SUPPORTING CONTINUOUS IMPROVEMENT: AN ACCOUNTING BASED CONTROL SYSTEM

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TABLE OF CONTENTS

Chapter Page

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

c	total cost of resources consumed to produce output
k	proportion of residual waste eliminated in a period
I	improvement (reduction in waste) observed in a period
p	price paid for each resource consumed to produce output
q	final product produced by the firm
r	activity rate
R	residual waste remaining after t improvements
w	total amount of waste to be eliminated
x	resource consumed to produce output
z	activity output consumed to produce the final product

CHAPTER I

THE RESEARCH PROBLEM

Introduction

Continuous improvement, which has emerged as a legitimate and widely accepted paradigm, provides a conceptually new approach for producing goods and services. Simply put, continuous improvement is the firm's desire to become always better. Elimination of waste is the motivation for continuous improvement (Suzaki, 1987).

Waste can exist for a variety of reasons. Inefficient operations can lead to the excess use of resources. Poor quality can lead to the excess use of labor and materials caused by inspection and rework and poor product design can cause the use of more costly production methods.

On the other hand, as waste in productive inputs is eliminated, output can be increased with no increase in cost or current output can be produced with less cost. In today's operating environment, controlling cost is essential for survival. By eliminating waste, the firm attempts to create or maintain a competitive advantage.

Continuous improvement is the elimination of waste throughout the organization. This waste elimination is achieved through small but systematic incremental changes that have the potential of creating a significant cumulative effect.

Zero waste is the ultimate goal. Thus, continuous improvement can be defined as systematic, incremental changes that lead to the complete elimination of waste.

Continuous improvement is a type of organizational change (Choi, 1995). Researchers in the area of organizational change have identified different types of changes. Two of these types of changes are referred to as "alpha changes" and "gamma changes" (Golembiewski et al., 1976; Van de Vliert et al., 1985). Bartunek and Moch (1987) note that alpha, or first-order, changes are "incremental modifications that make sense within an established framework or method of operating" and that gamma, or second-order, changes are "incremental modifications in the frameworks themselves." Alpha changes are gradual and incremental; gamma changes are abrupt, major changes that disrupt the entire organization. Process reengineering, for example, would be considered a gamma type change.

Choi (1995) notes that continuous improvement represents low risk, operational level change that has the potential of making a major change without disruptive effects. Thus, continuous improvement corresponds to alpha changes; it is characterized by the elimination of waste within an existing framework rather than any change in the framework itself. Over time, the incremental, systematic changes can accumulate to create a potentially large cumulative effect. Leavitt (1988), indicates that, "trying routinely to get better one step at a time is a far better way than shooting constantly for the moon . . . getting better and better one step at a time adds up. Sometimes a little step turns surprisingly into a big leap." Leavitt also concludes that "big prophetic leaps into sudden business successes are rare."

The potential for continuous improvement to produce a major change in organizational effectiveness depends strongly on the amount of waste present in an organization: the greater the amount of waste, the greater the opportunity for a major change. Yet the continuous improvement approach makes no effort to identify the total amount of waste present in the production process. Instead, efforts are made to identify some observable waste, eliminate it, and then repeat the process (Treece, 1993). Thus, waste elimination is achieved through a local search process that may be guided by some general, albeit vague, global sense of direction.

However, no well structured control system exists to support the firm's continuous improvement efforts (Mak and Roush, 1994; Greenwood and Reeve, 1992). As a result, McNair (1994) notes that, "Managers start taking on the look of zombies wandering from one continuous improvement seminar to another, juggling six different implementations at once and wondering where it will all end." If a control system can be developed, the firm will be able to successfully guide and evaluate waste elimination efforts. This, in turn, will support the firm's objective of creating or maintaining a competitive advantage by directing efforts to reduce cost.

The ability to control continuous improvement depends on the ability to identify the potential for improvement and how that improvement should be achieved. If the total waste to be eliminated were known, the potential benefits from continuous improvement would be revealed. This knowledge, in turn, could be used to guide and evaluate the firm's improvement efforts. Once the zero waste target is known, the firm can devise strategies for achieving this goal. Furthermore, the information could be used to facilitate choosing among competing continuous improvement options by identifying those that generate the greatest waste elimination relative to the cost of implementation.

Another issue of importance is the rate of waste elimination. Getting better one step at a time does not necessarily mean that it has to take a long time to achieve a large cumulative effect. In fact, a firm engaged in an orderly but rapid pace of change should have a competitive advantage over one with an orderly but slow pace of change. By guiding and evaluating improvement efforts, the control system can provide information to help the firm increase its rate of waste elimination by removing the uncertainty (and error) involved in waste elimination.

Finally, knowing the total potential for improvement may signal a need to search for gamma-type changes. This is because alpha changes may not accumulate to a point where they make a difference significant enough to maintain or improve a firm's competitive position. The potential for continuous improvement depends on the total amount of waste in the organization. Thus, the ability of continuous improvement to lead to the creation or maintenance of a competitive advantage can only be determined by identifying the total amount of waste present in the system.

Identifying the total waste, however, is not a trivial problem. Waste can be defined as the difference between the inputs actually used and the inputs that should have been used (optimal inputs) to produce a given output. The actual inputs are observable, but what are the optimal inputs? In theory, for a given output, the optimal inputs are found by minimizing the total cost of the inputs subject to an output constraint where output is produced according to a well defined production function.

By implementing the optimal input combination, the firm minimizes cost and achieves the zero waste state. Unfortunately, the production function that defines the output constraint is a theoretical construct and is not explicitly known to management. Since the underlying production function is unobservable, the zero waste state and total waste cannot be derived in a direct fashion and are also unobservable.

Purposes of this Study

This leads to the first of three purposes of this study. A model of continuous improvement is developed that uses observable data to identify the zero waste state. Although the optimal inputs and minimum cost that define the zero waste state are unobservable, the firm can observe the actual output, actual inputs and actual cost each period. As continuous improvement efforts eliminate waste, these observable data generate systematic input and cost sequences. To model continuous improvement, the conditions that must hold for the firm to achieve complete waste elimination are determined. Within a set of reasonable assumptions that are consistent with the notion of continuous improvement, these conditions and the systematic nature of the observable data are exploited to identify the zero waste state defined by an unknown production function. This information can be used to control the firm's continuous improvement efforts.

The next purpose of this study is to show how information gained from the model can be used to guide and evaluate the firm's waste elimination efforts. The observable data are used to ensure that continuous improvement efforts are progressing as intended. Also, the observable data provide feedback that is used to help the firm to increase the rate of waste elimination. By providing direction for continuous improvement, the control system should help the firm to create or maintain a competitive advantage.

The third and final purpose of this study is to incorporate the control system into an existing accounting structure. According to Choi (1995), waste elimination brought about through continuous improvement is accomplished by eliminating wasteful processes. A process is a series of activities linked to achieve a specific objective (Hansen and Mowen, 1997). Activity-based management is an accounting information system designed to focus the firm's attention of activities with the objective of eliminating wasteful practices. It seems, therefore, that activity-based management provides an ideal accounting structure to support continuous improvement. Thus, this study shows how the control system can be incorporated into a very basic activity-based management accounting structure.

The remainder of this study is organized as follows. Chapter II discusses the framework and objectives of the continuous improvement control system. Chapter III develops an input-based model of continuous improvement that uses observable data to identify the optimal input combination. In Chapter IV, the model is extended into a more general cost framework. This cost-based model is used in Chapter V to describe a control system capable of guiding and evaluating the firm's continuous improvement efforts. Chapter VI shows how the control system can be incorporated into a very basic activity-based management accounting structure. The final chapter summarizes the study, discusses the limitations and suggests extensions for future research.

CHAPTER II

THE FRAMEWORK OF A CONTINUOUS IMPROVEMENT CONTROL SYSTEM

Introduction

The purpose of any control system is to support the attainment of the firm's goals. One of the firm's goals is to minimize the cost of producing a product. This goal is operationalized through continuous improvements efforts that have the objective of completely eliminating waste, thereby minimizing cost. To support continuous improvement the control system should guide and evaluate the firm's waste elimination efforts to ensure that the cost minimizing zero waste state will be achieved. This chapter begins by describing the structure of a control system capable of supporting continuous improvement. Waste is defined and the structure of the control system is incorporated into the process of waste elimination.

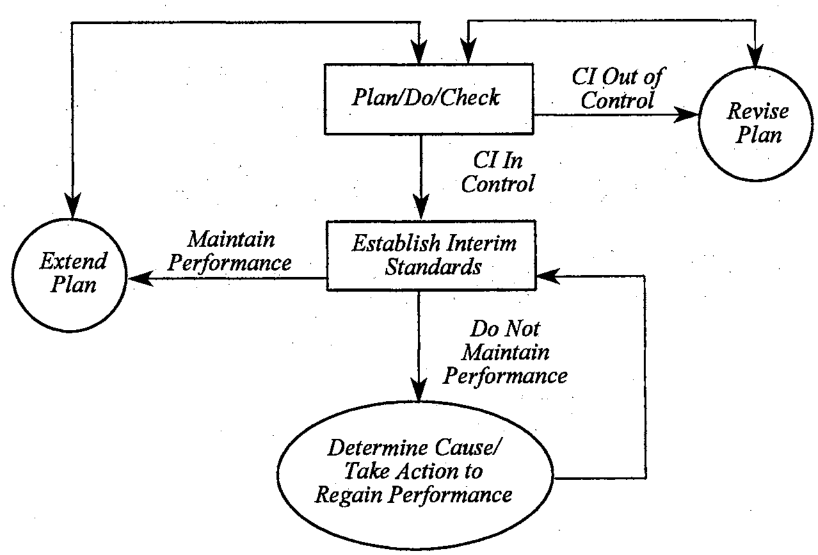
Structure of the Control System

Controlling is defined as the activity of monitoring a plan's implementation and taking corrective action as necessary. Ideally, a control system should evaluate the progress of a plan, provide insight into the root cause of the performance, focus attention on the biggest opportunity for improvement and suggest what actions should be taken to realize that opportunity (Drach, 1994; Nanni, et. al., 1990; Romano, 1989;

Mosconi and McNair, 1987). In a continuous improvement environment, McNair (1990) summarizes the control process as a "plan/do/check" loop.

The control system should identify the potential for improvement ("plan"), determine the actions necessary to implement the improvement ("do") and determine whether or not the intended improvements occur ("check") (Drach, 1994; Ostrenga, 1990). Such a system is illustrated in Figure 2.1. In implementing a control system to support continuous improvement efforts, the total amount of waste to be eliminated must be identified. Thus, the control system should guide and evaluate continuous improvement efforts so that this waste is completely eliminated. The waste elimination process can be guided by setting short term, interim standards designed to systematically eliminate waste. To evaluate the progress, actual results can then be compared to the standard amounts to determine if desired level of waste elimination is achieved.

Figure 2.1
Structure of a Continuous Improvement Control System



The established standards of the continuous improvement control system must be dynamic and theoretically based. As actual results conform to the desired outcome, the interim standards must be revised and extended. It is through the use of these interim standards that the control system guides and evaluates continuous improvement.

According to Choi (1995), the zero waste state should be the eventual outcome of continuous improvement. The control system is a critical part of this process. It reveals the potential for improvement, guides improvement efforts and evaluates actual progress. In short, the control system supports the attainment of the firm's goal of total waste elimination.

Waste Defined

The firm's objective is to minimize cost subject to the production function. The production function is expressed as $q = f(\mathbf{x})$ where q is the output of the firm and $\mathbf{x}$ is a 1 x n vector of inputs used to produce the output. (Bold characters denote vectors throughout the paper.) An input combination, $\mathbf{x}$, is technically efficient if it is impossible to produce more output using the same inputs. The optimal input combination, $\mathbf{x}^o$, is the technically efficient input combination that minimizes total cost of inputs for a given level of output. Let $\mathbf{x}^a$ be the actual input combination used to produce q. Total waste, $\mathbf{w}$, is defined as the difference between $\mathbf{x}^a$ and $\mathbf{x}^o$: $\mathbf{w} = \mathbf{x}^a - \mathbf{x}^o$. Thus, $w_i = x_i^a - x_i^o$ is the total waste for the ith input.

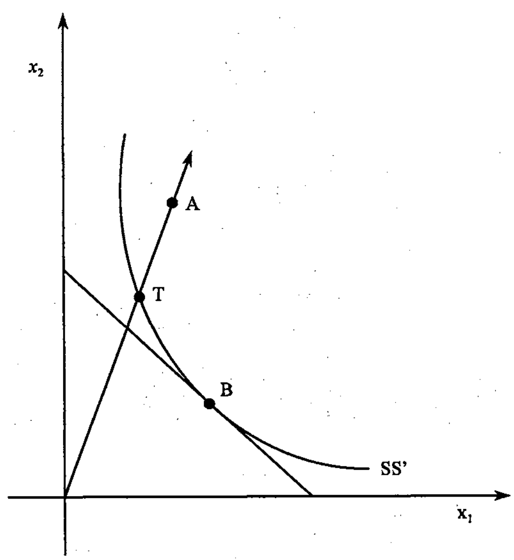
Letting $\mathbf{p}$ be the vector of input prices, $c^a = \sum_{i=1}^{n} p_i x_i^a$ is the total cost of the actual input combination. The minimum (optimal) cost to produce q is then

$$c^{o} = \sum_{i=1}^{n} p_i x_i^{o} \tag{2.1}$$

Given that input prices remain constant over time, the potential savings from the elimination of waste is the difference between c^a and c^o.

Using a two input production function, waste can be illustrated graphically. Let $q = f(x_1,x_2)$ be the production function. In Figure 2.2, the curve SS', commonly referred to as an isoquant, represents the locus of all technically efficient input combinations that can be used to produce a given output, q. Let $c = p_1x_1 + p_2x_2$ be the isocost curve for any combination of inputs. The point where the isocost curve is tangent to the isoquant, $\boldsymbol{x}^o$, defines the zero waste state (point B is Figure 2.2). The actual inputs used, $\boldsymbol{x}^a$, are represented by point A. Total waste is the difference between points A and B.

Figure 2.2
Waste in a Single Output/Two Input Setting



Total waste is made up of two types of waste: technical inefficiency and mix inefficiency. Technical inefficiency exists if the firm uses any input combination above the isoquant. By eliminating technical inefficiency, the firm is able to produce the same output using less inputs, thereby reducing cost. All input combinations that lie along the isoquant are technically efficient. Even though an infinite number of input combinations will eliminate technical inefficiency, only one of these combinations will minimize cost. Mix inefficiency exists if the technically efficient input combination that is identical in mix to the actual input combination does not minimize cost. By changing the relative mix of inputs used to produce the output, the firm can achieve further cost reductions.

Because point A lies above the isoquant, fewer inputs could have been used to produce the given output, q. The difference between points A and T is the portion of total waste attributable to technical inefficiency. Furthermore, the relative amounts of the inputs differ from the optimal inputs. This indicates that some waste is attributable to mix inefficiency (the difference between points T and B). Continuous improvement is a search process that moves the firm from point A to point B so that both technical and mix inefficiency are eliminated.

