THE EFFECTS OF CONCRETE, VIRTUAL, AND MULTIMODAL TANGRAM MANIPULATIVES ON SECOND GRADE ELEMENTARY STUDENTS' MATHEMATICS ACHIEVEMENT AND DEVELOPMENT OF SPATIAL SENSE: A CONVERGENT PARALLEL MIXED METHODS STUDY

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Graduate College of
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In partial fulfillment of
the requirements for the Degree of
DOCTOR OF PHILOSOPHY
May 2016

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ACKNOWLEDGMENTS

I would like to thank the many people who have helped me along this long, and at times, very difficult journey. First, thank you to Dr. Adrienne Sanogo, my dissertation chair, for helping to guide me through the dissertation process, for responding to my many texts, calls, and emails at all hours of the day and night. Her support through this process has been invaluable. I would also like to thank Dr. Juliana Utley, my graduate advisor, for always encouraging me to do my best, and pushing me to become a better student, teacher, and writer. To my committee members, Dr. Sheri Vasinda and Dr. Susan Stansberry, thank you for your time and feedback throughout the writing process.

I would like to thank the faculty of NSU, for believing in me before the Ph.D. was complete. I appreciate all the encouragement and support that you have all shown me through the dissertation process. Specifically, thank you Dr. Stan Sanders for always dropping by my office to check on my dissertation progress and for enabling me to complete my research.

Thank you to my family! I cannot thank you enough for the hours, days, weekends, months, and years, that you have all sacrificed your time, efforts, and energy so that I could attend classes and work on my dissertation. Thank you to my mother-in-law, for helping to watch the little ones and for dropping in with dinner. Thank you to my father-in-law, for never turning down a chance to discuss statistics over dinner and always ask how my research is coming along. To my niece Meghan, and nephew Matthew, thank you for babysitting and helping clean house when I was locked away in my room to study and write. Thank you to my sister, Lisa, for letting me vent over rough patches in life, work, and school.

To my father, he has always been my rock, taking every opportunity to let me know that he is proud of the woman I have become and for telling me from birth that I could do anything I set my mind to. To my mother, I do not think I could ever express in words how thankful I am of the many weekends she swooped in to entertain kids while I typed away. For the loads upon loads of laundry and dishes that she did while watching countless Disney movies on repeat with my children, I will be forever in her debt.

I would like to thank my children, Connor, Ali, Bodie, and Lily-bug. You have all grown up so much throughout this process. Thank you for all of your sweet hugs and kisses. You have all had to sacrifice so much for me. I am truly sorry for all the times I had to tell you, "Not now, I'm busy." Connor, thank you for stepping-up when I asked and helping with the little ones. Thank you for tolerating and loving me even when I was completely stressed out.

To my husband, Robert, thank you for loving me. Thank you for not ever complaining about a messy house or eating leftovers for dinner. Thank you for checking on me at two o'clock in the morning when I had not come upstairs to bed yet because I was still pounding away on my laptop. For squeezing me tight as I cried and felt like giving up, thank you for not letting me. I appreciate every meal you cooked, every bath you gave, every story you read and every other little thing you did to help give me more time to work. Thank you for letting me follow my dream! You and the kids are what keep me going.

Finally, I would like to thank my grandmother, Gigi, the most amazingly beautiful woman inside and out that I have ever met. I wish you were here to squeeze my neck, and tell me you love me a bushel and peck. You were and always will be my biggest fan. Thank you for inspiring me to shoot for the moon, and land amongst the stars. It took kidneys, Gigi, but I finally did it! But then again, you probably already knew that.

I finally made it…all because I stood on the shoulders of giants!

Acknowledgements reflect the views of the author and are not endorsed by committee members or Oklahoma State University.

Name: TRACY THOMPSON

Date of Degree: MAY, 2016

Title of Study: THE EFFECTS OF CONCRETE, VIRTUAL, AND MULTIMODAL TANGRAM MANIPULATIVES ON SECOND GRADE ELEMENTARY STUDENTS' MATHEMATICS ACHIEVEMENT AND DEVELOPMENT OF SPATIAL SENSE: A CONVERGENT PARALLEL MIXED METHODS STUDY

Major Field: EDUCATION

Abstract: This convergent parallel mixed methods study sought to determine what effects, if any, the use of multimodal tangrams have on second grade students' mathematical achievement and the development of spatial sense, as compared to concrete and virtual manipulatives. This research examined differences in math achievement and spatial development when using the three different types of manipulatives: concrete, virtual, and multimodal. While the quantitative findings showed no significant difference in achievement and spatial sense, the qualitative data suggested that students who had experiences with virtual and multimodal tangram manipulatives were more likely to experiment with the shapes on the screen. They saw the objects as being less permanent. Students in the physical tangram group were less likely to move pieces once they were set in place. Students in the virtual and multimodal groups also showed more persistence when solving difficult puzzles. However, this study found that experiences with tangram manipulatives, regardless of type, provided opportunities for students to develop geometric knowledge including composition and decomposition of shapes, shape congruence, and an understanding of transformations.

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CHAPTER I

INTRODUCTION

Educational researchers are constantly striving to keep up with new emerging technologies used in connection with historically tried and tested classroom practices. Although the concept of using manipulatives in the elementary classroom is not an innovative idea, emerging technology that could potentially change the way instructors teach with manipulatives has become available. Some mathematics manipulatives are known to date as far back as the early ancient Romans, Mayans, and Aztecs, who created counting boards, abacuses, and other counting tools (Boggan, Harper, & Whitmire, 2010). According to Boyer (1991), the ancient Chinese were known to carry around small bags of counting rods made of bamboo, ivory, or iron, used as a calculating device as early as 300 B.C. Since these early forms of manipulatives, there have been many additions to the ever growing population of manipulative choices available to today's classroom teachers, parents, and students.

Researchers agree that the use of manipulatives in mathematics increases mathematics achievement and plays a large part in student learning, understanding, and conceptualization of simple to complex concepts (Boggan, Harper, & Whitmire, 2010; Bouck & Flanagan, 2010; Burns & Hamm, 2011; Butler, 1994; Cooper, 2012; Kamii, Lewis, & Kirkland, 2001; Moyer, Bolyard, & Spikell, 2002; Puchner, Taylor, O'Donnell, & Fick, 2008; Siew, Chong, & Abdullah, 2013; Steen, Brooks, & Lyon, 2006). Educational theorists concur that manipulatives, when used correctly, can effectively aid students in progressing through different levels of cognitive development (Cooper, 2012; Kamii et al., 2001). Cooper (2012) specifies Piaget and Bruner as supporting the use of manipulatives to move between three stages of mathematical thinking – concrete, representational, and abstract (CRA). Cooper (2012) stated that, “Manipulatives have a central role in the CRA framework” (p105). Moreover, manipulatives play a large part in the constructivist view of learning. Siew et al.(2013) purported that the use of physical manipulatives was necessary for students to transition from one level of thinking to another within the Van Hiele’s constructivist view of knowledge.

With new emerging technologies, options for use of manipulatives in the classroom have become more varied. Although the purpose of manipulatives remains the same, teachers now have a wide variety of choices to use when selecting manipulatives for use within their classroom. However, which form of manipulative is most beneficial for the development of student cognition, specifically, for math achievement and spatial sense?

Foundation of the Problem

Research on new technological tools has not been able to keep up with the push for technology integration in the mathematics classroom. Many times teachers are using new software programs, apps, and other online resources without having any evidence that the new methods are in fact effective classroom tools. Additionally, there is little research to determine if there is a disadvantage to using technologically advanced virtual manipulatives in lieu of concrete manipulatives. Researchers must consider the differences in the way students interact with the different forms of manipulatives. Concrete manipulatives provide a tactile and kinesthetic interaction that is different than when using technologically advanced virtual manipulatives that rely on touch screen technology, controlled through slides or taps on a screen (Moyer et al., 2002).

Current students are generally accustomed to using technology at home and outside of the classroom on a near daily basis even as elementary students (Harris, Straker, & Pollock, 2013). According to Project Tomorrow (2014), 41% of K - 2nd grade students have access to non-school provided laptops, 41% have access to tablets, 22% have access to smart phones, and 18% have access to a digital reader. Previous non-digital learning styles of paper and pencil educational methods may seem archaic and may not hold students' attention nor motivate students to challenge themselves (Kiger, Herro, & Prunty, 2012).

Although there seems to be many effective uses for technology in the classroom, it may not necessarily always be the right choice for all students since items such as interactive white boards (IWB), smart phones, or tablets may restrict traditional face-to-face student collaboration (Kreijns, Kirschner, & Jochems, 2003; Palfrey & Gasser, 2008; Small & Vorgan, 2008;). Palfry & Gasser (2010) state that the changes in the way students collaborate within educational settings, learn, and interact socially all within a digital environment will not all be positive changes. Mobile digital smart devices may cause students to ignore other surroundings, suffocating the opportunity for learning face to face social cues (Small & Vorgan, 2008). Without these face-to-face collaborations, students may not have a chance to expand their zone of proximal development (ZPD) in terms of face to face social interaction with a more knowledgeable other (MKO), such as their ability to interpret facial expressions, gestures, and the tone or inflection in spoken words. Kreijns et al. (2003) states that rich social interactions require a MKO that can not only facilitate a digital classroom, but to also foster a community of learners within a digital environment where students interact in ways beyond the surface discussion. These socio-emotional interactions are pertinent to produce a successful interaction between students that is both collaborative and cooperative (Kreijns et al., 2003).

Multimodal manipulatives, manipulatives that combine both concrete and virtual manipulatives are new, emerging technology that has not yet made its way into mainstream educational settings. Products such as HP's Sprout (http://www.pcmag.com, 2014) and OSMO's (www.playosmo.com) gaming system have the potential to change the way people of all ages interact with the virtual and real-world. To date, there is little prior research other than anecdotal accounts of gamer enjoyment when using these products. Increased availability of emerging technology allows students to have exposure to these products outside of the classroom at some point during their personal and/or professional lives. Without educators employing the use of emerging technology in the classroom, will students be properly equipped when entering the professional workforce?

Statement of the Problem

Research on multimodal manipulatives such as OSMO gaming systems is still in its infancy. If the new technology is to be used in a classroom environment as anything more than an entertainment device, educators hold a responsibility to ensure the gaming system is at least a form of "edutainment" – for both educational and entertainment purposes, and not merely another gaming app. When using new multimodal manipulatives, educators need to consider if there is a prescribed benefit to the students within the classroom environment and what technological affordances OSMO may offer.

Purpose Statement

The purpose of this research study is to determine what effects, if any, the uses of concrete, virtual, and multimodal tangrams have on students' development of spatial sense and mathematical achievement. This research also examines any differences in spatial ability and math achievement when using the three different types of manipulatives: concrete, virtual, and multimodal, as categorized by gender and ability. Additionally, the researcher seeks to identify trends within student and teacher interviews and task-based interviews that seek to explain any differences between spatial development and math achievement when using any of the three types of tangram manipulatives.

Research Questions

The study addresses the following research questions:

1. What influences, if any, do the different tangram manipulatives (concrete, virtual, and multimodal) have on students' mathematics achievement?
2. What influences, if any, do the different tangram manipulatives (concrete, virtual, and multimodal) have on students' spatial sense?
3. What are second grade students' and their teacher's perceptions of the different types of manipulatives (concrete, virtual, and multimodal)?

Results of this study are fundamental within the body of knowledge on multimodal manipulative use. More specifically, results of this study are the first of its kind on the OSMO game system. The findings of this study may aid teachers when selecting effective manipulatives focusing on student mathematical gains and the development of spatial sense. Additionally, any feedback on this gaming system may lead to changes in the OSMO software apps or gaming pieces to better benefit users of this gaming system.

Assumptions

1. It was assumed that the participants in the study did not have any experience with a multimodal manipulative such as the OSMO prior to the study.
2. It was assumed that the participants responded to interview questions honestly and thoughtfully.
3. It was assumed that the observations of students' task-based interview sessions were accurate depictions of their current thought processes while manipulating tangram pieces.
4. It was assumed that the teacher participants involved in the study did not have any negative preconceived notions about concrete, virtual, or multimodal manipulatives.

Limitations

1. Due to the small sample size and single, Midwestern sample location of the elementary school that was used in the study, the study may not be generalizable to all second grade students.
2. Most recent reported demographics for the school stated enrollment was comprised of 61% Caucasian & other, 4% African-American, 1% Asian, 6% Hispanic, and 28% Native American students (Northeastern State University, 2015). The study may not be generalizable to other elementary schools with varying diversity ratings.
3. The interventions took place, one day a week for four weeks. Due to the limited number of tangram activity days, the study cannot account for effects of long-term exposure to transformation tasks.

Definition of Terms

Apps – short for “applets.”

Applets – Software applications generally used on mobile technology.

Concrete manipulatives – Puchner et al. (2008) defines concrete manipulative as hand-held tools used to create an external representation of a mathematical idea. Concrete manipulatives include items such as base ten blocks, Unifix cubes, and tangrams.

iPad – An electronic touch-screen tablet with an Apple operating system that utilizes smart technology to download, store, and play apps, and can be connected to the internet through Wi-Fi or data usage plans.

Manipulatives – physical objects that are used as teaching tools to engage students in the hands-on learning of mathematics” (Smith, 2009).

Mathematics achievement – Knowledge and mastery of a range of mathematical skills to include NCTM’s (2000a) Geometry Content Standards and Process Standards, and local state curriculum standards.

Mobile learning technologies – Mobile learning technologies include smart devices, such as smart phones, iPods, tablets, iPads, and laptops.

Multimodal manipulatives – Manipulatives that utilize both concrete and virtual applications of manipulatives simultaneously.

OSMO - a gaming system for use with an Apple iPad that includes a camera mirror to reflect images, a plastic iPad stand, a concrete manipulative wooden tangram set, a set of alphabet tiles in red lettering, a set of alphabet tiles in blue lettering, and three software apps; Tangrams, Words, and Newton, developed by Tangible Play, Inc.

Smart technology – hardware products that have artificial intelligence enhanced capabilities and/or the ability to access the internet. Some of these devices are designed to sense your actions or learn your patterns and alter their behavior accordingly (Khosrow-Pour, 2009).

Spatial awareness - also spatial sense.

Spatial reasoning – Battista (2007) defines spatial reasoning as, “The ability to ‘see,’ inspect, and reflect on spatial objects, images, relationships, and transformations” (p. 843). Spatial reasoning is when an individual considers one’s spatial sense (the ‘input’ of the spatial environment) and pairs it with one’s cognitive ability to form a formal geometric analysis.

Spatial sense – Copley (2010) states, “Children’*s spatial sense* is their awareness of themselves in relation to the people and objects around them” (p. 105). Wheatley (1990) states that spatial sense, “…has been known by a variety of other labels from spatial visualization, spatial reasoning, spatial perception, and visual imagery to mental rotations” (p. 10). However, Wheatley (1990) recommends that spatial sense be viewed from the lens of imagery.

Tangram – Lee, Lee, and Collins (2010) define a tangram as a seven piece geometric shape comprised of two small triangles, one medium triangle, two large triangles, one square, and one parallelogram that together form a perfect square. The tangram pieces, or tans, can also be arranged in a variety of figures such as birds, animals, numbers, or other shapes (Tian, 2012).

Technological affordances – interactions between the user and the technological tool (Wijekumar, Meyer, Wagoner, & Ferguson, 2006). Norman (1998) states, “Affordance is not a property, it is a relationship that holds between the object and the organism that is acting on the object. The same object might have different affordances for different individuals” (p.

123). Gagne, Wagner, Golas, and Keller (2004), define technology affordances are "the properties or functions of technology that extend our learning and perceptual capabilities" (p. 208).

Virtual dynamic manipulatives - Virtual dynamic manipulatives are defined as visual representations of concrete manipulatives that *can* be manipulated similar to concrete manipulatives (Moyer et al., 2002). Virtual dynamic manipulatives allow students to slide, flip, turn and manipulate the object with a mouse-click or touch screen much like a student would manipulate a concrete object. The dynamic description implies that students can manipulate the on-screen object as if it were a three-dimensional object (Moyer et al., 2002).

Virtual manipulatives – Moyer et al. (2002) defines virtual manipulatives as concrete manipulatives that are available on the World Wide Web. Bouck & Flanagan (2010) define virtual manipulatives as computer-based simulations of physical (concrete) manipulatives that are accessed via the Internet or computer software (p. 187).

Virtual static manipulatives - Virtual static manipulatives are defined as digital visual representations, much like pictures in a book (Moyer et al., 2002). These manipulatives are often just digital pictorial examples of what a concrete manipulative would look like, such as a picture of base ten blocks. Static virtual manipulatives *cannot* be used in the same way as a concrete manipulative or dynamic virtual manipulative since a student cannot physically slide, flip, turn, or manipulate the object (Moyer et al., 2002).

Organization of the Study

This dissertation is presented in a five chapter organizational format. Chapter I provides an introduction to the study, foundation of the problem, description of the problem, the purpose of the study, the research questions addressed in the study, assumptions and limitations, and definitions of terms that are used throughout the study. Chapter II includes a review of the literature relevant to the effects of the use of concrete, virtual, and multimodal manipulatives on second grade elementary students' mathematics achievement scores and their development of spatial sense. The methodology of the study is discussed in chapter III. Chapter IV presents the analysis of the data. Chapter V discusses the results of the study as well as a conclusion, implications, and opportunities for further research.

CHAPTER II

REVIEW OF LITERATURE

The purpose of this chapter is to depict research relevant to the effects of the use of concrete, virtual, and multimodal manipulatives on elementary students' math achievement and development of spatial sense. The research questions guiding this review were:

1. What influences, if any, did the different tangram manipulatives (concrete, virtual, and multimodal) have on students' mathematics achievement?
2. What influences, if any, did the different tangram manipulatives (concrete, virtual, and multimodal) have on students' spatial sense?
3. What were second grade students' and their teachers' perceptions of the different types of manipulatives (concrete, virtual, and multimodal)?

Research related to the current study is reported. Each section is addressed, with a summary of the relevant findings provided at the end of this chapter. Research areas that were examined include the following topics:

1. Constructivist thinking in mathematics education
2. Spatial sense
3. Integrating technology in the classroom
4. Concrete and virtual manipulatives
5. Tangram manipulatives

6. Mobile learning technology

7. Multimodal tangram manipulatives

Constructivist Thinking in Mathematics Education

According to Battista (2001), "All current major scientific theories describing mathematics learning agree that mathematical ideas must be personally constructed by students as they intentionally try to make sense of situations, including of course, communications from others" (p. 107). Educational theorists Jean Piaget and Lev Vygotsky are considered to have laid the groundwork for present day constructivist learning theory (Lourenco, 2012). This *constructivist* view is outlined by abstraction, perturbations, and reflections on the part of each student (Battista, 2001). Battista (2001) states:

> To be consistent with the constructivist view of learning, mathematics instruction not only must create a classroom culture of inquiry that encourages and supports students' personal construction of meaning, but it must also base the design of instructional tasks on appropriate scientific research (p. 109).

Consistent with the constructivist view of learning, Battista (2001) stated, "In geometry, the best research-based description of the development of student reasoning is known as the van Hiele theory" (p. 109). Van Hiele's (1986) model of geometric thinking describes students' processes as they move through knowledge construction in perceiving geometric shapes as whole units to then understanding the detailed attributes of each shape. Unlike Piaget's models of development, students do not have to reach a certain age to progress through the development levels that van Hiele proposes (Breyfogle & Lynch, 2010). Van Hiele (1986) outlines five levels of geometric understanding in his geometric model of understanding, beginning with the most basic level 0 - visualization, and continues to the highest level of understanding level 4 – rigor (Figure 1) as taken from Breyfogle & Lynch (2010, p. 234). Breyfogle & Lynch (2010) assert that children can only move through these levels when given the opportunity to experience mathematic construction which develops the necessary mathematic skills.

Level	Name	Description	Example	Teacher Activity
0	Visualization	See geometric shapes as a whole; do not focus on their particular attributes.	A student would identify a square but would be unable to articulate that it has four congruent sides with right angles.	Reinforce this level by encouraging students to group shapes according to their similarities.
1	Analysis	Recognize that each shape has different properties; identify the shape by that property.	A student is able to identify that a parallelogram has two pairs of parallel sides, and that if a quadrilateral has two pairs of parallel sides it is identified as a parallelogram.	Play the game "guess my rule," in which shapes that "fit" the rule are placed inside the circle and those that do not are outside the circle (see Russell and Economopoulos 2008).
2	Informal deduction	See the interrelationships between figures.	Given the definition of a rectangle as a quadrilateral with right angles, a student could identify a square as a rectangle.	Create hierarchies (i.e., organizational charts of the relationships) or Venn diagrams of quadrilaterals to show how the attributes of one shape imply or are related to the attributes of others.
3	Formal deduction	Construct proofs rather than just memorize them; see the possibility of developing a proof in more than one way.	Given three properties about a quadrilateral, a student could logically deduce which statement implies which about the quadrilateral (see **fig. 1**).	Provide situations in which students could use a variety of different angles depending on what was given (e.g., alternate interior or corresponding angles being congruent, or same-side interior angles being supplementary).
4	Rigor	Learn that geometry needs to be understood in the abstract; see the "construction" of geometric systems.	Students should understand that other geometries exist and that what is important is the structure of axioms, postulates, and theorems.	Study non-Euclidean geometries such as Taxi Cab geometry (Krause 1987).

Figure 1. The van Hiele model of geometric understanding
Note. From "Van Hiele Revisited," by M. Breyfogle and C. Lynch, 2010, Mathematics Teaching in the Middle School, 15 p.234. Copyright 2010 by National Council of Teachers of Mathematics. Reprinted with permission.

Van Hiele (1986) states that elementary students first learn to visually recognize shapes by their global appearance. When students are performing at the visual level of geometric understanding they are only able to recognize triangles, squares, parallelograms and so on by their shape, not the geometric properties of each shape (Siew et al., 2013). Students move to the second level of van Hiele's level of geometric thinking, analysis, when they begin to analyze the properties of figures and start to use the proper technical mathematical terminologies for describing the shapes (Siew et al., 2013). Siew et al. (2013)

also states that the third level of geometric thinking is reached by elementary students only after they can identify relationships between classes of shapes and discover different classes of shapes by simple logical deduction. According to Spear (1993), elementary students are capable of reaching the third level of geometric thinking, and should at least be capable of reaching the second level, analysis during elementary schooling. Fuys, Geddes, and Tishler (1988) affirm that van Hiele's theory stresses the use of hands-on manipulatives within the learning of geometry to effectively transition from one level of thinking to the next.

Van Hiele (1986) proposed a sequence of learning phases to guide teachers on how to instruct students through the different levels of understanding (Figure 2) as taken from Breyfogle and Lynch, 2010 (p. 235). Van Hiele (1986) stated, "The transition from one level to the following is not a natural process; it takes place under the influence of a teaching-learning program" (p. 50). Siew et al., (2013) reiterated, "Teachers hold the key to this transition from one level to the next" (p. 103). Most importantly, Siew et al., (2013) proposed that combining the use of hand-held manipulatives with van Hiele's five phases of learning promoted learning, geometric thinking, and a progression through at least the first two levels of geometric understanding.

Phase	Description
Information/inquiry	Teacher: Assess students' prior knowledge through discussion and allow questions to prompt topics to be explored
Directed orientation	Teacher and students: Explore sets of carefully sequenced activities
Explication	Students: Share explicit views and understandings about the activities
Free orientation	Teacher: Challenge students to solve problems related to the geometric concepts and make connections among them
Integration	Students: Reflect on observations and how they fit into the overall structure of the concepts

Source: Adapted from Geddes (1992)

Figure 2. The van Hiele sequence of phases of learning
Note. From "Van Hiele Revisited," by M. Breyfogle and C. Lynch, 2010, Mathematics Teaching in the Middle School, 15 p.235. Copyright 2010 by National Council of Teachers of Mathematics. Reprinted with permission.

Spatial Sense

Spatial Sense is a skill that both adults and children use in their everyday lives, whether it is simply interacting with their environment related to a job or play. Spatial sense is noted as an integral part of NCTM's content standards of helping students from pre-kindergarten to eighth grade to develop and learn mathematics (NCTM, 2000a). NCTM's (2000a) Geometry Content Standards state:

> Instructional programs from prekindergarten through grade 12 should enable all students to –
>
> - Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
>   - Pre-K - 2 Expectations: In pre-K through grade 2 all students should -

        - recognize, name, build, draw, compare, and sort two- and three-dimensional shapes;
        - describe attributes and parts of two- and three-dimensional shapes;
        - investigate and predict the results of putting together and taking apart two- and three-dimensional shapes.
- Specify location and describe spatial relationships using coordinate geometry and other representational systems
    - Pre-K - 2 Expectations: In pre-K through grade 2 all students should -
        - describe, name, and interpret relative positions in space and apply ideas about relative position;
        - describe, name, and interpret direction and distance in navigating space and apply ideas about direction and distance;
        - find and name locations with simple relationships such as “near to” and in coordinate systems such as maps.
- Apply transformations and use symmetry to analyze mathematical situations
    - Pre-K - 2 Expectations: In pre-K through grade 2 all students should -
        - recognize and apply slides, flips, and turns;
        - recognize and create shapes that have symmetry.
- Use visualization, spatial reasoning, and geometric modeling to solve problems
    - Pre-K - 2 Expectations: In pre-K through grade 2 all students should -

- create mental images of geometric shapes using spatial memory and spatial visualization;
- recognize and represent shapes from different perspectives;
- relate ideas in geometry to ideas in number and measurement;
- recognize geometric shapes and structures in the environment and specify their location. (NCTM, 2000a, p. 97)

Spatial sense is prevalent across the geometry strand. In her chapter on geometry and spatial sense in the early grades, Copley (2010) defines spatial sense as, “awareness of children themselves in reference to the people and objects around them” (p. 105) and points out that spatial sense develops over time and through many experiences. Researchers have also described spatial sense as “an intuition about shapes and the relationships between them” as well as “the ability to mentally visualize objects and spatial relationship – to turn things around in one’s mind” (Van de Walle, Karp, and Bay-Williams, 2015, p. 489) and more specifically as “[t]he ability to ‘see,’ inspect, and reflect on spatial objects, images, relationships, and transformations” (Battista, 2007, p. 843). Spatial reasoning is when an individual considers one’s spatial sense (the ‘input’ of the spatial environment) and pairs it with one’s cognitive ability to form a formal geometric analysis. Battista’s (2007) definition of spatial reasoning differs slightly from spatial sense in that spatial reasoning allows an individual to consider one’s spatial sense (the “input” of the spatial environment) and pair it with one’s cognitive ability to form a formal geometric analysis. Wheatley (1990) states that spatial sense, “…has been known by a variety of other labels from spatial visualization, spatial reasoning, spatial perception, and visual imagery to mental rotations” (p. 10). However, Wheatley (1990) recommends that spatial sense be viewed from the lens of imagery. Wheatley (2007) purports, “All meaningful mathematics learning is imaged-based. While there may be certain forms of mathematical reasoning that seem not to use imagery, most mathematical activity has a spatial component” (p. 1).

Wheatley (1990) identifies three components of imagery: construction of images, re-presentation, and transformation. He describes the construction of images as the mental image created within one’s mind through, “viewing objects, reading a passage, or just reflecting,” (Wheatley, 1990, p. 10). Wheatley further states that the mental images created, “may be concrete and limiting or dynamic and abstract. They are also unique” (p. 10). Re-presentation refers to the recall of formerly created mental constructions of images. Wheatley (1990) illustrates re-presentation as, “If you are asked to determine the number of windows in your house, you will likely re-present several images of your house as you mentally walk around it and ‘look’ at the windows” (p. 10). Transforming as a component of imagery describes the task of mentally transforming (rotating, sliding, or flipping an object) to compare different shapes.

Stanic and Owens (1990) depict the ability to identify spatial relationships as spatial ability. They drew from Bisphop (1983) and Halpern’s (1986) work to further explain that it is not a single spatial ability, but instead spatial abilities; both visualization and orientation. Stanic and Owens (1990) identifies, “[a] visualization factor, which includes the ability to imagine how pictorially presented objects will appear when they are rotated, twisted, or inverted” (p. 48). They define the second factor, orientation as, “[t]he ability to detect arrangements of elements within a pattern and the ability to maintain accurate perceptions in the face of changing orientations” (p. 10).

Many researchers agree that the development of spatial sense plays a crucial part in a students' mathematics success (Battista, 2001; Bohning & Althouse, 1997; Butler, 1994; Lee, et al. 2010; Reynolds & Wheatley, 1997;;). Most researchers also agree that another way children gain spatial sense is through the use of manipulatives (; Battista, 2001; Boggan, et al., 2010; Bohning & Althouse, 1997; Burns & Hamm, 2011;; Butler, 1994;;;; Lee et al. 2010; NCTM, 2000a; Thatcher, 2001). Manipulatives offer children the opportunity to construct their own knowledge through sorting, classifying, weighing, stacking and exploring (Boggan et al., 2010). Children who manipulate concrete materials are said to become more proficient in knowing positions, locations, and structures (Lee et al., 2010).

NCTM supports the development of children's spatial sense as part of elementary education curriculum. More specifically, NCTM's *Principles and Standards for School Mathematics* (2000a) for grades PK-2, state the professional standard:

> 4.4: Developing Geometry Understandings and Spatial Sense through Puzzle like Problems with Tangrams
>
> Describing figures and visualizing what they look like when they are transformed through rotation or flips or are put together or taken apart in different ways are important aspects of geometry in the lower grades (para. 1).

Spatial sense is also known to be an indicator of mathematical achievement as early as preschool and kindergarten. Guay and McDaniel's (1977) research on the relationship between mathematics achievement and spatial abilities among elementary school children concluded that high mathematical achievers have greater spatial ability than low mathematical achievers. Guay and McDaniel's (1977) research also purported that males have a greater high-level spatial ability than females, although male and females had similar low-level spatial ability.

Researchers Ehrlich, Levine, and Goldin-Meadow (2006) contended that spatial ability is not a fixed skill and that through practice or interventions, the development or refinement of spatial ability can occur. Lin, Shao, Wong, Lin and Niramitranon (2011) also agree that spatial reasoning is not a fixed ability and can be developed as demonstrated through significant results of pre and post-test scores after integrating a specific technological resource as a treatment. Results of Lin, et al.'s (2011) research also lends itself to the support of further integration of technology in education due to the nature of the virtual tools used within the study.

Integrating Technology in the Classroom

According to the National Center for Education Statistics (NCES), the most recent report stated that in 2009, roughly 97% of classrooms in the United States had at least one computer in the classroom (US Department of Education, n.d.). NCES also reported that approximately 23% of classrooms in 2009 were equipped with an interactive white board (IWB) (US Department of Education, n.d.). Increasingly available access to technology and internet services has created a demand for ways to efficiently and effectively use technology in the classroom to assist with student learning. The International Society for Technology in Education developed the National Education Technology Standards to help guide students, teachers and administrators in the proper implementation of technology in the classroom (ISTE, 2010). The NETS outline the following five standards and performance indicators for teachers to meet:

1. Facilitate and inspire student learning and creativity. Teachers use their expertise to guide students to challenges, creativity, and innovation in both face-to-face and virtual environments.
2. Design and develop digital age learning experiences and assessments. Teachers create digital tools, activities, and assessments aligned with technology content standards.
3. Model digital age work and learning. Teachers demonstrate their ability to use technology professionally through collaboration, correspondence, and continued research.
4. Promote and model digital citizenship and responsibility. Teachers instruct and model digital safety, encourage netiquette, and reinforce legal and ethical digital behavior.
5. Engage in professional growth and leadership. Teachers model and encourage lifelong learning and demonstrate leadership and effective use of technology (ISTE, 2008, http://www.iste.org/standards/ISTE-standards/standards-for-teachers).

Other supporters for technology in the classroom include NCTM's Technology-Supported Mathematics Learning Environments (2005). NCTM (2005) asserts, "Technology is an essential tool for teaching and learning mathematics effectively; it extends the mathematics that can be taught and enhances students' learning" (p. 1). In 2014, NCTM added to their stance on technology integration by including technology as an integral part of teaching and learning through the *Principles to Actions*. NCTM's *Principles to Actions*, recommended actions for educational leaders, policymakers, school and district administrators, teachers, mathematical coaches, and math specialists to guide the creation and instruction of high quality mathematics education (NCTM, 2014). The Principles to Actions by Huinker, Leinwand, and Brahier (2014) addresses six principles - teaching and learning, access and equity, curriculum, tools and technology, assessment, and professionalism. The most recent update of NCTM principles states that, "an excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking" (Huinker et. al, 2014, p. 534). The belief that technology can and should aid the learning of mathematics has existed since the early debut of computers.

In the early 1970's, prior to the abundance of technological tools available to today's students, Seymour Papert had already begun research on the effects of the integration of technology into mathematics classrooms. Papert (1972) stressed the differences between a student who does math and a mathematician. Papert (1972) stated, "Being a mathematician, again like being a poet, or a composer or an engineer, means *doing* rather than knowing or understanding" (p. 249). Papert (1972) viewed technology as a way for students to *do* math and to concurrently gain an understanding of the principles while allowing them to explore math creatively so that it was personal to them. This allows for students to become vested in their own learning. Even in the early 1970's, Papert viewed these opportunities of technology in mathematics as a way to engage and motivate audiences that may have previously lost interest, "students who fail to see any point or pleasure in bookish mathematics and who, under prevailing school conditions, simply drop out by labeling themselves 'not mathematically minded'" (p. 251). Papert's (1972) *doing rather than knowing* approach would prove to be of great importance as the 'project-based approach' gave way to current discovery learning methods. Papert's (1972) research studied students using Turtle Geometry – a computer program that allows students to program a "turtle" on-screen to move in directions relative to itself while leaving a line of its movements to create shapes similar to the current Geometers Sketchpad software capabilities. Papert (1972) concluded that his turtle geometry taught children to be mathematicians not only through their discovery when venturing away from set codes, but also through debugging, looking for their mistakes in codes, and learning what they needed to do to troubleshoot when a picture was not completed as originally intended. Papert (1972) felt that it was through these accidental discoveries that students were able to learn more.

Battista's (2001) beliefs on mathematics and the integration of technology echo a similar stance as Seymour Papert's (1972) ideas. Battista (2001) outlines three types of technology used in mathematics education to enhance student learning. These include general technological tools, technological tools for doing/performing mathematics, and technological tools for teaching mathematics. General technological tools include resources such as web-based communication. Technological tools for doing mathematics more easily and powerfully include items such as calculators and graphing programs (Battista, 2001). iPads or other mobile or tablet technology can be used in a classroom environment as a technological tool for doing mathematics. Technological tools for teaching mathematics include technology that enhances student mathematical learning, to include educational software packages and virtual dynamic manipulatives. Educational applets or "apps" for mobile technology geared towards instruction are considered a technological tool for teaching mathematics. There are many different genres of apps including entertainment, educational, and reference to name a few.

In today's society, technology is often used for entertainment purposes such as social networking and computer gaming. According to Hoffman and Ritchie (1997) and Reiser (2004) the use of technology can be used to increase the level of student motivation and engagement. When used for educational purposes, instructional games can create a new learning culture that better corresponds with students' habits and interests (Prensky, 2001). Prensky (2001) also postulated that instructional technology based games were effective as teaching tools when learning difficult and complex procedures because they (a) used action instead of explanation, (b) created personal motivation and satisfaction, (c) accommodated multiple learning styles and skills, (d) reinforced mastery skills, and (e) provided interaction. Kebritchi, Hirumi, and Bai's (2010) research on the effects of modern mathematics computer games on mathematics achievement reported that the use of technology in a mathematics classroom improved participants' mathematical understandings and skills. Kebritchi et al., (2010) also found that the technology integration of games in mathematics was an effective tool due to (a) the games were experiential in nature, (b) the games offered an alternative way of teaching and learning, (c) students were motivated to solve problems so that they could move on to another level, (d) students' mathematics phobias were lessened, and (e) students spent more time on task.

