

ANOMALY MEDIATED SUPERSYMMETRY BREAKING
AND NONSTANDARD NEUTRINO OSCILLATIONS

By

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CHAPTER 1

INTRODUCTION

In this section we give a brief description of the Standard Model of particle physics and reasons for going beyond it.

1.1 The Standard Model

The Standard Model (SM) of elementary particle physics has recorded remarkable success in describing physics at length scales ranging from atomic scales down to the shortest probed scale of about 10^{-18} m. It is a non-abelian gauge theory based on the gauge group [1]

$$SU(3)_C \times SU(2)_L \times U(1)_Y,$$

where $SU(3)_C$ is the color gauge group describing strong interactions and $SU(2)_L \times U(1)_Y$ is the electroweak gauge group describing weak and electromagnetic interactions.

The SM describes the interactions of quarks, leptons, gauge bosons and the Higgs boson. The field content and the transformation properties under the gauge symmetries are shown in Table 1.

It is important to note that the left- and the right-handed components of the matter fermions are assigned to different representations (doublets and singlets respectively) of the weak gauge group $SU(2)_L$, thereby allowing a chiral structure for the weak interactions.

The Yukawa and Higgs part of the SM Lagrangian is given by

$$\mathcal{L}_{Yukawa} = Y_{\alpha\beta}^{\ell} \ell_{\alpha} e_{\beta}^c \tilde{\phi} + Y_{\alpha\beta}^d Q_{\alpha} d_{\beta}^c \tilde{\phi} + Y_{\alpha\beta}^u Q_{\alpha} u_{\beta}^c \phi + \text{h.c.}, \quad (1.1)$$

where $\tilde{\phi} = i\sigma^2 \phi^* = \begin{pmatrix} \bar{\phi}^0 \\ -\phi^- \end{pmatrix}$. Here generation indices $\alpha, \beta = 1, 2, 3$ are explicitly displayed, while color and $SU(2)_L$ indices are suppressed.

Fields		$SU(3)_C$	$SU(2)_W$	$U(1)_Y$
Quarks	$Q_\alpha^i = \begin{pmatrix} u_\alpha^i \\ d_\alpha^i \end{pmatrix}$	3	2	$\frac{1}{6}$
	u_α^{ci}	$\bar{3}$	1	$-\frac{2}{3}$
	d_α^{ci}	$\bar{3}$	1	$\frac{1}{3}$
Leptons	$\ell_\alpha = \begin{pmatrix} \nu_\alpha \\ e_\alpha \end{pmatrix}$	1	2	$-\frac{1}{2}$
	e_α^c	1	1	2
Gluon	G_μ^a	8	1	0
Intermediate weak bosons	W_μ^r	1	3	0
Hypercharge gauge boson	B_μ	1	1	0
Higgs boson	$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	1	2	$\frac{1}{2}$

TABLE 1.1. Particle content of the SM and the charge assignment. Here $\alpha = 1, 2, 3$ is the generation index, $i = 1 - 3$ (color), $a = 1 - 8$ ($SU(3)_C$ generators) and $r = 1 - 3$ ($SU(2)_L$ generators).

1.2 Symmetry breaking via the Higgs mechanism

If we consider the $SU(2)_L \times U(1)_Y$ part of the Lagrangian, assuming that there is no Higgs field, all the fermions and the four gauge bosons (W_μ^r, B_μ) would be massless. This is unacceptable, for the weak interactions are short range, meaning that the mediators must be massive. We must then break the symmetry spontaneously which will ensure renormalizability. This is achieved through the scalar Higgs doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \quad (1.2)$$

The only observed unbroken local symmetry in Nature is the $U(1)_{em}$ (apart from $SU(3)_C$). Therefore the $SU(2)_L \times U(1)_Y$ symmetry should be broken down to $U(1)_{em}$. The renormalizable Higgs potential is given by

$$V_H \equiv \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2.$$

This has a minimum for $\mu^2 < 0$ at

$$\langle \phi^\dagger \phi \rangle = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2}. \quad (1.3)$$

We can choose the vacuum expectation value (VEV) after an $SU(2)_L$ transformation in the unitary gauge as

$$\langle \phi_0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \quad (1.4)$$

It is not difficult to see that the gauge boson associated with the $U(1)_{em}$ subgroup of $SU(2)_L \times U(1)_Y$ remains massless. The electric charge Q_{em} , the $U(1)_Y$ hypercharge and the third component of weak isospin T_{3L} are related by

$$Q_{em} = T_{3L} + \frac{Y}{2}, \quad (1.5)$$

and the gauge boson masses are given by

$$M_W = \frac{gv}{2}, \quad M_Z = \frac{M_W}{\cos \theta_W}, \quad M_A = 0. \quad (1.6)$$

Here g is the $SU(2)_L$ gauge coupling strength and $\tan \theta_W = g'/g$, where g' is the $U(1)_Y$ gauge coupling constant. These masses are obtained from the Lagrangian for the gauge and Higgs field, given by

$$L_{\text{gauge-Higgs}} = \left| \partial_\mu \phi - \frac{ig}{2} \vec{\tau} \cdot \vec{W}_\mu \phi - \frac{ig'}{2} B_\mu \phi \right|^2,$$

once the VEV of ϕ^0 is inserted.

It is worthwhile to note that the weak mixing angle θ_W is a parameter of the SM which has been measured to a very high accuracy. Another accurately measured quantity is the ρ parameter ($\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2 \theta_W}$) which is predicted to be 1 (at tree level) in the SM. New physics can also be severely constrained by the observed value of ρ .

After symmetry breaking, from the Yukawa interactions in Eq. (1.1), the fermions become massive with masses given by

$$M_u = Y_u v, \quad M_d = Y_d v, \quad M_\ell = Y_\ell v. \quad (1.7)$$

Here $Y_{u,d,\ell}$ are arbitrary 3×3 complex matrices in generation space.

Not all parameters in these matrices are observable in the SM. After fermion field redefinitions, the 3 eigenvalues of each of the matrices, 3 mixing angles and one phase entering in the charged W_μ^\pm interactions with quarks become physical quantities. One makes biunitary transformations, $U_L^u Y_u U_R^{u\dagger} = Y_u^{\text{diag}}$, $U_L^d Y_d U_R^{d\dagger} = Y_d^{\text{diag}}$, $U_L^\ell Y_\ell U_R^{\ell\dagger} = Y_\ell^{\text{diag}}$, in which case the charged W^\pm current takes the form

$$L_{\text{CC}}^{W_\mu^\pm} = \frac{g}{\sqrt{2}} \bar{u}_L \gamma_\mu V_{\text{CKM}} d_L W^{\mu+} + \text{h.c.},$$

where $V_{\text{CKM}} = U_L^{u\dagger} U_L^d$ is a unitary matrix, the Cabibbo–Kobayashi–Maskawa matrix or the quark mixing matrix.

Since there is no right-handed neutrino field ν_R , the neutrinos remain massless. The fermion masses are arbitrary since the Yukawa couplings Y are free parameters.

To find the Higgs boson mass, we write the complex field ϕ^0 in terms of real fields. The Higgs doublet then takes the form (in unitary gauge)

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \eta \end{pmatrix}, \quad (1.8)$$

where η is the physical Higgs scalar with mass

$$m_\eta^2 = 2\lambda v^2. \quad (1.9)$$

The Higgs mass is left undetermined since λ is a free parameter, with only its sign constrained to be positive.

There are several good features of the SM some of which are:

1. All the particles predicted by the SM have been observed except the Higgs boson.
2. Both baryon and lepton number are automatically conserved. This prevents rapid decay of the proton.
3. It has an extremely economical Higgs sector which is responsible for giving masses to all particles.
4. With only two independent parameters M_W and $\sin\theta_W$, all the electroweak processes at high energy are correctly described.

The SM also has several drawbacks. There are several free parameters in the SM Lagrangian: The Higgs coupling constant λ , the Higgs mass parameter μ^2 , three gauge couplings (g' , g , g_s), the number of generations (matter fields) and three Yukawa matrices $Y_{\alpha\beta}^u$, $Y_{\alpha\beta}^d$, $Y_{\alpha\beta}^\ell$. Despite the remarkable success of the SM, there are still several questions left unanswered. For example, does the Higgs boson exist? Do the gauge couplings unify? How is gravity incorporated?

An attempt to answer these numerous questions will take us to beyond the SM. For example, some earlier attempts tried to unify strong and electroweak forces by embedding the $SU(3)_C \times SU(2)_L \times U(1)_Y$ structure into higher groups such as $SU(5)$ and $SO(10)$. These ‘‘Grand Unified Theories’’ or GUT’s, were only partially successful.

Difficulties with the SM and GUT models concerning gauge hierarchy and fine tuning problems led to theoretical remedies such as technicolor, supersymmetry, string theory, etc. The most appealing of these theories is perhaps supersymmetry, which is the main focus of this thesis.

1.3 Gauge hierarchy problem

The hierarchy problem is one of the main reasons why we think supersymmetry has something to do with Nature, and that it might be broken at a scale comparable to the scale of weak interactions, rather than at some enormous energy such as the Planck scale $M_{Pl} \sim 10^{19}$ GeV. The mass hierarchy problem stems from the fact that masses, in particular scalar masses, are not stable to radiative corrections [2]. While fermion masses also receive radiative corrections from diagrams of the form in Figure 1.1, these are only logarithmically divergent (see for example [3]),

$$\delta m_f \simeq \frac{3\alpha}{4\pi} m_f \ln(\Lambda^2/m_f^2), \quad (1.10)$$

where Λ is an ultraviolet cutoff, where we expect new physics to play an important role. As one can see, even for $\Lambda \sim M_{Pl}$, these corrections are small, $\delta m_f \lesssim m_f$.

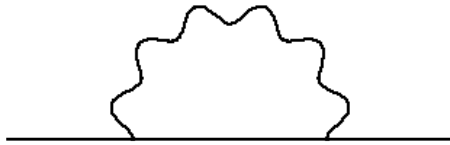


Figure 1.1. 1-loop correction to the mass of a fermion.

In contrast, scalar masses are quadratically divergent. 1-loop contributions to scalar masses, such as those shown in Fig. 1.2, are readily computed

$$\delta m_H^2 \simeq \{g_f^2, g^2, \lambda\} \int d^4k \frac{1}{k^2} \sim O\left(\frac{\alpha}{4\pi}\right) \Lambda^2, \quad (1.11)$$

due to contributions from fermion loops with coupling g_f , from gauge boson loops with coupling g^2 , and from quartic scalar-couplings λ .

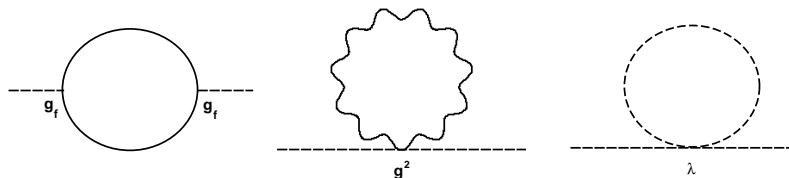


Figure 1.2. 1-loop corrections to a scalar mass.

An alternative and by far simpler solution to this problem exists if one postulates that there are new particles with similar masses and equal couplings to those responsible for the radiatively induced masses but with a difference (by a half unit) in spin. Then, because the contribution to δm_H^2 due to a fermion loop comes with a relative minus sign, the total contribution to the 1-loop corrected mass² is

$$\delta m_H^2 \simeq O\left(\frac{\alpha}{4\pi}\right) (\Lambda^2 + m_B^2) - O\left(\frac{\alpha}{4\pi}\right) (\Lambda^2 + m_F^2) = O\left(\frac{\alpha}{4\pi}\right) (m_B^2 - m_F^2). \quad (1.12)$$

If in addition, the bosons and fermions all have the same masses, then the radiative corrections vanish identically. The stability of the hierarchy only requires that the weak scale is preserved so that we need only require that

$$|m_B^2 - m_F^2| \lesssim 1 \text{ TeV}^2. \quad (1.13)$$

As we will see later, supersymmetry offers just the framework for including the necessary new particles and ensures the absence of these dangerous radiative corrections [4].

1.4 Gauge coupling unification

Another motivation for supersymmetry lies in the gauge coupling constant unification. In the SM, the gauge couplings do not unify. The solutions to the SM renormalization group equations to one loop accuracy are given by

$$\frac{1}{\alpha_i(Q)} = \frac{1}{\alpha_i(\mu)} + \frac{b_i}{2\pi} \log\left(\frac{\mu}{Q}\right),$$

where the b_i are

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + N_g \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} + N_h \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix}.$$

Here $N_g = 3$ is the number of generations and $N_h = 1$ is the number of Higgs doublets. The numerical values for the b_i coefficients are $b_i = (\frac{41}{10}, \frac{-19}{6}, -7)$. The three gauge coupling constants used as input are

$$\alpha_1 = 5\alpha/(3 \cos^2 \theta_W), \quad \alpha_2 = \alpha/\sin^2 \theta_W, \quad \alpha_3 = g_s^2/(4\pi),$$

where $\alpha^{-1}(M_Z) = 128.978$, $\sin^2 \theta_W = 0.23146$ and $\alpha_3 = 0.1184$.

On evolving the inverse of the three coupling constants as a function of logarithm of the unification scale Q , the result is shown in Fig. 3 (left). These couplings do not meet at a common point, hence unification does not occur. If we consider supersymmetric grand unified theory, the beta function coefficients are modified due to the quantum corrections involving the superpartners and are given in the Minimal Supersymmetric Standard Model (MSSM) by

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_g \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_h \begin{pmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{pmatrix}.$$

Here $N_g = 3$ and $N_h = 2$. The numerical value for b_i is $b_i = (\frac{33}{5}, 1, -3)$. If we assume that all the SUSY particle masses are around 1 TeV, on evolving the inverse coupling constants, they meet at a point (unify) as shown in Fig. 3 (right). The point at which these particles meet is around 10^{16} GeV. The SUSY particles are assumed to

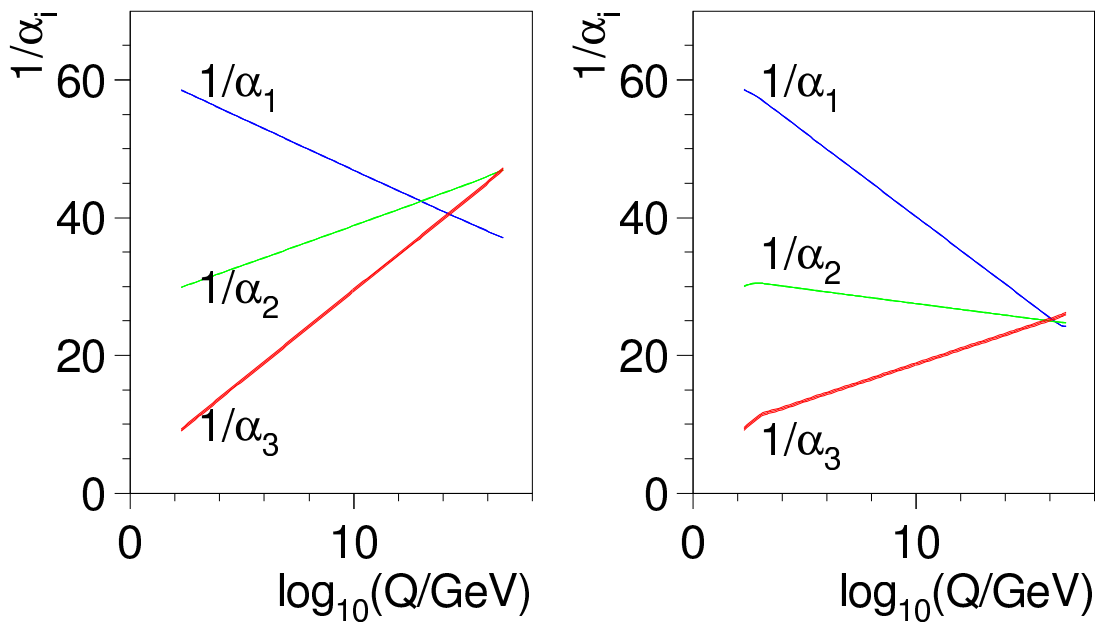


Figure 1.3. Running of the couplings in the SM (left) and its minimal supersymmetric version (right).

contribute only above the effective SUSY scale (~ 1 TeV) which causes the change of slope in the evolution of the couplings. This is another reason why most high energy physicist believe in supersymmetry.

The present thesis contains nine chapters. In the second chapter we review all the basics for Supersymmetry (SUSY), we define the SUSY algebra and introduce all the tools needed to write down the supersymmetric version of gauge field theories. In chapter 3, the minimal supersymmetric extension of the Standard Model is introduced, all the interactions and relevant mass matrices for our analysis are studied. In the fourth chapter we review various symmetry breaking models, here we introduce the Anomaly Mediated Supersymmetry Breaking (AMSB) and review the relevant literature. In chapter 5, we suggest TeV–Scale horizontal symmetry as a solution to the negative slepton mass squared problem of AMSB. In chapter 6, we suggest an $SU(2)_H$ model as a solution to the negative slepton mass problem. In chapter 7, we study a specific Z' model as a solution to the slepton mass problem of AMSB.

In Chapter 8, we suggest another model to solve this problem of AMSB with the quarks and the leptons transforming identically under two different $SU(3)$ symmetry group. Finally, we divert from the AMSB to Neutrino Physics, here we suggest a non-standard neutrino interaction as a solution to the neutrino oscillation problem.

CHAPTER 2

SUPERSYMMETRY

Supersymmetry (SUSY) is often called the last great symmetry of Nature. Rarely has so much effort, both theoretical and experimental, been spent to understand and discover a symmetry of Nature, which up to the present time lacks concrete evidence.

Why SUSY? If for no other reason, it would be nice to understand the origin of the fundamental difference between the two classes of particles distinguished by their spin, fermions and bosons. If such a symmetry exists, one might expect that it is represented by an operator which relates the two classes of particles. For example,

$$\begin{aligned} Q|\text{Boson}\rangle &= |\text{Fermion}\rangle, \\ Q|\text{Fermion}\rangle &= |\text{Boson}\rangle. \end{aligned} \tag{2.1}$$

However, without a connection to experiment, SUSY would remain a mathematical curiosity and a subject of a very theoretical nature as indeed it stood from its initial description in the early 1970's [5, 6] until its incorporation into a realistic theory of physics at the electroweak scale.

One of the first break-throughs came with the realization that SUSY could help resolve the difficult problem of mass hierarchies [2], namely the stability of the electroweak scale with respect to radiative corrections. With precision experiments at the electroweak scale, it has also become apparent that Grand Unification is not possible in the absence of SUSY [7].

Considering a new class of “fermionic” generators Q , that satisfy anti-commutation relations

$$[Q_\alpha, J^{\mu\nu}] = i\sigma_\alpha^{\mu\nu\beta} Q_\beta,$$

$$\begin{aligned}
[Q_\alpha, P^\mu] &= 0, \\
[\bar{Q}^{\dot{\alpha}}, J^{\mu\nu}] &= i\bar{\sigma}^{\mu\nu\dot{\alpha}\dot{\beta}}\bar{Q}^{\dot{\beta}}, \\
[\bar{Q}^{\dot{\alpha}}, P^\mu] &= 0,
\end{aligned} \tag{2.2}$$

where Q_α ($\bar{Q}^{\dot{\alpha}}$) is a symmetry operator (SUSY charge), P^μ is the energy–momentum operator and $J^{\mu\nu}$ is the angular momentum operator.

The Q 's are translationally invariant (no explicit x -dependence) and they satisfy anti–commutation relations

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma_{\alpha\dot{\beta}}^\mu P_\mu, \tag{2.3}$$

where the factor 2 is conventional and can be achieved by re–scaling the Q 's. There are three main properties of a supermultiplet: (1) All particles belonging to an irreducible representation of SUSY have the same mass, (2) there are equal number of fermionic (N_F) and bosonic (N_B) degrees of freedom in a supermultiplet, (3) the energy P_0 in a supersymmetric theory is always positive.

2.1 Supersymmetry algebra

Combined with the usual Poincaré and internal symmetry algebra the Super-Poincaré Lie algebra contains additional SUSY generators Q_α^i and $\bar{Q}_{\dot{\alpha}}^i$ [8]

$$\begin{aligned}
[P_\mu, P_\nu] &= 0, \\
[P_\mu, M_{\rho\sigma}] &= i(g_{\mu\rho}P_\sigma - g_{\mu\sigma}P_\rho), \\
[M_{\mu\nu}, M_{\rho\sigma}] &= i(g_{\nu\rho}M_{\mu\sigma} - g_{\nu\sigma}M_{\mu\rho} - g_{\mu\rho}M_{\nu\sigma} + g_{\mu\sigma}M_{\nu\rho}), \\
[B_r, B_s] &= iC_{rs}^t B_t, \\
[B_r, P_\mu] &= [B_r, M_{\mu\sigma}] = 0, \\
[Q_\alpha^i, P_\mu] &= [\bar{Q}_{\dot{\alpha}}^i, P_\mu] = 0, \\
[Q_\alpha^i, M_{\mu\nu}] &= \frac{1}{2}(\sigma_{\mu\nu})_\alpha^\beta Q_\beta^i, \quad [\bar{Q}_{\dot{\alpha}}^i, M_{\mu\nu}] = -\frac{1}{2}\bar{Q}_{\dot{\beta}}^i(\bar{\sigma}_{\mu\nu})_{\dot{\alpha}}^{\dot{\beta}}, \\
[Q_\alpha^i, B_r] &= (b_r)^i_j Q_\alpha^j, \quad [\bar{Q}_{\dot{\alpha}}^i, B_r] = -\bar{Q}_{\dot{\alpha}}^j (b_r)^i_j, \\
\{Q_\alpha^i, \bar{Q}_{\dot{\beta}}^j\} &= 2\delta^{ij}(\sigma^\mu)_{\alpha\dot{\beta}} P_\mu, \\
\{Q_\alpha^i, Q_\beta^j\} &= 2\epsilon_{\alpha\beta} Z^{ij}, \quad Z_{ij} = a_{ij}^r b_r, \quad Z^{ij} = Z_{ij}^+, \\
\{\bar{Q}_{\dot{\alpha}}^i, \bar{Q}_{\dot{\beta}}^j\} &= -2\epsilon_{\dot{\alpha}\dot{\beta}} Z^{ij}, \quad [Z_{ij}, \text{anything}] = 0, \\
\alpha, \dot{\alpha} &= 1, 2 \quad i, j = 1, 2, \dots, N.
\end{aligned} \tag{2.4}$$

Here P_μ and $M_{\mu\nu}$ are four-momentum and angular momentum operators, respectively, B_r are the internal symmetry generators, Q^i and \bar{Q}^i are the spinorial SUSY generators and Z_{ij} are the so-called central charges, while $\alpha, \dot{\alpha}, \beta, \dot{\beta}$ are the spinorial indices. In the simplest case one has one spinor generator Q_α (and the conjugated one $\bar{Q}_{\dot{\alpha}}$) that corresponds to an ordinary or N=1 SUSY. When $N > 1$ one has an extended SUSY.

The constraint on the number of SUSY generators comes from a requirement of consistency of the corresponding quantum field theory (QFT). The number of supersymmetries and the maximal spin of the particle in the multiplet are related by

$$N \leq 4S,$$

where S is the maximal spin. Since the theories with spin greater than 1 are non-renormalizable and the theories with spin greater than 5/2 have no consistent coupling to gravity, this imposes a constraint on the number of SUSY generators

$$\begin{aligned} N &\leq 4 && \text{for renormalizable theories (YM),} \\ N &\leq 8 && \text{for (super)gravity.} \end{aligned}$$

In what follows, we shall consider simple SUSY, or $N = 1$ SUSY, contrary to extended supersymmetries with $N > 1$. In this case, one has two types of supermultiplets: the so-called chiral multiplet, which contains two physical states (ϕ, ψ) with spin 0 and 1/2, respectively, and the vector multiplet with $\lambda = 1/2$, which also contains two physical states (λ, A_μ) with spin 1/2 and 1, respectively.

2.2 Superspace and superfields

An elegant formulation of SUSY transformations and invariants can be achieved in the framework of superspace [9]. Superspace differs from the ordinary Euclidean (Minkowski) space by the addition of two new coordinates, θ_α and $\bar{\theta}_{\dot{\alpha}}$, which are Grassmannian, i.e. anticommuting, variables

$$\{\theta_\alpha, \theta_\beta\} = 0, \quad \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = 0, \quad \theta_\alpha^2 = 0, \quad \bar{\theta}_{\dot{\alpha}}^2 = 0, \quad \alpha, \beta, \dot{\alpha}, \dot{\beta} = 1, 2.$$

Thus, we go from space to superspace

$$\begin{array}{ccc} \text{Space} & \Rightarrow & \text{Superspace} \\ x_\mu & & x_\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}} \end{array}$$

A SUSY group element can be constructed in superspace in the same way as an ordinary translation in the usual space

$$G(x, \theta, \bar{\theta}) = e^{i(-x^\mu P_\mu + \theta Q + \bar{\theta} \bar{Q})}. \quad (2.5)$$

It leads to a supertranslation in superspace

$$\begin{aligned} x_\mu &\rightarrow x_\mu + i\theta\sigma_\mu\bar{\varepsilon} - i\varepsilon\sigma_\mu\bar{\theta}, \\ \theta &\rightarrow \theta + \varepsilon, \\ \bar{\theta} &\rightarrow \bar{\theta} + \bar{\varepsilon}, \end{aligned} \quad (2.6)$$

where ε and $\bar{\varepsilon}$ are Grassmannian transformation parameters. From Eq. (2.6) one can easily obtain the representation for the supercharges Eq. (2.4) acting on the superspace

$$Q_\alpha = \frac{\partial}{\partial\theta_\alpha} - i\sigma_{\alpha\dot{\alpha}}^\mu\bar{\theta}^{\dot{\alpha}}\partial_\mu, \quad \bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} + i\theta_\alpha\sigma_{\alpha\dot{\alpha}}^\mu\partial_\mu. \quad (2.7)$$

Taking the Grassmannian transformation parameters to be local, or space-time dependent, one gets a local translation. As has already been mentioned, this leads to a theory of (super) gravity. To define the fields on a superspace, consider representations of the Super-Poincaré group Eq. (2.4) [10]. The simplest one is a scalar superfield $F(x, \theta, \bar{\theta})$ which is SUSY invariant. Its Taylor expansion in θ and $\bar{\theta}$ has only several terms due to the nilpotent character of Grassmannian parameters. However, this superfield is a reducible representation of SUSY. To get an irreducible one, we define a *chiral* superfield which obeys the equation

$$\bar{D}F = 0, \quad \text{where } \bar{D} = -\frac{\partial}{\partial\bar{\theta}} - i\theta\sigma^\mu\partial_\mu \quad (2.8)$$

is a superspace covariant derivative. In superspace (by Taylor expanding $y = x + i\theta\sigma\bar{\theta}$), a chiral superfield is written as

$$\begin{aligned} \Phi(y, \theta) &= A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \\ &= A(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu A(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\square A(x) \\ &+ \sqrt{2}\theta\psi(x) - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \theta\theta F(x). \end{aligned} \quad (2.9)$$

Here A is a complex scalar field (with two bosonic degrees of freedom), ψ is a Weyl spinor field (with 2 fermionic degrees of freedom) and F is the auxiliary field (with no physical meaning) which is needed to close the SUSY algebra (2.4). We see from here that a superfield contains an equal number of fermionic and bosonic degrees of freedom. Under a SUSY transformation with anticommuting parameter ε , the component fields transform as

$$\begin{aligned}\delta_\varepsilon A &= \sqrt{2}\varepsilon\psi, \\ \delta_\varepsilon\psi &= i\sqrt{2}\sigma^\mu\bar{\varepsilon}\partial_\mu A + \sqrt{2}\varepsilon F, \\ \delta_\varepsilon F &= i\sqrt{2}\bar{\varepsilon}\sigma^\mu\partial_\mu\psi.\end{aligned}\tag{2.10}$$

The antichiral superfield Φ^+ obey the equation

$$D\Phi^+ = 0, \quad \text{with } D = \frac{\partial}{\partial\theta} + i\sigma^\mu\bar{\theta}\partial_\mu.$$

The product of chiral (antichiral) superfields Φ^2, Φ^3 , etc., is also a chiral (antichiral) superfield, while the product of chiral and antichiral ones $\Phi^+\Phi$ is a general superfield.

For any arbitrary function of chiral superfields one has

$$\begin{aligned}\mathcal{W}(\Phi_i) &= \mathcal{W}(A_i + \sqrt{2}\theta\psi_i + \theta\theta F) \\ &= \mathcal{W}(A_i) + \frac{\partial\mathcal{W}}{\partial A_i}\sqrt{2}\theta\psi_i + \theta\theta\left(\frac{\partial\mathcal{W}}{\partial A_i}F_i - \frac{1}{2}\frac{\partial^2\mathcal{W}}{\partial A_i\partial A_j}\psi_i\psi_j\right).\end{aligned}\tag{2.11}$$

The \mathcal{W} is usually referred to as a superpotential which replaces the usual potential for the scalar fields. The vector superfield satisfies the condition $V = V^+$. They should be understood in terms of their power series expansion in θ and $\bar{\theta}$ as

$$\begin{aligned}V(x, \theta, \bar{\theta}) &= C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) \\ &+ \frac{i}{2}\theta\theta[M(x) + iN(x)] - \frac{i}{2}\bar{\theta}\bar{\theta}[M(x) - iN(x)] \\ &- \theta\sigma^\mu\bar{\theta}v_\mu(x) + i\theta\theta\bar{\theta}[\lambda(x) + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi(x)] \\ &- i\bar{\theta}\bar{\theta}\theta[\lambda + \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}(x)] + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}[D(x) + \frac{1}{2}\square C(x)].\end{aligned}\tag{2.12}$$

The component fields C, D, M, N and v_μ must be real for Eq. (2.12) to satisfy $V = V^+$. These vector supermultiplet contains 8 bosonic degrees of freedom (one

each for C , D , M , M , N and four from the real vector field v_μ) and 8 fermionic degrees of freedom (from the two component spinors χ and λ). The physical degrees of freedom corresponding to a real vector superfield V are the vector gauge field v_μ and the Majorana spinor field λ . All other components are unphysical and can be eliminated. We now define the supersymmetric generalization of an Abelian gauge transformation of the superfield V as

$$V \rightarrow V + \Phi + \Phi^+,$$

where Φ and Φ^+ are some chiral superfields. Under this transformation, the components transform as

$$\begin{aligned} C &\rightarrow C + A + A^*, \\ \chi &\rightarrow \chi - i\sqrt{2}\psi, \\ M + iN &\rightarrow M + iN - 2iF, \\ v_\mu &\rightarrow v_\mu - i\partial_\mu(A - A^*), \\ \lambda &\rightarrow \lambda, \\ D &\rightarrow D. \end{aligned} \tag{2.13}$$

We see that there is a special gauge known as the Wess-Zumino gauge [11] in which C , χ , M and N are all zero. Fixing this gauge breaks SUSY but still allows the usual gauge transformation $v_\mu \rightarrow v_\mu + \partial_\mu A$. In this gauge, the vector multiplet reduces to 4 bosonic degrees of freedom (1 for D and the three remaining components of v_μ) and 4 fermionic degrees of freedom (from the Majorana spinor λ). In this gauge the vector superfield takes the form

$$\begin{aligned} V &= -\theta\sigma^\mu\bar{\theta}v_\mu(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x), \\ V^2 &= -\frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}v_\mu(x)v^\mu(x), \\ V^3 &= 0, \\ V^n &= 0 \quad \text{for } n \geq 3. \end{aligned} \tag{2.14}$$

One can define also a field strength tensor (as analog of $F_{\mu\nu}$ in gauge theories)

$$W_\alpha = -\frac{1}{4}\bar{D}^2 e^V D_\alpha e^{-V},$$

$$\bar{W}_{\dot{\alpha}} = -\frac{1}{4}D^2 e^V \bar{D}_{\alpha} e^{-V}, \quad (2.15)$$

which is a polynomial in the Wess-Zumino gauge. (Here D s are the supercovariant derivatives.) The strength tensor is a chiral superfield

$$\bar{D}_{\dot{\beta}} W_{\alpha} = 0, \quad D_{\beta} \bar{W}_{\dot{\alpha}} = 0.$$

In the Wess-Zumino gauge it is a polynomial over component fields:

$$W_{\alpha} = T^a \left(-i\lambda_{\alpha}^a + \theta_{\alpha} D^a - \frac{i}{2} (\sigma^{\mu} \bar{\sigma}^{\nu} \theta)_{\alpha} F_{\mu\nu}^a + \theta^2 \sigma^{\mu} D_{\mu} \bar{\lambda}^a \right), \quad (2.16)$$

where

$$F_{\mu\nu}^a = \partial_{\mu} v_{\nu}^a - \partial_{\nu} v_{\mu}^a + g f^{abc} v_{\mu}^b v_{\nu}^c, \quad D_{\mu} \bar{\lambda}^a = \partial \bar{\lambda}^a + g f^{abc} v_{\mu}^b \bar{\lambda}^c.$$

In Abelian case eqs.(2.15) are simplified and take form

$$W_{\alpha} = -\frac{1}{4} \bar{D}^2 D_{\alpha} V, \quad \bar{W}_{\dot{\alpha}} = -\frac{1}{4} D^2 \bar{D}_{\dot{\alpha}} V.$$

2.3 Supersymmetric Action

Using the rules of Grassmannian integration:

$$\int d\theta_{\alpha} = 0 \quad \int \theta_{\alpha} d\theta_{\beta} = \delta_{\alpha\beta}$$

we can define the general form of a SUSY and gauge invariant Lagrangian as [10]:

$$\begin{aligned} \mathcal{L}_{SUSY}^{YM} &= \frac{1}{4} \int d^2\theta \operatorname{Tr}(W^{\alpha} W_{\alpha}) + \frac{1}{4} \int d^2\bar{\theta} \operatorname{Tr}(\bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}}) \\ &+ \int d^2\theta d^2\bar{\theta} \Phi_{ia}^{\dagger} (e^{gV})_b^a \Phi_i^b + \int d^2\theta \mathcal{W}(\Phi_i) + \int d^2\bar{\theta} \bar{\mathcal{W}}(\bar{\Phi}_i) \end{aligned} \quad (2.17)$$

Φ_i are chiral superfields which transform as:

$$\Phi_i \rightarrow e^{-ig\Lambda} \Phi_i$$

and

$$e^{gV} \rightarrow e^{ig\Lambda^{\dagger}} e^{gV} e^{-ig\Lambda}$$

where both Λ and V are matrices:

$$\Lambda_{ij} = \tau_{ij}^a \Lambda_a, \quad V_{ij} = \tau_{ij}^a V_a,$$

with τ^a the gauge generators. The supersymmetric field strength W_α is equal to

$$\mathcal{W}_\alpha = -\frac{1}{4} \bar{D} \bar{D} e^{-V} D_\alpha e^V$$

and transforms as: $\mathcal{W} \rightarrow e^{-i\Lambda} \mathcal{W} e^{i\Lambda}$.

\mathcal{W} is the superpotential, which should be invariant under the group of symmetries of a particular model. In terms of component fields the above Lagrangian takes the form [12]

$$\begin{aligned} \mathcal{L}_{SUSY}^{YM} &= -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - i\lambda^a \sigma^\mu D_\mu \bar{\lambda}^a + \frac{1}{2} D^a D^a \\ &+ (\partial_\mu A_i - igv_\mu^a \tau^a A_i)^\dagger (\partial_\mu A_i - igv^{a\mu} \tau^a A_i) - i\bar{\psi}_i \bar{\sigma}^\mu (\partial_\mu \psi_i - igv^{a\mu} \tau^a \psi_i) \\ &- D^a A_i^\dagger \tau^a A_i - i\sqrt{2} A_i^\dagger \tau^a \lambda^a \psi_i + i\sqrt{2} \bar{\psi}_i \tau^a A_i \bar{\lambda}^a + F_i^\dagger F_i \\ &+ \frac{\partial \mathcal{W}}{\partial A_i} F_i + \frac{\partial \bar{\mathcal{W}}}{\partial A_i^\dagger} F_i^\dagger - \frac{1}{2} \frac{\partial^2 \mathcal{W}}{\partial A_i \partial A_j} \psi_i \psi_j - \frac{1}{2} \frac{\partial^2 \bar{\mathcal{W}}}{\partial A_i^\dagger \partial A_j^\dagger} \bar{\psi}_i \bar{\psi}_j \end{aligned} \quad (2.18)$$

Integrating out the auxiliary fields D^a and F_i , one reproduces the usual Lagrangian. Contrary to the SM, where the scalar Higgs potential is arbitrary and is defined only by the requirement of the gauge invariance, in supersymmetric theories it is completely defined by the superpotential. It consists of the contributions from the D -terms and F -terms. The kinetic energy of the gauge fields yields the $\frac{1}{2} D^a D^a$ term, and the matter-gauge interaction yields the $g D^a \tau_{ij}^a A_i^* A_j$ one. Together they give

$$\mathcal{L}_D = \frac{1}{2} D^a D^a + g D^a \tau_{ij}^a A_i^* A_j. \quad (2.19)$$

The equation of motion reads

$$D^a = -g \tau_{ij}^a A_i^* A_j, \quad (2.20)$$

Substituting it back into Eq. (2.19) yields the D -term part of the potential

$$\mathcal{L}_D = -\frac{1}{2} D^a D^a \quad \Longrightarrow \quad V_D = \frac{1}{2} D^a D^a, \quad (2.21)$$

where D is given by Eq. (2.20). The F -term contribution can be derived from the matter field self-interaction. For a general type superpotential \mathcal{W} one has

$$\mathcal{L}_F = F_i^* F_i + \left(\frac{\partial \mathcal{W}}{\partial A_i} F_i + h.c. \right) \quad (2.22)$$

Using the equations of motion for the auxiliary field F_i

$$F_i^* = -\frac{\partial \mathcal{W}}{\partial A_i} \quad (2.23)$$

yields

$$\mathcal{L}_F = -F_i^* F_i, \quad \implies \quad V_F = F_i^* F_i, \quad (2.24)$$

where F is given by Eq. (2.23). The full potential is the sum of the two contributions

$$V = V_D + V_F. \quad (2.25)$$

Thus, the form of the Lagrangian is constrained by symmetry requirements. The only freedom is the field content, the value of the gauge coupling g , Yukawa couplings y_{ijk} and the masses. Because of the renormalizability constraint $V \leq A^4$ the superpotential should be limited by $\mathcal{W} \leq \Phi^3$. All members of a supermultiplet have the same masses, i.e. bosons and fermions are degenerate in mass. This property of SUSY theories contradicts phenomenology and requires SUSY breaking.

2.4 SUSY breaking

Since the SUSY algebra leads to mass degeneracy in a supermultiplet, it should be broken to explain the absence of superpartners at accessible energies. There are several ways of SUSY breaking. It can be broken either explicitly or spontaneously. In performing SUSY breaking one has to be careful not to spoil the cancellation of quadratic divergencies which allows one to solve the *hierarchy problem*. This is achieved by spontaneous breaking of SUSY. It is possible to show that in SUSY models the energy is always nonnegative definite. According to quantum mechanics the energy is equal to

$$E = \langle 0 | \hat{H} | 0 \rangle, \quad (2.26)$$

where \widehat{H} is the Hamiltonian and due to the SUSY algebra

$$\{Q_\alpha, \bar{Q}_\beta\} = 2(\sigma^\mu)_{\alpha\dot{\beta}} P_\mu. \quad (2.27)$$

Taking into account that $Tr(\sigma^\mu P_\mu) = 2P_0$ one gets

$$E = \frac{1}{4} \sum_{\alpha=1,2} \langle 0 | \{Q_\alpha, \bar{Q}_\alpha\} | 0 \rangle = \frac{1}{4} \sum_{\alpha} \|Q_\alpha | 0 \rangle\|^2 \geq 0. \quad (2.28)$$

Hence

$$E = \langle 0 | \widehat{H} | 0 \rangle \neq 0 \quad \text{if and only if} \quad Q_\alpha | 0 \rangle \neq 0.$$

Therefore, SUSY is spontaneously broken, i.e. the vacuum is not invariant under Q ($Q_\alpha | 0 \rangle \neq 0$), *if and only if* the minimum of the potential is positive (*i.e.* $E \geq 0$). Spontaneous breaking of SUSY is achieved in the same way as electroweak symmetry breaking. One introduces a field whose vacuum expectation value is nonzero and breaks the symmetry. However, due to the special character of SUSY, this should be a superfield whose auxiliary F or D component acquires nonzero VEVs. Thus, among possible spontaneous SUSY breaking mechanisms one distinguishes the F -type breaking and the D -type breaking.

i) Fayet-Iliopoulos (D -term) mechanism [12].

In this case the, the linear D -term is added to the Lagrangian

$$\Delta\mathcal{L} = \xi V|_{\theta\theta\bar{\theta}\bar{\theta}} = \xi \int d^2\theta d^2\bar{\theta} V. \quad (2.29)$$

It is $U(1)$ gauge and SUSY invariant by itself, however, it may lead to spontaneous breaking of both of them depending on the value of ξ . The drawback of this mechanism is the necessity of $U(1)$ gauge invariance. It can be used in SUSY generalizations of the SM but not in GUTs. The mass spectrum also causes some troubles since the following sum rule is always valid

$$S\text{Tr}\mathcal{M}^2 = \sum_J (-1)^{2J} (2J+1) m_J^2 = 0, \quad (2.30)$$

which is bad for phenomenology.

ii) O’Raifeartaigh (F -term) mechanism [12].

In this case, several chiral fields are needed and the superpotential should be chosen in such way that trivial zero VEVs for the auxiliary F -fields are forbidden. For instance, choosing the superpotential to be

$$\mathcal{W}(\Phi) = \lambda\Phi_3 + m\Phi_1\Phi_2 + g\Phi_3\Phi_1^2, \quad (2.31)$$

one gets the equations for the auxiliary fields

$$F_1^* = mA_2 + 2gA_1A_3, \quad (2.32)$$

$$F_2^* = mA_1, \quad (2.33)$$

$$F_3^* = \lambda + gA_1^2, \quad (2.34)$$

which have no solutions with $\langle F_i \rangle = 0$ and SUSY is spontaneously broken. The drawback of this mechanism is that there is a lot of arbitrariness in the choice of the potential. The sum rule (2.30) is also valid here. Unfortunately, none of these mechanisms explicitly works in SUSY generalizations of the SM. None of the fields of the SM can develop nonzero VEVs for their F or D components without breaking $SU(3)_C$ or $U(1)_Y$ gauge invariance since they are not singlets with respect to these groups. This requires the presence of extra sources for spontaneous SUSY breaking [13–18].

CHAPTER 3

THE MINIMAL SUPERSYMMETRIC STANDARD MODEL

The Minimal Supersymmetric Standard Model (MSSM) [19] respects the same gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ as does the SM. Here SUSY is somehow (softly) broken at the weak scale. The MSSM is the simplest phenomenologically viable supersymmetric theory beyond the SM in that it contains the fewest number of new particles and new interactions.

To construct the MSSM [20] we start with the complete set of chiral fermions, and add a scalar superpartner to each Weyl fermion so that each field represents a chiral multiplet. Similarly we must add a gaugino for each of the gauge bosons in the SM making up the gauge multiplets. The particles necessary to construct the MSSM are shown in Tables 3.1. and 3.2.

Superfield	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Particle Content
\hat{Q}	3	2	$\frac{1}{6}$	$(u_L, d_L), (\tilde{u}_L, \tilde{d}_L)$
\hat{U}^c	$\bar{3}$	1	$-\frac{2}{3}$	\bar{u}_R, \tilde{u}_R^*
\hat{D}^c	$\bar{3}$	1	$\frac{1}{3}$	\bar{d}_R, \tilde{d}_R^*
\hat{L}	1	2	$-\frac{1}{2}$	$(\nu_L, e_L), (\tilde{\nu}_L, \tilde{e}_L)$
\hat{E}^c	1	1	1	\bar{e}_R, \tilde{e}_R^*
\hat{H}_d	1	2	$-\frac{1}{2}$	(H_d, \tilde{H}_d)
\hat{H}_u	1	2	$\frac{1}{2}$	(H_u, \tilde{H}_u)

TABLE 3.1. Chiral superfields of the MSSM.

The MSSM is defined by its minimal field content (which accounts for the known SM fields) and minimal superpotential necessary to account for the known Yukawa

Superfield	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Particle Content
\hat{G}^a	8	1	0	G^μ, \tilde{g}^μ
\hat{W}^r	1	3	0	$W_r^\mu, \tilde{\omega}_r^\mu$
\hat{B}	1	1	0	B^μ, \tilde{b}^μ

TABLE 3.2. Vector Superfields of the MSSM.

mass terms. Notice that in Table 3.1. and 3.2., we have introduced a partner for every particle of the SM with the same internal quantum number and a spin differing by $\frac{1}{2}$.

We define the MSSM by the superpotential

$$W = \epsilon_{ij}[y_e H_d^j L^i e^c + y_d H_d^j Q^i d^c + y_u H_u^i Q^j u^c + \mu H_d^i H_u^j]. \quad (3.1)$$

Here, the indices, $\{ij\}$, are $SU(2)_L$ doublet indices and μ is the Higgs mass parameter. The Yukawa couplings, y , are all 3×3 matrices in generation space. Note that there is no generation index for the Higgs multiplets. Color and generation indices have been suppressed in the above expression. There are two Higgs doublets in the MSSM. This is a necessary addition to the SM which can be seen as arising from the holomorphic property of the superpotential. That is, there would be no way to account for all of the Yukawa terms for both up-type and down-type multiplets with a single Higgs doublet. To avoid a massless Higgs state, a mixing term $\epsilon_{ij}\mu H_d^i H_u^j$ must be added to the superpotential.

However, even if we stick to the minimal field content, there are several other superpotential terms which we can envision adding to Eq. (3.1) since they are consistent with all of the symmetries of the theory. We could have considered terms like

$$W_R = \mu'^i L_i H_u + \lambda^{ijk} L_i L_j e_k^c + \lambda'^{ijk} L_i Q_j d_k^c + \lambda''^{ijk} u_i^c d_j^c d_k^c, \quad (3.2)$$

where i, j and k are the generation indices and λ 's are the coupling constants.

In Eq. (3.2), the terms proportional to λ, λ' , and μ' , all violate lepton number by one unit. The term proportional to λ'' violates baryon number by one unit.

Each of the terms in Eq. (3.2) predicts new particle interactions and can be to some extent constrained by the lack of observed exotic phenomena. However, the combination of terms which violate both baryon and lepton number can be disastrous.

In order to avoid these unwanted terms, we impose a discrete symmetry on the theory called R -parity [21], which can be defined as

$$R = (-1)^{3B+L+2s}, \quad (3.3)$$

where B , L , and s are the baryon number, lepton number, and spin respectively. With this definition, it turns out that all of the known SM particles have R -parity $+1$, and all the superpartners of the known SM particles have $R = -1$, since they must have the same value of B and L but differ by $1/2$ unit of spin.

3.1 Electroweak symmetry breaking and the Higgs boson masses

We analyze the scalar potential in this section. It is derived from the superpotential and the terms involving the Higgs in the soft breaking Lagrangian.

The part of the scalar potential which involves only the Higgs bosons (H_u and H_d) is given by

$$\begin{aligned} V = & |\mu|^2(H_d^*H_d + H_u^*H_u) + \frac{1}{8}g'^2(H_u^*H_u - H_d^*H_d)^2 \\ & + \frac{1}{8}g^2(4|H_d^*H_u|^2 - 2(H_d^*H_d)(H_u^*H_u) + (H_d^*H_d)^2 + (H_u^*H_u)^2) \\ & + m_{H_d}^2H_d^*H_d + m_{H_u}^2H_u^*H_u + (B\mu\epsilon_{ij}H_d^iH_u^j + \text{h.c.}). \end{aligned} \quad (3.4)$$

Here the first term is the F -term, derived from $|(\partial W/\partial H_d)|^2$ and $|(\partial W/\partial H_u)|^2$ setting all sfermion VEV's equal to 0. The next two terms are D -terms, the first a $U(1)_Y$ D -term, recalling that the hypercharges for the Higgses are $Y_{H_d} = -\frac{1}{2}$ and $Y_{H_u} = \frac{1}{2}$, and the second is an $SU(2)_L$ D -term, taking $T^a = \sigma^a$ where σ^a are the three Pauli matrices. Finally, the last three terms are the soft SUSY breaking masses m_{H_d} and m_{H_u} , and the bilinear term $B\mu$. The Higgs doublets can be written as

$$\langle H_d \rangle = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \quad \langle H_u \rangle = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad (3.5)$$

where in Eq. (3.4) by $(H_d^*H_d)$, we mean $H_d^{0*}H_d^0 + H_d^{-*}H_d^-$ etc.

The neutral portion of Eq. (3.4) can be expressed more simply as

$$V = \frac{g^2 + g'^2}{8} (|H_d^0|^2 - |H_u^0|^2)^2 + (m_{H_d}^2 + |\mu|^2)|H_d^0|^2 + (m_{H_u}^2 + |\mu|^2)|H_u^0|^2 + (B\mu H_d^0 H_u^0 + \text{h.c.}). \quad (3.6)$$

For electroweak symmetry breaking, it will be required that either one (or both) of the soft masses ($m_{H_d}^2, m_{H_u}^2$) be negative (as in the SM).

From the minimization of the potential Eq. (3.6), we obtain the following two conditions

$$-2B\mu = (m_{H_d}^2 + m_{H_u}^2 + 2\mu^2) \sin 2\beta, \quad (3.7)$$

and

$$v^2 = \frac{4(m_{H_d}^2 + \mu^2 - (m_{H_u}^2 + \mu^2) \tan^2 \beta)}{(g^2 + g'^2)(\tan^2 \beta - 1)}, \quad (3.8)$$

where $\tan \beta = \frac{v_u}{v_d}$. From the potential and these two conditions, the masses of the physical scalars can be obtained. At the tree level,

$$m_{H^\pm}^2 = m_A^2 + m_W^2, \quad (3.9)$$

$$m_A^2 = m_{H_d}^2 + m_{H_u}^2 + 2\mu^2 = -B\mu(\tan \beta + \cot \beta), \quad (3.10)$$

$$m_{H,h}^2 = \frac{1}{2} \left[m_A^2 + m_Z^2 \pm \sqrt{(m_A^2 + m_Z^2)^2 - 4m_A^2 m_Z^2 \cos^2 2\beta} \right]. \quad (3.11)$$

Notice that these expressions and the above constraints limit the number of free inputs in the MSSM. First, from the mass of the pseudoscalar, we see that $B\mu$ is not independent and can be expressed in terms of m_A and $\tan \beta$. Furthermore from the conditions Eqs. (3.7) and (3.8), we see that if we keep $\tan \beta$, we can either choose m_A and μ as free inputs thereby determining the two soft masses, m_{H_d} and m_{H_u} , or we can choose the soft masses as inputs, and fix m_A and μ by the conditions for electroweak symmetry breaking. Both choices of parameter fixing are widely used in the literature.

The tree level expressions for the Higgs masses make some very definite predictions. The charged Higgs is heavier than M_W , and the light Higgs h , is necessarily lighter than M_Z . Note if uncorrected, the MSSM would already be excluded (from current accelerator limits). However, radiative corrections to the Higgs masses are

not negligible in the MSSM, particularly for a heavy top mass $m_t \sim 178$ GeV. The leading one-loop corrections to m_h^2 depend quartically on m_t and can be expressed as [22]

$$\begin{aligned} \Delta m_h^2 &= \frac{3m_t^4}{4\pi^2 v^2} \ln \left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right) \\ &+ \frac{3m_t^4 \hat{A}_t^2}{8\pi^2 v^2} \left[2h(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + \hat{A}_t^2 g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \right] + \dots \end{aligned} \quad (3.12)$$

where $m_{\tilde{t}_{1,2}}$ are the physical masses of the two stop squarks $\tilde{t}_{1,2}$ to be discussed in more detail shortly, $\hat{A}_t \equiv A_t + \mu \cot \beta$, (A_t is the SUSY breaking trilinear term associated with the top quark Yukawa coupling). The functions h and f are

$$h(a, b) \equiv \frac{1}{a-b} \ln \left(\frac{a}{b} \right), \quad g(a, b) = \frac{1}{(a-b)^2} \left[2 - \frac{a+b}{a-b} \ln \left(\frac{a}{b} \right) \right]. \quad (3.13)$$

Additional corrections to coupling vertices, two-loop corrections and renormalization-group resummations have also been computed in the MSSM [23]. With these corrections one can allow

$$m_h \lesssim 130 \text{ GeV}, \quad (3.14)$$

within the MSSM. While certainly higher than the tree level limit of M_Z , the limit still predicts a relatively light Higgs boson, and allows the MSSM to be experimentally excluded (or verified!) at the LHC.

