SUPPLY CHAIN COORDINATION UNDER SALES EFFORT FREE RIDING

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CHAPTER 1

INTRODUCTION

1.1 Motivation

With the rapid development of the internet and mobile technologies, *sales effort free riding* [see Shin, 2007, Bernstein et al., 2009] has caused a growing tension between brick-and-mortar retailers and online stores. In December 2011, Amazon released a smart phone app called PriceCheck to help shoppers seek out the online retailer's price for an item in a brick-and-mortar store by simply scanning its bar code. Amazon even offered special discounts for smart phone check-outs to encourage "showrooming", or so called scan and scram by brick-and-mortar shoppers [Indvik, 2011]. In January 2012, Target issued letters to its suppliers to seek help in curbing showrooming, according to the Wall Street Journal [Zimmerman, 2012] and the Time Magazine [Tuttle, 2012]. The letter, signed by Target executives, reads: "What we aren't willing to do is let online-only retailers use our brick-and-mortar stores as a showroom for their products and undercut our prices without making investments, as we do, to proudly display your brands." Suppliers are being asked to lower wholesale prices to enable Target to match the prices of its online competitors, and Target is also asking suppliers to create unique products only available in Target stores, helping avoid direct comparisons to online goods. Target is not the only retailer exploring every method to reduce the lost sales due to showrooming. Best Buy also offers price match to customers who find a cheaper price on their smartphones [Noguchi, 2011].

Besides the competition from independent online stores, brick-and-mortar retailers also face challenges from the manufacturer owned online channels. The rapid development of e-commerce technology has made it easier for manufacturers to engage in direct sales. For example, Estee Lauder selling its flagship Clinique brand directly over the Internet was considered a serious threat by the brick-and-mortar stores carrying Estee Lauder's products [Machlis, 1998a]. Similar conflicts have been reported by Avon Products Inc. [Machlis, 1998b], Bass Ale [Bucklin et al., 1997], IBM [Nasireti, 1998], and others. Some trade groups, such as the National Shoe Association and the National Sporting Goods Association, have gone to the point of urging members to reduce or eliminate purchases from manufacturers using online direct channels [Stern et al., 1996]. According to the survey by Tedeschi [2000] in The New York Times, about 42% of top suppliers (e.g., IBM, Pioneer Electronics, Cisco System, Estee Lauder, and Nike) in various industries had begun to sell directly to consumers through the Internet. At the same time, their brick-and-mortar retailer partners voice the belief that orders placed through manufacturers' direct channels are those should have been placed through them. In a cover story by Fortune magazine [Brooker, 1999], Home Depot issued a letter to all suppliers selling products over the Internet, saying "We recognize that a vendor has the right to sell through whatever distribution channels it desires. However, we too have the right to be selective in regard to the vendors we select and we trust that you can understand that a company may be hesitant to do business with its competitors."

No manufacturers can ignore the threats from brick-and-mortar retailers, as they are indispensable in selling the products. The physical shopping experience in brick-and-mortar stores is unmatchable by their online competitors. No matter how many photos, how extensive the description, and how many customer reviews are provided, buying online always comes with a higher degree of guesswork than buying in person. In addition, brick-and-mortar retailers can also insert efforts to stimulate the demand by mailing advertisement posters, providing attractive shelf space, offering trial samples, and educating customers about the product with sales representatives.

Showrooming reduces brick-and-mortar retailers sales efforts, thus it lowers the overall market demand and the manufacturer's sales revenue. Both the brick-and-mortar retailers and their suppliers want to resolve the problems caused by showrooming. However, the current tactics (simply price match or wholesale price discount) employed by the brick-and-mortar retailers are not effective in coordinating sales effort.

Apparently, the sales effort free riding problem dampens the brick-and-mortar retailers' desire to promote the manufacturer's products. If the manufacturer wants to retain the dual sales channel, he has to find a solution to mitigate the channel conflict caused by sales effort free riding. This is the inspiration of our research.

We examine the contracts under two supply chain structures, with independent or manufacturer-owned online channels. The manufacturer-owned online channels are favored by many industry leaders. For example, HP started online direct sales in the late 1990s, and by 2004, 26% of their orders were from the online direct sales [Burke, 2004]. Apple's online store sales grew by 90% in 2010, thanks to the strong demand for iPad2 and iPhone 4s. Independent on-line channels are usually adopted by manufacturers not in the leading positions, such as ASUS in the PC industry and CREATIVE in the consumer electronics industry. We show that the contracts proposed in this dissertation are effective in resolving the sales effort free riding conflicts between retail channels under both supply chain structures.

We also investigate the contracts under both deterministic and stochastic demands. The deterministic demand model depletes the cost and risk incurred by demand uncertainty. This helps us to focus the research effort on analyzing the impact of supply chain contracts on players' profit margin, action coordination, and system profit division. We are able to develop coordinating contract formula to achieve maximum supply chain system profit. In the second part of the dissertation, we also investigate the supply chain contracts under stochastic demand. The demand uncertainty is captured and analyzed in the profit models. We are able to show that the selective rebate with price match contract coordinates the sales effort.

The following sections present overviews of the major results under the two demand models.

1.2 Overview of the Supply Chain Contracts under Deterministic Demand

Under the deterministic demand, we analyze three contracts, namely the selective rebate contract with price match, the revenue sharing contract with price match and the selective target rebate contract with price match, in supply chain coordination under sales effort free riding. Under the selective rebate contract with price match, the brick-and-mortar retailer matches the online price if a customer shows the proof of the lower price. The manufacturer compensates the brick-and-mortar retailer partially with a rebate for each sale under price match. Notice that the brick-and-mortar retailer would not offer the lower online price to all customers, since the manufacturer's rebate compensates only part of the price difference. We show that this contract is mathematically equivalent to the revenue sharing contracts with price match. We show that both contracts can coordinate the supply chain, arbitrarily split the system profit, and achieve Pareto optimality. Furthermore, in the case where the manufacturer owns the online channel, there exists a solution regime on the Pareto-optimal frontier in which both the manufacturer and the brick-and-mortar retailer's profits are improved from the baseline case. As for the selective target rebate with price match, we show that such contract can coordinate the retailer's sales effort, but no profit division mechanism can guarantee Pareto improvement on online channel's profit. This is contrary to the results from the selective rebate and revenue sharing contracts.

1.3 Overview of the Supply Chain Contracts under Stochastic Demand

Under the stochastic demand, we propose a selective rebate contract with a price-match policy to help the manufacturer coordinate the brick-and-mortar retailer's sales effort. In this scheme, the brick-and-mortar retailer will match the online retailer's price if a customer shows the proof of the lower price. The manufacturer then offers a compensation rebate to the brick-and-mortar retailer based on the amount of sales with price match. We show that when demand is influenced by sales effort, a properly designed selective rebate contract with price match coordinates the brick-and-mortar retailer's sales effort.

We also examined other contracts, including the target rebate contract and the wholesale price discount contract, both with price match. The numerical analysis shows that the selective rebate outperforms other contracts in coordinating the brick-and-mortar retailer's sales effort and improving supply chain efficiency .

CHAPTER 2

LITERATURE REVIEW

This dissertation studies *supply chain contract design* to solve *sales effort free riding* between retail channels. In this section, we review the literature our research is related to: sales effort, free riding, supply chain coordination, supply chain contracts, and price match.

2.1 Sales Effort

There is a stream of research in operation management that considers the impact of a retailer's sales effort on demand expansion [see Chu and Desai, 1995, Lariviere and Padmanabhan, 1997, Netessine and Rudi, 2000, Taylor, 2002, Mukhopadhyay et al., 2008, Gilbert and Cvsa, 2003]. Gurnani et al. [2007] investigate the impact of the timing of investment decisions (sales effort, price, and product quality etc.) on the supply chain profitability, but they don't consider contractual incentives to coordinate the supply chain.

Some researchers show that sales effort can also affect various properties of the supply chain, such as the demand uncertainty analysis by Heese and Swaminathan [2003] and risk aversion analysis by Suo et al. [2005]. He et al. [2009] examine a supply chain facing stochastic demand that is dependent on both sales effort and retail price. They explore a variety of contracts including joint returns policy with revenue sharing contract, returns policy with sales rebate and penalty (SRP) contract, and revenue sharing contract with SRP. They find that only the returns policy with SRP contract can achieve supply chain coordination. Cachon [2003] surveys the recent literature on sales effort and supply chain coordination. He shows that sales effort, when incorporated with supply chain contracts such as sales rebate, buy back and revenue sharing, can achieve supply chain coordination. However, none of the past literature has investigated the sales effort free riding phenomenon.

There are also a number of papers in the marketing and franchising literatures elaborating on the basic retail effort model. For example, in Chu and Desai [1995] the supplier can also exert costly effort to increase demand, e.g., brand building advertising, but the impact of effort occurs only with a lag: they have a two period model and period one effort by the supplier increases only period two demand. They also enrich the retailer's effort model to include two types of effort, effort to increase short term (i.e., current period) sales and long term effort to increase long term customer satisfaction and demand (i.e., period two sales). They allow the supplier to compensate the retailer by paying a portion of her effort cost and/or by paying the retailer based on the outcome of her effort, i.e., a bonus for high customer satisfaction scores. The issue is the appropriate mix between the two types of compensation. Lal [1990] also includes supplier effort, but, effort again is non-enforceable. Although revenue sharing (in the form of a royalty payment) continues to distort the retailer's effort decision, it provides a useful incentive for the supplier to exert effort: the supplier will not exert effort if the supplier's profit does not depend directly on retail sales. Lal [1990] also considers a model with multiple retailers and horizontal spillovers: the demand enhancing effort at one retailer may increase the demand at other retailers. These spillovers can lead to free riding, i.e., one retailer enjoys higher demand due to the efforts of others without exerting her own effort. He suggests that the franchisor can control the problem of free riding by exerting costly monitoring effort and penalizing franchisees that fail to exert sufficient effort.

Some recent literature investigates the sales effort in a multi-channel supply chain. Xie and Neyret [2009] show that the co-op advertising and pricing strategies can maximize the system profit in a one-manufacturer-and-two-retailer supply chain. Karray [2010] investigates the effects of horizontal joint promotions among retailers and show that this cost sharing strategy can improve each channel member's profit through demand expansion and higher margins in all the channels. Xing and Liu [2012] study sales effort coordination with stochastic demand. They show that the selective rebate contract with price match solves the free riding problem and coordinates the brick-and-mortar retailer's sales effort. However, none of the literature has achieved maximum system profit while investigating sales effort free riding and coordination.

2.2 Free Riding

Free riding has been extensively studied in the fields of industrial organization and marketing (for a survey in a retail channel environment, see Carlton and Chevalier, 2001 and Antia et al., 2004). Sales effort free riding is first noticed by Telser [1960]. He points out that in a competitive retail industry, retailers may free ride other retailers' effort. Manufactures can discourage price competition to channel retailers' focus on such effort competition, which is to the manufacturer's liking. Jeuland and Shugan [1983] also analyze promotional retailer effort as an extension to their channel competition model. Manufacturer's effort is considered by Narayanan and Raman [1997] in a supply chain that the retailer stocks competing products while free rides the manufacturer's forecast effort. They investigate retailer-managed inventory, vendor-managed inventory, and buy-backs contract. They show that the best option depends on the market parameters, but none of them coordinates the supply chain. Carlton and Perloff [2004] examine how manufacturers take actions to avoid free riding, but their focus is on vertical restrictions.

There is some recent literature regarding free riding in the field of operation management. Shin [2007] shows that free riding may benefit both the free rider and the service provider in that it may soften price competition. Wu et al. [2004] examine a retail market where information service is provided by retailers to help consumers identify their ideal products. Their analysis suggests that a retailer in this setting needs to develop the capability of and reputation for service provision to obtain positive profits. A retailer who free rides all the time loses the market share. However, none of the papers has studied how to coordinate retailer effort with free riding, as we do in our dissertation. Bernstein et al. [2009] design a supply chain contract to increase channel competition to improve the manufacturer's profit. Sigua and Chintagunta [2009] study a problem of sharing advertising cost among the franchisor and the franchisees, which resembles a supply chain with a manufacturer and multiple retailers. They show that franchisor's compensation to coordinate the franchisees' advertising efforts can maximize the supply chain system profit. However, none of the papers above consider sales effort free riding among asymmetric retailers, e.g., brick-and-mortar/onine retailers.

2.3 Supply Chain Coordination

According to Cachon and Lariviere [2001], supply chain coordination is "to achieve the optimal performance if the firms coordinate by contracting on a set of transfer payments such that each firms objective becomes aligned with the supply chain's objective."

Supply chain coordination is a classic topic in the literature. Cachon [2003] analyzes every contract with their ability to coordinate the players' decisions across the supply chain. When coordination is achieved, the system profit of the supply chain is maximized. The question following supply chain coordination is the division of system profit. Some contracts have been proved to possess the ability of arbitrary division of system profit. For example, Cachon and Lariviere [2005] show that the revenue sharing contract can achieve supply chain coordination while arbitrarily splitting the system profit between the manufacturer and the retailer. Cai [2010] studies channel coordination in a dual channel supply chain. He shows that revenue sharing contract can coordinate both the retailer-retailer and retailer-direct channel supply chains, but at different supply chain efficiencies. To our best knowledge, none of the previous literature has discussed arbitrary profit division in a multi-channel supply chain.

The significance of supply chain coordination

Supply chain coordination is to achieve the maximum profit for all the supply chain players, from manufacturer to retailer. Theoretically, if a contract in a decentralized supply chain can achieve the same system profit as of a centralized supply chain, this supply chain contract is coordinating. The major reason that makes supply chain contract design so appealing is its potential to achieve the maximum system profit.

The values of contract design beyond supply chain coordination

Very few contracts, such as revenue sharing and buy back, can effectively coordinate the supply chain [see Cachon and Lariviere, 2001]. Many contracts targeting real world problems, such as the selective rebate contract in Xing and Liu [2012], is not designed to achieve supply chain coordination but a solution to free riding problem.

There are some other perspectives to demonstrate the values of a contract.

- *Supply chain efficiency* is the ratio of system profit between the decentralized supply chain and the centralized supply chain. If the supply chain efficiency is close to 100%, the contract is comparable to a coordinating contract.

- *Pareto improvement* means that none of the players is worse off, and at least one of the player is better off. Such property can guarantee that the contract can improve the bottom line of the business operations, while all of the supply chain players are willing to accept the contract.

Game theory in supply chain coordination

From the prospective of traditional operations research, if the profit functions in a contract model is jointly concave with respect to its decision variables, the contract can coordinate the supply chain. But joint concavity is not easy to prove when the number of variables are large. From the perspective of game theory, if the best response functions (derived from the first order derivatives of the profit functions) construct a Nash Equilibrium, then the supply chain contract is coordinating. It means that if the players rationally optimize their profits, they will spontaneously choose the Nash Equilibrium point from their respective best response functions and have no incentive to deviate from it. Comparing with the traditional operations research methods, there are many simpler game theory methods to prove the existence of a Nash Equilibrium, such as the quasi-concavity of payoff functions leading to a pure strategy Nash Equilibrium [Debreu, 1952], and the supermodular game having at least one Nash Equilibrium [Topkis, 1998].

2.4 Supply Chain Contracts

In this dissertation, wholesale price, channel rebate and revenue sharing contracts are studied to coordinate the newsvendor and to divide the supply chain's profit. Each contract coordinates by inducing the retailer to order more than he would with just a wholesale price contract. Revenue sharing does this by giving the retailer some downside protection. The target rebate contract does this by giving the retailer upside incentive: if demand is greater than the target level; the retailer effectively purchases the units sold above the level for less than their cost of production.

The various coordinating contracts may not be equally costly to administer. The wholesale price contract is easy to describe and requires a single transaction between the firms. The revenue sharing and rebate contracts are more costly to administer: the supplier must monitor the number of units the retailer has left at the end of the season, or the remaining units must be transported back to the supplier, depending on where the units are salvaged; the supplier also has to monitor and verify the sales at the retailer to calculate rebate or shared revenue.

In the following sections, we review the literature on the three supply chain contracts.

2.4.1 Wholesale price contract

In this section, we will discuss the simplest contract–the wholesale price contract: the supplier merely charges the retailer a fixed wholesale price per unit ordered.

It is well known that the wholesale price contract generally does not coordinate the supply chain [Cachon, 2003]. It coordinates the channel only if the supplier earns a non-positive profit. So the supplier clearly prefers a higher wholesale price. As a result, the wholesale price contract is generally not considered a coordinating contract.

Even though the wholesale price contract does not coordinate the supply chain, it is worth studying because it is commonly observed in practice. That fact alone suggests that it has valuable qualities. For instance, the wholesale price contract is simple to administer. As a result, a supplier may prefer the wholesale price contract over a coordinating contract if the additional administrative burden associated with the coordinating contract exceeds the supplier's potential profit increase.

Another important feature about wholesale price contract is that the supplier's share of supply chain profit increases more quickly than supply chain efficiency, with the increase of demand variation. Intuitively, the larger the demand variation is, the larger profit share the supplier obtains [Cachon, 2003]. One explanation for this pattern is that the retailer's profit represents compensation for bearing risk: with the wholesale price contract there is no variation in the supplier's profit, but the retailer's profit varies with the realization of demand. As the coefficient of variation decreases the retailer faces less demand risk and therefore her compensation is reduced. However, the retailer is not compensated due to risk aversion. If the retailer were risk averse, the supplier would have to provide for yet more compensation. Instead, the retailer is being compensated for the risk that demand and supply do not match. Lariviere and Porteus [2001] demonstrate this argument holds for a broad set of demand distributions.

There are some interesting extensions to wholesale price contract in the literature. Anupindi et al. [2001] suppose the supplier sells to a retailer that faces an infinite succession of identical selling seasons. There is a holding cost on left over inventory at the end of a season but inventory can be carried over to the next season. The retailer submits orders between seasons and the supplier is able to replenish immediately. Within each season the retailer faces a newsvendor problem that makes the trade-off between lost sales and inventory holding costs. Hence, the retailer's optimal inventory policy is to order up to a fixed level that is the solution to a newsvendor problem. Cachon [2004] and Ferguson et al. [2006] provide another twist on this setting: each model allows the supplier to produce more than the retailer orders, thereby allowing the retailer to place a second order after some demand information is received. They find that both firms may be better off by allowing the retailer to place this second order.

2.4.2 Channel rebate contract

Channel rebate, a broadly used incentive contract in the retail business, has been applied to coordinate the retailer's effort [see Lariviere, 1998, Tsay et al., 1998, Cachon, 2003, for surveys on supply chain contracts]. Taylor [2002] first shows that the target rebate can coordinate a single channel supply chain, then includes the retailer's sales effort and shows that a target rebate and returns contract can achieve coordination. Krishnan et al. [2004] show that a buy back and manufacturer rebate contract can coordinate the supply chain. In that paper, the retailer decides her order quantity first, then makes her effort decision after detecting a signal of the market demand. Taylor and Xiao [2009] compare rebate and returns contracts in a single channel supply chain with retailer's forecasting effort. They show that the manufacturer can achieve supply chain coordination with the optimal menu of returns contracts. However, none of the papers mentioned above has studied a manufacturer rebate based on the sales to a specific customer group and applied it in a multi-channel supply chain with sales effort.

2.4.3 Revenue sharing contract

With a revenue sharing contract the supplier charges a fixed amount per unit purchased plus the retailer gives the supplier a percentage of her revenue. Assume all revenue is shared, i.e., salvage revenue is also shared between the firms. (It is also possible to design coordinating revenue sharing contracts in which only regular revenue is shared.) Revenue sharing contracts have been applied effectively in the video rental industry. Cachon and Lariviere [2001] provide an analysis of these contracts in a more general setting.

Revenue sharing contract has been recognized as an effective supply chain coordinating contract in the literature. The following are the articles that build the foundation of revenue sharing contracts. Dana and Spier [2001] study revenue sharing contract in the context of a perfectly competitive retail market. Pasternack [2005] studies a single retailer newsvendor model in which the retailer can purchase some units with revenue sharing and other units with a wholesale price contract. He does not consider supply chain coordination in his model. Cachon and Lariviere [2005] study revenue sharing contract in supply chain coordination. They find that revenue sharing contract alone cannot coordinate retail effort, so they develop a variation on revenue sharing, a quantity discount contract, for this setting. Mortimer[2000] provides a detailed econometric study of the impact of revenue sharing contracts in the video rental industry. She finds that the adoption of these contracts increased supply chain profits by seven percent. Gerchak, Cho and Ray [2001] consider a video retailer that decides how many tapes to purchase and how much time to keep them. Revenue sharing coordinates their supply chain, but only provides one division of profit. They redistribute profits with the addition of a licensing fee. However, none of the papers have studied revenue sharing contract in the context of sales effort free riding.

2.5 Price Match

The impact of price match has been examined in the economics and marketing literature. The earliest analysis about price match is in Png and Hirshleifer [1987], in which a retailer discriminates between two classes of customers who have different cost of information. The retailer couples a retail price with an offer to match the lower price of other retailers. They show that the retail price of each retailer is increasing in the number of retailers, and the total sales are decreasing in the number of retailers. Furthermore, if retailers coordinate, they discriminate more aggressively and increase their profits by increase their total sales. Hess and Gerstner [1991] show that price match helps to avoid price competition since the retailer becomes cautious to use price cut to compete with her price matching rivals. Chen and Narasimhan [2001] argue that price-match guarantees generate not only a competition-alleviating effect, but also a competition-enhancing effect. The former case accords to Hess and Gerstner [1991]. The latter effect comes from the fact that price match encourages consumers' price search behavior and thus exaggerates price competition. Corts [1996] studies the fluctuations in equilibrium prices caused by price match policy, and showed that price match facilitates customer segmentation according to the extent of the customers' information about rivals' prices. However, the previous studies mostly considered the price match policy as a marketing tactic, and focused on its impact on price competition. So far we have not noticed any literature which considers price match as a tactic to coordinate retailer effort. To the best of our knowledge, this is the first research effort using price match for sales effort coordination with free riding between retailers.

CHAPTER 3

RESEARCH OBJECTIVES AND CONTRIBUTIONS

The objective of the research is to help the manufacturer solve sales effort free riding problem in a dual channel supply chain.

