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A COMPARISON OF TWO MODES OF INSTRUCTION UTILIZING ASPECTS OF INDIVIDUALIZED INSTRUCTION

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A COMPARISON OF TWO MODES OF INSTRUCTION UTILIZING ASPECTS OF INDIVIDUALIZED INSTRUCTION

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INTRODUCTION

A mode of instruction receiving considerable attention in the early seventies is that of individualized instruction. The National Council of Teachers of Mathematics devoted nearly the entire May, 1972 issue of The Mathematics Teacher to this topic. In this issue the editorial panel commented,

> The idea of fitting educational procedures and content to the capabilities and interests of the many individual children in our classes is neither new nor debated. However, there are new methods for doing this and new stresses on the importance of individualization which are being debated.

Often, when one mode of instruction receives considerable attention, we have a tendency to be more concerned with the process of teaching rather than the success of our teaching. There appears to be far too much emphasis upon "how to teach" rather than "how to teach so that students learn." Whatever method of instruction is employed, it must be remembered that student learning is the ultimate goal of the instruction. Wilder (1970) comments,

> One should never forget that teaching unaccompanied by students' learning is hardly deserving of being called teaching. This fact seems to be overlooked in much of the discussion of what constitutes "good" teaching. The responsibility in the teaching process is as much

the student's as the teacher's, and it may be a good idea now and then to remind the student tactfully of this fact.

Even the ultimate form of individualized instruction, namely self-education, usually requires a considerable effort and organization on the part of an instructor. Verner (1964) says,

> Self-education is possible when an individual has sufficient insight and skill to define objectives clearly, to select and arrange a sequence of developmental tasks for himself, and to manage and effectively direct his own progress with objectivity. . . . Such sophistication is not ordinarily characteristic of individuals in need of learning; consequently the education constructed by an external agent to make systematic achievement possible is still required in most cases in order for an individual to accomplish the needed learning.

This study, then was based upon the following premises. The teacher's role in the learning process is major; however it is necessary not to lose sight of the student's responsibility. It must never be assumed that the mode of instruction will eliminate the student's responsibility if learning is actually to occur.

Statement of the Problem

The main purpose of this study was to determine if there were any significant differences in the attitude or achievement of college algebra students regarding mathematics when elements of individualized instruction were employed both in an independent study program and in an intact classroom situation.

A secondary purpose was to determine if these students actually preferred independent study, which aspects of individualized instruction were preferred, and which of these students took greater advantage of additional learning opportunities.

Scope of the Problem

The study was conducted at St. Gregory's College, a private, liberal arts, transfer-oriented junior college in Shawnee, Oklahoma. Seventy-two students from three sections of Mathematics 1513, College Algebra, were involved in this study. The study was conducted during a nine week period of the 1973 fall semester.

Operational Definitions

Intact class refers to the treatment group meeting as a regular class but utilizing the following aspects of individualized instruction: behavioral objectives, degree of accomplishment commensurate with student ability, minimum rate of accomplishment determined by instructor, diagnostic testing and achievement testing.

Independent study refers to the treatment group studying individually but utilizing the following aspects of individualized instruction: behavioral objectives, degree of accomplishment commensurate with student ability, minimum rate of accomplishment determined by instructor, scheduled appointments with instructor, diagnostic testing and achievement testing.

Ability levels (high, medium, and low) were determined from the results of an achievement test over review material prior to the experimental period.

The learning center was a room staffed from 8:00 A.M. to 5:00 P.M. Monday through Friday to provide aid for the students in their study. The learning center contained a moderate library of books and film strips.

Procedure

All students received a packet for each unit of instruction. This packet contained a set of behavioral objectives, supplementary material and problem references for related readings, a diagnostic test, and an answer sheet with references for additional work on that unit. All lectures in the intact classes were based on the same units used by students in the independent study program. All classes were responsible for completion of each unit at approximately the same time. A learning center, staffed by instructors and/or student help, was available to all students.

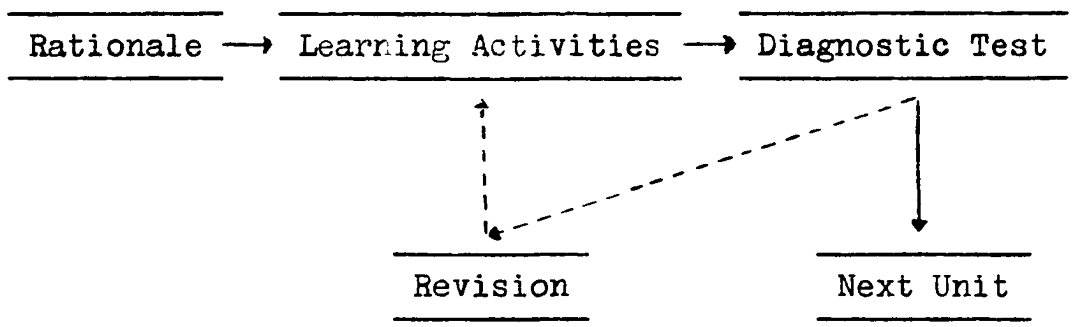
All classes used the same diagnostic tests. The diagnostic test grades were not used in determining the students' grades. However, these tests were evaluated and used to determine whether or not the student should progress to the next unit. Parallel forms of the diagnostic tests were available to any student who chose to continue his work and be re-tested on a given unit. All students who scored below 70 per cent on any diagnostic test were strongly advised to continue work in that unit. Also, any student, regardless of score, who felt he could improve his grade by additional work was permitted to do so. On the other hand, no student was required to take a make-up test.

Post-tests were administered to all students at the same time. Any student who was absent from this exam was given a parallel form of the examination at the earliest possible time. These tests were graded by the instructors and the sum of their total point value was equivalent to the point value of the pre-test. There was no "re-take" of the post-tests.

The first four weeks of the semester were used to allow the students to become familiar with the type of units that were to be used during the experiment. The material covered during this period was review material that allowed all students the opportunity to begin the experimental period with the knowledge of the prerequisites needed for the material covered during the experiment. It was also felt that this period would serve as some control over the Hawthorne Effect.

The basic paradigm as suggested by Herrscher (1971) for both treatments is illustrated in Figure 1. Implementation of this paradigm to each treatment group is discussed in Chapter III of this study.

Figure 1: Instructional System



Collection of Data

The data for this investigation, as related to achievement were collected from parallel forms of a pre-test and a post-test. The pre-test was administered immediately following the period of review. The test was conducted in the evening with no time limit being placed on the students. The post-test was administered in three parts at three-week intervals.

Although the analysis was performed on a sample size of seventy-two students, a total of eighty-five students enrolled for the course. Of the thirteen students omitted from the study, three never met a class, two dropped before the experiment began, two were dismissed from school, and six withdrew on the advice from their instructor.

Data were also collected to determine any attitudinal changes on the part of the students. At the first class meeting of the semester, a semantic differential was administered to each student. At the end of the semester the semantic differential was again administered.

This sample consisted of only sixty-nine cases as opposed to the previously mentioned seventy-two cases. This change in sample size was due to the enrollment of foreign students in these classes. Three of these students were unable to take the pre-test because of language difficulties.

Data were collected from a third source after the experimental period in order to determine which elements of individualized instruction students preferred.

Finally data were gathered for this research from the records of each instructor concerning the number of students who took advantage of the additional resources available to implement their learning. These records indicated the treatment group to which each individual was assigned, and the amount of time he spent working in the learning center. The regularly scheduled appointments with students on independent study programs were not included in these data.

Treatment of Data

Analysis of covariance was used to study the effects on achievement and attitude. A two treatment by three level factorial design was employed. The treatments were intact classes and independent study and the ability levels were low, medium, and high. The ability levels were determined by the results of the test covering the review material. The low group consisted of those students who failed to achieve an acceptable level of 60 per cent, the medium group had received a grade in the range of 60 per cent to 89 per cent, and the high group had received a grade above 90 per cent on this test. The size of each cell is illustrated in Table 1 for achievement and Table 2 for attitude.

Table 1: Cell Size: Achievement

	L	M	H
Intact class	7	24	5
Independent study	12	12	12

Table 2: Cell Size: Attitude

	L	M	H
Intact class	7	24	4
Independent study	11	12	11

The analysis of achievement was based on the data from the pre-test and the three part post-test. The pre-test and post-test used to determine the student's attitude toward mathematics was a semantic differential developed by McCallon and Brown (1971). The semantic differential was administered the first day of the semester and immediately following the experimental period. The .05 level of significance was used for all tests.

The subsequent tests employed were the following:

1. If there was a significant difference between levels the Duncan's Range Test was applied to analyze the difference.
2. If there was significant interaction then at least two of the levels were influenced by the treatment. Thus tests for simple main effects, as recommended by Bruning and Kintz (1968), were used to compare all interactions.

Questions to be Answered

1. Was there any difference in achievement between the treatment groups?
2. Was there any difference in achievement of the students in different ability levels?
3. Do the treatments significantly affect any level relative to achievement?
4. Was there any difference in attitude between the treatment groups?
5. Was there any difference in attitude of the students in different ability levels?
6. Do the treatments significantly affect any level relative to attitude?
7. Which elements of individualized instruction do students prefer?
8. Does student use of the learning center warrant its inclusion in the instructional design?

CHAPTER II

BACKGROUND AND RELATED LITERATURE

Background to the Problem

All educational institutions have been subjected to criticism of traditional college teaching methods. Johnson and Pennypacker (1971) contend these criticisms are centered about five main points. The first of these is class size. It is not unusual for a student to be enrolled in a class of fifty students and college classes may consist of well over a hundred students. A second criticism is that of inadequate physical facilities, especially in the science areas where laboratories are required. A third criticism, and the one of greatest concern in this research, is the manner in which the instructional material is presented. All too often a student feels the instructor is lecturing to an empty room though filled with students. The fourth criticism is the normal curve system of grading. Students generally know only a certain percentage of them will receive acceptable grades. As Bloom (1968) points out:

> Each teacher begins a new term (or course) with the expectations that about a third of his students will adequately learn what he has to teach, but not enough to be regarded as "good students." This set of

> expectations, supported by the school policies and practices on grading, becomes transmitted to the students through the grading procedures and through the methods and materials of instruction. The system creates a self-fulfilling prophecy such that the final sorting of students through the grading process becomes approximately equivalent to the original expectations.

The final criticism mentioned by Johnson and Pennypacker is that of determination of content. In many cases the only rationale for learning is fear of failure, rather than a rationale that indicates a pragmatic need for learning the content of a particular course.

In an attempt to quell these criticisms, educators of the sixties began to experiment with teaching innovations. Weisgerber (1972) points out some trends which took place in the sixties: modular scheduling, primarily in high schools; team teaching; variable grouping; non-gradeness; and yielding some independence to students through group paced study based on variables of achievement, interest, and work study skills instead of ability grouping. There were also trends toward physical flexibility which included the establishment of language laboratories and learning centers. Also, behavioral objectives were being utilized more frequently. Methods for identifying individual differences, assessing learner status and prescribing learner tasks of appropriate difficulty were being based more on assessment instruments and less on teacher intuition. Finally, the trend was to view the instructional approach as a formative, evolutionary process subject to revision and improvement based on feed back from the learners. All these trends led many educators to develop courses for individualized instruction.

Although today, many schools claim to make use of individualized instruction no common definition of this term has emerged. Suara (1972) reviewed five definitions of individualized instruction used in different studies. He then surveyed thirty community and junior colleges who claimed to be using this method of instruction to learn the nature of their programs. Seventy per cent of these programs included prescribed objectives, and partially set the time of classes. The programs did not agree on the limits of the test time. Also 45 per cent of these schools established a minimum rate of student accomplishment. Consequently, all literature pertaining to individualized instruction must be judged in terms of its operational definition.

According to Heather (1971), individualized instruction may be implemented in a group context. He states:

> Individualized instruction is not limited to independent learning or learning in a tutor-student dyad. Depending on the learning goal and learner characteristics, individualized instruction also can occur in group contexts.

