CONSTRUCTION OF DESIGN AIDS FOR BIAXIAL BENDING OF LONG RECTANGULAR REINFORCED CONCRETE COLUMNS

By

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CONSTRUCTION OF DESIGN AIDS FOR BIAXIAL BENDING OF LONG RECTANGULAR REINFORCED CONCRETE COLUMNS

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PREFACE

This study presents a refined approach to the analysis and design of rectangular tied reinforced concrete columns subjected to axial thrust and biaxial bending. One of the several techniques for the design of concrete columns currently in use is thoroughly examined, organized into a logical procedure and converted into graphical form to be used as design aids. Due to the character of the resulting charts the scope of this study is limited to the construction and illustration of design charts only so far as to convey the process by which they were formulated. It is intended for the future that a complete set of design charts be constructed for use over a large range of design parameters to serve as functional design aids for the structural engineer.

I wish to express my appreciation to my principal adviser, Professor Louis O. Bass, for his guidance, advice, and assistance during this study.

And in special recognition, sincere gratitude is extended to my wife, Janet, for her encouragement, respect, and many sacrifices.

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LIST OF SYMBOLS

A_r Area of reinforcement in compression area, sq. in.

A_s Area of tension reinforcement, sq. in.

A'_s Area of compression reinforcement, sq. in.

b Width of compression face of member, in.

c Distance from extreme compression fiber to the neutral axis, in.

c_b Distance from extreme compression fiber to the neutral axis under balanced loading conditions, in.

C_m A factor relating the actual moment diagram to an equivalent uniform moment diagram when a member is subject to buckling

d Distance from extreme compression fiber to the centroid of tension reinforcement, in.

d' Distance from extreme compression fiber to the centroid of the compression reinforcement, in.

e Eccentricity of design load parallel to axis measured from the centroid of the section, in.

E_c Modulus of elasticity of concrete, psi

E_s Modulus of elasticity of steel, psi

f'_c Specified compressive strength of concrete, psi

f_y Specified yield strength of reinforcement, psi

F_c Calculated resisting force of concrete, lb.

F_s Calculated resisting force of steel, lb.

h Overall thickness of member, in.

I_c Moment of inertia of gross cross section of columns, $in.^4$

I_s Moment of inertia of reinforcement about the centroidal axis of the column cross section, in.^{4}

k Effective length factor for compression members

l_u Unsupported length of compression member, ft.

M_1 Value of smaller design end moment on compression member calculated from a conventional elastic frame analysis, positive if member is bent in single curvature, negative if bent in double curvature, in-kips.

M_2 Value of larger design end moment on compression member calculated from a conventional elastic frame analysis, always positive, in-kips.

M_b' Maximum moment resistance of section under a balanced loading condition (not multiplied by ϕ), in-kips.

M_c Moment to be used for design of compression member, in-kips.

M_{ox}' Maximum moment resistance of section to bending about major axis (not multiplied by ϕ), in-kips.

M_{oy}' Maximum moment resistance of section to bending about minor axis (not multiplied by ϕ), in-kips.

M_u' Total maximum resistance of section to bending about both axes (not multiplied by ϕ), in-kips.

P Applied external (design) axial load, kips.

P_{cr} Critical buckling load, kips.

P_b' Axial load capacity at balanced loading conditions (not multiplied by ϕ), kips.

P_{ox}' Axial load capacity when subjected to bending about major axis only (not multiplied by ϕ), kips.

P_{oy}' Axial load capacity when subjected to bending about minor axis only (not multiplied by ϕ), kips.

P_o' Axial load capacity in the absence of bending (not multiplied by ϕ), kips.

P_u' Total axial load capacity under biaxial bending (not multiplied by ϕ), kips.

r Radius of gyration of the cross section of a compression member, in.

w Weight of concrete, pcf.

β_d Ratio of maximum design dead load moment to maximum design total load moment, always positive

δ Moment magnification factor for columns

ϕ Capacity reduction factor

CHAPTER I

INTRODUCTION

A column is defined as an upright compression member with a length of at least three times its least lateral dimension.[1] Columns may be short or long in which case the study of column behavior becomes necessary. A short concrete column subjected to an axial load will undergo a longitudinal deflection which is more or less uniform. And if failure occurs, it will be by shearing action on a plane of maximum shear. Any lateral deflections which may occur are usually very small in comparison to the longitudinal deflections and can practically be ignored as far as failure is concerned. Most columns in reinforced concrete structures are longer in comparison with their lateral dimensions than are the short columns mentioned above. The slenderness of a column is determined by its slenderness ratio which is defined as the ratio of its length to the radius of gyration of its cross sectional area with respect to the principal bending axis of the column.[2] Due to the column's length, as an axial load is introduced and increased in magnitude, some lateral

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[1] Building Code Requirements for Reinforced Concrete (ACI 318-71) (Michigan, 1973), p. 6.

[2] Ibid.

deflection will occur in the column if not restrained. This lateral deflection is normally caused by an eccentricity (loads not concentric), an initial curvature in the column or imperfections in the material. If lateral deflections such as these are ignored, appreciable errors will occur in the analysis.

The failure of a slender column does not usually involve shearing action as in the case of short columns, but rather a bending action. As an axial load is applied a lateral deflection occurs and increases as additional load is applied. While in this state of elastic deformation, the strain in the compression fibers increases to some critical value where sufficient yielding occurs to suddenly reduce the column's strength and cause it to collapse or buckle. It is this failure that necessitates a method of predicting a column's behavior under the influence of a given load.

For very simple cases in which only axial load is considered, several relationships have been found to predict accurately enough the behavior of slender rectangular columns. However, accuracy is lost when a bending moment is taken into consideration. And column analysis becomes even more complicated with the presence of two bending moments, one about each of the centroidal axes of the cross section. Several methods of analysis and design for these loading conditions have been introduced in recent years, for the most part as a result of testing and empirical data, and most give satisfactory results. Only those methods more commonly in use today will be discussed in this study. Other methods of analysis and a more detailed study of column behavior may be found in most texts dealing with the design of concrete columns.

The use of design aids has been exploited over the past several years and many have been introduced for a variety of situations. Conditions may arise such as in the case of a multistory building that requires long and time consuming calculations to determine the dimensions of all the columns in a structure. This involves numerous repetitive procedures working through the same set of computations many times. By presenting the relationships of column behavior in graphical or tabular form, the effort required to analyze and design a number of columns is significantly reduced. Although the design aids are somewhat limited in scope and application, those situations that commonly occur are adequately represented and only those rare and unique conditions are left to calculations.

The ideal design aid for reinforced concrete columns would yield an economical and adequate set of dimensions with minimum calculations in only a short time. Due to the many factors which influence column behavior the construction and use of design aids will remain limited by the various combinations of the parameters. It appears that any progress in the formulation of design aids will require that existing relationships be manipulated and combined into simpler and more compact forms which account for as wide an application as is possible. As the design aids become more refined, the analysis and design become simpler, faster and more efficient.

Due to the large volume of work encountered by designers today, it is imperative that efficient methods be utilized in the analysis and design of structures. But such efficiency must not be substituted or mistaken for accuracy and, to a certain extent, economy of design. Design aids must be simple and quick, but they must also maintain some degree of accuracy.

CHAPTER II

STATEMENT AND PURPOSE

The analysis and design of reinforced concrete columns require only simple assumptions and calculations in order to determine a safe capacity for axial loads. Since a condition of pure thrust is unlikely, the presence of bending moments must also be considered in the analysis. Such moments may arise from an eccentricity of the thrust with respect to the centroid of the column or from end restraints in monolithic frames. The relationships necessary to introduce bending moments into the analyses become complex and require time consuming calculations, especially for large numbers of columns subjected to a variety of loads. There exist a number of design aids in the form of tables and charts, such as interaction diagrams, which reduce the repetitive efforts required for the selection of adequate column cross sections. However, the applications of these design aids are either limited to specified loads, dimensions, and reinforcing, or they require a series of unique calculations in order to use them. Once a cross section has been selected, it must be further analyzed for the influences of slenderness and sidesway. A few design aids do account for these effects but considerable computations are required and some trial and error techniques must be used.[1] Another complication arises with the presence of bending moments about two axes of a square or rectangular cross section. Design aids are also available for biaxial bending, but these charts also are of limited application in that they do not account for slenderness or sidesway effects.[2] Even with the number of design aids available to increase the efficiency of column design, considerable repetitive efforts are still required to consider all the significant factors which influence column behavior.

It is, therefore, the purpose of this thesis to assemble the assumptions and relationships of current column design methods into a compact system of graphical design aids which will enable the engineer to efficiently select with minimum calculations column cross sections which will meet the requirements for strength, slenderness, creep, sidesway and biaxial bending. Although these design aids will still have some limitations on their application, they are not as strict as most. The techniques may easily be applied to increase the ranges of the parameters and include column sizes and material strengths not given in this study.

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[1]The most complete of these design aids are presented by Richard W. Furlong, "Column Slenderness and Charts for Design," <u>ACI Journal</u> (1971), pp. 9-17.

[2]L. O. Bass, J. S. Ford, and R. L. Pinc, <u>Design-Analysis Graphs for USD Tied Columns With Biaxial Bending</u> (Stillwater, Oklahoma, 1971).

The relationships necessary to construct the charts will be initiated by first accounting for all assumptions to be made regarding the materials and their behavior. Most general assumptions applied in this thesis are given in Chapter III along with the major provisions and requirements of the current edition of the "Building Code Requirements for Reinforced Concrete (ACI 318-71)" pertaining to compression members. Other assumptions and references to the code will be given throughout the text whenever necessary.

The discussion in Chapter IV deals with slenderness effects on reinforced concrete columns. A column is intended to support axial loads but since concrete properties may not be consistent within the same member and since cross sections of reinforced concrete members are not homogeneous, the length of the column and inaccuracies in construction and loading significantly affect a column's ability to withstand an axial load. The theory of buckling is investigated and applied to rectangular columns. Since concrete structures are subject to the effects of long term deformations, creep and its effects on concrete strength and behavior are also treated. If a column is but one of a series of columns in one story of a building, the stiffness of the column will affect the behavior of the other columns. Some consideration must be given to the effect of sidesway on the column's strength. This is also discussed in Chapter IV and forms the basis for the design aids presented in the appendix.

The ability of a column to withstand bending moment as well as axial load is a major consideration in analysis and design since almost all columns encountered will be subject to some form of bending moment. Chapter V presents the design considerations and formulas for uniaxial bending of rectangular concrete columns. The material will prepare for the construction of the familiar load-moment interaction diagram. The application of the relationships from Chapter IV dealing with slenderness effects will be incorporated into the uniaxial design equations yielding a thorough design procedure for uniaxial bending.

Because bending is not necessarily limited to only one axis, consideration must be given to the possibility of bending about two different axes simultaneously. The discussion in Chapter VI illustrates the theory of biaxial bending and presents the more common methods for designing columns subjected to axial load and biaxial bending. A simple method is adopted for use in the design aids and is combined with the relationships of previous chapters into a complete design procedure for biaxial bending. This procedure will determine the capacity of a given column for axial load and bending moments about both axes and will ensure that the column will withstand the effects of slenderness and sidesway. The set of equations presented here form the foundation for the design aids discussed in Chapter VII.

If all relationships necessary for the design of rectangular reinforced concrete columns are given and related to one another by a set of common parameters, many of the calculations required to solve the group of equations can be eliminated by transforming the equations into graphical relationships that can be solved visually. And since many of the variables in the equations are dependent on each other the iteration required to determine an economical cross section becomes less of a task. Chapter VII outlines the procedures involved in reducing the given equations and relationships into a set of graphical design aids which are presented in the appendix. Example problems are given in Chapter VIII to prove the validity of the graphs and a guide for the use of the charts is also included. The examples presented in Chapter VIII cover common situations to be encountered.

With the assistance of these design aids, column design can be expedited and more economic design can be realized. It is intended for the future that the scope of these charts be extended to include a larger range of parameters not given by this study.

CHAPTER III

ASSUMPTIONS AND CODE PROVISIONS

Before useful relationships can be constructed some general assumptions must be made with respect to the properties and behavior of reinforced concrete when subjected to various stress conditions: (1) For any strain produced in a reinforcing bar, the surrounding concrete will undergo an equal strain. In other words, it is assumed that the concrete and reinforcing steel produce a perfect bond and any deformation in one material must be accompanied by an identical deformation of the other material. (2) Cross sections that are plane before loading remain plane after loading. Although this is not actually true, when a section is loaded to near failure the error is insignificant.[1] (3) Concrete offers no resistance to tension stresses. When using ultimate strength design methods the concrete section is cracked throughout the area of tension stresses and the tension reinforcement carries all of the tension stresses. (4) The resisting stresses of the concrete at its ultimate strength is a function of the stress-strain relationship for a slow rate of loading. This has been shown to be reliable since during

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[1]Phil M. Ferguson, Reinforced Concrete Fundamentals (New York, 1973), pp. 32-33.

construction most loads are either sustained loads or accumulated at a slow rate over the duration of construction.[2]
(5) The Whitney Stress Block will define the compressive stress distribution over the cross section of a member.
Other assumptions dealing with specific relationships will be introduced as necessary throughout the text.

Since the design procedures and equations must comply with the provisions of the code, a summary of these requirements is given here. Those provisions dealing with reinforcing steel are not included in the summary. The discussion is limited to the requirements for the design and analysis of the column itself. References to the ACI Code (318-71) are by a decimal system. The first number indicates the chapter, the second number indicates the section, the third number indicates the paragraph and so on.

According to Section 9.2.1 the capacity of a compression member must be reduced by a factor ϕ given as 0.7 for tied rectangular columns. For reinforcement with a yield strength of 60,000 psi or less, ϕ may be increased linearly to 0.9 as the axial design load P_u decreases from $0.1f'_cA_g$ to zero providing the reinforcement is symmetrical and the quantity $(h - d' - d_s)/h$ is not less than 0.7. The term d_s is the distance between centroids of the outer rows of reinforcement.

[2]George Winter and Arthur H. Nilson, Design of Concrete Structures (New York, 1972), p. 41.

Section 10.3.1 requires that the cross section be designed on the basis of the assumption that strain in the steel and concrete is proportional to the distance from the neutral axis. If reinforcing steel with a yield stress greater than 60,000 psi is used, the maximum percentage of reinforcement is limited by Section 10.3.2 to 75 percent of the steel required to produce balanced loading under flexure without axial load. The design loads for columns must include an accompanying moment, Section 10.3.4, and the resulting eccentricity e, Section 10.3.6, which must be at least one inch or one-tenth of the depth of the column. Slenderness effects must also be considered. Section 10.9.1 permits from one to eight percent reinforcement with a minimum of four bars.

The guidelines for evaluation of slenderness effects are given in Section 10.11. The provisions are applicable in lieu of the structural analysis of Section 10.10.1 and are the basis of procedures used in this study. The unsupported length of a column l_u is defined as the clear distance between lateral supports. If a column is braced against sidesway, slenderness effects may be neglected when kl_u/r is less than $(34 - 12M_1/M_2)$ where k is an effective length factor, r is the radius of gyration, M_1 is the smaller of the two end design moments on the column and M_2 is the larger. The code permits r to be taken as 0.3 times the overall dimension perpendicular to the axis of bending. The effective length factor k may be taken as one unless an analysis, which is discussed in Chapter IV, yields a smaller value. If the column is not braced against sidesway, slenderness effects may be neglected when kl_u/r is less than 22, where k is to be determined by analysis. For any column with kl_u/r greater than 100, the analysis of Section 10.10.1 must be made.

The actual design of columns is controlled by Section 10.11.5. The design loads are to consist of the design axial load from a conventional frame analysis and a moment that is magnified by a factor δ. The magnification factor is a function of the two design end moments on the column and the ratio of design axial load to the critical buckling load. These relationships are discussed in Chapter VI.

CHAPTER IV

SLENDERNESS EFFECTS

When considering only the axial load capacity of a column, for a given length the column will have some critical value of concentric axial load above which the column will undergo inelastic buckling. This critical load is given by the Euler formula:

$$P_{cr} = \frac{\pi^2 EI}{(kl_u)^2} \tag{4.1}$$

where P_{cr} is the buckling load, E is the modulus of elasticity at buckling, I is the moment of inertia of the cross section about its centroid, and kl_u is the effective length of the column. Since concrete columns contain reinforcing steel the section is not homogeneous. Creep and tension cracks also affect the rigidity EI of the section. Therefore, EI cannot simply be determined from Young's Modulus. The code provides two empirical equations for EI:[1]

$$EI = \frac{(1/5)E_c I_g + E_s I_s}{1 + \beta_d} \tag{4.2}$$

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[1]Section 10.11.5, p. 32.

$$EI = \frac{E_c I_g}{2.5(1 + \beta_d)} \tag{4.3}$$

where E_c is the modulus of elasticity of concrete in psi, I_g is the moment of inertia of the gross cross section of the column in in.^{4}, E_s is the modulus of elasticity of steel, I_s is the moment of inertia of the reinforcing steel about the centroid of the cross section in in.^{4}, and β_d is the ratio of maximum design dead load moment to maximum design total load moment. This ratio is always positive.

Since a fixed ratio of stress to strain does not necessarily exist during initial loading of concrete members and because linear stresses are assumed, an exact modulus of elasticity cannot be defined.[2] Therefore, an approximate value for E_c is given by the code as[3]

$$E_c = w^{1.5}\ 33\ \sqrt{f'_c} \tag{4.4}$$

where w is the weight of the concrete in pcf for those weights between 90 and 155 pcf, and f'_c is the 28 day cylinder strength. E_c represents the secant modulus of elasticity which is the slope of a chord from zero to about $f'_c/2$ on the stress-strain diagram.[4] The term β_d in equations (4.2) and (4.3) accounts in part for the effect of creep. Creep deformations and curvature become larger as the

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[2]Ferguson, p. 9.

[3]Section 8.3.1, p. 22.

[4]Ferguson, pp. 9-10.

moments from sustained loads increase and the rigidity of the member decreases. To correct the stiffness of the column, the sum of the stiffnesses for the concrete and steel is reduced by $(1 + \beta_d)$.[5]

Both equations (4.2) and (4.3) give conservative values for rigidity. The proximity of each to the actual rigidity depends primarily upon the percentage of reinforcing steel ρ contained in the section. Comparison of the theoretical EI's obtained from tests to the EI's computed by equations (4.2) and (4.3) show that equation (4.3) is economical over only a small range at low values of ρ.[6,7] Equation (4.2) is somewhat more conservative in this same range but for larger percentages of steel yields a more economical value for EI. In order to avoid complicating the relationships, only the equation (4.2) will be used to determine column stiffnesses.