We study supply chain contracts with price match to solve this business problem, including selective rebate, wholesale price, revenue sharing and target rebate contracts. We focus on the selective rebate contract with price match, in which the brick-and-mortar retailer matches the online retailer's price if the customers are able to show proofs of the lower price. The manufacturer then offers a compensation rebate to the brick-and-mortar retailer based on the volume of sales with price match. Our purpose is to show that when demand is influenced by sales effort, a properly designed selective rebate contract with price match coordinates the brick-and-mortar retailer's sales effort. We also show that the other contracts are either equivalent or inferior to the selective rebate contract. Specifically, the revenue sharing contract is equivalent to the selective rebate contract. The target rebate contract is less efficient than the selective rebate contract, while the wholesale price contract alone cannot coordinate the supply chain.

The selective rebate contract also distinguishes from the classical manufacturer rebate contracts (e.g. linear rebate and target rebate) in the sense that the rebates are only given to the sales with price match. The manufacturer uses such a contract to encourage the brick-and-mortar retailer to exert sales effort.

This research addresses the following issues:

- **Sales effort**: Sales effort is important in stimulating market demand. For example, retailers can influence demand by providing attractive shelf space and hiring sales representatives to promote the products. However, sales effort is hard to analyze because of the lack of tractability. We employ a model in which a retailer makes quantity and effort decisions and then observes demand.

- **Free riding**: Sales effort free riding has become an important issue with the increasing popularity of online stores. The online retailer free-rides the brick-and-mortar retailer' sales effort. As a result, the brick-and-mortar retailer reduces her sales effort, and the manufacturer' total demand also decreases. In this research, we provide the manufacturer with a solution to solve this sales effort free riding problem.

These issues are addressed through a supply chain contracting framework that can be analyzed in deterministic and stochastic demand. Determining the best combination of supply chain contracts to maximize the total supply chain profit is the focus of this research.

The contributions of this dissertation are listed below:

- Study the coordination in a dual channel supply chain under sales effort free riding.

- Study the effectiveness of the selective rebate, target rebate, revenue sharing, and wholesale price contracts with price match in coordinating the supply chain.

- Study the effectiveness of the aforementioned contracts with price match under deterministic and stochastic demand.

- Study the properties of the aforementioned contracts, such as flexible division of system profit and Pareto improvement.

CHAPTER 4

THE SUPPLY CHAIN CONTRACTS UNDER DETERMINISTIC DEMAND

4.1 Model with Online Channel Owned by the Manufacturer

4.1.1 Assumptions and notations

In this section, we consider that the online channel is owned by the manufacturer. We assume exogenous retail price, similar to Cachon and Lariviere [2001] who also use fixed retail prices to analyze the demand forecast effort. Such assumption is recommended by Lariviere and Porteus [2001], "A fixed retail price keeps the underlying inventory problem sufficiently straightforward that one can study many aspects of supply chain interactions and incentives."

We assume that the demand is a linear function of sales effort, as in Cachon and Lariviere [2005], Chu and Desai [1995] and Desiraju and Moorthy [1997]. The linear demand function has been widely used in the literature of marketing science and economics. It derives from the empirical regression model which is suited with most of the normal goods with constant price elasticity of demand.

We also assume that the cost function of sales effort is in the form of $V(\theta) = h\theta^2$, as in Taylor [2002]. h is a nonnegative cost coefficient of the sales effort. The increasing and convex property of the cost function of sales effort is based on the theory of diminishing return of investment. To output the same level of marginal sales effort effectiveness, the retailer has to commit more investment.

The market demand is categorized into three customer groups: (1) The traditional customers who only shop in the brick-and-mortar stores, has a demand function of $D_b = a_b + \tau_b\theta - p_b$, where p_b is the brick-and-mortar retailer's retail price, and τ_b is the coefficient summarizing the demand boosting effect by sales effort on the traditional consumers; (2) The free-riding customers who take advantage of the brick-and-mortar retailer's sales effort but purchase online at a lower price, has a demand function of $D_f = a_f + \tau_f\theta - p_o$, where p_o is the online retailer's retail price, and τ_f is for the free-riding consumers; (3) The online shoppers who only purchase through online stores and are barely affected by the brick-and-mortar retailer's sales effort, has a demand function of $D_o = a_o + \tau_o\theta - p_o$, where τ_o is for the online only consumers.

We also assume the base demand for the three customer groups satisfy $a_b - p_b \geq a_f - p_o \geq 0$ and $a_o - p_o \geq a_f - p_o \geq 0$. This means that, without the influence of sales effort, the number of the free riding customers should be no greater than the traditional shoppers and the online shoppers.

In addition, we assume the traditional shoppers' demand is more sensitive to the brick-and-mortar retailer's sales effort than the free-riding and online only customers, i.e., $\tau_b > \tau_f > \tau_o$. This assumption is reasonable in that the traditional shoppers visit the brick-and-mortar retailer more frequently and are loyal to such retailers. On the other hand, the brick-and-mortar retailer understands the traditional shoppers better than free riding customers and tends to provide more effective sales effort to them, e.g., membership discounts are given to Sam's club members, while free riding customers are hard to receive such discounts because their purchases are realized online.

In the following, we study five cases: the selective rebate (S), the target rebate (T), the revenue sharing (R), the baseline case (B), and the centralized supply chain (C). We denote Π^{case}_{player} as the profit, where $case = \{T, R, S, B, C\}$, and $player = \{m, b, o, j\}$, representing the manufacturer, the brick-and-mortar retailer, the online retailer, and the manufacturer-online retailer joint venture. This notation system is applied throughout the chapter. A summary of notations can be found in Table B.1.

4.1.2 The baseline case

The baseline case is the primitive setting of the story. There is no incentive in the supply chain and the online retailer free rides the brick-and-mortar retailer's sales effort. The free-riding customers buy from the online stores.

The brick-and-mortar retailer's profit

Considering that the demand is deterministic, the brick-and-mortar retailer's only decision variable is her sales effort:

$$\Pi_b^B(\theta) = (p_b - w)(a_b + \tau_b\theta - p_b) - h\theta^2. \tag{4.1}$$

The first order and second order derivatives of $\Pi_b(\theta)$ are:

$$\frac{\partial \Pi_b^B(\theta)}{\partial \theta} = (p_b - w)\tau_b - 2h\theta,$$

$$\frac{\partial^2 \Pi_b^B(\theta)}{\partial \theta^2} = -2h < 0.$$

Thus $\Pi_b(\theta)$ is concave in θ and we can obtain the optimal θ as follows:

$$\theta^* = \frac{(p_b - w)\tau_b}{2h}. \tag{4.2}$$

The manufacturer's profit

The manufacturer's sole decision is the wholesale price. His profit is:

$$\Pi_m^B(w) = (w - c)(a_b + \tau_b\theta - p_b + a_o - p_o + a_f + \tau_f\theta - p_o). \tag{4.3}$$

After substituting θ with equation (4.2), the first order and second order derivatives of $\Pi_m^B(w)$ are:

$$\frac{\partial \Pi_m^B(w)}{\partial w} = a_b + a_o + a_f - p_b - 2p_o + \frac{(p_b - 2w + c)(\tau_b + \tau_f)\tau_b}{2h},$$

$$\frac{\partial^2 \Pi_m^B(w)}{\partial w^2} = -\frac{1}{h} < 0.$$

Thus $\Pi_m^B(w)$ is concave in w.

The optimal w^* should be in the form of:

$$w^{D*} = \frac{p_b + c}{2} - \frac{\overline{D}h}{\tau_b(\tau_b + \tau_f)}, \tag{4.4}$$

where $\overline{D} = a_b - p_b + a_o - p_o + a_f - p_o$. $\overline{D}$ can be considered as the initial market demand. The optimal effort can be obtained:

$$\theta^{D*} = \frac{(p_b - c)\tau_b}{4h} + \frac{\overline{D}}{2(\tau_b + \tau_f)}. \tag{4.5}$$

4.1.3 The centralized supply chain

Before investigating the selective rebate contract, we first look at the centralized supply chain, which will serve as a benchmark to evaluate the efficiency of the decentralized system. The results of the centralized supply chain also facilitate the analysis of supply chain coordination under the selective rebate contract with price match.

In the centralized system, the central planner only needs to decide the sales effort:

$$\Pi^C(\theta) = (p_b - c)(a_b + \tau_b\theta - p_b) + (p_o - c)(a_o + (\tau_f + \tau_o)\theta - 2p_o + a_f) - h\theta^2. \tag{4.6}$$

The first order and second order derivatives of $\Pi^C(\theta)$ are:

$$\frac{\partial \Pi^C(\theta)}{\partial \theta} = (p_b - c)\tau_b + (p_o - c)(\tau_f + \tau_o) - 2h\theta,$$

$$\frac{\partial^2 \Pi^C(\theta)}{\partial \theta^2} = -2h < 0.$$

Thus $\Pi^C(\theta)$ is concave in θ and we can obtain the optimal θ as follows:

$$\theta^{C*} = \frac{(p_b - c)\tau_b + (p_o - c)(\tau_f + \tau_o)}{2h}. \tag{4.7}$$

Plugging equation (4.7) into equation (4.6), we get the following close-form solution for Π^{C*}:

$$\begin{aligned}\Pi^{C*} =& (p_b - c)(a_b - p_b) + (p_o - c)(a_o - 2p_o + a_f) \\ &+ \frac{((p_b - c)\tau_b + (p_o - c)(\tau_f + \tau_o))^2}{4h}.\end{aligned} \tag{4.8}$$

Equation (4.7) defines the optimal sales effort that maximizes the system profit to the level of Equation (4.8). These two equations also represent the coordinated sales effort and system profit, and they are also the upper bounds of the optimal solutions of the decentralized models.

4.1.4 The selective rebate contract with price match

In this contract, the brick-and-mortar retailer matches the online channel's price if a customer shows the proof of the lower price. At the end of the selling season, the manufacturer who owns the online channel (the joint venture hereafter) offers a compensation rebate to the brick-and-mortar retailer for each sale under price match. The manufacturer decides the wholesale price (w_s) and rebate (u) to maximize his profit.

The brick-and-mortar retailer's profit

Because of the price match policy, the free-riding customers purchase in the brick-and-mortar store. The brick-and-mortar retailer's profit is:

$$\begin{aligned} \Pi_b^S(\theta) &= (p_b - w_s)(a_b + \tau_b\theta - p_b) \\ &\quad + (p_o - w_s + u)(a_f + \tau_f\theta - p_o) - h\theta^2. \end{aligned} \tag{4.9}$$

The first order and second order derivatives of $\Pi_b^S(\theta)$ are:

$$\frac{\partial \Pi_b^S(\theta)}{\partial \theta} = (p_b - w_s)\tau_b + (p_o - w_s + u)\tau_f - 2h\theta,$$

$$\frac{\partial^2 \Pi_b^S(\theta)}{\partial \theta^2} = -2h < 0.$$

Thus $\Pi_b^S(\theta)$ is concave in θ and we can obtain the optimal θ as follows:

$$\theta^* = \frac{(p_b - w_s)\tau_b + (p_o - w_s + u)\tau_f}{2h}. \tag{4.10}$$

Equation (4.10) shows that the brick-and-mortar retailer's optimal sales effort is independent of the initial market size of the product. With the same sales effort level, the retailer gains more demand by investing in a product of a larger initial market size.

The online retailer and manufacturer joint venture's profit

Since there is price match, the free-riding customers purchase in the brick-and-mortar stores. The joint venture's profit is:

$$\begin{aligned}\Pi_j^S(w_s,u) =& (p_o-c)(a_o+\tau_o\theta-p_o)+(w_s-c-u)(a_f+\tau_f\theta-p_o)\\ &+(w_s-c)(a_b+\tau_b\theta-p_b).\end{aligned} \tag{4.11}$$

After substituting θ with equation (4.10), equation (4.11) can be rewritten as:

$$\begin{aligned}\Pi_j^S(w_s,u) &= (p_o-w_s)(a_o-p_o)\\ &+(w_s-c-u)(a_f+\frac{(p_b-w_s)\tau_b(\tau_f+\tau_o)+(p_o-w_s+u)\tau_f^2}{2h}-p_o)\\ &+(w_s-c)(a_b+\frac{(p_b-w_s)\tau_b^2+(p_o-w_s+u)\tau_b(\tau_f+\tau_o)}{2h}-p_b+a_o-p_o).\end{aligned} \tag{4.12}$$

The system profit of the supply chain in terms of w and u is:

$$\begin{aligned}\Pi_j^S(w_s,u)+\Pi_b^S(w_s,u) =& (w_s-c-u)(a_f+\frac{(p_b-w_s)\tau_b(\tau_f+\tau_o)+(p_o-w_s+u)\tau_f^2}{2h}-p_o)\\ &+(w_s-c)(a_b+\frac{(p_b-w_s)\tau_b^2+(p_o-w_s+u)\tau_b(\tau_f+\tau_o)}{2h}-p_b+a_o)\\ &+(p_o-w_s)(a_o-p_o)\\ &+(p_b-w_s)(a_b+\frac{(p_b-w_s)\tau_b^2+(p_o-w_s+u)\tau_b(\tau_f+\tau_o)}{2h}-p_b)\\ &+(p_o-w_s+u)(a_f+\frac{(p_b-w_s)\tau_b\tau_f+(p_o-w_s+u)\tau_f^2}{2h}-p_o)\\ &-h(\frac{(p_b-w_s)\tau_b+(p_o-w_s+u)\tau_f}{2h})^2.\end{aligned} \tag{4.13}$$

Equation (4.13) doesn't reveal any information about the optimal decisions for the manufacturer, rather, it shows how the manufacturer's decisions affect the system profit. We will design a coordinating decision formula based on Equation (4.13) in the following section.

Supply chain coordination

In this section, we design the contract to achieve supply chain coordination. The supply chain achieves the maximum system profit, same as the centralized supply chain, but the individual player's profit is not guaranteed to be improved. We will investigate the solution regime under Pareto improvement in a later section.

Here we introduce variable λ. Define $u = \frac{\tau_b+\tau_f+\tau_o}{\tau_f+\tau_o}\lambda$ and $w_s = \lambda + c$. Note that by such design, we actually construct $u = \frac{\tau_b+\tau_f+\tau_o}{\tau_f+\tau_o}(w - c)$.

Theorem 4.1 *The selective rebate with price match contract achieves supply chain coordination when* $u = \frac{\tau_b+\tau_f+\tau_o}{\tau_f+\tau_o}(w_s - c)$. *It obtains the same system profit as the centralized supply chain.*

Proof. By substituting $u = \frac{\tau_b+\tau_f+\tau_o}{\tau_f+\tau_o}\lambda$ and $w_s = \lambda + c$ into equation (4.13), we can rewrite (4.13) as follows:

$$\begin{aligned}
\Pi_j^S(\lambda) + \Pi_b^S(\lambda) =& -\frac{\tau_b}{\tau_f+\tau_o}\lambda\big(a_f + \frac{(p_b-c)\tau_b + (p_o-c)\tau_f}{2h}\tau_f - p_o\big) \\
&+ \lambda\big(a_b + \frac{(p_b-c)\tau_b + (p_o-c)\tau_f}{2h}\tau_b - p_b + a_o - p_o\big) \\
&+ (p_o - c - \lambda)(a_o - p_o) \\
&+ (p_b - c - \lambda)\big(a_b + \frac{(p_b-c)\tau_b + (p_o-c)\tau_f}{2h}\tau_b - p_b\big) \\
&+ (p_o - c + \frac{\tau_b}{\tau_f}\lambda)\big(a_f + \frac{(p_b-c)\tau_b + (p_o-c)\tau_f}{2h}\tau_f - p_o\big) \\
&- \frac{((p_b-c)\tau_b + (p_o-c)\tau_f)^2}{4h} \\
=&(p_b - c)(a_b - p_b) + (p_o - c)(a_o - 2p_o + a_f) \\
&+ \frac{((p_b-c)\tau_b + (p_o-c)(\tau_f+\tau_o))^2}{4h}.
\end{aligned} \tag{4.14}$$

Thus equation (4.14) equals (4.8). ■

Theorem 4.1 shows that the manufacturer's optimal rebate only depends on the manufacturer's margin $(w_s - c)$ and the sales effort-demand sensitivities (τ_b, τ_f, τ_o).

It is independent of the market sizes, retail prices, and price differences. Particularly, if the free-riding and online only customers are very responsive to the brick-and-mortar retailer's sales effort, the manufacturer can reduce his incentive to achieve optimality. This implies that the manufacturer should consider the quality of the sales effort while considering offering the selective rebate contract with price match. The manufacturer benefits more by identifying the brick-and-mortar retailers whose sales efforts are more responsive among the free-riding and online only customers.

Arbitrary split of the system profit

Arbitrary allocation of system profit is a property shared by several efficient supply chain contracts, e.g., buy back contract and revenue sharing contract. In this section, we show that the selective rebate with price match contract also possesses a similar property, though each player has a reserved level of profit share.

Theorem 4.2 *Under selective rebate with price match contract, the system profit can be arbitrarily split by varying λ among the supply chain players. Especially, the joint venture attains his highest profit when:*

$$\lambda = \frac{(p_b - c)(a_b - p_b) + (p_o - c)(a_f - p_o) + \frac{((p_b-c)\tau_b+(p_o-c)(\tau_f+\tau_o))^2}{4h}}{a_b - p_b + p_o - a_f + (\tau_b - \tau_f)\frac{(p_b-c)\tau_b+(p_o-c)(\tau_f+\tau_o)}{2h}}. \tag{4.15}$$

Proof. We now transform the optimal decisions in terms of λ. Equation (4.9) can be changed to:

$$\begin{aligned}\Pi_b^{S*}(\lambda) =&\Pi^{C*} - (p_o - c)(a_o - p_o) \\ &- \lambda\left(a_b - p_b - a_f + p_o + (\tau_b - \tau_f)\frac{(p_b - c)\tau_b + (p_o - c)(\tau_f + \tau_o)}{2h}\right),\end{aligned} \tag{4.16}$$

where Π^{C*} is the centralized system profit as equation (4.8).

Equation (4.11) can be changed to:

$$\begin{aligned}\Pi_j^{S*}(\lambda) =&(p_o - c)(a_o - p_o) \\ &+ \lambda\left(a_b - p_b + p_o - a_f + (\tau_b - \tau_f)\frac{(p_b - c)\tau_b + (p_o - c)\tau_f}{2h}\right).\end{aligned} \tag{4.17}$$

Equations (4.16) and (4.17) indicate that there is a Pareto-optimal frontier in the optimal decision regime, determined by the intermediate decision variable λ through $u = \frac{\tau_b+\tau_f+\tau_o}{\tau_f+\tau_o}\lambda$ and $w_s = \lambda + c$. On the Pareto-optimal frontier, the solutions are Pareto-optimal, which means we cannot find a solution to improve either player's profit without undermining the other's. Though the system profit $(\Pi_b^{S*}(\lambda) + \Pi_j^{S*}(\lambda))$ has been maximized and equates Π^{C*}, the individual player's profit share is variable depending on λ.

From assumption $a_b - p_b > a_f - p_o$, we know that $a_b - p_b - a_f + p_o > 0$, thus in equation (4.16), the brick-and-mortar retailer's profit is a decreasing function of λ. Obviously, the joint venture's profit is increasing with λ as shown in (4.17). Equation (4.15) is obtained by equating (4.16) to zero. Notice that the nominator and denominator of equation (4.15) are both positive, thus there always exists a value of λ that makes the brick-and-mortar retailer obtain zero share of the system profit. ■

Pareto improvement

Though we have shown that the selective rebate contract can achieve arbitrary profit division and Pareto optimality, it is interesting to find out under what condition the selective rebate contract can improve the profitability for both the manufacturer joint venture and the brick-and-mortar retailer.

Naturally, after shifting the free-riding customers' demand from the online channel to the brick-and-mortar retailer, the brick-and-mortar retailer's profit will always be higher in the selective rebate contract than in the baseline case. The intriguing question is, under what condition, the joint venture will be better off.

