RATES OF CONCENTRATE-ROUGHAGE SUBSTITUTION AND ECONOMIC OPTIMUMS IN FEEDING YEARLING STEERS AND HEIFERS

By

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CHAPTER I

INTRODUCTION

The Problem Investigated

An infinite number of combinations of concentrates and roughage can be fed in alternative beef systems to produce a given amount of beef. A common self-fed feedlot mixture in the Southwest is milo and cottonseed meal as concentrates and cottonseed hulls and alfalfa hay as roughage. The concentrate-roughage ratio in the self-fed mixture varies considerably from one feedlot to another. Some feeders feel that a high concentrate-roughage ratio is necessary, /especially to reach a satisfactory degree of finish on the inside (marbling, etc)7 while other feeders prefer greater bulk in their mixtures.

There has been an increase in feedlot operations in the Southwest. Commercial feeders and farmers (who feed a few cattle as a supplementary enterprise) are utilizing more of the state grown feeds to produce a feedlot-finished animal for the market. Various types of roughage and concentrates are produced in Oklahoma. Alfalfa hay and sorghum silage are two of the major roughages grown. Grain sorghum is the primary feed grain grown in Oklahoma.

Since the optimum combination of concentrates and roughage may be of great economic importance to the Oklahoma farmer and feeder, it appears essential that the farmer should be better informed relative to the choice of the optimum ration. If the farmer were better informed of the rate of substitution of concentrates and roughage, he would be better qualified to choose the optimum combination of feeds to market through cattle each year.

Comparison of Steers and Heifers

Considerable interest has developed in feeding heifers in the Southwest due largely to the price relationships existing during recent years. It is possible that the optimum ratio (concentrate-roughage) might be different for steers and heifers. If this were the case, feeding the two sexes the same ration might not be economical.

When heifers and steers are fed for equal periods of time without regard to differences attained in finish, heifers generally make slower and less efficient gain than steers. However, when fattened to the same slaughter grade, there may be little difference in economy of gain. Also, the feeding period for heifers is shorter.[1]

[1] A. J. Dyer, L. A. Weaver, Fattening Comparisons Steers vs. Heifers, Missouri Agricultural Experiment Station Bulletin No. 646, February, 1955.

CHAPTER II

PROBLEM STATEMENT AND THEORETICAL SOLUTION TO THE PROBLEM

A production function is a means of describing an input-output relationship. A certain amount of input is required to produce a given quantity of product. The amount of output produced is dependent upon the quantity and quality of input applied. A production function can be helpful in the analysis of the transformation of feed to beef. The gain in weight of an animal depends upon several factors (feed, management, initial weight and others); therefore, as the combination and level of these factors vary, weight gain will also vary. A production function representing beef production may be expressed as follows:

(1) $Y = f(X_1, X_2, X_3, \cdots X_n)$

Y = Beef production in weight

X_1 = Concentrates

X_2 = Roughage

$X_3 \cdots X_n$ = Management, labor, initial weights and other relevant factors. This equation states that Y depends upon the application of $X_1, X_2, X_3, \cdots X_n$. A change in the combination and level of the independent variables ($X_1, X_2, X_3, \cdots X_n$) will result in a change of output (Y).

In the production of beef some of the inputs may be fixed at a given level. Thus, with management, labor, initial weight and other factors ($X_3 \cdots X_n$) fixed, we may be interested in the effect on output resulting from varying concentrates (X_1) and roughage (X_2).

A production function with some of the inputs fixed may be expressed as follows:

(2) $Y = f(X_1, X_2/X_3 \cdot\cdot\cdot X_n)$

The vertical line between X_2 and X_3 indicates that all factors to the right of the line were fixed in quantity while the inputs to the left of the line were variables. By holding a portion of the inputs constant and varying X_1 and X_2, it was possible to determine the rate of gain resulting from changes in the variable inputs.

A diagramatic presentation of equation (2) is shown in Figure 1. Total feed inputs were varied but the remainder of the inputs ($X_3 \cdot\cdot\cdot X_n$) were utilized in fixed amounts.[2] The inputs (X_i) were represented on the horizontal axis, and the outputs (Y_i) were represented on the vertical axis. The upward sloping curve illustrates the production function. This functional relationship indicates the relationship between feed consumption and weight with the other inputs held constant. This function is often called a response curve or a growth curve. Under this relationship diminishing returns would be expected. The law of diminishing returns states that

> ...if the input of one resource is increased by equal increments per unit of time while the inputs of other resources are held constant, total product output will increase, but beyond some point the resulting output increases will become smaller and smaller.[3]

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[2] X_1 and X_2 were fed in a fixed proportion and the level of inputs was varied.

[3] Richard H. Leftwich The Price System and Resource Allocation, Rinehart and Company. Inc., New York, 1955.

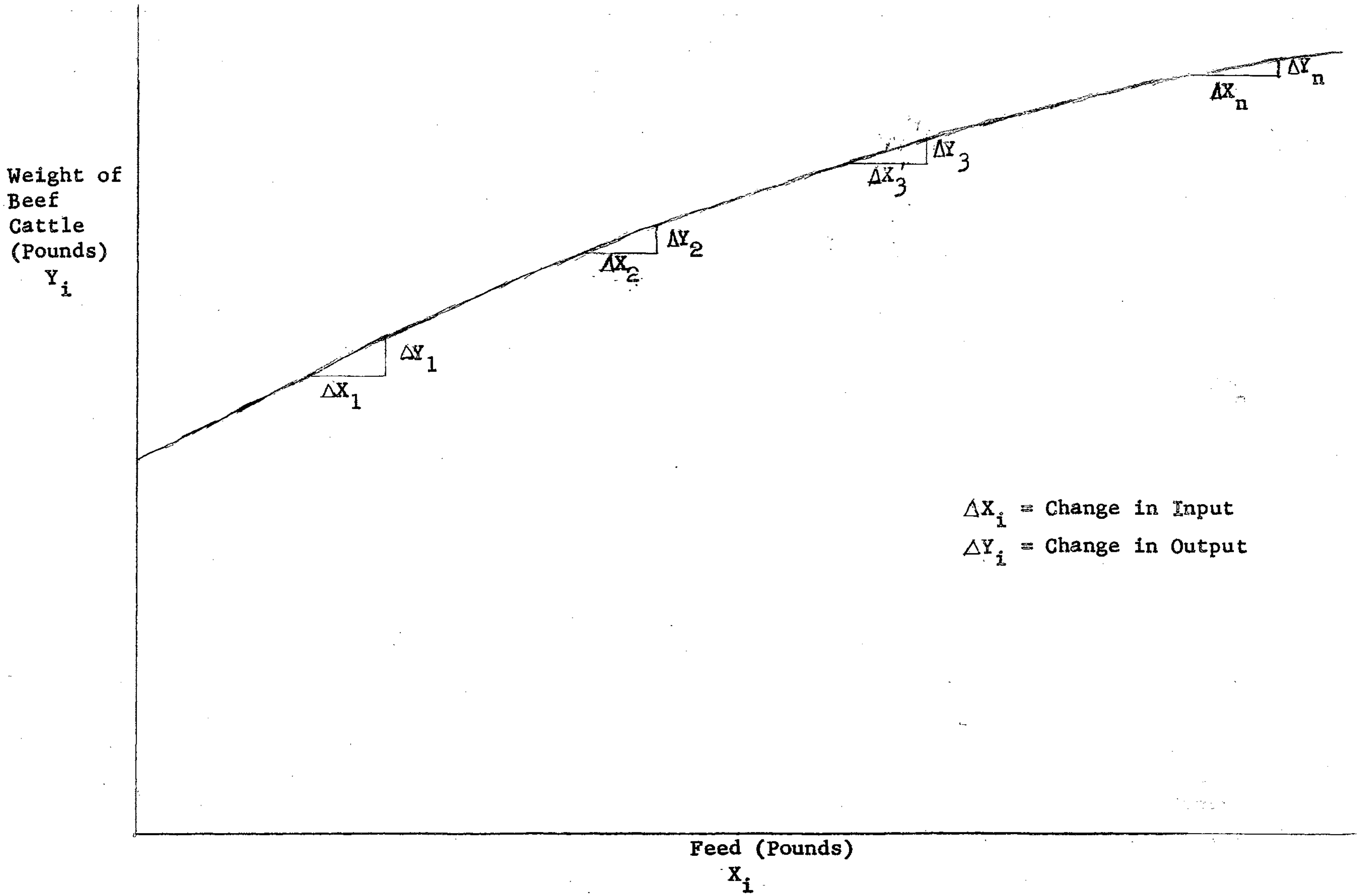

Figure 1. Hypothetical Production Function for Beef Production (Transformation of Feed to Beef with Other Inputs Constant)

The law of diminishing returns is demonstrated by the hypothetical production function in Figure 1. The function indicates that the transformation rate of feed into beef diminishes throughout the feeding period. The change in weight ($\triangle Y$) diminishes ($\triangle Y_1 > \triangle Y_2 > \cdots > \triangle Y_n$) for equal changes in feed inputs ($\triangle X_1 = \triangle X_2 = \triangle X_3 \cdots = \triangle X_n$). Therefore, the marginal rate of transformation of feed to beef diminishes throughout the function. Since the $\triangle X$'s are equal and each additional $\triangle Y$ becomes smaller and smaller, it is obvious that

$$(3) \quad \frac{\triangle Y_1}{\triangle X_1} > \frac{\triangle Y_2}{\triangle X_2} > \frac{\triangle Y_3}{\triangle X_3} \cdots > \frac{\triangle Y_n}{\triangle X_n}$$

Figure 1 indicates the gain in weight resulting from only one combination of concentrates and roughage. It is important for the feeder to know the weight gain resulting from different concentrate-roughage ratios.

An isoquant is a curve displaying equal outputs throughout. The isoquant shown in Figure 2 represents a given weight of beef which can be produced by alternative combinations of concentrates and roughage. Assuming that the isoquant represents 100 pounds gain, this output may result from a wide range of combinations of inputs (X_1, X_2). For example, point C' and C are equal outputs, but produced from different combinations of concentrates and roughage. Quantities "a" of roughage and "b" of concentrates are transformed into 100 pounds of gain while quantities "c" of roughage and "d" of concentrates are converted into the same gain. The slope of the isoquant at any given point is the marginal rate of technical substitution of concentrates and roughage in the ration.

