	0	0	0	85.0	9.5	5.0	0.5
improper	80	12	7	1	86	9	5	0	83.0	10.5	6.0	0.5
proper	83	9	7	1	86	9	5	0	84.5	9.0	6.0	0.5
invert	69	16	13	2	62	14	24	0	65.5	15.0	12.5	1.0
lowest terms	83	7	9	1	81	19	0	0	82.0	13.0	4.5	0.5
mixed numbers	83	9	7	1	95	0	5	0	89.0	4.5	6.0	0.5
ratio	19	48	30	3	90	10	0	0	54.5	29.0	15.0	1.5
recipe(mixture)	33	52	15	0	57	33	10	0	45.0	42.5	12.5	0.0

TABLE IX, Contd.

Percentage of (1) essential, (2) desirable, (3) unimportant, (4) blank.

Concepts	Teachers				Experts				Average			
	1	2	3	4	1	2	3	4	1	2	3	4
Decimals												
decimal point	89	7	3	1	100	0	0	0	94.5	3.5	1.5	0.5
cents point	44	29	23	4	52	28	5	15	48.0	28.5	14.0	9.5
tenths	85	7	7	1	100	0	0	0	92.5	3.5	3.5	0.5
hundredths	83	7	10	0	100	0	0	0	91.5	3.5	5.0	0.0
per cent	52	29	19	0	71	24	5	0	61.5	26.5	12.0	0.0
thousandths	61	22	15	2	90	10	0	0	75.5	16.0	7.5	1.0
mixed	63	21	13	2	86	0	9	5	74.5	10.5	11.0	4.0

Miscellaneous Applications. None of the concepts relating to a family budget were "essential" to the elementary teachers as shown in Table X. Concepts of a family budget which were "desirable" were income, clothing, food, health, recreation, savings, shelter or rent, save, and spend. Those concepts rated "unimportant" were advancement, insurance,

TABLE X

THE IMPORTANCE OF THE MATHEMATICAL CONCEPTS OF A FAMILY BUDGET FOR THE ELEMENTARY TEACHER

Percentage of (1) essential, (2) desirable, (3) unimportant, (4) blank.

Concepts	Teachers				Experts				Average			
	1	2	3	4	1	2	3	4	1	2	3	4
Budget (family)												
income	33	53	13	1	48	33	14	5	40.5	43.0	13.5	3.0
advancement	10	48	35	7	24	52	19	5	17.0	50.0	27.0	6.0
clothing	36	49	12	3	48	33	14	5	42.0	41.0	13.0	4.0
food	42	43	12	3	62	19	14	5	52.0	31.0	13.0	4.0
health	36	44	16	4	62	19	14	5	49.0	31.5	15.0	4.5
insurance	11	56	30	3	24	47	24	5	17.5	51.5	27.0	4.0
recreation	27	51	16	6	33	48	14	5	30.0	49.5	15.0	5.5
savings	40	47	11	2	52	33	10	5	46.0	40.0	10.5	3.5
shelter (rent)	28	51	16	5	43	33	19	5	35.5	42.0	17.5	5.0
miscellaneous	12	51	23	14	24	24	24	28	18.0	37.5	23.5	21.0
save	43	39	13	5	67	19	4	10	55.0	29.0	8.5	7.5
spend	52	36	7	5	67	19	5	9	59.5	27.5	6.0	7.0

and miscellaneous. These are not strictly mathematical concepts. However, the study of a family budget is found in most of the arithmetic textbooks.

The concepts of a family budget were "desirable." The majority, or 9 concepts of a family budget, were rated "desirable," and 3 concepts were rated "unimportant."

Table XI lists the concepts relating to business and the importance of these concepts as rated by the elementary teachers and the experts. None of the concepts of business were rated "essential." The concepts of business rated "desirable" were prices, cost price, list price, marked price, sale price, selling price, retail price, save, sell, spend, and cost. The two concepts net price and wholesale price were rated "unimportant."

TABLE XI

THE IMPORTANCE OF CERTAIN MATHEMATICAL CONCEPTS OF BUSINESS FOR THE ELEMENTARY TEACHER

Percentage of (1) essential, (2) desirable, (3) unimportant, (4) blank.

Concepts	Teachers				Experts				Average			
	1	2	3	4	1	2	3	4	1	2	3	4
Business												
prices	53	34	9	4	62	19	9	10	57.5	26.5	9.0	7.0
cost	51	38	9	2	62	24	9	5	56.5	31.0	9.0	3.5
list	16	54	27	3	38	33	24	5	27.0	43.5	25.5	4.0
marked	25	47	25	3	38	33	24	5	31.5	40.0	24.5	4.0
net	22	44	30	4	28	29	33	10	25.0	36.5	31.5	7.0
sale	39	43	17	1	48	33	9	10	43.5	38.0	13.0	5.5
selling	36	44	17	3	57	28	10	5	46.5	36.0	13.5	4.0
retail	25	51	23	1	29	38	28	5	27.0	44.5	22.5	3.0
wholesale	21	48	28	3	14	48	33	5	17.5	48.0	30.5	4.0
save	43	39	13	5	67	19	4	10	55.0	29.0	8.5	7.5
sell	49	38	10	3	57	29	5	9	53.0	33.5	7.5	6.0
spend	52	36	7	5	67	19	5	9	59.5	27.5	6.0	7.0
cost	42	27	4	27	76	19	0	5	59.0	23.0	2.0	16.0

The category on business was rated "desirable" because a majority of the items were so rated.

Table XII lists the concepts pertaining to graphs and their importance as rated by the teachers and experts. Only the concept bar graph was rated "essential." The concepts of graphs rated "desirable" were divided bar, horizontal bar, vertical bar, circle, line, picture or pictograph, scale, scale drawing, scale model, and scale maps.

TABLE XII

THE IMPORTANCE OF THE MATHEMATICAL CONCEPTS OF GRAPH FOR THE ELEMENTARY TEACHER

Percentage of (1) essential, (2) desirable, (3) unimportant, (4) blank.

Concepts	Teachers				Experts				Average			
	1	2	3	4	1	2	3	4	1	2	3	4
Graphs												
bar	44	44	5	7	81	14	0	5	62.5	29.0	2.5	6.0
divided	30	41	21	8	34	33	14	19	32.0	37.0	17.5	13.5
horizontal	36	47	13	4	62	24	5	9	49.0	35.5	9.0	6.5
vertical	38	47	12	3	62	24	9	5	50.0	35.5	10.5	4.0
circle	44	39	13	4	52	24	19	5	48.0	31.5	16.0	4.5
line	52	36	8	4	62	33	5	0	57.0	34.5	6.5	2.0
picture (pictograph)	43	43	11	3	67	24	9	0	55.0	33.5	10.0	1.5
scale	39	34	14	13	48	28	5	19	43.5	31.0	9.5	16.0
drawing	34	50	14	2	52	48	0	0	43.0	49.0	7.0	1.0
model	28	49	18	5	48	52	0	0	38.0	50.5	9.0	2.5
maps	52	38	9	1	62	33	0	5	57.0	35.5	4.5	3.0

The concepts of graphs were rated "desirable."

Table XIII lists the concepts pertaining to problems which are essential for understanding and solving problems. The "essential" concepts were verbal story problems, one-step problems, two-step problems, hidden facts, questions, answers, approximate answers, estimate answers, round (off) answers, checking, check (work), number stories, problems,

and round. All concepts under problems were rated "essential."

Certain concepts pertaining to verbal problems were rated "essential."

TABLE XIII

THE IMPORTANCE OF CERTAIN MATHEMATICAL CONCEPTS OF VERBAL PROBLEMS FOR THE ELEMENTARY TEACHER

Percentage of (1) essential, (2) desirable, (3) unimportant, (4) blank.

Concepts	Teachers				Experts				Average			
	1	2	3	4	1	2	3	4	1	2	3	4
Problems												
verbal (story)	85	7	1	7	62	14	0	24	73.5	10.5	0.5	15.5
one-step	91	8	0	1	76	24	0	0	83.5	16.0	0.0	0.5
two-step	85	15	0	0	76	24	0	0	80.5	19.5	0.0	0.0
hidden facts	68	22	8	2	76	10	9	5	72.0	16.0	8.5	3.5
questions	90	8	0	2	86	14	0	0	88.0	11.0	0.0	1.0
answers	80	15	2	3	81	19	0	0	80.5	17.0	1.0	1.5
approximate	48	45	6	1	76	19	5	0	62.0	32.0	5.5	0.5
estimate	51	43	6	0	81	19	0	0	66.0	31.0	3.0	0.0
round(off)	39	49	9	3	81	14	5	0	60.0	31.5	7.0	1.5
checking	75	15	0	10	71	19	0	10	73.0	17.0	0.0	10.0
check (work)	54	37	7	2	67	28	0	5	60.5	32.5	3.5	3.5
number stories	75	18	3	4	57	29	9	5	66.0	23.5	6.0	4.5
problems	66	21	8	5	81	9	5	5	73.5	15.0	6.5	5.0
round	71	20	5	4	67	24	0	9	69.0	22.0	2.5	6.5

Table XIV lists some other mathematical concepts which are important. Those concepts rated "essential" were same, equals, circle, squares, each, both and fewer. Other mathematical concepts rated "desirable" were air mail, stamps or postage, alike (as many as), and triangle.

The category was rated "essential" because a majority of the concepts were so rated.

TABLE XIV

THE IMPORTANCE OF OTHER MATHEMATICAL CONCEPTS FOR THE ELEMENTARY TEACHER

Percentage of (1) essential, (2) desirable, (3) unimportant, (4) blank.

Concepts	Teachers				Experts				Average			
	1	2	3	4	1	2	3	4	1	2	3	4
air mail	25	50	20	5	38	38	19	5	31.5	44.0	19.5	5.0
stamps (postage)	43	41	13	3	62	24	5	9	52.5	32.5	9.0	6.0
alike (as many as)	66	33	1	0	52	43	0	5	59.0	38.0	0.5	2.5
same	80	17	2	0	81	14	0	5	80.5	15.5	1.0	3.0
equals	78	18	3	1	86	9	0	5	82.0	13.5	1.5	3.0
circle	76	22	2	0	66	24	5	5	71.0	23.0	3.5	2.5
squares	61	27	9	3	71	19	0	10	66.0	23.0	4.5	6.5
triangle	28	38	27	7	67	24	4	5	47.5	31.0	15.5	6.0
each	73	23	3	1	86	9	0	5	79.5	16.0	1.5	3.0
both	72	23	3	2	86	9	0	5	79.0	16.0	1.5	3.5
fewer	65	28	4	3	86	9	0	5	75.5	18.5	2.0	4.0

Table XV lists the number of concepts which were checked (1) essential, (2) desirable, (3) unimportant, and (4) no rating. Per cents have been calculated on the total for each group and have been included in the table.