Theorem 4.3 *The joint venture's profitability is affected by the cost coefficient of sales effort h,*

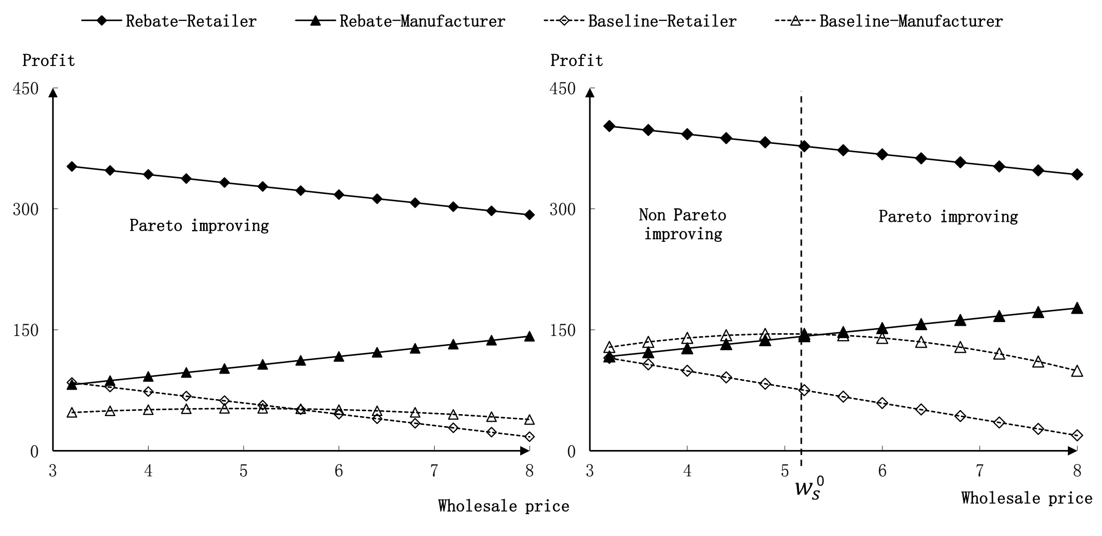
- *When $h > \frac{\tau_f(c\tau_b - p_b\tau_b + c\tau_f - p_o\tau_f)}{-2(a_f - p_o)}$, the joint venture is always better off in the selective rebate than in the baseline;*
- *When $h \leq \frac{\tau_f(c\tau_b - p_b\tau_b + c\tau_f - p_o\tau_f)}{-2(a_f - p_o)}$, if $p_b - p_o \leq \frac{2h\tau_b\tau_f}{a_f - p_o}$, there exists a threshold wholesale price $w_s^0 = p_b - \frac{2h(\tau_b\tau_f)}{a_f - p_o}$, such that the manufacturer joint venture's profit is higher with the selective rebate contract than in the baseline case when $w_s^* \geq w_s^0$; otherwise the manufacturer is always worse off.*

Proof. The difference between the joint venture's profit under the selective rebate and the baseline case is:

$$\begin{aligned}
\Pi_{diff} = &- (a_o - p_o)(-w_s + p_o) + (-w_s + p_o)\left(a_f + a_o - 2p_o - \frac{(w_s - p_b)\tau_b\tau_f}{2h}\right) \\
&+ (-c + w_s)\left(a_b + a_f + a_o - p_b - 2p_o - \frac{(w_s - p_b)\tau_b^2}{2h} - \frac{(w_s - p_b)\tau_b\tau_f}{2h}\right) \\
&- (-c + w_s)(a_b + a_o - p_b - p_o) \\
&- (-c + w_s)\left(\frac{\tau_b(-w_s\tau_b + p_b\tau_b - w_s\tau_f + p_o\tau_f + (-c + w_s)(\tau_b + \tau_f))}{2h}\right) \\
&- \left(-c + w_s - \frac{(-c + w_s)(\tau_b + \tau_f)}{\tau_f}\right)(a_f - p_o) \\
&- (-c + w_s)\left(\frac{\tau_f(-w_s\tau_b + p_b\tau_b - w_s\tau_f + p_o\tau_f + (-c + w_s)(\tau_b + \tau_f))}{2h}\right)
\end{aligned} \tag{4.18}$$

Set the joint venture's profit under the selective rebate contract and the baseline case equal, with condition $w_s^* \geq c$, then we have $w_s^0 = p_b - \frac{2ha_f - 2hp_o}{\tau_b\tau_f}$. Notice that $w_s^0 < p_b$. When $p_b - p_o \leq \frac{2h\tau_b\tau_f}{a_f - p_o}$, w_s^0 is also smaller than p_o. ■

Theorem 4.3 shows that when $p_b - p_o \leq \frac{2h\tau_b\tau_f}{a_f - p_o}$, there always exists a solution regime



(a) h=3

(b) h=1.5

Figure 4.1: The profit share with $c = 3, a_b = 30, a_f = 11, a_o = 26, p_b = 10, p_o = 8, \tau_b = 3, \tau_f = 1$, under selective rebate.

that guarantees at least one of the player's profit is improved from the baseline case, and none of the players is worse off. Intuitively, the wider the gap of $p_b - p_o$, the harder it is to find Pareto-improving solutions. This can be easily understood as follows: in order to stimulate the brick-and-mortar retailer, the manufacturer needs to provide partial compensation for the price difference $p_b - p_o$. The larger $p_b - p_o$ implies a larger profit transfer from the manufacturer to the brick-and-mortar retailer, thus it is harder to find a solution that doesn't undermine the manufacturer's profit share.

Among the parameters that affect the Pareto improvement of the supply chain players' profitability, the cost coefficient of sales effort (h) dominates the trend. Figure 4.1 shows the Pareto improvement scenario when $c = 3, a_b = 30, a_f = 11, a_o = 26, p_b = 10, p_o = 8, \tau_b = 3, \tau_f = 1$, with $h = 3$ in Figure 4.1(a) and $h = 1.5$ in Figure 4.1(b). Under this setting, the manufacturer joint venture's profit is not always higher in the selective rebate than in the baseline case. The Pareto-improving solution regime is $w_s^* \geq w_s^0$. Figure 4.1 shows that when h increases from 1.5 to 3, the manufacturer joint venture's profit in the baseline case moves downwards and w_s^0 becomes smaller.

Intuitively, when h increases, it is more costly for the brick-and-mortar retailer to exert sales efforts, thus the market demand and the manufacturer join-venturer's profit become smaller. However, with selective rebate, the manufacturer joint venture's profit decreases much slower than in the baseline case. When h increases, it is easier for the manufacturer joint venturer to achieve Pareto improvement by using selective rebate, thus the Pareto-improving regime becomes larger. In Figure 4.1, we can also see that the summation of the retailer and manufacturer's profits under the selective rebate is constant, an indication of Pareto-optimality.

4.1.5 The revenue sharing contract with price match

Under the revenue-sharing contract, a retailer pays a supplier a wholesale price for each unit purchased, plus a portion of the revenue the retailer generates. The revenue sharing contract has become more prevalent in the videocassette rental industry and among airline alliances relative to the traditional wholesale price contract.

The model

In a supply chain consisted of one brick-and-mortar retailer and one manufacturer owning an online retail channel, transactions between the retailer and manufacturer are governed by a revenue sharing contract. This contract contains two decision terms, μ and w_r. μ is the share of retail revenue the manufacturer receives, i.e., given retail revenues Π_b, the retailer must transfer $\mu\Pi_b$ to the manufacturer but retains the remaining $(1-\mu)\Pi_b$. It is natural to assume $\mu \in [0, 1]$, even though that restriction is not strictly required. We do not include the administrative costs associated with monitoring revenues and collecting transfers. In other words, we assume the cost of implementation has no impact on the contract the supplier oers or the quantity the retailer purchases. (Implementation costs, of course, may impact whether revenue sharing is adopted at all.) w_r is the wholesale price. Note that a standard wholesale-price contract is a revenue-sharing contract with $\mu = 0$.

The brick-and-mortar retailer's profit is:

$$\begin{aligned}\Pi_b^R(\theta) = &((1-\mu)p_b - w_r)(a_b + \tau_b\theta - p_b) \\ &+ ((1-\mu)p_o - w_r)(a_f + \tau_f\theta - p_o) - h\theta^2\end{aligned} \tag{4.19}$$

The approach to obtain the optimal decisions based on the first and second order derivatives is the same as in §4.1.4, thus we omit it.

$$\theta^*(w_r, \mu) = \frac{-w_r\tau_b + p_b\tau_b - \mu p_b\tau_b - w_r\tau_f + p_o\tau_f - \mu p_o\tau_f}{2h} \tag{4.20}$$

The joint venture's profit is:

$$\begin{aligned}\Pi_j^R(w_r, \mu) = &(w_r - c)(a_o + \tau_o\theta - p_o) + (w_r - c + \mu p_o)(a_f + \tau_f\theta - p_o) \\ &+ (w_r - c + \mu p_b)(a_b + \tau_b\theta - p_b)\end{aligned} \tag{4.21}$$

Equivalence to the selective rebate contract

In this section, we show that revenue sharing and selective rebate contracts are equivalent. For any selective contract there exists a revenue sharing contract that generates the same cash flows between the manufacturer and the brick-and-mortar retailer.

In the selective rebate contract, the brick-and-mortar retailer pays $w_s - u$ (w_s is the wholesale price in the selective rebate contract) for each unit sold to the free-riding customers under price match, and w_s for each unit sold to the traditional customers. In the revenue sharing contract, the brick-and-mortar retailer pays $w_r + \mu p_o$ (w_r is the wholesale price in the revenue sharing contract) for each unit sold to the free-riding customers under price match, and $w_r + \mu p_b$ for each unit sold to the traditional customers.

If the transactions from the brick-and-mortar retailer to the manufacturer are the same in both scenarios, that is:

$$\begin{aligned} w_s - u &= w_r + rp_o, && (4.22) \\ w_r + rp_b &= w_s, && (4.23) \end{aligned}$$

the two contracts result in the same profits for the retailer and the manufacturer for any combination of free riding and traditional customers' demand.

Theorem 4.4 *The revenue sharing contract with price match coordinates the supply chain with* $r = \frac{(\tau_b+\tau_f+\tau_o)}{\tau_f(p_b-p_o)}\lambda$ *and* $w_r = \lambda + c - \frac{(\tau_b+\tau_f+\tau_o)p_b}{\tau_f(p_b-p_o)}\lambda$.

Proof. Solving equations (4.22) and (4.23), we get $w_r = w_s - \frac{p_b u}{p_b - p_o}$. Since the selective rebate contract with price match coordinates the supply chain with $u = \frac{\tau_b+\tau_f+\tau_o}{\tau_f+\tau_o}\lambda$ and $w_s = \lambda + c$, the revenue sharing contract with price match coordinates the supply chain with $w_r = \lambda + c - \frac{(\tau_b+\tau_f+\tau_o)p_b}{\tau_f(p_b-p_o)}\lambda$ and $r = \frac{(\tau_b+\tau_f+\tau_o)}{\tau_f(p_b-p_o)}\lambda$. ■

Notice that $w_r - c = \lambda(1 - \frac{(\tau_b+\tau_f)p_b}{\tau_f(p_b-p_o)})$ and $1 - \frac{(\tau_b+\tau_f)p_b}{\tau_f(p_b-p_o)} < 0$. Considering the revenue sharing rate $\mu \geq 0$, we observe that $w_r \leq c$, which means that in the coordinating revenue sharing contract, the wholesale price should not be higher than the production cost. The manufacturer shares the cost of sales effort with the brick-and-mortar retailer. The manufacturer's revenue comes from the brick-and-mortar retailer's sales revenue, and thus is directly affected by the retailer's sales effort. In addition, the manufacturer shares the risk of demand uncertainty with the brick-and-mortar retailer.

4.1.6 The target rebate contract with price match

In this contract, the manufacturer decides wholesale price (w) and target rebate (r) to maximize his profit. The rebate is given to the brick-and-mortar retailer based on its overall sales beyond the target level T, namely $a_b + \tau_b\theta - p_b + a_f + \tau_f\theta - p_o - T$.

The brick-and-mortar retailer's profit

Because of the price match policy, the free-riding customers purchase in the brick-and-mortar store. The brick-and-mortar retailer's profit is hence changed to:

$$\begin{aligned}\Pi_b^T(\theta|T) =& (p_b - w)(a_b + \tau_b\theta - p_b) + (p_o - w)(a_f + \tau_f\theta - p_o) \\ &+ r(a_b + \tau_b\theta - p_b + a_f + \tau_f\theta - p_o - T)^+ - h\theta^2.\end{aligned} \tag{4.24}$$

Then the retailer's profit can be written in two cases as follows:

$$\Pi_b^T(\theta|T) = (p_b-w)(a_b+\tau_b\theta-p_b)+(p_o-w)(a_f+\tau_f\theta-p_o)+r(a_b+\tau_b\theta-p_b+a_f+\tau_f\theta-p_o-T)-h\theta^2,$$

when $a_b + \tau_b\theta - p_b + a_f + \tau_f\theta - p_o \geq T$.

$$\Pi_b^T(\theta|T) = (p_b - w)(a_b + \tau_b\theta - p_b) + (p_o - w)(a_f + \tau_f\theta - p_o) - h\theta^2,$$

when $a_b + \tau_b\theta - p_b + a_f + \tau_f\theta - p_o < T$.

For the case when $a_b + \tau_b\theta - p_b + a_f + \tau_f\theta - p_o < T$, the problem becomes trivial and thus we omit it.

For the case when $a_b + \tau_b\theta - p_b + a_f + \tau_f\theta - p_o \geq T$, the first order and second order derivatives of $\Pi_b^S(\theta)$ are:

$$\frac{\partial \Pi_b^T(\theta)}{\partial \theta} = (p_b - w + r)\tau_b + (p_o - w + r)\tau_f - 2h\theta,$$

$$\frac{\partial^2 \Pi_b^T(\theta)}{\partial \theta^2} = -2h < 0.$$

Thus $\Pi_b^T(\theta)$ is concave in θ and we can obtain the optimal θ as follows:

$$\theta^* = \frac{(p_b - w + r)\tau_b + (p_o - w + r)\tau_f}{2h}. \tag{4.25}$$

The brick-and-mortar retailer's profit function can be transformed into:

$$\begin{aligned}\Pi_b^T(w,r,T) =& \frac{1}{4h}(-4hTr + 4hra_b - 4hwa_b + 4hra_f - 4hwa_f - 4hrp_b \\ & + 4hwp_b + 4ha_bp_b - 4hp_b^2 - 4hrp_o + 4hwp_o + 4ha_fp_o - 4hp_o^2 \\ & + r^2\tau_b^2 - 2rw\tau_b^2 + w^2\tau_b^2 + 2rp_b\tau_b^2 - 2wp_b\tau_b^2 \\ & + p_b^2\tau_b^2 + 2r^2\tau_b\tau_f - 4rw\tau_b\tau_f + 2w^2\tau_b\tau_f \\ & + 2rp_b\tau_b\tau_f - 2wp_b\tau_b\tau_f + 2rp_o\tau_b\tau_f - 2wp_o\tau_b\tau_f \\ & + 2p_bp_o\tau_b\tau_f + r^2\tau_f^2 - 2rw\tau_f^2 + w^2\tau_f^2 + 2rp_o\tau_f^2 - 2wp_o\tau_f^2 \\ & + p_o^2\tau_f^2).\end{aligned} \tag{4.26}$$

The manufacturer's profit (joint venture)

The manufacturer's decisions are the wholesale price and the rebate. His profit is:

$$\begin{aligned}\Pi_j^T(w,r,T) =& (w-c-r)(a_f+\tau_f\theta-p_o+a_b+\tau_b\theta-p_b-T) \\ & + (p_o-c)(a_o-p_o).\end{aligned} \tag{4.27}$$

After substituting θ with equation (4.25), equation (4.27) can be rewritten as:

$$\begin{aligned}\Pi_j^T(w,r,T) = & (a_o-p_o)(-w+p_o) + (-c-r+w)(-T+a_b+a_f-p_b-p_o \\ & + \frac{\tau_b(r\tau_b - w\tau_b + p_b\tau_b + r\tau_f - w\tau_f + p_o\tau_f)}{2h} \\ & + \frac{\tau_f(r\tau_b - w\tau_b + p_b\tau_b + r\tau_f - w\tau_f + p_o\tau_f)}{2h}).\end{aligned} \tag{4.28}$$

The system profit in the supply chain in terms of w, r and T is:

$$
\begin{aligned}
\Pi_j^T + \Pi_b^T = -\frac{1}{4h}(&-4chT + 4hTw + 4hwa_o + 4ha_b\left(c - p_b\right) - 4chp_b + 4hp_b^2 \\
&+ 4ha_f\left(c - p_o\right) - 4chp_o - 4hwp_o - 4ha_op_o \\
&+ 8hp_o^2 + 2cr\tau_b^2 + r^2\tau_b^2 - 2cw\tau_b^2 \\
&- 2rw\tau_b^2 + w^2\tau_b^2 + 2cp_b\tau_b^2 - p_b^2\tau_b^2 + 4cr\tau_b\tau_f + 2r^2\tau_b\tau_f \\
&- 4cw\tau_b\tau_f - 4rw\tau_b\tau_f + 2w^2\tau_b\tau_f + 2cp_b\tau_b\tau_f + 2cp_o\tau_b\tau_f \\
&- 2p_bp_o\tau_b\tau_f + 2cr\tau_f^2 + r^2\tau_f^2 - 2cw\tau_f^2 - 2rw\tau_f^2 + w^2\tau_f^2 + 2cp_o\tau_f^2 - p_o^2\tau_f^2)
\end{aligned}
\tag{4.29}
$$

Coordinating decisions

We hereby define $r = \frac{2hT+p_b\tau_b+p_o\tau_f}{\tau_f+\tau_b}$ and $w = \frac{p_b\tau_b+p_o\tau_f+2hT}{\tau_f+\tau_b} + c$.

Theorem 4.5 *The target rebate with price match contract achieves supply chain coordination. It obtains the same system profit as the centralized supply chain.*

Proof. We obtain the coordinating formula by solving the following equations:

$$
\begin{aligned}
&-T + a_b + a_f - p_b - p_o + \frac{\tau_b\left(r\tau_b - w\tau_b + p_b\tau_b + r\tau_f - w\tau_f + p_o\tau_f\right)}{2h} \\
&= \frac{\tau_f\left(r\tau_b - w\tau_b + p_b\tau_b + r\tau_f - w\tau_f + p_o\tau_f\right)}{2h},
\end{aligned}
\tag{4.30}
$$

and

$$
\frac{r\tau_b - w\tau_b + p_b\tau_b + r\tau_f - w\tau_f + p_o\tau_f}{2h} = \frac{-c\tau_b + p_b\tau_b - c\tau_f + p_o\tau_f}{2h}. \tag{4.31}
$$

Equation (4.30) is the first order condition of optimal T derived from the manufacturer's profit function, and equation (4.31) is the condition for sales effort coordination. We have two equations and three variables, so the solutions are functions of T, $r = \frac{2hT+p_b\tau_b+p_o\tau_f}{\tau_f+\tau_b}$ and $w = \frac{p_b\tau_b+p_o\tau_f+2hT}{\tau_f+\tau_b} + c$.

By substituting $r = \frac{2hT+p_b\tau_b+p_o\tau_f}{\tau_f+\tau_b}$ and $w = \frac{p_b\tau_b+p_o\tau_f+2hT}{\tau_f+\tau_b} + c$ into equation (4.29), we can rewrite (4.29) as follows:

$$
\begin{aligned}
\Pi_j^T(T) + \Pi_b^T(T) =& -\frac{\tau_b}{\tau_f}T(a_f + \frac{(p_b - c)\tau_b + (p_o - c)\tau_f}{2h}\tau_f - p_o) \\
& + T(a_b + \frac{(p_b - c)\tau_b + (p_o - c)\tau_f}{2h}\tau_b - p_b + a_o - p_o) \\
& + (p_o - c - T)(a_o - p_o) \\
& + (p_b - c - T)(a_b + \frac{(p_b - c)\tau_b + (p_o - c)\tau_f}{2h}\tau_b - p_b) \\
& + (p_o - c + \frac{\tau_b}{\tau_f}T)(a_f + \frac{(p_b - c)\tau_b + (p_o - c)\tau_f}{2h}\tau_f - p_o) \\
& - \frac{((p_b - c)\tau_b + (p_o - c)\tau_f)^2}{4h} \\
=&(p_b - c)(a_b - p_b) + (p_o - c)(a_o - 2p_o + a_f) \\
& + \frac{((p_b - c)\tau_b + (p_o - c)\tau_f)^2}{4h}.
\end{aligned}
\tag{4.32}
$$

Thus equation (4.32) equals (4.8), the system profit in the centralized supply chain. ■

We can transform other optimal results in term of T.

Equation (4.26) can be changed to:

$$
\begin{aligned}
\Pi_b^T(T) =&\Pi^{C*} - (p_o - c)(a_o - p_o) \\
& - (T - a_b + p_b - a_o + a_f - \frac{p_b\tau_b + p_o\tau_f}{\tau_b + \tau_f})\frac{(p_b - c)\tau_b + (p_o - c)\tau_f}{2h},
\end{aligned}
\tag{4.33}
$$

where Π^{C*} is the centralized system profit as equation (4.8).

Equation (4.28) can be changed to:

$$
\Pi_j^T(T) = (T - a_b + p_b - a_o + a_f - \frac{p_b\tau_b + p_o\tau_f}{\tau_b + \tau_f})\frac{(p_b - c)\tau_b + (p_o - c)\tau_f}{2h}. \tag{4.34}
$$

Voluntary compliance

Voluntary compliance requires the profits of all players are nonnegative.

Theorem 4.6 *The target rebate with price match contract satisfies voluntary compliance when* $T \leq \frac{2h(\Pi^{C*}-(p_o-c)(a_o-p_o))}{(p_b-c)\tau_b+(p_o-c)\tau_f} + a_b - p_b + a_o - a_f + \frac{p_b\tau_b+p_o\tau_f}{\tau_b+\tau_f}$ *and* $T \geq a_b - p_b + a_o - a_f + \frac{p_b\tau_b+p_o\tau_f}{\tau_b+\tau_f}$.

Proof. First, from equation (4.33), we can see $\Pi_b^{S*}(T)$ is positive when

$$T \leq \frac{2h(\Pi^{C*} - (p_o - c)(a_o - p_o))}{(p_b - c)\tau_b + (p_o - c)\tau_f} + a_b - p_b + a_o - a_f + \frac{p_b\tau_b + p_o\tau_f}{\tau_b + \tau_f}.$$

When $T \geq a_b - p_b + a_o - a_f + \frac{p_b\tau_b+p_o\tau_f}{\tau_b+\tau_f}$, $\Pi_m^{S*}(T) \geq 0$. ■

4.2 Models with an Independent Online Channel

In this section, we discuss the supply chain with an independent online retailer. The manufacturer sells products through both retail channels with the same wholesale price. The reason to extend the previous model to include an independent online retail is twofold: 1) practically, many manufacturers sell their products through independent online retails, such as Amazon.com. By analyzing the behavior of the selective rebate contract in this extended scenario, we can increase the practical value of the selective rebate since it can be applied in a wider market context; 2) theoretically, no contracts have been revealed to coordinate a supply chain with channel conflicts on sales effort free riding. In this section, we show that the selective rebate contract is the first contract to achieve the aforementioned property.

4.2.1 The selective rebate with price match contract

The model

The manufacturer decides wholesale price (w_s) and target rebate (u) to maximize his profit. The brick-and-mortar retailer's profit is identical to that in §4.1.4 thus omitted.

Since there is price match, the free-riding customers purchase in the brick-and-mortar stores. The online retailer's profit becomes:

$$\Pi_o^S = (p_o - w_s)(a_o + \tau_o\theta - p_o). \tag{4.35}$$

The manufacturer's decisions are the wholesale price and the rebate. His profit is:

$$\Pi_m^S(w_s, u) = (w_s - c - u)(a_f + \tau_f\theta - p_o) + (w_s - c)(a_b + \tau_b\theta - p_b + a_o - p_o). \tag{4.36}$$

Supply chain coordination

We introduce variable λ. Define $u = \frac{\tau_b+\tau_f+\tau_o}{\tau_f+\tau_o}\lambda$ and $w_s = \lambda + c$. Note that by such design, we actually construct $u = \frac{\tau_b+\tau_f+\tau_o}{\tau_f+\tau_o}(w - c)$.