The Euler formula (4.1) is valid for a concentric load P_{cr} applied to a column with unsupported length l_u and an effective length kl_u. In frames that are braced against sidesway, the coefficient k will vary from 0.5 to 1.0 and for frames that are not laterally braced against sidesway, k varies from 1.0 to ∞. The coefficient k is a function of the rotational end restraints at each end of the column. The

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[5] Commentary on Building Code Requirements for Reinforced Concrete (ACI 318-71) (Michigan, 1971), Section 10.11.5, p. 42.

[6] J. G. MacGregor, J. E. Breen, and E. O. Pfrang, "Design of Slender Columns," ACI Journal (1970), p. 6.

[7] Commentary, Section 10.11.5, p. 41.

effect of k on the design of columns may be illustrated by dividing structures into three classes.[8] First are the very tall buildings, the lateral movements of which require lateral bracing or shear walls to restrict sidesway. Slenderness effects become critical in structures of this type. The second class includes those buildings that are tall enough to be subject to considerable lateral movement but not to the extent that lateral bracing is required. Slenderness effects require attention but do not dominate the design of columns. Most buildings fall into a third category. They are short enough that lateral movements are minor. In this case slenderness effects are usually of minor concern and the approximations for k given by the code are sufficient. The first two classes usually require analyses to determine k.

Several charts are available for determining the effective length factor k. The simplest of these, published by Jackson and Moreland, use an end restraint coefficient ψ for each end of the column.[9] The coefficient ψ is the ratio of the sum of the stiffnesses EI/L_c of the columns at the joint in the plane of bending to the sum of the stiffnesses EI/L_b of the beams at the joint, where L_c is the column length and L_b is the beam length. A coefficient is found for each end of the column and plotted on a nomograph. The factor k is read directly from the nomograph.

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[8]Paul F. Rice and Edward S. Hoffman, Structural Design Guide to the ACI Building Code (New York, 1972), pp. 291-297.

[9]MacGregor, Breen, and Pfrang, p. 6.

In determining the effective length factor k the code requires that the effects of cracking and reinforcement on the relative stiffness must be considered.[10] Since the members are designed and dimensioned according to their ultimate strengths, as the members approach failure, tension cracks form, deflections and curvatures increase, reinforcement yields and the rigidities of the members change. Therefore, the question arises as to what constitutes acceptable rigidities for the beams and columns at a joint. Since k must be known to properly dimension a column, and since k also depends on the rigidity of the column, an iterative process must be used. Member sizes must be assumed, k values computed and member sizes adjusted with the k values to find new k values, and so on. But again, even if member sizes are assumed, what should be used for a member's rigidity? Several approaches are presented of which the simplest is to use one-half of the gross moment of inertia on the column cross section in order to determine an initial relative stiffness ψ.[11] Other methods present more accurate results but since charts will be used for the iteration process, little extra time should be required for a less accurate initial guess.

The above procedure was given for a single column. But usually in a frame not braced against sidesway, there will exist more than one column in a given frame or story of a

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[10] Section 10.11.3, p. 32.

[11] Ferguson, pp. 523-525.

building. When this is the case any sidesway will involve the simultaneous lateral deflection of all the columns in that story. Assuming no torsional loading is introduced, all columns will deflect an equal amount and the shear and moments distributed among them will be functions of each column's stiffness relative to the total stiffness of all the columns. Whereas the stiffness analysis for a single column is used to find an appropriate factor δ by which the design moment is increased, when all columns in a story are considered, a δ will be found which, if greater than that δ for any of the individual columns, will be applied to all columns in the story.

Once P_{cr} is found for all the columns to be considered, the moment magnifier δ may be found from

$$\delta = \frac{C_m}{1 - \frac{P}{\phi P_{cr}}} \tag{4.5}$$

where $P/\phi P_{cr}$ is $\Sigma P/\Sigma \phi P_{cr}$ for all the columns in the story or $P/\phi P_{cr}$ if only a single column is being considered. C_m is given as

$$C_m = 0.6 + 0.4 \frac{M_1}{M_2} \geq 0.4 \tag{4.6}$$

where M_1/M_2 is the ratio of end moments, M_2 being the larger of the two and M_1 the smaller. When actual eccentricities are less than the minimum specified by the code, M_2 must be based on the minimum eccentricity. If no eccentricity is present at either end, M_1/M_2 must be taken as one. Where eccentricities are present but less than the minimum, the actual moments should be used to calculate C_m. The ratio M_1/M_2 is positive in the case of single curvature of the column and negative for double curvature. If $M_1 = M_2$ then $C_m = 1$ and the maximum moment will occur at mid-height of the column.[12]

If the column is in an unbraced frame and has a length to thickness ratio of $kl_u/r \leq 22$, it is a short column and the code allows C_m to be taken as one.[13] For braced frames a short column is one with a $kl_u/r \leq (34 - 12M_1/M_2)$ and C_m may again be taken as one. The ratio M_1/M_2 represents the same ratio as in equation (4.6).

Once δ has been computed for both an individual column and that column as one in a story (if applicable) the larger δ shall be used. The design moment shall be determined by

$$M_c = \delta M_2 \tag{4.7}$$

where M_2 is the larger of the two end moments. The column is then designed for an axial load of P and a moment of M_c.

Size and reinforcement are determined and the design procedure is repeated using a new stiffness based on the new dimensions. The procedure is iterated until the changes in size and reinforcement are small enough to be satisfactory.

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[12]Rice and Hoffman, p. 294.

[13]Section 10.11.4, p. 32.

CHAPTER V

UNIAXIAL BENDING

If a column were loaded axially with a force acting at the centroid of the column's cross section, some determinate strength would be available to resist the force. Using the assumptions made in Chapter III and assuming that the stress is uniformly distributed over the cross sectional area, it is possible to predict the value of load at which the column would fail. This failure due to strength should not be confused with failure due to buckling discussed in the last chapter. Since the load is concentric, all the steel in the section will yield at the same value of load. The load will be resisted by the stress developed in the steel and the stress developed in the concrete. See Figure 1. The stress in the steel at failure is given by f_y, the yield stress of the reinforcing steel. Since f_y is the same for all bars,

$$F_{s1} = f_y A_s' \quad \text{and} \quad F_{s2} = f_y A_s \tag{5.1}$$

The resisting force of the concrete is

$$F_c = 0.85\ f_c'$$

Summing the vertical forces,

$$P_o' = 0.85\ f_c'bh + f_y A_s' + f_y A_s$$

or $$P_o' = 0.85\ f_c'bh + f_y(A_s + A_s') \qquad (5.2)$$

The term P_o' represents the theoretical failure load for pure axial load with no bending. But since the code requires that

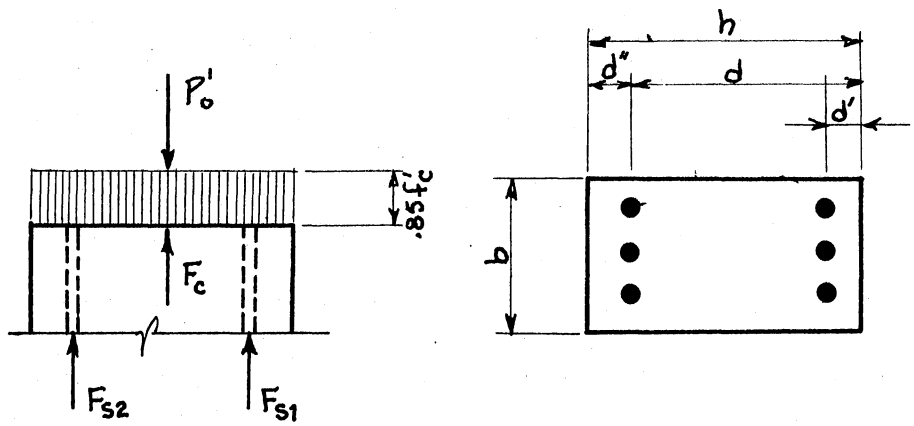

Figure 1. Resisting Forces of Column Section Under Pure Thrust.

columns be designed for some moment capacity at a minimum eccentricity, if the applied force is moved to one side, the magnitudes of the resisting forces are changed as shown in Figure 2. Since an eccentricity is present, the section must resist a moment in addition to the axial load. Again summing forces, if the maximum strain in the concrete is assumed to be 0.003 in/in, and since a linear stress distribution was assumed, a triangle can be used to illustrate the strain of the section.[1] Point N represents the neutral axis. From

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[1]ACI Code, Section 10.2.3, p. 30.

Figure 2b,

$$\epsilon_{s1} = \frac{(.003)(c - d')}{c} \quad \text{and} \quad F_{s1} = A_s'E_s\frac{(.003)(c - d')}{c} \qquad (5.3)$$

$$\epsilon_{s2} = \frac{(.003)(c - d)}{c} \quad \text{and} \quad F_{s2} = A_sE_s\frac{(.003)(c - d)}{c} \qquad (5.4)$$

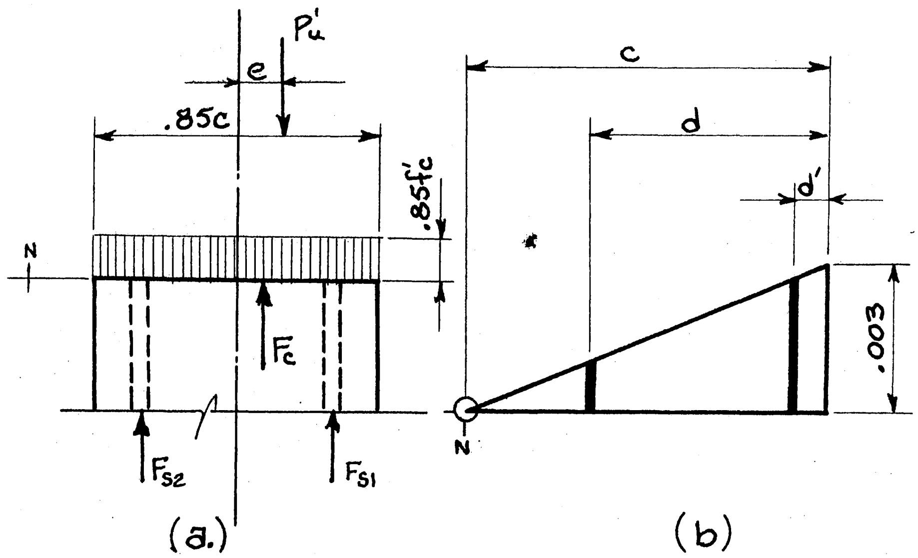

Figure 2. Resisting Forces of Column Section Under Eccentric Load.

Then with F_c we have

$$P_u' = (.85f_c')(.85)A_n + A_s'E_s\frac{(.003)(c - d')}{c} + A_sE_s\frac{(.003)(c - d)}{c}$$

$$\text{or} \quad P_u' = 0.7225f_c'A_nc + \frac{.003E_s}{c}\left[A_s'(c - d') + A_s(c - d)\right] \qquad (5.5)$$

The maximum resisting moment of the section is given by summing moments about the center of the section:

$$M_u' = P_u'e = F_c\left[\frac{h}{2} - \frac{.85c}{2}\right] + F_{s1}\left[\frac{h}{2} - d'\right] - F_{s2}\left[d - \frac{h}{2}\right]$$

$$\text{or } M_u' = 0.36125f_c'A_nc(h - .85c) + A_s'E_s\left[\frac{h}{2} - d'\right]\frac{(.003)(c - d')}{c}$$

$$- A_sE_s\left[d - \frac{h}{2}\right]\frac{(.003)(c - d)}{c} \qquad (5.6)$$

In the above equations the distance to the neutral axis c is not known but can be determined by simple statics. A trial and error method may be used by assuming a value for c and calculating the capacity of the column then comparing the eccentricity obtained with the actual eccentricity. If they differ, a new value is selected for c and the process repeated until they agree.

Equations (5.3) through (5.6) are valid for the range $d/.85 < c < h/.85$. As the eccentricity becomes larger, the neutral axis moves toward the load and c decreases. For the range $c_b < c < d/.85$ the following equations should be used. The distance to the neutral axis under a balanced loading condition is represented by c_b. Figure 3 shows that F_{s2} is now a tension force.

$$\epsilon_{s1} = \frac{(.003)(c - d')}{c} \quad \text{and} \quad F_{s1} = A_s'E_s\frac{(.003)(c - d')}{c} \qquad (5.8)$$

$$\epsilon_{s2} = \frac{(.003)(d - c)}{c} \quad \text{and} \quad F_{s2} = A_sE_s\frac{(.003)(d - c)}{c} \qquad (5.9)$$

$$\text{Then } P_u' = 0.7225\ f_c'A_nc + \frac{.003E_s}{c}\left[A_s'(c-d') - A_s(d-c)\right] \qquad (5.10)$$

The resisting moment is then:

$$M_u' = P_u'e = 0.36125\ f_c'\ A_n\ c\ (h - .85c)$$
$$+ A_s'E_s\left[\frac{h}{2} - d'\right]\frac{(.003)(c - d')}{c}$$
$$- A_sE_s\left[d - \frac{h}{2}\right]\frac{(.003)(d - 1)}{c} \qquad (5.11)$$

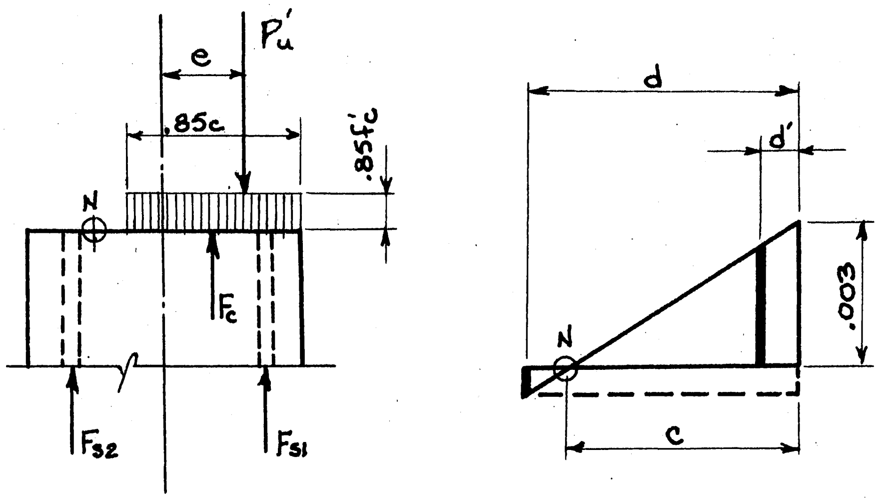

Figure 3. Resisting Forces of Column Section Under Eccentric Load.

The balanced loading condition exists when the maximum compressive strain in the concrete occurs at the same time the tension reinforcing steel begins yielding.[2] This condition is given by equations (5.12a) and (5.12b).

$$P_b' = 0.85\ f_c'(.85bc_b - A_s') + \frac{.003E_s}{c_b}\left[A_s'(c_b - d')\right] - A_sf_y \qquad (5.12a)$$

[2]ACI Code, Section 10.3.3, p. 30.

$$M_b' = 0.85\ f_c'(.85bc_b - A_s')\left[\frac{h}{2} - \frac{.85c_b}{2}\right]$$

$$+ A_s'E_s\left[\frac{h}{2} - d'\right]\ \frac{(.003)(c_b - d')}{c_b}$$

$$- A_sf_y\left[d - \frac{h}{2}\right] \qquad (5.12b)$$

$$c_b = \frac{.003\ d}{.003 + .00207}$$

For all values of c smaller than c_b the stress in the tension steel is f_y and $F_{s2} = A_sf_y$. With this value for F_{s2} substituted into equations (5.10) and (5.11), they will yield the failure loads P_u' and M_u' for the range $c < c_b$.

Over the first range from the minimum eccentricity to the point where balanced loading exists, failure in the section is controlled by compressive stress. As the eccentricity increases beyond balanced loading, failure is controlled by tension. Also, as e increases, P_u' decreases and M_u' increases up to some maximum value and then decreases. This is best illustrated in a load moment-interaction diagram as in Figure 4.

If P_u' is plotted versus M_u' (or $P_u'e$), the curve will represent the locus of the maximum theoretical allowable axial loads and moments P_u' and M_u' for any eccentricity e as shown in Figure 4. This curve is unique for some percentage and configuration of reinforcement. Normally the diagram is a family of curves for various percentages of steel. This type of diagram is to be used as the basis for the design aids in the appendix. Its application will be discussed in greater detail in Chapters VII and VIII and can be found in most texts dealing with the design of reinforced concrete columns.

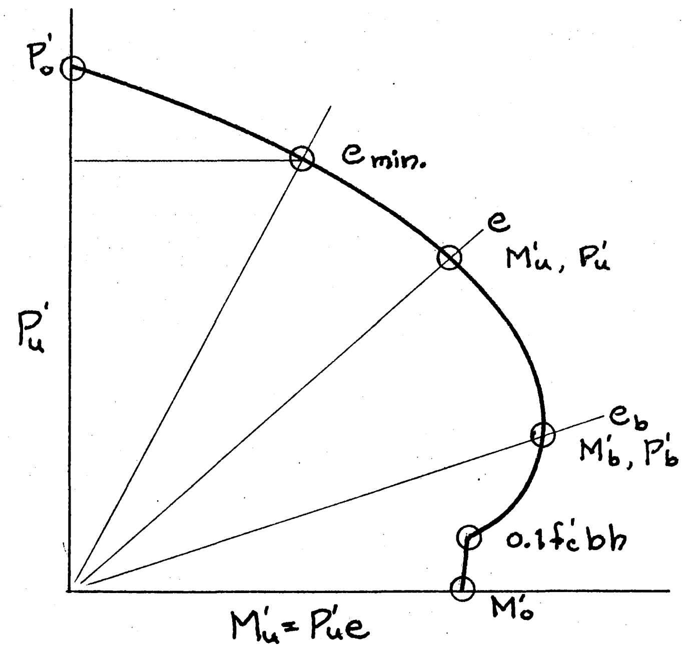

Figure 4. Load-Moment Interaction Diagram.

Another method of design better suited to longhand calculations is to construct a portion of an interaction diagram with several values of c giving allowable loads near the design loads.[3] The capacity of the column can then be taken directly from the curve.