The isoquant in Figure 2 displays a diminishing marginal rate of substitution. Thus, the marginal rate of substitution of concentrates and roughage is greater at point C' (a relatively high roughage ration)

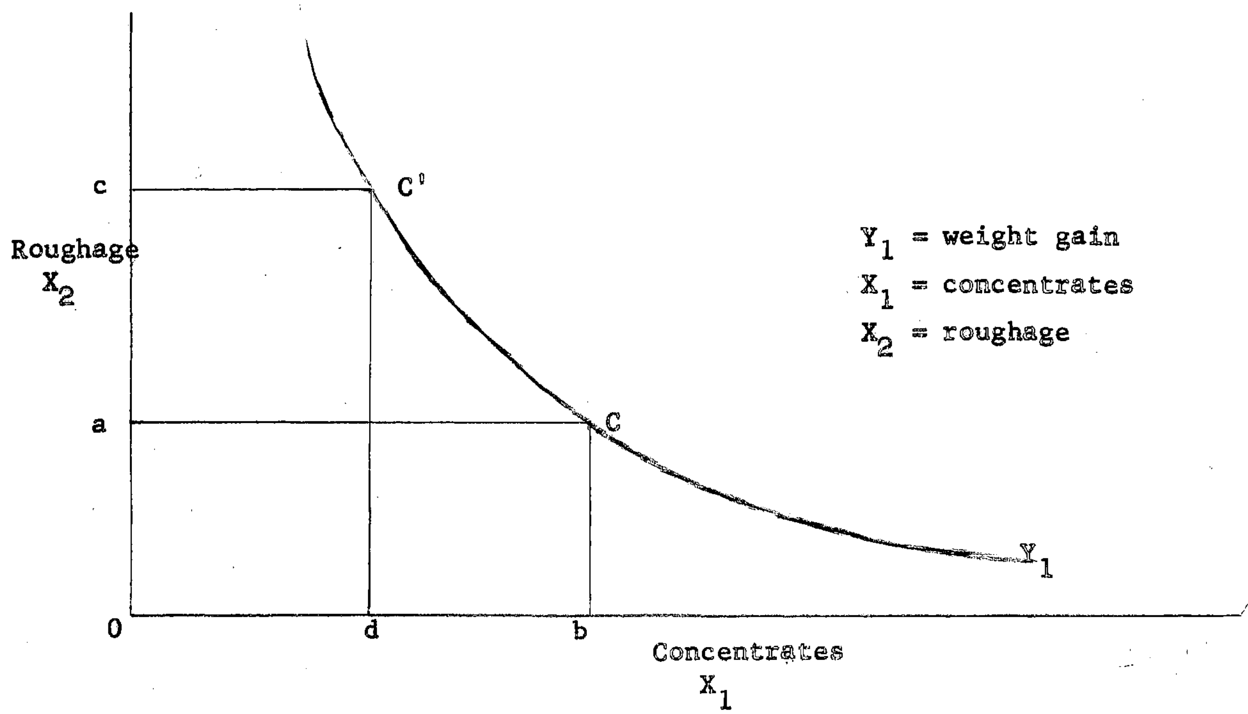

Figure 2. Diagram of an Isoquant Showing a Given Output (Y_1) Produced by Different Combinations of Inputs (X_1, X_2)

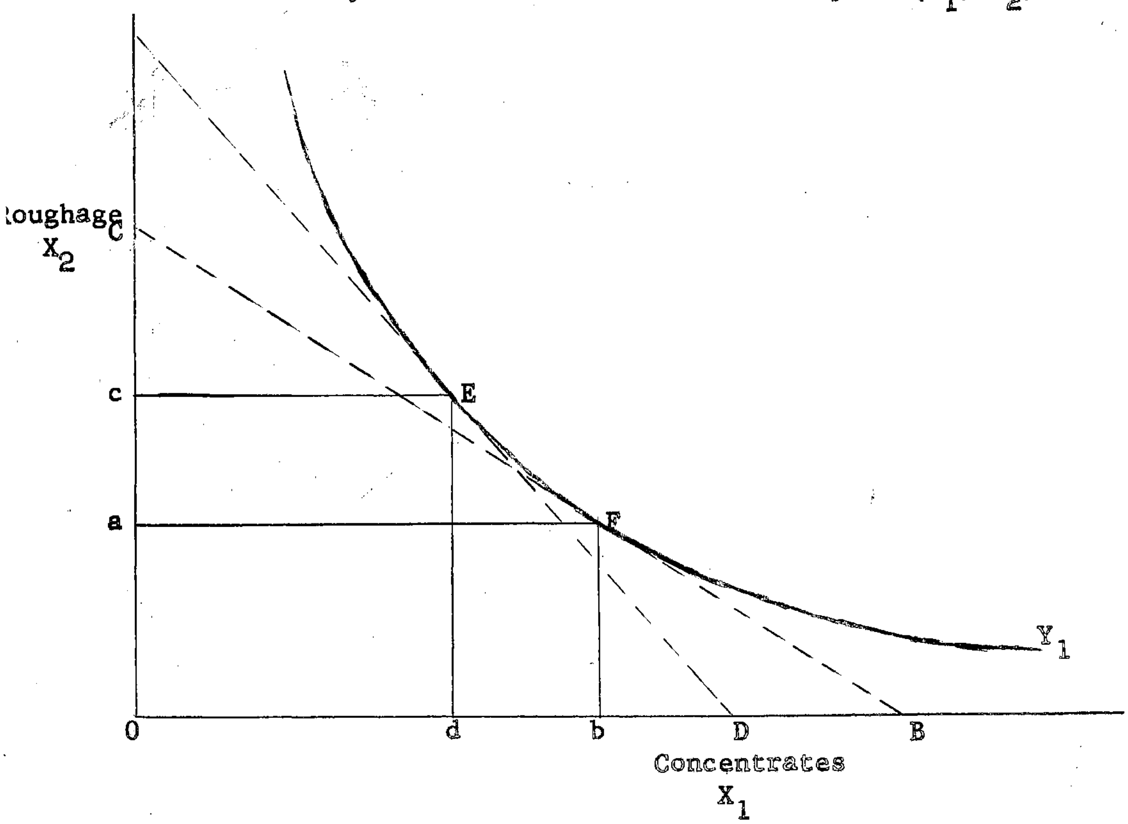

Figure 3. Illustration of Optimum Combinations of Inputs Required to Produce a Given Output Under Given Prices

than at point C (a relatively low roughage ration). In other words, when a ration consists of a relatively small amount of concentrates, a unit of concentrates will substitute for a larger amount of roughage than in a ration relatively high in concentrates.

Criteria for Determining the Optimum Combination of Two Variable Inputs

The optimum combination of the two inputs occurs at the point of tangency of the isoquant and the price line.[4] At this point, the marginal rate of substitution of X_1 and X_2 is equal to the inverse of the price ratio. Points "E" and "F" in Figure 3 are points representing the optimum combinations of the two variable inputs for two price situations.[5] Thus, with changes in the relative prices of X_1 and X_2, the optimum combination of the variable inputs change.

The feeder needs basic information relative to the marginal rate of substitution of concentrates and roughage, and he needs a choice guide which will aid him in selecting the least cost (optimum) ration under various price relationships.

The Effect of Isoquant Curvature on the Optimum Combination of Resources

To emphasize the importance of the curvature of the isoquants, the extreme cases of near perfect substitutability among inputs and near

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[4]This price line is a line representing different combinations of concentrates and roughage which have the same total cost.

[5]At these points $\frac{\Delta X_2}{\Delta X_1} = \frac{PX_1}{PX_2}$.

perfect complementarity among inputs will be considered. Important general rules can then be derived from this approach. Consider first the near perfect substitutability of inputs. Diminishing marginal productivity exists since there are two variable resources applied to fixed factors.

The isoquant (Y_1) in Figure 4 represents a given level of output. This isoquant results from two variable inputs (X_1, X_2) which are near perfect substitutes. Thus, the isoquant has a relatively small degree of curvature.

Two price lines have been plotted to the isoquant. One of the price lines is tangent with the isoquant at point A. The other price line is tangent with the isoquant at point B. Points A and B show optimum combinations of X_1 and X_2 required to produce Y_1 output under two different price relationships of X_1 and X_2. The important point that Figure 4 demonstrates is that with a relatively small change in the prices of X_2 and X_1 the optimum combination of X_1 and X_2 changes a great deal.

In summary, it can be said that when two variable inputs are good substitutes for each other (near perfect substitutability), a relatively small change in prices of the variable factors requires large changes in the optimum combination of the variable resources.[6]

The next case considered is that of two inputs which are near perfect complements. Figure 5 shows an isoquant which represents the output of Y_1 produced by two resources which are near perfect complements. Comparing Figures 4 and 5 it is easy to see the difference in the slopes

[6] Bradford and Johnson, Farm Management Analysis, John Wiley and Sons, Inc., 1953, pp. 138-139.

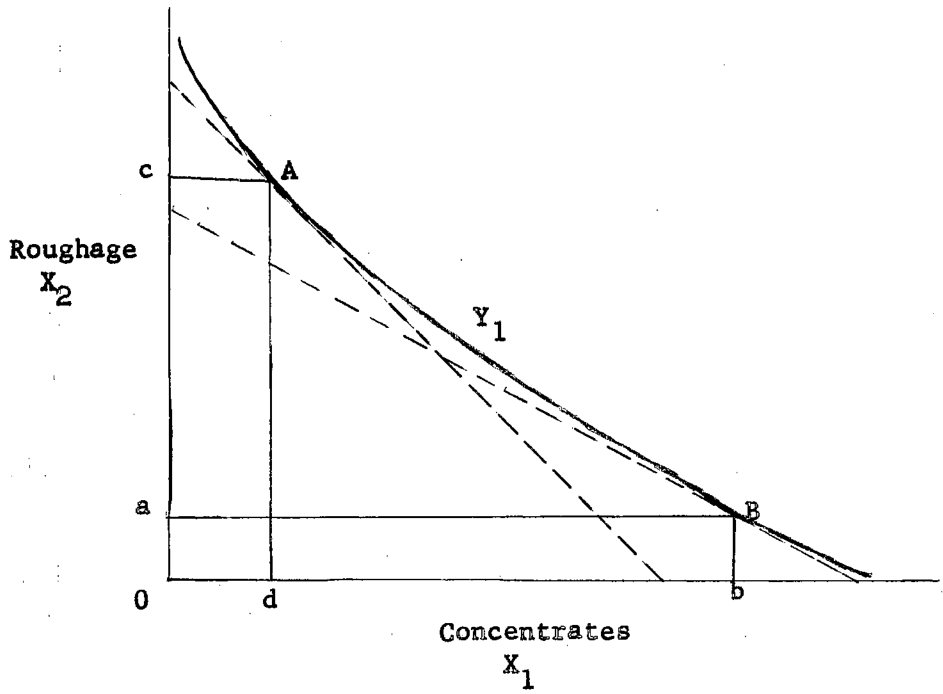

Figure 4. Illustration of Optimum Combinations of Inputs Which are Near Perfect Substitutes

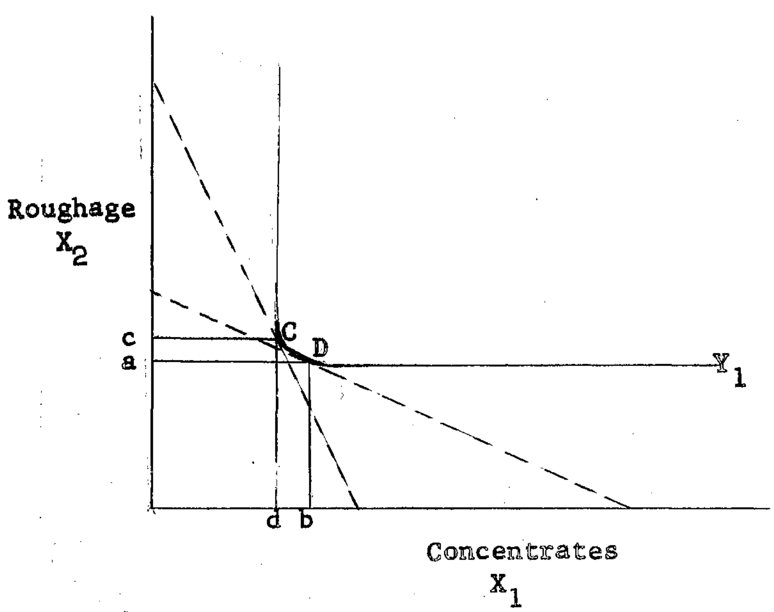

Figure 5. Illustration of Optimum Combinations of Inputs Which are Near Perfect Complements

of the isoquants of near perfect complements and near perfect substitutes. The portion of the isoquant in Figure 5 which is horizontal has a slope of zero, and the portion of the isoquant which is vertical has a slope infinitely large.