The Role of a Control System in Continuous Improvement

Continuous improvement is a waste elimination process. Once some amount of waste is identified, efforts are undertaken to rid the firm of that waste and thereby reduce cost. The first objective of the control system is to reveal the potential cost savings from continuous improvement by identifying the total amount of waste to be eliminated as defined by the firm's production function.

To create or maintain a competitive advantage, the firm must not only eliminate waste, but do so as rapidly as possible. The control system must guide the firm so that the rate of waste elimination increases. This must be done within the logical constraints of continuous improvement and the economic constraints of the firm's production function. These constraints are discussed in detail in Chapter III.

In addition, the control system should evaluate past and current continuous improvement efforts. This is necessary to ensure that waste elimination is progressing as intended and that the zero waste state will ultimately be achieved. A control system capable of guiding and evaluating continuous improvement efforts is described in Chapter V.

Knowledge of the zero waste state derived from the production function is the crucial first step for the control system. Unfortunately, the production function is a theoretical construct and, therefore, is not empirically observable. Thus, a need exists for a model to use observable data to identify the zero waste state. Such a model will be developed in Chapters III and IV.

Summary

The goal of continuous improvement is complete waste elimination. A control system which supports this goal should take the form of a dynamic "plan/do/check" loop. The opportunity for improvement will be revealed when the zero waste state is identified ("plan"). The firm's waste elimination efforts should be guided so that the firm achieves the zero waste state as rapidly as possible ("do"). Finally, actual progress toward the zero waste state must be evaluated to ensure that waste elimination efforts are progressing as planned ("check").

To control continuous improvement efforts, the zero waste state must be identified. However, the zero waste state is derived from the unknown production function. Thus, a need exists for a model which uses observable data to identify the total amount of waste to be eliminated even though the production function is unobservable.

CHAPTER III

REVELATION OF THE ZERO WASTE STATE: A MODEL OF CONTINUOUS IMPROVEMENT

Introduction

Continuous improvement is the firm's search for the zero waste state. This chapter describes the conditions sufficient for this search to be successful. Observable date are used to determine if these conditions exist. If the conditions exist, the data are then used to reveal the total amount of waste to be eliminated and, therefore, the zero waste state.

A Model of Continuous Improvement

Choi (1995) states that the "most direct driving force for continuous improvement is the organization's desire to eliminate waste." He goes on to quote Suzaki as saying that these changes come about through "ceaseless repetition." Thus, continuous improvement is characterized by incremental, ongoing changes that systematically eliminate waste on a period-by-period basis. In a continuous improvement environment, waste is systematically eliminated on a period-by-period basis. Over time, these changes align together to create the potentially large cumulative effect of total waste elimination.

Incremental and Systematic Waste Elimination

Let $\boldsymbol{w}$ be the initial waste present at the beginning of period 1, where period 1 is the first period in which continuous improvement is implemented. Let $\boldsymbol{I_t}$ be the observable improvement (waste eliminated) in period t. After t periods, the residual waste is expressed by the following

$$\boldsymbol{R_t = w - \sum_{v=1}^{t} I_v} \tag{3.1}$$

Equation 3.1 implies that the proportion of waste eliminated for the ith input in period t is defined as follows:

$$k_{it} = \frac{I_{it}}{R_{i,t-1}} \tag{3.2}$$

where $0 < k_{it} < 1$.

The requirement that $k_{it} > 0$ means that some waste is eliminated each period, consistent with the conceptual definition of continuous improvement. The upper bound, $k_{it} < 1$, implies that only a portion of the residual waste is eliminated in any given period. This is consistent with the notion of incremental improvement.

From Equation 3.2, $I_{it} = k_{it}R_{i,t-1}$. Thus, for period 1, the waste eliminated for input i is $I_{i1} = k_{i1}R_{i0} = k_{i1}w_i$. From Equation 3.1, the residual waste for input i at the end of period 1 is expressed as $R_{i1} = w_i - k_{i1}w_i = (1 - k_{i1})w_i$. Repeating this process for each period yields two equivalent equations for residual waste:

$$R_{it} = w_i - w_i\left[k_{i1} + k_{i2}(1 - k_{i1}) + \ldots + k_{it}\prod_{v=1}^{t-1}(1 - k_{iv})\right] \tag{3.3}$$

$$R_{it} = w_i \prod_{v=1}^{t} (1 - k_{iv}). \tag{3.4}$$

Note that the negative component of the right side of Equation 3.3 corresponds to $\sum_{v=1}^{t} I_{iv}$.

The residual waste equations and the notion of "ceaseless repetition" imply that the zero waste state can be described by any of the following three conditions:

$$\lim_{t\to\infty} R_{it} = \lim_{t\to\infty} \prod_{v=1}^{t} (1 - k_{iv}) w_i = 0 \tag{3.5}$$

$$\lim_{t\to\infty} \sum_{v=1}^{t} I_{iv} = w_i \tag{3.6}$$

$$\lim_{t\to\infty} \left[k_{i1} + \sum_{v=2}^{t} k_{iv} \prod_{m=1}^{v-1} (1 - k_{im}) \right] = 1. \tag{3.7}$$

The following lemma, stated without proof, describes the relationship among the above three equations.

Lemma 3.1: If any of the zero waste state equations holds (Equations 3.5 - 3.7), then the other two must also hold.

In order to reach the goal of total waste elimination, any given continuous improvement program must satisfy Lemma 3.1.

Systematic and Total Waste Elimination

Convergence of each of the three series described by Equations 3.5 through 3.7 is possible because of underlying economic constraints. The series, $\sum_{t=1}^{\infty} I_{it}$, is bounded from above and below such that $0 < \sum_{t=1}^{\infty} I_{it} \le w_i$. The upper bound, $\sum_{t=1}^{\infty} I_{it} \le w_i$, states that it is impossible to eliminate more waste than what exists. The lower bound follows from the definition of continuous improvement. These boundary conditions also imply that $0 < \lim_{t\to\infty}\left[k_{i1} + \sum_{v=2}^{t-1} k_{iv}\prod_{m=1}^{v-1}(1 - k_{im})\right] \le 1$ and $\lim_{t\to\infty}\prod_{v=1}^{t}(1 - k_{iv})w_i = sw_i$, where $0 \le s < 1$. Thus, convergence must occur for all three series. Unfortunately, there is no guarantee that any of the series converges to the zero waste state. However, by imposing certain restrictions on the behavior of k_{it}, convergence to the total waste elimination is assured. The following proposition describes the conditions for zero waste convergence.

Proposition 3.1: If $k_{it} \le k_{i,t+1}$ for all t, then $\lim_{t\to\infty}\left[k_{i1} + \sum_{v=2}^{t} k_{iv}\prod_{m=1}^{v-1}(1 - k_{im})\right] = 1.$

Proof: Let $k_{it} = k$ for all t. From Equations 3.3 and 3.4,

$$\lim_{t\to\infty} R_{it} = w_i \lim_{t\to\infty}(1 - k_i)^t = w_i \lim_{t\to\infty}\left[1 - \sum_{v=0}^{t} k_i(1 - k_i)^v\right]$$

$$\Rightarrow \lim_{t\to\infty}\sum_{v=0}^{t} k_i(1 - k_i)^v = 1.$$

Now let $k_{it} < k_{i,t+1}$. This implies that $1 - k_{it} > 1 - k_{i,t+1}$. Thus,

$$w_i(1 - k_{i1})^t \ge w_i\prod_{v=1}^{t}(1 - k_{iv})$$

$$\lim_{t\to\infty}(1 - k_{i1})^t \ge \lim_{t\to\infty}\prod_{v=1}^{t}(1 - k_{iv})$$

$$0 \ge \lim_{t\to\infty}\prod_{v=1}^{t}(1 - k_{iv}).$$

Similarly, since $1 - k_{it} > 0$ for every t, $\lim_{t\to\infty}\prod_{v=1}^{t}(1 - k_{iv}) \ge 0$. Thus, $\lim_{t\to\infty}\prod_{v=1}^{t}(1 - k_{iv}) = 0$,

and, by Lemma 3.1, $\lim_{t\to\infty}\left[k_{i1} + \sum_{v=2}^{t} k_{iv}\prod_{m=1}^{v-1}(1 - k_{im})\right] = 1.$

Q.E.D.

Example 3.1. Assume that $k_{it} = k$ for all t. This produces the following series:

$$k_{i1} + \sum_{v=2}^{t} k_{iv} \prod_{m=1}^{v-1} (1 - k_{im}) = k + k(1-k) + k(1-k)^2 + \ldots + k(1-k)^{t-1}.$$

This is a geometric series with a ratio of $(1 - k)$. Thus, the sum is $k/[1 - (1 - k)] = 1$.

Behavior of k_i

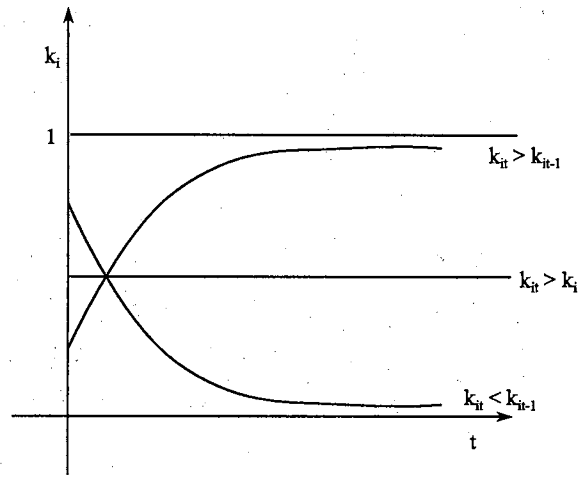
A constant or increasing k_i is a sufficient condition for convergence to the zero waste state. A constant k_i means that the firm can eliminate the same proportion of residual waste in future periods as it can in the current period. An increasing k_i implies that the rate of waste elimination increases over time so that the firm "gets better at getting better."

This is consistent with the objective of establishing a competitive advantage. *Ceteris paribus*, firms that eliminate waste more rapidly will prevail over those that do not. It makes sense that competition in the continuous improvement environment provides an incentive to produce k_is that increase over time.

There is some logical support for this possibility. As improvement occurs, management should be increasing its understanding of the true nature of the underlying production function. Over time, a cumulative information effect is created that may enable acceleration of the waste elimination process.

However, there are natural economic and logical limits to the concept of accelerating improvement. No more waste than exists can be eliminated. This implies that $\lim_{t \to \infty} k_{it} \leq 1$. As shown in Figure 3.1, k_i can increase over time but must increase at a decreasing rate. Increasing at a constant or increasing rate implies that $\lim_{t \to \infty} k_{it} \to \infty$, an economic impossibility. For example, if $k_{it} = t/(t + 1)$, the zero waste state will be achieved. It is easy to show that $k_{it}' > 0$ and $k_{it}'' < 0$.

Figure 3.1
The Behavior of k_i



Considering the cumulative information and competitive effects, a decreasing k_i makes the least sense. Additionally, it can be shown by example that a decreasing k_i may not produce a zero waste state. As with an increasing k_i, the type of decreasing behavior is restrained by the nature of continuous improvement and economic logic.

Specifically, $\lim_{t \to \infty} k_{it} \leq 0$ is required. This implies that a decreasing k_i must decrease at an increasing rate as shown in Figure 3.1. Decreasing at a constant or decreasing rate suggests that $\lim_{t \to \infty} k_{it} \to -\infty$, an economic impossibility. If $k_{it} = 1/t$, the waste will be completely eliminated. Obviously, $k_{it}' < 0$ and $k_{it}'' > 0$.

Thus, an increasing or decreasing k_i within a continuous improvement framework is defined as increasing at a decreasing rate or decreasing at an increasing rate. Clearly, knowing the behavior of k_i reveals much about the success of a continuous improvement program. Yet k_i is unobservable, and any inferences about k_i must be made from observable data.

The waste eliminated each period, I_{it}, is observable even though the terms k_{it} and $R_{i,t-1}$ that define I_{it} are unobservable. The observable waste elimination series, I_{iv}, $v = 1, 2, ..., t$, enables some specific statements to be made about k_i. These statements are expressed in Proposition 3.2.

Proposition 3.2: The following relationships hold between observable improvements and k_i:

A. If $I_{i,t+1}/I_{it} > I_{i,t+2}/I_{i,t+1}$, then k_i is increasing.

B. If $I_{i,t+1}/I_{it} < I_{i,t+2}/I_{i,t+1}$, then k_i is decreasing.

C. If $I_{i,t+1}/I_{it} = I_{i,t+2}/I_{i,t+1}$, then k_i is constant.

Proof: A. Assume k_i is either constant or decreasing. It must be at an increasing rate.

$$\frac{k_{i,t+1}}{k_{it}} - 1 \leq \frac{k_{i,t+2}}{k_{i,t+1}} - 1$$

$$\Rightarrow \frac{I_{i,t+1}}{I_{it}(1-k_{it})} \leq \frac{I_{i,t+2}}{I_{i,t+1}(1-k_{i,t+1})}$$

$$\Rightarrow \frac{I_{i,t+1}}{I_{it}} \leq \frac{I_{i,t+2}}{I_{i,t+1}}.$$

a contradiction.

B. Assume k_i is constant or increasing. It must be at a decreasing rate. Thus,

$$\frac{k_{i,t+1}}{k_{it}} - 1 \geq \frac{k_{i,t+2}}{k_{i,t+1}} - 1$$

$$\Rightarrow \frac{I_{i,t+1}}{I_{it}(1-k_{it})} \geq \frac{I_{i,t+2}}{I_{i,t+1}(1-k_{i,t+1})}$$

$$\Rightarrow \frac{I_{i,t+1}}{I_{it}} \geq \frac{I_{i,t+2}}{I_{i,t+1}}.$$

a contradiction.

C. From part A, assuming a strictly decreasing k_i produces a contradiction. From part B, assuming a strictly increasing k_i produces a contradiction. Thus, k_i is constant.

Q.E.D.

Thus, even though the exact value of k_i is unobservable, the behavior of k_i can be inferred from observable data.

<u>Example 3.2</u>. Suppose that $k_{it} = t/t + 1$ and $w_i = 40$. The unobservable sequence for k_{it} is ½, ⅔, ¾, ..., and the observable improvement series is 20, 13.3, 5, ... The following observable ratios can be computed.

$$I_2/I_1 = 13.3/20 = 0.665,\ I_3/I_2 = 5/13.3 = 0.376$$

Thus, $I_{i,t+1}/I_t > I_{i,t+2}/I_{i,t+1}$, which implies k_i is increasing. An increasing k_i implies, by Proposition 3.1, that the continuous improvement efforts will eventually lead to the zero waste state. However, what the zero waste state is and how long it will take to achieve it are still unknown.

Revelation of the Zero Waste State

The key to identifying the zero waste state is producing a convergent series that satisfies Lemma 3.1. The zero waste inputs are defined as the actual inputs minus the waste: $\boldsymbol{x}^o = \boldsymbol{x}^a - \boldsymbol{w}$. Since $\boldsymbol{x}^a$ is observable, knowing $\boldsymbol{w}$ reveals $\boldsymbol{x}^o$. Of course, $\boldsymbol{w}$ is unobservable; waste must be inferred from observable data.