Theorem 4.7 *The selective rebate with price match contract achieves supply chain coordination. It obtains the same system profit as the centralized supply chain.*

Proof. The proof is similar to that of Theorem 1, thus omitted. ■

The brick-and-mortar retailer's profit as a function of λ is:

$$\begin{aligned}\Pi_b^{S*}(\lambda) =&\Pi^{C*} - (p_o - c)(a_o - p_o)\\ &- \lambda(a_b - p_b - a_f + p_o + (\tau_b - (\tau_f + \tau_o))\frac{(p_b - c)\tau_b + (p_o - c)(\tau_f + \tau_o)}{2h}),\end{aligned} \tag{4.37}$$

where Π^{C*} is the centralized system profit as equation (4.8).

The online retailer's profit is:

$$\Pi_o^{S*}(\lambda) = (p_o - c)(a_o - p_o) - \lambda(a_o - p_o). \tag{4.38}$$

The manufacturer's profit is:

$$\Pi_m^{S*}(\lambda) = \lambda(a_b - p_b + a_o - a_f + (\tau_b - (\tau_f + \tau_o))\frac{(p_b - c)\tau_b + (p_o - c)(\tau_f + \tau_o)}{2h}). \quad (4.39)$$

Division of the system profit

Theorem 4.8 *Under selective rebate with price match contract, the system profit can be arbitrarily split by varying λ among the supply chain players. Especially, when $\lambda = p_o - c$, the manufacturer attains his highest profit.*

Proof. From assumption $a_b - p_b > a_f - p_o$, we know that $a_b - p_b - a_f + p_o > 0$, thus in equation (4.37), the brick-and-mortar retailer's profit is a decreasing function of λ. Obviously, the online retailer's profit is also decreasing with λ as shown in (4.38). Since we assume $a_o - p_o \geq a_f - p_o$, thus $a_o > a_f$, then the manufacturer's profit is increasing in λ as shown in (4.39). ■

In the selective rebate with price match contract, by increasing λ, the manufacturer increases w_s and u altogether and obtains a higher profit, based on equation (4.39).

Let's consider two borderline cases: $\lambda = 0$ and $\lambda = p_o - c$. When $\lambda = 0$, $u = 0$ and $w_s = c$, then we have $\Pi_b^{S*} = \Pi^{C*} - (p_o - c)(a_o - p_o)$, $\Pi_o^{S*} = (p_o - c)(a_o - p_o)$, $\Pi_m^{S*} = 0$. In this case, manufacturer sells at the marginal cost, eliminating double marginalization, and naturally the supply chain coordinates. But the manufacturer's share of system profit is 0. When $\lambda = p_o - c$, $u = \frac{\tau_b + \tau_f + \tau_o}{\tau_f + \tau_o}(p_o - c)$ and $w_s = p_o$, then we have $\Pi_b^{S*} = \Pi^{C*} - (p_o - c)(a_b - p_b + a_o - a_f + (\tau_b - \tau_f)\frac{(p_b - c)\tau_b + (p_o - c)\tau_f}{2h})$, $\Pi_o^{S*} = 0$, $\Pi_m^{S*} = (p_o - c)(a_b - p_b + a_o - a_f)$. We can see that the division of system profit is not as flexible as from 0 to 100%. Though the manufacturer and the online retailer could get a zero share of system profit (but not at the same time), the brick-and-mortar retailer is reserved for a minimum profit as $\Pi^{C*} - (p_o - c)(a_b - p_b + a_o - a_f + (\tau_b -$

$\tau_f)\frac{(p_b-c)\tau_b+(p_o-c)\tau_f}{2h})$, thanks to the sales to the traditional shoppers who always bring net profit to the brick-and-mortar retailer.

4.2.2 The revenue sharing contract with price match

This section considers a revenue sharing contract with an independent online retailer. The online retailer loses her free riding customers to the brick-and-mortar stores due to price match. The manufacturer doesn't directly compensate the online stores for her demand drain, but by coordinating wholesale price and sales effort to achieve so.

The model

The revenue sharing contract with price match contains two decision terms, μ and w_r. μ is the share of retail revenue the manufacturer receives, and w_r is the wholesale price.

The brick-and-mortar retailer's profit is identical to that in §4.1.5, thus omitted.

The online retailer's profit is:

$$\Pi_o^R = (p_o - w_r)(a_o + \tau_o\theta - p_o). \tag{4.40}$$

The manufacturer's profit is:

$$\begin{aligned}\Pi_m^R(w_r,\mu) =& (w_r - c + \mu p_o)\,(a_f + \tau_f\theta - p_o)\\ &+ (w_r - c + \mu p_b)\,(a_b + \tau_b\theta - p_b) + w_r(a_o + \tau_o\theta - p_o)\end{aligned} \tag{4.41}$$

Equivalence to the selective rebate contract

To get $w_s - u = w_r + \mu p_o$ and $w_r + \mu p_b = w_s$, we introduce $\mu = \frac{\tau_b+\tau_f+\tau_o}{\tau_f(p_b-p_o)}\lambda$ and $w_r = \lambda + c - \frac{(\tau_b+\tau_f+\tau_o)p_b}{\tau_f(p_b-p_o)}\lambda$ into $w_r + \mu p_o$ and $w_r + \mu p_b$, and $u = \frac{\tau_b+\tau_f+\tau_o}{\tau_f+\tau_o}\lambda$ and $w_s = \lambda + c$ into $w_s - u$ and w_s, thus the two contracts result in the same profits for the retailer and the manufacturer for any combination of free riding and traditional customers'demand.

By now, we have shown that the revenue sharing contract is equivalent to the selective rebate contract when the online channel is independent of the manufacturer. However, such equivalence only effects the coordinated sales effort and the system profit.

4.2.3 The target rebate contract with price match

In this contract, the manufacturer decides wholesale price (w) and target rebate (r) to maximize his profit. The rebate is given to the brick-and-mortar retailer based on its overall sales beyond the target level T, namely $a_b + \tau_b\theta - p_b + a_f + \tau_f\theta - p_o - T$. The independent online retailer orders to satisfy the demand from the online-only customers.

The retailers' profit functions

Because of the price match policy, the free-riding customers purchase in the brick-and-mortar store. The brick-and-mortar retailer's profit is:

$$\begin{aligned}\Pi_b^T(\theta|T) =&(p_b - w)(a_b + \tau_b\theta - p_b) + (p_o - w)(a_f + \tau_f\theta - p_o) \\ &+ r(a_b + \tau_b\theta - p_b + a_f + \tau_f\theta - p_o - T)^+ - h\theta^2.\end{aligned} \tag{4.42}$$

Notice that equation (4.42) is the same as equation (4.24) when the manufacturer owns the online channel. For simplicity, we omit the analysis process since it is the same as in §4.1.6, and only show the optimal sales effort below:

$$\theta^* = \frac{(p_b - w + r)\tau_b + (p_o - w + r)\tau_f}{2h}. \tag{4.43}$$

The online retailer's profit is:

$$\Pi_o^T = (p_o - w)(a_o + \tau_o\theta - p_o). \tag{4.44}$$

The manufacturer's profit

The manufacturer's decisions are the wholesale price, the rebate and target level. His profit is:

$$\begin{aligned}\Pi_m^T(w, r, T) =& (w - c - r)(a_f + \tau_f\theta - p_o + a_b + \tau_b\theta - p_b - T) \\ &+ (w - c)(a_o - p_o).\end{aligned} \tag{4.45}$$

The analysis process for the manufacturer's profit function is the same as in §4.1.6, thus omitted. We only show the system profit in the supply chain in terms of w, r and T:

$$\begin{aligned}\Pi_m^T + \Pi_o^T + \Pi_b^T =& -\frac{1}{4h}(-4chT + 4hTw + 4hwa_o + 4ha_b\,(c - p_b) \\ &+ 4ha_f\,(c - p_o) - 4chp_o - 4hwp_o - 4ha_op_o - 2cw\tau_b^2 \\ &- 2rw\tau_b^2 + w^2\tau_b^2 + 2cp_b\tau_b^2 - p_b^2\tau_b^2 + 2r^2\tau_b\tau_f \\ &- 4cw\tau_b\tau_f - 4rw\tau_b\tau_f + 2w^2\tau_b\tau_f + 2cp_b\tau_b\tau_f + 2cp_o\tau_b\tau_f \\ &- 2p_bp_o\tau_b\tau_f - 2cw\tau_f^2 - 2rw\tau_f^2 + w^2\tau_f^2 + 2cp_o\tau_f^2 - p_o^2\tau_f^2)\end{aligned} \tag{4.46}$$

Coordinating decisions

We hereby define $r = \frac{2hT + p_b\tau_b + p_o\tau_f}{\tau_f + \tau_b}$ and $w = \frac{p_b\tau_b + p_o\tau_f + hT}{\tau_f + \tau_b} + c$.

Theorem 4.9 *The target rebate with price match contract achieves supply chain coordination with an independent online retailer. It obtains the same system profit as the centralized supply chain.*

Proof. We obtain the coordinating formula by solving the following equations:

$$\begin{aligned}&-T+a_b-p_b+\frac{\tau_b\left(r\tau_b-w\tau_b+p_b\tau_b+r\tau_f-w\tau_f+p_o\tau_f\right)}{2h}\\&=\frac{\tau_f\left(r\tau_b-w\tau_b+p_b\tau_b+r\tau_f-w\tau_f+p_o\tau_f\right)}{2h},\end{aligned}\tag{4.47}$$

and

$$\frac{r\tau_b-w\tau_b+p_b\tau_b+r\tau_f-w\tau_f+p_o\tau_f}{2h}=\frac{-c\tau_b+p_b\tau_b-c\tau_f+p_o\tau_f}{2h}.\tag{4.48}$$

Equation (4.47) is the first order condition of optimal T derived from the manufacturer's profit function, and equation (4.48) is the condition for sales effort coordination. We have two equations and three variables, so the solutions are functions of T, $r=\frac{2hT+p_b\tau_b+p_o\tau_f}{\tau_f+\tau_b}$ and $w=\frac{p_b\tau_b+p_o\tau_f+hT}{\tau_f+\tau_b}+c$. Notice that equation (4.47) is slightly different from equation (4.30) in §4.1.6 because of the difference in the manufacturer's profit functions.

By substituting $r=\frac{2hT+p_b\tau_b+p_o\tau_f}{\tau_f+\tau_b}$ and $w=\frac{p_b\tau_b+p_o\tau_f+hT}{\tau_f+\tau_b}+c$ into equation (4.46), we can rewrite (4.46) as follows:

$$\begin{aligned}\Pi_m^T(T)+\Pi_b^T(T)+\Pi_o^T(T)=&-\frac{\tau_b}{\tau_f}T(a_f+\frac{(p_b-c)\tau_b+(p_o-c)\tau_f}{2h}\tau_f-p_o)\\&+T(a_b+\frac{(p_b-c)\tau_b+(p_o-c)\tau_f}{2h}\tau_b-p_b+a_o-p_o)\\&+(p_o-c-T)(a_o-p_o)\\&+(p_b-c-T)(a_b+\frac{(p_b-c)\tau_b+(p_o-c)\tau_f}{2h}\tau_b-p_b)\\&+(p_o-c+\frac{\tau_b}{\tau_f}T)(a_f+\frac{(p_b-c)\tau_b+(p_o-c)\tau_f}{2h}\tau_f-p_o)\\&-\frac{((p_b-c)\tau_b+(p_o-c)\tau_f)^2}{4h}\\=&(p_b-c)(a_b-p_b)+(p_o-c)(a_o-2p_o+a_f)\\&+\frac{((p_b-c)\tau_b+(p_o-c)\tau_f)^2}{4h}.\end{aligned}\tag{4.49}$$

Thus equation (4.49) equals (4.8), the system profit in the centralized supply chain. ■

Notice that the coordinating formula $w = \frac{p_b\tau_b + p_o\tau_f + hT}{\tau_f + \tau_b} + c$ is smaller than that in §4.1.6, because the manufacturer does not own the online channel. The manufacturer has to transfer profit to the retailers as a result of less market power over the retail channels, comparing with the case that the manufacturer owns the online channel.

Pareto improvement

Though we have shown that the target rebate contract can achieve supply chain coordination, it is interesting to find out under what condition the target rebate contract can improve the profitability for all of the supply chain players.

Naturally, after shifting the free-riding customers' demand from the online channel to the brick-and-mortar retailer, the brick-and-mortar retailer's profit will be higher under the target rebate with price match contract. The intriguing question is, under what condition, the manufacturer and online retailer will be better off.

Theorem 4.10 *There exists a range of T that achieves pareto improvement for all of the supply chain players.*

Proof. The difference between the manufacturer's profit under the target rebate and the baseline case is:

$$
\begin{aligned}
\Pi_{diff} = &- (a_o - p_o)(-T + p_o) + (-T + p_o)\left(a_f + a_o - 2p_o - \frac{(T - p_b)\tau_b\tau_f}{2h}\right) \\
&+ (a_b - p_b + T)\left(a_b + a_f + a_o - p_b - 2p_o - \frac{(T - p_b)\tau_b^2}{2h} - \frac{(T - p_b)\tau_b\tau_f}{2h}\right) \\
&- (a_b - p_b + T)(a_b + a_o - p_b - p_o) \\
&- (a_b - p_b + T)\left(\frac{\tau_b(-w\tau_b + p_b\tau_b - w\tau_f + p_o\tau_f + (-c + w)(\tau_b + \tau_f))}{2h}\right) \\
&- (-c + T)\left(\frac{\tau_f(-w\tau_b + p_b\tau_b - w\tau_f + p_o\tau_f + (-c + w)(\tau_b + \tau_f))}{2h}\right).
\end{aligned} \tag{4.50}
$$

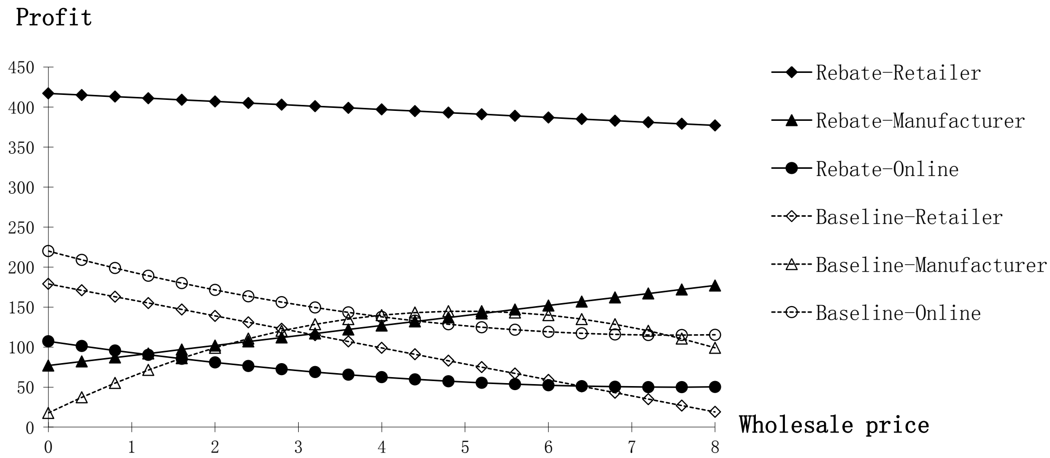


Figure 4.2: The profit share with $h = 1.4, c = 3, a_b = 30, a_f = 11, a_o = 26, p_b = 10, p_o = 8, \tau_b = 3, \tau_f = 1$, under target rebate

Let equation (4.50) equate 0, there are two solutions for T^*, $a_f + a_o - 2p_o + \frac{c\tau_b + c\tau_f}{2h}$ and $a_b + a_o + \frac{2ha_f - 2hp_o + p_b\tau_b\tau_f}{h}$. The larger one $a_b + a_o + \frac{2ha_f - 2hp_o + p_b\tau_b\tau_f}{h}$ is defined as T_x.

The difference between the online retailer's profit under the target rebate and the baseline case is:

$$\Pi_{diff} = \frac{(a_b + a_f + a_o - T)\tau_b}{2h} - \frac{(p_b - p_o)\tau_b + (p_o - c)\tau_f}{2h} \tag{4.51}$$

So when $T \leq a_b + a_f + a_o + \frac{(p_b - p_o)\tau_b + (p_o - c)\tau_f}{\tau_b}$, $\Pi_{diff} \geq 0$, and the online retailer is better off under the target rebate contract. ■

The following numerical analysis aims to further elaborate the pareto-improving domain. The standard setting for the parameters in the numerical analysis is as follows: $c = 2, a_b = 30, a_f = 11, a_o = 26, p_b = 10, p_o = 8, \tau_b = 3, \tau_f = 1$.

Figure 4.2 shows the pareto improvement scenario when $h = 1.4$. The x axis is w other than T because the baseline does not have T. The T values in the rebate is converted to w values using the coordinating formula. There is a short range of w values that achieves pareto improvement for all of the three players. The brick-and-mortar retailer's profit is always better in the target rebate. When $w \geq 5.5$, the manufacturer's profit is better off in the target rebate.

4.3 Summary of Chapter 4

This chapter examines the effectiveness of selective rebate contract with price match in coordinating a supply chain with retail channel conflicts caused by sales effort free riding, under deterministic demand. The price match policy diverts the demand of the free-riding customers from the online channel to the brick-and-mortar retailer. By doing so, the brick-and-mortar retailer's sales effort is rewarded with the free-riding customer's demand. In addition, the manufacturer provides partial compensation to the brick-and-mortar retailer to offset her loss due to price match. Such selective rebate boosts the brick-and-mortar retailer's order quantity and sales effort, achieves supply chain coordination and maximum system profits, and thus increase the supply chain efficiency.

In addition, revenue sharing contract with price match is also studied in this chapter. The deep root of equivalence between the selective rebate and revenue sharing contract with price match explains why both contracts can coordinate the supply chain with the manufacturer owning the online channel, and arbitrarily split the system profit. For any selective contract there exists a revenue sharing contract that generates the same cash flows. However, if the administrative costs associated with monitoring revenues and collecting transfers are considered in implementing the revenue sharing contract, the manufacturer will prefer selective rebate contract over revenue sharing contract. In addition, we show that the wholesale price contract is a special case of the revenue sharing contract, but the coordinating term requires the wholesale price equates the production cost, thus the manufacturer will not use wholesale price contract to tackle the sales effort free riding problem.

This chapter also demonstrates the superiority of the selective rebate over a traditional target rebate in achieving supply chain coordination in a way that is attractive to the supply chain players involved. Under a target rebate, the manufacturer induces the retailer to exert additional effort and order a larger quantity by increasing the retailer's marginal revenue. However, the manufacturer compensates for the entire sales if the retailer's order quantity exceeds the target level. A selective rebate offers an advantage to the manufacturer. The manufacturer only compensates for the sales to the free-riding customers.

CHAPTER 5

THE SUPPLY CHAIN CONTRACTS UNDER STOCHASTIC DEMAND

In this chapter, we study sales effort coordination for a supply chain with one manufacturer and two retail channels, where an online retailer offers a lower price and free-rides a brick-and-mortar retailer's sales effort. The free riding effect reduces brick-and-mortar retailer's desired effort level, and thus hurts the manufacturer's profit and the overall supply chain performance. To resolve the free riding problem, we design a contract with price match and selective compensation rebate.

We also examine other contracts, including the target rebate contract and the wholesale price discount contract, both with price match. The numerical analysis shows that the selective rebate outperforms other contracts in coordinating the brick-and-mortar retailer's sales effort and improving supply chain efficiency .

5.1 Assumptions and Notations

We consider a manufacturer who sells one product through an online retailer and a brick-and-mortar retailer, in a single selling season. The manufacturer produces prior to the selling season at a unit cost (c), and offers the same wholesale price (w) to the retailers. The brick-and-mortar retailer's unit retail price (p_b) is greater than the online retailer's (p_o) due to the former's higher operational cost. The wholesale price and the retail prices are exogenous. We use θ to represent the brick-and-mortar retailer's sales effort and $V(\theta)$ for the effort cost.

Assumption 5.1 *$V(\theta)$ is an increasing and convex function of θ, $\theta \geq 1$.*

When $\theta = 1$, the retailer does not insert extra sales efforts. The brick-and-mortar and the online retailers decide their order quantities, Q_b and Q_o, before the selling season. A summary of all the notations can be found in Appendix B.

The customers are classified into three groups according to their shopping habits:

- Online only customers are not affected by the brick-and-mortar retailer's sales effort. This is a reasonable assumption in that the online only shopper don't visit local stores and hence the local stores' effort has little to ignorable effect on online only shoppers. Their demand is denoted by D, a random variable with density $\psi(\cdot)$ and distribution $\Psi(\cdot)$.

- Traditional customers only shop in the brick-and-mortar stores. Their demand is $D_b = k_b\theta^{r_b}D$, where k_b is the scaling factor for the relative initial market size compared with the online only customers, and r^b is the scaling factor of market expansion by extra sales efforts.

- Free-riding customers are internet savvy. They visit the local brick-and-mortar store to experience the product, and then search for a cheaper price online. However, if the brick-and-mortar retailer matches the lower price, free-riding customers will purchase in the brick-and-mortar store to avoid waiting for the shipment and the risk of online payment. Their demand is defined by $D_f = k_f\theta^{r_f}D$.

The major assumptions regarding the market demand are:

Assumption 5.2 *The demand is an increasing and concave function of the sales effort, i.e.,* $r^b \leq 1$.

Such multiplicative form of demand function is commonly used in the literature (see Agrawal and Seshadri, 2000, and a survey by Petruzzi and Dada, 1999).

Assumption 5.3 *The market expansion effect on the free-riding customers is no higher than the traditional customers, i.e.,* $r^f \leq r^b$.

Assumption 5.4 *The market size of the free-riding customers is no bigger than that of the traditional customers, i.e.,* $k_f < k_b$.

In the following, we study five cases: the selective rebate (S), the target rebate (T), the wholesale price discount (D), the baseline case (B), and the integrated supply chain (I). We denote Π_{player}^{case} as the profit, where $case = \{T, D, S, B, I\}$, and $player = \{m, b, o, a\}$, representing the manufacturer, the brick-and-mortar retailer, the online retailer, and the overall supply chain. This notation system is applied throughout the chapter. A summary of notations can be found in Table B.2.

5.2 The Baseline Case

The baseline case is the primitive setting of this research. There is no incentive in the supply chain and the online retailer free rides the brick-and-mortar retailer's sales effort. The internet savvy (free-riding) customers buy from the online stores.