The point to be emphasized is that even though there are large changes in the price of X_1 relative to the price of X_2, the optimum combination of resources changes only very slightly. In other words, even though the relative prices of the variable inputs change a great deal, the change in the proportions of the two resources required for an optimum combination is small.[7]

In conclusion, the two cases show that it depends greatly upon the shape of the isoquant whether changes in price of the variable factors will dictate small or large changes in the combination of resources in order to utilize the variable inputs in the optimum proportions.

Isoquant Map

Figure 6 displays an isoquant map consisting of a family of five isoquants (Y_1, Y_2, Y_3, Y_4, Y_5). Y_1 is the lowest level of output and Y_5 is the highest level of output represented by the map. In symbolic language $Y_1 < Y_2 < Y_3 < Y_4 < Y_5$. This results from the varying level of factor employment or level of resource input.

The marginal productivity concept relates changes in one input to changes in output. The marginal physical product of an input is defined as the addition in total output resulting from an increase of one unit

[7] *Ibid.*, p. 141

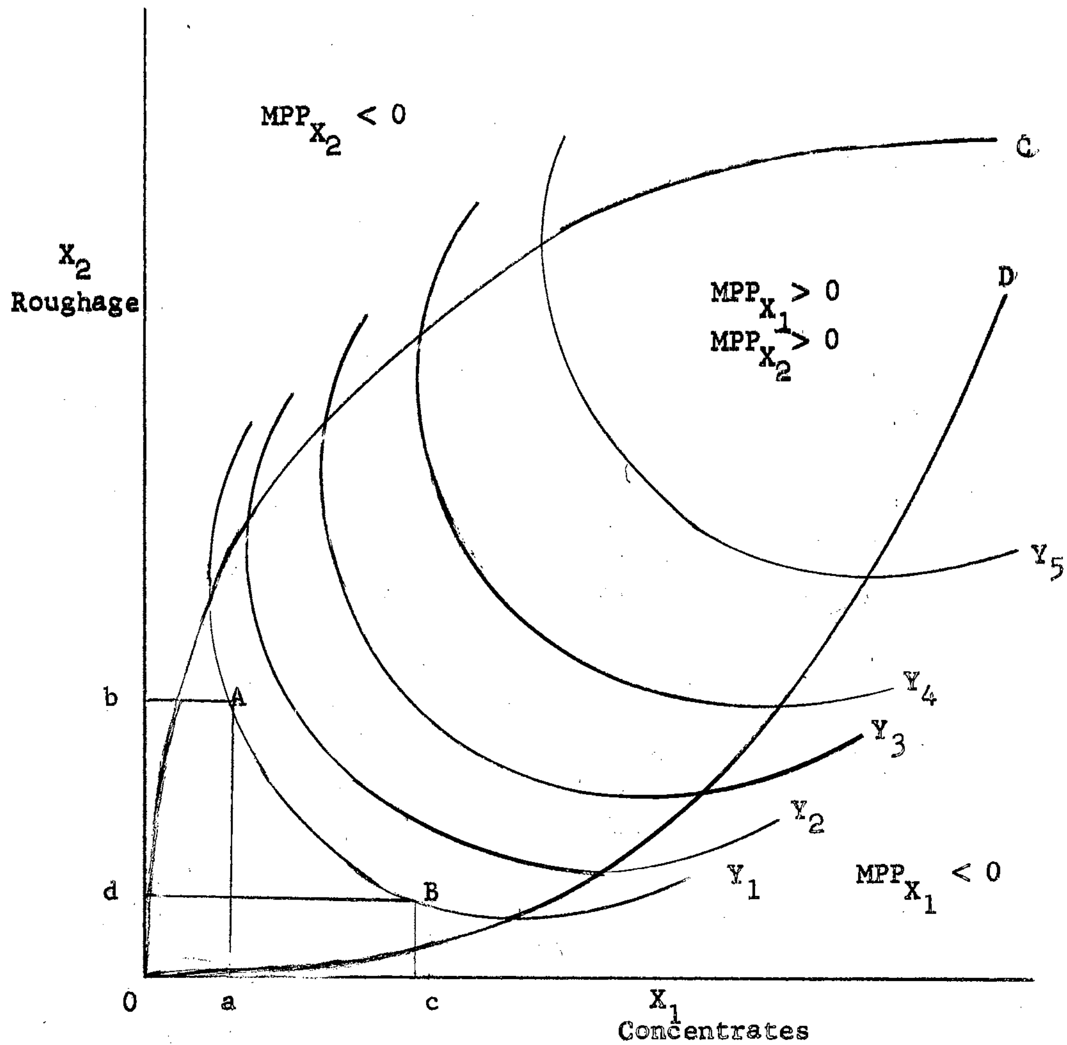

Figure 6. A Hypothetical Isoquant Map

in the quantity of the input while the other factors are held constant.[8] For instance, the MPP X_1 is the increase in Y resulting from a one unit increase in X_1, with the other inputs held constant. Likewise, the MPP X_2 is the increase in Y resulting from a one unit increase in X_2 while the other inputs remain constant. The MPP X_1 and MPP X_2 vary considerably as different combinations of X_1 and X_2 are used. Holding X_2 constant and increasing the amount of X_1 will eventually result, if increased far enough, in a decrease in the MPP X_1. The same condition will hold true if the amount of X_2 is varied and the amount of X_1 is held constant.[9]

The concept of marginal rate of technical substitution($MRS_{X_1X_2}$)is directly related to the isoquant. The marginal rate of technical substitution refers to the amount by which one input may be decreased as the other input is increased by one unit and output remains the same. The marginal rate of substitution of X_1 for X_2 at any point is equal to the slope of the isoquant. Therefore, the $MRS_{X_1X_2}$ is negative when the slope of the isoquant is negative (relevant range) and positive when the slope is positive (irrelevant range). The marginal rate of substitution can also be thought of as the ratio of the marginal physical product of X_1 (MPP X_1) to the marginal product of X_2 (MPP X_2) or $\frac{\text{MPP } X_1}{\text{MPP } X_2}$. In symbolic

[8]Marginal physical product will be referred to as MPP in this thesis. MPP X_1 is the marginal physical product resulting from the input X_1. In the same manner, MPP X_2 indicates the marginal physical product resulting from the input X_2.

[9]When all inputs are held constant except one and this input is increased far enough, the total production will reach a maximum.

terms:[10]

$$(4) \quad \text{Slope} = \frac{dX_2}{dX_1} = \frac{\text{MPP } X_1}{\text{MPP } X_2} = \text{MRS}_{X_1X_2}$$

The slope of the isoquant will be referred to as the $\text{MRS}_{X_1X_2}$ since the slope = $\text{MRS}_{X_1X_2}$. The isoquant is downward sloping to the right and has a negative slope in the range of technical substitution.[11] The inputs are technical complements when the $\text{MRS}_{X_1X_2} \geqq 0$.[12]

The area enclosed by the isoclines OC and OD in Figure 6 is the relevant range of technical substitutes ($\text{MRS}_{X_1X_2}$ has a negative slope). The area on or above OC and on or below OD constitutes the range of technical complements ($\text{MRS}_{X_1X_2} \geqq 0$). The MPP X_2 is equal to 0 at the points where the isoquants ($Y_1 \cdots Y_5$) intersect the line OC. Thus, the $\text{MRS}_{X_1X_2}$ at these points is undefined.[13] The MPP X_1 = 0 at the point

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[10]The symbol for slope of the isoquant is $\frac{dX_2}{dX_1}$. The MPP X_1 is equal to the change in Y resulting from an incremental change in X_1 ($\frac{dY}{dX_1}$). The MPP X_2 is equal to the change in Y resulting from an incremental change in X_2($\frac{dY}{dX_2}$). Therefore:

$$\frac{\text{MPP } X_1}{\text{MPP } X_2} = \frac{\frac{dY}{dX_1}}{\frac{dY}{dX_2}} = \frac{dY}{dX_1} \cdot \frac{dX_2}{dY} = \frac{dX_2}{dX_1} = \text{MRS}_{X_1X_2}.$$

[11]When resources are technical substitutes and one input is reduced in quantity, the other factor must always be increased in order to maintain the given output.

[12]When resources are technical complements reduction in the amount of one input cannot be replaced by an increase in another input.

[13]MPP $X_1 > 0$ and MPP $X_2 = 0$; hence $\frac{\text{MPP } X_1}{\text{MPP } X_2} = \infty$

where the isoquants ($Y_1 \cdot \cdot \cdot Y_5$) intersect the line OD. Consequently, the $MRS_{X_1X_2}$ at these points is equal to zero.[14]

The fact that the MPP of inputs X_1 and X_2 is equal to zero at these given points means simply that the last unit of the input (X_1 or X_2) adds nothing to total product. The area enclosed by the isoclines OC and OD is the only range where a producer can logically operate. In the relevant range the MPP of both X_1 or X_2 is greater than zero. As stated previously, the MPP of X_1 and X_2 varies as different combinations of inputs are used; therefore, the $MRS_{X_1X_2}$ changes as the combination of inputs vary. In this area the slope diminishes from left to right on the isoquant. The $MRS_{X_1X_2}$ is greater at point A in Figure 6 (Y_1 output resulting from "a" amount of X_1 and "b" amount of X_2) than at point B (Y_1 output resulting from "c" amount of X_1 and "d" amount of X_2). Thus, as a relatively greater quantity of X_1 is used to produce output Y_1 the $MRS_{X_1X_2}$ decreases.

[14]The MPP $X_1 = 0$ and MPP $X_2 > 0$; hence $\frac{MPP\ X_1}{MPP\ X_2} = 0$.

CHAPTER III

STATISTICAL PROCEDURES

Comparison of the Available Data and Ideal Data

The ideal data needed to solve the problem are such that a relatively complete segment of the surface could be derived which would have a wide range of ratios. The wide range would allow a better statistical fit to the data and a wider segment of the surface is more likely to include the economically optimum combination of feeds under relevant prices.