Most researchers agree that the integration of technology in the classroom with inquiry-based learning activities enhances the student learning process (Battista, 2001; Bouck & Flanagan, 2010; Delen & Belut, 2011; Wang, Kinzie, McGuire, & Pan, 2009;). Wang et al. (2009) defines inquiry-based learning as, "a problem solving process during which students answer research questions, construct their own knowledge, and develop their understanding with support of the teacher and peers" (p. 382). Wang and colleagues (2009) concluded that the integration of technology facilitated a higher potential for children's knowledge construction within an inquiry-based learning environment in a study on applying technology to inquiry-based learning in early childhood settings. In a pretest/posttest comparison of first graders' geometry test scores when using laptops and virtual geometric manipulatives, Steen, Brooks, and Lyon (2006) reported a significant gain in mathematical achievement. Steen et al. (2006) used daily journal entries by teacher participants to determine increased student motivation and positive student attitudes. The technology in Steen et al.'s (2006) research included more than the mere presence and use of a laptop, but also incorporated the treatment group's use of the increasingly popular virtual manipulatives to aid in the understanding of identifying, copying, and transforming shapes and other related activities while the control group utilized concrete manipulatives.

Concrete and Virtual Manipulatives

Manipulatives are a way that many educators have found to help children experience mathematical constructions and to aid children to bridge the gap between informal mathematics and formal mathematics as they navigate the various levels of development (Boggan, et al., 2010). According to Boggan et al. (2010), German educator Friedrich Froebel created 'Froebel Gifts' to be introduced to the world's first kindergarten. These *Frobelgaben*, included geometric building blocks and pattern blocks for student use. Later, Italian educator Maria Montessori began with Froebel's idea and continued on to design several more manipulatives to help students learn mathematics (Boggan et al., 2010). Today, manipulatives are commonly defined as, "physical objects that are used as teaching tools to engage students in the hands-on learning of mathematics" (Smith, 2009). However, a distinction of the different types of manipulatives used is needed, specifically, a difference between concrete manipulatives and virtual manipulatives.

Concrete Manipulatives

Puchner et al.(2008) define concrete manipulative as hand-held tools used to create an external representation of a mathematical idea. Concrete manipulatives include items such as base ten blocks, Unifix cubes, and tangrams. Puchner et al., (2008) reported that hands-on teaching with manipulatives tends to be appealing to teachers. Burns and Hamm (2011) completed a study on the comparison of concrete versus virtual manipulatives in third and fourth grade mathematics classes learning fractions and symmetry. They suggested that long-term use of concrete manipulatives at the early elementary level creates higher levels of performance than students who do not use manipulatives. Additionally, they found the concrete manipulative group attained slightly higher point gains on pre/posttest results than the virtual manipulative group, although not statistically significant.

Advantages of concrete manipulatives. Researchers have agreed that the correct use of concrete manipulatives can increase students' mathematics achievement and decrease students' mathematics anxiety (Boggan et al., 2010; Lee, et al., 2010; NCTM, 2000a). Other advantages to concrete manipulatives include the ability for students to use concrete manipulatives in collaborative, hands-on, group settings. Collaborative, group settings allow mathematical communication among students through words such as top, below, under, right, flip, turn, and slide – which are important to clarifying students' understanding of mathematical concepts (NCTM, 2000a). Lee et al. (2010) also highlights the act of *copying* to explore shapes and spatial relationships with concrete manipulatives as being beneficial to their spatial sense growth. With hand-held pieces students can create pictures through tracing, cutting, and pasting of figures onto larger pieces of construction paper. Each picture can then be hung and displayed in the classroom or sent home to show the student's progress with tangrams. Lastly, when students use hand-held concrete manipulatives, students can work with more than the given number of pieces per manipulative set by combining sets for more complex findings.

Kamii et al.'s (2001) research on when manipulatives are useful focused on manipulatives *with which* children can learn mathematics and *from which* children can learn mathematics. Manipulatives in Kamii et al.'s (2001) research included tangrams, counters, card games, base-ten blocks, and Unifix cubes. They concluded that the benefits of manipulatives lie within the quality of thinking the manipulatives stimulate. When manipulatives are viewed as a tool that can aid student learning, their possibilities are endless in the hands of a creative teacher to help students construct the logicomathematical knowledge to reach constructive abstraction – to *think* (Kamii, et. al, 2001). Puchner et al. (2008) reported that teacher knowledge increased when using manipulatives to aid in mathematical learning; however, their study of teachers' use of manipulatives revealed that the success of concrete manipulatives to aid learning lies mainly with the teachers' presentation, use, and experience with correctly presenting manipulative use within a lesson. Puchner et al. (2008) stressed that manipulatives do not automatically lead to understanding, but if presented correctly, can be a useful tool for properly trained teachers.

Disadvantages of concrete manipulatives. Some disadvantages that may arise from the use of concrete manipulatives include the limited availability of classroom sets (Moyer et al., 2002). If schools face funding issues along with overcrowded classes, it may not be possible for students to experience individual work with concrete manipulative sets or even small group interaction (Puchner et al., 2008). Other issues that could cause problems with concrete sets include sets that have missing or broken pieces (Kamii et al., 2001). Students could become very frustrated and not understand why they are not able to replicate certain figures like other classmates when their set has missing or broken pieces (Kamii, et al., 2001).

Students who show an interest in continuing to work with concrete manipulatives outside of the classroom may want to check-out manipulative sets to take home (Puchner et al., 2008). Unfortunately, the number of manipulative sets available and cost of replacement pieces may prevent teachers from allowing student check-outs for further exploration past the classroom (Puchner et al., 2008). Lastly, with class curricula overflowing from mandated class times, the distribution or creation of concrete manipulative sets along with the necessary clean up time may be viewed by some teachers as outweighing the advantages of their use (Moyer et al., 2002).

Virtual manipulatives

Moyer et al. (2002) defines virtual manipulatives as concrete manipulatives that are available on the World Wide Web. Since Moyer et al.'s (2002) article however, the creation of smart phone applications, SMART board resources, and computer programs have expanded virtual manipulatives past the sole location of the internet. A more recent study by Bouck & Flanagan (2010) defines virtual manipulatives as computer-based simulations of physical (concrete) manipulatives that are accessed via the Internet or computer software (p. 187). Virtual manipulatives are generally referred to by the same name as their concrete manipulative counterparts; however, virtual manipulatives are presented in an interactive manner either on-line or through a computer software environment (Bouck & Flanagan, 2010).

Commonly used virtual manipulatives in the elementary classroom include base ten blocks, pattern blocks, fraction circles, fraction bars, and symmetry activities (Burns & Hamm, 2011; Boggan et al., 2010; Mildenhall, Swan, Northcote, & Marshall, 2008; Moyer et al., 2002;). Many instructors are using IWBs for whole-class, small groups, pairs and individual work with virtual manipulatives (Mildenhall et al., 2008). Bouck and Flanagan (2010) listed four ways to use virtual manipulatives in mathematics classes: 1) introducing or reviewing a mathematical idea, 2) developing understanding of mathematical concepts by visually representing those that are abstract, 3) scaffolding student learning, and 4) actively engaging students in learning.

Virtual manipulatives are beginning to be used to not only assist in teaching mathematics material, but also to assess understanding of mathematical concepts. Johnson et al. (2008) wrote that instructors have the capability to expose students' thinking and understanding while working with virtual manipulatives. Johnson, Campet, and Zuidema(2008) suggested asking facilitating questions such as:

- How did you reach that conclusion?
- Can you use the pieces on the screen to show what you are thinking?
- Tell me why you are…(shading that in, moving that part, selecting that portion, and so on).
- Why do you think that answer was incorrect?
- What if the problem started with ___ instead of with ____?

- Do you think that strategy will work with the next problem? Why, or why not? (p. 203)

According to Moyer et al. (2002) virtual manipulatives can be categorized into two different types of virtual manipulatives – virtual static manipulatives and virtual dynamic manipulatives.

Virtual static manipulatives are defined as visual representations, much like pictures in a book (Moyer et al., 2002). These manipulatives are often just digital pictorial examples of what a concrete manipulative would look like, such as a picture of base ten blocks. Static virtual manipulatives *cannot* be used in the same way as a concrete manipulative or dynamic virtual manipulative since a student cannot virtually or haptically, slide, flip, turn, or manipulate the object (Moyer et al., 2002). Due to this distinction, static manipulatives are not considered true manipulatives and do not provide the same opportunities for student development as their concrete and virtual dynamic counterparts.

Virtual dynamic manipulatives are defined as visual representations of concrete manipulatives that *can* be manipulated similar to concrete manipulatives (Moyer et al., 2002). Virtual dynamic manipulatives allow students to slide, flip, turn and manipulate the object with a mouse-click or touch screen much like a student would manipulate a concrete object. The dynamic description implies that students can manipulate the on-screen object as if it were a three-dimensional object (Moyer et al., 2002). Although there are some very good interactive virtual dynamic manipulative sites to use in the classroom, some considerations need to be made when selecting a site to use. Moyer et al. (2002) suggests evaluating sites based on ease of use, accessibility of the site, appearance of images, and the clarity of instructions to name a few.

Advantages to virtual dynamic manipulatives. Virtual dynamic manipulatives have many advantages. Obvious advantages of virtual dynamic manipulatives include that students and teachers do not need to worry about misplacing pieces to multiple-piece manipulative sets (Bouck & Flanagan, 2010). Items such as an in-class IWB as a central tool for working with virtual dynamic manipulatives helps to create a classroom environment of a community of learners (Mildenhall et al., 2008). Children using IWBs for use with virtual manipulatives showed a higher level of focus among the students (Mildenhall et al., 2008).

Virtual dynamic manipulatives may allow students, teachers, and parents to have free access on-line, twenty-four hours a day when internet access is available. Virtual dynamic manipulatives are thought to be more appealing than concrete manipulatives for older students, since they feel that computer versions of the manipulatives are more sophisticated (Moyer et al., 2002; Mildenhall et al., 2008; Burns & Hamm, 2011). The Speak Up National Research Project reported that 31% of students in grades 3-5 had access to a school provided tablet or laptop, with 75% of those students reporting access to a device that could be taken home (Project Tomorrow, 2013). Thirty-one percent of students in grades 6-8 reported access to school provided tablets or laptops, with 58% reporting the ability to take the device home (Project Tomorrow, 2013). Additionally, 33% of students in grades 9-12 reported access to school provided tablets or laptops, with 64% of those being available for home use (Project Tomorrow, 2013). Project Tomorrow (2014) reports that these numbers continue to increase. The increases in available technology, in turn increases student access to virtual manipulatives with accessibility both in- and out-side of school.

Researchers agree that students enjoy having immediate feedback on how their actions affect the problems, something that is not always possible with one teacher and twenty-plus students in the classroom working with concrete manipulatives (Bouck & Flanagan, 2010; Burns & Hamm, 2011; Johnson et al., 2012;). According to Bouck & Flanagan, (2010), teachers are more likely to differentiate student manipulative activities when several different levels of activities are already accessible on-line through the same site. Some on-line or software based virtual manipulatives allow teachers to enter their own problems or to tweak the original virtual manipulative to better fit their education objectives. Several studies have indicated that students not only enjoy using virtual manipulatives, but they are just as effective as their concrete counterparts (Lin et al., 2011; Steen et al., 2006; Wang et al., 2009). Steen et al. (2006) researched the impact of virtual manipulatives on first grade geometry instruction and learning. Steen et al.'s (2006) research concluded that the treatment group with virtual bean stick picture manipulatives as compared to physical manipulatives showed significant improvement on mathematics pre/posttest achievement scores for first and second graders. Steen et al. (2006) also stated that the use of the virtual manipulative allowed students to see a pictorial representation of abstract concepts. Steen et al. (2006) proposed that students did not necessarily need a physical concrete manipulative, that a picture of the concrete manipulative could serve as a concrete model, while the manipulation could demonstrate abstract ideas.

Disadvantages to virtual manipulatives. Disadvantages of virtual dynamic manipulatives include the possibility of students' imaginations and creativity being restricted through the use of pre-programmed results or a select number of puzzles/figures. Once an individual has mastered a virtual manipulative game or program, new, challenging games or programs must be sought elsewhere. Additionally, some virtual tangram programs may not allow the use of more than the typical seven-piece puzzle thus restricting students from creating more complex shapes. Students would not be able to display their work in class or take their constructions home for parents to see without printing a hard copy (Puchner et al., 2008).

The use of dynamic virtual manipulatives in the classroom of course requires the use and availability of technological devices and connectivity, as well as the education and capability of the teacher to operate the technology. Johnson et al., (2012) studied three preservice teachers and their journey to use technological assessments tools throughout their one-on-one clinical interviews with elementary-age students. Johnson et al., (2012) highlighted a major potential drawback for virtual dynamic manipulatives – the unwillingness or inability to use technology in the classroom. According to Johnson et al., (2012) preservice teachers in their study noticed that, "Their supervising teachers rarely, if ever, used technology when teaching math. They conjectured that the veteran teachers did not have enough time or felt uneasy about integrating technology into their lessons" (p. 205). Koehler and Mishra (2007) suggest an unwillingness or inability on a lack of knowledge in technology, pedagogy, and content knowledge (TPACK) – the framework of interaction among content knowledge, pedagogy, and technology. According to Koehler and Mishra, "The interaction of these bodies of knowledge, both theoretically and in practices, produces the types of flexible knowledge needed to successfully integrate technology use into teaching" (2007, p. 60). They also point out that many teachers in the education field who struggle with technology integration obtained their degrees prior to when computers or other technologies made their way into the classroom.

According to the most recent report from the National Center for Education Statistics (NCES), in 2009, approximately 3% of classrooms in the United States still do not have a computer in the classroom. Not all schools that do have computers have enough computers to support the number of students enrolled. If a school has a large number of students, a small number of available computers, or worse, both, students may have difficulty working in groups of anymore than two with virtual dynamic manipulatives. In addition to the basic technology needed, Internet access or wireless Internet capability for mobile devices is needed, which again can be a strain on district funds.

Even though there has been an increase in tablets and smart phone use, this does not guarantee that all students will have these available. According to Project Tomorrow (2013), 58% of students in grades 3-5, 68% of students in grades 6-8, and 82% of students in grades 9-12 had personal access to tablets outside of school. Schools with “bring your own device” (BYOD) policies or schools that allow students to use personal electronics could highlight socio-economic differences of students within the classroom. Specifically, 70% of principals cite concerns for student equity in device access (Project Tomorrow, 2013). However, it is also reported that ten percent of schools have policies allowing BYOD, up from only 3% in 2010 (Project Tomorrow, 2013). Other issues often faced with technology include the inevitable technical difficulties of dead batteries and dropped Internet connections (Burns & Hamm, 2011).

Lastly, teachers may view the set-up and clean up of dynamic virtual manipulatives even more complicated than concrete manipulatives (Moyer et al., 2002). Since virtual manipulatives would require equipment set-up, possible movement of equipment if not stored in the same classroom, or if student relocation to a computer lab is necessary for the class period, these can all cut into already priceless instruction time (Burns & Hamm, 2011).

Clean-up may be time-consuming as well, ensuring that all computers are logged-off, clean, and in some cases returned to media departments.

Tangram Manipulatives

One specific manipulative that has been utilized in the elementary classrooms is the tangram. Lee et al., (2010) defines a tangram as a seven piece geometric shape comprised of two small triangles, one medium triangle, two large triangles, one square, and one parallelogram that together form a perfect square. The tangram pieces, or tans, can also be arranged in a variety of figures such as birds, animals, numbers, or other shapes (Tian, 2012). The tangram is purported to be an ancient Chinese puzzle that was, "specifically designed to enhance children's spatial sense and understanding of relationships between shapes" (Lee et al., 2010, p. 93). Butler (1994) reported that ivory tangram sets appeared in Britain during the late 1700's.

One of the reported benefits for using tangrams is that tangram puzzles can be made difficult enough to challenge a student but not frustrating to the student (Butler, 1994). Kamii et al. (2001) wrote that tangram manipulatives were a good intervention strategy to aid students to make a lower-level relationship needed to move on to the higher level constructive abstraction relationships. After students are familiar with the basic identification of the shapes, tangrams can be used to create figures, copy figures and even illustrate stories such as *Grandfather Tang's Story* by Ann Tompert (1990) for an even more enriching activity through the use of children's literature in mathematics (NCTM, 2000a). Not only do tangrams help students with making whole figures out of multiple pieces, they also teach dissection skills, sometimes referred to as composition and decomposition of complex shapes (Butler, 1994).

A recent study by Siew et al., (2013) on facilitating students' geometric thinking through van Hiele's phase-based learning suggested that using tangrams as manipulative teaching and learning aids allowed low achieving students to transition easily from van Hiele's level 0, visualization, to level 2 of geometric thinking, analysis. Additionally, Siew et al., (2013) found that the third-grade students enjoyed using the tangrams to enhance their cognitive ability while allowing them to be creative as well. Another study by Lin et al. (2011) postulated that the collaborative learning within the virtual tangram activity could help to bridge the gap between high-ability and low-ability students when researching the impact of using synchronous collaborative virtual tangrams in children's geometric learning. Lin et al., (2011) also argued that the virtual tangram "enabled resource sharing and formed the interdependent learning environment" (p. 256).

Mobile learning technology

John Dewey once said, "If we teach today's students as we taught yesterday's, we rob them of tomorrow" (1944, p. 167). Palfrey and Gasser (2008) state that current adolescents work, write, and interact with each other in ways very different from other generations such as multi-tasking on several different digital devices at once, writing in text-based abbreviations, and socializing through digital media. Small and Vorgan (2008) concluded that constant exposure to all the varied digital technologies affects neural pathways within the brain which supports claims that people who have high exposure to digital technology think and learn differently than other generations (Palfrey & Gasser, 2008).

Mobile learning technology is defined as smart devices such as smart phones, iPods, tablets, iPads, and laptops (Prasertsilp & Olfman, 2014). Unfortunately, according to White and Martin (2014), mobile learning technology is often seen as a threat to learning due to the many features on devices that could be potentially distracting from the learning process. White, Booker, Martin & Ching (2012) classified mobile device usage into four basic practices: (1) capturing and collecting information; (2) communicating and collaborating; (3) consuming and critiquing; and (4) constructing and creating. When using mobile learning technology to communicate and collaborate, students are able to participate in collaborative learning by sharing learning experiences through digital media. The constructivist learning theory suggests that when mobile learning technologies are used for constructing and creating, the learner is able to generate new knowledge from interactions between their new ideas and experiences in a digital environment. Armstrong (2014) stated, “In essence, technology is transforming students into explorers and teachers into guides” (p. 41).

Prior research states that a teacher’s use of technology in the classroom can affect student involvement in the subject, decrease learning times, and provide opportunities to learn in non-traditional ways (Pilgrim, Bledsoe, & Reiley, 2012; Rice, Cullen, & Davis, 2011; White & Martin, 2014). Additionally, Zaranis, Kalogiannakis, and Papadakis’s (2013) research on using mobile devices for teaching realistic mathematics in kindergarten education concluded that the use of tablets in the classroom aided learning and resulted in better learning outcomes for the students when compared to traditional teaching methods. Other research purported that the use of mobile devices increased student enjoyment of and participation in learning activities when the technology was integrated in meaningful ways (Aronin & Floyd, 2013; Mayberry, Hargix, Boles, Dugas, O’Neill, Rivera, & Meler, 2012; Zaranis, et al., 2013).

Current research in the educational field of the effects of touch screen technology on children’s development is still in its infancy. Research that was completed in 2003 by

Romeo, Edwards, McNamara, Walker, and Ziguras on issues related to the use of touch screen technology in early childhood education is now already outdated by today's standards of touch screen technology due to tablet technology released in 2010 with the release of the first generation iPads. Romeo et al.'s (2003) study found that young children had difficulties touching and dragging items on a screen. Romeo et al. (2003) suggested that, "the mouse may be a more appropriate input device for children at this stage of their development" (p. 333). Romeo et al. (2003) offered the explanation for this outcome that children were more familiar with mouse usage. However, since the release of the Apple iPad in 2010 and other tablet devices soon after, touch screen technology is used in most all mobile technology interfaces in the forms of tapping or swiping a finger. These movements evolved into a new set of digital affordances used when interacting with most mobile digital devices. Wijekumar et al.(2006) defined technology affordances as, "the interaction between users and tool" (p. 192). Recent studies on the digital affordances of mobile technology examine the use of touch screen technology and its use by young children (Walsh & Simpson, 2013, Merchant, 2015).

More current research in 2011 by the Micheal Cohen Group, LLC (MCG), suggests that touch screen technology as found in Apple iPads is simplistic enough for children as young as two years of age to master. MCG (2011) acted as principal investigators and evaluation team for the evaluation of activities of the U.S. Department of Education, Ready to Learn Program, to investigate young children, apps, and iPad usage. MCG's (2011) qualitative research findings indicated that several types of learning occurred during children's play with gaming apps, creative apps, and e-books. MCG (2011) described the different types of learning as tacit learning – the learning of how the game worked, mastering

– mastering learning of explicit learning tasks such as matching or counting, and the use of skills and models learned – applying learned skills to other types of games or other levels of play. MCG (2011) also found that when students were engaged with the app, the child was no longer focused on winning or losing, but instead, more interested in improving their own previous scores. MCG (2011) noted that if the app was deemed well-designed, then the child was capable of progressing from novice to mastery quickly. The idea that a student can not only progress quickly, but also be self-motivated to outperform themselves when using touch screen technology could be very advantageous to tablets in the educational field.

Multimodal Tangram Manipulatives

New technology has recently emerged that combines both concrete and virtual dynamic manipulatives into a multimodal manipulative to be used in conjunction with touch screen technology and mobile devices. Specifically, the company Tangible Play Inc./OSMO, has developed OSMO, an interactive multimodal gaming system (https://www.playosmo.com/). OSMO uses concrete manipulatives styled out of wooden blocks, coated alphabet tiles, and any other real-world objects that a player can place in front of the gaming system. OSMO combines these concrete manipulatives with a virtual dynamic manipulative by projecting the image of the concrete manipulatives onto a digital screen using an iPad, camera mirror piece, and a plastic stand to position the iPad. OSMO gaming kits also include access to four downloadable software apps: Tangrams, Words, Newton, and Masterpiece. Together, the concrete manipulatives, virtual dynamic manipulatives shown on the iPad, and the gaming software create a single learner or collaborative learning environment that links the real-world and virtual world. This integration of gaming and education is not a new idea. Virtual edutainment became a trend beginning in the early

1990's; however, the OSMO, is the first known gaming platform that combines concrete and virtual manipulatives into a multimodal manipulative to be used with touch screen technology on a mobile device for educational and entertainment purposes.

Summary

Individually, the topics of constructivist thinking in mathematics education, spatial sense, technology integration in the classroom, concrete and virtual manipulatives, tangram manipulatives, mobile learning technology, and multimodal tangram manipulatives have been researched, in some instances in-depth; however, multimodal manipulatives research is still in the early stages. Currently, there are no educational research findings on multimodal manipulatives, specifically, the OSMO gaming system and the effects of OSMO on elementary students' spatial sense development and math achievement. Additionally, there is very little research on the development of spatial sense and math achievement for students at the second grade level.

Chapter three discusses the methodology used to explore the research questions. The characteristics of the participants of the study are discussed in addition to a description of the setting for the study. The description of the study addresses classroom details, course activities, and instructional classroom procedures. Instruments used during the research process, research procedures, and data analysis are outlined. Lastly, there is a discussion of ethical considerations related to the study.

CHAPTER III

METHODOLOGY

This convergent parallel mixed methods research study used both quantitative data and qualitative data to explore the effects of concrete, virtual, and multimodal tangram manipulatives on elementary students' mathematics achievement and development of spatial sense. Chapter three describes the sampling procedures, participants, and setting as well as provides a detailed description of the instruments, design of the study and procedures that were used to collect and analyze the data. The study addresses the following research questions:

1. What influences, if any, do the different tangram manipulatives (concrete, virtual, and multimodal) have on students' mathematics achievement?
2. What influences, if any, do the different tangram manipulatives (concrete, virtual, and multimodal) have on students' spatial sense?
3. What are second grade students' and their teachers' perceptions of the different types of manipulatives (concrete, virtual, and multimodal)?

Research Design

The researcher conducted a convergent parallel mixed methods study to explore the effects of concrete, virtual, and multimodal manipulatives on elementary students' mathematics achievement and development of spatial sense. The researcher's questions in this study sought to determine a difference in the development of spatial sense based on the use of concrete tangram manipulatives, virtual dynamic tangram manipulatives, or multimodal tangram manipulatives. Also, the researcher sought to gain an in-depth understanding of both teacher and student perceptions when using the OSMO Tangram gaming system, a multimodal manipulative. Therefore, the research questions in this study guided the researcher to collect both quantitative data and qualitative data.

This study used a convergent parallel mixed methods design. The convergent parallel design allowed the primary researcher to collect both quantitative and qualitative data simultaneously (Creswell, 2011). According to Creswell and Plano Clark (2011) the convergent parallel design gives equal consideration to both the quantitative and qualitative strands of data. The convergent parallel design calls for the quantitative and qualitative strands to remain independent from each other until the interpretation of the results (Figure 3). Use of the convergent parallel design is recommended when there is limited time to collect data (Creswell & Plano Clark, 2011).

The convergent parallel design method enabled the researcher to gain a broader perspective by using both quantitative test results and qualitative observations. Philosophical assumptions behind the convergent parallel design include incorporating a pragmatic approach to the study (Creswell & Plano Clark, 2011). Creswell and Plano Clark (2011) recommend using a worldview such as the "umbrella" paradigm in lieu of trying to mix different paradigms (p. 78).

Strengths within the convergent parallel design include that the design makes "intuitive sense" (Creswell & Plano Clark, 2011, p,78). The convergent parallel design also tends to be very efficient when the researcher has limited time to collect data since both strands of data can be collected simultaneously in a single phase (Creswell & Plano Clark, 2011). Morse (1991) stated the use of a convergent design provided the opportunity, "to obtain different but complimentary data on the same topic" (p122). Analyzing both data sets will help the researcher better understand the questions under study.

Challenges of using a convergent parallel study design can include the increased effort and expertise required of the researcher since the researcher must preplan both the quantitative and qualitative strands of data collection, as well as be able to accurately interpret results acquired from mixing both data sets at the end of the study (Creswell & Plano Clark, 2011). Other difficulties that may arise include the possibility of different samples and different sample sizes between the quantitative and qualitative data strands (Creswell & Plano Clark, 2011). At the completion of the study, there is a possibility that the quantitative and qualitative data may contradict each other (Creswell & Plano Clark, 2011).

Figure 3. Research design.

Participants

A purposeful sample was used to include three elementary school teachers and three second grade classes comprised of a total of sixty-one (N = 61) students at a public Midwestern suburban elementary school. All student participants were enrolled in the second grade mathematics course with the teacher participants. The student participants spent approximately eighty minutes per day, five days per week, in their mathematics class with a highly-qualified certified elementary education teacher. Each teacher participant taught only mathematics courses for three separate grade levels per day from K – 2nd grade. Although the elementary education teachers were not specifically mathematics certified, they were selected by the school principal to teach the mathematics course by their mathematical ability and their interest in teaching the subject. Students that were absent for all or a portion of the quantitative data collections were not included in the analysis.

Most recent reported demographics for the school reported enrollment that was comprised of 61% Caucasian & other, 4% African-American, 1% Asian, 6% Hispanic, and 28% Native American students (Public school site diversity demographic data, 2015). Student participant demographics were representative of the school demographics.

Teacher participant information is presented by classroom. Pseudonyms are used for all participant names to ensure confidentiality. Teachers are currently arranged by cluster (named as an animal group), a group of three teachers that each specialize in a specific subject reading, writing, and mathematics, and teach across grades kindergarten, first, and second. Each animal group cluster contains approximately twenty-five kindergarten, twenty-five first grade, and twenty-five second grade students. For example, the Owl's cluster contains a reading, a writing, and a mathematics teacher that each teach three class periods per day – one kindergarten lesson, one first grade lesson, and one second grade lesson.

Classroom One

Classroom 1 teacher participant, Mrs. Peterson, holds a Bachelors of Science degree in Education from a regional institution in the Midwest. She holds a dual teaching certificate in early childhood and elementary education. She has been teaching for a total of seventeen years. Mrs. Peterson has been employed by the current elementary school for fourteen years. She has worked within the same district for all seventeen years. Her teaching experience includes thirteen years of teaching self-contained first grade and four years experience as the kindergarten, first, and second grade math teacher within her cluster. Mrs. Peterson has participated in various professional development and continuing education sessions through her school to further her education and training. Twenty students in this classroom returned their parental consent and student assent forms. Three students were removed from the study due to incomplete pre/posttest data due to school absences on the testing day. One student was removed from the study due to incomplete posttest data due to failing to respond to several pages of posttest questions. It was assumed that the student did not see questions on the back side of the double-sided posttest pages. Total student participants included in the quantitative data analysis for Mrs. Peterson's class were seventeen (n = 16).

Classroom Two

Classroom 2 teacher participant, Mrs. Edwards, holds a Bachelors of Science degree in Education from a regional university in the Midwest. She is a state certified early childhood education teacher. She has been teaching for a total of nine years. Mrs. Edwards has been employed by the current elementary school for all nine years of teaching. Her teaching experience includes one year of teaching kindergarten, five years of teaching self contained second grade, and three years as a mathematics instructor for grades kindergarten, first, and second. She has participated in various professional development and continuing education sessions through her school to further her education and training. Twenty-four of Mrs. Edwards' students returned their parental consent and student assent forms. One student was removed from the study due to incomplete posttest data due to a school absence. Twenty-three (n = 23) student participants were included in the quantitative data analysis.

Classroom Three

Classroom 3 teacher participant, Mrs. Green, holds a Bachelors of Science degree in Elementary Education from a large land-grant four year university in the Midwest. She is a certified elementary and early childhood education teacher. She has been teaching for a total of twelve years. Mrs. Green has been employed by the current elementary school for three years. She has one year experience teaching half-day kindergarten, seven years experience teaching full-day kindergarten, one year experience teaching first grade self-contained classes, and three years teaching experience with the K-2 math specific content courses. Mrs. Green has participated in various professional development workshops through outside vendors and her elementary school to further her education and training. Mrs. Green had twenty-two students return their parental consent and student assent forms. She did not have any students absent during the quantitative data collection phase. Total student participants for Mrs. Green’s class included in the quantitative data analysis were twenty-two (n = 22).

Setting

The study took place in a single Midwestern suburban elementary school and focused on three second grade classrooms: Classroom 1, 2, and 3. This particular elementary school offered a unique environment for elementary school students in that they traveled among three core subject teachers and three elective subject teachers. The three core subject teachers each specialized in one of the following subjects: mathematics, reading, and writing. The elective classes offered at the school include art, music, and physical education. The study focused on three elementary classroom teachers and their second grade mathematics course only.

Mrs. Peterson's classroom was decorated in a kangaroo theme. The classroom was lighted with overhead fluorescent lights within the tiled ceiling. A small bit of light shone through the small window, with blue curtains on the west side of the classroom. Desks were arranged in a large U-shape open to the front of the classroom. Inside the U-shape, there were four rows of four desks each, with a walkway through the middle, creating two columns of four rows, with two desks each (Figure 4). The front of the classroom was on the west wall of the room. The west wall contained Mrs. Peterson's desk, the IWB, a small window, and a small table with a desktop computer. The north wall of the classroom was lined with two large whiteboards, two bulletin boards on either side of the whiteboards, and a small round table with neatly labeled bins for students to turn in their homework. The bulletin boards were covered in brightly colored educational posters and important announcements. The east wall of the classroom housed a wall of upper and lower half-lockers. The south wall of the classroom had two small tables with two desktop computers and a small single-basin sink, framed with two large armoire cabinets on either side, cabinets over the sink, and cabinets and drawers under the sink. The teacher's desk was a small and cluttered space with many papers, books, and file folders stacked on the workspace.

Students in Mrs. Peterson's class were aware of the prescribed daily procedures for entering the classroom and policies and procedures during class time. Mrs. Peterson's class is typically a moderately quiet classroom with little talking allowed amongst students. Students in Mrs. Peterson's class generally only work in groups when at play stations during downtime. Otherwise, students are expected to work quietly and independently at their assigned seats.

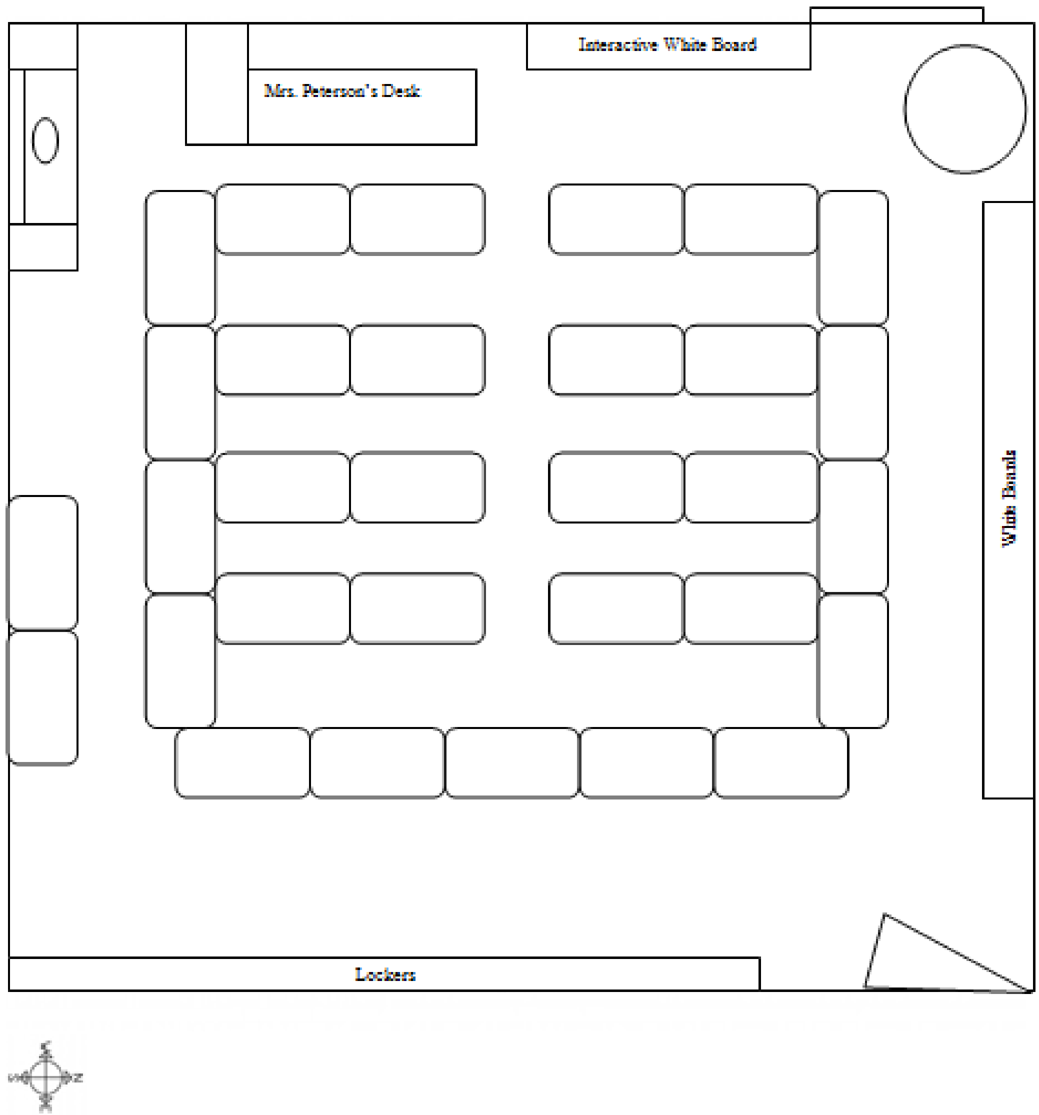

Figure 4. Diagram of Mrs. Peterson's classroom – Classroom 1.