3.2 The sfermions masses

We turn next to the discussion of scalar partners of the quarks and leptons. The mixing matrices for \tilde{m}_t^2 , \tilde{m}_b^2 and \tilde{m}_τ^2 are

$$\begin{pmatrix} \tilde{m}_{tL}^2 & m_t(A_t + \mu \cot \beta) \\ m_t(A_t + \mu \cot \beta) & \tilde{m}_{tR}^2 \end{pmatrix}, \quad (3.15)$$

$$\begin{pmatrix} \tilde{m}_{bL}^2 & m_b(A_b + \mu \tan \beta) \\ m_b(A_b + \mu \tan \beta) & \tilde{m}_{bR}^2 \end{pmatrix}, \quad (3.16)$$

$$\begin{pmatrix} \tilde{m}_{\tau L}^2 & m_\tau(A_\tau + \mu \tan \beta) \\ m_\tau(A_\tau + \mu \tan \beta) & \tilde{m}_{\tau R}^2 \end{pmatrix}, \quad (3.17)$$

with

$$\begin{aligned}
\tilde{m}_{tL}^2 &= \tilde{m}_Q^2 + m_t^2 + \frac{1}{6}(4M_W^2 - M_Z^2) \cos 2\beta, \\
\tilde{m}_{tR}^2 &= \tilde{m}_U^2 + m_t^2 - \frac{2}{3}(M_W^2 - M_Z^2) \cos 2\beta, \\
\tilde{m}_{bL}^2 &= \tilde{m}_Q^2 + m_b^2 - \frac{1}{6}(2M_W^2 + M_Z^2) \cos 2\beta, \\
\tilde{m}_{bR}^2 &= \tilde{m}_D^2 + m_b^2 + \frac{1}{3}(M_W^2 - M_Z^2) \cos 2\beta, \\
\tilde{m}_{\tau L}^2 &= \tilde{m}_L^2 + m_\tau^2 - \frac{1}{2}(2M_W^2 - M_Z^2) \cos 2\beta, \\
\tilde{m}_{\tau R}^2 &= \tilde{m}_E^2 + m_\tau^2 + (M_W^2 - M_Z^2) \cos 2\beta.
\end{aligned}$$

The first terms here (\tilde{m}^2) are the soft ones, which are calculated using the Renormalization Group (RG) equations starting from their values at the GUT (Planck) scale. The second ones are the usual masses of quarks and leptons and the last ones are the D terms of the potential.

The off-diagonal mixing term in the mass matrix is negligible for all but the third generation sfermions. The physical sfermion states and their masses are determined by diagonalizing the sfermion mass matrix.

3.3 Neutralinos

There are four new neutral fermions in the MSSM which not only receive mass but mix as well. These are the gauge fermion partners of the neutral B and W^3 gauge bosons, and the partners of the Higgs. The two gauginos are called the bino, \tilde{B} , and wino, \tilde{W}^3 respectively. The latter two are the Higgsinos, \tilde{H}_d and \tilde{H}_u . In addition to the SUSY breaking gaugino mass terms, $-\frac{1}{2}M_1\tilde{B}\tilde{B}$, and $-\frac{1}{2}M_2\tilde{W}^3\tilde{W}^3$, there are supersymmetric mass contributions of the type $W^{ij}\psi_i\psi_j$, giving a mixing term between \tilde{H}_d and \tilde{H}_u , $\frac{1}{2}\mu\tilde{H}_d\tilde{H}_u$, as well as terms of the form $g(\phi^*T^a\psi)\lambda^a$ giving the following mass terms after the appropriate Higgs VEVs have been inserted, $\frac{1}{\sqrt{2}}g'v_d\tilde{H}_d\tilde{B}$, $-\frac{1}{\sqrt{2}}g'v_u\tilde{H}_u\tilde{B}$, $-\frac{1}{\sqrt{2}}gv_d\tilde{H}_d\tilde{W}^3$, and $\frac{1}{\sqrt{2}}gv_u\tilde{H}_u\tilde{W}^3$. These latter terms can be written in a simpler form noting that for example, $g'v_d/\sqrt{2} = M_Z \sin \theta_W \cos \beta$.

Thus we can write the neutralino mass matrix as (in the $(\widetilde{B}, \widetilde{W}^3, \widetilde{H}_d^0, \widetilde{H}_u^0)$ basis) [24]

$$\begin{pmatrix} M_1 & 0 & -M_Z s_{\theta_W} \cos \beta & M_Z s_{\theta_W} \sin \beta \\ 0 & M_2 & M_Z c_{\theta_W} \cos \beta & -M_Z c_{\theta_W} \sin \beta \\ -M_Z s_{\theta_W} \cos \beta & M_Z c_{\theta_W} \cos \beta & 0 & -\mu \\ M_Z s_{\theta_W} \sin \beta & -M_Z c_{\theta_W} \sin \beta & -\mu & 0 \end{pmatrix}, \quad (3.18)$$

where $s_{\theta_W} = \sin \theta_W$ and $c_{\theta_W} = \cos \theta_W$. The mass eigenstates (a linear combination of the four neutralino states) and the mass eigenvalues are found by diagonalizing the mass matrix Eq. (3.18).

3.4 Charginos

There are two new charged fermionic states which are the partners of the W^\pm gauge bosons and the charged Higgs scalars, H^\pm , which are the charged gauginos, \widetilde{W}^\pm and charged Higgsinos, \widetilde{H}^\pm , or collectively charginos. The chargino mass matrix is composed similarly to the neutralino mass matrix. The result for the mass term is

$$-\frac{1}{2} (\widetilde{W}^-, \widetilde{H}^-) \begin{pmatrix} M_2 & \sqrt{2} m_W \sin \beta \\ \sqrt{2} m_W \cos \beta & \mu \end{pmatrix} \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}^+ \end{pmatrix} + \text{h.c.} \quad (3.19)$$

Note that unlike the case for neutralinos, two unitary matrices must be constructed to diagonalize Eq. (3.19). The result for the mass eigenstates of the two charginos is

$$m_{\widetilde{c}_1}^2, m_{\widetilde{c}_2}^2 = \frac{1}{2} \left[M_2^2 + \mu^2 + 2M_W^2 \pm \sqrt{(M_2^2 + \mu^2 + 2M_W^2)^2 - 4(\mu M_2 - M_W^2 \sin 2\beta)^2} \right] \quad (3.20)$$

Some additional resources on supersymmetry used in this preliminary introduction are the classic by Bagger and Wess on supersymmetry [25], the book by Ross on Grand Unification [26] and some other good reviews by Martin and others [27–33].

CHAPTER 4

ANOMALY MEDIATED SUPERSYMMETRY BREAKING

Understanding the origin of Supersymmetry breaking has been one of the main focuses of SUSY phenomenologists. It is highly non-trivial to construct models which break supersymmetry in a generally acceptable way.

The most common scenario for producing low-energy Supersymmetry breaking is called the *hidden sector*. The usual SM matter fields reside in the visible sector and the fields that break supersymmetry reside in the hidden sector. There are no (small) direct couplings between the two sectors. The symmetry breaking which occurs in the hidden sector is communicated to the visible sector via “messenger” fields.

Some of the several competing proposals on what the mediating interaction might be are Gravity mediation (SUGRA), Gauge mediation, Gaugino mediation and Anomaly mediation.

Any successful supersymmetry breaking scenario should at least satisfy the following conditions:

- The theory should give correct masses to the superpartners ~ 1 TeV, and the scalar mass-squared should be positive,
- The μ parameter should be between 100 GeV – 1 TeV and the $B\mu$ parameter should not be too much larger than μ^2 ,
- There are no large flavor changing neutral currents,
- CP should be approximately conserved (A & B phase should be small, as required by the measurement of the electric dipole moments of neutron and electron),

- The model should be simple enough such that it can be tested experimentally.

This thesis is based on the Anomaly mediation scenario of SUSY breaking. Before going into any details of the proposed models, I will briefly review the other three scenarios and what others have done on anomaly mediation.

4.1 Gravity mediation

In this scenario, the messenger is gravity. Supersymmetry is broken in the hidden sector by a VEV $\langle F \rangle$. The moduli field T , which appears as a result of compactification from higher dimensions and the dilaton field S , which is part of the SUGRA supermultiplet develop a non-zero VEV for their F components which in turn leads to spontaneous SUSY breaking. The soft mass term in the visible sector is roughly

$$m_{soft} \sim \frac{\langle F \rangle}{M_{Pl}}. \quad (4.1)$$

These soft masses should vanish as $\langle F \rangle \rightarrow 0$ where SUSY remains unbroken.

In this scenario, the SUSY sector is completely described by 5 input parameters: Higgs mass parameter (μ), common scalar mass (m_0), common gaugino mass ($m_{1/2}$), common trilinear coupling (A_0) and the Higgs mixing parameter (B).

When SUSY is broken at a scale $\sqrt{\langle F \rangle}$, the graviton will also obtain a mass

$$m_{soft} \sim m_{3/2} \sim \frac{\langle F \rangle}{M_{Pl}}. \quad (4.2)$$

Since we argued earlier that for SUSY to solve the hierarchy problem the mass scale should be $m_{soft} \sim 1$ TeV, therefore SUSY should be broken at a scale $\sqrt{\langle F \rangle} \sim 10^{11}$ GeV.

Some of the good features of the models are

- Extremely predictive– because the entire low energy spectrum is predicted in terms of few input parameters (m_0 , $m_{1/2}$, A_0 , $\tan \beta(B)$ and $sign(\mu)$), where all phenomenological limits can be expressed in terms of these parameters,

- Gauge couplings are unified and the gaugino masses are predicted to be the ratios of the gauge couplings,
- The μ problem is solved through Giudice–Masiero mechanism, where a singlet field Σ in the Kahler potential $\int d^4\theta \Sigma^* H_d H_u / M_{Pl}$ breaks SUSY,
- It is easy to generate positive scalar mass-squared.
- H_u mass-squared turns negative due to large top Yukawa coupling even if it starts of being positive at the Planck scale.

Despite the success of the theory, there are still some problems which are: CP is generally a problem, large freedom of parameters, absence of automatic suppression of flavor violation, lack of consistent theory of quantum supergravity (local symmetry).

4.2 Gauge mediation

In this scenario the Supersymmetry breaking is communicated from the hidden sector to the visible sector via gauge interactions. The main idea is to introduce new chiral multiplets (messengers) which couple indirectly to the MSSM fields through the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge interactions.

The particles ((s)quarks and (s)leptons) gets large mass by coupling to a gauge singlet chiral supermultiplet S . The superpotential for a typical gauge mediation can be written as

$$W = \lambda_1 S \ell \bar{\ell} + \lambda_2 S q \bar{q}. \quad (4.3)$$

The singlet scalar S and the auxiliary component of S (F_s) acquires a VEV by putting the scalar field into an O’Raifeartaigh–type model or a dynamical mechanism.

The gauginos get mass at 1-loop

$$M_i \sim \frac{\alpha_i}{4\pi} \Lambda \quad (i = 1, 2, 3), \quad (4.4)$$

where $\Lambda = F_s / \langle S \rangle$.

The MSSM scalars do not get any radiative corrections to their masses at 1-loop. Their masses arise at 2-loop level from those diagrams involving the gauge

fields and the messengers. The scalar masses are given by

$$\tilde{m} \sim \left(\frac{\Lambda}{4\pi} \right)^2 \{ \alpha_3^2 C_3 + \alpha_2^2 C_2 + \alpha_1^2 C_1 \}, \quad (4.5)$$

where C_i are the quadratic Casimir operators for the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group. This implies that the sparticles with the same gauge quantum number will have equal masses (for example: $\tilde{m}_e = \tilde{m}_\mu = \tilde{m}_\tau$).

In order for the gauginos and scalar soft masses to be ~ 1 TeV (as needed for the hierarchy problem) requires $\Lambda \sim 10^4 - 10^5$ GeV. In most of the gauge mediation models, the slepton and squark masses depend only on their gauge quantum numbers. This leads to the degeneracy of squark and slepton mass which results in the suppression of flavor changing neutral currents (FCNC's). The Lightest Supersymmetric Particle (LSP) is usually the gravitino, with mass $m_{3/2} \sim \Lambda^2/M_{pl} \sim 10^{-10}$ GeV, which can be crucial both for cosmology and collider physics.

In summary:

- gauge mediated supersymmetry breaking (GMSB) solves the FCNC problem,
- gaugino mass arise at 1-loop while the scalar mass-squared arise at two loop level,
- there is still a problem in the Higgs sector (offers no compelling solution to the μ problem),
- it does not offer any solution to the SUSY CP problem.

4.3 Gaugino mediation

In this scenario the SM quark and lepton fields are localized on a ‘3-brane’ in extra dimensions, while the gauge and Higgs fields propagate in the bulk. SUSY breaking masses for the gauginos and Higgs fields are generated by higher-dimensional contact terms between the bulk fields and the hidden sector fields, assumed to arise from a more fundamental theory such as string theory [34]. The leading contribution to the SUSY breaking for visible sector fields arises from loops of bulk gauge and Higgs fields as shown in Fig. 4.1

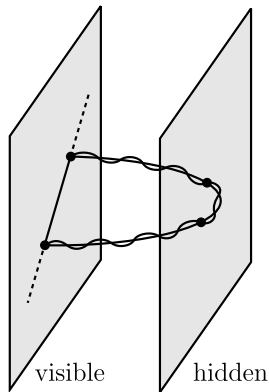


Figure 4.1. Leading diagram that contributes to SUSY–breaking scalar masses. The bulk line is a gaugino propagator.

The minimal version of gaugino mediation has only three high energy parameters μ , $m_{1/2}$ and M_c . Here $m_{1/2}$ is the universal gaugino mass at the unification scale and M_c is the compactification scale where the higher dimensional theory is matched onto the effective four–dimensional theory. For $\sin^2 \theta_W$ prediction to be preserved from gauge coupling unification requires $M_c > M_{GUT}$. In some other models of gaugino mediation [35] the μ parameter is determined by fitting to the Z mass. Such model requires only two free parameters $m_{1/2}$ and M_c .

The gaugino mediation scenario is the least developed in the literature. It does not offer any real solution to the μ problem.

4.4 Anomaly mediation

This scenario assumes that supersymmetry breaking takes place in a hidden or sequestered sector. The MSSM superfields are confined to a 3–brane in a higher dimensional bulk space–time separated from the sequestered sector by extra dimensions. A rescaling super–Weyl anomaly generates coupling of the auxiliary field of the gravity multiplet to the gauginos and the scalars of the MSSM, with the couplings determined by the SUSY renormalization group equations (RGE) [36].

Before going into much details, it is important to give a brief review on how this scenario address the numerous problems associated with the other three scenarios addressed earlier.

- The μ parameter can be generated without generating excessively large $B\mu$ due to the constraints from the coupling of the gravitational multiplet.
- The dominant anomaly-mediated contribution to the squark and slepton masses suppresses flavor violation automatically.
- There are no new phases in the A and B terms. This implies a natural solution to the SUSY CP problem. In other words CP can be violated on our 3-brane and nowhere else.
- The model is straightforward in the sense that the basic assumption is that SUSY breaking is derived from higher dimensional theory.
- These SUSY breaking models are very predictive. The ratio of the gaugino masses depends on the beta functions rather than the gauge couplings. The A -terms are predicted to be proportional to the corresponding Yukawa coupling and there is a nearly degenerate Wino/Zino LSP, of which the Zino is the lighter.
- The gaugino and scalar masses are comparable.
- Since the rescaling anomaly is UV insensitive, the pattern of SUSY breaking masses at any energy scale is governed only by the physics at that scale [36–38]. An arbitrary flavor structure in the SUSY scalar spectrum at high energies gets washed out at low energies. This Ultraviolet (UV) insensitivity provides an elegant solution to the SUSY flavor problem.
- It can naturally solve the cosmological gravitino abundance problem which tends to destroy the success of big bang cosmology in generic supergravity models [39].

- The decay of the moduli fields present in the model (as well as the gravitino) will produce neutralinos, especially the neutral Winos, with the right abundance to make it a viable cold dark matter candidate [40, 41].

We see from above that this model seems to be a viable (promising) model for understanding the MSSM supersymmetry breaking. It turns out that there is a major problem in this model which is discussed below.

4.4.1 The negative slepton mass problem of anomaly mediated supersymmetry breaking

In anomaly mediated supersymmetry breaking models (AMSB), the masses of the scalar components of the chiral supermultiplets are given by [36, 37]

$$(m^2)_{\phi_i}^{\phi_j} = \frac{1}{2} M_{aux}^2 \left[\beta(Y) \frac{\partial}{\partial Y} \gamma_{\phi_i}^{\phi_j} + \beta(g) \frac{\partial}{\partial g} \gamma_{\phi_i}^{\phi_j} \right]. \quad (4.6)$$

In the above equation summations over the gauge couplings g and the Yukawa couplings Y are assumed. $\gamma_{\phi_i}^{\phi_j}$ are the one-loop anomalous dimensions, $\beta(Y)$ is the beta function for the Yukawa coupling Y , and $\beta(g)$ is the beta function for the gauge coupling g . M_{aux} is the vacuum expectation value of a “compensator superfield” [36] which sets the scale of SUSY breaking. The gaugino mass M_g associated with the gauge group with coupling g is given by [36, 37]

$$M_g = \frac{\beta(g)}{g} M_{aux}. \quad (4.7)$$

The trilinear soft supersymmetry breaking term A_Y corresponding to the Yukawa coupling Y is given by [36, 37]

$$A_Y = -\frac{\beta(Y)}{Y} M_{aux}. \quad (4.8)$$

In the simplest scenario for generating the μ term for a special class of models, the contribution to the Higgs mixing parameter (the B -term) is given by [36]

$$B = -(\gamma_{H_u} + \gamma_{H_d}) M_{aux}. \quad (4.9)$$

Here γ_{H_u} and γ_{H_d} are the one-loop anomalous dimensions of the H_u and H_d fields. Similar relations hold for other bilinear terms in the SUSY breaking Lagrangian.

In the minimal scenario, it turns out that AMSB induces a **negative mass-squared for the sleptons**. Such a scenario is excluded since it would break electromagnetism. The reason for the negative mass-squared can be understood as follows. There are two sources for slepton masses in AMSB, the Yukawa part and the gauge part (cf: Eq. (4.6)). For the first two families the Yukawa couplings are negligible and the dominant contributions arise proportional to the gauge beta function. Since in the MSSM the $SU(2)_L$ and the $U(1)_Y$ gauge couplings are not asymptotically free, their gauge beta functions are positive. This makes the slepton mass-squared negative. In the squark sector, the masses are positive because $SU(3)_C$ gauge theory is asymptotically free.

4.4.2 Suggested solutions to the AMSB slepton mass problem

Several possible ways of avoiding the slepton mass problem of AMSB have been suggested. A non-decoupling universal bulk contribution to all the scalar masses is a widely studied option [36–42]. While this will make the minimal model phenomenologically consistent, the UV insensitivity of AMSB is no longer guaranteed. It is therefore interesting to investigate variations of the minimal model which maintain the UV insensitivity but provide positive mass-squared for the sleptons from physics at the TeV scale.

One way to avoid the negative slepton mass problem with TeV scale physics is to increase the Yukawa contributions in Eq. (4.6). This can be achieved by introducing new particles at the TeV scale with large Yukawa couplings to the lepton fields. This possibility was studied in Ref. [43] where the MSSM spectrum was extended to include 3 pairs of Higgs doublets, four singlets and a vector-like pair of color-triplets near the weak scale. The Yukawa contributions can also be enhanced by invoking R -parity violating couplings in the MSSM [44]. Unfortunately such a theory would generate unacceptably large neutrino masses. Yet another possibility is to make use of the positive D -term contributions from a $U(1)$ gauge symmetry broken at the weak scale. This was achieved by adding TeV scale Fayet-Iliopoulos terms explicitly to the theory in Ref. [45]. New D -term contributions generated in a controlled fashion

by the breaking of $U(1)_{B-L}$ at an arbitrary high scale may also provide positive contributions to the slepton masses [46, 47]. A low scale ancillary $U(1)$ as a solution to the problem has been studied in Ref. [48]. Effective supersymmetric theories which are devoid of the negative slepton mass problem of AMSB with new dynamics at the 10 TeV scale have been studied in Ref. [49]. Non-decoupling effects of heavy fields at higher orders have been analyzed in AMSB models in Ref. [50] as an attempt to solve the slepton mass problem.

CHAPTER 5

TeV-SCALE HORIZONTAL SYMMETRY AND THE SLEPTON MASS PROBLEM OF ANOMALY MEDIATION

5.1 Introduction

As noted in chapter 4, supersymmetry provides an elegant solution to the gauge hierarchy problem of the standard model. To be realistic, it must however be a broken symmetry. There are several ways of achieving supersymmetry (SUSY) breaking. Anomaly mediated SUSY breaking (AMSB) is an attractive and predictive scenario which has the virtue that it can solve the SUSY flavor problem [36, 37]. This scenario assumes that SUSY breaking takes place in a hidden or *sequestered* sector. The MSSM superfields are confined to a 3-brane in a higher dimensional bulk space-time separated from the sequestered sector by extra dimensions. A rescaling super-Weyl anomaly generates coupling of the auxiliary field of the gravity multiplet to the gauginos and the scalars of the MSSM, with the couplings determined by the SUSY renormalization group equations (RGE). Since the rescaling anomaly is UV insensitive, the pattern of SUSY breaking masses at any energy scale is governed only by the physics at that scale [36–38]. Arbitrary flavor structure in the SUSY scalar spectrum at high energies gets washed out at low energies. This ultraviolet insensitivity provides an elegant solution to the SUSY flavor problem.

The purpose of this thesis is to suggest and investigate the possibility of solving the negative slepton mass problem by making the gauge contribution in Eq. (4.6) positive. This can only be achieved by introducing a new non-Abelian gauge symmetry for leptons with negative gauge beta function. We point out that an $SU(3)_H$ horizontal symmetry acting on the lepton multiplets has all the desired properties

for achieving this. We show that such an $SU(3)_H$ horizontal symmetry broken at the TeV scale is consistent with rare leptonic processes owing to the emergence of approximate global symmetries.

The specific AMSB model we study is quite predictive. The lightest Higgs boson mass is predicted to be $m_h \lesssim 120$ GeV, and the parameter $\tan\beta$ is found to be $\tan\beta \simeq 4$. The model predicts the existence of new particles associated with the $SU(3)_H$ symmetry breaking sector. The $SU(3)_H$ vector bosons have masses of order 1–4 TeV. These particles should be accessible experimentally at the LHC.

The plan of the chapter is as follows. In section 5.2 we introduce our model. In section 5.3 we analyze the Higgs potential of the model. Here we derive the limits on $\tan\beta$ and m_h . In section 5.4 we present the SUSY spectrum of the model and show how the sleptons acquire positive masses. Numerical results for the full spectrum of the model are given in section 5.5. In section 5.6 we outline the most significant experimental consequences of the model. In section 5.7 we comment on the possible origins of the μ and the $B\mu$ terms. We summarize in section 5.8. In Appendix A, we give the relevant beta functions, anomalous dimensions as well as the soft masses.

5.2 $SU(3)_H$ horizontal symmetry

In this section we present our model. Since our aim is to have positive contributions to the slepton masses from the gauge sector, we are naturally led to a leptonic horizontal symmetry that is asymptotically free. Our model is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(3)_H$, where $SU(3)_H$ is a horizontal symmetry acting on the leptons. The left-handed lepton doublets and the antilepton singlets transform as fundamental representations of the $SU(3)_H$ gauge symmetry. The theory is made anomaly free by introducing three Higgs multiplets (Φ_1, Φ_2, Φ_3) which transform as antifundamental representations of $SU(3)_H$ and as singlets of the standard model. These fields are sufficient for breaking the $SU(3)_H$ symmetry completely near the TeV scale. The particle content of the model and the transformation properties under the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(3)_H$ are presented in Table 5.1. It turns out that the Higgs potential involving these Φ_i fields exhibits

a global $SU(3)_G$ symmetry. We take advantage of this global symmetry to suppress potentially large flavor changing neutral current processes mediated by the $SU(3)_H$ gauge bosons. The last column in Table 5.1 lists the transformation properties under the global $SU(3)_G$ symmetry (The Yukawa couplings of the model reduce the global $SU(3)_G$ down to $U(1)$.) The fields η_i and $\bar{\eta}_i$ are introduced to facilitate $SU(3)_H$ symmetry breaking within our AMSB framework.

Superfield	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$SU(3)_H$	$SU(3)_G$
Q_i	3	2	$\frac{1}{6}$	1	1
u_i^c	$\bar{3}$	1	$-\frac{2}{3}$	1	1
d_i^c	$\bar{3}$	1	$\frac{1}{3}$	1	1
L_α	1	2	$-\frac{1}{2}$	3	1
e_α^c	1	1	1	3	1
H_u	1	2	$\frac{1}{2}$	1	1
H_d	1	2	$-\frac{1}{2}$	1	1
Φ_i^α	1	1	0	$\bar{3}$	3
η_i	1	1	0	$\bar{3}$	3
$\bar{\eta}_i$	1	1	0	3	$\bar{3}$

TABLE 5.1. Particle content and charge assignment of the model. $SU(3)_G$ in the last column is a softly broken global symmetry present in the model. The indices i and α take values $i, \alpha = 1 - 3$.

Note that the quarks are neutral under $SU(3)_H$. This is necessitated by the requirements that $SU(3)_H$ be asymptotically free. A separate $SU(3)_{H'}$ acting on the quarks is a possible quark–lepton symmetric extension of the model. But we do not pursue such an extension here.

The superpotential of the model consistent with the gauge symmetries and the global $SU(3)_G$ symmetry is given by:

$$\begin{aligned}
W &= (Y_u)_{ij} Q_i H_u u_j^c + (Y_d)_{ij} Q_i H_d d_j^c + \mu H_u H_d \\
&+ \kappa \Phi_1^\alpha \Phi_2^\beta \Phi_3^\gamma \epsilon_{\alpha\beta\gamma} + \lambda \eta_a^\alpha \eta_b^\beta \Phi_c^\gamma \epsilon_{\alpha\beta\gamma} \epsilon^{abc} + M_\eta \eta_a \bar{\eta}_a.
\end{aligned} \tag{5.1}$$

Here $\alpha, \beta, \gamma = 1, 2, 3$ are $SU(3)_H$ indices, $i, j = 1, 2, 3$ are family indices, and $a, b, c = 1, 2, 3$ are $SU(3)_G$ indices. The mass parameters μ and M_η are of order TeV, which has a natural origin in AMSB [36]. We will comment on possible origin of these terms in Sec. 5.7.

In the $SU(3)_H$ symmetric limit the leptons are all massless. They obtain their masses from the effective operators

$$L_{eff}^l = \frac{L_\alpha e_\alpha^c \Phi_i^\alpha \Phi_i^\alpha H_d}{M_i^2}. \quad (5.2)$$

Such operators can be obtained by integrating fields shown in Fig. 1, for example. The masses of the heavy fields break $SU(3)_G$ symmetry softly (the $\bar{\psi}_i \psi_i$ and the $\bar{E}_i E_i$ mass terms in Fig. 5.1). Note that the mass scale M_i in Eq. (5.2) is of order 5

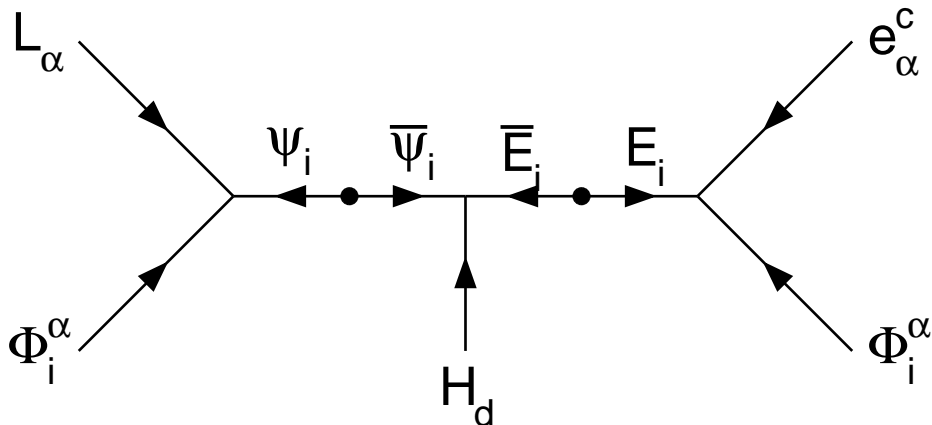


Figure 5.1. Effective operators inducing charged lepton masses.

TeV for generating realistic τ -lepton mass, of order 20 TeV for the μ mass and of order 300 TeV for the electron mass (assuming that all relevant Yukawa couplings are of order one). Since these masses are all much heavier than the effective SUSY breaking scale of order 1 TeV, these heavy fields will have no effect in the low energy SUSY phenomenology within AMSB. Note that no generation mixing is induced by these effective operators, which will guarantee the approximate conservation of electron number, muon number and tau lepton number. This is what makes the model consistent with FCNC data even when $SU(3)_H$ is broken at the TeV scale. Since the Higgs potential respects $SU(3)_H \times SU(3)_G$ symmetry, after spontaneous

symmetry breaking, the diagonal subgroup $SU(3)_{G+H}$ remains as an unbroken global symmetry. This subgroup contains e , μ and τ lepton numbers.

Since right-handed neutrinos are not required to be light for $SU(3)_H$ anomaly cancellation, they acquire heavy masses and decouple from the low energy theory. Small neutrino masses are then induced through the seesaw mechanism. Specifically, the following effective nonrenormalizable operators emerge after integrating out the heavy right-handed neutrino fields:

$$L_{eff}^\nu = \frac{\lambda_{ij}^{\alpha\beta} L_\alpha L_\beta H_u H_u \Phi_i^\alpha \Phi_j^\beta}{M_N^3}. \quad (5.3)$$

Here M_N represents the masses of the heavy right-handed neutrino fields. For $M_N \sim 10^7$ GeV and $\langle \Phi_i \rangle \sim$ TeV, neutrino masses in the right range for oscillation phenomenology are obtained. Note that Eq. (5.3) arises from integrating neutral leptons with their masses assumed to break all global symmetries. This enables generation of large neutrino mixing angles, as needed for phenomenology.

5.3 Symmetry breaking

The $SU(3)_H$ model has two sets of Higgs bosons: the usual MSSM Higgs doublets H_u and H_d , and the $SU(3)_H$ Higgs antitriplets Φ_i ($i = 1, 2, 3$). The Higgs potential is derived from the superpotential of Eq. (5.1) and includes the soft terms and the D terms. The tree level potential splits into two pieces:

$$V(H_u, H_d, \Phi_i) = V(H_u, H_d) + V(\Phi_i), \quad (5.4)$$

enabling us to analyze them independently. The first part, $V(H_u, H_d)$, is identical to the MSSM potential which is well studied. There are however significant constraints on the parameters in our AMSB extension, which we now discuss.

5.3.1 Constraints on $\tan\beta$ and m_h

Minimization of $V(H_u, H_d)$ gives

$$\sin 2\beta = \frac{2B\mu}{2\mu^2 + m_{H_u}^2 + m_{H_d}^2}, \quad \mu^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{M_Z^2}{2}. \quad (5.5)$$

Here $m_{H_u}^2$ and $m_{H_d}^2$ are the Higgs soft masses and are given in the Appendix for the AMSB model (see Eqs. (A.19)–(A.20).) The constraints on m_h and $\tan\beta$ arise since these soft masses and the B parameter are determined in terms of a single parameter M_{aux} in our framework.

We eliminate M_{aux} in favor of M_2 , the Wino mass ($M_2 = \frac{b_2 g_2^2}{16\pi^2} M_{aux}$). We see from Eqs. (4.9), (5.5) as well as from Eqs. (A.6)–(A.7) and Eqs. (A.19)–(A.20) of the Appendix that $\tan\beta$ is fixed in terms of M_2 . In Fig. 5.2 we plot $\tan\beta$ as a function of M_2 . For the experimentally interesting range of $M_2 \gtrsim 100$ GeV, we find that $\tan\beta \simeq 3.8 - 4.0$. In obtaining the limit on $\tan\beta$, we followed the following procedure. As inputs at M_Z we chose [51]

$$\alpha_3(M_Z) = 0.119, \quad \sin^2\theta_W = 0.2312, \quad \alpha(M_Z) = \frac{1}{127.9}. \quad (5.6)$$

Using the central value of $M_t = 174.3$ GeV, we obtain the running mass $m_t(M_t)$ with the 2-loop QCD correction as [52]

$$\frac{M_t}{m_t(M_t)} = 1 + \frac{4}{3} \frac{\alpha_3(M_t)}{\pi} + 10.9 \left(\frac{\alpha_3(M_t)}{\pi} \right)^2. \quad (5.7)$$

Using 5-flavor SM QCD beta functions we extrapolated $\alpha_3(M_Z)$ and obtained $\alpha_3(M_t) = 0.109$. The top quark Yukawa coupling is then found to be (for $M_t = 174.3$ GeV) $Y_t^{SM}(M_t) = 0.935$ corresponding to $m_t(M_t) = 162.8$ GeV. This coupling is then evolved from M_t to 1 TeV where we minimize the MSSM Higgs potential. Using standard model beta function we obtain $Y_t^{SM}(1 \text{ TeV}) = 0.851$. The corresponding MSSM coupling is $Y_t(1 \text{ TeV}) = Y_t^{SM}(1 \text{ TeV})/\sin\beta$, which for $\tan\beta \simeq 4.0$ (justified a-posteriori) is $Y_t(1 \text{ TeV}) = 0.824$. The gauge couplings at 1 TeV are found to be $g_1(1 \text{ TeV}) = 0.466$, $g_2(1 \text{ TeV}) = 0.642$ and $g_3(1 \text{ TeV}) = 1.098$. With these values of couplings at 1 TeV we obtained Fig. 5.2. Uncertainties in the prediction for $\tan\beta$ are estimated to be ± 0.5 , arising from the error in top quark mass and from the precise scale at which the Higgs potential is minimized. We conclude that $\tan\beta = 3.5$ – 4.5 in this model.

Since $\tan\beta$ is fixed and since the A_t parameter is not free in AMSB, there is a nontrivial prediction for the lightest Higgs boson mass m_h . We use the 2-loop

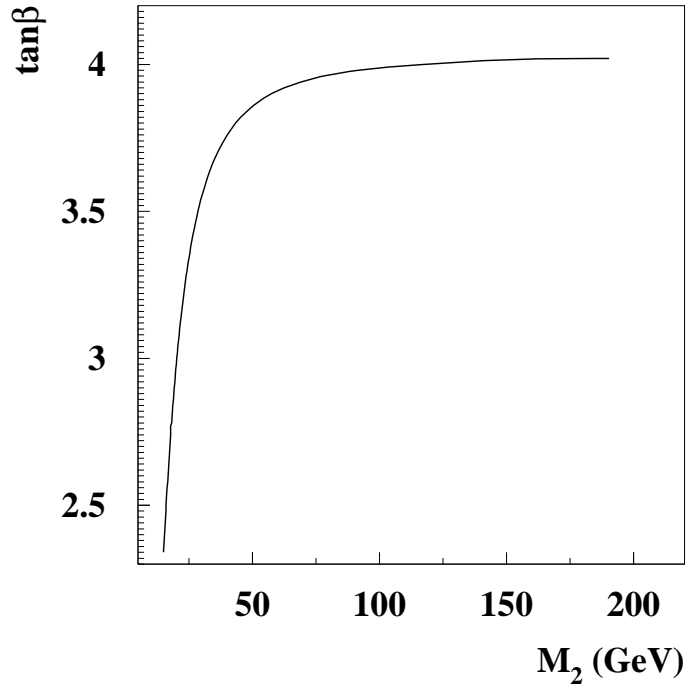


Figure 5.2. Plot of $\tan\beta$ as a function of M_2

radiatively corrected expression for $m_h^2 = (m_h^2)_o + \Delta m_h^2$, where $(m_h^2)_o$ is the tree-level value of the mass and the radiative correction is given by [53]

$$\Delta m_h^2 = \frac{3m_t^4}{4\pi^2 v^2} \left[t + X_t + \frac{1}{16\pi^2} \left(\frac{3m_t^2}{2v^2} - 32\pi\alpha_3(M_t) \right) (2X_t t + t^2) \right]. \quad (5.8)$$

Here

$$X_t = \frac{\tilde{A}_t^2}{m_{\tilde{t}}^2} \left(1 - \frac{\tilde{A}_t^2}{12m_{\tilde{t}}^2} \right), \quad \tilde{A}_t = A_t - \mu \cot\beta, \quad (5.9)$$

and $t = \log(\frac{m_{\tilde{t}}^2}{M_t^2})$, $v = 174$ GeV. $m_{\tilde{t}}^2$ is the arithmetic average of the diagonal entries of the squared stop mass matrix and A_t is the soft trilinear coupling associated with the top Yukawa coupling in the superpotential of Eq. (5.1). In these expressions, m_t is the one-loop QCD corrected running mass, $m_t = \frac{M_t}{1 + \frac{4}{3} \frac{\alpha_3(M_t)}{\pi}}$, which equals 166.7 GeV for $M_t = 174.3$ GeV. We find that $m_h \simeq 113$ GeV – 120 GeV, depending on the choice of M_{aux} . The larger value $m_h \simeq 120$ GeV is realized only for larger $M_t \simeq 180$ GeV. We list in Tables 5.2–5.4 the value of m_h , along with the other sparticle masses.

5.3.2 $SU(3)_H$ symmetry breaking

Let us now analyze the $SU(3)_H$ symmetry breaking sector of the potential. The potential $V(\Phi_i)$ is given by:

$$\begin{aligned}
V(\Phi_i) &= m_\phi^2(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2 + \Phi_3^\dagger\Phi_3) + \kappa A_\kappa \left(\Phi_1^\alpha\Phi_2^\beta\Phi_3^\gamma\epsilon_{\alpha\beta\gamma} + c.c \right) \\
&+ \kappa^2 [(\Phi_1\Phi_2)^\dagger(\Phi_1\Phi_2) + (\Phi_1\Phi_3)^\dagger(\Phi_1\Phi_3) + (\Phi_2\Phi_3)^\dagger(\Phi_2\Phi_3)] \\
&+ \frac{g_4^2}{8} \sum_{a=1}^8 |\Phi_1^\dagger\lambda^a\Phi_1 + \Phi_2^\dagger\lambda^a\Phi_2 + \Phi_3^\dagger\lambda^a\Phi_3|^2.
\end{aligned} \tag{5.10}$$

Here g_4 is the gauge coupling of the $SU(3)_H$, A_κ is the trilinear A -term corresponding to the coupling κ , m_ϕ^2 is the soft mass squared for the Φ_i fields. These soft SUSY breaking parameters are given in the Appendix (Eqs. (A.17), (A.23)). The κ^2 term in Eq. (5.10) is the F -term contribution and the last term in Eq. (5.10) is the $SU(3)_H$ D -term with λ^a being the $SU(3)_H$ generators.

The Higgs potential, Eq. (5.10), has an $SU(3)_H \times SU(3)_G$ symmetry, with the Φ_i fields ($i = 1 - 3$) transforming as $(\bar{\mathbf{3}}, \mathbf{3})$. This allows for a vacuum which preserves an $SU(3)_{H+G}$ diagonal subgroup. The VEVs of the Φ_i fields are then given by:

$$\langle \Phi_1 \rangle = \begin{pmatrix} u \\ 0 \\ 0 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ u \\ 0 \end{pmatrix} \quad \text{and} \quad \langle \Phi_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}. \tag{5.11}$$

Using these VEVs the potential becomes

$$\langle V(\Phi) \rangle = 3m_\phi^2 u^2 + 3\kappa^2 u^4 + 2\kappa A_\kappa u^3. \tag{5.12}$$

Minimization of Eq. (5.12) leads to the condition

$$u = \frac{-A_\kappa \pm \sqrt{-8m_\phi^2 + A_\kappa^2}}{4\kappa}. \tag{5.13}$$

The argument in the square root of Eq. (5.13), which should be positive for a consistent symmetry breaking, is given by

$$-8m_\phi^2 + A_\kappa^2 = \frac{M_{aux}^2}{(16\pi^2)^2} [15\kappa^4 + 56\kappa^2\lambda^2 + 304\lambda^4 - 8\kappa^2 g_4^2 - 32\lambda^2 g_4^2]. \tag{5.14}$$

Positivity of Eq. (5.14) leads to constraints on the parameters $\{\lambda, \kappa\}$. It can be shown that Eq. (5.14) implies $0 \leq |\kappa| \leq 0.731g_4$ and $0 \leq |\lambda| \leq 0.324g_4$. Furthermore, positivity of the slepton masses, along with the experimental limit $m_{slepton}^2 \gtrsim (100 \text{ GeV})^2$, require $g_4 \geq 0.5$. This essentially fixes the parameter space of the model. We get the right minimum by choosing the negative sign of the square root in Eq. (5.13) (for positive M_{aux}), with this choice, all the Higgs masses-squared will be positive.

Since the symmetry breaking chain is $SU(3)_H \times SU(3)_G \rightarrow SU(3)_{H+G}$, we can classify the masses of all scalars and fermions as multiplets of $SU(3)_{H+G}$. The complex $\Phi(\bar{3}, 3)$ scalar multiplet decomposes into 2 octets and two singlets of $SU(3)_{H+G}$. One octet gets eaten by the Higgs mechanism. A physical octet remains in the spectrum with a mass given by

$$\mathcal{M}_{octet}^2 = -2\kappa^2 u^2 - 2\kappa u A_\kappa + g_4^2 u^2. \quad (5.15)$$

There are two singlets, one scalar (ϕ_s) and one pseudoscalar (ϕ_p) with masses given by

$$m_{\phi_s}^2 = 4\kappa^2 u^2 + \kappa u A_\kappa, \quad (5.16)$$

$$m_{\phi_p}^2 = -3\kappa u A_\kappa. \quad (5.17)$$

In the fermionic sector, the octet Higgsino mixes with the octet gaugino with a mixing matrix given by

$$\mathcal{M}'_{octet} = \begin{pmatrix} m_4 & g_4 u \\ g_4 u & \kappa u \end{pmatrix}. \quad (5.18)$$

In addition, there is a Majorana fermion, a singlet of $SU(3)_{H+G}$, with a mass of

$$m_{\tilde{\phi}} = 2\kappa u. \quad (5.19)$$

Finally the gauge bosons form an octet with a mass

$$M_V = g_4 u. \quad (5.20)$$

5.4 The SUSY spectrum

We are now ready to discuss the full SUSY spectrum of the model. We will see that the tachyonic slepton problem is cured by virtue of the positive contribution from the $SU(3)_H$ gauge sector.

5.4.1 Slepton masses

The slepton mass-squareds are given by the eigenvalues of the mass matrices ($\alpha = e, \mu, \tau$)

$$M_i^2 = \begin{pmatrix} m_{\tilde{L}_\alpha}^2 & m_{E_\alpha} (A_{Y_{E_\alpha}} - \mu \tan \beta) \\ m_{E_\alpha} (A_{Y_{E_\alpha}} - \mu \tan \beta) & m_{\tilde{e}_\alpha^c}^2 \end{pmatrix}. \quad (5.21)$$

Here

$$m_{\tilde{L}_\alpha}^2 = \frac{M_{aux}^2}{(16\pi^2)} \left[Y_{E_\alpha} \beta(Y_{E_\alpha}) - \left(\frac{3}{2} g_2 \beta(g_2) + \frac{3}{10} g_1 \beta(g_1) + \frac{8}{3} g_4 \beta(g_4) \right) \right] + m_{E_\alpha}^2 + \left(-\frac{1}{2} + \sin^2 \theta_W \right) \cos 2\beta M_Z^2, \quad (5.22)$$

$$m_{\tilde{e}_\alpha^c}^2 = \frac{M_{aux}^2}{(16\pi^2)} \left[2Y_{E_\alpha} \beta(Y_{E_\alpha}) - \left(\frac{6}{5} g_1 \beta(g_1) + \frac{8}{3} g_4 \beta(g_4) \right) \right] + m_{E_\alpha}^2 - \sin^2 \theta_W \cos 2\beta M_Z^2. \quad (5.23)$$

The off diagonal terms in Eq. (5.21) are rather small as they are proportional to the lepton masses. The SUSY soft masses are calculated from the RGE give in the Appendix. The last terms of Eqs. (5.22)–(5.23) are the D -terms. Note the positive contribution from the $SU(3)_H$ gauge sector in Eqs. (5.22)–(5.23), given by the term $-\frac{8}{3}g_4\beta(g_4)$. In our model g_4 is asymptotically free with $\beta(g_4) = -\frac{3}{16\pi^2}g_4^3$. This contribution makes the mass-squared of all sleptons to be positive for $g_4 \geq 0.5$.

The left handed sneutrino mass is given by

$$m_{\tilde{\nu}_i}^2 = \frac{M_{aux}^2}{(16\pi^2)} \left[- \left(\frac{3}{2} g_2 \beta(g_2) + \frac{3}{10} g_1 \beta(g_1) + \frac{8}{3} g_4 \beta(g_4) \right) \right] + \frac{1}{2} \cos 2\beta M_Z^2, \quad (5.24)$$

where $i = e, \mu, \tau$.

5.4.2 Squark masses

The mixing matrix for the squark sector is similar to the slepton sector. The diagonal entries of the up and the down squark mass matrices are given by [27]

$$m_{\tilde{U}_i}^2 = (m_{soft}^2)_{\tilde{Q}_i} + m_{U_i}^2 + \frac{1}{6} (4M_W^2 - M_Z^2) \cos 2\beta,$$

$$\begin{aligned}
m_{\tilde{U}_i^c}^2 &= (m_{soft}^2)_{\tilde{U}_i^c} + m_{U_i}^2 - \frac{2}{3} (M_W^2 - M_Z^2) \cos 2\beta, \\
m_{\tilde{D}_i}^2 &= (m_{soft}^2)_{\tilde{Q}_i} + m_{D_i}^2 - \frac{1}{6} (2M_W^2 + M_Z^2) \cos 2\beta, \\
m_{\tilde{D}_i^c}^2 &= (m_{soft}^2)_{\tilde{D}_i^c} + m_{D_i}^2 + \frac{1}{3} (M_W^2 - M_Z^2) \cos 2\beta.
\end{aligned} \tag{5.25}$$

Here m_{U_i} and m_{D_i} are quark masses of different generations, $i = 1, 2, 3$. The squark soft masses are obtained from the RGE as

$$(m_{soft}^2)_{\tilde{Q}_i} = \frac{M_{aux}^2}{16\pi^2} \left(Y_{u_i} \beta(Y_{u_i}) + Y_{d_i} \beta(Y_{d_i}) - \frac{1}{30} g_1 \beta(g_1) - \frac{3}{2} g_2 \beta(g_2) - \frac{8}{3} g_3 \beta(g_3) \right) \tag{5.26}$$

$$(m_{soft}^2)_{\tilde{U}_i^c} = \frac{M_{aux}^2}{16\pi^2} \left(2Y_{u_i} \beta(Y_{u_i}) - \frac{8}{15} g_1 \beta(g_1) - \frac{8}{3} g_3 \beta(g_3) \right), \tag{5.27}$$

$$(m_{soft}^2)_{\tilde{D}_i^c} = \frac{M_{aux}^2}{16\pi^2} \left(2Y_{d_i} \beta(Y_{d_i}) - \frac{2}{15} g_1 \beta(g_1) - \frac{8}{3} g_3 \beta(g_3) \right). \tag{5.28}$$

5.4.3 η fermion and η scalar masses

The fields η and $\bar{\eta}$ transform as $(3, \bar{3})$ and $(\bar{3}, 3)$ under $SU(3)_H \times SU(3)_G$. After symmetry breaking, η and $\bar{\eta}$ both transform as $\mathbf{8} + 1$ of the diagonal $SU(3)_{H+G}$. The octet from η mixes with the octet from $\bar{\eta}$, and similarly for the singlets.

In the fermionic sector, the octet and the singlet mass matrices are given by

$$M_{octet}^\eta = \begin{pmatrix} -2\lambda u & M_\eta \\ M_\eta & 0 \end{pmatrix}, \tag{5.29}$$

$$M_{singlet}^\eta = \begin{pmatrix} 4\lambda u & M_\eta \\ M_\eta & 0 \end{pmatrix}. \tag{5.30}$$

In the scalar sector, there are 4 real octets and 4 real singlets from η and $\bar{\eta}$ fields. The two scalar octets are mixed, as are the two pseudoscalar octets. The mass squared matrices for the octet are

$$M_{s-octet}^2 = \begin{pmatrix} (\tilde{m}_{soft}^2)_\eta^\eta + M_\eta^2 + 2\lambda u(-A_\lambda - \kappa u + 2\lambda u) & M_\eta(B_\eta - 2\lambda u) \\ M_\eta(B_\eta - 2\lambda u) & (\tilde{m}_{soft}^2)_{\bar{\eta}}^{\bar{\eta}} + M_\eta^2 \end{pmatrix} \tag{5.31}$$

$$M_{p-octet}^2 = \begin{pmatrix} (\tilde{m}_{soft}^2)_\eta^\eta + M_\eta^2 + 2\lambda u(A_\lambda + \kappa u + 2\lambda u) & -M_\eta(B_\eta + 2\lambda u) \\ -M_\eta(B_\eta + 2\lambda u) & (\tilde{m}_{soft}^2)_{\bar{\eta}}^{\bar{\eta}} + M_\eta^2 \end{pmatrix} \tag{5.32}$$

The singlet scalar mass matrices are

$$M_{s-singlet}^2 = \begin{pmatrix} (\tilde{m}_{soft}^2)_\eta^\eta + M_\eta^2 + 4\lambda u(A_\lambda + \kappa u + 4\lambda u) & M_\eta(B_\eta + 4\lambda u) \\ M_\eta(B_\eta + 4\lambda u) & (\tilde{m}_{soft}^2)_{\bar{\eta}} + M_\eta^2 \end{pmatrix} \quad (5.33)$$

$$M_{p-singlet}^2 = \begin{pmatrix} (\tilde{m}_{soft}^2)_\eta^\eta + M_\eta^2 - 4\lambda u(A_\lambda - \kappa u - 4\lambda u) & -M_\eta(B_\eta - 4\lambda u) \\ -M_\eta(B_\eta - 4\lambda u) & (\tilde{m}_{soft}^2)_{\bar{\eta}} + M_\eta^2 \end{pmatrix} \quad (5.34)$$

The soft masses $(\tilde{m}_{soft}^2)_\eta^\eta$ and $(\tilde{m}_{soft}^2)_{\bar{\eta}}$ are given in Eqs. (61)–(62) of the Appendix.

5.5 Numerical results

We are now ready to present our numerical results for the SUSY spectrum. The scale of SUSY breaking, M_{aux} , should be in the range 40–120 TeV for the MSSM particles to have masses in the range 100 GeV – 2 TeV. Note that there is a large hierarchy in the masses of the gluino and the neutral Wino, $\frac{M_3}{M_2} \simeq 7.1$ (after taking account of radiative corrections), in AMSB models. Furthermore the lightest chargino is nearly mass degenerate with the neutral Wino, so $M_2 \gtrsim 100$ GeV is required to satisfy the LEP chargino mass bound.

The $SU(3)_H$ gauge coupling g_4 is chosen so that the sleptons have positive mass squared ($g_4 \geq 0.5$). We allow g_4 to take two values, $g_4 = 0.55$ (Tables 5.2 and 5.4) and $g_4 = 1.0$ (Table 5.3). Symmetry breaking considerations constrain the couplings κ and λ as discussed in Sec. 5.3 after Eq. 5.14. In Tables 5.2 and 5.4 we have taken $M_{aux} = 47.112$ TeV corresponding to a light spectrum, while in Table we have $M_{aux} = 66.695$ TeV with an intermediate spectrum. Other input parameters are listed in the respective Table captions. The mass parameter M_η cannot be much larger than 1 TeV, as that would decouple the effects of η , $\bar{\eta}$ fields which are needed for consistent symmetry breaking.

We see from Table 5.2 that the lightest Higgs boson mass is $m_h \simeq 118$ GeV. This is very close to the current experimental limit. If $M_t = 180$ GeV is used (instead of $M_t = 176$ GeV), for the same set of input parameters, m_h will be 119 GeV. m_h being close to the current experimental limit is a generic prediction of our framework. It holds in the spectra of Tables 5.3 and as well. We conclude that $m_h \lesssim 120$ GeV in this model.

The masses of the sleptons will depend sensitively on the choice of g_4 . The sleptons are relatively light, $m_{slep} \lesssim 300$ GeV, with $g_4 = 0.55$, while they are heavy, $m_{slep} \simeq 800$ GeV, when $g_4 = 1.0$. Note however that there is a correlation in the slepton masses and the $SU(3)_H$ gauge boson masses (M_V), with the lighter sleptons corresponding to lighter $SU(3)_H$ gauge bosons. It is worth noting that very light sleptons, below the current experimental limits of about 100 GeV, would be inconsistent with the limits on M_V arising from $e^+e^- \rightarrow \mu^+\mu^-$ type processes (see Sec. 6). Note also that the left-handed and the right-handed sleptons are nearly degenerate to within about 10 GeV in this model. This is a numerical coincidence having to do with the values of g_1 and g_2 and the MSSM beta functions (see the last paper of Ref. [39]). The new $SU(3)_H$ gauge boson contributions to the slepton masses are the same for the left-handed and the right-handed sleptons.