Data were available on three different ratios of roughage and concentrate. The available ratios were 1:1, 1:2, and 1:4.[15] It is apparent from the previous discussion that the available data may fall short of what is necessary for the "best" analysis. It is possible that the economically optimum combination may fall outside the range of the three available ratios (1:1, 1:2, 1:4). The available data will not allow as good a statistical fit to data as would be possible with a wider ratio variation.

The cattle in the experiment were marketed as each individual reached a given grade instead of marketing all the cattle at a given date. This limitation of the data made it necessary to take an average marketing date. Thus, the input-output relationship of the final weigh period

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[15]The ratios are expressed in terms of roughage to concentrate by weight pounds. For instance, 1:4 is one part roughage and four parts concentrates.

was determined by averaging the time periods in which the animals were fed during the final interval.

The other major limitation of the data was the slow gains of the cattle. The average daily gain of the steers ranged from 1.55 to 1.72 pounds. The average daily gain of the heifers ranged from 1.26 to 1.43 pounds. The steers made the greatest gain on the 1:2 ratio, while the heifers produced the highest gain from the 1:1 ratio. The average daily gains of the various lots are shown in Table I, Appendix A. A higher rate of gain was expected from the cattle of this feeding trial. It is possible that a slow rate of gain could influence the marginal rate of technical substitution of roughage and concentrates. However, it is assumed that such was not the case.

Experimental Procedure[16]

> Sixty, good-to-choice, Hereford calves were selected in August from a commercial herd near Ringling, in the southern part of the state. These calves were dropped in the fall and early winter of 1954-55 and were approximately 8 to 9 months of age. The drove contained an equal number of steers and heifers, selected to be as near alike in grade as possible. They were charged into the feeding pens at Ft. Reno at 22 cents per lb. for the steer cattle and 19 cents for heifers--the current price for a uniform group of calves of this quality.
>
> The calves were started on feed September 28th, at which time a shrunk weight (16 hours off feed and water) was obtained. Within each sex, they were divided into 3 lots of 10 calves each on the basis of shrunk weight and feeder grade, and one lot of each sex was self-fed one of the three rations shown in Table 1.[17] Further, each lot was divided into two duplicates of 5 calves each. A mineral mixture of 2 parts salt and one part steamed bone meal was available, free choice.

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[16]L. S. Pope, et al. "Fattening Steers and Heifers on Rations Containing Different Levels of Concentrate" Feeding and Breeding Tests 30th Annual Livestock Feeders' Day Report, Oklahoma Agricultural Experiment Station Miscellaneous Publication No. MP-45, June, 1956.

[17]The composition of the rations is shown in Table II, Appendix A and the chemical composition of the feeds is shown in Table III, Appendix A.

It was planned to market the cattle as they reached a grade of top good to low choice--a desirable slaughter grade for young cattle in this area. Accordingly, the slaughter grades of the cattle were estimated from time to time by a committee composed of a commission man from the Oklahoma City yards, a meats specialist from the Animal Husbandry staff and the project leader. The cattle were shipped to market when it was felt they had reached the desired carcass grade regardless of treatment.[18]

Selection of the Statistical Model

Eight different algebraic forms of equations (statistical models) were selected. These equations were:

(1) $Y = a X_1^{b_1} X_2^{b_2}$

(2) $Y = a + b_1\sqrt{X_1} + b_2\sqrt{X_2} + b_3X_1X_2$

(3) $Y = a + b_1\sqrt{X_1} + b_2\sqrt{X_2}$

(4) $Y = a + b_1X_1 + b_2\sqrt{X_1} + b_3X_2 + b_4\sqrt{X_2} + b_5X_1X_2$

(5) $Y = a + b_1X_1 + b_2\sqrt{X_1} + b_3X_2 + b_4\sqrt{X_2}$

(6) $Y = a + b_1X_1 + b_2X_1^2 + b_3X_2 + b_4X_2^2 + b_5X_1X_2$

(7) $Y = a + b_1X_1 + b_2X_1^2 + b_3X_2 + b_4X_2^2$

(8) $Y = a + b_1X_1 + b_2X_2$

Certain restrictions were specified by the economic model. Hence, it was necessary to make the following assumptions about these equations:[19] (1) diminishing returns to the variable factors, (2) diminishing MPP of each variable factor, and (3) diminishing marginal rate of substitution

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[18] The carcass data is shown in Table IV, Appendix A.

[19] These assumptions do not apply to equation (8).

between the variable factors. Also, it was possible to show complementarity of inputs with four of the equations.[20]

Equation (1) is the Cobb-Douglas equation which is linear in logarithms. This equation allows: (1) consistency with the economic model, (2) complementarity of the variable factors, and (3) no maximum. It also specifies constant elasticity of production. These conditions (1-3) will hold if: (a) each b value is greater than zero and less than 1.0, and (b) the sum of b_1 and b_2 values is greater than zero and less than 1.0. This equation under these conditions is consistent with the economic model.

Equation (2) is a square root equation with a cross-product term. If values of b_1, b_2 and b_3 are positive the function will allow: (1) consistency with the economic model, (2) complementarity of the variable factors, and (3) no maximum.

Equation (3) is a square root equation without a cross-product term. If the b_1 and b_2 values are positive the function allows: (1) consistency with the economic model, and (2) no maximum.

Equation (4) is a square root equation with a cross-product term. If the b_1 and b_3 values are negative and the b_2, b_4 and b_5 values are positive or all the b_i values are positive, the function allows: (1) consistency with the economic model, (2) complementarity of the variable factors, and (3) a possible maximum.

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[20] If the marginal physical product of one input is dependent upon the level of another input, there is complementarity of inputs. In statistics this relationship is called interaction. If the coefficient of the cross-product term is negative, there is negative complementarity, and if the coefficient is positive, there is positive complementarity. Negative complementarity is not consistent with the restrictions specified by the economic model.

Equation (5) is a square root equation without a cross-product term. If b_1 and b_3 values are negative and the b_2 and b_4 values are positive or all the b_i values are positive, the function allows: (1) consistency with the economic model, and (2) a possible maximum.

Equation (6) is a second degree polynomial. If the b_1, b_3 and b_5 values are positive and the b_2 and b_4 values are negative, the function allows: (1) consistency with the economic model, (2) complementarity of the variable factors, and (3) a maximum.

Equation (7) is a second degree polynomial without the cross-product term. If the b_1 and b_3 values are positive and the b_2 and b_4 values are negative, the function allows: (1) consistency with the economic model, and (2) a maximum.

Equation (8) is a linear equation. This equation is listed only for the purpose of contrasting functional behavior. It is not considered as a relevant equation since it fails to be consistent with the restrictions specified by the economic model.

Criteria for Selecting a Statistical Model

The t_{bi}, R^2 and S^2 values are the statistical criteria that will be used to determine goodness of fit of the selected equations. If an equation fails to be consistent with restrictions specified by the economic model, there is no advantage in applying the statistical test.

The t_{bi} and the R^2 will be the primary components of the statistical test. The t_{bi} is the symbol for the student t-test of the b_i values. This is a test to determine whether the b_i values are significantly different from zero at a given probability level.

The symbol R^2 is the coefficient of determination.[21] The size of the R^2 indicates how well a given equation fits the available data. The statistical test is based primarily upon the size of the R^2, once the significance of the b_i values has been determined. The goodness of fit is improved as the R^2 value approaches 1.0. If $R^2 = 1.0$ the equation characterizes the data perfectly. Hence, the equation passes through every observed point. The R^2 was selected instead of the correlation coefficient (R) since it represents the percentage of regression due to treatment.

[21] For a discussion of the meaning and calculation of R^2 see: Elmer B. Mode, Elements of Statistics, Prentice-Hall, pp. 239-241.

CHAPTER IV

THE RESULTS

The eight selected equations were fitted to the available data. The equations fitted to the steer data and related statistics are shown in Table I. The equations fitted to the heifer data and related statistics are shown in Table II.

In selecting the final equation from those fitted, the following tests were employed:

1. Consistency of the statistical model with the economic model. That is, the fitted models had to be consistent with the restrictions specified by the economic model.

2. The models that passed the above test were examined for goodness of statistical fit.

Of the first seven equations fitted to steer data, only the coefficients of equations 1, 3, 5 and 7 were consistent with the economic model.[22] For heifers the coefficients of equations 1, 2, 3, 5, 6 and 7 were consistent with the economic model. The statistical tests were relevant only for the equations which were consistent with the economic model.

The need for the first test has been emphasized again by the statistics of the linear equation (8). If the statistical test had been

[22] It is possible that stage I is present in the relevant equations.

TABLE I

SELECTED STATISTICS RELATED TO ALTERNATIVE EQUATIONS FOR STEERS

Equation	Consistent With Economic Model	"a" value		b_i	s_{b_i}	t_{b_i}	R^2	F	s^2
1. $\hat{Y} = aX_1^{b_1}X_2^{b_2}$	yes	.9454	b_2	.218*	.0390	5.570	.9954	216.366*	54.030
			b_1	.557*	.0361	15.400			
2. $\hat{Y} = a + b_1\sqrt{X_1} + b_2\sqrt{X_2}$	no[a]	-12.4654	b_3	.005*	.0007	7.331	.9720	34.763*	336.999
			b_2	-.967	.6372	-1.517			
$+ b_3X_1X_2$			b_1	5.741*	.3817	15.040			
3. $\hat{Y} = a + b_1\sqrt{X_1} + b_2\sqrt{X_2}$	yes	-34.7840	b_2	2.250*	.7094	3.172	.9325	13.806*	793.111
			b_1	5.518*	.5834	9.458			
4. $\hat{Y} = a + b_1X_1 + b_2\sqrt{X_1}$	no[a]	-.7105	b_5	-.002	.0015	1.499	.9878	81.087*	154.980
			b_4	.438	1.8140	.241			
$+ b_3X_2 + b_4\sqrt{X_2}$			b_3	.077	.0471	1.641			
			b_2	.142	1.2359	.115			
$+ b_5X_1X_2$			b_1	.130*	.0174	7.450			
5. $\hat{Y} = a + b_1X_1 + b_2\sqrt{X_1}$	yes	-1.9954	b_4	1.786	1.6000	1.116	.9871	76.290*	160.149
			b_3	.021	.0289	.731			
$+ b_3X_2 + b_4\sqrt{X_2}$			b_2	.146	1.2564	.116			
			b_1	.117*	.0189	6.213			

Table I (Continued)

Equation	Consistent With Economic Model	"a" value		b_i	s_{b_i}	t_{b_i}	R^2	F	S^2
6. $\hat{Y} = a + b_1X_1 + b_2X_1^2$	No[a/]	.4419	b_5	-.002	.0028	-.616	.9818	54.053*	231.083
$+ b_3X_2 + b_4X_2^2$			b_4	-.001	.0019	-.548			
$+ b_5X_1X_2$			b_3	.096*	.0285	3.363			
			b_2	-.0004	.0013	-.298			
			b_1	.138*	.0202	6.826			
7. $\hat{Y} = a + b_1X_1 + b_2X_1^2$	yes	.2890	b_4	-.002	.0010	-1.866	.9877	80.107*	152.612
$+ b_3X_2 + b_4X_2^2$			b_3	.087*	.0200	4.355			
			b_2	-.001	.0006	1.651			
			b_1	.143*	.0147	9.799			
8. $\hat{Y} = a + b_1X_1 + b_2X_2$	no[b/]	9.1988	b_2	.055*	.0064	8.683	.9830	59.377*	194.497
			b_1	.128*	.0046	28.005			

* Significant at the .01 level.

a/ Fail to be consistent with the economic model due to the wrong sign on the b value.

b/ The correct signs of the b value are present, but the equation does not conform to logic.