This concept is further substantiated by Tosti and Harmon (1972) when they say:

> The distinction between individualized and non-individualized instruction is not made on the basis of whether or not 100 students are experiencing the same learning activity at the same time, since it is possible that everyone of them should be engaged in this activity at this time. Nor should the distinction be made on the basis of whether the instructional system allows a

> student to progress at his own pace or not. A book can do this. Instead the degree of individualization must be defined in terms of instructional management. This means that individualized instruction is a function of the frequency with which the decision to change the instructional presentation is made as a result of the assessment of an individual student's achievement, needs, or goals.

This idea implies that it may be possible and easier to change the instructional presentation for a greater number of students if the original presentation takes place in a group where the assessment of an individual student's achievements, needs or goals are the format of the initial instructional presentation.

Carroll (1963) presents a conceptual model of school learning which implies that ". . . the learner will succeed in learning a given task to the extent that he spends the amount of time that he needs to learn the task." He postulates this under the following definition of learning task:

> The learner's task of going from ignorance of some specified fact or concept to knowledge or understanding of it or of proceeding from incapability of performing some special act to capability of performing it is a learning task.

He states further:

> It is required, however that the task be unequivocally described and that means can be found for making a valid judgement as to when the learner has accomplished the learning task, that is, has achieved the learning goal which has been set for him.

The complete model as proposed by Carroll is composed of five factors: 1) aptitude--the amount of time needed to learn the task under optimal instructional conditions;

2) ability to understand instruction--when instruction is less than optimal, then obviously the learner will need more time to accomplish the learning task and the additional time needed is an inverse function of his ability to understand instruction; 3) perseverance--the amount of time the learner is engaged actively in learning; 4) opportunity--time allowed for learning; 5) the quality of instruction--the instructional process will be optimal when he needs no more additional time to complete the learning task beyond that required in view of aptitude.

Clearly, the aptitude of each student resides in the individual. Furthermore, if it is assumed that each student receives the same initial instruction and that the instructor provides opportunities for additional or modified instruction for those students with difficulties, the differences in ability to understand instruction would be minimized. A further consequence of this assumption would be to equate the quality of instruction for each individual. This would allow the instructional process to approach optimization.

Justification of the Problem

As a consequence of the previously mentioned criticisms many instructors, in their enthusiasm to improve instruction, have attempted to implement individualized instruction. In many instances there are no built in controls to check on student progress; there is little management of instruction for individuals; and there are few considerations of the consequences of allowing students to proceed at their own rate. As a result many students fail to complete the course. Furthermore, the students' attitude toward individualized instruction may well be just the opposite of that which the instructor had hoped it would be.

Future considerations must encompass these facts. First, there presently is no unique definition of individualized instruction and consequently instructors must determine which aspects of individualized instruction they wish to implement and the procedure they will use to aid the student in accomplishing his goal. Secondly, the implementation of an independent study program presents inherent difficulties. It is unreasonable to expect students to do well without a great deal of understanding and direction as they proceed through the program. This involves much more time than the traditional lecture mode of instruction.

An effective program must provide several ways to improve procedures for implementing individualized instruction. A consideration of grading procedures, a definition of individualized instruction, a thorough knowledge of the construction of behavioral objectives, the utilization of a learning center, and a means for controlling such factors as perseverance would form a basis for such a program.

This study considered the implementation of aspects of individualized instruction in different modes of instruction. Such experimentation is needed to provide instructors using some form of individualized instruction the guidelines necessary to insure them the greatest amount of success.

Related Literature

For the purpose of this investigation the related literature deals with research conducted in three areas. The first of these areas is individualized instruction. It is shown that this method of instruction is a valid alternative to the traditional approach and that there are different forms of individualized instruction. Secondly, independent study programs are investigated. This literature shows how independent study programs have advanced from a program for honor students only to a program which may be available to all students. Studies of students' attitude constitute the third area of concern.

Individualized Instruction

In dealing with individualized instruction one must understand that this concept is not any newer to pedagogy than modern mathematics is new to mathematics. Shane (1962) contends that since 1850 there have been at least thirty-five programs attempting to deal with the individual differences that exist between students. Bloom (1968) developed a program of mastery learning. This plan was used to implement the conceptual paradigm of Carroll (1968). All of these programs were attempts to individualize instruction.

Fernald and DuNann (1972) implemented a form of individualized instruction program for large college classes. One group was taught by the traditional lecture method while the experimental group, in addition to attending one class per week for lecture, was divided into small groups of approximately fifteen for diagnostic testing purposes. After the tests were graded, all questions were answered, and student assistants were provided for those students having difficulty. Their results indicated that the performance of students receiving individualized instruction was superior to that of students in the traditional group on the hourly and final examinations. It was also established that students receiving individualized instruction indicated more positive attitudes toward the course than those students in the conventional group.

An experiment using a different form of individualized instruction was conducted by Bloomberg (1971) on a non-credit remedial mathematics course at Essex Community College. In this form of individualized instruction the instructional materials consisted of programmed materials, tutors, and self-tests. However, in this case all students in the individualized program were given placement tests and allowed to start at their appropriate level. These students were then allowed to work at their own pace until the course was completed. They were required to attend a regularly scheduled class which was staffed by both professionals and para-professionals. The only function of the staff members was to aid students seeking help. A mastery level was set at 80 per cent. Evaluation of this group compared with the traditional remedial course resulted in several conclusions:

1. Students in the individualized program achieved significantly higher in subsequent mathematics courses than those students in the traditional course.
2. Students failing in the individualized program failed to achieve above a "D" grade in subsequent mathematics courses.
3. Withdrawals were fewer in the individualized program.
4. A smaller percentage of students passed the individualized course than the traditional course.
5. A greater percentage of students in the individualized program re-enrolled in the course after failing it than those who failed the traditional course.

Another form of individualized instruction was reported by Moody (1972). In this experiment the object was to master ten mathematic objectives. The sample consisted of twenty groups of one, ten groups of two, four groups of five and one group of twenty-three in each of three elementary schools. Results indicated that all small groups had significantly greater achievement than did the group of twenty-three. Also, one to one instruction was significantly superior to the five to one instruction. The subsequent tests however, produced some interesting results. Examination of the means of the four groups indicated that although small group instruction is superior when compared to large group instruction, large group instruction is much more efficient in terms of total learning produced. Moody implies that for this reason it is tempting to suggest that personnel such as teacher aides might be efficaciously employed to instruct small groups of academically needy students at the same time that the regular instructor works with the remaining students.

Connolly and Sepe (1972) designed a study to determine three facts concerning student involvement in an individualized instruction program. The purpose of the study was to:

1. Measure student acceptance of the concept of individualized instruction.
2. Identify the positive and negative factors of individualized instruction as perceived by the student.
3. Identify characteristics of students selecting individualized instruction and traditional methods.

The results indicated that only 50 per cent of the students actually preferred individualized instruction; however, a majority preferred almost all the characteristics of individualized instruction including self-pacing, emphasis on the individual, grading based on achievement of objectives. Students, however, preferred teacher control rather than student control of the learning situation. The students were not willing to accept the responsibility for their learning.

Independent Study

Like individualized instruction, independent study programs are not new to education. One of the first surveys regarding this topic was reported in 1924 by the Division of Educational Relations of the National Research Council and edited by Aydelotte. (Bonthius, et al.,1957) The survey discussed honor courses described in selected college and university catalogs. These students worked independently, but under the supervision of an advisor. Thirty-five colleges indicated that they required honors work in addition to the usual work for a degree; nine others allowed honors work to replace regular requirements, or planned to do so. Aydelotte regarded this practice as an improvement and predicted that honor students would be allowed to do individual work for at least two of their four years.

In 1934 Sinclair and Taylor (Bonthius, et al. 1957) found that 103 colleges appeared to be offering honors work. Upon inquiry they found that only 81 of the institutions were in fact doing so. Further results indicated that of all the institutions offering honors work only 6 had 81 or more students, while 31 had 20 or less, and 13 of these had fewer than 6 students. These results were based on replies to inquiries and were estimates and not complete. So the numbers of participants were small and the freedom allowed was minimal. Yet Sinclair and Taylor reported:

> The evidence seems to indicate that the honors program, as a method of independent study, has definitely established itself in this country and is making satisfactory progress, although generally still in the experimental stage. . . . The past five years show considerable progress away from the earlier idea of honors courses to a broader program of independent study. (Bonthius, et al., 1957)

One of the most complete studies on independent study, summarized by Bonthius, Davis and Drushal (1957) was conducted by the Committee on Educational Inquiry on the College of Wooster Program:

1. Provide knowledge of the detailed structure and functioning of an independent study program which is required of all upperclass students.
2. Provide the faculty with as thorough an appraisal as possible of the strengths and problems of its programs, with a view toward improvement.

A questionnaire was used to obtain the reactions of the seniors in the Class of 1953. The interview method was used to obtain faculty reactions. This was not a program for honor students alone and a student's program for the last two years was apportioned as follows: approximately two-fifths of his work in courses in his major field, approximately two-fifths of his work in courses outside his major field, and one-fifth or three hours of credit per semester required of all students in independent study in his major field. The students were free to choose professors with whom they wished to work whenever possible, considering the constraints of teacher load and committee assignments. Eight independent study students were considered equal to three hours of teaching in class per week. The summary comparison of faculty and student opinion showed rather close agreement. They believed the program should be required of nearly all students and departments, but that a few exceptions should be possible. Majority sentiment of both was against leaving participation up to the option of the student. The category of changes mentioned by far the most frequently by students, but seldom by faculty, concerned the advisory process. The students felt a student may do poorly because of circumstances beyond his control, primarily inadequate advising. In general, students and faculty listed the same drawbacks for the teacher, both mentioning frequently that the program is a heavy burden on the time and energy of the advisors.

The major recommendations made by the committee included the establishment of an administrative board of three members to exert administrative control over the entire program of independent study within the framework of legislation enacted by the faculty. It recommended a standardized procedure for limited exemptions of individuals and special groups. They endorsed the utilization of seminars for the junior year. And finally, they recommended the separation of senior comprehensive examinations from independent study.

Dressel and DeLisle (1969) found in their study of catalogs that independent study is among those individualizing aspects of the curriculum which have shown the most marked change. They reported a trend toward making independent study available early for all college students rather than restricting it to superior advanced students. They also found that independent study had been extended to off campus experiments such as experimental colleges and interim terms, They reported that they found more tinkering, than profound innovation in higher education.

Dressel and Thompson (1973) in their book Independent Study state,

> The fact also remains that there has been no research which has clearly defined the objectives of independent study or proven either its superiority or economy over other types of learning.

Attitude

Wofford and Willoughby (1968) designed a study to determine the relationship between attitude variables and between scholastic behavior variables, and the interrelationship among scholastic behavior variables for college students. Their results indicate that the best predictor of scholastic behavior is the composite attitude scores with the effects of general attitudes toward life removed.

It was also stated that grades are significantly related to the attitudes toward the course, but not significantly related to the attitudes toward college. A final point of interest is that tardiness scores were significantly negatively correlated with course grades; absence scores were not.

Anttenen (1969) performed a longitudinal study concerning the relationship of attitudes toward mathematics and achievement in mathematics. The study was conducted over a six year period from late elementary to the late secondary school level. His findings indicated an overall low positive relationship between early and late mathematics attitude scores. He also found that attitude scores at the elementary level were not as good a predictor of mathematics achievement as were the attitude scores at the secondary level. However, significant positive correlations existed between all measures of attitude and achievement.