The brick-and-mortar retailer's profit model is:

$$\Pi_b^B\big(Q_b, \theta\big) = -wQ_b + p_b \max E[min\big(Q_b, k_b\theta^{r_b}D\big)] - V\big(\theta\big).$$

Theorem 5.1 *In the baseline case, the brick-and-mortar retailer's profit is jointly concave in her order quantity and sales effort. The optimal order quantity and sales effort satisfy:*

$$Q_b^* = k_b\theta^{r_b}\Psi^{-1}\big(\frac{p_b - w}{p_b}\big)$$

$$V'\big(\theta\big)|_{\theta^*} = \frac{k_b r_b \theta^{r_b - 1} p_b}{\big(k_b\theta^{r_b}\big)^2} E[\min\big(Dk_b\theta^{r_b}, Q_b\big)].$$

And the online retailer's profit equation is:

$$\Pi_o^B(Q_o) = -wQ_o + p_o E[\min\big(Q_o, (1 + k_f\theta^{r_f})D\big)].$$

The optimal order quantity is $Q_o^* = \big(1 + k_f\theta^{r_f}\big)\Psi^{-1}\big(\frac{p_o - w}{p_o}\big)$. Given the optimal orders of both retailers, we can obtain the manufacturer's profit as follows:

$$\begin{aligned}\Pi_m^B &= (w - c)(Q_b^* + Q_o^*) \\ &= (w - c)\Big(\big(1 + k_f\theta^{r_f}\big)\Psi^{-1}\big(\frac{p_o - w}{p_o}\big) + k_b\theta^{r_b}\Psi^{-1}\big(\frac{p_b - w}{p_b}\big)\Big).\end{aligned}$$

5.3 The Integrated Supply Chain

The integrated supply chain is also called the centralized supply chain, in which there is a central planner determines the decision variables to optimize the overall supply chain profit. The overall supply chain profit can be modeled as:

$$\begin{aligned}\Pi_a^I(Q_b, Q_o, \theta) = -c(Q_b + Q_o) &+ p_b E[\min\big(Q_b, k_b\theta^{r_b}D\big)] \\ &+ p_o E[\min\big(Q_o, (1 + k_f\theta^{r_f})D\big)] - V(\theta).\end{aligned} \tag{5.1}$$

Theorem 5.2 *In the integrated supply chain, and the system profit is concave in the sales effort. The optimal θ satisfies:*

$$V'(\theta)|_{\theta^*} = p_b k_b r_b \theta^{r_b - 1}\int_0^{\frac{Q_b^*}{k_b\theta^{r_b}}} D\psi(D)dD + p_o k_f r_f \theta^{r_f - 1}\int_0^{\frac{Q_o^*}{1 + k_f\theta^{r_f}}} D\psi(D)dD. \tag{5.2}$$

5.4 The Selective Rebate Contract with Price Match

Under the selective rebate and price match contract, the brick-and-mortar retailer sells the product to the traditional customers at p_b. As for free-riding customers who provide an evidence of a lower online price (p_o), the brick-and-mortar retailer will match the online price. For each unit sold at p_o, she will get a unit rebate u

$(0 \leq u \leq p_b - p_o)$ from the manufacturer as a compensation for her loss due to price match. We also assume $u \leq w - c$ such as the manufacturer will not have negative net revenue due to the rebate.

We study a Stackelberg game, where the manufacturer is the leader and the retailers are the followers. The sequence of events is as follows: 1) The manufacturer offers a rebate contract to the brick-and-mortar retailer; 2) The brick-and-mortar retailer chooses her order quantity and sales effort. The online retailer also chooses the order quantity. The manufacturer produces to meet the retailers' demand; 3) Demand is realized and payments are made according to the contract.

5.4.1 The brick-and-mortar retailer's profit function

We assume that the customers arrive homogenously and the brick-and-mortar retailer satisfies demands on a FCFS basis. Though traditional customers pay a higher price, the brick-and-mortar retailer will not reject free-riding customers' demand to reserve the products for traditional customers. The brick-and-mortar retailer's profit is as follows:

$$\begin{aligned} \Pi_b^S\big(Q_b, \theta\big) = &-wQ_b + \frac{p_b k_b \theta^{r_b - r_f} + \big(p_o + u\big)k_f}{k_b \theta^{r_b - r_f} + k_f} E[\min\big(D\big(k_b\theta^{r_b} + k_f\theta^{r_f}\big), Q_b\big)] \\ &- V\big(\theta\big). \end{aligned} \tag{5.3}$$

The first term $-wQ_b$ is the purchasing cost. The second term summarizes the sales revenue from free-riding and traditional customers, where $E[\min\big(D\big(k_b\theta^{r_b} + k_f\theta^{r_f}\big), Q_b\big)]$ is the volume of the overall sales. The margin for selling to traditional customers is p_b, while $p_o + u$ is the margin to sell to free-riding customers. $V\big(\theta\big)$ is the cost of effort.

Theorem 5.3 *The brick-and-mortar retailer's profit function is jointly quasi-concave, and furthermore, unimodal in Q_b and θ. The optimal order quantity and sales effort* *satisfy:*

$$Q_b^* = \big(k_b\theta^{r_b} + k_f\theta^{r_f}\big)\Psi^{-1}\Big(1 - \frac{k_b\theta^{r_b - r_f} + k_f}{p_b k_b \theta^{r_b - r_f} + \big(p_o + u\big)k_f} w\Big), \tag{5.4}$$

$$V'\big(\theta\big)|_{\theta^*} = \frac{k_f k_b\big(r_b - r_f\big)\theta^{r_b - r_f}\big(p_b - p_o - u\big)}{\big(k_b\theta^{r_b - r_f} + k_f\big)^2} E[\min\big(D\big(k_b\theta^{r_b} + k_f\theta^{r_f}\big), Q_b\big)]. \tag{5.5}$$

Please refer to Appendix A for all the proofs. Equation (5.5) is an implicit function. The optimal θ^* can be attained by satisfying (5.5), given explicit forms of demand distribution and $V(\theta)$.

5.4.2 The online retailer's profit model

Due to price match, free-riding customers' demand is diverted to the brick-and-mortar retailer. The online only shoppers are the sole customers of the internet retailer. The internet retailer's demand is D, and she offers no sales effort. Her profit model is as follows:

$$\Pi_o^S\big(Q_o\big) = -wQ_o + p_o E[\min\big(D, Q_o\big)].$$

It's a standard newsvendor problem and the optimal order quantity is $Q_o^* = \Psi^{-1}\big(\frac{p_o - w}{p_o}\big)$. The optimal order quantity for the online retailer also holds in the target rebate and wholesale price discount contracts with price match.

5.4.3 The manufacturer's profit model

With Q_b^* and Q_o^*, we can now model the manufacturer's profit:

$$\begin{aligned}\Pi_m^S\big(u\big) &= \big(w - c\big)\big(Q_b^*\big(u\big) + Q_o^*\big(u\big)\big) - u\frac{k_f Q_b^*\big(u\big)}{k_b\theta^{*r_b - r_f} + k_f}, \\ &= \big(w - c\big)\Psi^{-1}\big(\frac{p_o - w}{p_o}\big) + \big(w - c - u\big)\Psi^{-1}\Big(1 - \frac{k_b\theta^{r_b - r_f} + k_f}{p_b k_b\theta^{r_b - r_f} + \big(p_o + u\big)k_f} w\Big).\end{aligned} \tag{5.6}$$

Theorem 5.4 *The manufacturer's profit function is concave in u.*

Theorem 5.4 guarantees that there is a unique optimal solution to the manufacturer's rebate.

5.4.4 Analytical results

To obtain more analytical results, in this section, we assume the cost function of sales effort as $V(\theta) = a\theta^2$, and the demand is uniformly distributed as $U(\alpha - \beta, \alpha + \beta)$, such assumptions have appeared in several articles such as Taylor [2002].

Order quantity and sales effort analysis

Different from supply chain coordination which pursues the same system profit as the centralized supply chain, action coordination instead focuses on optimizing a single action for one player. Action coordination is non-trivial in supply chain contract analysis [Cachon, 2003]. Lariviere [2002] shows that a manufacturer can obtain a large share of supply chain profit by coordinating the forecasting effort of its dual channel retailers, although at the cost of supply chain efficiency. We find that the selective rebate with price match achieves action coordination for sales effort.

Theorem 5.5 *The optimal sales effort in the selective rebate and price match contract is the same as in the integrated supply chain. The optimal decisions satisfy:*

$$\theta^{S*} = \left(\frac{(p_b - c)k_b r_b + (p_o - c)k_f r_f}{4a}\right)^{\frac{1}{2\left(r_f + r_b\right)}}, \tag{5.7}$$

$$u^{S*} = \frac{k_b(p_o - w)\left(\frac{(p_b-c)k_b r_b + (p_o-c)k_f r_f}{4a}\right)^{\frac{r_b - r_f}{2(r_f + r_b)}} + k_f p_o}{\left(p_b k_b\left(\frac{(p_b-c)k_b r_b + (p_o-c)k_f r_f}{4a}\right)^{\frac{r_b - r_f}{2(r_f + r_b)}} + p_o k_f\right)^2}. \tag{5.8}$$

Theorem 5.5 implicates that under the selective rebate, the brick-and-mortar retailer provides the optimal sales effort as in the centralized supply chain. Next, we compare the brick-and-mortar retailer's order quantity and sales effort in the following scenarios: the integrated supply chain and the selective rebate contract with price match. We will see how the selective rebate contract affects the brick-and-mortar retailer's decisions regarding effort and order quantity.

Theorem 5.6 *With the same sales effort, the brick-and-mortar retailer will order less under the selective rebate contract with price match than in the integrated supply chain.*

Since the brick-and-mortar retailer's profit function is unimodal in the order quantity and the sales effort, we can obtain the following corollary from Theorem 5.5 and 5.6 directly.

Corollary 5.1 *Given the same order quantity as in the integrated supply chain, the brick-and-mortar retailer will exert more sales effort under the selective rebate contract with price match than in the integrated supply chain.*

Impact of demand variation

In this section, we examine the relationship between order quantity and demand variation under the selective rebate. With a uniform distribution $U\big(\alpha - \beta, \alpha + \beta\big)$, the demand variation can be measured by β. Denote $\frac{C_u}{C_o + C_u}$ as the critical ratio, where C_u and C_o represent the underage and overage costs respectively. We first introduce Proposition 5.1 to discuss the impact of demand variation on the optimal order quantities in the basic newsvendor model with one retailer only and no sales effort involved.

Proposition 5.1 *In a basic newsvendor model with a uniform demand distribution $U\big(\alpha - \beta, \alpha + \beta\big)$, when the critical ratio $<$ ($>$) 0.5, the optimal order decreases* *(increases) in β.*

Next, let's examine the impact of demand variation on the brick-and-mortar retailer's optimal order quantity.

Theorem 5.7 *Under the selective rebate with price match contract, the brick-and-mortar retailer's optimal order quantity is affected by the demand variation as follows:*

(1) When $w \in (0, \frac{p_b+p_o+u}{4}]$, the brick-and-mortar retailer's optimal order quantity increases with β;

(2) When $w \in (0.5p_b, p_b)$, the optimal order quantity decreases with β;

(3) When $w \in (\frac{p_b+p_o+u}{4}, 0.5p_b]$, the optimal order quantity increases (decreases) with β if $\big(0.5p_b - w\big)k_b\theta^{r_b-r_f} + \big(0.5p_o + 0.5u - w\big)k_f$ is positive (negative).

Theorem 5.7 can be explained by the impact of critical ratio based on Proposition 5.1. In the selective rebate contract, the brick-and-mortar retailer offers two retail prices (p_o and p_b) to the two customer groups respectively. Thus, conceptually, her total order could be considered as the summation of the orders from these two customer groups. The critical ratios for the traditional shoppers and the internet savvy shoppers are $\frac{p_b-w}{p_b}$ and $\frac{p_o+u-w}{p_o}$ respectively. When $w > 0.5p_b$, the critical ratios of both customer groups are smaller than 0.5, hence the optimal order quantity decreases in β. When $w < \frac{p_o+u}{2}$, the critical ratios of both are bigger than 0.5, thus the optimal order quantity increases in β. Because the assumption of $k_b\theta^{r_b} \geq k_f\theta^{r_f}$, when $w \in (\frac{p_o+u}{2}, \frac{p_b+p_o+u}{4}]$, the total optimal order quantity still increases in β. However, when $w \in (\frac{p_b+p_o+u}{4}, 0.5p_b]$, the impact of β is not easy to see and needs to be determined numerically by $\frac{\partial Q_b^*}{\partial \beta}$.

5.5 The Revenue Sharing Contract with Price Match

Under a revenue-sharing contract, the brick-and-mortar retailer pays a manufacturer a fixed wholesale price w for each unit purchased, plus a percentage μ of the revenue the retailer generates.

5.5.1 The brick-and-mortar retailer's profit function

We assume that the customers arrive homogenously and the brick-and-mortar retailer satisfies demands on a FCFS basis. Though traditional customers pay a higher price, the brick-and-mortar retailer will not reject free-riding customers' demand to reserve the products for traditional customers. The brick-and-mortar retailer's profit is as follows:

$$\begin{aligned}\Pi_b^R\big(Q_b,\theta\big) =& - wQ_b + \frac{(1-\mu)p_b k_b\theta^{r_b-r_f} + p_o(1-\mu)k_f}{k_b\theta^{r_b-r_f} + k_f}E[\min\big(D\big(k_b\theta^{r_b} + k_f\theta^{r_f}\big), Q_b\big)] \\ & - V\big(\theta\big).\end{aligned} \tag{5.9}$$

The first term $-wQ_b$ is the purchasing cost. The second term summarizes the sales revenue from free-riding and traditional customers, where $E[\min\big(D\big(k_b\theta^{r_b} + k_f\theta^{r_f}\big), Q_b\big)]$ is the volume of the overall sales. The margin for selling to traditional customers is $p_b(1-\mu)$, while $p_o(1-\mu)$ is the margin to sell to free-riding customers. $V\big(\theta\big)$ is the cost of effort.

Theorem 5.8 *The brick-and-mortar retailer's profit function is jointly quasi-concave, and furthermore, unimodal in Q_b and θ. The optimal order quantity and sales effort satisfy:*

$$Q_b^* = \big(k_b\theta^{r_b} + k_f\theta^{r_f}\big)\Psi^{-1}\Big(1 - \frac{k_b\theta^{r_b-r_f} + k_f}{p_b(1-\mu)k_b\theta^{r_b-r_f} + p_o(1-\mu)k_f}w\Big), \tag{5.10}$$

$$V'\big(\theta\big)|_{\theta^*} = \frac{k_f k_b\big(r_b - r_f\big)\theta^{r_b-r_f}\big(p_b - p_o\big)(1-\mu)}{\big(k_b\theta^{r_b-r_f} + k_f\big)^2}E[\min\big(D\big(k_b\theta^{r_b} + k_f\theta^{r_f}\big), Q_b\big)]. \tag{5.11}$$

Please refer to Appendix A for all the proofs of the dissertation. Equation (5.11) is an implicit function. The optimal θ^* can be attained by satisfying (5.11), given explicit forms of demand distribution and $V(\theta)$.

5.5.2 The online retailer's profit model

Due to price match, free-riding customers' demand is diverted to the brick-and-mortar retailer. The online only shoppers are the sole customers of the internet retailer. The internet retailer's demand is D, and she offers no sales effort. Her profit model is as follows:

$$\Pi_o^R(Q_o) = -wQ_o + p_oE[\min(D, Q_o)].$$

It's a standard newsvendor problem and the optimal order quantity is $Q_o^* = \Psi^{-1}\big(\frac{p_o-w}{p_o}\big)$. The optimal order quantity for the online retailer also holds in the target rebate and wholesale price discount contracts with price match.

5.5.3 The manufacturer's profit model

With Q_b^* and Q_o^*, we can now model the manufacturer's profit:

$$\begin{aligned}\Pi_m^R(\mu) &= (w-c)\big(Q_b^*(\mu) + Q_o^*(\mu)\big) + \frac{\mu p_b k_b \theta^{r_b - r_f} + p_o \mu k_f}{k_b \theta^{r_b - r_f} + k_f} Q_b^*(\mu) \\ &= (w-c)\Psi^{-1}\big(\frac{p_o - w}{p_o}\big) + (w-c)\Psi^{-1}\Big(1 - \frac{k_b\theta^{r_b-r_f} + k_f}{p_b k_b \theta^{r_b-r_f} + \mu p_o k_f} w\Big).\end{aligned} \tag{5.12}$$

Theorem 5.9 *The manufacturer's profit function is concave in μ. We assume the cost function of sales effort as $V(\theta) = a\theta^2$, and the demand is uniformly distributed as $U(\alpha-\beta, \alpha+\beta)$, and the optimal μ^* is:*

$$\mu^* = \frac{k_f k_b \theta^{*r_b - r_f} + p_b k_b \theta^{*r_b - r_f}}{4a(p_o k_f)^2}. \tag{5.13}$$

Theorem 5.9 guarantees that there is a unique optimal solution to the manufacturer's rebate.

5.5.4 Equivalence to the selective rebate contract

Consider that the remaining products owned by the brick-and-mortar retailer at the end of the selling season have zero savage value. In other words, the manufacturer doesn't share demand uncertainty risk with the brick-and-mortar retailer. Thus we can use similar tactic to relate the selective rebate and revenue sharing contracts. To get $w_s - u = w_r + \mu p_o$ and $w_r + \mu p_b = w_s$, we need to assume the cost function of sales effort as $V\big(\theta\big) = a\theta^2$, and the demand is uniformly distributed as $U\big(\alpha - \beta, \alpha + \beta\big)$, thus we can introduce Equations (5.13) and (5.8) into $w_r + \mu p_o$ and $w_r + \mu p_b$, and we show that

$$
\begin{aligned}
u - \mu(p_b - p_o) =& -(p_b - p_o)\frac{k_f k_b \theta^{*r_b - r_f} + p_b k_b \theta^{*r_b - r_f}}{4a(p_o k_f)^2} \\
&+ \frac{k_b p_o \left(\frac{(p_b - c)k_b r_b + (p_o - c)k_f r_f}{4a}\right)^{\frac{r_b - r_f}{2(r_f + r_b)}} + k_f p_o}{\left(p_b k_b \left(\frac{(p_b - c)k_b r_b + (p_o - c)k_f r_f}{4a}\right)^{\frac{r_b - r_f}{2(r_f + r_b)}} + p_o k_f\right)^2} \\
=&0.
\end{aligned}
$$

Thus the two contracts result in the same profits for the retailer and the manufacturer for any combination of free riding and traditional customers'demand.

By now, we have shown that the revenue sharing contract is equivalent to the selective rebate contract when the online channel is independent of the manufacturer. However, since the manufacturer doesn't share demand risk with the brick-and-mortar retailer, the brick-and-mortar retailer orders less than optimal and the supply chain efficiency is lost.

5.6 The Target Rebate Contract with Price Match

Target rebates are paid by the manufacturer to the retailer for the sales quantity beyond a specified target level. In Taylor [2002], a target rebate and returns contract is designed to coordinate the supply chain with retailer effort. In this section, we study the contract with target rebate and price match, without returns.

The sequence of events is as follows: (1) The manufacturer offers a target rebate (r) contract with a threshold (T) to the brick-and-mortar retailer; (2) The brick-and-mortar retailer chooses her order quantity and effort. The online retailer also chooses her order quantity. The manufacturer produces to meet the retailers' order; (3) Demand is realized and payments are made according to the contract. If the brick-and-mortar retailer's order quantity exceeds the target threshold, the manufacturer will pay a unit rebate (r) to the retailer.

5.6.1 The brick-and-mortar retailer's profit

The brick-and-mortar retailer's profit is:

$$\begin{aligned}\Pi_b^T\big(Q_b,\theta|T\big) =& -wQ_b + p_b E[\min\big((k_f\theta^{r_f}+k_b\theta^{r_b})D, Q_b\big)] \\ &+ rE[\min\big((k_f\theta^{r_f}+k_b\theta^{r_b})D, Q_b\big) - T]^+ - V\big(\theta\big),\end{aligned}$$

where T is the target rebate threshold.

Theorem 5.10 *For a given T, $\Pi_b^T\big(Q_b,\theta|T\big)$ is concave in Q_b in intervals $[0,T]$ and (T,∞), respectively. The optimal order quantity is uniquely determined by $Q_b^* = (k_f\theta^{r_f}+k_b\theta^{r_b})\Psi^{-1}\big(\frac{p_b-w+r}{p_b+r}\big)$.*

5.6.2 The manufacturer's profit model

The manufacturer's profit is:

$$\Pi_m^T\big(r|T\big) = \begin{cases} (w-c)\big(Q_b+Q_o\big) & Q_b < T \\ (w-c)\big(Q_b+Q_o\big) - rE[min\big(Q_b, \big(k_f\theta^{r_f}+k_b\theta^{r_b})D\big) - T]^+ & Q_b > T. \end{cases}$$

Proposition 5.2 $\Pi_m^T\big(r|T\big)$ *is concave in* r*, and* $r^*\big(T\big) = \big(p_b - w\big)\frac{T-\Psi^{-1}\left(\frac{p_b-w}{p_b}\right)}{\Psi^{-1}(\frac{p_b-c}{p_b})}$.

Theorem 5.11 $\Pi_m^T\big(T\big)$ *is concave in* T.

As shown in Theorem 5.11, the optimal decision of u is a function of T, and the manufacturer's profit can be converted into a single decision T. The concavity guarantees that there is a unique solution to maximize his profit.

5.7 The Wholesale Price Discount Contract with Price Match

Wholesale price discount contract is commonly used in the retail business. Though wholesale price discount alone cannot coordinate the supply chain, it is still favored by the management due to its ease of implementation. Thus we include the wholesale price discount into this contract comparison.