TABLE XV

AVERAGE RATINGS OF CONCEPTS

Data taken from Tables IV - XIV (per cent is figured on total)

	Essential		Desirable		Unimportant		No Rating		Total
	No.	%	No.	%	No.	%	No.	%	
Concepts	232	69.46	66	19.76	36	10.78	0	0.00	334

A total of 334 mathematical concepts were checked by the elementary teachers and experts. Two hundred thirty-two concepts or 69.46 per cent were rated "essential." Sixty-six mathematical concepts or 19.76 per

cent were rated "desirable." Thirty-six mathematical concepts or 10.78 per cent were rated "unimportant."

The experts generally rated the mathematical concepts higher than the elementary teachers rated them. The trends in checking were quite similar between the two groups of respondents. An item which was rated low in the "essential" column by the experts generally was rated low by the elementary teachers. Also, those items which were rated high in the "essential" column by the experts were rated high by the teachers.

The tables showed four sets of per cents for a total of 100 per cent for each concept. The fourth per cent represents those respondents who did not check the concept. The average per cent of omission checks for all the concepts was 2.56.

SUMMARY

Approximately 69 per cent of the 334 mathematical concepts checked by the elementary teachers and experts were rated "essential," 20 per cent of the mathematical concepts were rated "desirable" and 11 per cent of the mathematical concepts were rated "unimportant."

Seven of eleven categories of mathematical concepts were "essential" to the elementary teachers according to the check sheets of the elementary teachers and experts. These were the categories on the concepts of order, synthesis, comparison, measure, number system, verbal problems, and other mathematical concepts. Four categories were "desirable" for the elementary teachers. These categories were on the concepts of analysis, budget, business, and graphs. Of the subheadings under the eleven categories 18 were rated "essential," 5 were rated "desirable," and 1 was rated "unimportant."

Reference was made in Chapter II to the "importance of meanings," and to "meanings teachers should develop." The important concepts discussed in this chapter are adequately representative of the categories which Brownell suggests.[2] A definition is not sufficient for most of these concepts. As an example, the dictionary defines foot as a unit of length of 12 inches. A few pages further, it defines an inch as a small unit of measure, one twelfth of a foot. This has little or no meaning. Thus, meanings and understandings of the important concepts of mathematics should be part of the training of the elementary teacher.

[2] Supra, p. 15.

CHAPTER V

ANALYSIS OF DATA, PROCESSES

The objective of this chapter was to analyze the data which concerns the processes of elementary arithmetic in terms of the categories as mentioned in the hypothesis. The processes were rated in the same manner as the concepts in Chapter IV. An average rating of sixty per cent or higher in any check space indicated the importance of the process. If none of the three check spaces received a sixty per cent rating, then a combination of two check spaces was used to indicate the importance of the process.

Process is an operation, a course of procedure, a series of actions. Processes are the actions, operations, procedures, and doings of the learner in making knowledge a part of his being or existence.

The Categories of Processes

The categories of processes were based upon the topics listed by Stipanowich[1] and the topics included in the six sets of elementary arithmetic textbooks which were listed on pages 21-22. The following list of processes continues the categories of the questionnaire begun in Chapter IV[2]:

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[1] J. J. Stipanowich, "The Development and Appraisal of a Course in Basic Mathematics for Prospective Elementary School Teachers," (Unpublished Ed. D. dissertation, Northwestern University, 1956), pp. 85-87.

[2] Supra, pp. 34-36.

XII. The processes of counting and numerating.

(a) Counting.

(b) Numerating.

(c) Recognizing quantities.

XIII. The processes of grouping and regrouping.

(a) Adding.

(b) Subtracting.

(c) Multiplying.

(d) Dividing.

XIV. The processes of comparing.

(a) Comparing relatively.

(b) Measuring.

XV. The processes involved in graphing.

XVI. The processes involved in solving problems.

(a) Reading with understanding.

(b) Choosing mathematical processes and the operating of them.

(c) Checking mathematical computations.

Treatment of Data

A table which corresponds to each category of processes has been constructed, and lists the data, after treatment, from the questionnaires. The tables express in per cent the opinions of the teachers and experts, and an average between the two groups concerning the mathematical processes of elementary arithmetic. Each process was checked by the elementary teachers in one of three spaces as follows: (1) essential, (2) desirable, (3) unimportant. These check points under each classification were totaled and per cents, based on the total answered questionnaires, were calculated. Similar data were given for the experts. An average

was figured between the per cents of the teachers and the experts. All analyses were based upon the average rankings. The conclusions for the category as a whole were based upon the ratings of a majority of the subheadings in each category. Some of the subheadings were also rated by this method.

Counting, Numerating, and Recognizing Quantities in Mathematics. Counting is progressively giving numbers as 1, 2, 3, 4 . . . or 5, 10, 15 . . . . Numerating is working with numbers. Recognizing quantities is associating names and symbols, and identifying groups.

Table XVI lists the processes of counting, numerating, and recognizing quantities in mathematics and their related subheadings according to

TABLE XVI

THE IMPORTANCE OF THE MATHEMATICAL PROCESSES OF COUNTING AND NUMERATING FOR THE ELEMENTARY TEACHER

Percentage of (1) essential, (2) desirable, (3) unimportant, (4) blank.

Processes	Teachers				Experts				Average			
	1	2	3	4	1	2	3	4	1	2	3	4
counting	66	2	0	32	90	0	0	10	78.0	1.0	0.0	21.0
by ones	93	7	0	0	100	0	0	0	96.5	3.5	0.0	0.0
by groups	83	14	2	1	81	14	0	5	82.0	14.0	1.0	3.0
numerating	36	6	7	51	86	0	0	14	61.0	3.0	3.5	32.5
learning about numbers (meaning)	86	5	2	7	100	0	0	0	93.0	2.5	1.0	3.5
reading numbers	95	4	1	0	100	0	0	0	97.5	2.0	0.5	0.0
using numbers	82	4	2	12	100	0	0	0	91.0	2.0	1.0	6.0
writing numbers	93	4	3	0	95	0	0	5	94.0	2.0	1.5	2.5
Arabic numbers	93	6	0	1	90	5	0	5	91.5	5.5	0.0	3.0
Roman numbers	48	49	3	0	52	43	0	5	50.0	46.0	1.5	2.5
position	89	10	1	0	90	0	5	5	89.5	5.0	3.0	2.5
recognizing quantities	38	22	0	40	81	0	0	19	59.5	11.0	0.0	29.5
associating number names, words, symbols	82	17	0	1	95	0	0	5	88.5	8.5	0.0	3.0
identifying groups	73	22	1	4	95	0	0	5	84.0	11.0	0.5	4.5

their importance as rated by the elementary teachers and the experts. Counting, counting by ones, and counting by groups were rated "essential." Numerating and related processes of learning about numbers (meaning), reading numbers, using numbers, writing numbers, Arabic numbers, and positions were rated "essential." Numerating in Roman numbers was rated "desirable." Recognizing quantities also was rated "desirable." Associating number names, words, symbols, and identifying groups which are subheadings under recognizing quantities were rated "essential."

The category of counting and numerating was rated "essential" because 13 items were rated "essential" and 2 items were rated "desirable."

Grouping and Regrouping. Grouping is assembling objects according to some common characteristic. Regrouping is separating a group into smaller groups.

Table XVII shows the processes of grouping and regrouping and their ratings by the elementary teachers and experts. Adding and related processes of carrying and putting together were rated "essential." Subtracting and related processes of changing large groups into smaller groups, counting change, how many left, take away, and borrowing or carrying back were rated "essential." Counting away was rated "desirable."

Multiplying and carrying, a subheading under multiplying, were rated "essential." Also, dividing was rated "essential." That fifty-four per cent of the elementary teachers did not check this process probably indicated a misunderstanding.

The category on grouping and regrouping was rated "essential" because a majority, or 15 processes, was rated "essential" and 2 processes were rated "desirable."

TABLE XVII

THE IMPORTANCE OF THE MATHEMATICAL PROCESSES OF GROUPING AND REGROUPING IN ELEMENTARY ARITHMETIC FOR THE ELEMENTARY TEACHER

Percentage of (1) essential, (2) desirable, (3) unimportant, (4) blank.

Processes	Teachers				Experts				Average			
	1	2	3	4	1	2	3	4	1	2	3	4
Adding	73	2	2	23	71	0	0	29	72.0	1.0	1.0	26.0
carrying	51	17	21	11	90	0	10	0	70.0	8.5	15.5	5.5
putting together	72	16	6	6	86	14	0	0	79.0	15.0	3.0	3.0
Subtracting	46	2	1	51	81	0	0	19	63.5	1.0	0.5	35.0
changing larger groups into smaller groups	48	24	18	10	86	9	5	0	67.0	16.5	11.5	5.0
counting away	39	28	18	15	76	5	19	0	57.5	16.5	18.5	7.5
counting change	63	27	6	4	71	24	5	0	67.0	25.5	5.5	2.0
how many left	89	9	2	0	100	0	0	0	94.5	4.5	1.0	0.0
take away	87	9	2	2	86	14	0	0	86.5	11.5	1.0	1.0
borrowing (carrying back)	99	0	0	1	76	10	0	14	87.5	5.0	0.0	7.5
Multiplying	35	25	18	22	86	9	0	5	60.5	17.0	9.0	13.5
carrying	98	0	0	2	100	0	0	0	99.0	0.0	0.0	1.0
Dividing	32	13	1	54	81	0	0	19	56.5	6.5	0.5	36.5
separating into equal groups	64	23	8	5	95	5	0	0	79.5	14.0	4.0	2.5
cutting into equal parts	61	31	4	4	90	10	0	0	75.5	20.5	2.0	2.0
borrowing (carrying back)	98	1	1	0	81	9	5	5	89.5	5.0	3.0	2.5
getting an average (addition)	89	10	0	1	86	9	0	5	87.5	9.5	0.0	3.0

Comparing. Table XVIII lists the processes of comparing and their ratings as given by the elementary teachers and experts. Comparing and the subheading measuring were rated "essential." Comparing quantities was rated "desirable." Subheadings under comparing quantities rated "essential" were fractions (relative size), values in purchases, and comparing by difference. One process, comparing quantities by ratio, was rated "desirable."