For value of k_i that satisfy Proposition 3.1, an observable improvement series, $\boldsymbol{I}_1, \boldsymbol{I}_2, \ldots, \boldsymbol{I}_t$, will be created that sums to $\boldsymbol{S}$. It is assumed that for each of the i inputs, S_i can be deduced after a finite number of periods. Given certain assumptions, S_i will reveal w_i.

Assumption 3.1: Input prices remain constant so that $p_{it} = p_i$ for all t, where p_i is the price of the ith input.

Assumption 3.2: The underlying unknown production function remains constant over time so that $\frac{\partial f}{\partial t} = 0$.

Assumption 3.3: Output remains constant over time so that $q_t = q$ for all t.

Assumptions 3.1 and 3.2 ensure that $\boldsymbol{x}^o$ lies along a fixed expansion path. By assuming output to be constant, Assumption 3.3 limits $\boldsymbol{x}^o$ to a fixed point along this path. This last assumption will be relaxed later to allow for varying output. The following proposition shows how the waste can be identified.

Proposition 3.3: Given Assumptions 3.1 - 3.3, if $k_{it} \geq k_{i,t-1}$, then S_i reveals w_i.

Proof: Let x_{i0} be the input usage with waste of w_i. The observable waste sequence can be represented as follows:

$$k_{i1}w_i,\ k_{i2}(1-k_{i1})w_i,\ ...,\ k_{it}\prod_{v=1}^{t-1}(1-k_{iv})w_i$$

which has a sum of

$$S_i = \lim_{t\to\infty}\left[k_{i1} + \sum_{v=2}^{t} k_{iv}\prod_{m=1}^{v-1}(1-k_{im})\right]w_i$$

Thus, $S_i = w_i$.

Q.E.D.

The process in Proposition 3.3 can be repeated until waste has been identified for each of the i inputs. Once waste has been revealed, the zero waste state can be identified by subtracting $\boldsymbol{w}$ from $\boldsymbol{x}^a$.

Example 3.3. Suppose that a firm's unknown production function is $q = x_1^{½}x_2^{½}$, $q = 20$, and $\boldsymbol{p}$ = (\$1,\$1). Thus, $\boldsymbol{x}^o$ = (20,20). Let $\boldsymbol{x}_0^a$ = (15,60) so that $\boldsymbol{w}$ = (-5,40). Assume k_1 = ½ and k_2 = ¼. The observable improvement series are given below.

Improvement series:

x_1: -2.5, -1.25, -0.625, ...

x_2: 10, 7.5, 5.625, ...

The I_{t+1}/I_t ratios are constant (½ for the first input and ¾ for the second input). By Proposition 3.2, k_i is constant, and by Proposition 3.1, the series converges such that the zero waste state is achieved.

The observable improvement series suggest geometric series with ratios of ½ and ¾, respectively. Thus, S_1 = -5, S_2 = 40 and $\boldsymbol{w}$ = (-5,40). Thus, $\boldsymbol{x}^0$ = (20,20).

Nonconstant Output

The assumption of constant output (Assumption 3.3) can be relaxed provided the following restriction is placed on the behavior of the production function.

Assumption 3.4: $f(x)$ is homogeneous of degree one.

Linear homogeneity implies that $f(m\mathbf{x}) = mf(\mathbf{x})$, where m is any positive real number. Additionally, assume that the homogeneous property extends to include waste: $f(m(\mathbf{x} + \mathbf{w}) = mf(\mathbf{x} + \mathbf{w})$. For example, if the output doubles, then the waste also doubles so that if output increases from q to $2q$, the actual inputs will increase from $\mathbf{x}^a$ to $2\mathbf{x}^a$.

Now, consider the following observable sequences: $\{q_1, q_2, \ldots, q_t\}$ and $\{I_{i1}, I_{i2}, \ldots, I_{it}\}$, where q_t is allowed to change from one period to the next. By Assumption 3.4, $m_{jk} = q_j / q_k$, $j, k = 1, 2, ..., t, j \neq k$. The homogeneity parameter, m_{jk}, can be used to restate the observable improvement sequence such that waste elimination is measured with respect to a constant q, and thus a constant $\mathbf{w}$.

Suppose, for example, that waste is to be measured with respect to q_1. In this case, the improvement that would have been realized had q_k been equal to q_1 is $m_{1k}I_{ik}$. Thus, the restated sequence, $\{I_{i1}, m_{12}I_{i2}, m_{13}I_{i3}, \ldots, m_{1t}I_{it}\}$, provides a sequence of observable improvements using q_1 as the output standard for each period. This restated series converges to w_i if k_i is constant or increasing. This leads to the following corollary.

Corollary 3.1: Given a production function homogeneous of degree one and Assumptions 3.1 and 3.2, the observable improvement series reveals $\mathbf{w}$.

Proof: Follows directly from the property of homogeneity and Proposition 3.3.

Example 3.4. Suppose $x_{i0} = 100$ and $w_i = 80$. Assume that three periods of observable data exist:

q: 20, 25, 30

I_i: 40, 25, 15

Using q_1 as the standard, the I_i series can be restated as: 40, (20/25)25, (20/30)15, yielding 40, 20, 10. Since 20/40 = 10/20, k_i is constant and the restated series converges. Furthermore, k_i appears to have a value of ½. The sum of the geometric series is 40/(1 - ½) = 80 and $x_i^o = 2 - (100 - 80)$. By the homogeneity property, the value for any q_j is simply $m_{j1}q_j$. Thus, for $q = 25$, $x_i^o = (25/20)20 = 25$ and, similarly, for $q = 30$, $x_i^o = 30$.

Summary

Continuous improvement is characterized by incremental, systematic and total elimination of waste. As such, a portion of residual waste is eliminated so that some improvement (reduction in waste) is observed each period. In the limit, waste is completely eliminated, and the firm reaches the zero waste state as defined by the underlying production function.

The proportion of residual waste eliminated each period for each input, k_{it}, is unobservable. The firm can observe only the total improvement determined by the change in input consumption. However, by comparing the ratio of observable improvements over time, the behavior of k_{it} can be inferred.

If the proportion of residual waste eliminated each period is constant or increasing at a decreasing rate, waste will be completely eliminated in the limit. Even though the total amount of waste to be eliminated is unobservable, the optimal input combination can be revealed from the observable improvement series. If the behavior of k_{it} is such that the zero waste state will be achieved, the improvement series will converge such that the sum of the series is equal to the total amount of unobservable waste.

Thus, from the observable improvement series, the behavior of k_{it} can be determined. If k_{it} is constant or increasing at a decreasing rate, waste will be completely eliminated. The amount of waste can then be inferred by determining the sum of the observable improvement series. By subtracting total waste from the actual inputs consumed, the zero waste state is revealed.

The definition of continuous improvement states only that some improvement occurs each period. This model of continuous improvement assumes that some improvement is realized for every input in every period. Thus, the model assumes a more strict definition so that the firm always moves directly towards the zero waste state. For this to be true, the firm must have prior knowledge about the direction in which the zero waste state lies. It may not be reasonable to assume that such knowledge exists.

If the firm moves away from the optimal level of any input in any period, the observed change in input consumption will not represent an improvement. The assumption that some waste is eliminated for every input will be violated, but the observable improvement series can provide no signal that such a violation has occurred. Thus, the input-based model of continuous improvement developed in this chapter is limited in its practical application. This is discussed more fully in Chapter IV. A cost-based model that relaxes the assumption of periodic improvements is developed. This cost-based model overcomes the limitations of the input-based model.

CHAPTER IV

A COST-BASED MODEL OF CONTINUOUS IMPROVEMENT

Introduction

This chapter extends the model of continuous improvement to cost-based instead of input-based information. By using cost information, the definition of continuous improvement and the model can be generalized. The more general cost-based model is used in the development of the control system.

A Cost-Based Model

The control system that supports continuous improvement must guide the firm towards the zero waste state and ensure that the desired improvements are realized. A system based solely on physical measures of inputs may not perform these functions because the observable physical improvements can provide misleading signals. A system based on cost information will overcome the limitations of the physical measures.

Limitation of Input-Based Model

The definition of continuous improvement in Chapter III is very restrictive. It is required that k_{it} is bounded such that $0 < k_{it} < 1$ for all i and t. This severely restricts the area in which the firm may search for the optimal input combination. Furthermore, this continuous improvement search area changes based on the nature of waste. For the two input case, the firm must pursue one of the three alternatives listed below to eliminate waste.

Alternative waste elimination strategies:

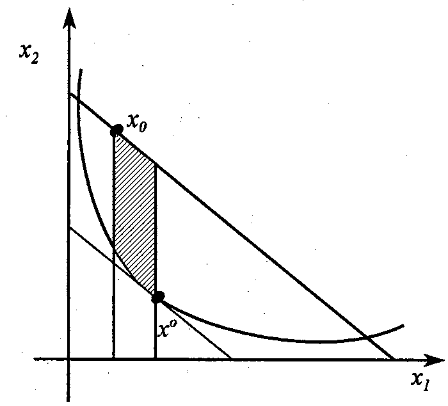
1. increase x_1, decrease x_2
2. decrease x_1, increase x_2
3. decrease x_1, decrease x_2

The continuous improvement search area for each of the three alternatives is shown by the shaded regions in Figure 4.1.

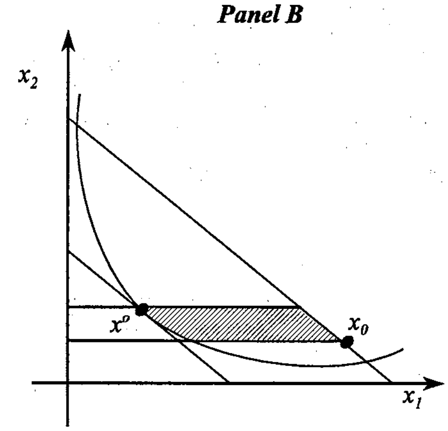
For the first case, the search area is bounded by the isocost curve at $t = 0$, the vertical lines passing through $\boldsymbol{x}^o$ and $\boldsymbol{x}_0$ and the isoquant. This is shown by the shaded region in Panel A of Figure 4.1. All points below the isoquant are infeasible. Any movement to the left of the vertical line passing through $\boldsymbol{x}_0$ moves the firm away from, rather than towards, $\boldsymbol{x}^o$ (ie. waste increases). Thus, $w_1 - I_{1t} > w_1$ so that $I_{1t} < 0$ and $k_{1t} < 0$. Similarly, any movement to the right of the vertical line passing through $\boldsymbol{x}^o$ moves the firm beyond the optimal input combination. Thus, $I_{1t} > R_{1t-1}$ so that $k_{1t} > 1$. Even though some movements above the isocost curve satisfy the requirement that $0 < k_{it} < 1$ for all i, the isocost curve is the appropriate bound. Any movement above the isocost curve increases cost and, therefore, would not be considered by a rational firm.

Figure 4.1
Input-Based Continuous Improvement Search Area

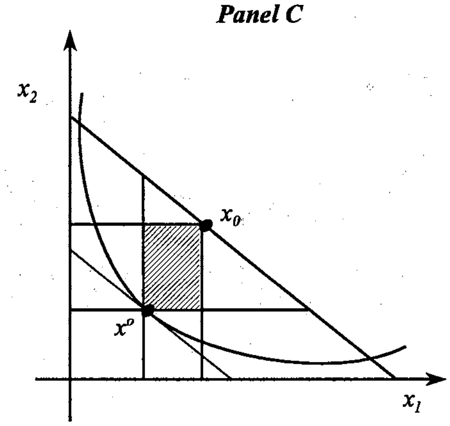
Panel A



Panel B



Panel C



If the firm must decrease x_1 and increase x_2, the continuous improvement search area will be bounded by the isocost curve, the horizontal lines passing through

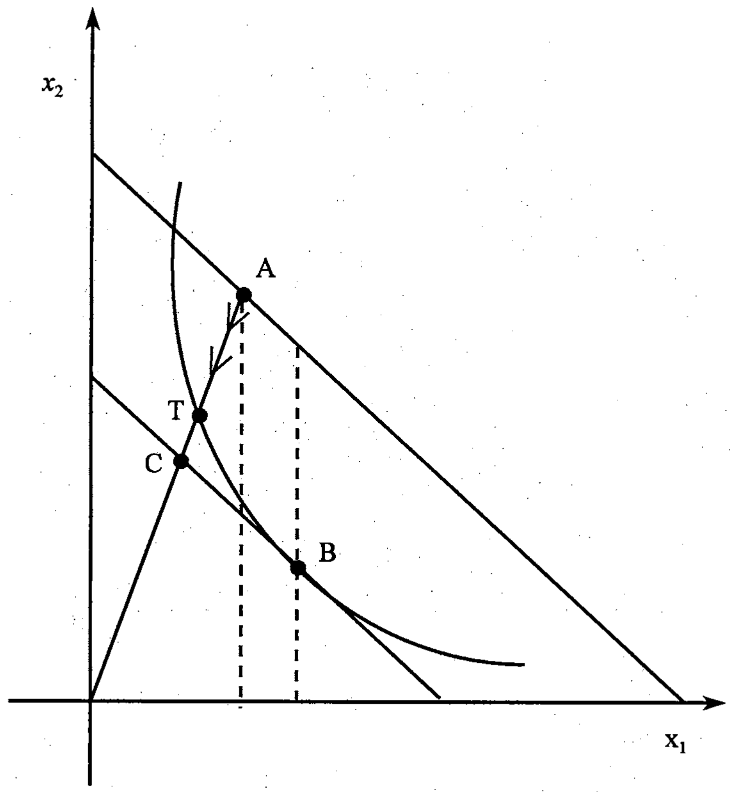
x^o and x_0 and the isoquant as shown in Panel B of Figure 4.1. Obviously, the isocost and isoquant bounds hold as in the first case. Any movement below the horizontal line passing through x_0 moves the firm away from x^o so that $k_{2t} < 0$. Likewise, any movement above the horizontal line passing through x^o moves the firm beyond the optimal input combination so that $k_{2t} > 1$.

Finally, if both inputs must be decreased, the continuous improvement search area is bounded by the horizontal and vertical lines passing through x^o and x_0 as shown in Panel C of Figure 4.1. Any movement to the right of the vertical line passing through x_0 yields $k_{1t} < 0$; any movement above the horizontal line passing through x_0 yields $k_{2t} < 0$. Similarly movements to the left of the vertical line or below the horizontal line passing through x^o yield $k_{1t} > 1$ or $k_{2t} > 1$, respectively.

To be within these narrow continuous improvement search areas, the firm must know at $t = 0$ in what direction x^o lies. It is not reasonable to assume that such knowledge exists. If the firm moves outside the area so that $k_{it} < 0$ or $k_{it} > 1$ for any input, the observable improvements will mask its true behavior. Consequently, a series that appears to satisfy Propositions 3.2 - 3.3 can be generated which converges to a nonoptimal input combination. Consider the following example.