Mrs. Edwards' classroom was adorned with tiger themed decorations. Lighting in the classroom was provided from the overhead fluorescent lights housed within the white ceiling tiles. A small window on the east side of the classroom was covered by a curtain and let in little light. Desks were arranged in large group clusters throughout the room. Each cluster of desks had five desks, two rows of two desks facing each other, with one desk pushed up to the end of the group (Figure 5). Four of the desk clusters were angled away from the front of the classroom, creating two inverse V-shapes in relation to the front of the classroom. The front of the classroom was located on the east wall. The east wall housed Mrs. Edwards' neat and tidy desk. To the right of Mrs. Edwards' desk, was a small table with her desktop computer and printer. There was a small round child-sized table in the southwest corner of the room. The west side of the classroom had two large whiteboards with bulletin boards on either side. The bulletin boards contained several educational posters and classroom instructions and procedures. The entrance door to the classroom was located on the south end of the west wall. The teacher typically kept the door shut and locked during class time. The west wall also contained a wall of upper and lower half-lockers, a small round table at the rear of the classroom, and two rectangular tables at the rear of the classroom. The south wall of the classroom had a small table with a single desktop computer, a small single-basin sink, two large armoire cabinets on either side, and cabinets above as well as cabinets and drawers below.

The atmosphere in Mrs. Edwards' class is undeniably structured and consistently monitored for sound or movement. Students in Mrs. Edwards' class were expected to enter the room silently and immediately put away any belongings and began working silently at their desks. Mrs. Edwards was often heard reminding her students to stay "Silent" and busy working. Students rarely worked in groups; however students were almost always working diligently at their assigned seats.

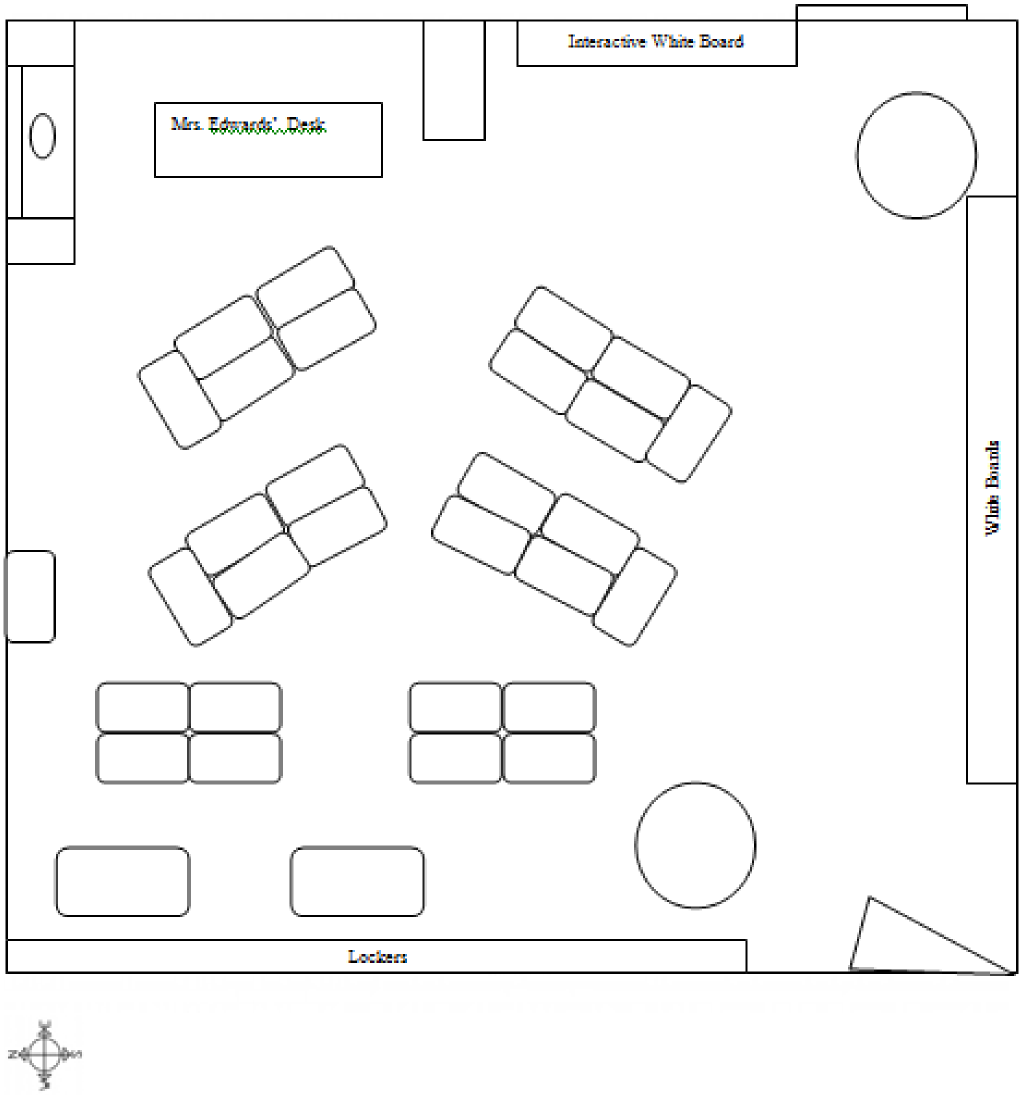

Figure 5. Diagram of Mrs. Edwards' classroom – Classroom 2.

Mrs. Green's classroom was brightly decorated in a bunny theme. There were curtains on the two small windows, bulletin boards with various elementary appropriate educational posters. The front of the classroom was located on the south side of the room and had a whiteboard, a table with boxes for turning in homework labeled by grade, and a lamp. The classroom had a small window on the south end of the west wall, an IWB to the right of the window with an area for students to sit on the rug placed on the floor, surrounded by bean bag chairs, pillows, and a short bookshelf, and Mrs. Green's white rocking chair in the corner. Just to the right of the whiteboard was Mrs. Green's desk that is neatly arranged with a pencil cup, stapler, tape, a desktop computer, and several filing stands. The north wall of the classroom had a small single-basin sink with cabinets above and cabinets and drawers below. The east wall of the classroom was lined with upper and lower half-lockers, neatly labeled with bunny themed cut-outs containing students' names.

The majority of the desks were arranged in two columns, each containing four rows of three desks pushed together at the sides (Figure 6). There were three desks offset from the rest, placed in a single column along the whiteboards on the south wall. The back of the classroom had three work station tables with four chairs or stools at each. There were two desktop computers at two small tables facing the east side of the classroom. There was not much room between desk rows to walk behind or around students.

Mrs. Green's room had a noticeably relaxed atmosphere. Students were often speaking loudly to each other while entering the classroom and continued to speak as they unhurriedly gathered their supplies needed for the day. Some students would go directly to their assigned desk, while others would sit in the bean bag chairs in the corner of the room or the tables and stools in the back of the room. Mrs. Green would often announce what the students were expected to be working on several minutes into the class period. Once students began their work, they often spoke quietly to each other while they worked. Students moved freely around the classroom throughout the class period, but did not require an excessive amount of reminders to stay on task.

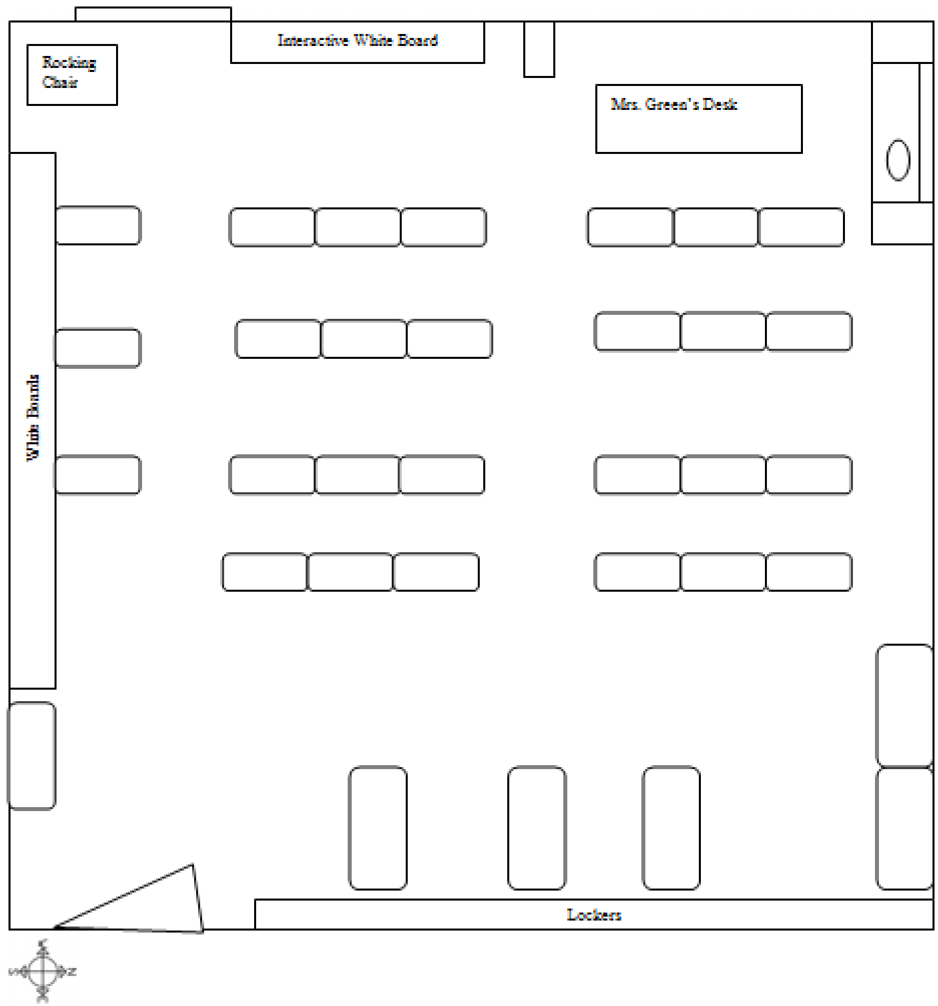

Figure 6. Diagram of Mrs. Green's classroom – Classroom 3.

The study took place over the course of six weeks. The study focused on three second grade mathematics classroom teachers and three second grade mathematics classes. Each of the classes was assigned a single type of tangram manipulative to use throughout the duration of the study: (1) Concrete manipulative – classroom 1; (2) Virtual manipulative – Classroom 2; and (3) Multimodal manipulative – classroom 3.

Students within the second grade classroom followed the curriculum standards as outlined by the local state department of education. The particular second grade standard addressed for the unit of study was the local state geometry standard of geometric properties and relationships used to recognize and describe shapes. Additionally, students are expected to be able to identify symmetric and congruent shapes and figures, as well as be able to investigate and predict the results of combining and decomposing two-dimensional shapes. The use of tangrams is specifically suggested by the state standards within the second grade suggested materials kit.

Classes 1, 2, and 3 participating in the study received the same tangram introductory lesson presented by the classroom teacher participants. Teacher participants underwent a brief training session of approximately one hour on tangrams facilitated by the researcher prior to data collection from the students. A copy of the lesson plan that the teachers used is located in Appendix A. Prior to beginning tangram manipulative activities within their classroom, students were placed in ability-based groups by the classroom teacher as determined by their pre-test mathematics achievement and mental transformation task pre-test results.

Control Class/Concrete Manipulative – Classroom 1

Classroom one had 27 second grade students enrolled. The researcher assigned the Concrete Manipulatives, wooden tangrams, to this class. Students worked collaboratively throughout the tangram activities to solve each concrete tangram puzzle and to complete the "Concrete Tangram" activity worksheets (Appendix B).

Virtual Manipulative Class – Classroom 2

Classroom two had 26 second grade students enrolled. Students in this classroom used an iPad and the JiuzhangTech Ltd free tangram app, version 1.1, (www.jiuzhangtech.com) for the classroom activity. Students worked collaboratively throughout the tangram activities to solve the virtual tangram puzzle and to complete the "Virtual Tangram" activity worksheets (Appendix C).

Multimodal Manipulative – Classroom 3

Classroom three had 26 second grade students enrolled. Students in this classroom used an iPad, the OSMO Tangram gaming system, and the Tangram app by Tangible Play, Inc. for the classroom activity. Students worked collaboratively throughout the tangram activities to complete the multimodal tangram puzzles and the corresponding "OSMO Tangram" activity sheets (Appendix D).

Tangram Lesson Cycle

Activities took place one day per week for a total of six weeks. The researcher was present with the classroom teacher during each activity-day class period. The researcher attended all six activity days for each of the three classroom groups.

Day 1 of the research, the classroom teacher handed each student a copy of the pre-test of mathematical achievement and spatial ability (see Appendices E & F). Students used upright file folders on their desks as dividers between themselves and their neighboring classmates to prevent the incidence of cheating. This was routine practice in all three second grade classrooms during testing times. The teacher read through the directions of the test with the students, fielded any student questions, and instructed students to begin on their tests. When students completed their tests, they raised their hand and their teacher retrieved the test from them. Students were given the option to either put their head down on their desk after completing the test, or to get out their library book from inside their desk cubby to read. Several students asked if they could get a book off of the bookshelf in the classroom to read. The teacher allowed some students to get up from their desk to get a book to read.

The second day of the study, the teacher presented the lesson on tangrams to the classroom (Appendix A). The teacher began the lesson by reading out loud to the class *The Warlord's Puzzle* by Virginia Walton Pilegard. After the story was complete, the teacher began a classroom discussion about the book. The teacher used the Teacher Tangram Lesson Plan (Appendix A) to facilitate the lesson. Students discussed vocabulary terms such as flip, slide, and rotate. After approximately ten minutes of classroom discussion, the teacher called out student names and student assigned identification numbers to arrange them into their ability-based groups as previously determined by their pre-test scores. Students quickly moved to other desks to sit with their assigned partners.

Once seated with their group, the teacher asked for one person from each group to line up near the front of the classroom to pick up their supplies to be used for the activity. Students were handed their assigned manipulative, one bag of crayons, and the Week 1 Activity worksheets (see Appendices B, C, & D). The students then began working in their groups. While the students were working, the teacher and the researcher walked around the classroom and observed the students. When students asked for help, the teacher or the researcher would try to guide the students to discover the answer. Approximately fifteen minutes prior to the end of the class period students were asked to begin cleaning up their supplies and to return their supplies to the small table at the front of the classroom. Once all supplies were collected, the teacher used the remaining class time to lead a class discussion on the students' experiences during the activity. She facilitated the discussion with questions such as, "What did you discover today? Was there anything that you discovered that surprised you?"

The third and fourth days of the study the students were told to get into their work groups while lined up in the hallway prior to entering the classroom for their class period. Once in groups, the students entered the classroom and picked up their supplies from the small front table before sitting at the desks. Again, students picked up their assigned manipulative, a bag of crayons, and the activity worksheet for the day – Week 2 Activity for day 3, and Week 3 Activity for day 4 (see Appendices B, C, & D). Students sat in the same area as previously used for their group. Once seated, the students immediately began working. While the students were working, the teacher and the researcher observed the students while walking around the classroom. The teacher and the researcher helped guide students to discover a solution if they asked for help. Approximately fifteen minutes prior to the end of class, the teacher asked students to begin cleaning up, and to return their supplies to the small table at the front of the classroom. Once all supplies were collected, the teacher led a classroom discussion about the students' experiences during the activity.

The fifth day of the study, the students entered the classroom in their groups, gathered their supplies, and began to work. While they began working in their groups, the researcher asked specific students if they would like to participate in a task-based interview and a semi-structured interview. The researcher used a purposive sample of three students, one from each ability level as determined from their pre-test scores. The researcher conducted the task-based and semi-structured interviews with each selected student while the other group members continued working in their groups. Task-based interviews and semi-structured interviews were conducted at a table in the corner of the classroom. The teacher observed the students and fielded questions while walking around the classroom. Approximately five minutes prior to the end of class, students were asked to clean up their supplies before leaving for their next class.

The sixth day of the study the students were instructed prior to entering the classroom to sit in their regular assigned seats and to place all of their belongings under their desk/seat. Students walked into the room quickly and quietly and sat in their seats. The teacher asked a single student to pass out the ‘privacy tents’ – the upright file folders used to stand on the desks to prevent students from cheating. While the folders were passed out, the teacher and the researcher passed out the post-tests (see Appendices E & F). Students were asked to be sure and write their assigned number on their post-test. The teacher read the directions aloud and instructed the students they could begin their test. Students were told to raise their hand with they finished and she would pick up the test. Students who finished prior to everyone else were allowed to read a book or to put their head down on their desk.

While the students were taking the post-test, the researcher and the teacher sat at the small table at the front of the classroom and completed the semi-structured interview about the teacher’s experiences during the study. Once the interview was complete, the teacher and the researcher observed the students until they completed the post-test.

Instrumentation

The quantitative data strand was comprised of test results from two sets of pre- and post- assessments. The first pre/post assessment was a standards-based second grade mathematics achievement test administered to classrooms 1, 2, and 3. The second pre/post assessment was a spatial skill, mental transformations test that was administered to classrooms 1, 2, and 3. The qualitative data strand was comprised of field notes, task-based interviews, semi-structured interviews detailing participants' personal accounts of their experience during tangram activities, and audio/video recordings of participants throughout the activities and interviews.

Quantitative Data Sources

Mathematics achievement. All second grade participants were administered a pre/post condensed version of the Second Grade California Standards Mathematics Test (Appendix E). This particular test is part of the Standardized Testing and Reporting (STAR) Program under policies set by the California State Board of Education. All questions from the California Standards Tests have been evaluated by committees of content experts to ensure their appropriateness for measuring the content standards in Grade 2 Mathematics. According to the California State Department of Education (http://www.cde.ca.gov/ta/tg/sr/resources.asp) all test items have been previously reviewed and approved to ensure fairness in regards to gender, ethnicity, and language.

The test used in this study contained released test questions from the California Standards Test forms from 2003, 2004, 2005, 2006, and 2007. An outline of items contained in the original testing document composed by the California State Department of Education (CSDE) is shown in Table 1.

Table 1

CSDE Math Achievement Test Items

Strand/Reporting cluster	Number of questions on exam	Number of released test questions
Number Sense – Place value, addition and subtraction	15	19
Number Sense – Multiplication, division, and fractions	23	27
Algebra and Functions	6	7
Measurement and Geometry	14	18
Statistics, Data Analysis, and Probability	7	9
TOTAL	65	80

Table 2 outlines the condensed version of the CSDE that was used in the current study. All test questions included in the pre/post mathematics achievement test given to study participants were preapproved by the participating second grade mathematics teacher to ensure appropriateness of questions.

Table 2

Description of CSDE and CSDE condensed

Strand/Reporting cluster	Number of questions on original exam	Number of questions on condensed exam
Number Sense – Place value, addition and subtraction	15	7
Number Sense – Multiplication, division, and fractions	23	12
Algebra and Functions	6	3
Measurement and Geometry	14	7
Statistics, Data Analysis, and Probability	7	3
TOTAL	65	32

The condensed mathematics achievement test was administered as a two part pre/post test that contained 16 questions each. Questions included on the pre and post assessment were evenly distributed by mathematical content standards. Samples of three of the assessment questions are shown in Figure 7.

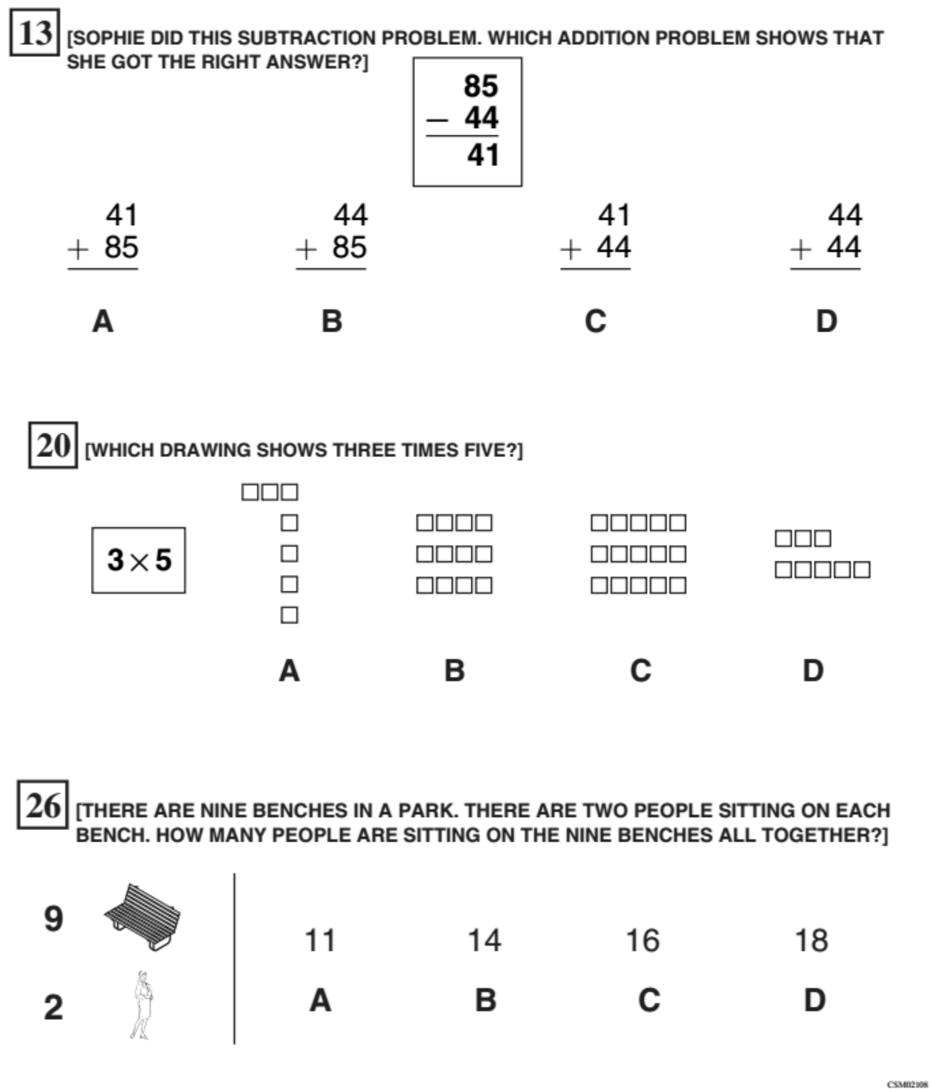

Figure 7. Mathematics achievement pre/post-test sample questions.

Spatial sense. All second grade participants were administered a pre/post test of spatial ability/mental transformations test. The instrument used was the Children's Mental Transformation Task (CMTT) test (Appendix F) as developed by Dr. Susan Levine, University of Chicago. This particular instrument is designed for children ages 4-7 years of age. The test is designed as a two-part 32 item test to be administered in two settings, a pre- and post- assessment of sixteen questions each.

> This task requires children to choose which shape would be made by moving two separate pieces together. It includes four types of items, all of which tap 2-D mental transformations: 1) horizontal translation, 2) diagonal translation, 3) horizontal rotation, and 4) diagonal rotation. (http://www.spatiallearning.org/index.php/resources/testsainstruments).

Test instructions and a sample of one of the spatial ability mental transformation task test questions are shown in Figure 8.

Instructions for the test: Circle which shape on the left would be made from moving the two pieces on the right together.

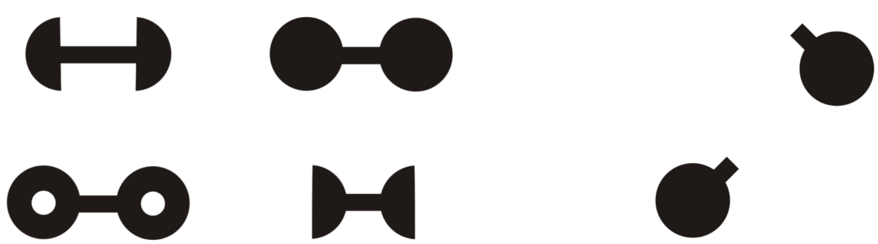

Figure 8. CMTT instructions and sample of test problem.

Permission to reproduce copies of the test and for use for the study was acquired from Dr. Susan Levin (levine@uchicago.edu) and is included in Appendix G.

Reliability and validity of the CMTT test were determined through previous studies that utilized the Levine CMTT test. Levine, Ratliff, Huttenlocher, and Cannon (2011) claimed reliability and validity of the CMTT, and when referring to the CMTT stated:

> Prior studies have shown this task to be a reliable measure of spatial transformation skill and have found that boys outperform girls on both rotation and translation items, with no significant interaction of gender and problem type (Ehrlich et al., 2006; Levine et al., 1999). A split-half reliability test of odd and even items on this abbreviated 10-item task showed that it maintains reliability, $r = .55$ adjusted using the Spearman–Brown formula. (p. 4).

Qualitative Data Sources

Observational field notes. Throughout the six-week data collection, the researcher did at times interact with the participants assuming the role of observer as participant (Glense & Peshkin, 1992). The researcher conducted a series of six observations over the course of six weeks with one observation per week. During the observations, the researcher compiled field notes concerning the teacher's approach to the lesson, questions the students asked about the activity, the level of engagement of the students with activity, team collaboration during the activity, and any other student comments as related to the activity. These observations lent valuable insight into the human behavior associated with the quantitative data strand and helped expand and clarify quantitative findings.

Task-based interviews. Students were asked to verbally describe their actions while completing the activities with the assigned manipulative (Appendix H). Students were allowed to choose a single puzzle from the Week 4 Practice worksheets for each manipulative type. The researcher asked student participants questions about their decisions and thinking. These interactions took approximately no more than 15 minutes and were recorded for audio and video and transcribed.

The task-based interview participants were selected based off of their second grade mathematics achievement pre-test: one student from each high, average, and low ability were selected from each manipulative group. The researcher examined the scores as a whole to determine the range of scores. The researcher then examined the scores as percentages, and categorized the scores based on traditional letter grade assignments (0-59.5% = F, 60 – 69.5% = D, 70 – 79.5% = C, 80 – 89.5% = B, 90 – 100% = A). The researcher then carefully chose the following ability based groupings:

- High-level ability: 13/16 – 16/16 (80-100%)
- Medium- level ability: 10/16 – 12/16 (60-79%)
- Low-level ability: 0/16 – 9/16 (0-59%)

Three students per class - one low, one medium, and one high- level ability were selected to take part in the student task-based and semi-structured interviews. Pseudonyms are used for student names ensure confidentiality in the study. The student math achievement pretest scores and student participant descriptions are outlined below:

Classroom 1, concrete manipulatives, low ability: Mary (43.75%). Mary was a very shy quiet girl. She tended to work slowly, but deliberately. Throughout the study Mary often sat quietly while allowing her partner to manipulate the pieces. Mary would often point to pieces for her partner to move, or move them without making any verbal communications. When she or her partner was struggling to solve a puzzle, they would generally wait for the teacher or the researcher to walk by their desks before asking for help. She did not complain about the activities; however, if they finished the activities, she would get out a book and read. When asked if she would like to participate in the interview she simply nodded her head and showed little emotion.

Classroom 1, concrete manipulatives, medium ability: Caleb (68.75%). Caleb was a soft-spoken child and was not socially out-going in class. Caleb generally worked quickly, but methodically. Caleb and his partner were very quiet during their interactions, mostly taking turns moving pieces, waiting for each other to complete their piece placement before they tried to add their own piece. Caleb would let his partner rearrange the pieces without protesting. Caleb would encourage his partner to raise his hand if they could not figure out the puzzle at the same rate of their peers around them. When Caleb and his partner completed the puzzles, they would often ask if the teacher could verify if they were right. They would then clean up their materials and clear off their desks. Caleb did not continue to play with the pieces after the activities were completed. Several times during the study, Caleb would ask to go to the restroom when his group was done with their worksheet. When asked if he would like to participate in the interview he simply stated, “Sure,” but showed little excitement.

Classroom 1, concrete manipulatives, high ability: Colby (93.75%). Colby was an outgoing, child that was vocal in class and often volunteered answers. Colby was well-liked by his peers and would often speak to other classmates during class. He was known to joke-around with his teacher and peers; however, he was quiet and engaged when his teacher, Mrs. Peterson was speaking. Colby and his partner both worked very quickly and would debate on piece placement throughout solving a puzzle. Colby would watch his partner’s movement of pieces intently when he was not the one arranging the tangrams. He was actively participating in solving the puzzle throughout each activity. Caleb and/or his partner would often announce their completion of each puzzle out loud to the teacher and his surrounding peers throughout the study. When asked if he would like to participate in the interview, Colby jumped out of his seat and hurried to the table with the researcher. Colby was obviously excited to be chosen to participate in the interviews.

Classroom 2, virtual manipulatives, low ability: Max (37.5%). Max appeared to be a happy child who followed directions in class, and listened intently to his teacher. Max had not portrayed any behavior problems throughout the study, and got along with his partner with few disagreements. Max's partner reported to the teacher on two separate occasions that Max was not allowing his partner a turn on the iPad during tangram activities. Max was noticed pushing back his partners arm or protesting if his partner relocated a piece that he had moved. Max smiled and agreed when asked if he would like to participate in the interview.

Classroom 2, virtual manipulatives, medium ability: Susie (68.75%). Susie was a slightly withdrawn student that did not seem to have many connections with her peers in the classroom. It was observed throughout the study that she rarely asked the teacher for help, or spoke-up during class discussions. Susie would listen to her partner's suggestions on where a piece should be placed, and would occasionally offer suggestions herself. Susie had no problem with letting her partner move the pieces on the iPad. Occasionally, her partner would become frustrated and slide the iPad to Susie for her to try a different strategy. Susie would make a few movements and then seek guidance from her partner on what she should do next. When asked if she would like to participate in an interview with the researcher she only nodded her head.

Classroom 2, virtual manipulatives, high ability: Ethan (93.75%). Ethan was a very loud student that would often argue with his partner, and tattle if he did not feel that he was given equal opportunity to move pieces. Ethan was very confident in his math abilities, even when he was not correct in his responses during class discussions. Ethan would often move his partner's hand, or push his elbow over so he could be in charge of moving the pieces. Ethan would also try to solve the puzzles without allowing his partner to move any of the pieces. Ethan's group was often the last group to turn in their supplies at the end of the activity. When asked if he would like to participate in the interview, Ethan responded that he knew he was going to be picked.

Classroom 3, multimodal manipulatives, low ability: McKenna (25%). McKenna was a well-liked student in her class with many friends. She was a well-behaved student who often volunteered to help her teacher when done with her work. McKenna rarely sat in her seat, and would stand by her desk while completing the puzzle. She and her partner worked together, often speaking and laughing while solving their puzzles. She was excited to participate in the interview; however, she showed signs of being slightly nervous and would smile and giggle when speaking to the researcher.

Classroom 3, multimodal manipulatives, medium ability: Toby (68.75%). Toby was a very articulate boy that showed no signs of nervousness or intimidation when asked if he would like to participate in the interview. He had no problems volunteering answers and often gave very well thought out responses during class discussions. Toby worked well with his equally articulate partner throughout the study. They were overheard on several occasions offering suggestions of 'flip' or 'turn' to each other. Toby and his partner would often finish before others in the class and would then continue playing with the tangram pieces. During the task-based interview, Toby was eager to participate and begin the task.

Classroom 3, multimodal manipulatives, high ability, Kevin (93.75%). Kevin was identified by the teacher as being a 'gifted' student that rarely struggled with mathematical concepts. When speaking to his peers, he was often perceived as being rude since he gave very direct responses. Kevin was overheard throughout the study stating if he thought each puzzle was easy or hard. Kevin's partner would often ask Kevin He was happy to have been chosen to speak with the researcher.

A summary of the students selected, and their respective scores and ability level is detailed in table 3.

Table 3

Student interview participants by manipulative group and score.

Maniuplative Group	High Ability	Med Ability	Low Ability
Classroom 1: Concrete	Colby (93.75%)	Caleb (68.75%)	Mary (43.75%)
Classroom 2: Virtual	Ethan (93.75%)	Susie (68.75%)	Max (37.5%)
Classroom 3: Multimodal	Kevin (93.75%)	Toby (68.75%)	McKenna (25%)

Student semi-structured interviews. The researcher interviewed the same student participants for the semi-structured interviews who were selected for the task-based interviews. During the interviews, students were asked to explain why they chose certain actions (flip, turn, rotate) during the activity. Questions used in the interview process were preapproved by the second grade teacher to insure age appropriate language (Appendix H). Interviews were video and audio recorded and transcribed.

Teacher semi-structured interviews. Semi-structured interviews were conducted with the three teacher participants (Appendix H). The interviews were video and audio recorded and transcribed. Additionally, the researcher asked teacher interview participants what the perceived benefits were of using the manipulative assigned to their classroom.

Procedures

This study was conducted in two phases; phase one – the quantitative phase in which the pre/post data were collected, and phase two – the qualitative phase where the task-based interview and semi-structured interview participants were selected based on results of the quantitative data. Qualitative data were interpreted together with the quantitative results to explain and add insight into the quantitative findings. Figure 9 provides a flowchart for the convergent parallel design. Prior to completing the study, Institutional Review Board (IRB), the school board of education and site principal approvals were obtained (Appendix I). Proper parental/guardian consent, student assent, and adult consent forms were collected before data collection began (Appendix J). Each student and teacher participant was assigned a confidential, unique identifying number. Only the researcher had access to the identifying information.

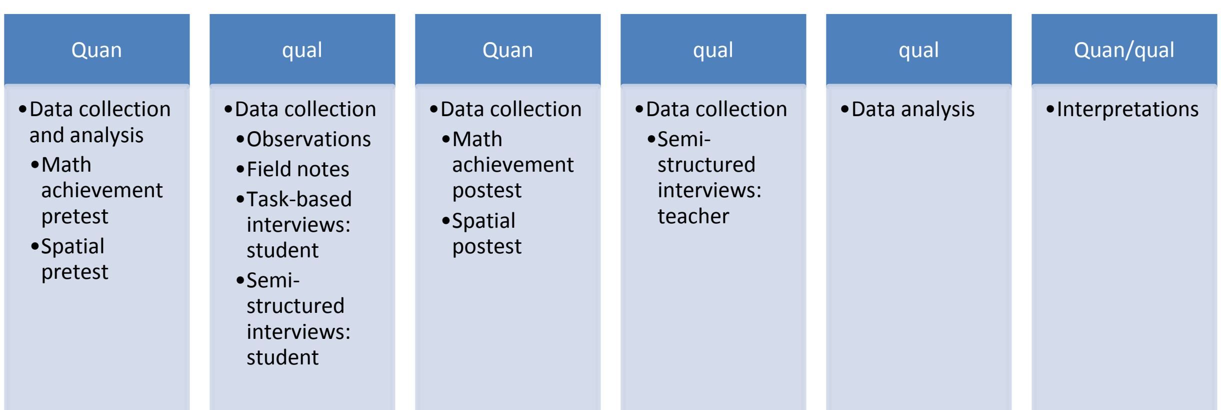

Figure 9. Flowchart of convergent parallel design.