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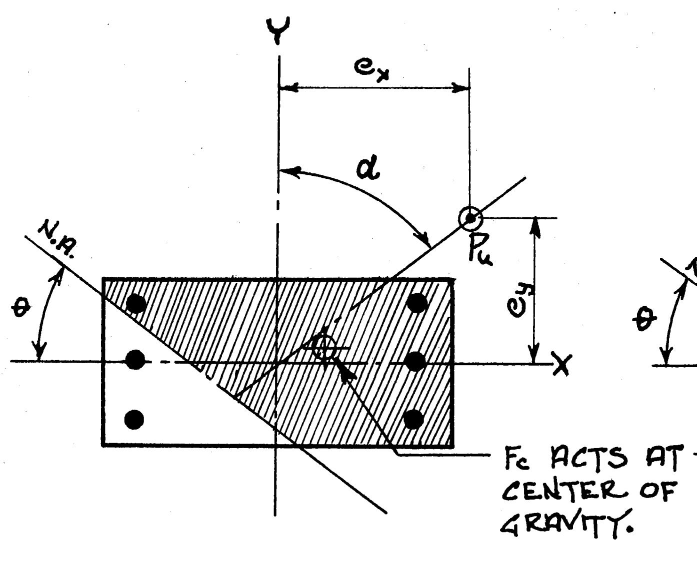
[3]Rice and Hoffman, pp. 265-274.

CHAPTER VI

BIAXIAL BENDING

In comparison to bending about one axis of a reinforced concrete column, biaxial bending presents an entirely different and more complex situation. As a second bending moment is introduced, the neutral axes are no longer parallel to the centroidal axes of the section, but lie at some angle θ from them. Due to the rectangular shape of the cross section, as θ increases, the area of the cross section under compression



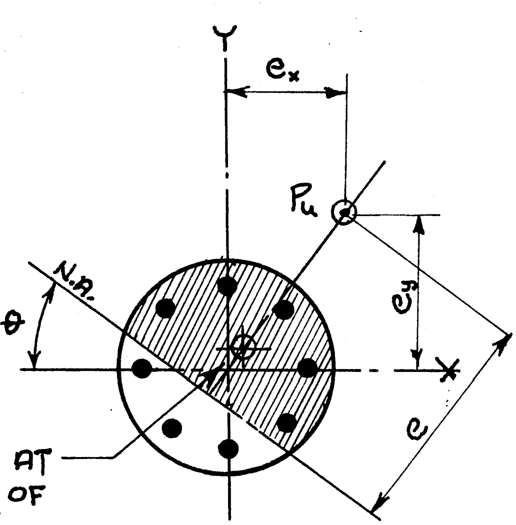


Figure 5. Compression Area of Rectangular Section Under Biaxial Bending.

Figure 6. Compression Area of Circular Section Under Biaxial Bending.

becomes triangular as shown in Figure 5. If this area maintained the same shape as θ changed, as in the case of a circular cross section, it would be a simple matter to find the relationship of one moment to the other, as illustrated in Figure 6. However, the shape of the cross section does vary with θ, but no simple and exact relationship is to be found between θ and the load capacity of the column. It is therefore necessary to rely on empirical relationships developed from biaxial bending tests on rectangular concrete columns.

In recent years several methods for designing biaxially loaded columns have been published. Most methods have in common some form of an interaction surface as shown in Figure 7. The curve ABCD represents the load-moment interaction diagram for the X axis of the column and the curve DEFG represents the load-moment interaction diagram for the Y axis. For any given value of axial load P_u' a horizontal plane may be passed through the interaction surface defining the maximum allowable moment the column will withstand when applied simultaneously with P_u'. For example we shall assume that point H in Figure 7 represents a value of axial load to be applied to a column. A horizontal plane passed through this point is represented by plane BHF. If this plane were to be removed from the diagram and viewed in plan it would appear as in Figure 8. For this value of P_u', if a moment occurred about only the X axis, the maximum allowable moment the column could withstand would be represented by point M_{ox}'. Likewise, if a moment occurred only about the Y axis, the

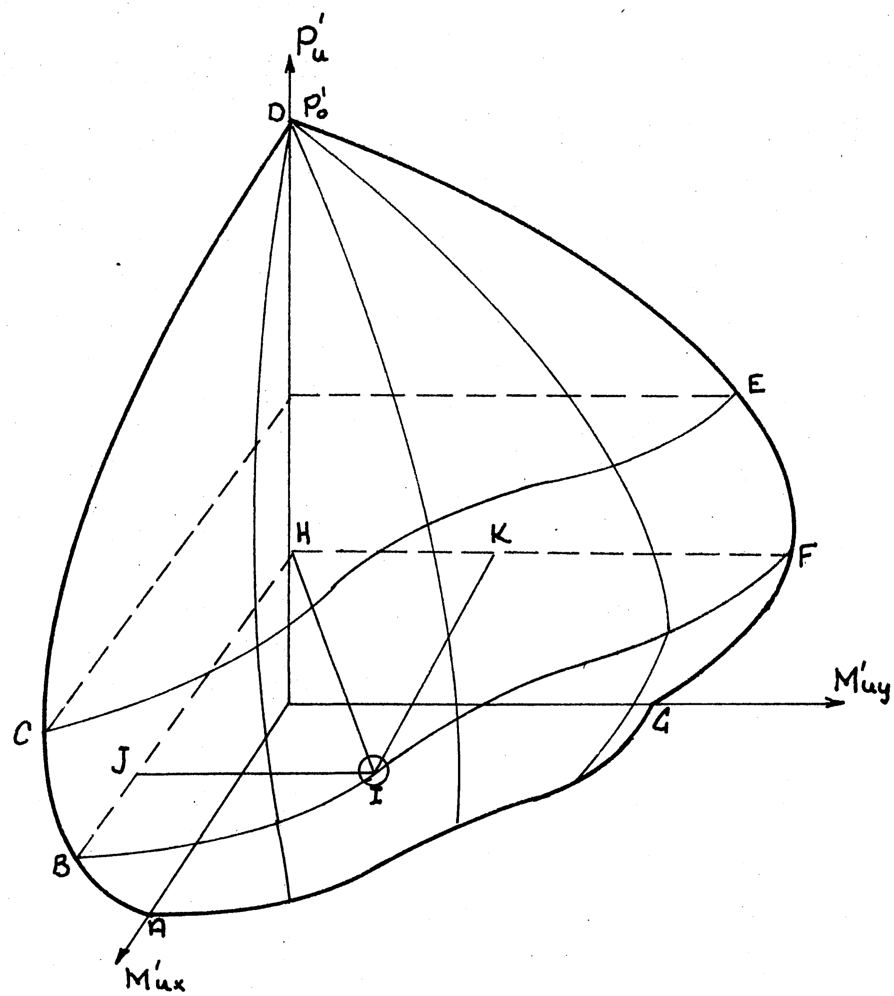


Figure 7. Biaxial Load-Moment Interaction Diagram.

maximum moment capacity would be given by point M'_{oy}. However, if the resultant moment applied to the column is about an axis at some angle α from the Y axis, its maximum allowable value is given by M'_u. Ideally, point M'_u would lie on the dashed elliptical curve, but in the case of rectangular columns, the fact that the compression area becomes triangular, as in Figure 5, alters the boundary similar to the solid

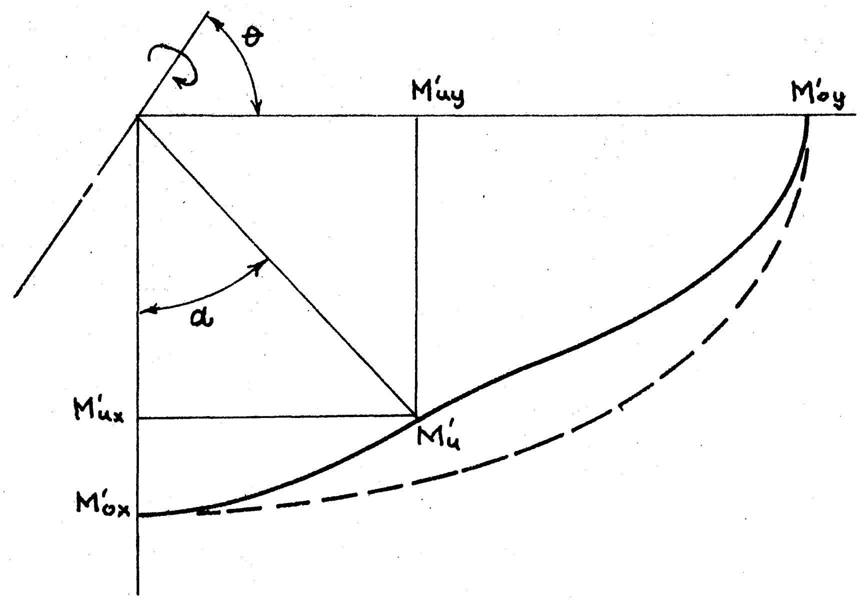


Figure 8. Section Through Biaxial Load-Moment Interaction Diagram at Constant Load P.

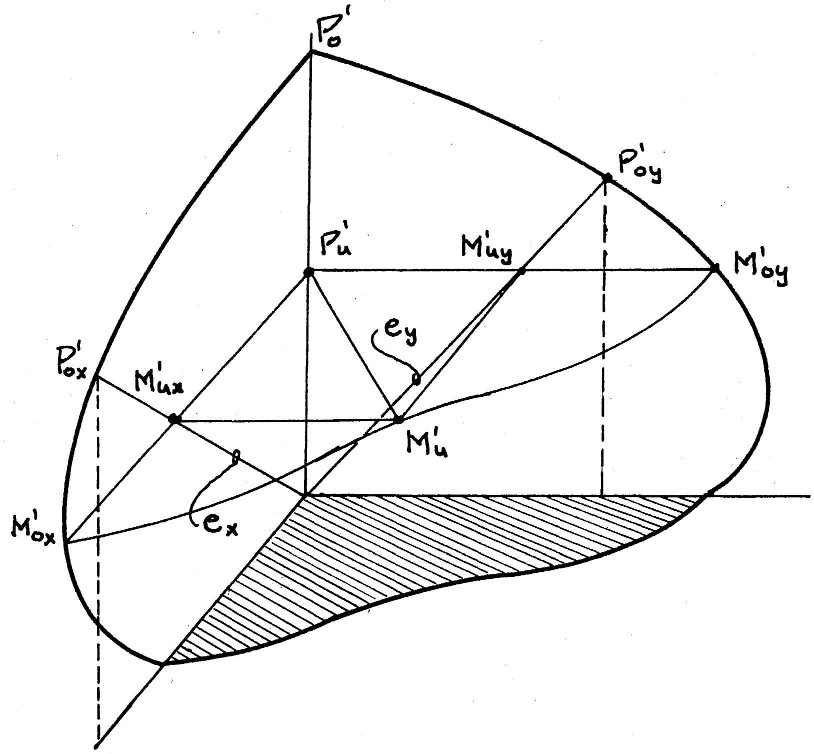


Figure 9. Relation of Uniaxial Capacities to Biaxial Load-Moment Interaction Diagram.

curve in Figure 8. If the maximum allowable moment M'_u is divided into X and Y components M'_{ux} and M'_{uy}, it becomes a simpler matter to formulate a relationship between the axial load P'_u and each of the moment components.

This problem is expanded in Figure 9 where P'_o is the ultimate concentric load with no eccentricity. At points M'_{ux} and M'_{uy}, an equivalent moment may be produced by the axial load P'_u acting at eccentricities e_x and e_y from the X and Y axes, respectively. If the eccentricity e_x were fixed and the load allowed to vary, the maximum allowable axial load would be P'_{ox}. If no bending occurred about the Y axis, the maximum axial load would be P'_{oy}. If the analysis of Figure 9 is approached from the opposite direction, that is if P'_{ox}, P'_{oy}, e_x and e_y are known, a relationship can be found between P'_{ox}, P'_{oy} and P'_u. One of the simplest relationships was developed by Bresler.[1] Tests and investigations of biaxial bending have shown his equation to be satisfactorily accurate under most of the range of axial load and bending moments. If a given cross section as in Figure 10 is subjected to bending moments M'_{ux} and M'_{uy} and an axial load P'_u as shown, the system of forces can be reduced to a single load acting at equivalent eccentricities obtained from

$$e_x = \frac{M'_{ux}}{P'_u} \qquad e_y = \frac{M'_{uy}}{P'_u}$$

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[1]Boris Bresler, "Design Criteria For Reinforced Columns Under Axial Load and Biaxial Bending," <u>ACI Journal</u> (1970), pp. 481-490.

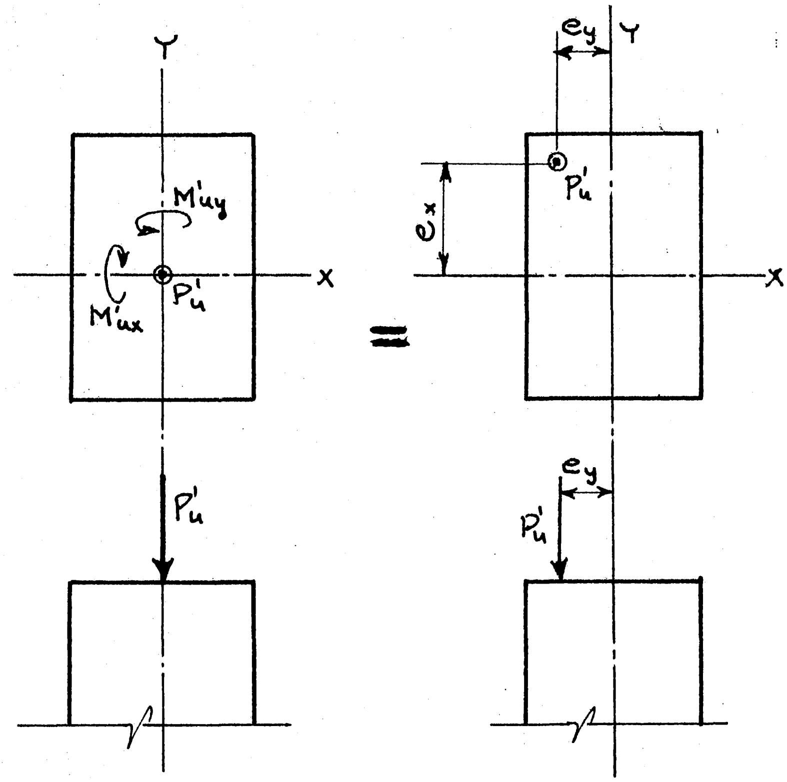

Figure 10. Equivalent Eccentricities of Axial Load.

Bresler's equation is

$$\frac{1}{P'_u} = \frac{1}{P'_{ox}} + \frac{1}{P'_{oy}} - \frac{1}{P'_o} \tag{6.1}$$

where P'_u is the ultimate axial capacity of the cross section and P'_o is the ultimate axial capacity under a concentric load, P'_{ox} is the ultimate axial capacity if moment occurs only about the X axis and P'_{oy} is the ultimate axial capacity if moment occurs only about the Y axis. P'_o can of course be found by simple statics once a column size and reinforcement is assumed. The capacity P'_{ox} is determined by considering the column subjected only to P'_u and M'_{ux}, and accounting for slenderness effects if applicable. Then P'_{ox} can be similarly determined. Finally, a value is obtained for P'_u and compared with the actual load applied to the column.

The origin of Bresler's equation stems from a failure surface obtained by plotting the failure load P'_u as a function of eccentricities x and y as shown in Figure 11. The values of x and y also serve to illustrate the relationship of P'_u and the bending moment components M'_{ux} and M'_{uy}. As can be seen in the diagram as the eccentricities increase,

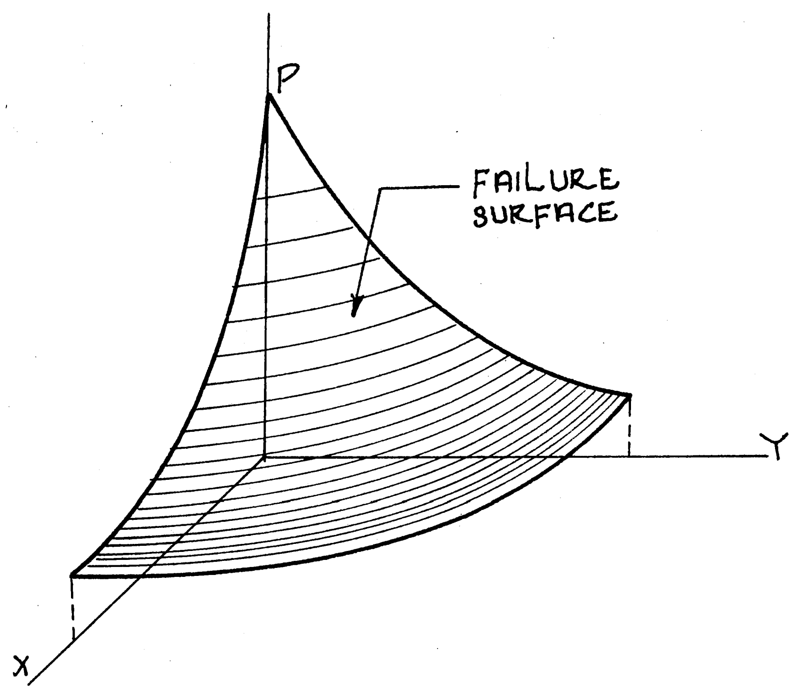

Figure 11. Failure Surface for Load vs Eccentricity.

bending moments increase and the failure load P_u' decreases to some limit at the bottom of the curve where axial load becomes negligible and the section is considered to be in pure bending. If the reciprocal of the failure load is plotted as a function of the eccentricities, a surface such as that in Figure 12 will be obtained. It is this surface from which Bresler's equation is actually derived. The surface being somewhat flat resembles a slightly warped plane. For a given

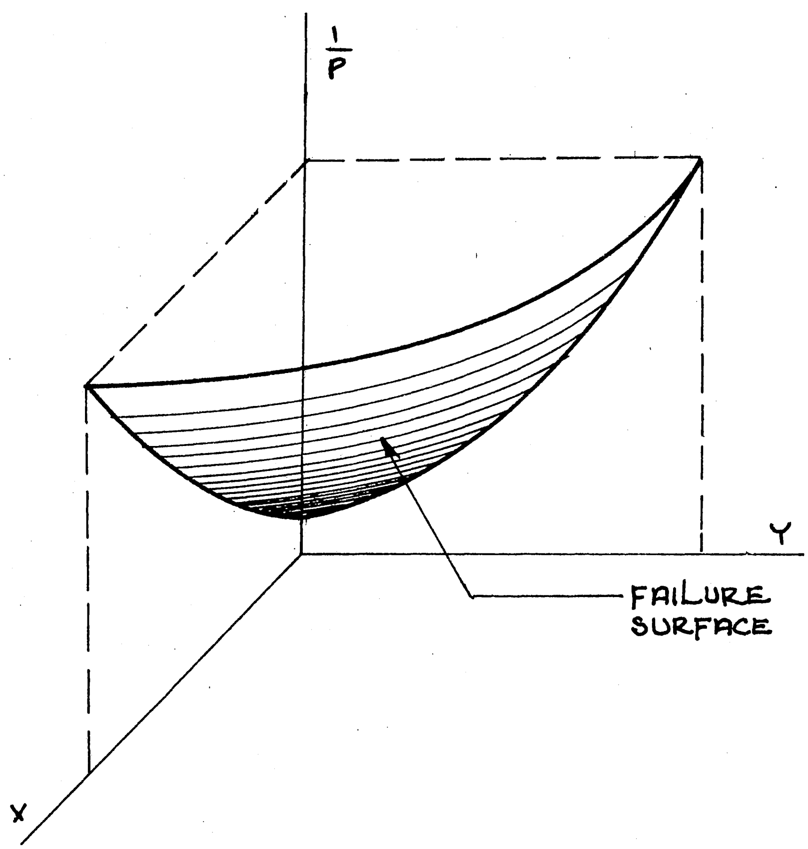

Figure 12. Failure Surface for Reciprocal of Load vs Eccentricity.