<u>Example 4.1</u>. Assume the problem is the same as described in Example 3.3. The problem is shown graphically in Figure 4.2 where points A and B represent x_o and x^o, respectively. Instead of moving within the continuous improvement search area, the firm moves down the expansion path passing through A. Thus, $k_{1t} < 0$ so that waste increases for x_1 (i.e., the firm is moving *away* from the optimal level of x_1).

Figure 4.2
A Violation of the Input-Based Model



The following input sequences are observed.

Input sequences:

x_1: 15, 14.3, 13.67, 13.103

x_2: 60, 57.2, 54.68, 52.412

resulting in the following improvement series.

Improvement series:

I_1: 0.7, 0.63, 0.567, ...

I_2: 2.8, 2.52, 2.268, ...

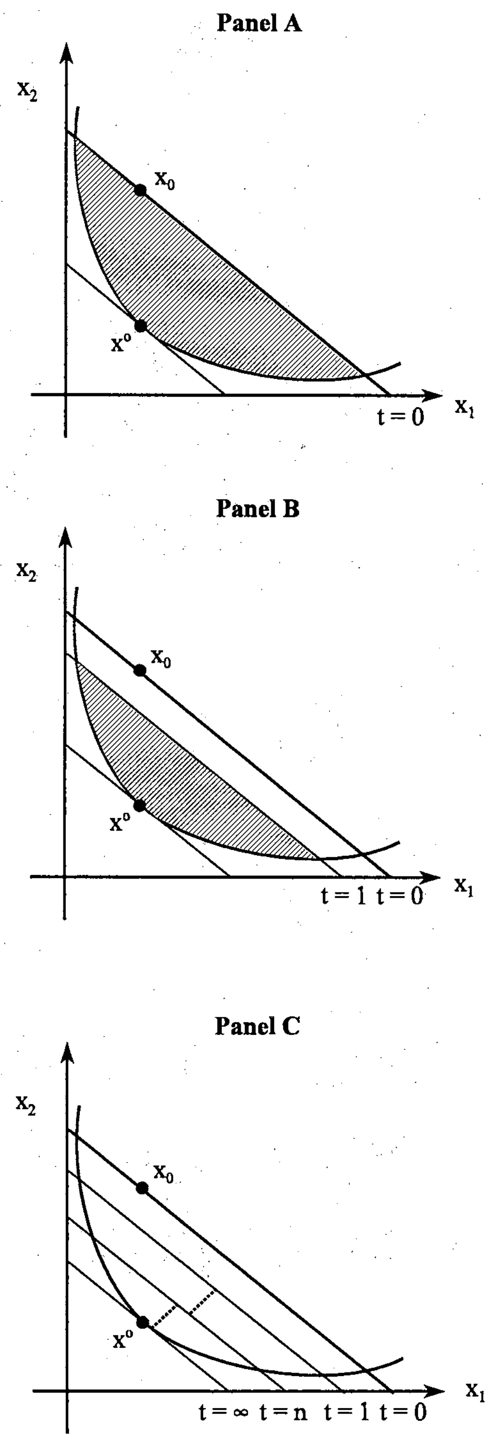
From Proposition 3.2, the I_{it} ratio test indicates a constant k_{it} for each of the inputs. The series sum to 7 for x_1 and 28 for x_2. This suggests that $\boldsymbol{x^o}$ = (8,32) which is infeasible (point C in Figure 4.2).

Cost Improvements

The isocost bound on the continuous improvement search area assumes rational, cost minimizing behavior by the firm. By eliminating waste, the firm is attempting to minimize cost subject to the unknown production function. Efforts to do so stem from the desire to create or maintain a competitive advantage. Thus, any decrease in cost will be viewed as an improvement.

This expands the continuous improvement search area to the region bounded by the isocost curve at $t = 0$ and the isoquant as shown by the shaded area in Panel A of Figure 4.3. Any movement below the isocost curve will decrease cost, regardless of the value of k_{it}. As an improvement (cost decrease) is realized in period 1, the upper bound becomes the isocost curve at $t = 1$. This shrinks the search area to the shaded region in Panel B. The region continues to shrink until, in the limit, it becomes a point and no further improvements (cost reductions) are possible within the existing technology. This is shown in Panel C of Figure 4.3.

Figure 4.3
Cost-Based Continuous Improvement Search Area



Cost is minimized only at one point, the zero waste state, $\boldsymbol{x}^o$. The firm plans improvements within the expanded cost-based search area until the zero waste state is achieved. Thus, cost is an alternative, but more general way, to view continuous improvement.

The actual cost in period t is defined as

$$c_t = \sum_{i=1}^{n} p_i x_{it}. \tag{4.1}$$

The minimum (zero waste) cost is given by Equation 2.1. The total dollar value of waste to be eliminated, w_c, through continuous improvement is the difference between the actual and minimum cost.

$$w_c = c_t - c^o. \tag{4.2}$$

Since any cost reduction is viewed as an improvement, the improvement in period t is given by the following equation.

$$I_{ct} = c_{t-1} - c_t \tag{4.3}$$

Assuming that input prices remain constant, the improvement can only be realized because of waste elimination. (The assumption of constant input prices holds throughout this study.) Consistent with the notion of continuous improvement, only a portion of the dollar value of residual waste will be eliminated each period. The dollar value of residual waste is

$$R_{ct} = c_t - c^o = w_c - \sum_{v=1}^{t} I_{cv}. \tag{4.4}$$

Letting k_{ct} be the proportion of the dollar value of residual waste eliminated in period t,

$$I_{ct} = k_{ct} R_{ct-1}. \tag{4.5}$$

Thus, in period t,

$$R_{ct} = w_c - w_c\left[k_{c1} + \sum_{v=2}^{t} k_{cv} \prod_{m=1}^{v-1} (1 - k_{cm})\right]. \tag{4.6}$$

Notice that Equation 4.6 is parallel to Equation 3.3 and that the definitions of w_c, R_{ct}, I_{ct} and k_{ct} are all parallel to those of w_i, R_{it}, I_{it} and k_{it}. Because of this parallel structure, the following similar propositions and corollary can be stated for the cost framework (proofs follow directly from Propositions 3.1 - 3.3 and Corollary 3.1).

Proposition 4.1: If $k_{ct} \leq k_{ct+1}$ for all t, then $\lim_{t\to\infty}\left[k_{c1} + \sum_{v=2}^{t} k_{cv} \prod_{m=1}^{v-1} (1 - k_{cm})\right] = 1.$

Proposition 4.2: The following relationships hold between observable cost improvements and k_c.

A. If $I_{c,t+1}/I_{ct} > I_{c,t+2}/I_{c,t+1}$, then k_c is increasing.

B. If $I_{c,t+1}/I_{ct} < I_{c,t+2}/I_{c,t+1}$, then k_c is decreasing.

C. If $I_{c,t+1}/I_{ct} = I_{c,t+2}/I_{c,t+1}$, then k_c is constant.

Proposition 4.3: Given Assumptions 3.1 - 3.3, if $k_{ct} \geq k_{c,t-1}$, then S_c reveals w_c.

Corollary 4.1: Given a production function homogeneous of degree one and Assumptions 3.1 and 3.2, the observable cost improvement series reveals w_c.

It is through the use of cost information that the firm can begin to guide and evaluate continuous improvement efforts. Unlike w_i, w_c is always greater than or equal to zero because $c_t \geq c^o$. As a result, the sign of k_{ct} is unambiguous. If cost decreases, $k_{ct} > 0$; if cost increases, $k_{ct} < 0$. If $k_{ct} > 1$, more waste than exists will be eliminated - an economic impossibility. Thus, when cost improvements are observed, k_{ct} is appropriately bounded such that $0 < k_{ct} < 1$, and the firm can know with certainty that they are within the cost continuous improvement search area given in Figure 4.3.

From Propositions 4.1 - 4.3, the behavior of k_{ct} can be inferred from observable improvements (cost decreases) and that if k_{ct} is constant or increasing at a decreasing rate, waste will be completely eliminated. The observable improvements can be used to infer the value of w_c and, therefore, c^o. This is illustrated in the following example.

<u>Example 4.2</u>. Consider the same problem described in Examples 3.3 and 4.1. From Example 3.3, $\boldsymbol{x}^o = (20,20)$ and $\boldsymbol{p} = (\$1,\$1)$. Thus, $c_0 = \$75.00$, $c^o = \$40.00$ and $w_c = \$35.00$. The expansion path intersects the isoquant at $\boldsymbol{x} = (10,40)$. From Figure 2.2, the total dollar value of waste, w_c, can be divided into technical inefficiency, w_c^T, and mix inefficiency, w_c^M: $w_c^T = \$75.00 - \$50.00 = \$25.00$ and $w_c^M = \$50.00 - \$40.00 = \$10.00$.

As the firm moves down the expansion path, the same output is produced using less of all inputs. This result has a great deal of intuitive appeal. Indeed, even though waste is increasing for x_1 ($k_{1t} < 0$), technical inefficiency decreases as follows.

Technical inefficiency:

w_c^T: \$25.00, \$21.50, \$18.35, \$15.52

However, along the expansion path, mix inefficiency remains constant. When the changes in technical and mix inefficiency are aggregated, the net result is a decrease in total waste. By using cost information, the firm can value tradeoffs between reductions in technical and mix inefficiency.

The input-based model ignores these potentially beneficial effects. To be within the input-based continuous improvement search area for this example, the firm must decrease both technical and mix inefficiency. Thus, cost provides a more complete measure of improvement.

Even though the firm is not within the continuous improvement search area described by the input-based model, it is within the cost-based search area because cost is decreasing. From the observed input usage, the following cost sequence will be observed.

Cost sequence:

c_t: \$75.00, \$71.50, \$68.35, \$65.52, ...

This yields the following cost improvement series.

Cost improvement series:

I_c: \$3.50, \$3.15, \$2.83, ...

The I_{ct} Ratio Test indicates a constant k_{ct} By the Convergence Property of Proposition 4.1, the optimal cost will be achieved. The sum of the series is \$35.00 which suggests that the optimal cost is \$40.00. This is the cost at $\boldsymbol{x}^o$.

The Search Process

With the cost-based system, the continuous improvement search area increases dramatically. The firm is not assumed to have any prior knowledge about the direction in which the optimal input combination lies. The firm knows only that cost will be reduced as waste is eliminated and that the zero waste state must be achieved to minimize total cost. This increases the need for a control system to guide the firm's continuous improvement efforts. It is unreasonable to expect the control system to guide the firm's search so that waste will be simultaneously eliminated in all inputs; this would put the firm in the restrictive search area of the input-based model. The control system must, however, guide the firm through a rational, yet conceivable, search process.

A Linear Search

Initially, the firm may choose to eliminate waste from one input at a time. For example, they may begin by eliminating waste in the most costly input, the input that with the greatest usage or the input in which the most waste is believed to exist. Such a process creates linear search patterns. By focusing on the most wasteful input, several improvements can be realized quickly and easily.

These linear search patterns are unlikely to lead to the optimal input combination. However, if k_{ct} is constant or increases at a decreasing rate, the Convergence Property ensures that the optimal cost will be revealed. Proposition 4.2 shows this result.

Proposition 4.4: For $q = f(x_1, x_2)$, if $k_{1t} = 0$ and $k_{ct} \geq k_{ct-1}$, then

$$p_1 x_{10} + p_2\left(x_{20} - w_2\left[k_{21} + \sum_{v=2}^{t} k_{2v} \prod_{m=1}^{v-1} (1 - k_{2m})\right]\right) = c^o.$$

Proof: From Equation 4.1,

$$c_t = p_1 x_{1t} + p_2 x_{2t} \tag{4.7}$$

Equation 4.7 can be rewritten so that

$$c_0 - w_c\left[k_{c1} + \sum_{v=2}^{t} k_{cv} \prod_{m=1}^{v-1} (1 - k_{cm})\right] =$$

$$p_1\left(x_{10} - w_1\left[k_{11} + \sum_{v=2}^{t} k_{1v} \prod_{m=1}^{v-1} (1 - k_{1m})\right]\right) +$$

$$p_2\left(x_{20} - w_2\left[k_{21} + \sum_{v=2}^{t} k_{2v} \prod_{m=1}^{v-1} (1 - k_{2m})\right]\right)$$

But, since $k_{1t} = 0$ for all t,

$$c_0 - w_c\left[k_{c1} + \sum_{v=2}^{t} k_{cv} \prod_{m=1}^{v-1} (1 - k_{cm})\right] =$$

$$p_1 x_{10} + p_2\left(x_{20} - w_2\left[k_{21} + \sum_{v=2}^{t} k_{2v} \prod_{m=1}^{v-1} (1 - k_{2m})\right]\right)$$

Taking the limit of both sides yields

$$c_0 - w_c \lim_{t \to \infty}\left[k_{c1} + \sum_{v=2}^{t} k_{cv} \prod_{m=1}^{v-1} (1 - k_{cm})\right] =$$

$$p_1 x_{10} + p_2\left(x_{20} - w_2 \lim_{t \to \infty}\left[k_{21} + \sum_{v=2}^{t} k_{2v} \prod_{m=1}^{v-1} (1 - k_{2m})\right]\right)$$

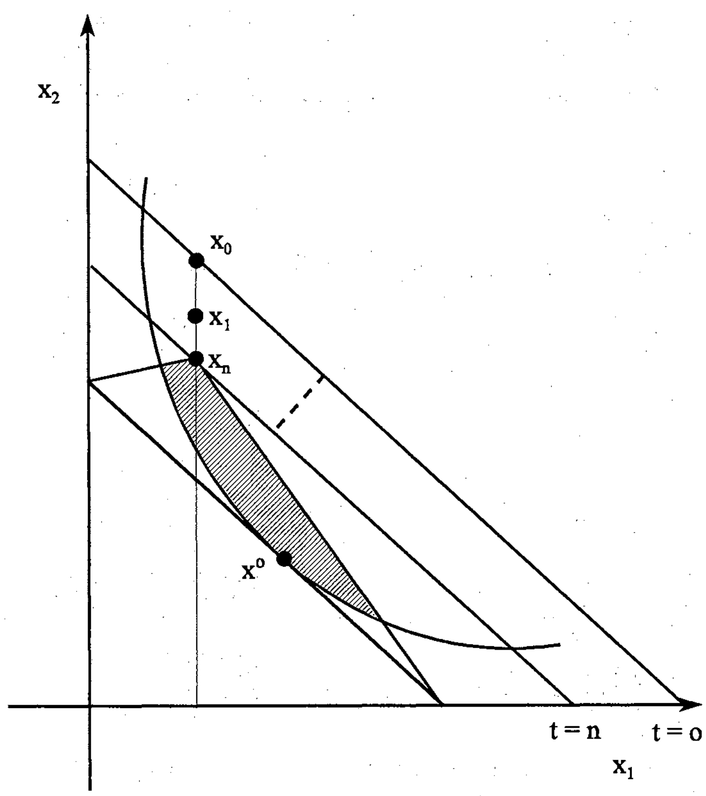
Since $k_{ct} \geq k_{ct-1}$, the Convergence Property ensures that

$$c^o = p_1 x_{10} + p_2\left(x_{20} - w_2 \lim_{t \to \infty}\left[k_{21} + \sum_{v=2}^{t} k_{2v} \prod_{m=1}^{v-1} (1 - k_{2m})\right]\right)$$

Q.E.D.