The process of measuring and the subheadings of English system,

TABLE XVIII

THE IMPORTANCE OF THE MATHEMATICAL PROCESS OF COMPARING IN ELEMENTARY ARITHMETIC FOR THE ELEMENTARY TEACHER

Percentage of (1) essential, (2) desirable, (3) unimportant, (4) blank.

Processes	Teachers				Experts				Average			
	1	2	3	4	1	2	3	4	1	2	3	4
Comparing	54	31	4	11	90	5	0	5	72.0	18.0	2.0	8.0
measuring	51	38	6	5	95	5	0	0	73.0	21.5	3.0	2.5
Comparing quantities	47	10	1	42	67	0	0	33	57.0	5.0	0.5	37.5
fractions (relative size)	85	11	2	2	100	0	0	0	92.5	5.5	1.0	1.0
values in purchases	68	25	4	3	86	14	0	0	77.0	19.5	2.0	1.5
by difference	65	30	3	2	100	0	0	0	82.5	15.0	1.5	1.0
by ratio	16	48	17	19	95	5	0	0	55.5	26.5	8.5	9.5
Measuring	47	23	3	27	76	5	0	19	61.5	14.0	1.5	23.0
English System	74	11	10	5	95	0	0	5	84.5	5.5	5.0	5.0
metric system	16	35	43	6	38	48	9	5	27.0	41.5	26.0	5.5
areas	64	25	9	2	86	9	0	5	75.0	17.0	4.5	3.5
capacity	40	30	28	2	81	9	5	5	60.5	19.5	16.5	3.5
length	52	39	6	3	95	5	0	0	73.5	22.0	3.0	1.5
money, identifying and value	71	21	4	4	90	10	0	0	80.5	15.5	2.0	2.0
money, reading, writing, and meaning	72	15	2	11	86	0	0	14	79.0	7.5	1.0	12.5
time	87	4	1	8	90	5	0	5	88.5	4.5	0.5	6.5
in hours and half hours	81	15	2	2	81	19	0	0	81.0	17.0	1.0	1.0
in days	66	27	5	2	86	14	0	0	76.0	20.5	2.5	1.0
in months	54	33	8	5	81	19	0	0	67.5	26.0	4.0	2.5
in years	89	9	1	1	95	5	0	0	92.0	7.0	0.5	0.5
in decades	49	39	11	1	57	38	5	0	53.0	38.5	8.0	0.5
in centuries	48	33	9	10	71	24	5	0	59.5	28.5	7.0	5.0
volume	32	41	19	8	67	28	0	5	48.5	34.5	9.5	6.5
weight	84	12	2	2	95	0	0	5	89.5	6.0	1.0	3.5

areas, capacity, length, money (identifying and value), money (reading, writing, and meaning), time, time in hours and half hours, time in days, time in months, time in years, and measuring were rated "essential." Measuring in metric system, measuring in centuries, and measuring

volume were rated "desirable."

The category on comparing was rated "essential" because a majority of the processes were so rated.

Graphing and Problem Solving. Table XIX lists the processes of graphing and their importance. Graphing showed 42.5 per cent "essential" and 42.5 per cent blank, hence no rating could be made. Interpreting (understanding) graphs, reading graphs and maps, and using graphs were rated "essential". Drawing to scale and mapping were rated "desirable."

TABLE XIX

THE IMPORTANCE OF THE MATHEMATICAL PROCESSES OF GRAPHING FOR THE ELEMENTARY TEACHER

Percentage of (1) essential, (2) desirable, (3) unimportant, (4) blank.

Processes	Teachers				Experts				Average			
	1	2	3	4	1	2	3	4	1	2	3	4
Graphing	28	22	3	47	57	5	0	38	42.5	13.5	1.5	42.5
drawing to scale	27	56	14	3	67	24	0	9	47.0	40.0	7.0	6.0
interpreting (understanding)	54	37	5	4	95	5	0	0	74.5	21.0	2.5	2.0
reading graphs, maps	67	28	3	2	95	5	0	0	81.0	16.5	1.5	1.0
using	56	33	7	4	86	14	0	0	71.0	23.5	3.5	2.0
mapping	33	50	12	5	67	33	0	0	50.0	41.5	6.0	2.5

The processes involved in graphing were rated "essential" since three items were rated "essential" and two items were rated "desirable."

Table XX lists the processes involved in solving problems in elementary arithmetic and their ratings as given by the elementary teachers and experts. Solving problems, and subheadings, reading problems, understanding problems, deciding (thinking), were rated "essential." The subheadings under thinking, planning and deciding of solving number stories, and solving problems were rated "essential." Checking was rated

"desirable." Approximating in round numbers, a subheading under checking, was rated "essential."

TABLE XX

THE IMPORTANCE OF THE MATHEMATICAL PROCESSES INVOLVED IN SOLVING PROBLEMS IN ELEMENTARY ARITHMETIC FOR THE ELEMENTARY TEACHER

Percentage of (1) essential, (2) desirable, (3) unimportant, (4) blank.

Processes	Teachers				Experts				Average			
	1	2	3	4	1	2	3	4	1	2	3	4
Solving problems	62	0	0	38	67	0	0	33	64.5	0.0	0.0	35.5
reading	98	2	0	0	95	0	0	5	96.5	1.0	0.0	2.5
understanding	99	0	1	0	95	0	0	5	97.0	0.0	0.5	2.5
deciding (thinking)	98	1	1	0	95	0	0	5	96.5	0.5	0.5	2.5
Thinking, planning, deciding	54	9	0	37	81	0	0	19	67.5	4.5	0.0	28.0
solving number stories	80	15	3	2	62	29	9	0	71.0	22.0	6.0	1.0
solving problems	74	17	7	2	81	14	0	5	77.5	15.5	3.5	3.5
Checking	36	7	0	57	81	5	0	14	58.5	6.0	0.0	40.5
approximating in round numbers	57	35	3	5	95	5	0	0	76.0	20.0	1.5	2.5

The category on solving problems was rated "essential" because a majority of the processes were so rated. Eight items were rated "essential," and one item was rated "desirable."

Table XXI lists the number of processes which were checked (1) essential, (2) desirable, (3) unimportant, and (4) no rating. Per cents have been calculated on the total for each group and have been included in the table.

TABLE XXI

AVERAGE RATINGS OF PROCESSES

Data taken from Tables XVI - XX (per cent is figured on total)

	Essential No.	%	Desirable No.	%	Unimportant No.	%	No Rating No.	%	Total
Processes	56	80.00	13	18.57	0	0.00	1	1.43	70

A total of 70 mathematical processes which included 14 main headings and 56 subheadings was checked by the elementary teachers and experts. Fifty-six, or 80 per cent, of the mathematical processes were rated "essential." These included 9 major headings and 47 subheadings. Thirteen or 18.57 per cent of the mathematical processes were rated desirable. These thirteen processes included four major headings and nine subheadings. One process, graphing, could not be rated individually by the methods set forth at the beginning of this chapter as it received an "essential" rating of 42.5 per cent and an "omission" rating of 42.5 per cent. However, the category was rated "essential" because three subheadings were rated "essential" and two subheadings were rated "desirable."

SUMMARY

Eighty per cent of the 70 mathematical processes checked by the elementary teachers and experts were rated "essential." Approximately 19 per cent of the mathematical processes were rated "desirable," and 1 per cent of the mathematical processes was not rated. All categories of mathematical processes were rated "essential" to the elementary teacher. These categories were the processes of counting and numerating, grouping and regrouping, comparing, graphing, and solving problems.

These processes adequately cover those recommended from the Guidance Report on the Commission on Post War Plans.[3] Meanings and understandings have great significance in the mathematical processes of arithmetic for the elementary teacher. An example which is inadequately treated in the textbooks and the teacher education books on arithmetic is the "inversion" rule in division of fractions. Most authors simply state the rule and give little or no explanation. It is simply an application of the principle that both numerator and denominator of a fraction can be multiplied by the same number without changing the value of the fraction. The multiplier is so chosen that the denominator becomes one.

These meanings and understandings of all mathematical processes of arithmetic should be a part of the training of the elementary teacher.

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[3] Supra, p. 2.

CHAPTER VI

SUMMARY OF FINDINGS

This investigation was concerned with the problem of identifying the concepts and processes of mathematics needed by an elementary teacher to teach arithmetic in grades one to six.

The need for this investigation has been supported by research which has shown a lack of understanding of meanings in elementary arithmetic by the teachers. Many teachers are incompetent in mathematical processes. Many teachers exhibit unfavorable attitudes toward arithmetic because of incompetency and a lack of understanding.

The design of the investigation was based upon three assumptions:

1. Meeting various classroom situations which require making decisions concerning the use of text material requires an understanding of the concepts and processes of mathematics on the part of the elementary teacher.

2. To teach arithmetic adequately for pupil learning with meaning and understanding, the teacher himself must understand the underlying mathematical concepts and processes.

3. Elementary teacher needs involving concepts and processes of arithmetic are of prime importance in their training.

The purposes of the investigation were (1) to identify the concepts and processes of elementary arithmetic, and (2) to ascertain from the opinions of a selected group of elementary teachers and experts the

importance of these concepts and processes.

The first purpose was obtained by analyzing thirty-eight elementary arithmetic textbooks which included six complete sets of books from grades one to six, and two extra first grade books. The second purpose was obtained through a questionnaire which was checked by a group of elementary teachers and a group of experts. A careful selection of both the elementary teachers and the experts was made. The teachers were selected from approved independent schools of more than twelve teachers and upon recommendation of their principals. The experts were selected from authors of elementary arithmetic textbooks, authors of books on elementary arithmetic, authors of articles on arithmetic, and mathematics educators. These included college teachers, and elementary supervisors in both teacher colleges and large city school systems.

The "essential" topics in elementary arithmetic have been fairly well established. The categories of concepts and processes were based on these established lists of topics, and especially on the six sets of arithmetic textbooks which were used in the survey for this study. Thus, the categories and questionnaire were closely related since they came mainly from the same sources.

The Findings

The basis for identification of mathematical concepts and processes was the importance attached to these items by the elementary teachers and by the experts.

Those categories and mathematical concepts rated "essential" from the checks on the questionnaire of both the elementary teachers and the experts were:

I. The concepts of order which included the natural numbers: one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety, and one hundred; and the ordinal numbers: first, second, third, fourth, fifth, sixth, seventh, eighth, ninth, and tenth.

II. Synthesis which included the following concepts of addition and multiplication: add, and, plus, how many, altogether, together, carrying, multiplier, multiply, product, tables, and twice.