The last research for our consideration is a paper by H. Eugene Hollick (1971). The paper represented the results of the "Coatesville Project," conducted during the academic year 1968-69. There were 250 high school students participating in one to four different disciplines: mathematics, English, social studies and journalism. The primary interest of this particular project was in student attitude. It was an ambitious program employing a system approach to learning and a complete learning center. The teacher's job was one of educational diagnostician and prescriber to fit individual needs. The teacher was available full time to meet students in both student initiated and scheduled conferences. Though they were not able to tell in advance who would profit from the individualized learning program, they found no student whose academic achievement decreased as a result of his participation. From this program, the school system experienced a 25 per cent decrease in dropout rate. A questionnaire answered by all students in the four subjects indicated that they found it a positive experience. The questionnaire indicated in part that 67 per cent felt the rate of progress was faster; 75 per cent had a higher interest level; 93 per cent had a positive relationship with the teacher; 80 per cent would like to use the system again, and overall the students felt they learned how to learn.

Summary

In summary, then the related literature indicates that indeed the notion of individualized instruction and independent study, though not necessarily disjoint concepts, need not be the same thing. There are indications that individualized instruction can be implemented in a group context. Considerably more investigation of the values of independent study programs is needed. Thus, the literature indicates a comparison of an intact class with an independent study program, both implementing aspects of individualized instruction would be beneficial to a better understanding of the relationship between these two concepts.

CHAPTER III

DESIGN OF THE STUDY

Individualized Instruction: Selection of Elements

For purposes of this research, it was necessary to identify and describe specifically the elements which would be used. The elements of individualized instruction included the following:

1. Behavioral objectives.
2. Degree of accomplishment.
3. Rate of accomplishment.
4. Diagnostic testing.
5. Achievement testing.

Behavioral objectives, based upon the guidelines of Mager's Preparing Instructional Objectives, (1962) were distributed to each student. These objectives were designed to clarify in the student's mind precisely what was expected of him, both in terms of content and acceptable levels of competency. They included the core concepts of Mathematics 1513, College Algebra, at St. Gregory's College. Samples of the objectives may be found in Appendix A.

The degree of accomplishment concerning these objectives was left to the prerogative of the student. He was told that he could choose his own level of accomplishment. At the same time, he was told that if he achieved the levels of competency as stated in the objectives, he need not have any fear of failing the course.

The rate of accomplishment for the students of the independent study program was intended to correspond with the intact class. However, those students capable of progressing at a more rapid rate than the intact class were encouraged to do so. Superior students of the intact class were often utilized by the instructor to provide assistance for those students who were having difficulty. Students in the intact class were neither encouraged nor discouraged to work ahead. Students having difficulty maintaining this minimum pace were encouraged to take advantage of the support sources available to them.

Within each unit the diagnostic test allowed the student to determine his own level of competency within that unit as it was completed. If the diagnostic test indicated a lack of competency concerning the concepts of a particular unit, the student was encouraged to continue his study concerning these concepts. He was encouraged to utilize the learning center as well as to take full advantage of the individual consultation which was available to him. But most important, he was told that his grade on the achievement test would be related to his ability to achieve the levels of competency described within the behavioral objectives. A secondary purpose of the diagnostic tests was to reinforce his expectation that the questions which would appear on the achievement tests would be related directly to the stated objectives.

Three achievement tests were given in the course, and these tests covered three, five, and four units respectively. Since some of the test items utilized more than one concept in the response, these tests did not correspond directly to the levels of competency of the stated objectives. Whereas the diagnostic tests for each unit dealt separately with the concepts of that unit, the achievement tests combined the unit concepts and were designed to measure the student's ability to recognize and respond to more than one concept if necessary. Of course, each test question was related directly to the stated objectives of the units.

In addition to the diagnostic tests, review sessions were scheduled prior to each achievement test. During these sessions, sample tests were available to all students who wished to take them as further preparation for the test. These sessions were primarily for those students having difficulty, but were open to any student choosing to attend. These sample tests were simply another assignment. They were longer and more difficult than the actual test but covered the same objectives as the achievement test. These sessions were conducted in the learning center.

The content of the course focused upon sets and inequalities of one variable, linear forms, systems of linear equations, and quadratic equations of the form:

$$y = ax^2 + bx + c \text{ where } a \neq 0.$$

Following these core concepts, additional topics were chosen by the instructor to complete the course.

Selection and Implementation of Treatments

Virginia T. White (1972) points out that "Individualized instruction is not synonymous with independent study, although independent study might be a proper subset of the total methods used." An investigation comparing an intact class and an independent study program both employing the same aspects of individualized instruction could further substantiate that these concepts, individualized instruction and independent study, are not synonymous.

Intact Class

Since the use of elements of individualized instruction is usually applied only in independent study programs, the following description of these elements as used in an intact class is carefully detailed so that the reader may have a better understanding of this procedure.

Each student in this group received a unit of instruction as previously described, identical to the unit given to a student on an independent study program. The instructor also used these units as a basis for his lectures. He explained the rationale for the material that was studied.

Next he explained the concepts the student had to understand in order to achieve the objective. Following this, he worked some examples, usually different from the examples in the supplementary material that illustrated the concepts.

He then allowed the students to begin their learning activities by working a few problems pertaining to this concept and the attainment of the behavioral objective. These problems were but a sample of the problems pertaining to the particular objective, but they allowed the students to apply immediately the ideas expressed by the instructor. As the students worked these problems the instructor checked on their work and answered any questions the students had as a result of trying to solve these problems. If problems existed for the majority of the students, he extended his explanation of the material and then had the students return to the solutions of the problems. If only a few students had questions which seemed unresolved, the instructor immediately scheduled an appointment to help those students. This appointment was met before the next scheduled class.

When all the behavioral objectives with a unit had been treated in this manner the students took the diagnostic test over that unit. The students, themselves, graded these tests. The instructor then scheduled an appointment for those students who wished to discuss or be re-tested over the particular unit. As before, this "re-take" was scheduled before the next regularly scheduled class meeting. These students were also made aware of the hours when they could receive additional help at the learning center where no specific appointment was necessary. The instructor collected the completed and graded diagnostic exams. He inspected these examinations and recorded the grades. They were then returned to the students.

Independent Study Program

The constraints placed upon the students in the independent study program consisted of scheduled appointments to report on their progress. Each student who made a "D" or "F" on the test over the review material was scheduled for four appointments per week. Those who made a "C" were scheduled for three appointments per week, and "A" or "B" students were scheduled for two appointments per week. Each meeting was scheduled for fifteen minutes. During these meetings questions were answered, the quality and amount of work done by the student was checked, diagnostic tests were examined and revisions made when necessary.

In this treatment the encounter with the learning activities was predominantly self-imposed. Each student was expected to maintain a rate of progress compatible with students in the intact classes. Those students who had difficulty maintaining this pace were directed to seek additional help at the learning center. When a student completed the work within a unit, he then took the diagnostic test. These were generally self-graded and then presented to the instructor to discuss the results and determine the direction for continued work.

Since students were required to meet their regularly scheduled individual conferences with the instructor, instructors were required to give considerable blocks of their time. One instructor had seventy-two regularly scheduled appointments per week, and the other instructor had forty-eight appointments per week. The instructor, by having time scheduled, was aware of other students and could more easily compensate for the large number and be sure no one was ignored or overlooked.

Test Instruments

Diagnostic Tests

Designed to be the final assignment of each unit, the diagnostic tests were constructed from the stated objectives of a given unit. Strict adherence to these objectives was the primary criterion for design; thus, each question was specifically modeled after a stated objective and its corresponding level of competency. Since students usually completed these tests outside the classroom before their scheduled appointment, the time of completion was not a significant factor in test design. These tests were not used in determining a student's grade. Instead, they served simply as a guide for students to measure their progress.

Parallel forms of each test were provided for those students who chose to be re-tested over a particular unit. In order to be re-tested, however, a student was required to complete additional assignments related to that unit.

Achievement Tests

The achievement test related to this study was designed to exhibit a high correlation with the stated behavioral objectives. Unlike the diagnostic tests, these examinations contained test items which required student competency in more than one objective. These items reflected several levels of difficulty in order to provide separation and to increase validity. Their format was traditional, and in order to award partial credit, students were asked to show all work toward solving a given problem.

An effort to obtain validity was made by submitting sixty-two test items along with the complete set of objectives relating to these items to a panel of three faculty members of the instructional staff in the Division of Natural Sciences at St. Gregory's College. Each of these instructors had taught traditional courses in college algebra. The panel was instructed to read carefully the set of behavioral objectives. They were then asked to read each test item as it related to the behavioral objective and rate each item as "good," "fair," or "poor." These ratings were then returned to the researcher and each was awarded a rating of three if "good," two if "fair," and one if "poor."

Any item receiving a total score of less than six was discarded. As a result, the pre-test and post-test were parallel forms of twenty-three questions, and the number of parts to each question ranged from one to five.

In order to determine the reliability of the instrument, coefficient alpha, as recommended by Nunnley (1967), was used. This reliability factor was computed by use of a library computer program at Oklahoma University. This program was furnished by the Health Sciences Computing Facility of the University of California at Los Angeles, and revised January 28, 1970. The computed alpha coefficient of reliability was .868. Garrett (1958) states that "In order to differentiate between the means of two school grades of relatively narrow range, a reliability coefficient need be no higher than .50 or .60." Thus, it is shown that the reliability of this instrument is in the acceptable range. The complete three-part achievement test may be found in Appendix B of this paper.

Measurement of Student Attitude

Student attitudes toward mathematics were measured through an instrument devised by Earl L. McCallon and John D. Brown (1971). This instrument was designed to contrast with the Likert type attitude instrument constructed by L. R. Aiken and R. M. Dreger (1972). McCallon and Brown hypothesized that their instrument was not only easier to construct, but it was not subjected to constant revision.

This instrument revealed a high positive correlation ($r = .90$) between the total score on the semantic differential and the score on Aiken's Mathematics Attitude Scale. In addition, there was a high positive correlation ($r = .87$) between the evaluative scales of the semantic differential and the score on the Mathematics Attitude Scale. Thus, it was inferred that the semantic differential was as effective a measure of attitude toward mathematics as the Mathematics Attitude Scale of Aiken. It was also shown that people possessing favorable and unfavorable attitudes toward mathematics would differ to the greatest extent on the evaluation scales of the semantic differential, thus leading to the construct validity of the semantic differential. The original Likert type mathematics attitude scale has been tested for reliability and validity with the consequent results that the scale is considered somewhat more valid when testing high school and college students. (See Appendix C.)

Student Opinion

A forced-free format was utilized to collect secondary data concerning student opinion toward independent study and the separate components of individualized study. Given to all participants of the study, this questionnaire was based upon a similar questionnaire designed by John J. Connolly and Thomas Sepe (1972). (See Appendix D.)

Control of Teacher Variation

Two teachers were used to implement the treatments. Thus teacher variance and its effects upon the research study was a major concern. In order to control this variation, the researcher designed units of study (see Appendix A) to supplement the required text (Hart College Algebra). Each unit of study contained a statement of objectives, supplemental study material, assignments, a diagnostic test, and an answer sheet. Additional assignments were designed to be included for those students who did not achieve an acceptable level of competency. All components of the units of study were discussed and agreed upon by the participating instructors. These units were then used as sources for lectures in their intact classes and were the guidelines for students on independent study.

An additional control factor was built in by allowing students freedom in scheduling appointments. They were free to choose either of the participating instructors as well as the time for their appointments. As a result, both instructors participated in the independent study program and both taught intact classes.

A third control of teacher variance was established through grading. All achievement tests were graded by both participating instructors. Since all tests were administered to all students at the same time and location, both instructors were present. After the examinations were completed by all of the students, each instructor graded pre-designated problems of each test. The weight attributed to each problem as well as the amount of partial credit which would be given for each problem was agreed upon in advance of the grading session.

The Sample

Enrollment procedures at St. Gregory's College are similar to those of most colleges and universities, and, consequently, scientific random sampling was impossible.