TABLE II

SELECTED STATISTICS RELATED TO ALTERNATIVE EQUATIONS FOR HEIFERS

Equation	Consistent With Economic Model	"a" value		b_i	s_{b_i}	t_{b_i}	R^2	F	S^2
1. $\hat{Y} = aX_1^{b_1}X_2^{b_2}$	yes	.9571	b_2	.251*	.0332	7.547	.9964	280.962*	28.667
			b_1	.508*	.0306	16.564			
2. $\hat{Y} = a + b_1\sqrt{X_1} + b_2\sqrt{X_2}$	yes	-8.4516	b_3	.005*	.0006	8.512	.9773	43.209*	187.546
			b_2	.010	.4749	.022			
$+ b_3X_1X_2$			b_1	4.179*	.2823	14.802			
3. $\hat{Y} = a + b_1\sqrt{X_1} + b_2\sqrt{X_2}$	yes	-28.0314	b_2	2.781*	.5819	4.778	.9342	14.210*	531.140
			b_1	4.066	.4746	8.567			
4. $\hat{Y} = a + b_1X_1 + b_2\sqrt{X_1}$	no[a/]	-.7980	b_5	-.0003	.0015	-.192	.9872	76.856*	112.411
			b_4	.800	1.6018	.500			
$+ b_3X_2 + b_4\sqrt{X_2}$			b_3	.056	.0435	1.284			
			b_2	.642	1.0530	.610			
$+ b_5X_1X_2$			b_1	.085*	.0174	4.867			
5. $\hat{Y} = a + b_1X_1 + b_2\sqrt{X_1}$	yes	-.9223	b_4	.967	1.3280	.728	.9870	76.771*	109.492
			b_3	.049	.0246	1.990			
$+ b_3X_2 + b_4\sqrt{X_2}$			b_2	.629	1.0371	.607			
			b_1	.083*	.0159	5.254			

Table II (Continued)

Equation	Consistent With Economic Model	"a" value		b_i	s_{b_i}	t_{b_i}	R^2	F	S^2
6. $\hat{Y} = a + b_1X_1 + b_2X_1^2 + b_3X_2 + b_4X_2^2 + b_5X_1X_2$	yes	2.1755	b_5	.00005	.0021	.026	.9868	74.666*	115.664
			b_4	-.0009	.0016	-.564			
			b_3	.080*	.0205	3.911			
			b_2	-.001	.0008	-1.437			
			b_1	.122*	.0140	8.714			
7. $\hat{Y} = a + b_1X_1 + b_2X_1^2 + b_3X_2 + b_4X_2^2$	yes	2.1687	b_4	-.0009	.0010	-.868	.9868	74.665*	112.540
			b_3	.080	.0186	4.322			
			b_2	-.001	.0006	-2.090			
			b_1	.122*	.0134	9.108			
8. $\hat{Y} = a + b_1X_1 + b_2X_2$	no b/	20.8514	b_2	.004	.0006	6.619	.9592	23.500*	329.743
			b_1	.098*	.0070	13.963			

* Significant at the .01 level.

a/ Fails to be consistent with the economic model due to the wrong sign on the b value.

b/ The correct signs of the b value are present, but the equation does not conform to logic.

the only means of selecting the best fitting equation, the linear equation could have been selected rather than some of the relevant equations, although it failed to conform to economic logic. Thus, both tests are essential for determining the equation which best characterizes the relationship.

Best Fitting Equation for Steers

Equation (1) ($Y = .9454\ X_1^{.557} X_2^{.218}$) fitted the available data better than the other relevant equations. Each of the b_i values was significant at the .01 level. Equation (3) was the only other relevant equation with all the b_i values significant at the .01 level.

The R^2 of equation (1) (.9954) was larger than the R^2 of equation (3) (.9325). Equation (1) had an "a" value of .9454 while equation (3) had a negative "a" value (-12.4654).[23] If all the b_i values had been significant at the .01 or .05 level, equation (7) would have been comparable with equation (1) in goodness of fit. The R^2 of equation (1) was only slightly greater than the R^2 of equation (7) but the latter had a slightly lower "a" value.

Best Fitting Equation for Heifers

As with steers, equation (1) ($.9571\ X_1^{.508} X_2^{.251}$) fitted the available data better than the other relevant equations. This was the only relevant equation with each of the b_i values significant at the .01 level. If all of the b_i values had been significant at the .01 or .05 level, equation (5) would have been comparable to equation (1). The R^2

[23]The data is in the form of total gains; therefore the expected "a" value approaches zero.

of equation (1) (.9964) was only slightly larger than the R^2 of equation (5) (.9870). The "a" value for equation (1) was .9571 as compared with -.9223 for equation (5).

CHAPTER V

ECONOMIC INTERPRETATIONS AND EVALUATIONS

Characteristics of the Cobb-Douglas Regression Equation

The Cobb-Douglas equation was selected as the equation best fitting the available data of both steers and heifers; therefore, this equation will be subjected to economic interpretations.

Diminishing transformation of feed into beef was apparent in the regression coefficients (exponents) in the Cobb-Douglas production function for steers and heifers. The regression coefficients[24] for steers were .557 for concentrates (X_1) and .218 for roughage (X_2). These are elasticities of production (E_p) since they indicate the percentage increases in weight resulting from a 1.0 percent increase in feed consumed.[25] Thus, when concentrate intake is increased by 1.0 percent, beef production will be increased by .557 percent (E_p of X_1 = .557). The same principle will hold true for the roughage (X_2). If roughage intake is increased by 1.0 percent, beef production will be increased by .218 percent (E_p of X_2 = .218).

The production function for heifers was similar to the function for steers. The elasticity of production of concentrates (.508) was

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[24]The exponents of X_1 or X_2 are the regression coefficients of the data in logarithms.

[25]Elasticity of production (E_p) is defined as the percentage increase in weight resulting from a one percent increase in the input .

slightly smaller for heifers than the elasticity of production of concentrates for steers. A significantly smaller E_p of X_1 for heifers would mean that steers utilized concentrates more efficiently than did heifers.[26] The elasticity of production of roughage in the equation for heifers was .251, which was slightly larger than the elasticity of production of .218 in the equation for steers. Thus, heifers apparently utilized roughage more efficiently than steers. The sum of the elasticities of production for heifers was equal to .759, which was slightly smaller than the sum of the elasticities for steers (.775). Consequently, a 1.0 percent increase in the concentrates and roughage will increase the beef production for heifers by .759 percent while the function for steers indicated that a 1.0 percent increase in the feed will increase the beef production by .775. If this difference were significant, then steers use feed more efficiently than do heifers.

Estimated isoproduct equations (isoquants) were derived directly from the estimated production function listed above for steers and heifers. Equation (1) was derived from the original Cobb-Douglas equation for steers ($\hat{Y} = .9454\, X_1^{.557} X_2^{.218}$). Equation (2) was derived from the original equation for heifers ($\hat{Y} = .9571\, X_1^{.508} X_2^{.251}$).

Estimated Isoproduct Equations:

(1) Steers: $$X_2 = \left(\frac{\hat{Y}}{.9454\, X_1^{.557}} \right)^{\frac{1}{.218}}$$

(2) Heifers: $$X_2 = \left(\frac{\hat{Y}}{.9571\, X_1^{.508}} \right)^{\frac{1}{.251}}$$

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[26] A test of significance was not computed for the difference between regression coefficients for steers and heifers.

By holding Y constant and varying X_1 from a small to a relatively great amount, the corresponding quantity of X_2 can be computed for heifers and steers by using equations (1) and (2). This procedure gave an isoquant for each given level of beef production, which was the first step in determining the marginal rate of substitution of X_1 and X_2. These computations are shown in Tables III and IV. Equation (1) for steers was used to derive columns 1 and 2 of Table III. Likewise, columns 1 and 2 of Table IV were derived from equation (2).

A given amount of beef (100, 200 or 300 pounds) could be produced from a wide range of combinations of concentrates and roughage. For instance, in Table IV a 100-pound gain could be produced from 400 pounds of concentrates combined with 613 pounds of roughage or the 100-pound gain could be produced at the other extreme with 660 pounds of concentrates and 222.2 pounds of roughage.

Estimated Rate of Substitution Equations:

(3) Steers: $$\frac{dX_2}{dX_1} = \frac{.557X_2}{.218X_1}$$

(4) Heifers: $$\frac{dX_2}{dX_1} = \frac{.508X_2}{.251X_1}$$

The above equations represented the marginal rate of substitution of concentrates and roughage for the steers and heifers at given pounds. These equations were used to compute the data in Column 5 of Tables III and IV. Column 6 is the reciprocal of Column 5. These substitution equations yield the slope of the isoquant.

The slope of the isoquant is equal to the marginal rate of substitution of concentrates for roughage. Within the range of 45.5 to 83.3 percent concentrates for a 100-pound gain (Table III), there was

TABLE III

CONCENTRATE-ROUGHAGE COMBINATIONS AND MARGINAL RATES OF SUBSTITUTION OF BEEF PRODUCTION FOR STEERS AT 100, 200 and 300 POUNDS GAIN

(1) Concentrates (lbs)	(2) Roughage (lbs)	(3) Percentage Concentrates	(4) The average marginal rate of substitution a/ $(\frac{dX_2}{dX_1})$	(5) The exact marginal rate of substitution b/ $(\frac{dX_2}{dX_1})$	(6) The exact marginal rate of substitution b/ $(\frac{dX_1}{dX_2})$
			100-Pound Gain		
400	478.7	45.5	2.82	3.07	.33
420	422.4	49.9	2.38	2.58	.39
440	374.9	54.0	2.02	2.18	.49
460	334.6	57.9	1.73	1.86	.54
480	300.0	61.5	1.49	1.60	.62
500	270.2	64.9	1.29	1.38	.72
520	244.4	68.0	1.12	1.20	.83
540	221.9	70.9	.99	1.05	.95
560	202.1	73.5	.87	.92	1.08
580	184.7	75.8	.76	.82	1.23
600	169.4	78.0	.68	.72	1.38
620	155.7	79.9	.60	.64	1.56
640	143.6	81.7	.54	.58	1.74
660	132.7	83.3		.52	1.94
			200-Pound Gain		
980	1056	48.1	2.65	2.76	.36
1000	1003	49.9	2.48	2.57	.39
1020	953.3	51.7	2.31	2.39	.42
1040	907.1	53.4	2.16	2.23	.45
1060	863.8	55.1	2.02	2.09	.48
1080	823.5	56.7	1.89	1.95	.51
1100	785.7	58.3	1.78	1.83	.55
1120	750.1	59.9	1.66	1.72	.58
1140	716.9	61.4	1.56	1.61	.62
1160	685.7	62.8	1.47	1.51	.66
1180	656.3	64.3	1.30	1.42	.70
1240	578.0	68.2	1.10	1.19	.84
1300	512.1	71.7	.93	1.01	.99
1360	456.2	74.9	.80	.86	1.16
1420	408.4	77.7	.68	.74	1.36
1480	367.3	80.1	.59	.64	1.57
1540	331.8	82.3	.52	.55	1.81
1600	300.8	84.2		.48	2.08