In this section, a wholesale price discount (d) is provided to the brick-and-mortar retailer together with the price match policy. The profit model of the brick-and-mortar retailer is as follows:

$$\Pi_b^W\left(Q_b,\theta\right) = -\,(w-d)Q_b + \frac{p_b k_b \theta^{r_b - r_f} + p_o k_f}{k_b\theta^{r_b-r_f}+k_f}E[\min\left(D\big(k_b\theta^{r_b}+k_f\theta^{r_f}\big), Q_b\right)] - V\big(\theta\big).$$

Proposition 5.3 *Under the wholesale discount contract with price match, the brick-and-mortar retailer's profit function is unimodular in her order quantity and sales* *effort. The optimal order quantity and sales effort satisfy:*

$$Q_b^* = \big(k_b\theta^{r_b} + k_f\theta^{r_f}\big)\Psi^{-1}\Big(1 - \frac{k_b\theta^{r_b - r_f} + k_f}{p_b k_b \theta^{r_b - r_f} + \big(p_o + u\big)k_f}(w-d)\Big),$$

$$V'(\theta)|_{\theta^*} = \frac{k_f k_b\big(r_b - r_f\big)\theta^{r_b - r_f - 1}\big(p_b - p_o + d - w\big)}{\big(k_b\theta^{r_b - r_f} + k_f\big)^2} E[\min\big(D\big(k_b\theta^{r_b} + k_f\theta^{r_f}\big), Q_b\big)].$$

With Q_b^* and Q_o^*, we can model the manufacture's profit as follows:

$$\begin{aligned}\Pi_m^W(d) &= (w - c - d)Q_b^* + (w - c)Q_o^*,\\ &= (w-c)\Psi^{-1}\big(\frac{p_o - w}{p_o}\big) + (w - c - d)\Psi^{-1}\Big(1 - \frac{k_b\theta^{r_b - r_f} + k_f}{p_b k_b \theta^{r_b - r_f} + \big(p_o + u\big)k_f}(w-d)\Big).\end{aligned}$$

Theorem 5.12 *Under the wholesale discount contract with price match, the manufacturer's profit is concave in d.*

5.8 Numerical Analysis

In this section, we will examine the impacts of the sales effort cost coefficient (a) and the proportion of free riding consumers ($\frac{k_f}{k_b}$) on the supply chain performance and the optimal sales effort.

The standard setting for the parameters in the numerical analysis is as follows: $k_f = 1, k_b = 1.25$, $r_f = 0.25, r_b = 0.5$, $p_b = 10, p_o = 7, w = 4, c = 3, a = 0.5$, $\alpha = 1200$, and $\beta = 200$. In each following section, one parameter may vary from the standard setting to reveal its impact.

5.8.1 Impacts of the sales effort cost coefficient (a)

In this subsection, we study the impacts of the sales effort cost coefficient (a) on the supply chain performance, by increasing a from 0.1 to 1.

Figure 5.1 shows the impact of changing a on the optimal sales effort. Note

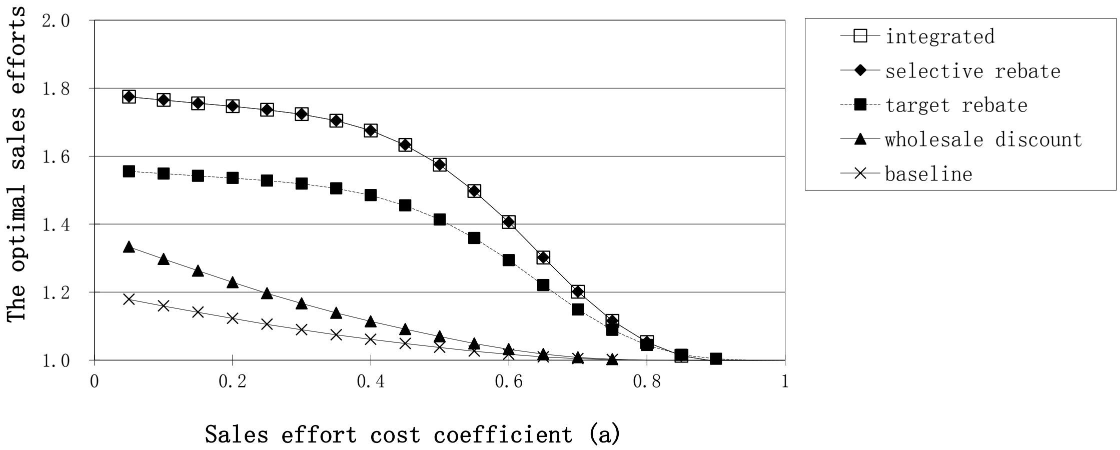


Figure 5.1: The impact of changing a on the optimal sales effort.

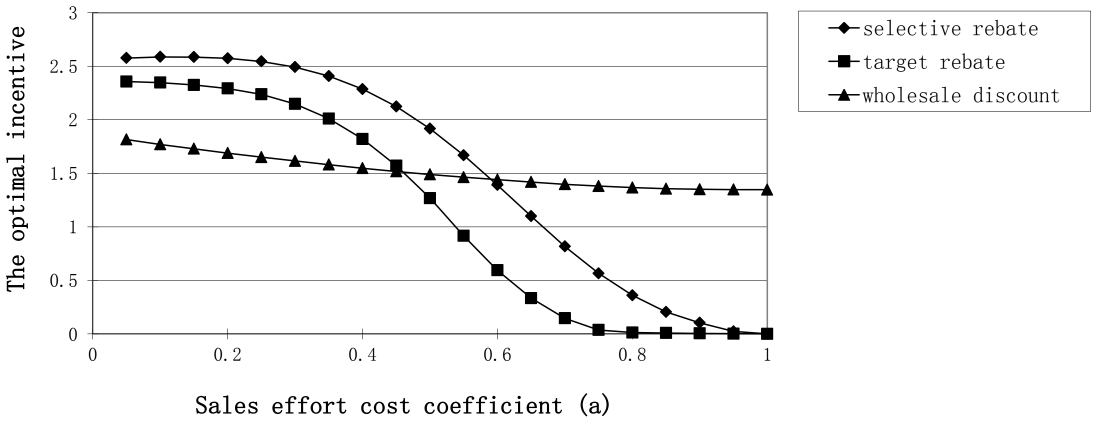


Figure 5.2: The optimal incentive values with increasing a.

that the curves of the integrated and the selective rebate scenarios overlap with each other. It confirms Theorem 5.5 that the selective rebate contract can coordinate sales effort. As a result, the selective rebate always achieves the highest optimal effort level, followed by the target rebate and wholesale price discount contracts. The selective rebate is more effective than the target rebate because it targets the free riding customers. Another observation is that the increase of a results in the decrease of the optimal sales effort among all the contracts charted. We also notice that the optimal sales effort under the selective and target rebate stay stable at the beginning and decline sharply when a hits a critical level, which can be explained by Figure 5.2.

Figure 5.2 shows the impact of the sales effort cost coefficient on the optimal contract incentives. With the increase of a, all the incentives are declining, though at different speeds. Contract incentives are provided by the manufacturer to boost the retailer's sales effort. When a increases, the incentive's effectiveness to boost sales effort declines and thus the manufacturer reduces his incentive. Shrinking manufacturer support will certainly dampen the retailer's desire to promote the products, thus sales effort diminishes as shown in Figure 5.1. Another observation in Figure 5.2 is that with the increase of a, the optimal selective and target rebates remain stable at first, then drop sharply after a hits a critical value, which explains Figure 5.1. On the other hand, the optimal wholesale discount decreases steadily. This reflects that the performance of the selective rebate and target rebate is more sensitive to a.

The impact of the sales effort cost coefficient on the supply chain efficiency is plotted in Figure 5.3. The supply chain efficiency is defined as the ratio of the system profit under each contract divided by the profit in the integrated supply chain.

Figure 5.3 shows that the supply chain efficiency decreases in a. Different from the integrated supply chain, the retailer in a decentralized system orders less than in the integrated supply chain due to double marginalization. The contract incentives increase the retailer's marginal revenue and thus increase her order quantity and sales

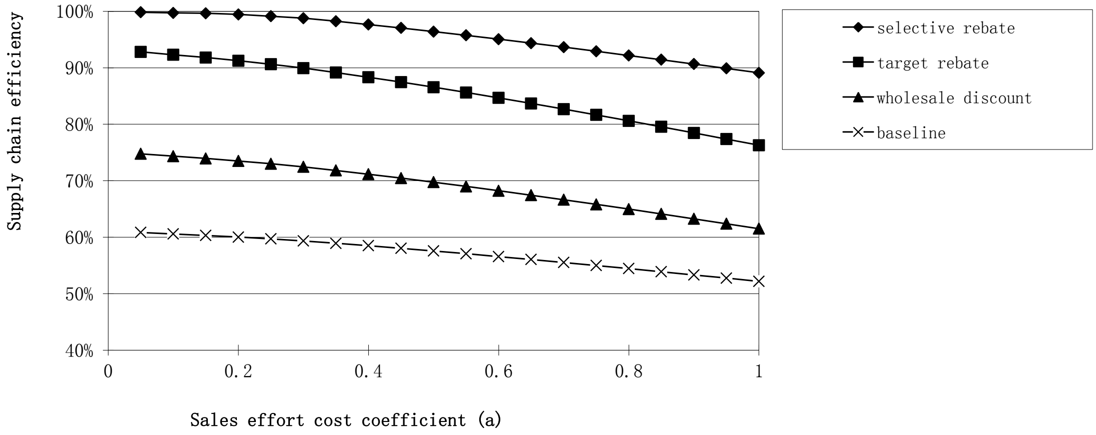


Figure 5.3: The impact of changing a on the supply chain efficiency.

effort. However, with a higher a, the incentives under all contracts decrease as shown in Figure 5.2. With a smaller incentive, the retailer has to order a smaller quantity than in the integrated case and the supply chain efficiency is lowered.

5.8.2 Impacts of the proportion of free riding customers

In this section, we will study the impact of the proportion of free riding customers on supply chain efficiency, sales effort, and incentives.

For this purpose, we set $k_f \in [0.125, 1.25]$, $k_b = 1.25$. The ratio k_f/k_b represents the proportion of free riding customers in the total demand for the brick-and-mortar retailer.

Figure 5.4 shows the impact of the proportion of free riding customers on the optimal sales effort. With the increase of k_f/k_b, the brick-and-mortar retailer's optimal sales effort increases in all the contracts, but steadily decreases in the baseline case. It seems counter-intuitive at first glance. When the scale of free riding increases, the brick-and-mortar retailer's sales effort might decrease. However, notice that all of the three contracts have employed price match policy. With the increase of k_f/k_b, the

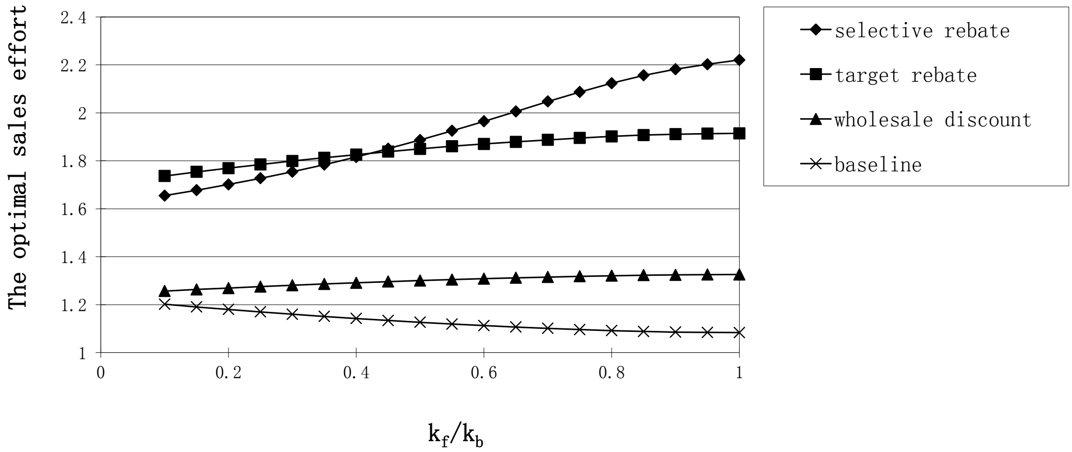


Figure 5.4: The optimal sales effort with changing k_f/k_b.

brick-and-mortar retailer's market share expands as well, thus the retailer is willing to boost her sales effort.

Figure 5.4 also shows that the optimal sales effort in the selective rebate is smaller than in the target rebate at the beginning. However, with the increase of k_f/k_b, the effort in the selective rebate surpasses the target rebate's effort curve. The major difference between the selective rebate and the target rebate is that the rebate is given for the sales to the internet savvy customers in the former case and the total sales in the latter case. When k_f/k_b is very small, the total incentive given to the retailer under the selective rebate contract is much smaller than in the target rebate. Thus the optimal sales effort under the selective rebate contract, which is affected by incentives, is smaller than that of the target rebate. However, with the increase of k_f/k_b, the effort curve of the selective rebate soon surpasses the target rebate.

Figure 5.5 plots the impact of the proportion of free riding customers on the optimal incentives. The selective rebate is always higher than the target rebate. Considering that the selective rebate is given for the sales to the internet savvy customers while the target rebate is given to both the internet savvy and traditional

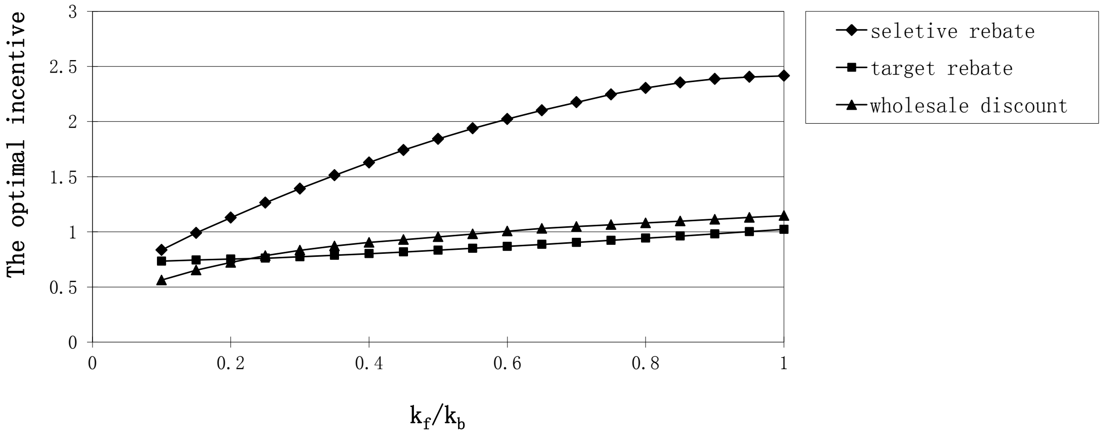


Figure 5.5: The optimal incentives with changing k_f/k_b.

shoppers, the manufacturer can afford a higher rebate rate in the selective rebate than in the target rebate.

Figure 5.6 compares the supply chain efficiency among the three contracts and the baseline case with changing ratio of k_f/k_b. The general trend is that the supply chain efficiency improves under all contracts with the increase of k_f/k_b, but decreases in the baseline case. Notice that all of the three contracts have applied price match policy. With the increase of the scale of free riding, a larger proportion of demand goes to the brick-and-mortar retailer, the sales effort contributor. Her sales effort is appropriately rewarded with market demand.

On another note, the efficiency of the selective rebate is much lower than the target rebate at the beginning. But it grows at the fastest speed until it crosses the curve of the target rebate. Then its increase rate falls to a moderate level but is still slightly higher than the target rebate. This is because the selective rebate is the most sensitive to the proportion of internet savvy customers. With the increase of k_f/k_b, the selective rebate with price match can most efficiently stimulate the brick-and-mortar retailer's sales effort, as shown in Figure 5.4. It also offers a higher incentive

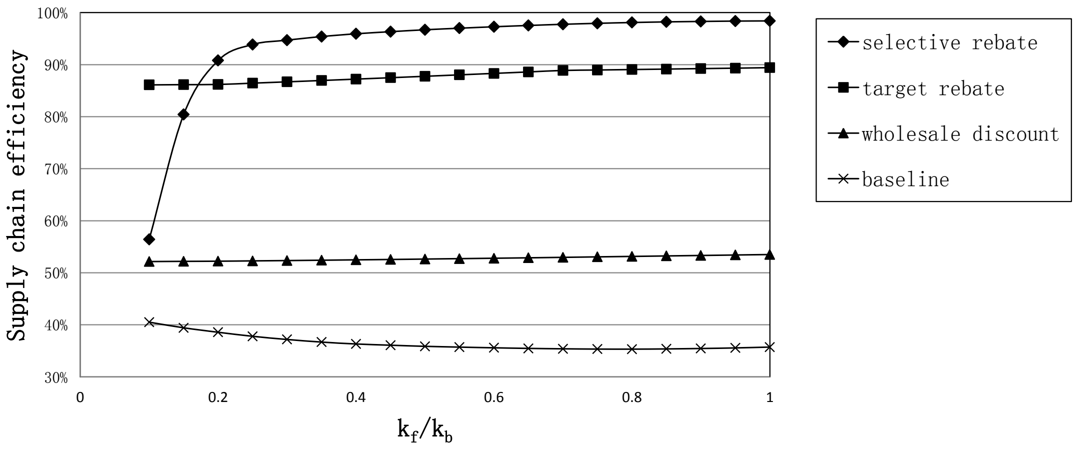


Figure 5.6: The supply chain efficiency with changing k_f/k_b.

than any other contract. Thus it excels in driving the retailer's decisions into the favored direction: more sales effort stimulates a larger demand; and a higher rebate increases the retailer's order quantity to better satisfy the expanded market.

5.9 Summary of Chapter 5

This chapter examines the effectiveness of different contracts in coordinating brick-and-mortar retailer's sale effort with free-riding online retailers, under stochastic demand. Among the contracts discussed, the selective rebate with price match contract has the best system performance, unless the proportion of free-riding customers is very small. Price match policy diverts the demand of the internet savvy customers from the online retailer to the brick-and-mortar retailer. By doing so, the brick-and-mortar retailer's sales effort is rewarded with the internet savvy customer's demand. In addition, the manufacturer provides partial compensation to the brick-and-mortar retailer to offset her loss due to price match. Such selective rebate boosts the brick-and-mortar retailer's order quantity and sales effort, and thus increase the supply chain efficiency. We also show that the selective rebate with price match coordinates the sales effort, and the contract performance is sensitive to the sales effort cost coefficient and the proportion of free-riding customers.

CHAPTER 6

CONCLUSIONS

This dissertation examines the effectiveness of different contracts in coordinating brick-and-mortar retailer's sale effort with free-riding online retailers. Our efforts are divided into two parts: supply chain contracts under deterministic and stochastic demand models.

Under the deterministic demand, we emphasize on the effectiveness of selective rebate contract with price match in coordinating a supply chain with retail channel conflicts caused by sales effort free riding. The price match policy diverts the demand of the internet savvy customers from the online channel to the brick-and-mortar retailer. By doing so, the brick-and-mortar retailer's sales effort is rewarded with the internet savvy customer's demand. In addition, the manufacturer provides partial compensation to the brick-and-mortar retailer to offset her loss due to price match. Such a selective rebate boosts the brick-and-mortar retailer's order quantity and sales effort, achieves supply chain coordination and maximum system profit, and thus increases the supply chain efficiency.

This dissertation also demonstrates the superiority of the selective rebate over a traditional linear rebate in achieving supply chain coordination in a way that is attractive to the supply chain players involved. Under a linear rebate, the manufacturer induces the retailer to exert additional effort and order a larger quantity by increasing the retailer's marginal revenue. However, the manufacturer fully bears the financial burden of increasing the retailer's marginal revenue. A selective rebate offers an advantage to the manufacturer. By setting the partial rebate value, the manufacturer can induce the retailer to exert more sales effort without covering the full cost of doing so.

Revenue sharing contract with price match is also studied. The deep root of equivalence between the selective rebate and revenue sharing contract with price match explains why both contracts can coordinate the supply chain with the manufacturer owning the online channel, and arbitrarily split the system profit. For any selective contract there exists a revenue sharing contract that generates the same cash flows. However, if the administrative costs associated with monitoring revenues and collecting transfers are considered in implementing the revenue sharing contract, the manufacturer will prefer selective rebate contract over revenue sharing contract.

We also show that the target rebate contract with price match is able to coordinate the supply chain. The manufacturer gives rebate to the brick-and-mortar retailer based on her overall sales beyond a target level. The system profit can be divided among the supply chain players if the target level is changed within a feasible range.

The second part of the dissertation studies the supply chain contracts under the stochastic demand. Among the selective rebate, target rebate and wholesale price contracts, the selective rebate with price match contract has the best system performance, unless the proportion of free-riding customers is very small. Price match policy diverts the demand of the internet savvy customers from the online retailer to the brick-and-mortar retailer. By doing so, the brick-and-mortar retailer's sales effort is rewarded with the internet savvy customer's demand. In addition, the manufacturer provides partial compensation to the brick-and-mortar retailer to offset her loss due to price match. The selective rebate contract with price match boosts the brick-and-mortar retailer's order quantity and sales effort, and solves the sales effort free riding problem.

There are several areas that can be further explored with the selective rebate contract. This dissertation captures a fundamental way that the retailer can influence her demand by exerting sales effort. However, we have taken the retail price to be exogenous. Exploring the manufacturer's use of rebates as an instrument that influences the retailer's pricing decision (e.g., to stimulate demand by driving down retail prices) may be a promising area for research. Finally, the analysis suggests that price matching, which is commonly used by retailers for price competition, can be used productively in a supplier-retailer rebate contract to mitigate channel conflicts. The idea of price competition can be extended to the areas of quality and service competition, and it may be fruitful for researchers to combine price/quality/service match with supply chain contracts to mitigate channel competition.

