CULTURAL PARTICLE SWARM OPTIMIZATION

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Nomenclature

M	Number of decision variables; dimension of decision variables
N	Number of particles; number of individuals; population size
m	Number of constraints
n	Number of objectives
P	Number of swarms; number of societies
L	Number of inequality constraints
δ	Tolerance for equality constraints
N_i	Population of the *i*-th swarm, number of individuals in the *i*-th society
$g(x) \leq 0$	Inequality constraint
$h(x) = 0$	Equality constraint
$pbest$	Personal best particle in PSO
$gbest$	Global best particle in PSO
$nbest$	Neighborhood best particle in PSO
$sbest$	Swarm best particle in PSO

$g(x) \leq 0$	Inequality constraint
$h(x) = 0$	Equality constraint
c_p	Personal acceleration in PSO
c_g	Global acceleration in PSO
c_n	Neighborhood acceleration in PSO
c_s	Swarm acceleration in PSO
w	Momentum in PSO

CHAPTER I

INTRODUCTION

Computational intelligence approaches based upon the psychosocial studies inspired from either the human or animal society have been the subject of the emerging research known as swarm intelligence. There has been some research in the area of swarm intelligence focused on optimization in the spirit of the particle swarm [1], ant colony system [2] and cultural algorithms [3]. While the population based heuristics adopted in swarm intelligence do not mathematically guarantee to always find the global optimum of the search space, they perform greatly well in different types of optimization problems. Particle swarm optimization (PSO) is an imitation of the collaborative behavior of the birds flying together with the means of their information exchange, while ant colony is based on the fact that individual ants interact with each other through their pheromone trails. Cultural algorithm (CA) is a dual inheritance system in which the collective behavior of the population of individuals constructs the belief space which will in turn be accessible to all individuals in the population space. Additionally, the multinational algorithm [4] solves difficult multimodal optimization problems by using heuristics imitating political interactions among nations.

In another heuristic, based on the relation between society and civilization [5] the intersociety versus intrasociety relationship among the individuals facilitates on building an optimization model. The whole population of individuals, called the civilization, is clustered into different societies based on their Euclidean closeness of the individuals. The performance of individuals will be a measure to decide which individuals are the leaders of the society. The rest of the individuals are to follow them in a way to improve themselves which leads to migration (*intrasocitey interaction*). From the civilization viewpoint, the leaders of the societies will improve themselves by migrating toward the best-performing leaders who are the civilization leaders (*intersociety interaction*). The weakness of this paradigm is its lack on using existing information from all of the individuals.

Particle swarm optimization is based on the changes of the positions and velocities of the particles in a manner that optimizes a goal function. PSO has demonstrated a promising performance for many optimization problems; yet its fast convergence often leads to premature convergence in which the local optima of the goal function are found instead of the global one. The tradeoff between fast convergence and being trapped in local optima is even more critical in multimodal functions. In order to escape from the local optima and avoid premature convergence, the search for global optimum should be diverse. Many researchers have improved the performance of the PSO by enhancing its ability with a more diverse search. Specifically, some have proposed to use multiple swarms each running PSO, and then exchange information

among them. The weakness of these algorithms is their lack on considering a diverse list of information to exchange, consequently premature convergence. Exchanging information among clusters has also been adopted as an important design in several computational methods. Distributed genetic algorithm [6] employs GA mechanism to evolve several subpopulations in parallel. At regular intervals, migration among subpopulations takes place. During the migration stage, a proportion of each subpopulation is selected and sent to another subpopulation. The migrant individuals will replace others based on a replacement policy.

Several population based heuristics have been developed to solve multiobjective optimization problems (MOPs) among which multiobjective evolutionary algorithm (MOEA) and multiobjective particle swarm optimization (MOPSO) are two popular paradigms. Although there exist many research on single objective PSO suggesting dynamic weights for the local and global acceleration, but most MOPSO researchers assume that all particles should move with the identical momentum, local, and global acceleration. To our best knowledge, there have not been any studies to consider a case in which particles fly with different "personalized" weights for the momentum, local, and global acceleration. Employing a personalized weight for each particle assigns a proper jump contributing to the effectiveness of the overall performance of the algorithm. One computational aspect is the difficulties of tuning proper value for the momentum, personal, and global acceleration in MOPSO in order to attain the best results for different test functions. From a biological point of view, work presented in [7] has also

shown that societies that can handle more complex tasks contain polymorphic individuals. Polymorphism is a significant feature of social complexity that results in differentiated individuals. The more differentiated the society, the easier it can handle complex tasks. Differentiation applies in principal to complex societies of prokaryotic cells, multicellular organisms, as well as to colonies of multicellular individuals such as ants, wasps, bees, and so forth. The colony performance is improved if individuals differentiate in order to specialize on particular tasks. As a result of differentiation, individuals perform functions more efficiently. In their study it has been shown the colony's ability to higher cooperative activity when tackling tasks is a direct consequence of differentiation among other factors.

There are few studies in the MOPSO research area that have tackled the issue of variable momentum for the particles although in all of them momentum is identical for all particles at a specific iteration. Some MOPSO paradigms have proposed simple strategies to adapt the momentum by simply decreasing the momentum throughout swarming while other MOPSO algorithms choose a random value for momentum at every iteration. To the best knowledge of the author, there is no noticeable study in MOPSO on adapting personalized dynamic momentum and acceleration based upon the need for the particles to exploration or exploitation.

Constrained optimization problem is another area that has been solved using population based paradigms during the last two decades. Swarm-based algorithms have recently been developed to handle constraints in these type of problems. Although there

are few research studies on PSO to solve constrained optimization problems, none of these studies adopt the information from all particles to perform communication within PSO in order to share common interest and to act synchronously. When particles share their information through communication with each other, they will be able to efficiently handle the constraints and optimize the objective function. From a sociological point of view, study has shown that human societies will migrate from one place to another in order to handle their own life constraints and limitations as well as to reach a better economical, social, or political life [8]. People living in different societies migrate in spite of the different value systems and cultural differences. Indeed the cultural belief is an important factor affecting the issues underlying the migration phenomena [9]. On the other hand, finding the appropriate information for communication within swarm can be computationally expensive. One computational aspect is the difficulties of finding the appropriate information to communicate within PSO in order to be able to simultaneously better handle the constraints and optimize the objective function.

The optimum solution for many real-world optimization problems changes over time. In such cases known as dynamic optimization problems, the heuristics should track the change as soon as it happens and responds promptly. For example, in job scheduling problems new jobs arrive or machines may break down during operations results a need for dynamic job schedules to accommodate the changes over time [10]. In another example, dynamic portfolio problem, the goal is to obtain an optimal allocation of assets to maximize profit and minimize investment risk [11].

There are four major categories of uncertainties that have been dealt with using population based evolutionary approaches: noise in the fitness function, perturbations in the design variables, approximation in the fitness function, and dynamism in optimal solutions [12]. While noise and approximation bring uncertainty in the objective function, perturbation introduces uncertainty in the decision space. The source of change can be because of the possible change in the objective function, constraints, environmental parameters, or problem representations during optimization process. These changes may affect the height, width, or location of optimum solution or a combination of these three parts [13].

The application of PSO to dynamic optimization problems has been studied by various researchers. There are some issues with the PSO mechanism that needs to be addressed. Maintaining outdated memory is one issue in dynamic optimization problems. When a problem changes, a previously good solution stored as neighborhood or personal best may no longer be good, and will mislead the swarm towards false optima. Diversity loss is another problem in which population normally collapses around the best solution. In dynamic optimization, the partially converged population after a change is detected should quickly re-diversify, find the new optimum and re-converge [10]. A number of adaptations have been applied to PSO in order to solve these difficulties. In general, a good evolutionary heuristic to solve DOPs should reuse as much information as possible from previous iterations to increase the optimization search. Among the researches performed in dynamic PSO none of these studies exploits information from all particles

to perform re-diversification through migration and repulsion. When particles share their information through migration process, they will be able to quickly re-diversify and move efficiently towards new optimum by re-converging around it. In order to construct the environment required for this re-divergence and re-convergence, we need to establish groundwork to assist us to utilize this information. The major groundwork is the belief space of cultural algorithm assisting the particles in an organized informational manner to locate the necessary information.

Discussed in psychosocial texts, attitudinal similarity is a leading factor to attraction among individuals while dissimilarity leads to repulsion in interpersonal relationship [14], as a result people often diverge from members of other social groups by selecting different cultural attitudes or behaviors [15]. Indeed different cultural beliefs lead to repulsion and increase the possibilities of divergence in ideas and in turn open up the doors to new opportunities.

One challenge is the difficulty to find the appropriate information to use so that it can be relied on for a quick re-diversification when a change happens in the environment. Using many concepts from the cultural algorithm, such as spatial knowledge, temporal knowledge, domain knowledge, normative knowledge and situational knowledge, the information will be organized competently and successfully in order to adopt in several steps of the PSO's updating mechanism in addition to re-diversification and repulsion among swarms. The special re-diversification problem to deal with the change in dynamic is an important task that can be solved more efficiently when we have access to

the knowledge throughout the search process that is performed by the cultural algorithm as the computational framework.

The remaining structure of this dissertation is as following. In Chapter II, a comprehensive literature survey is performed on related computational intelligence paradigms to prepare for the following chapters. Chapter III firstly elaborates on a paradigm based upon the intrasociety and intersociety interaction in order to simulate an algorithm to solve single objective optimization problems. Next the proposed modifications to this social-based heuristics will be introduced. This proposal has two aspects: one is based upon the idea of adopting information from all individuals in the society (i.e., not only the best performing individuals). The second proposal is based on the fact that different societies have different collective behavior. Politically speaking, the collective behavior of the societies have been quantified into a measure called the liberty rate. In the real sociological context, individuals in a democratic society will have more flexibility and freedom to choose a better environment to live. In contrast, individuals in a dictatorship society will suppress the politically environmental change. While individuals in a liberal society can freely move to be closer to the leaders, individual in a less liberal society will have restriction to move near the leaders. Hence the higher liberty rate a society has, the more flexibility an individual in such society can move. At the end of this chapter, simulation result for a real world mechanical problem is used to test the performance of two proposed modifications.

In Chapter IV, a heuristic is proposed to diversify the search space using a novel

three-level particle swarm optimization in a multiple swarm population space. The PSO mechanism is customized to incorporate three levels of searching process. In the lowest level, particles follow the best behaving particle in their own swarm; in next level, particles follow the best performing particle in the neighboring swarms, and finally in the highest level, particles track the whole population's best behaving particle. A novel algorithm is proposed to define the neighboring swarms based upon the closeness between representatives of each pair of swarms. After a specified number of iterations, the swarms communicate with each other. Each swarm assembles two lists, a sending list and a replacement list. To prepare these two sets of particles, diversity measure is considered as the primary goal instead of the performance of the particles alone. When particles are approaching the local optima, several of them will have similar positional information. This similar redundant information will be replaced by particles from other swarms to diversify the search space. At the end of this chapter, the simulated study is tested to solve benchmark multimodal optimization problems which demonstrate efficiency of the proposed heuristic and its potential to solve difficult optimization problems.

Chapter V proposes an innovative algorithm adopting the cultural information that exists in the belief space to adjust flight parameters of multiobjective particle swarm optimization (MOPSO) such as personal acceleration, global acceleration, and momentum. A belief space has been constructed containing three sections of knowledge as the groundwork to perform MOPSO and adapt the parameters. Every particle in

MOPSO will use its own adapted momentum and acceleration (local and global) at every iteration to approach the Pareto front. Cultural algorithm provides the required groundwork enabling us to employ the information stored in different belief space efficiently and effectively. The proposed cultural MOPSO is then evaluated against the state-of-the-art MOPSO models, showing very competitive and well performing outcome. Finally a comprehensive sensitivity analysis has been performed for the cultural MOPSO with respect to its tuning parameters.

In Chapter VI, a novel heuristics is proposed based upon the information extracted from belief space to facilitate the inter-swarm communication among multiple swarms in particle swarm optimization to solve constrained optimization problems. The cultural computational framework is to find the leading particles in the personal level, swarm level, and global level. Every particle will move using a three-level flight mechanism and then particles divide into several swarms and inter-swarm communication takes place to share the information. The performance of the proposed cultural constrained particle swarm optimization (CPSO) has been compared against ten state-of-the-art constrained optimization paradigms on 24 benchmark test problems. The comprehensive simulation results demonstrate cultural CPSO to be very effective and efficient.

Chapter VII proposes an innovative computational framework according to cultural algorithm to solve dynamic optimization problems using knowledge stored in the belief space in order to re-diversify and repel the population right after a change takes

place in the dynamic of the problem. Thus the algorithm can comfortably compute the repulsion factor for each particle and locate the leading particles in the personal level, swarm level and global level. Each particle in the proposed cultural-based dynamic PSO will fly through a mechanism of three level flight incorporated with a repulsion factor. After a change takes place, particles regroup into several swarms and a diversity-based migration among swarms along with repulsive mechanism implemented in repulsion factor will take place to increase the diversity as quickly as possible.

Finally, Chapter VIII discusses the concluding remarks on how swarm, culture, and society help in solving single objective, multiobjective, constrained, and dynamic optimization problems. The suggestions of the future work of this study are also proposed in this chapter.

CHAPTER II

LITERATURE REVIEW

In this chapter, we briefly review the related work that will assist in understanding the background concepts required for this dissertation. Population based computational intelligence heuristics has extensively evolved from natural evolutionary-based Genetic Algorithms (GA) [16-17] over decades of research work. Computational intelligence approaches based upon the psychosocial behavior inspired from either human or animal society have been the subject of the emerging research for a decade. Some concepts borrowed from sociology have shown great improvements in the performance of computational methods. Migration of individuals between concurrent evolving populations has shown its potential to improve the genetic algorithms mechanism [18]. In distributed GA [6] the sociologically inspired concept of communication shows great improvement in the performance of GA. The population is divided into several subpopulations each evolving an independently GA while at regular time intervals, these GAs communicate with each other.

Sociological researchers have constructed models to mimic the behavior of human and animal societies. Heppner and Grenander studied synchronization in groups of small birds like pigeons developing a flocking heuristics based upon the social interactions such

as attraction to a roost, attraction to flockmates and preserving the velocity [19]. Deneubourg and Goss has shown that the interaction between the individuals and their environment produces different collective patterns on decision making process by introducing a mathematical model [20] which is naturally observed to be essential in the schools of fishes, flocks of birds, groups of mammals, and many other social aggregates.

Millonas proposed a model of the collective behavior of a large number of locally acting organisms [21] in which organisms move probabilistically between local cells in space, but with different weights. The evolution and the flow of the organisms construct the collective behavior of the group. This model could successfully analyze movements of ants as swarming organism. Reynolds developed a computer animator of a simulated bird based upon the local perception of the dynamic environment, the laws of simulated physics ruling its motion, and a set of simulated behaviors [22].

Akhtar *et al.* proposes a socio-behavioral simulated model [23] based upon the concept that the behavior of an individual changes and improves due to social interaction with the society leaders who are identified using a Pareto rank scheme. On the other hand, the leaders of all societies themselves improve their own behavior which leads to a better civilization. Ursem introduced multinational evolutionary algorithm based on the relationship between different nations and their political interaction in order to optimize a profit function [4]. Ray and Liew adopted the intersociety and intrasociety relationship among the individuals and the leaders to optimize the single objective optimization problem [5]. The whole population, clustered into several groups, evolves in two stages.

Individuals within group follow the group's best performing individual, and in the whole population, the very best performing individual leads all groups' leaders. Ursem elaborates the idea of sharing among agents in a social entity as a means of maintaining multiple peaks in multimodal optimization problems [24].

Deneubourg and coauthors proposed a probabilistic model to explain behavior of ants as social agents [25] which was then followed by Goss *et al.* showing how sharing information among ants which was done by laying trail and following it could help to solve foraging problem in their societies [26]. Inspired by their research, Dorigo *et al.* introduced a new computational paradigm, Ant Colony Optimization (ACO) model, that could be adopted to solve engineering optimization problems. ACO's main characteristic was a positive feedback for rapid discovery of good solution of optimization problem, a distributed computation to avoid premature convergence, and a greedy heuristic to find acceptable solution in the early stages of the search process [2, 27]. The ACO model has been successfully applied to symmetric and asymmetric Travelling Salesman Problem (TSP) as a classical difficult combinatorial optimization problem [28-29], quadratic assignment problem [30], adaptive routing [31], job-scheduling problem [2]. Sahin *et al.* reported applying the ant-based swarm algorithm on forming different patterns through interaction among mobile robots [32].

Kennedy and Eberhart introduced the particle swarm optimization (PSO), an algorithm based on imitating behavior of flocking birds. It mimics grouping of birds as particles, their random movement, and regrouping them again to generate a model so that

it can solve engineering optimization problems [1, 33]. Particles are known with their positions and velocities and can be updated using:

$$v = wv + c_p r_1(pbest - x) + c_g r_2(gbest - x), \quad (2.1)$$

$$x = x + v,$$

where v is the velocity of the particle, x is the position of the particle, $pbest$ is the best position of each particle ever experienced, and $gbest$ is the best position among all particles. $r_1$and r_2 are random numbers uniformly generated in the range of $(0,1)$. c_p, c_g, and w are personal, social, and momentum coefficients [34] that are predefined constant values. The movement of the particles has been analyzed to understand the mechanism underlying the PSO and its relation to other population based heuristics [35]. The analysis of the particles' trajectory while moving [36] has led to a generalized model of the algorithm, containing a set of coefficients to control the system's convergence tendencies. The effects of various population structure and topologies on the performance of particle swarm algorithm have shown that von Neumann configuration consistently outperforms other types of topological configurations of particles' neighborhood [37-39].

Several versions of PSO have been developed. Discrete PSO was introduced [40] operating on discrete binary variables whose trajectories are defined as changes in the probability that a coordinate will take on a zero or one value. Comparing with GA on some multimodal optimization problems, discrete PSO showed competitive results [41-

42]. A modified PSO using constriction factor [43] performed well comparing with the original PSO. Particle swarms are also developed to track and optimize dynamic landscape systems [44]. Particle swarm optimization has also been modified to perform permutation optimization problems such as N-queens problem [45] by defining particles as permutations of a group of unique values and updating velocity based upon the similarity of two particles. The permutation of the particles change with a random rate defined by their velocities.

Clustering population into several swarms has been extensively studied. Stereotyping of the particles is investigated [46] in which substitution of cluster centers for $pbest$ shows better performance of the PSO suggesting that PSO is more effective when individuals are attracted toward the center of their own clusters. Al-Kazemi and Mohan divided the population into two sets at any given time, one set moving to the $gbest$ while another moving in opposite direction by selecting appropriate fixed values for (c_p, c_g, w) in each set [47]. After some iterations, if the $gbest$ would not improve, then the particles would switch their group. Baskar and Suganthan introduced a concurrent PSO consisting two swarms in order to search concurrently for a solution along with frequent passing of information, the $gbest$ of two swarms [48]. After each exchange, the two swarms had to track the better $gbest$ found. One of the swarms was using regular PSO while the other was using the Fitness-to-Distance ratio PSO [49]. Their approach improved the performance over both methods in solving single objective optimization problems. El-Abd and Kamel added a two-way flow of information between

two swarms improving its performance [50]. In their algorithm, when exchanging the best particle between two swarms, this particle is used to replace the worst particle in another swarm. The two swarms perform a fixed number of iterations, and then the best p particles inside each swarm will replace the worst p particles in the other swarm only if they have a better fitness. This makes it possible for both swarms to exchange new information from the other swarm's experience. Krohling *et al.* proposed co-evolutionary PSO in which two populations of PSO are involved [51]. One PSO runs for a specified number of iterations while the other remains static and serves as its environment. At the end of such period, $pbest$ values obtained in previous cycles have to be re-evaluated according to the new environment before starting evolution.

Particle swarm optimization has been widely applied for multiobjective optimization problems (MOPs) called multiobjective particle swarm optimization (MOPSO) to find a diverse set of potential solutions, known as Pareto front. There have been several algorithms to extend PSO to handle diversity issue in MOPs. Parspopoulos *et al.* [52] introduced vector evaluated particle swarm optimizer (VEPSO) to solve multiobjective problems. A VEPSO is a multi-swarm variant of PSO in which each swarm is evaluated using only one of the objective functions of the problem under consideration, and the information it possesses for this objective function is communicated to the other swarms through the exchange of their best experience. In VEPSO, the velocity of the particles in each swarm is updated using the best previous position, $gbest$, of another selected swarm. Selection of this swarm in the migration

scheme can be either random or in a sequential order. Ray and Liew [53] used Pareto dominance and combined concepts of evolutionary techniques with the particle swarm. This algorithm uses crowding distance to preserve diversity. Hu and Eberhart [54] in their dynamic neighborhood PSO proposed an algorithm to optimize only one objective at a time. The algorithm may be sensitive to the optimizing order of objective functions. Fieldsend and Singh [55] proposed an approach in which they used an unconstrained elite archive to store the nondominated individuals found along the search process. The archive interacts with the primary population in order to define local guides. Mostaghim and Teich [56] introduced a sigma method in which the best local guides for each particle are adopted to improve the convergence and diversity of the PSO. Li [57] adopted the main idea from NSGA-II into the PSO algorithm. Coello Coello *et al.* [58], on the other hand proposed an algorithm using a repository for the nondominated particles along with adaptive grid to select the global best of PSO. The algorithms proposed to solve MOPs using PSO are based upon promoting the nondominated particles at any given time, not exploiting the information of all particles in the population.

Many MOPSO paradigms are focused on the methods of selecting global best [53, 55-56, 58-64], or personal best [65]. Most MOPSOs adopt constant value for momentum and accelerations; however some MOPSOs use some simple dynamic to change the parameters. Indeed, one of the difficulties of the PSO and/or MOPSO is to deal with tuning the right value for the momentum, personal and global acceleration in order to get the best results for different test functions. Hu and Eberhart [54] in their dynamic

neighborhood MOPSO model and also Hu *et al.* [66] in the MOPSO with extended memory adopted a random number on the range (0.5,1) as the varying momentum. However both personal and global acceleration are constant values. Sierra and Coello Coello [62] in their crowding and ε-dominance based MOPSO used random value at the range (0.1,0.5) for the momentum and random values at the range (1.5,2.0) for the personal and global acceleration. They adopted this scheme to bypass the difficulties of fine tuning of these parameters for each test function.

Zhang *et al.* [64] introduced intelligent MOPSO based upon Agent-Environment-Rules model of artificial life. In their model, along with adopting some immunity clonal operator, the momentum was decreased linearly from 0.6 to 0.2, but the personal and global acceleration remained constant. Li [67] proposed an MOPSO based upon max-min fitness function. In his model, while the personal and global acceleration were set constant, the momentum was gradually decreased from 1.0 to 0.4. Zhang *et al.* [68] adopted a linearly-decreasing momentum from 0.8 to 0.4 for their MOPSO algorithm. However the personal and global acceleration were kept fixed. Mahfouf *et al.* [69] introduced adaptive weighted MOPSO in which they included adaptive momentum and acceleration. Using comparison study with other well-behaved algorithms, they demonstrated that the MOPSO search capability is enhanced by adding this adaptation. Ho *et al.* [63] noted the possible problem of selecting personal and global acceleration independently and randomly. He mentioned because of its stochastic nature they may both be too large or too small. In the former case, both personal and global experiences

are overused and as a result the particle will be driven too far away from the optimum. For the latter case, both personal and global experiences are not fully used and as a result the convergence speed of the algorithm is reduced. They used sociobiological activity such as hunting to assure that individuals balance between the weight of their own knowledge and the group's collective knowledge. In other words, they mentioned that the personal and global acceleration are somehow related to each other. When one acceleration is large, the other one should be small, and vice versa. Using this concept, they modified the main equation of PSO, Equation (2.1) to include a dependent acceleration and momentum [63].

Particle swarm optimization algorithms have been successfully developed to solve constrained optimization problems. Hu and Eberhart generated particles in PSO until the algorithm could find at least one particle in the feasible region and then adopted it to find best personal and global particles [70]. Parsopoulos and Vrahatis used a dynamic multi-stage penalty function to handle the constraints [71]. The penalty function consisted of weighted sum of all constraints violation with each constraint having a dynamic exponent and a multi-stage dynamic coefficient. A comparison of preserving feasible solution method [70] and dynamic penalty function [71] demonstrated that the convergence rate for dynamic penalty function algorithm was faster than that of feasible solution method [72].

Hu *et al.* modified the PSO mechanism to solve constrained optimization problems. PSO starts with a group of feasible solutions and a feasibility function is used

to check if the newly explored solutions satisfy all the constraints. Only feasible solutions are kept in the memory [73]. Linearly constrained optimization problems are the basis for a modified version of PSO in which the movement of the particles in the vector space is mathematically guaranteed by the velocity and position update mechanism to always find at least a local optimum [74]. In the ε constrained PSO, particles that satisfy constraints move to optimize the objective function while particles that violate constraints move in order to satisfy the constraints [75].

Krohling and Coelho adopted Gaussian distribution instead of uniform distribution for the personal and global term random weights of the PSO mechanism to solve constrained optimization problems formulated as min-max problems. They used two populations of the PSO simultaneously, first PSO focuses on evolving the variable vector while the vector of Lagrangian multiplier is kept frozen, and the second PSO is to concentrate on evolving the Lagrangian multiplier while the first population is maintained frozen. The use of normal distribution for the stochastic parameters of the PSO seems to provide a good compromise between the probability of having a large number of small amplitude around the current points, i.e., fine-tuning, and small probability of having large amplitudes, that may cause the particles to move away from the current points and escape from the local optima [76].

In master-slave PSO [77], master swarm is to optimize objective function while slave swarm is focused on constraint feasibility. Particles in the master swarm only fly toward the current better particles in the feasible region, and they will not fly toward

current better particles in the infeasible region. The slave swarm is responsible for searching feasible particles by flying through the infeasible region. Particles in slave swarm only fly toward current better particles in the infeasible region, and they will not fly toward current better particles in the feasible region. The feasible/infeasible leaders from swarm will then be communicated to lead the other swarm. By exchanging flight information between swarms, algorithm can explore a wider solution space.

Zheng *et al.* adopted an approach that congregates neighboring particles in the PSO to form multiple swarms in order to explore isolated, long and narrow feasible space [78]. They also applied a dynamic mutation operator with dynamic mutation rate to enhance flight of particles to feasible region more frequently. For constraint handling a penalty function has been adopted as to how far the infeasible particle is located from the feasible region. Saber *et al.* [79] introduced a version of PSO for constrained optimization problems. In their version of PSO, the velocity update mechanism uses a sufficient number of promising vectors to reduce randomness for better convergence. The coefficient velocity in the positional update equation is a dynamic rate depending on the error and iteration. They also reinitialized the idle particles if there are particles that are not improving for some iterations. Li *et al.* [80] proposed dual PSO with stochastic ranking to handle the constraints. One regular PSO evolves simultaneously along with a genetic PSO, a discrete version of PSO including a reproduction operator. The better of the two positions generated by these two PSOs is then selected as the updated position. Flores-Mendoza and Mezura-Montes [81] used Pareto dominance concept for constraint

handling technique on a bi-objective space, with one objective being sum of the inequality violation constraints and the second objective being sum of the equality violation constraints in order to promote better approach to feasible region. They also adopted a decaying parameter control for constriction factor and global acceleration of the PSO to prevent the premature convergence and to advance the exploration of the search space. Ting *et al.* [82] introduced a hybrid heuristic consisting PSO and genetic algorithm to tackle constraint optimization problem of load flow algorithm. They adopted two-point crossover, mutation, and roulette-wheel selection from genetic algorithms along with the regular PSO to generate the new population space. Liu *et al.* [83] incorporated discrete genetic PSO with differential evolution (DE) to enhance the search process in which both genetic PSO and DE update the position of the individual at every generation. The better position will then be selected.

Particle swarm optimization algorithms have been effectively developed to solve dynamic optimization problems (DOP) as well. Carlisle and Dozier [84] adjusted PSO mechanism to prevent making position/velocity decision according to the outdated memory by periodic resetting. Particles periodically replace their *pbest* vector with their current position, forgetting their past experiences. Eberhart and Shi [44] proposed that for small perturbation, the initialization of the swarm can start from old population, while large perturbation needs re-initialization. In detection and response paradigm [85] *gbest* and the second global best are evaluated to detect changes, then the positions of all particles are re-randomized to respond to the change. Charged swarm avoids collision

among particles based upon the force between electric charges which is inversely proportional to distance squared [86]. Atomic PSO [87] and quantum PSO [88] follow the structure of the chemical atom including a cloud of electrons randomly orbiting with a specific radius around the nucleolus.

An anti-convergence operator [89] assists interaction among swarms. Also an excluding operation defines a radius to include the best solution of the swarm. These close swarms compete with each other in order to promote diversity. The winner, the swarm with the best function value at its swarm attractor, will remain, while the loser will be re-initialized in the search space [89]. Swarms birth and death [90] was proposed by allowing multiple swarms to regulate their size by bringing new swarms to existence, or diminishing redundant swarms. This dynamic swarm size can be an alternative for anti-convergence and exclusion operators in the PSO mechanism.

In partitioned hierarchical PSO for dynamic optimization problems [91], the population is partitioned into some tree-form sub-hierarchies for a limited number of iterations after a change is detected. These sub-hierarchies continue to independently search for the optimum, resulting a wider spread-out of the search process after the change has occurred. The topmost level of tree-form hierarchies which contain the current best particle does not change, but all lower sub-hierarchies (sub-swarms) re-initialize the position and velocity and reset their personal best positions. These sub-hierarchies are rejoined again after a predefined number of iterations.

By adopting dynamic macro-mutation operator [92], PSO is able to maintain the diversity throughout the search process in order to solve DOPs. Every coordinate of each particle will undergo an independent mutation with a dynamic probability which possess its highest value when the change occurs in the dynamic landscape and gradually decreases till the next change takes place. The unified PSO in which the exploration and exploitation term of the PSO mechanism are unified into a unification factor has also been adopted for solving DOPs [93]. Zhang *et al*. [94] proposed a direct relation between the inertia weight of the particle and the change. In their model, the new *gbest* and *pbest* for each particle affect the inertia weight of the particle whenever a change in *gbest* or *pbest* occurs. Pan *et al*. [95] modified the PSO paradigm using a probability based movement of particles based upon the concept of energy change probability in Simulated Annealing (SA). The particle will move to the next position computed through traditional PSO heuristics only with a specific probability that exponentially depends on the difference between the objective values of the current and next iterations.

In species based PSO [96], the population is divided into some swarms, each surrounding a dominating particle called seed identified from the objective function values of the entire population. The new seed should not fall within the predefined radius of all previously found seeds in order to promote diversity. The seeds are then selected as the neighborhood best for different swarms. In multi-strategy ensemble PSO [97], particles are divided into two sections, part I uses a Gaussian local search to quickly seek global optimum in the current environment, while part II uses differential mutation to

explore the search space. The position of particles in part II do not follow the traditional PSO mechanism, instead each particle in part II is determined by the particle in part I through a mutation strategy.

Liu *et al*. [98] introduced a modified PSO to solve DOPs in which many compound particles exist. Each compound particle includes three single particles equilaterally distanced from each other in a triangular shape. A special reflection scheme is proposed to explore the search space more comprehensively in which the position of the worst particle among three in the compound will be replaced with the reflected one. In each compound particle, after reflection is performed, a representative among these three particles is probabilistically chosen based upon the objective function values and distance from other two member particles. The representative member particles will then participate in PSO update mechanism. The two non-representative particles will also move in the same distance/direction as representative particle has been moved in order to preserve the valuable information.

Recently a computational framework has been developed by Reynolds known as cultural algorithm (CA) based upon a dual inheritance system where information exists at two different levels: population level and the belief level [3]. Culture is defined as storage of information which does not depend on the individuals who generated and can be potentially accessed by all society members [3]. CA is an adaptive evolutionary computation method which is derived by cultural evolution and learning in agent-based societies [3, 99]. CA consists of evolving agents whose experiences are gathered into a

belief space consisting of various forms of symbolic knowledge. CA has shown its ability to solve different types of problems [3, 99-107] among which CAEP (cultural algorithm along with evolutionary programming) has shown successful results in solving MOPs [107]. Researchers have identified five basic sections of knowledge stored in belief space based upon the literature in cognitive science and semiotics: situational knowledge, normative knowledge, topographical knowledge [105], domain knowledge, and history knowledge [106]. *Situational knowledge* is a set of exemplary individuals useful for experiences of all individuals. Situational knowledge guides all individuals to move toward the exemplar individuals. *Normative knowledge* consists a set of promising ranges. Normative knowledge provides standard guiding principle within which individual adjustments can be made. Individuals jump into the good range using normative knowledge. *Topographical* or *spatial knowledge* keeps track of the best individuals which have been found so far in the promising region. Topographical knowledge leads all individuals toward the best performing cells in the search space [105]. *Domain knowledge* adopts information about the problem domain to lead the search. Domain knowledge about landscape contour and its related parameters guides the search process. *Historical* or *temporal knowledge* keeps track of the history of the search process and records key events in the search. It might be either a considerable move in the search space or a discovery of landscape change. Individuals use the history knowledge for guidance in selecting a move direction. Domain knowledge and history knowledge are useful on dynamic landscape problems [106]. The knowledge can swarm

between different sections of belief space [108-110] which in turn affect the swarming of population.

Becerra and Coello Coello [104] proposed cultured differential evolution for constrained optimization. The population space in their study was differential evolution (DE) while the belief space consist of situational, topographical, normative, and history knowledge. The variation operator in DE was influenced by the knowledge source of belief space. Yuan *et al.* [111] introduced chaotic hybrid cultural algorithm for constrained optimization in which population space as DE and belief space including normative and situational knowledge. They incorporated a logistic map function for better convergence of DE to use its chaotic sequence. Tang and Li [112] proposed a cultured genetic algorithm for constrained optimization problems by introducing a triple space cultural algorithm. The triple space includes belief space, population space in addition with anti-culture population consisting individuals disobeying the guidance of the belief space, and going away from the belief space guided individuals. The effect of disobeying behavior enhanced by some mutation operations makes the algorithm faster and less risky for premature convergence, by awarding the most successful individuals and punishing the unsuccessful population.

CHAPTER III

SOCIETTY AND CIVILIZAION FOR OPTIMIZATION

3.1 Introduction

Computational intelligence approaches based upon the psychosocial behavior inspired from either the human or animal society have been the subject of the emerging research for less than a decade. There has been some research in this area focused on optimization in the spirit of the particle swarm intelligence [1] or ant colony system [2]. Particle swarm optimization is an imitation of the collaborative behavior of the birds flying together with the means of information exchange, while ant colony is based on the fact that individual ants interact with each other through their pheromone trails. Additionally, Ursem [4] introduced another ideas based on the relationship between different nations and how to interact between the countries in order to optimize a profit function. More recently, in an attempt to mimic the interactional behavior between societies and within civilization, social algorithm had been proposed [5, 113]. Social algorithm adopts the intersociety and intrasociety relationship among the individuals and the leaders to optimize the single objective optimization problem. The whole population of individuals, called the civilization, is clustered into different societies based on the

Euclidean closeness of the individuals. The performance of individuals will be a measure to decide which individuals are the leaders of the society. The rest of the individuals are to follow them in a way to improve themselves which leads to migration (*intrasocitey interaction*). From the civilization viewpoint, the leaders of the societies will improve themselves by migrating to the best-performing leaders who are the civilization leaders (*intersociety interaction*) [114-115].

Ray and Liew have successfully demonstrated the performance of their model in single objective optimization problems [5]. Their model seems to be an alternative competitive paradigm to particle swarm heuristics. What was used in their model is mostly by throwing the information of the non-leader individuals away and replacing with those of the corresponding leaders. What is proposed in this chapter involves two aspects. Firstly, using the information of the individual, individual's talent is computed which equips each individual with different ability to invoke intra or intersociety interaction. Secondly, different society might have different collective behavior measure, called the liberty rate. In the real sociological relationship, a democratic society will have more flexibility and freedom to choose a better environment to live. In contrast, a dictatorship society will discourage individual to change the environment in reaching the leaders. While individuals in a liberal society can migrate easily to be closer to the leaders, individual in a less liberal society will have difficulty to move near the leaders. Hence the higher liberty rate a society has, the more flexibility an individual in such society can move.

The chapter is followed by Section 3.2 elaborating basics of social algorithms, including its motivation and how to build the societies in a civilization, how to identify the leaders of such societies, and how to migrate intra or inter-socially. It also proposes a novel modification which is based on the idea of using more information from the middle-class individuals. In Section 3.3 the proposed algorithm has been applied on single objective optimization problems to test its efficiency. In Section 3.4, the concluding remarks are discussed in applying social algorithm to solve optimization problems.

3.2 Social-based Algorithm for Optimization

In this section, the details of social-based algorithm are reviewed to solve single objective optimization problems and then the proposed methods on improving this heuristics are introduced. The general single objective function optimization problem is as the following form:

$$\text{Minimize: } f(x_1, x_2, \dots, x_M), \tag{3.1}$$

$$\text{Subject to: } g_k(x_1, x_2, \dots, x_M) \leq 0, \qquad k = 1,2, \dots, L, \tag{3.2}$$

$$h_k(x_1, x_2, \dots, x_M) = 0, \qquad k = L+1, \dots, m, \tag{3.3}$$

where L is the number of inequality constraints and m is the total number of inequality and equality constraints, respectively, $\mathbf{x} = (x_1, x_2, \dots, x_M)$ is the M-dimensional decision space variable. Because of limitation in computer simulations and accuracy of the variables considered, it is much easier to check the validity of an inequality than that of equality. As has been suggested by research in population based heuristics dealing with constraint handling, each equality constraint of $h_k = 0$ is originally transformed into a set of two simultaneous inequalities as $h_k \leq \delta$ and $h_k \geq -\delta$ where δ is an infinitesimal positive constant representing the accuracy of the algorithm. For example with $\delta = 0.0001$, the algorithm should proceed in a way that the following condition satisfies: $-0.0001 \leq h_k \leq 0.0001$ which will substitute $h_k = 0$ for the sake of accuracy. Therefore each equality constraint transforms to two inequalities constraints resulting total number of inequality constraints as $S = L + 2(m - L)$ as following:

$$g_k(x_1, x_2, \dots, x_M) = h_k(x_1, x_2, \dots, x_M) - \delta \leq 0, \qquad k = L + 1, \dots, m, \qquad (3.4)$$

$$g_{m-L+k}(x_1, x_2, \dots, x_M) = -\delta - h_k(x_1, x_2, \dots, x_M) \leq 0, \quad k = L + 1, \dots, m. \quad (3.5)$$

Now assume there are N individuals in the population as potential solutions for the constrained optimization problem. A constraint satisfaction factor, c_{kj}, is defined to quantify how much dissatisfied the k-th constraint ($k = 1,2,, \dots, S$) is made using the j-th individual, $\mathbf{x}_j$, ($j = 1,2, \dots, N$), and formulated as following:

$$c_{kj} = \begin{cases} 0, & k\text{th consraint is satisfied} \\ -g_k(\mathbf{x}_j), & k\text{th consraint is violated,} \end{cases} \quad k = 1,2,,\dots,S, \quad j = 1,2,\dots,N. \quad (3.6)$$

Based on this definition, when a constraint is satisfied by an individual, the assigned value for constraint violation factor, c_{kj}, is zero. If the k-th constraint is not met ($k = 1,2,\dots,S$) by the j-th individual, the negative-valued $-g_i(\mathbf{x}_j)$ is assigned as constraint violation factor, c_{kj}, to show how much the constraint is violated. Then a ranking scheme is performed for each constraint as to assign the rank of one to individuals who satisfy that constraint the most. Therefore, for the k-th constraint, individuals with the highest c_{kj} ($j = 1,2,\dots,N$) considering their sign will be assigned a rank of one, and individuals with the second highest c_{kj} ($j = 1,2,\dots,N$) again considering the sign will be assigned a rank of two, and so forth. After performing this nondominated ranking scheme for all constraints, a $S \times N$ matrix is formed as the rank matrix in which rank-one means that those individuals are nondominated for a specific constraint [5]. It can be seen that if there is one or more feasible individual for a specific constraint, those will be considered as rank-one individuals.

Figure 3.1 demonstrates the main flowchart of the social-based algorithm. The civilization is formed with N individuals that are initialized as uniform random numbers. Then each individual in the population of the civilization is evaluated using objective function value. The individuals are categorized into $P(t)$ societies using a non-supervised classification algorithm proposed by Ray and Liew [5] according to their closeness to each other. Notice that the number of societies may vary by time. Then the leaders of

each society are identified using a leader identification scheme which will be discussed in Figure 3.2. Next the individuals within the societies will move towards the nearest leader in their society using a migration scheme that will be discussed in Figure 3.3.

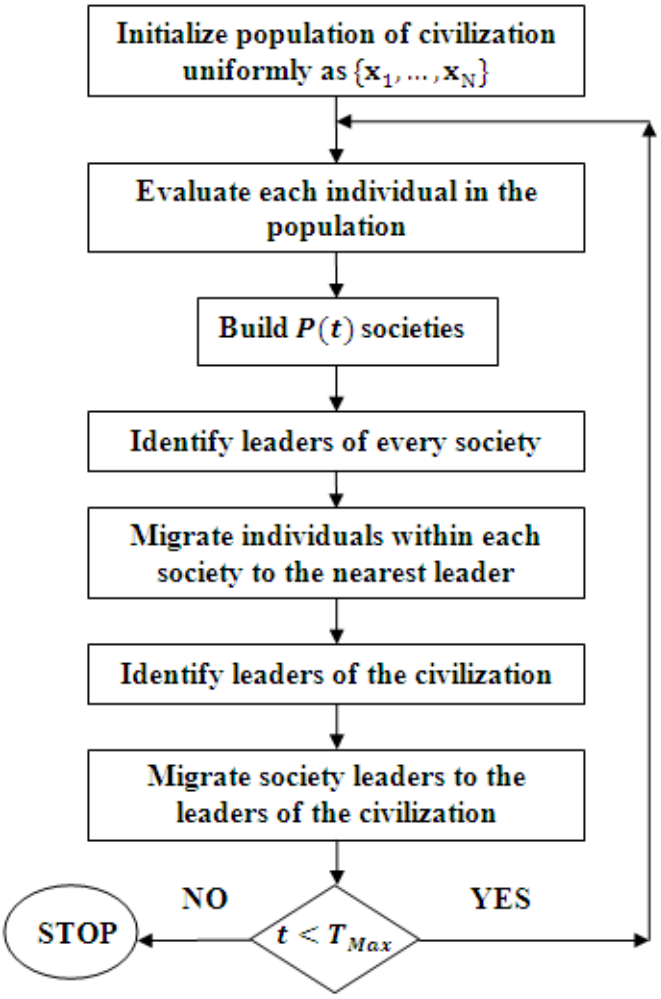


Figure 3.1 Flowchart for social-based single objective optimization

In the global level, the leaders of the civilization will then be identified through the same leader-identification scheme. Then the leaders of the societies will move

towards the global leaders of the civilization using the same migration scheme. This process continues until the termination criteria are met, i.e., the current iteration reaches a predefined maximum iteration, T_{Max}. In Figure 3.2, a flowchart is depicted to explain the leader identification scheme.

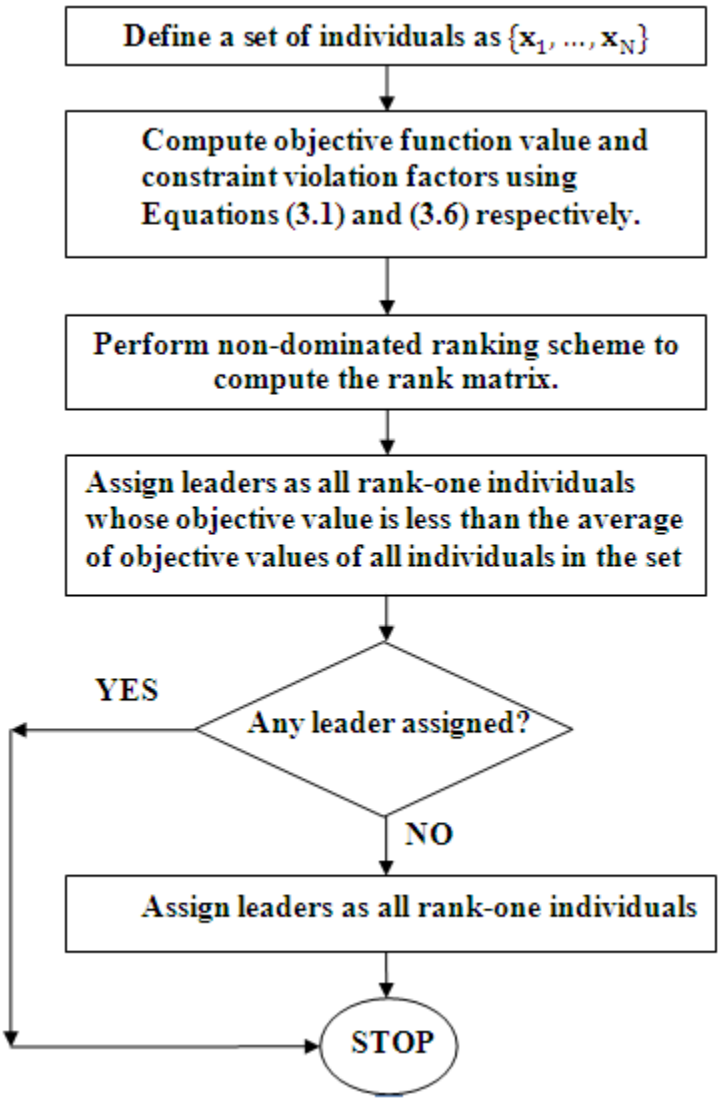


Figure 3.2 Flowchart for identifying leaders

As shown in this flowchart, a set of individuals are given to find their leaders. The leaders should be the best behaving individuals considering both objective values and constraints. The objective function value for each individual and constraint violation factors for each individual are computed using Equations (3.1) and (3.6), respectively. Through nondominated ranking scheme, the $S \times N$ rank matrix will be constructed. Leaders are identified among rank-one individuals whose objective function value is less than the average of objective function values of all individuals in the given set of individuals. This means that if there are any feasible individuals, the best ones shall be selected due to their objective function values. There might be a situation that there is no rank-one individual whose objective value is less than the average of all. In such case, simply all rank-one individuals will be assigned as leaders. The leader identification scheme is used for both society and civilization level.

Figure 3.3 shows the details of the migration scheme used in both intrasociety and intersociety level. Assume that an M-dimensional individual is given $\mathbf{x} = (x_1, x_2, \dots, x_M)$ along with a set of leaders, $L = \{\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \dots\}$. Before applying the migration scheme, it has to be noted that each dimension of the individual must be normalized as following:

$$\hat{x}_i = \frac{x_i - x_{i,\min}}{x_{i,\max} - x_{i,\min}}, \qquad i = 1,2,\dots,M, \tag{3.7}$$

where x_i is the i-th dimesnion of the M-dimensional decision variable (individual) which has the lowest limit of $x_{i,\min}$ and highest limit of $x_{i,\max}$, respectively. The normalized invidual, $\hat{x}_i$, will be in the range of (0,1). Next, Euclidian distance between the normalized individual, $\hat{\mathbf{x}} = (\hat{x}_1, \hat{x}_2, \dots, \hat{x}_M)$, and the k-th member of the leader set, $\boldsymbol{\rho}_k = (\rho_{k,1}, \rho_{k,2}, \dots, \rho_{k,M}) \in L$, will be computed as:

$$D_k = \sum_{i=1}^{M} (\hat{x}_i - \rho_{k,i})^2. \tag{3.8}$$

Next, the closest leader to the individual will be selected as $\tilde{\boldsymbol{\rho}} = (\tilde{\rho}_1, \tilde{\rho}_2, \dots, \tilde{\rho}_M)$ whose distance is:

$$\tilde{D} = \min_k D_k. \tag{3.9}$$

Then, each dimension of the normalized individual will be migrated using the above computed lowest distance through a random normal distribution value as following:

$$\hat{x}_i^{migrated} = \tilde{\rho}_i + N(0,\sigma)\tilde{D}, \qquad i = 1,2,\dots,M, \tag{3.10}$$

where $N(0,\sigma)$ is a random number with normal distribution with mean zero and a fixed standard deviation, σ , and $\hat{x}_i^{migrated}$ is the new location of the i-th dimension of the

individual. As the final step, the migrated position should be rescaled back to the original scale as following:

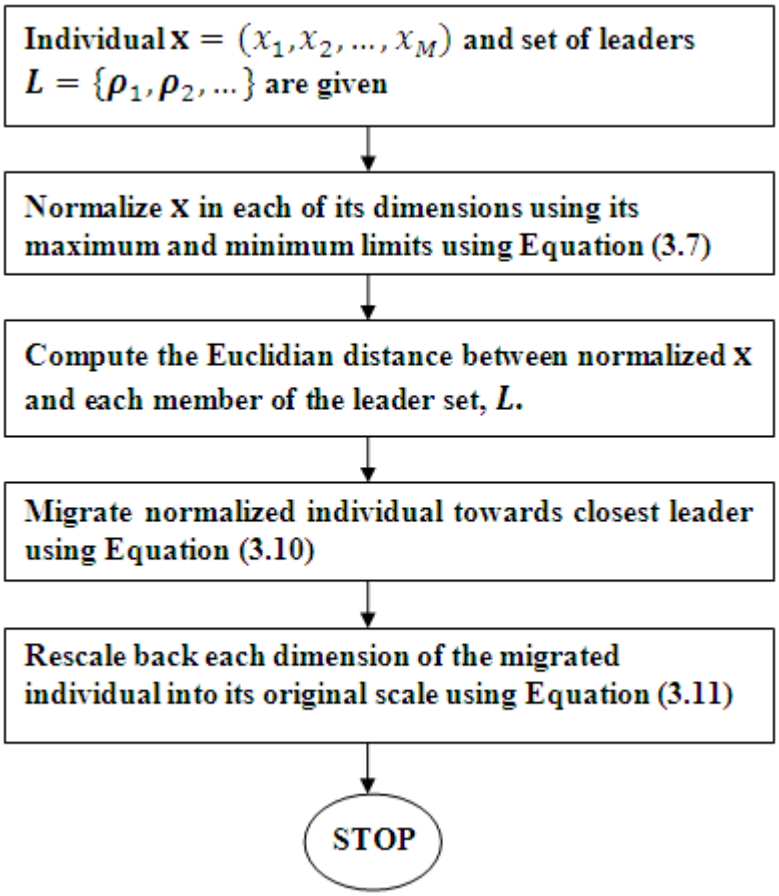


Figure 3.3 Flowchart on how to migrate individuals

$$x_i^{migrated} = \hat{x}_i^{migrated}\left(x_{i,\max} - x_{i,\min}\right) + x_{i,\min}, \qquad i = 1,2,\ldots,M. \qquad (3.11)$$

It should be noted that migration scheme explained here is adopted in both intrasociety level, migrating the individuals towards their society leaders, and intersociety

level, migrating society leaders towards the civilization leaders. Therefore performance of individuals will improve within each society by migrating towards the closest society leader, and in a global view, the performance of society leaders will also improve by migrating towards the best behaving leader in the whole civilization.

3.2.1 Proposed Modifications

In this subsection, two proposed modifications are presented. Social-based algorithm has shown its promise in some single objective optimization problems [5]. What is used in this model is mostly by throwing the information of the non-leader individuals away and replacing with that of the correspondent leaders, as shown in Equation (3.10). However, in the real life it occurs differently. Individuals keep their characteristics along with imitating from some good samples. In the real society, average individuals do not completely throw their past behavior away, but would continuously change it, keeping the history of their behavior. Having the history of the individuals in the local search (intrasociety interaction) helps the individuals keep the information that might be useful later. In the global search, the algorithm is leader-centric preventing to diverse chaotically. Therefore, the exploitation of the intrasociety migration is based on the importance of previous location of the individual. In the intersociety migration the rule remains leader-centric.

Figure 3.4 shows the pseudocode for the individuality importance in intrasociety migration. Individual $\mathbf{x} = (x_1, x_2, \dots, x_M)$ and set of leaders $L = \{\rho_1, \rho_2, \dots\}$ are given.

The individual is normalized using Equation (3.7) and then the Euclidian distance between normalized individual and each member of the leader set is computed using Equation (3.8) and the lowest distance is computed as Equation (3.9). Then the normalized individual will be migrated considering individuality importance as following:

$$\hat{x}_i^{migrated} = \hat{x}_i + N(0,\sigma)\widetilde{D}, \qquad i = 1,2,\dots,M. \tag{3.12}$$

- Individual $\mathbf{x} = (x_1, x_2, \dots, x_M)$ and set of leaders $L = \{\rho_1, \rho_2, \dots\}$ are given
- Normalize x in each of its dimensions using its maximum and minimum limits using Equation (3.7)
- Compute the Euclidian distance between normalized x and each member of the leader set, L
- Migrate normalized individual considering individuality importance using Equation (3.12)
- Rescale back each dimension of the migrated individual into its original scale using Equation (3.11)

Figure 3.4 Pseudocode for individuality importance in intrasociety migration

Finally each dimension of migrated individual should be rescale back into its original scale using Equation (3.11)

In another modification scheme, Liberty Rate is proposed. A democratic society has more flexibility and freedom to choose better situation to live. In contrast, a dictatorship society restricts change of the situation and reaching the leaders. While

individuals in a liberal society can migrate easily to be closer to the leaders, individual in a less liberal society will experience difficulty to move near the leaders. So giving preferences to approach to the leaders for different societies will improve the convergence to the optimized solution. Different society will have different collective behavior measure, called Liberty Rate. The higher liberty rate a society has, the more flexible individuals in such society can move.

The Liberty Rate of a society is proposed as the relative ratio of average objective functions of the society over the average objective functions of the civilization, formulated as following:

$$\mathbf{LibertyRate}_i = \alpha \frac{\mathcal{L}_i}{\boldsymbol{\mathcal{L}}}, \tag{3.13}$$

where α is a predefined normalization constant and $\mathcal{L}_i$ is the measure of the collective behavior of the i-th society, defined as the average of the objective values of the individuals who belong to the i-th society, formulated as:

$$\mathcal{L}_i = \frac{1}{N_i}\sum_{i=1}^{N_i} f(\mathbf{x}_i), \tag{3.14}$$

where N_i is number of individuals in the i-th society. $\boldsymbol{\mathcal{L}}$, the measure for the civilization's collective behavior is also defined as:

$$\boldsymbol{\mathcal{L}} = \frac{1}{N}\sum_{i=1}^{N} f(\mathbf{x}_i). \tag{3.15}$$

Then to migrate each individual in the i-th society, a liberty-based migration is performed as following:

$$\hat{x}_i^{migrated} = \hat{x}_i + N(0, \mathbf{LibertyRate_i})\widetilde{D}, \qquad i = 1,2,\dots,M. \tag{3.16}$$

3.3 Simulation Results

Spring Design is a mechanical design problem [116] to minimize the weight of a tension/compression spring as shown in Figure 3.5. There are nonlinear inequality constraints on minimum deflection, shear stress, surge frequency, limits on outside diameter and on design variables. The design variables are the mean coil diameter, x_1, the wire diameter, x_2, and the number of active coils, x_3, along with four inequality constraints. The mathematical formulation of the problem is as the following:

$$\text{Minimize: } f(\mathbf{x}) = (x_3 + 2)x_1x_2^2, \tag{3.17}$$

Subject to: (3.18)

$$1 - \frac{x_1^3 x_3}{71785\, x_2^4} \leq 0,$$

$$\frac{4x_1^2 - x_1x_2}{12566(x_1x_2^3 - x_1^3x_3)} - \frac{1}{5108\, x_2^2} - 1 \leq 0,$$

$$1 - \frac{140.45x_2}{x_1^2\, x_3} \leq 0,$$

$$\frac{x_1 + x_2}{1.5} - 1 \leq 0,$$

with the following limits on variables: $0.25 \le x_1 \le 1.3$, $0.05 \le x_2 \le 2.0$, $2 \le x_3 \le 15.$

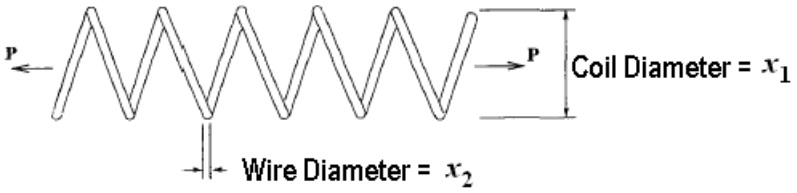


Figure 3.5 Schema for Spring Design problem [117]

Figures 3.6 demonstrate the simulation results for Spring Design problem using the proposed modifications compared with the original algorithm. Population size is 30 which is 10 times the number of decision variables as suggested in [5]. This result is after 50 independent runs are performed for all three algorithms. We can see the effect of defining liberty rate in comparison of two modifications. The convergence time and the best value for objective functions in the case that both modifications have been applied have been improved compared to the original method.

The comparison results are also shown in Table 3.1. It is noticeable that although two modifications give better results for best objective function, but the algorithm is not robust and the results for the worst objective function and mean objective function are not improved. For the original version, the standard deviation of algorithm discussed in Equation (3.10) has been considered as $\sigma = 1$.

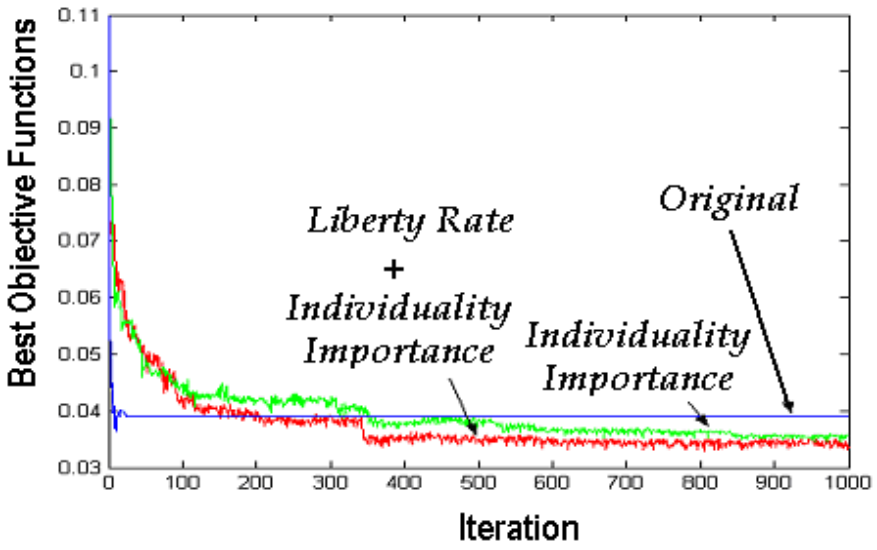


Figure 3.6 Comparison for best objective function for two proposed modifications: Original model (blue), modified by Individuality Importance (green) and modified by Liberty Rate and Individuality Importance (red)

Table 3.1 Comparison of results for Spring Design problem

Algorithms	Original Method	Individuality Importance	Liberty Rate and Individuality Importance
Best Objective Value (kg.m)	0.0464	0.0379	0.0331
Mean Objective Value (kg.m)	0.0464	6.2388	6.1516
Worst Objective Value (kg.m)	0.0464	32.2617	31.6833

3.4 Discussions

In this chapter, two modifications have been suggested for social-based algorithm. These modifications have been tested on a real world benchmark problem: the Spring Design problem. The simulation results demonstrate that adding two modifications facilitate the performance of the original algorithm resulting a better best objective

values. The first modified algorithm, individuality-based social algorithm outperforms the original social algorithm, while the liberty/individuality-based social algorithm outperforms both the original social algorithm and the individuality-based social algorithm in finding the best objective values. Both modified algorithms have migration policy better than the original social algorithm. The original algorithm is basically biased around the best performing individual which may result settling into a local optimum, while both modified versions are based upon individual's previous performance.