The detailed timeline for the study, including approximate activity time durations is outlined in Table 4.

Table 4

Timeline for data collection

Week	Participants	Duration	Classroom 1	Classroom 2	Classroom 3
1	Students	1 hr	Math achievement pre-test Spatial ability pre-test	Math achievement pre-test Spatial ability pre-test	Math achievement pre-test Spatial ability pre-test
	Teacher	1 hr	Facilitate Testing	Facilitate testing	Facilitate testing
2	Student	1 hr	Introductory tangram lesson Concrete Tangram Activity worksheet: Wk 1	Introductory tangram lesson Virtual Tangram Activity worksheet: Wk 1	Introductory tangram lesson OSMO Activity worksheet: Wk 1
	Teacher	1hr	Facilitate lesson Lead group discussion	Facilitate lesson Lead group discussion	Facilitate lesson Lead group discussion
3	Student	1 hr	Concrete Tangram Activity worksheet: Wk 2	Virtual Tangram Activity worksheet: Wk 2	OSMO Activity worksheet: Wk 2
	Teacher	1hr	Facilitate lesson Lead group discussion	Facilitate lesson Lead group discussion	Facilitate lesson Lead group discussion

Week	Participants	Duration	Classroom 1	Classroom 2	Classroom 3
4	Student	1hr	Concrete Tangram Activity worksheet: Wk 3	Virtual Tangram Activity worksheet: Wk 3	OSMO Activity worksheet: Wk 3
	Teacher	1hr	Facilitate lesson Lead group discussion	Facilitate lesson Lead group discussion	Facilitate lesson Lead group discussion
5	Student	1hr	Concrete Tangram Activity worksheet: Wk 4	Virtual Tangram Activity Worksheet: Wk 4	OSMO Activity Worksheet: Wk 4
	Student	(20 min)	Task-based interviews: High Average Low	Task-based interviews: High Average Low	Task-based interviews: High Average Low
		(20 min)	Semi-structured interviews: High Average Low	Semi-structured interviews: High Average Low	Semi-structured interviews: High Average Low
	Teacher	1hr	Facilitate lesson Lead group discussion	Facilitate lesson Lead group discussion	Facilitate lesson Lead group discussion
6	Student	1 hr	Math achievement post-test Spatial ability post-test	Math achievement post-test Spatial ability post-test	Math achievement post-test Spatial ability post-test
	Teacher	1 hr	Facilitate testing	Facilitate testing	Facilitate testing
		(20 min)	Semi-structured interview	Semi-structured interview	Semi-structured interview

On the first day of the study, the mathematics teacher administered the condensed CSDE (Appendix E) second grade mathematics achievement test and the Children's Mental Transformation Task Test (Appendix F) to all participating second grade students, classroom 1-3, prior to beginning the lesson on geometric properties and relationships.

The second day of the study, the classrooms 1, 2, and 3's teacher participant implemented the tangram lesson plan (Appendix A). Students discussed slides, flips, turns and demonstrated their understanding of these terms during whole class discussion. The instructor guided the students to understanding the terms translation, reflection, and rotation through interchanging the more complex words with slide, flip, and turn throughout the whole class discussion while using the appropriate manipulative for each classroom 1-3 to demonstrate terms.

After whole-class discussion was complete, each teacher arranged their students in the determined ability based groups of approximately two to three students per group. The teacher arranged each group around the appropriate manipulative for each classroom.

Classroom 1. Mrs. Peterson used handheld wooden tangram tiles with a tangram activity worksheet, "Concrete Tangrams – Week 1 Practice" (Appendix B) one class period per week over the course of four weeks. The wooden tangram tiles were accessible to students only during mathematics class time dedicated to tangram puzzles. The wooden tangram tiles were not issued for student checkout.

The third and fourth day of the study the students used the wooden tangram pieces to solve a series of four tangram puzzles with increasing difficulty and record their solutions on activity sheet "Week 2 Practice" and "Week 3 Practice" respectively for each day. Once all groups successfully solved all four puzzles, the students participated in whole-class discussion to review what they had learned. Mrs. Peterson asked students to reflect on their work. She guided students to discuss when it was necessary for students to use a flip, turn, or slide to solve their puzzle. Mrs. Peterson also asked students to consider other possible solutions, to compare and contrast solutions, and to consider if some of the puzzles were more challenging than others and why?

During Week Five, the fifth day of the study, one student from each ability based group high, average, and low, was selected to individually complete a task-based interview with a more difficult tangram puzzle, "Concrete Tangrams – Week 4 Practice" using the wooden tangram pieces that were audio and video recorded. The student was asked to verbalize their thinking while manipulating the pieces. The student then took part in a semi-structured interview regarding their perception of the concrete manipulative. The researcher conducted the student task-based and semi-structured interviews in the rear of the classroom at a small table with chairs. Students not selected to complete a puzzle individually continued working in groups to complete the "Week 4 Practice" activity sheet while using the wooden tangram tiles. During week six, the sixth day of the study the mathematics teacher administered the second part of the condensed CSDE second grade mathematics achievement and the post-test for the Children's Mental Transformation Task Test.

Classroom 2. Mrs. Edwards used the virtual dynamic tangram manipulatives with a tangram activity worksheet, "Virtual Tangrams – Week 1 Practice" (Appendix C) approximately one class period per week over the course of four weeks. Students used an iPad to complete tangram activities from JiuzhangTech Ltd's Apple iOS compatible free app (www.jiuzhangtech.com). The iPads were accessible to students only during mathematics class time dedicated to tangram puzzles. The iPads were not issued for student checkout.

The third and fourth day of the study the students used iPads and the Jiuzhangtech Ltd Tangram app to solve a series of four tangram puzzles with increasing difficulty and record their solutions on activity sheet “Week 2 Practice” and “Week 3 Practice” respectively, for each day. Once all groups successfully solved all four puzzles, the students participated in whole-class discussion to review what they have learned. Mrs. Edwards asked students to reflect on their work. She guided students to discuss when it was necessary for students to use a flip, turn, or slide to solve their puzzle. Mrs. Edwards also asked students to consider other possible solutions, to compare and contrast solutions, and to consider if some of the puzzles were more challenging than others and why?

The fifth day of the study one student from each ability-based group high, average, and low, was selected to complete a task-based interview with a more difficult tangram activity, “Week 4 Practice” using the iPad and the JiuzhangTech Ltd tangram app. The task-based interview session was audio and video recorded. The student was asked to verbalize their thinking while manipulating the pieces. The student then took part in a semi-structured interview regarding their perception of the virtual dynamic manipulative. Students not selected to complete a puzzle individually continued working in groups to complete the “Week 4 Practice” activity sheet while using the virtual tangrams and corresponding activity sheet. The sixth day of the study the mathematics teacher administered the second part of the condensed CSDE second grade mathematics achievement and the post-test for the Children’s Mental Transformation Task Test.

Classroom 3. Mrs. Green utilized the multimodal tangram manipulatives with the “OSMO Tangram” activity worksheet (Appendix D) approximately one class period per week over the course of four weeks. Students used an iPad to complete “OSMO Tangram -

Week 1 Practice" tangram activities on the OSMO gaming system from Tangible Play Inc. for the Tangram software app included with the gaming system. The iPads and OSMO gaming system were accessible to students only during mathematics class time dedicated to tangram puzzles. The iPads and OSMO gaming system were not issued for student checkout.

The third and fourth day of the study the students used iPads and the OSMO gaming system Tangram app to solve a series of four tangram puzzles with increasing difficulty and record their solutions on activity sheets "Week 2 Practice" and "Week 3 Practice" respectively, for each day. Once all groups successfully solved all four puzzles, the students participated in whole-class discussion to review what they had learned. Mrs. Green asked students to reflect on their work. She then guided students to discuss when it was necessary for students to use a flip, turn, or slide to solve their puzzle. Mrs. Green also asked students to consider other possible solutions, to compare and contrast solutions, and to consider if some of the puzzles were more challenging than others and why.

The fifth day of the study one student from each ability-based group high, average, and low, was selected to complete a more difficult tangram puzzle with the iPad and the OSMO Tangram app for the "Week 4 Practice" activity sheet that was audio and video recorded. The student was asked to verbalize their thinking while manipulating the pieces. The student then took part in a semi-structured interview regarding their perception of the virtual dynamic manipulative. Students not selected to complete a puzzle individually continued working in groups to complete the "Week 4 Practice" activity sheet while using the iPad and OSMO Tangram app and tiles. The sixth day of the study the mathematics teacher administered the second part of the condensed CSDE second grade mathematics achievement and the post-test for the Children's Mental Transformation Task Test.

Classrooms 1, 2, and 3. After the conclusion of the six week study, the second grade classes switched manipulatives until all classes had access to and experience with all three manipulative types. No additional quantitative data was collected during this time. All audio from the task-based interviews activities and semi-structured interviews was transcribed by the researcher within 60 days of recording. Table 5 shows the classroom activity timeline for the study.

Table 5

Timeline for classroom activities

Wk of study day	Classroom Activity
1	Teacher administered condensed CSDE 2nd grade mathematics pretest #1-16.
	Teacher administered Children's Mental Transformation Task pretest #1-16
2	Teacher guided instruction: Introduction to tangrams and geometric vocabulary
	High, average, and low ability based groups formed by teacher
	Students complete 4 puzzle activity, activity sheet, and classroom discussion
3	Students complete 4 puzzle activity, activity sheet, and classroom discussion
4	Students complete 4 puzzle activity, activity sheet, and classroom discussion
5	Selected students complete task-based interviews and individual semi-structured interviews
6	Teacher administered condensed CSDE 2nd grade mathematics pretest #17-32

Teacher administered Children’s Mental Transformation Task pretest #17-32
Teacher participant semi-structured interview over the manipulative type used in class

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Data Analysis

Quantitative Data Analysis

The quantitative data were analyzed to describe the research participants’ math achievement scores both before and after the spatial ability training. Using SPSS, both descriptive and inferential statistics were calculated. Descriptive statistics (e.g. means, standard deviations, standard error, with both lower and upper bound confidence levels of 95%) were computed for individual students and for each manipulative group. Data were tested for normality using the Shapiro-Wilks test which showed the data were not normally distributed. Thus, nonparametric statistics were used to analyze the data. The researcher used the Wilcoxon Signed Ranks Test with Z-test statistics to analyze changes in students’ mathematics achievement scores and spatial development scores. The Kruskal-Wallis Test with a median test of frequencies was used to explore differences in student scores based on manipulative type.

Qualitative Data Analysis

The researcher used the constant comparative method to analyze the qualitative data (Strauss & Corbin, 1998). The constant comparative method, “involves systematically comparing sections of text and noting similarities and differences between these sections” (Bloomberg & Volpe, 2012, p. 137). The researcher sorted and analyzed field observations and transcripts. The researcher used open coding to code all qualitative data by reading through transcripts line by line and jotting down the word that the researcher felt best described the action, tone, feeling, and overall meaning of each line. The researcher then used axial coding to identify an area of focus for the core phenomenon by thoughtfully examining the original codes and grouping them into categories (Strauss & Corbin, 1998). The researcher examined the categories for common threads such as statements, actions, theories, or feelings. Finally, the researcher narrated the findings in researcher memos and interpreted the results. The researcher organized the coding process with open codes, axial codes, and emergent themes as related to the research questions into a table (Appendix K).

Positionality. The researcher tried to reduce any bias by approaching the coding process with a clear and open mind to let the themes emerge from the data. The researcher brought certain biases to the study due to previous teaching experience and community involvement. The researcher has over eight years of experience teaching mathematics in secondary education, as well as two years of collegiate teaching experience in the fields of mathematics and teacher education. These experiences as a mathematics educator and an educator of future teachers have shaped her views on what effective mathematics practices should look like as well as views on effective teaching practices. The researcher is also a resident within the community in the school district where the study took place, and has two elementary aged children that attended the school under study; however, her children were not included as participants of the study. The researcher's ties to the school and the community have resulted in previous casual interactions with one of the teacher participants, as well as a handful of the student participants due to involvement in community and school sports functions. However, the researcher made an effort to bracket her experiences and let the lived experiences of the participants tell their story. The researcher ensured trustworthiness of results through triangulation of field notes, task-based interview data, and semi-structured interview data, as well as member checking teacher interviews.

Mixed Data Analysis

Within the convergent parallel design, “the researcher collects both qualitative and quantitative data during the same phase of the research process and then merges the two sets of results into an overall interpretation” (Creswell & Plano Clark, 2011, p. 77). Interpretation of the triangulated results discusses to what extent, and in what ways the qualitative and quantitative results converge or diverge (Creswell, 2009). The mixed data allowed for expansion on a sparse body of knowledge within the fast growing field.

Ethical Considerations

All task-based interview and semi-structured interview data were transcribed to ensure confidentiality of any identifying information of research participants. Pseudonyms were used when discussing specific student and/or teacher experiences to protect their identity, privacy, and confidentiality. The legal guardians of the research participants were made aware of the risks and benefits of participating in the study in advance and signed a consent form (Appendix J) that described in detail how confidentiality would be handled. Students were asked to sign an assent form prior to allowing their participation in the study (Appendix J). The students and legal guardians of the participants were also given the opportunity to withdraw from the study at any time.

Summary

A summary of the research questions, research instruments, and data analysis is provided below:

1. What influence, if any, do the different types of tangram manipulatives have on students' spatial sense and mathematics achievement? Inferential statistics computed for the study included the Wilcoxon Signed Ranks Test including the Z-test statistics, and the Kruskal-Wallis Test, which included a median test determined by frequencies. Qualitative data themes were examined for pertinence to geometry content standards and mathematical process standards to help inform results of the quantitative analysis.
2. What, if any, are the perceived benefits to students and teachers of using multimodal tangram manipulatives? The researcher used observational field notes, task-based interviews, and semi-structured interviews to gain qualitative data. Qualitative data were interpreted for themes, triangulated, and member checked to help inform results of the quantitative analysis.

CHAPTER IV

RESULTS

This convergent parallel mixed methods research study combined quantitative and qualitative data to examine the effects of concrete, virtual, and multimodal tangram manipulatives on 61 second grade elementary school students' mathematics achievement and spatial sense development scores. The study also examined the perceptions of the 9 elementary school students and 3 elementary school teachers regarding the effectiveness of the different types of manipulatives – concrete, virtual, and multimodal. The specific research questions guiding the study included:

1. What influences, if any, do the different tangram manipulatives (concrete, virtual, and multimodal) have on students' mathematics achievement?
2. What influences, if any, do the different tangram manipulatives (concrete, virtual, and multimodal) have on students' spatial sense?
3. What are second grade students' and their teacher's perceptions of the different types of manipulatives (concrete, virtual, and multimodal)?

First, findings on manipulative type and math achievement will be presented within this chapter. Second, findings based on manipulative type and spatial sense differences will be presented. Third, findings on the second grade students' and their teachers' perceptions of different types of manipulatives will be presented. Quantitative, qualitative, and connected findings will be addressed to include: quantitative findings from research data including pre/post CA Condensed STAR mathematics achievement scores, pre/post CMTT spatial sense scores, statistical analysis of manipulative group differences within math achievement and within spatial sense scores, as well as qualitative findings from research data collected from researcher field notes, task-based student interviews, semi-structured student interviews, and semi-structured teacher interviews.

Prior to gathering quantitative data on pre/post mathematics achievement and spatial sense, all second grade teacher participants addressed the same mathematics curriculum objectives. Students from all three participating classes had a basic understanding of two dimensional shapes to include identifying a triangle, square, and quadrilateral by the number of sides before the research study was begun. Students had not previously been introduced to the term, "parallelogram" or its identifying characteristics. The three teacher participants reported to the researcher that students had no previous experience with tangrams or any form of tangram manipulatives within the school setting.

The CA Condensed STAR mathematics achievement pre-test as well as the CMTT spatial sense pretest was administered at the start of the study to each student participant. The mathematics achievement and spatial sense pretests both contained 16 questions each, with each item worth a single point, for a total of 16 math achievement points and 16 spatial sense points.

Each student participant present for the last day of the study was also administered the 16 question mathematics achievement posttest and the 16 question spatial sense posttest. Each test item was worth a single point, for a total of 16 math achievement points and 16 spatial sense points. Since data from the mathematics achievement pre-posttest scores and spatial sense test were not normally distributed, SPSS was used to calculate non-parametric statistics.

Manipulative type and mathematics achievement

The first research question sought to determine if using different tangram manipulative types (concrete, virtual, or multimodal) had an effect on students' mathematics achievement and more specifically, their knowledge of geometric concepts. Participants were exposed to the assigned manipulative for one class period per week, over a period of four weeks, with two additional weeks, one at the beginning of the study, and one at the end of the study, dedicated to the mathematics achievement and spatial sense pre/posttests. Manipulative types were assigned as follows:

- Classroom 1, Mrs. Peterson: concrete manipulatives - wooden tangram blocks
- Classroom 2, Mrs. Edwards: virtual manipulatives - iPad tangram app by JiuzhangTech, Ltd.
- Classroom 3, Mrs. Green: multimodal manipulatives – OSMO gaming system with iPad and wooden blocks

Quantitative Results

Each group of second grade students was administered the mathematics achievement pretest. Data were separated into three groups by classroom manipulative type – concrete, virtual, and multimodal. Students from each class were identified as high, medium or low-level ability based on the CA condensed STAR mathematics achievement pretest scores (Table 6). Ability levels were determined by the following scores:

- High-level ability: 13/16 – 16/16 (80-100%)
- Medium- level ability: 10/16 – 12/16 (60-79%)

- Low-level ability: 0/16 – 9/16 (0-59%)

Table 6

Number and percentage of students per ability level based on math achievement pretest

Group	N	High-level	Medium-level	Low-level
Concrete	16	2 (12.5%)	9 (56.25%)	5 (31.25%)
Virtual	23	6 (26.09%)	11 (47.83%)	6 (26.09%)
Multimodal	22	3 (13.64%)	14 (63.64%)	5 (22.73%)

The researcher used the Kruskal-Wallis one-way ANOVA of Ranks Test, to determine if there were initial differences in the math achievement pre-test scores among the three manipulative groups as shown in the Mean Ranks Table (Table 7).

Table 7

Mean ranks of mathematics achievement pretest scores by manipulative groups.

Group	N	Mean Rank
Pre-test Concrete	16	28.59
Pre-test Virtual	23	33.72
Pre-test Multimodal	22	29.91

It was determined that there was not a statistically significant difference in second grade students' mathematics achievement pre-test scores among the different second grade classes ($H(2) = 0.942$, $p = .624$), with a mean rank of 28.59 for concrete manipulatives, 33.72 for virtual manipulatives, and 29.91 for multimodal manipulatives.

The researcher wanted to make sure that all student participants and groups had relatively the same ability level at the beginning of the study. Once it was determined that there were no previously existing differences among the three different manipulate groups, the researcher used the Wilcoxon Signed Ranks Test with Z-test statistics to analyze changes in second grade students' pre/posttest mathematics achievement scores for each group. The Wilcoxon Signed Ranks Test indicated that the median posttest scores were not significantly higher than the median pre-test score [$Z = -0.598$, $p > .05$] for the concrete manipulative group, [$Z = -0.750$, $p > .05$] for the virtual manipulative group, or [$Z = -1.617$, $p > .05$] for the multimodal manipulative group. The mean of ranks of posttest scores less than ranks of pre-test scores was 18.32. The mean of ranks of posttest scores were greater than ranks of pre-test scores was 29.73. Descriptive pre/post math achievement statistics for each manipulative group can be found in Table 8.

Table 8

Descriptive Statistics for Pre/Post Math Achievement (N = 61)

Group	Test	N	Mean	Median	St. Dev.	Min	Max
Concrete	PreMA	16	10.56	11	2.22	6	15
	PostMA	16	10.81	12	3.12	1	14
Virtual	PreMA	23	11.22	12	2.24	7	15
	PostMA	23	11.57	12	1.88	8	15
Multimodal	PreMA	22	10.50	11	2.61	4	15
	PostMA	22	11.23	11.5	3.12	4	15

In order to examine possible group differences in mathematics achievement between the three groups (concrete, virtual and multimodal), a Kruskal-Wallis Test was performed. It was determined that there were no significant differences in second grade students' mathematics achievement posttest scores among the different manipulative types – concrete, virtual, and multimodal ($H(2) = .244$, $p = .885$) see Table 9.

Table 9

Mean ranks of mathematics achievement posttest scores by group type

Group	N	Mean Rank
Posttest Concrete	16	29.19
Posttest Virtual	23	31.35
Posttest Multimodal	22	31.95

Qualitative Results

The influence of manipulative type on students' general mathematics achievement was also examined through the collection of qualitative data using field notes, video/audio recordings of classroom interactions, and task-based interviews. Emergent themes in the data were categorized into two major areas, geometric content and mathematical processes as they related to NCTM's (2000) standards as follows:

1. Geometry Content
    a. Shapes out of shapes
    b. Congruence
    c. Understanding of Transformations
2. Mathematical Processes
    a. Problem Solving
        i. Permanence

ii. Okay to be wrong

iii. Persistence

iv. Multiple solutions

b. Communication

Results are discussed by treatment group within each theme and then discussed as a whole.

Geometry content. NCTM's (2000a) Geometry Content Standards state:

> Instructional programs from prekindergarten through grade 12 should enable all students to –
>
> - Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
>   - Pre-K - 2 Expectations: In pre-K through grade 2 all students should -
>   - recognize, name, build, draw, compare, and sort two- and three-dimensional shapes;
>   - describe attributes and parts of two- and three-dimensional shapes;
>   - investigate and predict the results of putting together and taking apart two- and three-dimensional shapes.
> - Specify location and describe spatial relationships using coordinate geometry and other representational systems
>   - Pre-K - 2 Expectations: In pre-K through grade 2 all students should -
>   - describe, name, and interpret relative positions in space and apply ideas about relative position;

        - describe, name, and interpret direction and distance in navigating space and apply ideas about direction and distance;
        - find and name locations with simple relationships such as “near to” and in coordinate systems such as maps.
- Apply transformations and use symmetry to analyze mathematical situations
    - Pre-K - 2 Expectations: In pre-K through grade 2 all students should -
        - recognize and apply slides, flips, and turns;
        - recognize and create shapes that have symmetry.
- Use visualization, spatial reasoning, and geometric modeling to solve problems
    - Pre-K - 2 Expectations: In pre-K through grade 2 all students should -
        - create mental images of geometric shapes using spatial memory and spatial visualization;
        - recognize and represent shapes from different perspectives;
        - relate ideas in geometry to ideas in number and measurement;
        - recognize geometric shapes and structures in the environment and specify their location. (NCTM, 2000a, p. 96)

Shapes out of shapes. Student participants in all three groups demonstrated some extent of geometry content knowledge. Specifically, the students selected for the task based interview from each manipulative type were able to analyze characteristics and properties of two-dimensional geometric shapes by investigating and predicting the results of composing and decomposing two-dimensional shapes. The following specific instances portraying student analysis of characteristics were observed by the researcher during the task-based and semi-structured interviews.

While interviewing the concrete manipulative group participants, both Mary (low ability) and Colby (high ability) expressed knowledge of compositions of shapes. This was demonstrated by their responses when the researcher asked what they felt they learned about the shapes of the puzzle pieces. Mary stated, "…you could make other shapes out of the shapes." Whereas Colby discussed which shapes could be used to make other shapes, "the two small triangles can be used to create a square."

Within the virtual manipulative group, Susie (medium ability) began to struggle solving the tangram puzzle when she only had two shapes left – the two small triangles. The researcher asked her if she was stuck, to which she nodded her head, "Yes." The researcher asked Susie if the two remaining shapes would fit in the blank spot of the puzzle (which was a square shape), but she was unsure if they would work. The researcher asked Susie if she could fit the two small triangles together on the side of the screen (outside of the actual puzzle) to create the shape of the missing pieces (a square). Susie was able to quickly maneuver the two small triangles to create a square. Once she was able to see how to compose the square from the triangles, she was able to then move the pieces into the missing area of the activity to complete the puzzle. This observation showed that Susie was able to see the composition and decomposition of shapes outside of the puzzle, but had a more difficult time identifying the decomposed pieces when they were rotated from the original view within the puzzle.

One other student from the virtual group, Ethan (high ability), demonstrated an understanding of compositions and decompositions, but was also able to draw connections of his understanding to the initial tangram lesson. The researcher asked Ethan what he learned about the shapes used in the virtual tangram activities to which Ethan described not only the physical characteristics of the shapes and how many shapes were used in the puzzle, but he also referred back to the story, The Warlord's Puzzle, and stated, "you can make different designs with the pieces," showing that he viewed the complete puzzle pictures as compositions of all seven tangram pieces.

Within the multimodal group, McKenna (low ability) and Toby (medium ability) both exhibited understanding the ability to make 'shapes out of shapes.' Particularly, McKenna, when speaking about what she learned from working with the OSMO said, "[It] helps your brain a little bit, when you're doing these, you can make a square, you can use triangles." As she spoke, she moved the two small triangle pieces into the shape of a square to demonstrate to the researcher.

Toby's responses during the task-based and semi-structured interview indicated that he not only understood the geometry content, but that he was able to create new ideas based on his understanding of compositions and decompositions. Toby stated:

> "It would be cool if you could make your own shapes, like if the screen were blank, and you could make your own puzzles, and then, they would like appear on the screen instead of the black shapes. You could make houses, giant arrows, and it wouldn't matter which shape you used, as long as it matched the shape you wanted."

While he spoke he built a large arrow out of his tangram pieces. He then began to switch out the two smaller triangles for the square piece. His elaborate response and physical demonstration portrayed his understanding that pictures could be decomposed into smaller pieces of a whole.

Students in all three manipulate groups and across all three ability levels were observed investigating and predicting the results of putting together and taking apart two-dimensional shapes as illustrated by their completion of the tangram task-based interview, with the exception of Mary (concrete, low ability) who did not successfully complete the task. The researcher observed that all participants in the study were able to demonstrate at least a basic understanding of composition and decomposition since composition of shapes to create a new shape was inherent to the nature of the tangram puzzles themselves.

Congruence. Students from both the concrete and multimodal manipulative groups were observed showing an understanding of congruence through analyzing characteristics and properties of two-dimensional geometric shapes and comparing tangram pieces for likeness.

In the concrete manipulative group, Mary (low ability) exhibited a very basic understanding of congruence through her statement, "I realized that the shapes were exactly the same." Colby (high ability) physically manipulated his blocks to stack on top of each other while stating, "I'm looking to see if the shapes are the same as this one when I stick them together," when verbalizing his thought process during the task-based interview. The researcher noted that Colby had also identified both sets of congruent triangles in the puzzle during the semi-structured interview.

In the multimodal manipulative group, both Toby (medium ability) and Kevin (high ability) portrayed a solid understanding of congruence. Both participants were observed switching congruent pieces in the puzzle due to color preference. While solving the task-based interview tangram puzzle, Toby stated, "It's the same size, I could put it in either spot." Kevin also switched two of his congruent pieces in his puzzle while saying, "They were both the same size," and then muttered something about liking a certain color piece in the particular spot.

Both manipulative groups that indicated a strong understanding of congruence, the concrete and multimodal groups, shared a common thread of having physical pieces to manipulate that could be stacked to determine if the tangram pieces were exactly the same. The virtual manipulative group was unable to overlay pieces on the screen to determine congruence; however, the virtual group was able to place pieces side by side on the screen in the same orientation.

Understanding of transformations. Student participants in all three manipulative groups, concrete, virtual, and multimodal, were observed demonstrating the ability to apply transformations. Some students were able to verbalize their recognition of slides, flips, and turns, while others gestured the transformations with their hands.

Within the concrete group, Caleb (medium ability) stated that he knew how to, "turn, slide, and rotate," the shapes. Colby (high ability) did not verbalize his understanding of transformations; however, he was observed several times holding a wooden block over several other pieces in his puzzle, turning and flipping his piece trying to figure out where he should place it. Within the virtual group, Max (low ability) also demonstrated a knowledge of transformations by informing the researcher that he learned he could, "rotate, turn, and flip," the shapes. He also gestured small circle motions in the air with his index finger to for all three terms. Within the multimodal group, Toby (medium ability) verbalized his understanding of transformations by stating, "I need to rotate this one," while working the tangram puzzle for the task-based interview.

Although not all task-based interview student participants were able to verbally or nonverbally demonstrate recognition for slides, flips, and turns, they were all able to apply slides, flips, and turns to attempt to solve the tangram puzzles. This was demonstrated in their physical manipulation of the blocks to solve the concrete and multimodal puzzles, as well as the on-screen manipulation of shapes through screen taps and drags for the virtual puzzles.

Mathematical Processes. NCTM's (2000a) Mathematics Process Standards are broken into five categories of what instructional programs should enable students to perform including problem solving, reasoning and proof, communication, connections, and representation (p. 52). Emergent themes arose from the data that fell within the categories of problem solving and communication process standards to include: permanence, okay to be wrong, persistence, multiple solutions, and the communication of knowledge and processes (verbal and nonverbal).

Problem Solving. NCTM's (2000a) Mathematical Process Standard states:

> Instructional programs from prekindergarten through grade 12 should enable all students to –
>
> - Build new mathematical knowledge through problem solving
> - Solve problems that arise in mathematics and in other contexts
> - Apply and adapt a variety of appropriate strategies to solve problems
> - Monitor and reflect on the process of mathematical problem solving (p. 52).

Permanence. The researcher observed that students had a varying understanding of the permanence of their tangram tile placement choices. Students did not always recognize that their movement of tangram pieces did not have to be a permanent placement. Students' flawed perception of permanence of tangram pieces sometimes resulted in the necessity for the student to apply and adapt a different strategy to solve the tangram puzzles as demonstrated by Mary's experience. Mary (low ability), from the concrete manipulative group was observed starting completely over with her puzzle when she was down to one piece instead of simply moving or switching pieces within her puzzle.

Mrs. Peterson, the concrete manipulative group teacher, gave a more specific example of students' perceptions of permanence:

> "I think one of the disadvantages was that they were afraid to put that piece down, it was almost like it was a permanent decision. I don't think they realized that they could swap it out, because they felt like once they put that block there that it had to stay there. I think that was the biggest disadvantage for them, was that they felt like it was a permanent thing."

The researcher also observed this phenomenon on a whole-class level during the tangram activities in Mrs. Peterson's class, the concrete manipulative classroom. The researcher's field notes equated the misunderstanding of permanence with the concept of solving a jigsaw puzzle:

> "It's as if the students are treating solving the puzzles like solving a jigsaw puzzle, where there is only one possible placement for a piece, and you shouldn't force a piece where it doesn't belong."

The researcher also noted that high level ability students seldom viewed placement of a tangram piece as a permanent decision. High level ability students seemed more comfortable with using trial-and-error to solve a puzzle. Mrs. Peterson also felt that the high ability students were less likely to feel that tangram piece placement was permanent. Mrs.

Peterson was discussing several high ability students in her class that had been identified as 'gifted' when she stated:

> "They didn't think the pieces were permanent, they did have the concept more, more of moving the pieces, 'Oh, it didn't fit, it's not working, let's move this.' Whereas some of the other kids felt like once they put it down, that's where it had to stay."

Okay to be wrong. Students within all three manipulative groups were believed to have built new mathematical knowledge through problem solving. Caleb (medium ability), a student in the concrete manipulative group, informed the researcher that he learned, "Problem solving," to which the researcher asked him to clarify. Caleb stated, "It's okay to be wrong a couple of times." Colby (high ability) also demonstrated the belief that it was okay to be wrong when working with the concrete tangrams by continually placing and then moving his tangram pieces, never committing to a location until he was sure that piece belonged in the spot.

Ethan (high ability), a student in the virtual manipulative group, also demonstrated that it was okay to be wrong through his actions and verbal explanations while working the task-based interview tangram activity. Ethan stated, "No, that won't work, I'm going to try this one instead," while switching out tangram pieces. Ethan was observed using trial-and-error throughout his time solving the tangram puzzle.

Students within the virtual group seemed more comfortable with moving pieces around without a fear of being wrong. Mrs. Edwards illustrated her perception of the virtual manipulative group's casual approach to solving the tangram puzzles:

> "Mine weren't afraid to problem solve and try to figure it out. They just stuck the pieces there to see if they fit and if they didn't, they didn't worry about failure I

guess. They just tried to figure it out. Failure wasn't a problem, they didn't care. So let's try it again."

Mrs. Edwards also believed that the virtual tangram manipulatives were a benefit to her students' problem solving skills to show students that it was okay to be wrong.

> "They need problem solving in any skill they have, and math, they have to have it, and just trial and error. You're not going to succeed the first time, it's not going to be perfect and I think that they did fine with not being perfect. Then they would get it right, and then it would be perfect. Eventually it will happen."

Within the multimodal group, all three student interview participants displayed that they did not fear getting a "wrong" answer while solving the tangram puzzles. McKenna (low ability) demonstrated that she was okay with placing pieces in the wrong area by placing and removing pieces often, reflecting on her choice placements between moves when she stopped to examine the screen. Toby (medium ability) was also observed using the trial-and-error method often placing and removing his pieces until he completed the puzzle. Whereas Kevin (high ability), verbally expressed his comfort with switching out pieces and using trial-and-error by stating, "It's probably this one," as he continued to place and remove pieces until he successfully solved the puzzle. The students never displayed a feeling of defeat when their original choices of tangram piece placement did not work.

Mrs. Green described her students' reactions to incorrect solutions when trying to solve the tangram puzzles:

> "I think the OSMO helped the students with their spatial ability, and their thinking/problem solving skills. They would have to readjust or re-think the way they were solving a problem if they got down to the last two pieces and they wouldn't

work. They would try to trouble-shoot where they went wrong solving the puzzle, and fit the biggest pieces first."

Her observation of the multimodal group participants demonstrate that not only where the students okay with getting a "wrong" answer, but her students were also able to adapt their problem solving skills, and persevere, continuing to try new ways to solve the puzzles.

Persistence. Students were observed in all three manipulative groups of having varying levels of persistence to complete a puzzle; however, specific observations and statements by teachers and students are detailed to portray the level of persistence or lack of persistence for each manipulative group.

The researcher noted that Mary (concrete, low ability) was observed working slowly trying to solve the tangram puzzle and starting over twice after she was down to a single tangram piece when she determined it would not fit in the correct spot. Mary became frustrated and often asked for help from the researcher to solve the puzzle. After failing to successfully solve the puzzle, Mary asked the researcher, "Do I have to finish it?" Thus, demonstrating a lack of persistence to solve the puzzle.

The researcher also observed over the course of the study that some students within the concrete manipulative would group stop working a tangram puzzle that was challenging or frustrating and move on to another puzzle without completing the original puzzle. However, students within the virtual and multimodal manipulative groups would begin a tangram puzzle and continue trying to solve the puzzle until they were successful. It is important to note that students within the multimodal group would sometimes use the 'hints' on the OSMO program to work through a challenging puzzle.

Mrs. Green, the teacher for the multimodal manipulative group described her class's experiences with the OSMO and the students' persistence. She stated:

> "The students had fun going on to the next level and trying different puzzles. They even had to be stopped from going on when they were supposed to only be working specific puzzles on specific days. It was like a game."

Multiple solutions. None of the student interview participants in any of the three manipulate groups specifically stated that they noticed there were multiple solutions to the same puzzle; however, during whole class observations while they were working with the tangram puzzles, students in all three groups were observed by the researcher and teacher comparing answers and indicating different results.