Once the linear search path has revealed the optimal cost, the continuous improvement search area can be reduced to the shaded region of Figure 4.4. It is known with certainty that the zero waste state lies somewhere along the lower boundary which is the optimal isocost curve. The optimal isocost curve is the goal, target cost, for continuous improvement efforts.

Figure 4.4
Cost-Based Search Area After Optimal Cost is Identified



Example 4.3. As before, suppose that $\boldsymbol{x_0}$ = (15,60), $\boldsymbol{x^o}$ = (20,20) and $\boldsymbol{p}$ = (\$1,\$1) so that c_0 = \$75.00 and c^o = \$40.00. The firm chooses to focus waste elimination efforts on x_2 so that the continuous improvement process moves along the vertical line passing through $\boldsymbol{x_o}$. The following improvement series are observed.

Improvement series:

I_1: 0.00, 0.00, 0.00, ...

I_2: 8.75, 8.75, 6.56, ...

I_c: \$8.75, \$8.75, \$6.56, ...

The I_{ct} Ratio Test indicates an increasing k_{ct} so that the Convergence Property is satisfied. The series sum to 0, 35 and \$35.00 for x_1, x_2 and c, respectively. This suggests that c^o = \$40.00 and $\mathbf{x}^o$ = (15,25). This is the minimum cost, but (15,25) is not the zero waste state. However this point does lie along the optimal isocost curve.

Fluctuating k_{ct}

As the firm searches for the zero waste state, k_{ct} will not always be constant or increasing. If k_{ct} was always appropriately behaved, the linear path chosen by the firm would lead to the zero waste state. Consequently, there would be no need for a control system. However, the firm's search involves a certain amount of trial and error.

Initially, the waste to be eliminated is sufficiently large so that any effort to eliminate it should be successful. Thus, in the early periods of the search process, the firm should be able to successfully maintain or increase the rate of waste elimination. It is, therefore, reasonable to assume that the Convergence Property holds and the target (optimal) cost will be revealed. This assumption can be validated by the I_{ct} Ratio Test.

Unless the linear path chosen initially leads to the zero waste state, a nonoptimal cost will be achieved along the path so that $\lim_{t\to\infty}\left[k_{c1} + \sum_{v=2}^{t} k_{cv}\prod_{m=1}^{v-1}(1-k_{cm})\right] < 1.$

If k_{ct} is originally constant or increasing, it must eventually decrease for this to be true. Thus, it is likely that k_{ct} will fluctuate over time so that it increases and decreases.

From Proposition 4.1, the model of continuous improvement assumes a well behaved k_{ct}. To use the model in the control system, this must be relaxed. This leads to the following assumption.

Assumption 4.1: For all t, $k_{ct} \geq min(k_c)$.

Unlike the assumption that k_{ct} is well behaved, Assumption 4.1 states only that k_{ct} will not fall below some minimum value. Over time, k_{ct} can fluctuate above, but not below, this value. With this less restrictive assumption on the behavior of k_{ct}, the Convergence Property will still achieved. This is shown by the following proposition.

Proposition 4.5: If $k_{ct} \geq min(k_{ct})$ for all t, then $\lim_{t \to \infty} \left[k_{c1} + \sum_{v=2}^{t} k_{cv} \prod_{m=1}^{v-1} (1 - k_{cm}) \right] = 1.$

Proof: From Proposition 4.1, if $k_{ct} = min(k_c)$ for all t,

$$\lim_{t \to \infty} \left[k_{c1} + \sum_{v=2}^{t} k_{cv} \prod_{m=1}^{v-1} (1 - k_{cm}) \right] = 1.$$

If $k_{ct} > min(k_c)$, it must be that

$$\lim_{t \to \infty} \left[k_{c1} + \sum_{v=2}^{t} k_{cv} \prod_{m=1}^{v-1} (1 - k_{cm}) \right] \geq 1.$$

However, no more waste than exists can be eliminated. As a result,

$$\lim_{t \to \infty} \left[k_{c1} + \sum_{v=2}^{t} k_{cv} \prod_{m=1}^{v-1} (1 - k_{cm}) \right] \leq 1.$$

Thus,

$$\lim_{t \to \infty} \left[k_{c1} + \sum_{v=2}^{t} k_{cv} \prod_{m=1}^{v-1} (1 - k_{cm}) \right] = 1.$$

Q.E.D.

Example 4.4. Refer back to Example 4.3. Obviously, the firm can not continue along the linear path to reach the point (15,25) so that the cost is minimized. Even though k_{ct} is initially increasing, it must eventually decrease.

Extend the improvement series as follows.

Cost improvement series:

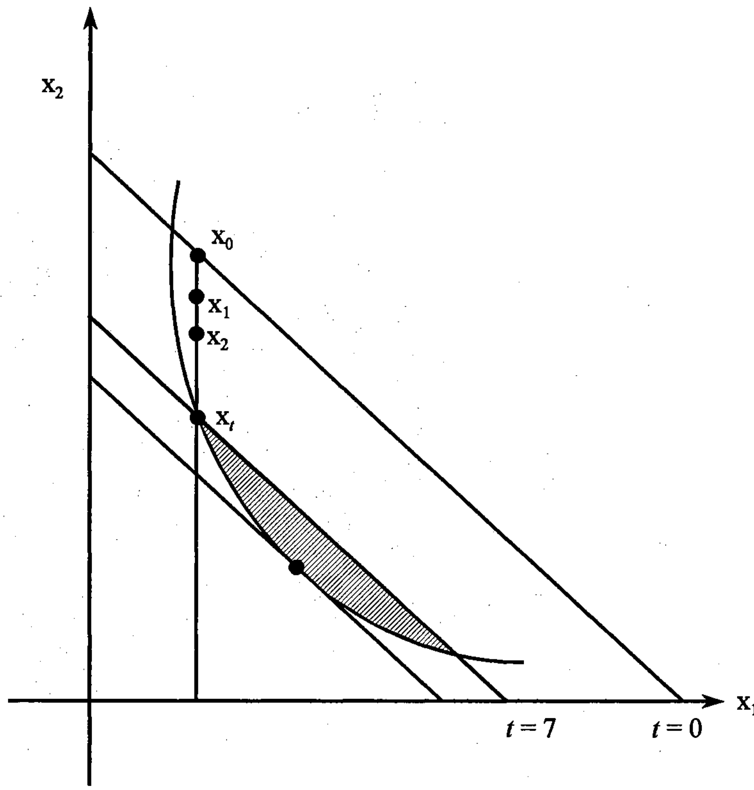
I_{ct}: \$8.75, \$8.75, \$6.56, \$4.38, \$2.73, \$1.64, ...

The I_{ct} Ratio Test indicates that k_{ct} is increasing each period.

However, the linear path intersects the isoquant where c = \$41.67. Thus, $I_{c7} \leq$ \$0.52. If I_{c7} = \$0.52, the I_{ct} Ratio Test indicates that k_{ct} has decreased. Even though k_{ct} is not well behaved, by Proposition 4.4, the firm can still achieve the zero waste state.

After period 7, the continuous improvement search area has been reduced to the shaded region in Figure 4.5. By redirecting the search into this area, the firm can continue to eliminate waste and the area will continue getting smaller. By appropriately directing and redirecting the search, waste will, in the limit, be completely eliminated. It is the function of the control system to provide this direction.

Figure 4.5
Reduced Cost-Based Search Area



Practical Convergence

Complete elimination of waste only happens in the limit. Since $k_{ct} < 1$ for all t, it is impossible to reach the optimal cost in a finite number of periods. The zero waste state is, therefore, a theoretical goal. After t periods,

$\left(1 - \left[k_{c1} + \sum_{v=2}^{t} k_{cv} \prod_{m=1}^{v-1} (1 - k_{cm})\right]\right) w_c$ amount of waste still exists.

To capture or maintain a competitive advantage, waste must be eliminated quickly instead of waiting an indefinite amount of time. Because of the competitive pressure to eliminate waste quickly, the firm should work towards a practical level of waste elimination rather that the theoretical goal of the zero waste state. Define m to be the target level of waste elimination, where $0 < m < 1$. The value of m is arbitrarily determined by the firm. For example, the firm may believe that a competitive advantage can be captured if 90% of the waste is eliminated. Thus, $m = 0.90$.

From Equation 4.4,

$$(1 - m)w_c = w_c - \sum_{v=1}^{t} I_{cv}, \tag{4.8}$$

where t is the number of periods required to reach the practical level of waste elimination. Equation 4.8 simplifies so that

$$mw_c = \sum_{v=1}^{t} I_{cv}. \tag{4.9}$$

Substituting Equation 4.5 into Equation 4.9 yields

$$mw_c = \sum_{v=1}^{t} k_{cv} R_{cv-1}. \tag{4.10}$$

Suppose that $k_{ct} = min(k_{ct})$ for all t. From Example 3.1, if k_{ct} is constant, the waste elimination series becomes a geometric series. After t periods, the sum of the series, S_c, is given by the following equation.

$$S_c = \frac{\min(k_{ct})w_c\left(1 - [1 - \min(k_{ct})]^t\right)}{\min(k_{ct})} \tag{4.11}$$

From Equation 4.10, $S_c = mw_c$. Equation 4.11 can be rewritten so that

$$t = \frac{\ln(1 - m)}{\ln(1 - \min(k_{ct}))}. \tag{4.12}$$

Even though w_c can only be eliminated in the limit, the firm can eliminate mw_c in a finite number of periods. Equation 4.12 shows that the number of periods required to achieve this level of waste elimination depends on the value of k_{ct}. Letting

m = 0.99, Table 4.1 shows the number of periods required to achieve the target level of waste elimination for different minimum values of k_{ct}. To capture or maintain a competitive advantage, the number of periods required to reach this goal must be minimized. Since the values in Table 4.1 assume a constant k_{ct}, they represent the maximum number of periods required to reach the target level of waste elimination.

Table 4.1
Periods Required to Reach Practical Convergence

$min(k_{ct})$	t
0.01	458
0.05	90
0.10	44
0.25	16
0.50	7
0.75	4
0.90	2

Example 4.5. Let c_0 = \$75 and w_c = \$35. Management feels that a competitive advantage can be gained if 99% of the waste is eliminated. Thus, m = 0.99 so that the target cost is \$40.35. Table 4.2 shows the cost sequence that would be observed for different minimum values of k_{ct} assuming $k_{ct} = min(k_{ct})$ for all t. Notice that the number of periods required to achieve the target cost corresponds to the values indicated in Table 4.1.

Table 4.2
Cost Sequence for Different Values of $min(k_{ct})$

	k_{ct}		
t	0.25	0.50	0.75
0	$75.00	$75.00	$75.00
1	66.25	57.50	48.75
2	59.69	48.75	42.19
3	54.77	44.38	40.55
4	51.07	42.19	40.14
5	48.31	41.09	
6	46.23	40.55	
7	44.67	40.27	
8	43.50		
9	42.63		
10	41.97		
11	41.48		
12	41.11		
13	40.83		
14	40.62		
15	40.47		
16	40.35		

Summary

The model of continuous improvement developed using physical input measures in Chapter III is very restrictive. The requirement that $0 < k_{it} < 1$ for all i and t forces the firm's search for the zero waste state into a very limited area. There is no reason to believe that this area can be identified in the initial period of the continuous improvement program.

Thus, it is not unreasonable to expect to firm to select an input combination outside the narrowly defined continuous improvement search area. If this happens, the constraint on the value of k_{it} will be violated. Consequently, the I_{it} Ratio Test may not reveal the true behavior of k_{it}, and the improvement series may suggest an optimal input combination which is infeasible.

By using cost as a measure of improvement, a model of continuous improvement which is parallel to the input-based model has been developed. The cost-based model, however, is more general. As a result, the behavior of k_{ct} is unambiguously revealed by the I_{ct} Ratio Test. If k_{ct} is appropriately behaved, the true optimal cost will be revealed regardless of the value of k_{it}.

As the firm moves along different search paths, k_{ct} will not, at all times, be appropriately behaved. It is more likely that k_{ct} will increase for several periods, decrease until a new search path is chosen, then increase again. However, even though k_{ct} is not well behaved, the optimal cost can be revealed and achieved. By moving toward the optimal cost, the firm must eventually achieve the zero waste state, $\boldsymbol{x}^o$.

The zero waste state can only be reached in the limit. It is, therefore, a theoretical goal. By setting a target level of waste elimination which is less than 100%, a target cost can be determined which can be achieved in a finite number of periods. The number of periods required to achieve this target cost depends on the value of k_{ct}.

It seems that the ability to control continuous improvement is derived from the ability to control k_{ct}. The cost-based model developed in this chapter provides the information necessary to establish this control. Chapter V details how information regarding k_{ct} is incorporated in the control system to guide and evaluate continuous improvement efforts.

CHAPTER V

CONTROLLING CONTINUOUS IMPROVEMENT

Introduction

The cost-based model of continuous improvement in Chapter IV provides the foundation for a control system which can guide and evaluate the firm's waste elimination efforts. This chapter first describes how the control system guides the firm's search for the zero waste state or the target waste state. Then, the control system is used to accelerate convergence to the practical waste level.

Guiding the Search Process

Guiding the firm's waste elimination efforts involves two steps. First, the control system must identify the target cost to be achieved. Second, it must ensure that the firm's continuous improvement efforts are sufficient to meet this goal.

Identifying the Target Cost

The first and most critical function of any guidance system is to identify the goal to be achieved. As such, a control system which guides continuous improvement must identify the optimal (target) cost as quickly as possible. Ideally, the target cost would be identified before the firm implemented its continuous improvement program. Since the production function is unobservable, this is impossible. However, using the cost-based model of continuous improvement developed in Chapter IV, this can be accomplished after a finite number of periods.

The Convergence Property ensures that if k_{ct} is constant or increasing, the improvement series will reveal the waste to be eliminated and, therefore, the optimal cost. As suggested in Chapter IV, the firm may initially concentrate its continuous improvement efforts on one input. By following a linear search path, a series of improvements that satisfy the Convergence Property should be generated quickly and easily.