Table III (Continued)

(1)	(2)	(3)	(4)	(5)	(6)
			300-Pound Gain		
1600	1939.0	45.2	2.90	3.10	.32
1660	1765.0	48.5	2.57	2.72	.37
1720	1611.0	51.6	2.27	2.40	.42
1780	1475.0	54.7	2.00	2.12	.47
1840	1355.0	57.6	1.77	1.89	.53
1900	1249.0	60.3	1.60	1.68	.59
1960	1153.0	63.0	1.43	1.51	.66
2020	1067.0	65.4	1.28	1.35	.74
2080	990.0	67.8	1.16	1.22	.82
2140	920.5	69.9	1.05	1.10	.91
2200	857.5	72.0	.95	1.00	1.00
2260	800.4	73.8	.87	.91	1.10
2320	748.4	75.6	.79	.83	1.21
2380	701.0	77.2	.72	.75	1.33
2440	657.8	78.8	.66	.69	1.45
2500	618.0	80.2	.61	.63	1.58
2560	581.6	81.5	.56	.58	1.72
2620	548.1	82.7	.52	.54	1.87
2860	517.2	83.8		.49	2.02

a/ The calculations were based on interval measurement and the values are negative.

b/ The calculations were based on point measurement and the values are negative.

TABLE IV

CONCENTRATE-ROUGHAGE COMBINATIONS AND MARGINAL RATES OF SUBSTITUTION OF BEEF PRODUCTION FOR HEIFERS AT 100, 200 and 300 POUNDS GAIN

(1) Concentrates (lbs)	(2) Roughage (lbs)	(3) Percentage Concentrates	(4) The average marginal rate of substitution a/ $(\frac{dX_2}{dX_1})$	(5) The exact marginal rate of substitution b/ $(\frac{dX_2}{dX_1})$	(6) The exact marginal rate of substitution b/ $(\frac{dX_1}{dX_2})$
			100-Pound Gain		
400	613.0	39.5	2.88	3.11	.32
420	555.3	43.1	2.50	2.68	.37
440	505.3	46.5	2.18	2.33	.43
460	461.8	49.9	1.91	2.03	.49
480	423.6	53.1	1.68	1.79	.56
500	390.0	56.2	1.49	1.58	.63
520	360.2	59.1	1.32	1.40	.71
540	333.7	61.8	1.18	1.25	.80
560	310.0	64.4	1.06	1.12	.89
580	288.7	66.8	.96	1.01	.99
600	269.5	69.0	.86	.91	1.10
620	252.2	71.1	.78	.82	1.21
640	236.5	73.0	.71	.75	1.34
660	222.2	74.8		.68	1.47
			200-Pound Gain		
1100	1254	46.7	2.13	2.31	.43
1160	1126	50.7	1.82	1.97	.51
1220	1017	54.5	1.58	1.69	.59
1280	922.4	58.1	1.36	1.46	.68
1340	840.7	61.4	1.19	1.27	.79
1400	769.3	64.5	1.04	1.11	.90
1460	706.6	67.4	.92	.98	1.02
1520	651.2	70.0	.82	.87	1.15
1580	602.0	72.4	.73	.77	1.30
1640	558.2	74.6	.65	.69	1.45
1700	519.1	76.6	.59	.62	1.62
1760	483.8	78.4	.53	.56	1.80
1820	452.0	80.1	.48	.50	1.99
1880	423.3	81.6		.46	2.19

Table IV (Continued)

(1)	(2)	(3)	(4)	(5)	(6)
			300-Pound Gain		
1940	2003	49.2	2.00	2.09	.48
2000	1883	51.5	1.83	1.91	.52
2060	1773	53.7	1.67	1.74	.57
2120	1673	55.9	1.53	1.60	.63
2180	1581	58.0	1.42	1.47	.68
2240	1496	60.0	1.30	1.35	.74
2300	1418	61.9	1.20	1.25	.80
2360	1346	63.7	1.10	1.16	.87
2420	1280	65.4	1.03	1.07	.94
2480	1218	67.1	.97	1.00	1.00
2540	1160	68.6	.90	.93	1.08
2600	1106	70.2	.83	.86	1.16
2660	1056	71.6	.77	.80	1.24
2720	1010	72.9	.73	.75	1.33
2780	966.1	74.2	.68	.70	1.42
2840	925.1	75.4	.64	.66	1.51
2900	886.8	76.6	.60	.62	1.61
2960	850.7	77.7	.56	.58	1.72
3020	816.8	78.7	.53	.55	1.82
3080	784.9	79.7	.50	.52	1.94
3140	754.8	80.6		.49	2.05

a/ The calculations were based on interval measurement and the values are negative.

b/ The calculations were based on point measurement and the values are negative.

considerable variation in the $MRS_{X_1X_2}$. At the point where a ration of 45.5 percent concentrates was fed the $MRS_{X_1X_2}$ was equal to 3.070. At the other extreme, where a ration of 83.3 percent concentrates was fed, the marginal rate of substitution of X_1 and X_2 was only .520. Thus, for a high roughage combination of the two feeds, one additional pound of concentrates substituted for approximately three pounds of roughage. But with a high concentrate ration (83.3 percent concentrates) one additional pound of concentrates replaced only approximately one-half pound of roughage.

Concentrates and roughage were adequate substitutes over a relatively wide range. However, a diminishing marginal rate of substitution of the two inputs was present.

It was stated in an earlier chapter that when two feeds are so combined that the marginal rate of substitution is equal to the inverse feed price ratio($\frac{dX_2}{dX_1} = \frac{PX_1}{PX_2}$) the optimum combination of the two feeds could be obtained. Inverse feed price ratios ($\frac{PX_1}{PX_2}$) for the substitution of concentrates for roughage are shown in Table V. The price of concentrates per pound divided by the price of roughage per pound ($\frac{PX_1}{PX_2}$) gave the inverse feed price ratio.

The exact marginal rate of substitution was equated to the inverse price ratio for an illustration. The optimum combination of the two feeds was determined for a given amount of gain by comparing the inverse feed price ratio for given feed prices with the $MRS_{X_1X_2}$. The price ratio was 1.20 with concentrates priced at \$1.50 per cwt. and roughage priced at \$25.00 per ton (Table V). What was the optimum combination

TABLE V

SELECTED FEED PRICE RATIOS[a/]

Price of Concentrates per 100 pounds	Roughage Price Per Ton						
	$10	$15	$20	$25	$30	$35	$40
	Price of Concentrate/Price of Roughage						
1.00	2.00	1.33	1.00	.80	.67	.57	.50
1.25	2.50	1.67	1.25	1.00	.83	.71	.62
1.50	3.00	2.00	1.50	1.20	1.00	.86	.75
1.75		2.33	1.75	1.40	1.17	1.00	.88
2.00		2.67	2.00	1.60	1.33	1.14	1.00
2.25		3.00	2.25	1.80	1.50	1.29	1.12
2.50			2.50	2.00	1.67	1.43	1.25
2.75			2.75	2.20	1.83	1.57	1.38
3.00			3.00	2.40	2.00	1.71	1.50
3.50				2.80	2.33	2.00	1.75
4.00					2.67	2.29	2.00
5.00					3.33	2.86	2.50

a/ Only price ratios corresponding to the rates of substitution included in the experimental data are given in this table.

of the two classes of feeds required to produce a 100-pound gain for steers under these feed prices? With reference to Table III, the optimum combination to produce a 100-pound gain for steers was a ration of 68 percent concentrates. Thus, with the assumed feed prices, the optimum combination would be 520 pounds of concentrates and 244.4 pounds of roughage (Columns 1 and 2).

CHAPTER VI

APPLICATION OF THE RESULTS

Cattle feeders in general, fail to adjust the proportions of concentrates and roughage to changing feed price ratios, although feed production in Oklahoma is diversified. Perhaps the lack of a simple method of determining the optimum combination of feeds is partially responsible for the failure of feeders to accept the feed price ratio as a choice rule for determining the optimum combination of feed.

The practical economic importance of feeding the optimum combination of these feeds are shown in Tables VI and VII. These tables display the total feed cost of producing 300 pounds of gain on steers resulting from various combinations of feeds with a wide range of feed prices.

For illustration of the use of these tables, concentrates priced at $2.00 per cwt. and roughage at $25.00 per ton will be assumed. These feed prices resulted in a price ratio of 1.60. This price relationship applied to steers (Column 4, Table III) indicated that a range from 60.3 to 63.0 percent concentrates must be fed to obtain an economic optimum combination. Referring to Table VI, the least total feed cost for this range of feed combinations would be $53.61 to produce 300 pounds of gain on steers. Feeding any combination of these two feeds outside the range 60.3 to 63.0 percent concentrates would result in an increase in

TABLE VI

TOTAL FEED COST OF VARIOUS COMBINATIONS OF CONCENTRATES AND ROUGHAGE REQUIRED TO PRODUCE 300 POUNDS OF GAIN FOR STEERS UNDER DIFFERENT FEED PRICES

Lbs. Concentrates	Lbs. Hay	Percentage of Concentrates	PX_1 a/ / PX_2 b/	1.00			2.00			3.00		
				15	25	35	15	25	35	15	25	35
1600	1939	45.2		30.54	40.24	49.93	46.54	56.24	65.93	62.54	72.24	81.93
1660	1765	48.5		29.84	38.66	47.49	46.44	55.26	64.09	63.04	71.86	80.69
1720	1611	51.6		29.28	37.34	45.39	46.48	54.54	62.59	63.68	71.74	79.79
1780	1475	54.7		28.86	36.24	43.61	46.66	54.04	61.41	64.46	71.84	79.21
1840	1355	57.6		28.56	35.34	42.11	46.96	53.74	60.51	65.36	72.14	78.91
1900	1249	60.3		28.37	34.61	40.86	47.37	53.61	59.86	66.37	72.61	78.86
1960	1153	63.0		28.25	34.01	39.78	47.85	53.61	59.38	67.45	73.21	78.98
2020	1067	65.4		28.20	33.54	38.87	48.40	53.74	59.07	68.60	73.94	79.27
2080	990	67.8		28.22	33.18	38.12	49.02	53.98	58.92	69.82	74.78	79.72
2140	920.5	69.9		28.30	32.91	37.51	49.70	54.31	58.91	71.10	75.71	80.31
2200	857.5	72.0		28.43	32.72	37.01	50.43	54.72	59.01	72.43	76.72	81.01
2260	800.4	73.8		28.60	32.60	36.61	51.20	55.20	59.21	73.80	77.80	81.81
2320	748.4	75.6		28.81	32.56	36.30	52.01	55.76	59.50	75.21	78.96	83.70
2380	701.0	77.2		29.06	32.56	36.07	52.86	56.36	59.87	76.66	80.16	83.67
2440	657.8	78.8		29.33	32.62	35.91	53.73	57.02	60.31	78.13	81.42	84.71
2500	618.0	80.2		29.64	32.72	35.82	54.64	57.72	60.82	79.64	82.72	85.82
2560	581.6	81.5		29.96	32.87	35.78	55.56	58.47	61.38	81.16	84.07	86.98

a/ Price of concentrates per cwt.

b/ Price of roughage per ton.