The 1973 Fall Schedule listed three sections of College Algebra as follows:

Section I:	8:00 A.M.	MWF
Section II:	10:00 A.M.	MWF
Section III:	2:00 P.M.	MWF

These three hour courses were listed without the names of the instructors, and the term STAFF was inserted. Consequently, students did not know at the time of enrollment who their instructors would be. Section designation was determined by a simple drawing after the students had completed enrollment, and they were identified as follows:

Section I:	Independent Study
Section II:	Intact Class
Section III:	Combination

It should be noted that certain students were selected from Section III to participate in the independent study program while the remainder constituted a second intact class. Those placed in the independent study program had received an "A" or "F" on the test at the conclusion of the review period. This procedure was used so that both instructors would have an intact class and to equate the number of students in both treatment groups.

At the conclusion of the review period, the three sections had the following characteristics:

	Total	Male	Female
Section I:	25	14	11
Section II:	23	13	10
Section III:	24	13	11
	72	40	32

A unique characteristic of the sample is that nineteen of the seventy-two students enrolled in college algebra during the 1973 fall semester were international students representing Hong Kong, Thailand, and the Bahamian Islands. Four of these students were included in Section I, seven in Section II, and eight in Section III. Consequently, there was no uniform test scores available for all students. Because of this, the review examinations were used as the only basis in determining ability levels used in this research.

All of the students included in the sample were required to take college algebra as course requirement for the Associates Degree at St. Gregory's College. Very few had taken a mathematics course at the college, but most had successfully completed at least two years of high school mathematics.

The Learning Center

To properly implement this program, it was imperative that the students would have a designated area which was always available to them in which they could receive the attention they needed. This learning center was a classroom adjacent to the offices of both instructors involved in the program. No classes were held in this room, and it was open to the students from 8:00 A.M. to 5:00 P.M., Monday through Friday. During these hours it was continuously staffed by instructors of the college and/or student personnel. In order to be eligible to serve as a student assistant, students had to be enrolled in analytic geometry or a course at a higher level.

Contained within the center was a modest mathematics library from which students were permitted to check-out books. In addition, film strips were available to the students for their use although they were not available for check-out. The list of books and filmstrips available are found in Appendix E. Ample blackboard space as well as the size of the room permitted students to work in small groups if they chose to do so.

Statistical Design

Analysis of covariance was used to study the effects on achievement and attitude. This is recommended by Campbell and Stanley (1963) when random assignment of subjects to treatments is not possible. Furthermore, according to Edwards (1968), analysis of covariance takes into account beginning differences by using a concomitant variable. In this instance the previous achievement in mathematics presented the greatest potential for biasing the evaluations. Consequently, the pre-tests administered for both achievement and attitude were used as the concomitant variable. Parallel forms of these pre-tests were used for the post-test data.

Since a two treatment by three level design was used, subsequent tests were planned to complete the analysis. Bruning and Kintz (1968) comment, "When multiple comparisons are planned before the experiment has been carried out, you can be sure that the tabled probability will be accurate for each of the tests." Therefore, Duncan's Mutiple Range test was used to determine which levels were significantly different. Also tests for simple main effects had been planned to anticipate significant interaction.

The results of the questionnaire on student preference were used to summarize student acceptance or rejection of the different aspects of individualized instruction employed in this research.

Finally, the summary of the records indicating student use of the learning center were used to determine which students took greatest advantage of the additional learning opportunities.

Assumptions and Limitations

One assumption of this study is that teacher variation had no effect on the outcome of this research. This is a serious assumption and considerable effort was spent in controlling the extraneous variable.

A second assumption made in this study was that the instrument used for evaluation of achievement was valid. An attempt was made to determine the validity of each item but it is still necessary to assume the validity of this instrument.

The most serious limitation of this study is that it was conducted at one junior college with a small group of students. This, then, restricts the generalization of the results of this experiment to a more general population.

CHAPTER IV

PRESENTATION AND ANALYSIS OF DATA

Hypothesis of Design

The design used in this experiment was a two treatment by three level factorial analysis of covariance. Since the design was used on both achievement and attitude the assumption that the slope of the within-cell regression lines not be significantly different, will be discussed before the presentation of the data.

Glass, et al. (1972) indicates that when dealing with assumptions, the important question is not whether the assumptions are exactly met or not, but whether the violations of the assumption have serious consequences on the validity of probability statements based on these assumptions.

Peckham in 1968 (Glass, et al. 1972) systematically varied population regression slopes for different combinations of number of treatment groups and number within each treatment group. The values of the covariate were fixed for the production of each empirical sampling distribution of the F-statistic. Peckham stated in his findings:

> That the empirical sampling distribution of F-statistic differed little from the theoretical sampling distribution unless the departure from homogeneous slopes was extreme. As the degree of heterogenity increased, the analysis became more conservative with respect to making a Type I error.

Kirk (1968) also indicates that the slopes can differ considerably when he recommends that a large level of significance (α = .10 or .25) should be used in testing the homogeneity of the regression slopes. Also Winer (1971) speaks of evidence to indicate that the analysis of covariance is robust with respect to homogeneity assumptions on regression coefficients.

In consideration of these findings and opinions, the assumption regarding the homogeneity of the within-cell regression lires will be regarded as satisfied in this research.

The analysis of both achievement and attitude were accomplished by using a library program available at the University of Oklahoma Computer Center on the Main Campus. This particular program, General Linear Hypothesis, was furnished by Health Sciences Computing Facility, at the University of California, Los Angeles and revised April 5, 1972.

Achievement Test

Pre-test scores were collected from all participants prior to the experimental period and following the review period. Post-test scores of achievement were obtained at three week intervals. The first segment of the achievement test dealt with concepts from the first three units of objectives, the second part dealt with concepts of the next five units of objectives, and the third part of the achievement test dealt with the final four units of objectives. A post-test score for the entire period was obtained for each student by combining the scores from the three parts of the achievement test.

The three part achievement test is found in Appendix B of this report. The highest possible score on part one was 52 points, on part two was 103 points and on part three 91 points were possible. Thus the total score for the complete achievement test was 246 points. Tables 3 and 4 indicate the means and standard deviations of each cell as well as overall scores. The overall mean of the post-test represents 71.4 per cent of the total of 246 points.

Table 3: Means for Pre-test and Post-test Scores: Achievement

	L	M	H	Overall
Intact Class				
Pre-test	12.71	24.50	22.60	21.94
Post-test	118.28	176.92	229.20	172.77
Independent Study				
Pre-test	5.25	21.08	46.50	24.28
Post-test	113.25	193.33	229.42	178.66
Overall				
Pre-test	8.00	23.36	39.47	23.11
Post-test	115.10	182.39	229.35	175.72

Table 4: Standard Deviations for Pre-test and Post-test Scores: Achievement

	L	M	H	Overall
Intact Class				
Pre-test	5.94	15.91	11.19	14.43
Post-test	21.37	31.70	12.79	42.64
Independent Study				
Pre-test	5.41	15.63	11.56	20.61
Post-test	34.84	26.40	10.29	55.30
Overall				
Pre-test	6.58	15.68	15.79	17.71
Post-test	30.00	30.68	10.66	49.12

The hypotheses for achievement, stated in null form, which were tested in this phase of the investigation were:

H_1: There are no significant differences in achievement test means between the two treatments.

H_2: There are no significant differences in achievement means among the three ability levels.

H_3: There are no significant interactions between the treatments and ability levels as measured by achievement test scores.

The computer results of this analysis are shown in Table 5.

Table 5: Analysis of Covariance: Achievement

Source	Sum of Squares	df	Mean Square	F
Treatment	118.36	1	118.36	0.16
Level	66078.72	2	33039.34	44.58
Interaction	1742.68	2	871.34	1.18
Covariance	519.53	1	519.56	0.70
Error	48176.37	65	741.17	
Total	116635.66	71		

The critical F-ratio for treatment with 1 and 65 degrees of freedom is 7.04, and the critical F-ratio for levels and interaction with 2 and 65 degrees of freedom is 4.95. Thus, H_2 will be rejected, but H_1 and H_3 will not be rejected. This indicates there is significant difference in achievement as measured by mean scores among the three ability levels, but that there is no significant difference in achievement as measured by mean scores between the treatments. Further there is no indication of significant interaction between treatments and ability levels.

To determine how the levels differ, Duncan's Multiple Range Test as recommended by Bruning and Kintz (1968) was applied. The standard error of the adjusted means was determined by the formula:

$$s_{\bar{x}}' = \frac{ms'_{error}}{n}$$

where ms'_{error} denotes the adjusted mean square error and n represents the harmonic mean of cell sizes. The standard error of the adjusted mean was computed to be 8.77. The results of this test are shown in Table 6.

Table 6: Duncan's Multiple Range Test*Applied to Ability Levels: Achievement

		L	M	H	Significant Range
	Adjusted Means	118.42	182.33	226.41	
Low	118.42		63.91	107.99	R_2=33.05
Medium	182.33			44.08	R_3=34.46

*α = .05: k = 2,3: 65 degrees of freedom

Because high minus low is the range of three means the difference must exceed 34.46 to be significant. Since this difference is actually 107.99 we conclude that high and low groups differ significantly. Similarly since medium minus low is 63.91 which exceeds $R_2 = 33.05$ we may conclude medium and low are significantly different. Also since high minus medium is the range of two means and 44.08 is greater than $R_2 = 33.05$ we conclude that high and medium are significantly different. Hence we conclude that all three levels are significantly different in achievement as measured by mean scores among ability levels. Further evidence of this difference is shown by distribution of grades. The post-test grades were awarded on the basis of 10 per cent grade ranges. That is 90 per cent for grade A, 80 per cent for grade B, 70 per cent for grade C, 60 per cent for grade D and below 60 per cent for grade F. The results of the achievement test are displayed in Table 7.

Table 7: Grade Distribution for the Achievement Test

Range	Grade	Intact Class			Independent Study		
		L	M	H	L	M	H
221-246	A	0	4	4	0	1	8
196-220	B	0	4	1	0	6	4
172-195	C	1	8	0	1	3	0
147-171	D	1	6	0	3	2	0
Below 147	F	5	2	0	8	0	0

Attitude Test

The data used for this analysis were collected from the pre-test and post-test scores of a semantic differential designed by McCallon and Brown (1971). The semantic differential consisted of fifteen bi-polar adjectives dealing with the concept "mathematics." A score of one to seven was awarded to each adjective depending on the placement of the individual's preference on the bi-polar sets. A copy of this instrument may be found in Appendix C. The means and standard deviations of each cell as well as overall scores are illustrated in Tables 8 and 9.

Table 8: Means for Pre-test and Post-test Scores: Attitude

	L	M	H	Overall
Intact Class				
Pre-test	57.00	66.75	70.25	65.20
Post-test	54.00	69.79	79.25	67.71
Independent Study				
Pre-test	66.82	61.58	69.18	65.74
Post-test	64.45	74.17	77.00	71.94
Overall				
Pre-test	63.00	65.03	69.47	65.46
Post-test	60.39	71.25	77.60	69.80

Table 9: Standard Deviation for Pre-test and Post-test Scores: Attitude

	L	M	H	Overall
Intact Class				
Pre-test	10.85	15.46	17.27	15.20
Post-test	9.88	15.03	7.14	15.23
Independent Study				
Pre-test	20.08	13.30	16.03	16.42
Post-test	13.63	12.85	8.17	12.68
Overall				
Pre-test	17.41	14.92	15.74	15.70
Post-test	13.09	14.31	7.72	14.09

It is interesting to note that the weaker students were not at all impressed with either treatment. They both had a slight drop from pre-test to post-test while every other cell showed some increase from pre-test to post-test.

The hypotheses for attitude, stated in the null form, which were tested in this phase of the investigation were:

H_4: There are no significant differences in attitude test score means between the two treatments.

H_5: There are no significant differences in attitude test score means among the three ability levels.

H_6: There are no significant interactions between the treatments and ability levels as measured by the attitude test scores.

The complete analysis of covariance table from the computer program is shown in Table 10.