In Tables 5.2–5.4 we have included the leading radiative corrections to the gaugino masses M_1 , M_2 and M_3 [54]. Including these radiative corrections we find (in Table 2) $M_1 : M_2 : M_3 = 3.0 : 1 : 7.4$. The lightest SUSY particle (LSP) is the neutral Wino, which is nearly mass degenerate with the charged Wino. In Tables 5.2–5.4 the mass splitting is about 60 MeV, but this does not take into account $SU(2)_L \times U(1)_Y$ breaking corrections [55]. These electroweak radiative corrections turn out to be very important, and we find $m_{\chi_1^\pm} - m_{\chi_1^0} \simeq 235$ MeV (with about 175 MeV arising from $SU(2)_L \times U(1)_Y$ breaking effects). The decay $\chi_1^\pm \rightarrow \chi_1^0 + \pi^\pm$ is then kinematically allowed, with the π^\pm being very soft. Once produced, the neutralino χ_1^0 will escape the detector without leaving any tracks. With the decay channel $\chi_1^\pm \rightarrow \chi_1^0 + \pi^\pm$ open, the lightest chargino will leave an observable track with a decay length of about a few cm. Search strategies for such a quasi-degenerate pair at colliders have been analyzed in Ref. [54, 56, 57].

In the $SU(3)_H$ sector, in Tables 5.2–5.4, the horizontal gauge boson has a mass of 1.5–4.0 TeV. The heavy Higgs bosons, Higgsinos, gauginos, squarks and the η fields all have masses $\lesssim (1 - 2)$ TeV.

MSSM Particles	Symbol	Mass (TeV)
Neutralinos	$\{m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}\}$	{0.146, 0.431}
Neutralinos	$\{m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}\}$	{0.876, 0.878}
Charginos	$\{m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}\}$	{0.146, 0.880}
Gluino	M_3	1.064
Higgs bosons	$\{m_h, m_H, m_A, m_{H^\pm}\}$	{0.118, 0.878, 0.877, 0.880}
R.H sleptons	$\{m_{\tilde{e}_R}, m_{\tilde{\mu}_R}, m_{\tilde{\tau}_1}\}$	{0.183, 0.183, 0.166}
L.H sleptons	$\{m_{\tilde{e}_L}, m_{\tilde{\mu}_L}, m_{\tilde{\tau}_2}\}$	{0.190, 0.190, 0.203}
Sneutrinos	$\{m_{\tilde{\nu}_e}, m_{\tilde{\nu}_\mu}, m_{\tilde{\nu}_\tau}\}$	{0.175, 0.175, 0.175}
R.H down squarks	$\{m_{\tilde{d}_R}, m_{\tilde{s}_R}, m_{\tilde{b}_1}\}$	{1.017, 1.017, 1.015}
L.H down squarks	$\{m_{\tilde{d}_L}, m_{\tilde{s}_L}, m_{\tilde{b}_2}\}$	{1.008, 1.008, 0.885}
R.H up squarks	$\{m_{\tilde{u}_R}, m_{\tilde{c}_R}, m_{\tilde{t}_1}\}$	{1.011, 1.011, 0.669}
L.H up squarks	$\{m_{\tilde{u}_L}, m_{\tilde{c}_L}, m_{\tilde{t}_2}\}$	{1.005, 1.005, 0.979}
New Particles	Symbol	Mass (TeV)
$SU(3)_H$ Gauge boson octet	M_V	2.213
Singlet Higgsino	$m_{\tilde{\phi}}$	0.402
Octet Higgsino/gaugino	$m_{\tilde{\phi}_{1,2}}$	{1.978, 2.450}
ϕ Higgs bosons	$\{m_{\phi_s}, m_{\phi_p}, m_{\phi\text{-octet}}\}$	{0.179, 0.624, 2.253}
Fermionic η (octet)	$m_{\eta_{1,2}}^{octet}$	{0.676, 1.480}
Fermionic η (singlet)	$m_{\eta_{1,2}}^{singlet}$	{0.479, 2.089}
Scalar η Higgs (octet)	$m_{\tilde{\eta}_{1,2}}^{s\text{-octet}}$	{0.454, 1.703}
Pseudoscalar η Higgs (octet)	$m_{\tilde{\eta}_{1,2}}^{p\text{-octet}}$	{0.908, 1.259}
Scalar η Higgs (singlet)	$m_{\tilde{\eta}_{1,2}}^{s\text{-singlet}}$	{0.717, 1.868}
Pseudoscalar η Higgs (singlet)	$m_{\tilde{\eta}_{1,2}}^{p\text{-singlet}}$	{0.264, 2.310}

TABLE 5.2. Sparticle masses for the choice $M_{aux} = 47.112$ TeV, $\tan \beta = 3.785$, $\mu = -0.873$ TeV, $y_b = 0.068$, $\lambda = 0.1$, $\kappa = 0.05$, $g_4 = 0.55$, $u = -4.024$ TeV, $M_\eta = 1.0$ TeV and $M_t = 0.176$ TeV.

MSSM Particles	Symbol	Mass (TeV)
Neutralinos	$\{m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}\}$	{0.198, 0.586}
Neutralinos	$\{m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}\}$	{1.179, 1.181}
Charginos	$\{m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}\}$	{0.198, 1.182}
Gluino	M_3	1.410
Higgs boson	$\{m_h, m_H, m_A, m_{H^\pm}\}$	{0.119, 1.179, 1.178, 1.181}
R.H sleptons	$\{m_{\tilde{e}_R}, m_{\tilde{\mu}_R}, m_{\tilde{\tau}_1}\}$	{0.245, 0.245, 0.227}
L.H sleptons	$\{m_{\tilde{e}_L}, m_{\tilde{\mu}_L}, m_{\tilde{\tau}_2}\}$	{0.254, 0.254, 0.267}
Sneutrinos	$\{m_{\tilde{\nu}_e}, m_{\tilde{\nu}_\mu}, m_{\tilde{\nu}_\tau}\}$	{0.242, 0.242, 0.242}
R.H down squarks	$\{m_{\tilde{d}_R}, m_{\tilde{s}_R}, m_{\tilde{b}_1}\}$	{1.373, 1.373, 1.193}
L.H down squarks	$\{m_{\tilde{d}_L}, m_{\tilde{s}_L}, m_{\tilde{b}_2}\}$	{1.361, 1.361, 1.370}
R.H up squarks	$\{m_{\tilde{u}_R}, m_{\tilde{c}_R}, m_{\tilde{t}_1}\}$	{1.365, 1.365, 0.940}
L.H up squarks	$\{m_{\tilde{u}_L}, m_{\tilde{c}_L}, m_{\tilde{t}_2}\}$	{1.359, 1.359, 1.276}
New Particles	Symbol	Mass (TeV)
$SU(3)_H$ Gauge boson octet	M_V	1.871
Singlet Higgsino	$m_{\tilde{\phi}}$	0.544
Octet Higgsino/gaugino	$m_{\tilde{\phi}_{1,2}}$	{1.553, 2.191}
ϕ Higgs bosons	$\{m_{\phi_s}, m_{\phi_p}, m_{\phi\text{-octet}}\}$	{0.247, 0.840, 1.955}
Fermionic η (octet)	$m_{\eta_{1,2}}^{octet}$	{0.716, 1.397}
Fermionic η (singlet)	$m_{\eta_{1,2}}^{singlet}$	{0.529, 1.890}
Scalar η Higgs (octet)	$m_{\tilde{\eta}_{1,2}}^{s\text{-octet}}$	{0.421, 1.699}
Pseudoscalar η Higgs (octet)	$m_{\tilde{\eta}_{1,2}}^{p\text{-octet}}$	{1.031, 1.098}
Scalar η Higgs (singlet)	$m_{\tilde{\eta}_{1,2}}^{s\text{-singlet}}$	{0.850, 1.593}
Pseudoscalar η Higgs (singlet)	$m_{\tilde{\eta}_{1,2}}^{p\text{-singlet}}$	{0.247, 2.189}

TABLE 5.3. Sparticle masses for the choice $M_{aux} = 63.695$ TeV, $\tan \beta = 4.02$, $\mu = -1.178$ TeV, $y_b = 0.0719$, $\lambda = 0.1$, $\kappa = 0.08$, $g_4 = 0.55$, $u = -3.402$ TeV, $M_\eta = 1.0$ TeV and $M_t = 0.1743$ TeV.

MSSM Particles	Symbol	Mass (TeV)
Neutralinos	$\{m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}\}$	{0.148, 0.436}
Neutralinos	$\{m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}\}$	{0.876, 0.878}
Charginos	$\{m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}\}$	{0.148, 0.878}
Gluino	M_3	1.064
Higgs boson	$\{m_h, m_H, m_A, m_{H^\pm}\}$	{0.118, 0.878, 0.877, 0.880}
R.H sleptons	$\{m_{\tilde{e}_R}, m_{\tilde{\mu}_R}, m_{\tilde{\tau}_1}\}$	{0.825, 825, 0.821}
L.H sleptons	$\{m_{\tilde{e}_L}, m_{\tilde{\mu}_L}, m_{\tilde{\tau}_2}\}$	{0.827, 0.827, 0.830}
Sneutrinos	$\{m_{\tilde{\nu}_e}, m_{\tilde{\nu}_\mu}, m_{\tilde{\nu}_\tau}\}$	{0.823, 0.823, 0.823}
R.H down squarks	$\{m_{\tilde{d}_R}, m_{\tilde{s}_R}, m_{\tilde{b}_1}\}$	{1.017, 1.017, 1.015}
L.H down squarks	$\{m_{\tilde{d}_L}, m_{\tilde{s}_L}, m_{\tilde{b}_2}\}$	{1.008, 1.008, 0.885}
R.H up squarks	$\{m_{\tilde{u}_R}, m_{\tilde{c}_R}, m_{\tilde{t}_1}\}$	{1.011, 1.011, 0.669}
L.H up squarks	$\{m_{\tilde{u}_L}, m_{\tilde{c}_L}, m_{\tilde{t}_2}\}$	{1.005, 1.005, 0.979}
New Particles	Symbol	Mass (TeV)
$SU(3)_H$ Gauge boson octet	M_V	3.779
Singlet Higgsino	$m_{\tilde{\phi}}$	1.058
Octet Higgsino/gaugino	$m_{\tilde{\phi}_{1,2}}$	{3.071, 4.495}
ϕ Higgs bosons	$\{m_{\phi_s}, m_{\phi_p}, m_{\phi\text{-octet}}\}$	{0.465, 1.646, 3.940}
Fermionic η (octet)	$m_{\eta_{1,2}}^{\text{octet}}$	{0.254, 2.521}
Fermionic η (singlet)	$m_{\eta_{1,2}}^{\text{singlet}}$	{0.137, 4.672}
Scalar η Higgs (octet)	$m_{\tilde{\eta}_{1,2}}^{s\text{-octet}}$	{0.588, 3.090}
Pseudoscalar η Higgs (octet)	$m_{\tilde{\eta}_{1,2}}^{p\text{-octet}}$	{1.058, 1.952}
Scalar η Higgs (singlet)	$m_{\tilde{\eta}_{1,2}}^{s\text{-singlet}}$	{0.964, 4.116}
Pseudoscalar η Higgs (singlet)	$m_{\tilde{\eta}_{1,2}}^{p\text{-singlet}}$	{0.711, 5.224}

TABLE 5.4. Sparticle masses for the choice $M_{aux} = 47.112$ TeV, $\tan \beta = 3.785$, $\mu = -0.873$ TeV, $y_b = 0.068$, $\lambda = 0.3$, $\kappa = 0.14$, $g_4 = 1.0$, $u = -3.779$ TeV, $M_\eta = 0.800$ TeV and $M_t = 0.176$ TeV.

5.6 Experimental signatures

The Lightest SUSY particle in the model is the neutral Wino (χ_1^0) which is nearly mass degenerate with the lightest chargino (χ_1^\pm), with a mass splitting of about 235 MeV. At the Tevatron Run 2 as well as at the LHC, the process $p\bar{p}$ (or pp) $\rightarrow \chi_1^0 + \chi_1^\pm$ will produce these SUSY particles. Naturalness suggest that $m_{\chi_1^0}, m_{\chi_1^\pm} \lesssim 300$ GeV (corresponding to $m_{gluino} \lesssim 2$ TeV). Strategies for detecting such a quasi-degenerate pair has been carried out in Ref. [54, 56, 57]. In the MSSM sector our model predicts $\tan\beta \simeq 4.0$ and $m_h \lesssim 120$ GeV, both of which can be tested at the LHC.

If the $SU(3)_H$ gauge coupling g_4 takes small values ($g_4 \simeq 0.55$), the slepton masses will be near the current experimental limit. For larger values of g_4 ($\simeq 1.0$) the slepton masses are comparable to those of the squarks.

The $SU(3)_H$ gauge boson masses are in the range $M_V = 1.5 - 4.0$ TeV. Although relatively light, these particles do not mediate leptonic FCNC, owing to the approximate $SU(3)_{H+G}$ global symmetries present in the model.

The most stringent constraint on M_V arises from the process $e^+e^- \rightarrow \mu^+\mu^-$. LEP II has set severe constraints on lepton compositeness [51, 58] from this process. We focus on one such amplitude, involving all left-handed lepton fields. In our model, the effective Lagrangian for this process is

$$L^{eff} = -\frac{2g_4^2}{3M_V^2}(\bar{e}_L\gamma_\mu e_L)(\bar{\mu}_L\gamma^\mu\mu_L). \quad (5.35)$$

Comparing with $\Lambda_{LL}^-(ee\mu\mu) > 6.3$ TeV [51, 58], we obtain $\frac{M_V}{g_4} \geq 2.05$ TeV. For $g_4 = 0.55$ (1.0) this implies $M_V \geq 1.129$ (2.052) TeV. From Tables 5.2–5.4 we find that these constraints are satisfied.

The model as it stands has an unbroken Z_2 symmetry (in addition to the usual R -parity) under which the superfields $\eta, \bar{\eta}$ are odd and all other superfields are even. If this symmetry is exact, the lightest of the η and $\bar{\eta}$ fields (a pseudoscalar singlet Higgs in the fits of Tables 5.2–5.3 and a singlet fermion in Table 5.4) will be stable. We envision this Z_2 symmetry to be broken by higher dimensional terms of the type $L_\alpha H_u \Phi^\alpha \bar{\eta}_\beta \Phi^\beta / \Lambda^2$. Such a term will induce the decay $\eta_1^{p-singlet} \rightarrow L + \chi_1^0$ with a lifetime less than 1 second for $\Lambda \leq 10^9$ GeV. This would make these η particles cosmologically

safe. It may be pointed out that the same effective operator, along with a TeV scale mass for the η fields, can provide small neutrino masses even in the absence of the operators given in Eq. 5.3.

5.7 Origin of the μ term

Any satisfactory SUSY breaking model should also have a natural explanation for the μ term (the coefficient of $H_u H_d$ term in Eq. (5.1)). In gravity mediated SUSY breaking models, there are at least three solutions to the μ problem. The Giudice–Masiero mechanism [59] which explains the μ term through the Kahler potential $\int H_u H_d Z^* d^4\theta / M_{pl}$ is not readily adaptable to the AMSB framework. The NMSSM extension which introduces singlet fields can in principle provide a natural explanation of the μ term in the AMSB scenario. We have however found that replacing $\mu H_u H_d$ by the term $S H_u H_d$ in the superpotential alone can not lead to realistic SUSY breaking. It is possible to make the NMSSM scenario compatible with symmetry breaking in the AMSB framework by introducing a new set of fields which couple to the singlet S . We do not follow this non–minimal alternative here.

There is a natural explanation for the μ parameter in the context of AMSB models, as suggested in Ref. [36]. It assumes a Lagrangian term $L \supset \alpha \int d^4\theta \frac{(\Sigma + \Sigma^\dagger)}{M_{Pl}} H_u H_d \Phi^\dagger \Phi$, where Σ is a hidden sector field which breaks SUSY and Φ is the compensator field. After a rescaling, $H_u \rightarrow \Phi H_u$, $H_d \rightarrow \Phi H_d$, this term becomes $L \supset \alpha \int d^4\theta \frac{(\Sigma + \Sigma^\dagger)}{M_{Pl}} H_u H_d \frac{\Phi^\dagger}{\Phi}$, which generates a μ term in a way similar to the Giudice–Masiero mechanism [59]. The $B\mu$ term is induced only through the super–Weyl anomaly and has the form given in Eq. (4.9). Our predictions for $\tan\beta$ and m_h depend sensitively on this assumption.

We now point out that the μ term may have an alternative explanation in the context of AMSB models. This is obtained by promoting $\mu H_u H_d$ in the superpotential to the following [60]:

$$W' = \frac{a H_u H_d S^2}{M_{Pl}} + \frac{b S^2 \bar{S}^2}{M_{Pl}}. \quad (5.36)$$

Here S and \bar{S} are standard model singlet fields. Including AMSB induced soft parameters for these singlets (which can arise in a variety of ways), this superpotential will have a minimum where $\langle S \rangle \simeq \langle \bar{S} \rangle \simeq \sqrt{M_{SUSY} M_{Pl}}$. This would induce μ term of order M_{SUSY} , as needed. From the effective low energy point of view, the superpotential will appear to have an explicit μ term. The B term will have a form as given in Eq. 4.9.

5.8 Summary

In this chapter we have suggested a new scenario for solving the tachyonic slepton mass problem of anomaly mediated SUSY breaking models. An asymptotically free $SU(3)_H$ horizontal gauge symmetry acting on the lepton superfields provides positive masses to the sleptons. The $SU(3)_H$ symmetry must be broken at the TeV scale. Potentially dangerous FCNC processes mediated by the $SU(3)_H$ gauge bosons are shown to be suppressed adequately via approximate global symmetries that are present in the model.

Our scenario predicts $m_h \lesssim 120$ GeV for the lightest Higgs boson mass of MSSM and $\tan\beta \simeq 4.0$. The lightest SUSY particle is the neutral Wino which is nearly degenerate with the lightest chargino and is a candidate for cold dark matter. The full spectrum of the model is given in Tables 5.2–5.4 for various choices of input parameters. The very few parameters of our model are highly constrained by the consistency of symmetry breaking.

CHAPTER 6

$SU(2)_H$ HORIZONTAL SYMMETRY AS A SOLUTION TO THE SLEPTON MASS PROBLEM OF ANOMALY MEDIATION

6.1 Introduction

Family symmetries may give a positive mass-squared contribution to the sleptons in AMSB. The simplest of such symmetry is an $SU(2)_H$ non-Abelian symmetry. This symmetry when acting on leptons only can be asymptotically free, hence their beta-function will be negative. This is very important because with this new symmetry, the sleptons enjoys the same freedom as the quarks and hence can solve the negative slepton mass problem of AMSB. The quarks are singlet of $SU(2)_H$ but it is possible that they transform under a different $SU(2)_H^q$ symmetry, so that there is an underlying quark-lepton symmetry. Here we will focus on a model where quarks carry no family symmetry.

In this chapter we suggest and investigate the possibility of solving the negative slepton mass problem of AMSB using this $SU(2)_H$ symmetry broken at the TeV scale. The leptons of the first two families transform as a doublet of $SU(2)_H$ and those of the third family transform as singlet under this new symmetry. The sleptons of the first two family gets a large positive contribution to their soft masses from the $SU(2)_H$ gauge sector. With e and μ forming a doublet of $SU(2)_H$, an important issue is how to split their masses, since in Nature $m_e \neq m_\mu$. We introduce two new vector-like fields that couples to the third family which will help to achieve $m_e \neq m_\mu$.

The model is quite predictive. The LSP is the Wino which is nearly mass degenerate with the chargino. The lightest Higgs boson mass is predicted to be $m_h \lesssim 135$ GeV, and the parameter $\tan\beta$ is found to be $\tan\beta \simeq 40$. This model

is completely different from the previous model because it also predicts a different mass hierarchy for the \tilde{e} , $\tilde{\mu}$ and $\tilde{\tau}$. In particular $m_{\tilde{e}}$, $m_{\tilde{\mu}}$ and $m_{\tilde{\tau}}$ are all quite different, which is a characteristic signature of this model. In addition, this model can easily be tested at the LHC by direct discovery of the gauge bosons associated with $SU(2)_H$.

The plan of the chapter is as follows. In section 6.2 we introduce our model. In section 6.3 we analyze the Higgs potential. The SUSY spectrum is presented in section 6.4. We discuss our numerical results in section 6.5. In section 6.6 we discuss the experimental implication of the model. We summarize in section 6.7.

6.2 $SU(2)_H$ horizontal symmetry

We define the gauge group symmetry of the model as

$$G_H \equiv SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)_H,$$

where $SU(2)_H$ is a horizontal symmetry that acts on the first two families of leptons. The third family is a singlet under this new $SU(2)_H$ symmetry. A pair of vector like leptons, E , E^c , which are $SU(2)_H$ singlets are needed to ensure $m_e \neq m_\mu$. The spectrum of the model is listed in Table. 6.1. The gauge group $SU(2)_H$ defined above is asymptotically free (β function is given in Eq. B.20) with this spectrum.

The superpotential of the model consistent with the gauge symmetries reads

$$\begin{aligned} W = & (Y_u)_{ij} Q_i H_u u_j^c + (Y_d)_{ij} Q_i H_d d_j^c + f_{e\mu} \psi_\alpha \psi_\alpha^c H_d + f_\tau \psi_\tau \tau^c H_d \\ & + f_{\tau E} \psi_\tau E^c H_d + f_{eE} E \psi^c \phi_u + \mu H_u H_d + \mu' \phi_u \phi_d + M_E E E^c \end{aligned} \quad (6.1)$$

It turns out that there is a Z_4 symmetry present in the Lagrangian, under which

$$\phi_u \rightarrow i\phi_u, \quad \phi_d \rightarrow -i\phi_d, \quad E \rightarrow -iE, \quad E^c \rightarrow iE^c, \quad \psi_\tau \rightarrow -i\psi_\tau, \quad \tau^c \rightarrow i\tau^c.$$

This Z_4 symmetry forbids the term $E\psi^c\phi_d$, which will be important to define an unbroken muon number. Since $SU(2)_H$ is broken at TeV, the gauge bosons of $SU(2)_H$ can potentially lead to large FCNC processes. The most dangerous of these are in the muon sector, eg; $\mu \rightarrow 3e$. Such process are forbidden by an unbroken muon number, making TeV scale horizontal symmetry phenomenologically consistent.

Superfield	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$SU(2)_H$
Q_i	3	2	$\frac{1}{6}$	1
u_i^c	$\bar{3}$	1	$-\frac{2}{3}$	1
d_i^c	$\bar{3}$	1	$\frac{1}{3}$	1
ψ_α	1	2	$-\frac{1}{2}$	2
ψ_α^c	1	1	+1	2
ψ_τ	1	2	$-\frac{1}{2}$	1
τ^c	1	1	+1	1
H_u	1	2	$\frac{1}{2}$	1
H_d	1	2	$-\frac{1}{2}$	1
ϕ_u	1	1	0	2
ϕ_d	1	1	0	2
E	1	1	-1	1
E^c	1	1	+1	1
Ψ_N	1	1	0	2

TABLE 6.1. Particle content and charge assignment of the model. The indices i and α take values $i = 1 - 3$ and $\alpha = 1 - 2$.

In the model, the ψ_α and ψ_α^c fields contain the first two family of leptons (e and μ) which transforms as a doublet under the $SU(2)_H$ gauge group, while the members of the third family (ψ_τ and τ^c) transform as singlets under the $SU(2)_H$ gauge group. The field Ψ_N , which transforms as a doublet under $SU(2)_H$ and as singlet under the SM gauge group, is introduced in order to cancel the Witten anomaly.

The neutinos in the model get masses from the following non-renormalizable operators:

$$\psi_\tau \psi_\tau \frac{H_u H_d}{M}, \quad \psi_\alpha \psi_\alpha \frac{H_u H_u}{M'^3} \phi_{u,d} \phi_{u,d}, \quad \psi_\alpha \psi_\tau \frac{H_u H_u}{M''} \phi_{u,d}. \quad (6.2)$$

These terms will lead to a consistent neutrino oscillation phenomenology.

6.3 Symmetry breaking

The symmetry breaking is achieved in the form

$$G_H \rightarrow G_{SM} \rightarrow SU(3)_C \times U(1)_{EM},$$

where $G_{SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$. The model has the possibility to be consistent with the known low energy physics. The new Higgs multiplets ϕ_u and ϕ_d are sufficient to break the $G_H \rightarrow G_{SM}$ completely near the TeV scale.

The tree level Higgs potential can be written as

$$\begin{aligned} V(H_u, H_d, \phi_u, \phi_d) &= (m_{H_u}^2 + \mu^2)|H_u|^2 + (m_{H_d}^2 + \mu^2)|H_d|^2 + B\mu(H_u H_d + \text{c.c.}) \\ &+ \frac{(g_2^2 + g_1^2)}{8}(|H_u|^2 - |H_d|^2)^2 + \frac{g_2^2}{2}|H_u H_d|^2 + \frac{g_4^2}{8}(|\phi_u|^2 - |\phi_d|^2)^2 \\ &+ \frac{g_4^2}{2}|\phi_u \phi_d|^2 + (m_{\phi_u}^2 + \mu'^2)|\phi_u|^2 + (m_{\phi_d}^2 + \mu'^2)|\phi_d|^2 \\ &+ B'\mu'(\phi_u \phi_d + \text{c.c.}). \end{aligned}$$

The soft masses $m_{H_u}^2$ and $m_{H_d}^2, m_{\phi_u}^2$ and $m_{\phi_d}^2$ parameters are determined in terms of the single parameter M_{aux} . The B and B' parameters are taken to be free in the model but in some special class of models, they are determined also by the same mass parameter M_{aux} . Upon symmetry breaking, the Higgs fields acquire VEV's

$$\langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \quad \langle H_d \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \quad \langle \phi_u \rangle = \begin{pmatrix} 0 \\ u_u \end{pmatrix}, \quad \langle \phi_d \rangle = \begin{pmatrix} u_d \\ 0 \end{pmatrix}. \quad (6.3)$$

It is desired that the VEVs obey $\langle \phi_u \rangle, \langle \phi_d \rangle \gg \langle H_u \rangle, \langle H_d \rangle$, in order for the symmetry breaking to be consistent.

Minimization of the Higgs potential $V(H_u, H_d, \phi_u, \phi_d)$ gives

$$\sin 2\beta = \frac{-2B\mu}{2\mu^2 + m_{H_u}^2 + m_{H_d}^2}, \quad \mu^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{M_Z^2}{2}, \quad (6.4)$$

$$\sin 2\beta' = \frac{-2B'\mu'}{2\mu'^2 + m_{\phi_u}^2 + m_{\phi_d}^2}, \quad \mu'^2 = \frac{m_{\phi_d}^2 - m_{\phi_u}^2 \tan^2 \beta'}{\tan^2 \beta' - 1} - \frac{M_{Z'}^2}{2}, \quad (6.5)$$

where we have introduced the notation $u_u = u \sin \beta'$, $u_d = u \cos \beta'$, $u^2 = u_u^2 + u_d^2$, $\tan \beta' = \frac{u_u}{u_d}$ and $M_{Z'}^2 = \frac{g_4^2}{2}(u_u^2 + u_d^2)$. $M_{Z'}$ is the mass of the gauge boson associated with the $SU(2)_H$.

To find the physical Higgs boson mass, we parameterize the Higgs fields (in the unitary gauge) as

$$\begin{aligned} H_u &= \begin{pmatrix} H^+ \sin \beta \\ v_u + \frac{1}{\sqrt{2}}(\phi_2 + i \cos \beta \phi_3) \end{pmatrix}, & \langle H_d \rangle &= \begin{pmatrix} v_d + \frac{1}{\sqrt{2}}(\phi_1 + i \sin \beta \phi_3) \\ H^- \cos \beta \end{pmatrix}, \\ \phi_u &= \begin{pmatrix} \phi^+ \sin \beta' \\ u_u + \frac{1}{\sqrt{2}}(\phi_4 + i \cos \beta' \phi_5) \end{pmatrix}, & \phi_d &= \begin{pmatrix} u_d + \frac{1}{\sqrt{2}}(\phi_6 + i \sin \beta' \phi_5) \\ \phi^- \cos \beta' \end{pmatrix}. \end{aligned} \quad (6.6)$$

The Higgs masses are obtained by expanding the Higgs potential and keeping only terms quadratic in the fields.

The masses of the CP-odd Higgs bosons $\{\phi_3, \phi_5\}$ are

$$m_A^2 = \frac{-2B\mu}{\sin 2\beta}, \quad m_{A'}^2 = -\frac{2B'\mu'}{\sin 2\beta'}. \quad (6.7)$$

The mass matrices for the CP-even neutral Higgs bosons $\{\phi_1, \phi_2\}$ and $\{\phi_4, \phi_6\}$ are decoupled. They are given by

$$(\mathcal{M}^2)_{cp\text{-}even} = \begin{pmatrix} m_A^2 \cos^2 \beta + M_Z^2 \sin^2 \beta & -\{m_A^2 + M_Z^2\} \frac{\sin 2\beta}{2} \\ -\{m_A^2 + M_Z^2\} \frac{\sin 2\beta}{2} & m_A^2 \sin^2 \beta + M_Z^2 \sin^2 \beta \end{pmatrix}, \quad (6.8)$$

$$(\mathcal{M}'^2)_{cp\text{-}even} = \begin{pmatrix} m_{A'}^2 \cos^2 \beta' + M_Z'^2 \sin^2 \beta' & -\{m_{A'}^2 + M_Z'^2\} \frac{\sin 2\beta'}{2} \\ -\{m_{A'}^2 + M_Z'^2\} \frac{\sin 2\beta'}{2} & m_{A'}^2 \sin^2 \beta' + M_Z'^2 \sin^2 \beta' \end{pmatrix}. \quad (6.9)$$

Finally, the charged Higgs boson mass (H^\pm and ϕ^\pm) is given by

$$m_{H^\pm}^2 = m_A^2 + M_W^2 \quad m_{\phi^\pm}^2 = m_{A'}^2 + M_Z'^2 \quad (6.10)$$

ϕ^\pm are electrically neutral, they are ‘‘charged’’ under $SU(2)_H$.

The Majorana mass matrix of the neutralinos $\{\tilde{B}, \tilde{W}_3, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{B}_H, \tilde{\phi}_d^0, \tilde{\phi}_u^0\}$ is

$$\mathcal{M}^{(0)} = \begin{pmatrix} M_1 & 0 & -\frac{v_d}{\sqrt{2}}g_1 & \frac{v_u}{\sqrt{2}}g_1 & 0 & 0 & 0 \\ 0 & M_2 & \frac{v_d}{\sqrt{2}}g_2 & -\frac{v_u}{\sqrt{2}}g_2 & 0 & 0 & 0 \\ -\frac{v_d}{\sqrt{2}}g_1 & \frac{v_d}{\sqrt{2}}g_2 & 0 & -\mu & 0 & 0 & 0 \\ \frac{v_u}{\sqrt{2}}g_1 & -\frac{v_u}{\sqrt{2}}g_2 & -\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_4 & \frac{u_d}{\sqrt{2}}g_4 & -\frac{u_u}{\sqrt{2}}g_4 \\ 0 & 0 & 0 & 0 & \frac{u_d}{\sqrt{2}}g_4 & 0 & -\mu' \\ 0 & 0 & 0 & 0 & -\frac{u_u}{\sqrt{2}}g_4 & -\mu' & 0 \end{pmatrix}, \quad (6.11)$$

where M_1 , M_2 and M_4 are the gaugino masses for $U(1)_Y$, $SU(2)_L$ and $SU(2)_H$ which are listed in Appendix B. The physical neutralino masses $m_{\tilde{\chi}_i^0}$ ($i=1-7$) are obtained as the eigenvalues of this mass matrix Eq. (6.11).

In the basis $\{\tilde{W}^+, \tilde{H}_u^+\}$, $\{\tilde{W}^-, \tilde{H}_d^-\}$, the chargino (Dirac) mass matrix is

$$\mathcal{M}^{(c)} = \begin{pmatrix} M_2 & g_2 v_d \\ g_2 v_u & \mu \end{pmatrix}. \quad (6.12)$$

Similarly, for the $SU(2)_H$ sector, we have

$$\tilde{\mathcal{M}}^{(c)} = \begin{pmatrix} M_4 & g_4 u_d \\ g_4 u_u & \mu' \end{pmatrix}. \quad (6.13)$$

The three $SU(2)_H$ gauge boson masses are given by

$$\mathcal{M}_V^2 = \frac{g_4^2}{2}(u_u^2 + u_d^2). \quad (6.14)$$

6.3.1 Lepton masses

Now we describe briefly how to obtain the masses of the ordinary leptons. We have introduced E and E^c fields in the superpotential Eq. (6.1) for the purpose of breaking $e - \mu$ degeneracy. These new fields mix with the usual leptons leading to the mass matrix

$$(e \quad \mu \quad \tau \quad E) \begin{pmatrix} f_\mu v_d & 0 & 0 & 0 \\ 0 & f_\mu v_d & 0 & 0 \\ 0 & 0 & f_\tau v_d & f_{\tau E} v_d \\ f_{eE} u_u & 0 & 0 & M_E \end{pmatrix} \begin{pmatrix} e^c \\ \mu^c \\ \tau^c \\ E^c \end{pmatrix}. \quad (6.15)$$

The muon field completely decouples with mass

$$m_\mu = f_\mu v_d. \quad (6.16)$$

We are then left with a 3×3 mass matrix for the e , τ and E fields. The eigenvalue equation can be easily solved using the hierarchy $m_e \ll m_\tau \ll m_E$ and the result is

$$\begin{aligned} m_\tau &\simeq f_\tau v_d \sqrt{\left\{1 + \frac{f_{\tau E}^2 f_{eE}^2}{f_\tau^2} \frac{u_u^2}{M_E^2 + f_{eE}^2 u_u^2}\right\}}, \\ m_e &\simeq \frac{m_\mu M_E}{\sqrt{M_E^2 + f_{eE}^2 u_u^2 + \frac{f_{\tau E}^2 f_{eE}^2 u_u^2}{f_\tau^2}}}, \\ m_E &\simeq \sqrt{M_E^2 + f_{eE}^2 u_u^2}. \end{aligned} \quad (6.17)$$

Note that $m_e \neq m_\mu$, showing consistency of the model.

6.4 The SUSY spectrum

We will show in this section that the tachyonic slepton problem is cured by virtue of the positive contribution from the $SU(2)_H$ gauge sector to the masses for the first two family and a large Yukawa coupling for the third family.

6.4.1 Slepton masses

The slepton masses are given by a 2×2 mass matrix for the smuon (since it decouples) and a 6×6 mass matrix for the $e, \tau, E, e^c, \tau^c, E^c$ fields. The smuon mass-squareds are given by the eigenvalues of the mass matrix

$$M_{\tilde{\mu}}^2 = \begin{pmatrix} m_{\tilde{\mu}}^2 & m_{\mu} (A_{f_{e\mu}} - \mu \tan \beta) \\ m_{\mu} (A_{f_{e\mu}} - \mu \tan \beta) & m_{\tilde{\mu}^c}^2 \end{pmatrix}, \quad (6.18)$$

where the diagonal entries are

$$\begin{aligned} m_{\tilde{\mu}}^2 &= \frac{M_{aux}^2}{(16\pi^2)} \left[2f_{e\mu}\beta(f_{e\mu}) - \left(\frac{3}{2}g_2\beta(g_2) + \frac{3}{10}g_1\beta(g_1) + \frac{3}{2}g_4\beta(g_4) \right) \right] \\ &+ m_{\mu}^2 + \frac{g_4^2}{4}(u_u^2 - u_d^2), \\ m_{\tilde{\mu}^c}^2 &= \frac{M_{aux}^2}{(16\pi^2)} \left[2f_{e\mu}\beta(f_{e\mu}) - \left(\frac{6}{5}g_1\beta(g_1) + \frac{3}{2}g_4\beta(g_4) \right) \right] \\ &+ m_{\mu}^2 + \frac{g_4^2}{4}(u_u^2 - u_d^2). \end{aligned}$$

Note that the positive contributions from the $SU(2)_H$ gauge sector are provided by the term $-\frac{3}{2}g_4\beta(g_4)$, with gauge beta function $\beta(g_4) = -\frac{3}{16\pi^2}g_4^3$. This contribution ensures that the mass-squareds of all sleptons are positive when $g_4 \geq 0.9$. It is important to point out that the $SU(2)_H$ D -term contributions to the diagonal entries of the mass matrix Eq. (6.18) can either be positive or negative but it must be such that its overall contribution is rather small compared to the soft mass term.

The mass matrix for the other sleptons is in the form

$$\begin{pmatrix} m_{\tilde{e}}^2 & 0 & f_{e\mu}f_{eE}v_d u_u & f_{e\mu}(A_{e\mu}v_d + \mu v_u) & 0 & 0 \\ 0 & m_{\tilde{\tau}}^2 & M_E f_{\tau E} v_d & 0 & f_{\tau}(A_{\tau}v_d + \mu v_u) & f_{\tau E}(A_{\tau E}v_d + \mu v_u) \\ f_{e\mu}f_{eE}v_d u_u & M_E f_{\tau E} v_d & m_{\tilde{E}}^2 & f_{eE}(A_{eE}u_u + \mu' u_d) & 0 & M_E B_E \\ f_{e\mu}(A_{e\mu}v_d + \mu v_u) & 0 & f_{eE}(A_{eE}u_u + \mu' u_d) & m_{\tilde{e}^c}^2 & 0 & M_E f_{eE} u_u \\ 0 & f_{\tau}(A_{\tau}v_d + \mu v_u) & 0 & 0 & m_{\tilde{\tau}^c}^2 & f_{\tau}f_{\tau E} v_d^2 \\ 0 & f_{\tau E}(A_{\tau E}v_d + \mu v_u) & M_E B_E & M_E f_{eE} u_u & f_{\tau}f_{\tau E} v_d^2 & m_{\tilde{E}^c}^2 \end{pmatrix},$$

where the diagonal entries of this mass matrix read

$$\begin{aligned}
m_{\tilde{e}}^2 &= \frac{M_{aux}^2}{(16\pi^2)} \left[2f_{e\mu}\beta(f_{e\mu}) - \left(\frac{3}{2}g_2\beta(g_2) + \frac{3}{10}g_1\beta(g_1) + \frac{3}{2}g_4\beta(g_4) \right) \right] \\
&+ f_{e\mu}^2 v_d^2 + \frac{g_4^2}{4}(u_d^2 - u_u^2), \\
m_{\tilde{e}^c}^2 &= \frac{M_{aux}^2}{(16\pi^2)} \left[2f_{e\mu}\beta(f_{e\mu}) - \left(\frac{6}{5}g_1\beta(g_1) + \frac{3}{2}g_4\beta(g_4) \right) \right] \\
&+ f_{e\mu}^2 v_d^2 + f_{eE}^2 u_u^2 + \frac{g_4^2}{4}(u_d^2 - u_u^2) \\
m_{\tilde{\tau}}^2 &= \frac{M_{aux}^2}{(16\pi^2)} \left[f_{\tau}\beta(f_{\tau}) + f_{\tau E}\beta(f_{\tau E}) - \left(\frac{3}{10}g_1\beta(g_1) + \frac{3}{2}g_2\beta(g_2) \right) \right] + (f_{\tau}^2 + f_{\tau E}^2)v_d^2, \\
m_{\tilde{\tau}^c}^2 &= \frac{M_{aux}^2}{(16\pi^2)} \left[2f_{\tau}\beta(f_{\tau}) - \left(\frac{6}{5}g_1\beta(g_1) \right) \right] + f_{\tau}^2 v_d^2 \\
m_{\tilde{E}}^2 &= \frac{M_{aux}^2}{(16\pi^2)} \left[f_{eE}\beta(f_{eE}) - \left(\frac{6}{5}g_1\beta(g_1) \right) \right] + m_E^2 + f_{eE}^2 u_u^2, \\
m_{\tilde{E}^c}^2 &= \frac{M_{aux}^2}{(16\pi^2)} \left[f_{\tau e}\beta(f_{\tau e}) - \left(\frac{6}{5}g_1\beta(g_1) \right) \right] + m_E^2 + f_{\tau E}^2 v_d^2
\end{aligned} \tag{6.19}$$

The requirement that the slepton masses are positive puts constraints on the couplings f_{τ} , f_{eE} , $f_{\tau e}$ and g_4 .

6.4.2 Squark masses

The mixing matrix for the squark sector is similar to the slepton sector, except that they receive no $SU(2)_H$ gauge contributions. The diagonal entries of the up and the down squark mass matrices are given by [61]

$$\begin{aligned}
m_{\tilde{U}_i}^2 &= (m_{soft}^2)_{\tilde{Q}_i}^{\tilde{Q}_i} + m_{U_i}^2 + \frac{1}{6} (4M_W^2 - M_Z^2) \cos 2\beta, \\
m_{\tilde{U}_i^c}^2 &= (m_{soft}^2)_{\tilde{U}_i^c}^{\tilde{U}_i^c} + m_{U_i}^2 - \frac{2}{3} (M_W^2 - M_Z^2) \cos 2\beta, \\
m_{\tilde{D}_i}^2 &= (m_{soft}^2)_{\tilde{Q}_i}^{\tilde{Q}_i} + m_{D_i}^2 - \frac{1}{6} (2M_W^2 + M_Z^2) \cos 2\beta, \\
m_{\tilde{D}_i^c}^2 &= (m_{soft}^2)_{\tilde{D}_i^c}^{\tilde{D}_i^c} + m_{D_i}^2 + \frac{1}{3} (M_W^2 - M_Z^2) \cos 2\beta,
\end{aligned} \tag{6.20}$$

where m_{U_i} and m_{D_i} are the quark masses of the different generations with $i = 1, 2, 3$.

The squark soft masses are obtained from the RGE as

$$(m_{soft}^2)_{\tilde{Q}_i}^{\tilde{Q}_i} = \frac{M_{aux}^2}{16\pi^2} \left(Y_{u_i}\beta(Y_{u_i}) + Y_{d_i}\beta(Y_{d_i}) - \frac{1}{30}g_1\beta(g_1) - \frac{3}{2}g_2\beta(g_2) - \frac{8}{3}g_3\beta(g_3) \right) \tag{6.21}$$

$$(m_{soft}^2)_{\tilde{U}_i^c} = \frac{M_{aux}^2}{16\pi^2} \left(2Y_{u_i}\beta(Y_{u_i}) - \frac{8}{15}g_1\beta(g_1) - \frac{8}{3}g_3\beta(g_3) \right), \quad (6.22)$$

$$(m_{soft}^2)_{\tilde{D}_i^c} = \frac{M_{aux}^2}{16\pi^2} \left(2Y_{d_i}\beta(Y_{d_i}) - \frac{2}{15}g_1\beta(g_1) - \frac{8}{3}g_3\beta(g_3) \right). \quad (6.23)$$

6.5 Numerical results

Here we present our numerical results for the SUSY spectrum. We first performed a one-loop accuracy numerical analysis to determine the sparticle and Higgs Spectrum. For experimental inputs for the SM gauge couplings we use the same procedure Ref. [61] for the g_1, g_2, g_3 with the central value of the top mass taken to be $M_t = 174.3$ GeV.

In the model presented, the scale of SUSY breaking, M_{aux} should be in the range 40 – 100 TeV for the MSSM particles to have masses in the range 0.1 – 2 TeV. The gauge coupling $g_4 \geq 0.9$ in order for the slepton masses for the first two families to be positive and in the right range. Since the positivity of the mass-squared of the slepton of the third family depends on the Yukawa couplings, we find that the couplings should obey $f_\tau, f_{\tau E} \geq 0.5$.

For a specific choice of parameters (Table. 6.2), we find the $m_{\mu_1}, m_{\mu_2} \sim 800$ GeV for the smuon. There is a significant mass splitting between the selectron and the stau. The lightest of the sleptons is the left-handed stau. If SUSY is discovered with a large mass hierarchy between the stau and the selectron (or smauon), this model will be a good candidate. The lightest Higgs mass is found to be around 128 GeV which is consistent with current experimental limit.

The lightest supersymmetric particle is Wino which is nearly mass degenerate with the lighter chargino of the SM. The $SU(2)_H$ gauge boson mass is found to be ~ 1.4 TeV. The heavy Higgs bosons, Higgsinos and squarks masses are in the range 0.7 – 2.0 TeV.

Particles	Symbol	Mass (TeV)
Neutralinos	$\{m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}\}$	{0.176, 274, 0.726, 1.080}
Neutralinos	$\{m_{\tilde{\chi}_5^0}, m_{\tilde{\chi}_6^0}, m_{\tilde{\chi}_7^0}\}$	{1.091, 1.096, 2.097}
Charginos	$\{m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}\}$	{0.176, 1.094}
Charginos ($SU(2)_H$)	$\{m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}\}$	{1.070, 2.102}
Gluino	M_3	1.556
Neutral Higgs bosons	$\{m_h, m_H, m_A\}$	{0.128, 0.922, 0.922}
Neutral Higgs bosons	$\{m_{h'}, m_{H'}, m_{A'}\}$	{0.143, 2.075, 1.554}
Charged Higgs bosons	m_{H^\pm}	0.925
Charged Higgs bosons $SU(2)_H$	m_{H^\pm}	2.080
R.H smuon	$\{m_{\tilde{\mu}_1}\}$	{0.867}
L.H smuon	$\{m_{\tilde{\mu}_L}\}$	{0.796}
R.H sleptons	$\{m_{\tilde{e}_R}, m_{\tilde{\tau}_1}, m_{E_R}\}$	{0.947, 0.176, 0.758}
L.H sleptons	$\{m_{\tilde{e}_L}, m_{\tilde{\tau}_2}, m_{\tilde{\mu}_L}\}$	{1.904, 0.533, 0.401}
R.H down squarks	$\{m_{\tilde{d}_R}, m_{\tilde{s}_R}, m_{\tilde{b}_1}\}$	{1.464, 1.464, 1.369}
L.H down squarks	$\{m_{\tilde{d}_L}, m_{\tilde{s}_L}, m_{\tilde{b}_2}\}$	{1.451, 1.451, 1.115}
R.H up squarks	$\{m_{\tilde{u}_R}, m_{\tilde{c}_R}, m_{\tilde{t}_1}\}$	{1.454, 1.454, 1.107}
L.H up squarks	$\{m_{\tilde{u}_L}, m_{\tilde{c}_L}, m_{\tilde{t}_2}\}$	{1.449, 1.449, 1.295}
$SU(2)_H$ gauge boson	M'_Z	1.382

TABLE 6.2. Sparticle masses in Model 1 for the choice $M_{aux} = 67.956$ TeV, $y_b = 0.8$, $f_\tau = 0.53$, $f_{eE} = 1.2$, $f_{\tau E} = 0.51$, $g_4 = 1.0$, $M_E = 10.4$ TeV and $M_t = 0.174$ TeV, $u = 1.955$ TeV, $\tan \beta = 57.4$, $\tan \beta' = 0.87$, $\mu = 1.088$ TeV, $\mu' = 0.276$ TeV, $B = 0.014$ TeV, $B' = 4.336$ TeV, $B_E = 0.009$ TeV.

6.6 Other experimental implications

The lightest SUSY particle in the model we considered is the wino ($\tilde{\chi}_1^0$) which is nearly mass degenerate with the chargino. This particle is stable and can be a candidate for cold dark matter. The model predicts the lightest Higgs mass $m_h \leq 135$ GeV which can be tested at the LHC.

Because the $SU(2)_H$ gauge bosons do not mix with the SM gauge bosons, electroweak precision data remains unchanged. Also the second family of leptons do not mix with the first and third family, this is because of the Z_4 symmetry present in the model. The processes $\mu \rightarrow 3e$ and $\mu \rightarrow e\gamma$ are not a problem in the model.

The $SU(2)_H$ gauge boson masses are degenerate with mass $M_V = 1.382$ TeV for the choice of parameters chosen in model 1. The most stringent constraint on M_V arising from the process $e^+e^- \rightarrow \mu^+\mu^-$. LEP II has set severe constraints on lepton compositeness [51, 58] from this process. The effective Lagrangian for the process is given by

$$L_{eff} = \frac{g_4^2}{2} \frac{(\bar{e}\gamma_\mu\mu)(\bar{\mu}\gamma^\mu e)}{M_V^2}.$$

Here M_V is the gauge boson mass. If we compare the above Lagrangian with the $\Lambda_{LL}^-(ee\mu\mu)$ [51, 58], we obtain the limit $M_V > 1.2$ TeV. This limit is satisfied in our model.

6.7 Summary

We have suggested in this chapter a new scenario for solving the tachyonic slepton mass problem of AMSB. An asymptotically free $SU(2)_H$ horizontal gauge symmetry acting on the lepton superfields provides positive masses to the sleptons of the first two families (\tilde{e} , $\tilde{\mu}$) while the Yukawa couplings associated with the third family (τ) field gives a large positive contribution to the $\tilde{\tau}$ mass. We have a large mass splitting between the \tilde{e} , $\tilde{\mu}$, and, $\tilde{\tau}$, due to the transformation properties under the new $SU(2)_H$ symmetry. This is how our model differs from the other models. The $SU(2)_H$ symmetry must be broken at the TeV scale for consistency and our model predicts $m_h \lesssim 135$ GeV for the lightest MSSM Higgs boson mass and $\tan\beta \simeq 40$. The

LSP is the neutral wino which is nearly mass degenerate with the lightest chargino and is a candidate for cold dark matter.

CHAPTER 7

CONSTRAINING Z' FROM SUPERSYMMETRY BREAKING

7.1 Introduction

One of the simplest extensions of the Standard Model (SM) is obtained by adding a $U(1)$ factor to the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge structure [62, 63]. Such $U(1)$ factors arise quite naturally when the SM is embedded in a grand unified group such as $SO(10)$, $SU(6)$, E_6 , etc. While it is possible that such $U(1)$ symmetries are broken spontaneously near the grand unification scale, it is also possible that some of the $U(1)$ factors survive down to the TeV scale. In fact, if there is low energy supersymmetry, it is quite plausible that the $U(1)$ symmetry is broken along with supersymmetry at the TeV scale. The Z'_χ and Z'_ψ models arising from $SO(10) \rightarrow SU(5) \times U(1)_\chi$ and $E_6 \rightarrow SO(10) \times U(1)_\psi$ are two popular extensions which have attracted much phenomenological attention [62–69]. Z' associated with the left–right symmetric extension of the Standard Model does not require a grand unified symmetry. Other types of $U(1)$ symmetries, which do not resemble the ones with a GUT origin, are known to arise in string theory, free–fermionic construction as well as in orbifold and D –brane models [70–72]. Gauge kinetic mixing terms of the type $B^{\mu\nu} Z'_{\mu\nu}$ [73] which will be generated through renormalization group flow below the unification scale can further disguise the couplings of the Z' .

The properties of the Z' gauge boson – its mass, mixing and couplings to fermions – associated with the $U(1)$ gauge symmetry are in general quite arbitrary [74]. This is especially so when the low energy theory contains new fermions for anomaly cancellation. In this chapter we propose and analyze a special class of $U(1)$ models wherein the Z' properties get essentially fixed from constraints of SUSY

breaking. We have in mind the anomaly mediated supersymmetric (AMSB) framework [36, 37]. In its minimal version, with the Standard Model gauge symmetry, it turns out that the sleptons of AMSB become tachyonic. We suggest the $U(1)$ symmetry, identified as $U(1)_x = xY - (B - L)$, where Y is the Standard Model hypercharge, as a solution to the negative slepton mass problem of AMSB. This symmetry is automatically free of anomalies with the inclusion of right-handed neutrinos. It is shown that the D -term of this $U(1)_x$ provides positive contributions to the slepton masses, curing the tachyonic problem. The consistency of symmetry breaking and the SUSY spectrum points towards a specific set of parameters in the Z' sector. For example, $1 < x < 2$ is needed for the positivity of the left-handed and the right-handed slepton masses. Furthermore, the $U(1)_x$ gauge coupling, g_x , is fixed to be between 0.4–0.5. The resulting Z' is found to be “leptophobic” [75] with $Br(Z \rightarrow \ell^+ \ell^-) \simeq (1 - 1.6)\%$ and $Br(Z \rightarrow q\bar{q}) \simeq 44\%$.