column, at least three points on the surface are known for some particular value of P'_u and are coordinates for the failure load for pure axial load P'_o, the e_x corresponding to the failure load P'_{ox} were moment to occur only about the Y axis, and the e_y similarly corresponding to the failure load P'_{oy}. These points can be plotted as $(1/P'_o, 0,0)$, $(1/P'_{ox}, e_x,0)$ and $(1/P'_{oy}, e_y, 0)$ as shown in Figure 13. If a plane were passed through the three points, any point on the failure

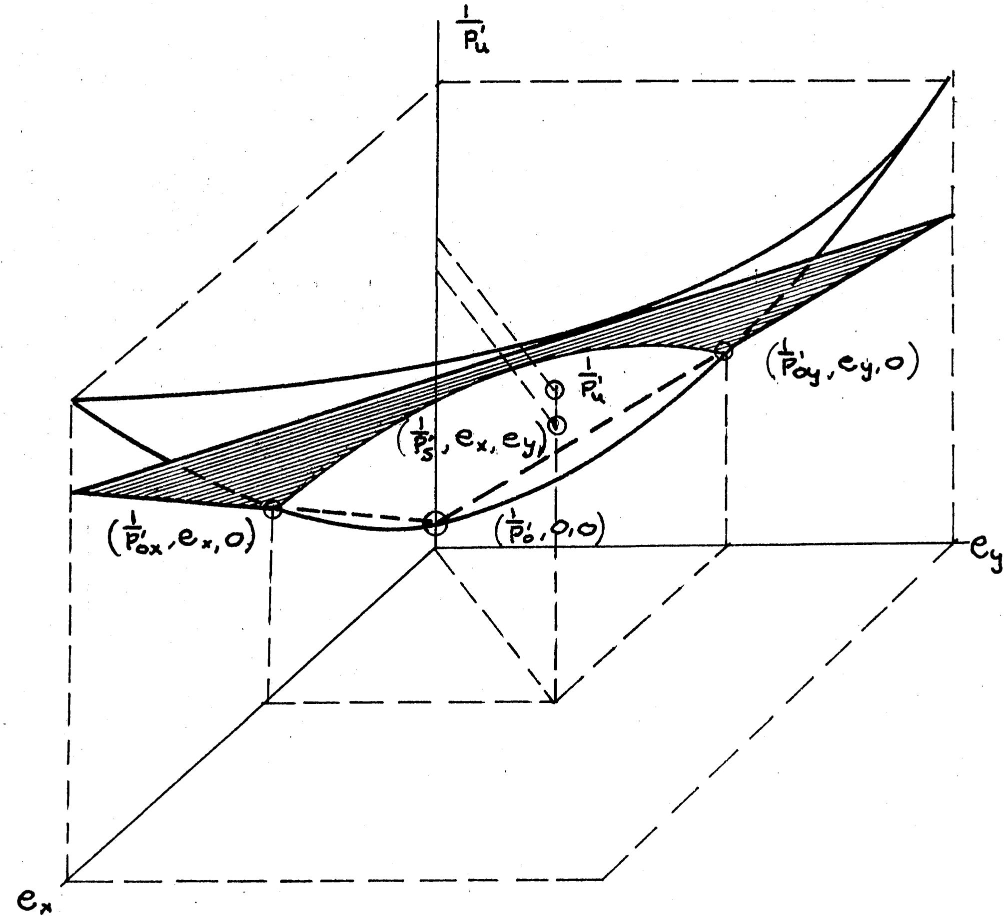

Figure 13. Bresler's Approximation of the Failure Plane 1/P vs e.

surface $(1/P'_s, e_x, e_y)$ can be approximated by a point $(1/P'_u, e_x, e_y)$ on the vertical projection to the plane. If some point $1/P'_s$ were defined by the three coordinates $(1/P'_s, e_x, e_y)$, see Figure 13, the location of $1/P'_s$ falls very near the intersection of the failure surface and the plane where the error in approximation is zero.

Since the plane is unique in that each value of $1/P'_{ox}$ and $1/P'_{oy}$ will yield unique values for e_x and e_y, the error in the approximations will very nearly be the same for all positions of the plane. The error will increase slightly, however, for very large values of $1/P'_{ox}$ and $1/P'_{oy}$. Results of the approximation were compared with theoretical results in Bresler's paper and found to be in excellent agreement, the average error being 3.3 percent.

In order to apply the approximation the plane must be defined by the three known points. The equation of the plane for some eccentricities e'_x and e'_y is

$$\begin{vmatrix} 0 & 1/P'_{ox} & 1 \\ e_y & 1/P'_{oy} & 1 \\ 0 & 1/P'_o & 1 \end{vmatrix} e'_x + \begin{vmatrix} 1/P'_{ox} & e_x & 1 \\ 1/P'_{oy} & 0 & 1 \\ 1/P'_o & 0 & 1 \end{vmatrix} e'_y + \begin{vmatrix} e_x & 0 & 1 \\ 0 & e_y & 1 \\ 0 & 0 & 1 \end{vmatrix} 1/P'_u = \begin{vmatrix} e_x & 0 & 1/P'_{ox} \\ 0 & e_y & 1/P'_{oy} \\ 0 & 0 & 1/P'_o \end{vmatrix}$$

where $(1/P'_{oy}, e'_x, e'_y)$ are the coordinates of the failure load for biaxial bending. Simplifying the equation we obtain:

$$e'_x\left(\frac{1}{P'_o} - \frac{1}{P'_{ox}}\right) + \frac{e'_y\, e_x}{e_x}\left(\frac{1}{P'_o} - \frac{1}{P'_{oy}}\right) + e_x\left(\frac{1}{P'_u} - \frac{1}{P'_o}\right) = 0$$

For biaxial bending the eccentricities will be the same as those for uniaxial bending, see Figure 10. Therefore, if $e'_x = e_x$ and $e'_y = e_y$, then

$$\frac{1}{P'_u} = \frac{1}{P'_{ox}} + \frac{1}{P'_{oy}} - \frac{1}{P'_o}$$

which is Bresler's formula. It is perhaps the simplest and most widely applicable relationship that has been developed.

Another method introduced by Bresler is of the form

$$\left(\frac{M_x}{M_{ox}}\right)^{\alpha} + \left(\frac{M_y}{M_{oy}}\right)^{\beta} = 1$$

where $M_x = P_u y$, $M_{ox} = P_u y_o$ when $x = M_y = 0$; and $M_y = P_u x$, $M_{oy} = P_u x_o$ when $y = M_x = 0$. Looking at the failure surface formed by the load-moment interaction diagram, Figure 14, the surface is formed by a family of curves at constant values of P_u. Bresler refers to these curves as load contours. If a plan bounded by a load contour at some P_u is examined as in Figure 15, and if the load contour is assumed to be a straight line, the equation of the load contour is given by

$$M_y = M_{oy} - M_x\left(\frac{M_{oy}}{M_{ox}}\right)$$

The equation can be written as

$$\frac{M}{M_{ox}} + \frac{M}{M_{oy}} = 1$$

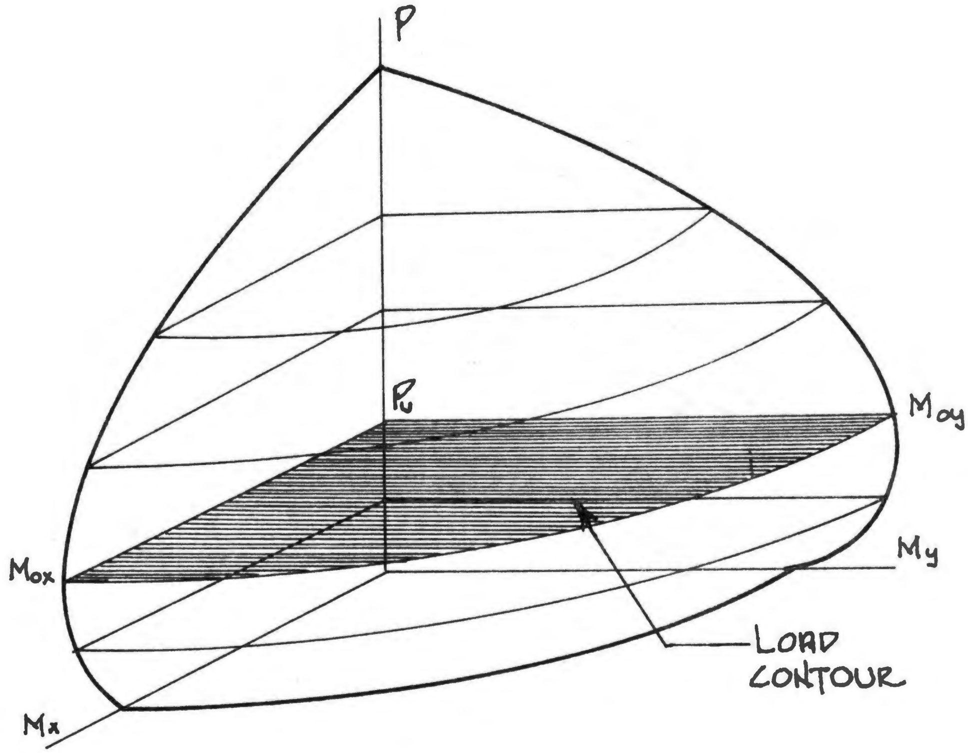

Figure 14. Load Contour of Biaxial Interaction Surface.

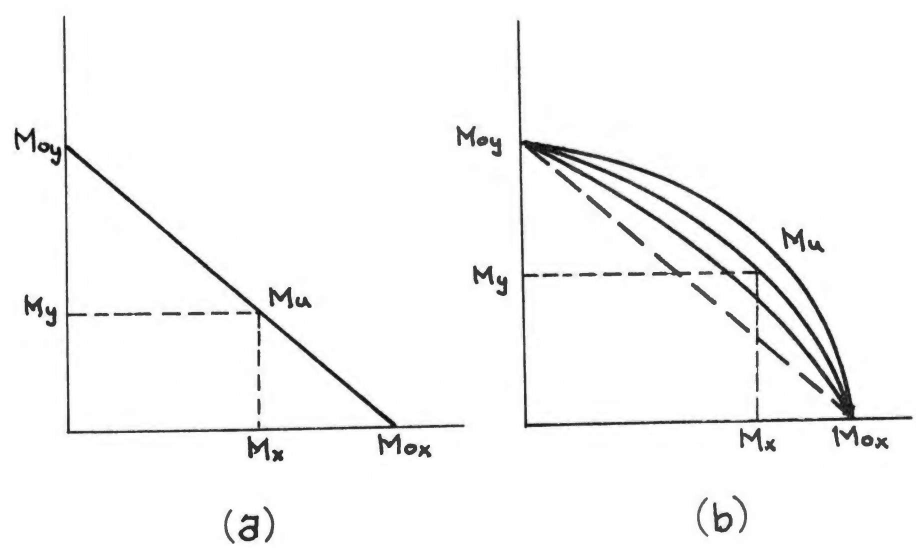

Figure 15. Approximation of Load Contour From Biaxial Interaction Surface.

If the load contour is curved instead of straight its equation is approximated by

$$\left(\frac{M_x}{M_{ox}}\right)^{\alpha} + \left(\frac{M_y}{M_{oy}}\right)^{\beta} = 1$$

in which α and β are dependent on the dimension of the column, steel reinforcing, stress-strain behavior of the materials, the concrete cover and lateral ties. Tests showed that this equation provided good approximations of analytical results but no one value of α or β can be assigned to accurately represent the load contour for all cases.[2] Therefore, the determination of α and β would add undesired complexity to the design procedure.

Other approximations have been derived, among them a method by Pannell based on a failure surface as in Figure 14.[3] These methods are either less accurate or require additional functions making the design procedure more complex. For this reason the first method by Bresler will be used in the design charts to be constructed here.

The one limitation of Bresler's equation is its applicable range. For small axial loads tension has more influence on a column's capacity. The relationships on which Bresler's equation is based are no longer valid due to the

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[2]Ibid.

[3]F. N. Pannell, "Biaxially Loaded Reinforced Concrete Columns," Proceedings, ASCE, Vol. 85, ST6 (June, 1959), pp. 47-54.

absence of the failure surface, Figure 13, in this range. Therefore, the equation will not be used for values of P_u less than $P_o/10$.[4] Another equation must be used for this lower range of axial loads.

If a load contour of Figure 14 is examined for a square column with equal steel in all faces at some P_u, the biaxial load-moment capacity will be represented by a near circular curve, Figure 16a, where M_{ox} and M_{oy} are the uniaxial moment

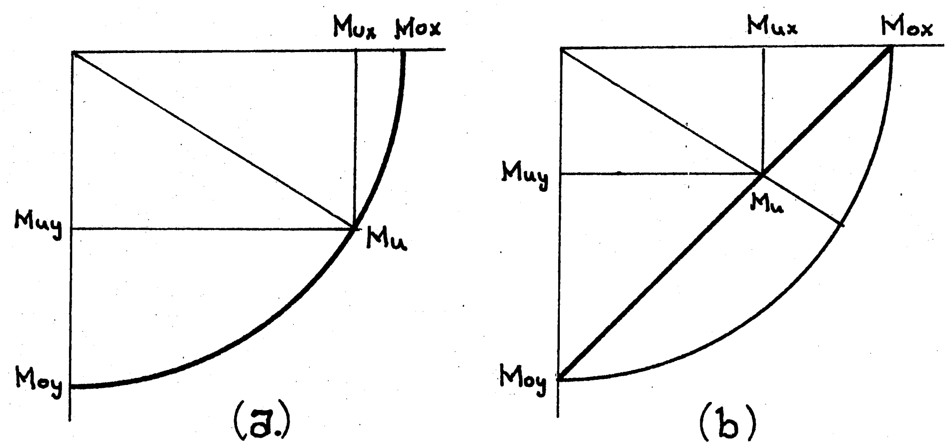

Figure 16. Load Contour for Square Section With Equal Reinforcement in All Faces.

capacities for the X and Y axes, respectively, and are equal. The design moments M_{ux} and M_{uy} are bounded by their intersection with the load contour M_u. An approximation of this limit can be made by assuming a straight line between M_{ox}

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[4]Rice and Hoffman, p. 290.

and M_{oy} as in Figure 16b. Then M_u will lie on the line and inside the curve for any combination of M_{ux} and M_{uy} giving a more conservative solution. The design moment M_u is simply the vector sum of $\overline{M}_{ux}$ and $\overline{M}_{uy}$ but should not be larger than M_{ox}.[5] This equation is suitable

$$\overline{M}_u = \overline{M}_{ux} + \overline{M}_{uy} \leq \overline{M}_{ox} \tag{6.2}$$

for a square column with equal reinforcement in all faces. However, if the reinforcement is not symmetrical or the column is rectangular, M_{ox} and M_{oy} are not equal and the load contour will resemble Figure 17a. A similar approximation can be made in this case as shown in Figure 17b. If equation

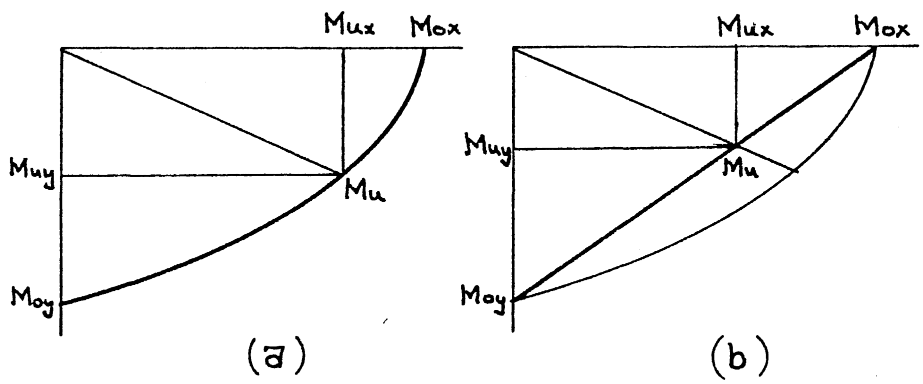

Figure 17. Load Contour for Rectangular Section With Symmetrical Reinforcement.

[5]Ibid., pp. 286-289.

(6.2) is written as

$$\frac{\bar{M}_{ux}}{\bar{M}_{ox}} + \frac{\bar{M}_{uy}}{\bar{M}_{oy}} \leq 1 \tag{6.3}$$

where $\bar{M}_{ox}$ and $\bar{M}_{oy}$ are assumed equal to $\bar{M}_u$, the relationship also represents the approximation in Figure 17b. Since $\bar{M}_{ox}$ was the upper limit of equation (6.2), when the equation is divided by $\bar{M}_{ox}$, one is the upper bound of equation (6.3). This equation was given by the previous code (ACI 318-63, Section 1407c, Eqn. 14-14) and limited to situations where tension controls the design.

Because of its simplicity, equation (6.3) will be used to determine biaxial capacity for the design charts to be presented here. Although the conservative error is significant, simplicity is considered to be important and since most columns encountered are not controlled by tension, the use of equation (6.3) should not provide unreasonable design as far as economy is concerned. Further discussion of the errors involved may be found in Paul F. Rice and Edward S. Hoffman, <u>Structural Design Guide to the ACI Building Code</u>, New York, 1972, Chapter 10.

CHAPTER VII

TRANSFORMATION INTO GRAPHICAL FORM

The preceding chapters have presented all the equations and procedures necessary for the complete design of a common reinforced concrete column. In those areas having several accepted techniques and theories, one method was selected for use in constructing the design aids. The equations used in this chapter will be repeated and referenced to their introduction in the text.

An outline of the series of operations required to design a column is given in Figure 18. The flow chart follows the column design procedure from the initial assumptions to final sizing. This procedure represents the fundamental approach to column design. Numerous short cuts and approximations have been introduced in many texts for preliminary checks to make the trial and error process less cumbersome. The reader is referred to Phil M. Ferguson, Reinforced Concrete Fundamentals, New York, 1973, for some of the more common methods of approximating column design.