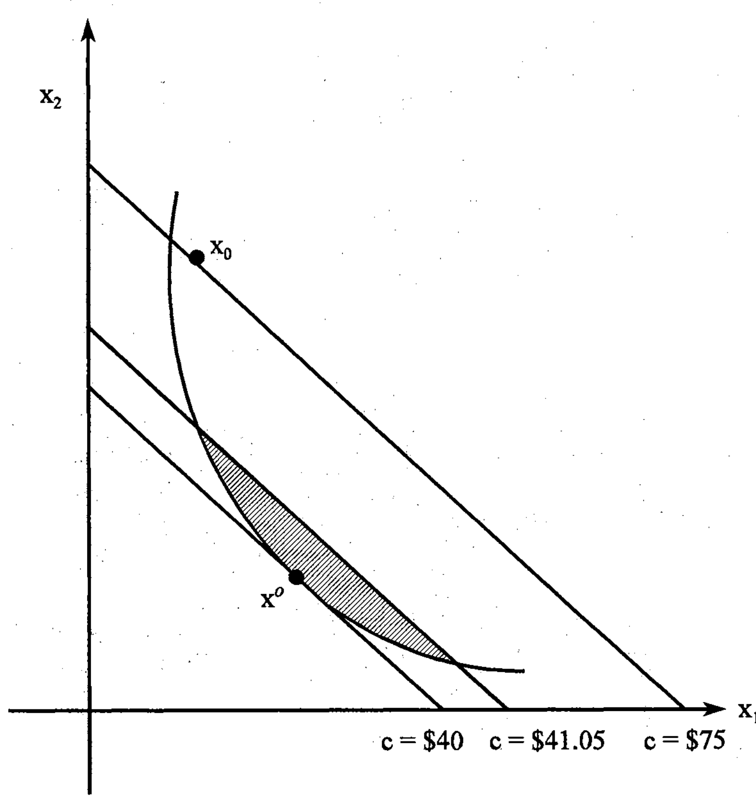
This linear search technique needs to continue only until number of improvements sufficient to determine the sum of the series have been observed. The number of periods required to reveal the optimal cost depends on the complexity of the series. For example, if k_{ct} is constant, the improvement series is a geometric series. The sum can be easily determined after three periods. As the improvement series becomes more complex, more periods will be needed to determine the structure and sum of the series.

The complexity of the improvement series depends on the behavior of k_{ct}. The behavior of k_{ct} is revealed by the I_{ct} Ratio Test. This knowledge is important to help determine the pattern of the series and to ensure that the Convergence Property is satisfied.

Once the waste to be eliminated has been revealed, the optimal cost will be known. This optimal cost is the theoretical goal of continuous improvement. In addition to this theoretical goal, the firm should set a practical target objective that can be reached in a finite number of periods. Setting a target cost of less than 100% waste elimination will increase number of feasible solutions that satisfy the objectives of continuous improvement. Consider the following example.

Example 5.1. Suppose the problem is the same as described in Example 4.2. Instead of complete waste elimination, the firm believes that a competitive advantage can be captured if 97% of the waste is eliminated. Thus, m = 0.97 and mw_c = \$33.95 so that the practical target cost is \$41.05. The feasible number solutions that satisfy the objectives of continuous improvement increases from one point, the zero waste state, to all of those shown by the shaded area in Figure 5.1.

Figure 5.1
Feasible Solutions for Target Cost



Guiding Waste Elimination

Setting the target cost is only the first step. The control system must also ensure that this goal will be reached. To do so, the continuous improvement efforts must be evaluated to ensure that they are achieving the intended results. The Convergence Test coupled with the I_{ct} Ratio Test provide the necessary means to evaluate current waste elimination efforts.

The Convergence Property ensures that if k_{ct} is constant or increasing, the zero waste state will be achieved. Obviously, if the firm is on course toward the optimal cost, the practical target cost will be achieved. Thus, the firm needs only to determine the behavior of k_{ct} to know if current continuous improvement efforts are functioning as intended.

Once the optimal cost becomes known, the value and, therefore, behavior of k_{ct} can be determined directly from Equation 4.5. The behavior of k_{ct} can also be determined by the I_{ct} Ratio Test. Because the optimal cost can not be known until after a finite number of periods, the latter method must be used in the early periods of the continuous improvement program. It is this knowledge of the behavior of k_{ct} that provides the necessary feedback to guide and evaluate continuous improvement.

If k_{ct} is shown to be constant or increasing, the Convergence Property is satisfied, and continuous improvement is known to be in control. On the other hand, a decreasing k_{ct} indicates that current continuous improvement efforts are out of control. The waste elimination process must then be revised. A new search path should be chosen until the Convergence Property is again satisfied. By Proposition 4.5, repeating this process will ensure that the target cost is achieved. This is illustrated by the following example.

Example 5.2. Extend Examples 4.4 and 5.1. If the firm continues down the initial linear path, in period 7, the I_{ct} Ratio Test will reveal that k_{ct} is decreasing. Thus, the current continuous improvement process is out of control, and a new search path must be chosen.

From Figure 4.5, it is evident that it is impossible to continue decreasing x_2 without increasing x_1. It may take several periods for this to become evident to the firm. However, the appropriate direction will eventually be chosen. The input sequences may appear as follows.

Input sequences:

x_1: ..., 15.00, 15.00, 15.00, 15.00, 15.10, 15.19, 15.27, ...

x_2: ..., 27.19, 26.67, 26.67, 26.67, 26.57, 26.16, 25.95, ...

This yields the following cost improvement series.

Cost improvement series:

I_c: ..., \$0.52, \$0.00, \$0.00, \$0.17, \$0.15, \$0.13, ...

To satisfy Proposition 4.5, any period where $k_{ct} = 0$ is removed from the series. In the revised series, k_{ct} initially increases, then decreases, then becomes constant. Once the constant k_{ct} is revealed, the Convergence Property will again be satisfied. This signals that continuous improvement efforts are in control.

Along the revised path, the input improvement series sum to suggest that $\boldsymbol{x}^0$ = (16,24), an infeasible point. Thus, along this path, k_{ct} must eventually decrease. This will again signal that continuous improvement efforts are out of control and that the firm needs to search for a new improvement path. This process continues until the target cost is achieved.

Accelerating Waste Elimination

To capture a competitive advantage, the number of periods required to achieve the target cost must be minimized. By Assumption 4.1, the values in Table 4.1 represent the *maximum* number of periods required to reach this goal. This puts an upper limit on the time horizon. By increasing k_{ct}, the firm can decrease the number of periods required to reach the target cost.

Equations 4.6 and 4.9 show that as waste is eliminated, R_{ct-1} decreases. Thus, early increases in k_{ct} result in larger improvements that later decreases. Several early increases in k_{ct} can create a large cumulative effect of rapid waste elimination. This is illustrated in the following example.

<u>Example 5.3</u>. Consider Examples 4.4 and 5.1. The firm's optimal cost is $40.00, and the target cost is $41.05. By Equation 4.5, k_{c1} = ¼. Letting $k_{c1} = min(k_{ct})$, Equation 4.12 shows that it will take a maximum of 12 periods to achieve the target cost. Because k_{ct} is increasing, the firm achieves a cost of $41.67, $0.62 from the target cost, after only 7 periods. Thus, there is a significant benefit from increasing the value of k_{ct} as much as possible and as early as possible. The control system should guide continuous improvement efforts so that this effect is achieved.

Guiding the Behavior of k_{ct}

The number of periods required to achieve the target cost is maximized when k_{ct} is constant. From the I_{ct} Ratio Test, if k_{ct} is constant, $I_{ct+1}/I_{ct} = I_{ct+2}/I_{ct+1}$. Let $D = I_{ct+1}/I_{ct} - I_{ct+2}/I_{ct+1}$. It must also be true that $D = 0$ if k_{ct} is constant.

If k_{ct} is increasing at a decreasing rate, $I_{ct+1}/I_{ct} > I_{ct+2}/I_{ct+1}$ so that $D > 0$. It seems that as the rate of increase in k_{ct} increases, the value of D increases. This information can be used to guide and evaluate the rate of waste elimination. Consider the following example.

Example 5.4. As before, $w_c = \$35.00$. Consider three alternatives for the behavior of k_{ct}.

Alternative 1:

$k_{ct} = 1/2$

Alternative 2:

$k_{ct} = t/(t + 1)$

Alternative 3:

$k_{ct} = (2t - 1)/2t$

The following cost improvement series will be observed for each of the three alternatives.

Cost improvement series:

Alternative 1:

I_{ct}: \$17.50, \$8.75, \$4.38, ...

Alternative 2:

I_{ct}: \$17.50, \$11.67, \$4.37, ...

Alternative 3:

I_{ct}: \$17.50, \$13.13, \$3.65, ...

As expected, waste is eliminated most quickly under Alternative 3. Notice the differences between the I_{ct} ratios.

$D = I_{ct+1}/I_{ct} - I_{ct+2}/I_{ct+1}$:

Alternative 1: D = \$8.75/\$17.50 - \$4.38/8.75 = 0.00

Alternative 2: D = \$11.67/\$17.50 - \$4.38/\$11.67 = 0.29

Alternative 3: D = \$13.13/\$17.50 - \$3.65/\$13.13 = 0.47

By guiding continuous improvement efforts so that the difference between the I_{ct} ratios increases, the rate of waste elimination increases. Thus, the target cost is achieved more quickly.

A Necessary Tradeoff

The continuous improvement control system has two objectives. The potential for improvement and the target cost must be identified. The control system must guide the firm's waste elimination efforts toward that goal. By influencing the behavior of k_{ct}, the control system forces a tradeoff between these two objectives.

If k_{ct} is constant, the improvement series is known to take the form of a geometric series, and the sum can be easily calculated. This behavior of k_{ct} will be revealed by the I_{ct} Ratio Test after three periods. A constant k_{ct} will minimize the number of periods required to identify the target cost.

However, if k_{ct} is constant, the number of periods required to achieve the target cost is maximized. By increasing k_{ct}, waste elimination is accelerated. As the rate of waste elimination is changed, the number of periods required to identify the target cost increases because the improvement series becomes more complex.

To create or maintain a competitive advantage, the firm must reduce cost by eliminating waste as quickly as possible. Example 5.3 shows that early increases in k_{ct} significantly reduces the number of periods required to achieve the target level of waste elimination. Even though the number required to identify the target cost increases, the Convergence Property ensures that the firm is on course toward the goal. Thus, in terms of the firm's competitive advantage, the benefit gained by accelerating the rate of waste elimination exceeds the benefit lost by not knowing the final goal of continuous improvement efforts as early as possible.

Summary

The cost-based model of continuous improvement provides the necessary framework for a control system capable of guiding and evaluating the firm's waste elimination efforts. The I_{ct} Ratio Test reveals the behavior of k_{ct}. If k_{ct} is constant or increasing, the Convergence Property ensures that continuous improvement efforts are in control. If k_{ct} is decreasing, continuous improvement efforts are out of control and must be revised until a constant or increasing k_{ct} is again achieved.

The control system should guide continuous improvement efforts in such a way that waste is eliminated as quickly as possible. This is accomplished by increasing the rate of waste elimination as much and as early as possible. Doing so will allow the firm to create or maintain a competitive advantage.

As the rate of waste elimination becomes more variable, the number of periods required to identify the target cost increases. Even though the target cost will remain unobservable for a longer period of time, the Convergence Property ensures that it will be achieved. The key to success in a competitive environment is to achieve the target cost as quickly as possible. Thus, the tradeoff between the time required for waste elimination and target cost identification is beneficial.

The control system must function within an accounting information system. The cost-based model of continuous improvement relies only on observable resource spending which is recorded by any cost management system. Chapter VI explicitly incorporates the continuous improvement control system into the existing accounting structure of activity-based management.

CHAPTER VI

ACTIVITY-BASED MANAGEMENT: AN ACCOUNTING STRUCTURE FOR A CONTINUOUS IMPROVEMENT CONTROL SYSTEM

Introduction

The model of continuous improvement and the subsequent control system developed in Chapters IV and V have assumed an unknown technology consistent the classical economic theory of the firm. In activity-based management, the firm operates according to a system of production functions rather than a single production function. Assuming a very basic activity-based management problem[1], this chapter will show that the technologies described by the classical and activity-based frameworks have the same properties, the same zero waste state and same level of waste. By incorporating earlier developments into activity-based management, this chapter will also show that the control system can be used to guide and evaluate continuous improvement in an activity-based environment and that the activity-based information provides insights into the nature and cause of waste.

The Structure of Activity-Based Management

Choi (1995) states that a culture conducive to continuous improvement is "process oriented" and designed to eliminate "wasteful practices". Activity-based

[1]It is assumed that the firm produces a single product and that all activities are unit level.

management is defined as an information system that focuses on activities with the intent of eliminating wasteful practices. In short, activity-based management is continuous improvement and, thus, provides an ideal accounting structure for the control system.

The control system discussed in Chapter V is designed to guide the firm's search for the optimal cost as defined by the underlying production function and evaluate the progress realized in this search. If the control system is to function within an activity-based management framework, it must be that activity-based management can be described by a production function. This allows the model of continuous improvement upon which the control system builds to be used to describe waste elimination effort within the activity-based management accounting structure.

The Production System

In the classical economic theory of the firm, the firm consumes resources to produce a product. Letting $\boldsymbol{x}$ be a vector of resources, the production function $q = f(\boldsymbol{x})$ defines the process that transforms these resources into the final product. The developments in Chapters II, III and IV implicitly assume that the waste elimination problem is structured according to this classical theory.

In an activity-based environment, the firm consumes the outputs of activities to produce a product and consumes resources to provide those activity outputs. Thus, letting $\boldsymbol{z}$ be a vector of activity outputs, the firm operates according to a system of production functions such that $q = h(\boldsymbol{z})$ and $z_j = g_j(\boldsymbol{x})$ for $j = 1 \ldots$ m activity outputs. The product level function, $h(\cdot)$, defines the process that transforms the activity outputs into the final product, q. The activity level functions, $g_j(\cdot)$, define the transformation of resources into those activity outputs, z_j.

It is easy to see that the system of production functions can be rewritten as a composite function of resources: $q = h(g(x))$. Assume that the following identity holds between the classical production function and the system of production functions in the activity-based environment.

$$f(\boldsymbol{x}) \equiv h(\boldsymbol{g}(\boldsymbol{x})) \tag{6.1}$$

Thus, the activity-based framework is nothing more than a more detailed way of describing the firm's technology. This is similar to the approach taken by Becker (1965) in his discussion of utility.

Even though the earlier developments assume that the firm's production function is unknown, some structure is imposed on that technology. Specifically, it is assumed that the firm's unknown production function is homogeneous of degree one. To extend these developments into the activity-based management accounting structure, the classical and activity-based technologies must both have this structure. The following proposition shows that such a relationship can exist.

Proposition 6.1: If $h(\cdot)$ and $g_j(\cdot)$ are homogeneous, then $f(\cdot)$ is also homogeneous.

Proof: Let $q = h(z)$ be homogeneous of degree α and $z_j = g_j(\boldsymbol{x})$ be homogeneous of degree β for $j = 1, \ldots, m$ activity outputs and $\boldsymbol{x} = (x_1, \ldots, x_2)$. By Equation 6.1,

$$q = f(\boldsymbol{x}) \equiv h[\boldsymbol{g}(\boldsymbol{x})] \equiv h[g_1(\boldsymbol{x}), \ldots, g_m(\boldsymbol{x})]$$

so,

$$\sum_{j=1}^{m} h_j z_j = \alpha\, q \tag{6.2}$$

$$\sum_{i=1}^{n} \frac{\partial g_j}{\partial x_i} \cdot x_i = \beta z_j \text{ for } j = 1, \ldots, m \tag{6.3}$$

$$f(x) \equiv h(g(x)),\ \sum_{j=1}^{m} h_j z_j = \alpha q \Rightarrow \sum_{j=1}^{m} j_j \cdot \left(\sum_{i=1}^{n} \frac{\partial g_j}{\partial x_i} \cdot x_i \right) = \alpha\beta q \tag{6.4}$$

Since $f_i = \sum_{j=1}^{m} h_j \frac{\partial g_j}{\partial x_i}$,

$$f(x) \equiv h(g(x)),\ \sum_{j=1}^{m} h_j z_j = \alpha q \Rightarrow \sum_{i=1}^{n} x_i f_i = \alpha\beta q$$

Thus $f(\cdot)$ is homogeneous of degree $\alpha\beta$.