The results of modified version of social algorithm is based upon two hypotheses, one is that information from all individuals must be collected and exploited to migrate to the best leader, while the other is that the rate of convergence in different societies is not necessarily the same and depends on the relative collective behavior of the individuals in the society with respect to the civilization. Indeed the result in this case is improved because of giving more weight to diversity to the search in the individual space. If we just throw away all the non-leaders individuals, we lose a lot of information that might be critical in the search process, however getting information from the other non-leaders individuals might add to the convergence time.

The best objective values obtained in both modified versions are better than original social algorithm; however the worst and mean values are not better than original algorithm, since we are keeping the diversity while evolving. This also implies room for further improvements in the future research.

CHAPTER IV

DIVERSITY-BASED INFORMATION EXCHANGE FOR PARTICLE SWARM OPTIMIZATION

4.1 Introduction

Particle swarm optimization (PSO) is based on the changes of the positions and velocities of the particles in a manner that optimizes a goal function. PSO has demonstrated a promising performance for many problems; yet its fast convergence often leads to premature convergence in local optima. The tradeoff between fast convergence and being trapped in local optima will be even more critical in multimodal functions having many local optima very close to each other. In order to escape from the local optima and avoid premature convergence, the search for global optimum should be diverse. Many researchers have improved the performance of the PSO by enhancing its ability with a more diverse search. Specifically, some have proposed to use multiple swarms each running independent PSO, and then exchange information among them.

Exchanging information among clusters has also been adopted as an important design in several computational methods. Distributed genetic algorithm [6] employs GA mechanism to evolve several subpopulations in parallel. During frequent migration

among subpopulations, some individuals from each subpopulation will be sent to another subpopulation to replace other individuals based upon a replacement policy. In another algorithm known as society and civilization model [5], individuals from multiple societies would cooperate with each other in order to enhance their performance. The migration in this model occurs in two levels; first, the migrating of individuals inside each society toward the society leaders (intrasociety level), and second, the migrating of society leaders toward the civilization leaders (intersociety level).

In this study, a method borrowed from distributed genetic algorithm is employed to exchange information among multiple swarms in PSO. At regular intervals, each swarm prepares two sets of particles. One set is the particles that must be sent to another swarm and another set is the particles that must be replaced by individuals from other swarms. To prepare these two sets of particles, diversity measure is considered as the primary goal instead of only performance of the particles. When particles are approaching the local optima, several of them will have similar positional information. This similar redundant information will be replaced by particles from other swarms. This algorithm also proposes a new paradigm to find each swarm's neighbors. The neighborhood between swarms is defined by the use of Hamming distance between representatives of each pair of swarms. The particle's movement in the space is based on one variation of PSO with three basic terms, each one leading the particles toward the best particles in its own swarm, in its swarm's neighborhood, and in the whole population.

The structure of this chapter is organized as follows. Section 4.2 reviews the related studies in this field. In Section 4.3, the proposed algorithm is explained in detail. The main ideas of the proposed method are shown. In Section 4.4, the simulation of the proposed algorithm is performed on a set of hard benchmark problems. Section 4.5 summarizes the benefits of the proposed paradigm on PSO and outlines the future work for multiobjective optimization problems due to the nature of the diversity promotion proposed.

4.2 Review of Related Work

Kennedy and Eberhart [1] introduced the particle swarm optimization, an algorithm based on imitating behavior of flocking birds. It mimics grouping of birds as particles, their random movement, and regrouping them again to generate a model so that it can solve engineering optimization problems. Particles are known with their positions and velocities and can be updated using:

$$v = wv + c_p r_1(pbest - x) + c_g r_2(gbest - x), \tag{4.1}$$

$$x = x + v,$$

where v is the velocity of the particle, x is the position of the particle, $pbest$ is the best position ever experienced of each particle, and $gbest$ is the best position ever attained

among all particles. $r_1$and r_2 are random numbers uniformly generated in the range of (0,1). c_p, c_g, and w are personal, social, and momentum coefficients that are predefined. The main problem for PSO is its fast convergence to local optima. Later, Kennedy [46] introduced the stereotyping of the particles in which substitution of cluster centers for $pbest$ showed appreciable improvement of the PSO performance. His research suggested that PSO is more effective when individuals are attracted toward the center of their own clusters.

In multimodal problems, the search effort needs to be diverse in order to find the global optimum among a set of many local optima. The fast converging behavior of the PSO makes this issue so critical for multimodal problems. To achieve a more diverse search, Al-Kazemi and Mohan [47] divided the population into two sets at any given time, one set moving to the $gbest$ while another moving in opposite direction by selecting appropriate fixed values for (c_p, c_g, w) in each set. After some iterations, if the $gbest$ is not improved, then the particles would switch their group. Baskar and Suganthan [48] in their concurrent PSO used two swarms to search concurrently for a solution along with frequent passing of information, which was the $gbest$ of two cooperating swarms. After each exchange, the two swarms had to track the better $gbest$ found. One of the swarms was using regular PSO, and the other was using the Fitness-to-Distance ratio PSO [49]. Their approach improved the performance over both methods in solving single objective optimization problems. El-Abd and Kamel [50] further improved the previous algorithm by adding a two-way flow of information between two swarms. In

their algorithm, when exchanging the best particle between two swarms, this particle is used to replace the worst particle in another swarm. The two swarms perform a fixed number of iterations, and then the best p particles inside each swarm will replace the worst p particles in the other swarm only if they have a better fitness. This makes it possible for both swarms to exchange new information from the other swarm's experience. Krohling *et al.* [51] proposed co-evolutionary PSO in which two populations of PSO are involved. One PSO runs for a specified number of iterations while the other remains static and serves as its environment. At the end of such period, $pbest$ values obtained in previous cycles have to be re-evaluated according to the new environment before starting evolution. Although these algorithms used information exchange among swarms, but none of them adopted specific paradigm based on promoting diversity in selecting and sending particles from one swarm to another.

On the other hand, one of the main concerns in multiobjective optimization problems (MOP) is also to search for a diverse set of potential solutions, known as Pareto front. There have been several algorithms to extend PSO to handle diversity issue in MOPs. Parspopoulos *et al.* [52] introduced vector evaluated particle swarm optimizer (VEPSO) to solve multiobjective problems. A VEPSO is a multi-swarm variant of PSO in which each swarm is evaluated using only one of the objective functions of the problem under consideration, and the information it possesses for this objective function is communicated to the other swarms through the exchange of their best experience. In VEPSO, the velocity of the particles in each swarm is updated using the best previous

position, $gbest$, of another selected swarm. Selection of this swarm in the migration scheme can be either random or in a sequential order. Ray and Liew [53] used Pareto dominance and combined concepts of evolutionary techniques with the particle swarm. This algorithm uses crowding distance to preserve diversity. Hu and Eberhart [54] in their dynamic neighborhood PSO proposed an algorithm to optimize only one objective at a time. The algorithm may be sensitive to the optimizing order of objective functions. Fieldsend and Singh [55] proposed an approach in which they used an unconstrained elite archive to store the nondominated individuals found along the search process. The archive interacts with the primary population in order to define local guides. Mostaghim and Teich [56, 60] introduced a sigma method in which the best local guides for each particle are adopted to improve the convergence and diversity of the PSO. Li [57] adopted the main idea from NSGA-II into the PSO algorithm. Coello Coello *et al.* [58], on the other hand, proposed an algorithm using a repository for the nondominated particles along with adaptive grid to select the global best of PSO. The algorithms proposed to solve MOPs using PSO are based upon promoting the nondominated particles at any given time, not exploiting the information of all particles in the population.

The information exchange through migration in order to increase the search ability of the algorithm has been used in some other innovated paradigms. Ray and Liew [5] introduced their society and civilization model for optimization in accordance with simulation of social behavior. Individuals in a society interact with each other in order to

improve. Such improvement is done by information acquisition from the better-performing individuals or leaders in that society. This intrasociety interaction will improve the individual's performance, but cannot improve the leader's performance. The leaders do communicate externally with the leaders of other societies to improve. This intersociety communication leads to migration of the leaders to developed societies, which in turn, moves the overall poor-performing societies toward better-performing ones. At first, population is clustered into several mutually exclusive ones based on their distance in parametric space. Then objective functions along with constraints (if any) lay down a ranking system to choose the leaders in each cluster, and then migration in two levels will take place. Society and civilization model showed competitive results on single objective constrained optimization problems with respect to GAs.

The concept of having multiple sets had been originally introduced and used in distributed genetic algorithm (DGA) [6]. In DGA, population is divided into several subpopulations each running its own GA independently. At regular time intervals, inter-processor communication will happen. During this migration stage, a proportion of each subpopulation is selected and sent to another subpopulation. The migrant individuals will replace others based on replacement policy. In another kind of distributed evolutionary algorithm, Ursem [4] adopted his multinational evolutionary algorithm using a spatially separated model. He applied a fitness-topology function, instead of clustering, to decide on the relationship between a point and a cluster. The algorithm was to find all peaks of a multimodal function in unconstrained optimization problems.

In DGA, there are different policies on selection of migrants and replacement of individuals within each of the subpopulations. Cantu-Paz [118] showed that sending the fittest individuals of the population and replacing individuals with low fitness produces the best results. Denzinger and Kidney [18] used a diversity measure to select individuals for migration. Power *et al.* [119] used a method for selection based on a diverse set of individuals rather than the highly fit ones. The reason is to avoid like information to be sent to another subpopulation. Sometimes the majority of individuals can be located very close to each other, especially in the last steps of convergence. Therefore, by selecting the fittest individuals, the similar individuals from a small area will be sent to the next subpopulation. In case the algorithm is likely to be trapped in a local optima, this similar information is useless to diversify the search and get away from the local optima. Instead, the basis is to choose a diversified list of individuals to send to the other GAs. The sending list will be filled by the following individuals in this order: (1) an average individual of the subpopulation as representative of the population, (2) *m* individuals based on closeness to this representative whose fitness is better than representative, (3) *m* individuals based on closeness to this representative whose fitness is poorer than representative, and finally (4) the fittest individual in the subpopulation. There will also be a replacement list that will be filled in the following order: (1) individuals having similar genetic information, by order of fitness, with least fit ones being replaced before better fit ones, and (2) individuals with lowest fitness values. Their method was applied to single objective multimodal optimization and showed significantly better results when

compared to standard DGA with send-best-replace-worst strategy.

4.3 Diversity-based Information Exchange among Swarms in PSO

The underline principle of the proposed algorithm is based upon the idea of exploiting the information of all particles in the population. The population will be divided into P swarms, and each swarm will perform a PSO paradigm. After some predefined iterations, the swarms will exchange information based on a diversified list of particles. Each swarm prepares a list of sending particles to be sent to the next swarm, and also prepares a list of replacement particles to be replaced by particles coming from other swarms. Each swarm chooses the leaders of the next generation from the updated swarm after exchange of particles. To select the list of particles to send, algorithm uses a strategy according to the locations of the particles in the swarm and their objective values instead of their objective values alone. A list is prepared in the following order.

Priority S1: The higher priority in the selection of particles is given to a particle that has the least average Hamming distance from others. This particle is considered as the representative of the swarm. The average Hamming distance between each pair of particles in the swarm is calculated and then the least among them is found.

Priority S2: The closest M particles to the representative particle whose objective value is greater than that of the representative will be chosen. M is a value that depends on the rate of information exchange, r, (a predefined value between 0 and 1) among

swarms, and population of each swarm, N_S:

$$M = \frac{rN_S}{2} - 1. \tag{4.2}$$

Priority S3: The closest M particles to the representative particle whose objective value is less than that of the representative will be chosen.

Priority S4: The best performing particle in the swarm will be chosen.

Depending on the predefined fixed value for allowable number of the sending list, the sending list will be filled in each swarm. There will also be a replacement list that each swarm prepares, based on the similar positional information of particles in the swarm. When swarms are approaching local optima, many locations of particles are similar to each other. Each swarm will then remove this excess information through its replacement list. The replacement list in each swarm is prepared in the following order.

Priority R1: Particles with identical parametric space information, by the order of their objective values, with the least objective values will be replaced first.

Priority R2: Particles with the lowest objective values will be replaced when all particles of the last priority have already been in the replacement list.

This information exchange among swarms can happen in a ring sequential or random order between each pair of swarms as shown in Figure 4.1. Each swarm accepts the sending list from other swarm and will replace it with its own replacement list. After

information exchange completes, the $pbest$ and $gbest$ will be selected. This algorithm is shown in Figure 4.2.

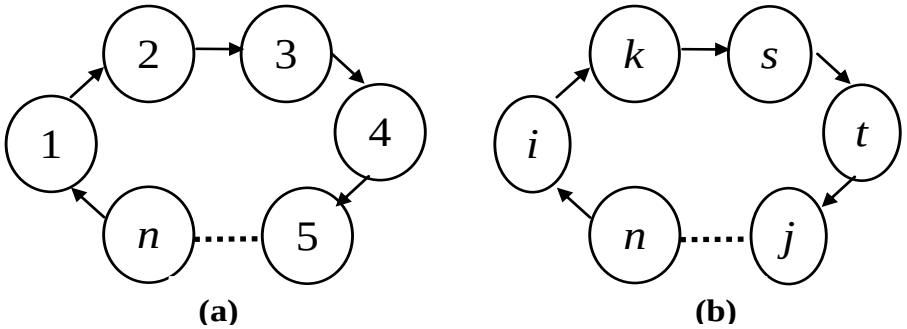


Figure 4.1 Ring and random sequential migration: Migration can be (a) in ring sequential order between swarms1 and 2, then between swarm 2 and 3, etc. or (b) in a random order between swarms. *i, k, s, t, j* are random numbers between 1 and *n*.

- Initialize population at time $t = 1$.
- Cluster population into *P* swarms using *k*-means clustering.
- If $t = t_{Migration}$, then:
  a. Prepare the sending list and replacement list for each swarm;
  b. Exchange particles between pairs of swarms, using sending and replacement lists of each swarm;
  c. Perform the PSO on new swarms using Equation (4.1).
- Else:
  Perform PSO on each swarm using Equation (4.1).
- Repeat the above steps until stopping criteria are met. ($t = t_{\max}$)

Figure 4.2 Main algorithm for diversity-based multiple PSO (DMPSO)

To further overcome the premature convergence problem, especially in multimodal objective optimization, and to increase the ability of communication among particles about common interest information, a concept of neighborhood is proposed to

promote the particles in a neighborhood to utilize and share information among themselves. For the PSO schema, a three-level mechanism is adopted. In personal level, particle in a swarm will follow the leader of the swarm that is the best behaving particle in that swarm. In neighborhood level, the particle will simultaneously follow the best behaving particle in its neighborhood to achieve a synchronized behavior in the neighborhood and to share the information, and finally in the global level, particles of each swarm will follow the best behaving particle in the whole population, seeking a global goal. This paradigm of PSO is formulated as:

$$v = wv + c_p r_1(pbest - x) + c_g r_2(gbest - x) + c_n r_3(nbest - x), \quad (4.3)$$
$$x = x + v,$$

where v is the velocity of the particle, x is the position of the particle, $pbest$ is the best position in the cluster, $gbest$ is the best position among all particles and $nbest$ is the best position among the particles' neighborhood. r_1, r_2, and r_3 are random numbers uniformly generated in the range of (0,1). Thus particles always move statistically towards the direction of $pbest$, $gbest$, and $nbest$ in order to use the past experience in the search process. c_p, c_g, and c_n are constant values representing the weight of each of the terms of personal, global, and neighborhood behavior and w is the momentum for previous velocity. It should be noted that the unified PSO [120] integrates the local best and global best PSOs into a single equation to update the velocity of particles based on the global

best particle, the neighboring best particle, and the particle's own best position, while in the proposed paradigm, velocity updates using the best particle in the cluster of particles, the best particle in the neighboring swarms, and the best global particle with no restriction on the weights.

To find the neighborhood among particles in PSO, there have been different strategies used by researchers [37, 121]. Some have applied ring neighborhood, the von Neumann neighborhood, or some other topological neighborhoods. The proposed definition of neighborhood is to define neighboring swarms according to the average as representative of each swarm to decide whether the swarms are in neighborhood of each other. In the *i*-th swarm with the particles of $\{x_1, x_2, \dots, x_K\}$, the representative, R_i, is defined by centroid of all particles:

$$R_i = \frac{1}{K}\sum_{j=1}^{K} x_j. \tag{4.4}$$

The inter-swarm distance between swarms i and j, R_{ij}, is defined by the inner products of two vectors:

$$R_{ij} = \sum_k R_i^k R_j^k, \tag{4.5}$$

where R_i^k is the *k*-th element of the representative R_i.

Swarms are defined to be in a neighborhood if and only if all inter-swarm distances among them are less than the average inter-swarm distance: $R_{ij} < \bar{R}$, $R_{ik} < \bar{R}$, … and $R_{kt} < \bar{R}$, where $\bar{R} = \frac{2}{P(P-1)} \sum_{i,j=1,i\neq j}^{P} R_{ij}$, where P is the number of swarms.

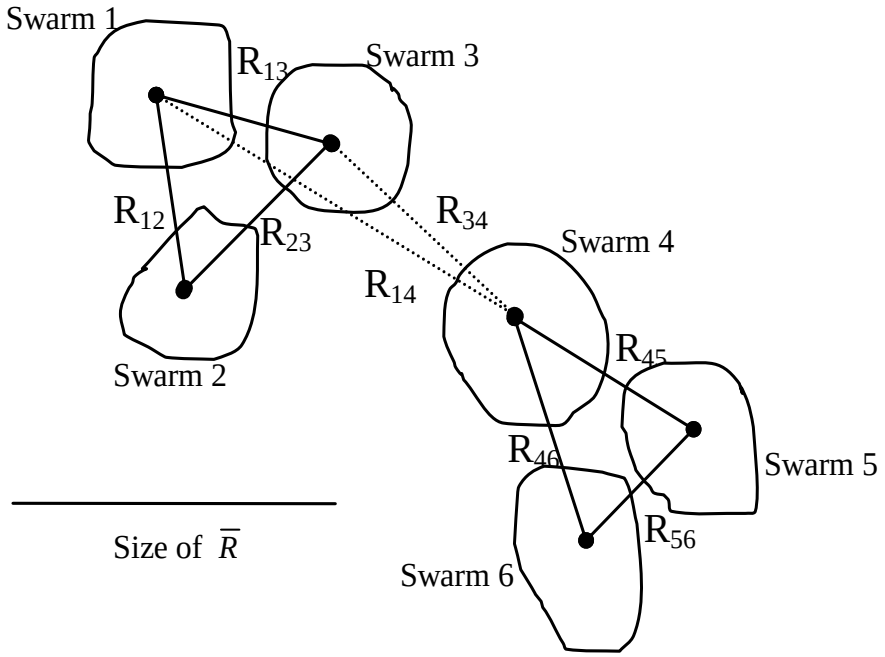


Figure 4.3 Schema of swarm neighborhood: Swarms 1, 2 and 3 are in a neighborhood, since $R_{12} < \bar{R}$, $R_{13} < \bar{R}$and $R_{23} < \bar{R}$ but swarm 4 does not belong to this neighborhood. Notice that even $R_{34} < \bar{R}$ but $R_{14} > \bar{R}$. Swarms 4, 5 and 6 form another neighborhood, because $R_{45} < \bar{R}$, $R_{46} < \bar{R}$ and $R_{56} < \bar{R}$. Swarm 3 does not belong to this neighborhood because even $R_{34} < \bar{R}$ but $R_{36} > \bar{R}$. (Solid circles denote the representative points of each swarm)

For example, for two of them, swarms *i* and *j* are in a neighborhood if and only if $R_{ij} < \bar{R}$. If $R_{ik} < \bar{R}$ but $R_{jk} > \bar{R}$, then swarm *k* does not belong to this neighborhood. In Figure 4.3, an example with six swarms is shown. Swarms 1, 2 and 3 are in a neighborhood because$R_{12} < \bar{R}$, $R_{23} < \bar{R}$ and $R_{13} < \bar{R}$ but swarm 4 does not belong to

this neighborhood. Notice that even $R_{34} < \bar{R}$ but $R_{14} > \bar{R}$. Swarms 4, 5 and 6 form another neighborhood because $R_{45} < \bar{R}$, $R_{46} < \bar{R}$ and $R_{56} < \bar{R}$. Swarm 3 does not belong to this neighborhood because even $R_{34} < \bar{R}$ but $R_{36} > \bar{R}$.

A brief explanation of the proposed algorithm is shown in Figure 4.4. The population is initialized and then clustered into *P* swarms using the *k*-means clustering method. Then the neighbor sets of each swarm will be found using the Equations (4.4) and (4.5) and the rule mentioned above as shown in Figure 4.3.

- Initialize population at time $t = 1$.
- Cluster population into *P* swarms.
- If $t = t_{Migration}$, then:
  a. Prepare the sending list and replacement list for each swarm;
  b. Exchange particles between pairs of swarms, using sending and replacement lists of each swarm;
  c. Find the neighbor sets of each swarm. $(N_i, i = 1,2,..., P)$;
  d. Perform the PSO on each new swarm:
    - Find the $pbest$, $gbest$, and $nbest$ for each new swarm,
    - Apply the modified version of PSO, Equation (4.3).
- Else:
  a. Find the neighbor sets of each swarm. $(N_i, i = 1,2,..., P)$;
  b. Perform the PSO on each swarm:
    - Find the $pbest$, $gbest$, and $nbest$ for each swarm,
    - Apply the modified version of PSO, Equation (4.3).
- Repeat the above steps until stopping criteria are met.
- $(t = t_{\max})$

Figure 4.4 Main algorithm for diversity-based multiple PSO with neighborhood (N-DMPSO)

To perform the PSO according to Equation (4.3), we have to find the best performing particle in each swarm, namely the $pbest$, the best performing particle in the all-neighbor sets of that swarm, namely the $nbest$, and the best performing particle in the whole population, $gbest$. This process will be iterated until the time for migration is reached. At regular fixed intervals, each swarm prepares a list of particles to send to the next swarm, a list of particles that must be replaced from other particles coming from other swarms; then exchange of particles between each of the two swarms will happen according to Figure 4.1. This algorithm including clustering, information exchange, and flight of particles will continue until the stopping criteria are met.

4.4 Simulation Results

The proposed algorithm was tested using some benchmark problems, which are often used to examine GA solving multimodal problems [4, 122]. These problems adopted from [119] vary in difficulty and dimension. In order to test the proposed algorithm, its performance has been compared with two distributed genetic algorithms [118-119]. One of them is DGA with a standard migration policy (SDGA), best-sent-worst-replaced [118]. The other one is a DGA with diversity-based migration policy (DDGA) [119]. In order to draw a fair comparison, the same rate of information exchange as their migration rate has been adopted. The main population for the proposed algorithms DMPSO and N-DMPSO was set as 50 particles. The *k*-means clustering

method was used with m=6 swarms. The coefficients of c_p, c_g, c_n, and w are selected as 1.4, 1.4, 1.4, and 0.8, respectively. The rate of information exchange is varied with the values of 0.05, 0.2, and 0.4. At the time interval of $T = 10$, particles are exchanged among swarms.

The first problem used to test the proposed algorithm is F_1 [119] with five peaks and four valleys between each of the two neighboring peaks. This function is depicted in Figure 4.5. Figure 4.5(a) shows a 3-D landscape while Figure 4.5(b) displays the contour map of the function F_1. The results of applying both proposed algorithms are shown in Table 4.1. The optimal solution found (in percentage) is calculated out of 30 independent runs for each algorithm. The solution is considered to be optimal when the optimal objective value of 2.5 is reached. The best objective values for the final solution is averaged over 30 runs to obtain the mean best objective reported in the table. Each algorithm is performed for three values of rate of information exchange, 0.05, 0.2, and 0.4. The best performing algorithm in each case is shown in bold face. The graphical view of the location of the best particles of the final solution is depicted in Figure 4.6.

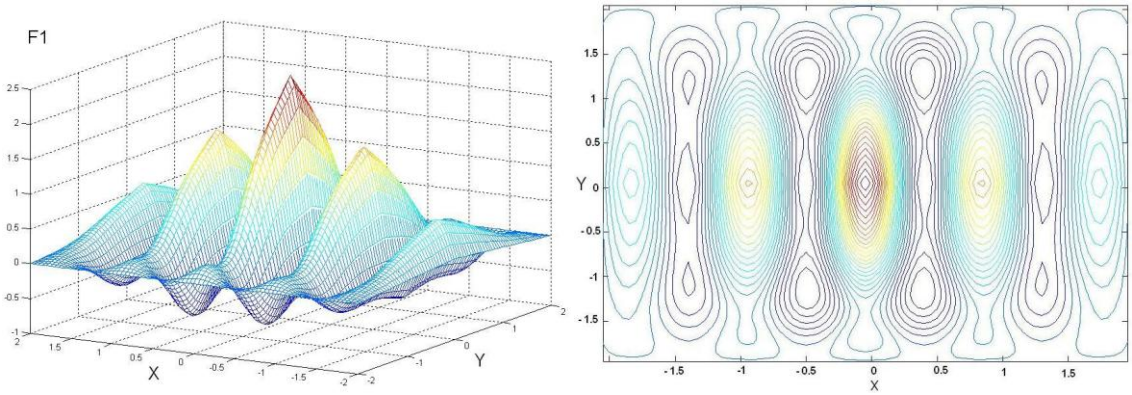
Figure 4.6 (a) and (b) are for DMPSO with rate of information exchange equal to 0.05 and 0.4, and (c) and (d) are for N-DMPSO with rate of information exchange equal to 0.05 and 0.4, respectively. Figure 4.6 shows that some of the particles in DMPSO will be trapped in local maxima (0.897,0) and (-0.897,0). In N-DMPSO, most particles approached toward (0,0), the global maximum. The results in Table 4.1 show that both proposed algorithms perform better and N-DMPSO outperforms all of them.

The second benchmark problem is F_2 [119] with 10 peaks shown in Figure 4.7. The results of the algorithms are also shown in Table 4.1. The graphical view of the best particles for both algorithms is obtained in Figure 4.8. This figure shows that some particles in DMPSO are trapped in a local maximum while in N-DMPSO most particles reach the global maximum. Results reported in Table 4.1 also show the better performance of N-DMPSO.

The next benchmark function is F_3 [119], shown in Figure 4.9, with two close peaks and a valley between them. The results of the algorithms are summarized in Table 4.1 as well, and the graphical view of the best particles is depicted in Figure 4.10. Figure 4.10 shows that in DMPSO some of the particles are trapped in local maximum at (-1.444,0), while in N-DMPSO most of the particles reached the global maximum at (1.697,0). Table 4.1 also illustrates that N-DMPSO is outperforming other algorithms. The next benchmark function is F_4 [119] with a total of five peaks, one global maximum and four local maxima in its neighborhood, shown in Figure 4.11. The results and the graphical presentation of the best particles in Table 4.1 and Figure 4.12 show once again that N-DMPSO has less particles trapped in four local maxima located at the corners of the variable space. The results obtained in Table 4.1 confirm a higher number of found optimal solutions. The benchmark function F_5 [119] has six peaks, two of which are global maxima as shown in Figure 4.13. The results of the algorithms are shown in Figure 4.14 and Table 4.1. The D-DGA in this problem outperforms the proposed algorithms when rate of information exchange is 0.05 and 0.2. On the other hand, with a

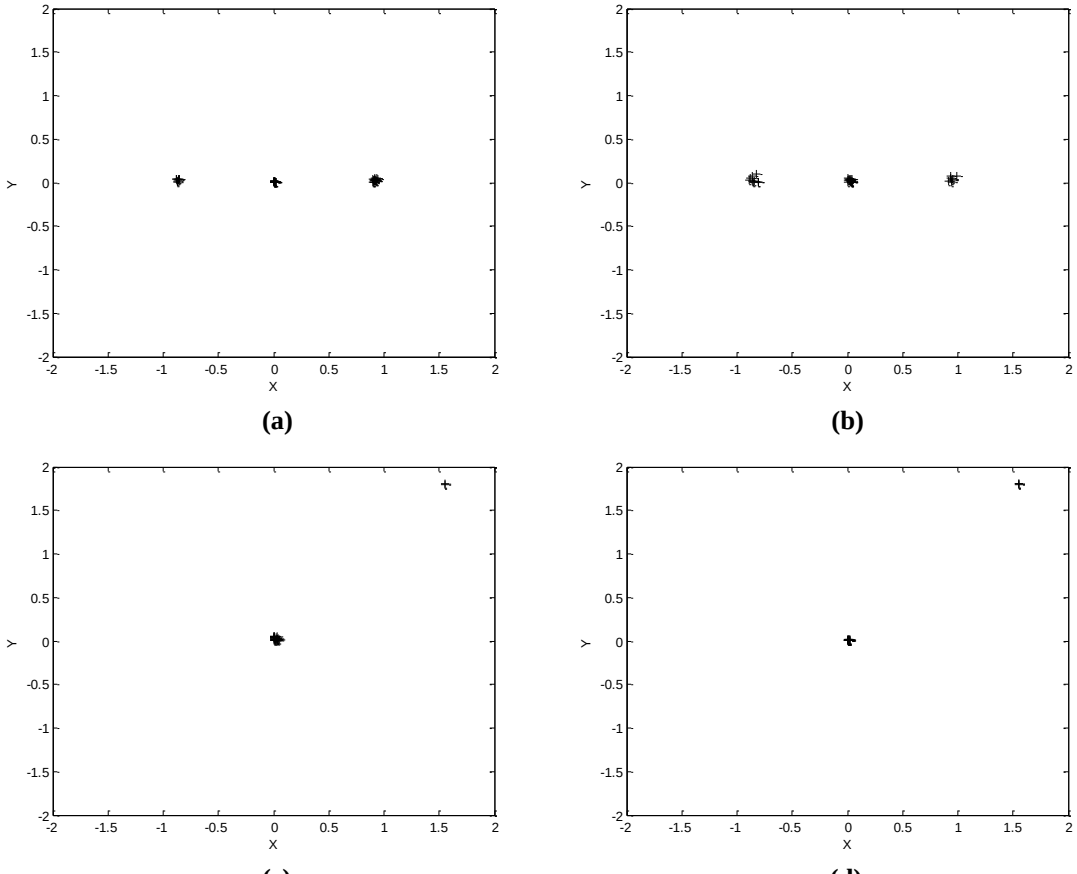
higher rate, the proposed algorithm performs better, i.e., when more particles are exchanged, PSO shows more superiority.

The benchmark function of F_6 [119] has a variable dimension. Three different dimensions of 25, 40 and 50 have been used here. The rate of the information exchange for this case and the remaining benchmark functions has been fixed at 0.1. The best objective value for the final solutions is averaged over 30 runs and shown in Table 4.2. The N-DMPSO outperforms the other three algorithms at dimensions 25 and 40 but at dimension 50, D-DGA performs better. The benchmark function of F_7 [119] has also a variable dimension, and three dimensions of 25, 40, and 50 have been adopted here. Results in Table 4.2 show that N-DMPSO outperforms the others at dimensions 25 and 40 but again, at dimension 50, D-DGA outperforms others. The next benchmark function, F_8 [119], has 10 variables. N-DMPSO also outperforms other algorithms in this case. And finally, the last benchmark function F_9 [119] has 40 variables. N-DMPSO performs better than other algorithms as well. In general, N-DMPSO outperformed other algorithms in several benchmark functions. It was outperformed in some cases, especially problems with very high dimension, by D-DGA. It might be due to the nature of GA that recombination demonstrates a better performance in high dimension; however it needs to be tested more.



(a) **(b)**

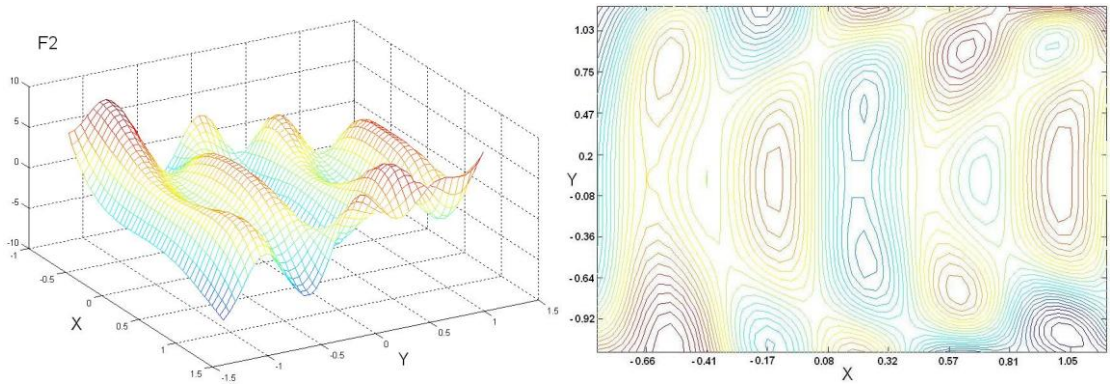
Figure 4.5 Benchmark function F1 with five peaks and four valleys: (a) 3-D landscape, (b) contour map.



(a) **(b)**

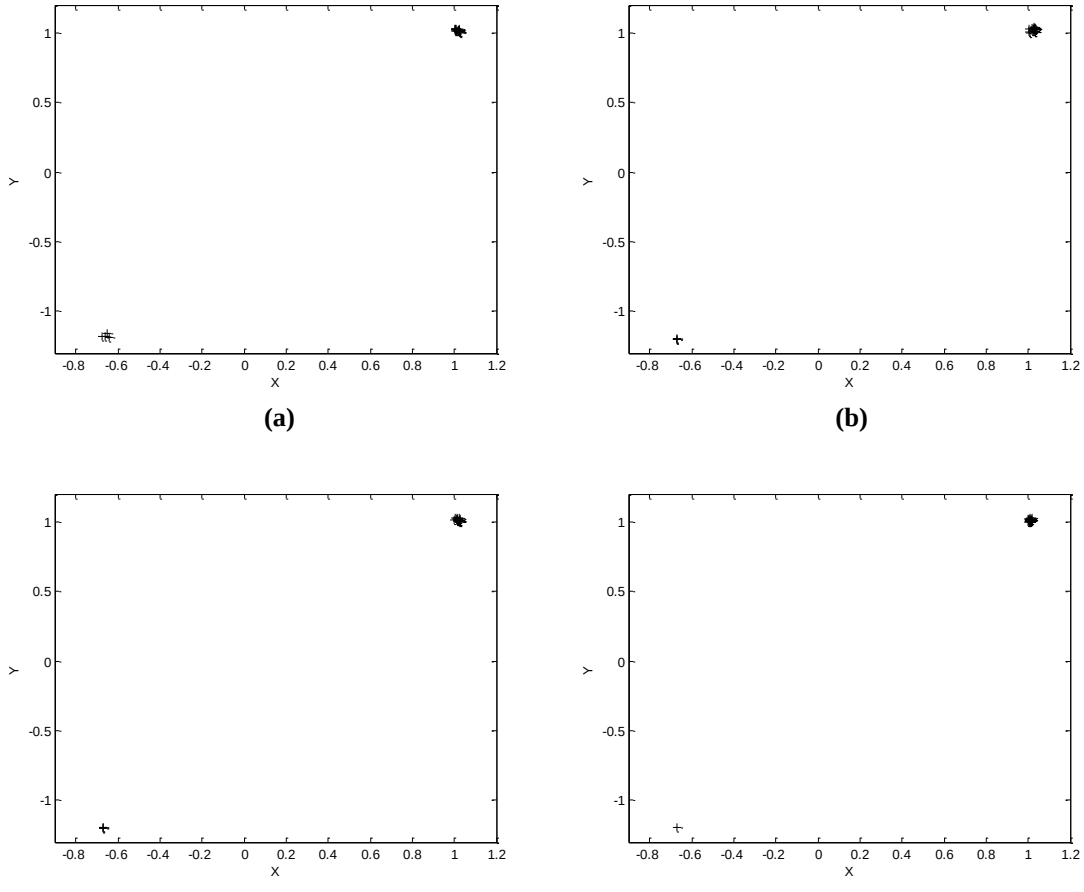
(c) **(d)**

Figure 4.6 Final best particles for F1: (a) DMPSO with *r* = 0.05, (b) DMPSO with *r* = 0.4, (c) N-DMPSO with *r* = 0.05, (d) N-DMPSO with *r* = 0.4.



(a) **(b)**

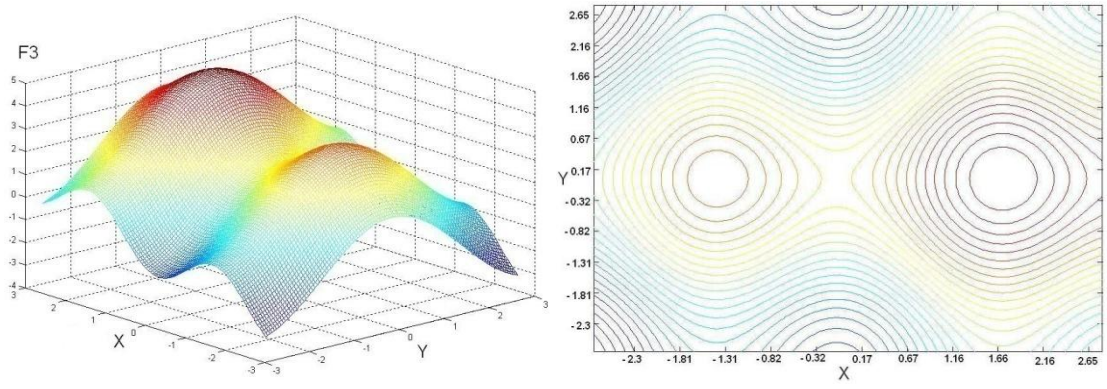
Figure 4.7 Benchmark function F2 with 10 peaks: (a) 3-D landscape, (b) contour map.



(a) **(b)**

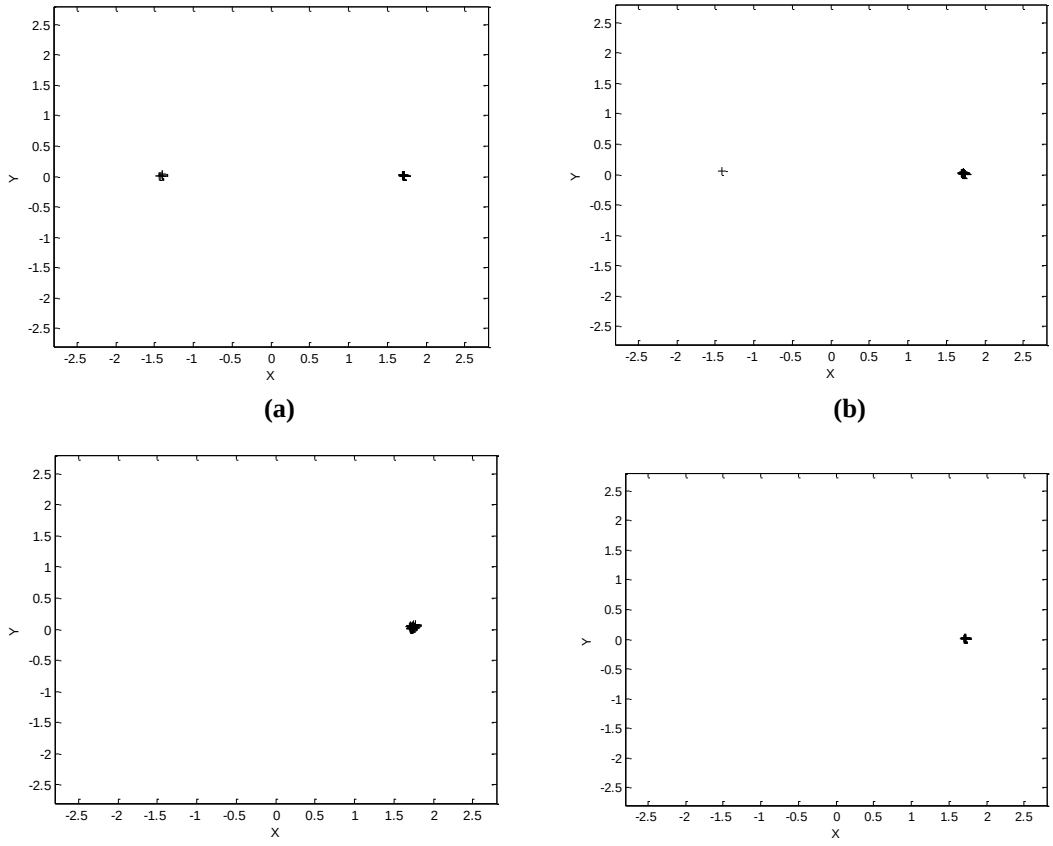
(c) **(d)**

Figure 4.8 Final best particles for F2: (a) DMPSO with *r* = 0.05, (b) DMPSO with *r* = 0.4, (c) N-DMPSO with *r* = 0.05, (d) N-DMPSO with *r* = 0.4.



(a) **(b)**

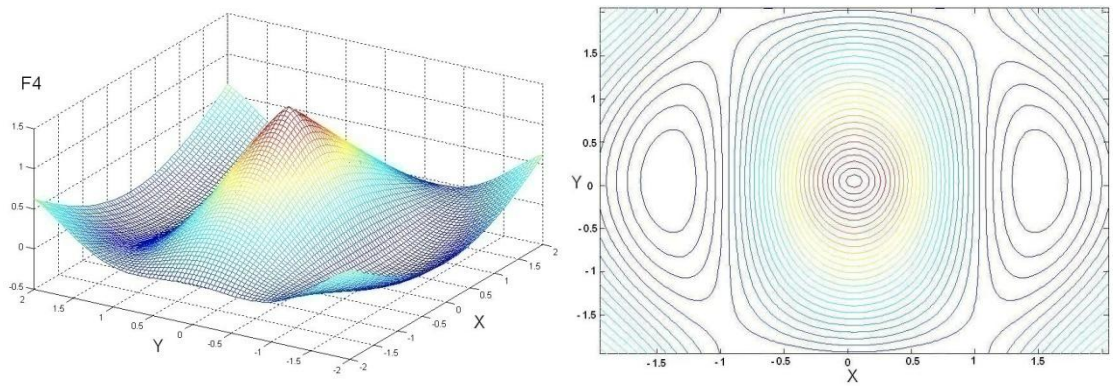
Figure 4.9 Benchmark function F3 with two peaks and one valley: (a) 3-D landscape, (b) contour map.



(a) **(b)**

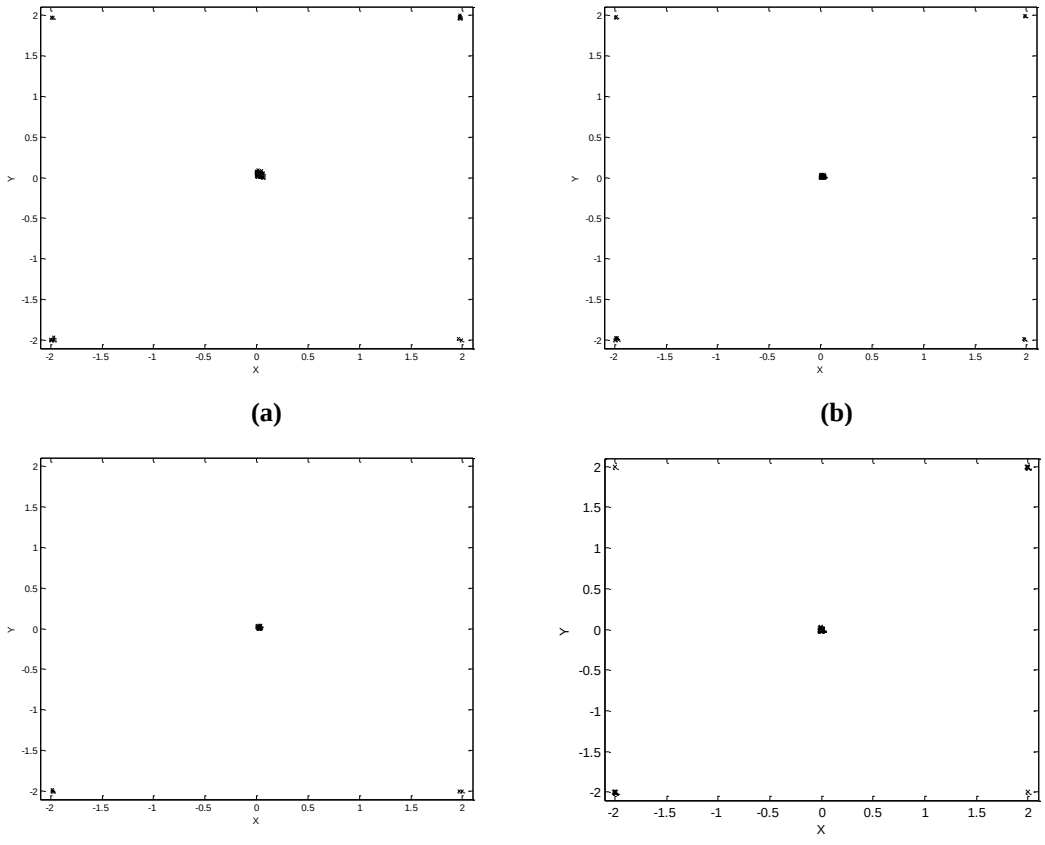
(c) **(d)**

Figure 4.10 Final best particles for F3: (a) DMPSO with *r* = 0.05, (b) DMPSO with *r* = 0.4, (c) N-DMPSO with *r* = 0.05, (d) N-DMPSO with *r* = 0.4.



(a) **(b)**

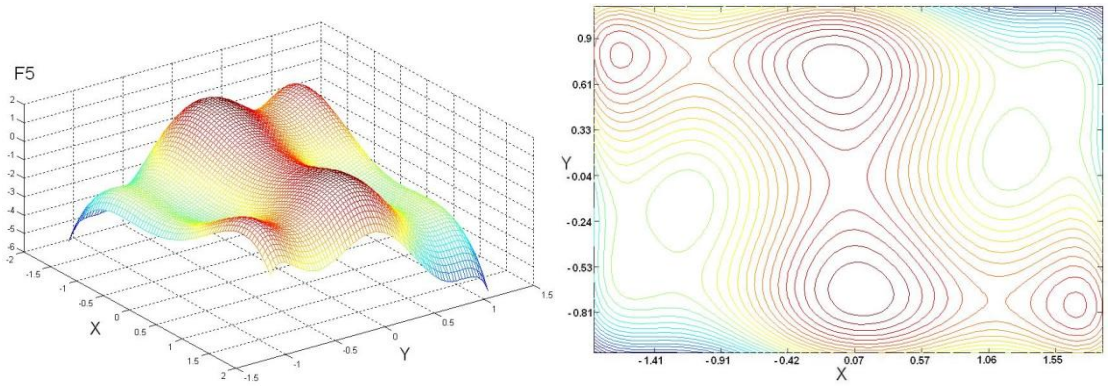
Figure 4.11 Benchmark function F4 with five peaks: (a) 3-D landscape, (b) contour map.



(a) **(b)**

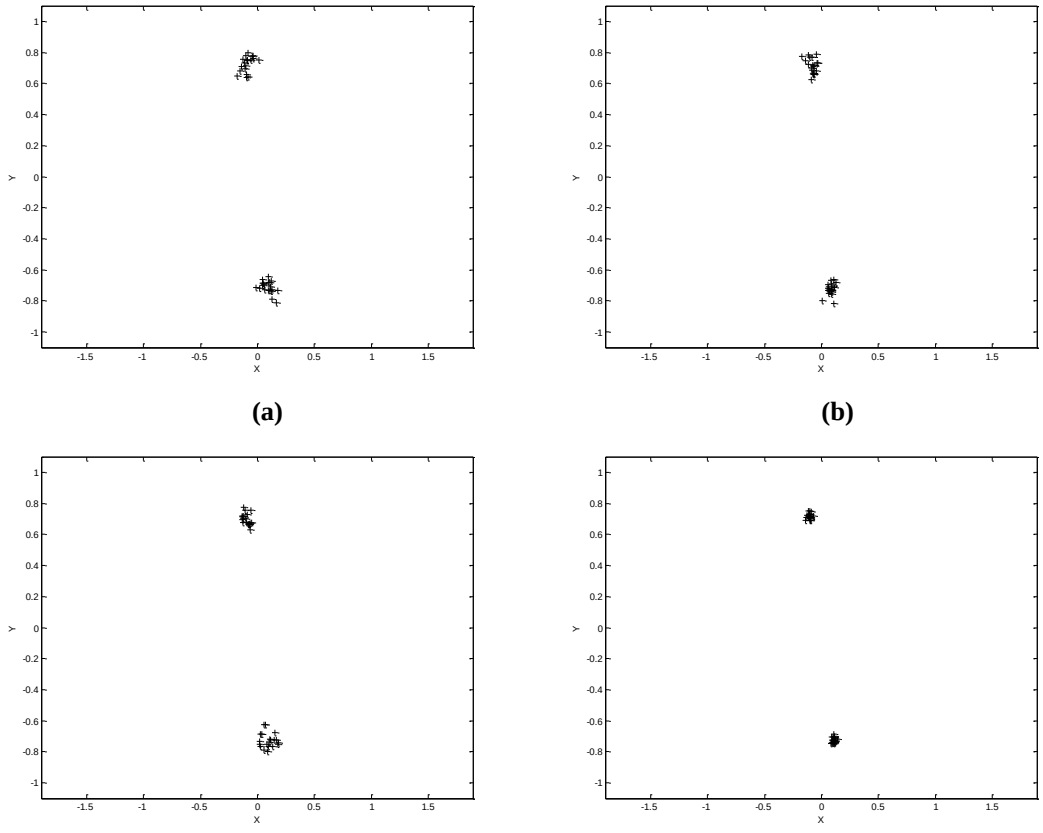
(c) **(d)**

Figure 4.12 Final best particles for F4: (a) DMPSO with *r* = 0.05, (b) DMPSO with *r* = 0.4, (c) N-DMPSO with *r* = 0.05, (d) N-DMPSO with *r* = 0.4.



(a) **(b)**

Figure 4.13 Benchmark function F5 with six peaks, two of which are global maxima: (a) 3-D landscape, (b) contour map.



(a) **(b)**

(c) **(d)**

Figure 4.14 Final best particles for F5: (a) DMPSO with *r* = 0.05, (b) DMPSO with *r* = 0.4, (c) N-DMPSO with *r* = 0.05, (d) N-DMPSO with *r* = 0.4.

Table 4.1 Results for optimal found (%) and mean best objective for F1, F2, F3 and F5

Algorithms			S-DGA	D-DGA	DMPSO	N-DMPSO
Max F1	Optimal Found (%)	$r = 0.05$	0%	0.8%	10.0%	**13.3%**
		$r = 0.2$	0%	13.3%	13.3%	**20.0%**
		$r = 0.4$	0%	15.8%	16.7%	**23.3%**
	Mean best objective	$r = 0.05$	1.98898	2.48217	2.4801	**2.4863**
		$r = 0.2$	1.97855	2.4605	2.4745	**2.4793**
		$r = 0.4$	2.02455	2.48811	2.4891	**2.4905**
Max F2	Optimal Found (%)	$r = 0.05$	0%	22.5%	33.3%	**53.3%**
		$r = 0.2$	0.8%	5.8%	13.3%	**26.6%**
		$r = 0.4$	0%	17.5%	23.3%	**33.3%**
	Mean best objective	$r = 0.05$	6.73371	8.58322	8.6739	**8.6783**
		$r = 0.2$	6.70137	8.63548	8.6532	**8.6621**
		$r = 0.4$	7.31735	8.68075	8.6923	**8.6953**
Max F3	Optimal Found (%)	$r = 0.05$	0%	3.3%	16.7%	**20.0%**
		$r = 0.2$	0%	20%	23.3%	**33.3%**
		$r = 0.4$	0%	23.3%	43.3%	**53.3%**
	Mean best objective	$r = 0.05$	4.67853	4.812	4.8121	**4.8127**
		$r = 0.2$	4.7159	4.810	4.8117	**4.8136**
		$r = 0.4$	4.73849	4.81496	4.8151	**4.8155**
Max F4	Optimal Found (%)	$r = 0.05$	3.3%	4.2%	16.7%	**26.6%**
		$r = 0.2$	0%	42.5%	43.3%	**53.3%**
		$r = 0.4$	0%	35%	36.7%	**43.3%**
	Mean best objective	$r = 0.05$	1.34999	1.48242	1.49016	**1.49127**
		$r = 0.2$	1.33163	1.49341	1.49281	**1.49332**
		$r = 0.4$	1.29936	1.49123	1.49178	**1.49341**
Max F5	Optimal Found (%)	$r = 0.05$	11.7%	**67.5%**	43.3%	53.3%
		$r = 0.2$	14.2%	**69.2%**	33.3%	36.7%
		$r = 0.4$	0.8%	22.5%	36.6%	**43.3%**
	Mean best objective	$r = 0.05$	0.970634	**1.03**	1.0283	1.0297
		$r = 0.2$	0.975198	**1.03006**	1.0281	1.0288
		$r = 0.4$	0.941727	1.02874	1.0293	**1.0297**

Table 4.2 Mean best objectives for F6, F7, F8, and F9

Algorithms	Dimension	S-DGA	D-DGA	DMPSO	N-DMPSO
Max F6	25	8456.46	8935.65	8745.8	**8947.3**
	40	12578	13131.1	13002.4	**13135.1**
	50	15004.6	**15554.9**	15387.5	15423.6
Max F7	25	8682.31	9093.61	9026.5	**9098.4**
	40	12796.1	13324.3	13304.5	**13331.3**
	50	15124	**15810.5**	15723	15799
Max F8	10	1.9513	1.97217	1.9673	**1.97221**
Max F9	10	605.201	627.921	616.436	**628.142**

4.5 Discussions

A paradigm for particle swarm optimization is presented in order to increase its ability to search widely and to overcome its premature convergence problem. The proposed algorithm uses multiple swarms and exchanges particles among them in regular intervals. The exchanged particles are selected according to the locations of the particles based on a promotional diversity strategy and their correspondence objective values. Furthermore, the PSO was modified using a new neighborhood term that helps the neighboring swarms share the common interest information. The neighborhood for each swarm is found using an unsupervised algorithm according to the inter-swarm distances between representatives of each pair of swarms. The proposed algorithms, N-DMPSO, showed a great performance compared to DMPSO and two versions of distributed genetic algorithm that have similar conceptual basis with the proposed algorithm. The DMPSO showed competitive results compared to DGAs. The N-DMPSO showed better

performance compared to DMPSO, indicating that sharing information in the neighborhood of swarms helps them to escape from local optima and locate the global optimum.

As a drawback of both proposed algorithms, they show dependence of their performance on the rate of information exchange. A range of rate has been selected from 0.05 to 0.4 which reveals no conclusion on what rate is better for a specific application. Further work is needed to find an optimum exchange rate. Due to the nature of the diversity promotion of the proposed algorithm that works well for multimodal problems, it can be a promising candidate as a basis for solving multiobjective optimization.

CHAPTER V

CULTURAL-BASED MULTIOBJECTIVE PARTICLE SWARM OPTIMIZATION

5.1 Introduction

Population based heuristic for solving multiobjective optimization problems (MOPs) has gained much attention. Multiobjective evolutionary algorithm (MOEA) and multiobjective particle swarm optimization (MOPSO) are two popular population based paradigms introduced within the last decade. MOPSO adopts the particle swarm optimization (PSO) paradigm [1] which in turn mimics behavior of the flocking birds. Although there exist many researches on single objective PSO suggesting dynamic weights for the local and global acceleration [123], most MOPSO researchers assume that all particles should move with the constant momentum, local, and global acceleration.

However there have not been many studies to consider a possibility in which particles fly with different "personalized" weights for the momentum, local, and global acceleration. Some may argue that there is no need to have a personalized weight for each particle. Even if an algorithm applies the same weight for all particles, for some particles requiring smaller weight, they will unnecessarily jump far away from the optimum, while

for some other particles that need greater weight, they will unsatisfactorily move slowly, resulting in both situations an inefficient design. On the other hand employing a personalized weight for each particle assigns an appropriate amount of jump and contributes to the effectiveness of the performance of the algorithm.

From a biological point of view, study [7] has shown that societies that can handle more complex tasks contain polymorphic individuals. Polymorphism is a significant feature of social complexity that results in differentiated individuals. The more differentiated the society, the easier it can handle complex tasks. Differentiation applies in principal to complex societies of prokaryotic cells, multicellular organisms, as well as to colonies of multicellular individuals such as ants, wasps, bees, and so forth. The colony performance is improved if individuals differentiate in order to specialize on particular tasks. As a result of differentiation, individuals perform functions more efficiently. In the study it has been shown the colony's ability to higher cooperative activity when tackling tasks is a direct consequence of differentiation among other factors.

There are few studies in the MOPSO that have tackled the issue of variable momentum for the particles although in all of them momentum is identical for all particles at a specific iteration. Some MOPSO paradigms have proposed simple strategies to adapt the momentum by decreasing the momentum throughout swarming [57, 64, 67-68, 124], while other MOPSOs choose a random value for momentum [54, 62-63, 66, 69] at every iteration.

The MOPSO, similar to PSO, is based upon a simple flight of the particle:

$$v_i^d(t+1) = \omega v_i^d(t) + c_p r_1 \left(pbest_i^d(t) - x_i^d(t)\right) + c_g r_2 \left(gbest^d(t) - x_i^d(t)\right), \quad (5.1)$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1), \quad (5.2)$$

where $x_i^d(t)$ is the *d*-th dimension of the position of the *i*-th particle at time t ($i = 1,2,\dots,N$ and $d = 1,2,\dots,M$). $v_i^d(t)$ is the *d*-th dimension of the velocity of the *i*-th particle at time t. $pbest_i^d(t)$ is the *d*-th dimension of the personal best position of the *i*-th particle at time t , and $gbest^d(t)$ is the *d*-th dimension of the global best position at time t. c_p and c_g are the constant values that are called personal and global acceleration which give different importance to personal or global term of Equation (5.1). r_1 and r_2 are uniform random numbers from $(0,1)$ to give stochastic characteristics to the flight of particles. ω is the velocity momentum of the particles. In Figure 5.1, it can be seen how three vectors which affect the flight of particles depend so much on the momentum, global, and local acceleration. When particles need to be used as exploiter or explorer the emphasis on each term in Equation (5.1) should be different. Therefore not all particles should have the same values for momentum, local, and global acceleration.

To the best knowledge of the author, there is no appreciable work in MOPSO on adapting personalized momentum and acceleration based upon the need for the particles to exploration or exploitation. Adaptation of these important factors in the flight of particles is an important task that cannot be solved unless we have access to the knowledge throughout the search process. In this study, a computational framework is

proposed based on cultural algorithm (CA) [3, 125] adopting knowledge stored in belief space in order to adapt flight parameters of MOPSO. Cultural algorithms have been frequently used to vary parameters of individual solution in optimization problems [126-127]. Proposed paradigm resorts different types of knowledge in belief space to personalize the parameters of the MOPSO for each particle. Every particle in MOPSO will use its own adapted momentum and acceleration (local and global) at every iteration to approach the Pareto front. Cultural algorithm provides required groundwork enabling one to employ the information stored in different sections of belief space efficiently and effectively. By incorporating CA into the optimization process, we categorize the information of the belief space and adopt it in a systematic manner. Information in the belief space provide required parameters needed for the optimization process whenever it is needed. As a result the optimization process will be more competent and successful.