AMSB models are quite predictive as regards the SUSY spectrum. The masses of the scalar components of the chiral supermultiplets in AMSB scenario are given by [36, 37]

$$(m^2)_{\phi_i}^{\phi_j} = \frac{1}{2} M_{aux}^2 \left[\beta(Y) \frac{\partial}{\partial Y} \gamma_{\phi_i}^{\phi_j} + \beta(g) \frac{\partial}{\partial g} \gamma_{\phi_i}^{\phi_j} \right], \quad (7.1)$$

where summations over the gauge couplings g and the Yukawa couplings Y are assumed. $\gamma_{\phi_i}^{\phi_j}$ are the one-loop anomalous dimensions, $\beta(Y)$ is the beta function for the Yukawa coupling Y , and $\beta(g)$ is the beta function for the gauge coupling g . M_{aux} is the vacuum expectation value of a “compensator superfield” [36] which sets the scale of SUSY breaking. The gaugino mass M_g , the trilinear soft supersymmetry breaking term A_Y and the bilinear SUSY breaking term B are given by [36, 37]

$$M_g = \frac{\beta(g)}{g} M_{aux}, \quad A_Y = -\frac{\beta(Y)}{Y} M_{aux}, \quad B = -M_{aux}(\gamma_{H_u} + \gamma_{H_d}). \quad (7.2)$$

We see that the SUSY masses are completely fixed in the AMSB framework once the spectrum of the theory and M_{aux} are specified.

The negative slepton mass problem arises in AMSB because in Eq. 7.1 the gauge beta functions for $SU(2)_L$ and $U(1)_Y$ are positive, $\gamma_{\phi_i}^{\phi_j}$ are negative, and the Yukawa

couplings are small for the first two families of sleptons. In our Z' models, there are additional positive contributions from the $U(1)_x$ D -terms which render these masses positive.

In Ref. [38] the negative slepton mass problem of AMSB has been solved with explicit Fayet–Iliopoulos terms added to the theory. In contrast, in our models, the D -term is calculable, which makes the Z' sector more predictive. We find $M_{Z'} = 2 - 4$ TeV and the $Z - Z'$ mixing angle $\xi \simeq 0.001$. Constraints from the electroweak precision observables are satisfied, with the Z' model giving a slightly better fit compared to the Standard Model.

Other attempts to solve the negative slepton mass problem of AMSB generally assume TeV-scale new physics [41, 43–50, 61] or a universal scalar mass of non AMSB origin [39]. In Ref. [61] we have shown how a non-Abelian horizontal symmetry which is asymptotically free solves the problem. Some of the techniques we use here for the symmetry breaking analysis are similar to Ref. [61].

The plan of this chapter is as follows. In section 7.2 we introduce our model. In section 7.3 we analyze the Higgs potential of the model. In section 7.4 we present formulas for the SUSY spectrum. Section 7.5 contains our numerical results for the SUSY spectrum as well as for the Z' mass and mixing. In section 7.6 we analyze the partial decay modes of the Z' . In section 7.7 we analyze other experimental test of the model. Here we show the consistency of our models with the precision electroweak data. Section 7.8 has our summary. In Appendix ?? we give the relevant expressions for the beta functions, anomalous dimensions as well as for the soft masses.

7.2 $U(1)_x$ model

We present our model in this section. We consider adding an extra $U(1)$ gauge group to the Standard Model gauge structure of MSSM. The model is then based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_x$, where the $U(1)_x$ charge is given by the following linear combination of hypercharge Y and $B - L$:

$$U(1)_x = xY - (B - L). \quad (7.3)$$

The particle content of the model and the $U(1)_x$ charge assignment are shown in Table 1. Besides the MSSM particles, the model has new particles $\{\nu_i^c, \nu^c, \bar{\nu}^c, S_+$ and $S_-\}$ which are all singlets of the Standard Model gauge group.

Superfield	Q_i	u_i^c	d_i^c	L_i	e_i^c	H_u	H_d	ν_i^c	ν^c	$\bar{\nu}^c$	S_+	S_-
$U(1)_x$	$\frac{x}{6} - \frac{1}{3}$	$-\frac{2x}{3} + \frac{1}{3}$	$\frac{x}{3} + \frac{1}{3}$	$-\frac{x}{2} + 1$	$x - 1$	$\frac{x}{2}$	$-\frac{x}{2}$	-1	-1	1	2	-2

TABLE 7.1. Particle content and charge assignment of the $U(1)_x$ model. Here $i = 1 - 3$ is the family index.

In order for \tilde{L}_i and \tilde{e}_i^c sleptons to have positive mass-squared from the $U(1)_x$ D -term, the charges of L_i and e_i^c must be of the same sign. This is possible only for $1 < x < 2$. We shall confine to this range of x , which is an important restriction on this class of models. The ν_i^c fields are needed for $U(1)_x$ anomaly cancellation. S_+ and S_- are the Higgs superfields responsible for $U(1)_x$ symmetry breaking. The $\nu^c + \bar{\nu}^c$ pair facilitates symmetry breaking within the AMSB framework. The superpotential of the model consistent with the gauge symmetries is given by:

$$\begin{aligned}
W = & (Y_u)_{ij} Q_i H_u u_j^c + (Y_d)_{ij} Q_i H_d d_j^c + (Y_l)_{ij} L_i H_d e_j^c + \mu H_u H_d \\
& + \mu' S_+ S_- + \sum_{i=1}^3 f_{\nu_i^c} \nu_i^c \nu_i^c S_+ + f_{\nu^c} \nu^c \nu^c S_+ + h \bar{\nu}^c \bar{\nu}^c S_- + M_{\nu^c} \nu^c \bar{\nu}^c. \quad (7.4)
\end{aligned}$$

Here $i, j = 1, 2, 3$ are the family indices. The mass parameters μ and μ' are of order TeV, which may have a natural origin in AMSB [36]. In general, one can write additional mass terms of the form $M_i \nu_i^c \bar{\nu}^c$ in the superpotential. Such terms will have very little effect on the symmetry breaking analysis that follows. We forbid such mass terms by invoking a discrete symmetry (such as a Z_2) which differentiates ν^c from ν_i^c .

Small neutrino masses are induced in the model through the seesaw mechanism. However, the ν_i^c fields, which remain light to the TeV scale, are not to be identified as the traditional right-handed neutrinos involved in the seesaw mechanism. The heavy fields which are integrated out have $U(1)_x$ -invariant mass terms. Specifically, the following effective nonrenormalizable operators emerge after integrating out the

heavy neutral lepton fields:

$$L_{eff}^\nu = \frac{Y_{\nu_{ij}}^2}{M_N^2} L_i L_j H_u H_u S_-. \quad (7.5)$$

Here M_N represents the masses of the heavy neutral leptons. For $M_N \sim 10^9$ GeV and $\langle S_- \rangle \sim$ TeV, sub-eV neutrino masses are obtained. Note that we have not allowed neutrino Dirac Yukawa couplings of the form $h_{\nu_{ij}} L_i \nu_j^c H_u$, which would generate Majorana masses of order MeV for the light neutrinos. We forbid such terms by a global symmetry G , either discrete or continuous. In our numerical examples we shall assume this symmetry to be non-Abelian, with ν_i^c transforming as a triplet [for example, G can be $O(3)$, S_4 , A_4 , etc.]. Such a symmetry would imply that $f_{\nu_i^c}$ in Eq. (7.4) are equal for $i = 1 - 3$.

7.3 Symmetry breaking

The scalar potential (involving H_u , H_d , S_+ , S_- fields) of the model is given by:

$$\begin{aligned} V = & (M_{H_u}^2 + \mu^2)|H_u|^2 + (M_{H_d}^2 + \mu^2)|H_d|^2 + (M_{S_+}^2 + \mu'^2)|S_+|^2 + (M_{S_-}^2 + \mu'^2)|S_-|^2 \\ & + B\mu(H_u H_d + h.c.) + B'\mu'(S_+ S_- + h.c.) + \frac{1}{8}(g_1^2 + g_2^2)(|H_u|^2 - |H_d|^2)^2 \\ & + \frac{1}{2}g_2^2|H_u H_d|^2 + \frac{1}{2}g_x^2 \left(\frac{x}{2}|H_u|^2 - \frac{x}{2}|H_d|^2 + 2|S_+|^2 - 2|S_-|^2 \right)^2, \end{aligned} \quad (7.6)$$

where the last term is the $U(1)_x$ D term. The B and the B' terms for the model are given by

$$B = -(\gamma_{H_u} + \gamma_{H_d})M_{aux} \quad \text{and} \quad B' = -(\gamma_{S_+} + \gamma_{S_-})M_{aux}, \quad (7.7)$$

where the γ 's are the one-loop anomalous dimensions given in the Appendix, Eqs. (115)–(116), (120)–(121).

We parameterize the VEVs of H_u , H_d , S_+ and S_- as

$$\langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \quad \langle H_d \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \quad \langle S_+ \rangle = z, \quad \langle S_- \rangle = y. \quad (7.8)$$

In minimizing the potential, we have to keep in mind the fact that the VEVs of $\langle S_+ \rangle$ and $\langle S_- \rangle$ should be much larger than the VEVs of $\langle H_u \rangle$ and $\langle H_d \rangle$ for a consistent

picture. In addition, the VEV of $\langle S_+ \rangle$ should be greater than the VEV of $\langle S_- \rangle$ in order for the D -term contribution to the slepton masses to be positive. We have checked explicitly that all the above – mentioned conditions are satisfied at the local minimum for a restricted choice of model parameters. The physical Higgs bosons as well as the sleptons acquire positive mass-squared, while generating a Z' mass and $Z - Z'$ mixing angle consistent with experimental constraints.

Minimization of the potential leads to the following conditions:

$$\sin 2\beta = \frac{2B\mu}{2\mu^2 + M_{H_u}^2 + M_{H_d}^2}, \quad (7.9)$$

$$\frac{M_Z^2}{2} = -\mu^2 + \frac{M_{H_d}^2 - M_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{x^2 g_x^2 v^2}{4} - \frac{x g_x^2 u^2 \cos 2\psi}{\cos 2\beta}, \quad (7.10)$$

$$\sin 2\psi = \frac{-2B'\mu'}{2\mu'^2 + M_{S_+}^2 + M_{S_-}^2}, \quad (7.11)$$

$$\frac{M_{Z'}^2}{2} = -\mu'^2 + \frac{M_{S_-}^2 - M_{S_+}^2 \tan^2 \psi}{(\tan^2 \psi - 1)} + \frac{x^2 g_x^2 v^2}{4} - \frac{x g_x^2 v^2 \cos 2\psi}{\cos 2\psi}. \quad (7.12)$$

Here $M_{Z'}^2 = \frac{x^2 g_x^2 v^2}{2} + 8g_x^2 u^2$, $\tan \beta = \frac{v_u}{v_d}$, $\tan \psi = \frac{z}{y}$, $\sqrt{v_u^2 + v_d^2} = v = 174$ GeV and $\sqrt{z^2 + y^2} = u$.

To see the consistency of symmetry breaking, we need to calculate the Higgs boson mass-squared and establish that they are all positive. We parameterize the Higgs fields (in the unitary gauge) as

$$\begin{aligned} H_u &= \begin{pmatrix} H^+ \sin \beta \\ v_u + \frac{1}{\sqrt{2}}(\phi_2 + i \cos \beta \phi_3) \end{pmatrix}, \quad \langle H_d \rangle = \begin{pmatrix} v_d + \frac{1}{\sqrt{2}}(\phi_1 + i \sin \beta \phi_3) \\ H^- \cos \beta \end{pmatrix}, \\ S_+ &= z + \frac{1}{\sqrt{2}}(\phi_4 + i \cos \psi \phi_5), \quad S_- = y + \frac{1}{\sqrt{2}}(\phi_6 + i \sin \psi \phi_5). \end{aligned} \quad (7.13)$$

The CP-odd Higgs bosons $\{\phi_3, \phi_5\}$ have masses given by

$$m_A^2 = \frac{2B\mu}{\sin 2\beta}, \quad m_{A'}^2 = -\frac{2B'\mu'}{\sin 2\psi}. \quad (7.14)$$

The mass matrix for the CP-even neutral Higgs bosons $\{\phi_1, \phi_2, \phi_4, \phi_6\}$ is given by

$$(\mathcal{M}^2)_{cp\text{-even}} = \begin{pmatrix} (\mathcal{M}^2)_{11} & (\mathcal{M}^2)_{12} & -2xg_x^2 v_d z & 2xg_x^2 v_d y \\ (\mathcal{M}^2)_{12} & (\mathcal{M}^2)_{22} & 2xg_x^2 v_u z & -2xg_x^2 v_u y \\ -2xg_x^2 v_d z & 2xg_x^2 v_u z & (\mathcal{M}^2)_{33} & (\mathcal{M}^2)_{34} \\ 2xg_x^2 v_d y & -2xg_x^2 v_u y & (\mathcal{M}^2)_{34} & (\mathcal{M}^2)_{44} \end{pmatrix}, \quad (7.15)$$

where

$$(\mathcal{M}^2)_{11} = m_A^2 \sin^2 \beta + M_Z^2 \cos^2 \beta + \frac{1}{2}(x^2 g_x^2 v^2 \cos^2 \beta), \quad (7.16)$$

$$(\mathcal{M}^2)_{12} = -m_A^2 \sin \beta \cos \beta - M_Z^2 \sin \beta \cos \beta - \frac{1}{2}x^2 g_x^2 v^2 \sin \beta \cos \beta, \quad (7.17)$$

$$(\mathcal{M}^2)_{22} = m_A^2 \cos^2 \beta + M_Z^2 \sin^2 \beta + \frac{1}{2}(x^2 g_x^2 v^2 \sin^2 \beta), \quad (7.18)$$

$$(\mathcal{M}^2)_{33} = m_{A'}^2 \cos^2 \psi + 8g_x^2 z^2, \quad (7.19)$$

$$(\mathcal{M}^2)_{34} = -m_{A'}^2 \sin \psi \cos \psi - 8g_x^2 yz, \quad (7.20)$$

$$(\mathcal{M}^2)_{44} = m_{A'}^2 \sin^2 \psi + 8g_x^2 y^2. \quad (7.21)$$

It is instructive to analyze the effect of the $U(1)_x$ D -term on the mass of the lightest MSSM Higgs boson h . Consider the upper left 2×2 sub sector of the CP-even Higgs boson mass matrix. It has eigenvalues given by

$$\begin{aligned} \lambda_{1,2} &= \frac{1}{2} \left[m_A^2 + M_Z^2 + \frac{x^2 g_x^2 v^2}{2} \right. \\ &\quad \mp \left. \sqrt{\left(m_A^2 + M_Z^2 + \frac{x^2 g_x^2 v^2}{2} \right)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta - 4m_A^2 \left(\frac{x^2 g_x^2 v^2}{2} \right) \cos^2 2\beta} \right] \end{aligned}$$

From the equation above, we obtain an upper limit on m_h

$$m_h \leq \sqrt{\frac{x^2 g_x^2 v^2}{2} + M_Z^2} |\cos 2\beta|. \quad (7.22)$$

The mixing between the doublets and the singlets will reduce the upper limit further. In fact, we find this mixing effect to be significant.

The lower 2×2 subsector of Eq. (7.15) has eigenvalues

$$\lambda'_{1,2} = \frac{1}{2} \left[8g_x^2 u^2 + m_{A'}^2 \mp \sqrt{(8g_x^2 u^2 + m_{A'}^2)^2 - 4m_{A'}^2 (8g_x^2 u^2) \cos^2 2\psi} \right]. \quad (7.23)$$

From Eq. (7.23) we obtain the upper bound of the lightest Higgs mass for the $SU(2)$ singlet sector:

$$m_{h'} \leq m_{A'} |\cos 2\psi|. \quad (7.24)$$

The above upper limit on $m_{h'}$ is affected only minimally by the mixing between the doublet and the singlet Higgs fields.

As in the MSSM, the mass of the charged Higgs boson H^\pm is given by

$$m_{H^\pm}^2 = m_A^2 + M_W^2. \quad (7.25)$$

We now turn to the supersymmetric fermion masses. The (Majorana) mass matrix of the neutralinos $\{\tilde{B}, \tilde{W}_3, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{B}', \tilde{S}_+, \tilde{S}_-\}$ is given by

$$\mathcal{M}^{(0)} = \begin{pmatrix} M_1 & 0 & -\frac{v_d}{\sqrt{2}}g_1 & \frac{v_u}{\sqrt{2}}g_1 & 0 & 0 & 0 \\ 0 & M_2 & \frac{v_d}{\sqrt{2}}g_2 & -\frac{v_u}{\sqrt{2}}g_2 & 0 & 0 & 0 \\ -\frac{v_d}{\sqrt{2}}g_1 & \frac{v_d}{\sqrt{2}}g_2 & 0 & -\mu & -\frac{v_d}{\sqrt{2}}xg_x & 0 & 0 \\ \frac{v_u}{\sqrt{2}}g_1 & -\frac{v_u}{\sqrt{2}}g_2 & -\mu & 0 & \frac{v_u}{\sqrt{2}}xg_x & 0 & 0 \\ 0 & 0 & -\frac{v_d}{\sqrt{2}}xg_x & \frac{v_u}{\sqrt{2}}xg_x & M'_1 & 2\sqrt{2}g_xz & -2\sqrt{2}g_xy \\ 0 & 0 & 0 & 0 & 2\sqrt{2}g_xz & 0 & \mu' \\ 0 & 0 & 0 & 0 & -2\sqrt{2}g_xy & \mu' & 0 \end{pmatrix} \quad (7.26)$$

where M_1 , M'_1 and M_2 are the gaugino masses for $U(1)_Y$, $U(1)_x$ and $SU(2)_L$. The physical neutralino masses $m_{\tilde{\chi}_i^0}$ ($i=1-7$) are obtained as the eigenvalues of this mass matrix. We denote the diagonalizing matrix as O :

$$O\mathcal{M}^{(0)}O^T = \text{diag}\{m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}, m_{\tilde{\chi}_5^0}, m_{\tilde{\chi}_6^0}, m_{\tilde{\chi}_7^0}\}. \quad (7.27)$$

In the basis $\{\tilde{W}^+, \tilde{H}_u^+\}$, $\{\tilde{W}^-, \tilde{H}_d^-\}$ the chargino (Dirac) mass matrix is

$$\mathcal{M}^{(c)} = \begin{pmatrix} M_2 & g_2v_d \\ g_2v_u & \mu \end{pmatrix}. \quad (7.28)$$

This matrix is diagonalized by a biunitary transformation $V^*\mathcal{M}^{(c)}U^{-1} = \text{diag}\{m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}\}$.

The $Z - Z'$ mixing matrix is given by

$$\mathcal{M}_{Z-Z'}^2 = \begin{pmatrix} M_Z^2 & \gamma M_Z^2 \\ \gamma M_Z^2 & M_{Z'}^2 \end{pmatrix}, \quad (7.29)$$

where

$$\gamma = \frac{-xg_x}{\sqrt{g_1^2 + g_2^2}}, \quad M_Z^2 = \frac{v^2}{2}(g_1^2 + g_2^2), \quad M_{Z'}^2 = \frac{x^2g_x^2v^2}{2} + 8g_x^2u^2. \quad (7.30)$$

The physical mass eigenstates Z_1 and Z_2 with masses M_{Z_1} , M_{Z_2} are

$$Z_1 = Z \cos \xi + Z' \sin \xi, \quad (7.31)$$

$$Z_2 = -Z \sin \xi + Z' \cos \xi, \quad (7.32)$$

where

$$M_{Z_1, Z_2}^2 = \frac{1}{2} \left[M_Z^2 + M_{Z'}^2 \pm \sqrt{(M_Z^2 - M_{Z'}^2)^2 + 4\gamma^2 M_Z^4} \right]. \quad (7.33)$$

The $Z - Z'$ mixing angle ξ is given by

$$\xi = \frac{1}{2} \arctan \left(\frac{2\gamma M_Z^2}{M_Z^2 - M_{Z'}^2} \right) \simeq -\gamma M_Z^2 / M_{Z'}^2. \quad (7.34)$$

We have ignored kinetic mixing of the form $B^{\mu\nu} Z'_{\mu\nu}$ in the Lagrangian [73, 74].

The masses of the heavy right-handed neutrinos are given by

$$m_{\nu_i^c} = f_{\nu_i^c} z, \quad (7.35)$$

where $i = 1 - 3$ is the family index. The fourth right-handed neutrino ν^c mixes with the $\bar{\nu}^c$ field forming two Majorana fermions. The masses are the eigenvalues of the mass matrix

$$M_{\nu^c \bar{\nu}^c} = \begin{pmatrix} f_{\nu^c} z & M_{\nu^c} \\ M_{\nu^c} & hy \end{pmatrix}, \quad (7.36)$$

where M_{ν^c} is the mass parameter that appears in the superpotential of Eq. (7.4). We denote the eigenstates of this matrix as ω_1, ω_2 and the mass eigenvalues as m_{ω_1} and m_{ω_2} .

7.4 The SUSY spectrum

7.4.1 Slepton masses

The slepton mass-squareds are given by the eigenvalues of the mass matrices

$$M_i^2 = \begin{pmatrix} m_{\tilde{l}_i}^2 & m_{e_i} (A_{Y_{l_i}} - \mu \tan \beta) \\ m_{e_i} (A_{Y_{l_i}} - \mu \tan \beta) & m_{\tilde{e}_i^c}^2 \end{pmatrix}, \quad (7.37)$$

where $i = e, \mu, \tau$, and

$$\begin{aligned} m_{\tilde{l}_i}^2 &= \frac{M_{aux}^2}{(16\pi^2)} \left[Y_{l_i} \beta(Y_{l_i}) - \left(\frac{3}{2} g_2 \beta(g_2) + \frac{3}{10} g_1 \beta(g_1) + 2 \left(1 - \frac{x}{2}\right)^2 g_x \beta(g_x) \right) \right] \\ &+ m_{e_i}^2 + \left(-\frac{1}{2} + \sin^2 \theta_W \right) \cos 2\beta M_Z^2 + 2g_x^2 \left(1 - \frac{x}{2}\right) (z^2 - y^2), \end{aligned} \quad (7.38)$$

$$\begin{aligned} m_{\tilde{e}_i^c}^2 &= \frac{M_{aux}^2}{(16\pi^2)} \left[2Y_{l_i} \beta(Y_{l_i}) - \left(\frac{6}{5} g_1 \beta(g_1) + 2(x-1)^2 g_x \beta(g_x) \right) \right] \\ &+ m_{e_i}^2 - \sin^2 \theta_W \cos 2\beta M_Z^2 + 2g_x^2 (x-1)(z^2 - y^2). \end{aligned} \quad (7.39)$$

The SUSY soft masses are calculated from the RGE given in the Appendix [Eqs. (C.15), (C.21)]. Note the positive contribution from the $U(1)_x$ D -terms in Eqs. (7.38)–(7.39), given by the terms $+2g_x^2(1-\frac{x}{2})(z^2-y^2)$ and $+2g_x^2(x-1)(z^2-y^2)$. There are also negative contributions proportional to $\beta(g_x)$, but in our numerical solutions, the positive D -term contributions are larger than the negative contributions. We seek solutions where $z = \langle S_+ \rangle$ and $y = \langle S_- \rangle$ are much larger than v_u, v_d , of order TeV, with $z \gtrsim y$.

The left-handed sneutrino masses are given by

$$m_{\tilde{\nu}_{L_i}}^2 = \frac{M_{aux}^2}{(16\pi^2)} \left[-\frac{3}{2}g_2\beta(g_2) - \frac{3}{10}g_1\beta(g_1) - 2\left(1-\frac{x}{2}\right)^2 g_x\beta(g_x) \right] + \frac{1}{2} \cos 2\beta M_Z^2 + 2g_x^2 \left(1-\frac{x}{2}\right) (z^2-y^2). \quad (7.40)$$

7.4.2 Squark masses

The mixing matrix for the squark sector is similar to the slepton sector. The diagonal entries of the up and the down squark mass matrices are given by

$$\begin{aligned} m_{\tilde{U}_i}^2 &= (m_{soft}^2)_{\tilde{Q}_i} + m_{U_i}^2 + \frac{1}{6} (4M_W^2 - M_Z^2) \cos 2\beta + 2g_x^2 \left(\frac{x}{6} - \frac{1}{3}\right) (z^2 - y^2), \\ m_{\tilde{U}_i^c}^2 &= (m_{soft}^2)_{\tilde{U}_i^c} + m_{U_i}^2 - \frac{2}{3} (M_W^2 - M_Z^2) \cos 2\beta + 2g_x^2 \left(-\frac{2x}{3} + \frac{1}{3}\right) (z^2 - y^2), \\ m_{\tilde{D}_i}^2 &= (m_{soft}^2)_{\tilde{Q}_i} + m_{D_i}^2 - \frac{1}{6} (2M_W^2 + M_Z^2) \cos 2\beta + 2g_x^2 \left(\frac{x}{6} - \frac{1}{3}\right) (z^2 - y^2), \\ m_{\tilde{D}_i^c}^2 &= (m_{soft}^2)_{\tilde{D}_i^c} + m_{D_i}^2 + \frac{1}{3} (M_W^2 - M_Z^2) \cos 2\beta + 2g_x^2 \left(\frac{x}{3} + \frac{1}{3}\right) (z^2 - y^2). \end{aligned} \quad (7.41)$$

Here m_{U_i} and m_{D_i} are quark masses of different generations, $i = 1, 2, 3$. The squark soft masses are obtained from the RGE as

$$(m_{soft}^2)_{\tilde{Q}_i} = \frac{M_{aux}^2}{16\pi^2} \left[Y_{u_i}\beta(Y_{u_i}) + Y_{d_i}\beta(Y_{d_i}) - \frac{1}{30}g_1\beta(g_1) - \frac{3}{2}g_2\beta(g_2) - \frac{8}{3}g_3\beta(g_3) - 2\left(\frac{x}{6} - \frac{1}{3}\right)^2 g_x\beta(g_x) \right], \quad (7.42)$$

$$(m_{soft}^2)_{\tilde{U}_i^c} = \frac{M_{aux}^2}{16\pi^2} \left[2Y_{u_i}\beta(Y_{u_i}) - \frac{8}{15}g_1\beta(g_1) - \frac{8}{3}g_3\beta(g_3) - 2\left(-\frac{2x}{3} + \frac{1}{3}\right)^2 g_x\beta(g_x) \right] \quad (7.43)$$

$$(m_{soft}^2)_{\tilde{D}_i^c} = \frac{M_{aux}^2}{16\pi^2} \left[2Y_{d_i}\beta(Y_{d_i}) - \frac{2}{15}g_1\beta(g_1) - \frac{8}{3}g_3\beta(g_3) - 2\left(\frac{x}{3} + \frac{1}{3}\right)^2 g_x\beta(g_x) \right] \quad (7.44)$$

7.4.3 Heavy sneutrino masses

The heavy right-handed sneutrinos ($\tilde{\nu}_i^c$) split into scalar ($\tilde{\nu}_{is}^c$) and pseudoscalar ($\tilde{\nu}_{ip}^c$) components with masses given by

$$m_{\tilde{\nu}_{is}^c}^2 = \frac{M_{aux}^2}{(16\pi^2)} [4f_{\nu_i^c}\beta(f_{\nu_i^c}) - 2g_x\beta(g_x)] - 2g_x^2(z^2 - y^2) + 2\mu' f_{\nu_i^c} y + 4f_{\nu_i^c}^2 z^2 + 2f_{\nu_i^c} A_{\nu_i} z, \quad (7.45)$$

$$m_{\tilde{\nu}_{ip}^c}^2 = \frac{M_{aux}^2}{(16\pi^2)} [4f_{\nu_i^c}\beta(f_{\nu_i^c}) - 2g_x\beta(g_x)] - 2g_x^2(z^2 - y^2) - 2\mu' f_{\nu_i^c} y + 4f_{\nu_i^c}^2 z^2 - 2f_{\nu_i^c} A_{\nu_i} z. \quad (7.46)$$

As for the fourth heavy sneutrino, there is mixing between the $\tilde{\nu}^c$ and the $\tilde{\nu}^c$ fields. This leads to two 2×2 mass matrices, one for the scalars, and one for the pseudoscalars. They are given by

$$M_{\tilde{\nu}_s^c}^2 = \begin{pmatrix} m_{\tilde{\nu}_s^c}^2 & 2M_{\nu^c} (f_{\nu^c} z + hy + \frac{B_{\nu^c \bar{\nu}^c}}{2}) \\ 2M_{\nu^c} (f_{\nu^c} z + hy + \frac{B_{\nu^c \bar{\nu}^c}}{2}) & m_{\tilde{\nu}_s^c}^2 \end{pmatrix}, \quad (7.47)$$

$$M_{\tilde{\nu}_p^c}^2 = \begin{pmatrix} m_{\tilde{\nu}_p^c}^2 & 2M_{\nu^c} (f_{\nu^c} z + hy + \frac{B_{\nu^c \bar{\nu}^c}}{2}) \\ 2M_{\nu^c} (f_{\nu^c} z + hy + \frac{B_{\nu^c \bar{\nu}^c}}{2}) & m_{\tilde{\nu}_p^c}^2 \end{pmatrix}, \quad (7.48)$$

where

$$m_{\tilde{\nu}_s^c}^2 = \frac{M_{aux}^2}{(16\pi^2)} (4f_{\nu^c}\beta(f_{\nu^c}) - 2g_x\beta(g_x)) - 2g_x^2(z^2 - y^2) + 2\mu' f_{\nu^c} y + 4f_{\nu^c}^2 z^2 + 2f_{\nu^c} A_{\nu^c} z + M_{\nu^c}^2, \quad (7.49)$$

$$m_{\tilde{\nu}_p^c}^2 = \frac{M_{aux}^2}{(16\pi^2)} (4f_{\nu^c}\beta(f_{\nu^c}) - 2g_x\beta(g_x)) - 2g_x^2(z^2 - y^2) - 2\mu' f_{\nu^c} y + 4f_{\nu^c}^2 z^2 - 2f_{\nu^c} A_{\nu^c} z + M_{\nu^c}^2, \quad (7.50)$$

$$m_{\tilde{\nu}_s^c}^2 = \frac{M_{aux}^2}{(16\pi^2)} (4h\beta(h) - 2g_x\beta(g_x)) + 2g_x^2(z^2 - y^2) + 2\mu' h z + 4h^2 y^2 + 2h A_h y + M_{\nu^c}^2, \quad (7.51)$$

$$m_{\tilde{\nu}_p^c}^2 = \frac{M_{aux}^2}{(16\pi^2)} (4h\beta(h) - 2g_x\beta(g_x)) + 2g_x^2(z^2 - y^2) - 2\mu' h z + 4h^2 y^2 - 2h A_h y + M_{\nu^c}^2, \quad (7.52)$$

$$B_{\nu^c \bar{\nu}^c} = -M_{aux}(\gamma_{\nu^c} + \gamma_{\bar{\nu}^c}). \quad (7.53)$$

Here s (p) stands for scalar (pseudoscalar). The beta functions, gamma functions and the A terms are given in the Appendix, Eqs. (C.8)–(C.22). We shall denote the mass eigenstates of the scalars as $\tilde{\omega}_{1s}$, $\tilde{\omega}_{2s}$ with masses $m_{\tilde{\omega}_{1s}}^2$, $m_{\tilde{\omega}_{2s}}^2$, and the pseudoscalars as $\tilde{\omega}_{1p}$, $\tilde{\omega}_{2p}$ with masses $m_{\tilde{\omega}_{1p}}^2$, $m_{\tilde{\omega}_{2p}}^2$.

7.5 Numerical results for the spectrum

As inputs at M_Z we choose the central values (in the \overline{MS} scheme) [51]

$$\alpha_3(M_Z) = 0.119, \quad \sin^2 \theta_W = 0.23113, \quad \alpha(M_Z) = \frac{1}{127.922}. \quad (7.54)$$

We keep the top quark mass fixed at its central value, $M_t = 174.3$ GeV. We follow the procedure outlined in Ref. [61] to determine the parameter $\tan \beta$ and the lightest Higgs boson mass m_h . The gauge couplings and the top quark Yukawa coupling are evolved from the lower momentum scale to $Q = 1$ TeV, where the Higgs potential is minimized. We use the Standard Model beta functions for this evolution. In determining the top quark Yukawa coupling $Y_t(m_t)$, we use 2-loop QCD corrections to convert the physical mass M_t into the running mass $m_t(m_t)$.

For the lightest Higgs boson mass of MSSM we use the 2-loop radiatively corrected expression for $m_h^2 = (m_h^2)_o + \Delta m_h^2$, where Δm_h^2 is given in Ref. [53].

We present numerical results for two models: Model 1 with $x = 1.3$, and Model 2 with $x = 1.6$. In Model 1, the left-handed sleptons are heavier than the right-handed sleptons, while the reverse holds for Model 2.

The value of M_{aux} should be in the range $M_{aux} = 40 - 100$ TeV if the SUSY particles are to have masses in the range 100 GeV – 2 TeV. In Table 7.2, corresponding to Model 1, we choose $M_{aux} = 56.398$ TeV. In Table 7.7 (for Model 2) we choose $M_{aux} = 59.987$ TeV. We have included the leading radiative corrections [54] to M_1 , M_2 and M_3 in our numerical study. In Model 1 we find $M_1 : M_2 : M_3 = 3.0 : 1 : 7.1$. The minimization conditions (Eqs. (7.9)–(10)) fix $\tan \beta = 4.39$ in this model. The choice of $g_x = 0.41$, $f_{\nu_i^c} = f_{\nu^c} = 0.28$, and $h = 0.921$ are motivated by the requirements of consistent symmetry breaking with $\langle S_+ \rangle \gtrsim \langle S_- \rangle \gg v_u, v_d$, and the positivity of slepton masses. We find that the model parameters are highly constrained. Only small deviations from the choice in Table 7.2 are found to be consistent.

From Table 7.2 we see that the lightest Higgs boson of the MSSM sector has mass of 121 GeV. The lightest SUSY particle is the neutralino $\tilde{\chi}_1^0$, which is approximately a neutral Wino. This is a candidate for cold dark matter [40]. Note that $\tilde{\chi}_1^0$ is nearly mass degenerate with the lighter chargino $\tilde{\chi}_1^\pm$ (which is approximately the charged

Wino). The mass splitting $m_{\tilde{\chi}_1^0} - m_{\tilde{\chi}_1^\pm} = 180$ MeV, where the bulk (173 MeV) arises from finite electroweak radiative corrections [55], not shown in Table 7.2.

In the $U(1)_x$ sector, there is a relatively light neutral Higgs boson h' with a mass of 60 GeV. This occurs since the parameter $\tan \psi = \frac{z}{y}$ is close to 1 – a requirement for consistent symmetry breaking [see Eq. (7.24)]. h' is an admixture of S_+ and S_- , and as such has no direct couplings to the Standard Model fields. Its mass being below 100 GeV is fully consistent with experimental constraints. The phenomenology of such a weakly coupled light neutral Higgs boson will be discussed in the section 7.

The mass of the Z' gauge boson and the $Z - Z'$ mixing angle are listed in Table 7.3 (for Model 1). In section 7 we show that these values are compatible with known experimental constraints.

Table 7.4 lists the eigenvectors of the neutralino mass matrix. These will become relevant in discussing the decays of the Z' gauge boson. Tables 7.5 and 7.6 give the eigenvectors of the chargino and the CP–even Higgs bosons, which will also be used in the study of Z' decays.

Tables 7.7–7.11 are analogous to Tables 7.2–7.6, except that they now apply to Model 2 (with $x = 1.6$). In this case, $\tan \beta = 5.83$ and $m_h = 126$ GeV. Here the right–handed sleptons are heavier than the left–handed sleptons. In fact, in this Model, the LSP is the left–handed sneutrino. This can also be a candidate for cold dark matter in the AMSB framework, as the decay of the moduli fields and the gravitino will produce $\tilde{\nu}_{Li}$ with an abundance of the right order [41, 76].

Particles	Symbol	Mass (TeV)
Neutralinos	$\{m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}\}$	$\{0.175, 0.517\}$
Neutralinos	$\{m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}\}$	$\{0.980, 0.980\}$
Neutralinos	$\{m_{\tilde{\chi}_5^0}, m_{\tilde{\chi}_6^0}, m_{\tilde{\chi}_7^0}\}$	$\{0.206, 1.644, 3.278\}$
Charginos	$\{m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}\}$	$\{0.175, 0.983\}$
Gluino	M_3	1.239
Neutral Higgs bosons	$\{m_h, m_H, m_A\}$	$\{0.121, 0.793, 0.792\}$
Neutral Higgs bosons	$\{m_{h'}, m_{H'}, m_{A'}\}$	$\{0.060, 2.394, 0.241\}$
Charged Higgs bosons	m_{H^\pm}	0.796
R.H sleptons	$\{m_{\tilde{e}_R}, m_{\tilde{\mu}_R}, m_{\tilde{\tau}_1}\}$	$\{0.215, 0.215, 0.205\}$
L.H sleptons	$\{m_{\tilde{e}_L}, m_{\tilde{\mu}_L}, m_{\tilde{\tau}_2}\}$	$\{0.249, 0.249, 0.257\}$
Sneutrinos	$\{m_{\tilde{\nu}_e}, m_{\tilde{\nu}_\mu}, m_{\tilde{\nu}_\tau}\}$	$\{0.220, 0.220, 0.220\}$
R.H down squarks	$\{m_{\tilde{d}_R}, m_{\tilde{s}_R}, m_{\tilde{b}_1}\}$	$\{1.284, 1.284, 1.284\}$
L.H down squarks	$\{m_{\tilde{d}_L}, m_{\tilde{s}_L}, m_{\tilde{b}_2}\}$	$\{1.186, 1.186, 1.028\}$
R.H up squarks	$\{m_{\tilde{u}_R}, m_{\tilde{c}_R}, m_{\tilde{t}_1}\}$	$\{1.098, 1.098, 0.644\}$
L.H up squarks	$\{m_{\tilde{u}_L}, m_{\tilde{c}_L}, m_{\tilde{t}_2}\}$	$\{1.184, 1.184, 1.099\}$
R.H scalar neutrinos	$\{m_{\tilde{\nu}_i^c}\} (i = 1 - 3)$	0.605
R.H pseudoscalar neutrinos	$\{m_{\tilde{\nu}_i^c}\} (i = 1 - 3)$	0.413
Heavy scalar neutrino ($\tilde{\nu}^c, \tilde{\bar{\nu}}^c$)	$\{m_{\tilde{\omega}_{1s}}, m_{\tilde{\omega}_{2s}}\}$	$\{1.142, 3.644\}$
Heavy pseudoscalar neutrino ($\tilde{\nu}^c, \tilde{\bar{\nu}}^c$)	$\{m_{\tilde{\omega}_{ps}}, m_{\tilde{\omega}_{2p}}\}$	$\{0.595, 1.439\}$
R.H neutrinos	$\{m_{\nu_i^c}\}$	0.455
Heavy neutrinos ($\nu^c, \bar{\nu}^c$)	$\{m_{\omega_1}, m_{\omega_2}\}$	$\{0.933, 1.635\}$

TABLE 7.2. Sparticle masses in Model 1 ($x = 1.3$) for the choice $M_{aux} = 56.398$ TeV, $\tan \psi = -1.295$, $u = 2.054$ TeV, $f_{\nu_i^c} = 0.28$, $f_{\nu^c} = 0.28$, $h = 0.921$, $g_x = 0.41$, $M_{\nu^c} = 1$ TeV and $M_t = 174.3$ GeV. This corresponds to $\tan \beta = 4.39$, $\mu = -0.977$ TeV, $\mu' = 0.214$ TeV, $y_b = 0.03$.

Z' boson mass	$M_{Z'}$	2.383 TeV
$Z - Z'$ mixing angle	ξ	0.001

TABLE 7.3. Z' mass and $Z - Z'$ mixing angle in Model 1 for the same set of input parameters as in Table 7.2.

Fields	$\tilde{\chi}_1^0$	$\tilde{\chi}_2^0$	$\tilde{\chi}_3^0$	$\tilde{\chi}_4^0$	$\tilde{\chi}_5^0$	$\tilde{\chi}_6^0$	$\tilde{\chi}_7^0$
\tilde{B}	-0.003	0.998	0.051	0.025	0.000	-0.001	0.000
\tilde{W}_3^0	-0.997	0.001	-0.052	-0.058	0.000	0.002	0.000
\tilde{H}_d^0	0.078	0.054	-0.703	-0.704	-0.002	0.030	0.001
\tilde{H}_u^0	-0.004	0.019	-0.707	0.706	0.001	-0.042	0.016
\tilde{B}'	0.000	0.000	-0.004	-0.023	-0.026	-0.612	-0.790
\tilde{S}_+	0.000	0.000	-0.011	0.039	-0.597	0.642	-0.479
\tilde{S}_-	0.000	0.000	-0.009	0.026	0.802	0.458	-0.382

TABLE 7.4. Eigenvectors of the neutralino mass matrix in Model 1. The unitary matrix O in Eq. (7.84) is the transpose of this array.

U_{11}	U_{12}	U_{21}	U_{22}	V_{11}	V_{12}	V_{21}	V_{22}
0.994	0.110	-0.110	0.994	1.000	0.006	-0.006	1.000

TABLE 7.5. Eigenvectors of the chargino mass matrix in Model 1, where U, V are the unitary matrices that diagonalize the chargino mass matrix ($V^* M^{(c)} U^{-1} = M_{diag}^{(c)}$).

Fields	h	h'	H	H'
H_d^0	0.226	-0.025	0.974	-0.007
H_u^0	0.967	-0.110	-0.227	0.027
S_+	-0.050	-0.612	-0.010	-0.790
S_-	0.104	0.783	-0.008	-0.613

TABLE 7.6. Eigenvectors of the CP-even Higgs boson mass matrix in Model 1. This array corresponds to X used in Eqs. (7.80) – (7.82) and Eq. (7.107) of the text.

Particles	Symbol	Mass (TeV)
Neutralinos	$\{m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}\}$	{0.186, 0.550}
Neutralinos	$\{m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}\}$	{1.049, 1.050}
Neutralinos	$\{m_{\tilde{\chi}_5^0}, m_{\tilde{\chi}_6^0}, m_{\tilde{\chi}_7^0}\}$	{0.498, 2.840, 4.539}
Charginos	$\{m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}\}$	{0.186, 1.051}
Gluino	M_3	1.298
Neutral Higgs bosons	$\{m_h, m_H, m_A\}$	{0.126, 0.625, 0.625}
Neutral Higgs bosons	$\{m_{h'}, m_{H'}, m_{A'}\}$	{0.023, 3.436, 0.125}
Charged Higgs bosons	m_{H^\pm}	0.630
R.H sleptons	$\{m_{\tilde{e}_R}, m_{\tilde{\mu}_R}, m_{\tilde{\tau}_1}\}$	{0.383, 0.383, 0.385}
L.H sleptons	$\{m_{\tilde{e}_L}, m_{\tilde{\mu}_L}, m_{\tilde{\tau}_2}\}$	{0.213, 0.213, 0.210}
Sneutrinos	$\{m_{\tilde{\nu}_e}, m_{\tilde{\nu}_\mu}, m_{\tilde{\nu}_\tau}\}$	{0.174, 0.174, 0.174}
R.H down squarks	$\{m_{\tilde{d}_R}, m_{\tilde{s}_R}, m_{\tilde{b}_1}\}$	{1.370, 1.370, 1.369}
L.H down squarks	$\{m_{\tilde{d}_L}, m_{\tilde{s}_L}, m_{\tilde{b}_2}\}$	{1.267, 1.267, 1.087}
R.H up squarks	$\{m_{\tilde{u}_R}, m_{\tilde{c}_R}, m_{\tilde{t}_1}\}$	{1.031, 1.031, 0.406}
L.H up squarks	$\{m_{\tilde{u}_L}, m_{\tilde{c}_L}, m_{\tilde{t}_2}\}$	{1.264, 1.264, 1.1141}
R.H scalar neutrinos	$\{m_{\tilde{\nu}_i^c}\}(i = 1 - 3)$	1.583
R.H pseudoscalar neutrinos	$\{m_{\tilde{\nu}_i^c}\}(i = 1 - 3)$	1.129
Heavy scalar neutrino ($\tilde{\nu}^c, \tilde{\bar{\nu}}^c$)	$\{m_{\tilde{\omega}_{1s}}, m_{\tilde{\omega}_{2s}}\}$	{1.852, 4.700}
Heavy pseudoscalar neutrino ($\tilde{\nu}^c, \tilde{\bar{\nu}}^c$)	$\{m_{\tilde{\omega}_{ps}}, m_{\tilde{\omega}_{2p}}\}$	{1.398, 2.586}
R.H neutrinos	$\{m_{\nu_i^c}\}$	0.829
Heavy neutrinos ($\nu^c, \bar{\nu}^c$)	$\{m_{\omega_1}, m_{\omega_2}\}$	{1.174, 2.070}

TABLE 7.7. Sparticle masses in Model 2 ($x = 1.6$) for the choice $M_{aux} = 59.987$ TeV, $\tan \psi = -1.202$, $u = 2.697$ TeV, $f_{\nu_i^c} = 0.4$, $f_{\nu^c} = 0.4$, $h = 1.0$, $g_x = 0.45$, $M'_1 = 2.197$ TeV, $M_{\nu^c} = 1$ TeV and $M_t = 174.3$ GeV. This corresponds to $\tan \beta = 5.83$, $\mu = -1.046$ TeV, $\mu' = -0.505$ TeV, $y_b = 0.06$.

7.6 Z' decay modes and branching ratios

The Z' gauge boson of our model has substantial coupling to the quarks. With its mass in the range 2–4 TeV, it will be produced copiously at the LHC via the process $pp \rightarrow Z'$. The reach of LHC is about 5 TeV for a Z' with generic quark

Z' boson mass	$M_{Z'}$	3.433 TeV
$Z - Z'$ mixing angle	ξ	0.00068

TABLE 7.8. Z' mass and $Z - Z'$ mixing angle in Model 2 for the same set of input parameters as in Table 7.7.

Fields	$\tilde{\chi}_1^0$	$\tilde{\chi}_2^0$	$\tilde{\chi}_3^0$	$\tilde{\chi}_4^0$	$\tilde{\chi}_5^0$	$\tilde{\chi}_6^0$	$\tilde{\chi}_7^0$
\tilde{B}	-0.001	0.998	-0.052	0.023	0.000	0.000	0.000
\tilde{W}_3^0	-0.997	0.002	0.053	-0.052	0.000	-0.001	0.000
\tilde{H}_d^0	-0.074	-0.052	-0.703	0.705	-0.002	0.011	0.001
\tilde{H}_u^0	0.000	-0.020	-0.707	-0.707	-0.001	-0.021	0.016
\tilde{B}'	0.000	0.000	0.006	-0.004	0.023	0.0563	0.826
\tilde{S}_+	0.000	0.000	0.011	0.018	-0.648	-0.620	0.441
\tilde{S}_-	0.000	0.000	0.007	0.017	0.761	-0.546	0.350

TABLE 7.9. Eigenvectors of the neutralino mass matrix in Model 2. The unitary matrix O in Eq. (7.84) is the transpose of this array.

U_{11}	U_{12}	U_{21}	U_{22}	V_{11}	V_{12}	V_{21}	V_{22}
0.994	0.105	-0.105	0.994	1.000	0.000	-0.000	1.000

TABLE 7.10. Eigenvectors of the chargino mass matrix in Model 2, where U , V are the unitary matrices that diagonalize the chargino mass matrix ($V^* M^{(c)} U^{-1} = M_{diag}^{(c)}$).

Fields	h	h'	H	H'
H_d^0	0.176	0.002	0.984	0.005
H_u^0	0.984	0.010	-0.176	-0.025
S_+	-0.012	-0.640	0.007	-0.768
S_-	-0.023	0.768	0.006	-0.640

TABLE 7.11. Eigenvectors of the CP-even Higgs boson mass matrix in Model 2. This array corresponds to X used in Eqs. (7.80) – (7.82) and Eq. (7.107) of the text.

and lepton couplings [77]. Our model will then be directly tested at the LHC. Once produced, the Z' will decay into various channels. It is important to identify the

dominant decay modes of the Z' and calculate the corresponding branching ratios. This is what we do in this section. We will see that our Z' is almost leptophobic, with $Br(Z' \rightarrow e^+e^-) = (1 - 1.5)\%$. Direct limits on such a Z' are rather weak, however, the $Z - Z'$ mixing which occurs in our models at the level of 0.001 does provide useful constraints.

We now turn to the dominant 2-body decays of Z' . In this analysis we can safely ignore the small $Z - Z'$ mixing for the most part.

The Lagrangian for Z' coupling to the Standard Model fermions can be written as

$$\mathcal{L} = g_x \bar{f} \gamma^\mu (v_f - a_f \gamma_5) f Z'_\mu. \quad (7.55)$$

The Z' decay rate into a fermion-antifermion pair is then

$$\Gamma(Z' \rightarrow \bar{f}f) = C_f \frac{g_x^2}{12\pi} M_{Z'} \left[v_f^2 \left(1 + 2 \frac{m_f^2}{M_{Z'}^2} \right) + a_f^2 \left(1 - 4 \frac{m_f^2}{M_{Z'}^2} \right) \right] \sqrt{1 - 4 \frac{m_f^2}{M_{Z'}^2}}. \quad (7.56)$$

Here $C_f = 3$ (1) for quarks (leptons), $M_{Z'}$ is the Z' mass and g_x is the $U(1)_x$ gauge coupling. The vector and the axial-vector couplings (v_f , a_f) are related to the $U(1)_x$ charges of the fermions as

$$v_f = \frac{1}{2} (Q(f_L) + Q(f_R)), \quad (7.57)$$

$$a_f = \frac{1}{2} (Q(f_L) - Q(f_R)). \quad (7.58)$$

Here Q is the $U(1)_x$ charge of f_L (listed in Table 7.1) and $Q(f_R) = -Q(f_L^c)$.

The decay width for $Z' \rightarrow \bar{\nu}_{Li} \nu_{Li}$ and $Z' \rightarrow \bar{\nu}_i^c \nu_i^c$ are:

$$\Gamma(Z' \rightarrow \bar{\nu}_{Li} \nu_{Li}) = \frac{g_x^2}{24\pi} Q_{\nu_{Li}}^2 M_{Z'}, \quad (7.59)$$

$$\Gamma(Z' \rightarrow \bar{\nu}_i^c \nu_i^c) = \frac{g_x^2}{24\pi} Q_{\nu_i^c}^2 M_{Z'} \left(1 - 4 \frac{m_{\nu_i^c}^2}{M_{Z'}^2} \right)^{\frac{3}{2}}. \quad (7.60)$$

There is mixing between the heavy vector-like ν^c and the $\bar{\nu}^c$ [Cf: Eq. (7.36)], with the mass eigenstates (ω_1, ω_2) given by

$$\begin{pmatrix} \nu^c \\ \bar{\nu}^c \end{pmatrix} = \begin{pmatrix} \cos \theta_{\nu^c} & \sin \theta_{\nu^c} \\ -\sin \theta_{\nu^c} & \cos \theta_{\nu^c} \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix}. \quad (7.61)$$

Since $Q_{\bar{\nu}^c} = -Q_{\nu^c}$, the Lagrangian for the Z' coupling to these neutrino is given by

$$\begin{aligned} \mathcal{L} &= \frac{g_x}{2} Q_{\nu^c} (\cos 2\theta_{\nu^c} \bar{\omega}_1 \gamma^\mu \gamma_5 \omega_1 - \cos 2\theta_{\nu^c} \bar{\omega}_2 \gamma^\mu \gamma_5 \omega_2 - \sin 2\theta_{\nu^c} \bar{\omega}_1 \gamma^\mu \gamma_5 \omega_2 \\ &\quad - \sin 2\theta_{\nu^c} \bar{\omega}_2 \gamma^\mu \gamma_5 \omega_1) Z'_\mu. \end{aligned} \quad (7.62)$$

This leads to the decay rates

$$\Gamma(Z' \rightarrow \omega_1 \omega_1) = \frac{g_x^2}{24\pi} M_{Z'} Q_{\nu^c}^2 \cos^2 2\theta_{\nu^c} \left(1 - 4 \frac{m_{\omega_1}^2}{M_{Z'}^2}\right)^{\frac{3}{2}}, \quad (7.63)$$

$$\Gamma(Z' \rightarrow \omega_2 \omega_2) = \frac{g_x^2}{24\pi} M_{Z'} Q_{\nu^c}^2 \cos^2 2\theta_{\nu^c} \left(1 - 4 \frac{m_{\omega_2}^2}{M_{Z'}^2}\right)^{\frac{3}{2}}, \quad (7.64)$$

$$\begin{aligned} \Gamma(Z' \rightarrow \omega_1 \omega_2) &= \frac{g_x^2}{12\pi} M_{Z'} Q_{\nu^c}^2 \sin^2 2\theta_{\nu^c} \left[1 - \frac{(m_{\omega_1}^2 + m_{\omega_2}^2)}{2M_{Z'}^2} - \frac{(m_{\omega_1}^2 - m_{\omega_2}^2)^2}{2M_{Z'}^4} - 3 \frac{m_{\omega_1} m_{\omega_2}}{M_{Z'}^2}\right] \\ &\quad \times \sqrt{\left(1 - \frac{(m_{\omega_1} + m_{\omega_2})^2}{M_{Z'}^2}\right) \left(1 - \frac{(m_{\omega_1} - m_{\omega_2})^2}{M_{Z'}^2}\right)}. \end{aligned} \quad (7.65)$$

Here m_{ω_1} (m_{ω_2}) are the masses of the physical Majorana fermions.