Any method of column design requires some initial assumptions as to column size, reinforcement, or loading conditions. The procedure used here requires the selection of a trial size and percentage of reinforcement for a column. The

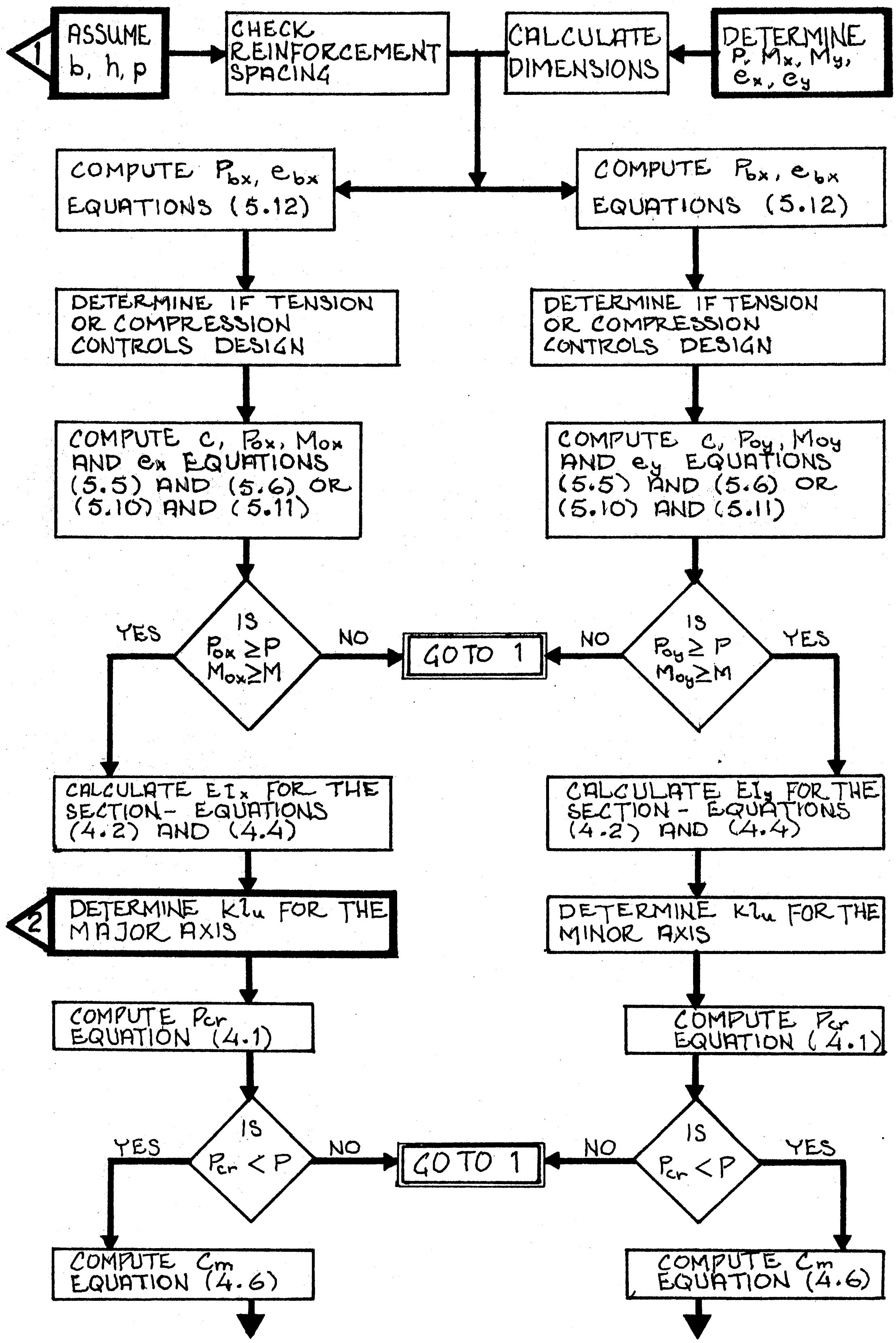

Figure 18. Flow Chart for Conventional Design of Reinforced Concrete Columns.

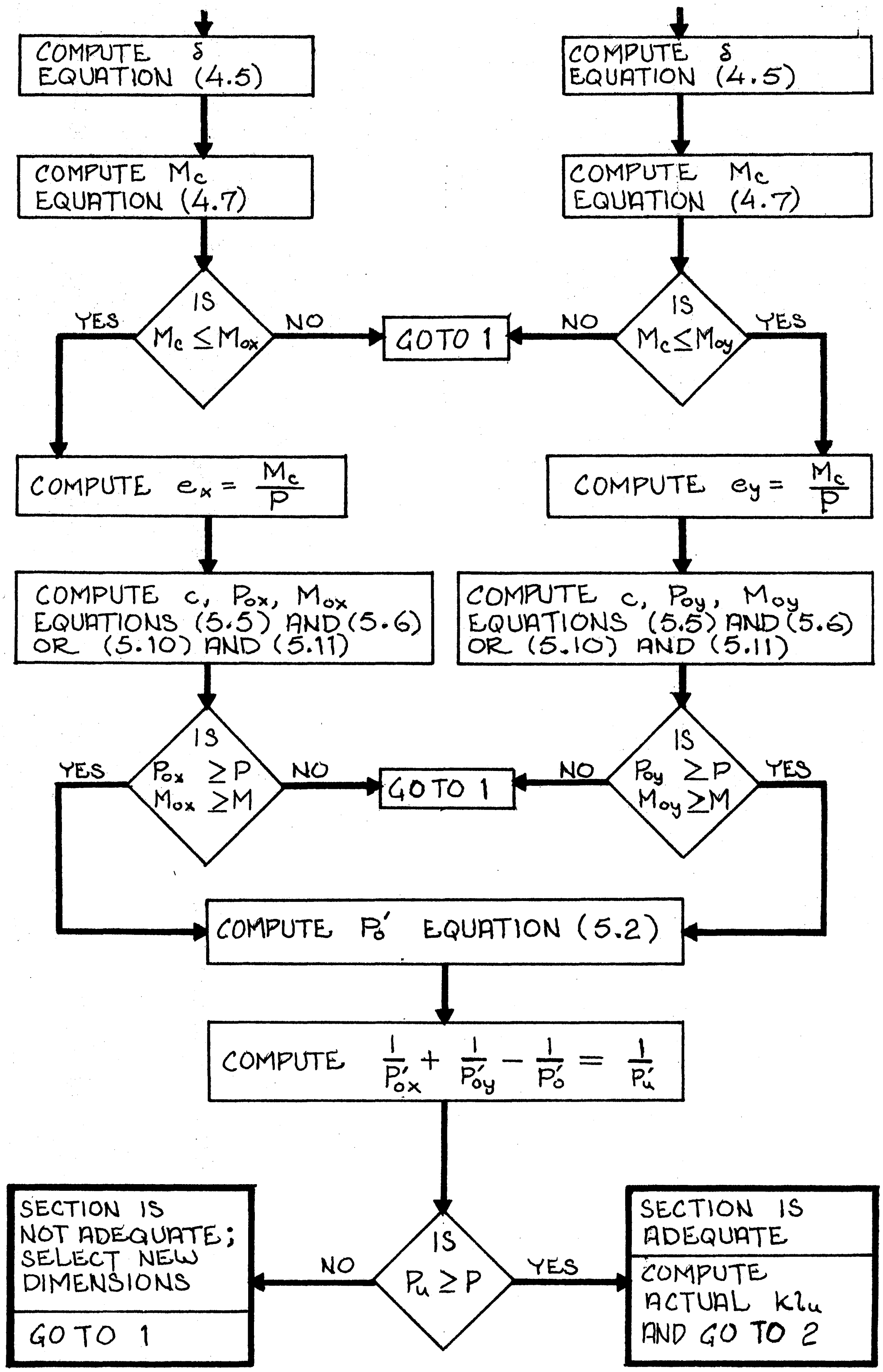

Figure 18 Continued.

capacity of the column is then determined and compared to the given load. Once the dimensions have been selected, the remaining unknown parameters lend themselves very well to graphical presentation. As can be seen from Figure 18 the section is first checked for uniaxial load-moment capacity. Then slenderness effects are checked for each axis and finally the biaxial capacity is determined. The general form of the design charts in the appendix follows these three steps. Design charts for column slenderness have been presented based on the previous code (ACI 318-63) and are assembled in terms of dimensionless parameters.[1] This requires that several preliminary calculations be made prior to using the charts. The sections of the charts dealing with slenderness effects also require more than simple calculations. The charts to be constructed here will follow a similar pattern except that relationships will conform to the current code and the parameters will be separated so that only simple calculations will be necessary. Dimensionless parameters will not be used, but rather a chart for each given column size. Also the relationships within the charts will be extended to include biaxial relationships. It is possible that since the exactness of the equations is to be sacrificed for the convenience of graphical solutions, some accuracy will be lost. However, the equations themselves are but empirical approximations and any errors in the solution will depend primarily

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[1]Richard W. Furlong, "Column Slenderness and Charts for Design," <u>ACI Journal</u> (1971), pp. 9-17.

on judgement and accuracy of plotting. A simple check by statics at the end of the design process should be sufficient to identify any significant errors made during the design procedure.

The basis of the design aids is the load-moment interaction diagram as shown in Figure 4. With the equations given in Chapter V, a diagram may be constructed for each axis of a given column size and percentage of steel. If the dimensions, reinforcing, load and properties are known and if an interaction diagram is available, it will be obvious whether failure is controlled by tension or compression and the column's capacity for any combination of load and moment may be found instantly. The upper boundary of the diagram, point P_o', is the capacity of the section under pure axial load and from statics is given by equation (5.2).

$$P_o' = 0.85\ f_c'\ A_n + f_y(A_s + A_s') \tag{5.2}$$

As moment is introduced the position of the neutral axis, distance c from the compression edge of the column, shifts toward the compression edge. As the eccentricity increases, c decreases. The capacity for axial load also decreases, but the moment resisting capacity increases. This curve between P_o' and P_b' is defined by

$$P_u' = 0.7225 f_c' A_n c + \frac{.003 E_s}{c}\left[A_s'(c-d') - A_s(d-1)\right] \tag{5.10}$$

$$M_u' = 0.36125f_c'A_nc(h- .85c) + A_s'E_s(h/2 -d')\frac{.003(c-d')}{c}$$

$$- A_sE_s(d- h/2)\frac{.003(d-1)}{c} \qquad (5.11)$$

When the neutral axis falls outside the section on the tension side ($.85c > h$), equations (5.5) and (5.6) must be used.

As the balanced loading condition P_b is passed, the moment capacity begins to decrease. This portion of the curve is given by

$$P_u' = 0.7225f_c'A_nc + A_s'f_y - A_sf_y \qquad (7.1)$$

$$M_u' = 0.36125f_c'A_nc(h- .85c) + A_s'f_y(h/2 -d')$$

$$- A_sf_y(d- h/2) \qquad (7.2)$$

The code provides that when the load capacity P_u' is less than $f_c'bh/10$ the safety factor ϕ may be increased from 0.7 to 0.9 as P_u' decreases to zero.[2] This puts an outward bend in the bottom of the interaction curve.

Since any column may contain several configurations and percentages of steel, a family of interaction curves may be drawn, one for each size bar group possible within the same size column. This will eliminate interpolation between percentages found on other diagrams. Lines of constant eccentricity are also plotted on the charts to aid in the selection of reinforcement. The uppermost line on the

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[2]Section 9.2.1.2, p. 26.

interaction diagram represents the minimum eccentricity allowed by the code, h/10.

The family of interaction curves is located in the upper right quadrant on each design chart. Another curve is superimposed on the graph and will be explained later in the chapter. The upper left quadrant consists of three families of curves. These three groups determine the ratio of applied load to critical load $P/\phi P_{cr}$. In order to determine this ratio, P_{cr} must be found from equation (4.1). This equation can be separated into a product of two terms in which the

$$P_{cr} = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2}{(kl_u)^2}(EI) \tag{4.1}$$

term EI is given by equation (4.2). If the equation for EI is separated into two terms, then the term $(E_cI_c/5 + E_sI_s)$

$$EI = \frac{1/5\ E_cI_c + E_sI_s}{1 + \beta_d} \tag{4.2}$$

has a unique value for each bar size group in a given size column. This term becomes one of the three slenderness parameters for equation (4.1). The second term $1/(1 + \beta_d)$ is the second parameter since its value is independent of the column properties and is a function of loading. The third slenderness parameter is $\pi^2/(kl_u)^2$ from equation (4.1). The product of these three parameters will yield P_{cr}, but $P/\phi P_{cr}$ is required to find the moment magnifier δ given by equation (4.5). In order to obtain the ratio of P to ϕP_{cr}, the parameters given above may be combined as shown in equation (7.3).

$$\delta = \frac{C_m}{1 - \frac{P}{\phi P_{cr}}} \tag{4.5}$$

$$\frac{P}{\phi P_{cr}} = \frac{P}{\phi(E_c I_c/5 + E_s I_s)} (1 + \beta_d) \frac{(kl_u)^2}{\pi^2} \tag{7.3}$$

Each of the three families of curves in the upper left quadrant of the design charts is the graphical representation of one of the three terms in equation (7.3). The lower left quadrant is a plot of $P/\phi P_{cr}$ vs δ for several values of C_m which is obtained from equation (4.6). In order to eliminate the calculation of C_m the actual parameter used will be M_1/M_2 and the family of curves is then defined by equation (7.4). Of course the code requires that C_m must not be less than 0.4 and that δ must not be less than 1.0.

$$C_m = 0.6 + 0.4 \left(\frac{M_1}{M_2}\right) \geq 0.4 \tag{4.6}$$

$$\delta = \frac{0.6 + 0.4 \left(\frac{M_1}{M_2}\right)}{1 - \frac{P}{\phi P_{cr}}} \geq 1.0 \tag{7.4}$$

The lower right quadrant corrects the initial design moment M_x by the moment magnifier δ and gives a new design moment M_c from equation (4.7). M_c can be projected vertically to intercept the line of constant eccentricity initially used and outward to the interaction curve and then a value of P_u can be read directly from the vertical axis of the interaction diagram. This value of P_u represents the failure load for the column axis under consideration and is the same as P_{ox} or P_{oy} in the biaxial bending equation (6.2). If this value is more than or equal to the initial P the design may be checked further for biaxial adequacy. If not, a new column size or reinforcement must be selected and the processes repeated.

Assuming the section is so far satisfactory, the additional curve superimposed on the interaction diagram is now used to invert the design load P_{ox} to obtain the appropriate terms of the biaxial equation.[3] Having completed the design

$$\frac{1}{P'_{ox}} = \frac{\phi}{P_{ox}} \tag{7.5}$$

for one axis of the column, the entire procedure is repeated for the other axis. If the column is square with the same reinforcement in all four faces the same chart may be used for both axes. If, however, the column is rectangular or square with unequal reinforcement the interaction diagram for moment about the minor axis will be of a smaller scale than that for the major axis. In order to avoid the confusion of additional curves on the charts and to maintain the accuracy of the larger scaled chart, a separate chart with an expanded moment scale has been constructed for the minor axis. The design procedure is identical except that the preliminary column size and reinforcement has already been determined by the design about the other axis. Upon obtaining

[3]If $P < P_o/10$, equation (6.3) must be used to determine biaxial capacity.

a satisfactory value for P_{oy} the reciprocal of P'_{oy} is found and the reciprocal of P'_o and by simple addition and subtraction the value of $1/P'_u$ is obtained from equation (6.2). By reversing the procedure for taking reciprocals, a value for P_u is found. If this P_u is greater than the actual load P used initially, the design is satisfactory. If the difference is too great a more economical section may be found quickly by adjusting the reinforcement or if necessary changing the column size. Note that if when repeating the design procedure for the minor axis a change is necessary in the size or reinforcement of the column, the major axis must be redesigned for the new conditions.

Once a graphical procedure is established, a method of converting the equations into graphical form must be determined and reasonable limits must be imposed on the range of values obtained from each equation. Obviously, without the use of a calculator or computer with plotting capabilities, an attempt to construct such numerous graphs would be time consuming to say the least. The charts in the appendix were drawn by a Hewlett-Packard 9830A calculator and a 9862A calculator plotter. A general program was written for most common condition to be encountered in the design of square and rectangular concrete columns. The compressive strength of the concrete is limited to 4000 psi, although the program will accept other strengths, and is assumed to be of normal weight, 145 pcf. The yield strength of the reinforcing steel is taken as 60,000 psi and its modulus of elasticity is

29,000,000 psi. The patterns of reinforcement are limited to square or rectangular patterns with a symmetrical arrangement of equal size bars on any two opposite faces. Lateral ties complying with the code provisions were used in all cases. A constant concrete cover of one and one-half inches is used in determining dimensions. These specifications are repeated in the appendix for reference.

Since a large range of column sizes is used, the capacities will also vary over a large range. If the same scale were used for all sizes, the interaction curves for the smaller sections would be too small to be used effectively. It was necessary to divide the scale into six ranges giving all diagrams approximately the same size. On any one chart for any one arrangement of reinforcement, all possible bar sizes are considered and an interaction curve plotted for each size, the largest plotted first to set the scale. The lines of constant eccentricity are plotted for the minimum eccentricity allowed by the code and at one inch intervals to four inches, a six inch eccentricity, and then at eight inch intervals to about one and one-fourth times the depth of the section. All units are in inches and kips.