IV. Comparison which included the following pairs of antonyms of mathematical significance: above-below, big-little, bottom-top, fast-slow, few-many, fewer-more, first-last, heavier-lighter, heavy-light, in-out, large-small, larger-smaller, largest-smallest, left-right, long-short, longer-shorter, longest-shortest, more-less, old-young, tall-short, taller-shorter, tallest-shortest, wide-narrow, high-low, higher-lower, highest-lowest, before-after, buy-sell, full-empty, east-west, north-south, increase-decrease, more-less, nearest-farthest, part-whole, share-keep, sum-difference, and upward-downward.

V. Measure which included the following useful units of the English system: teaspoon, tablespoon, cup, pint, quart, gallon, peck, bushel, units, pairs, dozen, zero, inch, foot or feet, foot ruler, yard, linear, mile, change, coins, cent, penny, nickel, dime, quarter, half-dollar, dollar bill,

halves, thirds, fourths, clock or o'clock, short (hour) hand, long (minute) hand, hour, half-hour, half-past, days (names), hours, minutes, calendar, week, months (names), year, leap year, decade, century, morning (A. M.), noon, afternoon (P. M.) midnight, daylight savings, table, standard zones, pound (lb.), average, dimensions, altitude, depth, distances, height, thickness, width, English system of measurement.

VI. The number system which included the following basic concepts for all numbers: Arabic, even, odd, whole, position, digits, groups, zero, units, ones' place, tens, tens place, hundreds, hundreds'place, thousands, millions, billions, part (of whole), denominator, equal parts, common, numerator, halves, thirds, fourths, fifths, sixths, sevenths, eighths, equivalent, improper, proper, invert, lowest terms, mixed numbers, like, (similar), unlike, decimal point, tenths, hundredths, per cent, thousandths, and mixed decimals.

X. Concepts pertaining to verbal problems which included: verbal story problems, one step problems, two step problems, hidden facts, questions, answers, approximate answers, estimate answers, round (off) answers, checking, check (work), number stories, problems, and round.

XI. Other, or miscellaneous concepts which included: same, equals, circle, squares, each, both, and fewer.

The following concepts from the four categories rated "desirable" were rated "essential": difference, how many left, minus, take away, fractions, borrowing, dividend, quotient, trial quotient, and bar graph.

Those categories and mathematical concepts rated "desirable" were:

III. Analysis which included the following concepts of subtraction and division: count change, left (over), remainder, exceeds, minuend, subtrahend, needs, and cancellation.

VII. Family budget which included: income, clothing, food, health, recreation, savings, shelter or rent, save, and spend.

VIII. Business which included the following general terms of retail selling: prices, cost price, list price, marked price, sale price, selling price, retail price, save, sell, spend, and cost.

IX. Graphs which included: divided bar, horizontal bar, vertical bar, circle, line, picture or pictograph, scale, scale drawing, scale model, and scale maps.

The following concepts from those categories rated "essential" were rated "desirable": column, sum, total, partial product, older-younger, oldest-youngest, half pint, barrel, cubic inches, cubic feet, cubic yards, yardstick, rod, silver dollar, sixths, length of months, ounce, hundred weight, abacus, census, Roman numbers, ratio, recipe (mixture), cents point, air mail, stamps or postage, alike (as many as), and triangle.

Those categories and mathematical processes rated "essential" from the checks on the questionnaires of both the elementary teachers and the experts were:

XII. Counting and numerating which included the following elementary basic understandings and manipulations: counting, counting by ones, counting by groups, numerating, learning about numbers (meaning), reading numbers, using numbers, writing numbers, Arabic numbers, positions, associating number names, words, symbols, and identifying groups.

XIII. Grouping and regrouping which included: adding, carrying, putting together, subtracting, changing large groups into smaller groups, counting change, how many left, take away, borrowing or carrying back, multiplying, and dividing.

XIV. Comparing which included: comparing, measuring, fractions (relative size) values in purchases, comparing by difference, measuring in the English system, measuring areas, capacity, length, money (identifying and value), money (reading, writing and meaning), time, time in hours and half hours, time in days, time in months, and time in years.

XV. Graphing which included: interpreting (understanding) graphs, reading graphs and maps, and using graphs.

XVI. Solving problems which included: reading problems, understanding problems, planning, deciding, thinking, solving, number stories, solving problems, and approximately in round numbers.

All the categories of mathematical processes were rated "essential." Some of the subheadings rated "desirable" were: numerating in Roman numbers, recognizing quantities, count away, comparing quantities, comparing by ratio, measuring in metric system, measuring in centuries, measuring volume, drawing to scale, mapping, checking (problems).

A rating of "essential" meant that an average of 60 per cent of both the elementary teachers and experts checked these items in the "essential" check space on the questionnaire. A rating of "desirable" meant either an average check of 60 per cent or more in the "desirable" check space or a combination of two check spaces to make 60 per cent or higher. Although the "essential" check space may have shown a higher rating than the "desirable" check space, the combination was rated "desirable."

A total of 334 mathematical concepts and a total of 70 mathematical processes were checked for importance by 245 elementary teachers and by 21 experts. Two hundred thirty-two concepts or 69.46 per cent were rated "essential." Sixty-six mathematical concepts or 19.76 per cent were rated "desirable." Thirty-six mathematical concepts or 10.78 per cent were rated "unimportant." Fifty-six or 80 per cent of the mathematical processes were rated "essential." Thirteen or 18.57 per cent of the mathematical processes were rated "desirable." None were rated "unimportant." One process was not rated.

A total of 404 mathematical concepts and processes were checked by the elementary teachers and experts. Approximately 71 per cent of these concepts and processes were rated "essential." Approximately 20 per cent of the concepts and processes were rated "desirable," and 9 per cent were rated "unimportant."

Thus the consensus concerning the concepts and processes of mathematics of both the experts and the elementary teachers as set forth in the hypothesis has been determined.

An arbitrary rating of 60 per cent, which was more than one-half of the votes, was chosen in making the final decision for the importance of each item. Although a more rigorous statistical treatment was possible, this treatment on the basis of 60 per cent or higher seemed adequate for this investigation.

This type of study has certain inherent weaknesses such as: (1) inability to communicate equally well to all respondents, (2) subjectivity in response, (3) failure to respond, and (4) inability of the investigator to check further on the responses.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

Two general conclusions seem evident from this investigation.

1. A clear consensus exists among veteran and competent elementary teachers, (a) concerning what concepts of elementary arithmetic are "essential" in the teaching of arithmetic, (b) concerning what processes of elementary arithmetic are "essential" in the teaching of arithmetic.

2. A clear consensus exists among the experts (a) concerning what concepts of elementary arithmetic are "essential" for the elementary teacher, (b) concerning what processes of elementary arithmetic are "essential" for the elementary teacher.

Recommendations

The concepts and processes which have been identified in this study do not include the totality of those which the teacher will likely need in the coming years. New and modern mathematical concepts are beginning to infiltrate the elementary school courses. Modern mathematics emphasizes structures, or patterns, which permeate all mathematics. The number system is based upon these structures. Structuring is an overall processing which extends throughout mathematics.

As a result of the findings in this study, the investigator makes the following recommendations:

1. All concepts and processes of elementary arithmetic which were rated "essential" should be strongly emphasized in the training of elementary teachers.
2. All concepts and processes of elementary arithmetic which were rated "desirable" should be included in the training of the elementary teacher.
3. All concepts and processes of elementary arithmetic which were rated "essential" along with their meanings and uses should be included in the textbooks for the training of elementary teachers.
4. These recommendations should be organized under the framework of the categories listed in Chapters IV and V and put into a bulletin for immediate use in the training of elementary teachers at both the pre-service and in-service levels.
5. College teachers should not only add new and modern mathematical concepts and processes, but should experiment to show whether these concepts and processes can be taught and, if so, whether they are an aid in understanding arithmetic and mathematics.
6. College teachers should explain and demonstrate to elementary teachers the structures as they permeate all mathematics, and should experiment to show whether these structures are an aid in understanding and development of mathematical thinking.

SELECTED BIBLIOGRAPHY

Anderson, G. Lester, and Arthur I. Gates. "The General Nature of Learning." Learning and Instruction. Forty-ninth Yearbook of the National Society for the Study of Education, Part I. Chicago: University of Chicago Press, 1950. Pp. 12-34.

Armstrong, W. Earl and T. M. Stinnett. A Manual on Certification Requirements for School Personnel in the United States. National Education Association, Washington, D. C., 1955.

Armstrong, W. Earl. "The Teacher Education Curriculum." The Journal of Teacher Education, VIII (September, 1957), 230-243.

Benezet, L. P. "Story of an Experiment." Journal of the National Education Association, XXIV (November 1935, January 1936), 241-244 and 301-303.

Breslich, E. R. "The Importance of Mathematics in General Education." Mathematics Teacher, XLIV (January, 1951), 1-6.

Brownell, William A. "The Place of Meaning in the Teaching of Arithmetic." Elementary School Journal, XLVII (January, 1947), 257-258.

Brownell, William A. "Psychological Consideration in the Learning and the Teaching of Arithmetic." The Teaching of Arithmetic. Tenth Yearbook of the National Council of Teachers of Mathematics. New York: Teachers College, Columbia University, Bureau of Publications, 1935. Pp. 1-32.

Brownell, William A. "Readiness and the Arithmetic Curriculum." Elementary School Journal, XXXVIII (January, 1938), 495-508.

Brownell, William A. "When Is Arithmetic Meaningful?" Journal of Educational Research, XXXVIII (March, 1945), 481-498.

Brownell, William A., and Harold E. Moser. Meaningful Versus Mechanical Learning: A Study in Grade II Subtraction. Duke University Research Studies in Education, No. 8. Durham, N. C.: Duke University Press, 1949.

Buckingham, B. R. Elementary Arithmetic, Its Meaning and Practice. Boston: Ginn and Company, 1947.

Buckingham, B. R. "Significance, Meaning, Insight - These Three." Mathematics Teacher, XXXI (January, 1938), 24-30.

Buckingham, B. R. and J. MacLathy. "The Number Abilities of Children When They Enter Grade One." Report of the Society's Committee on Arithmetic. Twenty-ninth Yearbook of the National Society for the Study of Education, Part II. Bloomington, Ill.: Public School Publishing Company, 1930. Pp. 473-524.

Buswell, G. T. "Introduction" (Chapter I). The Teaching of Arithmetic. Fiftieth Yearbook of the National Society for the Study of Education, Part II. Chicago: University of Chicago Press, 1951. Pp. 1-5.