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APPENDIX A

ADDITIONAL PROOFS

Proof of Theorem 5.1

The proof is similar to that of Theorem 5.3. To emphasize on the selective rebate, we omit the proof of the simpler baseline case here. ■

Proof of Theorem 5.2

The first and second order derivatives of equation (5.1) are:

$$\frac{\partial \Pi_a^I}{\partial Q_b} = p_b - c - p_b \Psi\big(\frac{Q_b}{k_b \theta^{r_b}}\big), \qquad \frac{\partial^2 \Pi_a^I}{\partial Q_b^2} = \frac{-p_b}{k_b \theta^{r_b}} \psi\big(\frac{Q_b}{k_b \theta^{r_b}}\big),$$
$$\frac{\partial \Pi_a^I}{\partial Q_o} = p_o - c - p_o \Psi\big(\frac{Q_o}{1 + k_f \theta^{r_f}}\big), \qquad \frac{\partial^2 \Pi_a^I}{\partial Q_o^2} = \frac{-p_o}{1 + k_f \theta^{r_f}} \psi\big(\frac{Q_o}{1 + k_f \theta^{r_f}}\big),$$

$$\frac{\partial \Pi_a^I}{\partial \theta} = p_b k_b r_b \theta^{r_b - 1} \int_0^{\frac{Q_b}{k_b \theta^{r_b}}} D\psi(D) dD + p_o k_f r_f \theta^{r_f - 1} \int_0^{\frac{Q_o}{1 + k_f \theta^{r_f}}} D\psi(D) dD - V'(\theta).$$

The optimal $Q_b^*(\theta)$ and $Q_o^*(\theta)$ are as follows:

$$Q_b^*(\theta) = k_b \theta^{r_b} \Psi^{-1}\big(\frac{p_b - c}{p_b}\big), \qquad Q_o^*(\theta) = \big(1 + k_f \theta^{r_f}\big) \Psi^{-1}\big(\frac{p_o - c}{p_o}\big).$$

By substituting $Q_b^*(\theta)$ and $Q_o^*(\theta)$, we can obtain the profit function of θ:

$$\begin{aligned}\Pi_a^I(\theta) = & - c\big(k_b \theta^{r_b} \Psi^{-1}\big(\frac{p_b - c}{p_b}\big) + \big(1 + k_f \theta^{r_f}\big) \Psi^{-1}\big(\frac{p_o - c}{p_o}\big)\big), \\ & + p_b k_b \theta^{r_b} \Psi^{-1}\big(\frac{p_b - c}{p_b}\big) + p_o\big(1 + k_f \theta^{r_f}\big) \Psi^{-1}\big(\frac{p_o - c}{p_o}\big) - V(\theta).\end{aligned}$$

Its first and second order derivatives w.r.t. θ are:

$$\frac{\partial \Pi_a^I}{\partial \theta} = p_b k_b r_b \theta^{r_b - 1} \int_0^{\frac{Q_b^*}{k_b \theta^{r_b}}} D\psi\big(D\big)dD + p_o k_f r_f \theta^{r_f - 1} \int_0^{\frac{Q_o^*}{1 + k_f \theta^{r_f}}} D\psi\big(D\big)dD - V'\big(\theta\big)|_{\theta^*}.$$

$$\begin{aligned}\frac{\partial^2 \Pi_a^I}{\partial \theta^2} =& \Psi^{-1}\big(\frac{p_b - c}{p_b}\big)\big(p_b - c\big)r_b\big(r_b - 1\big)k_b\theta^{r_b - 2} \\ &+ \Psi^{-1}\big(\frac{p_o - c}{p_o}\big)\big)\big(p_o - c\big)r_f\big(r_f - 1\big)k_f\theta^{r_f - 2} - V''\big(\theta\big) < 0,\end{aligned}$$

since $r_f, r_b < 1$.

Therefore $\Pi^W e_a\big(\theta\big)$ is concave in θ. The optimal θ satisfies:

$$V'\big(\theta\big)|_{\theta^*} = p_b k_b r_b \theta^{r_b - 1} \int_0^{\frac{Q_b^*}{k_b \theta^{r_b}}} D\psi\big(D\big)dD + p_o k_f r_f \theta^{r_f - 1} \int_0^{\frac{Q_o^*}{1 + k_f \theta^{r_f}}} D\psi\big(D\big)dD. \quad \blacksquare$$

Proof of Theorem 5.3

We use the bordered Hessian matrix to prove that $\Pi(Q_b, \theta)$ is jointly quasi-concave. The bordered Hessian of $\Pi\big(Q_b, \theta\big)$ is as follows:

$$B = \begin{pmatrix} \frac{\partial^2 \Pi}{\partial Q_b^2} & \frac{\partial^2 \Pi}{\partial Q_b \partial \theta} & \frac{\partial \Pi}{\partial Q_b} \\ \frac{\partial^2 \Pi}{\partial Q_b \partial \theta} & \frac{\partial^2 \Pi}{\partial \theta^2} & \frac{\partial \Pi}{\partial \theta} \\ \frac{\partial \Pi}{\partial Q_b} & \frac{\partial \Pi}{\partial \theta} & 0 \end{pmatrix}.$$

If $|B| = 2\frac{\partial \Pi}{\partial \theta}\frac{\partial \Pi}{\partial Q_b}\frac{\partial^2 \Pi}{\partial Q_b \partial \theta} - \big(\frac{\partial \Pi}{\partial \theta}\big)^2\frac{\partial^2 R}{\partial Q_b^2} - \big(\frac{\partial \Pi}{\partial Q_b}\big)^2\frac{\partial^2 \Pi}{\partial \theta^2} > 0$, then $\Pi\big(Q_b, \theta\big)$ is quasi-concave. Thus we need to know the signs of $\frac{\partial \Pi}{\partial \theta}\frac{\partial \Pi}{\partial Q_b}$, $\frac{\partial \Pi}{\partial \theta}$, $\frac{\partial^2 \Pi}{\partial Q_b \partial \theta}$, $\frac{\partial^2 R}{\partial Q_b^2}$, and $\frac{\partial^2 \Pi}{\partial \theta^2}$.

The first order and second order derivatives of $\Pi\big(Q_b, \theta\big)$ w.r.t. Q_b are:

$$\frac{\partial \Pi}{\partial Q_b} = -w + \frac{p_b k_b \theta^{r_b - r_f} + \big(p_o + u\big)k_f}{k_b \theta^{r_b - r_f} + k_f}\Big(1 - \Psi\Big(\frac{Q_b}{k_b \theta^{r_b} + k_f \theta^{r_f}}\Big)\Big), \tag{A.1}$$

$$\frac{\partial^2 \Pi}{\partial Q_b^2} = -\psi\Big(\frac{Q_b}{k_b\theta^{r_b} + k_f\theta^{r_f}}\Big)\frac{p_b k_b \theta^{r_b - r_f} + \big(p_o + u\big)k_f}{\big(k_b\theta^{r_b-r_f} + k_f\big)\big(k_b\theta^{r_b} + k_f\theta^{r_f}\big)} < 0.$$

The first order and second order derivatives of $\Pi(Q_b, \theta)$ w.r.t. θ are:

$$\begin{aligned}\frac{\partial \Pi}{\partial \theta} =& \frac{k_f k_b\big(r_b - r_f\big)\theta^{r_b-r_f}\big(p_b - p_o - u\big)}{\big(k_b\theta^{r_b-r_f} + k_f\big)^2} E[\min\big(D\big(k_b\theta^{r_b} + k_f\theta^{r_f}\big), Q_b\big)] \\ & - V'\big(\theta\big),\end{aligned} \tag{A.2}$$

$$\begin{aligned}\frac{\partial^2 \Pi}{\partial \theta^2} =& E[\min\big(\beta\big(k_b\theta^{r_b} + k_f\theta^{r_f}\big), Q_b\big)] \\ & \frac{k_f k_b\big(r_b - r_f\big)^2\big(p_b - p_o - u\big)\theta^{r_b-r_f}\big(k_f^2 - k_b^2\theta^{2(r_b-r_f)}\big)}{(k_b\theta^{r_b-r_f} + k_f)^4} - V''\big(\theta\big).\end{aligned}$$

Since $V''\big(\theta\big) > 0$, $r_f < r_b < 1$, $p_o + u \leq p_b$, and $k_f < k_b$, we can determine that $\frac{\partial^2 \Pi}{\partial \theta^2} < 0$.

$$\frac{\partial^2 \Pi}{\partial Q_b \partial \theta} = \Big(1 - \Psi\big(\frac{Q_b}{k_b\theta^{r_b} + k_f\theta^{r_f}}\big)\Big)\frac{k_f k_b\big(r_b - r_f\big)\theta^{r_b-r_f}\big(p_b - p_o - u\big)}{\big(k_b\theta^{r_b-r_f} + k_f\big)^2}.$$

It is easy to see that $\frac{\partial^2 \Pi}{\partial Q_b \partial \theta} \geq 0$, $\Pi(Q_b, \theta)$ is supermodular and thus $\frac{\partial \Pi}{\partial \theta}\frac{\partial \Pi}{\partial Q_b} \geq 0$ [Milgrom and Roberts, 1990].

As shown above, $\frac{\partial^2 \Pi}{\partial \theta^2} < 0$, $\frac{\partial^2 \Pi}{\partial Q_b^2} < 0$, $\frac{\partial \Pi}{\partial \theta}\frac{\partial \Pi}{\partial Q_b} \geq 0$ and $\frac{\partial^2 R}{\partial Q_b \partial \theta} \geq 0$, then $|B| > 0$, so $\Pi\big(Q, \theta\big)$ is quasi-concave.

Next, let's denote $Q_b^* = \big(k_b\theta^{r_b} + k_f\theta^{r_f}\big)\Psi^{-1}\Big(1 - \frac{k_b\theta^{r_b-r_f} + k_f}{p_b k_b\theta^{r_b-r_f} + \big(p_o+u\big)k_f}w\Big)$, such that $\frac{\partial \Pi}{\partial Q_b}|_{Q_b^*} = 0$. Since $\Psi^{-1}(\cdot)$ is a monotonically increasing function, we can easily verify from (A.1) that $\frac{\partial \Pi}{\partial Q_b} \geq 0$ if $Q_b \leq Q_b^*$, and $\frac{\partial \Pi}{\partial Q_b} < 0$ if $Q_b > Q_b^*$.

Hence $\Pi\big(Q_b, \theta\big)$ is not monotone, and since it is quasi-concave in Q_b and θ, $\Pi\big(Q_b, \theta\big)$ is unimodal. Thus Q_b^* is the optimal order quantity. From Equation (A.2), we can obtain the optimal effort which satisfies:

$$V'\big(\theta\big)|_{\theta^*} = \frac{k_f k_b \Big(r_b - r_f\Big)\theta^{r_b - r_f}\Big(p_b - p_o - u\Big)}{\Big(k_b\theta^{r_b - r_f} + k_f\Big)^2} E[\min\Big(D\Big(k_b\theta^{r_b} + k_f\theta^{r_f}\Big), Q_b\Big)]. \quad \blacksquare$$

Proof of Theorem 5.4

With the two retailers' optimal order quantities and the brick-and-mortar retailer's sales effort, we can obtain the manufacturer's profit function as follows:

$$\begin{aligned}\Pi\big(u\big) =& \big(w - c\big)\big(Q_b^* + Q_o^*\big) - u\frac{Q_b^*}{k_b\theta^{*r_b - r_f} + k_f},\\ =& \big(w - c - u\big)\Psi^{-1}\Big(1 - \frac{k_b\theta^{r_b - r_f} + k_f}{p_b k_b\theta^{r_b - r_f} + \big(p_o + u\big)k_f}w\Big) + \big(w - c\big)\Psi^{-1}\big(\frac{p_o - w}{p_o}\big).\end{aligned} \tag{A.3}$$

We denote the first term by $\big(w - c - u\big)\Psi^{-1}\Big(g(u)\Big)$, and the first and second order derivatives of $g(u)$ w.r.t u are:

$$\frac{\partial g(u)}{\partial u} = \frac{k_f k_b\theta^{*r_b - r_f} + k_f^2 w}{(p_b k_b\theta^{*r_b - r_f} + \big(p_o + u\big)k_f)^2} > 0,$$

$$\frac{\partial^2 g(u)}{\partial u^2} = -\frac{2k_f^3 w + 2k_b k_f^2\theta^{r_b - r_f}w}{(k_b p_b\theta^{r_b - r_f} + k_f(p_o + u))^3} < 0.$$

thus $g(u)$ is increasing and concave in u. Since $\Psi^{-1}\big(\cdot\big)$ is a monotonically increasing function, then $\Psi^{-1}\Big(g(u)\Big)$ is concave in u, thus $\frac{\partial^2\Psi^{-1}\Big(g(u)\Big)}{\partial u^2} < 0$. The second term in (A.3) is a constant, thus $\Pi\big(u\big)$ is concave in u. $\blacksquare$

Proof of Theorem 5.5

We have shown that $\Pi^W e_a\big(\theta\big)$ is concave in θ, thus we can obtain the optimal effort satisfying:

$$V'\big(\theta\big)|_{\theta^*} = p_b k_b r_b\theta^{r_b - 1}\int_0^{\frac{Q_b^*}{k_b\theta^{r_b}}} D\psi\big(D\big)dD + p_o k_f r_f\theta^{r_f - 1}\int_0^{\frac{Q_o^*}{1 + k_f\theta^{r_f}}} D\psi\big(D\big)dD.$$

After plugging in $Q_b^*(\theta) = k_b\theta^{r_b}\Psi^{-1}\big(\frac{p_b-c}{p_b}\big)$ and $Q_o^*(\theta) = \big(1 + k_f\theta^{r_f}\big)\Psi^{-1}\big(\frac{p_o-c}{p_o}\big)$, and uniform distribution, the closed form solution to the integrated supply chain sales effort is:

$$\theta^{I*} = \Big(\frac{\big(p_b - c\big)k_b r_b + \big(p_o - c\big)k_f r_f}{2a}\Big)^{\frac{1}{2\left(r_f+r_b\right)}}. \tag{A.4}$$

The manufacturer's profit equation (5.6) in the selective rebate can be simplified as:

$$\Pi_m^S = \big(w - c\big)\beta\big(\frac{p_o - w}{p_o}\big) + \big(w - c - u\big)\beta\Big(1 - \frac{k_b\theta^{r_b - r_f} + k_f}{p_b k_b\theta^{r_b-r_f} + \big(p_o + u\big)k_f}w\Big).$$

After obtaining the first order derivative, we can get the optimal rebate $u^{S*} = \frac{k_b\left(p_o-w\right)\theta^{*r_b-r_f}+k_f p_o}{\left(p_b k_b\theta^{*r_b-r_f}+p_o k_f\right)^2}$.

By plugging the optimal u^{S*} into equations (5.4) and (5.5), we can obtain:

$$\theta^{S*} = \Big(\frac{\left(p_b-c\right)k_b r_b+\left(p_o-c\right)k_f r_f}{4a}\Big)^{\frac{1}{2\left(r_f+r_b\right)}} = \theta^{I*}. \quad \blacksquare$$

Proof of Theorem 5.6

The optimal order quantity as a function of θ for the integrated supply chain is as follows.

In the integrated supply chain:

$$Q_b^{I*} = k_b\theta^{r_b}\Psi^{-1}\big(\frac{p_b - c}{p_b}\big). \tag{A.5}$$

In the selective rebate contract:

$$Q_b^{S*} = \big(k_b\theta^{r_b} + k_f\theta^{r_f}\big)\Psi^{-1}\Big(1 - \frac{k_b\theta^{r_b-r_f} + k_f}{p_b k_b\theta^{r_b-r_f} + \big(p_o + u\big)k_f}w\Big). \tag{A.6}$$

We compare Q_b^{I*} and Q_b^{S*}:

$$Q_b^{I*} - Q_b^{S*} = k_b\theta^{r_b}\Psi^{-1}\left(\frac{p_b - c}{p_b}\right) - \left(k_b\theta^{r_b} + k_f\theta^{r_f}\right)\Psi^{-1}\left(1 - \frac{k_b\theta^{r_b - r_f} + k_f}{p_b k_b\theta^{r_b - r_f} + \left(p_o + u\right)k_f}w\right)$$

Compare the two terms inside $\Psi^{-1}(\cdot)$,

$$\begin{aligned}&\frac{p_b - c}{p_b} - 1 + \frac{k_b\theta^{r_b - r_f} + k_f}{p_b k_b\theta^{r_b - r_f} + \left(p_o + u\right)k_f}\\ =&\frac{\left(p_b - c\right)p_b k_2\theta^{r_b - r_f} + \left(p_o\left(p_b - c\right) + u\left(p_b - c\right)\right)k_f}{p_b\left(p_b k_2\theta^{r_b - r_f} + \left(p_o + u\right)k_f\right)} > 0.\end{aligned}$$

Since $\Psi^{-1}\left(.\right)$ is an increasing function, and $k_b\theta^{r_b} > k_f\theta^{r_f}$, we have $Q_b^{I*} - Q_b^{S*} > 0$.

■

Proof of Proposition 5.1

In a basic newsvendor model, the optimal order is $Q^* = \Psi^{-1}(\frac{C_u}{C_u + C_o})$. Given the uniform distribution, the order formula can be further specified as $Q^* = \frac{C_u - C_o}{C_u + C_o}\beta + \alpha$, and $\frac{\partial Q^*}{\partial \beta} = \frac{C_u - C_o}{C_u + C_o}$. So when the critical ratio $\frac{C_u}{C_u + C_o} < (>)\ 0.5$, the optimal order decreases (increases) in β. ■

Proof of Theorem 5.7

The closed form solution of the brick-and-mortar retailer's optimal order quantity is:

$$Q_b^* = \left(k_b\theta^{r_b} + k_f\theta^{r_f}\right)\Psi^{-1}\left(1 - \frac{k_b\theta^{r_b - r_f} + k_f}{p_b k_b\theta^{r_b - r_f} + \left(p_o + u\right)k_f}w\right) \tag{A.7}$$

By applying uniform distribution $U(\alpha - \beta, \alpha + \beta)$ to the closed form solution, we have $\Psi(x) = \frac{x - \alpha + \beta}{2\beta}$ for $x \in [\alpha - \beta, \alpha + \beta]$. The inverse function is $x = 2\beta\Psi(x) + \alpha - \beta$.

Plug it into Equation (A.7), we have:

$$Q_b^* = 2\big(k_b\theta^{r_b} + k_f\theta^{r_f}\big)\Big(0.5\beta - \frac{k_b\theta^{r_b-r_f} + k_f}{p_b k_b\theta^{r_b-r_f} + \big(p_o+u\big)k_f} w\beta + 0.5\alpha\Big).$$

The first order derivative of Q_b^* w.r.t. β is:

$$\begin{aligned}\frac{\partial Q_b^*}{\partial \beta} &= 2\big(k_b\theta^{r_b} + k_f\theta^{r_f}\big)\Big(0.5 - \frac{k_b\theta^{r_b-r_f} + k_f}{p_b k_b\theta^{r_b-r_f} + \big(p_o+u\big)k_f} w\Big) \\ &= 2\big(k_b\theta^{r_b} + k_f\theta^{r_f}\big)\Big[\frac{\big(0.5p_b - w\big)k_b\theta^{r_b-r_f} + \big(0.5p_o + 0.5u - w\big)k_f}{p_b k_b\theta^{r_b-r_f} + \big(p_o+u\big)k_f}\Big]\end{aligned}$$

Let's consider 4 intervals of w: $(0, 0.5p_o{+}0.5u]$, $(0.5p_o{+}0.5u, \frac{p_b+p_o+u}{4}]$, $(\frac{p_b+p_o+u}{4}, 0.5p_b]$, and $(0.5p_b, p_b)$.

When $w \in (0, 0.5p_o + 0.5u]$, it's easy to see that $\frac{\partial Q_b^*}{\partial \beta} > 0$.

When $w \in (0.5p_o + 0.5w, \frac{p_b+p_o+u}{4}]$, $0 \le -(0.5p_o + 0.5u - w) \le 0.5p_b - w$, and also $k_b\theta^{r_b} \ge k_f\theta^{r_f}$ leads to $k_b\theta^{r_b-r_f} \ge k_f$, thus $\frac{\partial Q_b^*}{\partial \beta}$ is positive.

When $w \in (\frac{p_b+p_o+u}{4}, 0.5p_b]$, $-(0.5p_o + 0.5u - w) \ge 0.5p_b - w \ge 0$, thus the sign of $\frac{\partial Q_b^*}{\partial \beta}$ is dependant on the actual value of $\big(0.5p_b - w\big)k_b\theta^{r_b-r_f} + \big(0.5p_o + 0.5u - w\big)k_f$.

When $w \in (0.5p_b, p_b)$, clearly $\big(0.5p_b - w\big)k_b\theta^{r_b-r_f} + \big(0.5p_o + 0.5u - w\big)k_f < 0$, and thus $\frac{\partial Q_b^*}{\partial \beta}$ is negative. ∎

Proof of Theorem 5.8

We use a similar method as the proof of Theorem 5.3. First, we use the bordered Hessian matrix to prove that $\Pi^R(Q_b, \theta)$ is jointly quasi-concave. In the proof, we use Π to represent Π^R for simplicity. The bordered Hessian of $\Pi\big(Q_b, \theta\big)$ is as follows:

$$B = \begin{pmatrix} \frac{\partial^2\Pi}{\partial Q_b^2} & \frac{\partial^2\Pi}{\partial Q_b\partial\theta} & \frac{\partial\Pi}{\partial Q_b} \\ \frac{\partial^2\Pi}{\partial Q_b\partial\theta} & \frac{\partial^2\Pi}{\partial\theta^2} & \frac{\partial\Pi}{\partial\theta} \\ \frac{\partial\Pi}{\partial Q_b} & \frac{\partial\Pi}{\partial\theta} & 0 \end{pmatrix}.$$

If $|B| = 2\frac{\partial\Pi}{\partial\theta}\frac{\partial\Pi}{\partial Q_b}\frac{\partial^2\Pi}{\partial Q_b\partial\theta} - \big(\frac{\partial\Pi}{\partial\theta}\big)^2\frac{\partial^2 R}{\partial Q_b^2} - \big(\frac{\partial\Pi}{\partial Q_b}\big)^2\frac{\partial^2\Pi}{\partial\theta^2} > 0$, then $\Pi\big(Q_b,\theta\big)$ is quasi-concave. Thus we need to know the signs of $\frac{\partial\Pi}{\partial\theta}\frac{\partial\Pi}{\partial Q_b}$, $\frac{\partial\Pi}{\partial\theta}$, $\frac{\partial^2\Pi}{\partial Q_b\partial\theta}$, $\frac{\partial^2 R}{\partial Q_b^2}$, and $\frac{\partial^2\Pi}{\partial\theta^2}$.