Mrs. Peterson described her observation of the concrete manipulative class:

> "You could see them look at one, you know, they would be looking at people next to them, and they had solved it one way, and they had solved it a completely different way. I've been telling them this for three years. You don't always solve the problem the same way as your neighbor and they were able to see that."

Within the multimodal manipulative group, Toby (medium ability) and Kevin (high ability) both noticed that congruent shapes could be interchanged (as noted above in geometry content: congruence), and still successfully complete the puzzle. However, they did not articulate their perceptions as being an alternate solution to the puzzle, possibly due to the shapes being congruent, even though they were using different tangram tiles as indicated by the different colors.

Communication. The student groups were established to be collaborative in nature. However, it became apparent that students in the concrete and virtual manipulative classrooms were not accustomed to effectively communicating their mathematical knowledge or processes with their peers. During the semi-structured interview, the researcher asked each student participant, "How did your group work together to solve the puzzles?" Instead of working through the puzzles together and verbalizing their thoughts, two students from the concrete group, and one student from the virtual group made statements about having to 'take turns' moving the tangram pieces.

Students within both the concrete and virtual manipulative classes were observed expressing frustration stemming from communicating with their partner. Mrs. Peterson observed, "I think some of it [the challenge] was working with a partner and not being able to express to their partner, 'I'm frustrated and I can't figure this out.' That was some of it [the challenge]."

Although quantitative results on students' CA Condensed STAR mathematics achievement pre/posttest scores did not indicated a significant difference in students mathematical achievement, qualitative results from researcher field notes, video/audio recordings of classroom interactions, task-based interviews, and semi-structured interviews suggest that students' mathematics achievement was affected as shown in the emergent themes within geometric content and mathematical processes as related to NCTM's (2000a) standards. The qualitative results played an integral part of portraying student knowledge of geometric content to include their understanding of composition/decomposition of shapes, congruence, and transformations. Qualitative results also suggest that students' mathematical processes were affected in the form of problem solving skills and communication. The quantitative and qualitative data results will be further discussed in Chapter V of the study.

Manipulative type and spatial sense differences

The second research question sought to determine if using different tangram manipulative types (concrete, virtual, or multimodal) had an effect on students' spatial sense. Specifically, the researcher examined the data for differences in second grade students' spatial sense at the end of the study.

Quantitative Results

The researcher used the Kruskal-Wallis test to determine if there were significant differences in pretest student spatial sense scores among the three types of manipulatives – concrete, virtual, and multimodal. Pre-test spatial sense CMTT scores were calculated to determine any differences in group spatial sense prior to beginning the study (Table 10). It was determined that there were no significant differences in second grade students' pre-test spatial sense scores among the three types of manipulatives – concrete, virtual, and multimodal ($H(2) = .807$, $p = .668$).

Table 10

Mean ranks of spatial sense pretest scores by manipulative groups.

Group	N	Mean Rank
Pre-test Concrete	16	31.72
Pre-test Virtual	23	28.52
Pre-test Multimodal	22	33.07

The researcher used the Wilcoxon Signed Ranks Test to analyze changes in second grade students' CMTT pre/posttest scores for each treatment group. The Wilcoxon Signed

Ranks Test indicated that the median posttest scores were not statistically significantly higher than the median pre-test scores [$Z = -1.501$, $p > .05$] for the concrete manipulative group, and [$Z = -0.082$, $p > .05$] for the multimodal manipulative group. However, the CMTT spatial sense median posttest scores were statistically significantly higher than the median pre-test scores, [$Z = -2.753$, $p < .05$] for the virtual manipulative group. The effect size for this analysis ($d = .62$) was found to exceed Cohen's (1988) convention for a moderate effect ($d = .50$). The mean of ranks of posttest scores less than ranks of pre-test scores was 9.17 for the virtual manipulative group. The mean of ranks of posttest scores were greater than ranks of pre-test scores was 10.16 for the virtual manipulative group. Descriptive pre/post math achievement statistics for each manipulative group can be found in Table 11.

Table 11

Descriptive Statistics for Pre/Post Spatial Sense (N = 61)

Group	Test	N	Mean	Median	St. Dev.	Min	Max
Concrete	PreSP	16	13.75	14	1.438	11	16
	PostSP	16	14.50	14.5	1.211	12	16
Virtual	PreSP	23	13.22	14	1.953	9	16
	PostSP	23	14.35	15	1.668	10	16
Multimodal	PreSP	22	13.82	14	1.593	10	16
	PostSP	22	13.82	14	1.790	9	16

In order to analyze differences among the three different manipulative types – concrete, virtual, and multimodal, the Kruskal-Wallis test was repeated for second grade students' spatial sense posttest scores (Table 12). It was also determined that there were no significant differences in second grade students' posttest spatial sense scores among the three types of manipulatives – concrete, virtual, and multimodal ($H(2) = 1.629$, $p = .443$).

Table 12

Mean ranks of spatial sense posttest scores by manipulative groups.

Group	N	Mean Rank
Posttest Concrete	16	33.13
Posttest Virtual	23	33.13
Posttest Multimodal	22	27.23

Quantitative results from the CMTT spatial sense test displayed a statistically significant difference in students' spatial sense scores for the virtual manipulative group. Although the concrete manipulative group's mean and median scores both increased, the differences were not statistically significant in the study. Scores from the multimodal manipulative group showed no increase in mean or median scores from pre to posttest; however there was a change in the standard deviation of scores. These findings are further discussed in Chapter V to address any connections between the quantitative and qualitative data.

Students' and teachers' perceptions of the different manipulatives

The researcher conducted three semi-structured student participant interviews, and one semi-structured teacher participant interview in each of the three classrooms for a total of twelve responses to identify second grade students' and their teachers' perceptions of their assigned class manipulative. Field notes, audio/video recordings of student task-based interviews, and transcripts from the semi-structured student and teacher interviews were analyzed through open coding, axial coding, and identification of emergent themes.

During the open coding process, the researcher coded individual lines of the task-based and semi-structured student and teacher interview transcripts. Codes pertaining to student and teacher perceptions that arose from the initial reading of the transcripts included: challenge, frustrations, engaged, competitive, good attitude, eager, difficult, feedback from partner, feedback from teacher, feedback from iPad, frustrated with partner, pride, indifferent, faster, harder, easier, means to an end, finish, independent, focused on screen, iPads, and game. Next, during the axial coding process, the researcher reviewed the open codes multiple times and regrouped the open codes into the axial codes - feelings of frustration, problem solving, feedback, engagement, motivation, and hands-on/touch. Finally, the researcher sorted the axial codes into the major categories of: the need for feedback, engagement and motivation, and perceived advantages of each manipulative type. Results of the student and teacher perceptions data will be presented by treatment group within the following emergent themes:

- The need for feedback
- Engagement and motivation
- Perceived advantages of each manipulative type.

The need for feedback

The students' desire for immediate feedback was noticed not only by the researcher, but the teacher as well. The open coding process revealed several areas of feedback that were present in the study - feedback from peers, feedback from teacher and/or researcher, and feedback received from the iPads (in the virtual and multimodal manipulative groups).

Specific instances of teacher perceptions and student responses regarding feedback while working with the tangram puzzles are represented by manipulative group below.

Concrete. Within the concrete manipulative classroom, this desire for immediate feedback was often noticed when students were frustrated with the inability to successfully complete the tangram puzzle quickly, when students were not able to communicate with their partners on how the puzzle should be solved, and when students wanted the researcher or teacher to come by and see their actual puzzle to assess the correctness of the tangram puzzle. Mary (low ability) demonstrated a need for immediate feedback when she became frustrated with not being able to solve the tangram puzzle during the task-based interview. She often asked the researcher for help, eventually asking, "Do I have to finish it?"

Field notes and the video/audio recordings of the activity sessions showed that during completion of the first activity worksheet, there were no students asking for feedback from the teacher or researcher; however, week one included the easiest puzzles that also had a color representation of where each tangram piece should be placed (Appendix B). As the puzzles grew more difficult during Week 2 Practice, the researcher noted there were four instances of students requiring teacher feedback when frustrated, and three instances when students sought feedback from their peers other than their partner. During Week 3 Practice, the researcher noted ten instances of students asking for feedback from the teacher or researcher, and four instances of students asking peers other than their partner for feedback on how to complete the tangram puzzles. The need for feedback from peers other than their partners and from the teacher and/or researcher were generally during times when the students were not able to communicate with their partner on how to solve the puzzle, but some instances of feedback were to verify if they had the puzzle correct.

One particular instance of a student requesting immediate feedback involved Colby (high ability) during his task-based interview. Immediately after completing the concrete manipulative tangram puzzle, Colby was recorded saying, “There, I’m done, did I get it right?” to which the researcher replied that his puzzle looked very good. The researcher noted in the field notes that he seemed glad to have his puzzle confirmed as ‘correct’ by the researcher.

Virtual. The virtual manipulative group also initially desired immediate feedback by wanting to show the teacher or researcher their successful completion of a puzzle. However, their desire for feedback began to diminish after they completed a few puzzles and realized that the iPad would show the “Congratulations!” graphic each time a puzzle was completed correctly. Mrs. Edwards described her perception of the students’ need for immediate feedback when she stated:

> “Well, the first day you brought them [the iPads] and they all started figuring them out, they wanted us to see it. They wanted to come over and see that they had done it. So, then after a few times they did it, and they would hear the music, but a few of them wanted you to still come over and see what they did.”

The occurrence of the students’ need for feedback was also recorded by the researcher during review of the audio/video recordings. Similar to the concrete manipulative group, during the first week of tangram activities the students used the accompanying Week 1 Practice worksheet that included a color depiction of where each tangram piece should be placed to successfully complete the puzzle. Since students were able to self-check their work from their worksheets there were no instances of students requiring feedback from the teacher or other peers to verify if their puzzles were correct. However, when completing the

Week 2 Practice tangram activities the researcher counted two instances of students seeking feedback, and one instance of students seeking feedback from a peer other than their partner. The researcher also noted in her field notes that there was not a single instance of students asking the teacher or researcher if their puzzle was solved correctly for the virtual manipulative group throughout the study or the three task-based interview participants. Mrs. Green noticed the students' ability to work independent of feedback from the teacher or researcher. She commented, "Then they figured them out and they were, they would just go."

Multimodal. Students within the multimodal group rarely requested teacher feedback while working with the OSMO. The researcher noted that after the initial overview on how to operate the OSMO, there were no instances of students asking for help from the teacher or researcher while working on the Week 1 Practice worksheet (Appendix B). After reviewing the video/audio recordings, the researcher counted a total of five instances of students asking the teacher or researcher for feedback during the Week 2 Practice; however, of the five instances, two of the instances were related to technology issues – one iPad would not load the software program completely and one group was trying to complete puzzles in the 'Introduction' area of the program instead of in the 'Play' area. There were no instances of students requesting feedback during Week 3 Practice or during the task-based interview sessions of week 4. The OSMO Tangram App was programmed to give instant feedback in the form of flashing lights and music when a tangram puzzle is completed correctly. Additionally, the OSMO is capable of giving 'hints' on how to solve a puzzle if the 'hint' icon is selected by the user. Hints are stored in a 'hint bank' only after prior successful completion of other tangram puzzles. Students could only use as many hints as they had earned. It is important to note that some students in the class were observed using the 'hints' available within the OSMO program for feedback when they were struggling to determine a solution between their group members; however, none of the task-based interview participants used the hints while completing the tangram puzzle while observed by the researcher.

Immediate feedback was valued by both the students and the teachers as evidenced by specific statements during the interviews. When asked what she liked the most about the OSMO McKenna (low ability) stated, “It lights up when you have them in the right place on the other levels, but not on the blue level, because it was kind of hard.” The multimodal manipulative group teacher, Mrs. Green also valued the availability of the instant feedback on OSMO. She stated, “Students got very excited when they would complete a puzzle, and would jump up or exclaim, ‘YES!’ I didn’t have to go around and tell each student if they were right or not on the OSMO. It gave them instant feedback.”

Toby (medium ability) and Kevin (high ability) did not specifically state that they liked that the OSMO gave instant feedback, but they did show a physical reaction to OSMO’s feedback when it indicated that they had successfully completed the puzzle. When Toby correctly solved his tangram puzzle the OSMO lit up and played music while Toby exclaimed, “Alright! It finally worked!” as leaned back and looked up at the researcher with a smile. Kevin was making a few adjustments to the spacing in his puzzle when the OSMO indicated that he had successfully solved the puzzle by lighting up and playing a song. Kevin immediately reacted by squealing in a high-pitched voice, “I did it!” while pushing one arm into the air in victory.

Engagement and motivation

All three manipulative group teachers felt that the tangram manipulative assigned to their classroom both engaged and motivated their students. However, the researcher was able to observe all three manipulative group and note varying levels of student engagement and motivation. Teacher perceptions of their level of student engagement and motivation will be discussed by class as well as specific examples of the level of student engagement and motivation as taken from researcher field notes as well as task-based and semi-structured student interviews.

Concrete. The researcher had specifically asked Mrs. Peterson if she could describe the student engagement that she observed during her classes experiences with the concrete tangram manipulatives. Mrs. Peterson responded:

> "I felt the kids were very engaged, it was a new activity, something that they had never done before. I felt like they were eager to solve the problems kind of almost a little competitiveness came out in them. I think they stayed equally engaged the entire time, they wanted to complete the project."

Although her response indicated that the students were engaged in the manipulatives, the students did not indicate that concrete manipulatives were exciting to work with. Particularly, Mary (low ability), who showed an indifference to the manipulatives when asked what she thought about the concrete tangram puzzles. Mary shrugged her shoulders and implied that they were, "Okay," however, she did not appear excited about completing the activities. Caleb (medium ability) echoed her sentiments when he stated, "These would be more fun on an iPad because it would be easier." He went on to say they were fun and, "better than doing the other math." This attitude may be indicative of being engaged in the manipulative not because it was fun or intriguing, but rather, better than the procedural math done on a more regular basis.

The researcher noted that students would often begin cleaning up their supplies as soon as they were done with the required tangram activities for the day. Students were often overheard by the researcher asking to read a book instead of continuing to play with the blocks until they were instructed to pick them up. The researcher perceived these requests to do other work as the students viewing the puzzles as an activity to complete, a means to an end, instead of something in which they wanted to engage.

Virtual. Mrs. Edwards was asked by the researcher to describe the level of student engagement in her classroom while working with the virtual manipulatives. She responded, “I think it [student engagement] was above the norm. They were 100 percent involved. They loved it!” She went on to elaborate, “[There were] Very few redirections. They would stay on task.” When asked about student attitudes while working with the tangrams, Mrs. Edwards again mentioned student engagement, “They were engaged. They worked well with other partners so there were very few arguments or incidents.”

Throughout the study the researcher observed Susie (medium ability) as being a slightly withdrawn student with few connections with her peer in the classroom; however, during the task-based interviews the researcher noted that Susie stated, “I like that I get to work with a partner to help me.” Susie may have been motivated to be more engaged with the math activity since it enabled her to work collaboratively with a partner.

Multimodal. When asked to describe her class’s level of engagement with the OSMO manipulative, Mrs. Green responded:

> "The students were very excited to get to play the OSMOs. They looked forward to each day that they would be working with the block and iPads. The students did not need reminders to stay on task, and would try to work ahead if they finished the required activities for the day.

Mrs. Green elaborated that, "All students in the study actively participated. There was never a time when students sat-out or refused to participate. When asked to describe student attitudes she her response indicated that the OSMO encouraged student motivation when she said, "All of the students were excited to play with the OSMO. They were ready to solve the puzzles, and most of them looked forward to completing a level to be even more challenged on the next level." She continued, "A few students got frustrated with the harder level, but were ultimately even more excited when they were able to solve the puzzles on a hard level." Mrs. Green mentioned student engagement and motivation again when the researcher asked if she had noticed a change in motivation. She replied:

> "There was a slight change. Some of the students that were typically not excited to complete math work, were excited to get to work on the iPads and with the blocks. The students stayed on task until the task was complete.

At the conclusion of the interview, Mrs. Green was asked if there was anything else she would like to add, to which she chose to illustrate an instance where the OSMO increased motivation for several lower-level ability students that typically struggle with math, "The OSMO allowed them to feel successful in math, a subject they normally struggle in."

Although there were not any specific student statements regarding engagement or motivation there were researcher notes that described the level of student engagement and motivation while working with the OSMO. The researcher noted that all three student interview participants were completely engrossed in the tangram activity during the task-based interview. Additionally, throughout the study, students in the multimodal manipulative group continually displayed a desire to work with the OSMO, protesting when it was time to put them away, and continually asking if they could go on to another level of tangram puzzles.

Perceived advantages of each manipulative type

Concrete. A common theme that emerged from the concrete group regarding a perceived benefit for using the concrete manipulatives was that the students were able to touch and manipulate the tangram pieces. When asked what she perceived to be a benefit of the concrete manipulatives, Mrs. Peterson stated, “I guess just able to move, you know, they’re actually hands-on, they’re able to manipulate them.” Mrs. Peterson went on to describe some of her lower-level ability students’ experiences with the concrete manipulatives stating:

> “I have some that are lower-level that actually surprised me and were able to manipulate a little bit better than I thought they might do. I think just because it was hands-on, they were able to do that, manipulate them, rather than look at a piece of paper and say, this shape fits right here, they were able to manipulate a little bit more.”

Several students also indicated that they enjoyed the ability to touch the tangram pieces. Mary (low ability) said, “I like that we get to touch them.” Colby (high ability) also shared that he enjoyed being able to manipulate the pieces hands-on, “I like that we get to touch the pieces and move them around.” However, he also asked, “Are we going to get to work with the iPads?” indicating that he might enjoy working with technology as well.

In addition to the physical aspect of the concrete tangram puzzles, Mrs. Peterson stated that she would like to do the activity again in the future, "I think it would be nice, because I think it's just a different type of problem solving skill. Not just computation, but also their visual perception of how shapes fit into that outside." This indicated that she also saw value in the problem solving skills and spatial sense experienced by the students during the tangram activity.

Virtual. Within the virtual group, both the students and the teacher viewed the iPads as an advantage in working with the virtual tangram manipulatives. Mrs. Green demonstrated her view of technology as a benefit to her students as portrayed in her statement regarding the iPads, "The iPads were a hit in here. They loved it. I don't know what the other groups were like, but they liked the iPads because they're used to the electronics and they enjoyed it!" However, she was concerned that the student would not be able to continue doing the virtual tangram activities unless they had an iPad at home since the school did not have access to iPads for use in the classroom.

Without being prompted or asked about iPads at home, both Susie (medium ability) and Ethan (high ability) spoke to the researcher during their task-based interviews that they liked playing on iPads and that they had an iPad belonging to a parent in the home. Susie informed the researcher that she sometimes played games at home but not one like the app used in class. Ethan stated, "My mom has an iPad at home, and I get to play on it sometimes, but I don't have this game. I have an iPod and play games on it though, but not this one." Susie and Ethan's impromptu discussion of iPads led the researcher to believe that they viewed being able to use an iPad in math class as an exciting opportunity.

Other benefits from working with the virtual manipulatives from the teacher's perspective included the ability to use different problem solving skills. Mrs. Green stated:

> "Mine weren't afraid to problem solve and try to figure it out, they just stuck the pieces there to see if they did fit, and if they didn't. They didn't worry about failure, they just tried to figure it out. Failure wasn't a problem."

The researcher equated this viewpoint to the idea that students knew they could 'virtually undo' a piece if they did not think the puzzle was correct. Mrs. Green elaborated, "You can move it and it's done, and they can pull another piece up."

Multimodal. Mrs. Green shared her perceptions on the advantages of using the multimodal manipulative in her classroom. She felt the OSMO encouraged engagement as well as thinking skills as demonstrated in her statement during the semi-structured interview:

> "I think the OSMO was a great manipulative. I think it held their attention and made them use their critical thinking skills. They never felt like they were doing 'work' but at the same time the puzzles were very challenging. The students had fun going on to the next level and trying different puzzles. It was like a game."

Mrs. Green also viewed the instant feedback from OSMO and students' reactions to the OSMO as being an advantage to using the multimodal manipulative in her classroom. Mrs. Green stated, "Students got very excited when they would complete a puzzle, and would jump up or exclaim, 'YES!' I didn't have to go around and tell each student if they were right or not on the OSMO. It gave them instant feedback." McKenna (low ability) also showed an appreciation for the instant feedback as discussed previously by her statement, "It lights up when you have them in the right place on other levels, but not on the blue level, because it was kind of hard."

Mrs. Green also viewed the OSMO as a way to differentiate students' activities depending on their ability level. Mrs. Green expressed the OSMO was able to challenge Kevin (high ability), one of her students that had been identified as gifted. She stated that Kevin was very good at computing mathematical equations, but struggled with using hand-held manipulatives or anything that required fine motor skills and would get very frustrated when it came to using more than just his mind to complete a task.

Conclusion

This study examined 61 second grade students and three second grade mathematics teachers to explore the effects of concrete, virtual, and multimodal tangram manipulatives on the elementary students' mathematics achievements and development of spatial sense in addition to the experiences and perceptions of the use of the three different types of manipulatives as portrayed by nine of the 61 students' and the three teacher interviews. At the beginning of the study, quantitative data showed that there were no statistically significant differences in student mathematics achievement scores or their spatial sense scores among the classroom groups.

Both quantitative and qualitative data were collected to explain the results of the study. Although there were no quantitative differences among the post-test mathematics achievement scores, the qualitative data suggested that there may be specific advantages individual to each manipulative type. Themes were found within the qualitative data related to both geometric content standards and the mathematical process standards as outlined by NCTM's Principles and Standards (2000a). Themes examined within geometric content included: shapes out of shapes, congruence, and visualization of transformations. The emergent themes suggest that although a quantitative difference in mathematical achievement is not present, there are distinct portrayals of student mathematics achievement as categorized by geometric content and mathematical processes within the different manipulative groups. When examining the qualitative data for proof of mathematical achievement related to mathematical process standards, a major category of problem solving was used to detail the emergent themes within the data of: permanence, okay to be wrong, persistence, multiple solutions. Additionally, the ability (or lack of ability) for students to communicate within their groups was found to both an emergent theme in data as it pertains to the mathematics achievement of students based on the NCTM Mathematical Process Standards.

Quantitative data concluded there was a statistically significant difference in spatial sense scores for the virtual manipulative group; whereas, the concrete and multimodal manipulative groups did not show a statistically significant difference from pre to posttest scores. However, there were no statistically significant differences among the groups' posttest scores indicating that the pre to posttest difference in spatial sense scores for the virtual group was still not enough to set them apart from the concrete and multimodal group. Qualitative data collected from the three different manipulative type groups indicate that there are emergent themes of: need for feedback, engagement and motivation, and perceived advantages of each manipulative group that may suggest differences in spatial sense development not shown in the quantitative data.

Chapter V will discuss a summary of results, conclusions, implications, and recommendations for future research.

CHAPTER V

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Researchers agree that the use of manipulatives in mathematics increases mathematics achievement and plays a large part in student learning, understanding, and conceptualization of simple to complex concepts (Cooper, 2012; Kamii et al. 2001, Siew et al., 2013). However, they have yet to agree on which type of manipulative is the most effective as evidenced by Siew, et al.'s (2013) support of concrete manipulatives, while Lin, et al. (2011) support the use of virtual tangrams. Recent research by Siew, et al. (2013) suggests using physical manipulatives as a way to aid students to transition from one level of thinking to another within the van Hiele's constructivist view of knowledge. Whereas, Lin, et al. (2011) suggests that virtual tangrams support collaborative learning and aid to bridge the gap between high and low-level ability students. Additionally, Lin, et al. (2011) also suggested that virtual tangrams supported interdependent learning environments.

The goal of this research study was to address the use of emerging technological resources as manipulatives in the classroom and to determine if there was an advantage or disadvantage to using the new emerging technology of virtual manipulatives, specifically virtual tangram manipulatives and multimodal OSMO Tangrams. Differences in students experiences among the three types of manipulatives – concrete, virtual, and multimodal, were also explored.

The questions guiding this study were:

1. What influences, if any, do the different tangram manipulatives (concrete, virtual, and multimodal) have on students' mathematics achievement?
2. What influences, if any, do the different tangram manipulatives (concrete, virtual, and multimodal) have on students' spatial sense?
3. What are second grade students' and their teacher's perceptions of the different types of manipulatives (concrete, virtual, and multimodal)?

The convergent parallel mixed methods design used both quantitative and qualitative data. Participants in the study included 61 second grade elementary students who completed both the pre/post-test of mathematics achievement and the pre/post-test of spatial ability. Throughout the six week study, the researcher compiled field notes during class observations of the students' experiences with the three different types of manipulatives – concrete, virtual, and multimodal. Of the 61 student participants, nine students took part in task-based interviews and semi-structured interviews. Additionally, semi-structured interviews with the three participating second grade classroom teachers were also completed.

The discussion of the results of the study is organized first by the tangram type's influences on students' mathematics achievement, second, tangram type's influence on students' spatial sense, and third, teacher and student perceptions of manipulative type. Quantitative, qualitative and connected findings will be discussed.

Tangram type's influences on students' mathematics achievement

General Mathematics Achievement.

The first research question sought to explore the influence of different tangram manipulatives (concrete, virtual, and multimodal) on students' mathematics achievement.

Quantitative data were collected to answer this portion of the question. Data were analyzed using the Kruskall Wallace Test to explore group differences on the CA Condensed STAR Mathematics Achievement test both before and after the learning experience. The pre-post differences were investigated using the Wilcoxon Signed Ranks Test.

Quantitative data results indicated that there was no statistically significant difference between second grade students' CA Condensed STAR mathematics achievement pre and posttest scores at the $p < .05$ level. Given the short six week/four day instruction duration of the study, results of no statistical significance in this area are not a surprise. Research supports intensive interventions are available to increase mathematics achievement, but none specific to spatial sense due to children's beginning development of spatial skills through experiences over time, often before reaching school-age (Jacobse & Harskamp, 2011)

While there were not statistically significant gains in students' mathematics achievement as measured by the CA Condensed STAR pre/posttest scores, students did demonstrate gains in geometric content and mathematical processes not measured through the computational mathematics problems. Students' knowledge of geometric content and mathematical processes is of great importance since, "As students become familiar with shape, structure, location, and transformation and as they develop spatial reasoning they lay the foundation for understanding not only their spatial world, but also other topics in mathematics and in art, science, and social studies" (NCTM, 2000a, p. 97). Quantitative differences may be better measured with a different form of assessment that specifically addresses the communication and understanding of mathematical geometric concepts and terms, such as the geometric thinking test used in Siew et al.'s (2013) study on the facilitation of students' geometric thinking through van Hiele's phased based learning.

Knowledge of Geometric Content and Mathematical Process Standards

To assess students' knowledge of NCTM's (2000a) Geometric Content and Mathematical Process Standards during classroom instruction researcher field notes and subsequent task-based and semi-structured interviews were examined. The analysis of these findings resulted in emergent themes within two major categories – geometric content and mathematical processes. Themes identified within geometric content included – shapes out of shapes, congruence, and understanding of transformations. Themes identified within mathematical processes included problem solving and communication. The theme of problem solving included subcategories of permanence, okay to be wrong, persistence, and multiple solutions, Discussion of the results of the emergent themes and how they relate to prior research is addressed below.

Geometric content. Qualitative data suggested that the experiences with tangram manipulatives, regardless of type, provided opportunities for students to develop their geometric understanding from Level 0 – Visualization, all the way up to in some instances Level 2 – Informal Deduction, as outlined in van Hiele's model of geometric understanding (Breyfolgle & Lynch, 2010). This was evidenced by students' ability to not only see the geometric shapes as a whole – visualization, but to also recognize that each shape has different properties – analysis, and to see the interrelationships between figures – informal deduction.

Shapes out of shapes. The ability to decompose and compose shapes is an essential concept for students to master. It helps students develop the foundation for understanding geometry, part-whole relationships, and fractions (Clements & Sarama, 2014). Tangram puzzles help students develop this important skill specifically (Butler, 1994). In this study, the type of tangram manipulative made a difference in students' ability to decompose and compose shapes. Students within the multimodal and concrete groups who manipulated the physical tangram blocks were more likely than the virtual tangram group to be able to demonstrate the geometric concept of composing and decomposing shapes. Some students within the concrete and multimodal groups were able to physically demonstrate the composition and decomposition of shapes while communicating using the geometric academic language. Observations of students verbally describing specific transformations such as flip, turn, slide, while manipulating shapes are supported by Copley's (2010) suggestions that thinking spatially also includes the ability to use spatial vocabulary.

Congruence. Young children develop the ability to determine congruence of shapes at an early age by examining whether the shapes are mostly similar or not. However, Clements and Sarama (2014), indicated that "until about 7 years of age, students may not attend to the spatial relationships of all the parts of complex figures" (p. 149). Through guided experiences, such as OSMO and other tangram explorations, students can develop strategies for verifying congruence. In this study, it is postulated that deeper understandings of congruence of two-dimensional shapes were more pronounced within the concrete and multimodal manipulative groups due to their hands-on ability to physically manipulate a tangram tile while solving the puzzles. These findings agree with Fuys et al.'s (1988) study that concluded that the use of hands-on manipulatives within the learning of geometry are needed to effectively transition through the van Hiele's levels.

Understanding of transformations. Spatial skills, such as transformations, "support children's learning of specific topics, such as geometry and measurement, but they can also be applied to mathematical problem solving across topics (such as the use of a number line in arithmetic)" (Clements & Sarama, 2014, p. 127). Students that were able to touch, feel, turn, rotate, and flip the wooden tangram blocks not only used the correct terminology of the geometric movements of shapes such as "turn," "flip," and "rotate," and the use of "parallelogram," to identify specific quadrilaterals were also more likely to be able to physically demonstrate their understanding of such ideas by stacking blocks to test for congruence and shape composition/decomposition. This finding was similar to Moyer-Packenham, Salkind, and Bolyad's (2008) suggestion that experiences with tangram puzzles provide opportunities for learners to explore transformations such as translations, reflections, and rotations. Whereas, students within the virtual only manipulative group were found to gesture generally unrelated circling movements with the fingers when trying to communicate movements of transformations.

Mathematical processes.

Problem solving. Problem solving is an integral part of education as it allows student to, "use newly acquired knowledge in meaningful, real-life activities and assists them in working at higher levels of thinking" (Fredericks, 2005, p. 152). All three teachers in each of the manipulative groups viewed their assigned manipulative as beneficial to aid in student problem solving skills as demonstrated in their responses to the semi-structured interviews. When viewing the emergent themes as subcategories of problem solving, it was evident that the subcategories of permanence, okay to be wrong, persistence and multiple solutions each addressed various aspects of skills viewed important to problem solving. Students need to understand that tangram piece placement is not a permanent decision, that sometimes their first choice may not correctly solve the puzzle. Aiding students to explore problem solving as a process may allow them to understand that it is okay to be wrong, but when persevering through multiple solutions, they are bound to identify new strategies to complete the tangram.

Permanence. The psychological understanding of object permanence is described by Piaget as the knowledge of objects still being present even when a child cannot see them (1952). Piaget contends that children's knowledge is gained from interactions between the child and the object. Although the idea of permanence as observed within the study is not on objects hidden from view, the researcher postulates that students must learn of the opposite of permanence - an object's mobility, through interactions with the objects as well. The study suggests that students in the concrete manipulative group were less likely than the virtual or multimodal groups to experiment with tangram block placement, as indicated by a feeling of permanence once they arranged their blocks. This phenomenon is best described as students behaving as if they were solving a jigsaw puzzle: Students did not want to place a block in an incorrect position where it did not "fit." This finding has not been found in the extant literature on tangram manipulatives indicating the need to explore this finding in more depth in future studies.

Okay to be wrong. According to NCTM Principles and Standards (2000a), student must be able to "adapt a variety of appropriate strategies to solve problems" (p. 52). Even when students get incorrect answers they are building their mathematical knowledge on what did not work, so they can more forward and try another approach that may work. All three manipulative groups are believed to have built new mathematical knowledge through problem solving. Specifically, through realizing their answers were incorrect and having to trouble-shoot their original choices of tangram piece placements. The lower ability students within the concrete group struggled with the idea that tangram pieces were not permanent, and would show frustration when their first plan of action did not work. However, the medium and high ability students seemed more aware of the ability to switch out pieces if they were not correct. It is suggested that the use of manipulatives to learn geometry aids in the transition to higher level thinking as categorized in Van Hiele's theory of geometric thinking Fuys, et al. (1988). Practice with manipulatives could therefore allow the lower level students to progress to the same understandings of piece mobility as held by the higher level students.

Students within the virtual and multimodal group did not seem to become frustrated with wrong placements of tangram pieces. This could be attributed to viewing the tangram puzzles as games rather than mathematical tasks. Research on gaming technology by Kebritchi, et al. (2010) supports these findings of modern mathematics computer games as an effective tool for improving mathematical understanding and skills due to the games being experiential in nature.

Persistence. There is great value in persistence when problem solving as illustrated by Seeley (2015) when she stated, "As students engage in constructive struggling needed for some of these problems, they learn that perseverance, in-depth analysis, and critical thinking are valued in mathematics as much as recall, direct application, and instant intuition" (p. 114). It was found that students in the virtual and multimodal manipulative groups were more likely to persevere through more difficult puzzles, gaining more spatial practice than the concrete manipulative group as supported by Burns and Hamm's (2011) findings that virtual tangrams assisted students in moving through van Hiele's phase-based learning from level 0 of visualization, to level 2 of geometric thinking, analysis.

Multiple solutions. The processes involved in solving problems include enabling students to both, "apply and adapt a variety of strategies to solve problems," and, "monitor and reflect on the process of mathematical problem solving" (NCTM, 2000a, p. 52). Students within the concrete and multimodal manipulative group showed instances of identifying multiple solutions to single puzzles. These instances were particularly apparent within the concrete manipulative group as evidenced by Mrs. Peterson's statements regarding students witnessing their peers solving puzzles differently than they had. Students within the multimodal group indicated that they understood they could use different colored congruent pieces in alternating locations, but did not verbalize their recognition of this being an alternate solution. These findings agree with qualitative results from Siew and Abdullah's (2013) recent study regarding tangrams as beneficial to aid in students to view multiple solutions by their peers.

Communication. The importance of students' ability to, "specify locations and describe spatial relationships" is outlined in NCTM's Geometry Content Standards (2000a, p. 96). Without proper communication skills and knowledge of the academic language used within transformations, students may struggle with communicating while problem solving. Results of the study suggested that mainly lower-level ability students out of all three manipulative type groups struggled with communicating as a team to solve the puzzle. This could be attributed to their inability to express mathematical processes. Additionally, students who were using virtual only tangram manipulatives were forced to use whatever geometric academic language skills (no matter the skill level) they had to communicate since they did not have physical blocks to physically demonstrate what they were trying to communicate. This finding helps to explain how participants within Evans, Feenstra, Ryon,

& McNeill's (2011) multimodal approach to studying coding discourse developed a higher-level of group communication when working in a computer-based setting as compared to a physical setting when solving tangram puzzles.

Collaboration among student groups was reported in each manipulative type classroom by the teacher participant. All three teacher participants noted that they were surprised with how well their students worked within their specific manipulative group. However, the students viewed their group collaboration as taking turns moving the pieces. The students failed to view their group discourse as part of the group collaboration to solve a puzzle. Additionally, when students were asked how their group worked together to solve the puzzle, most students were, "not sure." Students such as Max and Ethan failed to see how their arguments or disagreements over shape placement played a part in their learning and understanding of geometric academic language, or movement though van Hiele's phases of learning.