The graphical conversion of equation (7.3) requires three steps. The first step deals with the first term

$$\frac{P}{\phi(E_c I_c/5 + E_s I_s)} \tag{7.6}$$

Since the scale of P is determined by the interaction diagram, the scales of the upper left quadrant will depend on

P. Since P is known, I_c and I_s must be calculated over their effective ranges. Since the size of the column is constant, I_c is constant and I_s is the only variable. An example graph is shown in Figure 19. The equation of a line can be calcu-

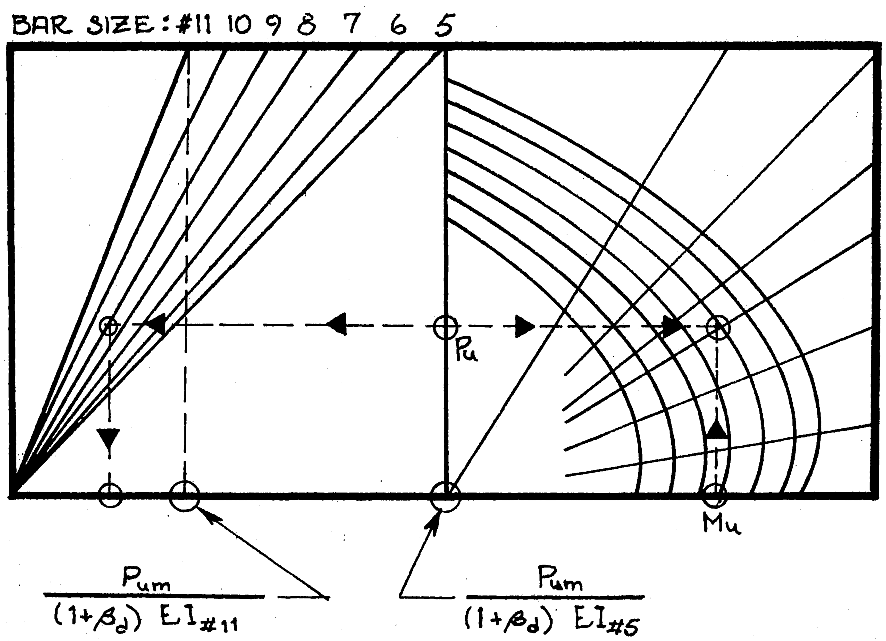

Figure 19. Graphical Representation of $P/(E_cI_c/5 + E_sI_s)$.

lated for each size bar used in the interaction diagram and can be plotted as a function of P. The upper limit of expression (7.6) will be a function of the upper limit of P for the diagram under consideration. If expression (7.6) is evaluated for the minimum stiffness possible within the given sizes of steel bars, a conservative upper bound will be obtained by using the upper limit on the vertical axis for P, which will be designated as P_{um}. This will plot a line from the lower left corner to the upper right corner of the graph, establishing the upper bound of the scale for this graph. Then using P_{um} a line may be constructed for each size bar given by the interaction diagram. If for any particular bar size a horizontal line is drawn from the design load P to intersect the appropriate bar size line, the vertical projection of this point to the horizontal axis gives the value of expression (7.6). The scale on this axis depends on the scale of P_u and the stiffnesses of the column. Since it will be different for each graph and since it is not necessary to know the value of expression (7.6), the axis is not scaled for this graph on the design charts.

The second step of the conversion is the multiplication by the term $(1 + \beta_d)$. The maximum value of β_d is one if dead load is the only load present. The lower limit of β_d approaches zero for a very small dead load moment compared to the total load moment. Therefore, $(1 + \beta_d)$ can vary from one to two. The lower bound of expression (7.7) is equal to that

$$\frac{P_{um}(1 + \beta_d)}{\phi(E_cI_c/5 + E_sI_s)} \tag{7.7}$$

of expression (7.6) while the upper bound doubles. Since the lower end of the scale for $P/(E_cI_c/5 + E_sI_s)\phi$ is zero, the lines for the $(1 + \beta_d)$ graph must originate at the upper left corner. The lines terminate within the lower half of the right side of the graph. As shown on the inverted vertical

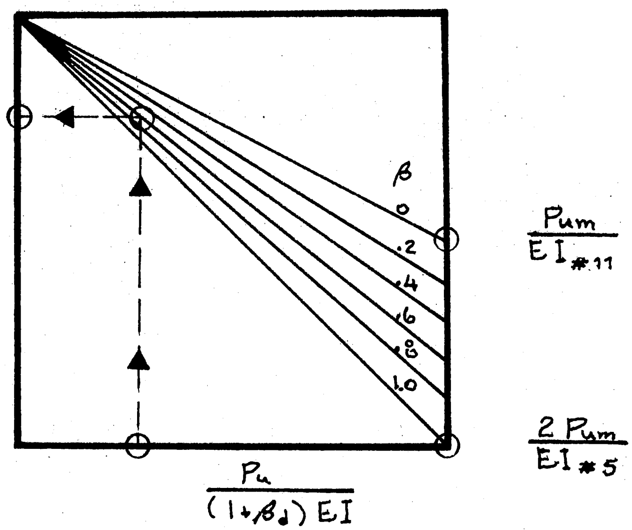

Figure 20. Graphical Representation of $(1 + \beta_d) \times P/(E_cI_c/5 + E_sI_s)$.

scale of Figure 20, the lines are plotted at intervals of β_d of 0.2. As is the case with the previous graph no scale will be required since only a relative value is necessary when plotting a solution.

The last step in the conversion multiplies the previous result by expression (7.8). Theoretically, k can vary from

$$\frac{(kl_u)^2}{\pi^2} \qquad (7.8)$$

0.5 to infinity. The shortest column to be considered is eight feet giving a minimum effective length kl_u of four feet. In order to give the charts a practical range, a maximum effective length of 100 feet will be used as the upper limit. A family of curves similar to Figure 20 is shown in Figure 21 for expression (7.8). The result will give a value

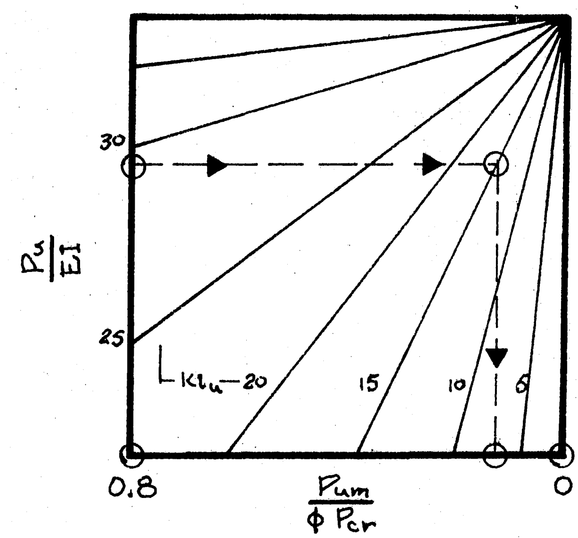

Figure 21. Graphical Representation of

$$\frac{(kl_u/\pi)^2 P(1 + \beta_d)}{\phi(E_c I_c/5 + E_s I_s)}$$

for $P/\phi P_{cr}$ which is scaled on the horizontal axis. Since any value of $P/\phi P_{cr}$ greater than one means that the critical buckling load has been exceeded, a practical range of 0.0 to 0.8 is used. Unlike the first two graphs, the scale for this quantity will remain constant and is given on each chart.

In order to obtain the moment magnifier δ, equation (7.4), the range of M_1/M_2 must be established. Its maximum value will be +1.0 when M_1 is equal to M_2. The minimum is zero when M_1 is zero. However, when M_1 is negative, the minimum value of M_1/M_2 is -1.0. The corresponding range for C_m is from 0.2 to 1.0, but the code requires that C_m is not to be less than 0.4. Therefore, the lower limit for M_1/M_2 must be -0.5. The code gives a lower boundary for δ as 1.0.

A reasonable upper limit for δ seems to be 3.0 since the curves in Figure 22 begin to approach a constant value near a δ of 3.0.

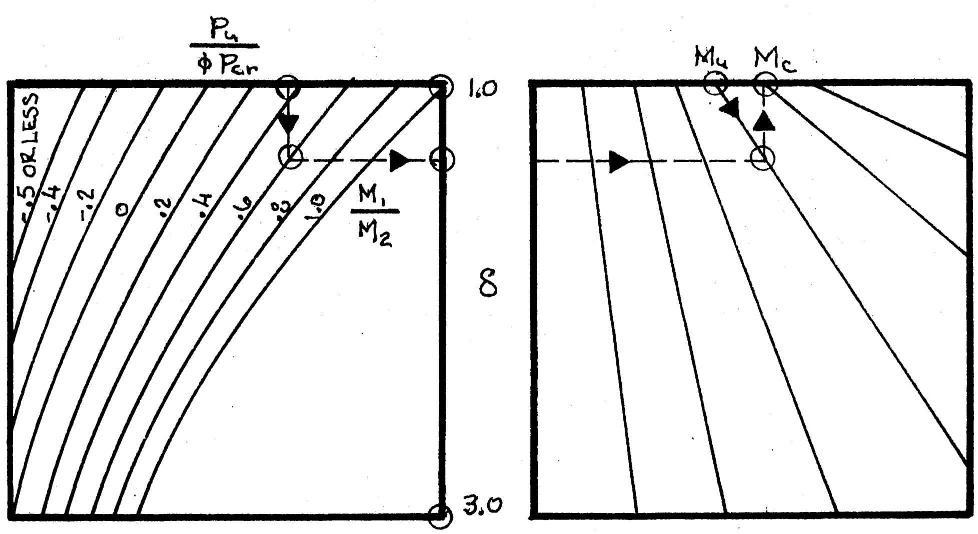

Figure 22. Graphical Representation of $\delta = C_m/(1 - P/\phi P_{cr})$ and $M_c = \delta M$.

Once the moment magnifier is determined, the curves in the right side of Figure 22 simply multiply the initial design moment M by δ. If the value obtained for δ is projected horizontally to intersect the line originating at the design moment M, the vertical projection of this point is the new design moment M_c of equation (4.7).

The last part of the chart presents an awkward situation. In order that the values for P_{ox} and P_o be compatible with Bresler's equation (6.2) they must be divided by 0.7 to obtain P'_{ox} and P'_{o} and then the reciprocal of each found. Multiplication and inversion are relatively simple graphical procedures. But in Figure 23 it can be seen that in the lower range of values for P_{ox} an accurate intersection of the reciprocal curve becomes almost impossible due to its decreasing slope. If when the curve becomes too flat the scale is increased over a short range until that curve becomes flat then the scale increased again. A curve such as in Figure 23 is formed where in this case three separate scales are used to define the curve. This procedure serves to eliminate the flatter part of the curve and increase the accuracy. Since

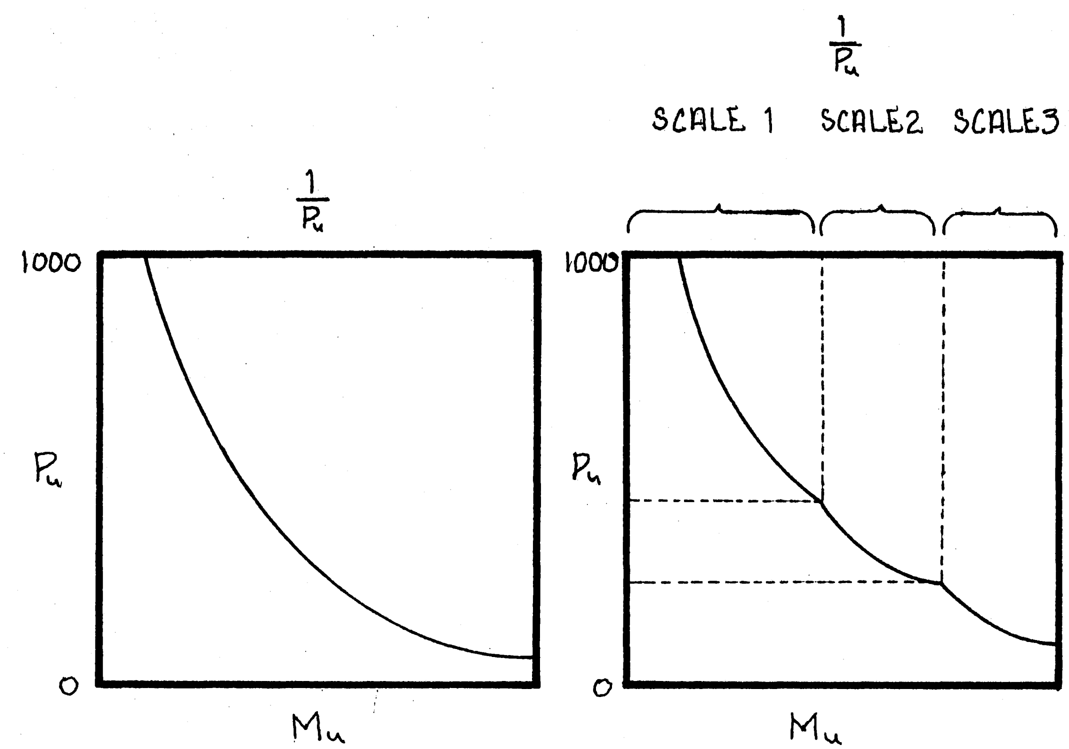

Figure 23. Graphical Representation of ϕ/P.

the curve is a reciprocal function, the lower end will approach infinity. However, for values of P_{ox} less than $P_o/10$ the curve is not valid so it is not necessary to carry the curve beyond this point. In programming the calculator to plot this curve, six possible scales are used for the P axis and each scale has a unique reciprocal curve with its own set of scales.

CHAPTER VIII

APPLICATION AND USE OF GRAPHS

The use of the design charts in the appendix requires only simple assumptions and calculations, the most difficult of which is the determination of the effective length factor k and the calculation of the moment magnification factor for one of several columns in a frame. A flow chart is given in Figure 28 which outlines the steps of procedure for using the charts. Though the flow chart appears complex and lengthy, the entire procedure is really quite simple and brief. An example problem will be given in the following paragraph to illustrate the use of the charts. Additional problems are given later dealing with uncommon situations which may require special procedures not given in Figure 28.

As an example, consider the problem where it is desired to design a column to carry the following design loads obtained from a structural analysis: the axial load is 400k, the moments about the major axis are 1100 "k and 780 "k at the top and bottom, respectively, and the moments about the minor axis are 775 "k and 310 "k at the top and bottom. Assume the effective length factor k is 0.75, and the unsupported length is 14 feet. Also, the ratio of dead load moment to total moment is 0.6 for both axes. Due to space requirements, the width of the column must be limited to 12 inches. The story δ will be considered to be insignificant for this problem and the column assumed to act individually.

As a trial size, a 12" x 18" section was selected as shown in Figure 24. The reinforcing will consist of eight number eight bars, two on each of the short sides of the column and four on each long side. Referring to Figure 29a if the applied load and moment about the major axis are plotted on the interaction diagram, the point falls well within the curve for number eight bars. However, slenderness effects and biaxial bending will reduce the allowable loads to the extent that even this section may not be adequate. With the applied load of 400 k, a line is projected from the 400 mark on the "P" scale to intercept the line representing a number eight bar size as shown in the upper left section of the chart. From this intersection a line is drawn vertically to the β = 0.6 line and from there horizontally to an imaginary line half-way between the two lines representing effective lengths kl_u of 10 and 11 feet for an effective length of 0.75 x 14 or 10.5 feet. Then a line drawn vertically to the $P/\phi P_{cr}$ scale gives a value of 0.17 for $P/\phi P_{cr}$. The line is continued across the axis to meet a line between 0.6 and 0.8 for M_1/M_2 of 0.71. From there a horizontal line yields the moment magnifier δ of 1.06. The line is continued horizontally to intercept the diagonal line originating at M equal to 1100 "k. This point is projected vertically across the M axis giving a new design moment M_c of 1170 "k and when carried up to P equal to 400 k, a new eccentricity is defined as 2.9 inches. The point is still within the boundary of the interaction curve for number eight bars. Therefore, the section will satisfy uniaxial bending about its major axis. If the point just plotted is extended to the interaction curve along the line of constant eccentricity, the maximum allowable axial load is found. This value is P_{ox}. From this point on the interaction curve a line is drawn horizontally to the reciprocal curve and then vertically for a value of $1/P'_{ox}$ of 0.00137.

The entire procedure is now repeated for the minor axis using Figure 29b. The values obtained are: $P/\phi P_{cr} = 0.34$, $\delta = 1.15$, $M_c = 890$ "k, e = 2.2 ", and $1/P'_{oy} = 0.00139$. From the table in the lower left quadrant of Figure 29a, for number eight bars, $1/P'_o$ is given as 0.00092. Then

$$0.00137 + 0.00139 - 0.00092 = 0.00184$$

which is Bresler's formula and the 0.00184 is the value of $1/P'_u$. The allowable load P_u is found by locating 0.00184 on the top scale and projecting the point down to the reciprocal curve and then left to the load scale. The intersection with this scale gives a value of P_u of 390 k which is less than the applied load of 400k. Even though the section is satisfactory for an applied moment about either axis, the presence of both moments simultaneously produces stresses in the section which it cannot safely resist. Therefore, either a new section must be selected or the reinforcement increased. Since the allowable load is exceeded by only a small amount, an increase in the section's dimensions would most likely result in an excess of capacity which is not needed. An increase of the size of reinforcing to number nine bars will be tried. Again progressing through the graphical design procedure the following values are obtained from the major axis: $P/\phi P_{cr} = 0.15$, $\delta = 1.05$, $M_c = 1140$ "k, e = 2.9", and $1/P'_{ox} = 0.0013$. Then about the minor axis, $P/\phi P_{cr} = 0.3$, $\delta = 1.1$, $M_c = 865$ "k, e = 2.1", and $1/P'_{oy} = 0.0013$. From the table at the lower left, $1/P'_o = 0.00084$. Then

$$0.0013 + 0.0013 - 0.00084 = 0.0018$$

Projecting this value through the reciprocal scale, a value of 400 k is obtained for P_u which is equal to the design load.

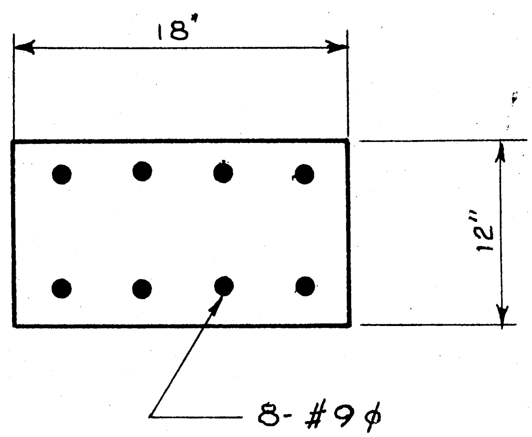

Figure 24. Column Section, Example 1.

To check the capacity of the column selected with the charts, a conventional analysis will be made using statics. An approximate method would be quicker, but greater accuracy is desired to check the results. From Figure 18 the first steps are to determine b, h, A_s, P, M_x, M_y, e_x and e_y.