The Z' interaction with the sfermions is described by the Lagrangian

$$\mathcal{L} = ig_x (v_f \pm a_f) \tilde{f}_{L,R}^* \overleftrightarrow{\partial}_\mu \tilde{f}_{L,R} Z'^\mu. \quad (7.66)$$

The rate for the decay Z' to sfermions is given by

$$\Gamma(Z' \rightarrow \tilde{f}_{L,R}^* \tilde{f}_{L,R}) = C_f \frac{g_x^2}{48\pi} M_{Z'} (v_f \pm a_f)^2 \left(1 - 4 \frac{m_{\tilde{f}_{L,R}}^2}{M_{Z'}^2}\right)^{\frac{3}{2}}, \quad (7.67)$$

where the $+$ ($-$) sign is for the left (right)-handed sfermions and $m_{\tilde{f}_{L,R}}$ is the left (right)-handed sfermion mass. v_f and a_f are as given in Eqs. (7.57)–(7.58).

In the top squark sector, there is non-negligible mixing between the left and the right-handed sfermions. This leads to the following modification of the Lagrangian:

$$\mathcal{L} = ig_x \left((v_f \pm a_f \cos 2\theta_{\tilde{f}}) \tilde{f}_{1,2}^* \overleftrightarrow{\partial}_\mu \tilde{f}_{1,2} - a_f \sin 2\theta_{\tilde{f}} (\tilde{f}_1^* \overleftrightarrow{\partial}_\mu \tilde{f}_2 + \tilde{f}_2^* \overleftrightarrow{\partial}_\mu \tilde{f}_1) \right) Z'^\mu, \quad (7.68)$$

where $\theta_{\tilde{f}}$ is the left-right sfermion mixing angle. The decay rate is given by

$$\Gamma(Z' \rightarrow \tilde{f}_{1,2}^* \tilde{f}_{1,2}) = C_f \frac{g_x^2}{48\pi} M_{Z'} (v_f \pm a_f \cos 2\theta_{\tilde{f}})^2 \left(1 - 4 \frac{m_{\tilde{f}_{1,2}}^2}{M_{Z'}^2}\right)^{\frac{3}{2}}, \quad (7.69)$$

$$\Gamma(Z' \rightarrow \tilde{f}_1^* \tilde{f}_2) = C_f \frac{g_x^2}{48\pi} M_{Z'} (a_f \sin 2\theta_{\tilde{f}})^2 \left[1 + 2 \frac{(m_1^2 + m_2^2)}{M_{Z'}^2} + \frac{(m_1^2 - m_2^2)^2}{M_{Z'}^4}\right]^{\frac{3}{2}} \quad (7.70)$$

The $\tilde{\nu}^c$ and $\tilde{\nu}^c$ splits into two scalar and two pseudoscalar which mix (see Eqs. (7.47)–(7.48)). The mass eigenstate $\tilde{\omega}_{is}$ and $\tilde{\omega}_{ip}$ are given as

$$\begin{pmatrix} \tilde{\nu}_s^c \\ \tilde{\nu}_s^c \end{pmatrix} = \begin{pmatrix} \cos \theta_{\omega s} & \sin \theta_{\omega s} \\ -\sin \theta_{\omega s} & \cos \theta_{\omega s} \end{pmatrix} \begin{pmatrix} \tilde{\omega}_{1s} \\ \tilde{\omega}_{2s} \end{pmatrix}, \quad (7.71)$$

$$\begin{pmatrix} \tilde{\nu}_p^c \\ \tilde{\nu}_p^c \end{pmatrix} = \begin{pmatrix} \cos \theta_{\omega p} & \sin \theta_{\omega p} \\ -\sin \theta_{\omega p} & \cos \theta_{\omega p} \end{pmatrix} \begin{pmatrix} \tilde{\omega}_{1p} \\ \tilde{\omega}_{2p} \end{pmatrix}. \quad (7.72)$$

The Lagrangian for the Z' coupling to the scalar–pseudoscalar pair is given by:

$$\begin{aligned} \mathcal{L} = & g_x \left[(Q_{\nu^c} \cos \theta_{\omega s} \cos \theta_{\omega p} + Q_{\bar{\nu}^c} \sin \theta_{\omega s} \sin \theta_{\omega p}) \tilde{\omega}_{1s} \overleftrightarrow{\partial}_\mu \tilde{\omega}_{1p} \right. \\ & + (Q_{\nu^c} \sin \theta_{\omega s} \sin \theta_{\omega p} + Q_{\bar{\nu}^c} \cos \theta_{\omega s} \cos \theta_{\omega p}) \tilde{\omega}_{2s} \overleftrightarrow{\partial}_\mu \tilde{\omega}_{2p} \\ & + (Q_{\nu^c} \cos \theta_{\omega s} \sin \theta_{\omega p} - Q_{\bar{\nu}^c} \sin \theta_{\omega s} \cos \theta_{\omega p}) \tilde{\omega}_{1s} \overleftrightarrow{\partial}_\mu \tilde{\omega}_{2p} \\ & \left. + (Q_{\nu^c} \sin \theta_{\omega s} \cos \theta_{\omega p} - Q_{\bar{\nu}^c} \cos \theta_{\omega s} \sin \theta_{\omega p}) \tilde{\omega}_{2s} \overleftrightarrow{\partial}_\mu \tilde{\omega}_{1p} \right] Z'^\mu. \quad (7.73) \end{aligned}$$

This leads to the decay rate

$$\Gamma(Z' \rightarrow \tilde{\omega}_{is} \tilde{\omega}_{jp}) = \frac{g_x^2}{48\pi} Q_{ij}^2 \left[1 - 2 \frac{(m_{\omega_{is}}^2 + m_{\omega_{jp}}^2)}{M_{Z'}^2} + \frac{(m_{\omega_{is}}^2 - m_{\omega_{jp}}^2)^2}{M_{Z'}^4} \right]^{\frac{3}{2}}, \quad (7.74)$$

where Q_{ij} is identified with the appropriate coupling to $\tilde{\omega}_{is} \tilde{\omega}_{jp}$ term in the Lagrangian of Eq. (7.73).

The supersymmetric partners of ν_i^c split into a scalar ($\tilde{\nu}_{is}^c$) and a pseudoscalar ($\tilde{\nu}_{ip}^c$). The decay of Z' to these fields is similar to those analyzed in Eq. (7.74):

$$\Gamma(Z' \rightarrow \tilde{\nu}_{is}^c \tilde{\nu}_{ip}^c) = \frac{g_x^2}{48\pi} Q_{\nu_i^c}^2 \left[1 - 2 \frac{(m_{\tilde{\nu}_{is}^c}^2 + m_{\tilde{\nu}_{ip}^c}^2)}{M_{Z'}^2} + \frac{(m_{\tilde{\nu}_{is}^c}^2 - m_{\tilde{\nu}_{ip}^c}^2)^2}{M_{Z'}^4} \right]^{\frac{3}{2}}, \quad (7.75)$$

where $m_{\tilde{\nu}_{is}^c}$ and $m_{\tilde{\nu}_{ip}^c}$ are the masses of the scalar and the pseudoscalar.

The Lagrangian for the Z' coupling to the charged Higgs bosons is given by

$$\begin{aligned} \mathcal{L} = & ig_x (Q_{H_d} \sin^2 \beta - Q_{H_u} \cos^2 \beta) H^+ \overleftrightarrow{\partial}_\mu H^- Z'^\mu \\ & + g_x (Q_{H_d} + Q_{H_u}) \sin \beta \cos \beta M_W (W_\mu^+ H^- + W_\mu^- H^+) Z'^\mu, \quad (7.76) \end{aligned}$$

where Q_{H_d} (Q_{H_u}) is the $U(1)_x$ charge of H_d (H_u) field. The decay rates of Z' to $H^+ H^-$ and $W^\pm H^\mp$ are given by

$$\Gamma(Z' \rightarrow H^+ H^-) = \frac{g_x^2}{48\pi} M_{Z'} (Q_{H_d} \sin^2 \beta - Q_{H_u} \cos^2 \beta)^2 \left(1 - 4 \frac{m_{H^\pm}^2}{M_{Z'}^2} \right)^{\frac{3}{2}}, \quad (7.77)$$

$$\begin{aligned}\Gamma(Z' \rightarrow W^\pm H^\mp) &= \frac{g_x^2}{192\pi} M_{Z'} (Q_{H_d} + Q_{H_u})^2 \left[1 + 2 \frac{(5M_W^2 - m_{H^\pm}^2)}{M_{Z'}^2} + \frac{(M_W^2 - m_{H^\pm}^2)^2}{M_{Z'}^4} \right] \\ &\times \sqrt{1 - 2 \frac{(M_W^2 + m_{H^\pm}^2)}{M_{Z'}^2} + \frac{(M_W^2 - m_{H^\pm}^2)^2}{M_{Z'}^4}}.\end{aligned}\quad (7.78)$$

Here m_{H^\pm} is the mass of the H^\pm Higgs boson and M_W is the mass of the W -boson.

The ZW^+W^- coupling of the Standard Model will induce, through $Z - Z'$ mixing, a $Z'W^+W^-$ coupling. The decay of Z' to a pair of W^+W^- is found to be [78]

$$\Gamma(Z' \rightarrow W^+W^-) = \frac{g_2^2}{192\pi} \cos^2 \theta_W \sin^2 \xi M_{Z'} \frac{M_{Z'}^4}{M_W^4} \left(1 + 20 \frac{M_W^2}{M_{Z'}^2} + 12 \frac{M_W^4}{M_{Z'}^4} \right) \left(1 - 4 \frac{M_W^2}{M_{Z'}^2} \right)^{\frac{3}{2}}$$

We now discuss the decays of $Z' \rightarrow Zh, ZH, Zh', ZH'$ as well as $Z' \rightarrow hA, h'A'$ etc.. The relevant Lagrangian is

$$\begin{aligned}\mathcal{L} &= 2g_x M_{Z'} \sum_{i=1}^4 (Q_{H_d} \cos \beta X_{1i} - Q_{H_u} \sin \beta X_{2i}) Z'^\mu Z_\mu H_i \\ &- g_x \sum_{i=1}^4 (Q_{H_d} \sin \beta X_{1i} + Q_{H_u} \cos \beta X_{2i}) Z'^\mu H_i^0 \overleftrightarrow{\partial}_\mu A \\ &- g_x \sum_{i=1}^4 (Q_{S_+} \cos \psi X_{3i} + Q_{S_-} \sin \psi X_{4i}) Z'^\mu H_i^0 \overleftrightarrow{\partial}_\mu A',\end{aligned}\quad (7.79)$$

where H_i^0 ($= h, h', H, H'$) are the neutral CP-even Higgs bosons, m_{H_i} are the masses of the corresponding Higgs boson, Q_{S_+} (Q_{S_-}) is the $U(1)_x$ charge of the S_+ (S_-) field and X_{ij} are the matrix elements of the unitary matrix that diagonalizes the CP-even mass matrix of Eq. (7.15). The decay rates are then

$$\begin{aligned}\Gamma(Z' \rightarrow ZH_i^0) &= \frac{g_x^2}{48\pi} M_{Z'} (Q_{H_d} \cos \beta X_{1i} - Q_{H_u} \sin \beta X_{2i})^2 \times \\ &\left[1 + 2 \frac{(5M_{Z'}^2 - m_{H_i}^2)}{M_{Z'}^2} + \frac{(M_{Z'}^2 - m_{H_i}^2)^2}{M_{Z'}^4} \right] \sqrt{1 - 2 \frac{(M_{Z'}^2 + m_{H_i}^2)}{M_{Z'}^2} + \frac{(M_{Z'}^2 - m_{H_i}^2)^2}{M_{Z'}^4}}\end{aligned}\quad (7.80)$$

$$\begin{aligned}\Gamma(Z' \rightarrow H_i A) &= \frac{g_x^2}{48\pi} M_{Z'} (Q_{H_d} \sin \beta X_{1i} + Q_{H_u} \cos \beta X_{2i})^2 \\ &\times \left[1 - 2 \frac{(m_A^2 + m_{H_i}^2)}{M_{Z'}^2} + \frac{(m_A^2 - m_{H_i}^2)^2}{M_{Z'}^4} \right]^{\frac{3}{2}}\end{aligned}\quad (7.81)$$

$$\begin{aligned}\Gamma(Z' \rightarrow H_i A') &= \frac{g_x^2}{48\pi} M_{Z'} (Q_{S_+} \cos \psi X_{3i} + Q_{S_-} \sin \psi X_{4i})^2 \\ &\times \left[1 - 2 \frac{(m_{A'}^2 + m_{H_i}^2)}{M_{Z'}^2} + \frac{(m_{A'}^2 - m_{H_i}^2)^2}{M_{Z'}^4} \right]^{\frac{3}{2}},\end{aligned}\quad (7.82)$$

where m_A and $m_{A'}$ are the pseudoscalar Higgs boson masses.

We parameterize the interactions between the neutralinos ($\tilde{\chi}_1^0, \tilde{\chi}_2^0, \dots, \tilde{\chi}_7^0$) and the Z' boson as

$$\mathcal{L} = \sum_{i,j} g_{ij} \bar{\tilde{\chi}}_i^0 \gamma^\mu \gamma_5 \tilde{\chi}_j^0 Z'_\mu. \quad (7.83)$$

Here the coupling g_{ij} is obtained from the eigenvectors of the neutralino mass matrix of Eq. (7.26) as

$$\hat{g} = \frac{g_x}{2} O \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{x}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{x}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 \end{pmatrix} O^T, \quad (7.84)$$

with $g_{ij} = (\hat{g})_{ij}$. Here O is the orthogonal matrix that diagonalizes the neutralino mass matrix. The Z' partial decay rates into neutralinos is found to be

$$\Gamma(Z' \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_i^0) = \frac{g_{ii}^2}{6\pi} M_{Z'} \left(1 - 4 \frac{m_i^2}{M_{Z'}^2}\right)^{\frac{3}{2}}, \quad (7.85)$$

$$\begin{aligned} \Gamma(Z' \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0) &= \frac{(g_{ij} + g_{ji})^2}{12\pi} M_{Z'} \left[1 - \frac{(m_i^2 + m_j^2)}{2M_{Z'}^2} - \frac{(m_i^2 - m_j^2)^2}{2M_{Z'}^4} - 3 \frac{m_i m_j}{M_{Z'}^2}\right] \\ &\times \sqrt{\left(1 - \frac{(m_i + m_j)^2}{M_{Z'}^2}\right) \left(1 - \frac{(m_i - m_j)^2}{M_{Z'}^2}\right)} \quad (i \neq j) \end{aligned} \quad (7.86)$$

where m_i are the neutralino masses. (Here our result disagrees with Eq. (48) of Ref. [64] by a factor of 2.)

The Lagrangian for the couplings of Z' to the charginos is given by [64]

$$\mathcal{L} = \frac{1}{2} g_x \sum_{i,j=1}^2 \bar{\tilde{\chi}}_i^\pm \gamma^\mu (v_{ij} + a_{ij} \gamma_5) \tilde{\chi}_j^\pm Z'_\mu. \quad (7.87)$$

The Z' decay rate into the chargino pair is then

$$\begin{aligned} \Gamma(Z' \rightarrow \tilde{\chi}_i^\pm \tilde{\chi}_j^\mp) &= \frac{g_x^2}{48\pi} M_{Z'} \left[(v_{ij}^2 + a_{ij}^2) \left(1 - \frac{(m_i^2 + m_j^2)}{2M_{Z'}^2} - \frac{(m_i^2 - m_j^2)^2}{2M_{Z'}^4}\right) + 3(v_{ij}^2 - a_{ij}^2) \frac{m_i m_j}{M_{Z'}^2} \right] \\ &\times \sqrt{\left(1 - \frac{(m_i + m_j)^2}{M_{Z'}^2}\right) \left(1 - \frac{(m_i - m_j)^2}{M_{Z'}^2}\right)}. \end{aligned} \quad (7.88)$$

Here m_i is the chargino mass, v_{ij} and a_{ij} are given in terms of the charges Q_{H_u} , Q_{H_d} and the matrices U and V which diagonalize the chargino mass matrix Eq. (7.28), can be explicitly written as [64]

$$v_{11} = Q_{H_d}V_{12}^2 - Q_{H_u}U_{12}^2, \quad (7.89)$$

$$a_{11} = Q_{H_d}V_{21}^2 + Q_{H_u}U_{21}^2, \quad (7.90)$$

$$v_{12} = v_{21} = Q_{H_d}V_{12}V_{11} - \delta Q_{H_u}U_{12}U_{11}, \quad (7.91)$$

$$a_{12} = a_{21} = Q_{H_d}V_{12}V_{11} + \delta Q_{H_u}U_{12}U_{11}, \quad (7.92)$$

$$v_{22} = Q_{H_d}V_{11}^2 - Q_{H_u}U_{11}^2, \quad (7.93)$$

$$a_{22} = Q_{H_d}V_{22}^2 + Q_{H_u}U_{22}^2, \quad (7.94)$$

where $\delta = \text{sgn}(m_{\tilde{\chi}_1^\pm}) \times \text{sgn}(m_{\tilde{\chi}_2^\pm})$.

In Table 7.12 we present the partial decay rates of Z' to two fermions and to two scalars in Model 1. The total width of Z' is 106 GeV (this ignores three body decays, which are more suppressed). One sees from Table 7.12 that the Z' decays dominantly to $q\bar{q}$ with $Br(Z' \rightarrow q\bar{q}) \simeq 43.93\%$. On the other hand, $Br(Z' \rightarrow e^+e^-) \simeq 1.16\%$ in this case. Thus this Z' is leptophobic. We also see that $Z' \rightarrow \tilde{\chi}_i^0\tilde{\chi}_j^0$ and $Z' \rightarrow \tilde{\chi}_i^\pm\tilde{\chi}_j^\mp$ are significant. There are also non-negligible decays into two Higgs particles, with $Z' \rightarrow h'A'$ being the dominant mode in this class. The decay of Z' into sfermions is a new production channel for supersymmetric particles. Decays into sneutrino pairs is the dominant mode in this category, with $Br(Z' \rightarrow \tilde{\nu}_L\tilde{\nu}_L) \simeq 7.74\%$. The signature will be $pp \rightarrow Z' \rightarrow \tilde{\nu}_{Li}\tilde{\nu}_{Li} \rightarrow \ell_i^-\ell_i^-\tilde{\chi}_1^+\tilde{\chi}_1^+$, where the sneutrino decays into $\ell_i^-\tilde{\chi}_1^+$, with the subsequent decay $\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 + \pi^\pm$, etc.

In Table 7.13 we list the Z' partial decay rates in Model 2. $Br(Z' \rightarrow e^+e^-) \simeq 1.60\%$ in this case. Other features are very similar to the case of Model 1 (Table 7.12).

Decay Modes of Z'	Width (GeV)
$Z' \rightarrow \{\bar{u}u, \bar{c}c, \bar{t}t\}$	{4.75, 4.75, 4.64}
$Z' \rightarrow \bar{d}d (\bar{s}s, \bar{b}b)$	9.59
$Z' \rightarrow \bar{e}e(\bar{\mu}\mu, \bar{\tau}\tau)$	1.13
$Z' \rightarrow \nu_{eL}\nu_{eL} (\nu_{\mu L}\nu_{\mu L}, \nu_{\tau L}\nu_{\tau L})$	0.65
$Z' \rightarrow \nu_{eR}\nu_{eR} (\nu_{\mu R}\nu_{\mu R}, \nu_{\tau R}\nu_{\tau R})$	4.19
$Z' \rightarrow \bar{\omega}_1\omega_1$	0.50
$Z' \rightarrow \{\tilde{\chi}_1\tilde{\chi}_3, \tilde{\chi}_1\tilde{\chi}_4, \tilde{\chi}_2\tilde{\chi}_4, \tilde{\chi}_3\tilde{\chi}_4\}$	{0.01, 0.01, 0.01, 3.38}
$Z' \rightarrow \{\tilde{\chi}_3\tilde{\chi}_5, \tilde{\chi}_4\tilde{\chi}_5, \tilde{\chi}_5\tilde{\chi}_5, \tilde{\chi}_5\tilde{\chi}_6\}$	{0.01, 0.05, 3.34, 5.65}
$Z' \rightarrow \{\tilde{\chi}_2^+\tilde{\chi}_2^-, \tilde{\chi}_1^+\tilde{\chi}_2^-, \tilde{\chi}_1^-\tilde{\chi}_2^+\}$	{3.36, 0.02, 0.02}
$Z' \rightarrow \tilde{u}_R^*\tilde{u}_R (\tilde{c}_R^*\tilde{c}_R)$	0.13
$Z' \rightarrow \{\tilde{t}_R^*\tilde{t}_R, \tilde{t}_L^*\tilde{t}_R, \tilde{t}_R^*\tilde{t}_L\}$	{0.88, 0.13, 0.13}
$Z' \rightarrow \tilde{e}_L^*\tilde{e}_L (\tilde{\mu}_L^*\tilde{\mu}_L, \tilde{\tau}_L^*\tilde{\tau}_L)$	0.30
$Z' \rightarrow \tilde{e}_R^*\tilde{e}_R (\tilde{\mu}_R^*\tilde{\mu}_R, \tilde{\tau}_R^*\tilde{\tau}_R)$	0.23
$Z' \rightarrow \tilde{\nu}_{eL}^*\tilde{\nu}_{eL} (\tilde{\nu}_{\mu L}^*\tilde{\nu}_{\mu L}, \tilde{\nu}_{\tau L}^*\tilde{\nu}_{\tau L})$	2.52
$Z' \rightarrow \tilde{\nu}_{1s}^c\tilde{\nu}_{1p}^c \{\tilde{\nu}_{2s}^c\tilde{\nu}_{2p}^c, \tilde{\nu}_{3s}^c\tilde{\nu}_{3p}^c\}$	1.94
$Z' \rightarrow \tilde{\omega}_{1s}\tilde{\omega}_{1p}$	0.36
$Z' \rightarrow Zh$	1.11
$Z' \rightarrow \{hA', HA, h'A'\}$	{0.03, 0.47, 0.62}
$Z' \rightarrow H^+H^-$	0.46
$Z' \rightarrow W^+W^-$	1.08
$Z' \rightarrow W^\pm H^\mp$	0

TABLE 7.12. Decay modes for Z' in Model 1 for the parameters used in Table 7.2. The total decay width is $\Gamma(Z' \rightarrow all) = 97.68$ GeV.

Decay Modes of Z'	Width (GeV)
$Z' \rightarrow \{\bar{u}u, \bar{c}c, \bar{t}t\}$	{15.00, 15.00, 14.86}
$Z' \rightarrow \bar{d}d (\bar{s}s, \bar{b}b)$	20.90
$Z' \rightarrow \bar{e}e(\bar{\mu}\mu, \bar{\tau}\tau)$	3.69
$Z' \rightarrow \nu_{eL}\nu_{eL} (\nu_{\mu L}\nu_{\mu L}, \nu_{\tau L}\nu_{\tau L})$	0.37
$Z' \rightarrow \nu_{eR}\nu_{eR} (\nu_{\mu R}\nu_{\mu R}, \nu_{\tau R}\nu_{\tau R})$	6.19
$Z' \rightarrow \{\bar{\omega}_1\omega_1, \bar{\omega}_1\omega_2\}$	{1.41, 0.06}
$Z' \rightarrow \{\tilde{\chi}_1\tilde{\chi}_3, \tilde{\chi}_1\tilde{\chi}_4, \tilde{\chi}_2\tilde{\chi}_4, \tilde{\chi}_3\tilde{\chi}_4\}$	{0.03 0.03, 0.03, 10.99}
$Z' \rightarrow \{\tilde{\chi}_3\tilde{\chi}_5, \tilde{\chi}_4\tilde{\chi}_5, \tilde{\chi}_5\tilde{\chi}_5, \tilde{\chi}_5\tilde{\chi}_6\}$	{0.01, 0.04, 1.63, 6.64}
$Z' \rightarrow \{\tilde{\chi}_2^+ \tilde{\chi}_2^-\}$	{10.96}
$Z' \rightarrow \tilde{u}_L^* \tilde{u}_L (\tilde{c}_L^* \tilde{c}_L)$	0.02
$Z' \rightarrow \tilde{u}_R^* \tilde{u}_R (\tilde{c}_R^* \tilde{c}_R)$	3.80
$Z' \rightarrow \{\tilde{t}_R^* \tilde{t}_R, \tilde{t}_L^* \tilde{t}_R, \tilde{t}_R^* \tilde{t}_L\}$	{5.93, 0.45, 0.45}
$Z' \rightarrow \tilde{d}_L^* \tilde{d}_L (\tilde{s}_L^* \tilde{s}_L, \tilde{b}_L^* \tilde{b}_L)$	0.02
$Z' \rightarrow \tilde{d}_R^* \tilde{d}_R (\tilde{s}_R^* \tilde{s}_R, \tilde{b}_R^* \tilde{b}_R)$	3.77
$Z' \rightarrow \tilde{e}_L^* \tilde{e}_L (\tilde{\mu}_L^* \tilde{\mu}_L, \tilde{\tau}_L^* \tilde{\tau}_L)$	0.18
$Z' \rightarrow \tilde{e}_R^* \tilde{e}_R (\tilde{\mu}_R^* \tilde{\mu}_R, \tilde{\tau}_R^* \tilde{\tau}_R)$	1.54
$Z' \rightarrow \tilde{\nu}_{eL}^* \tilde{\nu}_{eL} (\tilde{\nu}_{\mu L}^* \tilde{\nu}_{\mu L}, \tilde{\nu}_{\tau L}^* \tilde{\nu}_{\tau L})$	4.54
$Z' \rightarrow \tilde{\nu}_{1s}^c \tilde{\nu}_{1p}^c \{\tilde{\nu}_{2s}^c \tilde{\nu}_{2p}^c, \tilde{\nu}_{3s}^c \tilde{\nu}_{3p}^c\}$	1.04
$Z' \rightarrow \tilde{\omega}_{1s}\tilde{\omega}_{1p}$	0.91
$Z' \rightarrow Zh$	2.96
$Z' \rightarrow \{hA', HA, h'A'\}$	{0.01, 2.38, 0.60}
$Z' \rightarrow H^+H^-$	2.38
$Z' \rightarrow W^+W^-$	2.81
$Z' \rightarrow W^\pm H^\mp$	0

TABLE 7.13. Decay modes for Z' in Model 2 for the parameters used in Table 7.7. The total decay width is $\Gamma(Z' \rightarrow all) = 229.93$ GeV.

7.7 Other experimental signatures

In this section we discuss experimental signatures of the model other than Z' decays.

7.7.1 Z decay and precision electroweak data

The $Z - Z'$ mixing angle and the direct coupling of Z' to the Standard Model fermions leads to modification of Z decays. Precision electroweak data from LEP and SLC can be used to constrain such a Z' in the mass range of a few TeV. Typically one finds the $Z - Z'$ mixing angle ξ bounded to be less than a few $\times 10^{-3}$, which is satisfied in our models.

The mixing of Z with Z' shifts the mass of the Z boson from its SM value, while leaving the W mass unaffected. This leads to a positive shift in the ρ parameter:

$$\rho = \rho_{SM} \left(1 + \xi^2 \frac{M_{Z'}^2}{M_Z^2} \right). \quad (7.95)$$

The partial decay width $\Gamma(Z \rightarrow f\bar{f})$ is modified to

$$\Gamma(Z \rightarrow f\bar{f}) = \frac{\alpha M_Z}{12 \sin^2 \theta_W \cos^2 \theta_W} [(g_V \cos \xi + \kappa v_f \sin \xi)^2 + (g_A \cos \xi + \kappa a_f \sin \xi)^2] \quad (7.96)$$

where

$$g_V = (T_3 - 2q \sin^2 \theta_W), \quad g_A = T_3, \quad \kappa = \frac{2g_x \sin \theta_W \cos \theta_W}{e}, \quad (7.97)$$

with q being the electric charge of the fermion. v_f and v_a are given in Eqs. (7.57) and (7.58).

Partial widths of the Z will deviate from the Standard Model values owing to the shift in the coupling Z to fermions as well as due to a change in the derived value of $\sin^2 \theta_W$. We define

$$\Delta_f = \frac{\Gamma(Z \rightarrow f\bar{f})}{\Gamma(Z \rightarrow f\bar{f})_{SM}} - 1. \quad (7.98)$$

We use $\sin^2 \theta_W^{SM} = 0.23113$ (the best fit in the Standard Model) for evaluating $\Gamma(Z \rightarrow f\bar{f})_{SM}$. We do not perform a global fit to the available data, but we present a specific

fit which is at least as good as the Standard Model and perhaps slightly better. We choose to set $\Delta_\ell = 0$, which yields $\sin^2 \theta_W = 0.230717$ in Model 1. With this value of $\sin^2 \theta_W$ we find

$$\{\Delta_u, \Delta_d, \Delta_\nu\} = \{0.00100, 0.00171, 0.00206\} \quad (\text{Model 1}). \quad (7.99)$$

This leads to the following modifications of decay widths:

$$\Gamma_{had} = \Gamma_{had}^{SM} + \Delta_d(2\Gamma_d^{SM} + \Gamma_b^{SM}) + 2\Delta_u\Gamma_u^{SM} = 1.74545 \text{ GeV}, \quad (7.100)$$

$$\Gamma_{inv} = (1 + \Delta_\nu)\Gamma_{inv}^{SM} = 502.793 \text{ MeV}, \quad (7.101)$$

$$R_\ell = \frac{\Gamma_{had}}{\Gamma(Z \rightarrow \ell^+\ell^-)} = 20.7744. \quad (7.102)$$

We see that Γ_{had} is closer to the experimental value of 1.7444 GeV compared to the Standard Model value of 1.7429 GeV. Similarly R_ℓ is closer to the experimental value (20.767 ± 0.025) than the Standard Model value (20.744). On the other hand, Γ_{inv} is somewhat worse than the Standard Model fit (501.76 MeV) compared to the experimental value of (499.0 ± 1.5 MeV). This deviation is still within acceptable range. Here for our numerical fits we used the central values $\Gamma_d^{SM} = 0.383185$ GeV, $\Gamma_b^{SM} = 0.375926$ GeV and $\Gamma_c^{SM} = \Gamma_u^{SM} = 0.300302$ GeV [51].

The predicted value of M_W is modified as

$$M_W = \sqrt{\left[\left(1 + \xi^2 \frac{M_{Z'}^2}{M_Z^2} \right) \frac{1 - \sin^2 \theta_W}{1 - \sin^2 \theta_W^{SM}} \right]} M_W^{SM} = 80.4427 \text{ GeV}, \quad (7.103)$$

where $M_W^{SM} = 80.391$ GeV is used. This value is closer to the direct measurement $M_W = 80.446$ than the Standard Model value.

In Model 2 we find, following the same procedure, $\sin^2 \theta_W = 0.230783$, $\Delta_d = 0.00131$, $\Delta_u = 0.00089$, $\Delta_\nu = 0.00138$ and $\Gamma_{had} = 1.74493$ GeV, $\Gamma_{inv} = 502.453$ MeV, $R_\ell = 20.7682$, $M_W = 80.4356$ GeV.

The radiative correction parameter in μ decay, Δr , is slightly different in our model compared to the Standard Model. In the on shell scheme we have

$$\frac{M_W^2 \sin^2 \theta_W}{(M_W^2 \sin^2 \theta_W)_{SM}} = \frac{(1 - \Delta r)_{SM}}{(1 - \Delta r)}. \quad (7.104)$$

We obtain $\Delta r = 0.03501$ (in Model 1) using the Standard Model value of $\Delta r = 0.0355 \pm 0.0019$. Clearly, such a shift is consistent with experimental constraints ($(\Delta r)_{exp} = 0.0347 \pm 0.0011$).

7.7.2 Z' mass limit

The direct limit on the mass of Z' with generic couplings to quarks and leptons is $M_{Z'} > 600$ GeV. There is also a constraint on $M_{Z'}$ from the process $e^+e^- \rightarrow \mu^+\mu^-$. LEP II has set severe constraints on lepton compositeness [51, 58] from this process. We focus on one such amplitude, involving all left-handed lepton fields. In our model, the effective Lagrangian for this process is

$$L^{eff} = -g_x^2 \left(1 - \frac{x}{2}\right)^2 \frac{1}{M_{Z'}^2} (\bar{e}_L \gamma_\mu e_L) (\bar{\mu}_L \gamma^\mu \mu_L). \quad (7.105)$$

Comparing with $\Lambda_{LL}^-(ee\mu\mu) > 6.3$ TeV [51], we obtain $\frac{M_{Z'}}{g_x} \geq (1 - \frac{x}{2}) 2.51$ TeV. For $g_x = 0.41$ (0.45) and $x = 1.3$ (1.6) this implies $M_{Z'} \geq 361$ (226) GeV. For the choice of parameters in Tables 7.2 and 7.7, the above constraint is easily satisfied.

7.7.3 $h \rightarrow h'h'$ decay

Since the neutral Higgs boson h' is lighter than the Standard Model Higgs h , the decay $h \rightarrow h'h'$ can proceed for part of the parameter space. The decay rate is given by

$$\Gamma(h \rightarrow h'h') = \frac{G_{hh'}^2}{8\pi m_h} \sqrt{1 - 4 \frac{m_{h'}^2}{m_h^2}}, \quad (7.106)$$

where

$$\begin{aligned} G_{hh'}^2 &= \frac{(g_1^2 + g_2^2)}{4\sqrt{2}} [(v_d X_{11} - v_u X_{21})(X_{12}^2 - X_{22}^2) + 2(v_d X_{12} - v_u X_{22})(X_{11} X_{12} - X_{21} X_{22})] \\ &+ \frac{g_x^2}{4\sqrt{2}} [2(4X_{31} X_{32} - 4X_{41} X_{42} - xX_{11} X_{12} + xX_{21} X_{22}) \\ &\times (-xv_d X_{12} + xv_u X_{22} - 4yX_{42} + 4zX_{32}) \\ &- (4X_{32}^2 - 4X_{42}^2 - xX_{12}^2 + xX_{22}^2)(xv_d X_{11} - xv_u X_{21} + 4yX_{41} - 4zX_{31})]. \end{aligned} \quad (7.107)$$

Here X is the unitary matrix that diagonalizes the CP-even Higgs mass matrix of Eq. (7.15). In principle this can compete with the dominant decay $h \rightarrow b\bar{b}$. However we find that in Model 1 of Table 7.2 the decay is kinematically suppressed, while in Model 2 of Table 7.7 due to the small admixture of h in S_+ , S_- , this decay is

suppressed: $\Gamma(h \rightarrow h'h') = 1.48 \times 10^{-7}$ GeV (see Table 7.11). It is worth noting that if the mixings are as large as in Table 7.6 and if the decay is kinematically allowed, then $\Gamma(h \rightarrow h'h') \sim 0.1$ MeV is possible. Once produced, the dominant decays of h' will be $h' \rightarrow b\bar{b}$ and $h' \rightarrow c\bar{c}$ with comparable partial widths, as can be seen from H_u^0 and H_d^0 components in h' (see Table 7.6).

7.7.4 Signatures of SUSY particles

The supersymmetric particles, once produced in pp ($p\bar{p}$) collisions, will decay into the LSP. The LSP is $\tilde{\chi}_1^0$ (the neutral Wino) in Model 1 while it is the scalar neutrino $\tilde{\nu}_L$ in Model 2. In Model 1, $\tilde{\chi}_1^0$ is nearly mass degenerate with the lightest chargino $\tilde{\chi}_1^\pm$, with a mass splitting of about 180 MeV. The decay $\tilde{\chi}_1^0 \rightarrow \pi^\pm \chi_1^\mp$ will then occur within the detector. At the Tevatron Run 2 as well as at the LHC, the process $p\bar{p}$ (or pp) $\rightarrow \tilde{\chi}_1^0 + \tilde{\chi}_1^\pm$ will produce these SUSY particles. Naturalness suggest that $m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_1^\pm} \lesssim 300$ GeV (corresponding to $m_{gluino} \lesssim 2$ TeV). Strategies for detecting such a quasi-degenerate pair has been carried out in Ref. [56, 57]. In the case where the LSP is the left-handed sneutrino, the decay $\tilde{\chi}_1^\pm \rightarrow \ell^\pm \tilde{\nu}_L$ will be allowed. In this case $\tilde{\chi}_1^0$ will decay dominantly to $\tilde{\chi}_1^0 \rightarrow \tilde{\nu}_L \nu_L$.

7.8 Summary

We have suggested in this chapter a new class of supersymmetric Z' models motivated by the anomaly mediated supersymmetry breaking framework. The associated $U(1)$ symmetry is $U(1)_x = xY - (B - L)$, where Y is the Standard Model hypercharge. For $1 < x < 2$, the charges of the lepton doublets and the lepton singlets have the same sign. This implies that the $U(1)_x$ D -term can induce positive masses for both the doublet and the singlet sleptons and can cure the tachyonic problem of AMSB. We have shown explicitly that this is indeed possible in this class of models. In achieving this, the parameters of the model get essentially fixed. We have found that $M_{Z'} = 2-4$ TeV and the $Z-Z'$ mixing angle $\xi \simeq 0.001$. The phenomenologically

viable Z' turns out to be leptophobic – with $Br(Z' \rightarrow \ell^+\ell^-) \simeq (1 - 1.6)\%$. The dominant decay of Z' is to $q\bar{q}$ pair with $Br(Z' \rightarrow q\bar{q}) \simeq 44\%$. Decays into supersymmetric particles and Higgs bosons are also significant.

In Tables 7.2 and 7.7 we present our spectrum for two models, Model 1 (with $x = 1.3$) and Model 2 (with $x = 1.6$). The lightest SUSY particle is the neutral Wino (Model 1) or the sneutrino (Model 2). The partial decay widths of Z' are listed in Tables 7.12 and 7.13. These models are compatible with precision electroweak data, with the Z' models giving slightly better fits to the data than the Standard Model. This Z' should be within reach of LHC. The correlations between the Z' decays and the supersymmetric spectrum should make this class of models distinguishable from other Z' models.

CHAPTER 8

QUARK–LEPTON SUPERSYMMETRY

8.1 Introduction

In Nature it is a puzzle why some of the elementary fermions, viz; the quarks, feel strong interactions, while some others, the leptons do not. Perhaps at a higher scale the theory is manifestly quark–lepton symmetric and at low scale the disparity appears as a result of spontaneous symmetry breaking. By manifest quark-lepton symmetry we mean an interchange symmetry between quarks and leptons [79, 80]. The gauge symmetry of the SM and its spectrum does not admit such a symmetry. The simplest extension of the SM that achieves quark–lepton symmetry is obtained by postulating a new leptonic color force described by an $SU(3)_\ell$ gauge symmetry which acts on leptons, just as the $SU(3)_C$ force acts on the quarks. In this chapter we develop such a minimal supersymmetric quark–lepton symmetric model.

An interesting by–product of quark–lepton symmetric gauge sector is that if $SU(3)_\ell$ survives down to the TeV scale, anomaly mediated SUSY breaking can be consistently implemented without tachyonic sleptons. The gauge contributions from the $SU(3)_\ell$ sector render the sleptons with positive mass–squared, just as the $SU(3)_C$ contributions make quark mass–squared all positive. We show by explicit construction how this may be achieved and discuss the salient features of this model.

An interesting observation we make here is that gauge coupling unification works well within the minimal SUSY q – ℓ model, provided that the unification conditions are of string origin.

8.2 TeV scale quark–lepton symmetric model

The model is based on the gauge group

$$G_{ql} = SU(3)_\ell \times SU(3)_q \times SU(2)_L \times U(1)_x, \quad (8.1)$$

and is assumed to be supersymmetric.

The particle content of the model is shown in Table 8.1.

Superfield	$SU(3)_\ell$	$SU(3)_q$	$SU(2)_L$	$U(1)_x$
Q_L	1	3	2	$\frac{1}{6}$
u^c	1	$\bar{3}$	1	$-\frac{2}{3}$
d^c	1	$\bar{3}$	1	$\frac{1}{3}$
F_L	3	1	2	$-\frac{1}{6}$
E^c	$\bar{3}$	1	1	$\frac{2}{3}$
N^c	$\bar{3}$	1	1	$-\frac{2}{3}$
H_u	1	1	2	$\frac{1}{2}$
H_d	1	1	2	$-\frac{1}{2}$
χ_1	3	1	1	$\frac{1}{3}$
$\bar{\chi}_1$	$\bar{3}$	1	1	$-\frac{1}{3}$
χ_2	1	3	1	$-\frac{1}{3}$
$\bar{\chi}_2$	1	$\bar{3}$	1	$\frac{1}{3}$

TABLE 8.1. Particle content and charge assignment of the model.

The $SU(3)_\ell$ gauge group is the leptonic color group where the leptons F_L (E^c , N^c) transforms as triplet (antitriplets) while the $SU(3)_q$ gauge group is the usual color group. There is an exact interchange symmetry between the quarks and the leptons which is defined as:

$$Q \leftrightarrow F, \quad u^c \leftrightarrow E^c, \quad d^c \leftrightarrow N^c, \quad \chi_1 \leftrightarrow \chi_2, \quad \bar{\chi}_1 \leftrightarrow \bar{\chi}_2, \quad H_u \leftrightarrow H_d. \quad (8.2)$$

The model can be thought of as emerging from the quartification model proposed in Ref. [81] which has a higher gauge group

$$SU(3)_q \times SU(3)_\ell \times SU(3)_L \times SU(3)_R.$$

The superpotential of the model consistent with the gauge symmetries is given by *:

$$\begin{aligned}
W &= (Y_u)_{ij} Q_{L_i} H_u u_j^c + (Y_d)_{ij} Q_{L_i} H_d d_j^c + \mu H_u H_d \\
&+ (Y_\nu)_{ij} F_{L_i} H_u N_j^c + (Y_e)_{ij} F_{L_i} H_d E_j^c + \mu' \chi_1 \bar{\chi}_1 + \mu'' \chi_2 \bar{\chi}_2 \\
&+ (Y_N)_{ij} E_i^c N_j^c \bar{\chi}_1 + \frac{(Y_F)_{ij}}{2} F_{L_i} F_{L_j} \chi_1 \\
&+ (Y'_Q)_{ij} u_i^c d_j^c \bar{\chi}_2 + \frac{(Y_Q)_{ij}}{2} Q_{L_i} Q_{L_j} \chi_2.
\end{aligned} \tag{8.3}$$

The mass parameters μ , μ' and μ'' are of order TeV, which has a natural origin in AMSB [36]. The leptonic multiplets have the following structure:

$$F_\alpha = \begin{pmatrix} x_1 & x_2 & \nu \\ y_1 & y_2 & e \end{pmatrix}_\alpha, \quad E_\alpha^c = (y_1^c \quad y_2^c \quad e^c)_\alpha, \quad N^c = (x_1^c \quad x_2^c \quad \nu^c)_\alpha. \tag{8.4}$$

Here x, y are the exotic leptons needed to complete quark–lepton symmetry and $\alpha = 1, 2, 3$ is family indices. The electric charge generator is a linear combination of the diagonal generators of the gauge groups given by

$$Q = T_{3L} + X + \frac{T}{6}, \tag{8.5}$$

where $T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ is the $SU(3)_\ell$ (for the triplet representation of $SU(3)_\ell$) generator and X is the $U(1)_x$ charge. We identify the usual SM hypercharge as $Y = X + \frac{T}{6}$.

From Eq. (8.5) we find the relation between the electromagnetic coupling constant e and the other gauge coupling constant g_x , g_2 and g_ℓ as:

$$\frac{1}{e^2} = \frac{1}{g_2^2} + \frac{1}{g_x^2} + \frac{1}{3g_\ell^2}. \tag{8.6}$$

The $SU(3)_\ell$ gauge group acts on the leptons. This symmetry is broken by the VEVs of χ_1 and $\bar{\chi}_1$.

$$\langle \chi_1 \rangle = \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}, \quad \langle \bar{\chi}_1 \rangle = \begin{pmatrix} 0 \\ 0 \\ \bar{u} \end{pmatrix}.$$

*We do not impose q - ℓ interchange symmetry on the Yukawa couplings. If it is to be implemented, consistent with quark and lepton masses, a second pair of $H'_{u,d}$ will be needed.

The symmetry breaking pattern is

$$SU(3)_q \times SU(3)_\ell \times SU(2)_L \times U(1)_x \rightarrow SU(3)_q \times SU(2)_\ell \times SU(2)_L \times U(1)_Y.$$

The Higgs doublets H_u and H_d further breaks the symmetry $G_{SM} \times SU(2)_\ell$.

$$SU(3)_q \times SU(2)_L \times SU(2)_\ell \times U(1)_Y \rightarrow SU(3)_c \times U(1)_{em} \times SU(2)_\ell$$

8.2.1 Unification of gauge couplings

In order to check the prospect of unification in the low energy theory based on $G_{SM} \times SU(2)_\ell$, we use the solutions to the one-loop renormalization group equations

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu_0) + \frac{b_i}{2\pi} \ln \left(\frac{\mu_0}{\mu} \right), \quad (8.7)$$

where the gauge beta functions coefficients for the model are calculated to be

$$b_1 = \frac{47}{6}, \quad b_2 = 4, \quad b_3 = -2, \quad b_\ell = -2.$$

Using $\sin^2 \theta_W(M_Z) = 0.2315$, $\alpha^{-1}(M_Z) = 127.9$ as input and the condition for string unification

$$k_1 g_1^2 = k_2 g_2^2 = k_3 g_3^2 = k_\ell g_\ell^2, \quad (8.8)$$

where k_i are the Kac-Moody levels and with $k_1 = k_2 = 1$, $k_3 = k_\ell = 2$, we obtain to one loop accuracy

$$M_{GUT} = 1.6 \times 10^{16} \text{ GeV}, \quad \alpha_G^{-1} = 9.1 \quad \text{and} \quad \alpha_3(M_Z) = 0.123. \quad (8.9)$$

Note that when this model is embedded into $[SU(3)]^4$ quartification model, we have $k_1 = 1$ (as opposed to $k_1 = 5/3$ in $SU(5)$ or $SO(10)$ unification). The predicted value of $\alpha_3(M_Z)$ is in good agreement with experiment. Thus we see that the minimal quark-lepton supersymmetric model achieves unification of gauge couplings. We show the renormalization group evolution the inverse gauge couplings in Fig. 8.1.

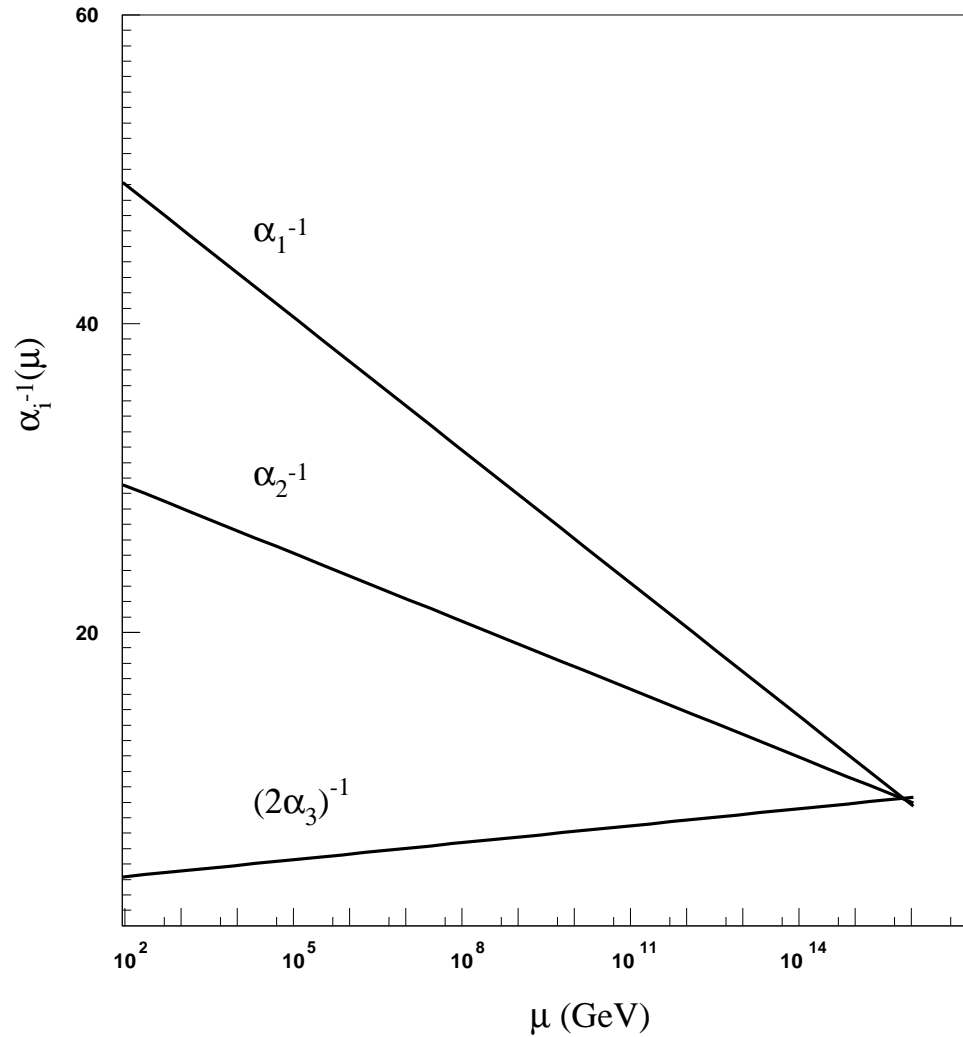


Figure 8.1. Renormalization group evolution the inverse gauge couplings.

8.3 Symmetry breaking

The model has two sets of Higgs bosons: the usual MSSM Higgs doublets H_u and H_d , and the $SU(3)_l$ Higgs triplets/antitriplets χ_1 and $\bar{\chi}_1$. The Higgs potential is derived from the superpotential of Eq. (8.3) and includes the soft terms and the

D terms. The Higgs potential cannot be split into two pieces because of the $U(1)$ D -term mixes the two Higgs sectors. The tree level potential for the model is given by:

$$\begin{aligned}
V(H_u, H_d, \chi_1, \bar{\chi}_1) &= (m_{H_u}^2 + \mu^2)|H_u|^2 + (m_{H_d}^2 + \mu^2)|H_d|^2 + B\mu(H_u H_d + c.c.) \\
&+ \frac{g_2^2}{8}(|H_u|^2 - |H_d|^2)^2 + \frac{g_2^2}{2}|H_u H_d|^2 + \frac{g_\ell^2}{2} \sum_a (\chi_1^\dagger \lambda^a \chi_1 - \bar{\chi}_1^\dagger \lambda^a \bar{\chi}_1)^2 \\
&+ (m_{\chi_1}^2 + \mu'^2)|\chi_1|^2 + (m_{\bar{\chi}_1}^2 + \mu'^2)|\bar{\chi}_1|^2 + B'\mu'(\chi_1 \bar{\chi}_1 + c.c.) \\
&+ \frac{g_x^2}{2} \left(\frac{|H_u|^2}{2} - \frac{|H_d|^2}{2} + \frac{|\chi_1|^2}{3} - \frac{|\bar{\chi}_1|^2}{3} \right)^2, \tag{8.10}
\end{aligned}$$

where the last term in Eq. (8.10) is the $U(1)_x$ D -term.