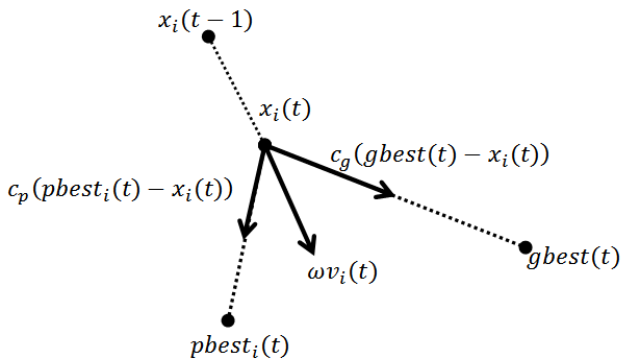


Figure 5.1 Schema of particle's movement in MOPSO: Vectors affecting how particle moves in MOPSO due to *gbest, pbest* and its velocity.

The remaining sections complete the presentation of this chapter. In Section 5.2, principles of cultural algorithm and related works in MOPSO are briefly reviewed. In

Section 5.3, the proposed cultural MOPSO is elaborated. In Section 5.4, simulation results are evaluated on the benchmark test problems in comparison with the state-of-the-art MOPSO models. This section also includes a sensitivity analysis for the proposed cultural MOPSO. Finally, Section 5.5 summarizes the concluding remarks and future work of this study.

5.2 Review of Literature

5.2.1 Related Works in Multiobjective PSO

Hu and Eberhart [54] in their dynamic neighborhood MOPSO model and also Hu *et al.* [66] in the MOPSO with extended memory adopted a random number on the range (0.5,1) as the varying momentum, however both personal and global acceleration are constant values. Sierra and Coello Coello [62] in their crowding and ε-dominance based MOPSO used random value at the range (0.1,0.5) for the momentum and random values at the range (1.5,2.0) for the personal and global acceleration. They adopted this scheme to bypass the difficulties of fine tuning these parameters for each test function.

Zhang *et al.* [64] introduced intelligent MOPSO based upon Agent-Environment-Rules model of artificial life. In their model, along with adopting some immunity clonal operator, the momentum was decreased linearly from 0.6 to 0.2, but the personal and global acceleration remained constant. Li [67] proposed an MOPSO based upon max-min

fitness function. In his model, while the personal and global accelerations were set constant, the momentum was gradually decreased from 1.0 to 0.4. Zhang *et al.* [68] adopted a linearly-decreasing momentum from 0.8 to 0.4 for their MOPSO algorithm. However the personal and global accelerations were kept fixed values.

Mahfouf *et al.* [69] introduced adaptive weighted MOPSO in which they included adaptive momentum and acceleration. Using comparison study with other well-behaved algorithms, they demonstrated that the proposed MOPSO search capability is enhanced by adding this adaptation. Ho *et al.* [63] noted the possible problem of selecting personal and global acceleration independently and randomly. He mentioned because of its stochastic nature they may both be too large or too small. In the former case, both personal and global experiences are overused and as a result the particle will be driven too far away from the optimum. For the latter case, both personal and global experiences are underused and as a result the convergence speed of the algorithm is reduced. They used sociobiological activity such as hunting to state that individuals balance between the weight of their own knowledge and the group's collective knowledge. In other words, the personal and global acceleration are somehow related to each other. When one acceleration is large, the other one should be small, and vice versa. Using this concept, they modified the main equation of PSO, Equation (5.1), to include a dependent acceleration and momentum [63].

It is a common belief that the need from every particle is different; they may need larger or smaller momentum, depending on which part of search process they are located.

They also need to have different emphasis on personal or global term in Equation (5.1) of MOPSO. Differentiated individual is a concept supported in the sociobiological studies [7]. As a result of differentiation, individuals perform functions more efficiently.

5.2.2 Related Work in Cultural Algorithm for Multiobjective Optimization

Cultural algorithm is an adaptive evolutionary computation method which is motivated by cultural evolution and learning in agent-based societies [3, 99]. CA consists of evolving agents whose experiences are gathered into a belief space consisting of various forms of symbolic knowledge. CA has shown its ability to solve different types of problems [3, 99-107] among which CAEP (cultural algorithm along with evolutionary programming) has shown successful results in solving MOPs [107]. Researchers have identified five basic sections of knowledge stored in belief space based upon the literature in cognitive science and semiotics: situational knowledge, normative knowledge, topographical knowledge [105], domain knowledge, and history knowledge [106]. The knowledge can swarm between different sections of belief space [108-110] which in turn affect the swarming of population. Furthermore, cultural algorithm has shown its ability [126-127] to optimize the control parameter of the optimization problem throughout the search process. In order to adjust the parameters of MOPSO, we need to store several types of required information, adopt this information in a proper manner, and update this data properly. All these needs can be satisfied by implementing cultural algorithm. CA provides groundwork for information repository through its belief space, use this

information by applying influence function to the knowledge space, and finally update the population and belief space simultaneously.

5.3 Cultural-based Multiobjective Particle Swarm Optimization

A summary of the pseudocode of the proposed algorithm is shown in Figure 5.2 and a block diagram of the algorithm is also shown in Figure 5.3. The population space (PSO) and its correspondent belief space (BLF) will be initialized at first. Then population space is evaluated using the fitness values. We apply acceptance function to select some particles which will be used to update belief space that consists of three sections: situational, normative, and topographical knowledge in the current version of implementation. Next we apply influence function and the belief space to adapt the parameters of the PSO for next iteration such as global acceleration, local acceleration, and momentum. We also use information on the belief space to select global best and personal best for next iteration. Afterward, particles in population space fly using personal and global best and newly adjusted momentum, local, and global acceleration. This process continues until the stopping criteria are met.

Initialize PSO and BLF at *t*=0

Repeat

- Evaluate PSO(t) using fitness.
- Apply ACCEPTANCE function to PSO(t) to select particles which affect BLF(t).
- Update BLF(t).
- Apply INFLUENCE function and BLF(t) to select *gbest*, *pbest* and to adapt the acceleration and momentum of particles in PSO(t).
- t=t+1.
- Update PSO(t) using new acceleration, momentum, *gbest*, and *pbest*.

Until Termination Criteria are met.

End

Figure 5.2 Pseudocode of the cultural MOPSO

In the remainder of this section, the acceptance function, different parts of belief space, and influence functions are thoroughly explained.

5.3.1 Acceptance Function

The belief space should be affected by the selected individuals. Therefore we apply Pareto nondomination as acceptance function to the current population of PSO. The nondominated set of particles at every iteration is chosen to update the belief space.

5.3.2 Belief Space

The belief space in the proposed cultural framework consists of three sections: situational, normative, and spatial (topographical) knowledge. Since the MOP problems of the interest have static landscapes, we only implement these three sections because the history and domain knowledge are mostly useful when fitness landscape is dynamic. In the following, type of information, the way to represent the knowledge, and the methodology to update the knowledge for each section of the belief space will be briefly explained.

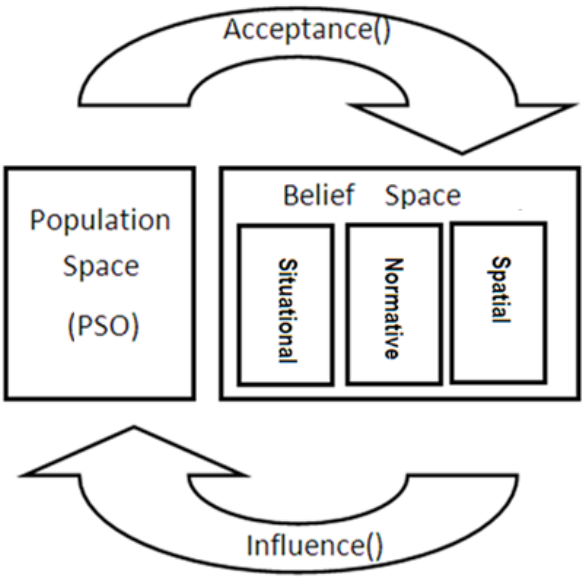


Figure 5.3 Schema of the adopted cultural framework, where the belief space consist situational knowledge, normative knowledge and spatial (topographical) knowledge

5.3.2.1 ***Situational Knowledge***

This part of belief space is used to archive the good exemplars of each individual.

Its representation is shown in Figure 5.4. $PA_i(t)$ $(i = 1,2, \dots, N$, where $N =$ number of particles) is the personal archive for the *i*-th particle that records nondominated set in the past history of the *i*-th particle. This means if we recall all past history of the positions of the *i*-th particle as $\Omega_i(t) = \{x_i(1), x_i(2), \dots, x_i(t)\}$, then for a given MOP with multiobjectives $\vec{f}(x)$, $PA_i(t)$ at time t is defined as following:

$$PA_i(t) := \left\{p \in \Omega_i(t) \middle| \nexists \acute{p} \in \Omega_i(t), \vec{f}(\acute{p}) \prec \vec{f}(p)\right\}, \qquad i = 1,2, \dots, N, \quad (5.3)$$

where $\vec{u} \prec \vec{v}$ means $\vec{u}$ dominates $\vec{v}$. Total number of personal archive PA_i is fixed and is equal to the number of particles, but the size of each $PA_i(t)$ varies in each time step. The situational knowledge will be used later to adapt local acceleration for MOPSO and also to select the personal best of each particle, $pbest_i$.

PA_1	PA_2	. . .	PA_N

Figure 5.4 Representation of situational knowledge

In order to update the situational knowledge we simply compare the current position of particle, $x_i(t)$, with its previously stored personal archive, $PA_i(t)$. If $x_i(t)$ dominates any member of $PA_i(t)$ then that member will be removed and the $x_i(t)$ will be placed in the archive. If $x_i(t)$ is dominated by all members of $PA_i(t)$, then $x_i(t)$ will not be added to the $PA_i(t)$. If $x_i(t)$ neither dominates nor is dominated by the members of

$PA_i(t)$, then $x_i(t)$ will be added to $PA_i(t)$. For $\forall p \in PA_i(t)$, the updating relation for personal archive will be as following:

$$PA_i(t+1) = \begin{cases} PA_i(t) \cup x_i(t) \backslash \{p\}, & \exists p, \vec{f}(x_i(t)) \prec \vec{f}(p) \\ PA_i(t), & \forall p, \vec{f}(p) \prec \vec{f}(x_i(t)). \\ PA_i(t) \cup x_i(t), & \vec{f}(p) \nprec \vec{f}(x_i(t)) \& \vec{f}(x_i(t)) \nprec \vec{f}(p) \end{cases} \quad (5.4)$$

Figure 5.5 shows a schematic view on how the personal archive is chosen. All the past history of the position of the *i*-th particle is shown in this figure. Among these positions, $x_i(1)$, $x_i(5)$ and $x_i(6)$, position at time $t = 1,5,$ and 6, will be selected as personal archive for the *i*-th particle, since these three positions, belong to the nondominated set as shown in Figure 5.5.

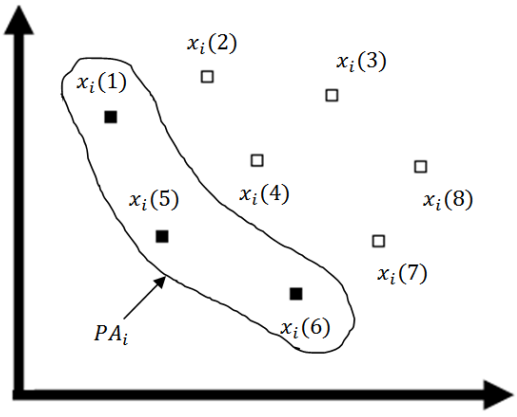


Figure 5.5 Schematic view of choosing the *i*-th element of situational knowledge, PA_i, among past history of position of the *i*-th particle. In this example, $PA_i = \{x_i(1), x_i(5), x_i(6)\}$. The schema is in objective space.

5.3.2.2 Normative Knowledge

Normative knowledge represents the best area in the objective space. It is represented as Figure 5.6 where $L(t) = [L_1(t)\, L_2(t) \ldots L_n(t)]$ and $U(t) = [U_1(t)\, U_2(t) \ldots\, U_n(t)]$ consist all lower and upper limits (in objective space) of the nondominated set of individuals that are generated by acceptance function at each iteration, respectively. This means that:

$$L_i(t) = \{\min_x(f_i(x)) \;\; |\forall x \in \Lambda(\mathrm{t})\}, \quad i = 1,2, \ldots, n, \tag{5.5}$$

$$U_i(t) = \{\max_x(f_i(x)) \;|\forall x \in \Lambda(\mathrm{t})\}, \quad i = 1,2, \ldots, n, \tag{5.6}$$

Figure 5.6 Representation of normative knowledge

where $\Lambda(\mathrm{t}) = \{x_1(t), x_2(t), \ldots, x_N(t)\}$, f_i is the *i*-th objective function in the objective vector of $\vec{f}$ and n is the number of objectives. Figure 5.7(a) demonstrates a schema of these two sections of normative knowledge for an example of two objective space. This section of normative space is used later to adapt global acceleration, also to find the global best of the MOPSO.

The other two elements of normative knowledge are $V_l(t) = [V_l^1(t)\, V_l^2(t) \ldots\, V_l^M(t)]$ and $V_u(t) = [V_u^1(t)\, V_u^2(t) \ldots\, V_u^M(t)]$ which are the lowest and highest values of velocity for the accepted individuals and M is the number of

decision space variables:

$$V_l^d(t) = \left\{\min_i \left(v_i^d\left(x_i(t)\right)\right),\ \forall x \in \Lambda(\mathrm{t})\right\}, \quad d = 1,2,\dots,M, \tag{5.7}$$

$$V_u^d(t) = \left\{\max_i \left(v_i^d\left(x_i(t)\right)\right),\ \forall x \in \Lambda(\mathrm{t})\right\}, \quad d = 1,2,\dots,M. \tag{5.8}$$

.

This section of the normative knowledge is later used to adapt momentum of the MOPSO. The normative knowledge is updated at each iteration based upon new nondominated set as follows (assuming all objectives are based on minimization problem).

$$L_i(t+1) = \begin{cases} f_i(x_k(t)), & \text{if } f_j(x_k(t)) < L_i(t),\ \exists k, x_k(t) \in \Lambda_{\mathrm{ns}}(\mathrm{t}) \\ L_i(t), & \text{otherwise} \end{cases}, \tag{5.9}$$

$$U_i(t+1) = \begin{cases} f_i(x_s(t)), & \text{if } f_i(x_s(t)) > U_i(t),\ \exists s, x_s(t) \in \Lambda_{\mathrm{ns}}(\mathrm{t}) \\ U_i(t), & \text{otherwise} \end{cases}, \tag{5.10}$$

where $x_k(t)$ and $x_s(t)$ are members of the nondominated set at time t, $\Lambda_{\mathrm{ns}}(\mathrm{t})$. Figure 5.7(b) shows the updating process of this section of normative knowledge. Note $L_1^t = L_1(t)$ and $L_1^{t+1} = L_1(t+1)$. Furthermore, $V_l(t)$ and $V_u(t)$ will be updated using the minimum and maximum velocities of the new set of nondominated individuals.

5.3.2.3 Topographical Knowledge

In order to represent topographical knowledge, we adopt the normative knowledge and then divide the space into grids of $s_1 \times s_2 \times \ldots \times s_n$ where s_i is the number of division in the *i*-th dimension of objective space, and n is the number of objectives. Each of the resultant cells will then be represented as shown in Figure 5.8 where $L_{cell} = [l_1 \ l_2 \ \ldots l_n]$ and $U_{cell} = [u_1 \ u_2 \ \ldots \ u_n]$ consist all lower and upper limits of the corresponding cell respectively, and N_{cell} is the number of nondominated individuals of the whole population located on that cell:

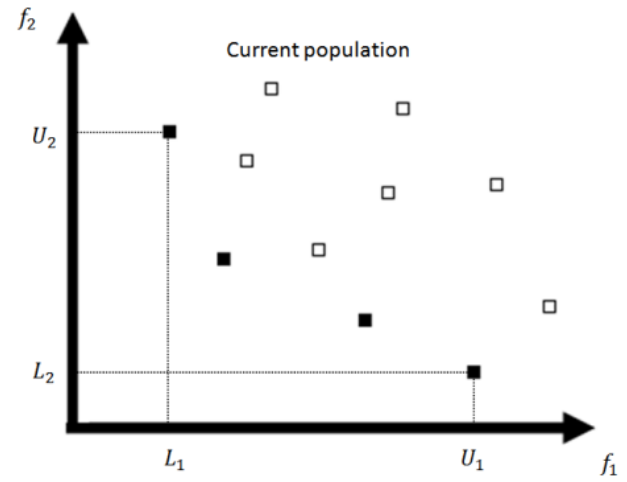
$$l_i(t) = L_i(t) + k\frac{U_i(t)-L_i(t)}{s_i}, \qquad k = 0,1,\ldots,s_i - 1, \quad i = 1,2,\ldots,n, \tag{5.11}$$

$$u_i(t) = l_i(t) + \frac{U_i(t)-L_i(t)}{s_i}, \qquad i = 1,2,\ldots,n, \tag{5.12}$$

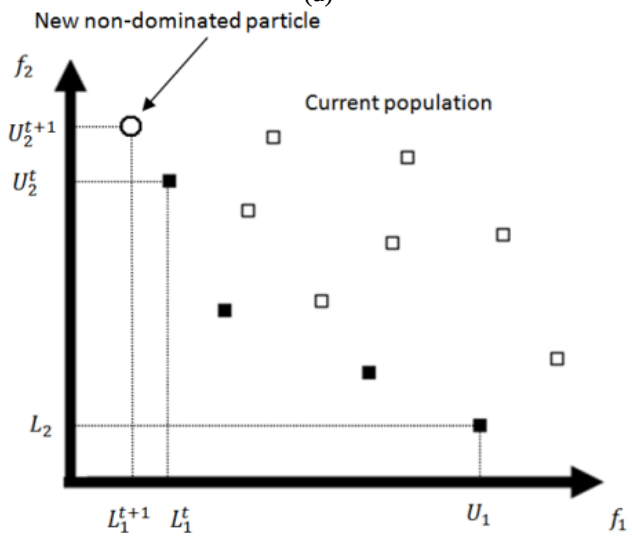
.

where $U_i(t)$ and $L_i(t)$ are given in Equations (5.5) and (5.6). Figure 5.9 demonstrates an example on how a cell will be represented.

At every iteration, the topographic knowledge will be updated. To do so, updated normative knowledge will be used to rebuild the cells and the nondominated points will be counted in each cell. Topographical knowledge will be used later to adapt global acceleration and also to find the global best, $gbest$.



(a)



(b)

Figure 5.7 Schema on how normative knowledge (a) can be found and (b) can be updated.

L_{cell}	U_{cell}	N_{cell}

Figure 5.8 Representation of knowledge in each cell.

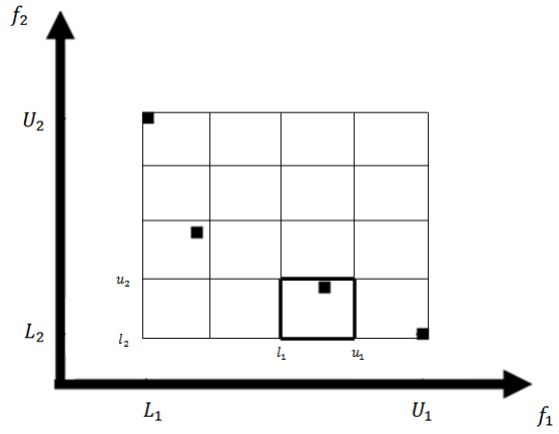


Figure 5.9 The cell representation for the highlighted cell in this example is $\langle \boldsymbol{L_{cell}} = [\boldsymbol{l_1}\ \boldsymbol{l_2}], \boldsymbol{U_{cell}} = [\boldsymbol{u_1}\ \boldsymbol{u_2}]\ , \boldsymbol{N_{cell}} = \boldsymbol{1}\rangle$, where $\boldsymbol{s_1} = \boldsymbol{s_2} = \boldsymbol{4}$.

5.3.3 Influence Functions

After the belief space is updated, the correspondent knowledge should be used to influence the MOPSO parameters. We propose to use the current knowledge in belief space to adapt PSO parameters, i.e., global acceleration, c_g, local acceleration, c_p, and momentum, ω.

5.3.3.1 Adapting Global Acceleration

We use topographical knowledge to adapt the global acceleration. It adjusts the direction and step size of the change in global acceleration. The motivation here is to give more or less weight to global search based upon the relative crowdedness of the cell in which *gbest* is located. If *gbest* moves from a very crowded cell to a less crowded one, we need to keep this direction, since it helps on preserving the diversity in the Pareto front, thus we increase the global acceleration. On the other hand, if *gbest* is moving from

a less crowded cell to a very crowded one, from one iteration to the next, then we need to decrease the weight of this direction. Finally if *gbest*'s cell population has not been changed, there is no need to either encourage or penalize its weight, therefore:

$$c_g(t+1) = \begin{cases} c_g(t) + \alpha.\left|N_g(t) - N_g(t+1)\right|, & \text{if } N_g(t) > N_g(t+1) \\ c_g(t) - \alpha.\left|N_g(t) - N_g(t+1)\right|, & \text{if } N_g(t) < N_g(t+1), \\ c_g(t), & \text{otherwise} \end{cases} \quad (5.13)$$

where $N_g(t)$ is the number of nondominated particles in the cell in which $gbest(t)$ is located, $N_g(t+1)$ is the number of nondominated particles in the cell in which $gbest(t+1)$ is located, $|.|$ denotes absolute value, and α is a normalization factor. Applying Equation (5.13) enforces a piecewise linear dynamic into variation of the global acceleration as a simple dynamic. The values of $N_g(t)$ and $N_g(t+1)$ are stored in topographical knowledge and can easily be used to adapt the global acceleration. The global acceleration will be limited in a range of $[c_{g,min}, c_{g,max}]$. Therefore $c_g(t+1)$ calculated in Equation (5.13) will then be compared to see if it is in this range:

$$c_g(t+1) = \begin{cases} c_{g,max}, & \text{if } c_g(t+1) > c_{g,max} \\ c_{g,min}, & \text{if } c_g(t+1) < c_{g,min} \\ c_g(t+1), & \text{otherwise} \end{cases}. \quad (5.14)$$

Equation (5.14) is required in order to keep the algorithm from being diverged.

We need to clarify that in regular MOPSO, c_g is remained constant in Equation (5.1), $\forall t, c_g(t+1) = c_g(t) = c_g$.

5.3.3.2 Adapting Local Acceleration

We use situational knowledge to build local grids in order to adjust local acceleration. We adjust the direction and the step size of the change in local acceleration. The procedure is similar to the one adopted for global acceleration. However in this case, we use the personal archive stored in the situational knowledge of the belief space. Therefore for each particle there will be different adjustment for its local acceleration based on the relative crowdedness of the cell in which $pbest_i$ is located. For each particle, we use its personal archive to build a local grid in order to find out the relative crowdedness of the $pbest_i$ from one iteration to the next. In Figure 5.10, a schema shows how a local grid is made using the situational knowledge for the *i*-th and *j*-th particle.

Each particle decides whether to increase or decrease its local acceleration separately based upon its personal archive. If the particle is moving from a less crowded cell to a more crowded one, we penalize its direction by decreasing the weight for local acceleration and if the particle is moving from a more crowded cell to a less crowded one, we need to keep that direction, thus increasing the weight for the local acceleration. However, if there is no change in crowdedness of the particle's cell, we should neither increase nor decrease the local acceleration. Thus:

$$c_{p,i}(t+1) = \begin{cases} c_{p,i}(t) + \beta.\left|N_{p,i}(t) - N_{p,i}(t+1)\right|, & \text{if } N_{p,i}(t) > N_{p,i}(t+1) \\ c_{p,i}(t) - \beta.\left|N_{p,i}(t) - N_{p,i}(t+1)\right|, & \text{if } N_{p,i}(t) < N_{p,i}(t+1), \\ c_{p,i}(t), & \text{otherwise} \end{cases} \quad (5.15)$$

where $N_{p,i}(t)$ is the number of nondominated particles located in the same cell of local grid of the *i*-th particle as $pbest_i(t)$, $N_{p,i}(t+1)$ is the number of nondominated particles located in the same cell of local grid of the *i*-th particle as $pbest_i(t+1)$, $|.|$ denotes absolute value, and β is a normalization factor. Piecewise linear behavior in Equation (5.15) imposes a simple dynamic to variation of the local acceleration. The local acceleration will also be restricted within a range of $[c_{p,min}, c_{p,max}]$. That means $c_{p,i}(t+1)$ calculated in Equation (5.15) will then be checked to see if it is in this range:

$$c_{p,i}(t+1) = \begin{cases} c_{p,max}, & \text{if } c_{p,i}(t+1) > c_{p,max} \\ c_{p,min}, & \text{if } c_{p,i}(t+1) < c_{p,min} \\ c_{p,i}(t+1), & \text{otherwise} \end{cases}. \quad (5.16)$$

Equation (5.16) is also required in order to keep the algorithm from being diverged. We also need to clarify in regular MOPSO, $c_{p,i}$ is kept constant in Equation (5.1), $\forall t, i: c_{p,i}(t+1) = c_{p,i}(t) = c_p$.

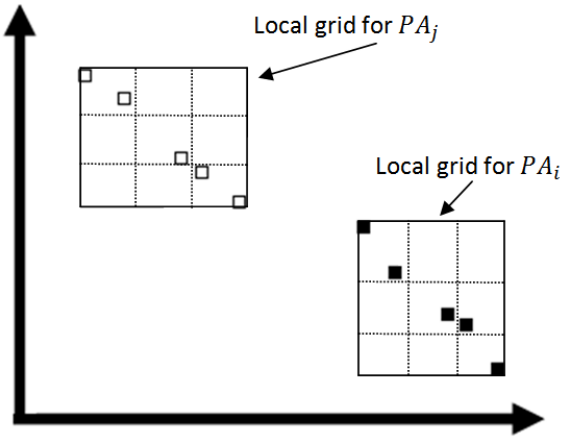


Figure 5.10 The schema of local grid for the personal archive of the *i*-th particle shown with ■, and local grid for the personal archive for the *j*-th particle shown with □.

5.3.3.3 Adapting Momentum

We use the normative knowledge to adapt the momentum of the particles. We adjust the direction of the momentum for each particle by adopting information of velocities of the best behaved particles. If any particle has velocity beyond the range of the best behaved particles we adjust it to be closer to this range:

$$\omega_i^d(t+1) = \begin{cases} \omega_i^d(t) + \Delta\omega, & \text{if } v_i^d(t) < V_l^d(t) \\ \omega_i^d(t) - \Delta\omega, & \text{if } v_i^d(t) > V_u^d(t) \\ \omega_i^d(t), & \text{otherwise} \end{cases}, \tag{5.17}$$

where $v_i^d(t)$ is the current velocity of the *i*-th particle in the *d*-th dimension. $V_l^d(t)$ and $V_u^d(t)$ are the information stored in normative knowledge section of belief space which are the lowest and highest velocity values for the current nondominated set of particles (see Equations (5.7) and (5.8)). $\Delta\omega$ is a predefined constant for step size of the

momentum. $\omega_i^d(t)$ is the velocity momentum for the *i*-th particle in the *d*-th dimension. The momentum needs to be limited in a range of $[\omega_{min}, \omega_{max}]$ to keep the algorithm from being diverged. That means $\omega_i^d(t+1)$ calculated in Equation (5.17) will need to be compared to see if it is in this range:

$$\omega_i^d(t+1) = \begin{cases} \omega_{max}, & \text{if } \omega_i^d(t+1) > \omega_{max} \\ \omega_{min}, & \text{if } \omega_i^d(t+1) < \omega_{min} \\ \omega_i^d(t+1), & \text{otherwise} \end{cases}. \quad (5.18)$$

Finally we need to clarify that in regular MOPSO, Equation (5.1), $\forall i, d, t: \omega_i^d(t+1) = \omega_i^d(t) = \omega$.

5.3.3.4 gbest Selection

We use the topographical knowledge stored in belief space to select *gbest* at each iteration. The method is borrowed from [58] which is based on selecting one nondominated point located in the least populated area of the objective space. We use roulette wheel selection to choose the appropriate cell which is more likely to be the least populated cell and then randomly choose a particle from that cell to be global leader of the particles. Each cell is assigned a fitness as [58]:

$$fitness_{cell} = \frac{10}{N_{cell}}, \quad (5.19)$$

where N_{cell} is the number of nondominated points located in that specific cell.

Probability of each cell to be selected in roulette wheel will then be proportional with this given fitness. Figure 5.11 shows the method to select *gbest* for an example with two objective functions. Any cell with one individual inside is twice more probable to be selected as *gbest* than any other cells consisting of two individuals.

5.3.3.5 $pbest$ Selection

In order to select the *pbest* we use the situational knowledge. Figure 5.12 shows the graphical representation of how *pbest* is selected. This algorithm has been shown experimentally to be one of the best methods to select *pbest* in order to preserve a good diversity of Pareto front [65]. In this figure, each square (□) represents a member of the personal archive for the *i*-th particle, $PA_i(t)$. $pbest_i(t)$ will be selected as a member of the archive that has the largest distance from all current population:

$$pbest_i(t) = \left\{p_m \in PA_i(t) \middle| \max_{m=1}^{N_i}\left(\sum_{i=1}^{N}(p_m - x_i(t))^2\right)\right\}, \tag{5.20}$$

where p_m is the member of personal archive of the *i*-th particle, $PA_i(t)$, and N_i is the number of particles in personal archive for the *i*-th particle, $PA_i(t)$.

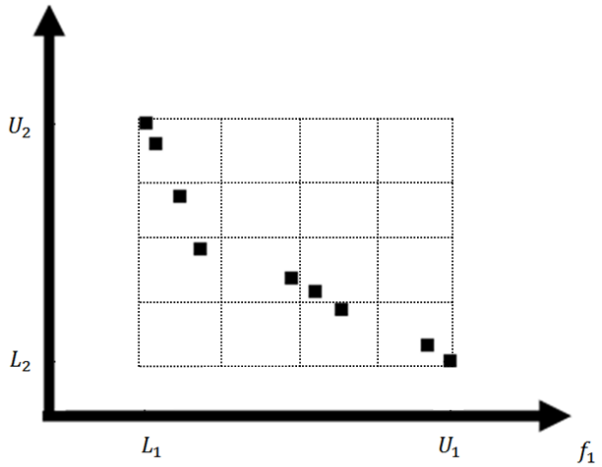


Figure 5.11 Method of selecting $gbest$ from topographical knowledge.

5.3.4 Global Archive

We preserve the best solution into a global archive which is limited in size. To update the global archive, each new nondominated solution will be compared with all members in the archive. This method is the same as method explained in [58]. If a new solution ($\acute{g}$) dominates any member of the global archive (GA) then that member will be deleted and $\acute{g}$ will be placed in the archive. If $\acute{g}$ is dominated by all members of the archive, then $\acute{g}$ will be disregarded. If $\acute{g}$ neither dominates nor is dominated by the members of the archive, then there will be two scenarios. If the size of the archive does not exceed the limit, the $\acute{g}$ will be added to the archive. However, if the archive is already full, then $\acute{g}$ will be added to the archive and another member which is located in the most populated area of the objective space will be deleted.

For $\forall g \in GA(t)$, the updating relation for global archive after receiving any new solution, $\acute{g}$, will be as following:

$$GA(t+1) = \begin{cases} GA(t) \cup \acute{g} \backslash \{g\}, & \exists g, \vec{f}(\acute{g}) \prec \vec{f}(g) \\ GA(t), & \forall g, \vec{f}(g) \prec \vec{f}(\acute{g}) \\ GA(t) \cup \acute{g}, & \vec{f}(g) \nprec \vec{f}(\acute{g}) \,\&\, \vec{f}(\acute{g}) \nprec \vec{f}(g) \,\&\, N_{GA} \leq GA_{size} \\ GA(t) \cup \acute{g} \backslash \{\tilde{g}\}, & \vec{f}(g) \nprec \vec{f}(\acute{g}) \& \vec{f}(\acute{g}) \nprec \vec{f}(g) \,\&\, N_{GA} > GA_{size} \end{cases}, \quad (5.21)$$

where N_{GA} is the current size of global archive, GA_{size} is the maximum size of global archive, and $\tilde{g}$ is located in the most populated area. To find $\tilde{g}$, we take advantage of grid structure using the members of global archive and then locate the most populated cell from that grid. $\tilde{g}$ will then be randomly selected from that cell and deleted.

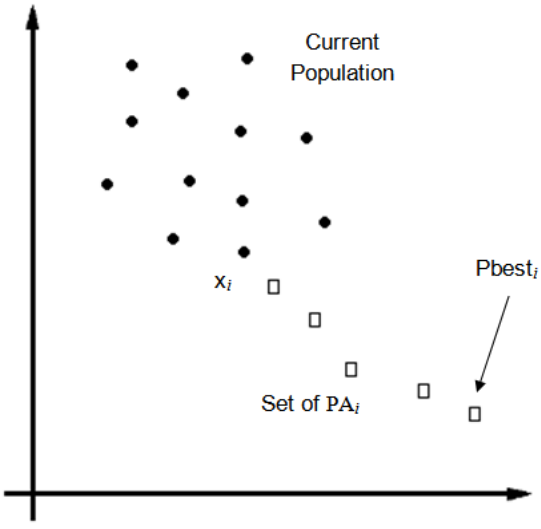


Figure 5.12 $pbest$ selection procedure from personal archive: The $pbest_i$ for particle x_i is selected among the set of personal archive, PA_i, in the objective space.

5.3.5 Time-decaying Mutation Operator

Due to tendency of immature convergence to local optima in PSO, a modified version of a mutation operator introduced in [58] is proposed for the particles which are not accepted through the acceptance function. The percentage of mutated particles, MP, is defined as following:

$$MP = (1 - \frac{t}{T})^{5/\mu}, \tag{5.22}$$

where μ is the mutation rate ($\mu > 0$), t is the current iteration and T is the final iteration. Adopting this form of mutation helps to scan a diverse region in the space at the beginning of the search process. As current time, *t*, increases, the percentage of the mutated particles approach to zero. This time-decaying mutation occurs in three ways:

(1) The number of particles that undergo the mutation is equal to:

$$N_{mutation} = MP.N_t, \tag{5.23}$$

where N_t is the number of dominated particles at the current iteration t which are not accepted through the acceptance function. These particles will be selected randomly.

(2) The range of mutation for each mutated particle will be time-decaying. For the *d*-th dimension of the particle, $x_i^d(t)$, this range is defined as follows:

$$R_{mutation} = MP.(x_{UP}^d - x_{LOW}^d), \tag{5.24}$$

where x_{UP}^d and x_{LOW}^d are the upper and lower limits of the particle in the *d*-th dimension. The mutated particle will then be a random number in the range of $x_i^d \pm R_{mutation}$. Incorporating the time-decaying *MP* into this equation results a wider search range for

every mutated particle at the beginning of the search. As the iteration increases, the search range for the mutated particle will be narrower.

(3) The mutation will happen for some dimensions of the selected particle. The number of dimensions of those selected particles is time-decaying as following:

$$N_{Dim} = \lfloor MP.M \rfloor, \tag{5.25}$$

where M is the number of decision variables and $\lfloor . \rfloor$ is the rounding operator. Similarly, incorporating *MP* into the number of dimensions for mutation will give one the benefit of having more number of dimensions to be mutated at the beginning of the search, while it approaches zero, as we reach the end of the process. These dimensions will be selected randomly.

In this design, in the beginning, most particles in the population are subjected to mutation (as well as the full range of the decision variables). This intends to produce a highly explorative behavior in the algorithm. As the number of iterations increases, the effect of the mutation decays.

5.4 Comparative Study and Sensitivity Analysis

This section consists of two experiments. In the first experiment, the performance of the cultural MOPSO is evaluated against selected MOPSOs, while the second experiment tests the sensitivity of the proposed algorithm with respect to its tuning parameters.

5.4.1 Comparison Experiment

In this experiment, five state-of-the-art MOPSOs have been chosen in order to compare their performance with that of the cultural MOPSO: sigma MOPSO [60], OMOPSO [62], NSPSO [57], cluster MOPSO [128] and MOPSO [58].

5.4.1.1 Parameter Settings

Each of the six algorithms used here perform 200 iterations (as suggested in most publications), and the archive size used is 100. The parameter settings for all of the MOPSOs are summarized in Table 5.1. All of the algorithms are implemented in Matlab using real-number representation for decision variables. However, binary representation of decision variables can also be adopted. For each experiment, 100 independent runs were conducted to collect the statistical results. All algorithms produced final Pareto fronts of fixed size population except for cluster MOPSO, which does not have a fixed archive size.

5.4.1.2 Benchmark Test Functions

To evaluate the performance of Cultural MOPSO against selected MOPSOs, six benchmark test problems are used [129-130]: ZDT1, ZDT2, ZDT3, ZDT4, DTLZ5, and DTLZ6.

Table 5.1 Parameter settings for all MOPSOs

	Population size	Archive size	No. of iterations	Other parameters or remarks
Cultural MOPSO	100	100	200	$\omega_{min} = 0.1,\ \omega_{max} = 0.9,\ c_{g,min} = 1, c_{g,max} = 3,\ c_{p,min} = 1, c_{p,max} = 3, \forall i, d: \omega_{i,d}^{1} = 0.4\ ;\ \ c_{g}^{1} = 2\ ;\ \forall i: c_{p,i}^{1} = 2$
Sigma MOPSO	100	100	200	Fixed inertial weight value, w = 0.4; Turbulence Factor, R is $[-1,1]$
OMOPSO	100	100	200	Mutation probability = $1/codesize$ and the values of w, c_1 and c_2 are random values $\varepsilon = 0.0075$ (Note: For ZDT6, $\varepsilon = 0.001$)
NSPSO	100	-	200	Fixed inertial weight value, w = 0.4
Cluster MOPSO	100	Not fixed	200	No. subswarms, $n_{swarm} = 4$; internal iterations, $stmax = 5$
MOPSO	100	100	200	50 divisions adaptive grid; mutation probability = 0.5

Test problems ZDT1, ZDT2, ZDT3, and ZDT4 are two-objective minimization problems with 150 decision variables each. Note that the number of decision variables has been increased from its standard size of 30 variables. This is to exploit all selected MOPSOs when encountered with a higher number of decision variables. Test problem ZDT1 has the convex Pareto fronts, while test problem ZDT2 has non-convex Pareto fronts. Both ZDT1 and ZDT2 test the ability of algorithm to find a fine spread of Pareto front. Test problem ZDT3 possesses a disconnected non-convex Pareto front. It is a good indicator to exploit the ability of algorithms to search for all of the disconnected regions and to maintain a uniform spread on those disconnected regions. Test problem ZDT4 presents a complexity with multi-modality characteristic. It has the difficulty of finding the global Pareto front in all of the 21^9 local segments. Test problem DTLZ5 is a three-

objective minimization problem with 150 decision variable. Note that the number of decision variables has been increased from its standard size of 12. This is again to exploit all MOSPOs when encountered with a higher number of decision variables. DTLZ5 has a three dimensional curve as Pareto front located on the surface of the unit sphere. Its difficulty is that the density of solutions closer to the Pareto front curve becomes much less than anywhere else in the search space. Test problem DTLZ6, a three-objective minimization problem with 22 decision variables, has four disconnected set of Pareto front regions. This problem tests an algorithm's ability to maintain subpopulation in multiple Pareto-optimal regions. The detailed formulation of these benchmark test functions are presented in Appendix A for reference.

5.4.1.3 Qualitative Performance Comparisons

For qualitative comparison, the plots of final Pareto fronts are presented for visualization. The resulted nondominated fronts (given the same initial population from a single run) of the six MOPSOs on all test functions are demonstrated in Figures 5.13 to 5.18. These figures show cultural MOPSO is able to find the well-extended, near-optimal Pareto fronts despite a very large number of decision variables for test functions ZDT1 to ZDT4 and DTLZ5. MOPSO [58] provides the second best results, where it can produce fine Pareto fronts similar to the ones produced by cultural MOPSO for most benchmark test functions. Cluster MOPSO, sigma MOPSO, and NSPSO produce the worst Pareto

fronts since they have difficulty in converging toward the true Pareto front, especially for functions ZDT1 to ZDT4 and DTLZ5 with high-dimensional decision spaces.

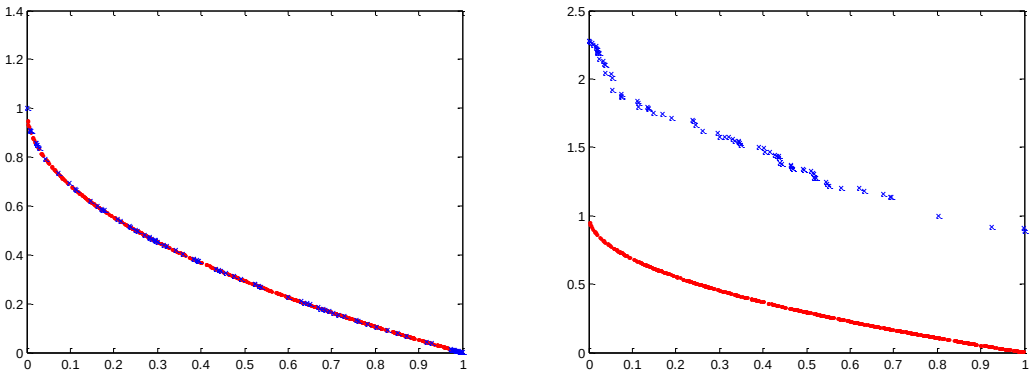
5.4.1.4 Quantitative Performance Evaluations

Two performance metrics are adopted to measure the performance of algorithms with respect to the dominance relations.

Hypervolume Indicator [131]: The hypervolume indicator is a measure to indicate how well the algorithm converges to the true Pareto front and how diversified the solution is. It calculates the size of the region covered by a defined reference point. For the minimization problems, a larger value indicates a better nondominated set. If hypervolume indicator for nondominated set of A, $I_H(A)$, is greater than hypervolume indicator for nondominated set of B, $I_H(B)$, then set B is not better than A for all pairs. This means a certain portion of objective space is dominated by A but not by B.

The performance metric for hypervolume indicator is computed for each selected MOPSOs along with cultural MOPSO on 100 independent runs. Figure 5.19 shows the box plots of I_H values for all MOPSOs for different test functions. This figure clearly indicates that cultural MOPSO outperforms sigma MOPSO, OMOPSO, NSPSO, and cluster MOPSO. However it does not provide conclusive relative performance of cultural MOPSO with respect to MOPSO due to their closeness of box plots in the scale of the figure. For further analysis, the Mann-Whitney rank-sum statistical test is conducted to evaluate the significant difference between two independent samples for all pairs [132] and the results are illustrated in Table 5.2. In this table, for each test function and each

MOPSO algorithm, 100 independent runs had been performed; therefore there are 100 I_H (hypervolume indicator) for each test function and each MOPSO algorithms. Then the rank-sum test (α=0.05) is performed between 100 I_H of the proposed algorithm with 100 I_H of another MOPSO algorithm (for each test function separately). As a result, Table 5.2 indicates that except for the test function ZDT4 in which both cultural MOPSO and MOPSO equally outperform other algorithms (i.e., based upon the p-values), cultural MOPSO performs better than all selected MOPSOs in all test functions.



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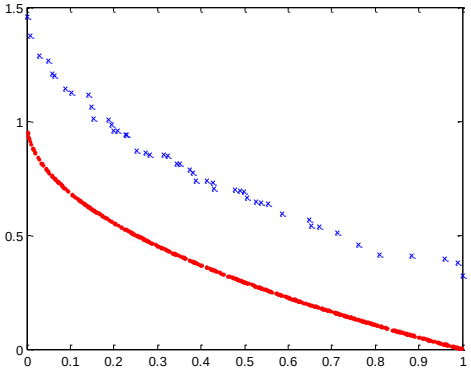

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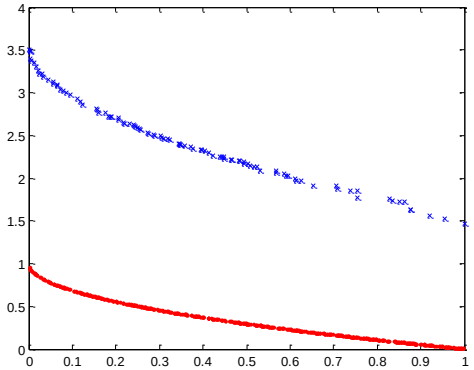

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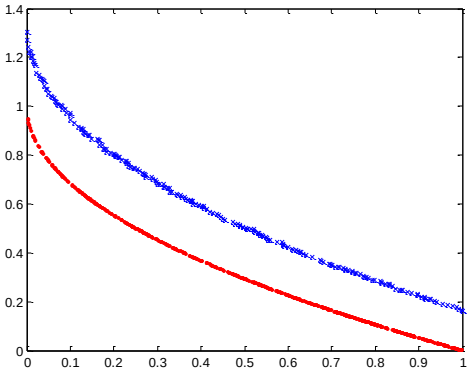

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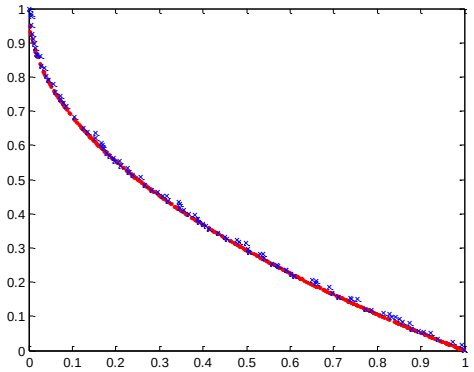
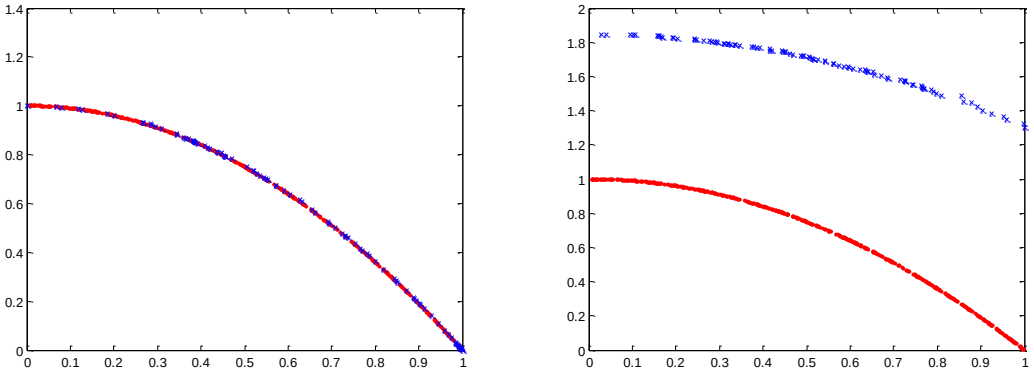


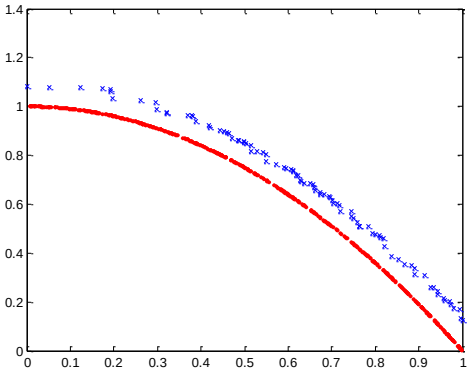
(f)

Figure 5.13 Pareto fronts produced by (a) cultural MOPSO, (b) sigma MOPSO, (c) OMOPSO, (d) NSPSO, (e) cluster MOPSO, and (f) MOPSO on test function ZDT1.

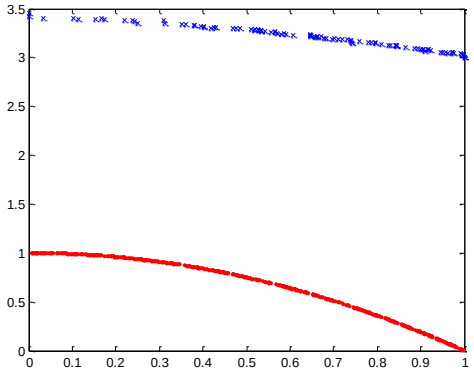


(a)

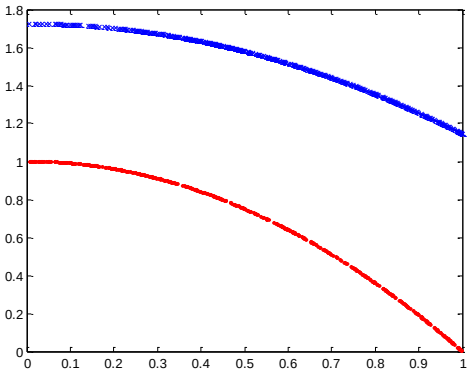
(b)



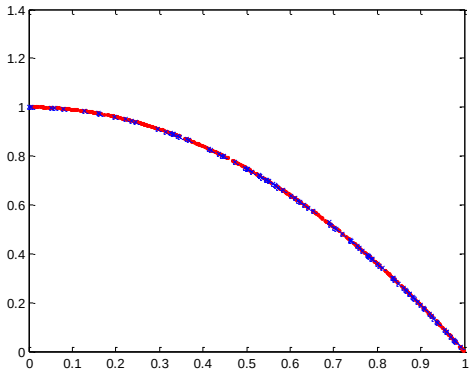
(c)



(d)

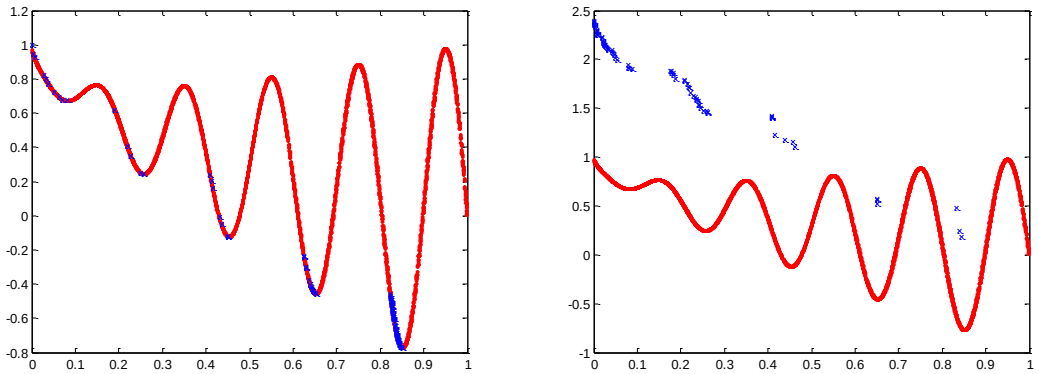


(e)



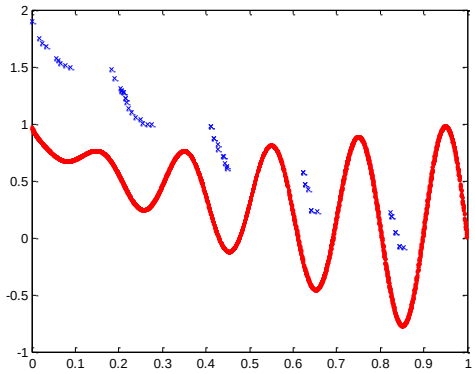
(f)

Figure 5.14 Pareto fronts produced by (a) cultural MOPSO, (b) sigma MOPSO, (c) OMOPSO, (d) NSPSO, (e) cluster MOPSO, and (f) MOPSO on test function ZDT2.

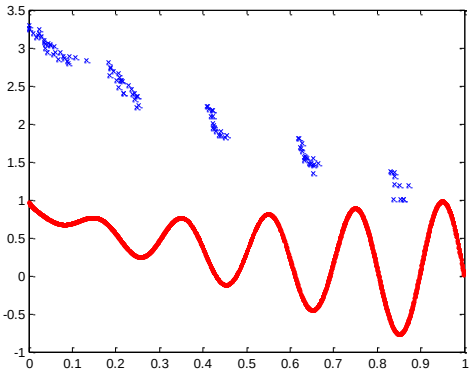


(a)

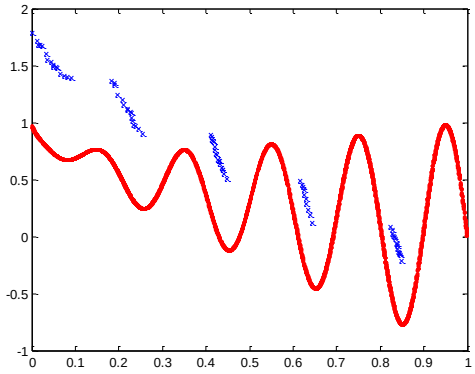
(b)



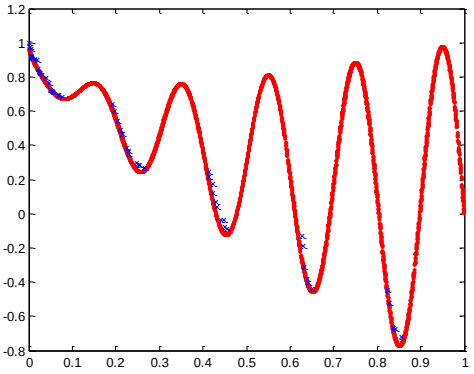
(c)



(d)



(e)



(f)

Figure 5.15 Pareto fronts produced by (a) cultural MOPSO, (b) sigma MOPSO, (c) OMOPSO, (d) NSPSO, (e) cluster MOPSO, and (f) MOPSO on test function ZDT3.

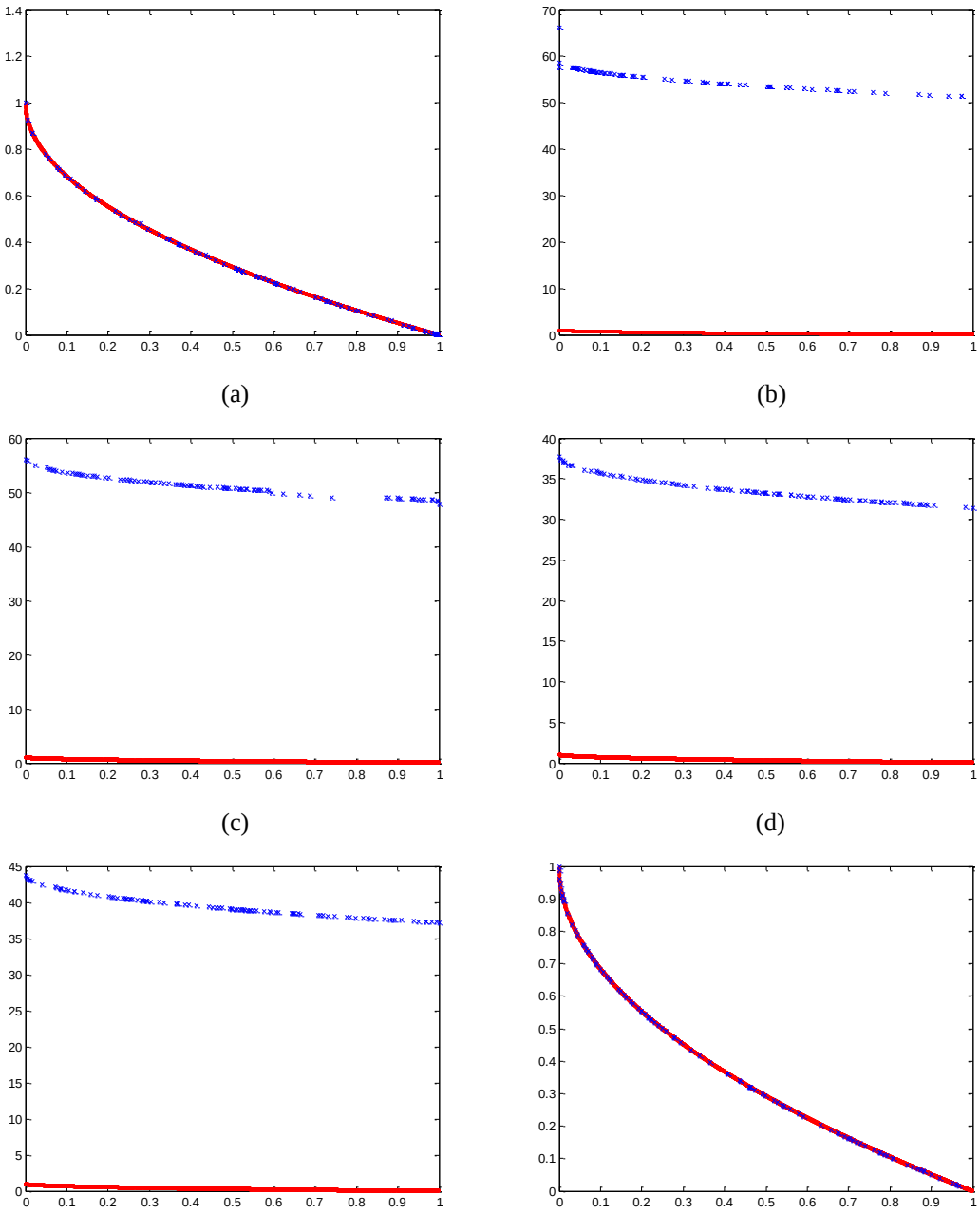


Figure 5.16 Pareto fronts produced by (a) cultural MOPSO, (b) sigma MOPSO, (c) OMOPSO, (d) NSPSO, (e) cluster MOPSO, and (f) MOPSO on test function ZDT4.