$b = 12"$	$M_{x1} = 780$ "k	$M_{y1} = 310$ "k
$h = 18"$	$M_{x2} = 1100$ "k	$M_{y2} = 775$ "k
A_s = 8 #9 bars	$e_x = 2.75"$	$e_y = 1.94"$
$P = 400$ k		

Also, $f'_c = 4$ ksi, $f_y = 60$ ksi, $E_s = 29 \times 10^3$ ksi and

$$E_c = w^{1.5}\ 33\ \ f'_c = (145\text{pcf})^{1.5}(33)\sqrt{4\text{ksi}}$$

$$E_c = 3.64 \times 10^3 \text{ ksi}$$

The section is shown in Figure 25 with dimensions with respect to bending about the major axis. In order to determine if compression or tension controls, the balanced condition will be analyzed. From equations (5.12), e_b is determined for one row of reinforcing on either side of the center line. This problem presents four rows and requires additional terms in the equation. The distance to the neutral axis is

$$c_b = \frac{(.003)(15.58)}{.003 + .00207} = 9.22"$$

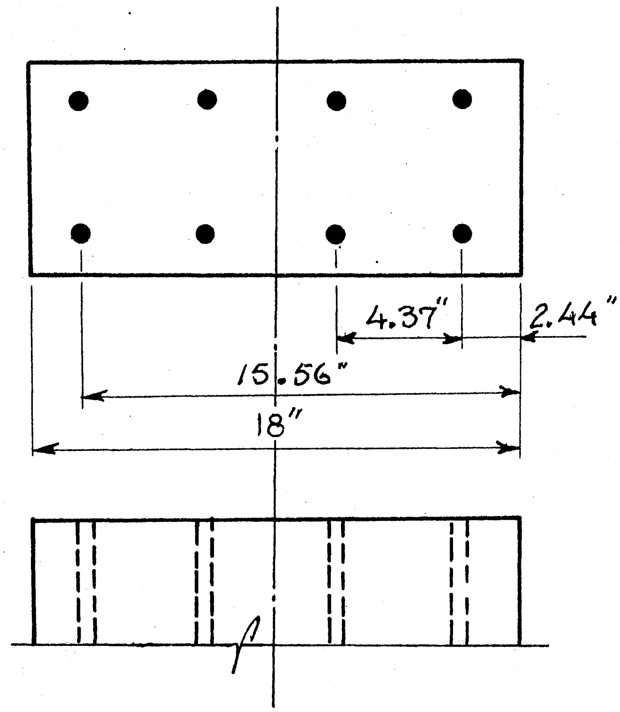

Figure 25. Column Section, Major Axis, Example 1.

Then: $P_b' = F_c + F_{s1} + F_{s2} + F_{s3} + F_{s4}$

$$F_c = 0.85f_c' \ (0.85\ bc_b - A_s')$$

$$F_c = (.85)(4)\ \left[(.85)(12)(9.22) - 4\right] = 306\ \text{k}$$

$$F_{s1} = \frac{(.003)(c_b - d')}{c_b} \frac{A_s}{2} E_s$$

$$F_{s1} = \frac{(.003)(9.22 - 2.44)(4)}{(9.21)(2)} (29\text{x}10^3)$$

$$F_{s1} = 128\ \text{k} > 2f_y \therefore \text{Use } F_{s1} = 120\ \text{k}$$

$$F_{s2} = \frac{(.003)(c_b - d' - s)}{c_b} \frac{A_s'}{2} E_s$$

$$F_{s2} = \frac{(.003)(9.22 - 2.44 - 4.37)(4)}{(9.21)(2)}(29\text{x}10^3) = 45 \text{ k}$$

$$F_{s3} = \frac{(.003)(d - s - c_b)}{c_b} \frac{A_s}{2} E_s$$

$$F_{s3} = \frac{(.003)(15.58 - 4.37 - 9.22)(4)}{(9.21)(2)}(29\text{x}10^3)$$

$$F_{s3} = -38 \text{ k}$$

$$F_{s4} = -120 \text{ k}$$

$$P_b' = 301 + 120 + 45 - 120 = 313 \text{ k}$$

$$M_b' = F_c\left[\frac{h}{2} - \frac{.85c_b}{2}\right] + F_{s1}\left[\frac{h}{2} - d'\right] + F_{s2}\left[\frac{s}{2}\right] - F_{s3}\left[\frac{s}{2}\right] - F_{s4}\left[\frac{h}{2} - d'\right]$$

$$M_b' = (306)\left[\frac{18}{2} - \frac{(.85)(9.22)}{2}\right] + (120)\left[\frac{18}{2} - 2.44\right] + (45)\left[\frac{4.37}{2}\right] - (-38)\left[\frac{4.37}{2}\right] - (-120)\left[\frac{18}{2} - 2.44\right]$$

$$M_b' = 3311 \text{ "k}$$

$$e_b = \frac{3311}{313} = 10.57"$$

Since the balanced eccentricity is greater than the actual eccentricity, compression controls the design. Now the capacity of the column in compression must be found. The distance to the neutral axis must be found by trial and error. As shown in Figure 26 the reactive force of the concrete is assumed to act between P and the center line of the column. The neutral axis is assumed to lie as shown. Since the strains in the steel caused by F_{s1} and F_{s4} may exceed the maximum steel strain of .00207 in/in, a check may be made to

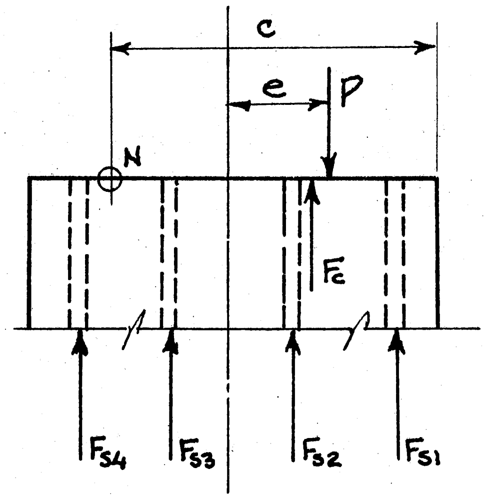

Figure 26. Resisting Forces of Section, Example 1.

determine whether the forces given below are valid. Since

$$F_c = .85f_c'(.85c_b - 3A_r)$$

$$F_c = (.85)(4)\left[(.85)(12)c - 6\right] = 34.68c - 20.4$$

$$F_{s1} = \frac{(.003)(c - d')}{c}A_rE_s$$

$$F_{s1} = \frac{(.003)(c - 2.44)}{c}(2)(29\text{x}10^3)$$

$$F_{s1} = 174 - \frac{424.56}{c}$$

$$F_{s2} = \frac{(.003)(c - s - d')}{c}A_rE_s$$

$$F_{s2} = \frac{(.003)(c - 4.37 - 2.44)}{c}(2)(29\text{x}10^3)$$

$$F_{s2} = 174 - \frac{1184.94}{c}$$

$$F_{s3} = \frac{(.003)(c - d + s)}{c} A_r E_s$$

$$F_{s3} = \frac{(.003)(c - 15.58 + 4.37)}{c}(2)(29x10^3)$$

$$F_{s3} = 174 - \frac{1950.54}{c}$$

$$F_{s4} = - \frac{(.003)(d - c)}{c} A_r E_s$$

$$F_{s4} = - \frac{(.003)(15.58 - c)}{c} (2)(29x10^3)$$

$$F_{s4} = 174 - \frac{2710.92}{c}$$

F_{s4} is close to the neutral axis, it will not be critical. The c required for F_{s1} to produce a strain of 0.00207 is given by

$$c = \frac{d'}{1 - \frac{0.00207}{0.003}} = \frac{2.44}{1 - \frac{.00207}{.003}} = 7.87"$$

Since c is assumed to be greater than 7.87", F_{s1} will produce a strain greater than .00207 and the maximum stress in the steel is 60 ksi. Therefore,

$$F_{s1} = f_y A_r = (60)(12) = 120 \text{ k}$$

Assuming a trial value for c of 15 inches,

$$F_c = 500 \text{ k}$$

$$F_{s1} = 120 \text{ k}$$

$$F_{s2} = 95 \text{ k}$$

$$F_{s3} = 44 \text{ k}$$

$$F_{s4} = -7 \text{ k}$$

and $$P'_{ox} = 762 \text{ k}$$

$$M'_{ox} = F_c \left[\frac{h}{2} - \frac{.85c}{2}\right] + F_{s1}\left[\frac{h}{2} - d'\right] + F_{s2}\left[\frac{s}{2}\right] - F_{s3}\left[\frac{s}{2}\right] - F_{s4}\left[\frac{h}{2} - d'\right]$$

$$M'_{ox} = (500)(9 - 6.38) + (120)(9 - 2.44) + (95)(2.19) - (44)(2.19) - (-7)(9 - 2.44) = 2278 \text{ "k}$$

$$e = \frac{2278}{762} = 2.96"$$

The e obtained from c = 15" is larger than e_x so a larger c will be tried. With c = 15.49", e = 2.75" which is correct.

$$P'_{ox} = 517 + 120 + 98 + 48 - 1 = 782 \text{ k}$$

$$P_{ox} = 0.7\ P'_{ox} = 547 \text{ k}$$

$$M'_{ox} = 517(9 - 6.49) + 120(9 - 2.44) + 98(2.19) - 48(2.19) + (9 - 2.44) = 2152 \text{ "k}$$

$$M_{ox} = 0.7\ M'_{ox} = 1506 \text{ "k}$$

P_{ox} and M_{ox} represent the maximum design loads that may be applied to the column with an eccentricity of 2.75".

The next step in the problem is to determine the moment of inertia of the section with respect to the major axis. From equation (4.2)

$$EI = \frac{\frac{1}{5}E_cI_c + E_sI_s}{1 + \beta_d}$$

$$I_c = \frac{bh^3}{12} = \frac{(12)(18)^3}{12} = 5832 \text{ in}^4$$

$$I_s = 4 \left[\frac{\pi(0.564)^4}{4} + (1)(9 - 2.44)^2\right]$$

$$+ 4 \left[\frac{\pi(0.564)^4}{4} + (1)\left(\frac{4.37}{2}\right)^2\right] = 191.86 \text{ in}^4$$

$$EI = \frac{\frac{1}{5}(3.64 \times 10^6)(5832) + (29 \times 10^6)(191.87)}{1 + 0.6}$$

$$EI = 6.131 \times 10^9 \text{ \#-in}^2$$

The critical buckling load is then

$$P_{cr} = \frac{EI\pi^2}{(kl_u)^2} = \frac{(6.131 \times 10^9)\pi^2}{(1000)\left[(.75)(14)(12)\right]^2} = 3811 \text{ k}$$

The moment magnifier is computed from equations (4.5) and (4.6).

$$C_m = 0.6 + 0.4\left(\frac{780}{1100}\right) = 0.88$$

$$\delta = \frac{C_m}{1 - \frac{P}{\phi P_{cr}}} = \frac{0.88}{1 - \frac{400}{(.7)(3811)}} = 1.04$$

The design moment then becomes

$$M_c = (1.04)(1100) = 1144 \text{ "k} < 1506 \text{ "k}$$

The new design moment is still within the allowable range of 1506 "k. With an increased design moment, the eccentricity must also increase.

$$e = \frac{1144}{400} = 2.86"$$

Then since the eccentricity has changed, a new value of c must be determined to find the capacity of the section. It was found that c = 15.25" satisfied the condition giving e = 2.87" and

$$P'_{ox} = 766 \text{ k} \qquad P_{ox} = 536 \text{ k}$$

$$M'_{ox} = 2202 \text{ "k} \qquad M_{ox} = 1541 \text{ "k}$$

The applied loads are still within the allowable range.

Before the analysis for biaxial bending is started, the column's capacity with respect to its minor axis should be checked. Repeating the procedure for the minor axis,

$$c_b = \frac{(.003)(9.56)}{.00507} = 5.66"$$

$$P'_b = F_c + F_{s1} + F_{s2}$$

$$F_c = 0.85f'_c(0.85c_b - A_r)$$

$$F_c = (.85)(4)\left[(.85)(18)(5.66) - 4\right] = 281 \text{ k}$$

$$F_{s1} = \frac{(.003)(c_b - d')}{c} A_r E_s$$

$$F_{s1} = \frac{(.003)(5.66 - 2.44)}{5.66}(4)(29 \times 10^3) = 198 \text{ k}$$

$$F_{s2} = -A_r f_y = -(4)(60) = -240 \text{ k}$$

$$P'_b = 281 + 198 - 240 = 239 \text{ k}$$

$$M'_b = F_c\left[\frac{h}{2} - \frac{.85c}{2}\right] + F_{s1}\left[\frac{h}{2} - d'\right] - F_{s2}\left[\frac{h}{2} - d'\right]$$

$$M'_b = (239)\left[6 - \frac{(.85)(5.66)}{2}\right] + (198)(6 - 2.44)$$

$$- (-240)(6 - 2.44) = 2418 \text{ "k}$$

$$e_b = \frac{2418}{239} = 10.12" > 2.13"$$

Compression will control the design. Figure 27 shows the assumed locations and directions of the resisting forces.

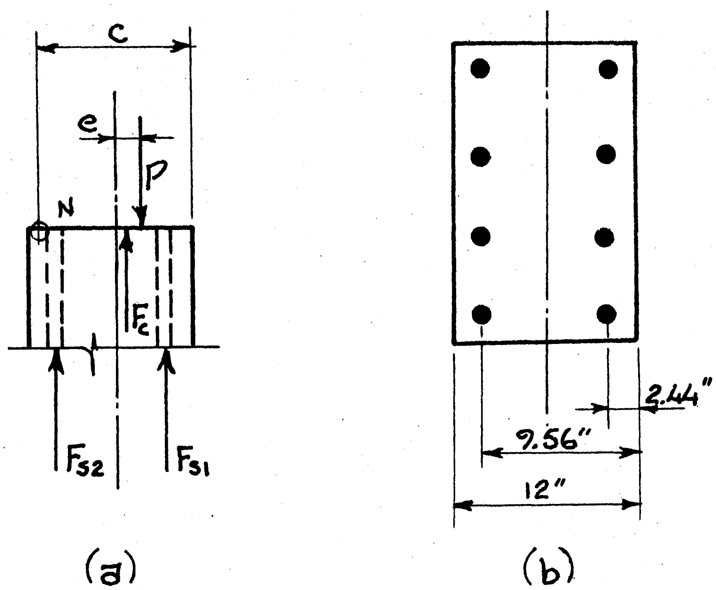

Figure 27. Resisting Forces, Minor Axis, Example 1.

Another trial and error procedure is required to determine the distance to the neutral axis. Having been illustrated once, the iteration is not given here. A value for c of 10.52" was found to be correct.

$$P'_{oy} = (.85)(4)\left[(.85)(18)(10.52) - 8\right] + 240$$

$$+ \frac{(.003)(10.52 - 9.56)}{10.52}(4)(29 \times 10^3)$$

$$P'_{oy} = 520 + 240 + 32 = 792 \text{ k}$$

$$P_{oy} = 0.7\, P'_{oy} = 554 \text{ k}$$

$$M'_{oy} = (520)(6 - 4.48) + (240)(6 - 2.44)$$

$$+ (-32)(6 - 2.44) = 1535 \text{ "k}$$

$$M_{oy} = 1075 \text{ "k}$$

$$e = \frac{1535}{792} = 1.94"$$

$$I_c = \frac{bh^3}{12} = \frac{(18)(12)^3}{12} = 2592 \text{ in}^4$$

$$I_s = 8\left[\frac{\pi(.564)^4}{4} + (1)(6 - 2.44)^2\right] = 102.2 \text{ in}^4$$

$$EI = \frac{\frac{1}{5}(3.64 \times 10^6)(2592) + (102.02)(29 \times 10^6)}{1 + 0.6}$$

$$EI = 3.028 \times 10^9 \text{ \#-in}^2$$

The critical buckling load is then

$$P_{cr} = \frac{\pi^2(3.029 \times 10^9)}{(1000)\left[(.75)(14)(12)\right]^2} = 1883 \text{ k}$$

$$C_m = 0.6 + 0.4\left(\frac{310}{775}\right) = 0.76$$

$$\delta = \frac{0.77}{1 - \frac{400}{(.7)(1883)}} = 1.11$$

$$M_c = \delta M_y = (775)(1.11) = 860 \text{ "k}$$

Again a new eccentricity is calculated as

$$e = \frac{860}{400} = 2.15"$$

It was found that a c of 10.13" satisfied the eccentricity of 2.15" and

$$P'_{oy} = 760 \text{ k} \qquad P_{oy} = 532 \text{ k}$$

$$M'_{oy} = 1631 \text{ "k} \qquad M_{oy} = 1142 \text{ "k}$$

Both values are greater than the design loads.

Biaxial bending will now be considered. The values required are summarized below.

$$P = 400 \text{ k}, \; P'_{ox} = 766 \text{ k}, \; P'_{oy} = 760 \text{ k}$$

The third term in Bresler's biaxial equation is the reciprocal of the axial capacity of the column in the absence of bending. From equation (5.2)

$$P'_o = (.85)(4)(12)(18) - 8 + (60)(8) = 1187 \text{ k}$$

$$\frac{1}{P'_{ox}} + \frac{1}{P'_{oy}} - \frac{1}{P'_o} = \frac{1}{P'_u}$$

$$\frac{1}{766} + \frac{1}{760} - \frac{1}{1187} = \frac{1}{562}$$

$$P_u = 0.7\ (562) = 393 \text{ k} \doteq 400 \text{ k}$$

The P_u obtained from Bresler's equation is just less than the design load, but the error is less than two percent:

$$\frac{400 - 393}{393} = 0.018 \text{ or } 1.8\%$$

This is close enough to be satisfactory and verifies the results of the design conducted with the design charts.

As a second example, assume the loads are a 300 k axial load, moments about one axis at either end are 640 "k and 430 "k, and moments about the other axis are 500 "k and -125 "k. The eccentricity for the larger moment will be 2.13". The eccentricity for the other axis is 1.67". For the first case $\beta = 0.9$ and $\beta = 0.4$ for the second.

A square section 14 inches deep will be tried, with four number eight bars in the corners. Assume kl_u is 11 feet for both axes. The ratios of end moments are 0.67 for the larger moments and -0.25 for the smaller moments. Using Figure 30 connect 300 on the load scale to the "bar size 8" line, then up to a point midway between the $\beta = 0.8$ and the $\beta = 1.0$ lines, then right to the "$kl_u = 11$" line and finally down to the "$P/\phi P_{cr}$" scale reading 0.305. Interpolating between $M_1/M_2 = 0.6$ and 0.8, connect the previous point on the $P/\phi P_{cr}$ scale to $M_1/M_2 = 0.67$ then right to find a δ of 1.26. Neglecting the story δ for the present and continuing to the right to meet the moment line of 640 "k then up through M_c = 800 "k to P = 300 k, an eccentricity of about 2.7" is defined and the point is still within the interaction boundary. Moving from the last point along a constant eccentricity of 2.7 to the interaction curve for number eight bars, then left to the reciprocal curve and up to the top scale, a value of 0.0018 is obtained for $1/P'_{ox}$. For the other direction, since the section is square the same chart may be used. The first plot from P = 300 to "bar size 8" gives the same point, but from there to $\beta = 0.4$, then right to $kl_u = 11$, and down gives $P/\phi P_{cr}$ as 0.235. With the ratio M_1/M_2 equal to -0.25 it is impossible to intercept this curve so a value of one is assumed for δ and the design moment remains unchanged. Actually, were the M_1/M_2 curve of -0.25 to be continued above the $P/\phi P_{cr}$ axis, the interception with $P/\phi P_{cr}$ of 0.235 lies at a δ less than one. Since the code requires a δ of at least one or greater, $\delta = 1$ must be used for the design. Since the design moment remains the same, the point at P = 300 and M = 500 lies within the interaction bound with an eccentricity of 1.67". This point projected to the interaction curve, right to the reciprocal curve and up gives $1/P'_{oy}$ as 0.0015. From the table in the lower left quadrant, $1/P'_o$ is 0.00118. Then

$$0.0018 + 0.0015 - 0.00118 = 0.00212$$

and from this point down to the reciprocal curve and left to the load scale gives an allowable axial load of 335 k which is adequate. However, if k were to be determined from different dimensions than those of the final section, a new value for k must be obtained and the section checked again.