The VEV's of H_u , H_d , χ_1 and $\bar{\chi}_1$ are parameterized as

$$\langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \quad \langle H_d \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \quad \langle \chi_1 \rangle = \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}, \quad \langle \bar{\chi}_1 \rangle = \begin{pmatrix} 0 \\ 0 \\ \bar{u} \end{pmatrix}. \tag{8.11}$$

Minimization of the potential Eq. (8.10) leads to the following conditions:

$$\begin{aligned}
(m_{H_u}^2 + \mu^2) &= -B\mu \frac{v_d}{v_u} + \frac{g_2^2}{4}(v_d^2 - v_u^2) + \frac{g_x^2}{12}(2\bar{u}^2 - 2u^2 + 3v_d^2 - 3v_u^2) \\
(m_{H_d}^2 + \mu^2) &= -B\mu \frac{v_u}{v_d} - \frac{g_2^2}{4}(v_d^2 - v_u^2) - \frac{g_x^2}{12}(2\bar{u}^2 - 2u^2 + 3v_d^2 - 3v_u^2) \\
(m_{\chi_1}^2 + \mu'^2) &= -B'\mu' \frac{\bar{u}}{u} - \frac{g_\ell^2}{3}(u^2 - \bar{u}^2) + \frac{g_x^2}{18}(2\bar{u}^2 - 2u^2 + 3v_d^2 - 3v_u^2) \\
(m_{\bar{\chi}_1}^2 + \mu'^2) &= -B'\mu' \frac{u}{\bar{u}} + \frac{g_\ell^2}{3}(u^2 - \bar{u}^2) - \frac{g_x^2}{18}(2\bar{u}^2 - 2u^2 + 3v_d^2 - 3v_u^2). \tag{8.12}
\end{aligned}$$

We now consider the scalar mass matrices. The mass matrix for the CP-even neutral Higgs bosons is given by

$$(\mathcal{M}^2)_{cp\text{-}even} = \begin{pmatrix} (\mathcal{M}^2)_{11} & (\mathcal{M}^2)_{12} & -\frac{g_x^2}{3}v_d u & \frac{g_x^2}{3}v_d \bar{u} \\ (\mathcal{M}^2)_{12} & (\mathcal{M}^2)_{22} & \frac{g_x^2}{3}v_u u & -\frac{g_x^2}{3}v_u \bar{u} \\ -\frac{g_x^2}{3}v_d u & \frac{g_x^2}{3}v_u u & (\mathcal{M}^2)_{33} & (\mathcal{M}^2)_{34} \\ \frac{g_x^2}{3}v_d \bar{u} & -\frac{g_x^2}{3}v_u \bar{u} & (\mathcal{M}^2)_{34} & (\mathcal{M}^2)_{44} \end{pmatrix}, \tag{8.13}$$

where

$$(\mathcal{M}^2)_{11} = -B\mu \frac{v_u}{v_d} + \frac{(g_x^2 + g_2^2)}{2}v_d^2$$

$$\begin{aligned}
(\mathcal{M}^2)_{22} &= -B\mu \frac{v_d}{v_u} + \frac{(g_x^2 + g_2^2)}{2} v_u^2 \\
(\mathcal{M}^2)_{33} &= -B'\mu' \frac{\bar{u}}{u} + \frac{2g_x^2}{9} u^2 + \frac{2g_\ell^2}{3} u^2 \\
(\mathcal{M}^2)_{44} &= -B'\mu' \frac{u}{\bar{u}} + \frac{2g_x^2}{9} \bar{u}^2 + \frac{2g_\ell^2}{3} \bar{u}^2 \\
(\mathcal{M}^2)_{12} &= B\mu - \frac{(g_x^2 + g_2^2)}{2} v_u v_d \\
(\mathcal{M}^2)_{34} &= B'\mu' - \frac{2g_x^2}{9} u_u u_d + \frac{8g_\ell^2}{3} u\bar{u}.
\end{aligned} \tag{8.14}$$

The CP-odd Higgs boson from H_u and H_d fields has a mass given by

$$M_A^2 = -\frac{B\mu}{v_u v_d} (v_u^2 + v_d^2). \tag{8.15}$$

The CP-odd Higgs bosons for χ_1 and $\bar{\chi}_1$ fields has a mass

$$M_{A'}^2 = -\frac{B'\mu'}{u\bar{u}} (u^2 + \bar{u}^2). \tag{8.16}$$

The charged Higgs boson mass from H_u and H_d fields is given by

$$(\mathcal{M}^2)_{CH} = -\frac{B\mu}{v_u v_d} (v_u^2 + v_d^2) + \frac{g_2^2}{2} (v_u^2 + v_d^2). \tag{8.17}$$

The $SU(2)_\ell$ ‘‘charged’’ Higgs boson mass from χ_1 and $\bar{\chi}_1$ fields is given by

$$(\mathcal{M}^2)_{CH'} = -\frac{B'\mu'}{u\bar{u}} (u^2 + \bar{u}^2) + \frac{g_\ell^2}{2} (u^2 + \bar{u}^2). \tag{8.18}$$

In the neutral fermion sector, the Higgsinos from H_u , H_d , χ_1 , $\bar{\chi}_1$ mix with the gauginos \tilde{X} , \tilde{W}_3 , \tilde{C}_8 (where \tilde{C}_8 is the gaugino associated with the λ_8 generator of $SU(3)_\ell$). The (Majorana) mass matrix of the neutralinos $\{\tilde{X}, \tilde{W}_3, \tilde{H}_u^0, \tilde{H}_d^0, \tilde{C}_8, \tilde{\chi}_1, \tilde{\bar{\chi}}_1\}$ is given by

$$\mathcal{M}^{(0)} = \begin{pmatrix} M_x & 0 & \frac{v_u}{\sqrt{2}} g_x & -\frac{v_d}{\sqrt{2}} g_x & 0 & \frac{\sqrt{2}}{3} g_x u & -\frac{\sqrt{2}}{3} g_x \bar{u} \\ 0 & M_2 & \frac{v_u}{\sqrt{2}} g_2 & -\frac{v_d}{\sqrt{2}} g_2 & 0 & 0 & 0 \\ \frac{v_u}{\sqrt{2}} g_x & \frac{v_u}{\sqrt{2}} g_2 & 0 & -\mu & 0 & 0 & 0 \\ -\frac{v_d}{\sqrt{2}} g_x & \frac{v_d}{\sqrt{2}} g_2 & -\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_\ell & -\sqrt{\frac{2}{3}} g_\ell u & \sqrt{\frac{2}{3}} g_\ell \bar{u} \\ \frac{\sqrt{2}}{3} g_x u & 0 & 0 & 0 & -\sqrt{\frac{2}{3}} g_\ell u & 0 & \mu' \\ -\frac{\sqrt{2}}{3} g_x \bar{u} & 0 & 0 & 0 & \sqrt{\frac{2}{3}} g_\ell \bar{u} & \mu' & 0 \end{pmatrix}, \tag{8.19}$$

where M_x , M_ℓ and M_2 are the gaugino masses for $U(1)_x$, $SU(3)_\ell$ and $SU(2)_L$. The physical neutralino masses $m_{\tilde{\chi}_i^0}$ ($i=1-7$) are obtained as the eigenvalues of this mass matrix. We denote the diagonalizing matrix as O such that:

$$OM^{(0)}O^T = \text{diag}\{m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}, m_{\tilde{\chi}_5^0}, m_{\tilde{\chi}_6^0}, m_{\tilde{\chi}_7^0}\}. \quad (8.20)$$

In the basis $\{\tilde{W}^+, \tilde{H}_u^+\}$, $\{\tilde{W}^-, \tilde{H}_d^-\}$ the chargino (Dirac) mass matrix is

$$\mathcal{M}^{(c)} = \begin{pmatrix} M_2 & g_2 v_d \\ g_2 v_u & \mu \end{pmatrix}. \quad (8.21)$$

This matrix is diagonalized by a biunitary transformation $V^* \mathcal{M}^{(c)} U^{-1} = \text{diag}\{m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}\}$.

There is also a doublet of $SU(2)_\ell$ ‘‘chargino’’ $\{\tilde{C}_{13}^+, \tilde{\chi}_1\}$, $\{\tilde{C}_{13}^-, \tilde{\chi}_1\}$ with mass matrix

$$\mathcal{M}_d^{(c)} = \begin{pmatrix} M_\ell & g_\ell \bar{u} \\ g_\ell u & \mu' \end{pmatrix}. \quad (8.22)$$

When $SU(3)_\ell$ breaks down to $SU(2)_\ell$, we have $8 = 3 + 2 + 2 + 1$. The triplet gaugino mass M_l is given by

$$M_\ell = \frac{\beta_{g_\ell}}{g_\ell} M_{aux}. \quad (8.23)$$

In the gauge boson sector, G_8 of $SU(3)_\ell$, X and W_3 of $SU(2)_L$ mix. We identify the mass eigenstates of A , Z and Z' as

$$A = \frac{g_2 g_\ell X + \frac{g_2 g_x}{\sqrt{3}} G_8 + g_\ell g_x W_3}{\sqrt{g_2^2 \left(g_\ell^2 + \frac{g_x^2}{3} \right) + g_\ell^2 g_x^2}} \quad (8.24)$$

$$Z = \frac{X + \frac{g_x}{\sqrt{3} g_\ell} G_8 - \frac{g_2}{g_x} \left(1 + \frac{g_x^2}{3 g_\ell^2} \right) W_3}{\sqrt{\left(1 + \frac{g_x^2}{3 g_\ell^2} \right) \left(1 + \frac{g_2^2}{g_x^2} + \frac{g_2^2}{3 g_\ell^2} \right)}} \quad (8.25)$$

$$Z' = \frac{\frac{g_x}{3} X - g_\ell G_8}{\sqrt{g_\ell^2 + \frac{g_x^2}{3}}}. \quad (8.26)$$

The $SU(2)_\ell$ doublet gauge boson mass is given by

$$M_{G_{13}^\pm}^2 = M_{G_{23}^\pm}^2 = \frac{g_\ell^2}{2} (u^2 + \bar{u}^2) \quad (8.27)$$

The $Z - Z'$ mixing matrix is given by

$$\mathcal{M}_{Z-Z'}^2 = \begin{pmatrix} M_Z^2 & \gamma M_Z^2 \\ \gamma M_Z^2 & M_{Z'}^2 \end{pmatrix}, \quad (8.28)$$

where

$$\begin{aligned} \gamma &= \frac{g_x}{\sqrt{3}g_\ell \sqrt{1 + \frac{g_x^2}{g_\ell^2} + \frac{g_x^2}{3g_\ell^2}}}, \quad M_Z^2 = \frac{v^2}{2} \left(\frac{g_x^2}{1 + \frac{g_x^2}{3g_\ell^2}} + g_\ell^2 \right) \\ M_{Z'}^2 &= \frac{2}{3} \left(g_\ell^2 + \frac{g_x^2}{3} \right) (u^2 + \bar{u}) + \frac{g_x^4}{6(g_\ell^2 + \frac{g_x^2}{3})} v^2. \end{aligned} \quad (8.29)$$

The physical mass eigenstates Z_1 and Z_2 with masses M_{Z_1} , M_{Z_2} are

$$Z_1 = Z \cos \xi + Z' \sin \xi, \quad (8.30)$$

$$Z_2 = -Z \sin \xi + Z' \cos \xi, \quad (8.31)$$

where

$$M_{Z_1, Z_2}^2 = \frac{1}{2} \left[M_Z^2 + M_{Z'}^2 \pm \sqrt{(M_Z^2 - M_{Z'}^2)^2 + 4\gamma^2 M_Z^4} \right]. \quad (8.32)$$

The $Z - Z'$ mixing angle ξ is given by

$$\xi = \frac{1}{2} \arctan \left(\frac{2\gamma M_Z^2}{M_Z^2 - M_{Z'}^2} \right) \simeq -\gamma M_Z^2 / M_{Z'}^2. \quad (8.33)$$

The Z' coupling to the quarks and leptons is given by

$$\begin{aligned} L_{Z'} &= \frac{Z'^\mu}{\sqrt{3g_\ell^2 - g'^2}} \left[\frac{g'^2}{6} \bar{Q} \gamma_\mu Q - \frac{2}{3} g'^2 \bar{u}^c \gamma_\mu u^c + \frac{1}{3} g'^2 \bar{d}^c \gamma_\mu d^c \right. \\ &+ (g_\ell^2 - \frac{g'^2}{2}) \bar{L} \gamma_\mu L - (g_\ell^2 - g'^2) \bar{e}^c \gamma_\mu e^c \\ &\left. - \frac{1}{2} \frac{g_\ell^2}{\sqrt{3g_\ell^2 - g'^2}} (\bar{x}_1 \gamma_\mu x_1 + \bar{x}_2 \gamma_\mu x_2 + \bar{y}_1 \gamma_\mu y_1 + \bar{y}_2 \gamma_\mu y_2) \right] \end{aligned} \quad (8.34)$$

where we used Eq. (8.6) to eliminate g_x in favor of g' ;

$$\frac{1}{g'^2} = \frac{1}{g_x^2} + \frac{1}{3g_\ell^2}. \quad (8.35)$$

8.4 The SUSY spectrum

We are now ready to discuss the full SUSY spectrum of the model. We will see that the tachyonic slepton problem is cured by virtue of the positive contribution from the $SU(3)_\ell$ gauge sector. We discuss the SUSY spectrum in the context of anomaly mediation where the B and the B' terms are in general free parameters but for a special class of models it takes the form

$$B = -(\gamma_{H_u} + \gamma_{H_d})M_{aux} \quad \text{and} \quad B' = -(\gamma_{\chi_1} + \gamma_{\bar{\chi}_1})M_{aux}, \quad (8.36)$$

where the γ 's are the one-loop anomalous dimensions given in the Appendix B.

8.4.1 Slepton masses

The slepton mass-squared are given by the eigenvalues of the mass matrices ($\alpha = e, \mu, \tau$)

$$M_l^2 = \begin{pmatrix} m_L^2 & m_E (A_{Y_E} - \mu \tan \beta) \\ m_E (A_{Y_E} - \mu \tan \beta) & m_{\tilde{e}^c}^2 \end{pmatrix}. \quad (8.37)$$

Here

$$\begin{aligned} m_L^2 &= \frac{M_{aux}^2}{(16\pi^2)} [2Y_E\beta(Y_E) + Y_\nu\beta(Y_\nu) + 2Y_F\beta(Y_F) \\ &\quad - \left(\frac{3}{2}g_2\beta(g_2) + \frac{1}{18}g_x\beta(g_x) + \frac{8}{3}g_\ell\beta(g_\ell) \right)] \\ &\quad + m_E^2 - \frac{g_x^2}{36}(3v_u^2 - 3v_d^2 + 2u^2 - 2\bar{u}^2) + \frac{g_2^2}{4}(v_d^2 - v_u^2) + \frac{g_\ell^2}{3}(u^2 - \bar{u}^2), \end{aligned} \quad (8.38)$$

$$\begin{aligned} m_{\tilde{e}^c}^2 &= \frac{M_{aux}^2}{(16\pi^2)} \left[2Y_E\beta(Y_E) + 2Y_N\beta(Y_N) - \left(+\frac{8}{9}g_x\beta(g_x) + \frac{8}{3}g_\ell\beta(g_\ell) \right) \right] \\ &\quad + m_E^2 + \frac{g_x^2}{9}(3v_u^2 - 3v_d^2 + 2u^2 - 2\bar{u}^2) - \frac{g_\ell^2}{3}(u^2 - \bar{u}^2). \end{aligned} \quad (8.39)$$

8.4.2 Squark masses

The mixing matrix for the squark sector is similar to the slepton sector. The diagonal entries of the up and the down squark mass matrices are given by [27]

$$m_{\tilde{U}_i}^2 = (m_{soft}^2)_{\tilde{Q}_i} + m_{U_i}^2 + \frac{g_x^2}{36}(3v_u^2 - 3v_d^2 + 2u^2 - 2\bar{u}^2) + \frac{g_2^2}{4}(v_d^2 - v_u^2),$$

$$\begin{aligned}
m_{\tilde{U}_i^c}^2 &= (m_{soft}^2)_{\tilde{U}_i^c} + m_{U_i}^2 - \frac{g_x^2}{9}(3v_u^2 - 3v_d^2 + 2u^2 - 2\bar{u}^2), \\
m_{\tilde{D}_i}^2 &= (m_{soft}^2)_{\tilde{Q}_i} + m_{D_i}^2 + \frac{g_x^2}{36}(3v_u^2 - 3v_d^2 + 2u^2 - 2\bar{u}^2) - \frac{g_2^2}{4}(v_d^2 - v_u^2), \\
m_{\tilde{D}_i^c}^2 &= (m_{soft}^2)_{\tilde{D}_i^c} + m_{D_i}^2 - \frac{g_x^2}{18}(3v_u^2 - 3v_d^2 + 2u^2 - 2\bar{u}^2).
\end{aligned} \tag{8.40}$$

Here m_{U_i} and m_{D_i} are quark masses of different generations, $i = 1, 2, 3$. The squark soft masses are obtained from the RGE as

$$\begin{aligned}
(m_{soft}^2)_{\tilde{Q}_i} &= \frac{M_{aux}^2}{(16\pi^2)} [Y_u\beta(Y_u) + Y_d\beta(Y_d) + 2Y_Q\beta(Y_Q) \\
&\quad - \left(\frac{3}{2}g_2\beta(g_2) + \frac{1}{18}g_x\beta(g_x) + \frac{8}{3}g_3\beta(g_3) \right)],
\end{aligned} \tag{8.41}$$

$$(m_{soft}^2)_{\tilde{U}_i^c} = \frac{M_{aux}^2}{(16\pi^2)} \left[2Y_u\beta(Y_u) + 2Y_{Q'}\beta(Y_{Q'}) - \left(\frac{8}{9}g_x\beta(g_x) + \frac{8}{3}g_3\beta(g_3) \right) \right] \tag{8.42}$$

$$(m_{soft}^2)_{\tilde{D}_i^c} = \frac{M_{aux}^2}{(16\pi^2)} \left[2Y_d\beta(Y_d) + 2Y_{Q'}\beta(Y_{Q'}) - \left(\frac{2}{9}g_x\beta(g_x) + \frac{8}{3}g_3\beta(g_3) \right) \right] \tag{8.43}$$

8.4.3 Exotic slepton masses

The exotic slepton mass-squared matrix reduces to a 4×4 matrix given by

$$\tilde{M}_{exotic}^{sl} = \begin{pmatrix} m_L^2 + Y_F^2 u^2 & Y_F(A_{Y_F} u + \mu' \bar{u}) & 0 & v_d u Y_e Y_F \\ Y_F(A_{Y_F} u + \mu' \bar{u}) & \tilde{m}_e^2 + v_d^2 Y_e^2 + u^2 Y_F^2 & -v_d \bar{u} Y_e Y_N & Y_e v_d (A_{Y_e} + \mu) \\ 0 & -v_d \bar{u} Y_e Y_N & m_{N^c}^2 + \bar{u}^2 Y_N^2 & -(A_{Y_N} Y_N \bar{u} + \mu' u Y_N) \\ v_d u Y_e Y_F & Y_e v_d (A_{Y_e} + \mu) & -(A_{Y_N} Y_N \bar{u} + \mu' u Y_N) & m_{e^c}^2 + v_d^2 Y_e^2 + Y_N^2 \bar{u}^2 \end{pmatrix}. \tag{8.44}$$

The A -terms A_{Y_F} , A_{Y_e} , A_{Y_N} are given in Appendix B and masses-squared $m_{N^c}^2$ is given by

$$m_{N^c}^2 = (\tilde{m}_{soft}^2)_{N^c} - \frac{g_x^2}{18}(3v_u^2 - 3v_d^2 + 2u^2 - 2\bar{u}^2) - \frac{g_\ell^2}{6}(u^2 - \bar{u}^2), \tag{8.45}$$

where the soft mass $(\tilde{m}_{soft}^2)_{N^c}$ is given in Appendix B.

8.4.4 Exotic lepton masses

The mass matrix for the exotic leptons in the basis $\{x_1 \ x_2 \ y_1^c \ y_2^c\}$ is given by

$$M_{exotic}^l = \begin{pmatrix} Y_F u & 0 & 0 & Y_\nu v_u \\ 0 & -Y_F u & Y_\nu v_u & 0 \\ 0 & Y_e v_d & Y_N \bar{u} & 0 \\ Y_e v_d & 0 & 0 & -Y_N \bar{u} \end{pmatrix}. \tag{8.46}$$

The physical mass is the eigenvalue the mass matrix Eq. (8.46).

8.5 Numerical results

We are now ready to present our numerical results for the SUSY spectrum. We first performed a one-loop accuracy numerical analysis to determine the sparticle and Higgs Spectrum. For experimental inputs for the SM gauge couplings we use the same procedure Ref. [61] for the g_1, g_2, g_3 with the central value of the top mass taken to be $M_t = 178$ GeV. In Tables 8.2 we have taken $M_{aux} = 70.492$ TeV, while in Table 8.7 we have $M_{aux} = 55.143$ TeV. Other input parameters are listed in the respective Table captions.

In the model presented, the LSP is not necessarily the neutral wino. In model 1, the LSP is the chargino of $SU(2)_\ell$ sector which decays to a lighter. This chargino when produced can decay to charged leptons as shown in the figure. The slepton masses are positive and the Z' constraints are all satisfied. The slepton mass is comparable with the squark mass because of the quark-lepton symmetry.

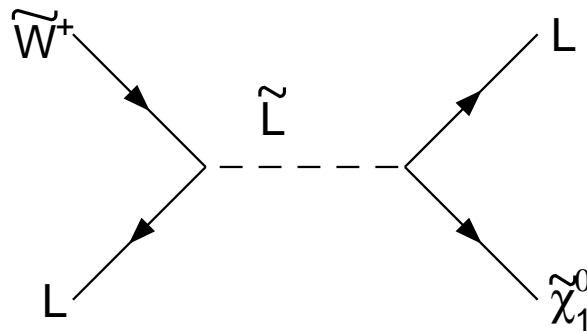


Figure 8.2. \tilde{W}^+ decay to two leptons and LSP.

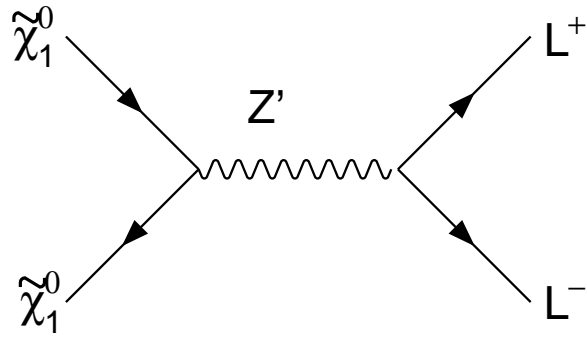


Figure 8.3. Neutralino annihilation to two charged leptons in the early universe.

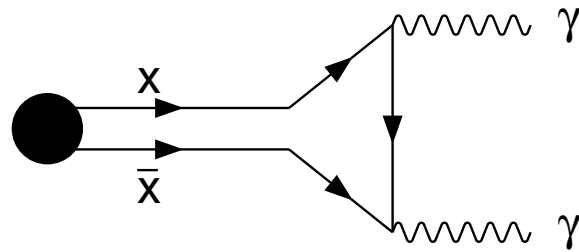


Figure 8.4. Bound state of two x leptons decay to two photons.

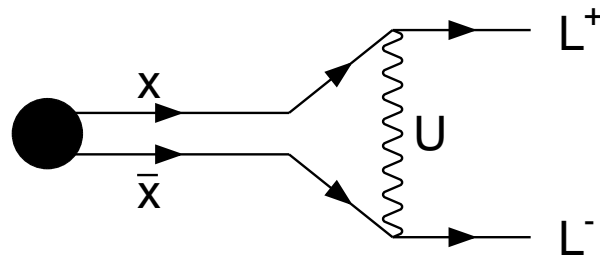


Figure 8.5. Bound state of two x leptons decay to two charged leptons via exchange of $SU(2)_H$ gauge boson.

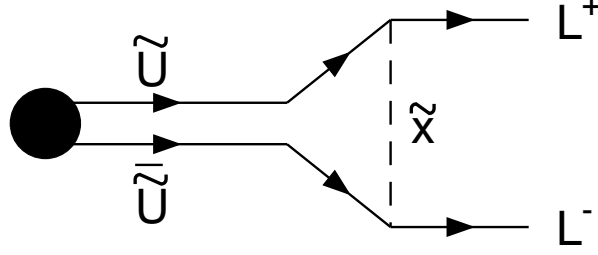


Figure 8.6. Doublet $SU(2)_H$ gauge boson decay to two charged leptons via exchange of neutralino LSP.

Neutralinos	$\{m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}\}$	$\{0.610, 0.729, 0.736, 1.363\}$
Neutralinos	$\{m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_6^0}, m_{\tilde{\chi}_7^0}\}$	$\{1.365, 1.687, 2.079\}$
Charginos	$\{m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}\}$	$\{0.729, 1.368\}$
Charginos ($SU(2)_\ell$)	$\{m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}\}$	$\{0.588, 2.275\}$
Gluino	M_3	1.089
Higgs bosons	$\{m_h, m_H, m_A, m_{H^\pm}\}$	$\{0.117, 1.304, 1.303, 1.305\}$
Higgs bosons	$\{m_{h'}, m_{H'}, m_{A'}, m_{H'^\pm}\}$	$\{0.028, 2.490, 1.854, 2.331\}$
R.H sleptons	$\{m_{\tilde{e}_R}, m_{\tilde{\mu}_R}, m_{\tilde{\tau}_1}\}$	$\{1.198, 1.198, 1.196\}$
L.H sleptons	$\{m_{\tilde{e}_L}, m_{\tilde{\mu}_L}, m_{\tilde{\tau}_2}\}$	$\{1.150, 1.150, 1.149\}$
R.H down squarks	$\{m_{\tilde{d}_R}, m_{\tilde{s}_R}, m_{\tilde{b}_1}\}$	$\{1.223, 1.223, 1.220\}$
L.H down squarks	$\{m_{\tilde{d}_L}, m_{\tilde{s}_L}, m_{\tilde{b}_2}\}$	$\{1.140, 1.140, 0.890\}$
R.H up squarks	$\{m_{\tilde{u}_R}, m_{\tilde{c}_R}, m_{\tilde{t}_1}\}$	$\{1.206, 1.206, 0.965\}$
L.H up squarks	$\{m_{\tilde{u}_L}, m_{\tilde{c}_L}, m_{\tilde{t}_2}\}$	$\{1.138, 1.138, 0.602\}$
Exotic sleptons	$\{\tilde{m}_{ex1}, \tilde{m}_{ex2}, \tilde{m}_{ex3}, \tilde{m}_{ex4}\}$	$\{0.864, 1.011, 1.357, 1.403\}$
Exotic leptons	$\{m_{ex1}, m_{ex2}\}$	$\{0.130, 0.127\}$
$SU(3)_\ell$ gauge boson	M_V	1.413

TABLE 8.2. Sparticle masses in Model 1 for the choice $M_{aux} = 70.492$ TeV, $\tan \beta = 3.82$, $M_t = 174.3$ GeV, $\mu = 1.360$ TeV, $\mu' = 0.610$ TeV $y_b = 0.07$, $Y_i = 0.1$, $u = -1.301$ TeV, $\bar{u} = 1.272$ TeV, $B = -0.306$ TeV and $B' = 2.816$ TeV.

Z' boson mass	$M_{Z'}$	1.662 TeV
$Z - Z'$ mixing angle	ξ	0.00032

TABLE 8.3. Z' mass and $Z - Z'$ mixing angle in Model 1 for the same set of input parameters as in Table 8.2.

Fields	$\tilde{\chi}_1^0$	$\tilde{\chi}_2^0$	$\tilde{\chi}_3^0$	$\tilde{\chi}_4^0$	$\tilde{\chi}_5^0$	$\tilde{\chi}_6^0$	$\tilde{\chi}_7^0$
\tilde{X}	-0.001	0.946	-0.027	0.037	-0.010	0.316	0.046
\tilde{W}_3^0	0.000	0.025	0.997	-0.061	-0.032	0.016	0.000
\tilde{H}_u^0	0.000	-0.030	-0.021	-0.680	0.707	0.191	-0.002
\tilde{H}_d^0	0.000	0.046	0.066	0.678	0.706	-0.187	-0.001
\tilde{C}^8	-0.009	0.190	-0.006	-0.128	-0.003	-0.426	-0.875
$\tilde{\chi}_1$	-0.700	0.180	-0.006	-0.169	0.000	-0.572	0.350
$\tilde{\tilde{\chi}}_1$	0.714	0.180	-0.006	-0.167	0.000	-0.566	0.331

TABLE 8.4. Eigenvectors of the neutralino mass matrix in Model 1. The unitary matrix O in Eq. (8.20) is the transpose of this array.

U_{11}	U_{12}	U_{21}	U_{22}	V_{11}	V_{12}	V_{21}	V_{22}
0.992	0.126	-0.126	0.992	0.996	0.088	-0.088	0.996

TABLE 8.5. Eigenvectors of the chargino mass matrix in Model 1, where U, V are the unitary matrices that diagonalize the chargino mass matrix ($V^* M^{(c)} U^{-1} = M_{diag}^{(c)}$).

U_{11}	U_{12}	U_{21}	U_{22}	V_{11}	V_{12}	V_{21}	V_{22}
0.780	-0.625	0.625	0.780	0.769	0.639	-0.639	0.769

TABLE 8.6. Eigenvectors of $SU(2)_\ell$ chargino mass matrix in Model 1, where U , V are the unitary matrices that diagonalize the $SU(2)_\ell$ chargino mass matrix ($V^* M_d^{(c)} U^{-1} = M_{d\text{diag}}^{(c)}$).

Neutralinos	$\{m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}\}$	{0.197, 0.520, 0.573, 1.027}
Neutralinos	$\{m_{\tilde{\chi}_5^0}, m_{\tilde{\chi}_6^0}, m_{\tilde{\chi}_7^0}\}$	{1.030, 1.140, 1.721}
Charginos	$\{m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}\}$	{0.569, 1.034}
Charginos ($SU(2)_\ell$)	$\{m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}\}$	{0.641, 1.682}
Gluino	M_3	0.844
Higgs bosons	$\{m_h, m_H, m_A, m_{H^\pm}\}$	{0.122, 0.923, 0.923, 0.926}
Higgs bosons	$\{m_{h'}, m_{H'}, m_{A'}, m_{H'^\pm}\}$	{0.023, 1.858, 1.318, 1.727}
R.H sleptons	$\{m_{\tilde{e}_R}, m_{\tilde{\mu}_R}, m_{\tilde{\tau}_1}\}$	{0.941, 0.941, 0.935}
L.H sleptons	$\{m_{\tilde{e}_L}, m_{\tilde{\mu}_L}, m_{\tilde{\tau}_2}\}$	{0.902, 0.902, 0.898}
R.H down squarks	$\{m_{\tilde{d}_R}, m_{\tilde{s}_R}, m_{\tilde{b}_1}\}$	{0.960, 0.960, 0.946}
L.H down squarks	$\{m_{\tilde{d}_L}, m_{\tilde{s}_L}, m_{\tilde{b}_2}\}$	{0.895, 0.895, 0.697}
R.H up squarks	$\{m_{\tilde{u}_R}, m_{\tilde{c}_R}, m_{\tilde{t}_1}\}$	{0.947, 0.947, 0.786}
L.H up squarks	$\{m_{\tilde{u}_L}, m_{\tilde{c}_L}, m_{\tilde{t}_2}\}$	{0.892, 0.892, 0.473}
Exotic sleptons	$\{\tilde{m}_{ex_1}, \tilde{m}_{ex_2}, \tilde{m}_{ex_3}, \tilde{m}_{ex_4}\}$	{0.653, 0.776, 1.078, 1.115}
Exotic leptons	$\{m_{ex_1}, m_{ex_2}\}$	{0.100, 0.103}
$SU(3)_\ell$ gauge boson	M_V	1.116

TABLE 8.7. Sparticle masses in Model 2 for the choice $M_{aux} = 55.143$ TeV, $\tan \beta = 7.87$, $M_t = 178.0$ GeV, $\mu = 1.024$ TeV, $\mu' = -0.197$ TeV $y_b = 0.14$, $Y_i = 0.1$, $u = 1.028$ TeV, $\bar{u} = 1.003$ TeV, $B = -0.104$ TeV and $B' = 4.414$ TeV.

Z' boson mass	$M_{Z'}$	1.311 TeV
$Z - Z'$ mixing angle	ξ	0.00048

TABLE 8.8. Z' mass and $Z - Z'$ mixing angle in Model 2 for the same set of input parameters as in Table 8.7.

8.5.1 Coupling of light Higgs to SM fermions

In order to determine the couplings of the light Higgs h' to the Standard Model fermions, we first determine the eigenvectors of the CP-even mass matrix as

$$\begin{pmatrix} H_d^0 \\ H_u^0 \\ \chi_1^0 \\ \bar{\chi}_1^0 \end{pmatrix} = O_H \begin{pmatrix} h \\ H \\ h' \\ H' \end{pmatrix}, \quad (8.47)$$

where O_H is the eigenvector that diagonalize the mass matrix. In model 1, O_H is given by

$$O_H = \begin{pmatrix} 0.255 & 0.967 & -0.007 & 0.001 \\ 0.966 & -0.255 & -0.027 & -0.002 \\ -0.019 & -0.001 & -0.708 & 0.706 \\ 0.021 & -0.001 & 0.706 & 0.708 \end{pmatrix}. \quad (8.48)$$

From the superpotential Eq. (8.3) we find the couplings of the third generation fermions to the light Higgs as

$$0.027Y_t h' t\bar{t}, \quad 0.007Y_b h' b\bar{b} \quad \text{and} \quad 0.007Y_\tau h' \tau\tau^c \quad (8.49)$$

In model 2, O_H is given by

$$O_H = \begin{pmatrix} 0.126 & 0.992 & -0.001 & 0.000 \\ 0.992 & -0.126 & -0.010 & 0.002 \\ -0.006 & 0.000 & -0.705 & -0.709 \\ 0.009 & 0.000 & 0.709 & -0.705 \end{pmatrix}. \quad (8.50)$$

From the superpotential Eq. (8.3) we find the couplings of the third generation fermions to the light Higgs as

$$0.010Y_t h' t \bar{t}, \quad 0.001Y_b h' b \bar{b} \quad \text{and} \quad 0.001Y_\tau h' \tau \tau^c \quad (8.51)$$

With these information, we can determine the decay width of the Higgs boson $h' \rightarrow b\bar{b}, \tau\tau^c$. Decay width of Z' to quarks and leptons

$$\Gamma(Z' \rightarrow \bar{u}u) = 0.164, \quad \Gamma(Z' \rightarrow \bar{d}d) = 0.048, \quad \Gamma(Z' \rightarrow \bar{t}t) = 0.161$$

$$\Gamma(Z' \rightarrow \bar{e}e) = 15.448$$

$$\Gamma(Z' \rightarrow \nu_L \nu_L) = 8.197, \quad \Gamma(Z' \rightarrow \nu_R \nu_R) = 7.875$$

$$\Gamma(Z' \rightarrow \bar{x}x) = 4.775, \quad \Gamma(Z' \rightarrow \bar{y}y) = 3.818$$

8.5.2 Neutralino s-channel annihilation

We calculate the thermal averaged cross section for s channel Z' boson contribution to the lightest neutralino annihilating into fermions. We show that the LSP is stable and is a candidate for cold dark matter. We begin by calculating the cross section for the process $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow f \bar{f}$. Here $\tilde{\chi}_1^0 \approx N_{11}\chi_1 + N_{12}\bar{\chi}_1$.

The cross section for this process is given by;

$$\sigma = \frac{s}{12\pi} \frac{(C_V^2 + C_A^2)(C_{V'}^2 + C_{A'}^2)}{s - M_{Z'}^2 + \Gamma_{Z'}^2 M_{Z'}^2} \left[1 - \frac{4M_{\tilde{\chi}_1^0}^2}{s} \right] \quad (8.52)$$

$$C_V = \frac{g'^2}{4\sqrt{3g_\ell^2 - g'^2}}$$

$$C_A = \frac{-4g_\ell^2 + 3g'^2}{4\sqrt{3g_\ell^2 - g'^2}}$$

$$C_{V'} = \frac{g_\ell^2}{\sqrt{3g_\ell^2 - g'^2}}$$

$$C_{A'} = 0.$$

We follow the same procedures as in [82]. We use the general formula

$$\langle \sigma v_{rel} \rangle = \frac{1}{M_\chi^2} \left[w - \frac{3}{2}(2w - w')x + O(x^2) \right]_{\frac{s}{4M_\chi^2}=1} \quad (8.53)$$

where the primes denote derivatives with respect to $\alpha (s/4M_\chi^2)$ and

$$w = \frac{3\alpha^2 M_\chi^2 (C_V^2 + C_A^2)(C_{V'}^2 + C_{A'}^2)}{4\pi (4\alpha M_\chi^2 - M_{Z'}^2)} \left(1 - \frac{1}{\alpha}\right). \quad (8.54)$$

Eq. (8.53) is to be evaluated at $s/4M_\chi^2 = 1$ and $x \equiv \frac{T}{M_\chi}$. The freeze – out temperature T is defined as the temperature at which the expansion rate of the co – moving volume becomes larger than the rate of annihilation. For a stable neutralinos $x \approx \frac{1}{20}$

The neutralino relic abundance through the rule of thumb [83]

$$\Omega_\chi h^2 \simeq 10^{-27} cm^3 s / \langle \sigma_{eff} v \rangle.$$

We find it to $\Omega_\chi h^2 \simeq 0.07$ in Model 1. and $\Omega_\chi h^2 \simeq 1.0$ in model 2.

8.6 Summary

In this chapter we have suggested a simple solution to the negative slepton mass problem of AMSB. The model we presented is a quark–lepton symmetric model based on a new leptonic color force described by an $SU(3)_\ell$ gauge symmetry. The model predicts the slepton masses to be ~ 1 TeV and they are of the same order as the squark mass. The model also predicts the lightest Higgs boson mass to be $m_h > 117$ GeV. There is a light Higgs present in the model, when produced they decay to $b\bar{b}$ and $\tau\tau^c$. We find the $M_{Z'} = 1.2 - 2.0$ TeV and the $Z - Z'$ mixing $\xi \simeq 0.0004$. The Z' turns out to be leptophobic.

The gauge coupling unification works well within the minimal SUSY quark–lepton model with the unification conditions of string origin. The LSP can either be the neutral wino or the chargino of $SU(2)_\ell$ which is a candidate for cold dark matter.

CHAPTER 9

CP VIOLATION IN NEUTRINO OSCILLATIONS FROM NONSTANDARD PHYSICS

9.1 Introduction

In recent years, the observation of solar [84–90] and atmospheric [91–94] neutrino deficit has provided strong evidence for neutrino oscillations. In particular, neutrino oscillation data suggest the mass-squared differences for the solar and atmospheric neutrinos to be $\Delta m_{\odot}^2 \sim 7.5 \times 10^{-5} \text{eV}^2$ and $\Delta m_{atm}^2 \sim 2.0 \times 10^{-3} \text{eV}^2$, respectively. Because of the hierarchy $\Delta m_{\odot}^2 \ll \Delta m_{atm}^2$, three flavors are needed to simultaneously explain the solar and atmospheric neutrino problem. However, the yet unconfirmed measurement from the Liquid Scintillator Neutrino Detector (LSND) experiment at Los Alamos indicates neutrinos oscillation with a mass squared difference $\Delta m_{LSND}^2 \sim 0.2 - 1 \text{eV}^2$. The LSND experiment has reported evidence for $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e$ and $\nu_{\mu} \rightarrow \nu_e$ oscillations and a range of possible mixing angles [95–97]. The probability for LSND oscillations with $\sin^2 2\theta_{LSND} \sim 3 \times 10^{-3}$, has drawn a lot of attention over the years. With only three neutrino, all observations cannot be explained by neutrinos oscillation including LSND. Several interesting papers have been written to explain this result.

There are two major ways one can explain the LSND result: (i.) by adding a sterile neutrino [98] or (ii.) by including New Physics (NP) [98–100]. The problem with alternative (i.) is that a sterile neutrino cannot be understood by the seesaw mechanism, so its mass is naturally of the order the Planck scale ($m_{\nu_s} \sim m_{Pl}$), but we want to explain the LSND result with $\Delta m_{LSND}^2 \sim 1 \text{eV}^2$. This implies that we will have a mass hierarchy problem, thus the possibility of explaining this result with a “sterile” neutrino may not be ideal. Although NP cannot explain the large solar

and atmospheric mixing angles with $\sin^2 2\theta_\odot \sim 1$ and $\sin^2 2\theta_{atm} \sim 1$, it might be responsible for small mixing $\sin^2[2\theta_{LSND}] \sim 3 \times 10^{-3}$ suggested by LSND.

To understand such angle θ_{LSND} from NP effects, a new physics amplitude of order 10 % would be needed. Currently the Mini Boone experiment [101] at Fermi Lab is in progress to check the LSND result, for a recent review see Refs. [102, 103].

with only three neutrino species, but allowing for arbitrary new physics effects we show that we can parameterize all the new physics effects in terms of 6 angles, 3 of which are CP violating. We show that a small amount of new physics gives rise to large CP and apparent CPT violation.

In section 9.2, we discuss the general formalism of neutrino oscillations both in two/three generations with/without new physics. For these cases we give explicit expressions for the probabilities and the CP violating asymmetry. We also give expressions for apparent CPT violating asymmetries. We also show the oscillation plots for different baselines. In section 9.4, we analyze the effect of new physics including matter effects using linearized approximations. We also show several plots for different choices of parameters and discuss how CP violation from matter effect can be distinguished from CP violation from new physics. We summarize in section 9.5. Finally, in Appendix E we present a realistic model for the two generation neutrino oscillations.

9.2 Neutrino oscillations including new physics

9.2.1 Neutrino mixing formalism

Here we first show how new physics effects change in neutrino oscillation probabilities. Consider the weak, the source and the detector eigenstates to be different. The weak eigenstate ($|\nu_\mu^w\rangle$) is a superposition of mass eigenstates ($|\nu_\mu^m\rangle$) given by

$$|\nu_\mu^w\rangle = \sum_\alpha U_{\mu\alpha}^w |\nu_\alpha^m\rangle, \quad (9.1)$$

where $\alpha = 1, 2, 3$. Similar equations hold for the source eigenstate ($|\nu_\mu^s\rangle$) and the detection eigenstate ($|\nu_\mu^d\rangle$) which is given by:

$$|\nu_\mu^s\rangle = \sum_\alpha U_{\mu\alpha}^s |\nu_\alpha^m\rangle, \quad (9.2)$$

$$|\nu_e^d\rangle = \sum_\alpha U_{e\alpha}^d |\nu_\alpha^m\rangle. \quad (9.3)$$

In the presence of new physics, the muon neutrino produced from π decay is the source eigenstate which is assumed to be different from the mass eigenstate and the electron neutrino detected from $\nu_e n \rightarrow ep$ is the detector eigenstate that also differs from the mass eigenstate. In the absence of new physics, the source, detection and weak eigenstates are all identical.

Generalizing this without specifying the neutrino flavor, we have $|\nu_\alpha^s\rangle$, $|\nu_\alpha^d\rangle$ and $|\nu_\alpha^w\rangle$. The amplitude for finding a $|\nu_n^d\rangle$ in the original $|\nu_l^s\rangle$ beam at time t is given by

$$\langle \nu_n^d | \nu_l^s \rangle(t) = \sum e^{-iE_\alpha t} U_{l\alpha}^s U_{n\alpha}^{d*}, \quad (9.4)$$

and the associated probability reads

$$P_{nl}(t) = |\langle \nu_n^d | \nu_l^s \rangle(t)|^2. \quad (9.5)$$

9.2.2 Two flavor neutrino mixing

The two generation example is always easier to analyze because of the simplicity of the neutrino mixing matrix, called MNS matrix. As discussed in Ref. [100], let us consider a muon neutrino beam produced by $\pi \rightarrow \mu\nu$ decay (source) and subsequent detection of an electron neutrino through the process $\nu n \rightarrow ep$ (inverse β - decay).

We parameterize the 2×2 unitary matrix as

$$U^s = \begin{pmatrix} e^{i\alpha_s} & 0 \\ 0 & e^{-i\alpha_s} \end{pmatrix} \begin{pmatrix} \cos \theta_s & \sin \theta_s \\ -\sin \theta_s & \cos \theta_s \end{pmatrix} \begin{pmatrix} e^{i\beta_s} & 0 \\ 0 & e^{-i\beta_s} \end{pmatrix} e^{i\gamma}, \quad (9.6)$$

and similarly for U^d with $(\theta_s, \alpha_s, \beta_s)$ replaced by $(\theta_d, \alpha_d, \beta_d)$. The phases α_s and α_d can be removed by an appropriate phase redefinition.

The probability of $\nu_e \rightarrow \nu_e$, $\bar{\nu}_e \rightarrow \bar{\nu}_e$, $\nu_e \rightarrow \nu_\mu$, $\nu_\mu \rightarrow \nu_e$, $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations in vacuum are found to be

$$P_{ee} = \cos^2(\theta_s - \theta_d) - \sin 2\theta_s \sin 2\theta_d \sin^2[\Delta m_{12}^2 t / 4E - (\beta_s - \beta_d)], \quad (9.7)$$

$$P_{\bar{e}\bar{e}} = \cos^2(\theta_s - \theta_d) - \sin 2\theta_s \sin 2\theta_d \sin^2[\Delta m_{12}^2 t/4E + (\beta_s - \beta_d)], \quad (9.8)$$

$$P_{e\mu} = P_{\mu e} = \sin^2(\theta_s - \theta_d) + \sin 2\theta_s \sin 2\theta_d \sin^2[\Delta m_{12}^2 t/4E - (\beta_s - \beta_d)], \quad (9.9)$$

$$P_{\bar{e}\bar{\mu}} = P_{\bar{\mu}\bar{e}} = \sin^2(\theta_s - \theta_d) + \sin 2\theta_s \sin 2\theta_d \sin^2[\Delta m_{12}^2 t/4E + (\beta_s - \beta_d)]. \quad (9.10)$$

In the above equations, we can introduce the parameter $\epsilon_\theta = \theta_d - \theta_s$.

The CP asymmetry can then be defined as

$$A_{\mu e} = \frac{P_{\mu e} - P_{\bar{\mu}\bar{e}}}{P_{\mu e} + P_{\bar{\mu}\bar{e}}}, \quad (9.11)$$

where $P_{\mu e}$ and $P_{\bar{\mu}\bar{e}}$ are given in Eqs. (9.9) and (9.10) respectively. If experiments are performed with both neutrino beam and antineutrino beam, $P_{\mu e}$ and $P_{\bar{\mu}\bar{e}}$ can be separately measured.

The plot of the CP asymmetry $A_{\mu e}$ as a function of the energy (GeV) for the two generation oscillation in vacuum is shown in Fig. 9.1 for fixed length $L = 2540$ km.

Remarks: We noticed that $P_{ee} \neq P_{\bar{e}\bar{e}}$ and $P_{\mu e} \neq P_{\bar{\mu}\bar{e}}$ because $\beta_s \neq \beta_d$, hence there is CP-asymmetry. If we go to the SM limit (absence of new physics) where $\theta_s = \theta_d$ and $\beta_s = \beta_d$, we obtain the usual two flavor vacuum oscillation probability. The new physics contribution to the two neutrino flavor oscillation may be of order 10%.

9.2.3 Three generation neutrino oscillation

9.2.3.1 General formalism in vacuum. We have seen that new physics implies a nonzero CP-asymmetry in the two generation neutrino oscillation in vacuum. Here we consider the three generation neutrino oscillation in vacuum, we develop a general formalism on how new physics effects can affect the known formalism. We will adopt the same notation used in the two generation case. Assume the source eigenstate is a superposition of the mass eigenstates given in Eq. (9.2) and detector eigenstate different from the source eigenstate given in Eq. (9.3). The time evolution equation for the detector eigenstate reads then

$$|\nu^d\rangle_t = U^d \hat{E} |\nu^m\rangle, \quad (9.12)$$

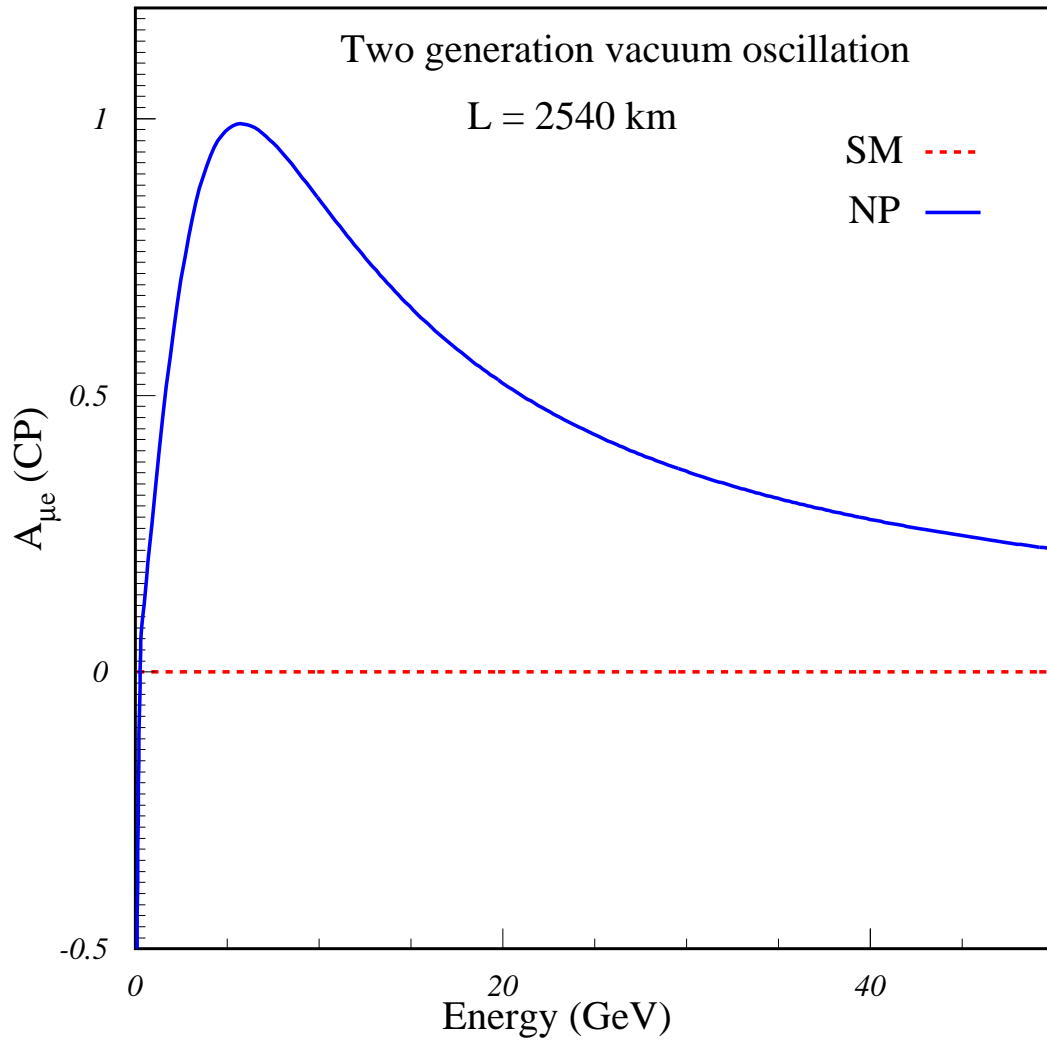


Figure 9.1. CP asymmetry $A_{\mu e} = \frac{P_{\mu e} - P_{\bar{\mu} \bar{e}}}{P_{\mu e} + P_{\bar{\mu} \bar{e}}}$ as a function of energy for two generation neutrino oscillation in vacuum for the choice $\Delta m_{21}^2 = 7.1 \times 10^{-5} \text{ eV}^2$, $L = 2540 \text{ km}$, $\theta_s = \frac{\pi}{5.6}$, $\epsilon_\theta = 0.005$, $\beta_d - \beta_s = 0.04$ (see text for definitions).

where \hat{E} is defined as

$$\hat{E} = \exp \left[-i \begin{pmatrix} E_1 t & 0 & 0 \\ 0 & E_2 t & 0 \\ 0 & 0 & E_3 t \end{pmatrix} \right]. \quad (9.13)$$

The probability amplitude for finding a ν_n^d in the original ν_l^s beam at time t is

$$\mathcal{A}_{nl} = \langle \nu_n^d | \nu_l^s \rangle_t = \sum_{\alpha, \beta} U_{n\alpha}^s | \nu_\alpha^m \rangle U_{l\beta}^{d*} \delta_{\alpha\beta} \hat{E}_\beta \langle \nu_\beta^m |, \quad (9.14)$$

which can be written in a simple form as $\mathcal{A} = U^s \hat{E} U^{d\dagger}$. Parameterizing our unitary matrix as

$$U^s = e^{i\gamma^s} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\chi^s} & 0 \\ 0 & 0 & e^{i\eta^s} \end{pmatrix}}_{P^s} V^s \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha^s} & 0 \\ 0 & 0 & e^{i\beta^s} \end{pmatrix}}_{Q^s} = e^{i\gamma^s} P^s V^s Q^s, \quad (9.15)$$

where V^s is given by

$$V^s = \begin{pmatrix} \cos \omega^s \cos \phi^s & \cos \phi^s \sin \omega^s & e^{-i\delta^s} \sin \phi^s \\ -\sin \omega^s \cos \psi^s - e^{i\delta^s} \cos \omega^s \sin \phi^s \sin \psi^s & \cos \omega^s \cos \psi^s - e^{i\delta^s} \sin \omega^s \sin \phi^s \sin \psi^s & \cos \phi^s \sin \psi^s \\ \sin \psi^s \sin \omega^s - e^{i\delta^s} \cos \omega^s \sin \phi^s \cos \psi^s & -e^{i\delta^s} \sin \omega^s \sin \phi^s \cos \psi^s - \cos \omega^s \sin \psi^s & \cos \phi^s \cos \psi^s \end{pmatrix}. \quad (9.16)$$

A similar definition holds for V^d . Using Eqs. (9.15) in (9.14), we find the probability amplitude to be

$$\mathcal{A} = P^s V^s Q^s \hat{E} Q^{d\dagger} V^{d\dagger} P^{d\dagger} e^{i(\gamma^s - \gamma^d)}. \quad (9.17)$$

For the probability $P_{ij} = |\mathcal{A}_{ij}|^2$ we obtain

$$P_{ij} = \left| \sum_k P_i^s V_{ik}^s R_k \hat{E}_k V_{jk}^{d*} P_j^{d*} \right|^2 = \left| \sum_k V_{ik}^s R_k \hat{E}_k V_{jk}^{d*} \right|^2, \quad (9.18)$$

where

$$R \hat{E} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\Delta_{21} + i\alpha} & 0 \\ 0 & 0 & e^{-i\Delta_{31} + i\beta} \end{pmatrix}. \quad (9.19)$$

Here $\alpha = \alpha^s - \alpha^d$ and $\beta = \beta^s - \beta^d$ are new physics parameters. In arriving at Eq. (9.19), we used the definitions

$$\Delta_{21} = (E_2 - E_1)t \simeq \frac{\Delta m_{21}^2 L}{2E} \quad \text{and} \quad \Delta_{31} = (E_3 - E_1)t \simeq \frac{\Delta m_{31}^2 L}{2E}. \quad (9.20)$$

We express the unitary matrix for detector eigenstate in terms of the source eigenstate using the following definitions;

$$\epsilon_\omega = \omega^d - \omega^s, \quad \epsilon_\phi = \phi^d - \phi^s, \quad \epsilon_\psi = \psi^d - \psi^s, \quad \epsilon_\delta = \delta^d - \delta^s. \quad (9.21)$$

We then express the detector unitary matrix in terms of the original source parameters and the small epsilon corrections, these epsilon parameters are also new physics parameters.