Table 10: Analysis of Covariance: Attitude

Source	Sum of Squares	df	Mean Square	F
Treatment	156.89	1	156.89	1.20
Level	1875.20	2	937.60	11.91
Interaction	174.45	2	87.22	1.11
Covariance	5400.43	1	5400.43	68.60
Error	4880.67	62	78.72	
Total	12487.64	68		

The critical F-ratio for treatments with 1 and 62 degrees of freedom is 7.06 and the critical F-ratio for levels and interaction with 2 and 62 degrees of freedom is 4.97. Thus again, H_2 will be rejected, but H_1 and H_3 will not be rejected. This indicates there is significant difference in attitude as measured by adjusted mean scores among the three ability levels, but that there are no significant difference in treatment as measured by mean scores between the two treatments. Neither is there an indication of significant interaction between treatments and ability levels.

Duncan's Multiple Range Test was applied to the varying ability levels. In this instance the standard error of the adjusted mean was computed to be 1.98. The results of this test are found in Table 11.

Table 11: Duncan's Multiple Range Test* Applied to Ability Levels: Attitude

		L	M	H	Significant Range
	Adjusted Means	62.58	71.62	74.07	
Low	62.58		9.04	11.49	$R_2=5.58$
Medium	71.62			2.45	$R_3=5.88$

*α = .05: k = 2,3: 62 degrees of freedom

Because high minus low and medium minus low are greater than R_3 = 5.88 and R_2 = 5.58 respectively, we conclude high and medium are both significantly different from low. However since the difference between high and medium is less than R_2 = 5.58 these two ability levels are not significantly different. Thus in this instance, high and medium are not significantly different from each other, but both are significantly different from the low ability level.

Student Preferences

These data were collected on a questionnaire using a forced choice format. The questionnaire was given to the students during the spring semester, 1974. This was done so that the students could make their choices, already knowing their grade in the course. It was hoped that this procedure might elicit answers which would more accurately reflect their true opinion. The total number of responses returned were thirty-four or approximately 47 per cent of the sample.

The researcher had anticipated a considerably higher return. The results are summarized in Table 12. The data on each question represent the number of responses first, followed by the percentage in parenthesis. The purpose of the questionnaire was to determine student preferences toward the aspects of individualized instruction.

Table 12: Student Preference

1. I 34(100) do prefer stated objectives on each unit.
   0(0) do not prefer stated objectives on each unit.
2. The objectives 34(100) helped me prepare for tests.
   0(0) were of no use.
3. The objectives were 28(82) concise and easy to understand.
   6(18) difficult to understand.
4. I 32(94) do prefer the supplementary units used in addition to the text.
   2(6) do not prefer the supplementary units used in addition to the text.
5. I would prefer 21(62) independent study program.
   12(38) regularly scheduled classes.
6. I would prefer to advance at 20(59) my own pace.
   14(41) pace determined by instructor.
7. I 27(80) did take advantage of the additional learning opportunities provided.
   7(20) did not take advantage of the additional learning opportunities provided.
8. I prefer tests to be given 28(82) outside of regular class time to allow all the time needed.
   6(18) during regularly scheduled class.
9. I 34(100) would recommend this course.
   0(0) would not recommend this course.

The use of objectives and supplementary units was clearly accepted by the students and their relationship to the test seemed to be understood by the students. Some revision of the statement of the objectives seems to be indicated. Also, it seems to be fairly evident that students prefer the test outside of regularly scheduled class time.

The most controversial issues are the two treatments and the pace. It is very difficult to arrive at any obvious conclusion from these results. This seems to indicate that the factors for student success in either program need to be given more attention. A final point of interest is the students' acceptance of the course independent of their treatment group.

Learning Center

The records used to arrive at the data shown in Table 13 were obtained from time sheets the students were asked to sign upon entering the learning center. They were to indicate their name, date, treatment group, and total amount of time spent. Upon inspection of these data, it is the belief of the researcher that the total amount of time indicated represents no more than 75 per cent of the actual time. The time factors involved in this discussion do not include the scheduled appointments between instructors and students participating in the independent study program.

Table 13: Time Element Involved in Use of Learning Center

	Intact Class	Independent Study
Total Number of Student Visits		
L	13	46
M	56	55
H	15	11
Total	84	112
Total Number of Different Students Participating		
L	4	12
M	20	10
H	3	7
Total	27	29
Average Amount of Time per Pupil Rounded to Highest 5 Minutes		
L	1hr. 5min.	2hr.50min.
M	2hr.50min.	4hr.
H	1hr.35min.	50min.
Total Amount of Time Spent in Learning Center		
L	5hr.30min.	33hr.50min.
M	43hr. 5min.	39hr.30min.
H	11hr. 5min.	5hr.45min.
Total	59hr.40min.	79hr. 5min.

The primary purpose of this part of the research was to determine if the learning center would be used and to what extent. The data indicate that the center was used for additional consultation a total of 138 hours and 45 minutes during the experimental period. This is a considerable amount of time. In addition there were approximately, 972 appointments scheduled in offices of the instructors adjacent to the center. Approximately 800 of these were met. The average meeting was 10 minutes; this would indicate an additional 133 hours for a grand total of 272 hours. These figures indicate the learning center was an integral part of the implementation of this program.

CHAPTER V

SUMMARY, IMPLICATIONS, AND RECOMMENDATIONS

Summary

This study was conducted at St. Gregory's College in Shawnee, Oklahoma, during the fall semester of 1973. St. Gregory's is a private, liberal arts, transfer oriented, junior college.

Aspects of individualized instruction were used in two modes of instruction: intact class and independent study. Behavioral objectives, degree of accomplishment commensurate with student ability, minimum rate of accomplishment, diagnostic testing and achievement testing were applied to both treatment groups. The treatment groups differed in the scheduling of classes. The intact classes met as a regularly scheduled class. The independent study group had no scheduled classes, but each student did have scheduled appointments. A learning center was provided for the use of all the students involved. Supplementary materials were also provided for all students.

The course was listed in the schedule as Mathematics 1513, College Algebra. The material presented dealt with concepts involved in sets, inequalities in one unknown, functions, linear forms, systems of linear equations, systems of linear inequalities, quadratics, and quadratic inequalities.

The sample consisted of students in three different sections of college algebra. Included in this sample were nineteen students from three foreign countries.

The treatment groups were divided into ability levels determined by their performance on an achievement test prior to the experimental period. These ability levels were designated as low, medium, and high.

The main purpose of this study was to determine if there were any significant differences in the achievement or attitude of these treatment groups. A secondary purpose was to determine student preference of mode of instruction and the different aspects of individualized instruction. Finally records were maintained to determine if the learning center was necessary in the instructional design.

The design used to analyze the effects on achievement and attitude was a two treatment by three level factorial analysis of covariance. Subsequent tests in the design included Duncan's Multiple Range Test and tests for simple main effects. A questionnaire employing a forced choice format was used to determine student preferences.

The data for analysis of achievement were collected from pre-test and post-test scores of parallel forms of an achievement test constructed by members of the faculty.

The data for analysis of attitude were collected from pre-test and post-test scores of a semantic differential constructed by McCallon and Brown (1971).

Findings

The null hypotheses concerning achievement were as follows:

H_1: There are no significant differences in achievement test score means between the two treatments.

H_2: There are no significant differences in achievement test means among the ability levels.

H_3: There are no significant interactions between the treatments and ability levels as measured by achievement test scores.

The results of the analysis of covariance indicated that H_1 and H_3 cannot be rejected, but that H_2 must be rejected. Further analysis, resulting from rejection of H_2, using Duncan's Multiple Range Test indicated all three ability levels had significantly different mean scores for achievement.

The null hypotheses concerning attitude were as follows:

H_4: There are no significant differences in attitude test score means between the two treatments.

H_5: There are no significant differences in attitude test score means among the ability levels.

H_6: There are no significant interactions between the treatments and ability levels as measured by attitude test scores.

The results of the analysis of covariance indicated that H_4 and H_6 cannot be rejected, but that H_5 must be rejected. Further analysis, resulting from the rejection of H_5, using Duncan's Multiple Range Test indicated that the low ability attitude mean score was significantly different from both medium and high ability attitude mean scores, but that medium and high ability mean scores were not significantly different.

The questionnaire to determine student preference toward the aspects of individualized instruction indicated that students preferred the use of behavioral objectives, the use of supplementary units of instruction, and testing outside of the class where no time factor was imposed upon them. The greatest discrepancy of student opinion appeared with the choice of independent study versus regularly scheduled classes and self-pacing versus minimum rate of accomplishment determined by instructor. A slight majority was in favor of independent study and self-pacing.

The learning center was used by fifty-six of the seventy-two students for a total of 138 hours and 45 minutes. This was in addition to 133 hours spent during appointments.

Implications

The general implication of this study, understanding the previously stated assumptions and limitations, is that teaching procedures employing certain aspects of individualized instruction can be as effective in an intact class situation as in an independent study program. From the standpoint of teacher time, it appears that the intact class would be preferred over the independent study program, for instructors would have more time to participate in alternate types of activities. From an administrative point of view, the intact class instructor could as successfully deal with a considerably larger number of students than the instructor working with an independent study program. Thus, it would follow that the cost of operating the independent study program would be substantially greater than the cost of operating the intact class program.

The implementation of new pedagogical techniques does not immediately change the attitude of students toward mathematics. Each student who has an unfavorable attitude toward mathematics has attained this position after several years of being confronted by a situation he found to be unfavorable. It would seem to follow that if his position is to be altered to a positive direction, it would take more than a single semester; it would require the restructuring of an entire program over an extended period of time.

Student attitude and its effect on achievement is particularly important to the success of the weaker student, and this study seems to point very clearly to the fact that simply to change classroom procedures is not nearly enough; nor is it enough for the instructor to initiate the learning activities and be available to work with the student. These factors indicate the importance of effective placement procedures or the flexibility of an instructional system which permits students to begin courses at a level compatible with their past experiences.

The fact that students from both treatment groups sought individual help from the classroom instructors shows that the procedures used did encourage these students to take advantage of additional learning opportunities. Since this source of help is always available, it points to a distinct advantage of these teaching procedures.

Students definitely approve of being tested over stated objectives, and, in addition, they react positively to using supplementary materials. Such approval tends to enhance the teacher-pupil relationship by instilling assurance in the student that if he can master the stated objectives, then his test performance will be satisfactory. The supplementary material reinforces the same concept.

The procedures for diagnostic testing need be improved. It should follow that if each student used these tests properly, he should score higher on the achievement test. This testing should provide valuable feedback information permitting the instructor to better manage the instruction for each individual.

The students' lack of unanimity on the type of program and rate of accomplishment substantiates the belief that many students are not totally aware of their responsibility in learning. They must be made aware of this responsibility and work in conjunction with the instructor if they are to reach their goal.

Recommendations

A comparison of independent study programs is needed. One group with complete freedom from any type of scheduling constraints, compared with a group in an independent study program with constraints could be useful in determining more adequately the effect of time constraints on independent study programs. These results would be especially interesting if the comparison was conducted in classes required of students, but not in their major field of study.

In this study a minimum rate of accomplishment was imposed upon the students. The study could be replicated with the introduction of an element of self-pacing. The results would indicate the effect of this concept on student success. A similar replication introducing a re-testing procedure for achievement testing could yield further information regarding factors relating to the success of the student.

Considerable work is needed to determine factors which will allow any instructional process to be more effective when encountered by weaker students. That is, such things as ways to motivate these students, self-pacing, and proper placement should be studied so that the instructional procedure will begin to meet the needs of these students.

A final recommendation would be to compare a complete intact class with another intact class with specified students in an independent study program. This type of experiment would be concerned with the results of reduced class size and the greater flexibility permitted those students who work at a pace different from the intact class.

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APPENDIX A

LEARNING PACKETS FOR COLLEGE ALGEBRA 1513

The following are samples of the packets used during the experimental period. Each sample represents a part of the units from the three achievement tests covered. The entire set of units is not included for it is the researcher's belief that these packets should be used to supplement a text, or if not they should be designed by the instructor in accordance with the objectives of his particular course.