Q.E.D.

This result shows that homogeneity of $h(\cdot)$ and $g(\cdot)$ is sufficient for the homogeneity of $f(\cdot)$. Furthermore, if $h(\cdot)$ and $g(\cdot)$ are homogeneous of degree one, $f(\cdot)$ will be also. Since this is the only structure required by either the input or cost-based models of continuous improvement, they can be extended into activity-based management.

The Zero Waste State

Since the activity-based framework is simply a more detailed way of expressing the classical problem, then the resources consumed to provide the activity outputs must be the same resources used to produce the product in the classical problem so that

$$x_i = \sum_{j=1}^{m} x_{ij}. \tag{6.5}$$

Since the activity outputs are endogenous to the system, the firm will incur costs only for the purchase of resources. Thus, the firm's basic objective in activity-based management is the same as the classical problem: minimize the cost of resources subject to the underlying technology.

However, in activity-based management, the optimization problem faced by the firm is more complex than the classical problem. At the activity level, the firm must minimize the cost of resources used to provide each of the activity outputs demanded at the product level subject to $g_j(\cdot)$. Let $\mathbf{x}_j^o$ be the zero waste combination of resources that should be used to provide the actual amount of the *j*th activity output, z_j^a, consumed at the product level. The minimum activity level cost for the *j*th activity is given by the following equation:

$$c_j^o = \sum_{i=1}^{n} p_i x_{ij}^o. \tag{6.6}$$

At the product level, the cost of activity outputs used to produce the final product must be minimized subject to $h(\cdot)$. Since activity outputs are produced and consumed internally, they have no market price. The cost is determined by the activity rate. The activity rate, r_j, is the cost incurred to provide one unit of activity output. By the assumption of linear homogeneity, the optimal activity rate for the *j*th activity can be defined as follows.

$$r_j^o = \frac{c_j^o}{z_j^a} \tag{6.7}$$

To minimize cost, the firm must select the optimal combination of activity outputs, z^o. The optimal cost is given by the following equation.

$$c^o = \sum_{j=1}^{m} r_j^o z_j^o \tag{6.8}$$

If the firm minimizes cost, $z^a = z^o$. As a result, substituting Equation 6.7 into Equation 6.8 yields

$$c^o = \sum_{j=1}^{m} c_j^o = \sum_{j=1}^{m}\sum_{i=1}^{n} p_i x_{ij}^o. \tag{6.9}$$

By Equation 6.5, it must be that

$$\sum_{j=1}^{m} x_{ij}^o = x_i^o. \tag{6.10}$$

Thus, the zero waste state is the same for both the classical and activity-based management frameworks. The following example illustrates this point.

Example 6.1. Suppose that the firm produces $q = 20$ units of output according to a linear production function.

$$q = \min\left(\frac{x_1}{2}, \frac{x_2}{2}\right)$$

Thus, $\boldsymbol{x}^o = (40,40)$. The activity-based management system of production functions is as follows:

$$q = \min(z_1, z_2)$$

$$z_1 = \min(x_{11}, x_{21})$$

$$z_2 = \min(x_{12}, x_{22})$$

For $q = 20$, $\boldsymbol{z}^o = (20,20)$. If $z_1 = 20$, $\boldsymbol{x}_1^o = (x_{11}, x_{21}) = (20,20)$. Similarly, if $z_2 = 20$, $\boldsymbol{x}_2^o = (x_{12}, x_{22}) = (20,20)$. From Equation 6.10, $x_1^o = x_{11}^o + x_{12}^o = 20 + 20 = 40$ and $x_2^o = x_{21}^o + x_{22}^o = 20 + 20 = 40$ so that the zero waste state is the same for both the classical and activity-based problems.

Dual Nature of Waste

If the firm fails to minimize cost at either the activity or product level, waste will exist. Waste exists at the product level if the firm uses a nonoptimal combination of activity outputs to produce the product. Waste exists at the activity level if the firm consumes a nonoptimal combination of resources to provide any of the activity outputs demanded at the product level.

Let x_j^a be the actual resources consumed to provide the *j*th activity output. The actual cost to provide the activity output demanded at the product level, c_j, is the total cost of the resources consumed:

$$c_j = \sum_{i=1}^{n} p_i x_{ij}^a. \tag{6.11}$$

Waste for each of the activity outputs is the difference between the actual and optimal cost of providing the activity output demanded: $w_{cj}^A = c_j - c_j^o$. Total activity level waste is given by the following equation.

$$w_c^A = \sum_{j=1}^{m} w_{cj} = \sum_{j=1}^{m} \sum_{i=1}^{n} p_i (x_{ij}^a - x_{ij}^o) \tag{6.12}$$

Similarly, let z^a be the actual combination of activity outputs used to produce the final product, q. The product level waste is given by the following equation.

$$w_c^P = \sum_{j=1}^{m} r_j^o (z_j^a - z_j^o) \tag{6.13}$$

Continuous improvement calls for total waste elimination. For this to be accomplished, waste must be eliminated at both the activity and product levels. Thus, the total waste to be eliminated is the sum of the total activity level waste and the product level waste: $w_c = w_c^A + w_c^P$.

If waste is eliminated at the activity level, the firm will use $\boldsymbol{x}_j^o$ amount of resources to produce z_j^a. Assuming that $g_j(\cdot)$ is homogeneous of degree one, the optimal resource combination that should be used to provide the optimal level of the jth activity output, z_j^o, is $y\boldsymbol{x}_j^o$ where $y = z_j^o/z_j^a$. By substituting Equation 6.7 into Equation 6.13 and summing Equations 6.12 and 6.12, the total waste to be eliminated can be expressed as follows.

$$w_c = \sum_{j=1}^{m}\sum_{i=1}^{n} p_i(x_{ij}^a - yx_{ij}^o) \tag{6.14}$$

Thus, total waste can be expressed as a function of resource spending. Total waste is the difference between the cost of the actual resources used to provide the actual activity outputs consumed and the cost of the resources that should be used to provide the optimal combination of activity outputs. By Equations 6.10 and 6.14, total waste is the same for both the activity-based management and classical frameworks. The following example illustrates this point.

Example 6.2. Consider the same problem described in Example 6.1. Also, let $\boldsymbol{p}$ = (\$1,\$1). Suppose that $\boldsymbol{z}^a$ = (30,60), $\boldsymbol{x}_1^a = (x_{11}^a, x_{21}^a)$ = (40,60) and $\boldsymbol{x}_2^a = (x_{12}^a, x_{22}^a)$ = (80,120) so that $\boldsymbol{x}^a$ = (120,180).

Since $z_1^a = 30$ and $z_2^a = 60$, $\boldsymbol{x}_1^o$ = (30,30) and $\boldsymbol{x}_2^o$ = (60,60). From Equation 6.7, r_1^o = (\$30 + \$30)/30 = \$2 and r_2^o = (\$60 + \$60)/60 = \$2. The activity level waste for each of the activities is determined by Equation 6.12 as follows:

$$w_{c1}^{A} = (\$1)(40 - 30) + (\$1)(60 - 30) = \$40$$

$$w_{c2}^{A} = (\$1)(80 - 60) + (\$1)(120 - 60) = \$80$$

The total activity level waste is determined by summing over each of the activities: $w_c^A = w_{c1}^A + w_{c2}^A = \$120$. From Equation 6.13, the total product level waste is

$$w_c^P = (\$2)(30 - 20) + (\$2)(60 - 20) = \$100.$$

so that, $w_c = w_c^A + w_c^P = \$120 + \$100 = \$220$.

In the classical framework, since $\boldsymbol{x}^a = (120,180)$ and $\boldsymbol{x}^o = (40,40)$, $w_c = \$220$. Thus, the total waste to be eliminated is the same for both the classical and activity-based management frameworks. However, the latter approach provides a more detailed insight into the nature of waste.

The Control System in Activity-Based Management

Since the classical and activity-based management frameworks express the same problem and have the same optimal solution, either approach can be used to guide and evaluate continuous improvement. Activity-based management simply provides more detailed information. This detail provides valuable insights into the nature and cause of waste. This makes activity-based management a more ideal accounting structure for the control system.

The Information Content of Activity-Based Management

Activities are the central focus of activity-based management. They represent the work performed by the firm. By identifying what activities are performed, describing how they are performed and determining why they are performed, the firm gains an understanding of the process involved in producing the product.

Once they have been identified and described, activities are analyzed to determine the root cause of why they are performed. From this analysis, all activities are classified as either value-added or nonvalue-added. An activity is nonvalue-added if $h(z, z_j) = h(z)$ for $z_j \notin z$. In other words, an activity is considered to be nonvalue-added if the output of that activity is not included in any technically efficient combination of activity outputs. If the output of an activity does belong to some technically efficient combination, that activity is considered to be value-added.

A value-added activity can have a nonvalue-added component under either of two conditions. First, if $z_j^a > z_j^o$, the firm consumes a nonoptimal amount of activity output. The excess consumption is nonvalue-added. Also, if $\boldsymbol{x}_j^a \neq \boldsymbol{x}_j^o$, the firm consumes a nonoptimal combination of resources to provide the activity output demanded. Thus, a value-added activity has a nonvalue-added component if the activity output is not provided efficiently, not used efficiently or both.

The nonvalue-added activities and nonvalue-added components of value-added activities represent waste. Together, the total cost of this waste, given in Equation 6.14, is the nonvalue-added cost. Through continuous improvement, the firm systematically eliminates the nonvalue-added cost to achieve the value-added (optimal) cost.

To completely eliminate the nonvalue-added cost, the firm must perform all activities efficiently and consume only the zero waste combination of activity outputs. Knowing why the waste exists should help the firm to better understand how to proceed towards the zero waste state. By identifying, describing and analyzing activities, the firm learns whether waste exists because a nonoptimal set of activities is being performed, the activities are not being performed efficiently or both. This information can then be used to identify the areas for improvement and to guide the firm in its improvement efforts.

Example 6.3. Assume the same problem described in Example 6.2. In addition to z_1 and z_2, the firm performs a third activity, z_3, where $z_3 = min\ (x_{13}, x_{23})$. Let $z_3^a = 20$ and $\boldsymbol{x}_3^a = (30,40)$ so that $\boldsymbol{x}^a = (150,220)$. As before, $\boldsymbol{x}^o = (40,40)$ so that the firm has total waste of \$290.

Since $q = min\ (z_1, z_2)$, z_3 is not included in any technically efficient combination of activity outputs. It is a nonvalue-added activity. The total resource spending of \$70 represents waste. Knowing this, the firm should target this activity for complete elimination.

From Example 6.2, the outputs of activities 1 and 2 are not provided efficiently and the activity outputs are not used efficiently. Thus, even though each of these activities is value-added, each has a nonvalue-added component equal to the sum of the activity level and product level waste. To eliminate waste, the firm must select a more efficient combination of activity outputs 1 and 2 and provided these activity outputs more efficiently.

Waste Elimination in Activity-Based Management

The nonvalue-added cost to be eliminated is defined by the system of unobservable production functions. Thus, the activity analysis must rely on the cost-based model of continuous improvement and the control system developed in Chapters IV and V to guide and evaluate the firm's waste elimination efforts. These earlier developments extend in a straight forward way to the activity-based management framework.

In the cost-based model of continuous improvement, the firm realizes an improvement equal to the change in resource spending. From Equations 4.3 and 6.10, this improvement is given by the following equation.

$$I_{ct} = \sum_{j=1}^{m} \sum_{i=1}^{n} p_i(x_{ijt-1} - x_{ijt}) \tag{6.14}$$

This total cost improvement can occur as a result of a reduction in activity and/or product level waste. Through continuous improvement, the firm acts to simultaneously eliminate both activity and product level waste. To reflect the dual nature of waste in activity-based management and to identify the waste eliminated at each level, the total improvement must be disaggregated into activity and product level improvements.

An activity level improvement is realized when resources are used more efficiently to produce activity outputs. In period *t-1*, the firm incurs c_{jt-1} amount of spending to provide z_{jt-1} amount of activity output: $c_{jt-1} = \sum_{i=1}^{n} p_i x_{ijt-1}$. Similarly, the firm incurs actual cost of c_{jt} in period t to provide z_{jt} amount of activity output. By the assumption of linear homogeneity, if the output of the jth activity would have remained constant, the firm would have incurred resource spending in period t of $(z_{jt-1}/z_{jt})(c_{jt})$. The difference between the actual cost in period *t-1* and the cost that would have been incurred in period t if activity output would have remained constant is the activity level improvement. Thus, the improvement in period t for the jth activity output, I_{cjt}^{A}, is given by the following equation.

$$I_{cjt}^{A} = \sum_{i=1}^{n} p_i (x_{ijt-1} - \frac{z_{jt-1}}{z_{jt}} x_{ijt}) \tag{6.15}$$

At the product level, improvements are realized only because the activity outputs are used more efficiently. Operationally, the improvement is the decrease in spending that would have been realized if no activity level improvements occurred (ie. there was no change in the efficiency of the usage of resources, only the usage of activity outputs). If resources were not used more efficiently, the actual cost of each unit of activity output (the actual activity rate) would remain unchanged. The actual activity rate in period t for the jth activity output, r_{jt}, is given by the following equation.

$$r_{jt} = \frac{\sum_{i=1}^{n} p_i x_{ijt}}{z_{jt}} \tag{6.16}$$

The product level improvement, I_{cjt}^{P}, is then determined as follows.

$$I_{cjt}^{P} = r_{jt} (z_{jt-1} - z_{jt}) \tag{6.17}$$

It is easy to see that the total of the product level improvements and the activity level improvements for each of the j activity outputs sum to the total decrease in cost given by Equation 6.14. Thus, the activity and product level improvements provide a more detailed view of the same cost improvement described by the cost-based model of continuous improvement. This is illustrated by the following example.

Example 6.4. Extend the problem presented in Examples 6.1 - 6.3. Table 6.1 shows the resource spending in total and for each of the three activities and the actual activity outputs for the first three periods.

Table 6.1
Observable Resource Spending

		c_{jt}			z_j^a		
t	c_t	c_{1t}	c_{2t}	c_{3t}	z_{1t}	z_{2t}	z_{3t}
0	$370.00	$100.00	$200.00	$70.00	$30.00	$60.00	$20.00
1	225.00	70.00	120.00	35.00	26.25	45.00	12.73
2	152.50	55.00	80.00	17.50	24.06	36.00	8.10
3	116.25	47.50	60.00	8.75	22.86	30.86	5.15

The total improvement for activity 1, I_{c1t}, is the change in resource spending for that activity: $I_{c11} = c_{10} - c_{11} = \$100 - \$70 = \$30$. By Equations 6.15 and 6.17, this total improvement can be disaggregated into activity and product level improvements.

$$I^A_{c11} = \$100 - \frac{30}{26.25}(\$70) = \$20$$

$$I^P_{c11} = \frac{\$70}{26.25}(30 - 26.25) = \$10.$$

The total improvements for activities 2 and 3 can be disaggregated in a similar manner.