Buswell, G. T. "The Psychology of Learning in Relation to the Teaching of Arithmetic." The Teaching of Arithmetic. Fiftieth Yearbook of the National Society for the Study of Education, Part II. Chicago: University of Chicago Press, 1951. Pp. 143-154.

Buswell, G. T. "Needed Research in Arithmetic." The Teaching of Arithmetic. Fiftieth Yearbook of the National Society for the Study of Education, Part II. Chicago: University of Chicago Press, 1951. Pp. 282-297.

Developing Meaningful Practices in Arithmetic. A publication of the Central New York School Study Council. Syracuse, New York: School of Education, Syracuse University, 1951.

Dodes, I. A. "The Science of Teaching Mathematics." Mathematics Teacher, XLVI (March, 1952), 157-166.

Dutton, W. H. "Attitudes of Prospective Teachers Toward Arithmetic." Elementary School Journal, LII (October, 1951), 84-90.

Dyer, Henry S., Robert Kalin, and Frederic M. Lord. Problems in Mathematics Education. Princeton: Educational Testing Service, 1956.

Featherston, E. Glenn, and J. Dan Hull. Analysis of Research in the Teaching of Arithmetic, 1955 and 1956. U. S. Department of Health, Education and Welfare, Office of Education, Washington, D. C.

Fehr, H. F. "Present Research in the Teaching of Arithmetic." Teachers College Record, LII (October, 1950), 11-23.

Glenn, W. H. "Curriculum Development to Capitalize on the Interrelationships of the Many Areas of Mathematics." Emerging Practices in Mathematics Education. Twenty-second Yearbook of the National Council of Teachers of Mathematics. Washington, D.C.: National Council of Teachers of Mathematics, 1954. Pp. 9-15.

Glennon, Vincent J. "A Study of the Growth and Mastery of Certain Basic Mathematical Understandings of Seven Educational Levels." Mathematics Teacher, XLII (December, 1949), 389-396.

Glennon, Vincent J. "A Study in Needed Redirection in the Preparation of Teachers of Arithmetic." Mathematics Teacher, XLII (December, 1949), 389-396.

Glennon, Vincent J. and C. W. Hunnicutt. What Does Research Say about Arithmetic?" Association for Supervision and Curriculum Development, A Department of the National Education Association, Washington, D. C., 1953.

Grossnickle, Foster E. "The Training of Teachers of Arithmetic." The Teaching of Arithmetic. Fiftieth Yearbook of the National Society for the Study of Education, Part II. Chicago: University of Chicago Press, 1951. Pp. 203-231.

"Guidance Report of the Commission on Post War Plans." Report of the Commission on Postwar Plans of the National Council of Teachers of Mathematics. Mathematics Teacher, XL (November, 1947), 315-339.

Gunderson, A. G. "Number Concepts Held by Seven Year Olds." Mathematics Teacher, XXXIII (January, 1940), 18-24.

Harap, H. and C. E. Mapes. "The Learning of Fundamentals in an Arithmetic Activity Program." Elementary School Journal, XXXIV (March, 1934), 515-525.

Harding, L. W. and I. P. Bryan. "An Experimental Comparison of Drill and Direct Experience in Arithmetic Learning in a Fourth Grade." Journal of Educational Research, XXXVII (January, 1944), 321-337.

Hartung, M. L. "Modern Methods and Current Criticisms of Mathematical Education." School Science and Mathematics, LV (February, 1955), 85-90.

Havighurst, R. J. "Can Mathematicians and Educationists Cooperate?" Mathematics Teacher, XXX (May, 1937), 211-213.

Hightower, H. W. "Effect of Instructional Procedures on Achievement in Fundamental Operations in Arithmetic." Educational Administration and Supervision, XL (October, 1954), 336-348.

Hilgard, E. R. Theories of Learning. New York: Appleton-Century-Crofts, 1948.

Hilgard, E. R. Theories of Learning, Second Edition. New York: Appleton-Century-Crofts, 1956.

Hoel, L. "An Experiment in Clinical Procedures for Arithmetic." Emerging Practices in Mathematics Education. Twenty-second Yearbook of the National Council of Teachers on Mathematics. Washington, D. C.: National Council of Teachers of Mathematics, 1954. Pp. 222-232.

Horn, Ernest. "Arithmetic in the Elementary School Curriculum." The Teaching of Arithmetic. Fiftieth Yearbook of the National Society for the Study of Education, Part II. Chicago: University of Chicago Press, 1951. Pp. 6-21.

Improvement of the Teaching of Mathematics. Oklahoma State Department of Education, Oklahoma City, Oklahoma, 1957.

Kinsella, J. J. "Some Reflections on the General Mathematics Situation." School Science and Mathematics, LIV (June, 1954), 431-438.

Kinzer, J., and L. Kinzer. "College Chemistry Students Deficient in Arithmetic Academic Data." Educational Research Bulletin, XXVII (January, 1948), 8-10.

Koenker, R. H. "Arithmetic Readiness at the Kindergarten Level." Journal of Educational Research, XLII (November, 1948), 218-223.

Layton, W. I. "The Certification of Teachers of Mathematics." Mathematics Teacher, XLII (December, 1949), 377-380.

MacLane, S. "The Impact of Modern Mathematics." Bulletin of the National Association of Secondary School Principals, XXXVIII (May, 1954), 66-70.

Mallory, V. A. "Providing for Individual Needs in Mathematics." Mathematics Teacher, XXX (May, 1937), 214-220.

McConnell, T. R. "Recent Trends in Learning Theory." Arithmetic in General Education. Sixteenth Yearbook of the National Council of Teachers of Mathematics. New York: Teachers College, Columbia University, Bureau of Publications, 1941. Pp. 268-289.

McLellan, James A., and John Dewey. The Psychology of Numbers. New York: D. Appleton Century Company, 1916.

Morton, R. L. Teaching Arithmetic in the Elementary School, Volume II, Intermediate Grades. New York: Silver Burdett Company, 1938.

Morton, R. L. Teaching Arithmetic. What Research Says to the Teacher. No. 2. Washington D. C.: National Education Association, October, 1953.

Mueller, Francis, and Harold Moser. "Background Mathematics for Teachers of Arithmetic." Emerging Practices in Mathematics Education. Twenty-second Yearbook of the National Council of Teachers of Mathematics. Washington, D. C.: National Council of Teachers of Mathematics, 1954. Pp. 180-188.

Newsom, C. V. "Mathematical Background Needed by Teachers of Arithmetic." The Teaching of Arithmetic. Fiftieth Yearbook of the National Society for the Study of Education, Part II. Chicago: University of Chicago Press, 1951. Pp. 232-250.

Orleans, J. S. The Understanding of Arithmetic Processes and Concepts Possessed by Teachers of Arithmetic. New York: Office of Research and Evaluation, Division of Teacher Education, College of the City of New York, Publication No. 12, 1952.

Phillips, C. "Background and Mathematical Achievement of Elementary Education Students in Arithmetic for Teachers." School Science and Mathematics, LIII (January, 1953), 48-52.

Plank, E. N. "Observation on Attitudes of Young Children Toward Mathematics." Mathematics Teacher, XLIII (October, 1950), 252-263.

Potter, M. A. "Corralling the Wandering Decimal Point." Mathematics Teacher, XL (February, 1947), 51-57.

Pressey, Luella Cole. "The Determination of the Technical Vocabulary of the School Subjects." School and Society, XX (July 19, 1924), 91-96.

Rosenquist, L. L. Young Children Learn to Use Arithmetic. Boston: Ginn and Company, 1949.

Shannon, J. R. "Percentages of Returns on Questionnaires in Reputable Educational Research." Journal of Educational Research, XLII (October, 1948), 138-141.

Skinner, B. F. "The Science of Learning and the Art of Teaching." Harvard Educational Review, XXIV (Spring, 1954), 86-97.

Smith, R. "Meaningful Division." Mathematics Teacher, LIII (January, 1950), 12-18.

Smith, R. "Provisions for Individual Differences." The Learning of Mathematics, Its Theory and Practice. Twenty-first Yearbook of the National Council of Teachers of Mathematics. Washington, D. C.: National Council of Teachers of Mathematics, 1953. Pp. 271-203.

Spitzer, H. F. "Techniques for Evaluating Outcomes of Instruction in Arithmetic." Elementary School Journal, XLIX (September, 1948), 21-31.

Stipanowich, J. J. "The Development and Appraisal of a Course in Basic Mathematics for Prospective Elementary School Teachers." (Unpublished Ed. D. dessertation, Northwestern University, 1956).

Stokes, C. Newton. Teaching the Meanings of Arithmetic. New York: Appleton-Century-Crofts, 1951.

Storm, W. B. "Arithmetic Meanings That Should be Tested." Arithmetic 1948. Supplementary Educational Monographs, No. 66. Chicago: University of Chicago Press, 1948. Pp. 26-31.

Sueltz, Ben A., and John W. Beredick. "The Need for Extending Arithmetical Learning." Mathematics Teacher, XLIII (February, 1950), 69-73.

A Survey of Mathematical Education. A Study of the Causes of Student Drop-out, Failure, and Incompetence at the Elementary and Secondary Levels. Research Bulletin No. 55-27, Educational Testing Service, Princeton, N. J., 1955. Multilithed.

Swenson, E. J. "Arithmetic for Preschool and Primary-grade Children." The Teaching of Arithmetic. Fiftieth Yearbook of the National Society for the Study of Education, Part II. Chicago: University of Chicago Press, 1951. Pp. 53-75.

Thiele, C. L. "Arithmetic in the Middle Grades." The Teaching of Arithmetic. Fiftieth Yearbook of the National Society for the Study of Education, Part II. Chicago: University of Chicago Press, 1951. Pp. 76-102.

Thiele, C. L. "An Incidental or an Organized Program of Number Teaching?" Mathematics Teacher, XXXI (February, 1938), 63-67.

Thorndike, Edward L. New Methods in Teaching Arithmetic. New York: Rand-McNally and Company, 1921.

Thorndike, Edward L. The Psychology of Arithmetic. New York: Macmillan and Company, 1922.

Thorndike, Edward L., and Irving Lorge. The Teacher's Word Book of 30,000 Words. New York: Columbia University Press, 1944.

Thrum, M. E. "The Developments of Concepts of Magnitude." Child Development, VI (June, 1935), 120-140.

Trow, William Clark. Scientific Method in Education. Boston: Houghton Mifflin Company, 1925.