CHAPTER II

Unit I

Objectives

1. Correctly graph three of four numerical statements involving one variable.

2. Write the proof for each of the following theorems, x, y, and z any real numbers:

   a) $x < y$ if and only if $x + z < y + z$.

   b) For $z > 0$ we have $x < y$ if and only if $xz < yz$.

   c) For $z < 0$ we have $x < y$ if and only if $xz > yz$.

   d) If $x < y$ and $y < z$ then $x < z$.

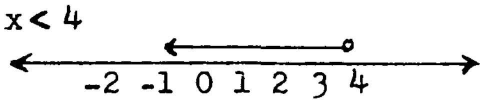
3. Solve and graph at least eight of ten inequalities by using the theorems stated in objective 2.

Assignment and Supplementary Material

(Remember numbers in this section correspond to the number of the objectives.)

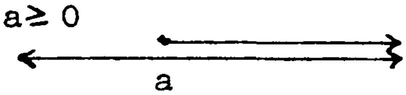
1. This technique is easy to master. Just remember that every real number corresponds to a point on the real number line and also every point on the real number line has a real number for its coordinate. In graphing these statements we shall employ the convention illustrated in the following examples.

   Example 1. $x < 4$

   The hollow point means all numbers less than four are included in the set but four itself does not belong to that set.

   Example 2. $a \geq 0$

   The solid point means that a and all numbers greater than a belong to the set.

Assignment:

Read Text Section 31 Chapter II.

Graph each of the following:

a) $x < 2$ b) $-3 \leq x$ c) $x = 0$

d) $5 > x$ e) $3/2 \geq x$

2. An important idea in studying these proofs is to understand statements of the form A if and only if B, where A and B are simple components. To prove any statement of this type we must prove two things: 1) If A then B; 2) If B then A. 1) is equivalent to saying if we assume A to be true can we prove B true. 2) is equivalent to saying if we assume B to be true can we prove A to be true. Both together imply A if and only if B.

What do these theorems really say? Theorem a) says if the same number is added to or subtracted from an inequality the inequality remains in the same direction. Theorem b) says as long as we multiply an inequality by a positive number the inequality remains in the same direction. Theorem c) is the one which really needs close attention. The theorem says that if an inequality is multiplied by any negative number the direction of the inequality is changed.

Example 1. $-5 < 8$. Now multiply by (-3)
Then we have $15 > -24$.

Assignment:

Read Text Section 32 Chapter II.

Now prove each of the theorems without use of text.

Assignment:

Exercise 9 page 57 numbers 3-15.

<u>Diagnostic Test</u>

1. Graph each of the following on the real number line.

a) $x = 2$ b) $2 \geq x$ c) $-5 < x$ d) $x > -3$

2. Prove each of the following theorems for x, y, and z any real numbers.

a) $x < y$ if and only if $x + z < y + z$.

b) If $x < y$ and $y < z$ then $x < z$.

3. Solve and graph on the real number line each of the following:

a) $x + 3 \geq 2$ b) $2x < 6$

c) $x + 5 \leq 2x$ d) $3x + 1 < 1$

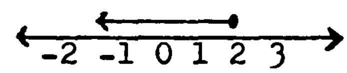
e) $-x \geq -2$ f) $(3/5)(3x - 2) - (1/10)(6x + 7) \leq 0$

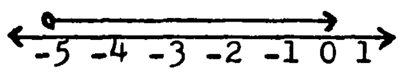
g) $(2/3)x + 1 \leq 4/7$ h) $(3/5)x \geq -2/5$

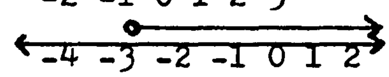
i) $3x + 2 \leq 5x - 7$ j) $2/5 + 3 < x - 2/5$

<u>Answer Sheet and Recommendations</u>

1. a) 

b) 

c) 

d) 

If more than one of these are incorrect see your instructor.

2. a) $x < y$ if and only if $x + z < y + z$.

Proof: Part I. If $x < y$ then $x + z < y + z$. $x < y$ implies $y - x = p$ where p is a positive number. Then $y + z - x - z = p$ and then $(y + z) - (x + z) = p$. Then by definition $x + z < y + z$.

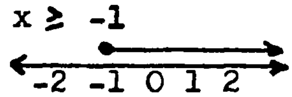
Part II. If $x + z < y + z$ then $x < y$. Now $x + z < y + z$ implies $(y + z) - (x + z) = p$ where p is a positive number. But now $(y + z) - (x + z) = y + z - x - z = y - x = p$. Thus, $x < y$.

b) If $x < y$ and $y < z$ then $x < z$.

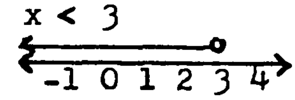
Proof: $x < y$ implies $y - x = p$ where p is a positive number. Also, $y < z$ implies $z - y = q$ where q is a positive number. Adding we have $(y - x) + (z - y) = p + q$. Thus $z - x = p + q$ and we conclude $x < z$.

If either of these are missed refer to text Section 32 Chapter II.

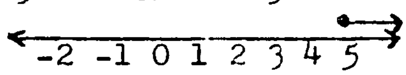
3. a) $x \geq -1$



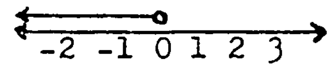
b) $x < 3$



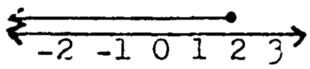
c) $5 \leq x$ or $x \geq 5$



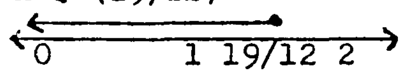
d) $x < 0$



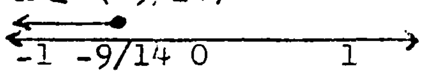
e) $x \leq 2$



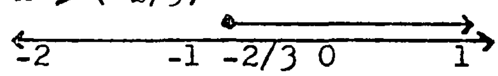
f) $x \leq (19/12)$



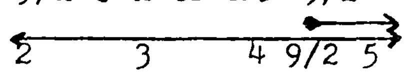
g) $x \leq (-9/14)$



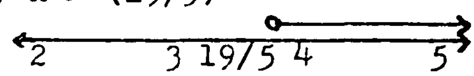
h) $x \geq (-2/3)$



i) $9/2 \leq x$ or $x \geq 9/2$



j) $x > (19/5)$



If more than two are incorrect refer to:

Beckenbach College Algebra Exercise 45 page 104 numbers 1-12.

Barnett Intermediate Algebra Exercise 19 page 85 numbers 1-14.

Drooyan Elementary Algebra: Structure and Skills page 236 numbers 1-24.

CHAPTER III

Unit IV

Objectives

1. Correctly write the equation of a line through two given points in at least three of four problems.

2. Correctly write the equation of a line given the slope of the line and a point on the line in at least three of four problems.

3. Write the equation of a line correctly, given the slope and y-intercept of the line, in at least three of four problems.

4. Given the equation of a line determine the slope and y-intercept of the line in at least three of four problems.

5. Write the equation of a line through a given point parallel to, and perpendicular to, a given line in at least three of four problems.

6. Find the appropriate constant in two given equations so that the lines are a) parallel; b) perpendicular. Do this correctly in two of three problems.

Assignment and Supplementary Material

1. Since two points determine one and only one line through those points, it seems reasonable that we should be able to determine the equation of that line. To do this we use the fact that the slope between any two points on the same line is the same. (Remember to show three points are collinear we show that the slopes between any two of those points are the same.)

   First we shall look at an example and then try to generalize.

   Example 1. Write the equation of a line through the points (1,3) and (3,7). Certainly we can compute the slope of the line between (1,3) and (3,7).

   $$m = \frac{7-3}{3-1} = \frac{4}{2} = 2$$

   Now an equation of a line must contain two variables, so we let (x,y) be any point on the line through (1,3) and (3,7).

   Then since (x,y) is on the same line as (1,3) and (3,7) the slope of the line through (1,3) and (x,y) must be equal to the slope of the line through (1,3) and (3,7).

Therefore $\frac{y-3}{x-1} = \frac{7-3}{3-1} = 2$.

That is $\frac{y-3}{x-1} = 2$. Now multiply both members of the equation by (x-1). Then we have $(x-1)(\frac{y-3}{x-1}) = 2(x-1)$

or $y-3 = 2(x-1)$ so $y-3 = 2x-2$ 1)

and finally $y = 2x+1$ by adding +3 to both sides of $y-3 = 2x-2$.

Let us agree that our final answer will be expressed as y equals something times x plus a constant. One more point about this example: again since (x,y) is on line through (1,3) and (3,7) we could say the slope between (x,y) and (3,7) must be equal to the slope of the line through (1,3) and (3,7)

Hence $\frac{y-7}{x-3} = 2$ which implies $y-7 = 2(x-3)$ or $y-7 = 2x-6$

so $y = 2x+1$. 2)

Note that this is the same as the first result. That is statement 1) is the same as statement 2).

Now to generalize from this example. Let $P_1:(x_1,y_1)$ and $P_2:(x_2,y_2)$ be two given points. And let $P:(x,y)$ be the coordinates of any other point on the line through P_1 and P_2.

The slope of the line between P_1 and P_2 is $m_{P_1P_2} = \frac{y_2-y_1}{x_2-x_1}$

Also the slope of the line between P_1 and P is $m_{P_1P} = \frac{y-y_1}{x-x_1}$

Since these two slopes are equal we write $\frac{y_2-y_1}{x_2-x_1} = \frac{y-y_1}{x-x_1}$

This is called the <u>two point</u> form for the equation of a line and is used whenever two points are given.

Assignment:

Read text page 84 Two Point Form.

Problems Exercise 18 page 87 numbers 19-23.

2. Now turn your attention to writing the equation of a line with a given slope through a given point. Again let us look at an example before we generalize.

Example 2. Write an equation of a line through (-3,3) with slope (-2/3). Now let (x,y) be any other point on the line through (-3,3) with slope (-2/3). Then the slope of the line between (x,y) and (-3,3) must be (-2/3).

That is $\frac{y-3}{x-(-3)} = \frac{-2}{3}$ or $\frac{y-3}{x+3} = \frac{-2}{3}$.

Multiplying both members of the equation by (x+3) we have

$[(x+3)]\ (\frac{y-3}{x+3}) = \frac{-2}{3}\ [(x+3)]$ or $(y-3) = \frac{-2}{3}\ (x+3)$,

which implies $y-3 = \frac{-2}{3}x-2$. Then adding (+3) to both

members we have $y = \frac{-2}{3}x+1$.

Now to generalize this idea let us assume we are given a point $P_1:(x_1,y_1)$ and a slope m. Then for (x,y) any

point on the line through P_1 with slope m we have,

$\frac{y-y_1}{x-x_1} = m$ or $y-y_1 = m(x-x_1)$.

This is called the point slope form for the equation of a line, and is used whenever we are given a point and slope.

Assignment:

Read text page 84 Point Slope Form.

Problems Exercise 18 numbers 13-16 and 18.

3. To write the equation of a line given the slope and y-intercept is just a special case of the point slope form. In this instance the point will always be of the form (0,b). Note that every point on the y-axis has first coordinate 0, thus (0,b) is called the y-intercept. Sometimes we will just say the y-intercept is b which means to consider the point (0,b). Hence if we are given slope m and y-intercept b we may substitute m and (0,b) in the point slope form. Thus we would have $y-b = m(x-0)$ or $y = mx+b$.
This is called the slope intercept form and is used whenever you are given the slope and y-intercept. This form is extremely important.

Assignment:

Read Text page 85 Slope Intercept Form.

Problems Exercise 18 numbers 1, 3, 4, 7, 8.

4. In each of objectives 1, 2, and 3 you were told to write your answer in y form. That is for example, $y = 2x+1$ or $y = -2/3x+1$. Now let us investigate the slope intercept form more closely. The slope intercept form for the equation of a line is: $y = mx+b$.