The activity level improvement indicates that a cost reduction was realized because resources were used more efficiently to provide the activity output. The product level improvement represents an additional cost savings because the activity output was used more efficiently to produce the product. Thus, the total improvement of $30 occurred because of the elimination of a portion of both activity and product level waste.

Guiding the Elimination of Nonvalue-Added Cost

To ensure that the total elimination of nonvalue-added cost will be accomplished, the firm must guide and evaluate the activity and product level waste elimination efforts. The control system developed in Chapter V can be extended to activity-based management to provide this guidance. Extension of the control system to the activity-based framework is straight forward.

As product and activity level improvements are realized, the residual waste decreases. After t periods, the residual product level waste, R_{cjt}^P, and activity level waste, R_{cjt}^A, for the jth activity output are given by the following equations.

$$R_{cjt}^{P} = w_{cj}^{P} - \sum_{v=1}^{t} I_{cjv}^{P} \tag{6.18}$$

and

$$R_{cjt}^{A} = w_{cj}^{A} - \sum_{v=1}^{t} I_{cjv}^{A} \tag{6.19}$$

A portion, k_{cjt}^P (k_{cjt}^A), of the residual product (activity) level waste will be eliminated in each period. The following equations determine the proportion of waste eliminated at each level in period t.

$$k_{cjt}^{P} = \frac{I_{cjt}^{P}}{R_{cjt-1}^{P}} \tag{6.20}$$

and

$$k_{cjt}^{A} = \frac{I_{cjt}^{A}}{R_{cjt-1}^{A}} \tag{6.21}$$

If it is observed that $I_{cjt}^P > 0$ ($I_{cjt}^A > 0$), then $k_{cjt}^P > 0$ ($k_{cjt}^A > 0$). By the assumptions of continuous improvement and economic feasibility, $k_{cjt}^P < 1$ ($k_{cjt}^A < 1$). Thus, k_{cjt}^P (k_{cjt}^A) is observably and unambiguously bounded such that $0 < k_{cjt}^P < 1$ ($0 < k_{cjt}^A < 1$).

Obviously, the elimination of nonvalue-added cost follows the same basic structure as the cost-based model of continuous improvement developed in Chapter IV. Thus, by the Convergence Property, if k_{cjt}^{P} (k_{cjt}^{A}) is constant or increasing, the product (activity) level waste will be completely eliminated in the limit. By Equations 6.15 and 6.17, the observable total cost improvement series (I_{c1}, I_{c2}, ... , I_{ct}) can be disaggregated into product and activity level improvement series. The behavior of k_{cjt}^{P} (k_{cjt}^{A}) can be inferred by applying the I_{ct} Ratio Test to each of these series individually. As discussed in Chapter V, this information can then be used to guide and evaluate continuous improvement efforts. Consider the following example.

Example 6.5. From the cost sequences in Table 6.1, the improvement series shown in Table 6.2 can be calculated. Applying the I_{ct} Ratio Test to the total cost improvement series shows that k_{ct} is constant. Thus, the Convergence Property is satisfied and, in the limit, waste will be completely eliminated. The series sums to reveal an optimal cost of $80.

Table 6.2
Cost Improvement Series

		I_{cjt}			I_{cjt}^{A}			I_{cjt}^{P}		
t	I_{ct}	I_{c1t}	I_{c2t}	I_{c3t}	I_{c1t}^{A}	I_{c2t}^{A}	I_{c3t}^{A}	I_{c1t}^{P}	I_{c2t}^{P}	I_{c3t}^{P}
1	$145.00	$30.00	$80.00	$35.00	$20.00	$40.00	$15.00	$10.00	$40.00	$20.00
2	72.50	15.00	40.00	17.50	10.00	20.00	7.50	5.00	20.00	10.00
3	36.25	7.50	20.00	8.75	5.00	10.00	3.75	2.50	10.00	5.00

Consider activity 1. Applying the I_{ct} Ratio Test to the activity and product level improvement series indicates that k_{c1t}^{A} and k_{c1t}^{P} are constant. The series sum to reveal activity level waste of $40 and product level waste of $20. If the activity level waste is eliminated, the firm would incur a cost of $60 to provide 30 units of activity output so that r_1^o = $2. Thus, from Equation 6.13, $z_1^o = 20$. At the zero waste state, the firm would incur a cost of $40 to provide 20 units of activity output for activity 1 for the production of 20 units of the final product.

The zero waste state for the other activities can be determined in a similar manner. By guiding and evaluating the behavior of k_{cjt}^A and k_{cjt}^P as discussed in Chapter V, the firm can accelerate waste elimination and create a competitive advantage. Thus, the continuous improvement control system can be easily incorporated into the existing accounting structure of activity-based management.

Summary

Continuous improvement is designed to eliminate wasteful practices. Thus, with a central focus on activities, activity-based management provides an ideal accounting structure for the control system that supports continuous improvement. The control system, described in Chapter V, is easily extended to the activity-based framework.

The cost-based model of continuous improvement upon which the control system builds assumes a classical production function wherein the firm consumes resources to produce a product. In activity-based management, the firm consumes resources to perform activities and consumes the outputs of those activities to produce the product. Thus, activity-based management is described by a system of production functions rather than a single production function.

The technology described by activity-based management gives a more detailed view of waste. By analyzing the root causes, the activities can be classified as either value-added or nonvalue-added. The total cost of nonvalue-added activities and the nonvalue-added components of value-added activities is waste. The total waste, the optimal cost and the zero waste state is the same for both the activity-based and classical frameworks.

As the firm eliminates nonvalue-added cost, waste will be simultaneously eliminated at both the activity and product levels. The total cost improvement can be disaggregated into activity and product level improvements. By applying the cost-based model of continuous improvement to each of the production functions in the activity-based management system independently, information gained from the activity and product level improvement series can be used to guide and evaluate the firm's waste elimination efforts as described in Chapter V.

The description of activity-based management presented in this chapter is limited to a very basic problem. Obviously, future research will benefit from a more complex and realistic model in which multiple products are produced and nonunit level activities are performed. These and other extensions to this study are discussed in Chapter VII.

CHAPTER VII

SUMMARY AND EXTENSIONS

Summary and Contributions

Continuous improvement is characterized by the incremental, systematic and complete elimination of waste. In physical measures, waste is the difference between the actual inputs used and optimal inputs that should have been used to produce a given quantity of output. In financial measures, waste is the difference between the cost of the actual inputs used and the cost of the optimal inputs. The zero waste state is found by minimizing cost subject to an output constraint given by a well defined production function.

In a continuous improvement environment, the control system must identify the total amount of waste to be eliminated. Then, the actions necessary to achieve the improvement must be determined. Finally, actual progress must be evaluated to determine whether or not the intended improvement occurs. In short, the control system must identify the amount of waste to be eliminated and then guide and evaluate the firm's progress toward the zero waste state.

In practice, identifying the waste to be eliminated is no trivial matter. Because the underlying production function is unobservable, the optimal inputs and minimum cost are also unobservable. The firm can only observe the actual inputs used and cost incurred each period. Assuming that some waste is eliminated from each input in each period, the differences in input consumption across time create observable improvement series. These series reveal much about the firm's continuous improvement efforts.

The waste that remains after each improvement is the residual waste. Because of the incremental nature of continuous improvement, only a portion of the residual waste will be eliminated in any given period. If the proportion of waste eliminated each period increases at a decreasing rate or remains constant over time, total waste will be eliminated in the limit. This is known as the Convergence Property. By comparing the ratios of the observable improvements over time, the firm can infer whether or not this property is satisfied.

If the Convergence Property is satisfied, the amount of waste to be eliminated for each input can be inferred from the observable improvement series. Due to the systematic nature of waste elimination in a continuous improvement environment, the improvement series for each input converges such that the sum of the series is equal to the total amount of waste to be eliminated for that input. By subtracting total waste for each from the actual amount of inputs initially consumed, the optimal input combination can be determined. Thus, under certain conditions defined by the Convergence Property, the nature of continuous improvement can be exploited so that the observable data can be used to identify the zero waste state after a finite number of periods. This is true even though the underlying production function is unobservable.

The assumption that some improvement is realized for each input in each period requires that the firm have prior knowledge of the direction in which the optimal input combination lies. It is not realistic to assume that such knowledge exists. If this assumption is violated, the observable improvement series will provide ambiguous signals about the total amount of waste to be eliminated and the proportion of residual waste eliminated each period. Thus, the input-based model of continuous improvement is limited in its practical application.

These limitations and ambiguities can be overcome by a cost-based model of continuous improvement. The input-based model extends to the cost framework in a parallel manner. However, any decrease in cost unambiguously signals an improvement. Thus, if the observable cost improvement series satisfies the Convergence Property the minimum cost will always be revealed. If the firm achieves the minimum cost, it must also achieve the optimal input combinations. It is this cost-based model that allows for the control of continuous improvement efforts.

The I_{ct} Ratio Test and the Convergence Property provide the foundation for the control system. If the I_{ct} Ratio Test reveals that the proportion of residual waste eliminated each period is constant or increasing at a decreasing rate, the Convergence Property is satisfied so that the firm is on course to the zero waste state and continuous improvement efforts are in control. If the I_{ct} Ratio Test reveals that the proportion of residual waste eliminated each period is decreasing, the firm cannot be sure that the zero waste state will be achieved and, therefore, continuous improvement efforts are out of control and must be revised. The I_{ct} Ratio Test evaluates the waste elimination efforts and guides the firm toward the zero waste state.

However, the zero waste state can only be achieved in the limit. It is, therefore, the theoretical goal of continuous improvement. For practical purposes, the firm should set some target level of waste elimination that can be achieved in a finite number of periods. If the firm is on course toward the zero waste state, it must also achieve this practical target cost.

To create or maintain a competitive advantage, the firm must reach the target cost more quickly than competitors. To do so, the rate of waste elimination must be accelerated, especially in the early periods. As the rate of waste elimination increases, the difference between the I_{ct} ratios will increase. By maximizing this difference, the firm will achieve the target cost as rapidly as possible. Thus, the I_{ct} ratios can be used to not only to determine if the target cost will be achieved but also to control the rate of waste elimination.

The data used to control continuous improvement should be provided by an accounting information system. Activity-based management is an accounting structure that focuses the firm's attention on activities with the objective of eliminating waste. Thus, activity-based management provides an ideal accounting structure for the control system.

In an activity-based framework, the firm consumes resources to provide activity outputs and consumes the activity outputs to produce the final product so that the firm operates according to a system of production functions. In the production system, waste exists at the activity level if the firm consumes a nonoptimal combination of resources to provide the actual activity outputs demanded at the product level. Waste exists at the product level if a nonoptimal combination of activity outputs is consumed to produce the product. To reach the zero waste state, waste must be completely eliminated at both the activity and product levels.

By disaggregating waste into activity and product levels, activity-based management provides detailed insight into the nature and cause of waste. As waste is eliminated, improvement series can be observed at each level for each activity. The I_{ct}

Ratio Test can be applied to each of these series individually to guide and evaluate waste elimination efforts. Thus, the control system can be incorporated in a straightforward way into the existing accounting structure of activity-based management.

This study develops a model of continuous improvement that, within a set of reasonable assumptions, uses observable data to ensure that the conditions necessary to achieve complete waste elimination exist and identifies the zero waste state defined by an unobservable technology. Information gained from this model is used to guide and evaluate continuous improvement efforts by determining if the firm is on course toward the zero waste state and by controlling the rate of waste elimination to ensure that the target cost is achieved rapidly. The resulting control system can be readily incorporated into the existing accounting structure of activity-based management. This represents the first step toward the creation of a comprehensive control system for continuous improvement. At the same time, the limitations of this study suggest many possible extensions for future research.

Limitations and Extensions

The control system developed in this study is built upon a model of continuous improvement. According to the model, the proportion of residual waste eliminated each period, k, must be constant or increasing at a decreasing rate if the zero waste state is to be achieved. In total, there are seven possibilities for the behavior of k: constant; increasing at a decreasing, constant or increasing rate; decreasing at a decreasing, constant or increasing rate. The I_{ct} Ratio Test identifies only three of these and the Convergence Property is satisfied for only two. These alternative behaviors open avenues for future research.

Obviously, waste will be completely eliminated in a finite number of periods if k increases at a constant or increasing rate. What about a decreasing k? It can be shown by example that the zero waste state can be achieved if k is decreasing, but this will not always be true. What conditions must hold for a decreasing k to create an improvement series that reveals the zero waste state?

The behavior of k is inferred from the I_{ct} Ratio Test. This test deals with only three of the seven possible behaviors for k. It is yet to be determined how the observable data be used to identify other alternative patterns. A more complete understanding of k is needed.

The observable improvement series of the input-based model fail to reveal when $k < 0$. Thus, in the model presented here, financial measures of performance are more complete and accurate than physical measures. This is counter to the current trend away from financial measures. Thus, continued research into the relative merits of each is warranted.

From an increased understanding of the relationship between financial and physical measures of performance and the information provided by each, the model of continuous improvement could be expanded. Using both measures together, it may be possible to separate waste into technical and mix inefficiency. This might provide further guidance for the firm's future improvement efforts. Even though neither measure is sufficient to reveal the optimal input combination, both together, along with an increased understanding of k, may completely identify the zero waste state. Future research should address these possibilities.

Throughout this study, it has been assumed that input prices remain constant. Except in certain special circumstances (ie. the firm produces output according to a linear production function), a change in prices will result in a change in the zero waste state. The model of continuous improvement should, therefore, be extended to include the possibility of changing prices.

Another assumption of the current model is that the underlying technology, although unknown, remains unchanged. A firm could implement alpha (continuous improvement) and gamma (reengineering) type changes simultaneously. Any change in the underlying technology will most likely result in a change in the zero waste state. Thus, the model should be further extended to identify these changes.

A model that included changes in technology and prices, separated waste into technical and mix inefficiency and identified both the minimum cost and optimal input combination would provide a great deal of information about the waste elimination problem facing the firm. With this information, it may be possible to identify the actual production function faced by the firm. Knowing this, a comprehensive control system to guide and evaluate continuous improvement can be created.

The control system is incorporated into the existing accounting structure of activity-based management. This study presents only a very basic description of activity-based management. A more complete modeling of the framework is needed. This model should include, at a minimum, multiple products and nonunit level activities. Incorporating the comprehensive control system into this accounting structure will provide the support necessary to help the firm to create or maintain a competitive advantage.

In conclusion, this study represents an important first step toward the creation of a comprehensive control system capable of supporting continuous improvement. However, much work remains to be done. The control system and the model upon which it is built must be more fully developed before they can be tested empirically and incorporated into practice.

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