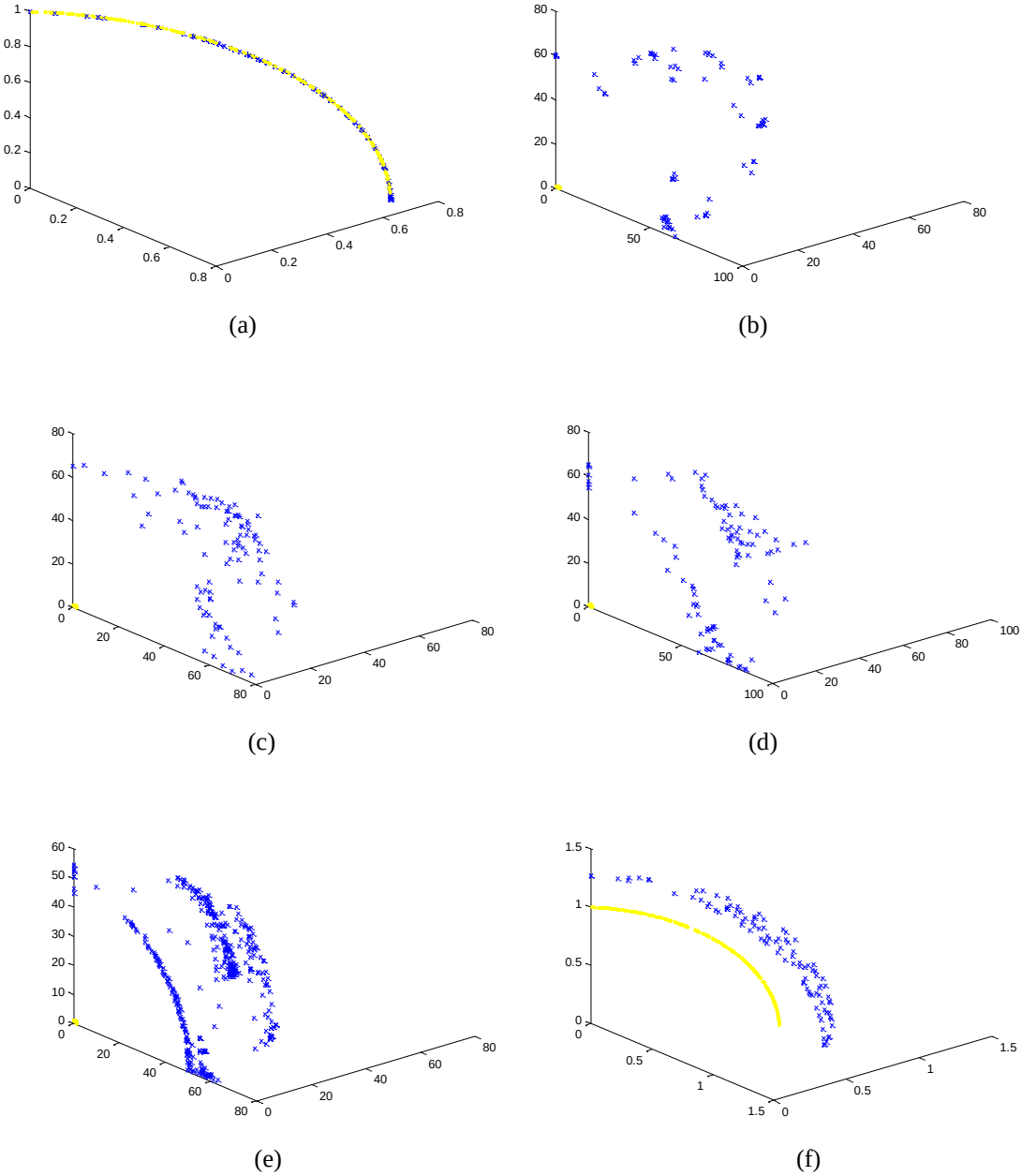
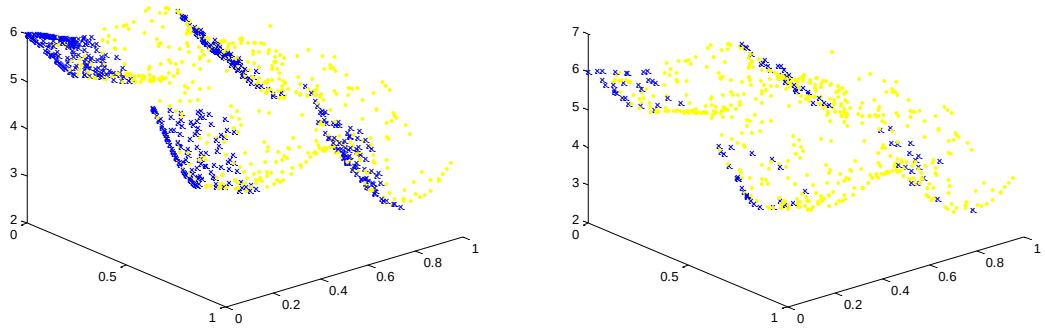
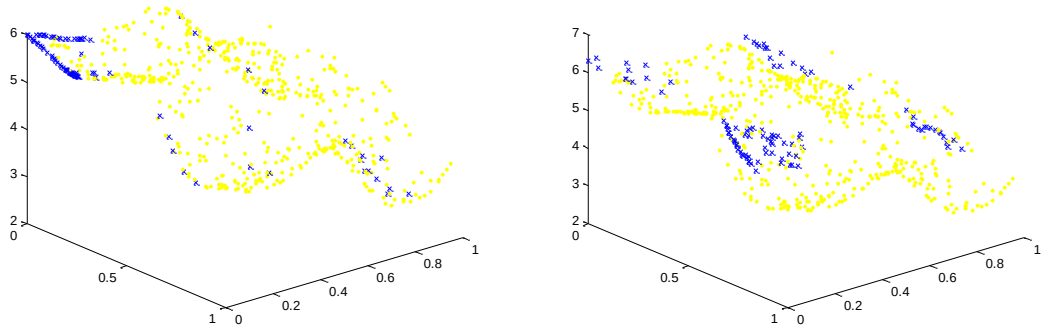


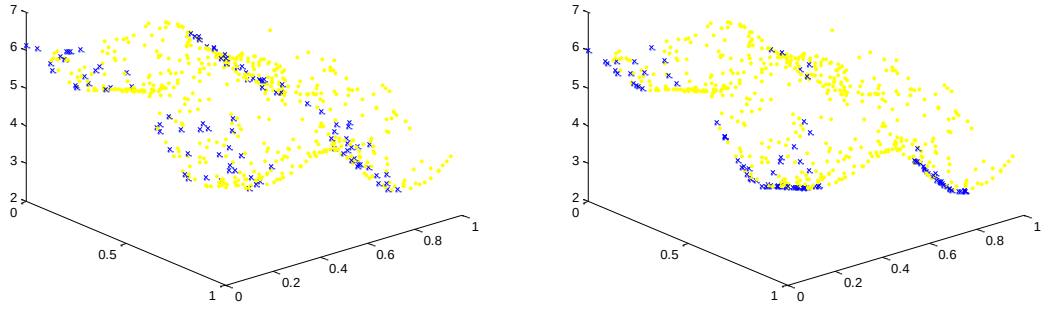
Figure 5.17 Pareto fronts produced by (a) cultural MOPSO, (b) sigma MOPSO, (c) OMOPSO, (d) NSPSO, (e) cluster MOPSO, and (f) MOPSO on test function DTLZ5.



(a) (b)



(c) (d)



(e) (f)

Figure 5.18 Pareto fronts produced by (a) cultural MOPSO, (b) sigma MOPSO, (c) OMOPSO, (d) NSPSO, (e) cluster MOPSO, and (f) MOPSO on test function DTLZ6.

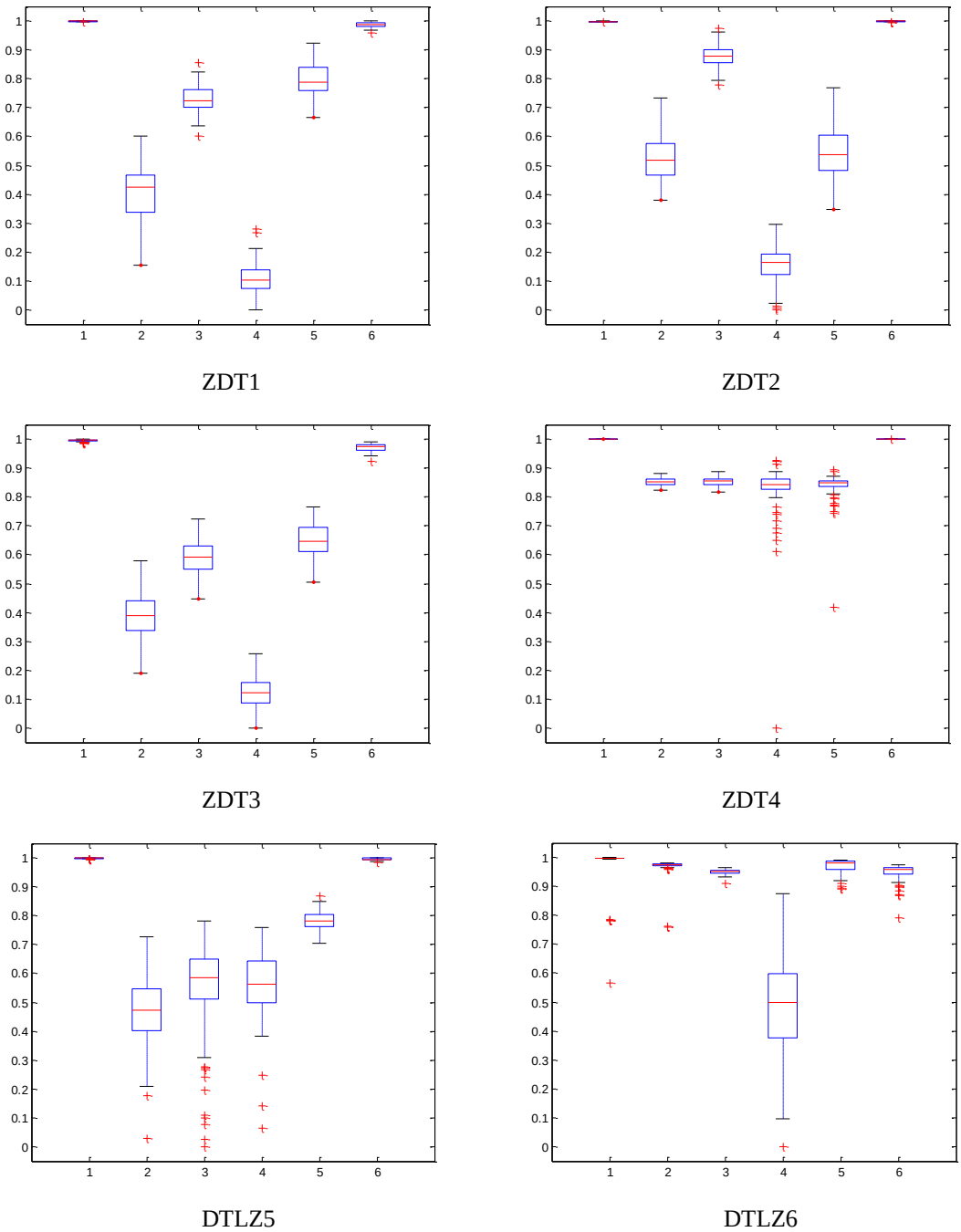


Figure 5.19 Box plot of hypervolume indicator for all test functions. Column numbers refer to (1) cultural MOPSO, (2) sigma MOPSO, (3) OMOPSO, (4) NSPSO, (5) cluster MOPSO, and (6) MOPSO.

Additive Binary Epsilon Indicator [133]: This binary indicator shows whether a nondominated set is better than another. Assume that the additive binary epsilon indicator for the nondominated sets of A and B are denoted as $I_{\varepsilon+}(A,B)$ and $I_{\varepsilon+}(B,A)$, respectively. If $I_{\varepsilon+}(A,B) < 0$ and $I_{\varepsilon+}(B,A) > 0$, then A is strictly better than B. If $I_{\varepsilon+}(B,A) \leq 0$ and $I_{\varepsilon+}(A,B) < I_{\varepsilon+}(B,A)$, then it concludes that A weakly dominates B. Finally, if $I_{\varepsilon+}(A,B) > 0$ and $I_{\varepsilon+}(B,A) > 0$, then A and B are incomparable. Again, Mann-Whitney rank-sum statistical test is conducted to check if there is significant difference between the two distributions for $I_{\varepsilon+}(A,B)$ and $I_{\varepsilon+}(B,A)$ [132].

Table 5.2 Testing of the distribution of I_H values using Mann-Whitney rank-sum statistical test. Each cell in the table presents the z-value and p-value as the form of (z-value, p-value) with respect to the alternative hypothesis (p-value < α=0.05) for pair of cultural MOPSO and a selected MOPSO. The distribution of cultural MOPSO is significantly different or better than those selected MOPSOs unless stated.

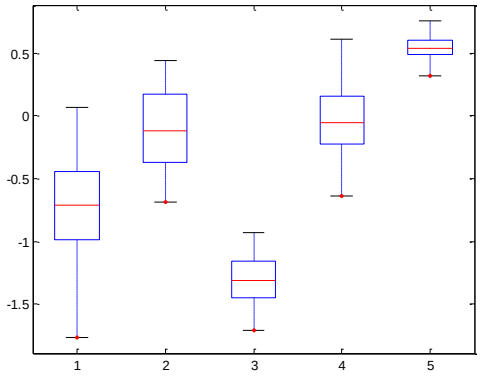
Test Functions	I_H (cultural MOPSO) AND				
	I_H (sigmaMOPSO)	I_H (OMOPSO)	I_H (NSPSO)	I_H (clusterMOPSO)	I_H (MOPSO)
ZDT1	(-12.2157, $2.6e^{-34}$)	(-12.2157, $2.6e^{-34}$)	(-12.2157, $2.6e^{-34}$)	(-12.2157, $2.6e^{-34}$)	(-11.5022, $1.3e^{-30}$)
ZDT2	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-5.6407, 1.7e-8)
ZDT3	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-12.0898, 1.2e-33)
ZDT4	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-1.3183,0.18) **No Difference**
DTLZ5	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-10.1520, 3.2e-24)
DTLZ6	(-11.0233, 3.0e-28)	(-10.9942, 4.1e-28)	(-12.0984, 1.1e-33)	(-10.9940, 4.1e-28)	(-10.9940, 4.1e-28)

Figures 5.20 to 5.25 illustrate the results for additive binary ε-indicator via box plots where each figure gives the results for a test function. Each figure consists two box plots of $I_{\varepsilon+}(A, X_{1-5})$ and $I_{\varepsilon+}(X_{1-5}, A)$, in which A denotes the cultural MOPSO and X_{1-5} represent sigma MOPSO, OMOPSO, NSPSO, cluster MOPSO, and MOPSO, respectively. For ZDT1 in Figure 5.20, $I_{\varepsilon+}(A, X_{1-4}) < 0$ and $I_{\varepsilon+}(X_{1-4}, A) > 0$, which indicates that cultural MOPSO is strictly better than sigma MOPSO, OMOPSO, NSPSO, and cluster MOPSO. It also shows that $I_{\varepsilon+}(A, X_5) > 0$ and $I_{\varepsilon+}(X_5, A) > 0$, which indicates that cultural MOPSO and MOPSO are incomparable. For ZDT2 and ZDT3 in Figures 5.21 and 5.22, $I_{\varepsilon+}(A, X_{1,3,4}) < 0$ and $I_{\varepsilon+}(X_{1,3,4}, A) > 0$ which indicates that cultural MOPSO is strictly better than sigma MOPSO, NSPSO and cluster MOPSO. It also shows that $I_{\varepsilon+}(A, X_2) \cong 0$ and $I_{\varepsilon+}(A, X_2) < I_{\varepsilon+}(X_2, A)$ which indicates that cultural MOPSO weakly dominates OMOPSO. Lastly, it shows that $I_{\varepsilon+}(A, X_5) > 0$ and $I_{\varepsilon+}(X_5, A) > 0$, which implies that cultural MOPSO and MOPSO are incomparable. For ZDT4 in Figure 5.23, $I_{\varepsilon+}(A, X_{1-4}) < 0$ and $I_{\varepsilon+}(X_{1-4}, A) > 0$, which indicates that cultural MOPSO is strictly better than sigma MOPSO, OMOPSO, NSPSO, and cluster MOPSO. It also shows that $I_{\varepsilon+}(A, X_5) > 0$ and $I_{\varepsilon+}(X_5, A) > 0$, which indicates that cultural MOPSO and MOPSO are incomparable.

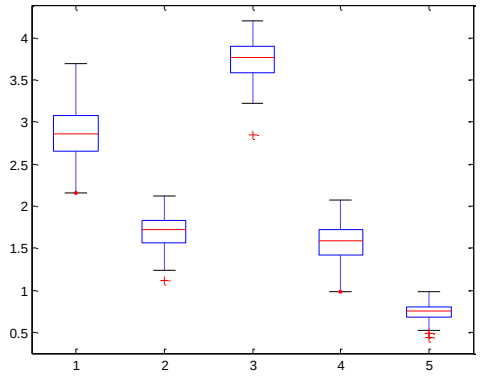
For DTLZ5 in Figure 5.24, $I_{\varepsilon+}(A, X_{1,2,3}) \cong 0$ and $I_{\varepsilon+}(X_{1,2,3}, A) > 0$, which indicates that cultural MOPSO weakly dominates sigma MOPSO, OMOPSO, and NSPSO. It also shows that $I_{\varepsilon+}(A, X_4) < 0$ and $I_{\varepsilon+}(X_4, A) > 0$, which indicates that

cultural MOPSO is strictly better than cluster MOPSO. Finally, it shows that $I_{\varepsilon+}(A, X_5) > 0$ and $I_{\varepsilon+}(X_5, A) > 0$, which implies that cultural MOPSO and MOPSO are again incomparable. Finally for DTLZ6 in Figure 5.25, it shows that $I_{\varepsilon+}(A, X_3) < 0$ and $I_{\varepsilon+}(X_3, A) > 0$, which indicates that cultural MOPSO is strictly better than NSPSO. It also shows that $I_{\varepsilon+}\left(A, X_{1,2,4,5}\right) > 0$ and $I_{\varepsilon+}\left(X_{1,2,4,5}, A\right) > 0$, which implies that cultural MOPSO is incomparable with sigma MOPSO, OMOPSO, cluster MOPSO, and MOPSO.

For further analysis, the distributions of additive binary ε-indicator values are tested using the Mann-Whitney rank-sum statistical test, which are illustrated in Table 5.3. In this table, for each test function and each MOPSO algorithm, 100 independent runs have been used to compute a pair of $I_{\varepsilon+}(A, B)$ and $I_{\varepsilon+}(A, B)$ between each run of the proposed algorithm with each run of another MOPSO algorithm (for each test function separately). As a result, only for test function ZDT2, there was no statistically significant difference between the proposed method and one of the chosen MOPSOs. The p-values for different test function in the rightmost column of Table 5.3 show that cultural MOPSO performs better than MOPSO except for the function ZDT2 where there is no difference between the two algorithms. Also looking at the p-values for the test function DTLZ6 in this table, it illustrates that cultural MOPSO outperforms other MOPSOs. Overall when the results in Table 5.3 is combined with the box plots in Figures 5.20 to 5.25, we can conclude that cultural MOPSO is statistically better than most MOPSOs.

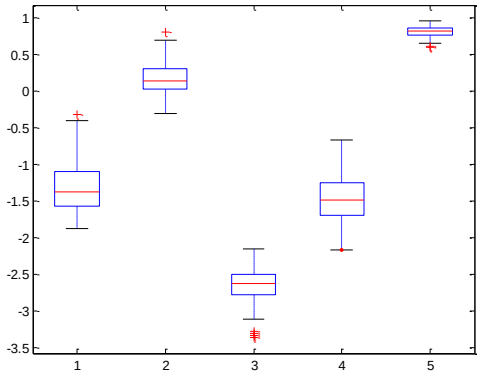


$I_{\varepsilon+}(cultural\ MOPSO, X_{1-5})$

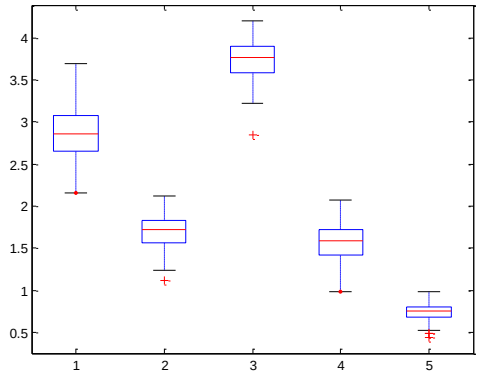


$I_{\varepsilon+}(X_{1-5}, cultural\ MOPSO)$

Figure 5.20 Box plot for additive binary epsilon indicator ($I_{\varepsilon+}$values) on test function ZDT1 (X_{1-5} refer to sigma MOPSO, OMOPSO, NSPSO, cluster MOPSO, and MOPSO respectively.)

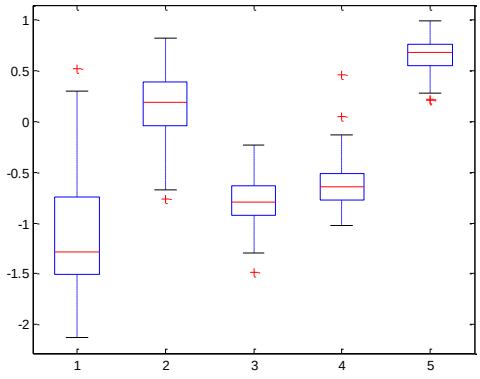


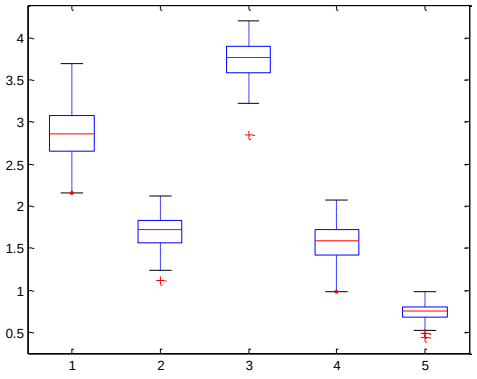
$I_{\varepsilon+}(cultural\ MOPSO, X_{1-5})$



$I_{\varepsilon+}(X_{1-5}, cultural\ MOPSO)$

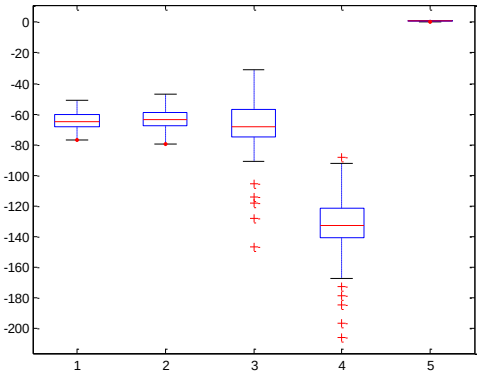
Figure 5.21 Box plot for additive binary epsilon indicator ($I_{\varepsilon+}$values) on test function ZDT2 (X_{1-5} refer to sigma MOPSO, OMOPSO, NSPSO, cluster MOPSO, and MOPSO respectively.)

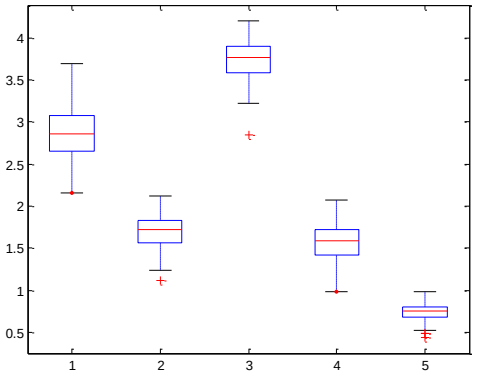




$I_{\varepsilon+}(cultural\ MOPSO, X_{1-5})$ $I_{\varepsilon+}(X_{1-5}, cultural\ MOPSO)$

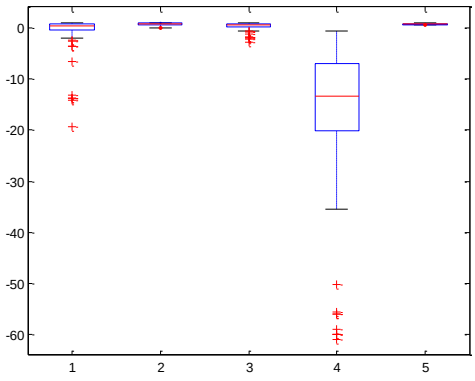
Figure 5.22 Box plot for additive binary epsilon indicator ($I_{\varepsilon+}$values) on test function ZDT3 (X_{1-5} refer to sigma MOPSO, OMOPSO, NSPSO, cluster MOPSO, and MOPSO respectively.)

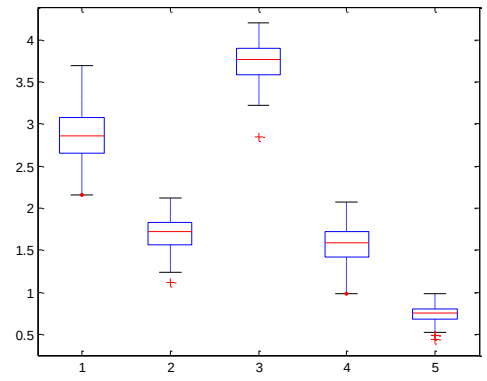




$I_{\varepsilon+}(cultural\ MOPSO, X_{1-5})$ $I_{\varepsilon+}(X_{1-5}, cultural\ MOPSO)$

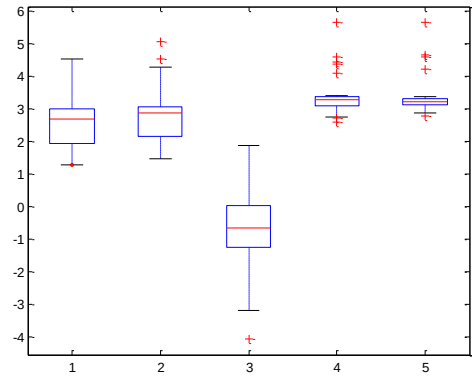
Figure 5.23 Box plot for additive binary epsilon indicator ($I_{\varepsilon+}$values) on test function ZDT4 (X_{1-5} refer to sigma MOPSO, OMOPSO, NSPSO, cluster MOPSO, and MOPSO respectively.)

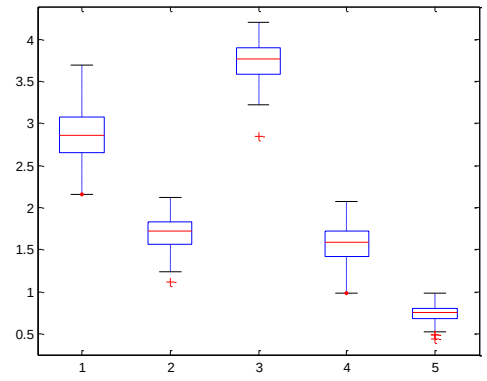




$I_{\varepsilon+}(cultural\ MOPSO, X_{1-5})$ $I_{\varepsilon+}(X_{1-5}, cultural\ MOPSO)$

Figure 5.24 Box plot for additive binary epsilon indicator ($I_{\varepsilon+}$values) on test function DTLZ5 (X_{1-5} refer to sigma MOPSO, OMOPSO, NSPSO, cluster MOPSO, and MOPSO respectively.)





$I_{\varepsilon+}(cultural\ MOPSO, X_{1-5})$ $I_{\varepsilon+}(X_{1-5}, cultural\ MOPSO)$

Figure 5.25 Box plot for additive binary epsilon indicator ($I_{\varepsilon+}$values) on test function DTLZ6 (X_{1-5} refer to sigma MOPSO, OMOPSO, NSPSO, cluster MOPSO, and MOPSO respectively.)

Table 5.3 Testing of the distribution of $I_{\varepsilon+}$ values using Mann-Whitney rank-sum statistical test. Each cell in the table presents the z-value and p-value as the form of (z-value, p-value) with respect to the alternative hypothesis (p-value < α=0.05) for pair of cultural MOPSO (shown by *A*) and other selected MOPSOs (shown by X_{1-5} referring to sigma MOPSO, OMOPSO, NSPSO, cluster MOPSO, and MOPSO respectively). The distribution of cultural MOPSO is significantly different or better than those selected MOPSOs unless stated.

Test Functions	$I_{\varepsilon+}(A,X_1)$ and $I_{\varepsilon+}(X_1,A)$	$I_{\varepsilon+}(A,X_2)$ and $I_{\varepsilon+}(X_2,A)$	$I_{\varepsilon+}(A,X_3)$ and $I_{\varepsilon+}(X_3,A)$	$I_{\varepsilon+}(A,X_4)$ and $I_{\varepsilon+}(X_4,A)$	$I_{\varepsilon+}(A,X_5)$ and $I_{\varepsilon+}(X_5,A)$
ZDT1	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-10.1852, 2.3e-34)
ZDT2	(-12.2157, 2.6e-34)	(-12.2084, 2.8e-34)	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-0.3506,0.73) **No Difference**
ZDT3	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-3.5783, 3.5e-4)
ZDT4	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-9.8701, 5.6e-23)
DTLZ5	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)	(-12.1766, 4.1e-34)	(-12.1766, 4.1e-34)
DTLZ6	(-11.7441, 7.6e-32)	(-12.1473, 5.9e-34)	(-12.2108, 2.7e-34)	(-12.2157, 2.6e-34)	(-12.2157, 2.6e-34)

5.4.2 Sensitivity Analysis

One may argue on many parameters associated with the cultural MOPSO and the difficulty of selecting appropriate set of parameters. There are several algorithms in the literature to find the optimum value for the parameters of optimization process. Fogel *et. al* [134] introduced meta-evolutionary programming by simultaneously evolving the parameters of the optimization problem such as mutation rate along with the potential solution of the problem. Self-adaptation as a step-size control mechanism was proposed [135-136] by applying evolutionary operator into object variables and control parameters

at the same time to optimize the control parameter along with finding the solution of the problem. In order to assess the robustness of the algorithm, a sensitivity analysis is conducted with respect to the lower and upper limit of personal acceleration, $c_{p,min}$ and $c_{p,max}$, lower and upper limit of global acceleration, $c_{g,min}$ and $c_{g,max}$, lower and upper limit of momentum, ω_{min} and ω_{max}, grid size, $s_i = s, \forall i$, population size, N, and mutation rate, μ. In Table 5.4, the values for these parameters are shown.

Table 5.4 Parameter selection for sensitivity analysis

Changing parameter	Other parameters
$c_{p,min} = 0.5, 1, 1.5$	$c_{p,max} = 3, c_{g,min} = 1, c_{g,max} = 3, \omega_{min} = 0.1, \omega_{max} = 0.9, s = 10, N = 100, \mu = 1$
$c_{p,max} = 2.5, 3, 3.5$	$c_{p,min} = 1, c_{g,min} = 1, c_{g,max} = 3, \omega_{min} = 0.1, \omega_{max} = 0.9, s = 10, N = 100, \mu = 1$
$c_{g,min} = 0.5, 1, 1.5$	$c_{p,min} = 1, c_{p,max} = 3, c_{g,max} = 3, \omega_{min} = 0.1, \omega_{max} = 0.9, s = 10, N = 100, \mu = 1$
$c_{g,max} = 2.5, 3, 3.5$	$c_{p,min} = 1, c_{p,max} = 3, c_{g,min} = 1, \omega_{min} = 0.1, \omega_{max} = 0.9, s = 10, N = 100, \mu = 1$
$\omega_{min} = 0.05, 0.1, 0.2$	$c_{p,min} = 1, c_{p,max} = 3, c_{g,min} = 1, c_{g,max} = 3, \omega_{max} = 0.9, s = 10, N = 100, \mu = 1$
$\omega_{max} = 0.8, 0.9, 1$	$c_{p,min} = 1, c_{p,max} = 3, c_{g,min} = 1, c_{g,max} = 3, \omega_{min} = 0.1, s = 10, N = 100, \mu = 1$
$s = 5, 10, 20$	$c_{p,min} = 1, c_{p,max} = 3, c_{g,min} = 1, c_{g,max} = 3, \omega_{min} = 0.1, \omega_{max} = 0.9, N = 100, \mu = 1$
$N = 50, 100, 150$	$c_{p,min} = 1, c_{p,max} = 3, c_{g,min} = 1, c_{g,max} = 3, \omega_{min} = 0.1, \omega_{max} = 0.9, s = 10, \mu = 1$
$\mu = 0.5, 1,$ and 10	$c_{p,min} = 1, c_{p,max} = 3, c_{g,min} = 1, c_{g,max} = 3, \omega_{min} = 0.1, \omega_{max} = 0.9, s = 10, N = 100$

For each value of one chosen parameter, 30 independent runs of cultural MOPSO were conducted. The additive binary epsilon indicator ($I_{\varepsilon+}$ values) is used to compare the Pareto set for each run. For example, to investigate the sensitivity of the algorithm with respect to $c_{p,min}$, three values of $c_{p,min} = 0.5, 1, 1.5$ are adopted. After 30 independent runs for the algorithm with each parameter setting on $c_{p,min}$, we calculate $I_{\varepsilon+}(A,B)$, $I_{\varepsilon+}(B,A)$, $I_{\varepsilon+}(A,C)$, $I_{\varepsilon+}(C,A)$, $I_{\varepsilon+}(B,C)$, $I_{\varepsilon+}(C,B)$ where A, B, and C refer to algorithm with $c_{p,min} = 0.5, 1$, and 1.5, respectively. Notice that for $I_{\varepsilon+}(X,Y)$, each singe run of X is compared against every single run of Y. Then box plots for these six pairs are constructed. Figures 5.26 to 5.34 show the box plot for all nine different parameters for sensitivity analysis. For further analysis, Mann-Whitney rank-sum statistical test is implemented to check if there is a significant difference between the two distributions for $I_{\varepsilon+}(X,Y)$ and $I_{\varepsilon+}(Y,X)$ [132]. The results are displayed in Tables 5.5 to 5.13.

Figure 5.26 along with Table 5.5 demonstrate that by changing the lower limit of personal acceleration, $c_{p,min}$, for all test functions there is no significant difference among the final Pareto fronts using different values of $c_{p,min}$, except for the test function DTLZ5 when comparing $I_{\varepsilon+}(A,B)$ and $I_{\varepsilon+}(B,A)$, where A and B refer to algorithm with $c_{p,min} = 0.5$ and 1.0, respectively. Figure 5.27 along with Table 5.6 illustrates that by changing the upper limit of personal acceleration, $c_{p,max}$, for all test functions there is no significant difference among the final Pareto fronts using different values of $c_{p,max}$.

Figure 5.28 along with Table 5.7 demonstrate that by changing the lower limit of global acceleration, $c_{g,min}$, for all test functions there is no significant difference among the final Pareto fronts using different values of $c_{g,min}$, except for the test function ZDT2 when comparing $I_{\varepsilon+}(A,C)$ and $I_{\varepsilon+}(C,A)$, and for test function DTLZ5 when comparing $I_{\varepsilon+}(B,C)$ and $I_{\varepsilon+}(C,B)$, where A,B and C refer to cultural MOPSO with $c_{g,min} = 0.5, 1.0$ and 1.5, respectively. Figure 5.29 along with Table 5.8 illustrates that by changing the upper limit of global acceleration, $c_{g,max}$, for all test functions there is no significant difference among the final Pareto fronts using different values of $c_{g,max}$.

Figure 5.30 along with Table 5.9 demonstrate that by changing the lower limit of momentum, ω_{min}, for all test functions there is no significant difference among the final Pareto fronts using different values of ω_{min}, except for the test function ZDT4 when comparing $I_{\varepsilon+}(A,B)$ and $I_{\varepsilon+}(B,A)$, where A and B refer to algorithm with $\omega_{min} = 0.05$ and 0.1, respectively. Figure 5.31 along with Table 5.10 show that by changing the upper limit of momentum, ω_{max}, for all test functions there is no significant difference among the final Pareto fronts using different values of ω_{max}, except for the test function ZDT1 when comparing $I_{\varepsilon+}(A,B)$ and $I_{\varepsilon+}(B,A)$, where A and B refer to algorithm with $\omega_{max} = 0.8$ and 0.9, respectively. Figure 5.32 along with Table 5.11 demonstrate that by changing the grid size, s, for all test functions there is no significant difference among the final Pareto fronts using different values of s, except for the test function ZDT1 when comparing $I_{\varepsilon+}(A,C)$ and $I_{\varepsilon+}(C,A)$, and test function DTLZ6 when comparing $I_{\varepsilon+}(A,B)$

and $I_{\varepsilon+}(B,A)$, where A, B and C refer to algorithm with $s = 5, 10$ and 20, respectively. Figure 5.33 along with Table 5.12 show that by changing the population size, N, for all test functions there is no significant difference among the final Pareto fronts using different values of N, except for the test function DTLZ6 when comparing $I_{\varepsilon+}(A,B)$ and $I_{\varepsilon+}(B,A)$, where A and B refer to algorithm with $N = 50$ and 100, respectively.

At last, Figure 5.34 along with Table 5.13 show that by changing the mutation rate, μ, for all test functions there is no significant difference among the final Pareto fronts using different values of μ, except for the test function ZDT4 when comparing $I_{\varepsilon+}(A,B)$ and $I_{\varepsilon+}(B,A)$, where A and B refer to algorithm with $\mu = 0.5$ and 1, respectively. Overall, for Tables 5.5 to 5.13 for each set of parameters and each test function, 30 independent runs have been performed, then a pair of $I_{\varepsilon+}(A,B)$ and $I_{\varepsilon+}(B,A)$ between every two algorithms with different set of tuning parameters are computed. The rank-sum test using α=0.05 shows that a few of these results are statistically significant different. Among various values of the parameters (i.e., totally 162 different cases), in only 9 cases, appreciable differences were observed which is about 5% of the cases tested. Hence, it is reasonable to say that the cultural MOPSO is a fairly robust design with respect to its parameter setting.

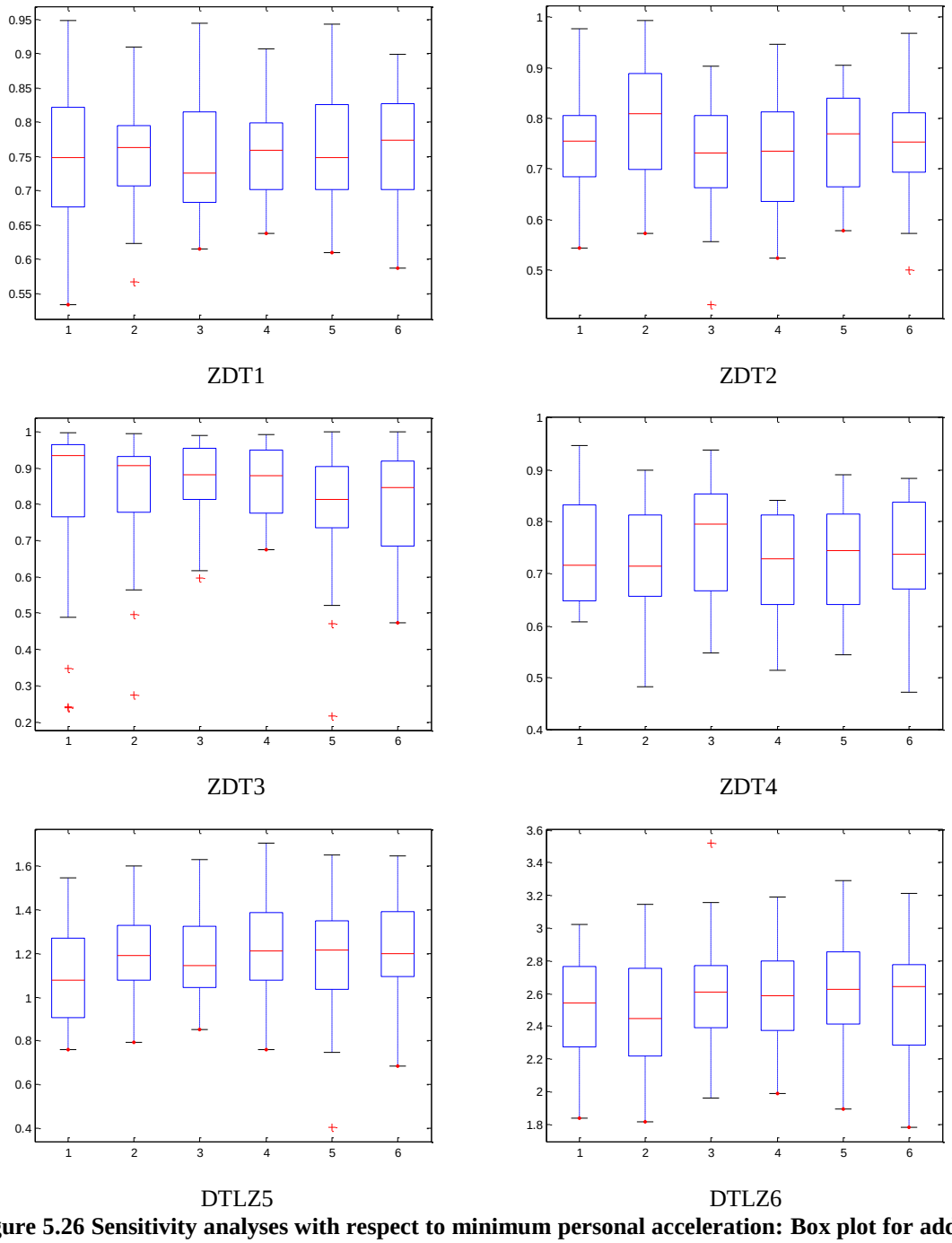
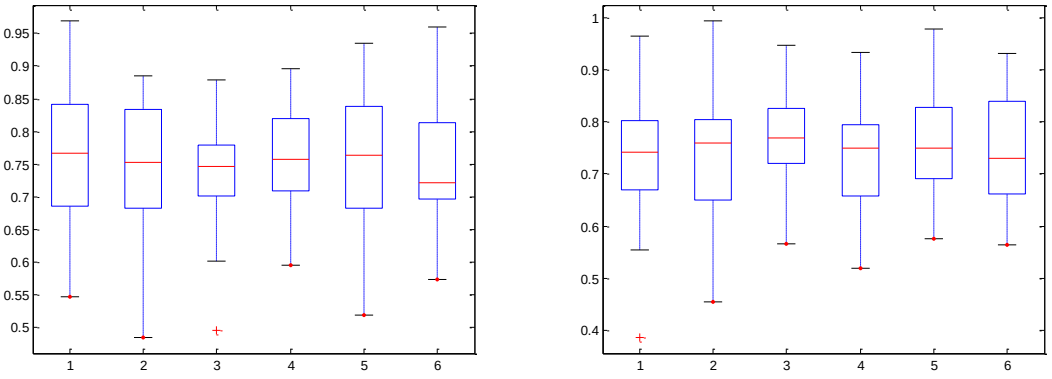
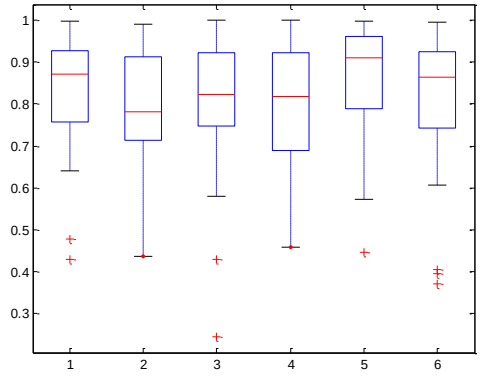


Figure 5.26 Sensitivity analyses with respect to minimum personal acceleration: Box plot for additive binary epsilon indicator ($I_{\varepsilon+}$ values) using different values for $c_{p,min}$ on the test functions. The column numbers refer to (1) $I_{\varepsilon+}(A,B)$, (2) $I_{\varepsilon+}(B,A)$, (3) $I_{\varepsilon+}(A,C)$, (4) $I_{\varepsilon+}(C,A)$, (5) $I_{\varepsilon+}(B,C)$, (6) $I_{\varepsilon+}(C,B)$ where A, B, and C refer to algorithm with $c_{p,min} = 0.5, 1$, and 1.5, respectively.

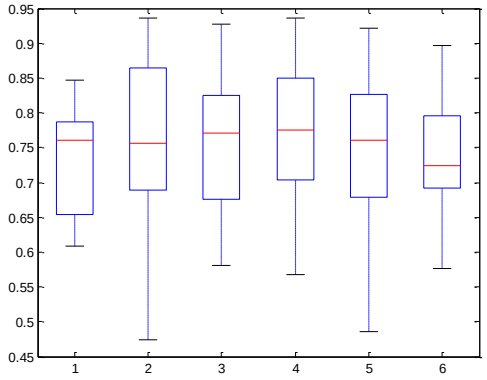


ZDT1

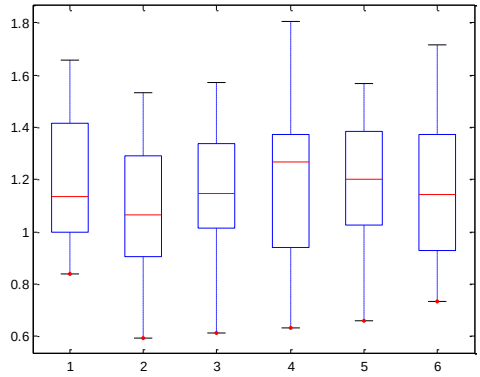
ZDT2



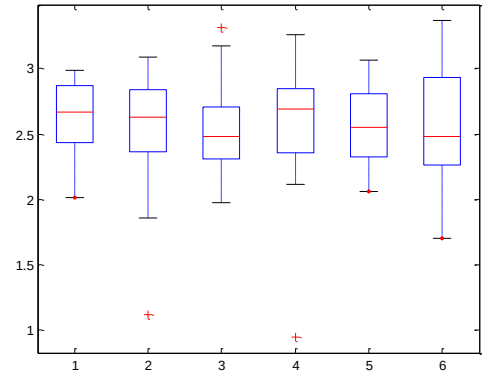
ZDT3



ZDT4

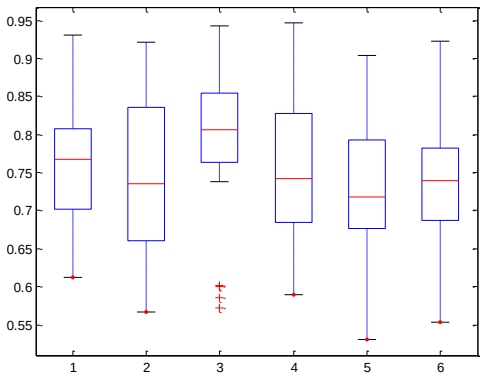


DTLZ5

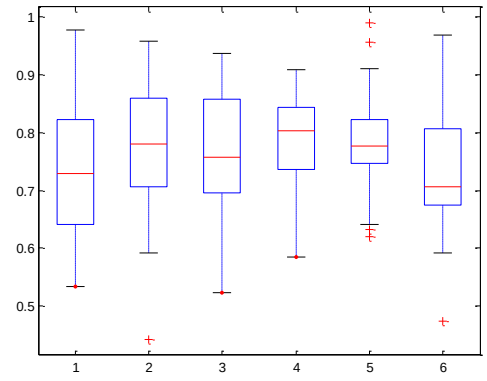


DTLZ6

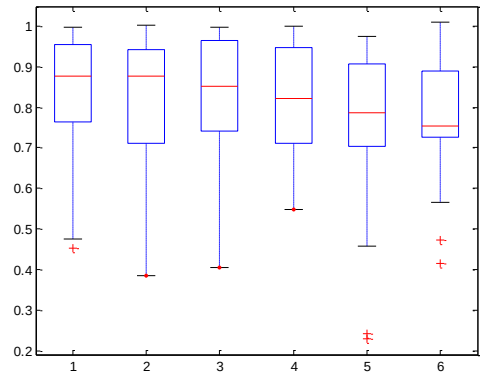
Figure 5.27 Sensitivity analyses with respect to maximum personal acceleration: Box plot for additive binary epsilon indicator ($I_{\varepsilon+}$ values) using different values for $c_{p,max}$ on the test functions. The column numbers refer to (1) $I_{\varepsilon+}(A,B)$, (2) $I_{\varepsilon+}(B,A)$, (3) $I_{\varepsilon+}(A,C)$, (4) $I_{\varepsilon+}(C,A)$, (5) $I_{\varepsilon+}(B,C)$, (6) $I_{\varepsilon+}(C,B)$ where A, B, and C refer to algorithm with $c_{p,max} = 2.5, 3$, and 3.5, respectively.



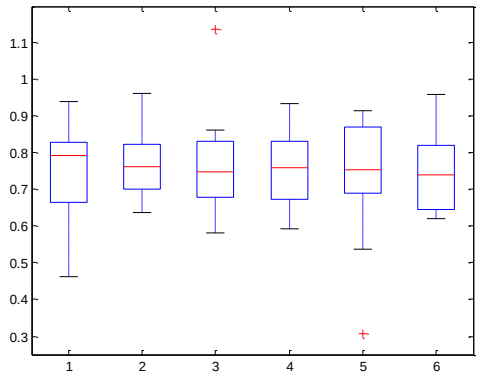
ZDT1



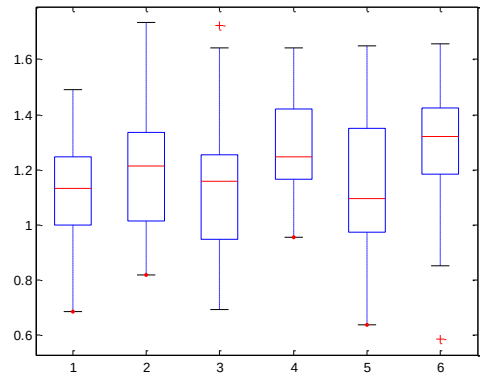
ZDT2



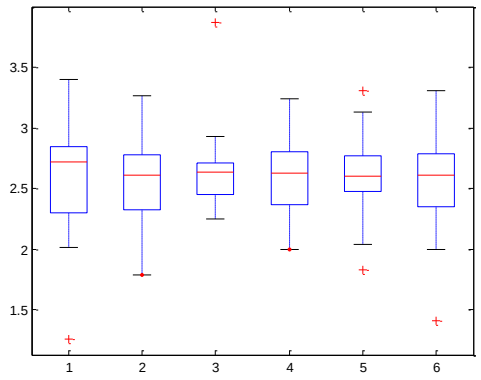
ZDT3



ZDT4



DTLZ5



DTLZ6

Figure 5.28 Sensitivity analyses with respect to minimum global acceleration: Box plot for additive binary epsilon indicator ($I_{\varepsilon+}$ values) using different values for $c_{g,min}$ on the test functions. The column numbers refer to (1) $I_{\varepsilon+}(A,B)$, (2) $I_{\varepsilon+}(B,A)$, (3) $I_{\varepsilon+}(A,C)$, (4) $I_{\varepsilon+}(C,A)$, (5) $I_{\varepsilon+}(B,C)$, (6) $I_{\varepsilon+}(C,B)$ where A, B, and C refer to algorithm with $c_{g,min} = 0.5, 1$, and 1.5, respectively.

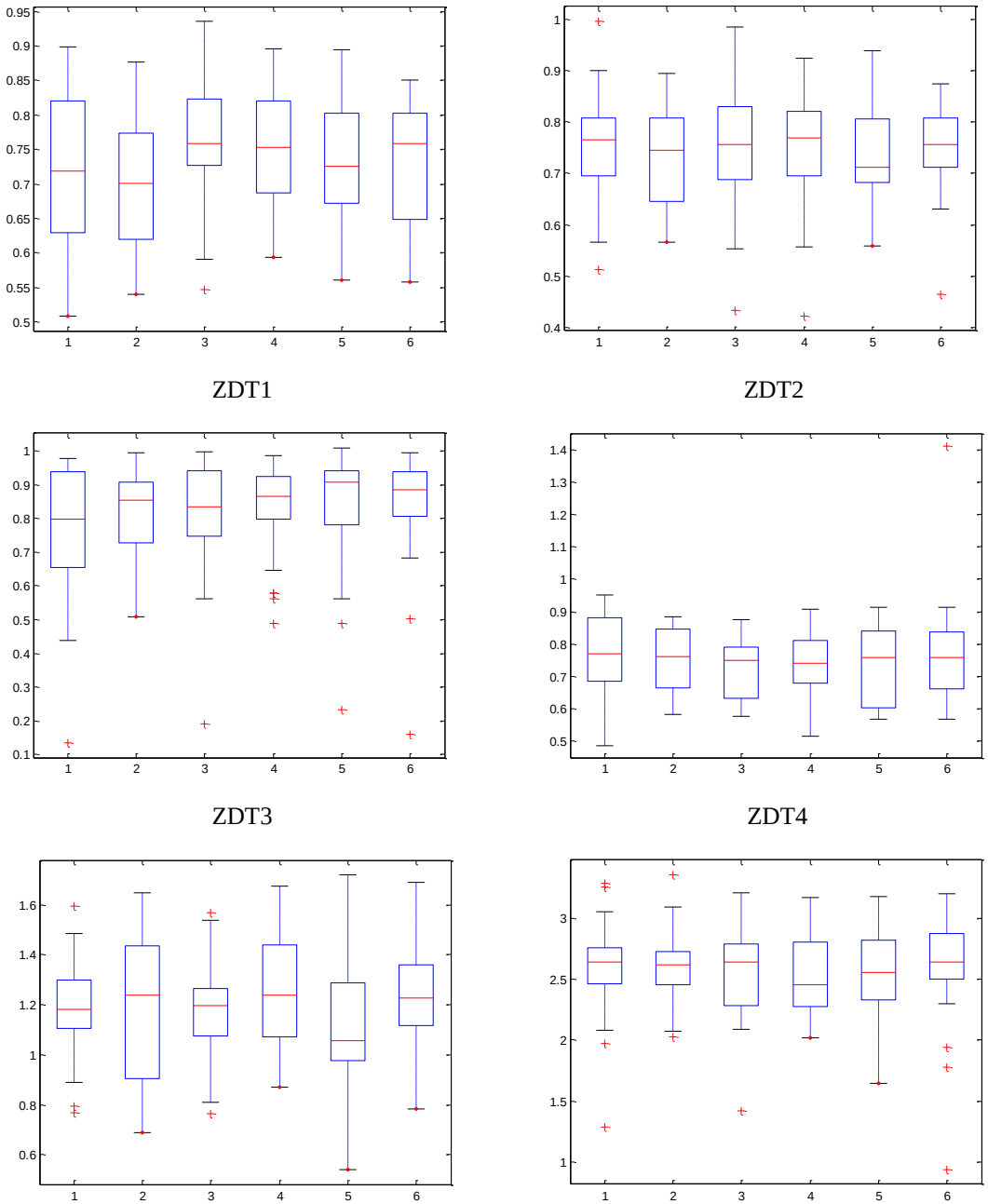


Figure 5.29 Sensitivity analyses with respect to maximum global acceleration: Box plot for additive binary epsilon indicator ($I_{\varepsilon+}$ values) using different values for $c_{g,max}$ on the test functions. The column numbers refer to (1) $I_{\varepsilon+}(A,B)$, (2) $I_{\varepsilon+}(B,A)$, (3) $I_{\varepsilon+}(A,C)$, (4) $I_{\varepsilon+}(C,A)$, (5) $I_{\varepsilon+}(B,C)$, (6) $I_{\varepsilon+}(C,B)$ where A, B, and C refer to algorithm with $c_{g,max} = 2.5, 3$, and 3.5, respectively.

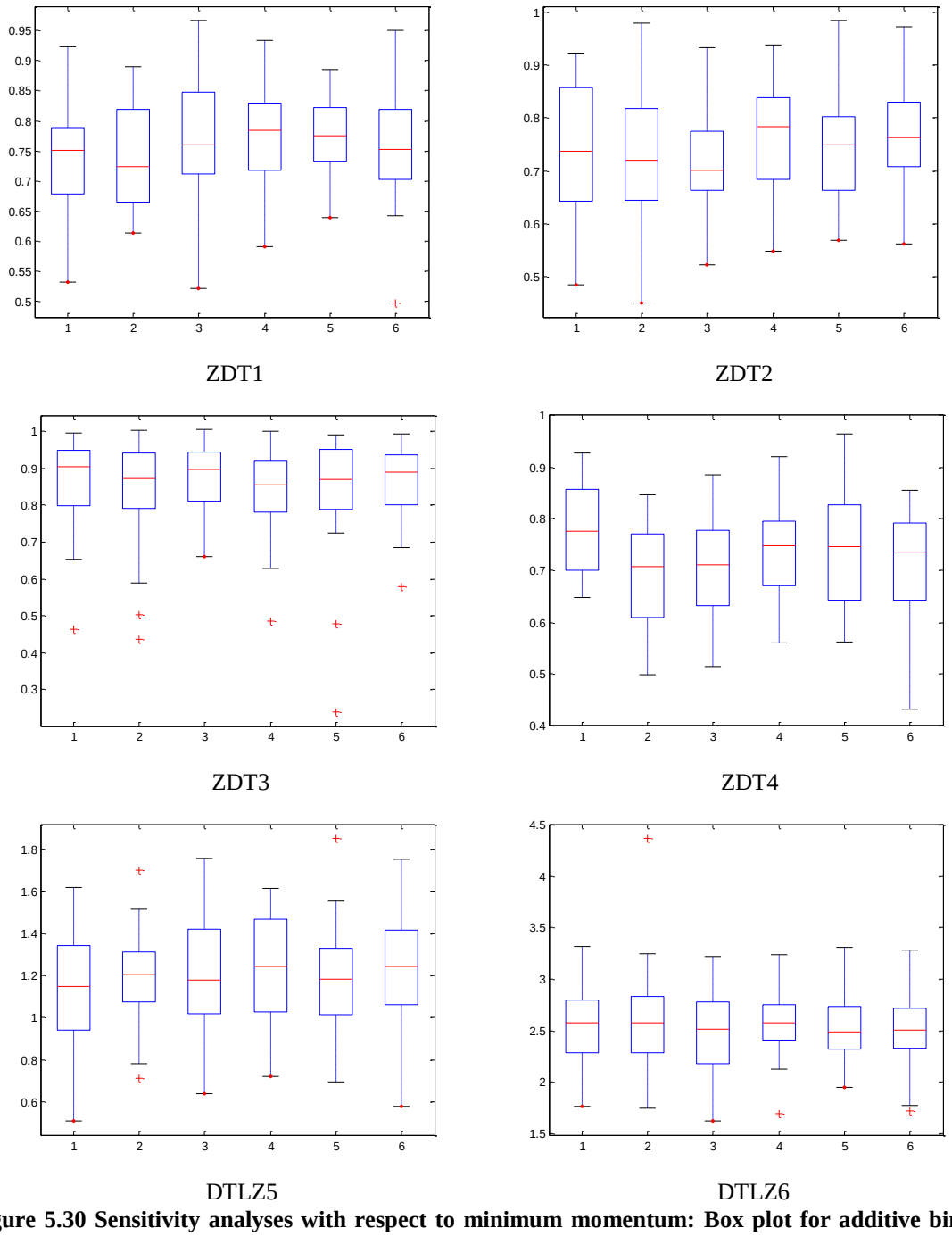
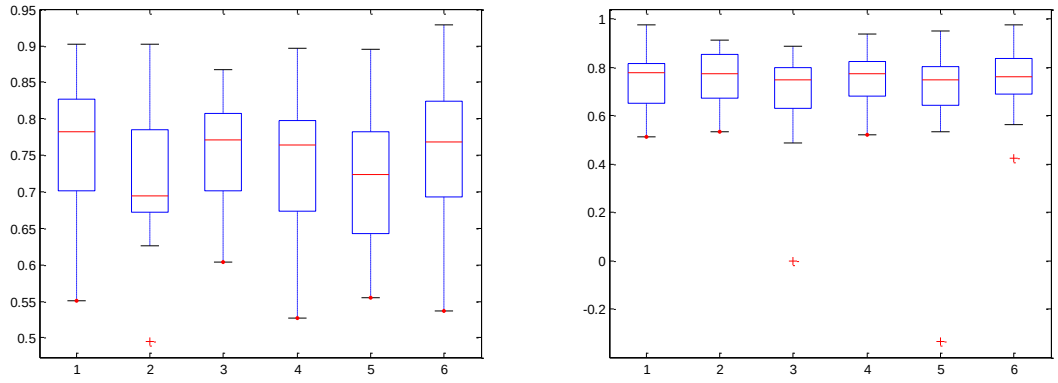
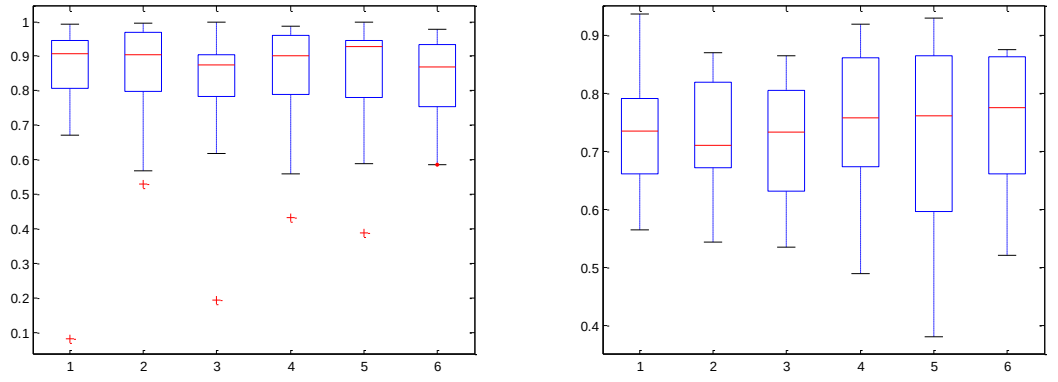


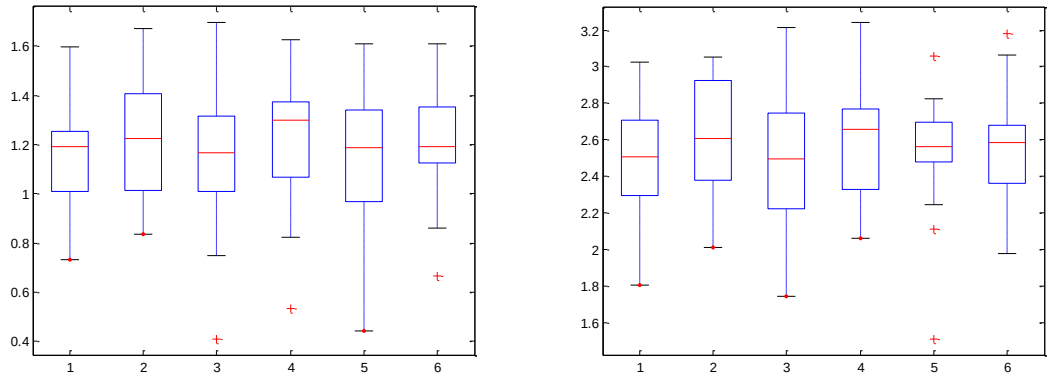
Figure 5.30 Sensitivity analyses with respect to minimum momentum: Box plot for additive binary epsilon indicator ($I_{\varepsilon+}$ values) using different values for ω_{min} on the test functions. The column numbers refer to (1) $I_{\varepsilon+}(A,B)$, (2) $I_{\varepsilon+}(B,A)$, (3) $I_{\varepsilon+}(A,C)$, (4) $I_{\varepsilon+}(C,A)$, (5) $I_{\varepsilon+}(B,C)$, (6) $I_{\varepsilon+}(C,B)$ where A, B, and C refer to algorithm with $\omega_{min} = 0.05, 0.1$, and 0.2, respectively.