TABLE VII

TOTAL FEED COST OF VARIOUS COMBINATIONS OF CONCENTRATES AND ROUGHAGE REQUIRED TO PRODUCE 300 POUNDS OF GAIN FOR HEIFERS UNDER DIFFERENT FEED PRICES

Lbs. Concen-trates	Lbs. Hay	Percentage of Concen-trates	PX_1 a/	1.00			2.00			3.00		
			PX_2 b/	15	25	35	15	25	35	15	25	35
1940	2003	49.2		34.42	44.44	54.45	53.82	63.84	73.85	73.22	83.24	93.25
2000	1883	51.5		34.12	43.54	52.95	54.12	63.54	72.95	74.12	83.54	92.95
2060	1773	53.7		33.90	42.76	51.63	54.50	63.36	72.23	75.10	83.96	92.83
2120	1673	55.9		33.75	42.11	50.48	54.95	63.31	71.68	76.15	84.51	92.88
2180	1581	58.0		33.66	41.56	49.47	55.46	63.36	71.27	77.26	85.16	93.07
2240	1496	60.0		33.62	41.10	48.58	56.02	63.50	70.98	78.42	85.90	93.38
2300	1418	61.9		33.64	40.72	47.82	56.64	63.72	70.82	79.64	86.72	93.82
2360	1346	63.7		33.70	40.42	47.16	57.30	64.02	70.76	80.90	87.62	94.36
2420	1280	65.4		33.80	40.20	46.60	58.00	64.40	70.80	82.20	88.60	95.00
2480	1218	67.1		33.94	40.02	46.12	58.74	64.82	70.92	83.54	89.62	95.72
2540	1160	68.6		34.10	39.90	45.70	59.50	65.30	71.10	84.90	90.70	96.50
2600	1106	70.2		34.30	39.82	45.36	60.30	65.82	71.36	86.30	91.82	97.36
2660	1056	71.6		34.52	39.80	45.08	61.12	66.40	71.68	87.72	93.00	98.28
2720	1010	72.9		34.78	39.82	44.88	61.98	67.02	72.08	89.18	94.22	99.28
2780	966.1	74.2		35.05	39.88	44.71	62.85	67.68	72.51	90.65	95.48	100.31
2840	925.1	75.4		35.34	39.96	44.59	63.74	68.36	72.99	92.14	96.76	101.39
2900	886.8	76.6		35.65	40.08	44.52	64.65	69.08	73.52	93.65	98.08	102.52
2960	850.7	77.7		35.98	40.23	44.49	65.58	69.83	74.09	95.18	99.43	103.69
3020	816.8	78.7		36.33	40.41	44.49	66.53	70.61	74.69	96.73	100.81	104.89
3080	784.9	79.7		36.69	40.61	44.54	67.49	71.41	75.34	98.29	102.21	106.14
3140	754.8	80.6		37.06	40.84	44.61	68.46	72.24	76.01	99.86	103.64	107.41

a/ Price of concentrates per cwt.

b/ Price of roughage per ton.

total feed cost required to produce 300 pounds of gain. Two combinations outside the optimum range should serve to make the point clear. If 81.5 percent concentrates were fed, the total feed cost of producing the given gain would be $58.47. Thus, the higher concentrate mixture would increase total feed cost by $4.86 over the least cost combination. At 45.2 percent concentrates a total feed cost of $56.24 which would represent an increase in cost of $2.63. With other price relationships, an even greater differential would appear in total feed cost. To simplify the determination of optimum combinations Tables VIII and IX were prepared.

Variations in the Prices of Grain and Hay

Experiments suggest that grain and hay are technical substitutes within rather wide limits. Therefore, farmers and feeders may substitute grain and hay as dictated by the price fluctuations. If further research shows that the rate of gain and the carcass finish are unaffected by the grain-hay ratio over a wide range, the problem of selecting the optimum ration is merely a question of selecting the combination of grain and hay that minimizes cost per pound of gain. If rate of gain and finish are affected, these factors must be considered in selecting the ration.

The grain-hay price ratio represents the pounds of hay that can be purchased with a pound of grain. Thus, when grain is two cents per pound and hay is one cent per pound the grain-hay price ratio is 2.0. The grain sorghum-alfalfa hay price ratios and the grain sorghum-prairie hay price ratios for the years 1940-54 are shown in Figure 7. During this period the trend for both grain-hay price ratios has been downward.

TABLE VIII

PERCENTAGE CONCENTRATES REQUIRED TO SATISFY THE EQUILIBRIUM CONDITION FOR STEERS FOR 300 POUNDS OF GAIN

	Prairie Hay						
Price	$10	$15	$20	$25	$30	$35	$40
Milo	Percentage Concentrates						
1.00	54.7-57.6	65.4-67.8	69.9-72.0	75.6-77.2	78.8-80.2	81.5-82.7	82.7-83.8
1.25	48.5-51.6	60.3-63.0	65.4-67.8	69.9-72.0	73.8-75.6	77.2-78.8	80.2-81.5
1.50	45.2-48.5	54.7-57.6	63.0-65.4	67.8-69.9	69.9-72.0	73.8-75.6	77.2-78.8
1.75		51.6-54.7	57.6-60.3	63.0-65.4	67.8-69.9	69.9-72.0	73.8-75.6
2.00		48.5-51.6	54.7-57.6	60.3-63.0	65.4-67.8	67.8-69.9	69.9-72.0
2.25		45.2-48.5	51.6-54.7	57.6-60.3	63.0-65.4	65.4-67.8	67.8-69.9
2.50			48.5-51.6	54.7-57.6	60.3-63.0	63.0-65.4	65.4-67.8
2.75			45.2-48.5	51.6-54.7	57.6-60.3	60.3-63.0	63.0-65.4
3.00			45.2-48.5	48.5-51.6	54.7-57.6	57.6-60.3	63.0-65.4
3.50				45.2-48.5	51.6-54.7	54.7-57.6	57.6-60.3
4.00					48.5-51.6	51.6-54.7	54.7-57.6
5.00					< 45.2	45.2-48.5	48.5-51.6

a/ This table was computed by equating the price ratio from Table V with the exact marginal rate of substitution in Table III.

b/ Only feed combinations corresponding to the rates of substitution included in the experimental data are given in this table.

TABLE IX

PERCENTAGE CONCENTRATES REQUIRED TO SATISFY THE EQUILIBRIUM CONDITION FOR HEIFERS FOR 300 POUNDS OF GAIN

	Priaire Hay						
Price	$10	$15	$20	$25	$30	$35	$40
Milo			Percentage Concentrates				
1.00	49.2-51.5	60.0-61.9	65.4-67.1	71.6-72.9	74.2-75.4	77.7-78.7	79.7-80.6
1.25	< 49.2	53.7-55.9	60.0-61.9	65.4-67.1	70.2-71.6	72.9-74.2	75.4-76.6
1.50	< 49.2	49.2-51.5	55.9-58.0	61.9-63.7	65.4-67.1	70.2-71.6	72.9-74.2
1.75		< 49.2	51.5-53.7	58.0-60.0	61.9-63.7	65.4-67.1	68.6-70.2
2.00		< 49.2	49.2-51.5	53.7-55.9	60.0-61.9	63.7-65.4	65.4-67.1
2.25		< 49.2	< 49.2	51.5-53.7	55.9-58.0	60.0-61.9	63.7-65.4
2.50			< 49.2	49.2-51.5	53.7-55.9	58.0-60.0	60.0-61.9
2.75			< 49.2	< 49.2	51.5-53.7	55.9-58.0	58.0-60.0
3.00			< 49.2	< 49.2	49.2-51.5	53.7-55.9	55.9-58.0
3.50				< 49.2	< 49.2	49.2-51.5	51.5-53.7
4.00					< 49.2	< 49.2	49.2-51.5
5.00					< 49.2	< 49.2	< 49.2

a/ This table was computed by equating the inverse price ratio from Table V with the exact marginal rate of substitution in Table IV.

b/ Only feed combinations corresponding to the rates of substitution included in the experimental data are given in this table.