9.2.3.2 Bilarge mixing. Recent results from the KamLAND experiment have further confirmed the large mixing angle (LMA) solution to the solar neutrino problem [104] in a terrestrial experiment. For three neutrinos, the neutrino mixing matrix V_{MNS} is specified by three rotation angles θ_{13} , θ_{12} and θ_{23} (ϕ , ω , ψ) and one CP-violating phase δ . Experiments suggest $\theta_{13} \lesssim 13^\circ$ for $|\Delta m_{atm}^2| = 2 \times 10^{-3}$ eV, and $\theta_{23} = 45^\circ \pm 10^\circ$. This suggests a simple form of the unitary matrix, where we take $\phi^s \equiv \phi^s + \epsilon_{13}$ and $\psi^s = \frac{\pi}{4} + \epsilon_{23}$ while keeping the solar angle ω^s and the CP-phase δ^s as free parameters. Using the parametrization Eq. (9.21), we make expansions in terms of the small parameters ϕ^s , ϵ_{13} , ϵ_ϕ , ϵ_{23} , ϵ_ω , ϵ_ψ , ϵ_δ , α , β and Δ_{21} . Keeping terms only up to second order, we find approximate expressions for the probabilities given by

$$P_{ee} \simeq 1 - 4(\phi^s)^2 \sin^2 \left[\frac{\Delta_{31}}{2} \right] - \frac{1}{4}(\Delta_{21} - \alpha)^2 \sin^2 2\omega^s - 4\phi^s \epsilon_\phi \sin^2 \left[\frac{\Delta_{31}}{2} \right] - \epsilon_\omega^2 - \epsilon_\phi^2, \quad (9.22)$$

$$P_{e\mu} \simeq 2(\phi^s)^2 \sin^2 \left[\frac{\Delta_{31}}{2} \right] + \frac{1}{8}(\Delta_{21} - \alpha)^2 \sin^2 2\omega^s + (\Delta_{21} - \alpha)\phi^s \sin 2\omega^s \left(\frac{1}{2} \cos \delta^s \sin \Delta_{31} - \sin \delta^s \sin^2 \frac{\Delta_{31}}{2} \right) + 2\phi^s \epsilon_\phi \sin^2 \left[\frac{\Delta_{31}}{2} \right] - \frac{1}{2}(\Delta_{21} - \alpha)\epsilon_\phi \sin \delta^s \sin 2\omega^s + \phi^s \epsilon_\omega (\sin \delta^s \sin \Delta_{31} + 2 \cos \delta^s \sin^2 \left[\frac{\Delta_{31}}{2} \right]) + \epsilon_\phi \epsilon_\omega \cos \delta^s + \frac{1}{2}(\epsilon_\phi^2 + \epsilon_\omega^2), \quad (9.23)$$

$$P_{\mu e} \simeq 2(\phi^s)^2 \sin^2 \left[\frac{\Delta_{31}}{2} \right] + \frac{1}{8}(\Delta_{21} - \alpha)^2 \sin^2 2\omega^s + (\Delta_{21} - \alpha)\phi^s \sin 2\omega^s \left(\frac{1}{2} \cos \delta^s \sin \Delta_{31} + \sin \delta^s \sin^2 \frac{\Delta_{31}}{2} \right) + 2\phi^s \epsilon_\phi \sin^2 \left[\frac{\Delta_{31}}{2} \right] + \frac{1}{2}(\Delta_{21} - \alpha)\epsilon_\phi \sin 2\omega^s (\cos \delta^s \sin \Delta_{31} - \sin \delta^s \cos \Delta_{31}) + \phi^s \epsilon_\omega (\sin \delta^s \sin \Delta_{31} - 2 \cos \delta^s \sin^2 \left[\frac{\Delta_{31}}{2} \right]) + \epsilon_\phi \epsilon_\omega (\cos \delta^s \cos \Delta_{31} + \sin \delta^s \sin \Delta_{31}) + \frac{1}{2}(\epsilon_\phi^2 + \epsilon_\omega^2), \quad (9.24)$$

$$P_{\mu\tau} \simeq \sin^2 \left[\frac{\Delta_{31}}{2} \right] - 2(\phi^{s2} + 2\epsilon_{23}^2) \sin^2 \left[\frac{\Delta_{31}}{2} \right] - \frac{1}{2}(\Delta_{21} - \alpha) \cos^2 \omega^s \sin \Delta_{31} + \frac{1}{4}(\Delta_{21} - \alpha)^2 \cos^2 \omega^s (\cos \Delta_{31} - \sin^2 \omega^s) - \phi^s (\Delta_{21} - \alpha) \sin \delta^s \sin 2\omega^s \sin^2 \left[\frac{\Delta_{31}}{2} \right] - \frac{1}{2}\beta \sin \Delta_{31} + \frac{1}{4}\beta (\beta + 2(\Delta_{21} - \alpha) \cos^2 \omega^s) \cos \Delta_{31} + 2\phi^s \epsilon_\omega \cos \delta^s \sin^2 \left[\frac{\Delta_{31}}{2} \right] + \frac{1}{2}\epsilon_\omega (\Delta_{21} - \alpha) \sin 2\omega^s \sin^2 \frac{\delta^s}{2} \sin \Delta_{31} - 2\phi^s \epsilon_\phi \sin^2 \left[\frac{\Delta_{31}}{2} \right] - 4\epsilon_{23} \epsilon_\psi \sin^2 \left[\frac{\Delta_{31}}{2} \right]$$

$$\begin{aligned}
& - \frac{1}{2}(\Delta_{21} - \alpha)\epsilon_\phi \sin \delta^s \sin 2\omega^s \sin^2\left[\frac{\Delta_{31}}{2}\right] - \frac{1}{2}(\epsilon_\phi^2 + \epsilon_\omega^2) \sin^2\left[\frac{\Delta_{31}}{2}\right] \\
& + \epsilon_\phi \epsilon_\omega (\cos \delta^s \sin^2\left[\frac{\Delta_{31}}{2}\right] - \sin \delta^s \sin\left[\frac{\Delta_{31}}{2}\right] \cos\left[\frac{\Delta_{31}}{2}\right]) + \epsilon_\psi^2 \cos \Delta_{31}.
\end{aligned} \tag{9.25}$$

Since we are interested in the LSND result, we also made an expansion in terms of Δ_{31} which is small for the baseline and energy chosen for the experiment. The probability $P_{\mu e}$ can be written as

$$P_{\mu e} \simeq \frac{1}{8}\alpha^2 \sin^2 2\omega + \frac{1}{2}\alpha\epsilon_\phi \sin \delta \sin 2\omega + \epsilon_\phi \epsilon_\omega \cos \delta + \frac{1}{2}(\epsilon_\phi^2 + \epsilon_\omega^2). \tag{9.26}$$

Here we ignored Δ_{21} and Δ_{31} terms since they are very small for LSND setup. For the choice of parameters $\alpha = 0.04$, $\epsilon_\phi = 0.03$, $\epsilon_\omega = 0.03$, $\delta = \frac{\pi}{4}$, $\omega = \frac{\pi}{5.6}$, we find $P_{\mu e} \simeq 0.0021$. This can consistently explain the LSND anomaly.

We can also write the probabilities Eqs. in (9.22–9.25) in terms of the detection parameters. For example the probability $P_{\mu e}$ when expressed in terms of the detector angles is given by

$$\begin{aligned}
P_{\mu e} & \simeq 2(\phi^d)^2 \sin^2\left[\frac{\Delta_{31}}{2}\right] + \frac{1}{8}(\Delta_{21} - \alpha)^2 \sin^2 2\omega^d \\
& + (\Delta_{21} - \alpha)\phi^d \sin 2\omega^d \left(\frac{1}{2} \cos \delta^d \sin \Delta_{31} + \sin \delta^d \sin^2 \frac{\Delta_{31}}{2}\right) \\
& - 2\phi^d \epsilon_\phi \sin^2\left[\frac{\Delta_{31}}{2}\right] - \frac{1}{2}(\Delta_{21} - \alpha)\epsilon_\phi \sin \delta^d \sin 2\omega^d \\
& + \phi^d \epsilon_\omega (\sin \delta^d \sin \Delta_{31} - 2 \cos \delta^d \sin^2\left[\frac{\Delta_{31}}{2}\right]) + \epsilon_\phi \epsilon_\omega \cos \delta^d + \frac{1}{2}(\epsilon_\phi^2 + \epsilon_\omega^2).
\end{aligned} \tag{9.27}$$

This is exactly symmetrical in form with the probability $P_{e\mu}$ written in terms of the source parameters of Eq. (9.23) which is not symmetrical with $P_{e\mu}$ of Eq. (9.24).

A number of neutrino experiments have been proposed, which aim to test several theoretical proposal on possible CP violation in the neutrino sector. Here we define CP violation as the difference $P_{ij} - P_{\bar{i}\bar{j}}$ which is nonzero in this model. We give various expressions for the CP asymmetry for different oscillation channels. We find the CP asymmetries to be

$$\begin{aligned}
\Delta P_{e\mu}(CP) & = P_{e\mu} - P_{\bar{e}\bar{\mu}} \\
& \simeq -2\phi^s \Delta_{21} \sin \delta^s \sin 2\omega^s \sin^2\left[\frac{\Delta_{31}}{2}\right] + 2\phi^s \epsilon_\omega \sin \delta^s \sin \Delta_{31}
\end{aligned}$$

$$\begin{aligned}
& - \epsilon_\phi \Delta_{21} \sin \delta^s \sin 2\omega^s - \frac{1}{2} \alpha \Delta_{21} \sin^2 2\omega^s \\
& - \phi^s \alpha \cos \delta^s \sin 2\omega^s \sin \Delta_{31}, \tag{9.28}
\end{aligned}$$

$$\begin{aligned}
\Delta P_{\mu e}(CP) &= P_{\mu e} - P_{\bar{\mu} \bar{e}} \\
&\simeq 2\phi^s \Delta_{21} \sin \delta^s \sin 2\omega^s \sin^2 \left[\frac{\Delta_{31}}{2} \right] + 2(\phi^s + \epsilon_\phi) \epsilon_\omega \sin \delta^s \sin \Delta_{31} \\
&- \epsilon_\phi \Delta_{21} \sin \delta^s \sin 2\omega^s \cos \Delta_{31} - \frac{1}{2} \alpha \Delta_{21} \sin^2 2\omega^s \\
&- (\phi^s + \epsilon_\phi) \alpha \cos \delta^s \sin 2\omega^s \sin \Delta_{31}, \tag{9.29}
\end{aligned}$$

$$\begin{aligned}
\Delta P_{\mu\tau}(CP) &= P_{\mu\tau} - P_{\bar{\mu}\bar{\tau}} \\
&\simeq -\Delta_{21}(\alpha - \beta) \cos^2 \omega^s \cos \Delta_{31} + \alpha \cos^2 \omega^s \sin \Delta_{31} - \beta \sin \Delta_{31} \\
&+ \frac{1}{4} \alpha \Delta_{21} \sin^2 2\omega^s - \epsilon_\phi \Delta_{21} \sin \delta^s \sin 2\omega^s \sin^2 \left[\frac{\Delta_{31}}{2} \right] \\
&+ \frac{1}{2} \epsilon_\phi \alpha \cos \delta^s \sin 2\omega^s \sin \Delta_{31} - \frac{1}{2} \epsilon_\omega \alpha \sin 2\omega^s \sin \Delta_{31} \\
&- 2\phi^s \Delta_{21} \sin \delta^s \sin 2\omega^s \sin^2 \left[\frac{\Delta_{31}}{2} \right] - \epsilon_\phi \epsilon_\omega \sin \delta^s \sin \Delta_{31}, \tag{9.30}
\end{aligned}$$

$$\begin{aligned}
\Delta P_{ee}(CP) &= P_{ee} - P_{\bar{e}\bar{e}} \\
&\simeq \alpha \Delta_{21} \sin^2 2\omega^s. \tag{9.31}
\end{aligned}$$

Similarly it turns out that there is apparent CPT asymmetry in our model, though there is no true CPT violation. This apparent CPT violation arises because of CP violation in new physics. In the standard scenario, there is no CPT violation. The apparent CPT asymmetries are defined as

$$\begin{aligned}
\Delta P_{e\mu}(CPT) &= P_{e\mu} - P_{\bar{\mu}\bar{e}} \\
&\simeq \epsilon_\omega (2\phi^s + \epsilon_\phi) (\cos \delta^s - \cos(\Delta_{31} + \delta^s)) \\
&+ \alpha \phi^s \sin 2\omega^s (\sin \delta^s - \sin(\Delta_{31} + \delta^s)) - \frac{\alpha \Delta_{21}}{2} \sin^2 2\omega^s \\
&- \frac{\epsilon_\phi}{2} \sin 2\omega^s [(\Delta_{21} - \alpha) \sin \delta^s + (\Delta_{21} + \alpha) \sin(\Delta_{31} + \delta^s)], \tag{9.32}
\end{aligned}$$

$$\begin{aligned}
\Delta P_{\mu\tau}(CPT) &= P_{\mu\tau} - P_{\bar{\tau}\bar{\mu}} \\
&\simeq \alpha \Delta_{21} \left(\frac{1}{4} \sin^2 2\omega^s - \cos^2 \omega^s \cos \Delta_{31} \right) + \frac{\beta \Delta_{21}}{2} \cos^2 \omega^s \cos \Delta_{31} \\
&- \frac{\epsilon_\phi \Delta_{21}}{2} \sin 2\omega^s \cos \delta^s \sin \Delta_{31} + \alpha (2\phi^s + \epsilon_\phi) \sin 2\omega^s \sin \delta^s \sin^2 \frac{\Delta_{31}}{2} \\
&- \beta \sin \Delta_{31} + \frac{\epsilon_\omega}{2} [4(2\phi^s + \epsilon_\phi) \cos \delta^s \sin^2 \frac{\Delta_{31}}{2} - \alpha \sin 2\omega^s \sin \Delta_{31}] \\
&+ \alpha \cos^2 \omega^s \sin \Delta_{31}. \tag{9.33}
\end{aligned}$$

Notice that the apparent CPT asymmetries ΔP_{ee} , $\Delta P_{\mu\mu}$ and $\Delta P_{\tau\tau}$ coincide with their corresponding CP asymmetries.

9.2.3.3 Exact analysis of three generation neutrino oscillation in vacuum. Here

we show the derivation for the three generation vacuum oscillation probabilities and change in probabilities including new physics effect without matter effect. we use the same general formalism as in section 9.2.3.1, with the unitary matrix as given in Eq. (9.16) and with the assumption that the source eigenstate is different from the detector eigenstate. The new physics effects comes from the α , β and the ϵ terms. The exact expressions for the probability of $\nu_e \rightarrow \nu_e$, $\nu_e \rightarrow \nu_\mu$ and $\nu_\mu \rightarrow \nu_\tau$ is

$$\begin{aligned}
P_{ee} = & \sin^2 \phi^s \sin^2 \phi^d + \frac{1}{2} \sin 2\phi^s \sin 2\phi^d (\cos \omega^s \cos \omega^d \cos(\Delta_{31} - \beta + \delta^s - \delta^d) \\
& + \sin \omega^s \sin \omega^d \cos(\Delta_{21} - \Delta_{31} - \alpha + \beta - \delta^s + \delta^d)) \\
& + \cos^2 \phi^s \cos^2 \phi^d \left(\cos^2(\omega^s - \omega^d) - \sin 2\omega^s \sin 2\omega^d \sin^2 \left(\frac{\Delta_{21} - \alpha}{2} \right) \right), \quad (9.34)
\end{aligned}$$

$$\begin{aligned}
P_{e\mu} = & \cos^2 \phi^s \sin^2 \phi^d \sin^2 \omega^s \sin^2 \omega^d \sin^2 \psi^d + \cos^2 \phi^s \cos^2 \omega^s \sin^2 \omega^d \cos^2 \psi^d \\
& + \cos^2 \phi^s \sin^2 \phi^d \cos^2 \omega^s \cos^2 \omega^d \sin^2 \psi^d + \cos^2 \phi^s \sin^2 \omega^s \cos^2 \omega^d \cos^2 \psi^d \\
& + \sin^2 \phi^s \cos^2 \phi^d \sin^2 \psi^d + \frac{1}{2} \cos^2 \phi^s \sin \phi^d \sin 2\omega^d \sin 2\psi^d \cos \delta^d (\cos^2 \omega^s - \sin^2 \omega^s) \\
& + \frac{1}{2} \cos^2 \phi^s \sin \phi^d \sin 2\omega^s \sin 2\psi^d \\
& \times (\sin^2 \omega^d \cos(\Delta_{21} - \alpha + \delta^d) - \cos^2 \omega^d \cos(\Delta_{21} - \alpha - \delta^d)) \\
& + \frac{1}{2} \cos^2 \phi^s \sin 2\omega^s \sin 2\omega^d (\sin^2 \phi^d \sin^2 \psi^d - \cos^2 \psi^d) \cos(\Delta_{21} - \alpha) \\
& - \frac{1}{2} \sin 2\phi^s \cos \phi^d \cos \omega^s \sin \omega^d \sin 2\psi^d \cos(\Delta_{31} - \beta + \delta^s) \\
& - \frac{1}{2} \sin 2\phi^s \sin 2\phi^d \cos \omega^s \cos \omega^d \sin^2 \psi^d \cos(\Delta_{31} - \beta + \delta^s - \delta^d) \\
& + \frac{1}{2} \sin 2\phi^s \cos \phi^d \sin \omega^s \cos \omega^d \sin 2\psi^d \cos(\Delta_{21} - \Delta_{31} - \alpha + \beta - \delta^s) \\
& - \frac{1}{2} \sin 2\phi^s \sin 2\phi^d \sin \omega^s \sin \omega^d \sin^2 \psi^d \cos(\Delta_{21} - \Delta_{31} - \alpha + \beta - \delta^s + \delta^d), \quad (9.35)
\end{aligned}$$

$$\begin{aligned}
P_{\mu\tau} = & \cos^2 \phi^s \cos^2 \phi^d \cos^2 \psi^d \sin^2 \psi^s + \cos^2 \psi^s \sin^2 \psi^d (\cos^2 \omega^s \cos^2 \omega^d + \sin^2 \omega^s \sin^2 \omega^d) \\
& + \sin^2 \phi^s \sin^2 \psi^s \sin^2 \psi^d (\sin^2 \omega^s \cos^2 \omega^d + \sin^2 \omega^d \cos^2 \omega^s) \\
& + \sin^2 \phi^s \sin^2 \phi^d \sin^2 \psi^s \cos^2 \psi^d (\cos^2 \omega^s \cos^2 \omega^d + \sin^2 \omega^s \sin^2 \omega^d) \\
& + \sin^2 \phi^d \cos^2 \psi^s \cos^2 \psi^d (\sin^2 \omega^s \cos^2 \omega^d + \sin^2 \omega^d \cos^2 \omega^s)
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \sin \phi^s \sin 2\omega^s \sin 2\psi^s \cos \delta^s (\sin^2 \phi^d \cos^2 \psi^d - \sin^2 \psi^d) (\cos^2 \omega^d - \sin^2 \omega^d) \\
& + \frac{1}{2} \sin \phi^d \sin 2\omega^d \sin 2\psi^d \cos \delta^d (\cos^2 \psi^s - \sin^2 \phi^s \sin^2 \psi^s) (\cos^2 \omega^s - \sin^2 \omega^s) \\
& - \frac{1}{2} \sin \phi^s \sin \phi^d \sin 2\omega^s \sin 2\omega^d \sin 2\psi^s \sin 2\psi^d \cos \delta^s \cos \delta^d \\
& + \frac{1}{2} \sin 2\omega^s \sin 2\omega^d (\sin^2 \psi^d - \sin^2 \phi^d \cos^2 \psi^d) \\
& \times (\cos^2 \psi^s - \sin^2 \phi^s \sin^2 \psi^s) \cos(\Delta_{21} - \alpha) \\
& - \frac{1}{2} \cos \phi^s \cos \phi^d \sin \omega^s \sin \omega^d \sin 2\psi^s \sin 2\psi^d \cos(\Delta_{31} - \beta) \\
& + \frac{1}{2} \sin \phi^s \cos^2 \omega^s \sin 2\omega^d \sin 2\psi^s (\sin^2 \psi^d - \sin^2 \phi^d \cos^2 \psi^d) \cos(\Delta_{21} - \alpha + \delta^s) \\
& + \frac{1}{2} \sin \phi^s \sin^2 \omega^s \sin 2\omega^d \sin 2\psi^s (\sin^2 \phi^d \cos^2 \psi^d - \sin^2 \psi^d) \cos(\Delta_{21} - \alpha - \delta^s) \\
& + \frac{1}{2} \sin \phi^d \sin 2\omega^s \sin^2 \omega^d \sin 2\psi^d (\cos^2 \psi^s - \sin^2 \phi^s \sin^2 \psi^s) \cos(\Delta_{21} - \alpha + \delta^d) \\
& + \frac{1}{2} \sin \phi^d \sin 2\omega^s \cos^2 \omega^d \sin 2\psi^d (\sin^2 \phi^s \sin^2 \psi^s - \cos^2 \psi^s) \cos(\Delta_{21} - \alpha - \delta^d) \\
& - \frac{1}{2} \sin 2\phi^s \cos \phi^d \cos \omega^s \sin \omega^d \sin^2 \psi^s \sin 2\psi^d \cos(\Delta_{31} - \beta + \delta^s) \\
& + \frac{1}{2} \sin \phi^s \sin \phi^d \cos^2 \omega^d \sin 2\psi^s \sin 2\psi^d (\sin^2 \omega^s \cos(\Delta_{21} - \alpha - \delta^s - \delta^d) \\
& - \cos^2 \omega^s \cos(\Delta_{21} - \alpha + \delta^s - \delta^d)) \\
& + \frac{1}{2} \sin \phi^s \sin \phi^d \sin^2 \omega^d \sin 2\psi^s \sin 2\psi^d (\cos^2 \omega^s \cos[\Delta_{21} - \alpha + \delta^s + \delta^d] \\
& - \sin^2 \omega^s \cos[\Delta_{21} - \alpha - \delta^s + \delta^d]) \\
& + \cos \phi^s \sin 2\phi^d \cos \omega^d \cos^2 \psi^d (\sin \omega^s \sin \psi^s \cos \psi^s \cos[\Delta_{31} - \beta - \delta^d] \\
& + \sin \phi^s \cos \omega^s \sin^2 \psi^s \cos[\Delta_{31} - \beta + \delta^s - \delta^d]) \\
& - \cos \phi^s \sin 2\phi^d \sin \omega^d \cos^2 \psi^d (\cos \omega^s \sin \psi^s \cos \psi^s \cos[\Delta_{21} - \Delta_{31} - \alpha + \beta + \delta^d] \\
& - \sin \phi^s \sin \omega^s \sin^2 \psi^s \cos[\Delta_{21} - \Delta_{31} - \alpha + \beta - \delta^s + \delta^d]) \\
& + \cos \phi^s \cos \phi^d \cos \omega^d \sin 2\psi^d (\sin \phi^s \sin \omega^s \sin^2 \psi^s \cos[\Delta_{21} - \Delta_{31} - \alpha + \beta - \delta^s] \\
& - \cos \omega^s \cos \psi^s \sin \psi^s \cos[\Delta_{21} - \Delta_{31} - \alpha + \beta]). \tag{9.36}
\end{aligned}$$

The oscillation probabilities for the other neutrino channels can be obtained using these three probabilities given above:

$$P_{e\tau} = 1 - (P_{ee} + P_{e\mu}), \tag{9.37}$$

$$P_{\mu e} = P_{e\mu} \{ \phi^s \leftrightarrow \phi^d, \omega^s \leftrightarrow \omega^d, \psi^s \leftrightarrow \psi^d, \delta^s \leftrightarrow -\delta^d \}, \tag{9.38}$$

$$P_{\mu\mu} = 1 - (P_{\mu e} + P_{\mu\tau}), \quad (9.39)$$

$$P_{\tau e} = 1 - (P_{ee} + P_{\mu e}), \quad (9.40)$$

$$P_{\tau\tau} = 1 - (P_{e\tau} + P_{\mu\tau}), \quad (9.41)$$

$$P_{\tau\mu} = 1 - (P_{\tau e} + P_{\tau\tau}). \quad (9.42)$$

We can also arrive at the antineutrino probabilities from the neutrino probabilities by the simple prescription

$$P_{\bar{a}\bar{b}} = P_{ab}\{\alpha \rightarrow -\alpha, \beta \rightarrow -\beta, \delta^s \rightarrow -\delta^s, \delta^d \rightarrow -\delta^d\}. \quad (9.43)$$

In the SM limit, the oscillation probabilities in Eqs. (9.34) – (9.42) reduce to

$$\begin{aligned} P_{ee} &= 1 - \sin^2 2\phi \sin^2 \frac{\Delta_{31}}{2} - (\cos^4 \phi \sin^2 2\omega + \sin^2 \omega \sin^2 2\phi) \sin^2 \frac{\Delta_{21}}{2} \\ &+ \sin^2 \omega \sin^2 2\phi (2 \sin^2 \frac{\Delta_{21}}{2} \sin^2 \frac{\Delta_{31}}{2} + \frac{1}{2} \sin \Delta_{21} \sin \Delta_{31}), \end{aligned} \quad (9.44)$$

$$\begin{aligned} P_{e\mu} &= \sin^2 \psi \sin^2 2\phi \sin^2 \frac{\Delta_{31}}{2} + 4J (\sin \Delta_{21} \sin^2 \frac{\Delta_{31}}{2} - \sin \Delta_{31} \sin^2 \frac{\Delta_{21}}{2}) \\ &- (\sin^2 \omega \sin^2 \psi \sin^2 2\phi - 4K) [2 \sin^2 \frac{\Delta_{21}}{2} \sin^2 \frac{\Delta_{31}}{2} + \frac{1}{2} \sin \Delta_{21} \sin \Delta_{31}] \\ &+ [\cos^2 \phi (\cos^2 \psi - \sin^2 \phi \sin^2 \psi) \sin^2 2\omega \\ &+ \sin^2 \omega \sin^2 \psi \sin^2 2\phi - 8K \sin^2 \omega] \sin^2 \frac{\Delta_{21}}{2}, \end{aligned} \quad (9.45)$$

$$\begin{aligned} P_{\mu\tau} &= \cos^4 \phi \sin^2 2\psi \sin^2 \frac{\Delta_{31}}{2} + 4J (\sin \Delta_{21} \sin^2 \frac{\Delta_{31}}{2} - \sin \Delta_{31} \sin^2 \frac{\Delta_{21}}{2}) \\ &- [\cos^2 \phi \sin^2 2\psi (\cos^2 \omega - \sin^2 \phi \sin^2 \omega) + 4K \cos 2\psi] \\ &\times (2 \sin^2 \frac{\Delta_{21}}{2} \sin^2 \frac{\Delta_{31}}{2} + \frac{1}{2} \sin \Delta_{21} \sin \Delta_{31}) + 4K \cos 2\psi \\ &+ [\sin^2 2\psi (\cos^2 \omega - \sin^2 \phi \sin^2 \omega)^2 + \sin^2 \phi \sin^2 2\omega (1 - \sin^2 2\psi \cos^2 \delta) \\ &+ \sin \phi \sin 2\omega \cos 2\omega \sin 2\psi \cos 2\psi (1 + \sin^2 \phi) \cos \delta] \sin^2 \frac{\Delta_{21}}{2}, \end{aligned} \quad (9.46)$$

where the quantities J and K are defined as

$$J = \frac{1}{8} \cos \phi \sin 2\phi \sin 2\psi \sin 2\omega \sin \delta, \quad (9.47)$$

$$K = \frac{1}{8} \cos \phi \sin 2\phi \sin 2\psi \sin 2\omega \cos \delta. \quad (9.48)$$

One of the ultimate goals of neutrino factories is to investigate the phenomenon of neutrino oscillations, which have so far been observed by the atmospheric and solar

neutrino experiments, with unprecedented accuracy. CP violation gives a nonzero difference between the oscillation probabilities $P_{ab} \neq P_{\bar{a}\bar{b}}$. Here we give the expressions for the CP asymmetries $\nu_e \rightarrow \nu_e$, $\nu_e \rightarrow \nu_\mu$, and $\nu_\mu \rightarrow \nu_\tau$, which are

$$\Delta P_{ab}(CP) = P_{ab} - P_{\bar{a}\bar{b}}, \quad (9.49)$$

$$\begin{aligned} \Delta P_{ee}(CP) &= \sin 2\phi^s \sin 2\phi^d \cos \omega^s \cos \omega^d \sin(\beta - \delta^s + \delta^d) \sin \Delta_{31} \\ &+ \sin 2\phi^s \sin 2\phi^d \sin \omega^s \sin \omega^d \sin(\alpha - \beta + \delta^s - \delta^d) \sin(\Delta_{21} - \Delta_{31}) \\ &+ \cos^2 \phi^s \cos^2 \phi^d \sin 2\omega^s \sin 2\omega^d \sin \alpha \sin \Delta_{21}, \end{aligned} \quad (9.50)$$

$$\begin{aligned} \Delta P_{e\mu}(CP) &= -\sin 2\phi^s \sin 2\phi^d \cos \omega^s \cos \omega^d \sin^2 \psi^d \sin(\beta - \delta^s + \delta^d) \sin \Delta_{31} \\ &- \sin 2\phi^s \cos \phi^d \cos \omega^s \sin \omega^d \sin 2\psi^d \sin(\beta - \delta^s) \sin \Delta_{31} \\ &- \cos^2 \phi^s \sin 2\omega^s \sin 2\omega^d (\cos^2 \psi^d - \sin^2 \phi^d \sin^2 \psi^d) \sin \alpha \sin \Delta_{21} \\ &- \cos^2 \phi^s \sin \phi^d \sin 2\omega^s \sin 2\psi^d \\ &\times (\cos^2 \omega^d \sin(\alpha + \delta^d) - \sin^2 \omega^d \sin(\alpha - \delta^d)) \sin \Delta_{21} \\ &+ \sin 2\phi^s \cos \phi^d \sin \omega^s \cos \omega^d \sin 2\psi^d \sin(\alpha - \beta + \delta^s) \sin(\Delta_{21} - \Delta_{31}) \\ &- \sin 2\phi^s \sin 2\phi^d \sin \omega^s \sin \omega^d \sin^2 \psi^d \sin(\alpha - \beta + \delta^s - \delta^d) \\ &\times \sin(\Delta_{21} - \Delta_{31}), \end{aligned} \quad (9.51)$$

$$\begin{aligned} \Delta P_{\mu\tau}(CP) &= \sin 2\omega^s \sin 2\omega^d (\sin^2 \psi^d - \cos^2 \psi^d \sin^2 \phi^d) \\ &\times (\cos^2 \psi^s - \sin^2 \phi^s \sin^2 \psi^s) \sin \alpha \sin \Delta_{21} \\ &- \cos \phi^s \cos \phi^d \sin \omega^s \sin \omega^d \sin 2\psi^s \sin 2\psi^d \sin \beta \sin \Delta_{31} \\ &- \sin \phi^d \sin 2\omega^s \sin 2\psi^d (\cos^2 \psi^s - \sin^2 \phi^s \sin^2 \psi^s) \\ &\times (\sin^2 \omega^d \sin(\alpha - \delta^d) - \cos^2 \omega^d \sin(\alpha + \delta^d)) \sin \Delta_{21} \\ &+ \sin \phi^s \cos^2 \omega^s \sin 2\omega^d \sin 2\psi^s (\cos^2 \psi^d - \sin^2 \phi^d \sin^2 \psi^d) \\ &\times (\cos^2 \omega^s \sin(\alpha - \delta^s) - \sin^2 \omega^s \sin(\alpha + \delta^s)) \sin \Delta_{21} \\ &+ \cos \phi^s \sin 2\phi^d \sin \omega^s \cos \omega^d \sin 2\psi^s \cos^2 \psi^d \sin(\beta + \delta^d) \sin \Delta_{31} \\ &- \sin 2\phi^s \cos \phi^d \cos \omega^s \sin \omega^d \sin^2 \psi^s \sin 2\psi^d \sin(\beta - \delta^s) \sin \Delta_{31} \\ &+ \sin 2\phi^s \sin 2\phi^d \cos \omega^s \cos \omega^d \sin^2 \psi^s \cos^2 \psi^d \sin(\beta - \delta^s + \delta^d) \sin \Delta_{31} \\ &+ \sin \phi^s \sin \phi^d \sin 2\psi^s \sin 2\psi^d (\sin^2 \omega^s \cos^2 \omega^d \sin(\alpha + \delta^s + \delta^d) \\ &+ \cos^2 \omega^s \sin^2 \omega^d \sin(\alpha - \delta^s - \delta^d)) \sin \Delta_{21} \end{aligned}$$

$$\begin{aligned}
& - \sin \phi^s \sin \phi^d \sin 2\psi^s \sin 2\psi^d (\cos^2 \omega^s \cos^2 \omega^d \sin(\alpha - \delta^s + \delta^d) \\
& + \sin^2 \omega^s \sin^2 \omega^d \sin(\alpha + \delta^s - \delta^d)) \sin \Delta_{21} \\
& - \cos \phi^s \cos \phi^d \cos \omega^s \cos \omega^d \sin 2\psi^s \sin 2\psi^d \sin(\alpha - \beta) \sin[\Delta_{21} - \Delta_{31}] \\
& + \sin 2\phi^s \cos \phi^d \sin \omega^s \cos \omega^d \sin^2 \psi^s \sin 2\psi^d \sin(\alpha - \beta + \delta^s) \\
& \times \sin[\Delta_{21} - \Delta_{31}] \\
& + \sin 2\phi^s \sin 2\phi^d \sin \omega^s \sin \omega^d \sin^2 \psi^s \cos^2 \psi^d \sin(\alpha - \beta + \delta^s - \delta^d) \\
& \times \sin[\Delta_{21} - \Delta_{31}] \\
& - \cos \phi^s \sin 2\phi^d \cos \omega^s \sin \omega^d \sin 2\psi^s \cos^2 \psi^d \sin(\alpha - \beta - \delta^d) \\
& \times \sin[\Delta_{21} - \Delta_{31}]. \tag{9.52}
\end{aligned}$$

In the SM limit (source=detector), the above expressions for change in probabilities reduce to

$$\begin{aligned}
\Delta P_{ee}(CP) &= \Delta P_{\mu\mu}(CP) = \Delta P_{\tau\tau}(CP) = 0 \\
\Delta P_{e\mu}(CP) &= \Delta P_{\mu\tau}(CP) \\
&= 2 \cos \phi \sin 2\phi \sin 2\omega \sin 2\psi \sin \delta \sin \frac{\Delta_{21}}{2} \sin \frac{\Delta_{31}}{2} \sin \left[\frac{\Delta_{21} - \Delta_{31}}{2} \right]
\end{aligned}$$

9.3 Numerical results

We now turn to the three generation oscillation in vacuum. Here we considered three different baselines, 730 km (Fermilab – Soudan, CERN – Gran Sasso), 295 km (SJHF – Super K) and 2540 km (BNL – Homestake) These are some of the proposed experiments. The probability plots as a function of energy at fixed length are shown in Figs. 9.2 and 9.3 (Figs. 9.8 and 9.9) for a particular choice of parameters. The dotted lines are the plots for the standard three generation vacuum oscillations without new physics and the solid lines are the plots including the new physics parameters. In these plots we choose the new physics parameters to be between 5 – 10 %. The plots of change in probabilities are shown in Figs. 9.4 and 9.5 (Figs. 9.10 and 9.11) and that of CPT asymmetries is depicted in Fig. 9.6 (Fig. 9.12). We also show the probability plots as a function of length for fixed energy (5 GeV) in Figs. 9.13 and 9.14, the CP asymmetries Figs. 9.15 and 9.16 and the CPT asymmetries in Fig. 9.17.

In Fig. 9.7, we plot the CP asymmetry for the same set of input parameters as in Fig. 9.1, with the exception that we set here $\delta = 0$ and $\epsilon_\delta = 0$. We see that significant deviation from standard oscillation arise with new physics at 5–10 % level.

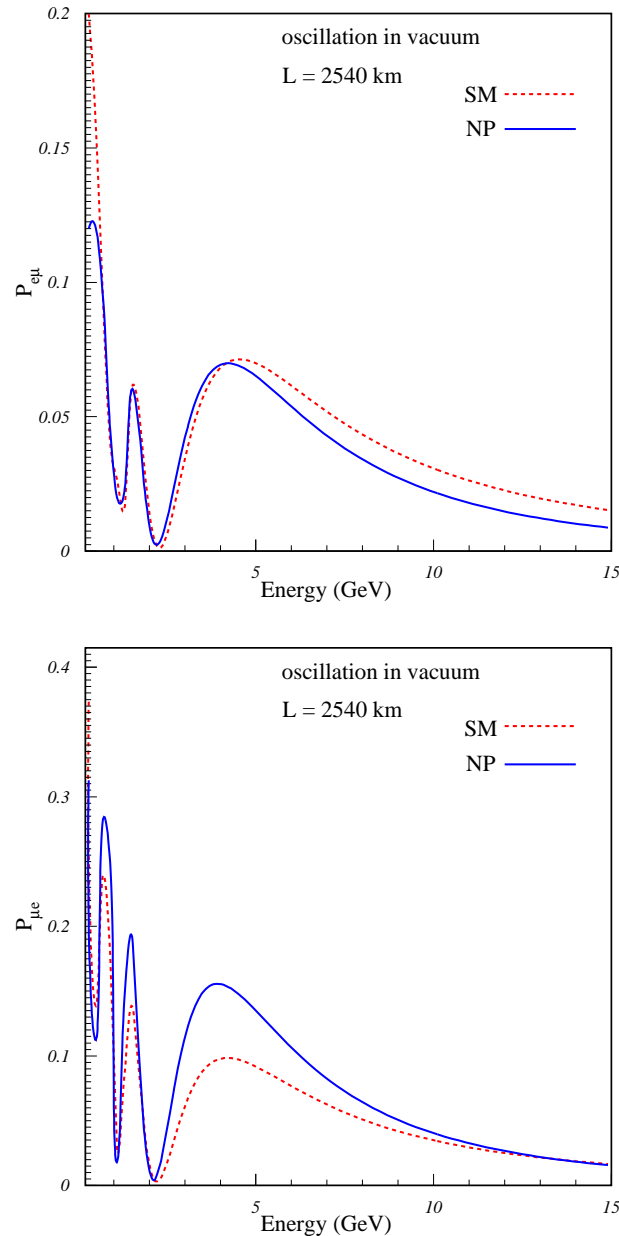


Figure 9.2. Oscillation probabilities $P_{e\mu}$ and $P_{\mu e}$ as a function of energy for the choice $\psi = \frac{\pi}{4}$, $\omega = \frac{\pi}{5.6}$, $\phi = \frac{\pi}{15}$, $\delta = \frac{\pi}{4}$, $\Delta m_{21}^2 = 7.1 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{31}^2 = 2.0 \times 10^{-3} \text{ eV}^2$ for a fixed baseline $L = 2540$ km. The dotted line is the Standard Model prediction and the solid line includes new physics for the choice $\beta = 0.1$, $\alpha = -0.1$, $\epsilon_\omega = -0.06$, $\epsilon_\phi = 0.05$, $\epsilon_\psi = 0.05$, $\epsilon_\delta = 0.05$.

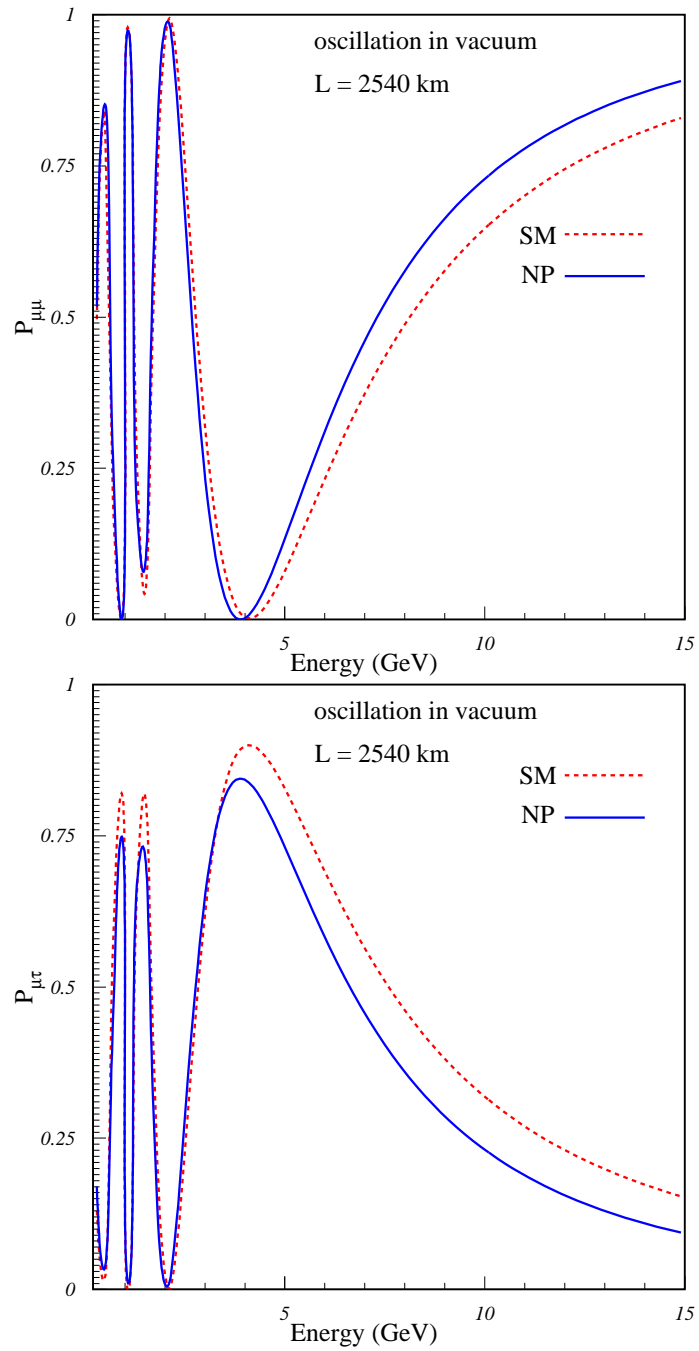


Figure 9.3. Oscillation probabilities $P_{\mu\mu}$ and $P_{\mu\tau}$ as a function of energy for the same choice of input parameters as in Fig. 9.2.

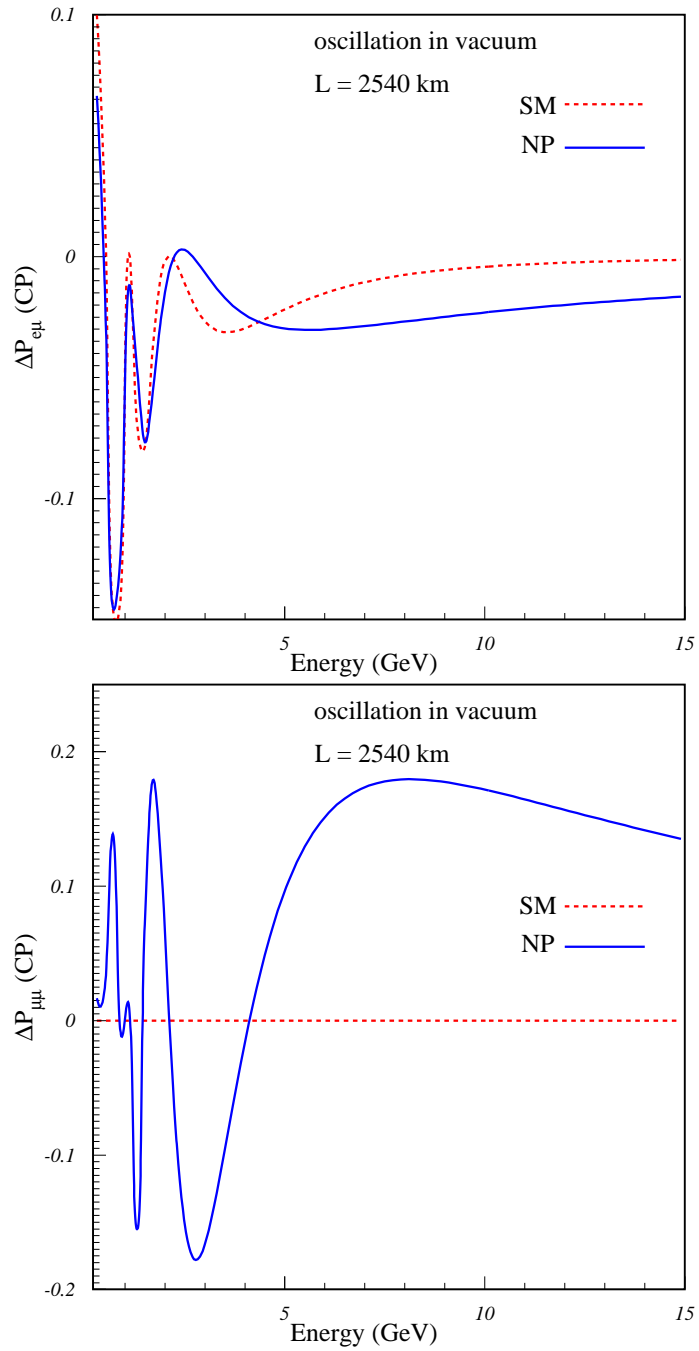


Figure 9.4. Change in oscillation probabilities $\Delta P_{e\mu} (CP) = P_{e\mu} - P_{\bar{e}\bar{\mu}}$ and $\Delta P_{\mu\mu} (CP) = P_{\mu\mu} - P_{\bar{\mu}\bar{\mu}}$ as a function of energy for the same choice of input parameters as in Fig. 9.2.

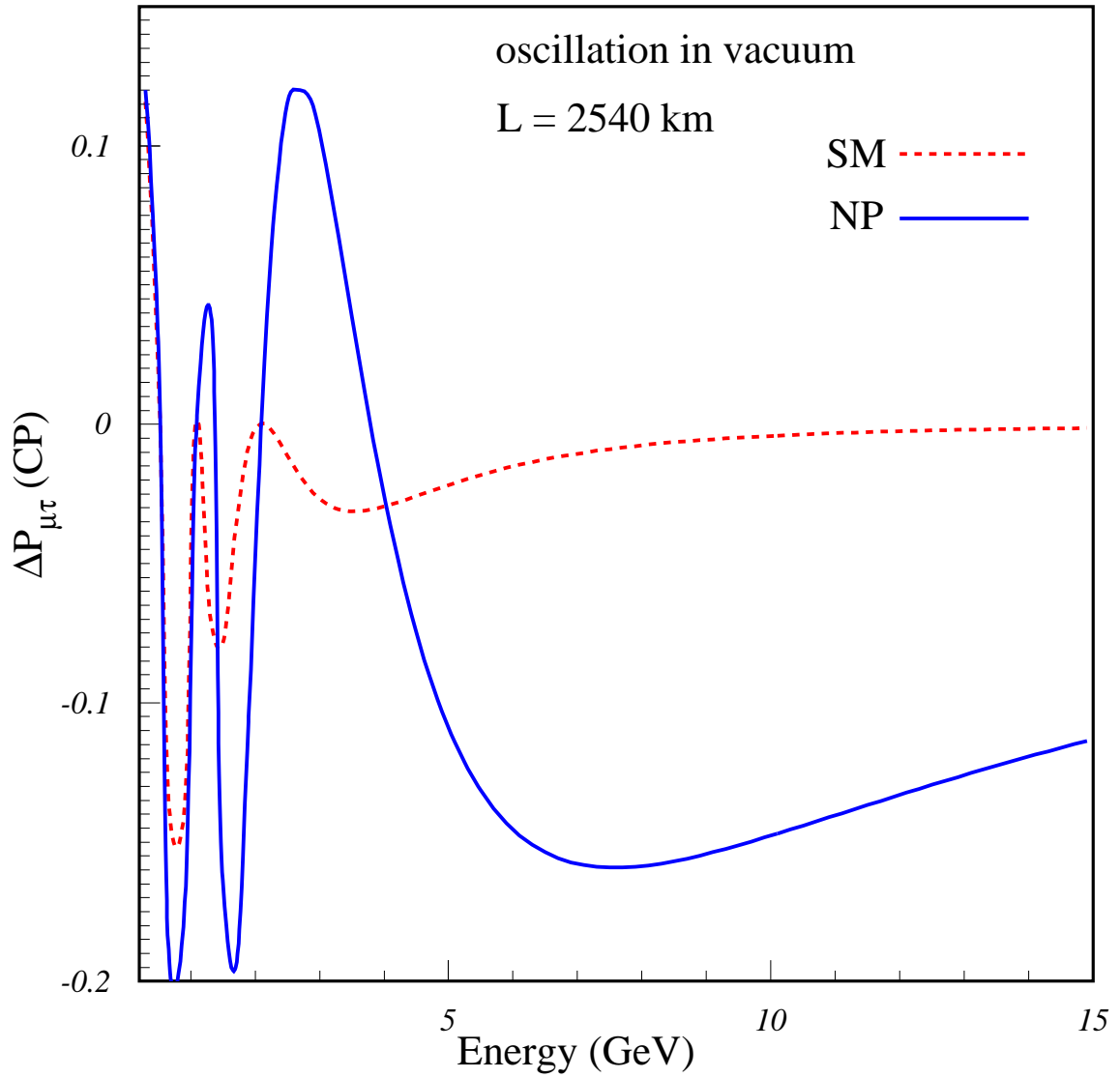


Figure 9.5. Change in oscillation probability $\Delta P_{\mu\tau} (CP) = P_{\mu\tau} - P_{\bar{\mu}\bar{\tau}}$ as a function of energy for the same choice of input parameters as in Fig. 9.2.