It was also noticed that both the concrete manipulative and virtual manipulative students struggled to learn to collaborate through speaking with their partners. Students in both groups verbalized taking-turns instead of working collaboratively on the puzzles. This could be due to the highly structured atmosphere of the concrete and virtual classes as generally quiet classrooms with little to no instances of peer to peer interaction encouraged prior to the study. Students within the multimodal manipulative group were accustomed to speaking with their peers due to the casual atmosphere of the class prior to the study. While learning through the various manipulative types is important, teacher practices could hinder students learning by limiting opportunities for students to communicate their thinking with their peers. Classrooms with high quality mathematics talk provide opportunities for students to “use others as resources, to share … ideas with others, and to participate in the joint construction of knowledge” (Smith & Stein, 2011, p, 1). Having the opportunity to collaborate and communicate with their peers in a shared problem-solving experience may have had an influence on their development of mathematical knowledge.

Tangram type’s influences on students’ spatial sense

Overall spatial sense. The importance of spatial sense lies within one’s ability to know and function within the world around us (Cross, Woods, & Schweingruber, 2009). Spatial skills such as transformations "support children's learning of specific topics, such as geometry and measurement, but they can also be applied to mathematical problem solving across topics (such as the use of the number line in arithmetic)" (Clements & Sarama, 2014, p. 127). The second research question sought to explore the influence of different tangram manipulatives (concrete, virtual, and multimodal) on students’ spatial sense and spatial development. The spatial sense CMTT scores were analyzed using the Kruskall Wallace Test to explore group differences both before and after the learning experiences. The pre-post differences were investigated using the Wilcoxon Signed Ranks Test. Results of the Kruskall Wallace Test indicate that prior to the beginning of the study, there were no significant differences in the three groups’ spatial sense.

At the conclusion of the intervention, the participants were provided with a post CMTT spatial sense test. Differences between groups at the conclusion of the intervention were explored using the Kruskall Wallace Test. No significant group differences were found among the concrete, virtual, and multimodal tangram groups. The researcher then conducted Wilcoxon Signed Ranks Tests for each manipulative group to explore for differences between pre- and posttest CMTT scores by group. Data suggests that there was significant difference between second grade students' pre/post CMTT spatial sense scores at the $p <$ .002 level for the virtual manipulative group, but not for the concrete of multimodal groups. These results are similar to Ehrlich et al.'s (2006) findings who postulated that spatial sense is not a fixed skill; that with proper training or practice, an individual can increase their spatial ability. Additionally, the results were in agreement with Lin et al.'s (2011) study, that spatial sense can be improved with virtual tools.

Due to the results indicating that the virtual group was the only manipulative group to post statistically significant gains, the research further examined the virtual manipulative classroom for qualitative data to support the group's gains. The researcher believes that the vast difference in classroom atmosphere prior to the study to the collaborative nature of ability based groups during the study could have affected the quantitative outcomes for the virtual manipulative group. Students within the virtual manipulative group were allowed the opportunity to socially interact with their peers during the study, which was against the norm of prior classroom management. Social interactions aid to create a community of learners as supported by Kreijns, et al. (2003).

Statistically significant results for the virtual manipulative group, and not the concrete or multimodal group could be attributed to the slightly larger number of participants. Other factors that could have affected the virtual manipulative group may include slightly higher ability levels of student participants prior to the beginning of the study, although not statistically significant enough to warrant a difference among groups in the pretest results.

Student and Teacher Perceptions of Manipulative Type

The third research question sought to explore second grade students' and their teachers' perceptions of the different types of manipulatives (concrete, virtual, and multimodal). Discussion of the results of the student and teacher perceptions data is presented by themes to include – the need for feedback, engagement and motivation, and perceived advantages of each manipulative types.

The Need for Feedback

Without feedback, students do not have any guidance on if their processes are correct and unknowingly practice flawed logic (Sun, 2012). Instant feedback as related to iPads and technology was identified as an important factor by Attard and Curry's (2012) study who stated, "The affordance of instant feedback highlights the iPad's potential for building students' confidence in terms of risk taking and feeling safe to make mistakes and try again, building persistence" (p. 80). Students' desire for immediate feedback was evidenced in the findings of the study. The students' desire for and lack of immediate feedback on tangram activities was only present in the concrete manipulative group. Students within this group were more apt to raise their hand, motion, or verbally ask the teacher or researcher to examine their work. Often, students expressed the need for feedback when they had become frustrated with trying to solve the puzzle or wanted confirmation that their puzzle was correct. It was noted that typically, lower-level ability students were the majority of students that perceived difficulties in the concrete and virtual manipulative types. Students within the virtual and multimodal groups were observed less often requesting immediate feedback from the teacher or researcher. Even though they did at times ask the teacher or researcher to look at their puzzle, it was generally viewed as students wanting the teacher or researcher to acknowledge their accomplishment, not to correct or assess the puzzle. These results confirm multiple other studies to include Burns and Hamm's (2011) findings comparing concrete and virtual manipulative, and Bouck and Flanagan's (2010) findings on virtual manipulates.

The virtual manipulative group and the multimodal group students were able to gain immediate feedback from the iPad without teacher involvement. Additionally, if a student could not solve a tangram puzzle, or was getting overly frustrated, immediate feedback in the form of 'hints' were available to the multimodal group. This could have potentially played a large role in reducing the student frustrations within the multimodal group as compared to the concrete and virtual groups. Strom's (2009) findings on the use of multi-sensory learning experiences allowing lower-level ability students to become more involved in class activities and conversations may also explain why students using the OSMO did not become overly frustrated with the task.

Engagement and Motivation

Examining results of the study indicate that the availability of immediate feedback may contribute to continued student engagement and motivation. As part of the constructivist view, research supports healthy perturbations; however, if students become overly frustrated, they will likely give up (Battista, 2001). When exploring the engagement and motivation within the manipulative groups, qualitative results showed that students within the virtual and multimodal manipulative group were more likely to persist in and remain engaged in solving the tangram puzzles. Additionally, the virtual and multimodal groups were more likely to practice their spatial ability skills through the completion of multiple puzzles – beyond the prescribed four activities each day. Since spatial sense has been found to be a skill that can be developed as supported by Ehrlich, et al. (2006) and Lin, et al. (2011), these findings suggest that students completing more puzzles may in fact have a better opportunity to further develop their spatial sense.

Perceived Advantages of Each Manipulative Type

The researcher found that both students and teacher viewed the ability to physically touch the manipulatives as an advantage of the concrete manipulatives. However, students within the concrete group also expressed a desire to work with iPads. The multimodal manipulative, OSMO, could satisfy the students' desire to both physically touch the manipulatives and to work with iPads simultaneously. Students who touch physical manipulatives rather than a virtual representation may benefit from hands-on manipulatives as illustrated by Clements and Sarama, "Sensory-concrete knowledge refers to knowledge that demands the support of concrete objects and children's knowledge of manipulating these objects" (p. 317). Sensory-concrete knowledge may aid the student in making the cognitive connections that virtual touch may not be able to satisfy.

Advantages of the both the virtual manipulatives and the multimodal manipulatives included the ability for the students to receive instant feedback. Immediate feedback on the correctness of tangram puzzles allowed students to apply different problem solving strategies, as supported by NCTM's Process Standards (2000a). The multimodal manipulative, OSMO, also offered feedback when solicited by the students in the form of hints. The hint feature could potentially further reduce the need for teacher interventions or teacher feedback as well as encourage continuation of a puzzle by preventing students from becoming overly frustrated. Virtual manipulatives are sometimes capable of giving immediate feedback in the form of visual cues or hints which help guide the student to a solution which is determined to be beneficial to student learning (Suh & Moyer, 2007).

Implications

The findings of this study have several implications for educators, administrators, and the field of education. First, this study adds to the knowledge base of research that suggests that spatial sense can be developed through training with the use of tangram manipulatives. Second, although there was not a statistically significant difference in math achievement, qualitative findings indicated that students demonstrated the development of geometric content knowledge as related to NCTM's Geometric Content Standards (2000a). Third, qualitative data illustrates specific benefits to students regarding immediate feedback and engagement and motivation when using virtual or multimodal manipulatives, thus justifying the push for the use of emerging technologies in the classroom.

Recommendations for Future Research

This study yielded several recommendations for improvement of the current study and for future research. Both methods of data collection, quantitative and qualitative allow for further examination and explanation of the study results. For the quantitative data collection, it is recommended to expand the longevity of the study to gain a better view of a measure of mathematics achievement after extended exposure to specific manipulative types. Qualitative data could be expanded upon by exposing all three classes with each manipulative and additional student and teacher participant interviews to explore which manipulative type was preferred and why.

To increase the generalizability of the study, it is recommended to complete the study on a larger scale and to include a more diverse population of participants. It is recommended to use Likert scale-based research to expand the study to explore motivation and engagement of students among different groups of manipulatives to further explain preferences and effectiveness among the different manipulative types. Research on student frustrations or perturbations while using the different manipulative types could allow a glimpse into the challenges of critical thinking while using the concrete, virtual, and multimodal manipulatives. It is recommended to study any differences in the role of the teacher as facilitator while examining individual students' and groups' ability to complete tasks independently through collaboration. These findings could potentially add to the knowledge of inquiry-based learning.

Implications from this study are also notably important when considering technology, pedagogy, and content knowledge (TPACK) since TPACK plays a large role on how and if teachers integrate technology into their lessons. Although TPACK was not examined and discussed in detail in this study, Mrs. Edwards mentioned in her interview that technology is rarely used in her classroom; however, she was excited to see her students learning and engaged. Further research exploring the impact of TPACK on technology integration using the tangram manipulatives such as the OSMO would prove important in the field.

Concluding remarks

The effectiveness of manipulative use to benefit students is well known and documented; however, some teachers may be unsure if their available or preferred method of manipulative type is effective for student improvement. The results of this research study support previous studies that conclude that spatial sense is not a fixed skill. With proper training and usage of tangram manipulatives, students have the capacity to increase their spatial sense. Also, the study provides evidence that virtual tangram manipulatives can be used as an effective tool to aid in the development of spatial sense. However, qualitative results indicate that emerging technologies, and new ways to learn, such as the OSMO, creates an enjoyable medium of developing spatial sense with obvious advantages of student engagement and motivation, student perseverance through challenging tasks, the ability of students to work independent of direct instruction by the classroom teacher, and the availability of lesson differentiation. Results suggest that the multimodal manipulative – OSMO, combines the undisputed benefits of concrete manipulatives, in addition to the benefits of student collaboration and the interdependent learning environment offered through the virtual environment.

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APPENDICES

Appendix A

Teacher Tangram Lesson Plan

1. The teacher participant will read a researcher provided book, The Warlord's Puzzle by Virginia Pilegard.
2. The teacher will have one set of concrete tangrams to demonstrate to the class what a tangram is.
3. The teacher will ask the students how many total pieces are in a tangram puzzle.
4. The teacher and students will identify the different shapes within the tangram puzzle together based on the shapes' physical characteristics.
5. The teacher will manipulate the shapes in turns, flips, and slides to show that the pieces move and can make different pictures.
6. The teacher will use a variety of vocabulary terms for turn, flip, and slide, such as rotate, reflect, and translate.
7. The teacher will ask students to describe the different characteristics of each 2 dimensional shape and to compare/contrast the seven shapes.

a. Are any of the shapes the same shape?

b. Are any of the shapes the same size?

c. Can a shape be the same, but a different size?

d. What kind of shapes can we make out of the shapes provided?

8. The teacher will ask students to postulate what they think the two small triangles could form.

9. The teacher will explain that shapes that are identical in size and form – such as the two small triangles – are said to be congruent. Shapes that are identical in form but not in size – such as the small, medium, and large triangles are said to be similar.

10. The teacher will place the students in ability based groups as determined by their pre-test mathematics achievement test scores.

11. Each group of students can practice manipulating their tangrams by completing the Tangram Activity Worksheet (Concrete, virtual, or OSMO as determined by treatment group/classroom number) for Week 1 Practice.

12. The teacher will facilitate a whole class discussion over the students' experiences and findings throughout the activity.

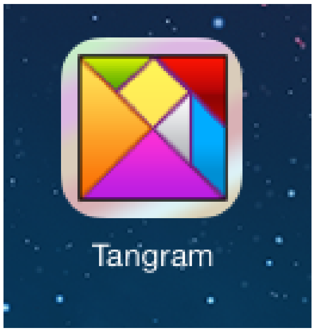
    a. Is there more than one way to solve each puzzle?

    b. Which way is the right way?

    c. Is there a right way?

Appendix B

Concrete Tangram Activity



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Concrete Tangrams

You will work with your partners to solve four tangram puzzles using the tangram tiles. Talk to your group members to decide where you think each piece goes.

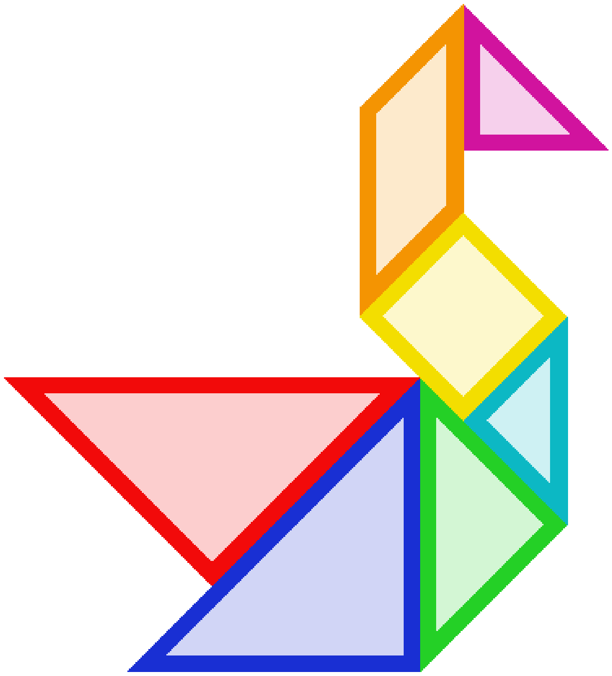
After you solve each puzzle, record your answers on the following answer sheet. Use a pencil to draw the shape outlines and crayons to color in the shapes you used to solve the puzzles.

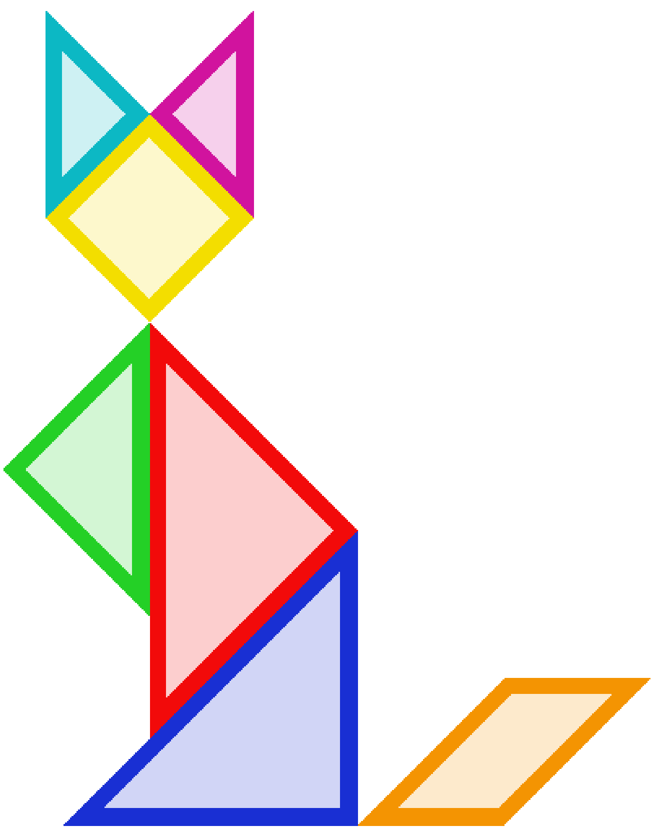
- You may slide, rotate, and flip your shapes to solve each puzzle.

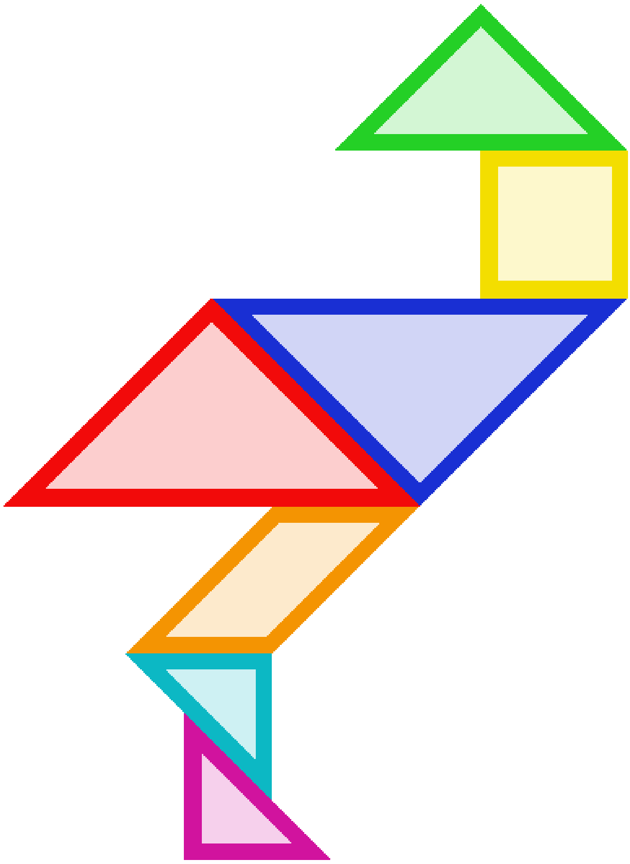
- You must use all seven pieces for each puzzle.

- You must place all tiles flat on the table.

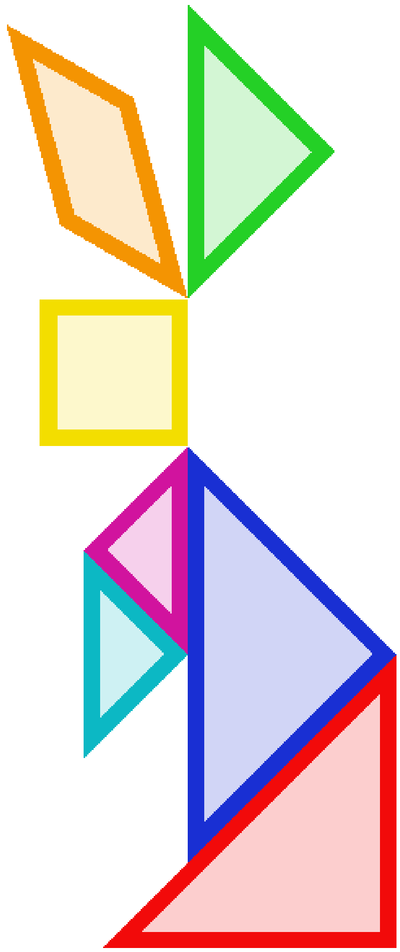
- You may not stack the tiles on top of each other.

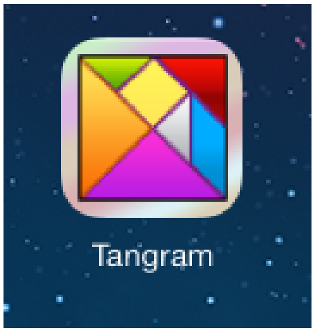






Week 1 Practice





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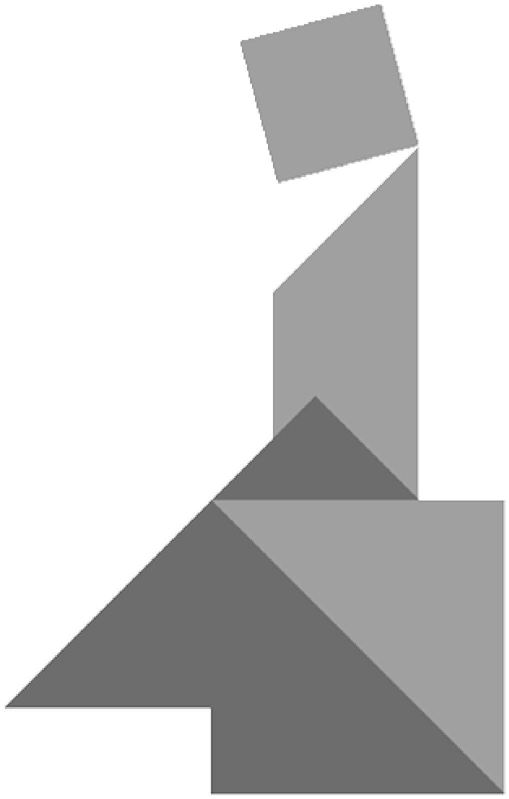
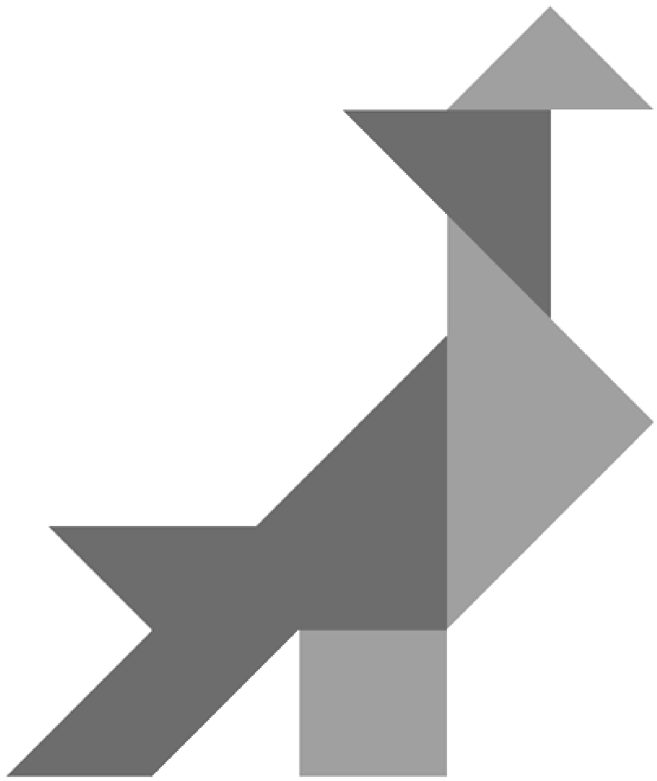
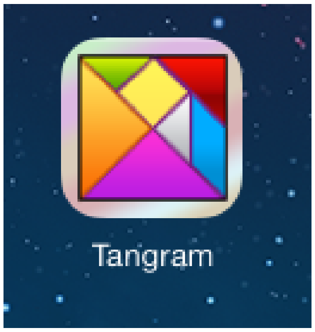


Week 2 Practice





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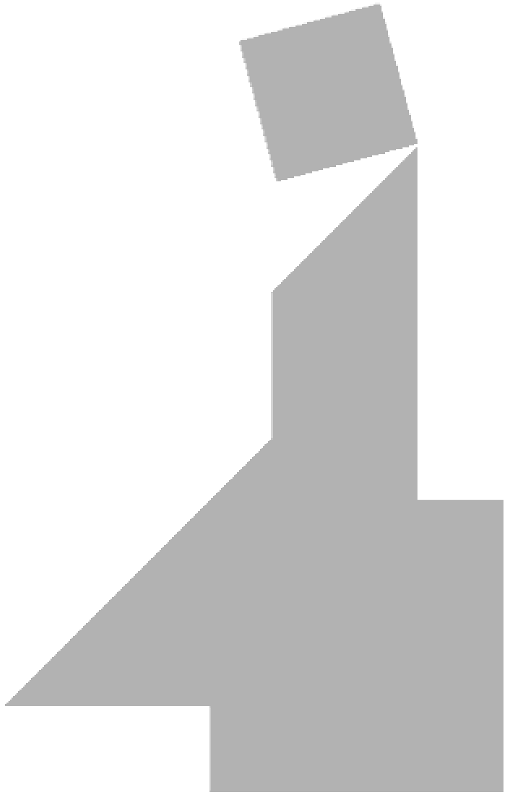

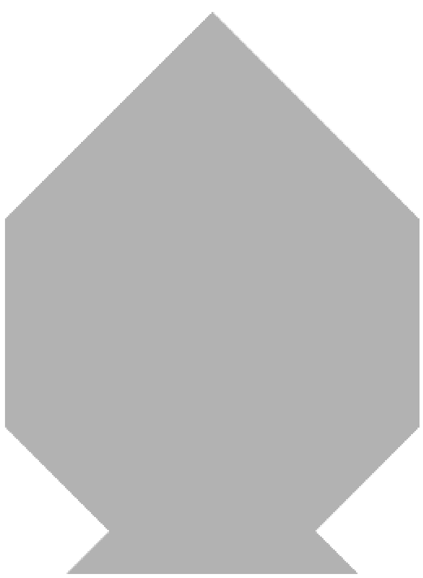

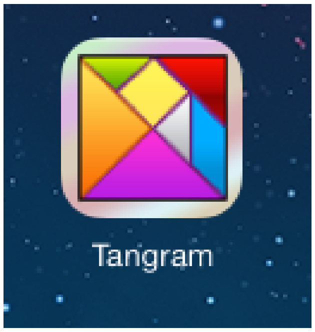

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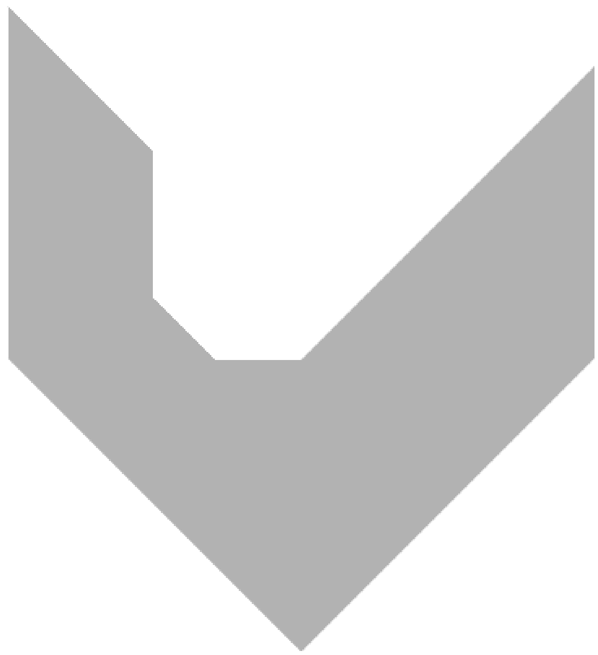
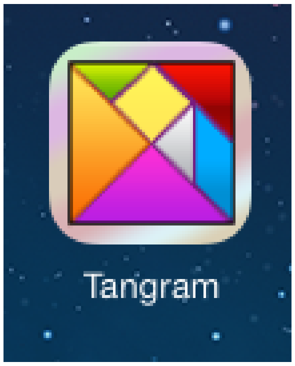


Appendix C

Virtual Tangram Activity



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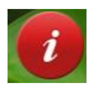
Virtual Tangrams

You will work with your partners to solve four tangram puzzles on the iPad. Talk to your group members to decide where you think each piece goes.

After you solve each puzzle, record your answers on the following answer sheet. Use a pencil to draw the shape outlines and crayons to color in the shapes you used to solve the puzzles.

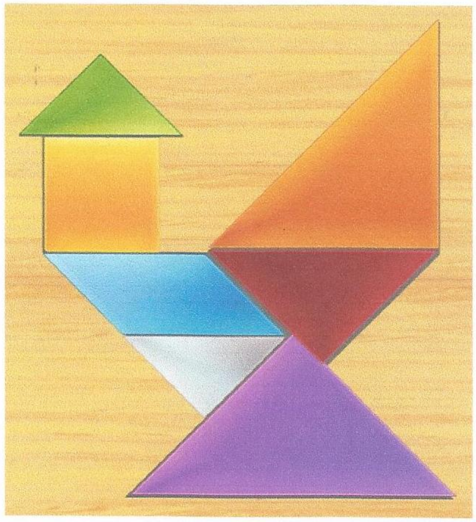
- To select a puzzle, click on the  button in the top right corner of the screen to view the puzzle options. You may view additional puzzles by clicking the arrows at the bottom of the puzzle pages.

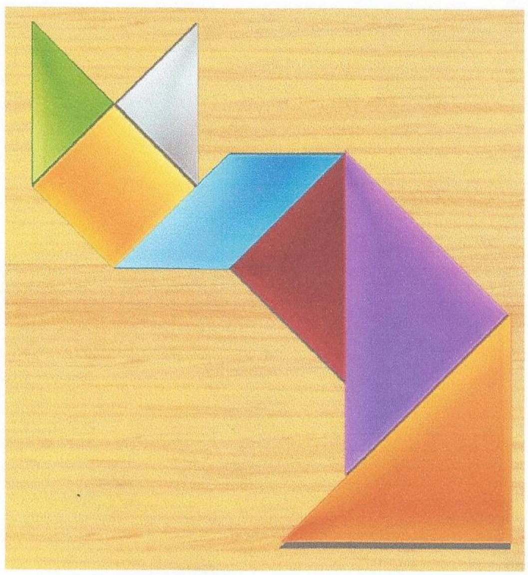
- To move each shape, gently hold your finger on the shape and slide your finger across the screen. When you have your shape where you want it, lift your finger off the screen.

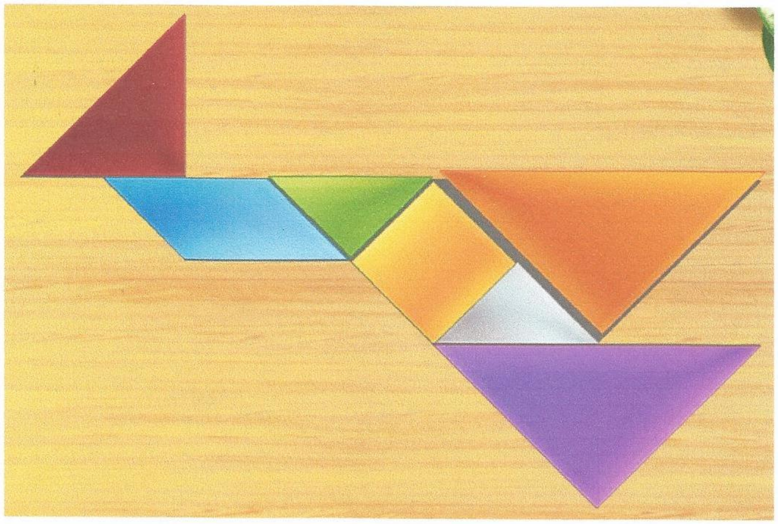
- To rotate or turn your shape, tap the  button below the shape.

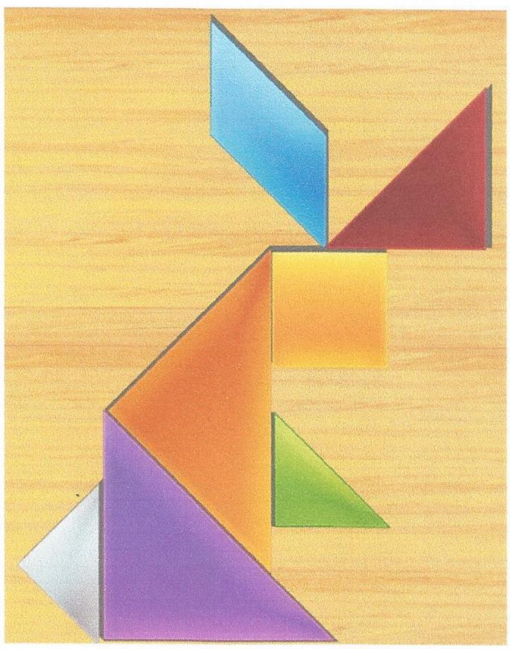
- To flip your shape, tap the  button above the shape.

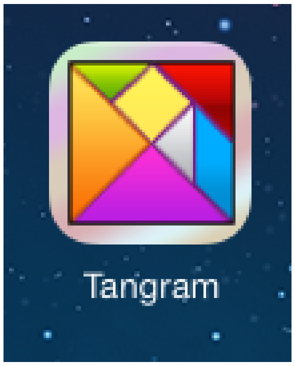
- When you have all of your pieces arranged correctly the game will tell you "Congratulations!" and you may go on to the next shape











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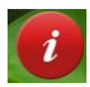
- To select a puzzle, click on the  button in the top right corner of the screen to view the puzzle options. You may view additional puzzles by clicking the arrows at the bottom of the puzzle pages.