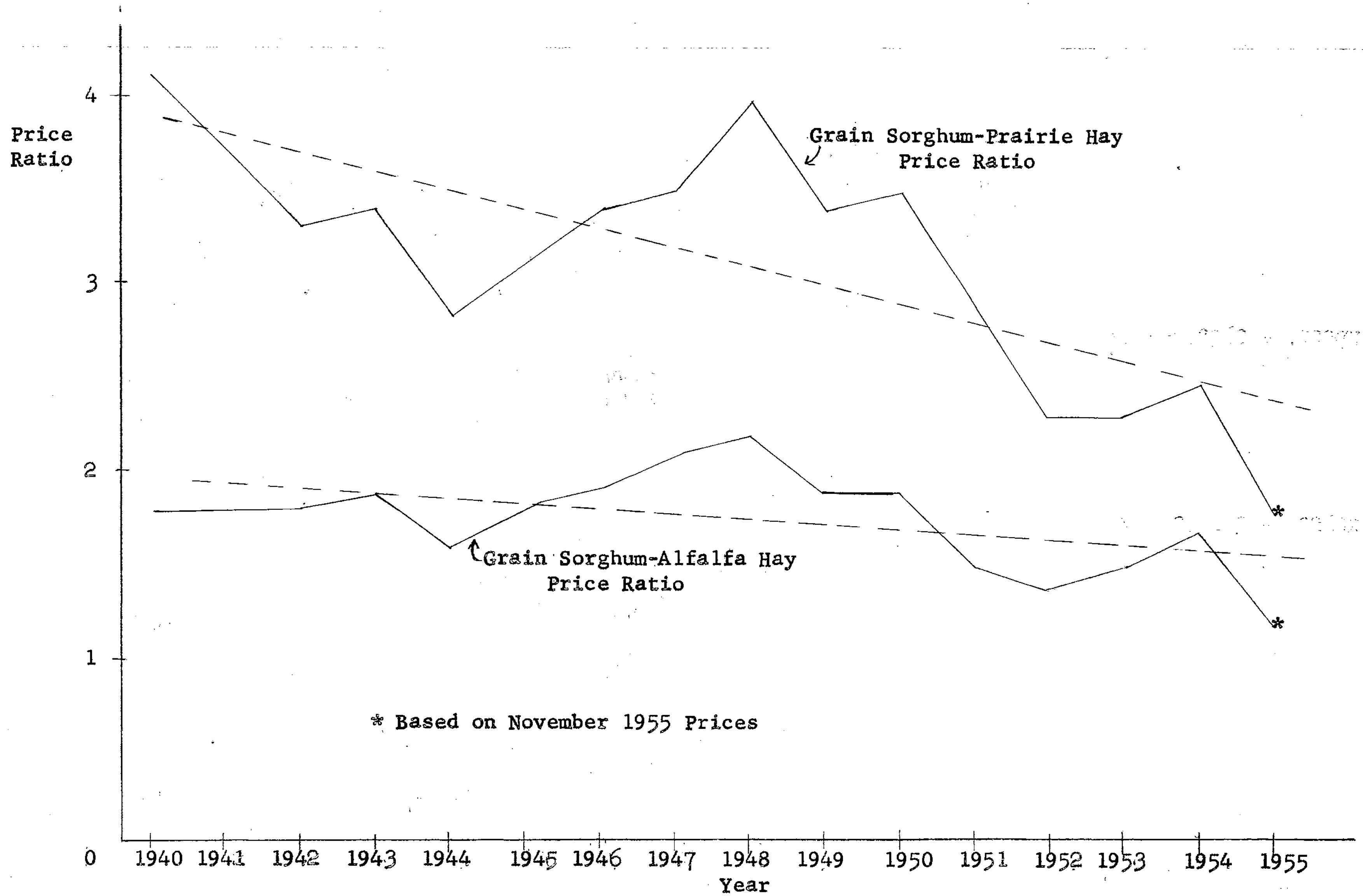

Figure 7. Oklahoma Grain Sorghum-Alfalfa Hay and Grain Sorghum-Prairie Hay Price Ratios, 1940-1955, with Trends

Source: Kenneth Tefertiller and James S. Plaxico, "Feed Outlook", Oklahoma Current Farm Economics Vol. 28, No. 6, December, 1955.

In other words, grain has become cheaper relative to hay. For example, in 1940 the grain sorghum-prairie hay price ratio was 4.1 while the 1954 ratio was 2.5. Thus 100 pounds of grain sorghum would have purchased approximately one-half as much hay in 1954 as it would have in 1940.

The fluctuation in the grain-hay price ratio rather than the trend, is the most important point relevant to the study. For instance, the grain sorghum-prairie hay price ratio was 3.5 in 1950 while in 1952 it dropped to 2.3. Thus, one pound of grain would purchase only 2.3 pounds of hay in 1952 while it would purchase 3.5 pounds in 1950. Thus the most economical ration in 1950 could hardly have been the optimum combination in 1952.

A significant change in the feed price ratio may be the only economic basis for varying the combination of concentrates and roughage. Therefore, a pertinent question arises. Is there significant variation in the price ratio of these feeds? With a wide variation between months and between years in the price ratio, the economic importance of adjustment is intensified.

The monthly price ratios of grain sorghum and alfalfa hay for the years 1950-1955 are shown in Table X. There was considerable difference in the yearly range for the five-year period. The year 1951 had the lowest yearly range (.59). The highest range was 1.22 found in 1954. The greatest monthly range was in June (.85), while the lowest monthly range was in December (.29). The larger yearly and monthly range in the ratios may create an incentive to vary the ratio so that $MRS_{X_1X_2}$ is equal to the inverse of the feed price ratios.

Thus, it should be apparent from the above discussion that a feeder fails to maximize profit by feeding a fixed combination of concentrates

TABLE X

PRICES OF GRAIN SORGHUM RELATIVE TO PRICES OF ALFALFA HAY BY MONTHS, OKLAHOMA, 1950-1955

Year	Months												Yearly Aver-age	Range
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.		
						Grain Sorghum - Alfalfa Hay Ratios								
1950	1.75	1.93	2.02	2.05	2.11	2.34	2.34	2.05	1.80	1.61	1.55	1.52	1.93	.82
1951	1.57	1.56	1.54	1.43	1.59	1.83	1.83	1.53	1.32	1.24	1.25	1.25	1.47	.59
1952	1.25	1.25	1.39	1.59	1.98	2.05	1.69	1.43	1.36	1.31	1.30	1.24	1.42	.81
1953	1.22	1.23	1.35	1.42	1.88	1.71	1.63	1.75	1.69	1.62	1.53	1.53	1.52	.66
1954	1.57	1.61	1.77	1.82	2.17	2.56	2.20	1.69	1.47	1.34	1.38	1.44	1.75	1.22
1955	1.39	1.44	1.44	1.44	1.69	2.15	2.00	1.57	1.37	1.32	1.32	1.29	1.57	.86
Monthly Ave.	1.46	1.50	1.58	1.62	1.90	2.11	1.95	1.67	1.50	1.41	1.39	1.38	1.61	
Range	.53	.70	.67	.63	.58	.85	.72	.62	.48	.38	.30	.29	.51	

Source: Prices Received by Farmers, Agricultural Marketing Service, USDA, Oklahoma City, Oklahoma.

and roughage. The feed price ratio variations between months and between years shown in Table X may indicate the economic importance of adjustment of these feeds so that the $MRS_{X_1X_2}$ is equal to the inverse of the feed price ratio. When a fixed combination of concentrates and roughage is continually fed, it is impossible to maintain a least cost combination.

CHAPTER VII

SUMMARY AND CONCLUSIONS

It is possible for feeders to produce beef by feeding a wide range of ratios of concentrates and roughage. Thus, choice criteria are needed to solve the problem of obtaining the optimum ration. This thesis provides a method of analysis and choice guides for solving the problem.

Experimental data were analyzed to solve the problem of optimum ration choice. The data were obtained from feeding trials conducted at the Ft. Reno Agricultural Experiment Station. Three different combinations of concentrates and roughage were fed in the feeding trial. The three rations consisted of 50:50, 65:35 and 80:20 ratios of concentrates and roughage.

Several equations were fitted to the data. The Cobb-Douglas regression equation was selected as the best fitting equation for both steers and heifers. Four equations for steers were consistent with the restrictions specified by the economic model, while six equations for heifers were consistent with these restrictions.

There was a relatively large variance in the feed-price ratios which affected the optimum ration and the profitability of feeding steers and heifers. The analysis of the experimental data showed that concentrates and roughage were adequate substitutes over a relatively wide range.

The results indicated that the marginal rate of substitution of concentrates and roughage was of economic importance in feed lot operations. Wide variation was found in total feed costs for producing 300 pounds of gain with various combinations of concentrates and roughage under different price relationships.

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APPENDIX A

APPENDIX TABLE I

WEIGHT GAIN OF STEERS AND HEIFERS RESULTING FROM THE DIFFERENT RATION[1]

	Steers			Heifers		
Lot No. and	1	3	5	2	4	6
Ratio Fed	50:50	65:35	80:20	50:50	65:35	80:20
No. of calves/lot	10	10	10*	10	10	10
Ave. days on feed	173	172	170	173	171	174
Ave. Weights (lbs)						
Initial 9/28/55	542	542	536	511	511	511
Gain to 134 days**	261	269	250	225	210	175
Gain to marketing	278	295	263	248	219	219
Ave. daily gain	1.61	1.72	1.55	1.43	1.28	1.26
Ave. ration consumed (lbs)						
Concentrates***	9.94	11.88	11.95	9.45	10.45	11.76
Roughage	9.90	6.42	3.10	9.40	5.66	3.00
Total feed/calf/day (lbs)	19.84	18.30	15.05	18.85	16.11	14.76
Feed per cwt. gain (lbs)						
Concentrates	358	403	463	381	477	537
Roughage	356	218	118	379	258	137

* One steer removed 12/8/55 for sickness of unknown cause and is not included in data for this lot.
** Last weight on all cattle before marketing was begun.
*** Includes small amount of $CaCo_3$ added to 65:35 and 80:20 ratios, plus small intake (.06-.10 lbs. per day) of 2-1 salt-bone meal mixture fed free choice. Cost of minerals also included in calculating feed cost per cwt. gain.

1/ An abstract from L. S. Pope, et al, "Fattening Steers and Heifers on Rations Containing Different Levels of Concentrate," 30th Annual Livestock Feeders' Day Report, Oklahoma Agricultural Experiment Station Miscellaneous Publication No. MP-45, June, 1956.

APPENDIX TABLE II

THE PHYSICAL COMPOSITION OF SELF-FED RATIONS (PERCENT)[2]

Lot Number	1 and 2	3 and 4	5 and 6
Ratio of Concentrates to Roughage	50:50	65:35	80:20
Ground Milo	36.5	53.0	69.3
Cottonseed Meal	8.5	6.7	5.0
Molasses	5.0	5.0	5.0
Chopped Alfalfa Hay	25.0	17.5	10.0
Cottonseed Hulls	25.0	17.5	10.0
Calcium Carbonate	--	.3	.7

2/ An abstract from L. S. Pope, et al, "Fattening Steers and Heifers on Rations Containing Different Levels of Concentrate," 30th Annual Livestock Feeders' Day Report, Oklahoma Agricultural Experiment Station Miscellaneous Publication No. MP-45, June, 1956.

APPENDIX TABLE III

THE CHEMICAL COMPOSITION OF THE FEEDS (PERCENT)[3]

	Moisture	Ash	Crude Protein	Fat	Crude Fiber	N.F.E.
Milo	11.39	1.16	10.38	3.12	1.49	72.46
Cottonseed Meal	6.64	9.04	41.16	2.41	11.32	29.43
Alfalfa Hay	8.36	9.59	18.85	2.47	27.05	33.68
Cottonseed Hulls	6.69	2.84	3.53	1.00	38.78	47.16
Molasses			3.75			

3/ An abstract from L. S. Pope, et al, "Fattening Steers and Heifers on Rations Containing Different Levels of Concentrate," 30th Annual Livestock Feeders' Day Report, Oklahoma Agricultural Experiment Station Miscellaneous Publication No. MP-45, June, 1956.

APPENDIX TABLE IV

THE CARCASS GRADE FOR STEERS AND HEIFERS FED DIFFERENT RATIONS[4]

	Steers			Heifers		
Lot No. and	1	3	5	2	4	6
Ratio Fed	50:50	65:35	80:20	50:50	65:35	80:20
Overnight shrink prior to final weight (percent)	4.04	3.35	3.81	4.23	3.95	3.82
Yield (percent)*	61.76	61.66	61.00	61.28	61.70	61.37
Ave. U.S. Carcass Grade	Gd+	Gd-Gd+	Gd+	Ch-	Gd+Ch-	Gd+
Numerical Score**	5.1	4.7	5.4	5.9	5.6	5.0

* Calculated from hot carcass weight minus 2.5 percent shrink, based on final weight at Ft. Reno.
** A numerical score ranging from 1 for average commercial to 8 for top choice.

4/ An abstract from L. S. Pope, et al, "Fattening Steers and Heifers on Rations Containing Different Levels of Concentrate," 30th Annual Livestock Feeders' Day Report, Oklahoma Agricultural Experiment Station Miscellaneous Publication No. MP-45, June, 1956.

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