This says that if the equation of a line is written in the y form the coefficient of x will be the slope and the constant will be the y-intercept. So given any linear equation if we write it in the slope intercept form we can identify the slope and the y-intercept.

Example 3. In each of the following equations identify the slope and y-intercept.

a) $y = -5x+2$

Hence, $m = -5$(the coefficient of x) and y-intercept is 2.

b) $y = -3+2x$

Now $m = 2$(the coefficient of x) and y-intercept is (-3).

c) $5x+2y = 4$

This implies $2y = -5x+4$ Why?

And therefore $y = (-5/2)x+2$ Why?

Hence, $m = -5/2$ and y-intercept is 2.

Assignment:

Problems Exercise 87 numbers 35-40.

5. Let us consider an example of what objective 5 requires of us. If you do not know what objective 5 is go back right now and read it.

Example 4. Find the line through (2,3) parallel to and perpendicular to $3x-2y = 1$. If we write $3x-2y = 1$ in slope-intercept form we have $y = (3/2)x-1/2$.

Part I. Equation of parallel line. Since parallel lines have equal slopes the slope of the desired line must be the same as the slope of $3x-2y = 1$. That is the slope must be $3/2$. Now the line is to go through $(2,3)$, so we are given the slope $3/2$ and a point $(2,3)$.

Substituting $m = 3/2$ and $(x,y) = (2,3)$ in point slope form we have $y-y_1 = m(x-x_1)$

$$y-3 = (3/2)(x-2)$$

$$y-3 = (3/2)x-3 \text{ or}$$

$$y = (3/2)x$$

So $y = (3/2)x$ is parallel to $3x-2y = 1$ and $y = (3/2)x$ goes through $(2,3)$. Note the y-intercept in the equation is the origin.

Part II. Equation of perpendicular line. If two lines are perpendicular the products of their slopes is -1. So if we let m be the slope of the line whose equation we are going to write we have: $(3/2)m = -1$ since slope of given line is $3/2$. Now solving for m by multiplying both members by $2/3$: $(2/3)(3/2)m = (-1)2/3$. Therefore $m = -2/3$.

So now using point slope form again we will write the equation of a line with slope $(-2/3)$ through $(2,3)$. $y-y_1 = m(x-x_1)$ implies $y-3 = (-2/3)(x-2)$ (Substitution) or $y-3 = (-2/3)x+4/3$ then $y = (-2/3)x+13/3$. (Since $3+4/3 = 9/3+4/3 = 13/3$.)

Thus $y = (2/3)x+13/3$ is perpendicular to $3x-2y = 1$ and goes through $(2,3)$.

Assignment:

Exercise 19 numbers 1, 2, 3, 5, 6.

6. An example will be used to illustrate objective 6. Be sure you have read objective 6.

Example 5. Find the constant h if the lines $2x+hy = 3$, $x+3y = 6$ are:

a) Parallel. If the lines are parallel they must have equal slopes, so write each equation in slope-intercept form and identify each slope. $2x+hy = 3$ implies $hy = -2x+3$ Why? Then $y = (-2/h)x+3/h$ Why? Therefore slope of $2x+hy = 3$ is $-2/h$. Now for $x+3y = 6$ then $3y = -x+6$ or $y = (-1/3)x+2$
Therefore slope of $x+3y = 6$ is $-1/3$.

Equate the slopes and solve for h. $-2/h = -1/3$ which implies $h = 6$. Hence, if $h = 6$ the two given lines will be parallel.

b) Perpendicular. Again using the fact that if two lines are perpendicular then the product of their slopes is (-1) we write the product of the slopes found in part a) of this example and solve for h. So $(-2/h)(-1/3) = -1$ which implies $2/3h = -1$ or $2 = -3h$ and $-2/3 = h$.
Thus if $h = -2/3$ the two given lines will be perpendicular.

Assignment:

Exercise 19 page 90 numbers 11-13.

Diagnostic Test

1. Write the equation of the line satisfying the conditions.

   a) (2,5);(4,3) b) $m = 2$;(-1,3)

   c) $m = -1$;y-intercept 2 d) (-2,-1);(4,0)

   e) $m = 0$;(-5,-2) f) $m = 4/3$;y-intercept -3

   g) (0,0);(5,4) h) $m = -3$;(4,6)

   i) $m = 5$;(0,5) j) (0,-3);(-3,0)

   k) $m = -2/3$;(0,0) l) $m = 0$;y-intercept 1

2. Write the equations in slope-intercept form and indicate the slope and y-intercept of each.

   a) $5x-2y = 2$ b) $x+y = 0$

   c) $-3x+2y = 6$ d) $5x = 12y+6$

3. Write the equation of a line through the given point parallel to, and perpendicular to each given line.

   a) $2x-y = 4$;(3,-2) b) $-x = 2y+6$;(4,0)

4. a) Find the constant h so that the given lines are parallel.

   $3x-5y = 2, \; hx+10y = 3$

   b) Find the constant h so that the given lines are perpendicular.

$3x+4y = 7$, $6x+hy = 5$

c) Find the constant h so that the given lines are parallel.

$x-hy = 1$, $y+2x = -3$

Answer Sheet and Recommendations

1. a) $y = -x+7$ b) $y = 2x+5$

c) $y = -x+2$ d) $y = x/6-2/3$

e) $y = -2$ f) $y = (4/3)x-3$

g) $y = (4/5)x$ h) $y = -3x+18$

i) $y = 5x+5$ j) $y = -x-3$

k) $y = (-2/3)x$ l) $y = 1$

If more than three are incorrect refer to:

Text Exercise 23 page 100 numbers 12-17.

Beckenbach College Algebra pages 130-31 numbers 1-18, 32.

Barnett Intermediate Algebra pages 97-98 numbers 5-20.

2. a) $y = 5/2x-1$ so $m = 5/2$; y-intercept

b) $y = -x$ so $m = -1$; y-intercept 0

c) $y = (3/2)x+3$ so $m = 3/2$; y-intercept 3

d) $y = (5/12)x-1/2$ so $m = 5/12$; y-intercept $(-1/2)$.

If more than one is incorrect refer to:

Beckenbach College Algebra page 130 numbers 13-18.

Barnett Intermediate Algebra page 97 numbers 1-4.

3. a) parallel line $y = 2x-8$; perpendicular $y = -x/2-1/2$

b) parallel line $y = -x/2+2$; perpendicular $y = 2x-8$

If the equation of more than one line is incorrect refer to:

Text Exercise 23 page 100 numbers 18,19.

Beckenbach *College Algebra* page 130 numbers 25-30.

4. a) $h = -6$ b) $h = -9/2$ c) $h = -1/2$

If more than one is incorrect refer to:

Text Exercise 23 page 100 number 25.

CHAPTER IV

Unit II

Objectives

1. Perform the operations of addition and multiplication correctly on six of eight problems involving complex numbers.

2. Correctly write the quadratic formula.

3. Solve five of six quadratic equations of the form $ax^2 + bx + c = 0$ where $a \neq 0$, by using the quadratic formula.

Assignment and Supplementary Material

1. How many real numbers are there? Certainly our answer would be an infinite number. Yet with all of these numbers we are not able to solve the equation $x^2 = -1$. In order to be able to solve an equation like $x^2 = -1$ we now will introduce the set of complex numbers. These numbers will be defined as follows using C to denote this set.

   C = (a+bi: a is any real number, b is any real number, and $i^2 = -1$)

   Now the implication that $i^2 = -1$ is that $i = \sqrt{-1}$. Furthermore note that if b = 0 and a is any real number then the real numbers are a subset of the complex numbers.

   In any complex number a+bi we refer to a as the real part of the complex number and b as the imaginary part of the complex number so that in the complex number 5+6i we say 5 is the real part 6 is the imaginary part.

   Further examples:

	Number	Real Part	Imaginary Part
1.	6i	0	6
2.	-3-4i	-3	-4
3.	2+5i	_____	_____
4.	6	_____	_____
5.	-6+2i	_____	_____

One more comment on i before we define the operations

Note: $i = \sqrt{-1}$

$i^2 = -1$

$i^3 = i^2 \cdot i = -1$

$i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$

Now any positive integral power of i is either i, -1. -i, or 1. Given any power of i simply divide the exponent by 4. Then

1) if remainder is 0 the answer is 1

2) if remainder is 1 the answer is i

3) if remainder is 2 the answer is -1

4) if remainder is 3 the answer is -i

From the following examples see if you can see why the above is true.

Example 1. i^{72} Now $72 \div 4 = 18$ thus $i^{72} = (i^4)^{18} = 1^{18} = 1$

2. i^{55} Now $55 \div 4 = 13$ with remainder 3 thus

$i^{55} = (i^4)^{13} i^3 = 1^{13} \cdot i^3 = -1$

3. i^{34} Now $34 \div 4 = 8$ with remainder 2 thus

$i^{34} = (i^4)^8 \cdot i^2 = 1^8 \cdot i^2 = -1$

4. i^{45} Now $45 \div 4 = 11$ with remainder 1 thus

$i^{45} = (i^4)$___ $\cdot$ i___ = ___

5. i^{97} Now $97 \div 4 =$ ___ with remainder ___

thus $(i)^{97} = (i^4)$___ $\cdot$ i___ = ___

Now let us define the operation of addition on complex numbers. For any two complex numbers a+bi and c+di we have:

$$(a+bi) + (c+di) = (a+c) + (b+d)i.$$

That is simply add real parts and imaginary parts.

Example 1) $(4+3i) + (7+10i) = 11 + 13i$

2) $(5-6i) + (-3+5i) = 2 - i$

3) $4i + (-7+3i) = -7 + 7i$

4) $(8+2i) + (5-7i) =$ _________

5) $6 + (-3-5i) =$ _________

Now the product of two complex numbers is found the same way you multiply any two binomials.

$(a+bi)(c+di) = (ac-bd) + (bc+ad)i$

Example 1) $(4+3i)(7+10i) = 28 + 61i + 30i^2 = -2 + 61i$

2) $(5-6i)(-3+5i) = -15 + 43i - 30i^2 = 13 + 43i$

3) $(8+2i)(5-7i) = 40 - 46i - 14i^2 = 54 - 46i$

4) $(-3+i)\ (-3-i) =$ _________

5) $(2-5i)^2 =$ _________

We are now ready to solve the equation $x^2 = -1$

$$x = \pm\sqrt{-1} = \pm i$$

Furthermore we can now take the square root of any negative number.

Example 1) $\sqrt{-64} = \sqrt{64}\sqrt{-1} = 8i$

2) $\sqrt{-20} = \sqrt{4}\sqrt{-1}\sqrt{5} = 2i\sqrt{5}$

3) $\sqrt{-81} =$ _________

4) $\sqrt{-200} =$ _________

Assignment:

Read Text Section 20 page 33.

Problems Exercise 5 page 40 numbers 114-41.

2,3. Returning now to the general form of the quadratic equation $ax^2+bx+c = 0$ where $a \neq 0$. We shall now derive what is known as the quadratic formula. This formula can be used to solve any quadratic equation of the form $ax^2+bx+c = 0$ where $a \neq 0$. The method used to derive this formula is to complete the square on $ax^2+bx+c = 0$.

Given: $ax^2+bx+c = 0$

$ax^2+bx = -c$ Why?

$a(x^2+\frac{b}{a}x) = -c$

Factoring by completing the square we have

$a(x^2+\frac{b}{a}x+\frac{b^2}{4a^2}) = -c + \frac{b^2}{4a}$ (Refer to Chap. IV Unit I)

$a(x+\frac{b}{2a})^2 = \frac{b^2-4ac}{4a}$ Why

$(x+\frac{b}{2a})^2 = \frac{b^2-4ac}{4a^2}$

Taking square roots we have

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}} = \pm\frac{\sqrt{b^2-4ac}}{2a}$$

Thus $x = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = \frac{-b \pm\sqrt{b^2-4ac}}{2a}$

(Memorize this formula)

Now let us put this formula to work solving some quadratic equations.