The first order and second order derivatives of $\Pi\big(Q_b,\theta\big)$ w.r.t. Q_b are:

$$\frac{\partial\Pi}{\partial Q_b} = -w + \frac{p_b(1-\mu)k_b\theta^{r_b-r_f} + p_o(1-\mu)k_f}{k_b\theta^{r_b-r_f}+k_f}\Big(1-\Psi\Big(\frac{Q_b}{k_b\theta^{r_b}+k_f\theta^{r_f}}\Big)\Big), \tag{A.8}$$

$$\frac{\partial^2\Pi}{\partial Q_b^2} = -\psi\Big(\frac{Q_b}{k_b\theta^{r_b}+k_f\theta^{r_f}}\Big)\frac{(1-\mu)p_bk_b\theta^{r_b-r_f}+p_o(1-\mu)k_f}{\big(k_b\theta^{r_b-r_f}+k_f\big)\big(k_b\theta^{r_b}+k_f\theta^{r_f}\big)} < 0.$$

The first order and second order derivatives of $\Pi(Q_b,\theta)$ w.r.t. θ are:

$$\begin{aligned}\frac{\partial\Pi}{\partial\theta} =& \frac{k_fk_b\big(r_b-r_f\big)\theta^{r_b-r_f}\big(p_b-p_o\big)(1-\mu)}{\big(k_b\theta^{r_b-r_f}+k_f\big)^2}E[\min\big(D\big(k_b\theta^{r_b}+k_f\theta^{r_f}\big),Q_b\big)] \\ &- V'\big(\theta\big),\end{aligned} \tag{A.9}$$

$$\begin{aligned}\frac{\partial^2\Pi}{\partial\theta^2} =& E[\min\big(\beta\big(k_b\theta^{r_b}+k_f\theta^{r_f}\big),Q_b\big)] \\ & \frac{k_fk_b\big(r_b-r_f\big)^2\big(p_b-p_o\big)(1-\mu)\theta^{r_b-r_f}\big(k_f^2-k_b^2\theta^{2(r_b-r_f)}\big)}{(k_b\theta^{r_b-r_f}+k_f)^4} - V''\big(\theta\big).\end{aligned}$$

Since $V''\big(\theta\big) > 0$, $r_f < r_b < 1$, $p_o \le p_b$, and $k_f < k_b$, we can determine that $\frac{\partial^2\Pi}{\partial\theta^2} < 0$.

$$\frac{\partial^2\Pi}{\partial Q_b\partial\theta} = \Big(1-\Psi\big(\frac{Q_b}{k_b\theta^{r_b}+k_f\theta^{r_f}}\big)\Big)\frac{k_fk_b\big(r_b-r_f\big)\theta^{r_b-r_f}\big(p_b-p_o\big)(1-\mu)}{\big(k_b\theta^{r_b-r_f}+k_f\big)^2}.$$

It is easy to see that $\frac{\partial^2\Pi}{\partial Q_b\partial\theta} \ge 0$, $\Pi(Q_b,\theta)$ is supermodular and thus $\frac{\partial\Pi}{\partial\theta}\frac{\partial\Pi}{\partial Q_b} \ge 0$.

As shown above, $\frac{\partial^2\Pi}{\partial\theta^2} < 0$, $\frac{\partial^2\Pi}{\partial Q_b^2} < 0$, $\frac{\partial\Pi}{\partial\theta}\frac{\partial\Pi}{\partial Q_b} \ge 0$ and $\frac{\partial^2 R}{\partial Q_b\partial\theta} \ge 0$, then $|B| > 0$, so $\Pi\big(Q,\theta\big)$ is quasi-concave.

Next, let's denote $Q_b^* = \left(k_b\theta^{r_b} + k_f\theta^{r_f}\right)\Psi^{-1}\left(1 - \frac{k_b\theta^{r_b-r_f}+k_f}{(1-\mu)p_bk_b\theta^{r_b-r_f}+p_o(1-\mu)k_f}w\right)$, such that $\frac{\partial\Pi}{\partial Q_b}|_{Q_b^*} = 0$. Since $\Psi^{-1}(\cdot)$ is a monotonically increasing function, we can easily verify from (A.8) that $\frac{\partial\Pi}{\partial Q_b} \geq 0$ if $Q_b \leq Q_b^*$, and $\frac{\partial\Pi}{\partial Q_b} < 0$ if $Q_b > Q_b^*$.

Hence $\Pi\left(Q_b, \theta\right)$ is not monotone, and since it is quasi-concave in Q_b and θ, $\Pi\left(Q_b, \theta\right)$ is unimodal. We can obtain the optimal effort which satisfies:

$$V'\left(\theta\right)|_{\theta^*} = \frac{k_fk_b\left(r_b-r_f\right)\theta^{r_b-r_f}\left(p_b-p_o\right)(1-\mu)}{\left(k_b\theta^{r_b-r_f}+k_f\right)^2}E[\min\left(D\left(k_b\theta^{r_b} + k_f\theta^{r_f}\right), Q_b\right)]. \quad \blacksquare$$

Proof of Theorem 5.9

In the proof, we use Π to represent Π^R for simplicity. With the two retailers' optimal order quantities and the brick-and-mortar retailer's sales effort, we can obtain the manufacturer's profit function as follows:

$$\begin{aligned}\Pi\left(\mu\right) =&\left(w-c\right)\left(Q_b^* + Q_o^*\right) + \frac{\mu p_bk_b\theta^{r_b-r_f} + p_o\mu k_f}{k_b\theta^{r_b-r_f} + k_f}Q_b^*\left(\mu\right),\\ =&\left(w-c\right)\Psi^{-1}\left(1 - \frac{k_b\theta^{r_b-r_f} + k_f}{p_bk_b\theta^{r_b-r_f} + \mu p_ok_f}w\right) + \left(w-c\right)\Psi^{-1}\left(\frac{p_o - w}{\mu p_o}\right).\end{aligned} \tag{A.10}$$

We denote the first term by $\left(w-c\right)\Psi^{-1}\left(g(\mu)\right)$, and the first and second order derivatives of $g(\mu)$ w.r.t u are:

$$\frac{\partial g(\mu)}{\partial\mu} = \frac{\mu k_fk_b\theta^{*r_b-r_f} + k_f^2w}{(p_bk_b\theta^{*r_b-r_f} + \mu p_ok_f)^2} > 0,$$

$$\frac{\partial^2 g(\mu)}{\partial\mu^2} = -\frac{2k_f^3w\mu^2 + 2k_bk_f^2\theta^{r_b-r_f}w}{(k_bp_b\theta^{r_b-r_f} + k_fp_o\mu)^3} < 0.$$

thus $g(\mu)$ is increasing and concave in μ. Since $\Psi^{-1}\left(\cdot\right)$ is a monotonically increasing function, then $\Psi^{-1}\left(g(\mu)\right)$ is concave in μ, thus $\frac{\partial^2\Psi^{-1}\left(g(\mu)\right)}{\partial\mu^2} < 0$. The second term in (A.10) is a constant, thus $\Pi\left(\mu\right)$ is concave in μ, We assume the cost function of sales effort as $V\big(\theta\big) = a\theta^2$, and the demand is uniformly distributed as $U\big(\alpha-\beta, \alpha+\beta\big)$ and we can obtain the optimal μ^*:

$$\mu^* = \frac{k_f k_b \theta^{*r_b - r_f} + p_b k_b \theta^{*r_b - r_f}}{4a(p_o k_f)^2}.$$

■

Proof of Theorem 5.10

We analyze the brick-and-mortar retailer's effort and order quantity by considering two cases for a given $T > 0$: $Q_b \le T$ and $Q_b > T$. In the following, we prove that $\Pi_b^T\big(Q_b, \theta|T\big)$ is concave in Q_b in each interval. Note that:

when $Q_b \le T$

$$\frac{\partial \Pi_b^T\big(Q_b, \theta|T\big)}{\partial Q_b} = -w + p_b\big(1 - \Psi\big(\frac{Q_b}{k_f\theta^{r_f} + k_b\theta^{r_b}}\big)\big),$$

$$\frac{\partial^2 \Pi_b^T\big(Q_b, \theta|T\big)}{\partial Q_b^2} = -p_b\psi\big(\frac{Q_b}{k_f\theta^{r_f} + k_b\theta^{r_b}}\big)\frac{1}{k_f\theta^{r_f} + k_b\theta^{r_b}} \le 0,$$

and when $Q_b > T$

$$\frac{\partial \Pi_b^T\big(Q_b, \theta|T\big)}{\partial Q_b} = p_b - w + r - \big(p_b + r\big)\Psi\big(\frac{Q_b}{k_f\theta^{r_f} + k_b\theta^{r_b}}\big),$$

$$\frac{\partial^2 \Pi_b^T\big(Q_b, \theta|T\big)}{\partial Q_b^2} = -\big(p_b + r\big)\psi\big(\frac{Q_b}{k_f\theta^{r_f} + k_b\theta^{r_b}}\big)\frac{1}{k_f\theta^{r_f} + k_b\theta^{r_b}} \le 0.$$

For the sales effort, we only give the first order derivative when $Q_b > T$, since we will rule out the trivial scenario of $Q_b \leq T$ later.

$$\frac{\partial \Pi_b^T(Q_b,\theta|T)}{\partial \theta} = k_f k_b(r_b - r_f)\theta^{r_b - r_f}(p_b + r)E[\min(\beta(k_b\theta^{r_b} + k_f\theta^{r_f}), Q_b)] - V'(\theta).$$

The optimal θ satisfies:

$$V'(\theta)|_{\theta^*} = k_f k_b(r_b - r_f)\theta^{r_b - r_f}(p_b + r)E[\min(\beta(k_b\theta^{r_b} + k_f\theta^{r_f}), Q_b)]. \quad \text{(A.11)}$$

Therefore $\Pi_b^T(Q_b,\theta|T)$ is concave in Q_b in each interval $[0,T]$ and (T,∞). In these two intervals, the optimal order quantity is: When $Q_b \leq T$, $Q_b^* = (k_f\theta^{r_f} + k_b\theta^{r_b})\Psi^{-1}(\frac{p_b-w}{p_b})$; when $Q_b > T$, $Q_b^* = (k_f\theta^{r_f} + k_b\theta^{r_b})\Psi^{-1}(\frac{p_b-w+r}{p_b+r})$. However, the first case indicates that the rebate is not used by the retailer, and the manufacturer should avoid offering such target level. To address this issue, we introduce Lemma A.1 as follows.

Lemma A.1 *There is a unique target threshold $\tau(T)$ such that: If $T > \tau(T)$, the retailer orders less than T, and hence the rebate will never be used; if $T < \tau(T)$, the retailer orders more than T; and if $T = \tau(T)$, the retailer feels indifferent on the target rebate.*

Proof. Denote $\underline{\cdot}$ to the cases when $Q_b^* \leq T$, and $\bar{\cdot}$ to the notations when $Q_b^* > T$.

When $Q_b^* \leq T$, the retailer's profit is:

$$\underline{\Pi_b^T}(\underline{Q_b},\underline{\theta}|T) = (p_b - w)(k_f\underline{\theta}^{r_f} + k_b\underline{\theta}^{r_b})\Psi^{-1}(\frac{p_b - w}{p_b}) - V(\underline{\theta}).$$

When $Q_b^* > T$, the profit is:

$$\overline{\Pi}_b^T(\overline{Q_b},\overline{\theta}|T) = (p_b - w + r)(k_f\overline{\theta}^{r_f} + k_b\overline{\theta}^{r_b})\Psi^{-1}(\frac{p_b - w + r}{p_b + r}) - V(\overline{\theta}) - rT.$$

By comparing the profit functions, the retailer chooses wether or not to order a higher quantity than T.

$$\begin{aligned}\overline{\Pi}_b^T\big(\overline{Q_b},\overline{\theta}|T\big)-\underline{\Pi}_b^T\big(\underline{Q_b},\underline{\theta}|T\big) =&(p_b-w+r)(k_f\overline{\theta}^{r_f}+k_b\overline{\theta}^{r_b})\Psi^{-1}\big(\frac{p_b-w+r}{p_b+r}\big)\\ &-(p_b-w)(k_f\underline{\theta}^{r_f}+k_b\underline{\theta}^{r_b})\Psi^{-1}\big(\frac{p_b-w}{p_b}\big)\\ &-V(\overline{\theta})+V(\underline{\theta})-rT.\end{aligned} \tag{A.12}$$

Divide both sides of (A.12) by r and denote:

$$\begin{aligned}\tau(T)=&\frac{(p_b-w+r)(k_f\overline{\theta}^{r_f}+k_b\overline{\theta}^{r_b})\Psi^{-1}\big(\frac{p_b-w+r}{p_b+r}\big)}{r}\\ &-\frac{(p_b-w)(k_f\underline{\theta}^{r_f}+k_b\underline{\theta}^{r_b})\Psi^{-1}(\frac{p_b-w}{p_b})-V(\overline{\theta})+V(\underline{\theta})}{r}.\end{aligned} \tag{A.13}$$

$\tau(T)$ is a function of T because r and θ are functions of T. Equation (A.12) can be rewritten as:

$$\frac{\overline{\Pi}_b^T\big(\overline{Q_b},\overline{\theta}|T\big)-\underline{\Pi}_b^T\big(\underline{Q_b},\underline{\theta}|T\big)}{r}=\tau(T)-T. \tag{A.14}$$

Now we can compare $\tau(T)-T$ instead of $\overline{\Pi}_b^T\big(\overline{Q_b},\overline{\theta}|T\big)-\underline{\Pi}_b^T\big(\underline{Q_b},\underline{\theta}|T\big)$. The sign of $\tau(T)-T$ is dependent on the setting of parameters. For example, when the cost of effort is ignorable, and $T=0$, then we have $T\le\tau(T)$; if r_f, r_b are extremely small and T is extremely large, then $\tau(T)<0$ and $\tau(T)<T$, intuitively speaking, if T is unreasonably large, the retailer will ignore the manufacturer's rebate offer. So for each T, there is a unique $\tau(T)$ such that: If $T>\tau(T)$, the retailer rejects the target rebate contract, i.e., ordering less than T; if $T<\tau(T)$, the retailer accepts the contract and makes an order quantity above T; and if $T=\tau(T)$, the retailer feels indifferent to the target rebate. ■

Lemma A.1 tells us that there is a upper bound of T for the manufacturer to offer an effective rebate contract to the brick-and-mortar retailer. To avoid trivial scenarios like the retailer refusing to participate, the manufacturer offers a target $T < \tau(T)$. By doing so, the brick-and-mortar retailer's profit is concave in Q_b and the optimal order quantity is uniquely determined by $Q_b^* = (k_f\theta^{r_f} + k_b\theta^{r_b})\Psi^{-1}\big(\frac{p_b-w+r}{p_b+r}\big)$. ■

Proof of Proposition 5.2

With $Q_b^* = k_f\theta^{r_f}\Psi^{-1}\big(\frac{p_b-w+r}{p_b+r}\big)$ and $Q_o^* = \Psi^{-1}\big(\frac{p_o-w}{p_o}\big)$, considering the case of $Q_b > T$, we have:

$$
\begin{aligned}
\Pi_m^T\big(r|T\big) =&\big(w-c\big)\big(k_f\theta^{r_f}\Psi^{-1}\big(\frac{p_b-w+r}{p_b+r}\big)\\
&+u\Psi^{-1}\big(\frac{p_o-w}{p_o}\big)\big)+r\big(k_f\theta^{r_f}\Psi\big(\frac{p_b-w+r}{p_b+r}\big)-T\big),\\
=&\big(w-c+r\big)k_f\theta^{r_f}\Psi^{-1}\big(\frac{p_b-w+r}{p_b+r}\big)\\
&-rT+\big(w-c\big)\Psi^{-1}\big(\frac{p_o-w}{p_o}\big).
\end{aligned}
\tag{A.15}
$$

Since $\frac{\partial^2\left(\frac{p_b-w+r}{p_b+r}\right)}{\partial r^2} = \frac{-w}{(p_b+r)^2} < 0$, $\frac{p_b-w+r}{p_b+r}$ is concave in r. $\Psi^{-1}\big(\frac{p_b-w+r}{p_b+r}\big)$ is concave in r because $\Psi^{-1}(\cdot)$ is an increasing function, thus $\Pi_m^T\big(r|T\big)$ is concave in r. Based on the first order condition, we can obtain r^* w.r.t. T as follows:

$$
r^*\big(T\big) = \big(p_b-w\big)\frac{T-\Psi^{-1}\big(\frac{p_b-w}{p_b}\big)}{\Psi^{-1}\big(\frac{p_b-c}{p_b}\big)}
\tag{A.16}
$$

Though (A.16) satisfies the first order condition, we are not sure that (A.16) is the optimal solution since r is also bounded by $[0, p_b-p_o]$. If $r^*\big(T\big) > p_b - p_o$, then the optimal is at the border point $r^* = p_b - p_o$, so $\frac{T-\Psi^{-1}\left(\frac{p_b-w}{p_b}\right)}{\Psi^{-1}\left(\frac{p_b-c}{p_b}\right)} > 1$, thus $T > \Psi^{-1}\big(\frac{p_b-c}{p_b}\big) + \Psi^{-1}\big(\frac{p_b-w}{p_b}\big)$. Since the system optimal is $\Psi^{-1}\big(\frac{p_b-c}{p_b}\big)$, if T is bigger than the system optimal, the retailer will reject the contract, contradicting the assumption that the manufacturer offers the contract to effectively seduce the retailer to order more than the target threshold. So $r^*\big(T\big) \le p_b - p_o$, and $r^*\big(T\big)$ is optimal. ■

Proof of Theorem 5.11

In Equation (A.16), obviously $T - \Psi^{-1}\left(\frac{p_b - w}{p_b}\right)$ is linear in T, thus $r^*(T)$ is linear in T. Considering that $\Pi_m^T\big(r|T\big)$ is concave in $r^*(T)$, so $\Pi_m^T\big(r|T\big)$ is concave in T. Substitute (A.16) into (A.15), the manufacturer's profit is a function of T only. Provided the explicit forms of demand distribution and $V(\theta)$, we can obtain T^* numerically.

Note that $\Psi^{-1}\left(\frac{p_b - w}{p_b}\right)$ approximates the brick-and-mortar retailer's order quantity in the baseline case. To ensure $r^*\big(T\big)$ is feasible, $T \geq \Psi^{-1}\left(\frac{p_b - w}{p_b}\right)$, which indicates that T has to be at least as large as the order quantity in the baseline case to ensure the manufacturer's profitability under the target rebate contract. ■

Remark A.1 *Considering that Lemma A.1 offers a upper bound of T to ensure the brick-and-mortar retailer's profitability, here we can summarize the boundaries of T as: For the target rebate threshold T, there exist an upper bound and a lower bound to ensure the brick-and-mortar retailer and the manufacturer's profitability.*

Proof of Proposition 5.3.

The proof is similar to that of Theorem 5.3, thus omitted. ■

Proof of Theorem 5.12

Note that $\Psi^{-1}\Big(1 - \frac{k_b\theta^{r_b - r_f} + k_f}{p_b k_b \theta^{r_b - r_f} + p_o k_f}(w-d)\Big)$ is increasing and quasi-convex in d, therefore $\big(w - c - d\big)\Psi^{-1}\Big(1 - \frac{k_b\theta^{r_b - r_f} + k_f}{p_b k_b \theta^{r_b - r_f} + p_o k_f}(w - d)\Big)$ is quasi-concave in d, and thus $\Pi_m^W\big(d\big)$ is quasi-concave in d. Since d is bounded by [0, w], there exists a unique d that maximizes the manufacturer's profit. The value of optimal d can be obtained given the explicit forms of demand distribution and $V(\theta)$. ■

APPENDIX B

TABLE OF NOTATIONS

Table B.1: Notations for the Deterministic Demand Model

Notation	Description
w	Wholesale price.
θ	Sales effort.
u	Selective rebate.
μ	Revenue sharing rate.
r	Target rebate.
T	Target level.
$V(\theta)$	Cost function of sales effort.
c	Unit production cost.
p_b, p_o	Retail prices
a_b, a_f, a_o	Base demand
h	Sales effort cost coefficient.
τ_b,τ_f	The scaling factor of sales effort on demand of the traditional (free-riding) customers.

Table B.2: Notations for the Stochastic Demand Model

Notation	Description
c	Unit production cost.
D	Base random demand.
w	Wholesale price.
θ	Sales effort.
$V\big(\theta\big)$	Cost function of sales effort.
a	Sales effort cost coefficient.
$p_b(p_o)$	Brick-and-mortar (online) retailer's unit retail price.
$Q_b(Q_o)$	Brick-and-mortar (online) retailer's order quantity.
$k_b(k_f)$	The scaling factor for the relative initial market size of the traditional (free-riding) customers.
$r_b(r_f)$	The scaling factor of market expansion by extra sales efforts of the traditional (free-riding) customers.
u	The selective rebate given by the manufacturer to the brick-and-mortar retailer.
r	The target rebate given by the manufacturer to the brick-and-mortar retailer.
T	The order threshold for the brick-and-mortar retailer to receive target rebate.
d	The wholesale price discount given by the manufacturer to the brick-and-mortar retailer.

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We study a supply chain with one manufacturer and two retail channels, where an online retailer offers a lower price and free-rides a brick-and-mortar retailer's sales effort. The free riding effect reduces brick-and-mortar retailer's desired effort level, and thus hurts the manufacturer's profit and the overall supply chain performance. To coordinate the efforts, we design and compare several supply chain contracts: a selective rebate contract with price match, a revenue sharing contract with price match, and a target rebate contract with price match, as well as a wholesale price contract with price match. We study the contract performance under both deterministic and stochastic demands. Under deterministic demand, the analysis goes with two cases: the online channel is owned by or independent of the manufacturer.

We show that the selective rebate contract coordinates the supply chain in both cases. It can also allocate the system profit arbitrarily between the supply chain players. Furthermore, in the case that the manufacturer owns the online channel, there exists a solution regime on the Pareto-optimal frontier in which both the manufacturer and the brick-and-mortar retailer are better off from the baseline case. We also show that the manufacturer's optimal rebate only depends on the manufacturer's marginal profit and the consumers' sales effort sensitivities. The optimal rebate is independent of the market size and retail prices. In addition, we show that the revenue sharing contract with price match is equivalent to the selective rebate contract. Under stochastic demand, we show that the selective rebate contract outperforms all other contracts by improving supply chain efficiency.

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