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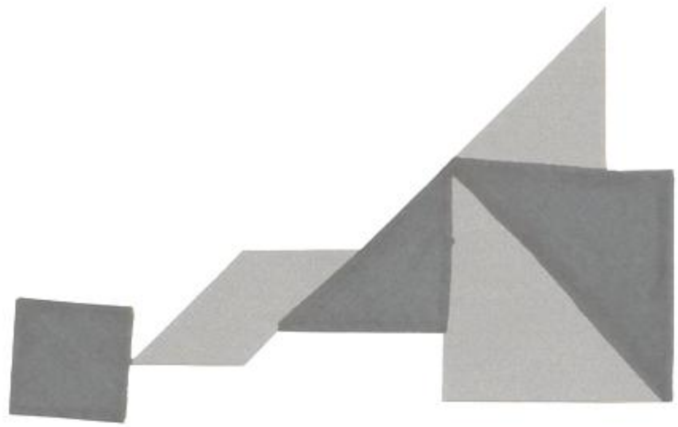
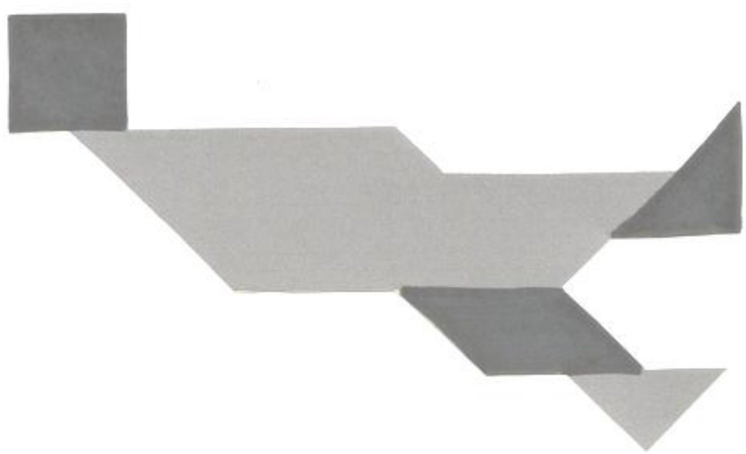
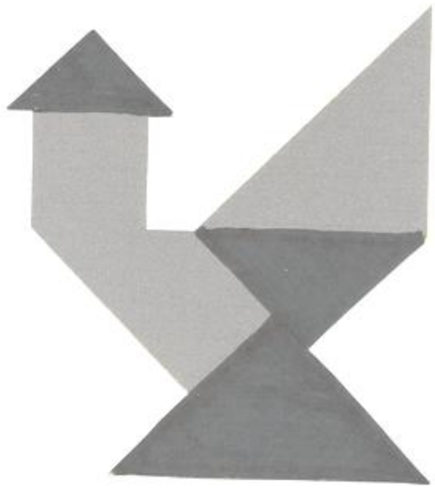
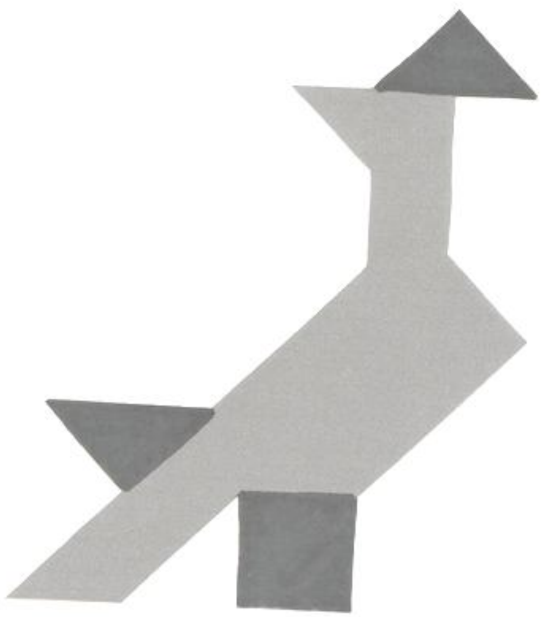
Week 2 Practice

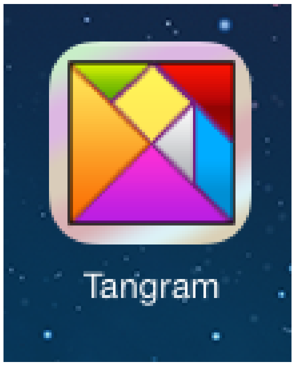




Week 2 Practice





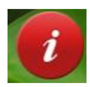


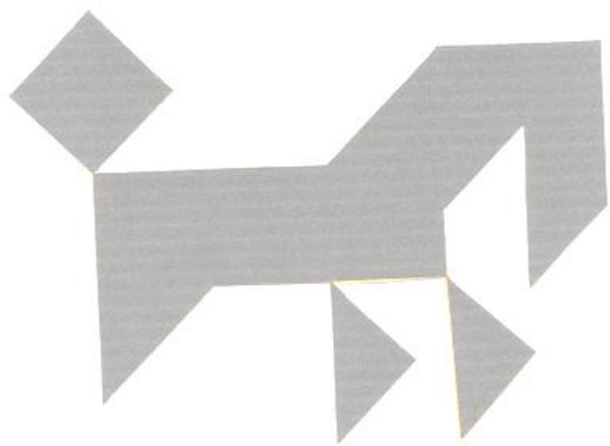
Names:__________________________________

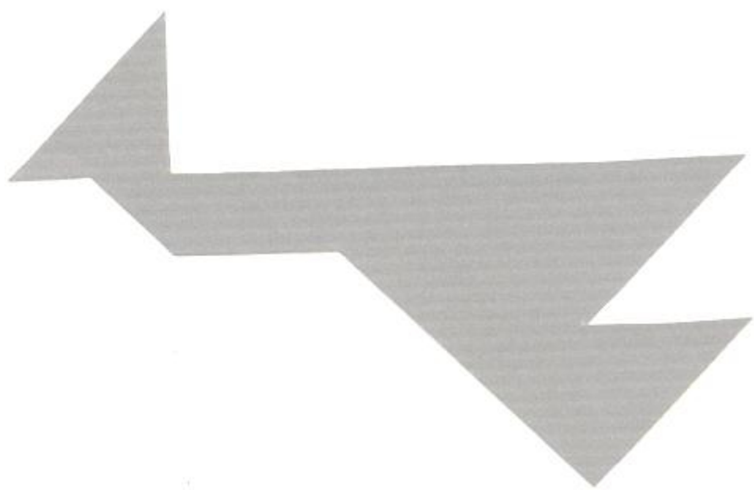
Virtual Tangrams

You will work with your partners to solve four tangram puzzles on the iPad. Talk to your group members to decide where you think each piece goes.

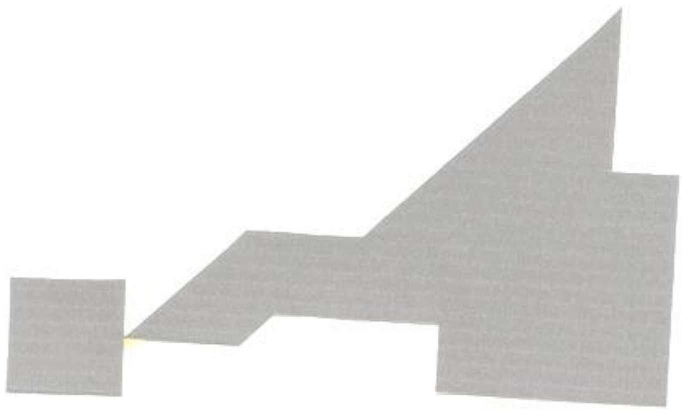
After you solve each puzzle, record your answers on the following answer sheet. Use a pencil to draw the shape outlines and crayons to color in the shapes you used to solve the puzzles.

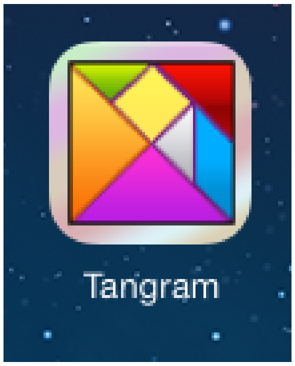
- To select a puzzle, click on the  button in the top right corner of the screen to view the puzzle options. You may view additional puzzles by clicking the arrows at the bottom of the puzzle pages.
- To move each shape, gently hold your finger on the shape and slide your finger across the screen. When you have your shape where you want it, lift your finger off the screen.
- To rotate or turn your shape, tap the  button below the shape.
- To flip your shape, tap the  button above the shape.
- When you have all of your pieces arranged correctly the game will tell you "Congratulations!" and you may go on to the next shape





Week 3 Practice





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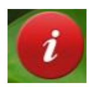
- To select a puzzle, click on the  button in the top right corner of the screen to view the puzzle options. You may view additional puzzles by clicking the arrows at the bottom of the puzzle pages.
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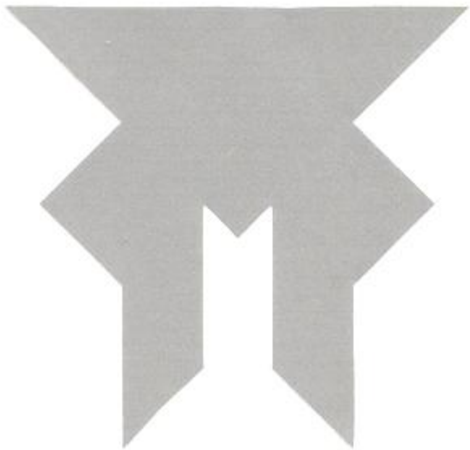
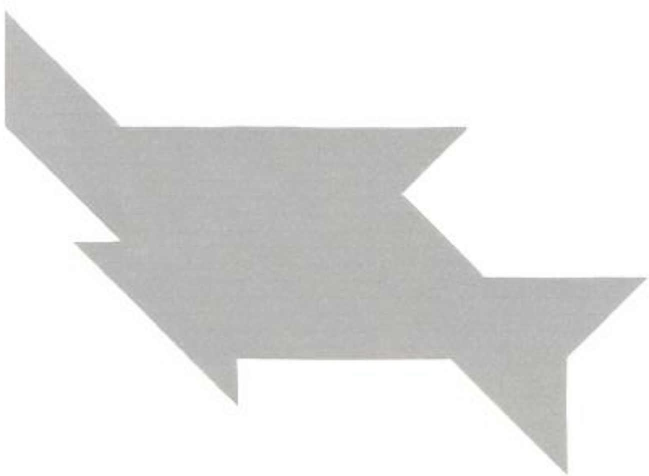
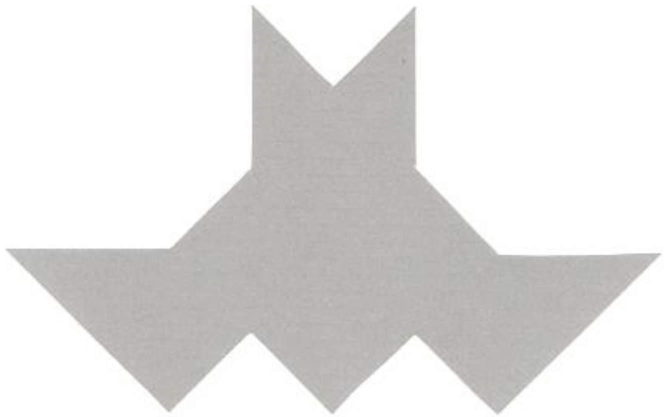
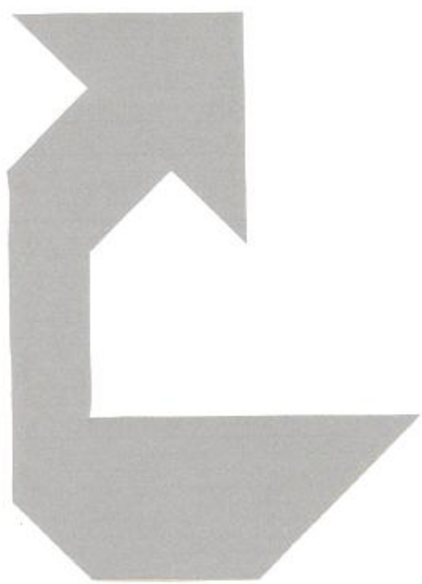


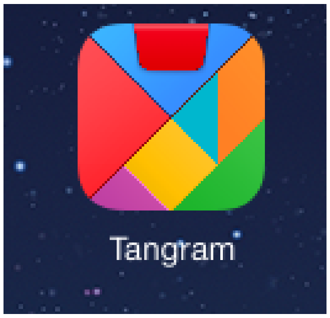






Appendix D

OSMO Tangram Activity



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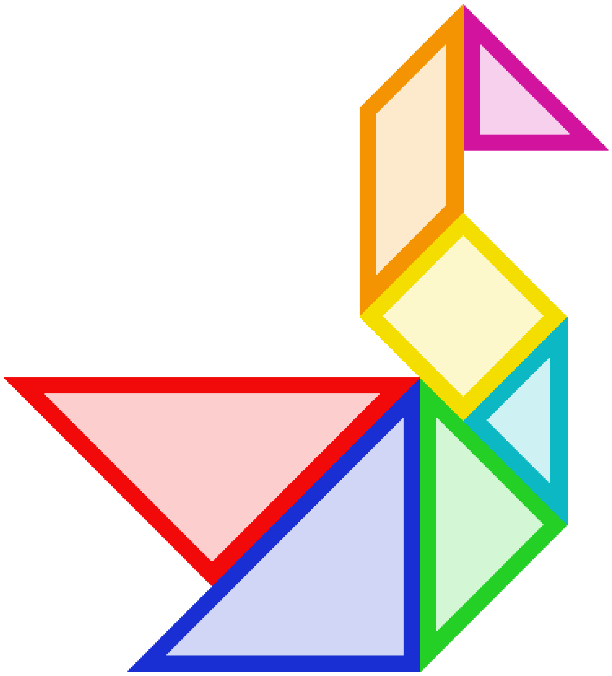
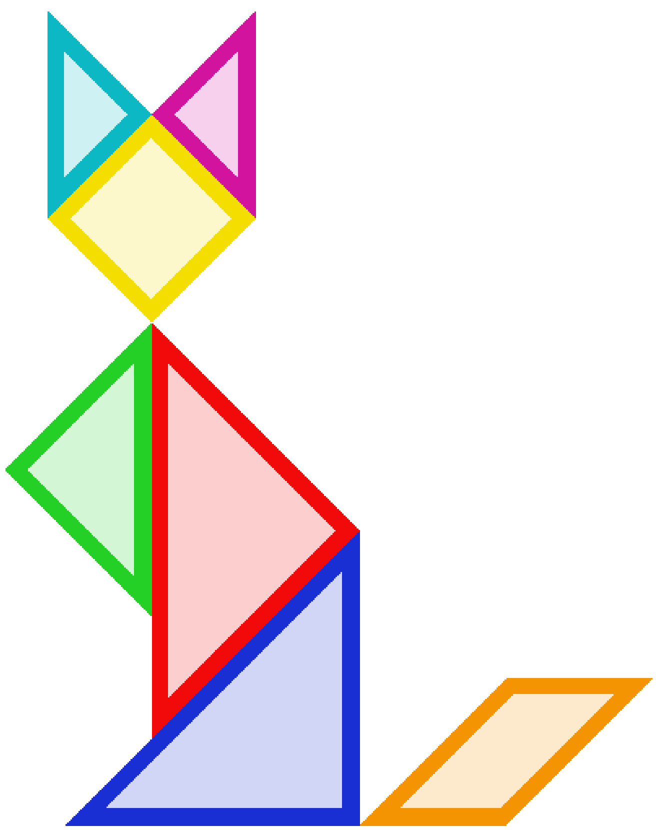
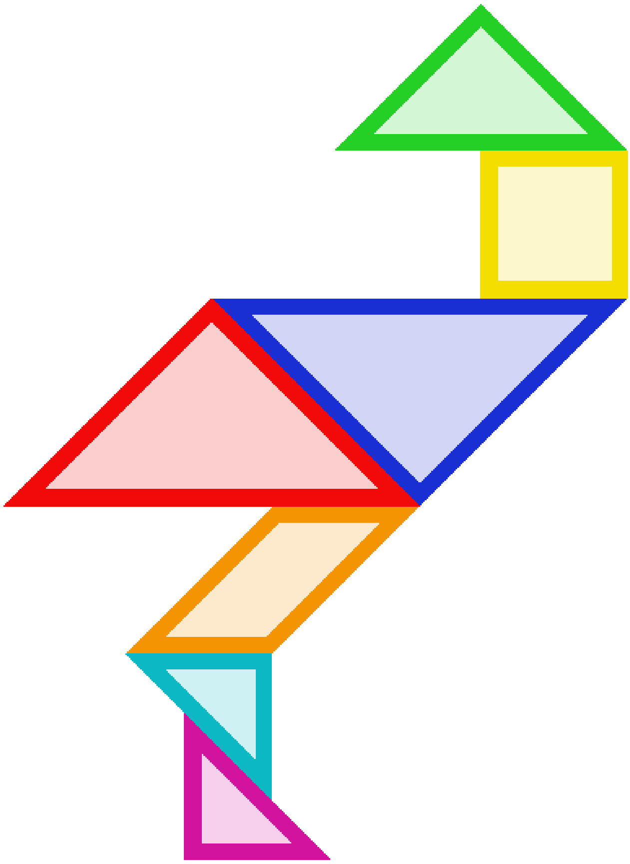
- You must use all seven shapes to complete each puzzle.

- When you complete the puzzle correctly the puzzle will light up and make music.

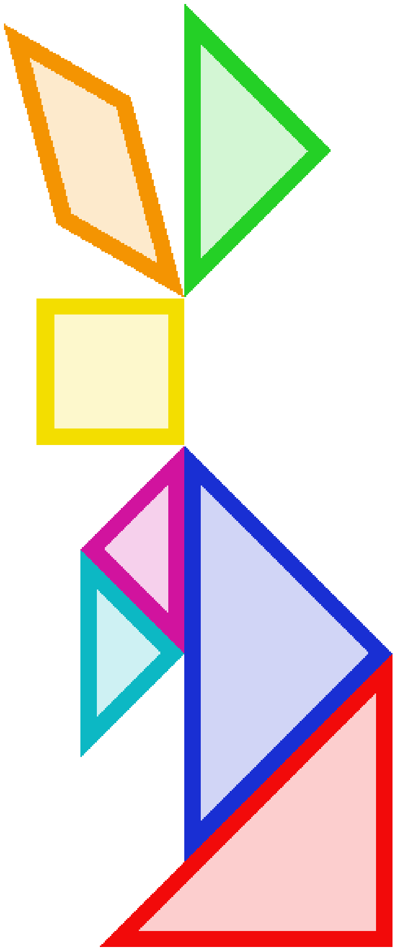
- Once you complete a puzzle, you may go on to the next puzzle on your worksheet.

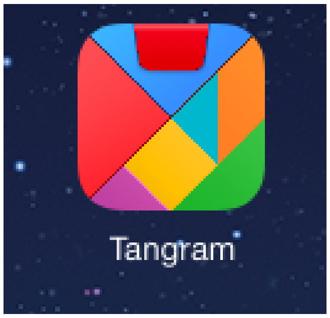






Week 1 Practice





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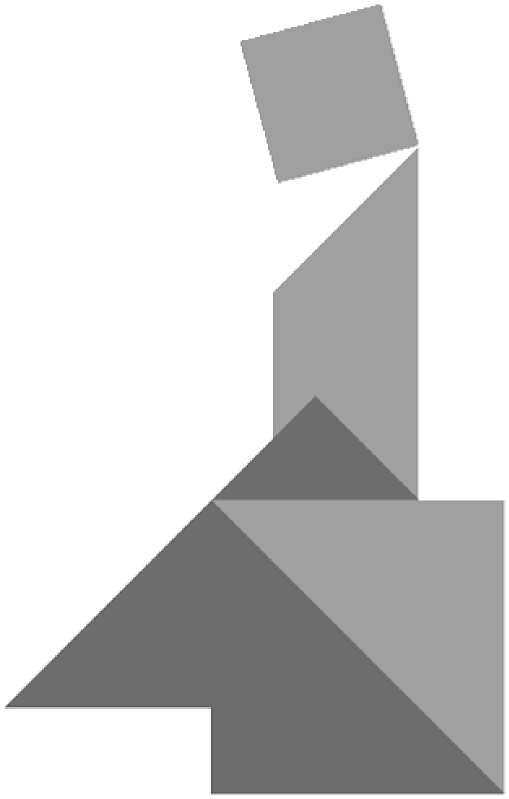

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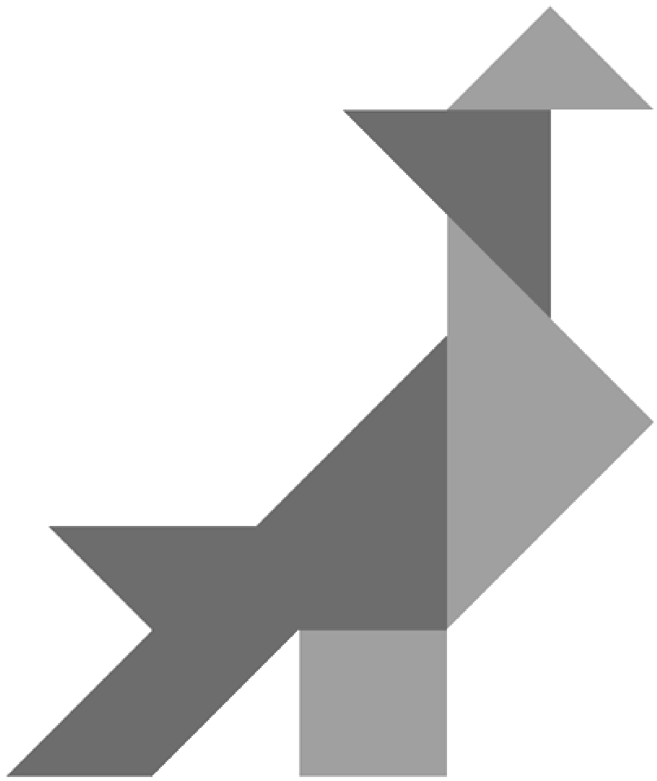

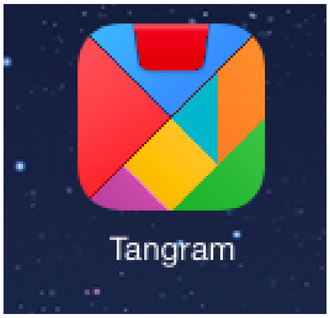

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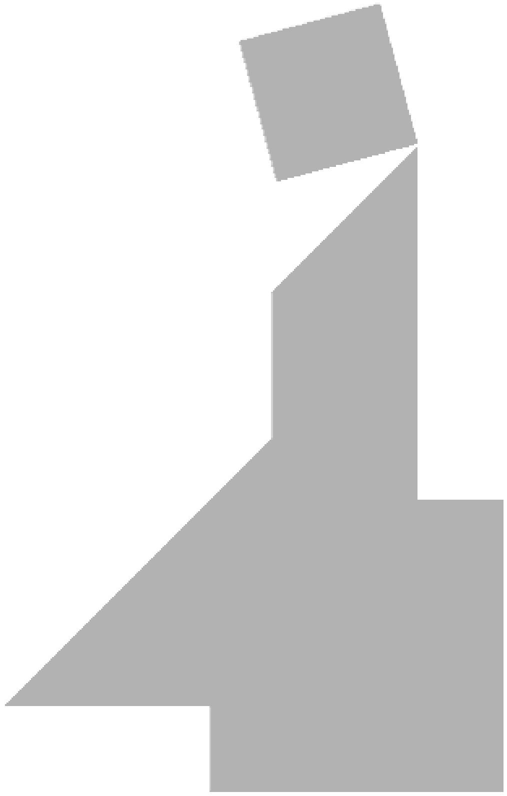

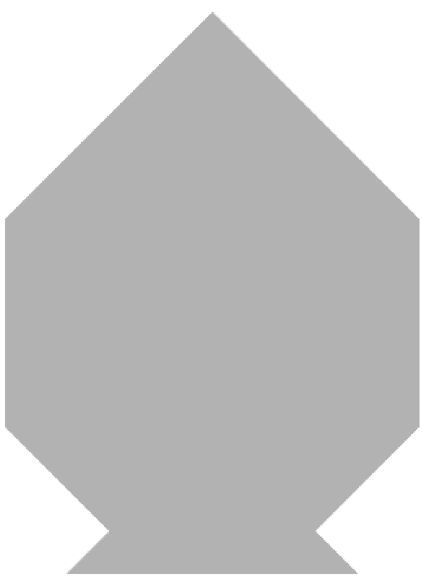

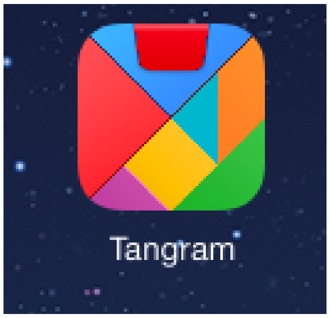

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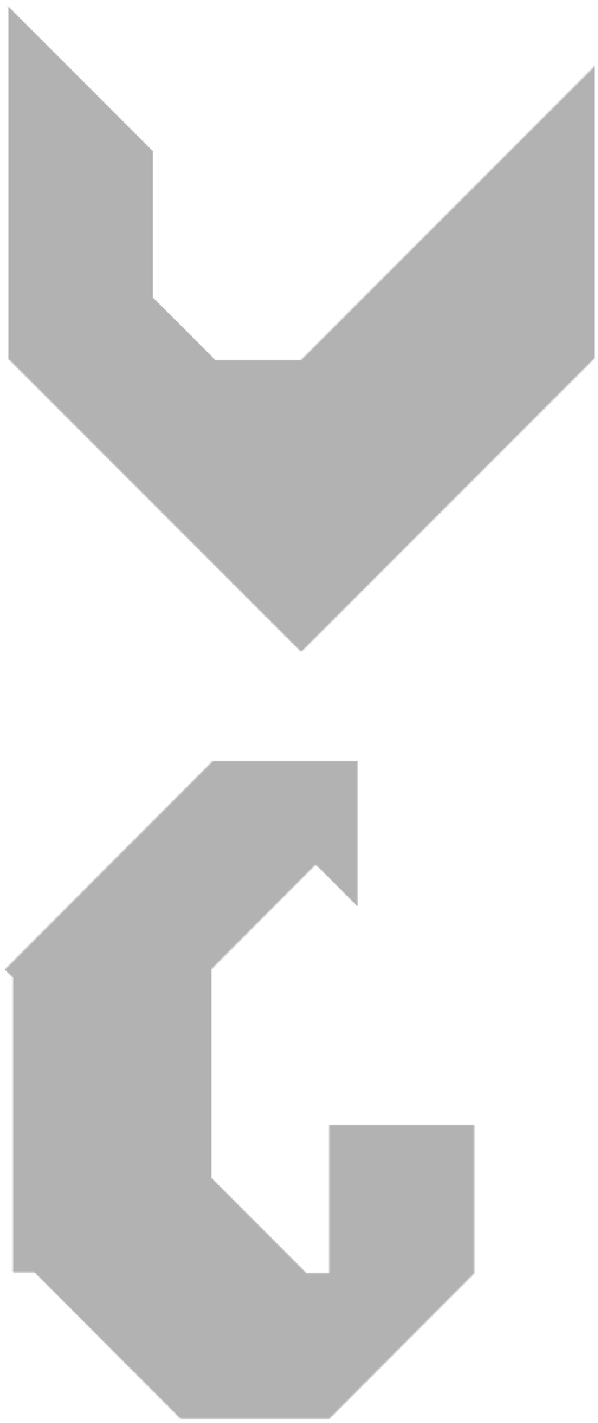


Appendix E

Condensed California State Department of Education STAR Math Achievement Test – 2nd Grade

Condensed STAR Pretest

The questions in brackets are not printed in the test booklet. The test administrator reads these questions aloud to students.

1 [A NUMBER HAS NINE ONES, SIX TENS, AND EIGHT HUNDREDS. WHAT IS THE NUMBER?]

869	896	968	986
A	**B**	**C**	**D**

CSM02136

7 [WHICH NUMBER SENTENCE IS TRUE?]

$359 < 375$	$359 > 375$	$359 < 359$	$359 > 359$
A	**B**	**C**	**D**

CSM01005

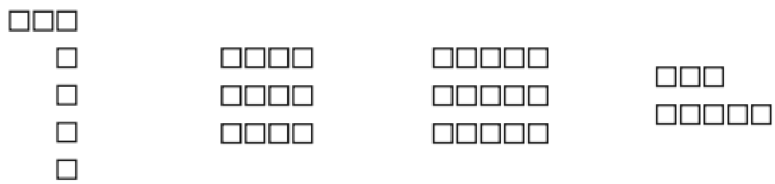
13 **[SOPHIE DID THIS SUBTRACTION PROBLEM. WHICH ADDITION PROBLEM SHOWS THAT SHE GOT THE RIGHT ANSWER?]**

$$\boxed{\begin{array}{r} 85 \\ -\ 44 \\ \hline 41 \end{array}}$$

A	B	C	D
$\begin{array}{r} 41 \\ +\ 85 \\ \hline \end{array}$	$\begin{array}{r} 44 \\ +\ 85 \\ \hline \end{array}$	$\begin{array}{r} 41 \\ +\ 44 \\ \hline \end{array}$	$\begin{array}{r} 44 \\ +\ 44 \\ \hline \end{array}$

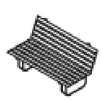
20 **[WHICH DRAWING SHOWS THREE TIMES FIVE?]**

$\boxed{3 \times 5}$



A B C D

26 **[THERE ARE NINE BENCHES IN A PARK. THERE ARE TWO PEOPLE SITTING ON EACH BENCH. HOW MANY PEOPLE ARE SITTING ON THE NINE BENCHES ALL TOGETHER?]**

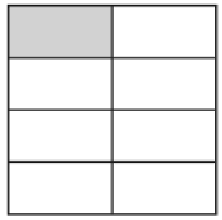
9 

2 

A	B	C	D
11	14	16	18

CSM02108

30 [WHAT FRACTIONAL PART OF THIS FIGURE IS SHADED?]



A	B	C	D
$\frac{1}{8}$	$\frac{1}{7}$	$\frac{1}{4}$	$\frac{1}{2}$

CSM02147

---

35 [WHICH FRACTION IS EQUAL TO ONE WHOLE?]

A	B	C	D
$\frac{1}{3}$	$\frac{1}{8}$	$\frac{2}{3}$	$\frac{8}{8}$

CSM01018

---

40 [SHAMIKA IS SAVING MONEY TO BUY A BOOK. SHE HAS SAVED ONE FIVE-DOLLAR BILL, THREE ONE-DOLLAR BILLS, ONE QUARTER, THREE DIMES, AND FOUR NICKELS. HOW MUCH MONEY DOES SHE HAVE SO FAR?]

$7.95
A

$8.75
B

$8.55
C

$7.75
D

CSM00571

47 [WHAT NUMBER GOES IN THE BOX TO MAKE THIS NUMBER SENTENCE TRUE?]

$$\boxed{15 + 8 = \square + 15}$$

7	8	15	23
A	**B**	**C**	**D**

CSM00091

---

48 [LOOK AT THE NUMBER SENTENCE IN THE BOX. WHICH OF THE FOLLOWING HAS THE SAME VALUE AS SIX PLUS FIVE?]

$$\boxed{6 + 5 = 11}$$

A $6 - 5 = \square$

C $5 \times 6 = \square$

B $5 + 6 = \square$

D $5 - 6 = \square$

CSM02148

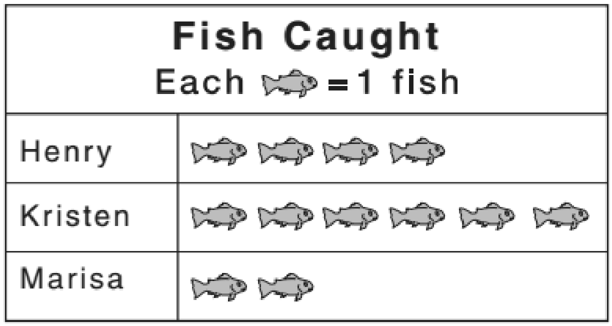
---

44 [JAMES HAS TWO DOLLARS AND FORTY-SIX CENTS. WHICH IS A CORRECT WAY TO WRITE THIS AMOUNT OF MONEY?]

\$2.46	\$2.46¢	\$2 and 4.6¢	\$2 and .46¢
A	**B**	**C**	**D**

CSM00027

53 [LOOK AT THE GRAPH. HOW MANY FISH DID HENRY AND KRISTEN CATCH ALL TOGETHER?]



4	6	10	12
A	**B**	**C**	**D**

CSM02090

55 [THIS COMB IS ABOUT 12 BUTTONS LONG. ABOUT HOW MANY TOOTHPICKS LONG IS THE COMB?]

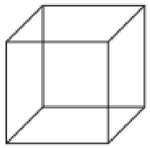
4	8	10	12
A	**B**	**C**	**D**

61 [SEAN IS GOING ON VACATION TO VISIT HIS GRANDPARENTS. HE WILL BE GONE ONE MONTH. ABOUT HOW MANY DAYS WILL SEAN BE GONE?]

7 days	30 days	52 days	365 days
A	**B**	**C**	**D**

CSM00373

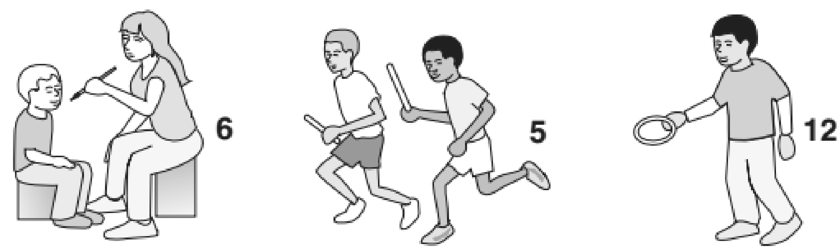
64 [HOW MANY FACES DOES A CUBE HAVE?]



4	5	6	8
A	**B**	**C**	**D**

CSM00096

72 [THE STUDENTS IN MRS. KIM'S CLASS ARE VOTING FOR THE BOOTH THEY WANT TO HAVE AT THE FUN FAIR. SIX STUDENTS WANT FACE PAINTING. FIVE STUDENTS WANT A RELAY RACE. TWELVE STUDENTS WANT THE RING TOSS. WHICH TALLY CHART SHOWS THESE RESULTS?]



Fun Fair	
Face Painting	𝍸 |
Relay Race	𝍸 𝍸 ||
Ring Toss	𝍸

A

Fun Fair	
Face Painting	𝍸
Relay Race	𝍸 𝍸 ||
Ring Toss	𝍸 |

C

Fun Fair	
Face Painting	𝍸 𝍸 ||
Relay Race	𝍸
Ring Toss	𝍸 |

B

Fun Fair	
Face Painting	𝍸 |
Relay Race	𝍸
Ring Toss	𝍸 𝍸 ||

D

CSM10414

Question Number	Correct Answer	Standard	Year of Release
1	*A*	2NS1.1	2004
2	*C*	2NS1.1	2005
3	*B*	2NS1.1	2006
4	*D*	2NS1.1	2007
5	*B*	2NS1.2	2004
6	*D*	2NS1.2	2005
7	*A*	2NS1.3	2003
8	*C*	2NS1.3	2004
9	*A*	2NS1.3	2005
10	*A*	2NS1.3	2006
11	*A*	2NS1.3	2007
12	*D*	2NS1.3	2007
13	*C*	2NS2.1	2003
14	*B*	2NS2.1	2004
15	*D*	2NS2.1	2007
16	*D*	2NS2.2	2003
17	*D*	2NS2.2	2004
18	*B*	2NS2.2	2005
19	*D*	2NS2.2	2006
20	*C*	2NS3.1	2004
21	*B*	2NS3.1	2005
22	*C*	2NS3.1	2007
23	*C*	2NS3.2	2003
24	*A*	2NS3.2	2005
25	*B*	2NS3.2	2006
26	*D*	2NS3.3	2003
27	*B*	2NS3.3	2004
28	*C*	2NS3.3	2006
29	*A*	2NS3.3	2007
30	*A*	2NS4.1	2003
31	*B*	2NS4.1	2004
32	*A*	2NS4.1	2005
33	*B*	2NS4.2	2003
34	*C*	2NS4.2	2005
35	*D*	2NS4.3	2003
36	*B*	2NS4.3	2005
37	*D*	2NS4.3	2007
38	*A*	2NS5.1	2003
39	*B*	2NS5.1	2004
40	*B*	2NS5.1	2006

Question Number	Correct Answer	Standard	Year of Release
41	*B*	2NS5.1	2007
42	*D*	2NS5.2	2003
43	*A*	2NS5.2	2005
44	*A*	2NS5.2	2006
45	*A*	2NS5.2	2007
46	*D*	2NS6.1	2004
47	*B*	2AF1.1	2003
48	*B*	2AF1.1	2004
49	*D*	2AF1.1	2005
50	*A*	2AF1.1	2007
51	*A*	2AF1.2	2003
52	*D*	2AF1.2	2005
53	*C*	2AF1.3	2004
54	*B*	2MG1.1	2006
55	*A*	2MG1.2	2004
56	*B*	2MG1.3	2004
57	*B*	2MG1.3	2006
58	*C*	2MG1.3	2006
59	*B*	2MG1.3	2007
60	*C*	2MG1.3	2007
61	*B*	2MG1.4	2003
62	*D*	2MG1.4	2005
63	*C*	2MG1.5	2005
64	*C*	2MG2.1	2003
65	*B*	2MG2.1	2003
66	*A*	2MG2.1	2006
67	*A*	2MG2.2	2004
68	*A*	2MG2.2	2006
69	*D*	2MG2.2	2006
70	*B*	2MG2.2	2007
71	*A*	2MG2.2	2007
72	*D*	2PS1.1	2005
73	*D*	2PS1.1	2006
74	*B*	2PS1.1	2007
75	*D*	2PS1.2	2003
76	*D*	2PS1.2	2006
77	*A*	2PS1.3	2005
78	*A*	2PS1.3	2006
79	*A*	2PS1.4	2004
80	*C*	2PS1.4	2007

5 [WHAT IS ANOTHER NAME FOR FOUR HUNDRED PLUS FORTY PLUS EIGHT?]