Example 1) Solve $2x^2-4x+5 = 0$

Note $a = 2$, $b = -4$, and $c = 5$

$$x = \frac{-(-4)\pm\sqrt{(-4)^2-4(2)(5)}}{2(2)} = \frac{4\pm\sqrt{16-40}}{4}$$

$$= \frac{4\pm\sqrt{-24}}{4} = \frac{4\pm\sqrt{4}\sqrt{-1}\sqrt{6}}{4} = \frac{4\pm2i\sqrt{6}}{4}$$

$$= \frac{2(2\pm i\sqrt{6})}{4} = \frac{2\pm i\sqrt{6}}{2}$$

Thus SS = $[\frac{2+i\sqrt{6}}{2}, \frac{2-i\sqrt{6}}{2}]$

2) Solve $3x^2-2 = 6x$

First rewrite as $3x^2-6x-2 = 0$

then $a = 3$, $b = -6$, and $c = -2$

$$x = \frac{-(-6)\pm\sqrt{(-6)^2-4(3)(-2)}}{2(3)} = \frac{6\pm\sqrt{36+24}}{6}$$

$$= \frac{6\pm\sqrt{60}}{6} = \frac{6\pm\sqrt{4}\sqrt{15}}{6} = \frac{6\pm2\sqrt{15}}{6}$$

$$= \frac{2(3\pm\sqrt{15})}{6} = \frac{3\pm\sqrt{15}}{3}$$

Thus SS = $[\frac{3+\sqrt{15}}{3}, \frac{3-\sqrt{15}}{3}]$

Things to remember:

1. Memorize the quadratic formula $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

2. Be sure to write the equation in the form $ax^2+bx+c = 0$ and then identify a, b, and c.

3. This procedure may be used to solve all quadratics of the given form.

Assignment:

Read Text Section 50 page 106.

Problems Exercise 25 page 108 odd 45-61.

Diagnostic Test

1. Perform the indicated operations:

a) $\sqrt{-25}$ b) $(5-2i) + (6+3i)$

c) $(5-3i)(-2+5i)$ d) $(\frac{3}{2} + \frac{5}{8}i) + (\frac{-1}{4} + \frac{1}{4}i)$

e) $(4i)(-3i)$ f) $(2i)^5$

g) $(5-3i)(i+2)$ h) $(3-2i)(4+i)$

2. Write the quadratic formula.

3. Solve the following quadratic equations using the quadratic formula.

a) $2x^2-x-10 = 0$ b) $8x-4x^2 = 9$

c) $25y^2+9 = 0$ d) $15x^2-x = 28$

e) $27+6x = -2x^2$ f) $1 = 12x-9x^2$

Answer Sheet and Recommendations

1. a) 5i b) 11+i c) 7-8i

d) $\frac{5}{4} + \frac{7}{8}i$ e) 12 f) 32i

g) 13-i h) 14-5i

If more than two are incorrect refer to:

Schaum's College Algebra pages 63-65.

2. Refer to Supplementary Material.

3. a) SS = [5/2,-2] b) $[\frac{2+i\sqrt{5}}{2}, \frac{2-i\sqrt{5}}{2}]$

c) SS = [(3/5)i, (-3/5)i]

d) SS = [-4/3, 7/5] e) $[\frac{-3+3i\sqrt{5}}{2}, \frac{-3-3i\sqrt{5}}{2}]$

f) SS = $[\frac{2+\sqrt{3}}{3}, \frac{2-\sqrt{3}}{3}]$

If more than one is incorrect refer to:

Text page 108 even problems 44-60.

APPENDIX B

THE ACHIEVEMENT TEST

PART I COLLEGE ALGEBRA MATH 1513

1. Solve and graph each of the following inequalities:

   a) $4x \leq 8$

   b) $3x + 1 > 1 - x$

   c) $x - 5 \geq 4$

2. Given the sets $A = [1,2,3,4,5]$, $B = [a,b,c,d,e]$, $C = [a,b,c,1,5]$, and $D = [0,2,4,6,8,10]$ find:

   a) $A \cup B$

   b) $B \cap C$

   c) C/D

   d) $(D/B) \cup C$

3. Solve and graph each of the following inequalities:

   a) $-4 \leq \frac{1}{4}x + 3 \leq 6$

   b) $|3 - x| \leq 4$

   c) $|2x - 1| > 4$

   d) $5/4 \leq 3x + \frac{1}{4} < \frac{1}{4}$

4. Prove for x, y, and z any real numbers that if $x < y$ then $x + z < y + z$.

PART II COLLEGE ALGEBRA MATH 1513

1. Given the points A, B, C, and D with coordinates (-5), (-2), 3, and 7 respectively. Compute:

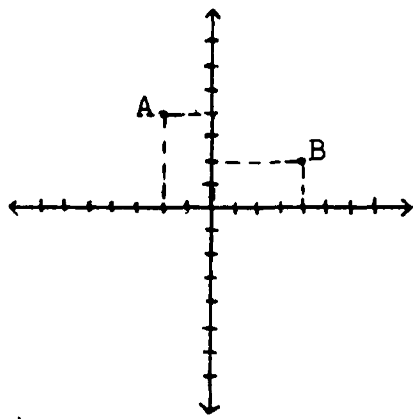
a) $\overline{AD}$

b) $\overline{DB}$

c) $|\overline{CB}|$

2. a) What are the coordinates of A:(,) and B:(,).

   b) Plot the points (3,6), (-5,0), (4,2), and (-3,-3).



3. Given the points A:(3,8) and B:(-3,0);

   a) compute the distance, $|\overline{AB}|$, from A to B.

   b) compute the slope of the line from A to B.

4. Determine whether or not the points A:(2,-2), B:(-8,-7), and C:(6,0) are collinear.

5. Write the equation of a line satisfying the given conditions. Graph each equation.

   a) Points (3,-4) and (-6,-5).

   b) Slope $\frac{1}{4}$ and point (9,4).

   c) y-intercept 2 and slope $(-\frac{1}{2})$.

6. Given the functions $f(x) = 2x + 1$ and $s(t) = t^2 - 2t$. Compute:

   a) f(3)

   b) s(4)

   c) 5f(-4)

   d) f(1)s(-3)

7. Write the equation of a line through (4,-1) perpendicular to $5x - 3y = 2$.

8. Find the constant h so that the lines $2y - hx = 4$

and $3y + 9 = 6x$ are parallel.

9. Solve the following systems of linear equations:

   a) $x - y = 4$ and $2x = -y + 2$

   b) $y = 4 - x$ and $2x - 12 = -2y$

   c) $3x + 2y = 18$ and $5x - 3y = 11$

10. Solve graphically the following system of linear inequalities; $2x + y \geq -3$ and $x - 2y > 6$.

PART III COLLEGE ALGEBRA MATH 1513

1. Graph the parabola whose equation is $y = -x^2 - 4x - 1$, identifying the axis of symmetry, vertex, and at least two other points.

2. Perform the indicated operations on the following complex numbers.

   a) i^{51}

   b) $(5 + 4i)(-3 + i)$

   c) $(5 + 4i) + (-3 + i)$

3. Find the value of the constant k in $kx^2 - x + 1 - k = 0$ so that the roots are equal.

4. Given the quadratic equation $2x^2 - 3x + 5 = 0$.

   a) Describe the characteristics of the roots by use of the discriminant.

   b) Determine the sum of the roots.

   c) Determine the product of the roots.

5. Solve the following equations.

   a) $16 = 4x^2$

   b) $x^2 + 4x - 12 = 0$

   c) $4x^2 - x = 4$

   d) $x^2 + 2x + 6 = 0$

6. Find the constant k in $x^2 - 6kx + 2k = 0$ so that the product of the roots is 3/2.

7. Form a quadratic equation for each set of given roots.

   a) $-4:1$

   b) $1 - 3i:1 + 3i$

8. Find the solution set for each of the following equations.

   a) $x^4 - 11x^2 + 28 = 0$

   b) $y^3 = 27$

   c) $z^6 - 7z^3 - 8 = 0$

   d) $x = -x + 6$

9. Solve and graph the inequality $x^2 - 2x - 8 < 0$.

APPENDIX C

ATTITUDE TEST

Please place an (x) on the blank which most appropriately describes how you feel about mathematics as described by each of the following adjectives:

Example:

Bad:____:____:_x_:____:____:____:____:Good

Be sure you mark each pair of adjectives.

Mathematics

1. Bad:____:____:____:____:____:____:____:Good
2. Hard:____:____:____:____:____:____:____:Soft
3. Afraid:____:____:____:____:____:____:____:Unafraid
4. Active:____:____:____:____:____:____:____:Passive
5. Valuable:____:____:____:____:____:____:____:Worthless
6. Strong:____:____:____:____:____:____:____:Weak
7. Love:____:____:____:____:____:____:____:Hate
8. Fast:____:____:____:____:____:____:____:Slow
9. Comfortable:____:____:____:____:____:____:____:Uncomfortable
10. Awful:____:____:____:____:____:____:____:Nice
11. Enjoyable:____:____:____:____:____:____:____:Unenjoyable
12. Light:____:____:____:____:____:____:____:Heavy
13. Varied:____:____:____:____:____:____:____:Repetitive

14. **Secure:____:____:____:____:____:____:____:Insecure**

15. **Pleasant:____:____:____:____:____:____:____:Unpleasant**

APPENDIX D

STUDENT QUESTIONNAIRE

Please take one moment of your time to answer the following questionnaire. I am very interested in your opinion of the college algebra course you took last semester after the notion of the grade has passed. Your responses will help me improve the course next semester.

1. I ____ do prefer stated objectives on each unit.

   ____ do not prefer stated objectives on each unit.

2. The objectives ____ helped me prepare for tests.

   ____ were of no use.

3. The objectives were ____ concise and easy to understand.

   ____ difficult to understand.

4. I ____ do prefer the supplementary units used in addition to the text.

   ____ do not prefer the supplementary units used in addition to the text.

5. I would prefer ____ independent study program.

   ____ regularly scheduled classes.

6. I would prefer to advance at ____ my own pace.

   ____ pace determined by instructor.

7. I ____ did take advantage of the additional learning opportunities provided.

   ____ did not take advantage of the additional learning opportunities provided.

8. I prefer tests to be given ____ outside of regular class time to allow all the time needed.

____ during regularly scheduled class time.

9. I ____ would recommend this course.

____ would not recommend this course.

I appreciate your taking the time to answer this questionnaire. Please either return by campus mail or drop it by my office.

APPENDIX E

MATERIALS AVAILABLE IN LEARNING CENTER

The following reference books and filmstrips are available at the learning center, room 406. The books may be checked out but the filmstrips must remain in the learning center.

Books

Ayres Theory and Problems of College Mathematics

Bear Essentials of Algebra and Elementary Functions

Beckenbach College Algebra, second edition

Bush Foundations of Mathematics

Cameron Algebra and Trigonometry with Analytic Geometry

Drooyan Elementary Algebra for College Students

Eulenberg Intermediate Algebra: A College Approach

Fischer Integrated Algebra and Trigonometry

Heimer Contemporary Algebra, Book 1-5

Heineman College Algebra

Keedy Algebra and Trigonometry

Nichols College Mathematics for General Education

Nichols Introductory Algebra for College Students

Nunem Beginning Algebra

Rich Elementary Algebra

Rosenbach *College Algebra*, 4th and 5th editions

Russell *Algebra Problems*

Spiegel *Theory and Problems of College Algebra*

Wright *Elementary Functions*

Filmstrips

Common Solution to Two Linear Equations

Equations and Equivalent Equations

Equations and Inequalities

Equations with Fractions

Graphing Inequalities in Two Variables

Graphs of Inequalities in One Variable

Nature of Roots of Quadratic Equations

Proof in Algebra: Solving Equations

Quadratic Equations and Their Solutions

Solving Inequalities

Solving Radical Equations

Solving Two Linear Equations Algebraically

Work Problems in Mathematics