ZDT1 ZDT2

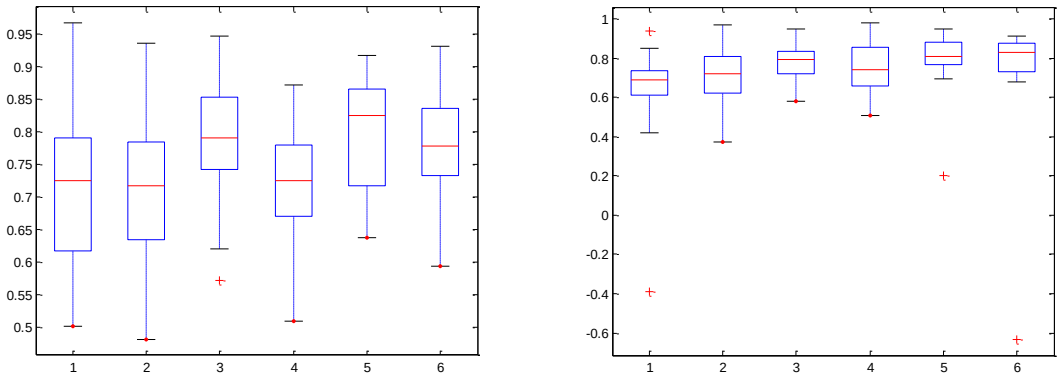


ZDT3 ZDT4



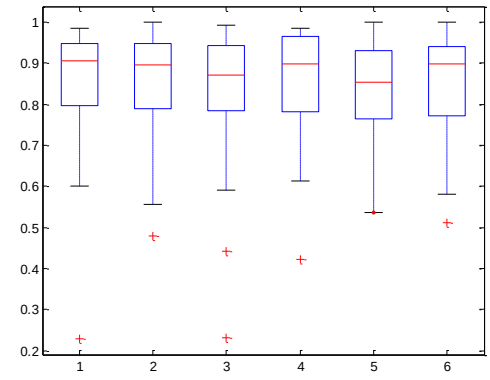
DTLZ5 DTLZ6

Figure 5.31 Sensitivity analyses with respect to maximum momentum: Box plot for additive binary epsilon indicator ($I_{\varepsilon+}$ values) using different values for ω_{max} on the test functions. The column numbers refer to (1) $I_{\varepsilon+}(A,B)$, (2) $I_{\varepsilon+}(B,A)$, (3) $I_{\varepsilon+}(A,C)$, (4) $I_{\varepsilon+}(C,A)$, (5) $I_{\varepsilon+}(B,C)$, (6) $I_{\varepsilon+}(C,B)$ where A, B, and C refer to algorithm with $\omega_{max} = 0.8, 0.9$, and 1, respectively.

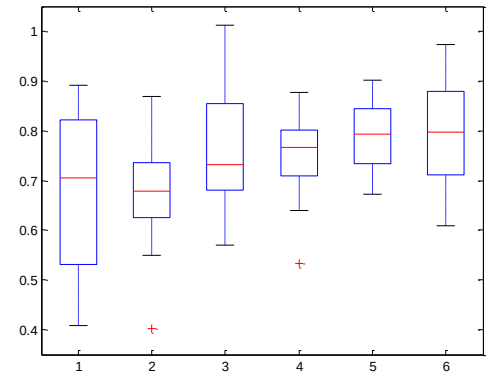


ZDT1

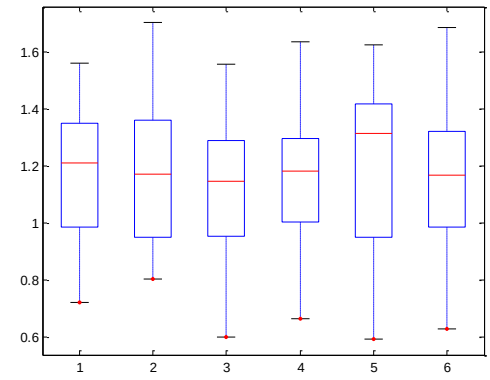
ZDT2



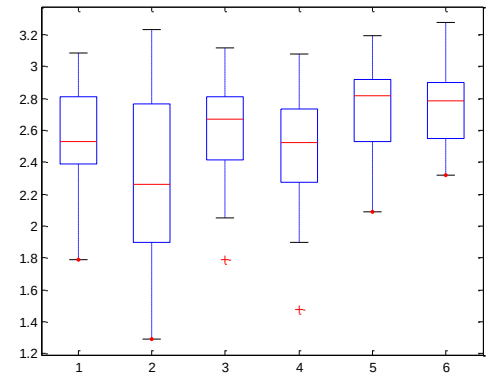
ZDT3



ZDT4

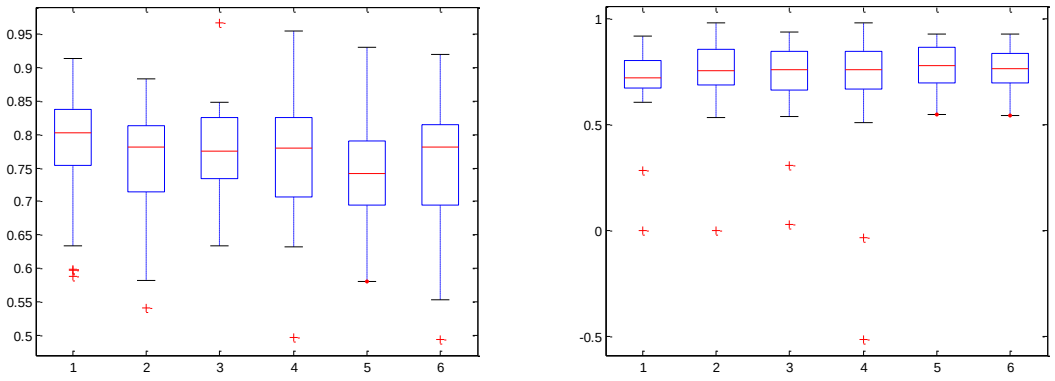


DTLZ5

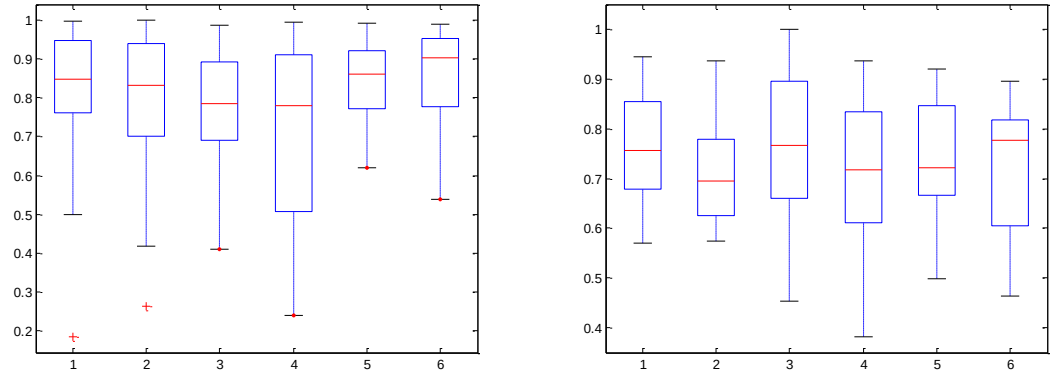


DTLZ6

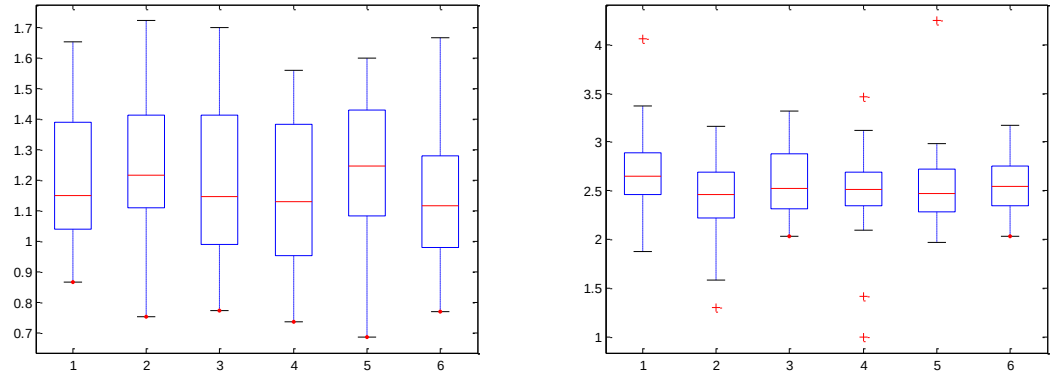
Figure 5.32 Sensitivity analyses with respect to grid size: Box plot for additive binary epsilon indicator ($I_{\varepsilon+}$ values) using different grid size, s, on the test functions. The column numbers refer to (1) $I_{\varepsilon+}(A,B)$, (2) $I_{\varepsilon+}(B,A)$, (3) $I_{\varepsilon+}(A,C)$, (4) $I_{\varepsilon+}(C,A)$, (5) $I_{\varepsilon+}(B,C)$, (6) $I_{\varepsilon+}(C,B)$ where A, B, and C refer to algorithm with $s = 5, 10$, and 20, respectively.



ZDT1 ZDT2

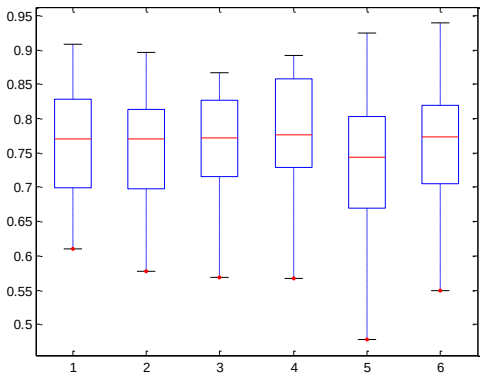


ZDT3 ZDT4

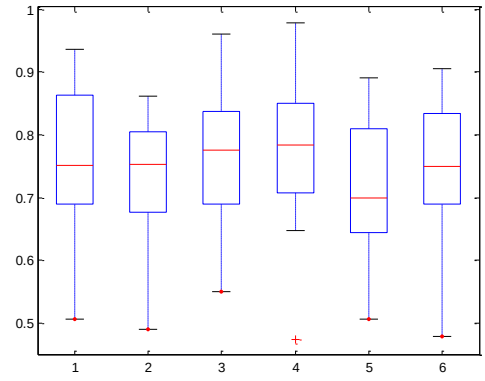


DTLZ5 DTLZ6

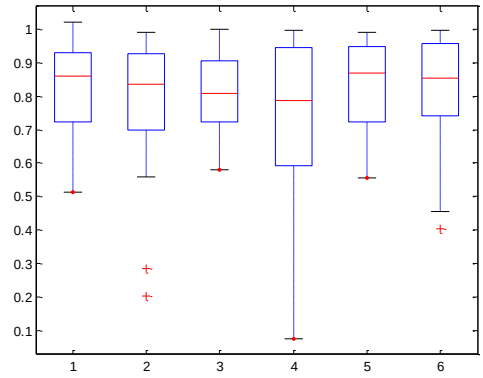
Figure 5.33 Sensitivity analyses with respect to population size: Box plot for additive binary epsilon indicator ($I_{\varepsilon+}$ values) using different population size, N, on the test functions. The column numbers refer to (1) $I_{\varepsilon+}(A,B)$, (2) $I_{\varepsilon+}(B,A)$, (3) $I_{\varepsilon+}(A,C)$, (4) $I_{\varepsilon+}(C,A)$, (5) $I_{\varepsilon+}(B,C)$, (6) $I_{\varepsilon+}(C,B)$ where A, B, and C refer to algorithm with $N = 50, 100$, and 150, respectively.



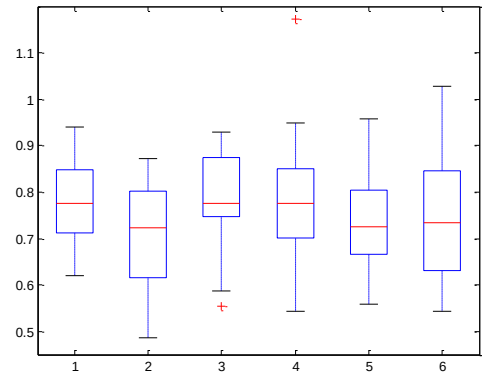
ZDT1



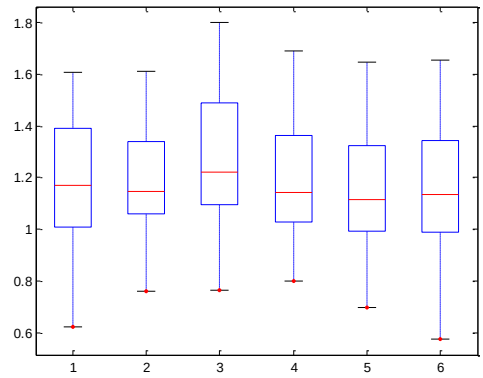
ZDT2



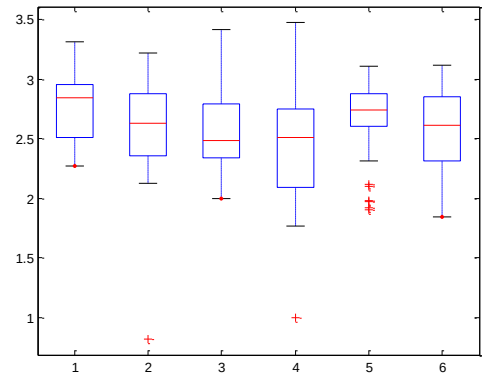
ZDT3



ZDT4



DTLZ5



DTLZ6

Figure 5.34 Sensitivity analyses with respect to mutation rate: Box plot for additive binary epsilon indicator ($I_{\varepsilon+}$ values) using different mutation rate, μ, on the test functions. The column numbers refer to (1) $I_{\varepsilon+}(A,B)$, (2) $I_{\varepsilon+}(B,A)$, (3) $I_{\varepsilon+}(A,C)$, (4) $I_{\varepsilon+}(C,A)$, (5) $I_{\varepsilon+}(B,C)$, (6) $I_{\varepsilon+}(C,B)$ where A, B, and C refer to algorithm with $\mu = 0.5, 1$, and 10, respectively.

Table 5.5 Statistical test to check sensitivity to minimum personal acceleration: Testing of the distribution of $I_{\varepsilon+}$ using Mann-Whitney rank-sum statistical test. Each cell in the table presents the z-value and p-value as the form of (z-value, p-value) with respect to the alternative hypothesis (p-value < α=0.05) for each combination pair of algorithms A, B, and C where A, B, and C refer to cultural MOPSO with $c_{p,min} = 0.5, 1$, and 1.5, respectively.

Test Functions	ZDT1	ZDT2	ZDT3	ZDT4	DTLZ5	DTLZ6
$I_{\varepsilon+}$ **(A,B) and** $I_{\varepsilon+}$ **(B,A)**	(-0.1996, 0.84)	(-1.6632,0.10)	(-1.0423,0.30)	(-0.1257,0.90)	(-2.0180,0.04) **Different**	(-0.4509,0.65)
$I_{\varepsilon+}$ **(A,C) and** $I_{\varepsilon+}$ **(C,A)**	(-0.8205,0.42)	(0,1)	(-0.1848,0.85)	(-1.3971,0.16)	(-1.2789,0.20)	(-0.1848,0.85)
$I_{\varepsilon+}$ **(B,C) and** $I_{\varepsilon+}$ **(C,B)**	(-0.2735,0.78)	(-0.0222,0.98)	(-0.0295, 0.97)	(-0.1109,0.91)	(-0.5100, 0.61)	(-0.5987,0.55)

Table 5.6 Statistical test to check sensitivity to maximum personal acceleration: Testing of the distribution of $I_{\varepsilon+}$ using Mann-Whitney rank-sum statistical test. Each cell in the table presents the z-value and p-value as the form of (z-value, p-value) with respect to the alternative hypothesis (p-value < α=0.05) for each combination pair of algorithms A, B, and C where A, B, and C refer to cultural MOPSO with $c_{p,max} = 2.5, 3$, and 3.5, respectively.

Test Functions	ZDT1	ZDT2	ZDT3	ZDT4	DTLZ5	DTLZ6
$I_{\varepsilon+}$ **(A,B) and** $I_{\varepsilon+}$ **(B,A)**	(-0.5101,0.61)	(-0.1700,0.86)	(-1.2493,0.22)	(-1.1606,0.25)	(-1.3084,0.19)	(-0.2883,0.77)
$I_{\varepsilon+}$ **(A,C) and** $I_{\varepsilon+}$ **(C,A)**	(-0.8205,0.41)	(-1.3676,0.17)	(-0.1922,0.85)	(-0.9092,0.36)	(-0.2587,0.80)	(-1.5154,0.13)
$I_{\varepsilon+}$ **(B,C) and** $I_{\varepsilon+}$ **(C,B)**	(-0.2144,0.83)	(0.3326,0.74)	(-1.1606,0.25)	(-0.2144,0.83)	(-0.2144,0.83)	(-0.3030,0.76)

Table 5.7 Statistical test to check sensitivity to minimum global acceleration: Testing of the distribution of $I_{\varepsilon+}$ using Mann-Whitney rank-sum statistical test. Each cell in the table presents the z-value and p-value as the form of (z-value, p-value) with respect to the alternative hypothesis (p-value < α=0.05) for each combination pair of algorithms A, B, and C where A, B, and C refer to cultural MOPSO with $c_{g,min} = 0.5, 1$, and 1.5, respectively.

Test Functions	ZDT1	ZDT2	ZDT3	ZDT4	DTLZ5	DTLZ6
$I_{\varepsilon+}$ (A,B) and $I_{\varepsilon+}$ (B,A)	(-1.0571,0.29)	(-1.2493,0.21)	(-0.5544,0.58)	(-0.1109,0.91)	(-1.3528,0.18)	(-0.6875,0.50)
$I_{\varepsilon+}$ (A,C) and $I_{\varepsilon+}$ (C,A)	(-1.7815,0.07)	(-0.8353,0.40)	(-0.3622,0.72)	(-0.2735,0.78)	(-2.2842,0.02) **Different**	(-0.1257,0.90)
$I_{\varepsilon+}$ (B,C) and $I_{\varepsilon+}$ (C,B)	(-0.2587,0.80)	(-2.1511,0.03) **Different**	(-0.2735,0.78)	(-0.6875,0.49)	(-1.9589,0.06)	(-0.2587,0.80)

Table 5.8 Statistical test to check sensitivity to maximum global acceleration: Testing of the distribution of $I_{\varepsilon+}$ using Mann-Whitney rank-sum statistical test. Each cell in the table presents the z-value and p-value as the form of (z-value, p-value) with respect to the alternative hypothesis (p-value < α=0.05) for each combination pair of algorithms A, B, and C where A, B, and C refer to cultural MOPSO with $c_{g,max} = 2.5, 3$, and 3.5, respectively.

Test Functions	ZDT1	ZDT2	ZDT3	ZDT4	DTLZ5	DTLZ6
$I_{\varepsilon+}$ (A,B) and $I_{\varepsilon+}$ (B,A)	(-0.4805,0.63)	(-0.8205,0.41)	(-0.5101,0.61)	(-0.8649,0.39)	(-0.2292,0.82)	(-0.3917,0.70)
$I_{\varepsilon+}$ (A,C) and $I_{\varepsilon+}$ (C,A)	(-0.4509,0.65)	(-0.2883,0.77)	(-0.2144,0.83)	(-0.3770,0.71)	(-1.0571,0.29)	(-0.3917,0.70)
$I_{\varepsilon+}$ (B,C) and $I_{\varepsilon+}$ (C,B)	(-0.2144,0.83)	(-1.0275,0.30)	(-0.1848,0.85)	(-0.0960,0.92)	(-1.6632,0.10)	(-1.2197,0.22)

Table 5.9 Statistical test to check sensitivity to minimum momentum: Testing of the distribution of $I_{\varepsilon+}$ using Mann-Whitney rank-sum statistical test. Each cell in the table presents the z-value and p-value as the form of (z-value, p-value) with respect to the alternative hypothesis (p-value < α=0.05) for each combination pair of algorithms A, B, and C where A, B, and C refer to cultural MOPSO with $\omega_{min} = 0.05, 0.1$, and 0.2, respectively.

Test Functions	ZDT1	ZDT2	ZDT3	ZDT4	DTLZ5	DTLZ6
$I_{\varepsilon+}$ (A,B) and $I_{\varepsilon+}$ (B,A)	(-0.1404,0.88)	(-0.2439,0.81)	(-0.0370,0.97)	(-1.9885,0.05) **Different**	(-0.4361,0.66)	(-0.2143,0.83)
$I_{\varepsilon+}$ (A,C) and $I_{\varepsilon+}$ (C,A)	(-0.1995,0.84)	(-1.6041,0.11)	(-0.9388,0.35)	(-0.8353,0.40)	(-0.2883,0.77)	(-0.6283,0.53)
$I_{\varepsilon+}$ (B,C) and $I_{\varepsilon+}$ (C,B)	(-0.4066,0.68)	(-0.7614,0.45)	(-0.3474,0.73)	(-0.2735,0.78)	(-0.7170,0.47)	(-0.1109,0.91)

Table 5.10 Statistical test to check sensitivity to maximum momentum: Testing of the distribution of $I_{\varepsilon+}$ using Mann-Whitney rank-sum statistical test. Each cell in the table presents the z-value and p-value as the form of (z-value, p-value) with respect to the alternative hypothesis (p-value < α=0.05) for each combination pair of algorithms A, B, and C where A, B, and C refer to cultural MOPSO with $\omega_{max} = 0.8, 0.9$, and 1 , respectively.

Test Functions	ZDT1	ZDT2	ZDT3	ZDT4	DTLZ5	DTLZ6
$I_{\varepsilon+}$ (A,B) and $I_{\varepsilon+}$ (B,A)	(-2.1511,0.03) **Different**	(-0.6136,0.54)	(-0.3622,0.72)	(-0.1109,0.91)	(-1.0275,0.30)	(-1.4415,0.15)
$I_{\varepsilon+}$ (A,C) and $I_{\varepsilon+}$ (C,A)	(-0.9388,0.35)	(-1.1310,0.26)	(-0.6283,0.53)	(-1.2345,0.22)	(-1.3676,0.17)	(-0.7318,0.46)
$I_{\varepsilon+}$ (B,C) and $I_{\varepsilon+}$ (C,B)	(-1.3823,0.16)	(-0.6579,0.51)	(-1.1605,0.25)	(-0.3622,0.72)	(-0.7614,0.45)	(0,1)

Table 5.11 Statistical test to check sensitivity to grid size: Testing of the distribution of $I_{\varepsilon+}$ using Mann-Whitney rank-sum statistical test. Each cell in the table presents the z-value and p-value as the form of (z-value, p-value) with respect to the alternative hypothesis (p-value < α=0.05) for each combination pair of algorithms A, B, and C where A, B, and C refer to cultural MOPSO with $s = 5, 10$, and 20, respectively.

Test Functions	ZDT1	ZDT2	ZDT3	ZDT4	DTLZ5	DTLZ6
$I_{\varepsilon+}$ (A,B) and $I_{\varepsilon+}$ (B,A)	(-0.0370,0.97)	(-0.9832,0.35)	(-0.0813,0.94)	(-0.1109,0.91)	(-0.2144,0.83)	(-2.1216,0.03) **Different**
$I_{\varepsilon+}$ (A,C) and $I_{\varepsilon+}$ (C,A)	(-2.9051,0.004) **Different**	(-0.3770,0.71)	(-0.5692,0.57)	(-0.3770,0.71)	(-0.0813,0.94)	(-1.2936,0.20)
$I_{\varepsilon+}$ (B,C) and $I_{\varepsilon+}$ (C,B)	(-0.9092,0.36)	(-0.0221,0.98)	(-0.7318,0.47)	(-0.1108,0.91)	(-0.9388,0.35)	(-0.2735,0.78)

Table 5.12 Statistical test to check sensitivity to population size: Testing of the distribution of $I_{\varepsilon+}$ using Mann-Whitney rank-sum statistical test. Each cell in the table presents the z-value and p-value as the form of (z-value, p-value) with respect to the alternative hypothesis (p-value < α=0.05) for each combination pair of algorithms A, B, and C where A, B, and C refer to cultural MOPSO with $N = 50, 100$, and 150, respectively.

Test Functions	ZDT1	ZDT2	ZDT3	ZDT4	DTLZ5	DTLZ6
$I_{\varepsilon+}$ (A,B) and $I_{\varepsilon+}$ (B,A)	(-1.1754,0.24)	(-0.8648,0.39)	(-0.6727,0.50)	(-1.5154,0.13)	(-0.5692,0.57)	(-2.3433,0.02) **Different**
$I_{\varepsilon+}$ (A,C) and $I_{\varepsilon+}$ (C,A)	(-0.0665,0.95)	(-0.1700,0.86)	(-0.5248,0.60)	(-1.0275,0.30)	(-0.4214,0.67)	(-0.4214,0.67)
$I_{\varepsilon+}$ (B,C) and $I_{\varepsilon+}$ (C,B)	(-0.9979,0.32)	(-0.6727,0.50)	(-0.4805,0.63)	(-0.2144,0.83)	(-1.7963,0.07)	(-0.5840,0.56)

Table 5.13 Statistical test to check sensitivity to mutation rate: Testing of the distribution of $I_{\varepsilon+}$ using Mann-Whitney rank-sum statistical test. Each cell in the table presents the z-value and p-value as the form of (z-value, p-value) with respect to the alternative hypothesis (p-value < α=0.05) for each combination pair of algorithms A, B, and C where A, B, and C refer to cultural MOPSO with $\mu = 0.5, 1$, and 10, respectively.

Test Functions	ZDT1	ZDT2	ZDT3	ZDT4	DTLZ5	DTLZ6
$I_{\varepsilon+}$ *(A,B)* and $I_{\varepsilon+}$ *(B,A)*	(-0.3179,0.75)	(-0.7466,0.45)	(-0.4361,0.66)	(-2.0476,0.04) **Different**	(-0.1109,0.91)	(-1.7076,0.09)
$I_{\varepsilon+}$ *(A,C)* and $I_{\varepsilon+}$ *(C,A)*	(-0.6283,0.53)	(-0.4805,0.63)	(-1.1754,0.24)	(-0.3918,0.70)	(-1.3380,0.18)	(-0.3918,0.70)
$I_{\varepsilon+}$ *(B,C)* and $I_{\varepsilon+}$ *(C,B)*	(-1.1458,0.25)	(-1.1310,0.26)	(-0.1552,0.87)	(-0.0369,0.97)	(-0.5396,0.59)	(-0.5692,0.57)

5.5 Discussions

In this chapter, we have proposed the cultural MOPSO, an algorithm to adapt parameters of the MOPSO using the knowledge stored in various sections of belief space. Cultural algorithm provides required groundwork through information stored in its belief space. Incorporating CA into the optimization process enables us to efficiently and effectively categorize the information and use it in a well-organized way. Information in the belief space facilitates the optimization process by providing required data whenever it is needed. As a result, the optimization process will be more knowledgeable and successful. The momentum, personal acceleration, and global acceleration are adapted

based upon the information in normative, situational, and topographical knowledge of belief space. Personal and global best are also computed using the information stored in belief space.

Several high dimensional bi-objective and tri-objective benchmark test problems with convex and non-convex Pareto fronts have been chosen to exploit the ability of the proposed algorithm to search for the optimized solutions in different case studies. Statistical results using Mann-Whitney rank-sum test for hypervolume indicator show that cultural MOPSO performs better than some well-regarded MOPSO algorithms, i.e., sigma MOPSO, OMOPSO, NSPSO, cluster MOPSO, and MOPSO except for the function ZDT4 where there is no difference between the proposed method and MOPSO [58]. Furthermore, statistical results using Mann-Whitney rank-sum test for additive binary epsilon indicator illustrate that cultural MOPSO performs better than other selected MOPSO algorithms, i.e., sigma MOPSO, OMOPSO, NSPSO, cluster MOPSO, and MOPSO except for the test function ZDT2 where there is no significant difference between the proposed method and MOPSO [58].

Further investigation of the cultural MOPSO is conducted to assess its robustness with respect to the algorithm's tuning parameters. In an extensive sensitivity analysis, based upon additive binary epsilon indicator, the analysis through rank-sum statistical test provides an assurance that the proposed cultural MOPSO is insensitive to the reasonable choices of nine design parameters. It suggests that we can revise the proposed algorithm in Section (5.3), by assigning random numbers for these nine tuning parameters: lower

and upper limit of personal acceleration, lower and upper limit of global acceleration, lower and upper limit of momentum, grid size, population size, and mutation rate.

As a proposed future work, the dynamics of the momentum and acceleration could be further investigated. In this work, we have simply assumed a simple piecewise linear dynamics for momentum and acceleration. Adopting self-adaptation [135-136] will assure the independence of the proposed algorithm from design parameters by incorporating the tuning parameters discussed in Subsection 5.4.2 into the optimization process which can be the future work of this study. Another interesting area is to exploit cultural MOPSO under dynamic environment when fitness landscape will change periodically or sporadically.

CHAPTER VI

CONSTRAINED CULTURAL-BASED OPTIMIZATION USING MULTIPLE SWARM PSO WITH INTER-SWARM COMMUNICAION

6.1 Introduction

Population based paradigms to solve constrained optimization problems have attracted much attention during the most recent years. Genetic-based algorithms and swarm-based paradigms are two popular population based heuristics introduced for solving constrained optimization problems [137-139]. Particle swarm optimization (PSO) [1] is a swarm intelligence design based upon mimicking behavior of the social species such as flocking birds, schooling fish, swarming wasps, and so forth. Constrained particle swarm optimization (CPSO) is a relatively new approach to tackle constrained optimization problems [70-72, 74-83]. What constitute the challenges of the constrained optimization problem are various limits on decision variables, the types of constraints involved, the interference among constraints, and the interrelationship between the constraints and the objective functions. In general constrained optimization problem can be formulated as:

$$\text{Optimize } f(\mathbf{x}) = f(x_1, x_2, \dots, x_M), \tag{6.1}$$

subject to inequality constraints:

$$g_k(\mathbf{x}) \leq 0\ , \qquad k = 1,2, \dots, L, \tag{6.2}$$

and equality constraints:

$$h_k(\mathbf{x}) = 0, \qquad k = L+1, \dots, m. \tag{6.3}$$

It should be noted that in this study minimization problems are considered without the loss of generality (due to duality principle). Individuals that satisfy all of the constraints are called feasible individuals while individuals that do not satisfy at least one of the constraints are called infeasible individuals. Active constraints are defined as the inequality constraints that satisfy $g_k(\mathbf{x}) = 0$ $(k = 1,2, \dots, L)$ at the global optimum solution, therefore all equality constraints, $h_k(\mathbf{x}) = 0$ $(k = L+1, \dots, m)$ are active constraints.

Although there are a few researches on PSO to solve constrained optimization problems, none of these studies fully explore the information from all particles to perform communication within PSO in order to share common interest and to act synchronously. When particles share their information through communication with each other, they will be able to efficiently handle the constraints and optimize the objective function. In order to construct the environment needed to share information, we need to build groundwork

to enable us to employ this information as needed. The main groundwork is the the belief space of cultural algorithm [3, 99] which can assist the particles in an organized informational environment to find the required information. Cultural algorithm has alone shown its own ability to solve engineering problems [99-106, 108-112, 125, 140-142] especially some constrained optimization ones [103-104, 111-112, 142].

From a sociological point of view, study has shown that human societies will migrate from one place to another in order to counter their own life constraints and limitations as well as to reach a better economical, social, or political life [8]. People living in different societies migrate in spite of the different value systems and cultural distinctions. Indeed the cultural belief is an important factor affecting the issues underlying the migration phenomena [9].

On the other hand, finding the appropriate information for communication within swarm can be computationally expensive. One computational aspect is the difficulties of finding the appropriate information to communicate within PSO in order to be able to simultaneously handle the constraints and optimize the objective function. Using many concepts inspired from the cultural algorithm, such as normative knowledge, situational knowledge, spatial knowledge, and temporal knowledge, we will be able to efficiently and effectively organize the knowledge acquired from evolutionary process to facilitate PSO's updating mechanism as well as swarm communications. The inter-swarm communication for the constrained optimization problems using PSO is an important duty that cannot be solved unless we have access to the knowledge throughout the search

process given the cultural algorithm as the computational framework.

In this study, a novel computational framework based on cultural algorithm is proposed by adopting knowledge stored in belief space in order to assist the inter-swarm communication, to search for the leading particles in the personal level, swarm level and global level. Every particle in CPSO will fly through a three level flight and then particles divide into several swarms and inter-swarm communication takes place to share the information. The remaining sections complete the presentation of this chapter as follows. In Section 6.2, principles of cultural algorithm and related works performed in CPSO are briefly reviewed. In Section 6.3, the proposed cultural CPSO is elaborated in details. In Section 6.4, simulation results are evaluated on the benchmark test problems in comparison with the state-of-the-art constraint handling models. Finally, Section 6.5 summarizes the concluding remarks and future study.

6.2 Review of Literature

6.2.1 Related Work in Constrained PSO

Relevant works of constrained particle swarm optimization algorithms are briefly reviewed in this subsection to motivate the proposed ideas. Particle swarm optimization [1] has shown its promise to solve the constrained optimization problems. Hu and Eberhart simply generated particles in PSO for the constrained optimization problems until they are located in the feasible region and then used these particles in feasible region for finding best personal and global particles [70]. Parsopoulos and Vrahatis used a

dynamic multi-stage penalty function for constraint handling [71]. The penalty function is weighted sum of all constraints violation with each constraint having a dynamic exponent and a multi-stage dynamic coefficient. Coath and Halgamuge presented a comparison of two constraint handling methods based upon preserving feasible solutions [70] and dynamic penalty function [71] to solve constrained nonlinear optimization problems using PSO [72]. It demonstrated that the convergence rate for penalty function based PSO was faster than that of feasible solution method.

Paquet and Engelbrecht proposed a modified PSO to solve linearly constrained optimization problems [74]. An essential characteristic of their modified PSO is that the movement of the particles in the vector space is mathematically guaranteed by the velocity and position update mechanism of PSO. They proved that their modified PSO is always assured to find at least a local optimum for linear constrained optimization problems. Takahama and Sakai in their ε-constrained PSO proposed an algorithm in which particles that satisfy the constraints move to optimize the objective function while the particles that violate the constraints move to satisfy the constraints [75]. In order to adaptively control the maximum velocity of the particles, particles are divided into some groups and their movement in those groups is compared.

Krohling and Coelho adopted Gaussian distribution instead of uniform distribution for the personal and global term random weights of the PSO mechanism to solve constrained optimization problems formulated as min-max problems [76]. They used two populations simultaneously; first PSO focuses on evolving the variable vector

while the vector of Lagrangian multiplier is kept frozen, and the second PSO is to concentrate on evolving the Lagrangian multiplier while the first population is kept frozen. The use of normal distribution for the stochastic parameters of the PSO seems to provide a good compromise between the probability of having a large number of small amplitude around the current points and small probability of having large amplitudes, that may cause the particles to move away from the current points and escape from the local optima.

Yang *et al.* [77] proposed a master-slave PSO in which master swarm is responsible for optimizing objective function while slave swarm is focused on constraint feasibility. Particles in the master swarm only fly toward the current better particles in the feasible region. The slave swarm is responsible for searching feasible particles by scouting through the infeasible region. The feasible/infeasible leaders from swarm will then communicate to lead the other swarm. By exchanging flight information between swarms, algorithm can explore a wider solution space.

Zheng *et al.* [78] adopted an approach that congregates neighboring particles in the PSO to form multiple swarms in order to explore isolated, long and narrow feasible space. They also applied a mutation operator with dynamic mutation rate to encourage flight of particles to feasible region more frequently. For constraint handling a penalty function has been adopted as to how far the infeasible particle is located from the feasible region. Saber *et al.* [79] introduced a version of PSO for constrained optimization problems. In their version of PSO, the velocity update mechanism uses a sufficient

number of promising vectors to reduce randomness for better convergence. The velocity coefficient in the positional update equation is a dynamic rate depending on the error and iteration. They also reinitialized the idle particles if there are not improvements for some iterations.

Li *et al.* [80] proposed dual PSO with stochastic ranking to handle the constraints. One regular PSO evolves simultaneously along with a genetic PSO which is a discrete version of PSO including a reproduction operator. The better of the two positions generated by these two PSOs is then selected as the updated position. Flores-Mendoza and Mezura-Montes [81] used Pareto dominance concept for constraint handling on a bi-objective space, with one objective being sum of the inequality constraint violations and the second objective being sum of the equality constraint violations in order to promote better approach to feasible region. They also adopted a decaying parameter control constriction factor and global acceleration of the PSO to prevent the premature convergence and to advance the exploration of the search space. Ting *et al.* [82] introduced a hybrid heuristic consisting PSO and genetic algorithm to tackle constraint optimization problem of load flow problems. They adopted two-point crossover, mutation, and roulette-wheel selection from genetic algorithms along with the regular PSO to generate the new population space. Liu *et al.* [83] incorporated discrete genetic PSO with differential evolution (DE) to enhance the search process in which both genetic PSO and DE update the position of the individual at every generation. The better position will then be selected.

In [143], the constraint handling techniques are embedded into the flight mechanism of PSO, including separate procedures to update infeasible and feasible personal bests in order to guide the infeasible individuals towards the feasible regions while promote search for optimal solutions. Additionally, storing infeasible nondominated solutions along with the best feasible solutions in global best archive is to assist the search for feasible regions and better solution. The adjustment of accelerated constants is based on the number of feasible personal bests and the constraint violations of personal bests and global best. Simulation study shows the proposed design is able to obtain quality solution in a very efficient manner.

6.2.2 Related Works in Cultural Algorithm for Constrained Optimization

Originated by Reynolds [3, 99], cultural algorithm (CA) is a dual inheritance system where information exists at two different space, population space and belief space, and can pass along to the next generation. CA has shown its ability to solve different types of problems among which Jin and Reynolds's algorithm [142] enhanced the performance of evolutionary programming as population space by adopting the belief space in order to solve constrained optimization problems.

Researchers have identified five basic sections of knowledge stored in belief space: situational knowledge, normative knowledge, spatial or topographical knowledge [105], domain knowledge, and temporal or history knowledge [106]. Becerra and Coello Coello proposed a cultured differential evolution for constrained optimization [104]. The

population space in their study was differential evolution (DE) while the belief space consists of situational, topographical, normative, and history knowledge. The variation operator in DE was influenced by the knowledge source of belief space. Yuan *et al.* introduced chaotic hybrid cultural algorithm for constrained optimization in which population space is DE and belief space includes normative and situational knowledge [111]. They incorporated a logistic map function for better convergence of DE. Tang and Li proposed a cultured genetic algorithm for constrained optimization problems by introducing a triple space cultural algorithm [112]. The triple space includes belief space, population space in addition to an anti-culture population consisting individuals disobeying the guidance of the belief space and going away from the belief space guided individuals. The effect of disobeying enhanced by some mutation operations appreciably makes the algorithm faster and less risky for premature convergence, by awarding the most successful individuals and punishing the most unsuccessful population.

6.3 Cultural Constrained Optimization Using Multiple-Swarm PSO

The pseudocode of the proposed design is shown in Figure 6.1 and a block diagram depicting the operation of the proposed algorithm is also shown in Figure 6.2. The population space (PSO) will be initialized and then divided into several swarms based upon the proximity of the particles. The correspondent belief space (BLF) will then be initialized. We then evaluate population space using the fitness values. Acceptance

function is applied to select particles which will be used for the belief space. Belief space consists of four sections: normative, spatial (topographical), situational, and temporal (or history) knowledge. This cultural framework plays a key role in the algorithm. Influence function is then applied to the belief space to adjust the key parameters of PSO for next iteration, i.e., personal best, swarm best and global best. After a predefined iteration, influence function manipulates to the belief space to perform communication among swarms which is done by preparing two sets of particles for each swarm to share with the other swarms. Afterward, particles in the population space fly using newly computed personal, swarm, and global best. This process continues until the stopping criteria are met.

Initialize PSO at t=0.
Initialize BLF at t=0
Repeat

- Evaluate PSO(t).
- Divide PSO(t) into several swarms using k-means.
- Apply ACCEPTANCE function to PSO(t) to select particles which affect BLF(t).
- Adapt BLF(t) including Normative, Spatial, Situational, and Temporal Knowledge.
- Apply INFLUENCE function to BLF(t) to select *pbest*(t), *sbest*(t), and *gbest*(t) of PSO(t).
- If $t=T_{migration}$, perform cultural-based inter-swarm communication.
- t=t+1.
- Update PSO(t) using new *pbest*(t), *sbest*(t), and *gbest*(t).

Until Termination Criteria are met.
End

Figure 6.1 Pseudocode of the cultural constrained particle swarm optimization

In the remainder of this section, the multi swarm population space, acceptance function, different parts of belief space, influence functions, and inter-swarm communication strategy are elaborated in details.

6.3.1 Multi-Swarm Population Space

The population space here consists of multiple swarms, each swarm performing a PSO paradigm. The particles are clustered into a predefined number of swarms using k-means clustering algorithm. In this study, the number of swarms, P, is chosen roughly 10% of the population size, N:

$$P = \lfloor 0.1N \rfloor, \tag{6.4}$$

where $\lfloor . \rfloor$ refers to a rounding operator. This multiple swarm PSO is a modified version of the algorithm introduced by Yen and Daneshyari [144-145]. To overcome the premature convergence problem of PSO and to promote the particles in a swarm sharing information among themselves, a three-level flight for PSO mechanism has been adopted. In personal level, particle will follow its best experienced behavior in its history. In swarm level, the particle will simultaneously follow the best behaving particle in its swarm to achieve a synchronal behavior among the neighboring particles, and finally in the global level, the entire population will follow the best known particle seeking a global goal. This modified paradigm of PSO is formulated as:

$$v_i^d(t+1) = wv_i^d(t) + c_p r_1 \left(pbest_i^d(t) - x_i^d(t)\right) + c_s r_2 \left(sbest_{i,j}^d(t) - x_i^d(t)\right) + c_g r_3 \left(gbest^d(t) - x_i^d(t)\right),$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1), \tag{6.5}$$

where $v_i^d(t)$ is the d-th dimension of velocity of the i-th particle at time t, $x_i^d(t)$ is the d-th dimension of position of the i-th particle at time t, $pbest_i^d(t)$ is the d-th dimension of best past position of the i-th particle at time t, $sbest_{i,j}^d(t)$ is defined as the d-th dimension of best particle from swarm j in which particle i belongs. $gbest^d(t)$ is the d-th dimension of the best particle of population at time t. r_1, $r_2$and r_3 are uniformly generated random numbers in the range of $(0,1)$, c_p, c_s and c_g are constant parameters representing the weights for personal, swarm, and global behavior and w is the momentum for previous velocity.

6.3.2 Acceptance Function

The belief space should be affected by a selection of best individuals. Therefore all particles located in the feasible space, along with p% of the infeasible particles that have the least violation of constraints are selected, where p is a predefined value. This allows infeasible individuals with minimum constraint violations to portray feasibility

landscape.

6.3.3 Belief Space

The belief space in this paradigm consists of four sections: normative, spatial, situational, and temporal knowledge. Since the constrained optimization problems of the interest have static landscapes, only these four sections have been implemented because the domain knowledge, the fifth element, is mainly useful when fitness landscape is dynamic. In the remainder of this section, type of information, the ways to represent the knowledge and methodology on how to update the knowledge for each section of the belief space are discussed thoroughly.

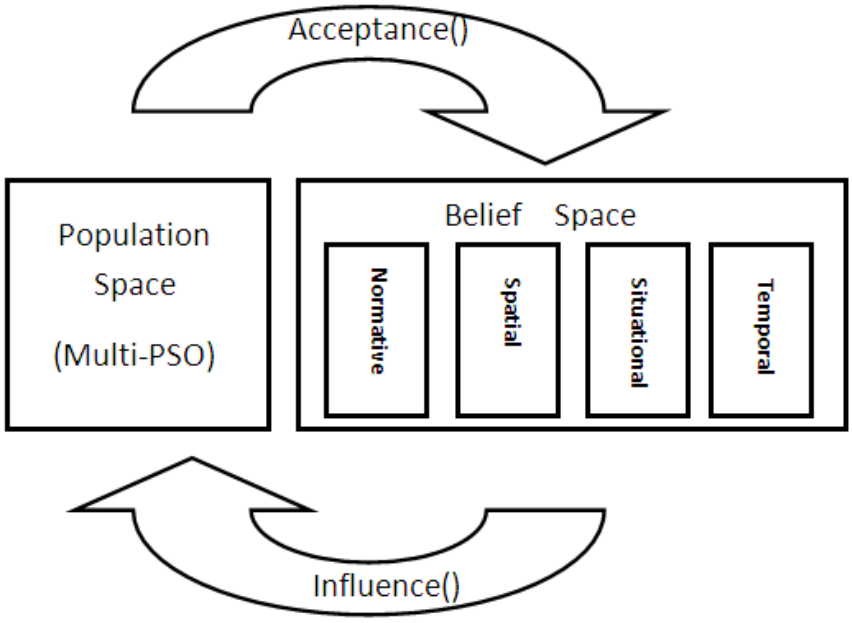


Figure 6.2 Schema of the cultural framework adopted, where belief space consists of normative knowledge, spatial (topographical) knowledge, situational knowledge, and temporal (history) knowledge, and population space is a multiple swarm PSO.

6.3.3.1 Normative Knowledge

Normative knowledge represents the best area in the objective space. It is represented as Figure 6.3 where $\mathbf{f}(t) = [\mathbf{f}_1(t)\ \mathbf{f}_2(t)\ ...\ \mathbf{f}_N(t)]$ and $V(t) = [v_1(t)\ v_2(t)\ ...\ v_N(t)]$ (N is the number of particles). $\mathbf{f}_j(t)$ is a normalized objective function defined as following:

$$\mathbf{f}_j(t) = \frac{f(\mathbf{x}_j) - f_j^{min}(t)}{f_j^{max}(t) - f_j^{min}(t)}, \quad j = 1,2, \dots, N, \tag{6.6}$$

where $f(\mathbf{x}_j)$ is the objective function value for particle $\mathbf{x}_j$, $f_j^{min}(t) = \min_{\mathbf{x}\in\mathbf{X}}(f(\mathbf{x}_j))$ is the lower bound of the objective function value on the j-th particle at time t, and $f_j^{max}(t) = \max_{\mathbf{x}\in\mathbf{X}}(f(\mathbf{x}_j))$ is the upper bound of the objective function value on the j-th particle at time t. $\mathbf{X}$ refers to the current population at time t.

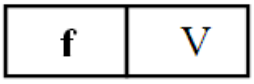


Figure 6.3 Representation for normative knowledge

$v_j(t)$ is a measure of violation of all constraints for particle $\mathbf{x}_j$ defined as following:

$$v_j(t) = \frac{1}{m}\sum_{k=1}^{m} \frac{c_k(\mathbf{x}_j)}{c_k^{max}}, \quad j = 1,2, \dots, N, \tag{6.7}$$

where m is the number of constraints and $c_k(\mathbf{x}_j)$ is related to the k-th constraint evaluated at particle $\mathbf{x}_j$ as following:

$$c_k(\mathbf{x}_j) = \begin{cases} \max\left(0, g_k(\mathbf{x}_j)\right), & k = 1,2,\dots,L \\ \max\left(0, \left|h_k(\mathbf{x}_j)\right| - \delta\right), & k = L+1,\dots,m \end{cases}, \tag{6.8}$$

and:

$$c_k^{max} = \max_{\mathbf{x}\in\mathbf{X}}(c_k(\mathbf{x}_j)). \tag{6.9}$$

In order to update the normative knowledge, new objective function values will be normalized using Equation (6.6), and constraint violation measures will be updated by the new position of the particles using Equation (6.7). The information in the normative knowledge is used to assemble the framework for spatial knowledge.

6.3.3.2 Spatial Knowledge

In order to represent spatial or topographical knowledge, the normative knowledge is adopted. The method used in this section is similar to the penalty function method to handle constraints introduced by Tessema and Yen [146]. The normalized objective functions, **f**, and violation measures, V, are set as the axes of a 2-D space as shown in Figure 6.4. Two particles are mapped in this space for visualization. Figure 6.5 shows spatial knowledge stored for every particle located in the **f**-V space where $D(t) = [D_1(t)\ D_2(t)\ \dots\ D_N(t)]$ and $\mathcal{F}(t) = [\mathcal{F}_1(t)\ \mathcal{F}_2(t)\ \dots \mathcal{F}_N(t)]$ (N is the number of

particles). $D_j(t)$ is the Euclidean distance from the origin of the **f**-V space defined as:

$$D_j(t) = (v_j(t)^2 + \mathbf{f}_j(t)^2)^{1/2}, \quad j = 1,2,\dots,N, \tag{6.10}$$

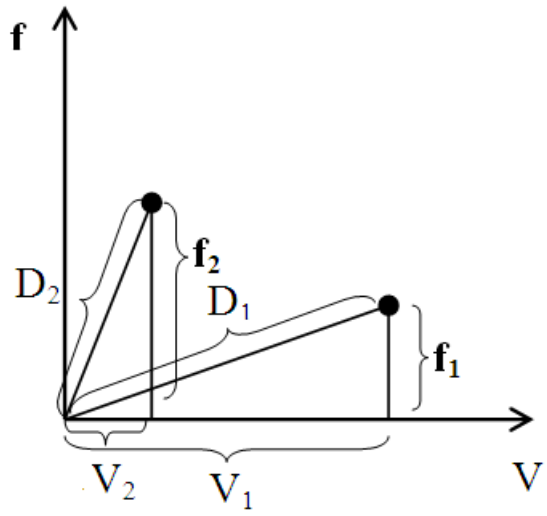


Figure 6.4 The schema to represent how the spatial knowledge is computed.

and $\mathcal{F}_j$ is the modified objective function value to handle constraints computed as a weighted sum of three spatial distances D, v, and **f**, as following:

$$\mathcal{F}_j(t) = \begin{cases} D_j(t) + (1 - r(t))v_j(t) + r(t)\mathbf{f}_j(t), & \text{if } r(t) \neq 0, \mathbf{x}_j \text{ is infeasible particle} \\ v_j(t), & \text{if } r(t) = 0, \mathbf{x}_j \text{ is infeasible particle} \\ \mathbf{f}_j(t), & \text{if } \mathbf{x}_j \text{ is feasible particle} \end{cases}, \tag{6.11}$$

$$(j = 1,2,\dots,N)$$

where $r(t)$ is the ratio of number of feasible particles over the population size, $\mathbf{f}_j(t)$ and $v_j(t)$ are defined in Equations (6.6) and (6.7), respectively. If $0 < r(t) \ll 1$, then $v_j(t)$ will be more important than $\mathbf{f}_j(t)$ in Equation (6.11), consequently $\mathcal{F}_1(t) > \mathcal{F}_2(t)$ in schema shown in Figure 6.4, which means particle 2 outperforms particle 1 for a minimization problem. But when $0 \ll r(t) < 1$, then $\mathbf{f}_j(t)$ will be more important than $v_j(t)$ in Equation (6.11), consequently $\mathcal{F}_1(t) < \mathcal{F}_2(t)$ in schema shown in Figure 6.4, which in turn means particle 1 outperforms particle 2.

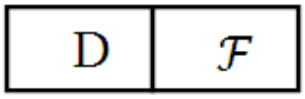


Figure 6.5 Representation of spatial knowledge for each particle

At every iteration, the spatial knowledge will be updated. To do so, updated normative knowledge will be used to rebuild the spatial distance for every particle using Equations (6.10) and (6.11). Spatial knowledge will be used later to find the global best particle of population space and to build a communication strategy among swarms.

6.3.3.3 Situational Knowledge

This part of belief space is used to keep the good exemplar particles for each swarm. Its representation is shown in Figure 6.6. $\hat{X}_i(t)$ $(i = 1,2, \dots, P)$ where P is the number of swarms defined in Equation (6.4), is the best particle in the i-th swarm based

upon information received from the spatial knowledge in accordance with both objective function value and constraints violation. Assume that at an arbitrary iteration the i-th swarm consists N_i particles as $\Omega_{\mathrm{i}} = \{z_1, z_2, \dots, z_{N_i}\}$ and that $\Im = \{\mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_{N_i}\}$ is a set consisting the modified objective values extracted from the spatial knowledge corresponding to $z_1, z_2, \dots$ and z_{N_i}, respectively. Then $\hat{X}_i(t) \in \Omega_{\mathrm{i}}$ is defined such that:

$$\mathcal{F}\left(\hat{X}_i(t)\right) = \min_{1<l<N_i} \mathcal{F}_l, \qquad i = 1,2, \dots, P, \tag{6.12}$$

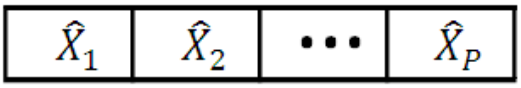


Figure 6.6 Representation for situational knowledge

where $\mathcal{F}\left(\hat{X}_i(t)\right)$ is the modified objective function value for the particle $\hat{X}_i(t)$. In order to update the situational knowledge, the updated position of the particles will be used to evaluate Equations (6.6) to (6.11) to compute updated modified objective function values, and then the particle corresponding to the least value in each swarm will be stored in situational knowledge. The situational knowledge will be used later to compute the swarm best particles and to facilitate the communication among swarms.

6.3.3.4 Temporal Knowledge

This part of belief space is used to keep the history of the individual's behavior. Its representation is shown in Figure 6.7 where $\mathcal{T}(t) = [\mathcal{T}_1(t)\, \mathcal{T}_2(t) \dots \mathcal{T}_N(t)]$ and

$\mathcal{P}(t) = [\mathcal{P}_1(t)\, \mathcal{P}_2(t) \ldots \mathcal{P}_N(t)]$ (N is the number of particles). $\mathcal{T}_j(t)$ is a set of past temporal pattern of the j-th particle which are collected at every time step from part of the spatial knowledge, $\mathcal{F}_j(t)$, and is defined as following:

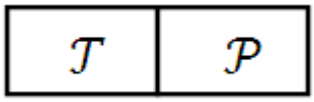


Figure 6.7 Representation for temporal knowledge

$$\mathcal{T}_j(t) = \{\mathcal{F}_j(1), \mathcal{F}_j(2), \ldots, \mathcal{F}_j(t)\}, \quad j = 1,2, \ldots, N, \tag{6.13}$$

where $\mathcal{F}_j(1), \mathcal{F}_j(2), \ldots$ and $\mathcal{F}_j(t)$ are the modified objective function values defined in Equation (6.11) for the time steps $1,2, \ldots,$ and t, respectively. $\mathcal{P}_j(t)$ is the set of all past positions of the j-th particle in the whole population defined as $\mathcal{P}_j(t) = \{\mathbf{x}_j(1), \mathbf{x}_j(2), \ldots, \mathbf{x}_j(t)\}$, $j = 1,2, \ldots, N$. The temporal knowledge will be updated at every iteration. To do so, the updated spatial knowledge, the updated position of the particle, and previously stored temporal knowledge will be adopted as following:

$$\begin{aligned} \mathcal{T}_j(t+1) &= \mathcal{T}_j(t) \cup \{\mathcal{F}_j(t+1)\}, \quad j = 1,2, \ldots, N, \\ \mathcal{P}_j(t+1) &= \mathcal{P}_j(t) \cup \{\mathbf{x}_j(t+1)\}, \quad j = 1,2, \ldots, N. \end{aligned} \tag{6.14}$$

The temporal knowledge will later be used to compute the personal best for every particle in the population space.

6.3.4 Influence Functions

After belief space is updated, the correspondent knowledge should be used to influence the flight of particles in PSO. We propose to use the knowledge in belief space to select the personal best, swarm best, and global best for the PSO flight mechanism. Furthermore, we propose to adopt the information in the belief space to perform a communication strategy among swarms.

6.3.4.1 pbest Selection

In order to select the personal best, we exploit information in the temporal knowledge section of the belief space. The best behaving particle's past history should be selected as following:

$$pbest_i(t) = \left\{\exists \mathbf{x}_i(\hat{t}) \in \mathcal{P}_i(t), \left|\mathcal{F}_i(\hat{t}) = \min_t\left(\mathcal{T}_i(t)\right)\right.\right\}, \qquad i = 1,2,\dots,N, \quad (6.15)$$

where $\mathcal{P}_i(t) = \{\mathbf{x}_i(1), \mathbf{x}_i(2), \dots, \mathbf{x}_i(t)\}$ is the set of all past positions of the i-th particle, and $\mathcal{T}_i(t) = \{\mathcal{F}_i(1), \mathcal{F}_i(2), \dots, \mathcal{F}_i(t)\}$ is the corresponding modified objective values for the past history of the i-th particle both extracted from the temporal knowledge section of the belief space.

6.3.4.2 sbest Selection

In order to select the swarm best particle, the situational knowledge is adopted. The information stored in the situational knowledge section of the belief space is simply copied into swarm best particles:

$$sbest_i(t) = \hat{X}_i(t), \qquad i = 1,2,\dots,P, \tag{6.16}$$

where P is the number of swarms and $\hat{X}_i(t)$ is the representation of the situational knowledge in the belief space.

6.3.4.3 $gbest$ Selection

The spatial knowledge stored in the belief space is used to compute *gbest*(t) at each iteration. The global best particle is found as following:

$$gbest(t) = \{\exists \mathbf{x}_j(t) \in \mathbb{P}(t), 1 \le j \le N \big| \mathcal{F}_j(t) = \min(\mathbb{F}(t))\}, \tag{6.17}$$

where $\mathbb{P}(t) = \{\mathbf{x}_1(t), \mathbf{x}_2(t), \dots, \mathbf{x}_N(t)\}$, is the entire population of particles at time t, and $\mathbb{F}(t) = \{\mathcal{F}_1(t), \mathcal{F}_2(t), \dots, \mathcal{F}_N(t)\}$ is a set consisting of the modified objective function values for all particles at time t.