Turner, I. S. The Training of Mathematics Teachers. Fourteenth Yearbook of the National Council of Teachers of Mathematics. New York: Teachers College, Columbia University, Bureau of Publications, 1939.

Twenty-first Yearbook of the National Council of Teachers of Mathematics. The Learning of Mathematics, Its Theory and Practice. Washington, D. C.: National Council of Teachers of Mathematics, 1953.

Van Engen, H. "An Analysis of Meaning in Arithmetic." Elementary School Journal, XLIX (February and March, 1949), 321-329 and 395-400.

Van Engen, H. "Some More Thoughts on Placing the Decimal Point in Quotients." Mathematics Teacher, XXXVIII (October, 1945), 243-245.

Washburne, C. W. "Mental Age and the Arithmetic Curriculum: A Survey of the Committee of Seven Grade Placement Investigations to Date." Journal of Educational Research, XXXIII (March, 1931), 210-231.

Washburne, C. W. "A Reply to Brownell's Critique of the Committee of Seven's Experiments." Elementary School Journal, XXXIX (February, 1939), 417-431.

Washburne, C. W. "When Should We Teach Arithmetic?" Elementary School Journal, XXVII (May, 1938), 659-665.

Washburne, M. F. "Mathematical Ability, Reasoning, and Academic Standing." American Journal of Psychology, L (November, 1937), 484-488.

Weaver, J. F. "Misconceptions about Rationalization in Arithmetic." Mathematics Teacher, XLIV (October, 1951), 377-381.

Weaver, J. F. "Some Areas of Misunderstanding about Meaning in Arithmetic." Elementary School Journal, LI (September, 1950), 35-41.

Wheat, H. G. How to Teach Arithmetic. Evanston, Illinois: Row, Peterson and Company, 1951.

Wheat, H. G. "The Nature and Sequence of Learning Activities in Arithmetic." The Teaching of Arithmetic. Fiftieth Yearbook of the National Society for the Study of Education, Part II. Chicago: University of Chicago Press, 1951. Pp. 22-39.

Wheat, H. G. The Psychology and Teaching of Arithmetic. Boston: D. C. Heath Publishing Company, 1937.

Wilson, J. D. "Trends in Elementary and Secondary School Mathematics, 1918-1948." (Unpublished doctoral dissertation, Stanford University, 1949).

Woody, C. "A General Educator Looks at Arithmetic Readiness." Mathematics Teacher, XXX (November, 1937), 314-321.

APPENDIX A

Directions for Scoring

The following concepts and processes of elementary arithmetic were selected on a frequency of occurrence basis from six major sets of elementary arithmetic books. These concepts and processes may vary in importance in the mathematical development of the child.

Concepts are the elements of knowledge. Concepts as used here are simply mathematical terms which are used by the teacher or textbook to develop the child in his mathematical understanding, thinking and reasoning. Processes are the operations such as adding, subtracting, multiplying, dividing, measuring, etc., which the child performs with numbers.

Please check in the space at the right according to the importance which you consider the concept or process to have in the mathematical development of understanding, reasoning, and thinking of the child. The ratings are (1) essential, (2) desirable, (3) unimportant.

CONCEPTS

PART I: FIRST, SECOND AND THIRD GRADES

1. Essential
2. Desirable
3. Unimportant

		1	2	3
	ADDITION			
1	add			
2	and			
3	column			
4	plus (+)			
5	how many			
6	sum			
7	total			
8	altogether			
9	together			
	SUBTRACTION			
10	count change			
11	cross (out)			
12	difference			
13	how many left			
14	left (over)			
15	minus (-)			
16	remainder			
17	take away			
	MEASURE			
	Time:			
18	clock (o'clock)			
19	short hand (hr)			
20	long hand (min)			
21	hour			
22	half-hour			
23	half-past			
24	days (names)			
25	hours			
26	minutes			
27	seconds			
28	calendar			
29	week			
30	months (names)			
31	months (length			
32	year			
33	Weight:			
34	ounce			
35	pound (lb)			
36	Length:			
37	inch			
38	foot (feet)			
39	foot ruler			
40	yard			

		1	2	3
41	yardstick			
42	Volume:			
43	half pint (cup)			
44	pint			
45	quart			
46	Parts			
47	halves			
48	thirds			
49	fourths			
50	sixths			
51	Money			
52	coins			
53	cent			
54	penny			
55	nickel			
56	dime			
57	quarter			
58	half-dollar			
59	dollar, silver			
60	dollar, bill			
	COMPARISONS			
61	above-below			
62	big-little			
63	bottom-top			
64	fast-slow			
65	few-many			
66	fewer-more			
67	first-last			
68	heavier-lighter			
69	heavy-light			
70	in-out			
71	large-small			
72	larger-smaller			
73	largest-smallest			
74	left-right			
75	long-short			
76	longer-shorter			
77	longest-shortest			
78	more-less			
79	old-young			
80	older-younger			
81	oldest-youngest			
82	tall-short			
83	taller-shorter			
84	tallest-shortest			
85	wide-narrow			
86	high-low			
87	higher-lower			
88	highest-lowest			
	NUMBERS			
89	1 one			
90	2 two			
91	3 three			

		1	2	3
92	4 four			
93	5 five			
94	6 six			
95	7 seven			
96	8 eight			
97	9 nine			
98	10 ten			
99	11 eleven			
100	12 twelve			
101	13 thirteen			
102	14 fourteen			
103	15 fifteen			
104	16 sixteen			
105	17 seventeen			
106	18 eighteen			
107	19 nineteen			
108	20 twenty			
109	30 thirty			
110	40 forty			
111	50 fifty			
112	60 sixty			
113	70 seventy			
114	80 eighty			
115	90 ninety			
116	100 one hundred			
	ORDINAL NUMBERS			
117	first			
118	second			
119	third			
120	fourth			
121	fifth			
122	sixth			
123	seventh			
124	eighth			
125	ninth			
126	tenth			
	OTHER TERMS			
127	air-mail			
128	alike (as many as)			
129	same			
130	answer			
131	both			
132	cents point			
133	change			
134	check (work)			
135	circle			
136	cost			
137	count			
138	dozen			
139	each			
140	equals (=)			
141	fewer			
142	group(s)			

		1	2	3
143	oneness			
144	twoness			
145	fiveness			
146	whole			
147	half			
148	middle			
149	next			
150	multiplication (X)			
151	multiply			
152	need(s)			
153	next			
154	ones place			
155	tens place			

		1	2	3
156	hundreds place			
157	number stories			
158	problems			
159	round			
160	save			
161	sell			
162	spend			
163	squares (figure)			
164	stamps (postage)			
165	think			
166	triangle			
167	zero			

PROCESSES

FIRST, SECOND AND THIRD GRADES

1. Essential
2. Desirable
3. Unimportant

		1	2	3
168	Adding			
169	carrying			
170	putting together			
171	Checking			
172	Comparing			
173	measuring			
174	Counting			
175	by ones			
176	by groups			
177	Dividing			
178	separating into equal groups			
179	cutting into equal parts			
180	Grouping			
181	Measuring			
182	length			
183	money (identifying and value)			

		1	2	3
184	reading, writing and meaning			
185	time, in hours and half-hours			
186	time, in days			
187	time, in months			
188	volume (liquid)			
189	Multiplying			
190	Numerating			
191	learning about numbers (meaning)			
192	reading numbers			
193	using numbers			
194	writing numbers			
195	Recognizing quantities			
196	associating number names, words, symbols			
197	identifying groups			
198	Subtracting			
199	changing larger units to smaller			
200	counting away			
201	counting change			
202	how many left			
203	take away			
204	Thinking (planning-deciding)			
205	solving number stories			
206	solving problems			

CONCEPTS

PART II: FOURTH, FIFTH AND SIXTH GRADES

1. Essential
2. Desirable
3. Unimportant

		1	2	3
	BUDGET (Family)			
1	income (salary, wages)			
2	advancement			
3	clothing			
4	food			
5	health			
6	insurance			
7	recreation			
8	savings			
9	shelter (rent)			
10	miscellaneous			
	BUSINESS			
11	prices			
12	cost			
13	list			
14	marked			
15	net			
16	sale			
17	selling			
18	retail			
19	wholesale			
	COMPARISON			
20	before-after			
21	buy-sell			
22	full-empty			
23	east-west			
24	north-south			
25	increase-decrease			
26	more-less			
27	nearest-farthest			
28	part-whole			
29	share-keep			
30	sum-difference			
31	upward-downward			
	GRAPHS			
32	bar			
33	divided			
34	horizontal			
35	vertical			
36	circle			
37	line			
38	picture (pictograph)			

1. Essential
2. Desirable
3. Unimportant

		1	2	3
39	scale			
40	drawing			
41	model			
42	maps			
	SUBTRACTION			
43	exceeds			
44	minuend			
45	subtrahend			
	MULTIPLICATION			
46	carrying			
47	multiplier			
48	product			
49	partial product			
50	tables			
51	twice			
	DIVISION			
52	fractions			
53	borrowing			
54	caret ( )			
55	cancellation			
56	dividend			
57	divisor			
58	quotient			
59	trial quotient			
	PROBLEMS			
60	verbal (story)			
61	one step			
62	two step			
63	hidden facts			
64	questions			
65	answers			
66	approximate			
67	estimate			
68	round (off)			
69	checking			
	NUMBERS			
70	Arabic			
71	even			
72	odd			
73	position			
74	digits			
75	units			

		1	2	3
76	tens			
77	hundreds			
78	thousands			
79	millions			
80	billions			
81	whole			
	FRACTIONS			
82	part (of whole)			
83	denominator			
84	equal parts			
85	common			
86	numerator			
87	halves			
88	thirds			
89	fourths			
90	fifths			
91	sixths			
92	sevenths			
93	eighths			
94	equivalent			
95	improper			
96	proper			
97	invert			
98	lowest terms			
99	mixed numbers			
100	ratio			
101	recipe (mixture)			
102	like (similar)			
103	unlike			
	DECIMALS			
104	decimal point			
105	tenths			
106	hundredths			
107	per cent			
108	thousandths			
109	mixed			
	MEASUREMENT			
110	English System			
111	Metric System			
	COUNTING			
112	units			
113	pairs			
114	dozen			
115	score			
116	gross			
117	quire			
118	ream			
119	zero			
	LENGTH			
120	linear			
121	rod			
122	mile			
123	millimeter			