So far the story δ has not been considered in the examples. As an illustration of its effect, assume that in the previous example an analysis of the frame acting in the direction of the minor axis (least moments) showed that for the story in which the column is located δ is equal to 1.9. Since this is larger than the δ of 1.0 used in the example, 1.9 must be used to give a design moment of 960 "k. The eccentricity is then about 3.2" and $1/P'_{oy}$ becomes 0.00205. Then

$$0.0018 + 0.00205 - 0.00118 = 0.00267$$

This gives an allowable axial load of 267 k in which case the section is inadequate for biaxial bending. Therefore, either the area of the section must be increased or larger bars may be tried in a new analysis.

For a third example a problem has been selected in which the axial load is less than the minimum required for application of the reciprocal curve on the design charts. Consider an axial load of 125 k acting with moments of 2275 "k and 1700 "k about the major axis and 2025 "k and 1215 "k about the minor axis. Assume the effective length in the direction of the major axis is 60 feet and 12.5 feet in the direction of the minor axis. The value of β is taken as one for both axes.

A rectangular section 20 inches by 24 inches will be tried with eight number nine bars, four bars in each short face. M_1/M_2 for the major axis is 0.75 and for the minor axis 0.6. Using Figure 31a, for number nine bars, $0.1P'_o$ is given as 192 k in the table in the lower left quadrant. Since the design load is only 125 k, the reciprocal curve may not be used for a biaxial analysis. Instead, equation (6.3) must be used. For this equation the required values are M_{cx}, M_{cy}, and M_u which is the vector sum of M_{ox} and M_{oy}.

For the major axis, $P/\phi P_{cr}$ is found to be 0.36, δ is

1.42, and M_{cx} is 3250 "k. From the interaction curve for number nine bars, M_{ox} is 4940 "k. Then for the minor axis $P/\phi P_{cr}$ is 0.03, δ is 1 since the M_1/M_2 curve is to the left of $P/\phi P_{cr}$, and M_{cy} is 2025 "k. M_{oy} is then read from the interaction curve as 3660 "k.

$$M_u = \sqrt{(4940)^2 + (3660)^2} = 6150 \text{ "k}$$

Then from equation (6.3)

$$\frac{3250}{6150} + \frac{2025}{6150} = 0.86 < 1.0$$

Any value of this equation less than one indicates a safe section for the given loads and conditions. The closer to one, the more efficient the section is. Since the equation is conservatively in error anyway, it may be desired to re-design for a smaller section or less reinforcing steel to obtain a value closer to one.

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APPENDIX

BIAXIAL DESIGN CHARTS

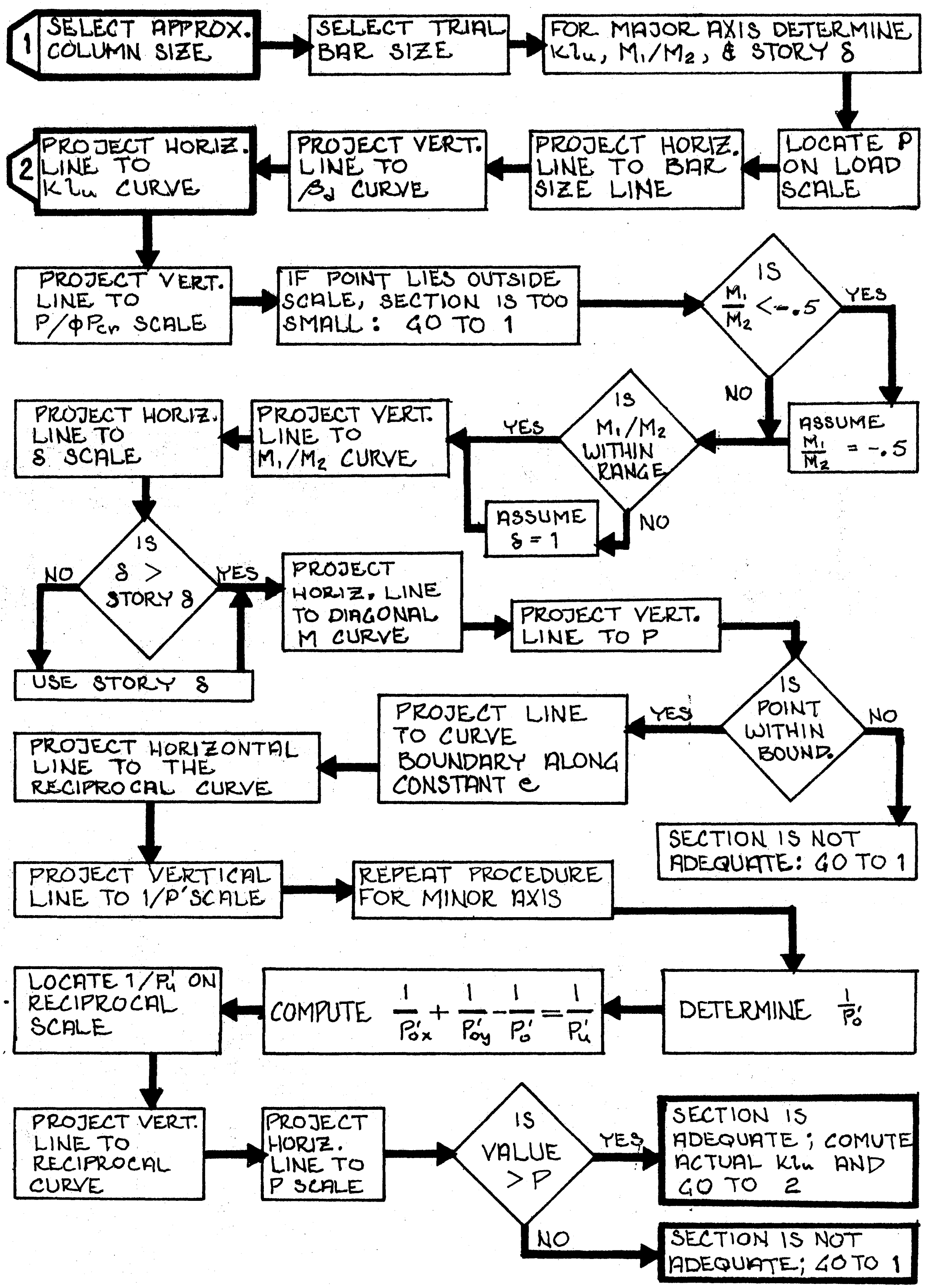

Figure 28. Flow Chart for Design of Reinforced Concrete Columns Using Design Charts.

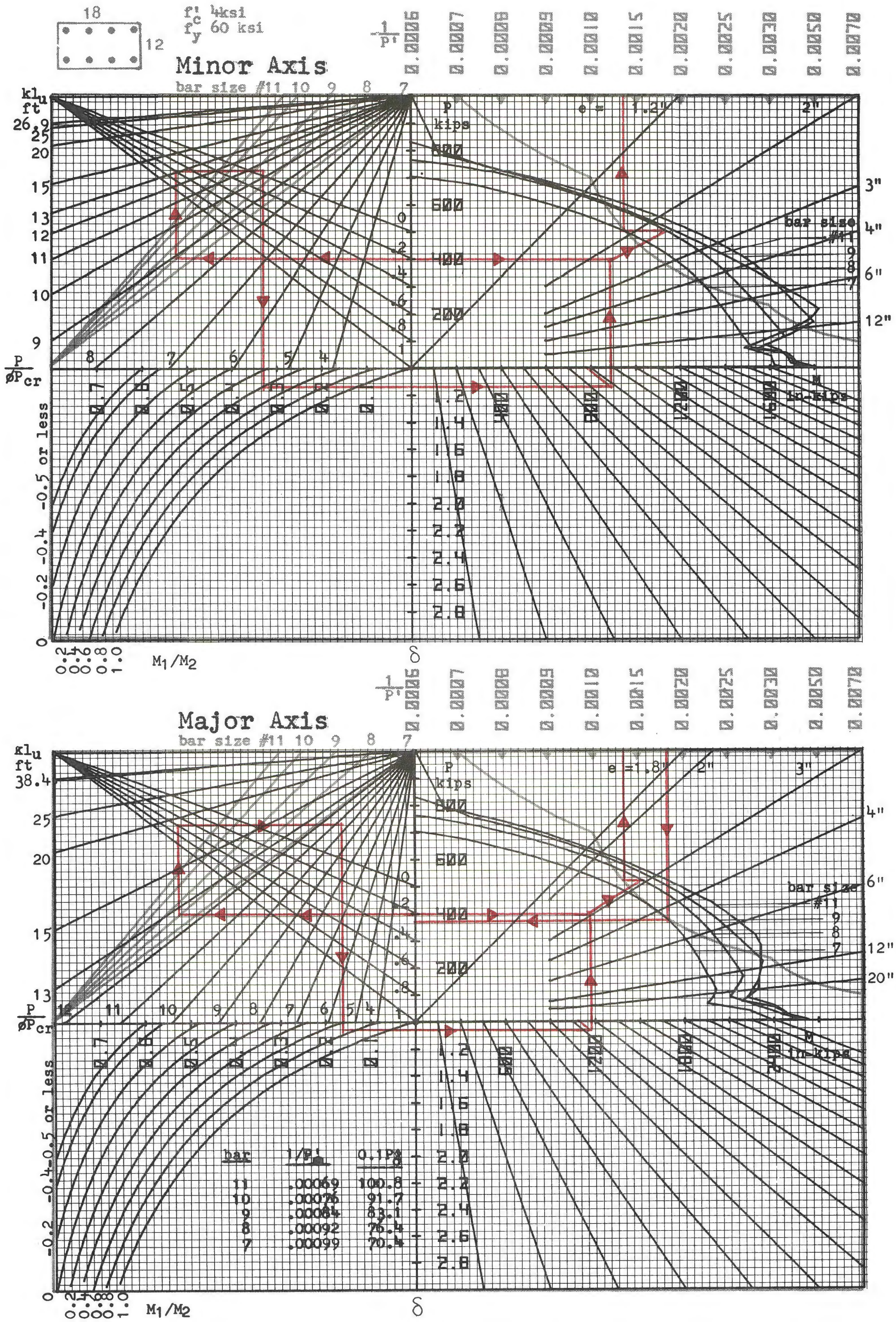

Figure 29. Biaxial Design Chart, Column Size 12" x 18".

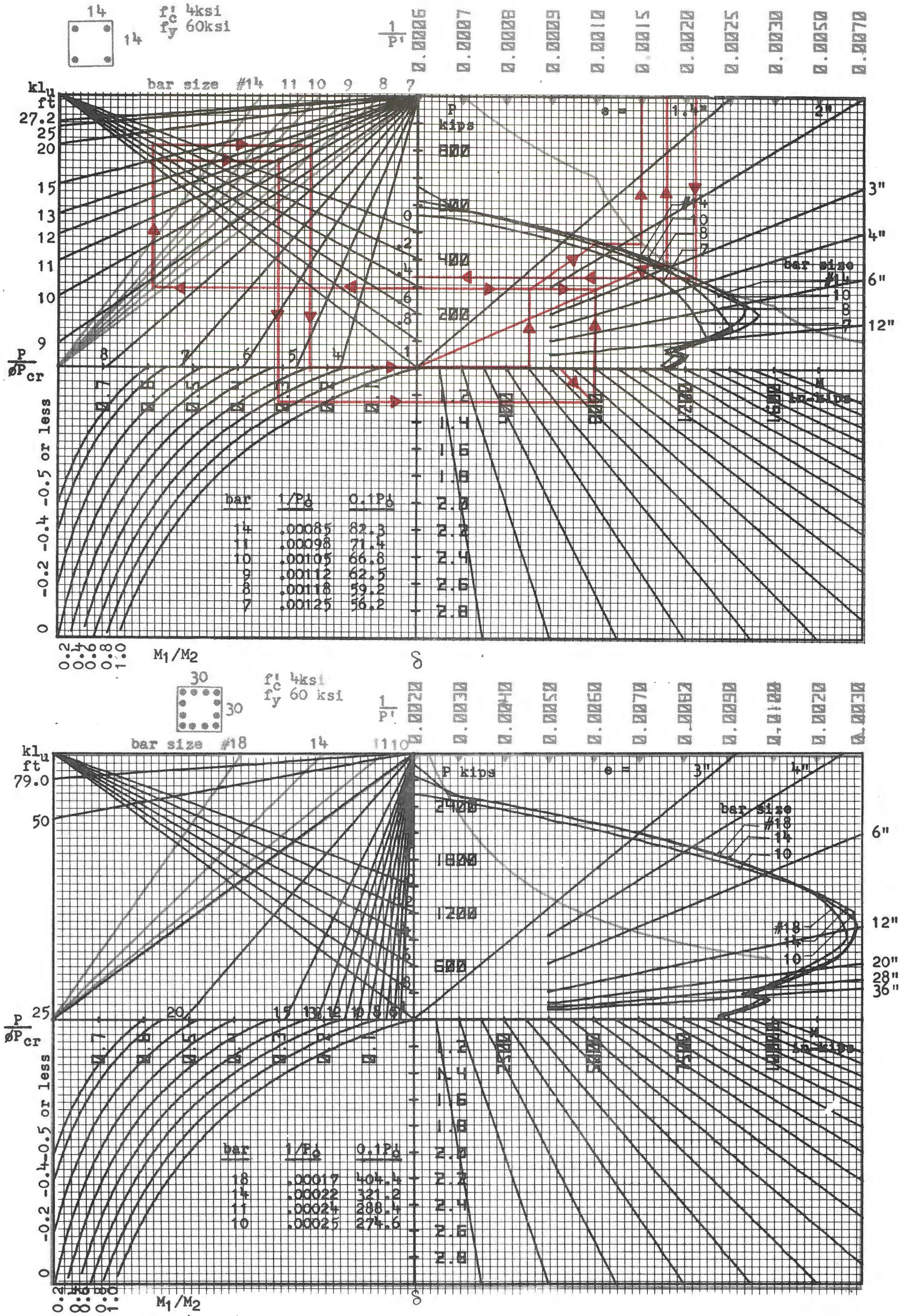

Figure 30. Biaxial Design Chart, Column Sizes 14" x 14" and 30" x 30".

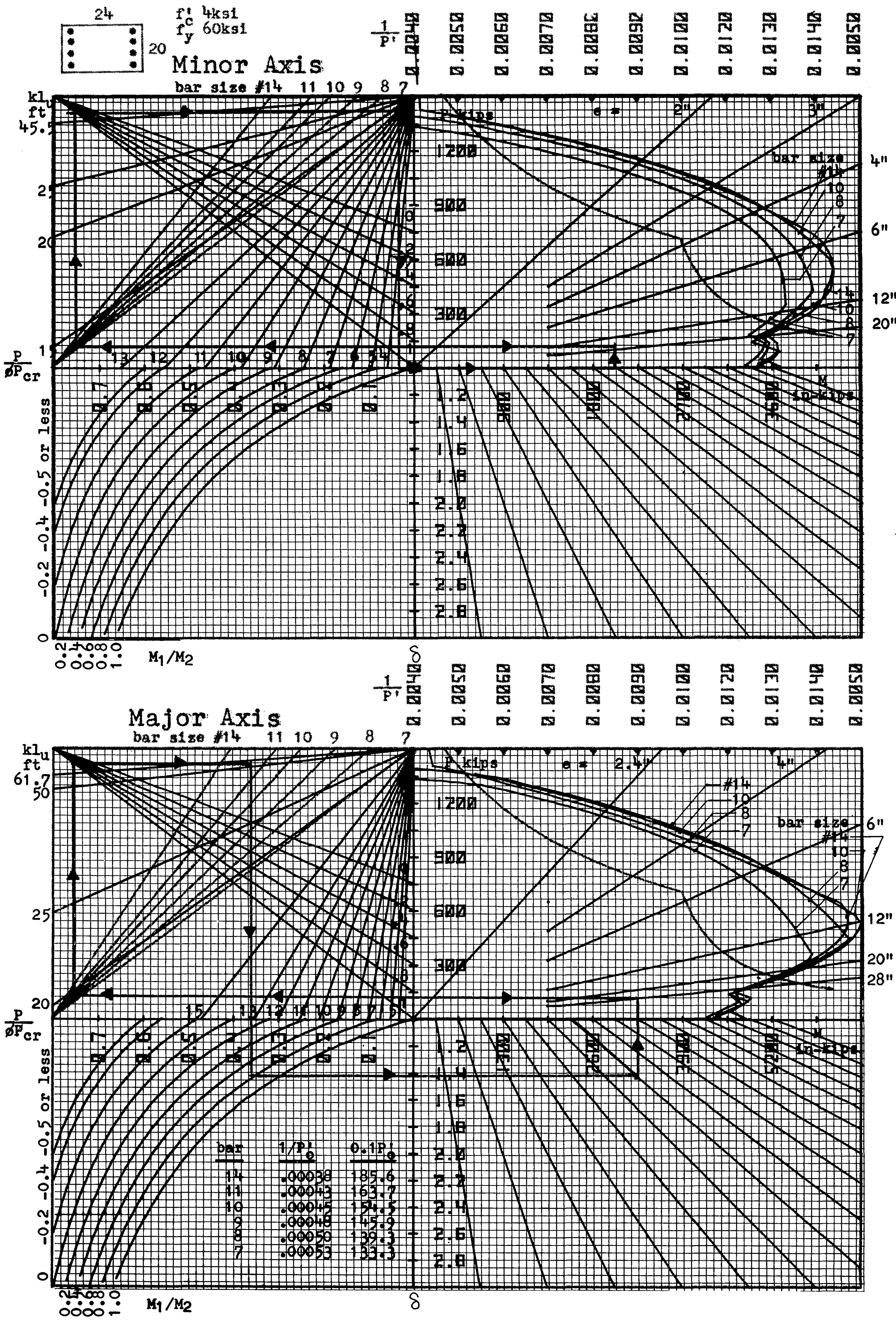

Figure 31. Biaxial Design Chart, Column Size 20" x 24".

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