6.3.4.4 Inter-Swarm Communication Strategy

After some predefined iterations, $T_{migration}$, the swarms will perform information exchange. Each swarm prepares a list of sending particles to be sent to the next swarm, and also assembles a list of replacement particles to be replaced by particles coming from other swarms. This communication strategy is a modified version of the algorithm adopted in [145]. We use the information stored in the belief space to perform communication among swarms. To do so, each swarm prepares two list of particles $\mathbb{S}_i$ and $\mathbb{R}_i$ $i = 1,2, \dots, P$, where P is the fixed number of swarms defined in Equation (6.4) . $\mathbb{S}_i$ is a list of particles in the i-th swarm to be sent to the next swarm and $\mathbb{R}_i$ is a list of particles in the i-th swarm to be replaced by particles coming from another swarm. The inter-swarm communication strategy is based upon the particles' locations in the swarm and their modified objective value which is stored in the belief space. The sending list for the swarm is prepared in the following order:

(1) The highest priority in the selection of particles is given to a particle that has the least average Hamming distance from others. This particle is considered as the representative of the swarm.

(2) The second priority is given to the closest M_i particles to the representative particle in the i-th swarm whose modified objective value stored in the spatial knowledge of the belief space is greater than that of the representative. M_i is defined as [144]:

$$M_i = \frac{rN_i}{2} - 1, \tag{6.18}$$

where r, rate of information exchange among swarms, is a predefined value between 0 and 1, N_i is the population of the i-th swarm.

(3) The third priority is given to the closest particles to the representative particle whose modified objective value extracted from the belief space is less than that of the representative.

(4) The fourth and last priority is given to the best performing particle in the swarm.

Note that depending on the predefined fixed value for allowable number of the sending list, $N_{migration}$, the sending list will be filled in each swarm using the above-mentioned priorities.

There will also be a replacement list that each swarm prepares, based upon the similar positional information of particles in the swarm. When swarms are approaching local optima, many particles' locations are the same. Each swarm will remove this excess information through its replacement list. The replacement list in each swarm is assembled in the following order:

(1) The first priority is given to the particles with identical decision space information in the order of their modified objective values extracted from the belief space, with the least modified objective values being replaced first.

(2) The second and last priority is given to the particles with the lowest modified objective values if all particles of the first priority have already been placed in the replacement list.

This information exchange among swarms happens in a ring sequential order between each pair of swarms. Each swarm accepts the sending list from other swarm and will replace it with its own replacement list.

6.4 Comparative Study

In this section, the performance of the cultural CPSO is evaluated against those of the selected state-of-the-art constrained optimization heuristics.

6.4.1 Parameter Settings

The parameters of the cultural CPSO are set as shown in Table 6.1. The tolerance for equality constraints in Equation (6.8), δ, is set as 0.0001. In the flight mechanism, the momentum, w, is randomly selected from the uniform distribution of (0.5, 1), the personal, swarm and global acceleration, c_p, c_s and c_g are all selected as 1.5.

Table 6.1 Parameter settings for cultural CPSO

δ	Tolerance for equality constraints in Equation (6.8)	0.0001
w	Momentum in Equation (6.5)	$rand(0.5, 1)$
c_p	Personal acceleration in Equation (6.5)	1.5
c_s	Swarm acceleration in Equation (6.5)	1.5
c_g	Global acceleration in Equation (6.5)	1.5
N	Population size	100
r	Rate of information exchange in Equation (6.18)	30%
$N_{migration}$	Allowable number of migrating particles	5

The population size is fixed at 100 particles. The maximum velocity for the particles in specific dimension, v_{max}^{d}, is set at half of the range of the particle's position in that dimension:

$$v_{max}^{d} = (x_{max}^{d} - x_{min}^{d})/2 \ . \tag{6.19}$$

The rate for information exchange among swarms affects how much swarms communicate with each other. The higher rate corresponds to more communication and better overall performance of the algorithm, but it does incur higher computational complexity, while a lower rate imposes less computational complexity and relatively poorer performance. The heuristic choice is set at 30%. The allowable number of migrating particles among swarms is set as 5% of the population size, which is $N_{migration} = 0.05N = 5$.

6.4.2 Benchmark Test Functions

The proposed cultural CPSO has been tested on 24 benchmark functions [147] to verify its performance. The characteristics on these test functions are summarized in Table 6.2. These problems include various types of objective functions such as linear, nonlinear, quadratic, cubic, and polynomial. These benchmark problems vary in the number of decision variables, M, between 2 and 24, and number of constraints, between 1 and 38. In this table, ρ is the estimated ratio of the feasible region over the search space which varies as low as 0.0000% to as high as 99.9971%. The numbers of different types of constraints are also shown for each test function: the number of linear inequality (LI),

the number of nonlinear inequality (NI), the number of linear equality (LE) and the number of nonlinear equality (NE). In this table, a is the number of active constraints at the known optimal solution, $\vec{\mathbf{x}}^*$, and $f(\vec{\mathbf{x}}^*)$ is the objective function of the known optimal solution [147]. The detailed formulation of these benchmark test functions are presented in Appendix B for reference.

6.4.3 Simulation Results

The experiments reported in this study are performed on a computer with 1.66 GHz Duel-Core Processor and 1GB RAM operating on a Windows XP Professional. The programs are written in Matlab. Extensive experiments have been performed on all 24 benchmark test functions based upon comparison methods suggested in [147] which are explicitly followed by researchers in the field in order to have meaningful comparison. For three different functions evaluations (FEs) of 5,000, 50,000, and 500,000, the objective function error values, $f(\vec{\mathbf{x}}) - f(\vec{\mathbf{x}}^*)$ are found, while $f(\vec{\mathbf{x}}^*)$ is the best known solution [147] presented in the rightmost column in Table 6.2. Notice when $f(\vec{\mathbf{x}}) - f(\vec{\mathbf{x}}^*) \leq 1e-10$, the final error is considered as zero. For each benchmark test problem, a total of 25 independent runs are performed.

The statistical measures including the best, median, worst, mean and standard deviations are then computed. These results are tabulated in Tables 6.3 to 6.6. For the best, median and worst solutions, the number of constraints that can not satisfy feasibility condition is found and shown as an integer inside parenthesis after the best, median, and

worst solution, respectively in these tables. The parameter c shows three different integers demonstrating the number of constrains including equality and inequality ones that are violated by more than 1, 0.01 and 0.0001, respectively for the median solution. The parameter $\bar{v}$ indicates the average value of the violations of all constraints at the median solution defined in [147]:

Table 6.2 Summary of 24 benchmark test functions

Prob.	M	Type of function	ρ	LI	NI	LE	NE	a	$f(\vec{\mathbf{x}}^*)$
$g01$	13	Quadratic	0.0111%	9	0	0	0	6	-15.0000000000
$g02$	20	Nonlinear	99.9971%	0	2	0	0	1	-0.8036191042
$g03$	10	Polynomial	0.0000%	0	0	1	1	1	-1.0005001000
$g04$	5	Quadratic	52.1230%	0	6	0	0	2	-30665.5386717834
$g05$	4	Cubic	0.0000%	2	0	3	3	3	5126.4967140071
$g06$	2	Cubic	0.0066%	0	2	0	0	2	-6961.8138755802
$g07$	10	Quadratic	0.0003%	3	5	0	0	6	24.3062090681
$g08$	2	Nonlinear	0.8560%	0	2	0	0	0	-0.0958250415
$g09$	7	Polynomial	0.5121%	0	4	0	0	2	680.6300573745
$g10$	8	Linear	0.0010%	3	3	0	0	6	7049.2480205286
$g11$	2	Quadratic	0.0000%	0	0	0	1	1	0.7499000000
$g12$	3	Quadratic	4.7713%	0	1	0	0	0	-1.0000000000
$g13$	5	Nonlinear	0.0000%	0	0	0	3	3	0.0539415140
$g14$	10	Nonlinear	0.0000%	0	0	3	0	3	-47.7648884595
$g15$	3	Quadratic	0.0000%	0	0	1	1	2	961.7150222899
$g16$	5	Nonlinear	0.0204%	4	34	0	0	4	-1.9051552586
$g17$	6	Nonlinear	0.0000%	0	0	0	4	4	8853.5396748064
$g18$	9	Quadratic	0.0000%	0	13	0	0	6	-0.8660254038
$g19$	15	Nonlinear	33.4761%	0	5	0	0	0	32.6555929502
$g20$	24	Linear	0.0000%	0	6	2	12	16	0.2049794002
$g21$	7	Linear	0.0000%	0	1	0	5	6	193.7245100700
$g22$	22	Linear	0.0000%	0	1	8	11	19	236.4309755040
$g23$	9	Linear	0.0000%	0	2	3	1	6	-400.0551000000
$g24$	2	Linear	79.6556%	0	2	0	0	2	-5.5080132716

Table 6.3 Error values for different function evaluations (FEs) on test problems $g01 - g06$

FEs \ Prob.		$g01$	$g02$	$g03$	$g04$	$g05$	$g06$
5×10^3	Best	1.4372e1(0)	3.5723e-1(0)	5.8738e-1(0)	1.2328e2(0)	6.4758e2(4)	6.9146e1(0)
	Median	9.8536 (4)	4.5976e-1(0)	8.9452e-1(0)	6.4738e2(0)	8.6193e2(4)	8.4628e2(0)
	Worst	1.9525(7)	5.4657e-1(0)	1.12648(0)	9.4384e2(0)	1.5495e3(4)	2.3859e3(0)
	c	(2, 4, 4)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(4, 4, 4)	(0, 0, 0)
	$\bar{v}$	3.4517e-1	0	0	0	2.53456e1	0
	Mean	8.3780	4.6576e-1	9.9473e-1	6.3810e2	8.5907e2	7.1844e2
	Std.	3.3715	3.7841e-2	1.4528e-1	1.4925e2	4.8496e2	5.6820e2
5×10^4	Best	2.4729e-10(0)	1.4365e-2(0)	0(0)	6.3404e-8(0)	8.4357e-7(0)	4.9348e-6 (0)
	Median	3.5467e-10(0)	3.1324e-2(0)	0(0)	2.3748e-7(0)	7.5597e-7(0)	6.9834e-6(0)
	Worst	4.0234e-10(0)	5.9435e-2(0)	0(0)	7.8263e-6(0)	4.9528e-6(0)	8.5197e-6(0)
	c	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
	$\bar{v}$	0	0	0	0	0	0
	Mean	3.6294e-10	3.1048e-2	0	2.9230e-7	7.7823e-7	7.0125e-6
	Std.	4.5637e-12	1.6403e-2	0	4.3839e-7	1.8347e-7	5.9238e-7
5×10^5	Best	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
	Median	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
	Worst	0(0)	1.9543e-2(0)	0(0)	0(0)	0(0)	0(0)
	c	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
	$\bar{v}$	0	0	0	0	0	0
	Mean	0	1.9659e-3	0	0	0	0
	Std.	0	4.7549e-3	0	0	0	0

Table 6.4 Error values for different function evaluations (FEs) on test problems $g07 - g12$

FEs \ Prob.		$g07$	$g08$	$g09$	$g10$	$g11$	$g12$
5×10^3	Best	4.3452e1(0)	7.6478e-8(0)	9.5829(0)	5.3675e3(0)	2.5643e-4(0)	4.5645e-8(0)
	Median	2.6788e2(0)	3.2784e-4(0)	5.3950e1(0)	6.8574e3(2)	5.8274e-3(0)	3.5965e-5(0)
	Worst	3.9643e3(1)	8,5367e-1(0)	4.7204e2(0)	7.4534e2(4)	3.9837e-2(0)	1.6754e-2(0)
	c	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 2, 3)	(0, 0, 0)	(0, 0, 0)
	$\bar{v}$	0	0	0	2.4545e-2	0	0
	Mean	2.8642e2	4.8947e-4	5.0025e1	8.3554e3	6.9445e-3	8.5645e-4
	Std.	4.8034e2	7.3674e-3	2.6584e1	5.8689e3	4.5685e-3	6.1904e-3
5×10^4	Best	0(0)	0(0)	0(0)	4.2219e-7(0)	5.9854e-9(0)	0(0)
	Median	0(0)	0(0)	0(0)	3.9540e-6(0)	4.0546e-7(0)	0(0)
	Worst	0(0)	0(0)	0(0)	6.4859e-6(0)	6.9434e-5(0)	0(0)
	c	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
	$\bar{v}$	0	0	0	0	0	0
	Mean	0	0	0	4.0143e-6	7.8687e-6	0
	Std.	0	0	0	1.9344e-7	8.9676e-6	0
5×10^5	Best	0(0)	0(0)	0(0)	1.3494e-9(0)	0(0)	0(0)
	Median	0(0)	0(0)	0(0)	4.6015e-8(0)	0(0)	0(0)
	Worst	0(0)	0(0)	0(0)	9.5246e-8(0)	0(0)	0(0)
	c	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
	$\bar{v}$	0	0	0	0	0	0
	Mean	0	0	0	4.5064e-8	0	0
	Std.	0	0	0	7.0345e-9	0	0

Table 6.5 Error values for different function evaluations (FEs) on test problems $g13 - g18$

FEs \ Prob.		$g13$	$g14$	$g15$	$g16$	$g17$	$g18$
5×10^3	Best	6.8764(3)	-4.3950e1(3)	3.4289(2)	2.4959e-1(0)	3.5859e2(4)	3.8494(12)
	Median	8.2840(3)	-2.0960e2(3)	4.3395(2)	4.5851e-1(0)	6.2048e2(4)	4.5005(12)
	Worst	1.3940e1(3)	-2.3849e2(3)	5.3859(2)	7.4930e-1(2)	9.8363e2(4)	6.0375(12)
	c	(0, 3, 3)	(3, 3, 3)	(0, 2, 2)	(0, 0, 0)	(4, 4, 4)	(10, 11, 11)
	$\bar{v}$	1.3947	7.0902	1.4759e-1	0	8.3839e1	9.3849
	Mean	7.3904	-2.0035e2	4.2174	4.3735e-1	5.9303e2	4.6720
	Std.	1.8473	6.2387e1	2.6102	1.8276e-1	9.9278e1	1.8494
5×10^4	Best	2.3894e-9(0)	0(0)	0(0)	4.4748e-8(0)	2.1273e1(0)	0(0)
	Median	4.9694e-6(0)	0(0)	0(0)	1.9323e-4(0)	6.2893e1(0)	0(0)
	Worst	6.3938e-1(0)	0(0)	3.5796e-5(0)	2.4385e-2(0)	8.4849e1(0)	1.4634e-7(0)
	c	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
	$\bar{v}$	0	0	0	0	0	0
	Mean	5.9404e-2	0	3.7594e-7	2.5782e-4	3.8373e1	8.7561e-9
	Std.	3.8949e-1	0	4.2893e-4	6.4839e-3	3.2394e1	6.9661e-2
5×10^5	Best	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
	Median	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
	Worst	6.8495e-8(0)	0(0)	0(0)	0(0)	0(0)	0(0)
	c	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
	$\bar{v}$	0	0	0	0	0	0
	Mean	4.8055e-9	0	0	0	0	0
	Std.	2.5855e-6	0	0	0	0	0

Table 6.6 Error values for different function evaluations (FEs) on test problems $g19 - g24$

FEs \ Prob.		$g19$	$g20$	$g21$	$g22$	$g23$	$g24$
	Best	3.9605e2(0)	5.6996 (12)	7.5479e1(5)	8.4563e3(19)	4.2033e2(4)	8.4834e-4(0)
	Median	5.0387e2(0)	1.3656e1(19)	1.8977e2(5)	9.7685e3(19)	6.2017e2(5)	9.5092e-3(0)
	Worst	6.4760e2(0)	1.9574e1(17)	5.7689e2(5)	9.9964e3(19)	9.3945e2(6)	6.9804e-2(0)
5×10^3	c	(0, 0, 0)	(5, 16, 16)	(1, 4, 6)	(19, 19, 19)	(2, 5, 6)	(0, 0, 0)
	$\bar{v}$	0	2.8796	4.8632	8.6785e7	1.8495	0
	Mean	4.8792e2	1.4098e1	2.6778e2	1.1205e4	5.6996e2	1.0034e-2
	Std.	9.7634e1	1.7860e1	3.6781e2	4.8754e3	3.4856e2	1.8075e-2
	Best	8.9457e-8(0)	3.6759e-1(16)	8.9865e-5(0)	6.657(4)	4.7893e-4(0)	0(0)
	Median	3.6790e-6(0)	3.6758(16)	4.6453e-3(0)	2.4567e3(16)	2.6778e-3(0)	0(0)
	Worst	1.9426e-5(0)	7.9865(20)	6.0965(0)	5.7685e4(19)	8.5623e-2(0)	0(0)
5×10^4	c	(0, 0, 0)	(2, 5, 8)	(0, 0, 0)	(3, 8, 16)	(0, 0, 0)	(0, 0, 0)
	$\bar{v}$	0	8.9863e-1	0	2.5673e1	0	0
	Mean	4.9453e-6	3.7396	7.8757e-1	7.5678e3	7.5610e-3	0
	Std.	5.8438e-6	1.1930	8.9868	6.9868e3	3.7609e-2	0
	Best	0(0)	-3.0694e-2(18)	6.9854e-8(0)	1.4568(0)	0(0)	0(0)
	Median	0(0)	-2.4096e-2(16)	6.7685e-6(0)	7.9653e1(0)	0(0)	0(0)
	Worst	0(0)	-2.0129e-2(19)	9.0956e-6(0)	1.3576e2(0)	0(0)	0(0)
5×10^5	c	(0, 0, 0)	(1, 4, 6)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
	$\bar{v}$	0	1.3459e-2	0	0	0	0
	Mean	0	-2.5001e-2	2.5609e-6	9.7685e1	0	0
	Std.	0	4.6950e-3	5.8796e-6	3.5475e1	0	0

$$\bar{v} = \frac{1}{m}\left[\sum_{k=1}^{L} G_k(\vec{\mathbf{x}}) + \sum_{k=L+1}^{m} H_k(\vec{\mathbf{x}})\right], \tag{6.20}$$

where:

$$G_k(\vec{\mathbf{x}}) = \begin{cases} g_k(\vec{\mathbf{x}}), & \text{if } g_k(\vec{\mathbf{x}}) > 0 \\ 0, & \text{if } g_k(\vec{\mathbf{x}}) \leq 0 \end{cases}, \qquad k = 1,2,\dots,L, \tag{6.21}$$

and:

$$H_k(\vec{\mathbf{x}}) = \begin{cases} |h_k(\vec{\mathbf{x}})|, & \text{if } |h_k(\vec{\mathbf{x}})| - \delta > 0 \\ 0, & \text{if } |h_k(\vec{\mathbf{x}})| - \delta \leq 0 \end{cases}, \qquad k = L+1,\dots,m, \tag{6.22}$$

For each independent run, the number of function evaluations to locate a solution satisfying $f(\vec{\mathbf{x}}) - f(\vec{\mathbf{x}}^*) \leq 0.0001$ is recorded. For each benchmark function, statistical measures of these 25 runs including the best, median, worst, mean, and standard deviations are then computed. These results are shown in Table 6.7. In the same table, Feasible Rate, Success Rate and Success Performance are also calculated for each test function. Feasible Rate is a ratio of feasible runs over total runs, where feasible run is defined as a run with maximum function evaluation of 500,000 during which at least one feasible solution is found. Successful Rate is a ratio of successful runs over the total runs, where successful run is defined as a run during which the algorithm finds a feasible solution, $\vec{\mathbf{x}}$, satisfying $f(\vec{\mathbf{x}}) - f(\vec{\mathbf{x}}^*) \leq 0.0001$. Success Performance is defined as [147]:

$$\text{Success Performance} = \frac{mean(\text{FEs for succesful Runs}) \times (\text{Number of Total Runs})}{\text{Number of Successful Runs}}. \tag{6.23}$$

Table 6.7 Number of function evaluations (FEs) to achieve the fixed accuracy level ($f(\vec{x}) - f(\vec{x}^*) \leq 0.0001$), Success Rate, Feasibility Rate, and Success Performance.

Prob.	Best	Median	Worst	Mean	Std	Feasible Rate	Success Rate	Success Performance
$g01$	24786	27348	49601	35834	11639	100%	100%	35834
$g02$	56392	93674	500000	184530	173487	100%	76%	242803
$g03$	26498	28564	29129	28602	673.86	100%	100%	28602
$g04$	25983	26934	27045	26903	403.91	100%	100%	26903
$g05$	29629	31897	32983	30961	693.52	100%	100%	30961
$g06$	27688	29549	30189	29429	503.59	100%	100%	29429
$g07$	26024	28388	30877	28109	458.15	100%	100%	28109
$g08$	2302	5280	8938	5418.4	1935.4	100%	100%	5418.4
$g09$	30178	31866	32353	31327	331.57	100%	100%	31327
$g10$	26356	27990	29234	28028	459.09	100%	96%	29196
$g11$	4589	10678	31878	12897	10558	100%	100%	12897
$g12$	3289	7580	10454	6738.1	1378.5	100%	100%	6738.1
$g13$	31897	36878	256891	47895	43788	100%	100%	47895
$g14$	24678	28512	48724	26980	3589.2	100%	100%	26980
$g15$	30219	31029	32064	30984	335.76	100%	100%	30984
$g16$	28373	31795	69374	42750	2647.3	100%	100%	42750
$g17$	158367	193045	273890	210454	42084	100%	92%	228754
$g18$	28504	30496	62567	37575	6467	100%	100%	37575
$g19$	21345	23768	27910	24502	1032	100%	100%	24502
$g20$	-	-	-	-	-	0%	0%	-
$g21$	37385	122705	197614	141639	39574	100%	96%	147541
$g22$	-	-	-	-	-	100%	0%	-
$g23$	62091	182065	500000	259393	112038	100%	100%	259393
$g24$	17364	19391	29047	18972	4283	100%	100%	18972

These tables show that feasible solutions can be reliably found within the maximum FEs for all benchmark problems except for function $g20$. The final solutions of all benchmark problems can be identified with an error of less than 0.0001 from the optimal solution within the maximum FEs except for functions $g20$ and $g22$. Most benchmark functions find the optimal solution with the error of less than 0.0001 before 50,000 FEs except for functions $g02$, $g17$, $g20$, $g22$ and $g23$. It can also be observed that cultural CPSO has 100% feasible rate for all benchmark problems except function $g20$, and 100% success rate for all benchmark problems except for functions $g02$, $g10$, $g17$, $g21$ and $g22$. However it should be noted that for functions $g10$, $g17$ and $g21$ the success rate is fairly high at 96%, 92% and 96%, respectively. Summary of statistical results for the best, median, mean, worst, and standard deviation obtained by cultural CPSO over 25 independent runs are summarized in Table 6.8. As it can be seen in this table, except for function $g20$, feasible solutions have been found for all other benchmark problems.

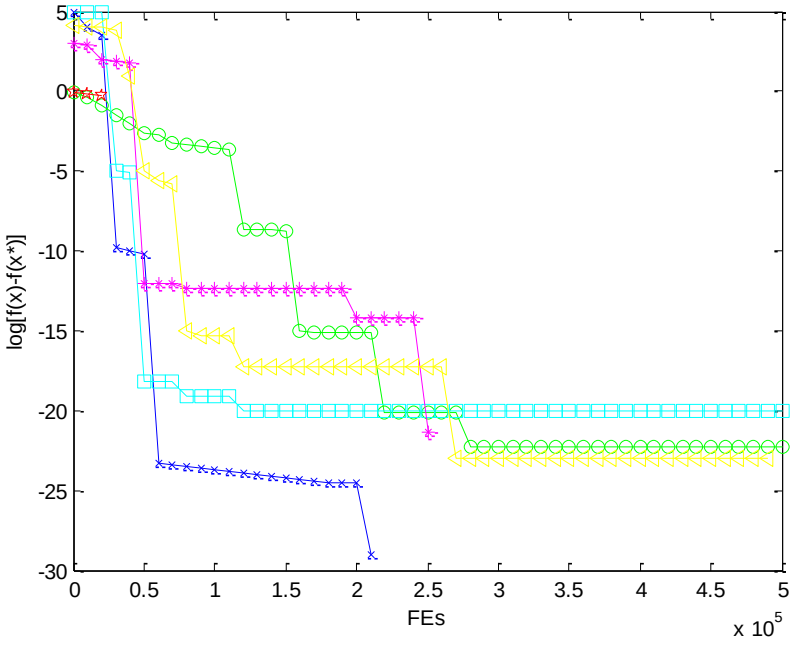
6.4.4 Convergence Graphs

For the median run for each test function with the function evaluations (FEs) of 500,000, two semi-log graphs are plotted for each test function. The first graph is $\log_{10}[f(\vec{\mathbf{x}}) - f(\vec{\mathbf{x}}^*)]$ vs. FEs, while $f(\vec{\mathbf{x}}^*)$ is given in the rightmost column of Table 6.2, and $f(\vec{\mathbf{x}})$ is the objective value for the best solution at the specific FE. The second graph

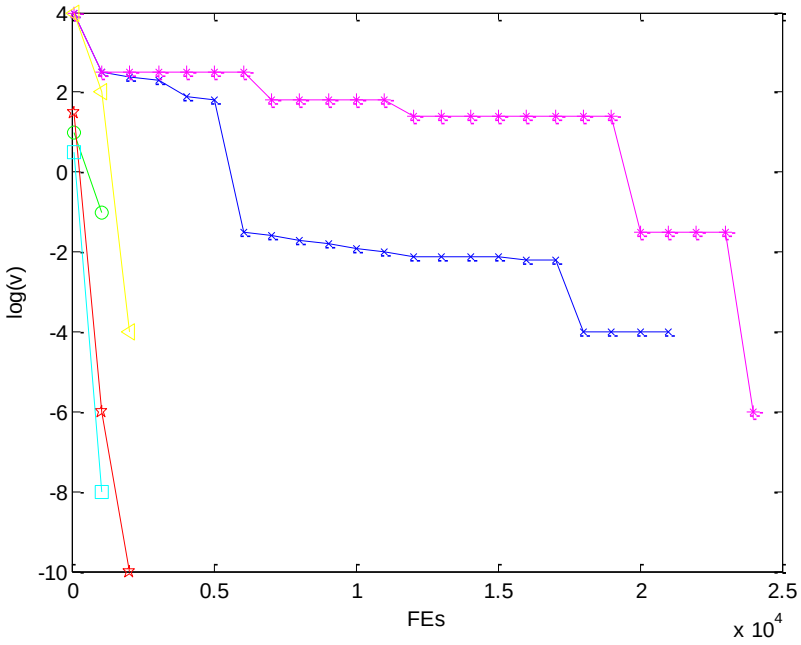
is $\log_{10}(\bar{v})$ vs. FEs, where $\bar{v}$ is the average value of the violations of all constraints at specific FE defined as Equations (6.20) to (6.22). For these two graphs, points which satisfy $f(\vec{\mathbf{x}}) - f(\vec{\mathbf{x}}^*) \leq 0$ are not plotted, since logarithm for zero or negative numbers cannot be computed. Figures 6.8 to 6.11 show these two graphs for all 24 benchmark problems.

Table 6.8 Summary of statistical results found by cultural CPSO (IS denotes for Infeasible Solution)

Prob.	Optimal	Best	Median	Mean	Worst	Std. Dev.
$g01$	-15.0000000000	-15.0000000000	-15.0000000000	-15.0000000000	-15.0000000000	0.0000e0
$g02$	-0.8036191042	-0.8036191042	-0.8036191042	-0.8016532042	-0.7840761042	4.6784e-3
$g03$	-1.0005001000	-1.0005001000	-1.0005001000	-1.0005001000	-1.0005001000	3.6759e-13
$g04$	-30665.5386717834	-30665.5386717834	-30665.5386717834	-30665.5386717834	-30665.5386717834	1.7890e-16
$g05$	5126.4967140071	5126.4967140071	5126.4967140071	5126.4967140071	5126.4967140071	6.0912e-12
$g06$	-6961.8138755802	-6961.8138755802	-6961.8138755802	-6961.8138755802	-6961.8138755802	3.8095e-11
$g07$	24.3062090681	24.3062090681	24.3062090681	24.3062090681	24.3062090681	1.3724e-12
$g08$	-0.0958250415	-0.0958250415	-0.0958250415	-0.0958250415	-0.0958250415	7.8088e-11
$g09$	680.6300573745	680.6300573745	680.6300573745	680.6300573745	680.6300573745	5.8797e-17
$g10$	7049.2480205286	7049.2480205299	7049.2480205746	7049.2480205736	7049.2480206238	6.9806e-7
$g11$	0.7499000000	0.7499000000	0.7499000000	0.7499000000	0.7499000000	4.6756e-17
$g12$	-1.0000000000	-1.0000000000	-1.0000000000	-1.0000000000	-1.0000000000	1.7648e-14
$g13$	0.0539415140	0.0539415140	0.0539415140	0.0539415188	0.0539415825	1.5409e-7
$g14$	-47.7648884595	-47.7648884595	-47.7648884595	-47.7648884595	-47.7648884595	6.7830e-11
$g15$	961.7150222899	961.7150222899	961.7150222899	961.7150222899	961.7150222899	2.6598e-16
$g16$	-1.9051552586	-1.9051552586	-1.9051552586	-1.9051552586	-1.9051552586	3.9578e-13
$g17$	8853.5396748064	8853.5396748064	8853.5396748064	8853.5396748064	8853.5396748064	1.5329e-11
$g18$	-0.8660254038	-0.8660254038	-0.8660254038	-0.8660254038	-0.8660254038	8.0934e-14
$g19$	32.6555929502	32.6555929502	32.6555929502	32.6555929502	32.6555929502	5.9083e-12
$g20$	0.2049794002	0.1742854002 (**IS**)	0.1435914002(**IS**)	0.1128974002(**IS**)	0.1848504002(**IS**)	7.3832e-2
$g21$	193.7245100700	193.7245101398	193.7245168385	193.7245126309	193.7245191656	4.6482e-5
$g22$	236.4309755040	237.887775504	316.083975504	334.115975504	372.190975504	1.5438e2
$g23$	-400.0551000000	-400.0551000000	-400.0551000000	-400.0551000000	-400.0551000000	6.2319e-11
$g24$	-5.5080132716	-5.5080132716	-5.5080132716	-5.5080132716	-5.5080132716	5.8794e-15

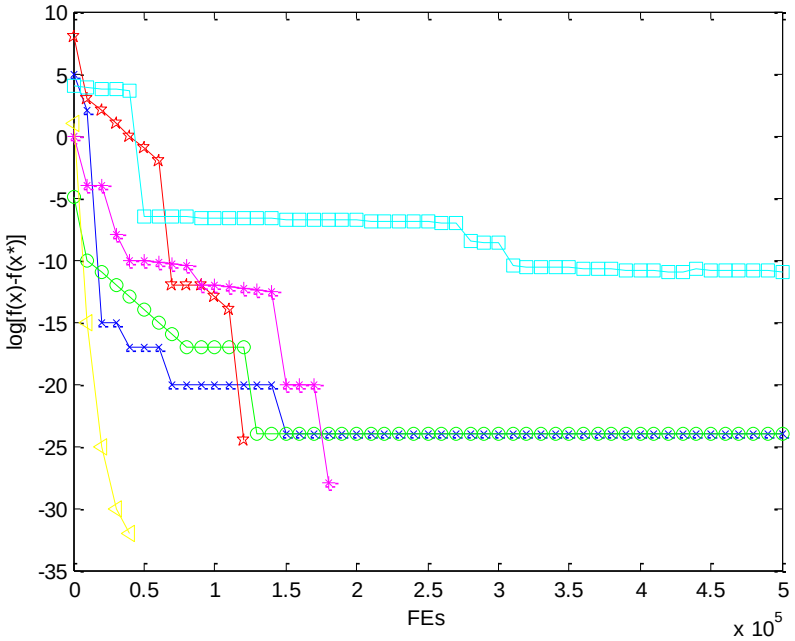


(a)

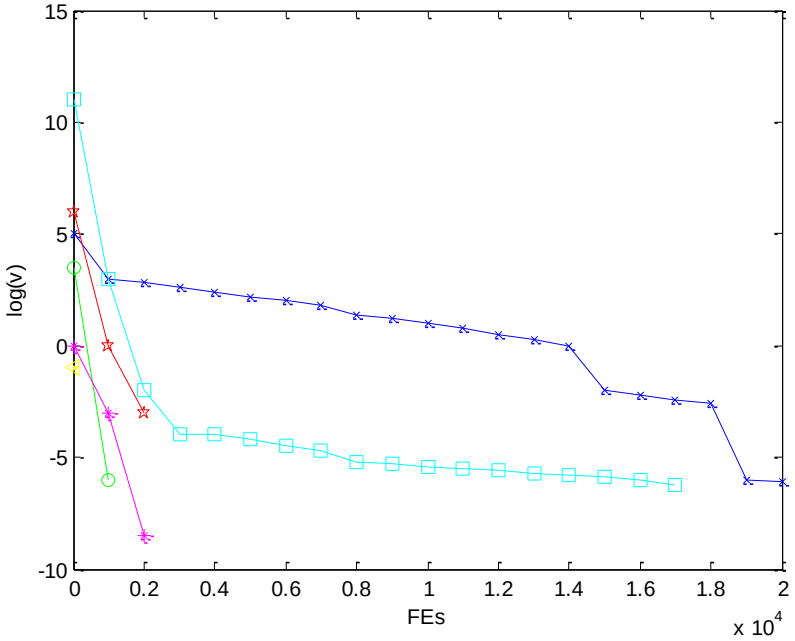


(b)

Figure 6.8 Convergence graphs for problems $g01$ (denoted as×), $g02$ (denoted asO), $g03$ (denoted as ☆), $g04$ (denoted as □), $g05$ (denoted as ∗) and $g06$ (denoted as ◁): (a) Function error values, (b) Mean constraint violations.

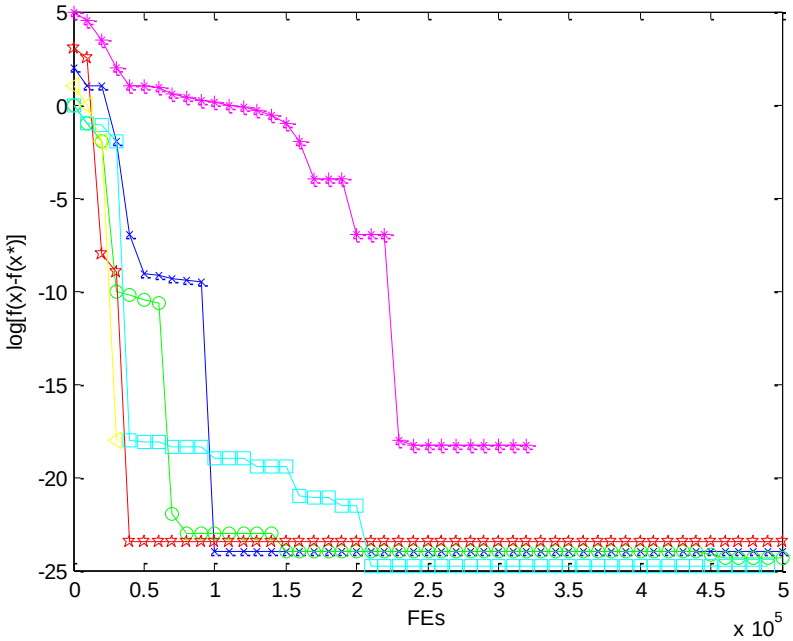


(a)

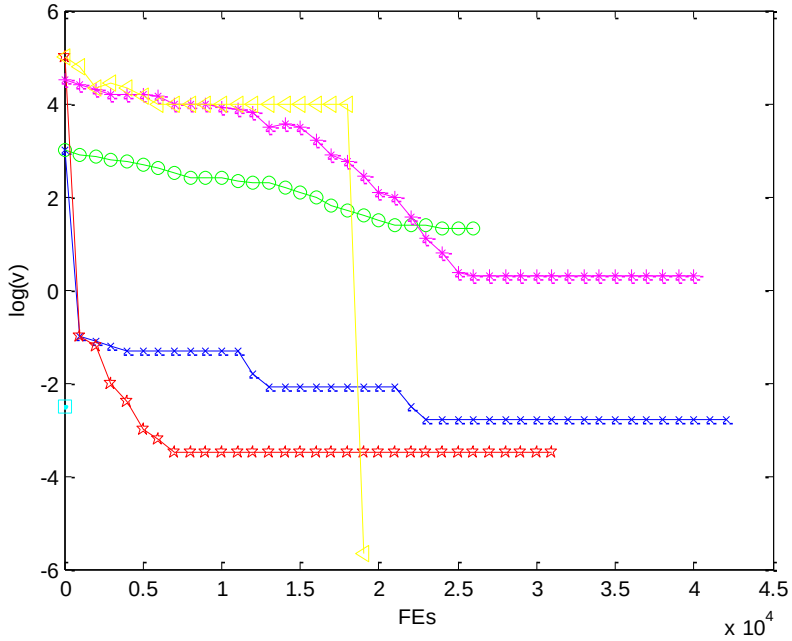


(b)

Figure 6.9 Convergence graphs for problems $g07$ (denoted as×), $g08$ (denoted asO), $g09$ (denoted as ☆), $g10$ (denoted as □), $g11$ (denoted as ∗) and $g12$ (denoted as ◁): (a) Function error values, (b) Mean constraint violations.

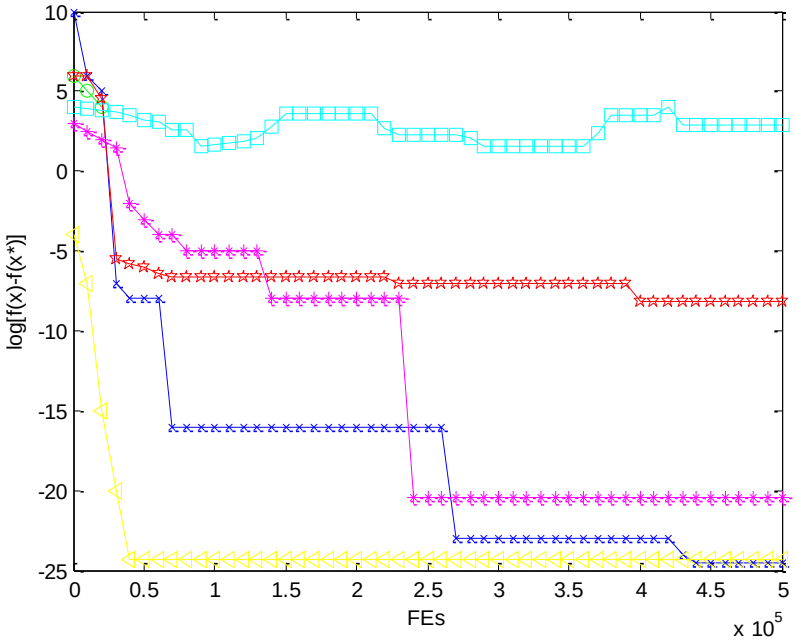


(a)

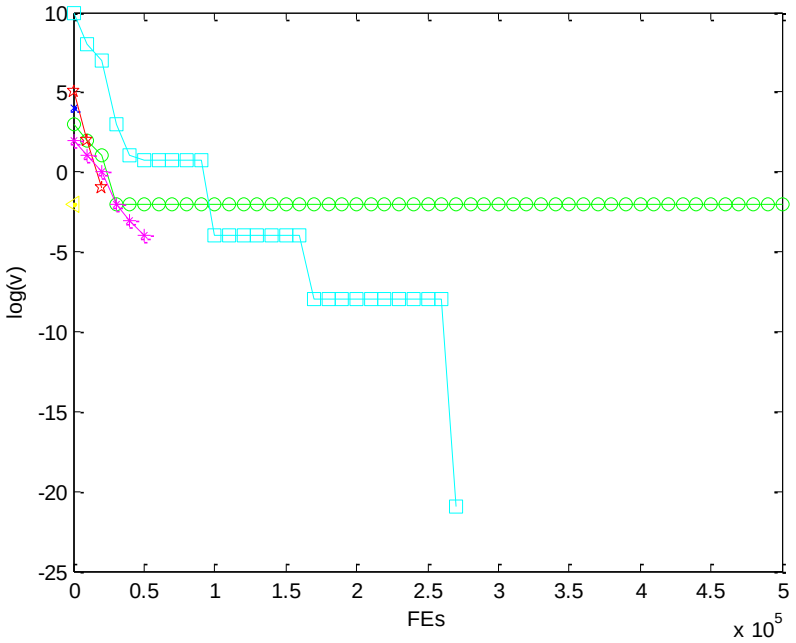


(b)

Figure 6.10 Convergence graphs for problems $g13$ (denoted as×), $g14$ (denoted asO), $g15$ (denoted as ☆), $g16$ (denoted as □), $g17$ (denoted as ∗) and $g18$ (denoted as ◁): (a) Function error values, (b) Mean constraint violations.



(a)



(b)

Figure 6.11 Convergence graphs for problems $g19$ (denoted as×), $g20$ (denoted asO), $g21$ (denoted as ☆), $g22$ (denoted as □), $g23$ (denoted as ∗) and $g24$ (denoted as ◁): (a) Function error values, (b) Mean constraint violations.

6.4.5 Algorithm Complexity

In Table 6.9, the algorithm's complexity corresponding to all 24 benchmark problems are shown. The computed times in seconds for complexity are $T1$, $T2$, and $(T2 - T1)/T1$ where $T1$ is defined as:

$$T1 = (\sum_{i=1}^{24} t1_i)/24, \tag{6.24}$$

where $t1_i$ is the computing time of 10,000 evaluations for problem i, and $T2$ is also defined as:

$$T2 = (\sum_{i=1}^{24} t2_i)/24, \tag{6.25}$$

where $t2_i$ is the complete computing time for the algorithm with 10,000 evaluations for problem i [147]. The running times shown in this table are related to the time spent in belief space, population space, acceptance function and influence functions.

Table 6.9 Computational complexity

$T1$	$T2$	$(T2 - T1)/T1$
6.2351	11.3280	0.8168

6.4.6 Performance Comparison

Furthermore, the performance of the cultural CPSO has been compared with ten state-of-the-art constrained optimization heuristics using their best-achieved reported results in terms of two performance indicators, feasible rate and success rate. The selected high-performance algorithms are PSO [148], DMS-PSO [149], ε_DE [150], GDE [151], jDE-2 [152], MDE [153], MPDE [154], PCX [155], PESO+ [156], SaDE

[157]. The comparative results are then demonstrated in Tables 6.10 and 6.11 for feasible rate and success rate, respectively. The average performance for each algorithm is also computed. Table 6.10 demonstrate that cultural CPSO has the average feasible rate of 95.83% on 24 benchmark problems that places it at top performing algorithm along with DMS-PSO [149], ε_DE [150] and SaDE [157]. Results in Table 6.11 indicate that proposed cultural CPSO has the average success rate of 90.00% on 24 benchmark problems placing it at the third best performing algorithm after ε_DE and PCX [155] with 91.67% and 90.17% of success rate, respectively.

6.4.7 Sensitivity Analysis

In this subsection, the sensitivity of the algorithm performance with respect to some parameters is briefly assessed. The parameters to be tuned in the proposed algorithm are the personal acceleration, c_p, swarm acceleration, c_s, global acceleration, c_g and the rate for information exchange among swarms, r. Notice that the allowance number of particles to migrate, $N_{migration}$, is a fraction of the population size and does not need to be tuned. The tolerance for equality constraints is considered a fixed number of 0.0001 to be able to fairly compare the results of the proposed algorithm with those of other algorithms. The flight momentum is also randomly selected from a uniform distribution and does not have tuning issue, and maximum velocity of the particles in specific dimension depends on the particle's positional range, consequently will not be adjusted either.

Table 6.10 Comparison of cultural CPSO with the state-of-the-art constrained optimization methods in terms of feasible rate

Prob.	PSO [148]	DMS-PSO [149]	ε_DE [150]	GDE [151]	jDE-2 [152]	MDE [153]	MPDE [154]	PCX [155]	PESO+ [156]	SaDE [157]	Cultural CPSO
$g01$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g02$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g03$	100%	100%	100%	96%	100%	100%	100%	100%	100%	100%	100%
$g04$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g05$	100%	100%	100%	96%	100%	100%	100%	100%	100%	100%	100%
$g06$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g07$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g08$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g09$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g10$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g11$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g12$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g13$	100%	100%	100%	88%	100%	100%	88%	100%	100%	100%	100%
$g14$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g15$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g16$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g17$	100%	100%	100%	76%	100%	100%	96%	100%	100%	100%	100%
$g18$	100%	100%	100%	84%	100%	100%	100%	100%	100%	100%	100%
$g19$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g20$	0%	0%	0%	0%	4%	0%	0%	0%	0%	0%	0%
$g21$	8%	100%	100%	88%	100%	100%	100%	100%	100%	100%	100%
$g22$	0%	100%	100%	0%	0%	0%	0%	0%	0%	100%	100%
$g23$	100%	100%	100%	88%	100%	100%	100%	100%	96%	100%	100%
$g24$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
Average	87.83%	95.83%	95.83%	88.17%	91.83%	91.67%	91.00%	91.67%	91.50%	95.83%	95.83%

A sensitivity analysis has been applied to a selected set of benchmark problems by varying one parameter at a time while the other parameters are set as values in Table 6.1. Test functions $g03$, $g10$, $g14$, $g18$, and $g21$ have been selected for which the feasibility and success rate are extremely well or very well, therefore the comparison can be done by changing tuning parameters. Tables 6.12 to 6.15 show the results of the

sensitivity analysis. For every set of parameters, 25 independent runs are performed. The mean statistical results for feasible solutions have been recorded along with the feasible rate and the success rate as defined earlier, for every set of parameters.

Table 6.11 Comparison of cultural CPSO with the state-of-the-art constrained optimization methods in terms of success rate

Prob	PSO [148]	DMS-PSO [149]	ε_DE [150]	GDE [151]	jDE-2 [152]	MDE [153]	MPDE [154]	PCX [155]	PESO+ [156]	SaDE [157]	Cultural CPSO
$g01$	72%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g02$	0%	84%	100%	72%	92%	16%	92%	64%	56%	84%	76%
$g03$	0%	100%	100%	4%	0%	100%	84%	100%	100%	96%	100%
$g04$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g05$	24%	100%	100%	92%	68%	100%	100%	100%	100%	100%	100%
$g06$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g07$	72%	100%	100%	100%	100%	100%	100%	100%	96%	100%	100%
$g08$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g09$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g10$	8%	100%	100%	100%	100%	100%	100%	100%	16%	100%	96%
$g11$	100%	100%	100%	100%	96%	100%	96%	100%	100%	100%	100%
$g12$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g13$	0%	100%	100%	40%	0%	100%	48%	100%	100%	100%	100%
$g14$	0%	100%	100%	96%	100%	100%	100%	100%	0%	80%	100%
$g15$	84%	100%	100%	96%	96%	100%	100%	100%	100%	100%	100%
$g16$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
$g17$	0%	0%	100%	16%	4%	100%	28%	100%	0%	4%	92%
$g18$	100%	100%	100%	76%	100%	100%	100%	100%	92%	92%	100%
$g19$	12%	100%	100%	88%	100%	0%	100%	100%	0%	100%	100%
$g20$	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
$g21$	0%	100%	100%	60%	92%	100%	68%	100%	0%	60%	96%
$g22$	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
$g23$	0%	100%	100%	40%	92%	100%	100%	100%	0%	88%	100%
$g24$	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
Aver	48.83	86.83	91.67%	74.17	76.67	84.00	84.00%	90.17	65.00%	83.50	90.00%

The results in Tables 6.12 to 6.14 show the effect of varying the personal, swarm and global acceleration on the algorithm's performance. It seems that the effect of varying the acceleration on algorithm's performance is by some extent, problem-dependent. This makes it difficult to identify the optimum parameters in order to achieve the best performance.

Table 6.12 Sensitivity analysis with respect to personal acceleration, c_p: Mean results of feasible solutions, Feasible Rate and Success Rate are computed over 25 independent runs.

c_p / Prob.	Mean results of feasible solutions, Feasible Rate, Success Rate			
	1.0	1.5	2.0	2.5
$g03$	-1.0005001000, 100%,100%	-1.0005001000, 100%,100%	-1.0005001000, 100%,100%	-1.0005001000, 100%,100%
$g10$	7049.248020570674, 100%,96%	7049.248020573664, 100%,96%	7049.248020573941, 100%,100%	7049.248020570062, 100%,96%
$g14$	-47.7648884595, 100%,100%	-47.7648884595, 100%,100%	-47.7648884595, 100%,100%	-47.7648884595, 100%,92%
$g18$	-0.8660254038, 100%,100%	-0.8660254038, 100%,100%	-0.8660254038, 100%,96%	-0.8660254038, 100%,100%
$g21$	193.7245128803, 100%,96%	193.7245126309, 100%,96%	193.7245121603, 100%,100%	193.7245139367, 100%,92%

We suggest further analyzing this issue and implementing an adaptive dynamic law based upon the need for exploration or exploitation in the **f**-v space discussed in spatial knowledge of the belief space. This approach is similar to the one introduced in [140-141]. The results in Table 6.16 show that by increasing the rate for information exchange, the success rate will be greatly improved for all selected benchmark problems. On the other hand by decreasing this rate, the success rate gets deteriorate.

Table 6.13 Sensitivity analysis with respect to swarm acceleration, c_s: Mean results of feasible solutions, Feasible Rate and Success Rate are computed over 25 independent runs.

c_s / Prob.	Mean results of feasible solutions, Feasible Rate, Success Rate			
	1.0	1.5	2.0	2.5
$g03$	-1.0005001000, 100%,100%	-1.0005001000, 100%,100%	-1.0005001000, 100%,100%	-1.0005001000, 100%,100%
$g10$	7049.248020574453, 100%,100%	7049.248020573664, 100%,96%	7049.248020579940, 100%,96%	7049.248020573296, 100%,96%
$g14$	-47.7648884595, 100%,100%	-47.7648884595, 100%,100%	-47.7648884595, 100%,100%	-47.7648884595, 100%,100%
$g18$	-0.8660254038, 100%,96%	-0.8660254038, 100%,100%	-0.8660254038, 100%,100%	-0.8660254038, 100%,96%
$g21$	193.7245126006, 100%,100%	193.7245126309, 100%,96%	193.7245124569, 100%,100%	193.7245124389, 100%,96%

6.5 Discussions

In this chapter, the cultural CPSO, a novel heuristic to solve constrained optimization problems has been proposed which incorporates information of objective function and constraints violation, to construct a cultural framework consisting two sections: a multiple swarm PSO with the ability of inter-swarm communication as population space and a belief space including four sections, normative knowledge, spatial knowledge, situational knowledge, and temporal knowledge. Each swarm assembles two lists of particles to share with other swarms based upon cultural information retrieved from different sections of the belief space. This cultural-based communication facilitates

the algorithm's performance on better handling the constraints along with optimizing the objective function simultaneously. Cultural CPSO shows competitive results when performing extensive experiments on 24 benchmark test functions. Comparison study with chosen state-of-the-art constrained optimization techniques indicate that cultural CPSO is able to perform well competitive in terms of commonly used performance metrics, feasible rate and success rate. Furthermore, sensitivity analysis was performed on the parameters of the paradigm, which shows that by increasing the rate of information exchange, the success rate is greatly improved. As future work, the proposed framework for single-objective optimization will be extended into a cultural-based multiobjective particle swarm optimization and to exploit its robust performance under dynamic environment when fitness landscape and constraints will change periodically or sporadically.

Table 6.14 Sensitivity analysis with respect to global acceleration, c_g: Mean results of feasible solutions, Feasible Rate and Success Rate are computed over 25 independent runs.

c_g / Prob.	Mean results of feasible solutions, Feasible Rate, Success Rate			
	1.0	1.5	2.0	2.5
$g03$	-1.0005001000, 100%,100%	-1.0005001000, 100%,100%	-1.0005001000, 100%,100%	-1.0005001000, 100%,100%
$g10$	7049.248020573377, 100%,96%	7049.248020573664, 100%,96%	7049.248020584087, 100%,100%	7049.248020593467, 100%,96%
$g14$	-47.7648884595, 100%,100%	-47.7648884595, 100%,100%	-47.7648884595, 100%,92%	-47.7648884595, 100%,100%
$g18$	-0.8660254038, 100%,96%	-0.8660254038, 100%,100%	-0.8660254038, 100%,96%	-0.8660254038, 100%,100%
$g21$	193.7245146753, 100%,96%	193.7245126309, 100%,96%	193.7245128903, 100%,92%	193.7245136098, 100%,100%

Table 6.15 Sensitivity analysis with respect to rate of information exchange, r: Mean results of feasible solutions, Feasible Rate and Success Rate are computed over 25 independent runs.

r / Prob.	Mean results of feasible solutions, Feasible Rate, Success Rate			
	10%	20%	30%	40%
$g03$	-1.0005001000, 100%,92%	-1.0005001000, 100%,100%	-1.0005001000, 100%,100%	-1.0005001000, 100%,100%
$g10$	7049.248020692614, 100%,92%	7049.248020579157, 100%,96%	7049.248020573664, 100%,96%	7049.248020550004, 100%,100%
$g14$	-47.7648884586, 100%,96%	-47.7648884595, 100%,100%	-47.7648884595, 100%,100%	-47.7648884595, 100%,100%
$g18$	-0.8660254017, 100%,96%	-0.8660254038, 100%,100%	-0.8660254038, 100%,100%	-0.8660254038, 100%,100%
$g21$	193.7245268306, 100%,92%	193.7245138506, 100%,96%	193.7245126309, 100%,96%	193.7245110215, 100%,100%

CHAPTER VII

DYNAMIC OPTIMIZATION USING CULTURAL-BASED PARTICLE SWARM OPTIMIZATION

7.1 Introduction

Many real-world optimization problems are dynamic thus the optimum solution changes in time. In such cases, the optimization algorithm should detect the change and respond to the change promptly. Examples of dynamic optimization problems include jobs scheduling, changing profits in portfolio optimization, and fluctuating demand. There are four major categories of uncertainties that have been dealt with using population based evolutionary approaches: noise in the fitness function, perturbations in the design variables, approximation in the fitness function, and dynamism in optimal solutions [12]. While noise and approximation bring uncertainty in the objective function, perturbation introduces uncertainty in the decision space. This study is focused on dynamic optimization problems (DOPs), formulated as:

Optimize $f(\mathbf{x}, e)$, (7.1)

where $\mathbf{x} = (x_1, x_2, ..., x_M)$ is the *M*-dimensional decision variable limited in each dimension as $x_{j,\min} \le x_j \le x_{j,\max}$ (for $j = 1,2,..., M$), f is the objective function, e represents the possible change in the objective function, constraints, environmental parameters, or problem representations during optimization process. As a result these changes represented by parameter e may affect the height, width, or location of optimum solution or a combination of these three parts [13]. For the simplicity purposes, this study is performed on the minimization problems. Note that a maximization problem can be converted to a minimization problem simply using multiplication by –1.

One common example of DOPs is job shop scheduling problems in which new jobs arrive or machines may break down during operations resulting a need for dynamic job schedules to accommodate the changes over time [10]. Another example of DOPs is dynamic portfolio problem in which the goal is to obtain an optimal allocation of assets to maximize profit and minimize investment risk [11]. Dynamic portfolio management can also be observed in coordinating different power stations in order to maximize profit and minimize risk. Some of the uncertainties here include spot market prices, load obligations, and strip/option prices. Practically speaking, optimization can be needed for the market price as often as every hour [11].

Population based heuristic had been adopted to solve optimization problems with dynamic landscape in the last few years. Particle swarm optimization (PSO) [1] is a popular population based paradigms introduced within the last decade. PSO mimics

behavior of the flocking birds by introducing a simple particle flight mechanism as:

$$v_i^d(t+1) = wv_i^d(t) + c_p r_1(pbest_i^d(t) - x_i^d(t)) + c_g r_2(gbest^d(t) - x_i^d(t)), \quad (7.2)$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1), \quad (7.3)$$

where $x_i^d(t)$ is the d-th dimension of the position of the i-th particle at time t ($i = 1,2,..., N$ and $d = 1,2,..., M$, where N is number of particles and M is the decision space dimension). $v_i^d(t)$ is the d-th dimension of the velocity of the i-th particle at time t. $pbest_i^d(t)$ is the d-th dimension of the personal best position of the i-th particle at time t, and $gbest^d(t)$ is the d-th dimension of the global best position at time t. c_p and c_g are the constant personal and global acceleration which give different importance weight to personal or global term of (7.2). r_1 and r_2 are uniform random numbers from (0,1) to give stochastic characteristics to the flight of particles. w is the velocity inertia weight of the particles. The application of PSO to dynamic optimization problems has been studied by various researchers [10, 44, 84-98, 158-163]. There are some issues with the PSO mechanism that needs to be addressed. One of them is the outdated memory in a sense that if the problem changes, a previously good solution stored as neighborhood or personal best may no longer be good, and will mislead the swarm towards false optima if the memory is not updated. The other issue is diversity loss. The population should

normally collapse around the best solution found during the optimization. In dynamic optimization, the partially converged population after a change is detected should quickly re-diversify, find the new optimum and re-converge quickly [10]. A number of adaptations have been applied to PSO in order to solve these deficiencies; memories can be refreshed or forgotten and swarms may be re-diversified through randomization, or exchange of information using multi-swarms.

In general, a good evolutionary heuristic to solve DOPs should be able to track the changing optimal solution even under high severity and frequency of change. It must reuse as much information as possible from previous generations to enhance the optimization search. Among the researches performed in dynamic PSO none of these studies use information from all particles to perform re-diversification through migration and repulsion. When particles share their information through migration process, they will be able to quickly re-diversify and move efficiently towards new optimum by re-converging around it. In order to construct the environment required for this re-divergence and re-convergence, we need to build groundwork to assist us to utilize this information. The major groundwork is the belief space of cultural algorithm assisting the particles in an organized informational manner to locate the necessary information.

Through psychosocial literature, studies show that attitudinal similarity leads to attraction while dissimilarity leads to repulsion in interpersonal relationship [14], consequently people often diverge from members of other social groups by selecting cultural tastes (e.g., possessions, attitudes, or behaviors) that distinguish them from

others. For example, a field study has found that students stopped wearing a particular wristband when members of a geeky dormitory next door started wearing them [15]. Indeed different cultural beliefs lead to repulsion and increase the possibilities of divergence in ideas and in turn open up the doors to new opportunities.

Computationally speaking, one difficulty is to find the proper information to adopt in order to rely on a quick re-diversification when a change happens in the environment. Using many concepts from the cultural algorithm, such as spatial knowledge, temporal knowledge, domain knowledge, normative knowledge and situational knowledge, we will be able to efficiently and effectively organize the available knowledge to adopt in several steps of the PSO's updating mechanism as well as re-diversification and repulsion among swarms. The special re-diversification problem to deal with the change in dynamic is an important task that cannot be solved unless we have access to the knowledge throughout the search process that is performed by the cultural algorithm as the computational framework.

In this study, a novel computational framework based on cultural algorithm has been proposed using knowledge stored in the belief space to re-diversify the population right after a change takes place in the dynamic of the problem. Thus the algorithm can comfortably compute the repulsion factor for each particle and locate the leading particles in the personal level, swarm level and global level. Each particle in the proposed cultural-based dynamic PSO will fly through a mechanism of three level flight incorporated with a repulsion factor. After a change takes place, particles regroup into several swarms and a

diversity-based migration among swarms along with repulsive mechanism implemented in repulsion factor will take place to increase the diversity as quickly as possible. The remaining sections of this chapter to complete the presentation are as following. In Section 7.2, related works in dynamic PSO and related research in cultural algorithm have been reviewed. Section 7.3 includes a detailed description of the proposed cultural-based dynamic PSO. In Section 7.4, simulation results are evaluated on the benchmark test problems in comparison with the state-of-the-art paradigms. Lastly, Section 7.5 summarizes the concluding remarks and future work of this study.