A	B	C	D
4408	448	400408	4048

CSM00361

9 [WHICH SIGN MAKES THE NUMBER SENTENCE TRUE?]

$$22 + 10 \square 32$$

A	B	C	D
=	+	>	<

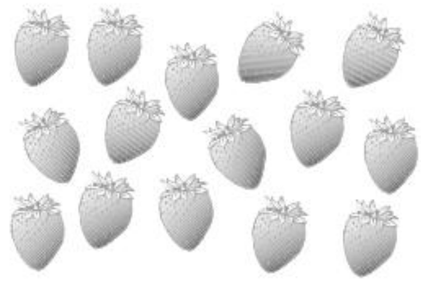
16 [WHAT IS THE SOLUTION TO THIS PROBLEM?]

$$\begin{array}{r} 419 \\ -\ \ 12 \\ \hline \end{array}$$

A	B	C	D
431	421	417	407

CSM02158

23 [KAYLA HAS THESE STRAWBERRIES. SHE WILL GIVE FOUR STRAWBERRIES TO EACH OF HER THREE FRIENDS. HOW MANY STRAWBERRIES WILL BE LEFT FOR KAYLA?]



1	2	3	4
A	**B**	**C**	**D**

CSM02110

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28 [WHICH NUMBER SHOWS THE ANSWER TO FIVE TIMES SIX?]

11	25	30	35
A	**B**	**C**	**D**

CSM10076

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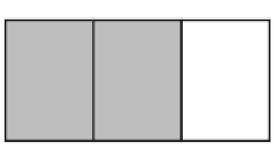
33 [WHAT FRACTION OF THIS SHAPE IS SHADED?]



$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{2}$	$\frac{3}{1}$
A	**B**	**C**	**D**

CSM01446

38 **[MONIQUE HAS FOUR QUARTERS, TWO DIMES, AND ONE NICKEL. HOW MUCH MONEY DOES SHE HAVE?]**

$1.25 **A**	$1.05 **C**
$0.75 **B**	$1.45 **D**

CSM00998

42 **[LEE HAS THE MONEY YOU SEE IN THE BOX. HOW MUCH MONEY IS THIS?]**

$2.15 **A**	$2.25 **C**
$2.20 **B**	$2.30 **D**

46 **[ABOUT HOW LONG IS A DOLLAR BILL?]**

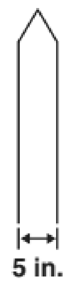
1 foot	1 inch	6 feet	6 inches
A	**B**	**C**	**D**

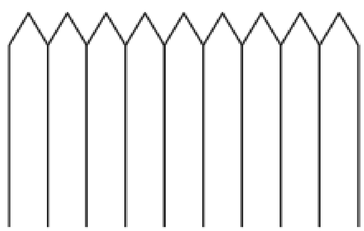
51 [ANDREW HAD FIFTEEN PENNIES. HE FOUND SOME MORE. NOW HE HAS THIRTY-THREE. WHICH NUMBER SENTENCE COULD BE USED TO FIND HOW MANY PENNIES HE FOUND?]

$15 + \square = 33$	$\square - 33 = 15$
A	**C**
$15 + 33 = \square$	$\square - 15 = 33$
B	**D**

CSM01477

54 [EACH FENCE POST IS FIVE INCHES WIDE. HOW WIDE IS THE FENCE IN THE PICTURE?]

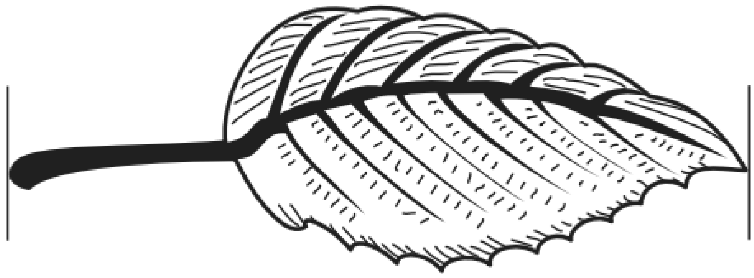




30 inches	45 inches	50 inches	65 inches
A	**B**	**C**	**D**

CSM10086

56 [LOOK AT THE PICTURE OF THE LEAF. MEASURE THE LENGTH OF THE LEAF AND STEM IN INCHES. ABOUT HOW LONG ARE THE LEAF AND STEM TOGETHER?]



4 inches	5 inches	6 inches	7 inches
A	**B**	**C**	**D**

CSM10032

63 **[A MOVIE STARTED AT ELEVEN O'CLOCK A.M. AND LASTED THREE HOURS. AT WHAT TIME DID THE MOVIE END?]**

12:00 p.m.	1:00 p.m.	2:00 p.m.	3:00 p.m.
A	**B**	**C**	**D**

CSM20060

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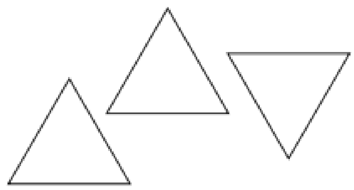
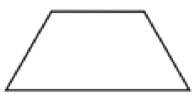
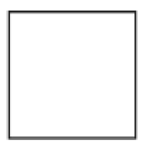
67 **[LOOK AT THE THREE TRIANGLES. WHICH OF THE FOLLOWING SHAPES CAN BE MADE FROM THE THREE TRIANGLES?]**

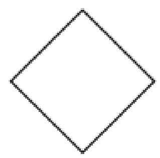




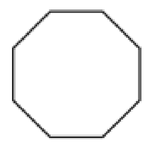
A



B

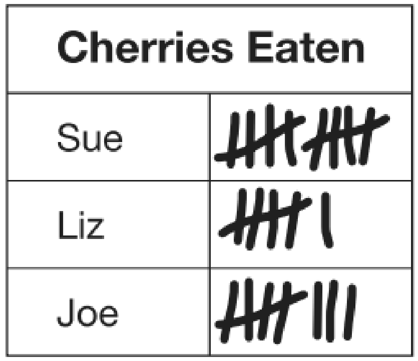


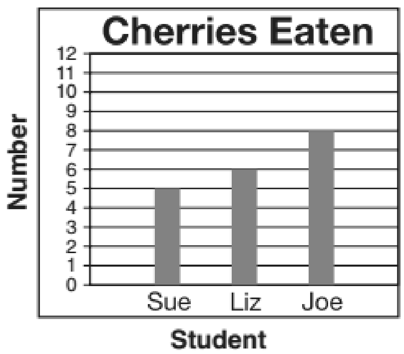
C



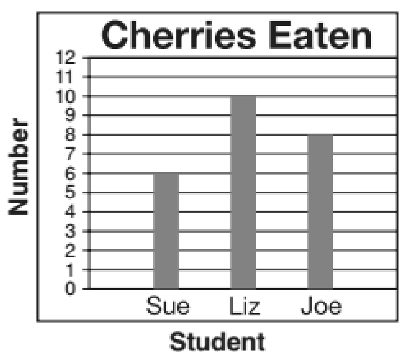
D

75 **[LOOK AT THE TALLY CHART AT THE TOP OF THE PAGE. THE TALLY CHART SHOWS THE NUMBER OF CHERRIES EACH STUDENT ATE. WHICH GRAPH MATCHES THE TALLY MARKS IN THE CHART?]**

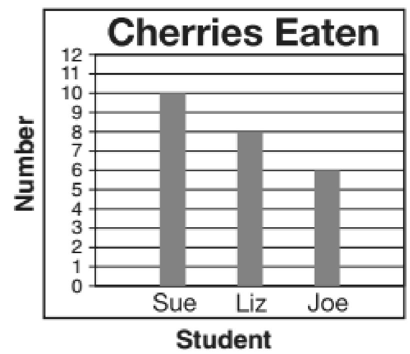




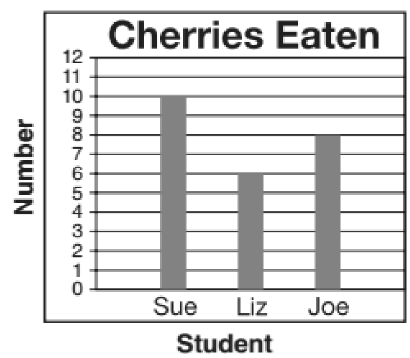
A



B



C



D

CSN00320

79 [CARRIE PRACTICES THE PIANO EACH DAY. THE TABLE SHOWS HOW LONG SHE PRACTICED EACH DAY LAST WEEK. HOW MANY MINUTES LONGER DID SHE PRACTICE ON WEDNESDAY THAN ON TUESDAY? MARK YOUR ANSWER.]

Piano Practice Times

Day	Minutes
Monday	26
Tuesday	24
Wednesday	30
Thursday	35
Friday	15

6	5	4	2
A	**B**	**C**	**D**

CSN00228

Question Number	Correct Answer	Standard	Year of Release
1	*A*	2NS1.1	2004
2	*C*	2NS1.1	2005
3	*B*	2NS1.1	2006
4	*D*	2NS1.1	2007
5	*B*	2NS1.2	2004
6	*D*	2NS1.2	2005
7	*A*	2NS1.3	2003
8	*C*	2NS1.3	2004
9	*A*	2NS1.3	2005
10	*A*	2NS1.3	2006
11	*A*	2NS1.3	2007
12	*D*	2NS1.3	2007
13	*C*	2NS2.1	2003
14	*B*	2NS2.1	2004
15	*D*	2NS2.1	2007
16	*D*	2NS2.2	2003
17	*D*	2NS2.2	2004
18	*B*	2NS2.2	2005
19	*D*	2NS2.2	2006
20	*C*	2NS3.1	2004
21	*B*	2NS3.1	2005
22	*C*	2NS3.1	2007
23	*C*	2NS3.2	2003
24	*A*	2NS3.2	2005
25	*B*	2NS3.2	2006
26	*D*	2NS3.3	2003
27	*B*	2NS3.3	2004
28	*C*	2NS3.3	2006
29	*A*	2NS3.3	2007
30	*A*	2NS4.1	2003
31	*B*	2NS4.1	2004
32	*A*	2NS4.1	2005
33	*B*	2NS4.2	2003
34	*C*	2NS4.2	2005
35	*D*	2NS4.3	2003
36	*B*	2NS4.3	2005
37	*D*	2NS4.3	2007
38	*A*	2NS5.1	2003
39	*B*	2NS5.1	2004
40	*B*	2NS5.1	2006

Question Number	Correct Answer	Standard	Year of Release
41	*B*	2NS5.1	2007
42	*D*	2NS5.2	2003
43	*A*	2NS5.2	2005
44	*A*	2NS5.2	2006
45	*A*	2NS5.2	2007
46	*D*	2NS6.1	2004
47	*B*	2AF1.1	2003
48	*B*	2AF1.1	2004
49	*D*	2AF1.1	2005
50	*A*	2AF1.1	2007
51	*A*	2AF1.2	2003
52	*D*	2AF1.2	2005
53	*C*	2AF1.3	2004
54	*B*	2MG1.1	2006
55	*A*	2MG1.2	2004
56	*B*	2MG1.3	2004
57	*B*	2MG1.3	2006
58	*C*	2MG1.3	2006
59	*B*	2MG1.3	2007
60	*C*	2MG1.3	2007
61	*B*	2MG1.4	2003
62	*D*	2MG1.4	2005
63	*C*	2MG1.5	2005
64	*C*	2MG2.1	2003
65	*B*	2MG2.1	2003
66	*A*	2MG2.1	2006
67	*A*	2MG2.2	2004
68	*A*	2MG2.2	2006
69	*D*	2MG2.2	2006
70	*B*	2MG2.2	2007
71	*A*	2MG2.2	2007
72	*D*	2PS1.1	2005
73	*D*	2PS1.1	2006
74	*B*	2PS1.1	2007
75	*D*	2PS1.2	2003
76	*D*	2PS1.2	2006
77	*A*	2PS1.3	2005
78	*A*	2PS1.3	2006
79	*A*	2PS1.4	2004
80	*C*	2PS1.4	2007

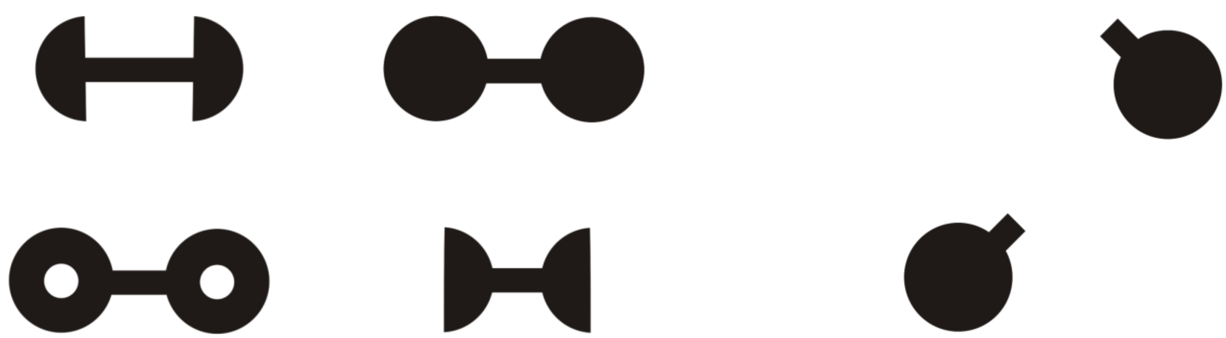
Appendix F

Children's Mental Transformation Task Test (CMTT)

- All participating students will be handed a paper version of the test.
- Students will be asked to complete #1-16 for the pretest and #17-32 for the posttest. Students will only receive questions for the portion of the test they are taking.
- All students will be using Form A-Order 1.
- The teacher participant will be asked to read the instructions out loud to the class.

Instructions for the test: Circle which shape on the left would be made from moving the two pieces on the right together.

The following sample problem will be shown on the Smartboard.



1

2

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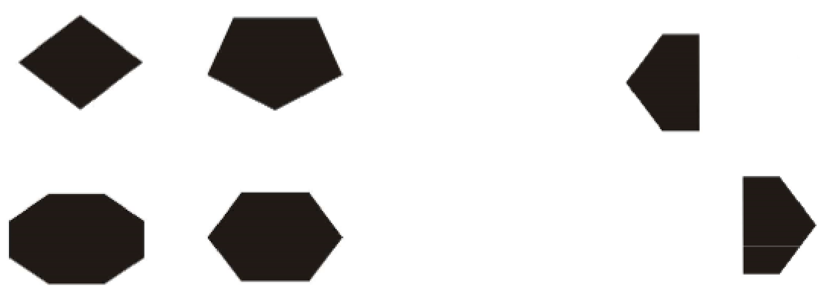

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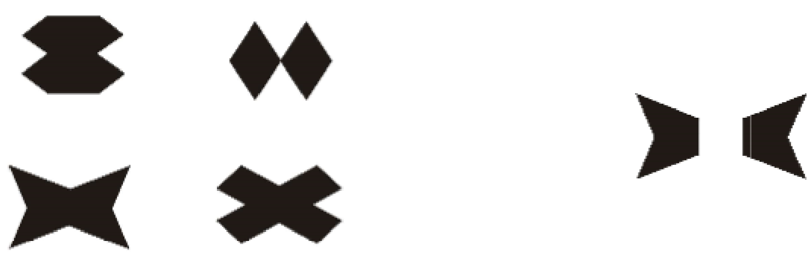
24



25

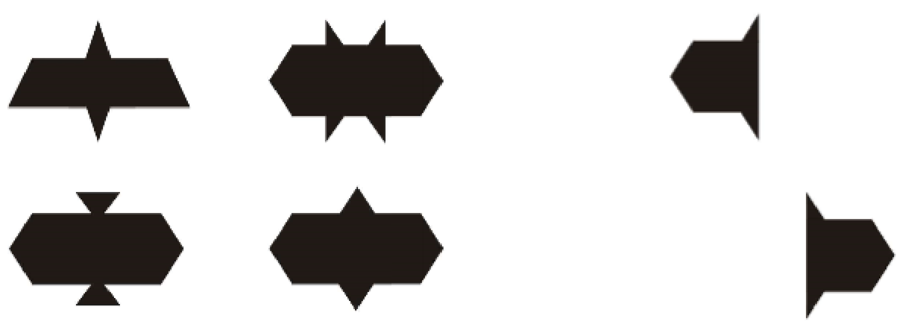
26

27

28

29

30



31

32

Appendix G

Permission for Reproduction

Tracy Thompson

From: Lynn Breyfogle [mbreyfog@bucknell.edu]
Sent: Monday, April 18, 2016 8:12 AM
To: Tracy Thompson; Courtney Lynch
Subject: Re: Permission for Use - Dissertation

Dear Tracy-
Yes, we approve the inclusion of both tables (p. 234 & p. 235).
-Lynn

On Sun, Apr 17, 2016 at 9:06 PM, Tracy Thompson <thompson.07@windstream.net> wrote:

Dr. Breyfogle and Mrs. Lynch –

Thank you for your quick reply and for your permission to use the table from page 234. I was so determined to send off my original email, that I forgot to also request permission to reproduce the accompanying table from page 235 as well, the van Hiele sequence of phases of learning. Since my original email was specific to state use of just the table from page 234, I would like to make sure I have permission to use both before electronically submitting my dissertation.

I would be happy to send you a link to the electronic document after publishing with my university. I will send a separate email to request permission of both so that I can provide a clean version in my appendix, pending your approval of course.

I apologize for the inconvenience.

Thank you,

Tracy Thompson

1

Permission for use of CMTT

Tracy Thompson

From: susanclevine@gmail.com
Sent: Saturday, November 01, 2014 11:06 AM
To: Tracy Thompson
Subject: Re: Permission for use of Children's Mental Transformation Task

Hi Tracy,

Thanks for your email. Happy to have you use the CMTT. If you have any questions let me know. Would love to hear how your project turns out.

Best, Susan

Sent from my iPhone

On Nov 1, 2014, at 10:46 AM, "Tracy Thompson" <thompson.07@windstream.net> wrote:

> Ms. Susan Levine-
>
> I am a student at Oklahoma State University pursing a Ph.D. in Education, Professional Studies of Mathematics. I am currently ABD and in the process of proposal approval. You were listed as the contact person for the Children's Mental Transformation Task test listed on http://www.spatiallearning.org . I will be submitting my dissertation proposal this month to begin collecting data in Spring 2015. I believe that this particular test would benefit my mixed methods study of the effects of dynamic virtual manipulatives on the development of spatial abilities in elementary students in second grade.
>
> Please contact me on the steps I need to take to obtain permission for use, instructions for use, and any other pertinent information I may need. I may be contacted via email at Thompson.07@windstream.net, scovilt@okstate.edu, or by phone at (918) 277-3670. I look forward to hearing from you soon.
>
> Thank you,
>
> Tracy Thompson, M.Ed.
> Assistant Professor of Education
> Northeastern State University

1

Appendix H

Student Interview Protocol

The interviewer will greet the interviewee. Interviewer will say the following: "Thank you for coming back and talking to me. While you are solving the puzzle I would like for you to say what you are thinking. You may choose not to answer a question or to stop the interview at any time. Is that okay with you?" (Wait for answer and respond to any questions that may arise.) "Is it okay for me to video record you while you work a tangram puzzle? (Wait on answer, start recording after permission has been given.) I will type a copy of everything you say on the video. No one will know what you say or do on the tape except for me." The interviewer will ask the interviewee to speak aloud while completing a puzzle from Activity Week 4. The interviewer will encourage the interviewee to verbalize his/her thoughts by asking questions such as:

- Why did you pick that piece?
- Why did you move that piece?
- What are you thinking about moving next? Why?
- Could you have used the pieces in other places and still had the same shape?

Discuss student perceptions of the manipulative used in his/her group.

- What did you think about the tangram puzzles you did in class?
- What did you like the most about the tangram puzzles you used?
- What did you like the least about the tangram puzzles you used?
- What would make you like the tangram puzzles more?
- What would make you like the tangram puzzle less?
- How did your group work together to solve the puzzles?

- If you could change anything about the activities that you did, what would you change?
- What did you learn about the shapes of the puzzle pieces?

Teacher Interview Protocol

The interviewer will greet the interviewee. Interviewer will say the following: "Thank you for allowing me to interview you. I would like to ask you a few questions about your experiences during this study. You may choose not to answer a question or to stop the interview at any time. Is that okay with you?" (Wait for answer and respond to any questions that may arise.) "I am planning on video recording the interview. I will transcribe your interview verbatim to use for this research project. The researcher is the only one that will have access to this data. Your name and all other identifying characteristics will be kept confidential. Is it okay for me to video record you? (Wait on answer, start recording after permission has been given.) I will type a copy of everything you say on the video. No one will know what you say or do on the tape except for me." The interviewer will ask the interviewee to discuss their experiences with the manipulative used in class such as:

- Describe the level of student engagement that was observed while using the manipulative.
- Describe student's attitudes observed during work with the manipulative.
- Did you notice any change in motivation in the students?
- What did the manipulative do for the students' mathematical ability?
- Describe group behavior when using the manipulatives.
- Describe any advantages you think the manipulative holds.
- Describe any disadvantages you may foresee with the use of the manipulative.
- Is there anything you would like to add about your experience?

Appendix I

OSU IRB Approval

Oklahoma State University Institutional Review Board

Date:	Thursday, March 19, 2015
IRB Application No	ED1535
Proposal Title:	The effects of concrete, virtual, and multimodal manipulatives on elementary students' development of spatial sense and mathematics achievement: A two-tier mixed methods study
Reviewed and Processed as:	Expedited

Status Recommended by Reviewer(s): Approved Protocol Expires: 3/18/2016

Principal Investigator(s):

Tracy Thompson	Adrienne Sanogo
23930 E. 148th St. South	230 Willard
Coweta, OK 74429	Stillwater, OK 74078

The IRB application referenced above has been approved. It is the judgment of the reviewers that the rights and welfare of individuals who may be asked to participate in this study will be respected, and that the research will be conducted in a manner consistent with the IRB requirements as outlined in section 45 CFR 46.

☒ The final versions of any printed recruitment, consent and assent documents bearing the IRB approval stamp are attached to this letter. These are the versions that must be used during the study.

As Principal Investigator, it is your responsibility to do the following:

1.Conduct this study exactly as it has been approved. Any modifications to the research protocol must be submitted with the appropriate signatures for IRB approval. Protocol modifications requiring approval may include changes to the title, PI advisor, funding status or sponsor, subject population composition or size, recruitment, inclusion/exclusion criteria, research site, research procedures and consent/assent process or forms
2.Submit a request for continuation if the study extends beyond the approval period. This continuation must receive IRB review and approval before the research can continue.
3.Report any adverse events to the IRB Chair promptly. Adverse events are those which are unanticipated and impact the subjects during the course of the research; and
4.Notify the IRB office in writing when your research project is complete.

Please note that approved protocols are subject to monitoring by the IRB and that the IRB office has the authority to inspect research records associated with this protocol at any time. If you have questions about the IRB procedures or need any assistance from the Board, please contact Dawnett Watkins 219 Cordell North (phone: 405-744-5700, dawnett.watkins@okstate.edu).

Sincerely,

Hugh Crethar, Chair
Institutional Review Board

Appendix J

Adult Consents/Student Assents

ADULT CONSENT FORM

OKLAHOMA STATE UNIVERSITY

PROJECT TITLE:

THE EFFECTS OF MULTIMODAL MANIPULATIVES ON ELEMENTARY STUDENTS' DEVELOPMENT OF SPATIAL SENSE THROUGH MATHEMATICS ACHIEVEMENT, AND GENDER: A MIXED METHODS STUDY

INVESTIGATORS: Tracy Thompson, M.Ed./Oklahoma State University

PURPOSE:

The purpose of this research study is to determine what effects, if any, the use of multimodal tangrams have on student mathematical achievement and the development of spatial sense, as compared to concrete and virtual manipulatives. This research will also examine any differences in math achievement and spatial development when using the three different types of manipulatives: concrete, virtual, and multimodal, as categorized by gender and ability. Additionally, the researcher seeks to identify trends within student and teacher interviews and speak-alouds that seek to explain any differences between math-achievement and spatial development when using any of the three types of tangram manipulatives.

PROCEDURES

You will administer two pre- and post-assessments. A pre- and post- mathematics achievement test will be administered to test understanding of a range of mathematical content skills. A pre- and post- spatial awareness test will be administered to test spatial sense development. You will conduct a teacher-guided lesson on tangrams using three different forms of manipulatives: concrete (wooden tangram tiles), virtual (digital tangram pieces using an iPad), and multimodal (OSMO gaming system – wooden tangram tiles projected onto an iPad). You will group students in ability based groups of low, average, and high. You will facilitate one group activity per week for a total of four weeks. Each weekly activity will consist of a given worksheet with four tangram puzzles. You will guide a whole class discussion of the students' results and experiences at the conclusion of each activity.

Your will be asked to discuss your experiences with each type of manipulative and to verbally describe your perception of student behaviors while working with the manipulatives. This study is designed to last a total of approximately 6 hours. The time will be divided into six weeks, with approximately one hour per week spent on completing the study.

You may be audio and/or video recorded during quantitative and/or qualitative data collection. You may have your picture taken while guiding/participating in activities related to the study.

At the end of the study, you will be asked to ensure that student participants will have an opportunity to use each of the concrete, virtual, and multimodal manipulatives that were not previously used in the students' treatment group. This will help to ensure that every student experiences the three different types of manipulatives and receives the same potential benefit of the other groups of participants.

RISKS OF PARTICIPATION:

There are no known risks associated with this project which are greater than those ordinarily encountered in daily life.

BENEFITS OF PARTICIPATION:

Benefits of participation in the study may include increased familiarity with iPad technology, increased student math achievement scores, increased spatial sense scores, and/or increased knowledge of geometric compositions and decompositions.

If you are interested, we will send you a copy of the results of the study when it is finished.

CONFIDENTIALITY:

The records of this study will be kept private. Any written results will discuss group findings and will not include information that will identify you or your students. Pseudonyms will be used in lieu of identifying names. Research records will be stored on a password protected computer in a locked office and only researchers and individuals responsible for research oversight will have access to the records.

Data will be destroyed five years after the study has been completed.

Video or audio tapes will be transcribed and destroyed within 90 days of the interview.

COMPENSATION:

Compensation for participation in the study will not be provided.

CONTACTS :

You may contact any of the researchers at the following addresses and phone numbers, should you desire to discuss your participation in the study and/or request information about the results of the study: Tracy Thompson, M.Ed., Bagley Hall 216, Dept. of Teacher Education, Northeastern State University, 600 N. Grand Ave., Tahlequah, OK 74464, (918) 444-3730 or Dr. Adrienne Sanogo, Ph.D., 230 Willard Hall, Dept. of Teaching and Curriculum Leadership, Oklahoma State University, Stillwater, OK 74078, (918) 744-9515. If you have questions about your rights as a research volunteer, you may contact the IRB Office at 219 Cordell North, Stillwater, OK 74078, 405-744-3377 or irb@okstate.edu

PARTICIPANT RIGHTS:

I understand that my participation is voluntary, that there is no penalty for refusal to participate, and that I am free to withdraw my consent and participation in this project at any time, without penalty.

CONSENT DOCUMENTATION:

I have been fully informed about the procedures listed here. I am aware of what I will be asked to do and of the benefits of my participation. I also understand the following statements:

I affirm that I am 18 years of age or older.

I have read and fully understand this consent form. I sign it freely and voluntarily. A copy of this form will be given to me. I hereby give permission for my participation in this study.

Signature of Participant Date

I certify that I have personally explained this document before requesting that the participant sign it.

Signature of Researcher Date

PARENT/GUARDIAN PERMISSION FORM

OKLAHOMA STATE UNIVERSITY

PROJECT TITLE:

THE EFFECTS OF MULTIMODAL MANIPULATIVES ON ELEMENTARY STUDENTS' DEVELOPMENT OF SPATIAL SENSE THROUGH MATHEMATICS ACHIEVEMENT, AND GENDER: A MIXED METHODS STUDY

INVESTIGATOR(S): Tracy Thompson, M.Ed./Oklahoma State University

PURPOSE:

The purpose of this research study is to determine what effects, if any, the use of multimodal tangrams have on student mathematical achievement and the development of spatial sense, as compared to concrete and virtual manipulatives. This research will also examine any differences in math achievement and spatial development when using the three different types of manipulatives: concrete, virtual, and multimodal, as categorized by gender and ability. Additionally, the researcher seeks to identify trends within student and teacher interviews and speak-alouds that seek to explain any differences between math-achievement and spatial development when using any of the three types of tangram manipulatives.

PROCEDURES:

Your child will complete a teacher-guided lesson on tangrams using one of three forms of manipulatives: concrete (wooden tangram tiles), virtual (digital tangram pieces using an iPad), or multimodal (OSMO gaming system – wooden tangram tiles projected onto an iPad).Your child will complete two pre- and post-assessments. A pre- and post- mathematics achievement test will be administered to test understanding of a range of mathematical content skills. A pre- and post- spatial awareness test will be administered to test spatial sense development.

Your child may be selected to discuss his/her experience with the manipulatives and to verbally describe their actions while working with the manipulatives. This study is designed to last a total of approximately 6 hours. The time will be divided into six weeks, with approximately one hour per week spent on completing the study.

Study participants may be audio and/or video recorded during quantitative and/or qualitative data collection. Study participants may have their picture taken while participating in activities related to the study.

At the end of the study, participants will have the opportunity to use each of the concrete, virtual, and multimodal manipulatives to ensure that every student experiences the three different types of manipulatives and receives the same potential benefit of the other groups of participants.

RISKS OF PARTICIPATION:

There are no known risks associated with this project which are greater than those ordinarily encountered in daily life.

BENEFITS OF PARTICIPATION:

Benefits of participation in the study may include increased familiarity with iPad technology, increased student math achievement scores, increased spatial sense scores, and/or increased knowledge of geometric compositions and decompositions.

If you are interested, we will send you a copy of the results of the study when it is finished.

CONFIDENTIALITY:

The records of this study will be kept private. Any written results will discuss group findings and will not include information that will identify you or your child. If results of a single child are included in the study, a pseudonym will be used in lieu of the child's identifying name. Research records will be stored on a password protected computer in a locked office and only researchers and individuals responsible for research oversight will have access to the records.

Data will be destroyed five years after the study has been completed.

Video or audio tapes will be transcribed and destroyed within 90 days of the interview.

COMPENSATION:

Compensation for participation in the study will not be provided.

CONTACTS:

You may contact any of the researchers at the following addresses and phone numbers, should you desire to discuss your participation in the study and/or request information about the results of the study: Tracy Thompson, M.Ed., Bagley Hall 216, Dept. of Teacher Education, Northeastern State University, 600 N. Grand Ave., Tahlequah, OK 74464, (918) 444-3730 or Dr. Adrienne Sanogo, Ph.D., 230 Willard Hall, Dept. of Teaching and Curriculum Leadership, Oklahoma State University, Stillwater, OK 74078, (918) 744-9515. If you have questions about your rights as a research volunteer, you may contact the IRB Office at 219 Cordell North, Stillwater, OK 74078, 405-744-3377 or irb@okstate.edu

PARTICIPANT RIGHTS:

I understand that my child's participation is voluntary, that there is no penalty for refusal to participate, and that I am free to withdraw my permission at any time. Even if I give permission for my child to participate I understand that he/she has the right to decline.

CONSENT DOCUMENTATION:

I have been fully informed about the procedures listed here. I am aware of what my child and I will be asked to do and of the benefits of my participation. I also understand the following statements:

I have read and fully understand this permission form. I sign it freely and voluntarily. A copy of this form will be given to me.

I hereby give permission for my child (insert child's name here)_____ __________ ______ _______ to participate in this study.

____ _________________________

Signature of Parent/Legal Guardian Date

____

Printed Name of Parent/Legal Guardian

I certify that I have personally explained this document before requesting that the participant sign it.

_____________________________________ ___________________________

Signature of Researcher Date

____

Printed Name of Researcher

ASSENT FORM

OKLAHOMA STATE UNIVERSITY

Dear Student,

We are interested in learning about your experience using several different types of tangram manipulatives. In order to understand this, we would like you to participate in a tangram activity using either plastic tangram pieces, computer tangram pieces on an iPad, or wooden tangram pieces that project onto an iPad. We would also like you to answer some questions regarding your experience with the different types of tangram pieces. We will need your permission to let us test your math skills before and after playing the gaming system. Your parent/guardian is aware of this project.

Please understand that you do not have to do this. You do not have to answer any questions that you do not want to. You may stop at any time.

Your name will not be on the forms you fill out, and you will be given a number that will be put on your answer sheet so no one will know whose answers they are. The only way anyone would know how you answered is if we are worried about you, and then we would call your parent/guardian. If you have any questions about the form or what we are doing, please ask us. Thank you for your help.

Sincerely,

Tracy Thompson, M.Ed.

Graduate Student Oklahoma State University

Adrienne Sanogo, Ph.D.

Associate Professor Oklahoma State University

I have read this form and agree to help with your project.

____

(your name**)**

____

(your signature)

____

(date)

Appendix K

Coding Process

Coding process for influences and perceptions of the different tangram types

Relevant Research Topic	Major Category	Emergent Themes	Associated Concepts
Influence of different manipulative type on math achievement and spatial sense.	Content	shapes out of shapes, congruence, visualization of transformations	composition/decomposition, turning blocks, motioning movement, spatial, visualize, perception, transformation vocab, flip, turn, rotate
	Problem solving	permanence/ flexibility, okay to be wrong, persistence, multiple solutions	started over, problem solving, jigsaw puzzle, start over, different ways to solve, more than 1 answer, never committed, skip to next puzzle, ask to be done, fear of wrong answer, trial and error, no assistance, hesitant
	Commun-ication	communicate knowledge and processes, verbal/non-verbal gestures, group communication or lack thereof	groups, small groups, talking, feedback by peer, feedback by teacher, feedback by manipulative, frustrated with not being able to communicate, not used to talking to each other, took turns,

Student and teacher perceptions by manipulative type	Need for feedback	feelings of frustration, feedback, problem solving, engagement, motivation, hands-on/touch	challenge, frustrations, competitive, difficult, feedback from partner, feedback from teacher, feedback from iPad, frustrated with partner, faster, harder, easier, independent,
	Engagement and motivation	Engagement, motivation	Engaged, good attitude, eager, pride, focused on screen, indifferent, means to an end, finish, game
	Advantages to each manip. type	Problem solving, hands on/touch	asked about iPads, get to touch them,

VITA

Tracy Ann Thompson

Candidate for the Degree of

Doctor o f Philosophy

Thesis: THE EFFECTS OF CONCRETE, VIRTUAL, AND MULTIMODAL TANGRAM MANIPULATIVES ON SECOND GRADE ELEMENTARY STUDENTS' MATHEMATICS ACHIEVEMENT AND DEVELOPMENT OF SPATIAL SENSE: A CONVERGENT PARALLEL MIXED METHODS STUDY

Major Field: Mathematics Education

Biographical:

Education:

Completed the requirements for the Doctor of Philosophy in Professional Studies at Oklahoma State University, Stillwater, Oklahoma in May, 2016.

Completed the requirements for the Master of Science in Mathematics Education at Northeastern State University, Broken Arrow, OK, in December 2009.

Completed the requirements for the Bachelor of Science in Business Administration at University of Tulsa, Tulsa, OK, in August, 2002.

Completed the requirements for the Associate of Arts in Liberal Arts at Tulsa Community College, Tulsa, OK, in July 2000.

Experience:

Assistant Professor of Education, Tenure Track
Northeastern State University, Tahlequah, OK, August 2014 - Present

Mathematics Instructor, 9^{th}-10^{th} grade
North Intermediate High School, Broken Arrow, OK, August 2007 – July 2014

Lecturer: Mathematics Structures
Oklahoma State University, Tulsa, OK, Spring 2013