		1	2	3
124	centimeter			
125	meter			
126	kilometer			
127	hand			
128	span			
129	pace			
130	fathom			
131	nautical mile			
132	knot			
	TIME			
133	morning (A.M.)			
134	noon			
135	afternoon (P.M.)			
136	midnight			
137	daylight savings			
138	year			
139	leap year			
140	decade			
141	century			
142	table			
143	zones, standard			
	AREA			
144	surface			
145	square units (in. ft. yd.)			
146	acre			
147	section (sq. mi.)			
	CAPACITY (volume)			
148	teaspoon			
149	tablespoon			
150	cup			
151	gill			
152	pint			
153	quart			
154	gallon			
155	peck			
156	bushel			
157	barrel			
158	milliliter			
159	liter			
160	kiloliter			
	VOLUME			
161	cubic units (in. ft. etc.)			
162	board feet			
163	cubic centimeters			
	WEIGHT			
164	hundredweight			
165	ton			
166	long ton			
167	grain			
168	milligram			
169	gram			

		1	2	3
170	kilogram			
171	carat			
	OTHER TERMS			
172	abacus			
173	average			
174	census			
175	dimensions			

		1	2	3
176	altitude			
177	depth			
178	distances			
179	height			
180	thickness			
181	width			

PROCESSES

FOURTH, FIFTH AND SIXTH GRADES

1. Essential
2. Desirable
3. Unimportant

		1	2	3
182	Dividing			
183	borrowing (carrying back)			
184	getting an average (addition)			
185	Multiplying			
186	carrying			
187	Numerating			
188	Arabic numbers			
189	Roman numbers			
190	positions			
191	Subtracting			
192	borrowing (carrying back)			
193	Solving Problems			
194	reading			
195	understanding			
196	deciding (thinking)			
197	Measuring			
198	English System			
199	Metric System			
200	areas			
201	capacity (volume)			

		1	2	3
202	time			
203	years			
204	decades			
205	centuries			
206	weight			
207	Graphing			
208	drawing to scale			
209	interpretating (understanding)			
210	reading graphs, maps			
211	using			
212	mapping			
213	Comparing quantities			
214	fractions (relative size)			
215	values in purchases			
216	by difference			
217	by ratio			
218	Checking			
219	approximating in round numbers			

PERSONAL DATA SHEET

Please fill in the following blanks:

Name _____ Address _____

School in which you work _____

Grade of teaching, if self contained _____

If departmentalized, what department _____

Studied mathematics as follows: (Give number of years)

High School	_____	years
General Math	_____	years
Algebra	_____	years
Geometry	_____	years
Other (Specify)		
_____	_____	years
_____	_____	years

Semester hours of college mathematics _____

Semester hours of special methods of mathematics in college _____

Number of years of teaching _____

APPENDIX B

NORTHEASTERN STATE COLLEGE

Tahlequah, Oklahoma

Department of Mathematics (date)

(Inside address)

I am sure that you, as a college teacher and leading educator, share my interest in the adequate preparation of elementary teachers. Since elementary teachers exert a tremendous influence upon children in developing attitudes, interests, enthusiasms, habits and ambitions, they should be carefully selected, excellently trained and most happy in their work. The purpose of this study is to identify the concepts and processes of mathematics needed by an elementary teacher to teach arithmetic adequately in grades one to six. It is hoped this may be of use in the preparation of elementary teachers.

Would you please take a few minutes to check the enclosed questionnaire? Also, if there are other concepts or processes which you consider important, I would sincerely appreciate your listing them.

In order that I might get an opinion from successful teachers on the job, would you please list the names and addresses of five or more elementary arithmetic teachers or supervisors in your area.

Dr. James H. Zant of Oklahoma State University is advising with me, and would appreciate your cooperation in making this survey.

Thank you very much for your time in helping me make this survey. A summary will be sent to you when the survey is completed.

Sincerely yours,

Raymond Carpenter
Associate Professor
of Mathematics

NORTHEASTERN STATE COLLEGE

Tahlequah, Oklahoma

Department of Mathematics (date)

(inside address)

As a mathematics teacher in college, I am very much interested in the preparation of elementary teachers of arithmetic. I have prepared a questionnaire on the concepts and processes of elementary arithmetic, and need the opinions of a number of elementary teachers on the importance of these concepts in the teaching of elementary students.

I want to solicit your help, and wish to ask you to list five or more of your better elementary arithmetic teachers within grades one to six, together with their addresses and the grade which each one teaches.

Please select the teachers with the following in mind:

1. Do they seem to be better teachers in comparison with others in their field?

2. Do pupils seem to rate higher in arithmetic after being taught by these teachers in comparison with other teachers?

3. Have these teachers been successful as teachers over a period of years?

I will in no way involve your name with these teachers. I want an honest, voluntary response to the questionnaire. I have already had responses from a number of college professors and would like very much to compare these with successful teachers in the field.

Dr. James H. Zant of Oklahoma State University, Stillwater, Oklahoma is assisting me with this survey, and joins with me in thanking you for your prompt and helpful cooperation.

Sincerely,

Raymond Carpenter
Associate Professor
of Mathematics

From: (Give your name and address)

NAME	ADDRESS	GRADE

NORTHEASTERN STATE COLLEGE

Tahlequah, Oklahoma

Department of Mathematics (date)

(inside address)

As a superior teacher in your field, your experience would be most valuable in the training of elementary teachers. And, as a teacher in a college whose main objective is the preparation of teachers, I am vitally interested in the preparation of elementary teachers of arithmetic.

In the pages which follow, there is a questionnaire (or that part of the questionnaire which embraces the grade or grades you are teaching) based on concepts and processes which children experience in grades one to six. The purpose of this study is to identify the concepts and processes of mathematics needed by an elementary teacher to teach arithmetic adequately in grades one to six. It is hoped that the results of this survey may be used in more adequate preparation of elementary teachers.

Will you please check the questionnaire, fill out the personal data sheet, and return them to me in the self addressed envelope? Dr. James H. Zant of Oklahoma State University is advising me with this survey, and joins with me in thanking you for your time. If you so indicate, a summary of the results will be mailed to you when it is completed.

Yours very truly,

Raymond Carpenter
Associate Professor
of Mathematics

NORTHEASTERN STATE COLLEGE

TAHLEQUAH, OKLAHOMA

Department of Mathematics (date)

(inside address)

Two weeks ago you received a questionnaire on the concepts and processes of elementary arithmetic for grades one to six. It is very important that I have your answer as soon as possible.

Would you please check the enclosed postcard, or the questionnaire, and mail it to me as soon as you can conveniently do so?

Thank you very much.

Sincerely,

Raymond Carpenter
Associate Professor
of Mathematics

APPENDIX C

ARITHMETIC BOOKS ANALYZED FOR LIST OF CONCEPTS AND PROCESSES

Brueckner, Leo J., F. E. Grossnickle, and Elda L. Merton. Arithmetic We Use (Grades One to Six). Chicago: John C. Winston Company, 1942.

Clark, John R., Charlotte W. Junge, and Harold E. Moser. Growth in Arithmetic (Grades One to Six). New York: World Book Company, 1952.

Mallory, Virgil S., Dennis H. Cooke, and Amanda Loughren. Using Arithmetic (Grades One to Six). Chicago: Benj. H. Sanborn and Company, 1946.

Morton, Robert Lee, Merle Gray, Elizabeth Springstun and William L. Schaaf. Making Sure of Arithmetic (Grades One to Six). New York: Silver Burdett Company, 1952.

Studebaker, J. W., W. C. Findley, G. M. Ruch, and F. B. Knight. Study Arithmetics (Grades one to Six). Chicago: Scott Foresman and Company, 1947.

Wheat, H.G., Geraldine Kauffman, and Harl R. Douglass. Row-Peterson Arithmetic (Books One to Six). Evanston: Row, Peterson and Company, 1954.

Two Other Books Used

Bartoo, G. C., Bess Stinson, and Jesse Osborn. Adventures with Numbers (Grade 1). St. Louis: Webster Publishing Company, 1952.

Stern, Catherine. Discovering Arithmetic (Grade 1). Boston: Houghton Mifflin Company, 1952.

APPENDIX D

LIST OF EXPERTS

Dr. Lee E. Boyer
Millersville State Teachers College
Millersville, Pennsylvania

Dr. Leo J. Brueckner
7267 Hollywood Blvd., Apt. 3
Hollywood, California

Dr. B. R. Buckingham
Editorial Department
Ginn and Company
Boston, Massachusetts

Dr. Sarah Burkhart
Office of the County Superintendent
Tulsa, Oklahoma

Dr. G. T. Buswell
Professor of Educational Psychology
University of California
Berkeley, California

Dr. John R. Clark
Mt. Road, Route 3, Box 149
New Hope, Pennsylvania

Dr. Chester K. Davis
Director of Training School
Arizona State College
Flagstaff, Arizona

Dr. Harold Fawcett
Chairman, Dept. of Education
Ohio State University
Columbus, Ohio

Dr. Bob Fouch
Florida State University
Tallahasse, Florida

Dr. William Gage
University of Florida
Gainesville, Florida

Dr. Glennadine Gibbs
Iowa State Teachers College
Cedar Falls, Iowa

Dr. Foster Grossnickle
State Teachers College
Jersey City, N. J.

Miss Frankie E. Harris
Northeastern State College
Tahlequah, Oklahoma

Dr. R. L. Morton
Professor of Education
Ohio University
Athens, Ohio

Dr. Francis Mueller
State Teachers College
Towson, Maryland

Dr. C. V. Newsom
President, New York University
New York City, N. Y.