7.2 Review of Literature

7.2.1 Related Work in Dynamic PSO

Relevant works of particle swarm optimization that had been adopted to solve DOPs are briefly discussed in this subsection in order to motivate the proposed ideas. Particle swarm optimization has demonstrated its ability to solve the dynamic optimization problems. Carlisle and Dozier [84] adjusted PSO mechanism so it avoids making position/velocity decision based on the outdated memory. They introduced periodic resetting by having the particles periodically replace their *pbest* vector with their current position, forgetting their past experiences. They also introduced triggered resetting in which particles reset when the goal moves some specific distance from its original position. Eberhart and Shi [44] proposed that when perturbation is small, the

initialization of the swarm can start from old population, while with large perturbation, it would be better to re-initialize and then compare the results with the old swarm and select the best one. Hu and Eberhart [85] introduced a detection and response paradigm for PSO to solve dynamic problems in which *gbest* and the second global best are evaluated to monitor the changes. As to respond, the whole particles' positions are re-randomized.

Blackwell and colleages proposed charged swarm to avoid collision among particles based upon the force between electric charges which is inversely proportional to distance squared [86]. In a later work, the atomic model of PSO [87] and quantum PSO [88] are introduced in which the particles follow the structure of the chemical atom including a cloud of electrons randomly orbiting with a specific radius around the nucleolus. They have applied their models into multiple swarm PSO to solve multiple peak dynamic function problem [88], outperforming other evolutionary algorithm based heuristics. An anti-convergence operator is introduced [89] for swarms to interact with each other. Also an excluding operation is performed on swarms with their best solutions within a predefined radius. The nearby swarms compete with each other in order to promote diversity. The winner, the swarm with the best function value at its swarm attractor, will remain, while the loser will be re-initialized in the search space [89]. Blackwell [90] proposed swarms birth and death by allowing multiple swarms to regulate their size by bringing new swarms to existence, or diminishing redundant swarms. This dynamic swarm size removes the need for anti-convergence and exclusion operators in the PSO mechanism.

Brabazon and colleagues [158] adapted particle swarm metaphor in the domain of organizational adaptation in the presence of uncertainty. Strategic adaptation is considered as an attempt to uncover peaks on a high-dimensional strategic landscape. Some strategic configurations produce high profits, others produce poor results. A model is also adopted to estimate the noise incorporated in the strategy fitness. Janson and Middendorf [91] proposed partitioned hierarchical PSO for dynamic optimization problems. In their model, the population is partitioned into some tree-form sub-hierarchies for a limited number of iterations after a change is detected. These sub-hierarchies continue to independently search for the optimum, resulting a wider spread-out of the search process after the change has occurred. The topmost level of tree-form hierarchies which contain the current best particle does not change, but all lower sub-hierarchies (sub-swarms) by re-initializing the position and velocity and resetting their personal best positions. These sub-hierarchies are rejoined again after a predefined number of iterations. In a later work [159] a function re-evaluation paradigm is added to handle the noise. In this work, change detection mechanism for noisy environment is also proposed based upon observing the changes occurring within the hierarchy.

Venayagamoorthy [160] adopted adaptive critic design (ACD) to handle DOP problems using particle swarm. The dynamic change in this study is caused in the inertia weight with the goal to optimize the objective function. Two neural networks of the ACD, namely Critic network and Action network, will receive the inputs as the inertia weight and the fitness value for *gbest* of the current iteration respectively. The objective

of the Action network is to minimize the output of the Critic network by varying the inertia weight to improve the *gbest* fitness. Esquivel and Coello Coello [92] proposed a dynamic macro-mutation operator along with PSO to maintain the diversity throughout the search process in order to solve DOPs. Every coordinate of each particle will undergo an independent mutation with a dynamic probability which possess its highest value when the change occurs in the dynamic landscape and gradually decreases till the next change takes place.

Parsopoulos and Vrahatis [93] adopted their proposed unified PSO in dynamic environments. The unified PSO combines the exploration and exploitation term of the PSO mechanism into a unification factor to balance the influence of the global and local search directions. Zhang *et al*. [94] proposed a direct relation between the inertia weight of the particle and the change. In their model, the new *gbest* and *pbest* for each particle affect the inertia weight of the particle whenever a change in *gbest* or *pbest* occurs. Pan *et al*. [95] modified the PSO paradigm using a probability based movement of particles based upon the concept of energy change probability in Simulated Annealing (SA). The particle will move to the next position computed through traditional PSO heuristics only with a specific probability that exponentially depends on the difference between the objective values of the current and next iterations.

Trojanowski proposed quantum particles in multi-swarm to solve dynamic optimization tasks. His two-phase paradigm includes computing an angle and a distance for the new location of the particles. The proposed method allows the locations to be

distributed over the entire search space. The angle is obtained from an angularly uniform distribution on the surface of a hyper-sphere while the distance is an α-stable random variate [161]. Parrott and Li [96] proposed species based PSO for solving dynamic optimization problems. The population is divided into some swarms, each surrounding a dominating particle called species seeds which are identified from the entire population based upon their objective function values. The new seed should not fall within the predefined radius of all previously found seeds in order to promote diversity. The seeds are then selected as the neighborhood best for different swarms. In a later work, Li and colleagues [10, 162-163] included quantum particles into species based PSO to promote more diversity along with the re-randomization of the worst species.

Du and Li [97] introduced multi-strategy ensemble PSO in which particles are divided into two sections, part I uses a Gaussian local search to quickly seek global optimum in the current environment, while part II uses differential mutation to explore the search space. The position of particles in part II do not follow the traditional PSO mechanism, instead each particle in part II is determined by the particle in part I through a mutation strategy. There is 50% chance of getting closer to a randomly chosen *pbest* particles or going farther away from that *pbest*. Liu *et al*. [98] introduced a modified PSO to solve DOPs. In the proposed model, PSO consists of many compound particles. Each compound particle includes three single particles equilaterally distanced from each other in a triangular shape. A special reflection scheme is proposed to explore the search space more comprehensively in which the position of the worst particle among three in the

compound will be replaced with the reflected one. In each compound particle, after reflection is performed, a representative among these three particles is probabilistically chosen based upon the objective function values and distance from other two member particles. The representative member particles will then participate in PSO update mechanism. The two non-representative particles will also move the same distance/direction as representative particle has been moved in order to preserve the valuable information.

7.2.2 Related Works in Cultural Algorithm for Dynamic Optimization

Reynolds [3] proposed cultural algorithm (CA) as a double interconnecting heritage system in which information passed along to the next iteration through two interconnecting spaces, population and belief space. Defining culture as information storage in a broader than individual level which is accessible by all society members, CA tries to mimic it through its belief space scheme [99]. CA has shown its ability to solve different types of problems including dynamic optimization problems [106, 164]. Cultural framework had also been successfully adopted to assist particle swarm optimization to solve multiobjective optimization problems [140-141], and constrained optimization problems [165].

7.3 Cultural Particle Swarm for Dynamic Optimization

A summary of the pseudocode of the proposed paradigm is depicted in Figure 7.1 and a block diagram representation of the proposed algorithm is demonstrated in Figure 7.2. The population space (PSO) will be initialized and then divided into several swarms according to the closeness of the particles. The belief space (BLF) is then initialized. We evaluate population space using the objective function values. Next we apply acceptance function to select some particles which will be later adopted for the belief space. Belief space consists of five sections, situational, temporal (or history), domain, normative and spatial (topographical) knowledge. This cultural framework plays a key role in the heuristics. Next we apply influence functions to the belief space in order to select the key parameters of PSO for next iteration, including the repulsion factor for each particle, personal best, swarm best and global best. Through a scheme using information from a belief space, the change in dynamic will be detected. As soon as the change is detected, influence function applies to the belief space to perform the repulsive diversity-promoted migration among swarms. This migration will take place using the information extracted from the belief space. Then particles in the population space fly using newly computed repulsion factor, personal, swarm, and global best. This process continues until the stopping criteria are met.

In the remainder of this section, thorough explanation of the multi-swarm divergence-promoted population space, acceptance function, different parts of belief space including situational, temporal, domain, normative and spatial knowledge, influence functions including change-driving diversity-based migration are presented.

Initialize PSO at t=0.

Initialize BLF at t=0

Repeat

- Evaluate PSO(t).
- Divide PSO(t) into several swarms using k-means.
- Apply ACCEPTANCE function to PSO(t) to select particles which affect BLF(t).
- Adjust BLF(t) including Situational, Temporal, Domain, Normative, and Spatial Knowledge.
- Apply INFLUENCE function to BLF(t) to select *pbest*(t), *sbest*(t), and *gbest*(t) and to compute the repulsion factor for each particle of PSO(t).
- If change is detected, perform the repulsive diversity-based migration among the swarms.
- t=t+1.
- Update PSO(t) using new repulsion factors *pbest*(t), *sbest*(t), and *gbest*(t).

Until Termination Criteria are met.

End

Figure 7.1 Pseudocode of the cultural-based dynamic PSO

7.3.1 Multi Swarm Population Space

The population space in the proposed algorithm includes several swarms in which each swarm performs a modified divergence-promoted PSO paradigm. The particles are clustered into a predefined number of swarms using k-means clustering algorithm. In this study, the number of swarms, P, is 0.1 of the population size, N:

$$P = \lfloor 0.1N \rfloor, \tag{7.4}$$

where $\lfloor . \rfloor$ is a rounding operator. In order to solve the diversity loss due to dynamic environment, a modification is added to the original three-level flight of PSO mechanism introduced by Yen and Daneshyari [144-145] based upon repulsion factor between particles. In the three-level flight, particle will follow the best attained experience in its history (personal level), and simultaneously follow the best behaving particle in its swarm to achieve a synchronal behavior in the neighboring particles and to share the information (swarm level), and finally also follow the best behaving particle in the whole population (global level). This paradigm of PSO has been formulated in [165] as:

$$v_i^d(t+1) = wv_i^d(t) + c_p r_1(pbest_i^d(t) - x_i^d(t)) + c_s r_2(sbest_i^d(t) - x_i^d(t)) + c_g r_3(gbest^d(t) - x_i^d(t)), \tag{7.5}$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1), \tag{7.6}$$

where $v_i^d(t)$ is the *d*-th dimension of velocity of the *i*-th particle at time *t*, $x_i^d(t)$ is the *d*-th dimension of position of the *i*-th particle at time *t*, $pbest_i^d(t)$ is the *d*-th dimension of the best past position of the *i*-th particle at time *t*, $sbest_i^d(t)$ is the *d*-th dimension of the best particle in the swarm in which the *i*-th particle belongs at time *t*, $gbest^d(t)$ is the *d*-th

dimension of the best particle of population at time *t*. r_1, r_2, and r_3 are uniformly generated random numbers in the range of (0,1), c_p, c_s, and c_g are constant values representing the weight for personal, swarm, and global behavior and w is the momentum for previous velocity. The swarm flight, Equations (7.5) and (7.6), has been modified to promote diversity after a change is detected as following:

$$v_i^d(t+1) = wv_i^d(t) + c_p r_1(pbest_i^d(t) - x_i^d(t)) + c_s r_2(sbest_i^d(t) - x_i^d(t)) + \\ + c_g r_3(gbest^d(t) - x_i^d(t)) + \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{x_i^d(t) - x_j^d(t)}{\left| x_i^d(t) - x_j^d(t) \right|^3} Q_i(t) Q_j(t), \tag{7.7}$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1). \tag{7.8}$$

The forth term in the above equation is called the repulsive term and is incorporated into the dynamic of the particles in the swarm based upon the psychosocial studies. The psychological research shows that dissimilarity leads to repulsion in interpersonal relationship [14]. As a result people often diverge from members of other social groups by selecting cultural tastes (e.g., possessions, attitudes, or behaviors) that distinguish them from others [15]. A repulsion factor is added to all particles in the population space as a modified version of charged PSO. In charged PSO, some particles are considered as charged with fixed charges that repel from other charged particles according to the coulomb law [86]. In the modified version proposed here, $Q_i(t)$ and

$Q_j(t)$ are the repulsion factors for particles i and j at time t, respectively. $x_i^d(t) - x_j^d(t)$ denotes the vector connecting current position of particle i, to that of particle j and $\frac{x_i^d(t) - x_j^d(t)}{\left|x_i^d(t) - x_j^d(t)\right|^3}$ is inspired from the inverse squared-distance proportionality of coulomb force. Repulsion factor follows a dynamic which is computed via the cultural information extracted from the belief.

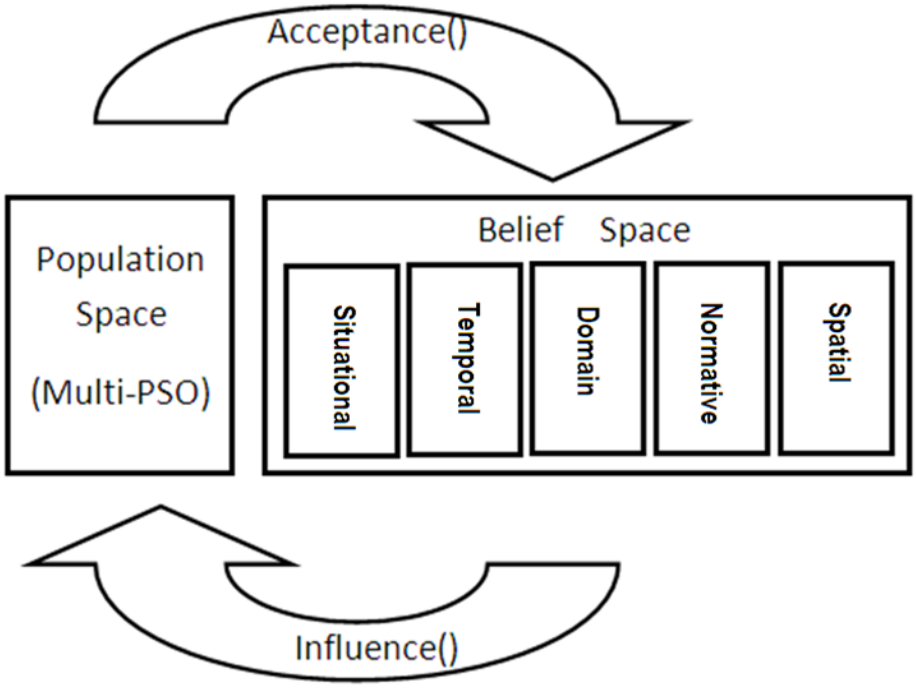


Figure 7.2 Schema of the cultural framework adopted here, where population space is a multiple swarm PSO and belief space consists of situational knowledge, temporal (history) knowledge, domain knowledge normative knowledge, and spatial (topographical) knowledge.

7.3.2 Acceptance Function

The acceptance function is to select the best individuals that affect the belief space. All particles in the population are sorted in order in terms of their objective

function values at the current iteration and p% of the particles starting from best to worst are selected, where p is a predefined constant value.

7.3.3 Belief Space

The belief space in this paradigm consists of five sections, situational, temporal, domain, normative and spatial knowledge. In the remainder of this section, the type of information, represent method of the knowledge and updating methodology of the knowledge in each section of the belief space are elaborated.

7.3.3.1 Situational Knowledge

This part of belief space is used to keep the good exemplar particles for each swarm. Its representation is shown in Figure 7.3. $\hat{\mathbf{x}}_i(t)\,(i=1,2,...,P)$ where P is the number of swarms defined in Equation (7.4), is the best particle in the i-th swarm based upon objective function evaluation. Assume that at an arbitrary iteration the i-th swarm consists N_i particles as $z_1, z_2, ..., z_{N_i}$ with correspondent objective functions values as $f_1, f_2, ..., f_{N_i}$ respectively. Then $\hat{\mathbf{x}}_i(t) \in \{z_1, z_2, ..., z_{N_i}\}$ is defined such that:

$$f(\hat{\mathbf{x}}_i(t), e) = \min_{1<l<N_i} f_l, \qquad i = 1,2,...,P, \tag{7.9}$$

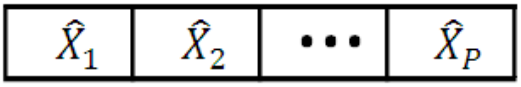


Figure 7.3 Representation for situational knowledge

where $f(\hat{\mathbf{x}}_i(t), e)$ is the objective function value for the particle $\hat{\mathbf{x}}_i(t)$. The situational knowledge does not update at every iteration, but only when a change is detected in the landscape. To do so, the objective values for the new positions of the particles will be adopted. Then the particle corresponding to the least value in each swarm will be stored in situational knowledge. The situational knowledge will be used by the domain knowledge, also to compute the swarm best particles for the flight, and to facilitate the communication among swarms.

7.3.3.2 Temporal Knowledge

This part of belief space is used to keep the history of the individual's behavior. Its representation is shown in Figure 7.4 where $\mathcal{T}(t) = \{\mathcal{T}_1(t), \mathcal{T}_2(t), ..., \mathcal{T}_N(t)\}$ and $\mathcal{P}(t) = \{\mathcal{P}_1(t), \mathcal{P}_2(t), ..., \mathcal{P}_N(t)\}$ (N is the number of particles). $\mathcal{T}_j(t)$ is a set of past temporal pattern up to time, t, of the j-th particle defined as follows:

Figure 7.4 Representation for temporal knowledge

$$\mathcal{T}_j(t) = \{f_j(1), f_j(2), ..., f_j(t)\}, \quad j = 1,2,..., N, \tag{7.10}$$

where $f_j(1), f_j(2),..., f_j(t)$ are the objective function values of the *j*-th particle for the time steps 1,2, … and *t*, respectively. $\mathcal{P}_j(t)$ is the set of all past positions of the *j*-th particle in the whole population defined as $\mathcal{P}_j(t) = \{\mathbf{x}_j(1), \mathbf{x}_j(2),..., \mathbf{x}_j(t)\}$, ($j = 1,2,..., N$). The temporal knowledge will be updated at every iteration. To do so, the updated position of the particle and previously stored temporal knowledge will be adopted as following:

$$\mathcal{T}_j(t+1) = \mathcal{T}_j(t) \cup \{f_j(t+1)\}, \quad j = 1,2,..., N,$$
$$\mathcal{P}_j(t+1) = \mathcal{P}_j(t) \cup \{\mathbf{x}_j(t+1)\}, \quad j = 1,2,..., N. \tag{7.11}$$

The temporal knowledge will later be used to compute the personal best for every particle in the population space.

7.3.3.3 Domain Knowledge

This part of belief space adopts information about the problem domain and its related parameters to lead the search process. This section keeps all the positional/objective values for *gbest* and *sbest* from the last migration till the current time. Its representation is shown in Figure 7.5 which consists of four parts: g , fg , S , and fs . The first part, $g(t)$, is defined as following:

$$\mathcal{g}(t) = \{gbest(t^*) \mid T_k^{Migration} \le t^* \le t\}, \tag{7.12}$$

where $T_k^{Migration}$ denotes the time for the *k*-th migration (and the last migration before the current iteration) among swarms ($k = 0,1,2,...$), and t is the current iteration, and P is the number of swarms given by Equation (7.4). $T_k^{Migration} \le t^* \le t$ denotes all the iterations, t^*, among the last migration , $T_k^{Migration}$, and the current time, t. By default it is assumed that $T_0^{Migration} = 1$. $gbest(t^*)$ is computed as following:

$$gbest(t^*) = \{\exists \mathbf{x}_j(t) \in \mathbb{P}(t^*), 1 \le j \le N \mid f_j(t^*) = \min(\mathbb{F}(t^*))\}, \tag{7.13}$$

where $\mathbb{P}(t^*) = \{\mathbf{x}_1(t^*), \mathbf{x}_2(t^*), ..., \mathbf{x}_N(t^*)\}$ is the whole population of particles at time t^*, and $\mathbb{F}(t^*) = \{f_1(t^*), f_2(t^*), ..., f_N(t^*)\}$ is a set consisting of the modified objective function values for all particles at time t^*. The second part of domain knowledge is $\mathcal{fg}(t)$ which is defined as objective values for each values of the $\mathcal{g}(t)$. Notice that since the objective function in Equation (7.1) is dynamic, therefore the objective function value for the same position may not necessarily be identical for two different iterations due to the possible change of environment. In the domain knowledge we preserve the objective value as:

$$\mathcal{fg}(t) = \{ f(gbest(t^*)) \Big| T_k^{Migration} \le t^* \le t \}, \tag{7.14}$$

where $f(.)$ is the objective function adopted for the time t^*, which may not be the same as $f(.)$ for the current time due to environmental change, e (see Equation (7.1)). The third section of the domain knowledge is $\mathcal{S}(t)$ computed as:

$$\mathcal{S}(t) = \{ \hat{\mathbf{x}}_i(t^*) \Big| T_k^{Migration} \le t^* \le t, i = 1,2,..., P \}, \tag{7.15}$$

Figure 7.5 Representation for the domain knowledge

where $\hat{\mathbf{x}}_i(t^*)$ is extracted from the situational knowledge for all such time t^* between the last migration, $T_k^{Migration}$ and the current time, t. Finally, the fourth section of the domain knowledge is $\mathcal{fs}(t)$ objective values for each values of the $\mathcal{S}(t)$ as following:

$$\mathcal{fs}(t) = \{ f(\hat{\mathbf{x}}_i(t^*)) \Big| T_k^{Migration} \le t^* \le t, i = 1,2,..., P \}, \tag{7.16}$$

where $f(.)$ is again the objective function used for the time t^*, not the current time, since due to dynamic nature of the problem $f(.)$ for the current time might have been different

from $f(.)$ for the time t^*.

The domain knowledge is then updated at every iteration and reset when a migration among swarms takes place as following:

$$\mathcal{g}(t+1) = \begin{cases} \mathcal{g}(t) \cup \{gbest(t+1)\}, & \text{if} \quad t+1 < T_{k+1}^{Migration} \\ gbest(t+1), & \text{if} \quad t+1 = T_{k+1}^{Migration} \end{cases}, \tag{7.17}$$

$$\mathcal{fg}(t+1) = \begin{cases} \mathcal{fg}(t) \cup \{f(gbest(t+1))\}, & \text{if} \quad t+1 < T_{k+1}^{Migration} \\ f(gbest(t+1)), & \text{if} \quad t+1 = T_{k+1}^{Migration} \end{cases}, \tag{7.18}$$

$$\mathcal{S}(t+1) = \begin{cases} \mathcal{S}(t) \cup \{\hat{\mathbf{x}}_i(t+1) | i = 1,2,..., P\}, & \text{if} \quad t+1 < T_{k+1}^{Migration} \\ \{\hat{\mathbf{x}}_i(t+1) | i = 1,2,..., P\}, & \text{if} \quad t+1 = T_{k+1}^{Migration} \end{cases}, \tag{7.19}$$

$$\mathcal{fs}(t+1) = \begin{cases} \mathcal{fs}(t) \cup \{f(\hat{\mathbf{x}}_i(t+1)) | i = 1,2,..., P\}, & \text{if} \quad t+1 < T_{k+1}^{Migration} \\ \{f(\hat{\mathbf{x}}_i(t+1)) | i = 1,2,..., P\}, & \text{if} \quad t+1 = T_{k+1}^{Migration} \end{cases}. \tag{7.20}$$

This means that if migration does not happen, the new *gbest* is computed using Equation (7.13) and will be added to the domain knowledge along with its correspondent objective values. Also the new information from situational knowledge for the current iteration, along with their correspondent objective values will be added to update the domain knowledge. On the other hand if migration takes place then the new *gbest* is computed using Equation (7.13) and will be placed as the first part of domain knowledge. The second part is the objective value this new *gbest*. The third part of the domain knowledge is extracted from the current situational knowledge, and finally the fourth part

is placed via the objective value for the third part. In this way, the domain knowledge will be constructed if migration has taken place. The domain knowledge will later be used to detect the changes of the dynamic landscape of the problem, and also to produce the required particles for particles' flights such as global best and swarm best.

7.3.3.4 Normative Knowledge

In this section of the belief space best areas are adopted to nominate and exchange among swarms. Its representation is demonstrated in Figure 7.6 which consists two parts $\mathbb{S} = \{\mathbb{S}_1, \mathbb{S}_2, ..., \mathbb{S}_P\}$ and $\mathbb{R} = \{\mathbb{R}_1, \mathbb{R}_2, ..., \mathbb{R}_P\}$ where $\mathbb{S}_i$ $(i = 1,2,..., P)$ denotes a send list of particles in the *i*-th swarm which will be selected to be sent to the next swarm, while $\mathbb{R}_i (i = 1,2,..., P)$ is a replace list of particles in the *i*-th swarm to be replaced by particles coming from another swarm.

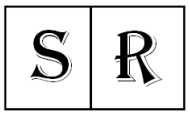


Figure 7.6 Representation of normative knowledge

This mechanism to increase diversity has been introduced and adopted by Yen and Daneshyari [144-145]. This mechanism is used to quickly regain the divergence after a change is detected in the landscape of the problem. To regain the divergence, each swarm prepares two lists of particles, a list to be sent to the next swarm and another to be replaced by particles coming from another swarm. These two sections of normative

knowledge is prepared according to the particles' locations in the swarm and the objective function values. Assume that at an arbitrary iteration the *i*-th swarm consists N_i particles as $\Omega_i = \{z_1, z_2, ..., z_{N_i}\}$. The sending list for the swarm is prepared in the following order [165]:

(1) The highest priority in the selection of particles is given to a particle that has the least average Hamming distance from others. This particle is considered as the representative of the swarm. The average Hamming distance between each pair of particles in the swarm is calculated and then the least among them is found. The least average Hamming distance, $\bar{z}$, is then formulated as:

$$\bar{z} = \min_{1<k<N_i} \bar{z}_k, \tag{7.21}$$

where $\bar{z}_k$ is the average distance from $z_k (1 < k < N_i)$ to other particles in the swarm. z_k is a particle of the *i*-th swarm such as $\Omega_i = \{z_1, z_2, ..., z_{N_i}\}$ at an arbitrary iteration. $\bar{z}_k$ is computed as following:

$$\bar{z}_k = \frac{1}{N_i - 1} \sum_{\substack{l=1 \\ l \neq k}}^{N_i} \langle z_k - z_l \rangle, \tag{7.22}$$

where:

$$\langle u \rangle = \sum_{d=1}^{M} |u^d|, \tag{7.23}$$

where u^d is the *d*-th dimension of vector u, *M* is the total dimension of the vector, and $|.|$ denotes the absolute value.

(2) The second priority is given to the closest particles to the representative particle whose objective value is greater than that of the representative. Assume that $f_1, f_2, ..., f_{N_i}$, and $\bar{f}$ are objective values corresponding to $z_1, z_2, ..., z_{N_i}$, and $\bar{z}$ respectively. Therefore the second priority is given to:

$$\{\forall \bar{y} | \bar{y} \in \Omega_i, f(\bar{y}, e) \in \mathcal{H}, \langle \bar{y} - \bar{z} \rangle \text{ is the } M_i \text{ - th smallest values}\}, \tag{7.24}$$

where $\mathcal{H} = \{ f_l | 1 < l < N_i, f_l > \bar{f} \}$, and M_i is a threshold value for the *i*-th swarm that depends on the rate of information exchange among swarms, r, (a predefined value between 0 and 1), and population of the *i*-th swarm, N_i, defined as following [144-145]:

$$M_i = \frac{rN_i}{2} - 1. \tag{7.25}$$

(3) The third priority is given to the closest particles to the representative particle whose modified objective value extracted from the belief space is less than that of the representative:

$$\{\forall \overline{w} \big| \overline{w} \in \Omega_i,\ f(\overline{w}, e) \in \mathcal{G}, \langle \overline{w} - \bar{z} \rangle \text{ is the } M_i \text{ - th smallest values}\}, \tag{7.26}$$

where $\mathcal{G} = \{ f_l \big| 1 < l < N_i,\ f_l < \bar{f} \}$.

(4) The fourth and last priority is given to the best performing particle in the swarm:

$$\{\bar{s} \big| \bar{s} \in \Omega_i, f(\bar{s}, e) = \min_{1<l<N_i} f_l\}. \tag{7.27}$$

Note that depending on the predefined fixed value for allowable number of the sending list, $N_{migration}$, the sending list will be filled in each swarm using the above-mentioned priorities.

The other section of the normative knowledge, replacement list $\mathbb{R}_i$ is also prepared by each swarm, based upon the similar positional information of particles in the swarm. When swarms are approaching local optima, many particles' locations are the same. Each swarm will remove this excess information through its replacement list. The replacement list in each swarm is assembled in the following order:

(1) The first priority is given to the particles with identical parametric space information, by the order of their modified objective values extracted from the belief space, with the least modified objective values being replaced first.

(2) The second and last priority is given to the particles with the lowest modified

objective values if all particles of the first priority have already been in the replacement list.

The normative knowledge is updated whenever a change is detected in the dynamic landscape. After a change is detected, the normative knowledge will be updated using the current positional information and their corresponding objective values. The normative knowledge later will be used to perfrom the migration among swarms and to give a jump start along with spatial knowledge to the search process of the changed landscape.

7.3.3.5 Spatial Knowledge

Spatial knowledge is discussed in this subsection. The spatial knowledge of the belief space, represented as Figure 7.7, consists of two sections, $Q(t) = \{Q_1(t), Q_2(t), ..., Q_N(t)\}$ and $\Theta(t) = \{\theta_1(t), \theta_2(t), ..., \theta_N(t)\}$, where N is the number of particles.

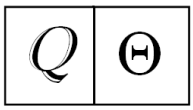


Figure 7.7 Representation for spatial knowledge

$\theta_j(t)$ ($j = 1,2,..., N$) is computed as a shifted and normalized objective function for the *j*-th particle defined as:

$$\theta_j(t) = \begin{cases} 2\dfrac{f(\mathbf{x}_j, e) - f_j^{\min}(t)}{f_j^{\max}(t) - f_j^{\min}(t)} - 1, & \text{When a change is not detected} \\ rand(-1,1), & \text{When a change is detected} \end{cases}, \quad j = 1,2,..., N, \quad (7.28)$$

where $f(\mathbf{x}_j, e)$ is the objective function value for the *j*-th particle, $\mathbf{x}_j$, and $f_j^{\min}(t) = \min_{\mathbf{x} \in X}(f(\mathbf{x}_j, e))$ is the lower bound of the objective function value on the *j*-th particle at time *t*, and $f_j^{\max}(t) = \max_{\mathbf{x} \in X}(f(\mathbf{x}_j))$ is the upper bound of the objective function value on the *j*-th particle at time *t*. $Q_j(t)$ ($j = 1,2,..., N$) called repulsion factor is then computed for all particles through a sigmoid function as shown in Figure 7.8 as following:

$$Q_j(t) = \frac{1}{1 + \exp(\alpha\theta_j(t))}, \qquad j = 1,2,..., N, \qquad (7.29)$$

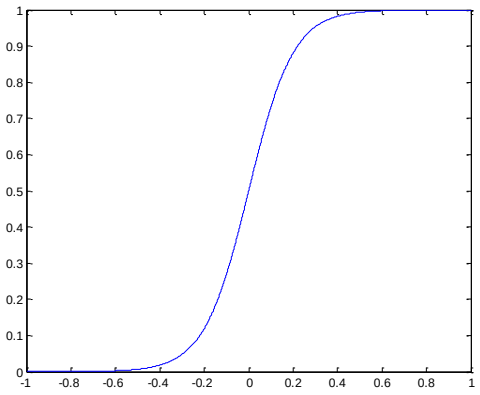


Figure 7.8 Sigmoid function to compute repulsion factor in spatial knowledge with $\alpha = 10$

where α is the rate for the sigmoid function. According to Equation (7.28), when a

change has not taken place in the environment, $\theta_j(t)$ gets a value closer to –1 as its objective function, $f(\mathbf{x}_j, e)$ gets smaller, and closer to 1 as $f(\mathbf{x}_j, e)$ gets larger. The sigmoid transformation of Equation (7.29) will then compute the repulsion factor, $Q_j(t)$ such that for the better half of the particles' population, there is no or very small repulsion factor and for the other half of the population space, the repulsion factor will be close to 1 (e.g., for the best particle in the population space, $Q_i(t) = 0$). Hence during the flight of particles of PSO, the better particles will not be repelled, so we do not lose information of the better particles while the particles will be repelled from the worse particles in the population space.

On the other hand, as soon as a change is detected, we do not want to force particles to still be close to the best particles because the environment has changed and they may not be any different from other particles. In this case, $\theta_j(t)$ is statistically assigned as a uniform random number between –1 and 1. It is then transformed through the sigmoid function to compute $Q_j(t)$. Statistically speaking, due to the performance of the sigmoid function, a random half of the particles will be assigned a value close to zero as repulsion factor, and the other half of the half will be assigned a value of 1. Although this process avoids particles from being stuck near optimum point, but it also helps preserve part of the evolutionary information stored in the search process and not completely forget all the evolutionary data and re-start fresh. This mechanism helps to re-diversify the search space quickly right after the change in the dynamic landscape is

detected and helps the algorithm with a jump start.

The spatial knowledge is updated at every iteration. After the PSO flight is performed, the new positions of the particles will be evaluated using the objective functions, and then the new spatial knowledge will be computed. Spatial knowledge will be used to compute the repulsion term in the the flight mechanism.

7.3.4 Influence Functions

After belief space is updated, the correspondent knowledge should be used to influence the flight of particles in PSO. We propose to use the knowledge in belief space to select the personal best, swarm best, and global best for the PSO mechanism, and to perform the repulsive diversity-based migration among swarms.

7.3.4.1 pbest Selection

In order to select the personal best, we use information in the temporal knowledge section of the belief space. The best behaving behavior in the particle's past history should be selected as following:

$$pbest_i(t) = \{\exists \mathbf{x}_j(\hat{t}) \in \mathcal{P}_j(t), 1 \le j \le N \,|\, f_j(\hat{t}) \in \mathcal{T}_j(t),\ f_j(\hat{t}) = \min(\mathcal{T}_j(t))\}, \qquad (7.30)$$

$$(i = 1,2,...,P)$$

where $\mathcal{P}_j(t) = \{\mathbf{x}_j(1), \mathbf{x}_j(2), ..., \mathbf{x}_j(t)\}$ is the set of all past positions of the *j*-th particle in the whole population, and $\mathcal{T}_j(t) = \{f_j(1), f_j(2), ..., f_j(t)\}$ ($j = 1,2,..., N$) is the objective values for the past history of the *j*-th particle, both extracted from the temporal knowledge section of the belief space.

7.3.4.2 sbest Selection

In order to select the swarm best particle, the situational knowledge is adopted. The information stored in the situational knowledge section of the belief space is simply copied into swarm best particles:

$$sbest_i(t) = \hat{\mathbf{x}}_i(t), \qquad i = 1,2,..., P, \tag{7.31}$$

where P is the number of swarms and $\hat{\mathbf{x}}_i(t)$ is the representative of the situational knowledge of the belief space.

7.3.4.3 gbest Selection

We use the domain knowledge stored in the belief space to copy the latest and current element of $\mathcal{g}(t)$ of Equation (7.12) as the $gbest(t)$.

7.3.4.4 Diversity based Migration Driven by Change

In order to perform repulsive change-driven diversity based migration, we use information in the domain knowledge and normative knowledge sections of the belief space. The information in the domain knowledge will be used to monitor the change in the dynamic, while information in the normative knowledge will be adopted to do the migration as a response to the detected change. The change has taken place if and only if there is at least one $\delta \in \Delta$ such that:

$$\{\exists \delta \in \Delta \big| \delta > \delta_0\}, \tag{7.32}$$

where Δ is defined as:

$$\Delta = \{\big| \mathit{fg}(t) - f(g(t)) \big|\} \cup \{\big| \mathit{fs}(t) - f(S(t)) \big|\}. \tag{7.33}$$

$\mathit{fg}(t)$, $g(t)$, $\mathit{fs}(t)$ and $S(t)$ are adopted from the domain knowledge of the belief space, and $|.|$ denotes the absolute value. The allowable change, δ_0, is set to a predefined small value. Notice that there is a difference between objective function in Equation (7.33), i.e., $f(g(t))$ and $f(S(t))$, with the objective function in Equations (7.14) and (7.16), i.e., $f(gbest(t^*))$ and $f(\hat{\mathbf{x}}_i(t^*))$, this difference is due to the possible environmental changes. To be more precise, $f(g(t)) = f(gbest(t^*), e(t))$, is the objective value for all *gbest* values (all previous iterations, t^*, in domain knowledge) computed using the "current" *e*. While $f(gbest(t^*)) = f(gbest(t^*), e(t^*))$ is the objective value for all *gbest* values (all previous iterations, t^*, in domain knowledge) computed under the environmental

parameter e, at the previous iteration, t^*.Therefore any difference that appears in Equation (7.33) should be due to the differences between the environmental parameter at the current iteration, $e(t)$, and the previous time, $e(t^*)$.

When the change is detected, as the response to the change, we have to quickly re-diversify because the previous optimum solutions are no longer valid for the new environment. This response is performed using a repulsive diversity based migration through the information in the normative knowledge of the belief space. As soon as the change is detected, the data from the normative knowledge will be adopted to exchange information among swarms. Each swarm accepts the sending list $\mathbb{S}$ from other swarm and will replace it with its own replacement list, $\mathbb{R}$ (Both $\mathbb{S}$ and $\mathbb{R}$ are extracted from the normative knowledge). This information exchange among swarms happens in a ring sequential order between each pair of swarms.

7.4 Experimental Study

In this section the performance of the proposed cultural-based dynamic particle swarm optimization is evaluated against those of the state-of-the-art dynamic particle swarm optimization (DPSO) heuristics.

7.4.1 Benchmark Test Problems

Six test functions as benchmark problems have been used to test the ability of the proposed cultural-based DPSO as following: MP1 (moving cone peaks benchmark problem) is a maximization problem which has components as moving competing cones with independently varying height, width and location [166]. DF2 (time-varying Gaussian peaks problem) is a maximization problem that adopts independently varying-dimensional Gaussian peaks. Each peak's amplitude, center, and variance can be varied independently [167]. DF3 is a minimization problem as moving parabola with linear translation in change [168-169]. DF4 is also a minimization problem of moving parabola but with random dynamics [168-169]. DF5 is a minimization problem of moving parabola with circular dynamic [168-169] and finally DF6 (oscillating peaks function) is a maximization problem that has been used in [170]. It has two landscapes with ten peaks each. The parameters of each peak can independently vary. The detailed formulation of these benchmark test functions are presented in Appendix C for reference.

For the simulations here, benchmark problems have the following parameter setting wherever applies, unless stated otherwise: number of peaks is set as default value of 10, every 5,000 evaluations the change takes place. The peak shape is cone (for MP1), Gaussian (for DF2), parabola (for DF3, DF4, and DF5), and bell curve (for DF6). Default decision space dimension is 5. Each decision variable is limited between 0 and 100. The height and width severity are set as 7 and 1 respectively. The height peak is limited

between 30 and 70. The width peak is also limited between 1 and 12. Finally, the peak shift length is set as 1.

7.4.2 Comparison Algorithms

The proposed algorithm has been compared against other state-of-the-art dynamic particle swarm optimization paradigms that are adopted to solve DOPs. These algorithms include DPSO [44], hybrid DPSO (h-DPSO) [92], modified DPSO (m-DPSO) [94] and speciation based DPSO (s-DPSO) [96]. DPSO [44] is a regular particle swarm optimization that adopts a simple strategy with a small perturbation, the initialization of the swarm can start from old population, while with large perturbation, it does re-initialize and then compare the results with the old swarm and select the best one. The selected parameters are given in Table 7.1. For the moment of inertia, as suggested in [44], a uniform random number with average of 0.75 is selected. In h-DPSO [92] a dynamic macro-mutation operator plays the role of maintaining diversity throughout the search process. The mutation is for every coordinate of each particle. The mutation will take place with a probability within the minimum and maximum value given in Table 7.1 as suggested in [92] and possess its highest value when the change occurs in the dynamic landscape and gradually decreases till the next change takes place. The swarm size and neighborhood radius size are also given in this table as suggested in the literature [92]. The next algorithm is m-DPSO [94] the changed local optimum and global optimum are

adopted to guide the movement of each particle and avoid making direction and velocity decisions in the basis of outdated memory. These two changes dynamically affect the inertia weight. The influence weight of *pbest* vs. *gbest* is set as 0.4 as suggested in [94]. The last heuristic for comparison is the s-DPSO [96] that divides population into species, each one surrounding a dominating particle, namely seed. The parameter settings are given in Table 7.1 as suggested by [96].

The parameter settings of cultural DPSO are also summarized in Table 7.1 as many of these settings are adopted in other paradigms of PSO [165]. The population size is 100. All of the algorithms are implemented in Matlab using real-number representation for decision variables. For each test function, 50 independent runs were conducted with a maximum objective function evaluation of 500,000.

Table 7.1 Parameter settings for different paradigms

Algorithm	Parameters Settings
Cultural DPSO	α=10, δ_0 =0.0001, c_p= c_s= c_g=1.5, w=rand(0.5,1) , r =30%, $N_{Migration}$ =5
DPSO	c_p= c_g= 1.5, w= rand(0.5,1)
h-DPSO	c_p= c_g = 1.5, w= 0.5, p_{min}=0.5, p_{max}=0.8, $Swarm_{size}$=50, $r_{Neighborhood}$=4
m-DPSO	c_p= c_g = 1.5, λ = 0.4
s-DPSO	φ_1= φ_2= 2.05, χ = 0.729844, r_s=0.5

7.4.3 Comparison Measure

To quantify the performance of the proposed paradigm, the offline error variation (OEV) index, $\overline{e_{offline}}$, defined as the average of the error between the true optimal fitness and the best fitness at each evaluation [171] is used:

$$\overline{e_{offline}} = \frac{1}{T}\sum_{i=1}^{T}(f_{True} - f_{Best}^{i}), \tag{7.34}$$

where i is the evaluation counter, T is the total number of evaluations, f_{True} is the true optimum solution updated after a change occurs, and f_{Best}^{i} is the best individual out of the evaluations starting from the last occurrence of change until the current evaluation. For perfect tracking of change the offline error variation should be zero.

7.4.4 Simulation Results

The number of evaluations computed as the product of the population size and the current iteration is used as the counter for comparing the paradigms against each other. Table 7.2 compares the performance of the proposed cultural DPSO with selected state-of-the-art DPSOs on test problem MP1 as a function of iterations elapsed between changes, peak numbers and decision space dimensions, respectively. Figures (7.9) to

(7.11) also depicts a graphical comparison of the OEV index of different algorithms on test function MP1 as a function of elapsed iterations between changes, peak numbers, and decision space dimensions, respectively. As can be observed from the first section of Table 7.2 and Figure 7.9 the proposed cultural-based DPSO performs better than all selected state-of-the-art DPSOs for different iteration intervals between the changes. When the iteration interval between changes is short, i.e., high frequency of change, the proposed algorithm performs much better than other algorithms. When the frequency of change decreases, the proposed algorithm performs better or equal to s-DPSO. From the second section of Table 7.2 and comparison graph in Figure 7.10, it can be seen that cultural DPSO can easily outperform other DPSOs with both small and large number of peaks suggesting that the algorithm can handle multiple peaks as well as a smaller number of peaks. Lastly the third section of Table 7.2 along with Figure 7.11 demonstrates that when decision space dimension increases, the proposed paradigm can retain its performance while PSO, h-DPSO and m-DPSO show difficulties.

However the proposed algorithm will still perform better than s-DPSO in higher dimensions. In Figure 7.9, it is shown that as the number of iterations elapsed between changes increases (lower frequency of change), algorithms usually perform better through lower values for OEV index. As can be seen from Figure 7.11, the offline error first increases by increasing peak number but then decreases for a higher number of peaks. Figure 7.12 also demonstrates that as the dimension of the decision space increases, the performances of the algorithms deteriorate.

Table 7.2 OEV index after 500,000 FEs on test problem MP1 as a function of elapsed iterations between changes, peak numbers and decision space dimension, respectively

Algorithms		DPSO	h-DPSO	m-DPSO	s-DPSO	Cultural DPSO
Elapsed Iterations (Peak no.=10) (Dimension=5)	1	21.5659	16.8633	18.9424	16.0456	**14.7615**
	5	20.0529	14.4976	16.8144	10.7378	**8.5748**
	10	18.9479	11.8323	13.4967	8.3500	**7.7687**
	25	17.2919	9.3947	10.9319	7.8238	**5.3367**
	50	15.8938	8.3067	9.8267	5.2829	**4.0949**
	100	12.3284	5.8512	7.9279	3.7739	**3.5965**
Peak Numbers (Dimension=5) (Elapsed Iterations=50)	1	10.6706	7.2796	8.3222	3.3266	**2.0110**
	10	15.8938	8.3067	9.8267	5.2829	**4.0949**
	20	17.7876	9.8267	10.5007	5.7245	**4.1584**
	30	21.7697	10.1928	11.2383	6.3762	**4.3696**
	40	20.5412	9.3868	10.9821	5.6689	**4.2820**
	50	18.8187	8.9244	9.7894	5.2037	**4.1987**
	100	17.3904	8.0668	9.3434	4.8239	**3.5810**
	200	16.0405	7.8382	8.5455	4.0527	**3.2445**
Dimension (Peak no.=10) (Elapsed Iterations=50)	5	15.8938	8.3067	9.8267	5.2829	**4.0949**
	10	19.2543	9.7779	12.5128	7.3182	**5.4785**
	20	25.5887	11.9483	18.5846	9.3703	**7.8644**
	50	30.7807	15.9640	20.3326	12.3574	**10.6298**

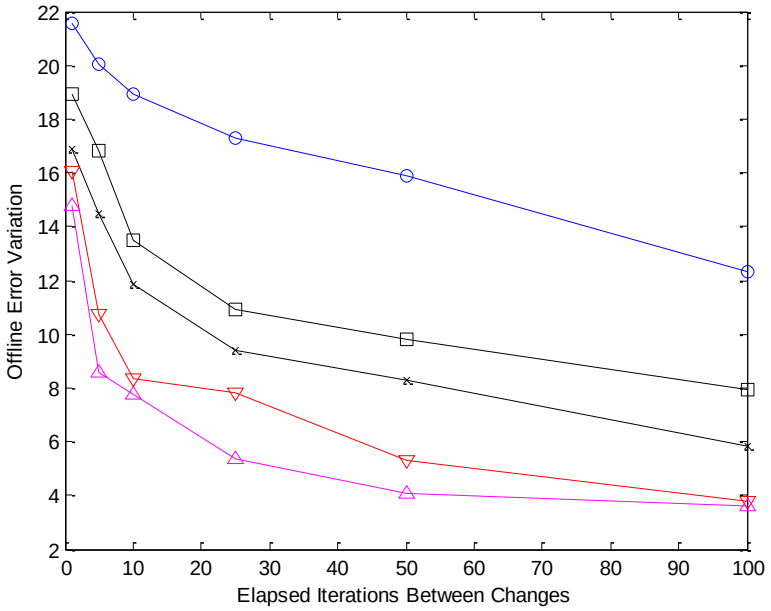


Figure 7.9 Comparison of OEV index as a function of elapsed iterations between changes on test function MP1 with 10 peaks (PSO, h-DPSO, m-DPSO, s-DPSO, and Cultural DPSO are denoted as O, ×, □, ∇ and Δ, respectively)

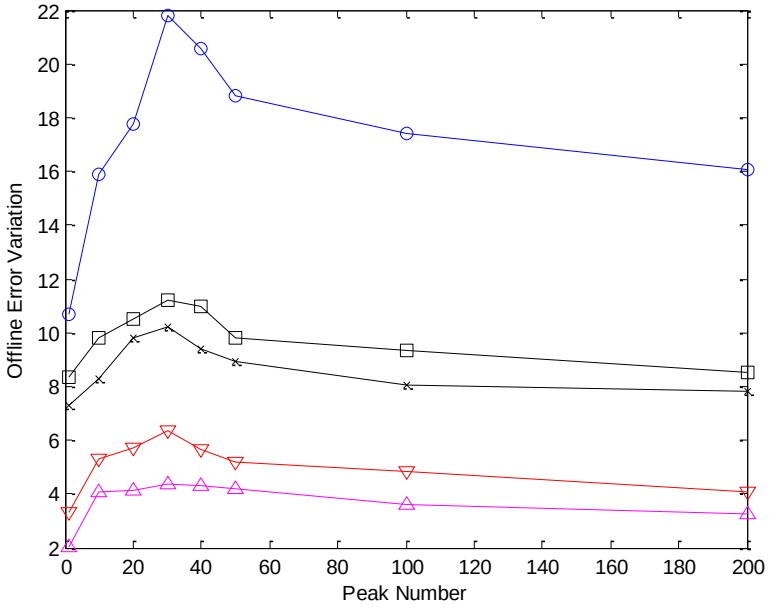


Figure 7.10 Comparison of OEV index as a function of peak numbers on test function MP1 (DPSO, h-DPSO, m-DPSO, s-DPSO, and Cultural DPSO are denoted as O, ×, □, ∇ and Δ, respectively)

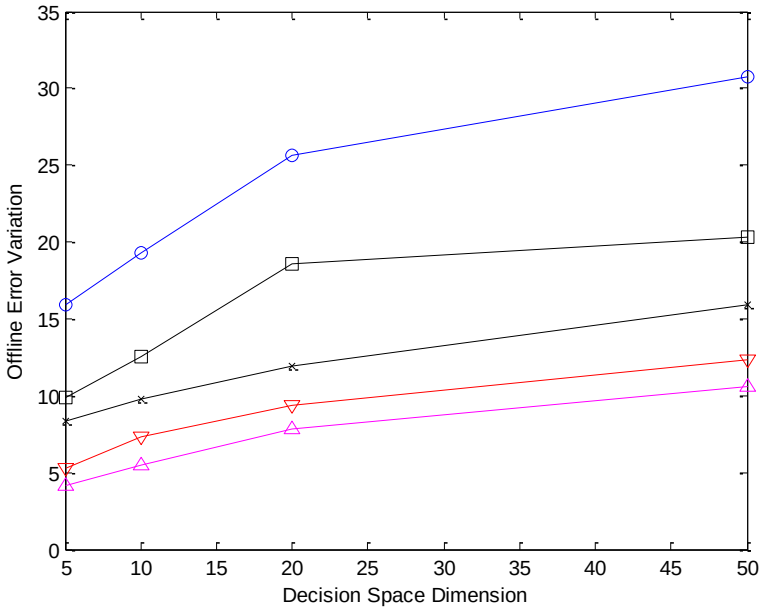


Figure 7.11 Comparison of OEV index as a function of decision space dimension on test function MP1 with 10 peaks (DPSO, h-DPSO, m-DPSO, s-DPSO, and Cultural DPSO are denoted as O, ×, □, ∇ and Δ, respectively)

Table 7.3 shows comparison result of different algorithms using the OEV index after 500,000 evaluations on test problem DF2 as a function of elapsed time between changes and peak numbers respectively. As can be seen from this table, the proposed heuristic has performed better than other selected DPSOs even when the number of iterations elapsed between two changes is small (high frequency of change), or large (low frequency of change). Table 7.3 also indicates better performance of cultural-based DPSO on high and low peak numbers.

Table 7.3 OEV index after 500,000 FEs on test problem DF2 as a function of elapsed iterations between changes and peak numbers, respectively

Algorithms		DPSO	h-DPSO	m-DPSO	s-DPSO	Cultural DPSO
Elapsed Iterations (Peak no.=10)	1	22.6567	17.8519	19.9326	15.4838	**11.8472**
	5	20.1790	16.0555	17.2925	13.6653	**10.0272**
	10	17.7310	14.1937	16.9677	12.1027	**8.1664**
	25	15.0887	13.7920	15.7640	10.8993	**6.9341**
	50	14.1655	11.5214	12.2221	9.4532	**5.1894**
	100	13.3858	11.3541	11.8883	7.2937	**4.3288**
Peak Numbers (Elapsed Iterations=50)	1	12.9645	8.5282	11.1864	5.8702	**2.6477**
	5	13.5484	9.7094	11.7487	7.6604	**3.8549**
	10	14.1655	11.5214	12.2221	9.4532	**5.1894**
	50	18.3276	12.7019	14.9619	10.8763	**5.8110**
	100	23.6404	15.2986	18.0566	11.2019	**6.0423**
	200	27.1843	18.5044	20.6374	12.7543	**6.6962**

Comparisons between the performances of cultural DPSO against selected algorithms are demonstrated by OEV index on moving parabola problems with linear dynamic (DF3), and random dynamic (DF4) and circular dynamic (DF5) in Tables (7.4) to (7.6), respectively. Each of these three tables consist the comparison as a function of cycle length evaluations and peak numbers. The results in these three tables show that cultural DPSO outperforms other DPSO paradigms in both low/high frequency and low/high peak numbers.

Table 7.4 OEV index after 500,000 FEs on test problem DF3 as a function of cycle length evaluations between changes and peak numbers, respectively

Algorithms		DPSO	h-DPSO	m-DPSO	s-DPSO	Cultural DPSO
Cycle length evaluations (Peak no.=10)	1,000	9.7765	6.7418	8.7616	4.4373	**1.8717**
	2,500	9.3827	6.5966	8.2517	4.2268	**1.6174**
	5,000	8.9235	6.2617	8.0357	3.8446	**1.4048**
	10,000	8.7330	6.0996	7.7455	3.5833	**1.1916**
	20,000	8.4857	5.7436	7.3795	3.3067	**0.9385**
	100,000	8.1117	5.4658	7.1713	3.1384	**0.7433**
Peak no. (Cycle length =5000)	1	8.4597	5.4109	7.3592	3.2203	**0.7167**
	5	8.7230	5.8472	7.8620	3.6865	**0.9576**
	10	8.9235	6.2617	8.0357	3.8446	**1.4048**
	50	9.6820	6.7730	8.3427	4.1612	**1.7498**
	100	10.5639	6.9577	8.6916	4.4093	**1.9764**
	200	10.9837	7.4803	8.9838	4.8666	**2.3176**

Table 7.5 OEV index after 500,000 FEs on test problem DF4 as a function of cycle length evaluations between changes and peak numbers, respectively

Algorithms		DPSO	h-DPSO	m-DPSO	s-DPSO	Cultural DPSO
Cycle length evaluations (Peak no.=10)	1,000	10.1820	6.9003	9.1100	4.9024	**1.9104**
	2,500	9.7682	6.7108	8.6871	4.6705	**1.7394**
	5,000	9.3995	6.5495	8.3202	4.1221	**1.5110**
	10,000	8.9101	6.1014	7.9551	3.8166	**1.3524**
	20,000	8.7339	5.9111	7.4018	3.5209	**1.0952**
	100,000	8.3882	5.6033	7.2759	3.2033	**0.8505**
Peak no. (Cycle length =5000)	1	8.6803	5.7203	7.5045	3.3105	**0.7850**
	5	8.9472	5.9475	7.9879	3.7624	**1.2661**
	10	9.3995	6.5495	8.3202	4.1221	**1.5110**
	50	10.0940	6.9105	8.5103	4.2105	**1.7320**
	100	10.9905	7.1383	8.7662	4.4995	**1.9410**
	200	11.5193	7.5776	9.1106	4.9195	**2.3952**

Table 7.6 OEV index after 500,000 FEs on test problem DF5 as a function of cycle length evaluations between changes and peak numbers, respectively

Algorithms		DPSO	h-DPSO	m-DPSO	s-DPSO	Cultural DPSO
Cycle length evaluations (Peak no.=10)	1,000	10.3495	7.2870	9.4110	5.1551	**1.9840**
	2,500	9.9478	6.9485	8.8660	4.8400	**1.7910**
	5,000	9.5980	6.8778	8.5229	4.3481	**1.6258**
	10,000	9.1336	6.3227	8.1593	3.9330	**1.4817**
	20,000	8.9105	6.1005	7.7206	3.6484	**1.2155**
	100,000	8.5119	5.7490	7.4605	3.3710	**0.9750**
Peak no. (Cycle length =5000)	1	8.9209	5.8103	7.7101	3.5103	**0.8929**
	5	9.1332	6.1854	8.1690	3.9004	**1.3820**
	10	9.5980	6.8778	8.5229	4.3481	**1.6258**
	50	10.2059	7.1100	8.6202	4.4820	**1.8720**
	100	11.2449	7.3665	8.9110	4.6114	**2.0973**
	200	11.7101	7.6505	9.5339	5.1776	**2.5191**

Test function DF6 has two landscapes with ten peaks as it has been used in [170]. The parameters of each peak can be varied independently. In Table 7.7, the performance of selected algorithms is compared for different cycle lengths. As can be observed from the table, the cultural DPSO shows better performance both in lower and higher frequencies of change compared to DPSO, h-DPSO and m-DPSO, while its performance is equal to or less than s-PSO in high frequency of change and equal to or better than s-PSO in low frequency of changes. For better quantitative comparison of the algorithms over all benchmark problems, the Mann–Whitney rank sum test has been conducted to examine the significance of the difference between the algorithms [132]. In Table 7.8, the p-values with respect to the alternative hypothesis (for p-values less than α=0.5) for each

pair of the cultural DPSO and a selected DPSO paradigm are presented. The distribution of the proposed algorithm has significant difference with respect to that of the selected DPSO, unless it is marked in the table. As can be seen from the table, the cultural DPSO outperforms other DPSOs for test problems DF2-DF5. For test problems MP1 and DF6, the proposed paradigm outperforms DPSO, h-DPSO, and m-DPSO appreciably. However the performance of the cultural DPSO is no different than s-DPSO on these two test functions and performs equally well with s-DPSO.

In Table 7.9, the performance of cultural DPSO along with other DPSOs for lower fitness evaluations at 50,000 evaluations has been investigated to check the relation among algorithms at an earlier stage in the search process. As shown in the table, at the earlier stage, the cultural DPSO outperforms DPSO, h-DPSO, and m-DSO for all six adopted test functions. In comparison between cultural DPSO and s-DPSO, the results in the table demonstrate that cultural DPSO outperforms s-DPSO for test functions MP1 and DF2-DF5, but is outperformed by s-DPSO for test function DF6. Notice that this table adopts the default value of 5,000 for cycle length. The result in Table 7.9 for comparison between s-DPSO and cultural DPSO on test function DF6 is similar to results from Table 7.7 for lower cycle length of 1,000, 2,500 and 5,000 fitness evaluations (higher frequencies). Furthermore, the results at earlier stages computed in Table 7.9 (50,000 FEs) follow the same pattern as previously discussed tables at the later stages, i.e., Tables (7.3) to (7.7) for 500,000 FEs. Therefore maximum number of fitness evaluations does not affect the relative performance of cultural DPSO compared to those of other DPSOs.

This suggests that even in the earlier stage, the proposed cultural DPSO can be relied on to obtain a relatively better performance compared to the other state-of-the-art DPSOs.

Table 7.7 OEV index after 500,000 FEs on test problem DF6 as a function of cycle lengths evaluations between changes

Cycle Length	DPSO	h-DPSO	m-DPSO	s-DPSO	Cultural DPSO
1,000	10.4950	7.4247	9.1277	2.8633	2.9176
2,500	9.8606	7.3043	8.9541	2.8522	2.8956
5,000	9.5561	7.1855	8.7864	2.8456	2.8641
10,000	8.3459	6.2464	8.3488	2.6884	2.6207
20,000	7.7967	6.1882	7.3398	2.6511	2.6034
100,000	7.4462	6.0789	7.0982	2.6368	2.5352

Table 7.8 P-values using Mann-Whitney rank-sum test with α=0.5. There is significant difference between a pair of comparing algorithms unless it is stated as no difference denoted as ND.

Test Problem	Cultural DPSO AND			
	DPSO	h-DPSO	m-DPSO	s-DPSO
MP1	4.44e-07	1.37e-04	1.79e-05	0.1031 (**ND**)
DF2	3.63e-05	1.95e-04	5.96e-05	0.0024
DF3	3.63e-05	3.63e-05	3.63e-05	3.63E-05
DF4	3.63e-05	3.63e-05	3.63e-05	3.63E-05
DF5	3.63e-05	3.63e-05	3.63e-05	3.63E-05
DF6	0.0022	0.0022	0.0022	1 (**ND**)

The experimental results presented in this section show that overall performance of the cultural DPSO is better than most of the selected DPSOs for all benchmark test functions chosen. However for test function DF6 and MP1, its performance shows no

difference when compared to s-DPSO (both in earlier and later stages of search process). However the proposed cultural DPSO still well outperforms s-DPSO on other four benchmark test problems as shown by Mann-Whitney statistical tests. This suggests that cultural algorithm with its abilities such as different sections of belief space prepares an organized informational storage that will help the process of quick re-divergence and re-convergence around new optimum points when a change happens in the dynamic of the problem.

Table 7.9 OEV index after 50,000 FEs using default parameters

Algorithms	DPSO	h-DPSO	m-DPSO	s-DPSO	Cultural DPSO
MP1	16.9269	9.4858	10.6862	4.9268	4.3686
DF2	15.1176	12.8552	13.9242	10.4435	5.4236
DF3	9.6942	7.4026	9.6903	4.9584	1.6409
DF4	10.0439	7.1154	9.8032	5.9512	1.6814
DF5	10.7485	7.3807	9.9352	5.7724	1.7366
DF6	10.9565	8.2904	9.6078	2.8526	2.9067

7.5 Discussions

In this study, the cultural-based dynamic particle swarm optimization has been proposed to solve DOP problems. This novel heuristic is built upon a cultural framework that consist two sections, a multiple swarm PSO as the population space and a belief space including five sections: situational knowledge, temporal knowledge, domain

knowledge, normative knowledge, and spatial knowledge. The required information are categorized properly in the belief space in such a manner that can be easily accessed to move toward the optimum solution in the population space, and to monitor and detect any possible changes in the environment, also to respond quickly to the occurred changes by a repulsive diversity-promoted migration. When particles share their information through migration process, they will be able to quickly re-diversify and move efficiently towards new optimum by re-converging around it. The cultural information stored in the belief space will assist the population space in selecting the leading particles in the PSO flight. The flight mechanism follows a three level movement along with a repulsive term that is effective when a change has taken place. The three-level flight happens in the personal level, swarm level, and global level for which all leading particles will be assessed through the information extracted from different sections of the belief space. The particles will also repel each other the most, when a change has happened through a sigmoid repulsion factor. This phenomenon is repeatedly observed in the psychosocial studies as repulsion in interpersonal relationship among individuals generating new cultural opportunities through cultural divergence.

The novel cultural-based dynamic PSO is evaluated against some selected state-of-the-art evolutionary algorithm and particle swarm based heuristics on different benchmark dynamic test functions. Comparison study through experimental results show that the novel cultural-based dynamic PSO outperforms the selected state-of-the-art dynamic PSOs in almost all benchmark test functions suggesting that the organized and

categorized cultural information stored in belief space assist in better performing the search process under dynamic environment. The information extracted from belief space drives the repulsive divergence-promoted migration to quickly re-diversify the particles in the search space after a change takes place in the dynamic landscape and re-converge them through a modified three-level flight mechanism around new optimum. As a future work, it is suggested that the personal, swarm and global acceleration will not be a fixed value as it is in this study, but follow a dynamic behavior and adapt based upon the particle's or swarm's needs in the spatial space of the belief space as it can be observed how dynamic acceleration can improve the result of PSO convergence [140-141].

CHAPTER VIII

CONCLUSION

In this dissertation, several innovative heuristics using sociologically inspired concepts such as society and civilization, migration, communication, culture, swarms and beliefs have been proposed to solve engineering single objective optimization, multi objective optimization, constrained optimization and dynamic optimization problems.

A politically inspired measure called liberty rate has been introduced to facilitate the optimization process in social-based optimization algorithm. The simulation results show the performance improvement attained by accumulating the liberty rate into the original heuristics. The second modification on social-based optimization algorithm is to collect information from all individuals for migration purposes.

A diversity-based migration process among swarms in particle swarm optimization has also been proposed to solve multimodal optimization problems. The proposed PSO flight mechanism includes three levels which in turn also diversify its search ability. In the lowest level, particles follow the best behaving particle in their own swarm; in next level, particles follow the best performing particle in the neighboring swarms, and finally in the highest level, particles track the whole population's best

behaving particle. Adopting a two-way communication among each pair of swarms, the particles do not fall prematurely stuck in the local optima. The exchanged particles are selected according to the location of the particles based on a diversity strategy and their correspondence objective values. Furthermore, the PSO was modified using a new neighborhood term that helps the neighboring swarms share the common interest information. The neighborhood for each swarm is found using an unsupervised algorithm according to the inter-swarm distances between representatives of each pair of swarms. Simulation results on multimodal problems demonstrate that the proposed algorithm N-DMPSO shows a great performance compared to DMPSO and two versions of distributed genetic algorithm that share similar basis with the proposed algorithm. The DMPSO showed competitive results compared to DGAs. The N-DMPSO showed better performance compared to DMPSO, assuming that sharing information in the neighborhood of swarms helps to escape from local optima and locate the global optimum. However N-DMPSO and DMPSO both are dependent to the rate of information exchange.

A novel heuristics of cultural MOPSO has also been proposed to adjust flight parameters such as personal acceleration, global acceleration and momentum. Cultural algorithm provides the required groundwork enabling us to employ the information stored in different sections of belief space efficiently and effectively. Using the knowledge stored in various parts of belief space such as normative, situational, and topographical knowledge, cultural MOPSO shows promising results when compared to

some well-regarded MOPSOs. The comparison study based upon the hypervolume indicator and additive binary epsilon indicator show that cultural MOPSO provides better solution when compared on different hard benchmark test functions with high dimension and complexity. Indeed cultural MOPSO outperforms all selected well-regarded MOPSOs, except in one test function there is no difference between cultural MOPSO and another MOPSO. Consequently cultural MOPSO is significantly better than most MOPSOs and weakly dominates all of the selected MOPSOs.

Further comprehensive investigation of the cultural MOPSO demonstrates its robustness with respect to the algorithm's tuning parameters. In an extensive sensitivity analysis, the final Pareto fronts of any pair of algorithms are compared when one parameter is changed. Using additive binary epsilon indicator, the analysis demonstrate an almost-robust algorithm when nine different parameters of the algorithm are varied, i.e., about 95% of the tests indicates no change of results by tuning the parameters.

Additionally, a new cultural constrained particle swarm optimization has been proposed to solve constrained optimization problems. The heuristics incorporates information of objective function and constraints violation, to construct a cultural framework consisting two sections: a multiple swarm PSO with the ability of inter-swarm communication as population space and a belief space including four parts, normative knowledge, spatial knowledge, situational knowledge, and temporal knowledge. Every particle will fly through a three-level flight and then particles divide into several swarms and inter-swarm communication takes place to exchange the information. The cultural

CPSO is evaluated against 10 state-of-the-art constrained optimization paradigms on 24 difficult benchmark test problems. The simulation results show that cultural CPSO has the average feasible rate of 95.83% on 24 benchmark problems that places it as top performing algorithm along with DMS-PSO [149], ε_DE [150] and SaDE [157]. It also indicates that the proposed cultural CPSO has the average success rate of 90.00% on all benchmark problems placing it at the third best performing algorithm after ε_DE and PCX [155] with 91.67% and 90.17% of success rate, respectively. Overall, cultural CPSO is able to perform well competing with other well-performing algorithms in the field in terms of feasible rate and success rate.

Furthermore, the novel cultural-based dynamic particle swarm optimization has been proposed in order to solve DOP problems. Built upon a cultural framework consisting a multiple swarm PSO as the population space and a belief space including five sections, situational knowledge, temporal knowledge, domain knowledge, normative knowledge, and spatial knowledge, the cultural-based DPSO categorizes effectively the required information in the belief space in such a manner that can be easily accessed.

The information extracted from the belief space assists on moving toward the optimum solution in the population space, and to detect any occurring changes in the environment, also to respond quickly to the occurred changes by a repulsive diversity-promoted migration. When particles share their information through migration process, they will be able to quickly re-diversify and move efficiently towards new optimum by re-converging around it. The cultural information stored in the belief space will assist the

population space in selecting the leading particles in the PSO flight. The flight mechanism follows a three level movement along with a repulsive term that is most-affective when a change has taken place. The three-level flight happens in the personal level, swarm level, and global level for which all leading particles will be assessed through the information extracted from different sections of the belief space. The particles will also repel each other the most, when a change has happened through a sigmoid repulsion formulation.

The cultural-based dynamic PSO has also been evaluated against some selected state-of-the-art dynamic PSO heuristics on different benchmark dynamic test functions. Comparison study demonstrates that the proposed cultural-based dynamic PSO performs better or equally when compared with the selected state-of-the-art dynamic PSOs in all benchmark test functions suggesting that the organized and categorized cultural information stored in belief space assist in better performing the search process in dynamic environment. The information extracted from belief space drives the repulsive divergence-promoted migration to quickly re-diversify the particles in the search space after a change takes place in the dynamic landscape and re-converge them through a modified three-level flight mechanism around new optimum.

Overall, in this dissertation, cultural-based particle swarm optimization has been proposed to solve different types of optimization problems ranging from a single objective, multiobjective, constrained and finally dynamic optimization problems. The incorporation of elements of culture through the well-organized belief space assists the

retrieving process of required information much easier. In all of these proposed heuristics, the main structure of the algorithm follows an identical framework as far as population space, acceptance function, and influence function and finally different sections of belief space such as normative knowledge, situational knowledge, spatial knowledge, temporal knowledge, and domain knowledge depending on the need of the proposed paradigms. The flourishing performance of cultural-based PSO can be understood due to its all-the-time monitoring and adjustment through the feedback process from the population space, via acceptance function to belief space, and back to the population space via the influence functions. This feedback fundamentally assists in adjusting the optimum parameters for the entire system. The cultural PSO proposed here has seen a great success when compared experimentally against other state-of-the-art heuristics in different types of optimization problems, suggesting its further potential to be developed and its potentially successful applications on developing optimization algorithms for real-world problems.

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APPENDIX A

BENCHMARK TEST FUNCTIONS FOR MULTIOBJECTIVE OPTIMIZATION PROBLEMS

Test functions *ZDT1* [129]:

$$\text{Minimize } \mathcal{F}(\mathbf{x}) = (f_1, f_2) \tag{A.1}$$

$$f_1(\mathbf{x}) = x_1 \text{ and } f_2(\mathbf{x}) = g(x_2, \dots, x_M)h(f_1, g)$$

$$g(x_2, \dots, x_M) = 1 + 9\sum_{i=2}^{M} x_i/(M-1) \text{ and } h(f_1, g) = 1 - \sqrt{f_1/g}$$

where $\mathbf{x} = (x_1, x_2, \dots, x_M)$, and $x_i \in [0,1]$ $(i = 1,2, \dots, M)$. $M = 30$ is the decision space dimension. The convex Pareto-optimal front is formed with $g(\mathbf{x}) = 1$

Test functions *ZDT2* [129]:

$$\text{Minimize } \mathcal{F}(\mathbf{x}) = (f_1, f_2) \tag{A.2}$$

$$f_1(\mathbf{x}) = x_1 \text{ and } f_2(\mathbf{x}) = g(x_2, \dots, x_M)h(f_1, g),$$

$$g(x_2, \dots, x_M) = 1 + 9\sum_{i=2}^{M} x_i/(M-1) \text{ and } h(f_1, g) = 1 - (f_1/g)^2$$

where $\mathbf{x} = (x_1, x_2, \dots, x_M)$, and $x_i \in [0,1]$ $(i = 1,2, \dots, M)$. $M = 30$ is the decision space dimension. The non-convex Pareto-optimal front is formed with $g(\mathbf{x}) = 1$

Test functions *ZDT3* [129]:

$$\text{Minimize } \mathcal{F}(\mathbf{x}) = (f_1, f_2) \tag{A.3}$$

$$f_1(\mathbf{x}) = x_1 \text{ and } f_2(\mathbf{x}) = g(x_2, \dots, x_M)h(f_1, g),$$

$$g(x_2, \dots, x_M) = 1 + 9\sum_{i=2}^{M} x_i/(M-1) \text{ and}$$

$$h(f_1, g) = 1 - \sqrt{f_1/g} - (f_1/g)\sin(10\pi f_1)$$

where $\mathbf{x} = (x_1, x_2, \dots, x_M)$, and $x_i \in [0,1]$ $(i = 1,2, \dots, M)$. $M = 30$ is the decision space dimension. The discrete Pareto-optimal front formed with $g(\mathbf{x}) = 1$, consists of several noncontiguous convert parts.

Test functions *ZDT4* [129]:

$$\text{Minimize } \mathcal{F}(\mathbf{x}) = (f_1, f_2) \tag{A.4}$$

$$f_1(\mathbf{x}) = x_1 \text{ and } f_2(\mathbf{x}) = g(x_2, \dots, x_M)h(f_1, g),$$

$$g(x_2, \dots, x_M) = 1 + 10(M-1) + \sum_{i=2}^{M}(x_i^2 - 10\cos(4\pi x_i)) \text{ and}$$

$$h(f_1, g) = 1 - \sqrt{f_1/g}$$

where $\mathbf{x} = (x_1, x_2, \dots, x_M)$, and $x_i \in [0,1]$ $(i = 1,2, \dots, M)$. $M = 10$ is the decision space dimension. It contains 21^9 local Pareto-optimal fronts. The global Pareto-optimal front is formed with $g(\mathbf{x}) = 1$.

Test function *DTLZ5* [130]:

$$\text{Minimize } \mathcal{F}(\mathbf{x}) = (f_1, f_2, f_3) \tag{A.5}$$

$$f_1(\mathbf{x}) = (1 + g(\mathbf{x}_M))\cos(\theta_1)\cos(\theta_2)$$

$$f_2(\mathbf{x}) = (1 + g(\mathbf{x}_M))\cos(\theta_1)\sin(\theta_2)$$

$$f_3(\mathbf{x}) = (1 + g(\mathbf{x}_M))\sin(\theta_1)$$

$$\theta_i = \frac{\pi}{4(1+g(\mathbf{x}_M))}(1 + 2g(\mathbf{x}_M)x_i)\ (i = 1,2) \text{ and } g(\mathbf{x}_M) = \sum_{x_i \in \mathbf{x}_M} x_i^{0.1}$$

where $x_i \in [0,1]$ $(i = 1,2, \dots, 10)$.

This test problem will test algorithm's ability to converge to the degenerated curve. The true Pareto-optimal front is a 3D curve on the surface of the unit-sphere. The size of $\mathbf{x}_M$ vector is chosen as 10.

Test function *DTLZ6* [130]:

Minimize $\mathcal{F}(\mathbf{x}) = (f_1, f_2, \dots, f_M)$ (A.6)

$$f_1(\mathbf{x}_1) = x_1$$

$$\vdots$$

$$f_{M-1}(\mathbf{x}_{M-1}) = x_{M-1}$$

$$f_M(\mathbf{x}) = (1 + g(\mathbf{x}_M))h(f_1, f_2, \dots, f_{M-1}, g)$$

$$g(\mathbf{x}_M) = 1 + \frac{9}{|\mathbf{x}_M|}10\sum_{x_i \in \mathbf{x}_M} x_i \quad \text{and} \quad h = M - \sum_{i=1}^{M-1}[\frac{f_i}{1+g}(1 + \sin(3\pi f_i))]$$

where $x_i \in [0,1]$ $(i = 1,2, \dots, n)$. This test problem has 2^{M-1} disconnected Pareto-Optimal regions in the search space. The functional g requires $k = |\mathbf{x}_M|$ decision variables and the total number of variables is $n = M + k - 1$. This problem tests algorithm's ability to maintain subpopulation in different Pareto-optimal regions.

APPENDIX B

BENCHMARK TEST FUNCTIONS FOR CONSTRAINED OPTIMIZATION PROBLEMS

All benchmark problems in this Appendix along with the best global minimum found have been reported from [147].

Test function $g01$

Minimize: $f(\mathbf{x}) = 5\sum_{i=1}^{4} x_i - 5\sum_{i=1}^{4} x_i^2 - \sum_{i=5}^{13} x_i$ (B.1)

Subject to:

$$g_1(\mathbf{x}) = 2x_1 + 2x_2 + x_{10} + x_{11} - 10 \leq 0$$

$$g_2(\mathbf{x}) = 2x_1 + 2x_3 + x_{10} + x_{12} - 10 \leq 0$$

$$g_3(\mathbf{x}) = 2x_2 + 2x_3 + x_{11} + x_{12} - 10 \leq 0$$

$$g_4(\mathbf{x}) = -8x_1 + x_{10} \leq 0$$

$$g_5(\mathbf{x}) = -8x_2 + x_{11} \leq 0$$

$$g_6(\mathbf{x}) = -8x_3 + x_{12} \leq 0$$

$$g_7(\mathbf{x}) = -2x_4 - x_5 + x_{10} \leq 0$$

$$g_8(\mathbf{x}) = -2x_6 - x_7 + x_{11} \leq 0$$

$$g_9(\mathbf{x}) = -2x_8 - x_9 + x_{12} \leq 0$$

where $0 \leq x_i \leq 1$ $(i = 1,2, \dots ,9)$, $0 \leq x_i \leq 100$ $(i = 10,11,12)$ and $0 \leq x_{13} \leq 1$

The optimum is at $\mathbf{x}^* = (1,1,1,1,1,1,1,1,1,3,3,3,1)$ with $f(\mathbf{x}^*) = -15$. Six constraints are active $(g_1, g_2, g_3, g_7, g_8, g_9)$.

Test function $g02$

$$\text{Minimize: } f(\mathbf{x}) = -\left| \frac{\sum_{i=1}^{M} \cos^4(x_i) - 2\prod_{i=1}^{M} \cos^2(x_i)}{\sqrt{\sum_{i=1}^{M} i x_i^2}} \right| \tag{B.2}$$

Subject to:

$$g_1(\mathbf{x}) = 0.75 - \prod_{i=1}^{M} x_i \leq 0$$

$$g_2(\mathbf{x}) = \sum_{i=1}^{M} x_i - 7.5M \leq 0$$

where $M = 20$ and $0 \leq x_i \leq 10$ $(i = 1,2, \dots , M)$.

The optimum is at $\mathbf{x}^* = (3.16246061572185, 3.12833142812967, 3.094792129887$ $91, 3.06145059523469, 3.02792915885555, 2.99382606701730, 2.95866871765285,$ $2.92184227312450, 0.49482511456933, 0.48835711005490, 0.48231642711865, .$ $0.47664475092742, 0.47129550835493, 0.46623099264167, 0.46142004984199,$ $0.45683664767217, 0.45245876903267, 0.44826762241853, 0.44424700958760,$ $0.44038285956317)$ with $f(\mathbf{x}^*) = -0.80361910412559$. Constraint g_1 is close to being active.

Test function $g03$

Minimize: $f(\mathbf{x}) = -(\sqrt{M})^M \prod_{i=1}^{M} x_i$ (B.3)

Subject to:

$$h_1(\mathbf{x}) = \sum_{i=1}^{M} x_i^2 - 1 = 0$$

where $M = 10$ and $0 \leq x_i \leq 1$ $(i = 1,2, \dots, M)$.

The optimum is at $\mathbf{x}^* = (0.3162435764728306 9, 0.3162435774143383 39, 0.3162435780125923 02, 0.3162435775739149 16)$ with $f(\mathbf{x}^*) = -1.00050010001000$.

Test function $g04$

Minimize: $f(\mathbf{x}) = 5.3578547x_3^2 + 0.8356891x_1x_5 + 37.293239x_1 - 40792.141$ (B.4)

Subject to:

$$g_1(\mathbf{x}) = 85.3344072 + 0.0056858x_2x_5 + 0.0006262x_1x_4 - 0.0022053x_3x_5 - 92 \leq 0$$

$$g_2(\mathbf{x}) = -85.3344072 - 0.0056858x_2x_5 - 0.0006262x_1x_4 + 0.0022053x_3x_5 \leq 0$$

$$g_3(\mathbf{x}) = 80.51249 + 0.0071317x_2x_5 + 0.0029955x_1x_2$$
$$+0.0021813x_3^2 - 110 \leq 0$$
$$g_4(\mathbf{x}) = -80.51249 - 0.0071317x_2x_5 - 0.0029955x_1x_2$$
$$-0.0021813x_3^2 + 90 \leq 0$$
$$g_5(\mathbf{x}) = 9.300961 + 0.0047026x_3x_5 + 0.0012547x_1x_3$$
$$+0.0019085x_3x_4 - 25 \leq 0$$
$$g_6(\mathbf{x}) = -9.300961 - 0.0047026x_3x_5 - 0.0012547x_1x_3$$
$$-0.0019085x_3x_4 + 20 \leq 0$$

where $78 \leq x_1 \leq 102$, $33 \leq x_2 \leq 45$, and $27 \leq x_i \leq 45$ ($i = 3,4,5$).

The optimum is at $\mathbf{x}^* = (78,33,29.995256025681598 5,45,36.77581290578820 73)$, with $f(\mathbf{x}^*) = -3.066553867178332\mathrm{e}+4$. Two constraints are active (g_1, g_6).

Test function $g05$

Minimize: $f(\mathbf{x}) = 3x_1 + 0.000001x_1^3 + 2x_2 + (0.000002)/3x_2^3$ (B.5)

Subject to:

$$g_1(\mathbf{x}) = -x_4 + x_3 - 0.55 \leq 0$$
$$g_2(\mathbf{x}) = -x_3 + x_4 - 0.55 \leq 0$$
$$h_3(\mathbf{x}) = 1000\sin(-x_3 - 0.25) + 1000\sin(-x_4 - 0.25) + 894.8 - x_1 = 0$$
$$h_4(\mathbf{x}) = 1000\sin(x_3 - 0.25) + 1000\sin(x_3 - x_4 - 0.25) + 894.8 - x_2 = 0$$
$$h_5(\mathbf{x}) = 1000\sin(x_4 - 0.25) + 1000\sin(x_4 - x_3 - 0.25) + 1294.8 = 0$$

where $0 \leq x_1 \leq 1200$, $0 \leq x_2 \leq 1200$, $-0.55 \leq x_3 \leq 0.55$ and $-0.55 \leq x_4 \leq 0.55$.

The optimum is at $\mathbf{x}^* = (679.945148297028709, 1026.06697600004691, 0.1188$

$76369094410433, -0.39623348521517826)$, with $f(\mathbf{x}^*) = 5126.4967140071$.

Test function $g06$

Minimize: $f(\mathbf{x}) = (x_1 - 10)^3 + (x_2 - 20)^3$ (B.6)

Subject to:

$g_1(\mathbf{x}) = -(x_1 - 5)^2 - (x_2 - 5)^2 + 100 \leq 0$

$g_2(\mathbf{x}) = (x_1 - 6)^2 + (x_2 - 5)^2 - 82.81 \leq 0$

where $13 \leq x_1 \leq 100$, and $0 \leq x_2 \leq 100$.

The optimum is at $\mathbf{x}^* = (14.09500000000000064, 0.8429607892154795668)$ with $f(\mathbf{x}^*) = -6961.81387558015$. Both constraints are active.

Test function $g07$

Minimize: $f(\mathbf{x}) = x_1^2 + x_2^2 + x_1 x_2 - 14x_1 - 16x_2 + (x_3 - 10)^2 + 4(x_4 - 5)^2 +$

$(x_5 - 3)^2 + 2(x_6 - 1)^2 + 5x_7^2 + 7(x_8 - 11)^2 + 2(x_9 - 10)^2 +$

$(x_{10} - 7)^2 + 45$ (B.7)

Subject to:

$g_1(\mathbf{x}) = -105 + 4x_1 + 5x_2 - 3x_7 + 9x_8 \leq 0$

$g_2(\mathbf{x}) = 10x_1 - 8x_2 - 17x_7 + 2x_8 \leq 0$

$g_3(\mathbf{x}) = -8x_1 + 2x_2 + 5x_9 - 2x_{10} - 12 \leq 0$

$$g_4(\mathbf{x}) = 3(x_1 - 2)^2 + 4(x_2 - 3)^2 + 2x_3^2 - 7x_4 - 120 \leq 0$$

$$g_5(\mathbf{x}) = 5x_1^2 + 8x_2 + (x_3 - 6)^2 - 2x_4 - 40 \leq 0$$

$$g_6(\mathbf{x}) = x_1^2 + 2(x_2 - 2)^2 - 2x_1x_2 + 14x_5 - 6x_6 \leq 0$$

$$g_7(\mathbf{x}) = 0.5(x_1 - 8)^2 + 2(x_2 - 4)^2 + 3x_5^2 - x_6 - 30 \leq 0$$

$$g_8(\mathbf{x}) = -3x_1 + 6x_2 + 12(x_9 - 8)^2 - 7x_{10} \leq 0$$

where $-10 \leq x_i \leq 10$ ($i = 1,2, \ldots ,10$).

The optimum is at $\mathbf{x}^* = (2.17199634142692,2.3636830416034,8.77392573913157,$ $5.09598443745173,0.990654756560493,1.43057392853463,1.32164415364306,$ $9.82872576524495,8.2800915887356,8.3759266477347)$ with $f(\mathbf{x}^*) = 4.3062090$ 6818. Six constraints are active ($g_1, g_2, g_3, g_4, g_5, g_6$).

Test function $g08$

Minimize: $f(\mathbf{x}) = -\frac{\sin^3(2\pi x_1)\sin(2\pi x_2)}{x_1^3(x_1+x_2)}$ (B.8)

Subject to:

$$g_1(\mathbf{x}) = x_1^2 - x_2 + 1 \leq 0$$

$$g_2(\mathbf{x}) = 1 - x_1 + (x_2 - 4)^2 \leq 0$$

where $0 \leq x_i \leq 10$ ($i = 1,2$).

The optimum is at $\mathbf{x}^* = (1.22797135260752599,4.24537336612274885)$ with $f(\mathbf{x}^*) = -0.0958250414180359$.

Test function $g09$

Minimize: $f(\mathbf{x}) = (x_1 - 10)^2 + 5(x_2 - 12)^2 + x_3^4 + 3(x_4 - 11)^2 + 10x_5^6 + 7x_6^2 + x_7^4 - 4x_6x_7 - 10x_6 - 8x_7$ (B.9)

Subject to:

$g_1(\mathbf{x}) = -127 + 2x_1^2 + 3x_2^4 + x_3 + 4x_4^2 + 5x_5 \leq 0$

$g_2(\mathbf{x}) = -282 + 7x_1 + 3x_2 + 10x_3^2 + x_4 - x_5 \leq 0$

$g_3(\mathbf{x}) = -196 + 23x_1 + x_2^2 + 6x_6^2 - 8x_7 \leq 0$

$g_4(\mathbf{x}) = 4x_1^2 + x_2^2 - 3x_1x_2 + 2x_3^2 + 5x_6 - 11x_7 \leq 0$

where $-10 \leq x_i \leq 10$ ($i = 1,2, \ldots,7$).

The optimum is at $\mathbf{x}^* = (2.33049935147405174, 1.95137236847114592, -0.4775$
$41399510615805, 4.36572624923625874, -0.624486959100388983, 1.038130994$
$10962173, 1.5942266780671519)$ with $f(\mathbf{x}^*) = 680.630057374402$.

Test function $g10$

Minimize: $f(\mathbf{x}) = x_1 + x_2 + x_3$ (B.10)

Subject to:

$g_1(\mathbf{x}) = -1 + 0.0025(x_4 + x_6) \leq 0$

$g_2(\mathbf{x}) = -1 + 0.0025(x_5 + x_7 - x_4) \leq 0$

$g_3(\mathbf{x}) = -1 + 0.01(x_8 - x_5) \leq 0$

$g_4(\mathbf{x}) = -x_1x_6 + 833.33252x_4 + 100x_1 - 83333.333 \leq 0$

$g_5(\mathbf{x}) = -x_2x_7 + 1250x_5 + x_2x_4 - 1250x_4 \leq 0$

$g_6(\mathbf{x}) = -x_3x_8 + 1250000 + x_3x_5 - 2500x_5 \leq 0$

where $100 \leq x_1 \leq 10000$, $1000 \leq x_i \leq 10000$, $(i = 2,3)$ and $10 \leq x_1 \leq 1000$, $(i = 4,5, \ldots,8)$.

The optimum is at $\mathbf{x}^* = (579.306685017979589,1359.97067807935605,5109.970$ $65743133317, 182.01769963061534,295.601173702746792, 217.98230036938$ $4632,286.41652592786852,395.601173702746735)$ with $f(\mathbf{x}^*) = 7049.24802052$ 867.

Test function $g11$

Minimize: $f(\mathbf{x}) = x_1^2 + (x_2 - 1)^2$ (B.11)

Subject to:

$h_1(\mathbf{x}) = x_2 - x_1^2 = 0$

where $-1 \leq x_i \leq 1$ ($i = 1,2$).

The optimum is at $\mathbf{x}^* = (-0.707036070037170616,0.500000004333606807)$ with $f(\mathbf{x}^*) = 0.7499$.

Test function $g12$

Minimize: $f(\mathbf{x}) = -(100 - (x_1 - 5)^2 - (x_2 - 5)^2 - (x_3 - 5)^2)/100$ (B.12)

Subject to:

$g(\mathbf{x}) = (x_1 - p)^2 + (x_2 - q)^2 + (x_3 - r)^2 - 0.0625 \leq 0$

where $0 \leq x_i \leq 10$ ($i = 1,2,3$) and $p, q, r = 1,2, \ldots, 9$. The feasible region of the search space consists of 9^3 disjointed spheres. A point is feasible if and only if there exist p, q, r such that the above inequality holds. The optimum is at $\mathbf{x}^* = (5,5,5)$ with $f(\mathbf{x}^*) = -1$. The solution lies within the feasible region.

Test function $g13$

Minimize: $f(\mathbf{x}) = e^{x_1 x_2 x_3 x_4 x_5}$ (B.13)

Subject to:

$h_1(\mathbf{x}) = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 - 10 = 0$

$h_2(\mathbf{x}) = x_2 x_3 - 5x_4 x_5 = 0$

$h_3(\mathbf{x}) = x_1^3 + x_2^3 + +1 = 0$

where $-2.3 \leq x_i \leq 2.3$ ($i = 1,2$) and $-3.2 \leq x_i \leq 3.2$ ($i = 3,4,5$).

The optimum is at $\mathbf{x}^* = (-1.71714224003, 1.59572124049468, 1.8272502406271, -0.763659881912867, -0.76365986736498)$ with $f(\mathbf{x}^*) = 0.053941514041898$.

Test function $g14$

Minimize: $f(\mathbf{x}) = \sum_{i=1}^{10} x_i \left(c_i + \ln \frac{x_i}{\sum_{j=1}^{10} x_j} \right)$ (B.14)

Subject to:

$h_1(\mathbf{x}) = x_1 + 2x_2 + 2x_3 + x_6 + x_{10} - 2 = 0$

$h_2(\mathbf{x}) = x_4 + 2x_5 + x_6 + x_7 - 1 = 0$

$h_3(\mathbf{x}) = x_3 + x_7 + x_8 + 2x_9 + x_{10} - 1 = 0$

where $0 \leq x_i \leq 10$ ($i = 1,2,\ldots,10$) and $c_1 = -6.089$, $c_2 = -17.164$, $c_3 = -34.054$, $c_4 = -5.914$, $c_5 = -24.721$, $c_6 = -14.986$, $c_7 = -24.1$, $c_8 = -10.708$, $c_9 = -26.662$, $c_{10} = -22.179$.

The optimum is at $\mathbf{x}^* = (0.0406684113216282, 0.147721240492452, 0.783205732104114, 0.00141433931889084, 0.485293636780388, 0.000693183051556082, 0.0274052040687766, 0.0179509660214818, 0.0373268186859717, 0.0968844604336845)$ with $f(\mathbf{x}^*) = -47.7648884594915$.

Test function $g15$

Minimize: $f(\mathbf{x}) = 1000 - x_1^2 - 2x_2^2 - x_3^2 - x_1x_2 - x_1x_3$ (B.15)

Subject to:

$h_1(\mathbf{x}) = x_1^2 + x_2^2 + x_3^2 - 25 = 0$

$h_2(\mathbf{x}) = 8x_1 + 14x_2 + 7x_3 - 56 = 0$

where $0 \leq x_i \leq 10$ ($i = 1,2,3$).

The optimum is at $\mathbf{x}^* = (3.5121281261179513, 0.21698751042955613, 3.552178549291799 21)$ with $f(\mathbf{x}^*) = 961.715022289961$.

Test function $g16$

Minimize: $f(\mathbf{x}) = 0.000117y_{14} + 0.1365 + 0.00002358y_{13} + 0.000001502y_{16} +$

$$0.0321y_{12} + 0.004324y_5 + 0.0001\frac{c_{15}}{c_{16}} + 37.48\frac{y_2}{c_{12}} + 0.0000005843y_{17} \quad \text{(B.16)}$$

Subject to:

$$g_1(\mathbf{x}) = \frac{0.28}{0.72}y_4 - y_5 \leq 0$$

$$g_2(\mathbf{x}) = x_3 - 1.5x_2 \leq 0$$

$$g_3(\mathbf{x}) = 3496\frac{y_2}{c_{12}} - 21 \leq 0$$

$$g_4(\mathbf{x}) = 110.6 + y_1 - \frac{62217}{c_{17}} \leq 0$$

$$g_5(\mathbf{x}) = 213.1 - y_1 \leq 0$$

$$g_6(\mathbf{x}) = y_1 - 405.23 \leq 0$$

$$g_7(\mathbf{x}) = 17.505 - y_2 \leq 0$$

$$g_8(\mathbf{x}) = y_2 - 1053.6667 \leq 0$$

$$g_9(\mathbf{x}) = 11.275 - y_3 \leq 0$$

$$g_{10}(\mathbf{x}) = y_3 - 35.03 \leq 0$$

$$g_{11}(\mathbf{x}) = 214.228 - y_4 \leq 0$$

$$g_{12}(\mathbf{x}) = y_4 - 665.585 \leq 0$$

$$g_{13}(\mathbf{x}) = 7.458 - y_5 \leq 0$$

$$g_{14}(\mathbf{x}) = y_5 - 584.463 \leq 0$$

$$g_{15}(\mathbf{x}) = 0.961 - y_6 \leq 0$$

$$g_{16}(\mathbf{x}) = y_6 - 265.916 \leq 0$$

$$g_{17}(\mathbf{x}) = 1.612 - y_7 \leq 0$$

$$g_{18}(\mathbf{x}) = y_7 - 7.046 \leq 0$$

$$g_{19}(\mathbf{x}) = 0.146 - y_8 \leq 0$$

$$g_{20}(\mathbf{x}) = y_8 - 0.222 \leq 0$$

$$g_{21}(\mathbf{x}) = 107.99 - y_9 \leq 0$$

$$g_{22}(\mathbf{x}) = y_9 - 273.366 \leq 0$$

$$g_{23}(\mathbf{x}) = 922.693 - y_{10} \leq 0$$

$$g_{24}(\mathbf{x}) = y_{10} - 1286.105 \leq 0$$

$$g_{25}(\mathbf{x}) = 926.832 - y_{11} \leq 0$$

$$g_{26}(\mathbf{x}) = y_{11} - 1444.046 \leq 0$$

$$g_{27}(\mathbf{x}) = 18.766 - y_{12} \leq 0$$

$$g_{28}(\mathbf{x}) = y_{12} - 537.141 \leq 0$$

$$g_{29}(\mathbf{x}) = 1072.163 - y_{13} \leq 0$$

$$g_{30}(\mathbf{x}) = y_{13} - 3247.039 \leq 0$$

$$g_{31}(\mathbf{x}) = 8961.448 - y_{14} \leq 0$$

$$g_{32}(\mathbf{x}) = y_{14} - 26844.086 \leq 0$$

$$g_{33}(\mathbf{x}) = 0.063 - y_{15} \leq 0$$

$$g_{34}(\mathbf{x}) = y_{15} - 0.386 \leq 0$$

$$g_{35}(\mathbf{x}) = 71084.33 - y_{16} \leq 0$$

$$g_{36}(\mathbf{x}) = -140000 + y_{16} \leq 0$$

$$g_{37}(\mathbf{x}) = 2802713 - y_{17} \leq 0$$

$$g_{38}(\mathbf{x}) = y_{17} - 12146108 \leq 0$$

where:

$$y_1 = x_2 + x_3 + 41.6$$

$$c_1 = 0.024x_4 - 4.62$$

$$y_1 = \frac{12.5}{c_1} + 12$$

$$c_2 = 0.0003535x_1^2 + 0.5311x_1 + 0.08705y_2x_1$$

$$c_3 = 0.052x_1 + 78 + 0.002377y_2x_1$$

$$y_3 = \frac{c_2}{c_3}$$

$$y_4 = 19y_3$$

$$c_4 = 0.04782(x_1 - y_3) + \frac{0.1956(x_1 - y_3)^2}{x_2} + 0.6376y_4 + 1.594y_3$$

$$c_5 = 100x_2$$

$$c_6 = x_1 - y_5 - y_4 - y_3$$

$$c_7 = 0.950 - \frac{c_4}{c_5}$$

$$y_5 = c_6c_7$$

$$y_6 = x_1 - y_5 - y_4 - y_3$$

$$c_8 = (y_5 + y_4)0.995$$

$$y_7 = \frac{c_8}{y_1}$$

$$y_8 = \frac{c_8}{3798}$$

$$c_9 = y_7 - \frac{0.0663y_7}{y_8} - 0.3153$$

$$y_9 = \frac{96.82}{c_9} + 0.321y_1$$

$$y_{10} = 1.29y_5 + 1.258y_4 + 2.29y_3 + 1.71y_6$$

$$y_{11} = 1.71x_1 - 0.452y_4 + 0.580y_3$$

$$c_{10} = \frac{12.3}{752.3}$$

$$c_{11} = (1.75y_2)(0.995x_1)$$

$$c_{12} = 0.995y_{10} + 1998$$

$$y_{12} = c_{10}x_1 + \frac{c_{11}}{c_{12}}$$

$$y_{13} = c_{12} - 1.75y_2$$

$$y_{14} = 3623 + 64.4x_2 + 58.4x_3 + \frac{146312}{y_9 + x_5}$$

$$c_{13} = 0.995y_{10} + 60.8x_2 + 48x_4 - 0.1121y_{14} - 5095$$

$$y_{15} = \frac{y_{13}}{c_{13}}$$

$$y_{16} = 148000 - 331000y_{15} + 40y_{13} + 48x_4 - 61y_{15}y_{13}$$

$$c_{14} = 2324y_{10} - 28740000y_2$$

$$y_{17} = 14130000 - 1328y_{10} - 531y_{11} + \frac{c_{14}}{c_{12}}$$

$$c_{15} = \frac{y_{13}}{y_{15}} - \frac{y_{13}}{0.52}$$

$$c_{16} = 1.104 - 0.72y_{15}$$

$c_{17} = y_9 + x_5$

and where $704.4148 \leq x_1 \leq 906.3855$, $68.6 \leq x_2 \leq 288.88$, $0 \leq x_3 \leq 134.75$, $193 \leq x_4 \leq 287.0966$, and $25 \leq x_5 \leq 84.1988$.

The optimum is at $\mathbf{x}^* = (705.174537070090537,68.5999999999999943,102.899999999999991,282.324931593660324,37.5841164258054832)$ with $f(\mathbf{x}^*) = -1.90515525853479$.

Test function $g17$

Minimize: $f(\mathbf{x}) = f_1(x_1) + f_2(x_2)$ (B.17)

where: $f_1(x_1) = \begin{cases} 30x_1, & 0 \leq x_1 < 300 \\ 31x_1, & 300 \leq x_1 < 400 \end{cases}$

$$f_2(x_2) = \begin{cases} 28x_2, & 0 \leq x_2 < 100 \\ 29x_2, & 100 \leq x_2 < 200 \\ 30x_2, & 200 \leq x_2 < 1000 \end{cases}$$

Subject to:

$$h_1(\mathbf{x}) = -x_1 + 300 - \frac{x_3 x_4}{131.078}\cos(1.48477 - x_6) + \frac{0.90798x_3^2}{131.078}\cos(1.47588) = 0$$

$$h_2(\mathbf{x}) = -x_2 - \frac{x_3 x_4}{131.078}\cos(1.48477 + x_6) + \frac{0.90798x_4^2}{131.078}\cos(1.47588) = 0$$

$$h_3(\mathbf{x}) = -x_5 - \frac{x_3 x_4}{131.078}\sin(1.48477 + x_6) + \frac{0.90798x_4^2}{131.078}\sin(1.47588) = 0$$

$$h_4(\mathbf{x}) = 200 - \frac{x_3 x_4}{131.078}\sin(1.48477 - x_6) + \frac{0.90798x_3^2}{131.078}\sin(1.47588) = 0$$

where $0 \le x_1 \le 400$, $0 \le x_2 \le 1000$, $340 \le x_3 \le 420$, $340 \le x_4 \le 420$, $-1000 \le x_5 \le 1000$, and $0 \le x_6 \le 0.5236$.

The optimum is at $\mathbf{x}^* = (201.784467214523659, 99.9999999999999005, 383.07103 4852773266, 420, -10.9076584514292652, 0.0731482312084287128)$ with $f(\mathbf{x}^*) = 8853.53967480648$.

Test function $g18$

Minimize: $f(\mathbf{x}) = -0.5(x_1x_5 - x_2x_3 + x_3x_9 - x_5x_9 + x_5x_8 - x_6x_7)$ (B.18)

Subject to:

$g_1(\mathbf{x}) = x_3^2 + x_4^2 - 1 \le 0$

$g_2(\mathbf{x}) = x_9^2 - 1 \le 0$

$g_3(\mathbf{x}) = x_5^2 + x_6^2 - 1 \le 0$

$g_4(\mathbf{x}) = x_1^2 + (x_2 - x_9)^2 - 1 \le 0$

$g_5(\mathbf{x}) = (x_1 - x_5)^2 + (x_2 - x_6)^2 - 1 \le 0$

$g_6(\mathbf{x}) = (x_1 - x_7)^2 + (x_2 - x_8)^2 - 1 \le 0$

$g_7(\mathbf{x}) = (x_3 - x_5)^2 + (x_4 - x_6)^2 - 1 \le 0$

$g_8(\mathbf{x}) = (x_3 - x_7)^2 + (x_4 - x_8)^2 - 1 \le 0$

$g_9(\mathbf{x}) = x_7^2 + (x_8 - x_9)^2 - 1 \le 0$

$g_{10}(\mathbf{x}) = x_2x_3 - x_1x_4 \le 0$

$g_{11}(\mathbf{x}) = -x_3x_9 \le 0$

$g_{12}(\mathbf{x}) = x_5 x_9 \leq 0$

$g_{13}(\mathbf{x}) = x_6 x_7 - x_5 x_8 \leq 0$

where $-10 \leq x_i \leq 10$ ($i = 1,2, \ldots ,8$) and $0 \leq x_9 \leq 20$.

The optimum is at $\mathbf{x}^* = (-0.657776192427943163, -0.153418773482438542, 0.323413871675240938, -0.946257611651304398, -0.657776194376798906, -0.753213434632691414, 0.323413874123576972, -0.346462947962331735, 0.599794662852175 42)$, with $f(\mathbf{x}^*) = -0.866025403784439$.

Test function $g19$

Minimize: $f(\mathbf{x}) = \sum_{j=1}^{5} \sum_{i=1}^{5} c_{ij} x_{(10+i)} x_{(10+j)} + 2 \sum_{j=1}^{5} d_j x_{(10+j)}^3 - \sum_{i=1}^{10} b_i x_i$ (B.19)

Subject to:

$g_j(\mathbf{x}) = -2 \sum_{i=1}^{5} c_{ij} x_{(10+i)} - 3 d_j x_{(10+j)}^2 - e_j + \sum_{i=1}^{10} a_{ij} x_i \leq 0, \quad j = 1,2, \ldots ,5$

where $0 \leq x_i \leq 10$ $(i = 1,2, \ldots ,15)$, $\vec{b} = [-40, -2, -0.25, -4, -4, -1, -40, -60,5,1]$ and the remaining data is represented in Table B.1.

The optimum is at

$\mathbf{x}^* = (1.66991341326291344e-17, 3.95378229282456509e-16, 3.9459904 5143233784, 1.06036597479721211e-16, 3.2831773458454161, 9.999999 99999999822, 1.12829414671605333e-17, 1.2026194599794709e-17, 2.50706276000769697e-15, 2.24624122987970677e-15, 0.3707648474 17013987, 0.278456024942955571, 0.523838487672241171, 0.388620152$

510322781,0.298156764974678579) with $f(\mathbf{x}^*) = 32.6555929502463$.

Table B.1 Data set for test problem g19

j	1	2	3	4	5
ej	-15	-27	-36	-18	-12
c_{1j}	30	-20	-10	32	-10
c_{2j}	-20	39	-6	-31	32
c_{3j}	-10	-6	10	-6	-10
c_{4j}	32	-31	-6	39	-20
c_{5j}	-10	32	-10	-20	30
d_j	4	8	10	6	2
a_{1j}	-16	2	0	1	0
a_{2j}	0	-2	0	0.4	2
a_{3j}	-3.5	0	2	0	0
a_{4j}	0	-2	0	-4	-1
a_{5j}	0	-9	-2	1	-2.8
a_{6j}	2	0	-4	0	0
a_{7j}	-1	-1	-1	-1	-1
a_{8j}	-1	-2	-3	-2	-1
a_{9j}	1	2	3	4	5
a_{10j}	1	1	1	1	1

Test function $g20$

Minimize: $f(\mathbf{x}) = \sum_{i=1}^{24} a_i x_i$ (B.20)

Subject to:

$$g_i(\mathbf{x}) = \frac{\left(x_i + x_{(i+12)}\right)}{\sum_{j=1}^{24} x_j + e_i} \leq 0, \quad i = 1,2,3$$

$$g_i(\mathbf{x}) = \frac{\left(x_{(i+3)} + x_{(i+15)}\right)}{\sum_{j=1}^{24} x_j + e_i} \leq 0, \quad i = 4,5,6$$

$$h_i(\mathbf{x}) = \frac{x_{(i+12)}}{b_{(i+12)} \sum_{j=13}^{24} \frac{x_j}{b_j}} - \frac{c_i x_i}{40 b_i \sum_{j=1}^{12} \frac{x_j}{b_j}} = 0, \quad i = 1,2,\ldots,12$$

$$h_{13}(\mathbf{x}) = \sum_{i=1}^{24} x_i - 1 = 0$$

$$h_{14}(\mathbf{x}) = \sum_{i=1}^{12} \frac{x_i}{d_i} + k \sum_{i=13}^{24} \frac{x_i}{b_i} - 1.671 = 0$$

where $0 \le x_i \le 10$ $(i = 1,2, \dots ,24)$, $k = (0.7302)(530)(\frac{14.7}{40})$ and the remaining data is demonstrated in Table B.2.

Table B.2 Data set for test problem g20

i	a_i	b_i	c_i	d_i	e_i
1	0.0693	44.094	123.7	31.244	0.1
2	0.0577	58.12	31.7	36.12	0.3
3	0.05	58.12	45.7	34.784	0.4
4	0.2	137.4	14.7	92.7	0.3
5	0.26	120.9	84.7	82.7	0.6
6	0.55	170.9	27.7	91.6	0.3
7	0.06	62.501	49.7	56.708	
8	0.1	84.94	7.1	82.7	
9	0.12	133.425	2.1	80.8	
10	0.18	82.507	17.7	64.517	
11	0.1	46.07	0.85	49.4	
12	0.09	60.097	0.64	49.1	
13	0.0693	44.094			
14	0.0577	58.12			
15	0.05	58.12			
16	0.2	137.4			
17	0.26	120.9			
18	0.55	170.9			
19	0.06	62.501			
20	0.1	84.94			
21	0.12	133.425			
22	0.18	82.507			
23	0.1	46.07			
24	0.09	60.097			

The optimum is at

$$\mathbf{x}^* = (1.28582343498528086e-18, 4.83460302526130664e-34, 0, 0, 6.3045$$

$9929660781851e-18$,$7.57192526201145068e-34$, $5.0335069837284043 7e-34$,$9.28268079616618064e-34$,0, $1.767233845255473 59e-17$,$3.556 86101822965701e-34$, $2.994138500834713 46e-34$, $0.158143376337580 827$, $2.296017741616998 33e-19$, $1.061069386110429 47e-18$, $1.319683 44319506391e-18$,0.530902525044209539,0, $2.891483102577735 35e-18$,$3.348921261806661 59e-18$,0, 0.310999974151577319,$5.41244666317 833561e-05$, $4.849931652469595 53e-16$) . This solution is a little infeasible and no feasible solution is found so far.

Test function $g21$

Minimize: $f(\mathbf{x}) = x_1$ (B.21)

Subject to:

$$g_1(\mathbf{x}) = -x_1 + 35x_2^{0.6} + 35x_3^{0.6} \leq 0,$$

$$h_1(\mathbf{x}) = -300x_3 + 7500x_5 - 7500x_6 - 25x_4x_5 + 25x_4x_6 + x_3x_4 = 0$$

$$h_2(\mathbf{x}) = 100x_2 + 155.365x_4 + 2500x_7 - x_2x_4 - 25x_4x_7 - 15536.5 = 0$$

$$h_3(\mathbf{x}) = -x_5 + \ln(-x_4 + 900) = 0$$

$$h_4(\mathbf{x}) = -x_6 + \ln(x_4 + 300) = 0$$

$$h_5(\mathbf{x}) = -x_7 + \ln(-2x_4 + 700) = 0$$

where $0 \leq x_1 \leq 1000,\ 0 \leq x_2, x_3 \leq 40, 100 \leq x_4 \leq 300, 6.3 \leq x_5 \leq 6.7,\ 5.9 \leq x_5 \leq 6.4$, and $4.5 \leq x_5 \leq 6.25$.

The optimum is at $\mathbf{x}^* = (193.724510070034967, 5.56944131553368433e-27,$ $17.3191887294084914, 100.047897801386839, 6.68445185362377892, 5.99$ $168428444264833, 6.21451648886070451)$ with $f(\mathbf{x}^*) = 193.724510070035$.

Test function $g22$

Minimize: $f(\mathbf{x}) = x_1$ (B.22)

Subject to:

$g_1(\mathbf{x}) = -x_1 + x_2^{0.6} + x_3^{0.6} + x_4^{0.6} \leq 0,$

$h_1(\mathbf{x}) = x_5 - 100000x_8 + 1 \times 10^7 = 0$

$h_2(\mathbf{x}) = x_6 + 100000x_8 - 100000x_9 = 0$

$h_3(\mathbf{x}) = x_7 + 100000x_9 - 5 \times 10^7 = 0$

$h_4(\mathbf{x}) = x_5 + 100000x_{10} - 3.3 \times 10^7 = 0$

$h_5(\mathbf{x}) = x_6 + 100000x_{11} - 4.4 \times 10^7 = 0$

$h_6(\mathbf{x}) = x_7 + 100000x_{12} - 6.6 \times 10^7 = 0$

$h_7(\mathbf{x}) = x_5 - 120x_2x_{13} = 0$

$h_8(\mathbf{x}) = x_6 - 80x_3x_{14} = 0$

$h_9(\mathbf{x}) = x_7 - 40x_4x_{15} = 0$

$h_{10}(\mathbf{x}) = x_8 - x_{11} + x_{16} = 0$

$h_{11}(\mathbf{x}) = x_9 - x_{12} + x_{17} = 0$

$h_{12}(\mathbf{x}) = -x_{18} + \ln(x_{10} - 100) = 0$

$$h_{13}(\mathbf{x}) = -x_{19} + \ln(-x_8 + 300) = 0$$

$$h_{14}(\mathbf{x}) = -x_{20} + \ln(x_{16}) = 0$$

$$h_{15}(\mathbf{x}) = -x_{21} + \ln(-x_9 + 400) = 0$$

$$h_{16}(\mathbf{x}) = -x_{22} + \ln(x_{17}) = 0$$

$$h_{17}(\mathbf{x}) = -x_8 - x_{10} + x_{13}x_{18} - x_{13}x_{19} + 400 = 0$$

$$h_{18}(\mathbf{x}) = x_8 - x_9 - x_{11} + x_{14}x_{20} - x_{14}x_{21} + 400 = 0$$

$$h_{19}(\mathbf{x}) = x_9 - x_{12} - 4.60517x_{15} + x_{15}x_{22} + 100 = 0$$

where $0 \le x_1 \le 20000,\ 0 \le x_2, x_3, x_4 \le 1 \times 10^6,\ 0 \le x_5, x_6, x_7 \le 4 \times 10^7$, $100 \le x_8 \le 299.99,\ 100 \le x_9 \le 399.99,\ 100.01 \le x_{10} \le 300, 100 \le x_{11} \le 400,\ 100 \le x_{12} \le 600,\ 0 \le x_{13}, x_{14}, x_{15} \le 500,\ 0.01 \le x_{16} \le 300,\ 0.01 \le x_{17} \le 400, -4.7 \le x_{18}, x_{19}, x_{20}\ x_{21}, x_{22} \le 6.25$

The optimum is at $\mathbf{x}^* = (236.430975504001054, 135.82847151732463, 204.818152544824585, 6446.54654059436416, 3007540.83940215595, 4074188.65771341929, 32918270.5028952882, 130.075408394314167, 170.817294970528621, 299.924591605478554, 399.258113423595205, 330.817294971142758, 184.51831230897065, 248.64670239647424, 127.658546694545862, 269.182627528746707, 160.000016724090955, 5.29788288102680571, 5.13529735903945728, 5.59531526444068827, 5.43444479314453499, 5.07517453535834395)$ with $f(\mathbf{x}^*) = 236.430975504001$.

Test function $g23$

Minimize: $f(\mathbf{x}) = -9x_5 - 15x_8 + 6x_1 + 16x_2 + 10(x_6 + x_7)$ (B.23)

Subject to:

$g_1(\mathbf{x}) = x_9x_3 + 0.02x_6 - 0.025x_5 \leq 0$

$g_2(\mathbf{x}) = x_9x_4 + 0.02x_7 - 0.015x_8 \leq 0,$

$h_1(\mathbf{x}) = x_1 + x_2 - x_3 - x_4 = 0$

$h_2(\mathbf{x}) = 0.03x_1 + 0.01x_2 - x_9(x_3 + x_4) = 0$

$h_3(\mathbf{x}) = x_3 + x_6 - x_5 = 0$

$h_4(\mathbf{x}) = x_4 + x_7 - x_8 = 0$

where $0 \leq x_1, x_2, x_6 \leq 300,\ 0 \leq x_3, x_5, x_7 \leq 100$,$0 \leq x_4, x_8 \leq 200,$ and $0.01 \leq x_9 \leq 0.03$.

The optimum is at $\mathbf{x}^* = (0.0051000000000259465, 99.9947000000000514, 9.01920162996045897e-18, 99.9999000000000535, 0.0001000000000270860$ $86, 2.75700683389584542e-14, 99.9999999999999574, 2000.01000001000$ $00100008)$ with $f(\mathbf{x}^*) = -400.055099999999584$.

Test function $g24$

Minimize: $f(\mathbf{x}) = -x_1 - x_2$ (B.24)

Subject to:

$g_1(\mathbf{x}) = -2x_1^4 + 8x_1^3 - 8x_1^2 + x_2 - 2 \leq 0$

$$g_2(\mathbf{x}) = -4x_1^4 + 32x_1^3 - 88x_1^2 + 96x_1 + x_2 - 36 \leq 0,$$

where $0 \leq x_1 \leq 3$, and $0 \leq x_2 \leq 4$. The feasible region consist two disconnected sub-regions.

The optimum is at $\mathbf{x}^* = (2.329520197477623.17849307411774)$ with $f(\mathbf{x}^*) = -5.50801327159536.$

APPENDIX C

BENCHMARK TEST FUNCTIONS FOR DYNAMIC OPTIMIZATION PROBLEMS

Test functions *MP1* [166]:

Moving Cone Peaks Benchmark Problem is a maximization problem which has components as moving competing cones with independently varying height, width and location formulated as:

$$f_{MP1}(\mathbf{x},t) = \max(B(\mathbf{x}), \max_{i=1,2,\ldots,M} P(\mathbf{x}, h_i(t), w_i(t), \mathbf{p}_i(t))) \quad \text{(C.1)}$$

where $B(\mathbf{x})$ is a time-invariant basis landscape and P is a function that defines cone-shaped peaks with M peaks whose height (h_i), width (w_i), and location ($\mathbf{p}_i$) are time-varying.

Test functions *DF2* [167]:

Time-Varying Gaussian Peaks Problem is a maximization problem that adopts independently varying-dimensional Gaussian peaks. Each peak's amplitude, center, and variance can be varied independently, formulated as:

$$f_{DF2}(\mathbf{x},t) = \max_{i=1,2...,N}[A_i(t)Exp(\frac{-d(\mathbf{x},C_i(t))^2}{2\sigma_i^2(t)})] \quad \text{(C.2)}$$

where $A_i(t)$, $C_i(t)$ and $\sigma_i(t)$ are the amplitude, the center and width of the *i*-th peak ($i = 1,2,..., N$) in the *M*-dimensional Gaussian peak, respectively.

Test functions *DF3* [168-169]:

Moving Parabola with Linear Translation is formulated as:

$$f_{DF3}(\mathbf{x},t) = \min\sum_{i=1}^{M}(x_i + \delta_i(t))^2 \qquad \delta_i(t) = \begin{cases} 0, & t = 0 \\ \delta_i(t-1) + s, & t \neq 0 \end{cases} \qquad i = 1,2,..., M \quad \text{(C.3)}$$

Test functions *DF4* [168-169]:

Moving Parabola with Random Dynamics is described as:

$$f_{DF4}(\mathbf{x},t) = \min\sum_{i=1}^{M}(x_i + \delta_i(t))^2 , \delta_i(t) = \begin{cases} 0, & t = 0 \\ \delta_i(t-1) + s \times N_i(0,1), & t \neq 0 \end{cases} \quad i = 1,2,..., M \quad \text{(C.4)}$$

Test functions *DF5* [168-169]:

Moving Parabola with Circular Dynamics is expressed as:

$$f_{DF5}(\mathbf{x},t)=\min\sum_{i=1}^{M}(x_i+\delta_i(t))^2 \tag{C.5}$$

$$\delta_i(t)=\begin{cases}0, & t=0, i=\text{odd}\\ \delta_i(t-1)+s\times\sin(\frac{2\pi t}{\gamma}), & t\neq 0, i=\text{odd}\end{cases}\quad \delta_i(t)=\begin{cases}s, & t=0, i=\text{even}\\ \delta_i(t-1)+s\times\cos(\frac{2\pi t}{\gamma}), & t\neq 0, i=\text{even}\end{cases}$$

Test functions *DF6* [170]:

Oscillating Peaks Function is a maximization problem which is similar to the moving peaks function in that the landscape consists of l (usually l=2) landscapes generated by the moving peaks function. The problem oscillates between the l landscapes according to a cosine function formulated below. The parameters of each peak can independently vary.

$$f_i(t)=\min\omega(t)\,f_i(0)\qquad \omega(t)=\frac{1}{3}\cos(\frac{2t\pi}{steps}+2\pi\frac{i-1}{l})+\frac{2}{3},\qquad i=1,2,...,l \tag{C.6}$$

where *steps* defines the number of intermediate steps in one cycle.(*steps*=10).

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