Dr. Edna E. Parker
Associate Professor of Education
Florida State University
Tallahasse, Florida

Dr. Ann Peters
Keene Teachers College
Keene, New Hampshire

Miss Mildred E. Randels
Northeastern State College
Tahlequah, Oklahoma

Dr. C. C. Richtmeyer
Central Michigan College
Mt. Pleasant, Michigan

Dr. Herbert F. Spitzer
Principal, University Elementary School
Iowa City, Iowa

Dr. C. Newton Stokes
Professor of Mathematics
Temple University
Philadelphia 22, Pennsylvania

Dr. Ben A. Sueltz
Professor of Mathematics
State Teachers College
Cortland, New York

Dr. Robert L. Swain
New York State Teachers College
New Platz, New York

Dr. Esther J. Swenson
Professor of Elementary Education
University of Alabama
University, Alabama

Dr. C. L. Thiele
Divisional Director, Exact Sciences
Detroit Public Schools
Detroit, Michigan

Dr. Vaud Travis
Northeastern State College
Tahlequah, Oklahoma

Dr. Henry VanEngen
University of Wisconsin
Madison, Wisconsin

Dr. Harry G. Wheat
Professor of Education
West Virginia University
Morgantown, West Virginia

Dr. Mary Witt
University School
Florida State University
Tallahasse, Florida

Dr. Clifford Woody (deceased)
School of Education
University of Michigan
Ann Arbor, Michigan

Dr. F. Lynwood Wren
Professor of Mathematics
George Peabody College of Teachers
Nashville, Tennessee

APPENDIX E

PRINCIPALS AND SUPERVISORS WHO RECOMMENDED ELEMENTARY TEACHERS TO CHECK THE QUESTIONNAIRE

ALABAMA

Miss Flora Mary Pearson
Westlawn School
Mobile, Alabama

Mrs. Dorthia Taube
City and County Supervisor
of Elementary Education
Mobile, Alabama

Miss Sara Davis
Verner School
Tuscaloosa, Alabama

Miss Katie Williams
Tuscaloosa County Schools
Tuscaloosa, Alabama

Miss Margaret Strickland
West End School
Tuscaloosa, Alabama

CALIFORNIA

Miss Sagie M. Ostendorf
Williams Elementary School
Bakersfield, California

KANSAS

Miss Ida M. Tinnin
Frances Willard Elementary School
Arkansas City, Kansas

Jim Harris
Central Elementary School
Baxter Springs, Kansas

Miss Vida M. Williams
Whittier Elementary School
Coffeyville, Kansas

James Yates
Park Elementary School
Columbus, Kansas

Miss Vera Clark
Kansas Ave. Elementary School
Emporia, Kansas

Miss Margaret Stinsman
Central Elementary School
Hutchinson, Kansas

W. A. Culp
Washington Elementary School
Independence, Kansas

Mapes Davis
L. M. Alcott Elementary School
Kansas City, Kansas

W. L. Duby
Washington Elementary School
Newton, Kansas

Ralph Loyd
Hawthorne Elementary School
Ottawa, Kansas

George L. Dove
Washington Elementary School
Parsons, Kansas

Joe Heitz
Eugene Field Elementary School
Pittsburg, Kansas

C. A. Brooks
Lincoln Elementary School
Salina, Kansas

Miss Althea Smith
Classen Elementary School
Wichita, Kansas

Walter W. Smith
College High Elementary School
Wichita, Kansas

Ralph E. Jones
Franklin Elementary School
Wichita, Kansas

Miss Jessie Thompson
Willard Elementary School
Wichita, Kansas

MISSOURI

Marvin Thomas
Central School
Boonville, Missouri

Mrs. Lyda Gibbs
Jefferson Elementary School
Cape Girardeau, Missouri

Joe M. Barnes
Ridgeway School
Columbia, Missouri

Paul G. Fleeman
Grant Elementary School
Columbia, Missouri

Mrs. Mildred Kearnes
Benton and Oldham Elementary Schools
Independence, Missouri

Miss Bess N. Dahl
Hale H. Cook Elementary School
Kansas City, Missouri

Eugene P. Wheeler
Humboldt Elementary School
Kansas City, Missouri

William L. Wynn
J. Milton Turner Elementary School
Kirkwood 22, Missouri

Inez M. Harrison
Central Elementary School
Neosho, Missouri

C. E. Coursey
Wheatley Elementary School
Popular Bluff, Missouri

Miss Virginia Renshaw
Boyd Elementary School
Springfield, Missouri

Mrs. Emma Gann
Holland Elementary School
Springfield, Missouri

Charles R. Swan
Robberson Elementary School
Springfield, Missouri

Herbert F. Church
Adams Elementary School
St. Louis, Missouri

Lloyd L. Glenn
South Park Elementary School
St. Joseph, Missouri

Elizabeth J. Watson
Eugene Field School
Webb City, Missouri

Harold T. Downs
Lockwood Elementary School
Webster Groves, Missouri

OKLAHOMA

Miss Bonnie M. Allen
Irving Elementary School
Ada, Oklahoma

S. G. Hove
Wilson Elementary School
Altus, Oklahoma

Paul Bailey
Sunset Elementary School
Anadarko, Oklahoma

Mrs. Irene McGoodwin
Elementary Coordinator
Ardmore Public Schools
Ardmore, Oklahoma

J. H. McBride
Garfield Elementary School
Bartlesville, Oklahoma

Paul Starks
Highland Park Elementary School
Bartlesville, Oklahoma

Ernest B. Godley
Huston Elementary School
Blackwell, Oklahoma

Miss Linnie Wood
Edison Elementary School
Bristow, Oklahoma

M. Cecil Rhoades
Southside Elementary School
Broken Arrow, Oklahoma

Bill J. Anthis
Claremont Elementary School
Claremore, Oklahoma

Clarence Stringer
Elementary School
Coweta, Oklahoma

Clyde Bowen
Wilson Elementary School
Cushing, Oklahoma

David L. Williams
Washington Irving Elementary School
Durant, Oklahoma

Miss Leona Kennedy
Coordinator Elementary School
Edmond, Oklahoma

Miss Hazel Kirbie
Longfellow Elementary School
Elk City, Oklahoma

Homer Stout
Lincoln Elementary School
El Reno, Oklahoma

Miss Esther Hinshawe
Adams Elementary School
Enid, Oklahoma

Miss Ethel MacGoddard
Coolidge Elementary School
Enid, Oklahoma

Mrs. Irene Sloan
Harrison Elementary School
Enid, Oklahoma

Miss Frances Leeper
Central Elementary School
Guthrie, Oklahoma

Harrison Steele
Central Elementary School
Idabel, Oklahoma

Miss Thelma Talla
Elementary Coordinator
Lawton Public Schools
Lawton, Oklahoma

Delbert Wolf
Wilson Elementary School
Miami, Oklahoma

Herbert Flowers
Country Estates Elementary School
Midwest City, Oklahoma

Miss Virginia Rose
Sooner Elementary School
Midwest City, Oklahoma

Arthur Toon
Houston Elementary School
Muskogee, Oklahoma

Eiland Rainwater
Irving Elementary School
Muskogee, Oklahoma

Eschol R. Haley
Longfellow Elementary School
Muskogee, Oklahoma

Miss Cordia V. Callihan
Whittier Elementary School
Muskogee, Oklahoma

Miss Foy Runyan
Coordinator of Elementary Schools
Norman Public Schools
Norman, Oklahoma

Howard C. Thompson
Elementary and Junior High School
Nowata, Oklahoma

Wade Davenport
Adams Elementary School
Oklahoma City, Oklahoma

Dalton L. Eads
Buchanan Elementary School
Oklahoma City, Oklahoma

Miss Clara M. Wade
Coolidge Elementary School
Oklahoma City, Oklahoma

Miss Lila G. Quinn
Fillmore Elementary School
Oklahoma City, Oklahoma

Earl Martin
Johnson Elementary School
Oklahoma City, Oklahoma

Darrell McFeaters
Nichols Hills Elementary School
Oklahoma City, Oklahoma

Leon C. Nance
Shields Heights Elementary School
Oklahoma City, Oklahoma

Miss Nina Birkhead
Stan Watie Elementary School
Oklahoma City, Oklahoma

E. O. Davis
Westwood Elementary School
Oklahoma City, Oklahoma

Miss Mildred Nelson
Coordinator of Elementary Education
Okmulgee, Oklahoma

Eldon Wagner
Elementary Schools
Pawhuska, Oklahoma

L. E. States
Elementary School
Perry, Oklahoma

Miss Cleo Melton
Elementary Supervisor
Ponca City Public Schools
Ponca City, Oklahoma

Miss Ruth Stanford
Central Elementary School
Pryor, Oklahoma

F. N. Shields
Elementary Grade School 1-3
Putnam City, Oklahoma

Dan Davis
Elementary Grade School 4-6
Putnam City, Oklahoma

J. W. Fleming
Jefferson Elementary School
Sapulpa, Oklahoma

H. B. Smith, Jr.
Roosevelt Elementary School
Seminole, Oklahoma

Woodrow Floyd
Elementary School
Stigler, Oklahoma

Miss Nora Hinrichs
Eugene Field and Westwood Elementary Schools
Stillwater, Oklahoma

J. H. King
Jefferson and Will Rogers Elementary Schools
Stillwater, Oklahoma

N. S. Hopkins
Lincoln & Highland Park Elementary Schools
Stillwater, Oklahoma

Gerald Bowers
Elementary and Jr High School
Stilwell, Oklahoma

Leonard W. Rainwater
Elementary Schools
Tahlequah, Oklahoma

Arley U. Garrett
Alcott Elementary School
Tulsa, Oklahoma

Curtis Turner
Eugene Field Elementary School
Tulsa, Oklahoma

W. E. Hagar
Franklin Elementary School
Tulsa, Oklahoma

James S. Elledge
Houston Elementary School
Tulsa, Oklahoma

George J. Hooper
Lanier Elementary School
Tulsa, Oklahoma

R. D. Rutherford
Lowell Elementary School
Tulsa, Oklahoma

A. M. Calloway
Elementary School
Wagoner, Oklahoma

Mrs. Beatrice Taylor
Elementary School
Walters, Oklahoma

Terry McCarty
Elementary School
Weatherford, Oklahoma

Raymond G. Fleming
Central Elementary School
Wetumka, Oklahoma

Ellis DeWeese
Elementary School
Wilburton, Oklahoma

WEST VIRGINIA

Everett Bailey
Knob School
Princeton, West Virginia

VITA

Raymond Carpenter

Candidate for the Degree of

Doctor of Education

Thesis: IDENTIFYING CONCEPTS AND PROCESSES IN MATHEMATICS NEEDED FOR THE ADEQUATE PREPARATION OF ELEMENTARY TEACHERS

Major Field: Higher Education

Minor Field: Mathematics

Biographical:

Personal data: Born at Ozark, Arkansas, October 8, 1907.

Education: Attended grade school at a rural school in Franklin County, Arkansas; graduated from Conway, Arkansas High School in 1925; received the Bachelor of Arts degree from Hendrix College, Conway, Arkansas in 1928; received the Master of Arts degree from Columbia University in 1937; completed requirements for the degree of Doctor of Education in May, 1959.

Professional experience: Taught in high schools in Louann, Arkansas, 1928-1930, Idabel, Oklahoma, 1930-31, Pawhuska, Oklahoma 1931-1942 (high school principal, 1938-1942), U. S. Army 1942-1945. Associate professor of mathematics, Northeastern State College, 1946-1959.

Professional Organizations: Kappa Mu Epsilon; Mathematics Association of America; National Council of Teachers of Mathematics; National Education Association; Oklahoma Education Association.