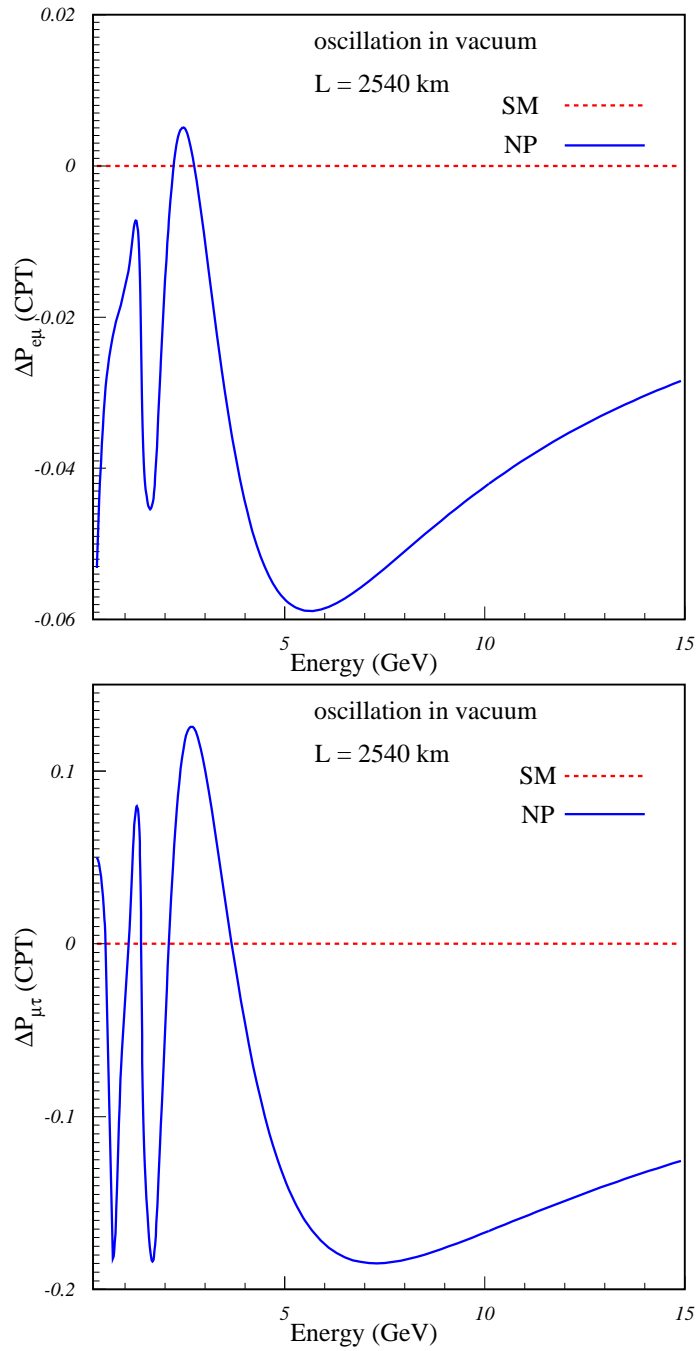


Figure 9.6. Apparent CPT violation parameters $\Delta P_{e\mu} (CPT) = P_{e\mu} - P_{\bar{\mu}e}$ and $\Delta P_{\mu\tau} (CPT) = P_{\mu\tau} - P_{\bar{\tau}\bar{\mu}}$ as a function of energy for the same choice of input parameters as in Fig. 9.2.

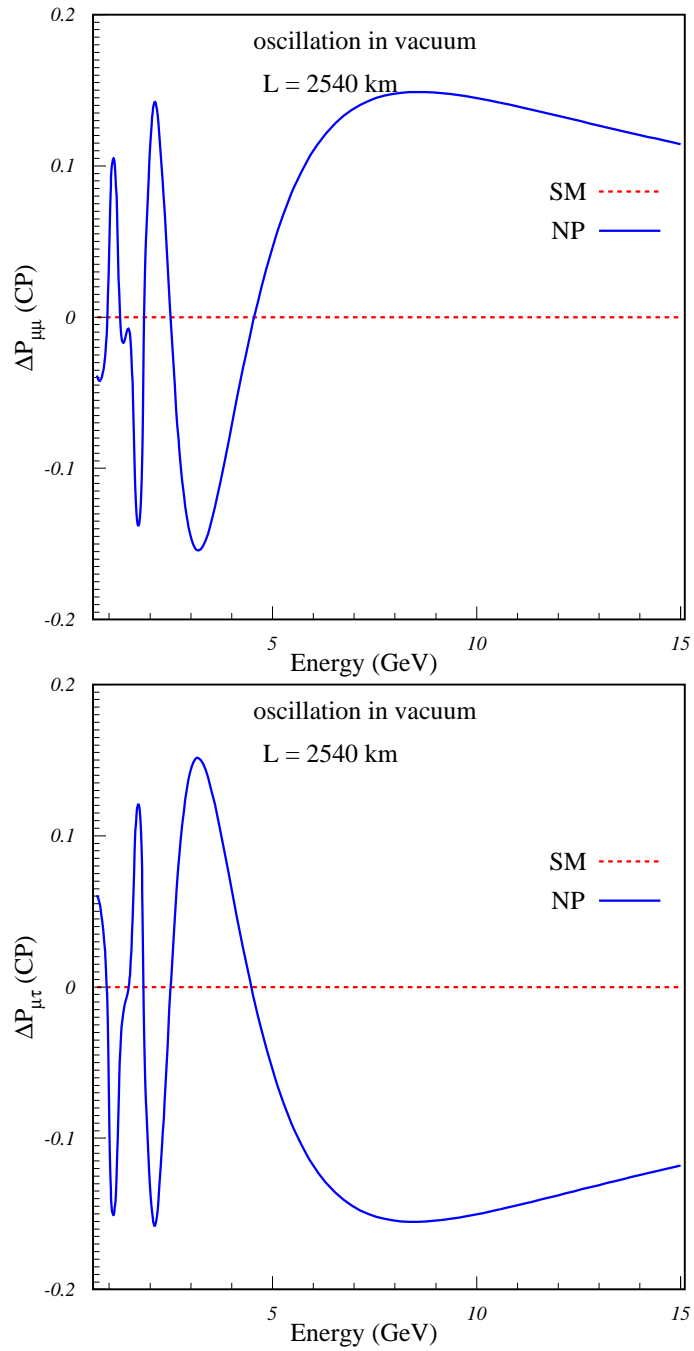


Figure 9.7. Change in oscillation probabilities $\Delta P_{\mu\mu} (CP) = P_{\mu\mu} - P_{\bar{\mu}\bar{\mu}}$ and $\Delta P_{\mu\tau} (CP) = P_{\mu\tau} - P_{\bar{\mu}\bar{\tau}}$ as a function of energy for the same choice of input parameters as in Fig. 9.2, except that $\delta = 0$ and $\epsilon_\delta = 0$.

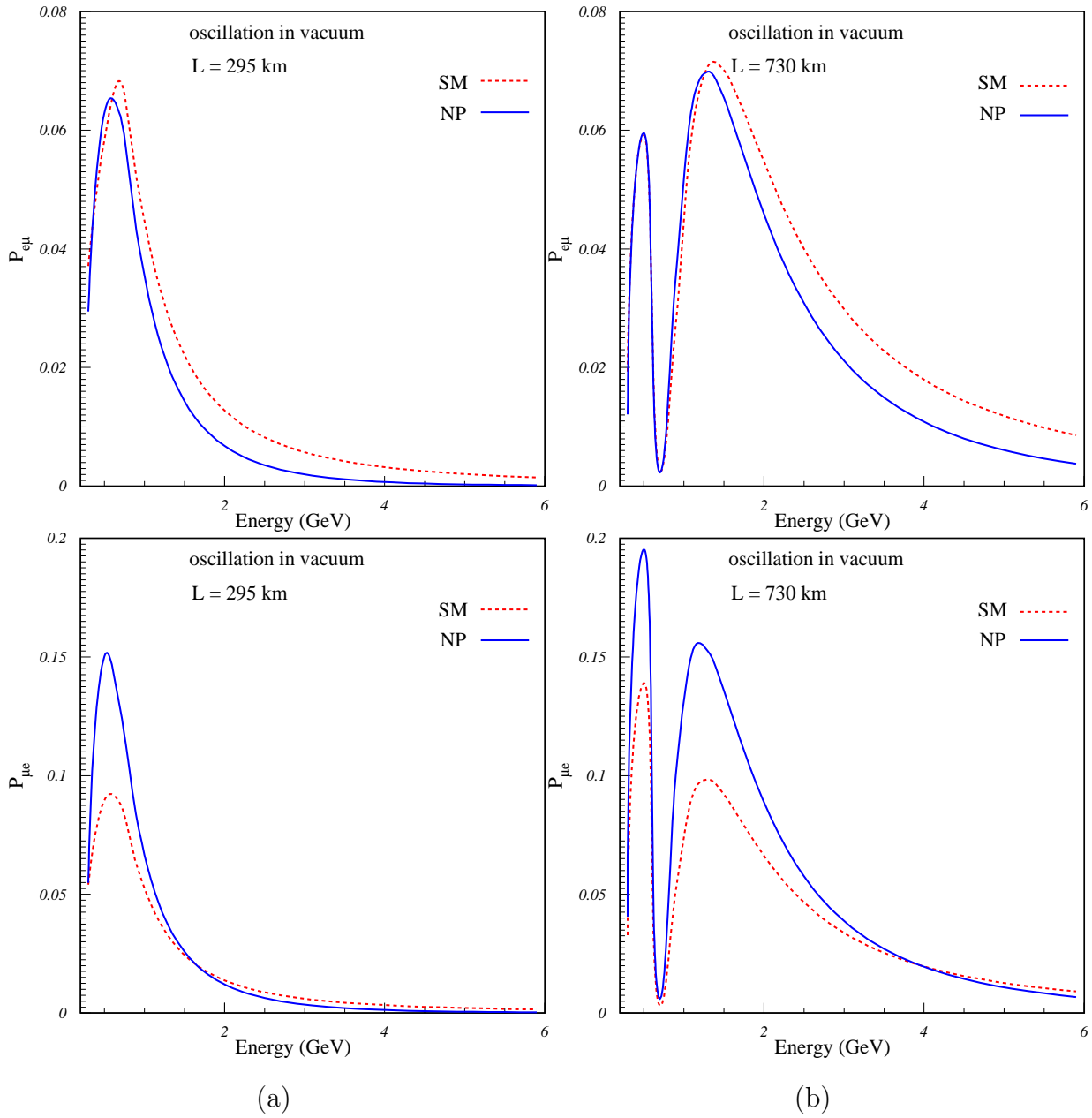


Figure 9.8. Oscillation probabilities $P_{e\mu}$ and $P_{\mu e}$ as a function of energy for a fixed baseline $L = 295$ km (a) and $L = 730$ km (b). All other parameters are as in Fig. 9.2.

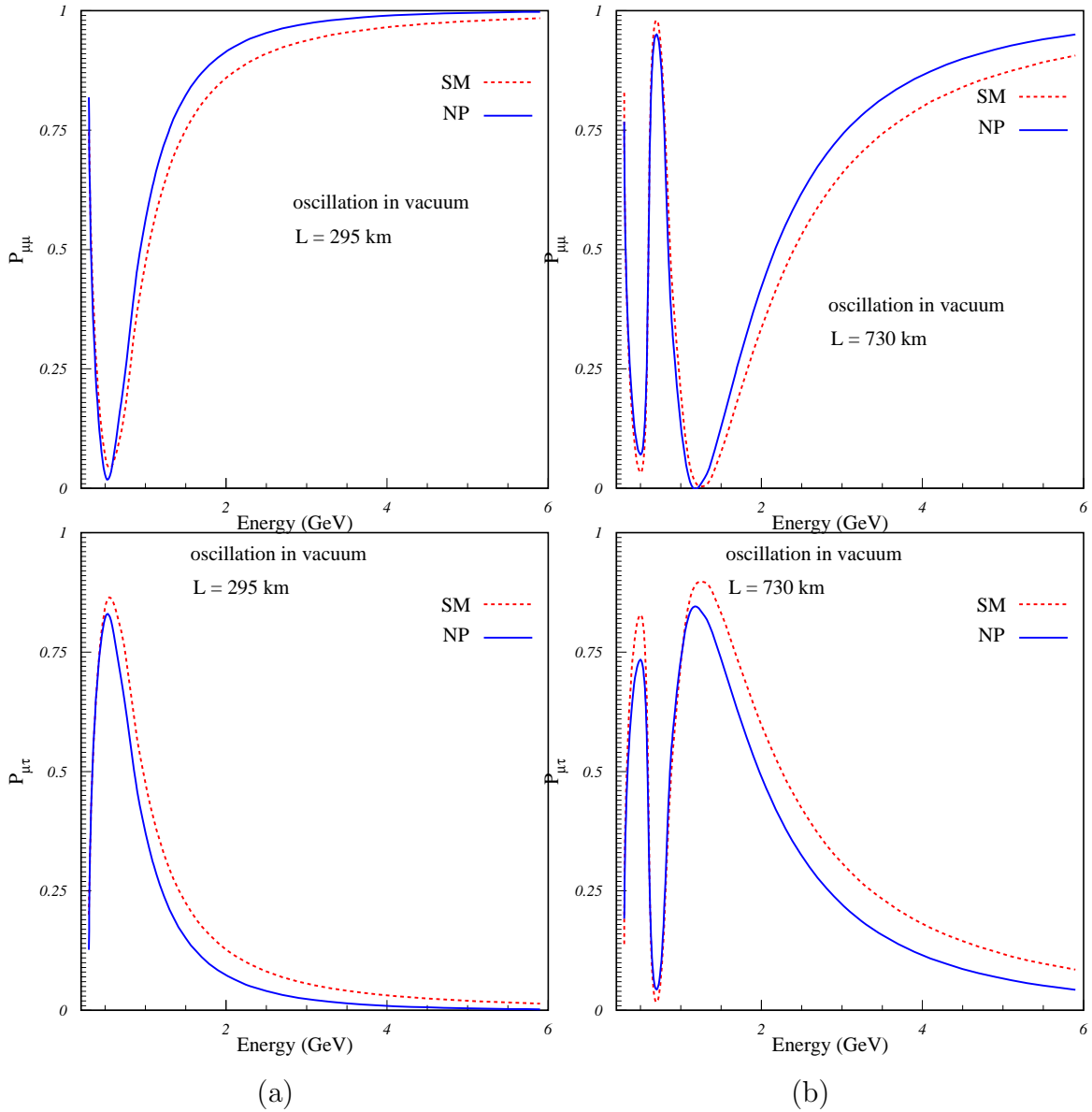


Figure 9.9. Oscillation probabilities $P_{\mu\mu}$ and $P_{\mu\tau}$ as a function of energy for fixed baseline $L = 295$ km (a) and $L = 730$ km (b). Input parameters are as in Fig. 9.2.

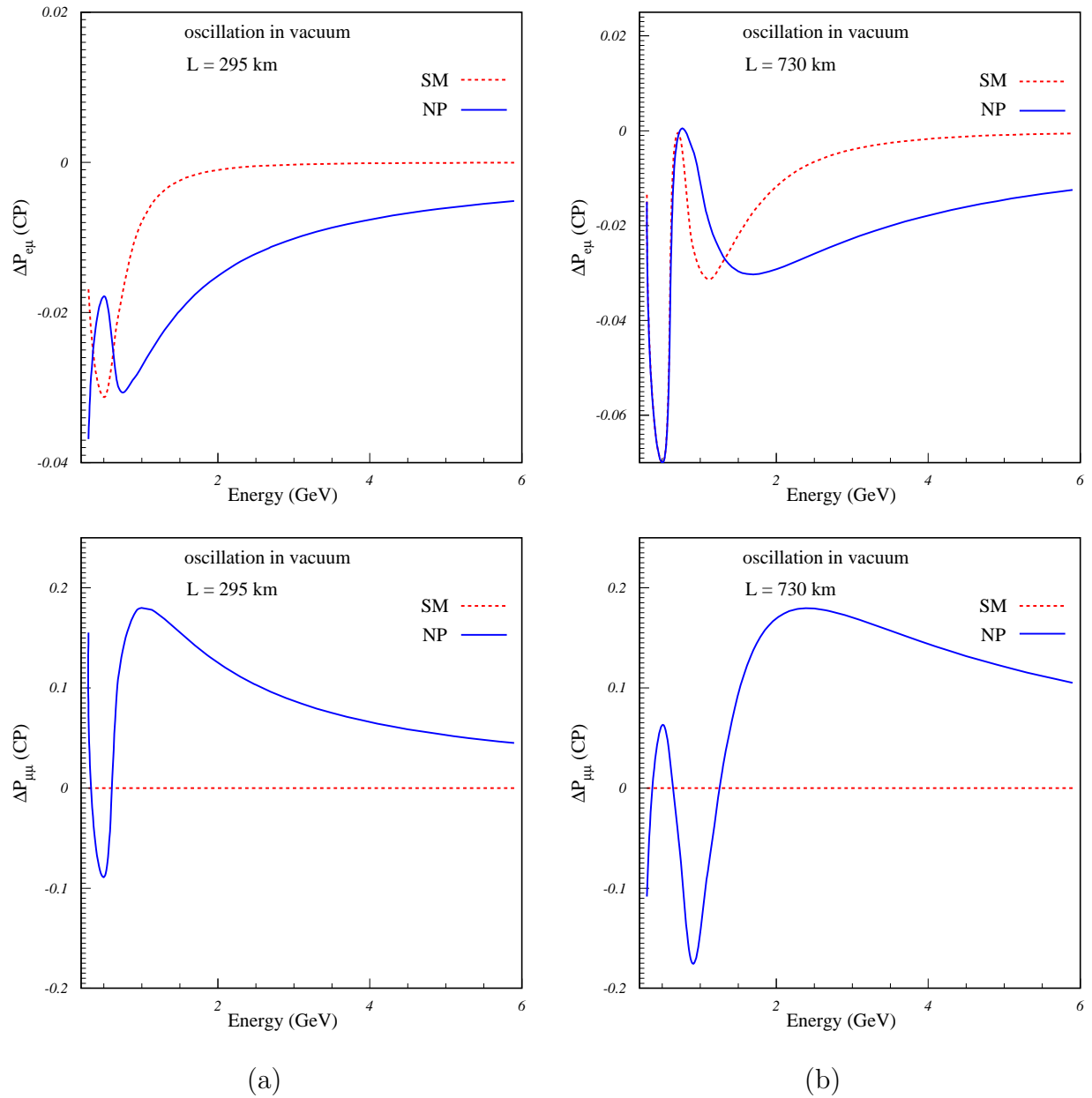


Figure 9.10. Change in oscillation probabilities $\Delta P_{e\mu} (CP) = P_{e\mu} - P_{\bar{e}\bar{\mu}}$ and $\Delta P_{\mu\mu} (CP) = P_{\mu\mu} - P_{\bar{\mu}\bar{\mu}}$ as a function of energy for the same choice of input parameters as in Fig. 9.2.

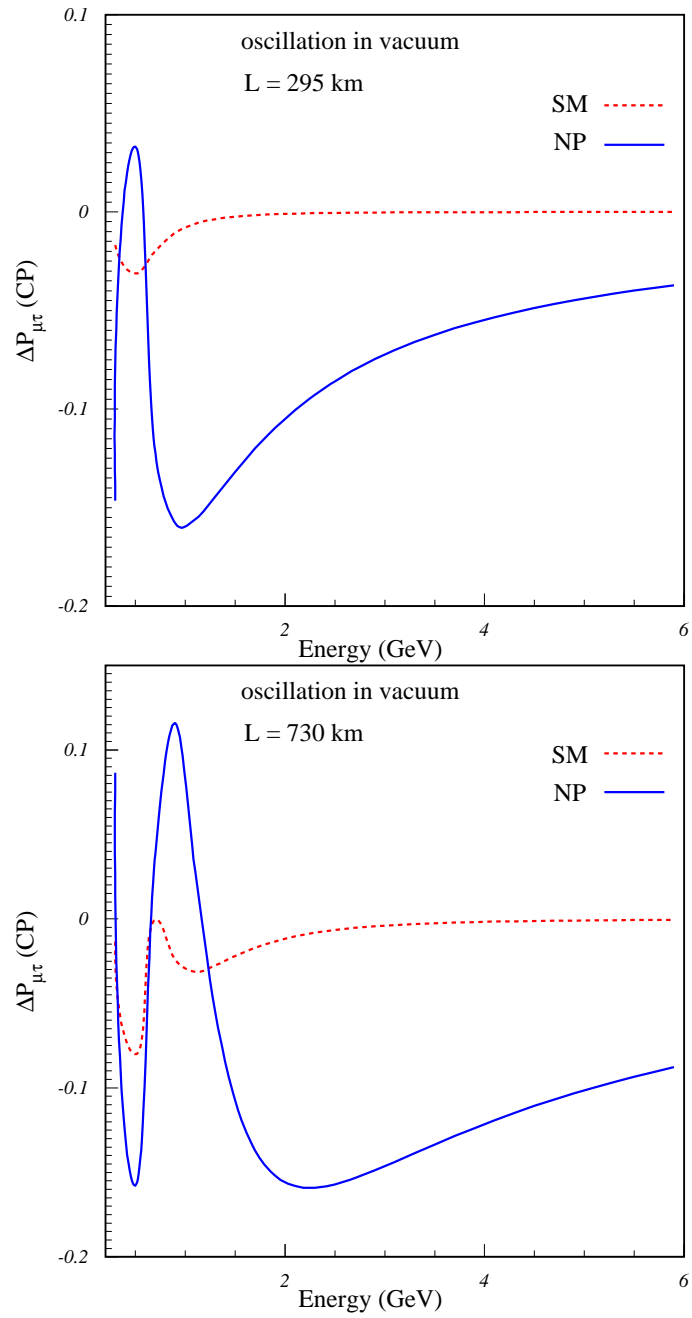


Figure 9.11. Change in oscillation probability $\Delta P_{\mu\tau}$ (CP) = $P_{\mu\tau} - P_{\bar{\mu}\bar{\tau}}$ as a function of energy for the same choice of input parameters as in Fig. 9.2.

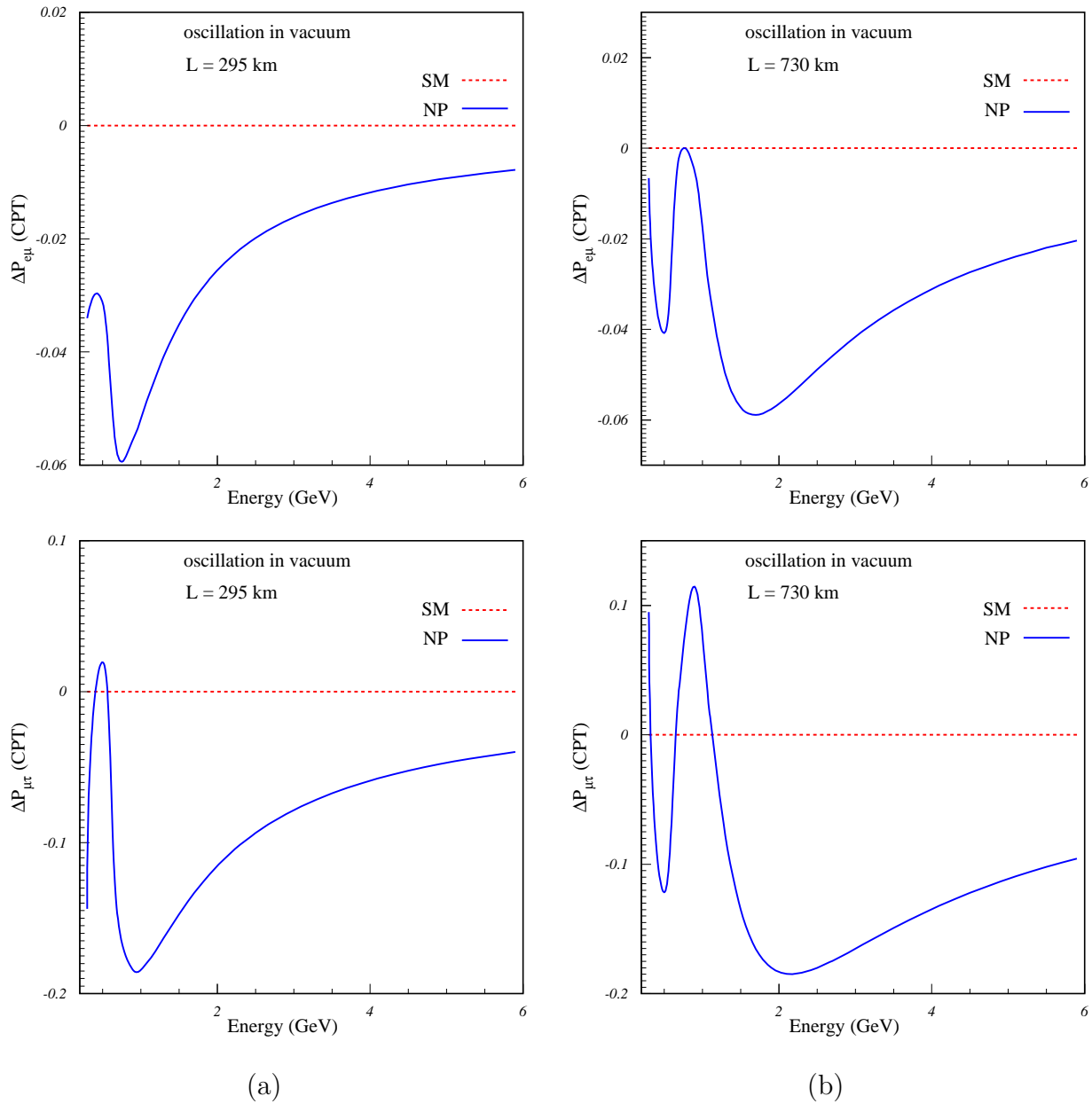


Figure 9.12. Apparent CPT violation parameters $\Delta P_{e\mu} (CPT) = P_{e\mu} - P_{\bar{\mu}e}$ and $\Delta P_{\mu\tau} (CPT) = P_{\mu\tau} - P_{\bar{\tau}\bar{\mu}}$ as a function of energy for the same choice of input parameters as in Fig. 9.2.

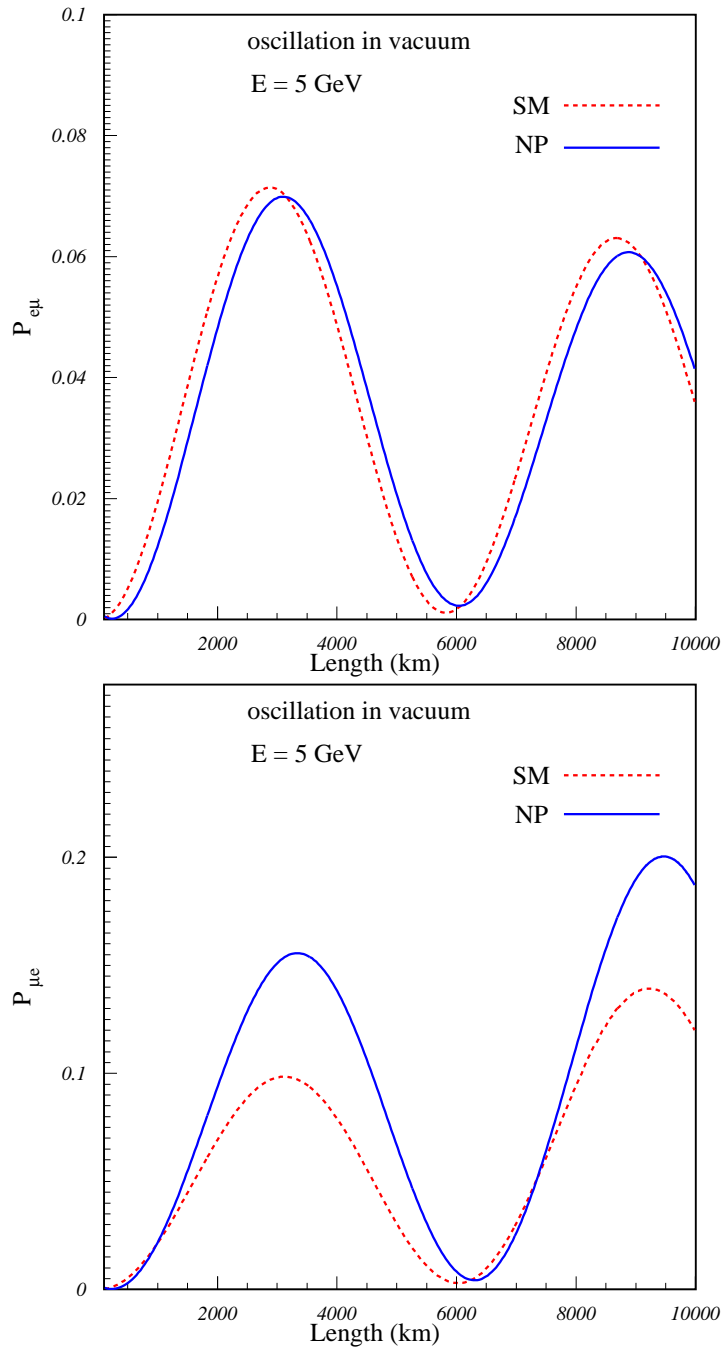


Figure 9.13. Oscillation probabilities $P_{e\mu}$ and $P_{\mu e}$ as a function of Length for fixed energy $E = 5$ GeV. All other parameters are the same as in Fig. 9.2.

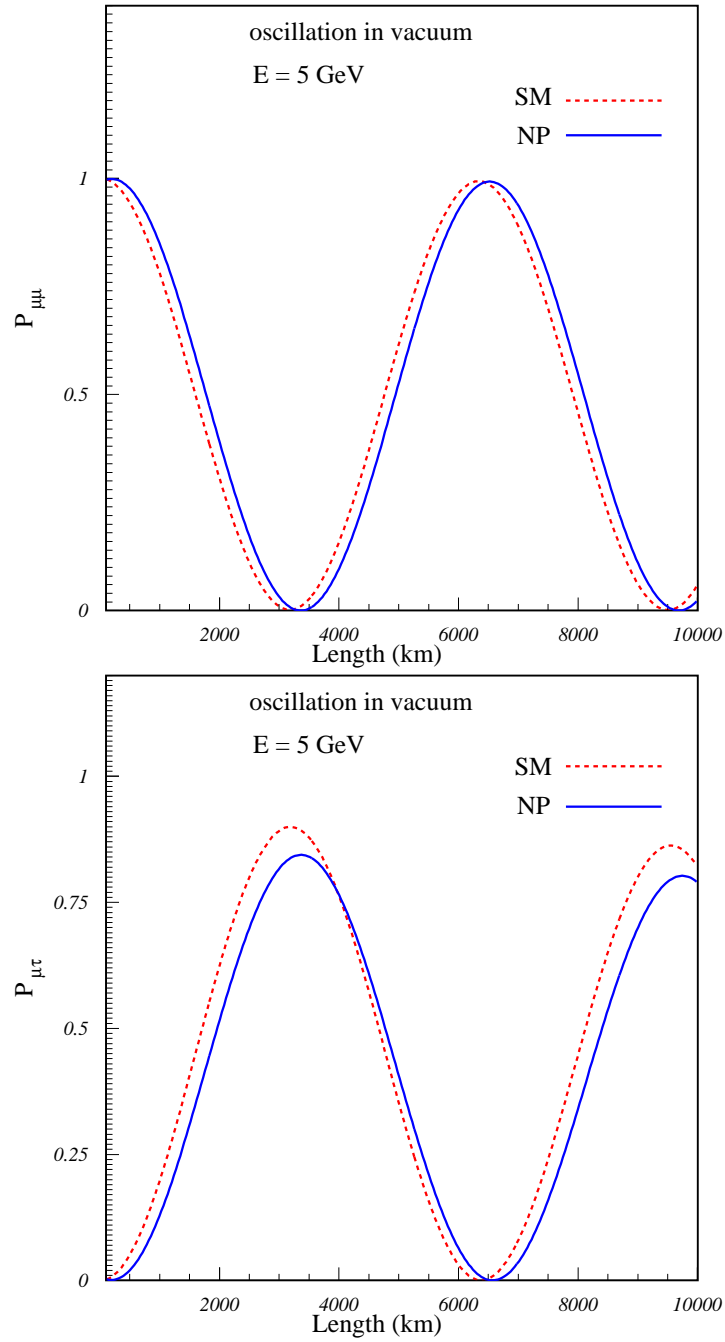


Figure 9.14. Oscillation probabilities $P_{\mu\mu}$ and $P_{\mu\tau}$ as a function of length for the same choice of input parameters as in Fig. 9.2.

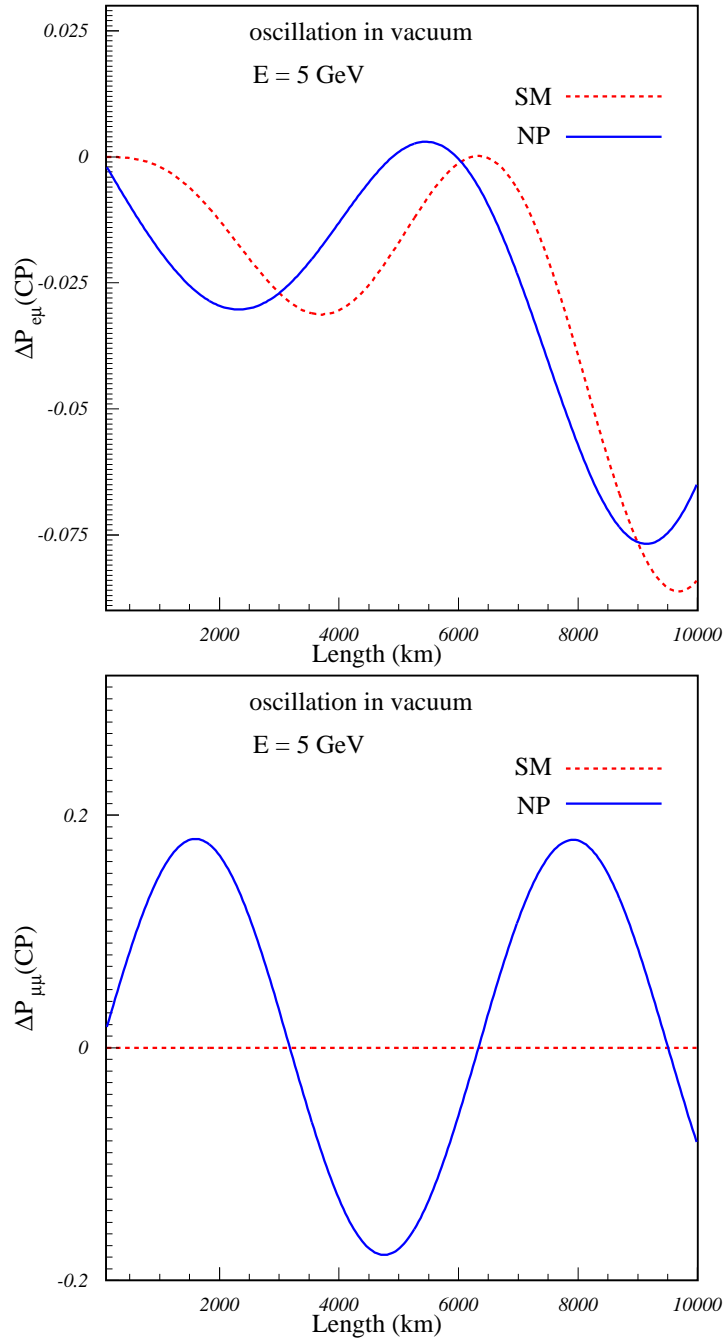


Figure 9.15. Change in oscillation probabilities $\Delta P_{e\mu}(CP) = P_{e\mu} - P_{\bar{e}\bar{\mu}}$ and $\Delta P_{\mu\mu}(CP) = P_{\mu\mu} - P_{\bar{\mu}\bar{\mu}}$ as a function of length for the same choice of input parameters as in Fig. 9.2.

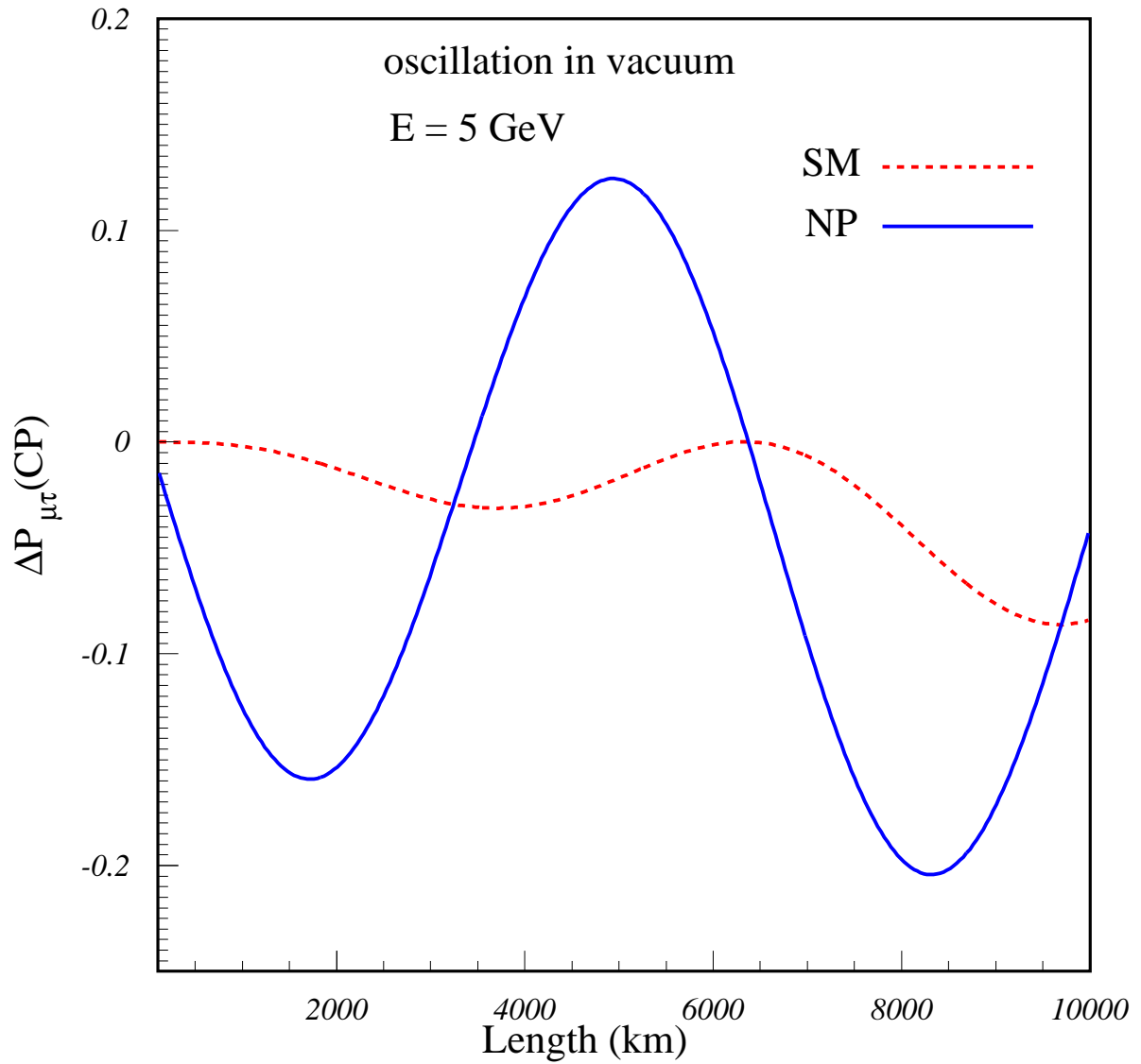


Figure 9.16. Change in oscillation probability $\Delta P_{\mu\tau} (CP) = P_{\mu\tau} - P_{\bar{\mu}\bar{\tau}}$ as a function of length for the same choice of input parameters as in Fig. 9.2.

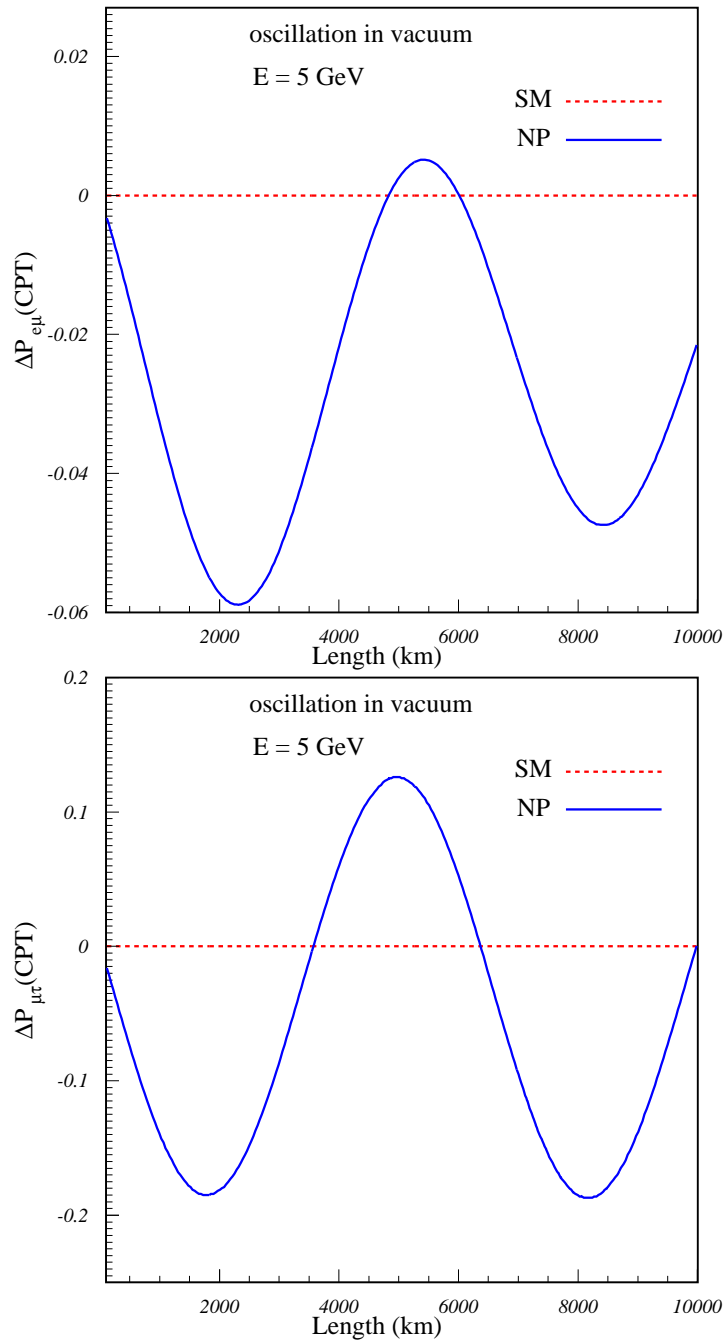


Figure 9.17. Apparent CPT violation parameters $\Delta P_{e\mu}(CPT) = P_{e\mu} - P_{\bar{\mu}e}$ and $\Delta P_{\mu\tau}(CPT) = P_{\mu\tau} - P_{\bar{\tau}\mu}$ as a function of Length for the same choice of input parameters as in Fig. 9.2.

9.4 Three neutrino oscillations including matter effects

There have been a number of attempts to find simple and exact analytic formulas for three generation neutrino oscillations including matter effects for long baselines [105–110]. In one of the attempts, corrections to the neutrino mixing parameters in the presence of constant matter density (2.8 g/cm^3) were calculated in Ref. [111] using a series expansion in terms of the mass hierarchy $\frac{\Delta m_{21}^2}{\Delta m_{31}^2}$ ($\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2}$) and small mixing angle ϕ . They obtained the expressions for a one to one correspondence to the vacuum case, which are valid for energies above the solar resonance ($\sim 0.5 \text{ GeV}$). The parameter mappings were used to find simple and accurate formulas for oscillation probabilities in matter including CP violating effects. We use these parameter mappings to justify the effect of new physics and at the end show some numerical plots obtained for the probabilities as a function of energy at fixed length 2540 km. We also show plots of $\Delta P(CP)$ as a function of energy and demonstrate how it is possible to measure the asymmetry in the near future. At the end of this section, we will combine these plots and outline the differences between pure new physics effects and SM CP effects. The CP asymmetry has considerable importance in CP-violation studies, the problem is that matter effects cause contributions to the CP-asymmetry, which can not be easily distinguished from intrinsic CP-violation. We show that the low energy option is not the best solution to measure effects from the CP-phase δ . The SNO experiment [112] favors the MSW LMA [113] solution to the solar neutrino problem, long baseline experiments such as JHF and neutrino factory experiments are planned in the near future.

9.4.1 Formalism

We define the parameters $\lambda = \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \ll 1$, $\Delta m_{atm}^2 = \Delta$, $\Delta m_{\odot}^2 = \lambda\Delta$ and $\Delta m_{32}^2 = (1 - \lambda)\Delta$. In matter, the effective Hamiltonian in the flavor basis is given by

$$H = \frac{1}{2E} \left[U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right], \quad (9.53)$$

where the matter effect term is given by

$$A \equiv 2\sqrt{2}G_F n_e E_\nu = 7.56 \times 10^{-5} \text{eV}^2 \frac{\rho}{\text{g cm}^{-3}} \frac{E}{\text{GeV}}. \quad (9.54)$$

The approximate expressions for the eigenvalues and eigenvectors can be found in Ref. [111]. Two different resonances occur: (i) $\hat{A} = \lambda$ (solar resonance) and (ii) $\hat{A} = \cos 2\phi$ (atmospheric resonance), where $\hat{A} = \frac{A}{\Delta}$. Here we focus on $|\hat{A}| > \lambda$ which is appropriate for neutrino energies above 1 GeV in matter density of 2.8g/cm³. As was pointed out in Ref. [111], the expressions obtained will not show the correct convergence for $\hat{A} \rightarrow 0$ and the result will hence not be good for the resonance $\hat{A} \simeq 1$.

9.4.2 Parameter mapping

The one-to-one correspondence for the parameter mapping to the vacuum case is given by the following expressions [111]

$$\sin \phi' = \frac{\sin 2\phi}{\sqrt{2\hat{C}(\mp\hat{A} + \hat{C} \pm \cos 2\phi)}} \pm \frac{\lambda\hat{A} \sin^2 \omega \sin^2 2\phi}{2\hat{C}\sqrt{2\hat{C}^2(\pm\hat{A} + \hat{C} \mp \cos 2\phi)}}, \quad (9.55)$$

$$\sin \omega' = \frac{\lambda\hat{C} \sin 2\omega}{|\hat{A}| \cos \phi \sqrt{2\hat{C}(\mp\hat{A} + \hat{C} \pm \cos 2\phi)}}, \quad (9.56)$$

$$\sin \psi' = \sin \psi + \frac{\lambda \cos \delta \hat{A} \sin 2\omega \sin \phi \cos \psi}{\pm 1 + \hat{C} \mp \hat{A} \cos 2\phi}, \quad (9.57)$$

$$\sin \delta' = \sin \delta \left(1 - \lambda \frac{\cos \delta}{\tan 2\psi} \frac{2\hat{A} \sin 2\omega \sin \phi}{\pm 1 + \hat{C} \mp \hat{A} \cos 2\phi} \right), \quad (9.58)$$

where

$$\hat{C} = \sqrt{(\hat{A} - \cos 2\phi)^2 + \sin^2 2\phi}. \quad (9.59)$$

Higher order terms in λ are ignored. The upper sign is valid for $\hat{A} < \cos 2\phi$ and the lower sign is valid for $\hat{A} > \cos 2\phi$. For the case $\hat{A} < \cos 2\phi$, the mass squared difference is

$$\Delta m_{\odot}^{\prime 2} = \frac{1}{2}(-1 - \hat{A} + \hat{C})\Delta + \lambda\Delta(\cos^2 \omega - \frac{(1 + \hat{C} - \hat{A} \cos 2\phi) \sin^2 \omega}{2\hat{C}}), \quad (9.60)$$

$$\Delta m_{atm}^{\prime 2} = \hat{C}\Delta + \lambda\Delta \frac{(-1 + \hat{A} \cos 2\phi) \sin^2 \omega}{\hat{C}}, \quad (9.61)$$

for the case $\hat{A} > \cos 2\phi$, the mass squared difference is

$$\Delta m_{\odot}^{2'} = -\frac{1}{2}(1 + \hat{A} + \hat{C})\Delta - \lambda\Delta(\cos^2 \omega - \frac{(-1 + \hat{C} + \hat{A} \cos 2\phi) \sin^2 \omega}{2\hat{C}}) \quad (9.62)$$

$$\Delta m_{atm}^{2'} = -\hat{C}\Delta - \lambda\Delta \frac{(-1 + \hat{A} \cos 2\phi) \sin^2 \omega}{\hat{C}}. \quad (9.63)$$

Using these parameter mappings to replace each of the parameters in Eq. (9.14), we have a one-to-one correspondence to the vacuum oscillation giving rise to a new unitary matrix as in Eq. (9.16) with the above parameter mapping used to replace the terms in V^s . We also make the replacement $\Delta m_{\odot}^2 \equiv \Delta m_{\odot}^{2'}$ and $\Delta m_{atm}^2 \equiv \Delta m_{atm}^{2'}$.

Note that in deriving the results above, the source eigenstate is assumed to be equal to the detection eigenstate. Using the same formalism as outlined in section 9.2 and the same procedure as in section 9.18, we assume here that the source eigenstate is equal to the detection eigenstate. The probability is then given by $P = |V' \cdot \hat{E}'_p \cdot V'^{\dagger}|^2$, where

$$\hat{E}'_p = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\Delta'_{21}} & 0 \\ 0 & 0 & e^{-i\Delta'_{31}} \end{pmatrix},$$

and

$$\Delta'_{21} = 2.53 \times \frac{L}{E} \Delta m_{\odot}^{2'}, \quad \Delta'_{31} = 2.53 \times \frac{L}{E} \Delta m_{atm}^{2'}. \quad (9.64)$$

In the three generation vacuum oscillations analyzed in section 9.2, we can separate out the new physics effect from the usual three generation probabilities. This term is then added to the expressions for the three generation matter effect. Because of the complicated nature of the unitary matrix in matter, it is very difficult to come up with a simple analytic expressions for the probabilities, the change in probability (ΔP) and CP-asymmetry. We show the numerical plots of the probabilities as a function of Energy in Figs. 9.16 and 9.17 at fixed length 2540 km for different choices of new physics parameters (ϵ_{ω} , ϵ_{ϕ} , ϵ_{ψ} , ϵ_{δ} , α and β). The plots of change in the probability as a function of energy is shown in Figs. 9.19 and 9.20 and the CPT asymmetry is shown in Fig. 21. In these figure, the dotted line denote the pure standard three generation oscillations in matter with no new physics and the solid line includes new physics.

It is worth noting that there is a term in the expansion which is of order $(\Delta m_{\odot}^2/\Delta m_{atm}^2)^2$ that will contribute to the probability at a very low value of $\sin^2 2\phi$ which in our case will not contribute significantly for the value $\phi = \frac{\pi}{15}$ that we have chosen here.

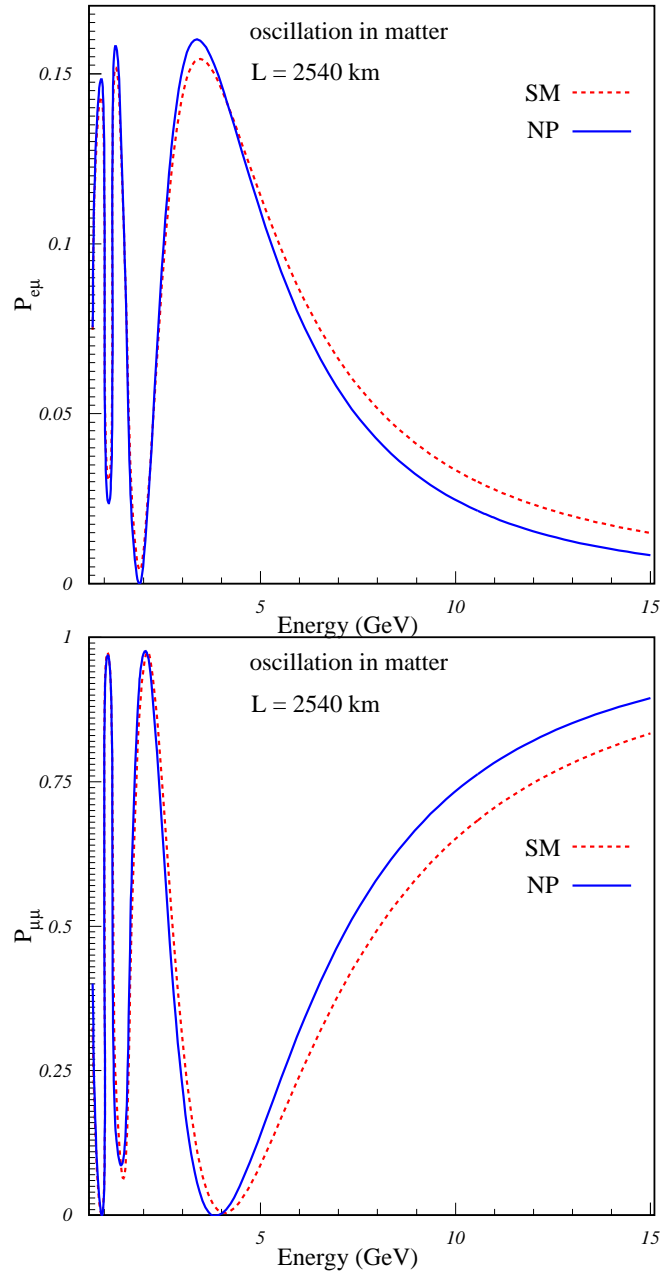


Figure 9.18. Oscillation probabilities $P_{e\mu}$ and $P_{\mu\mu}$ in matter (assuming constant matter density $\rho = 2.8 \text{ g/cm}^3$) as a function of energy for fixed length $L = 2540 \text{ km}$. All other parameters are the same as in Fig. 9.2.

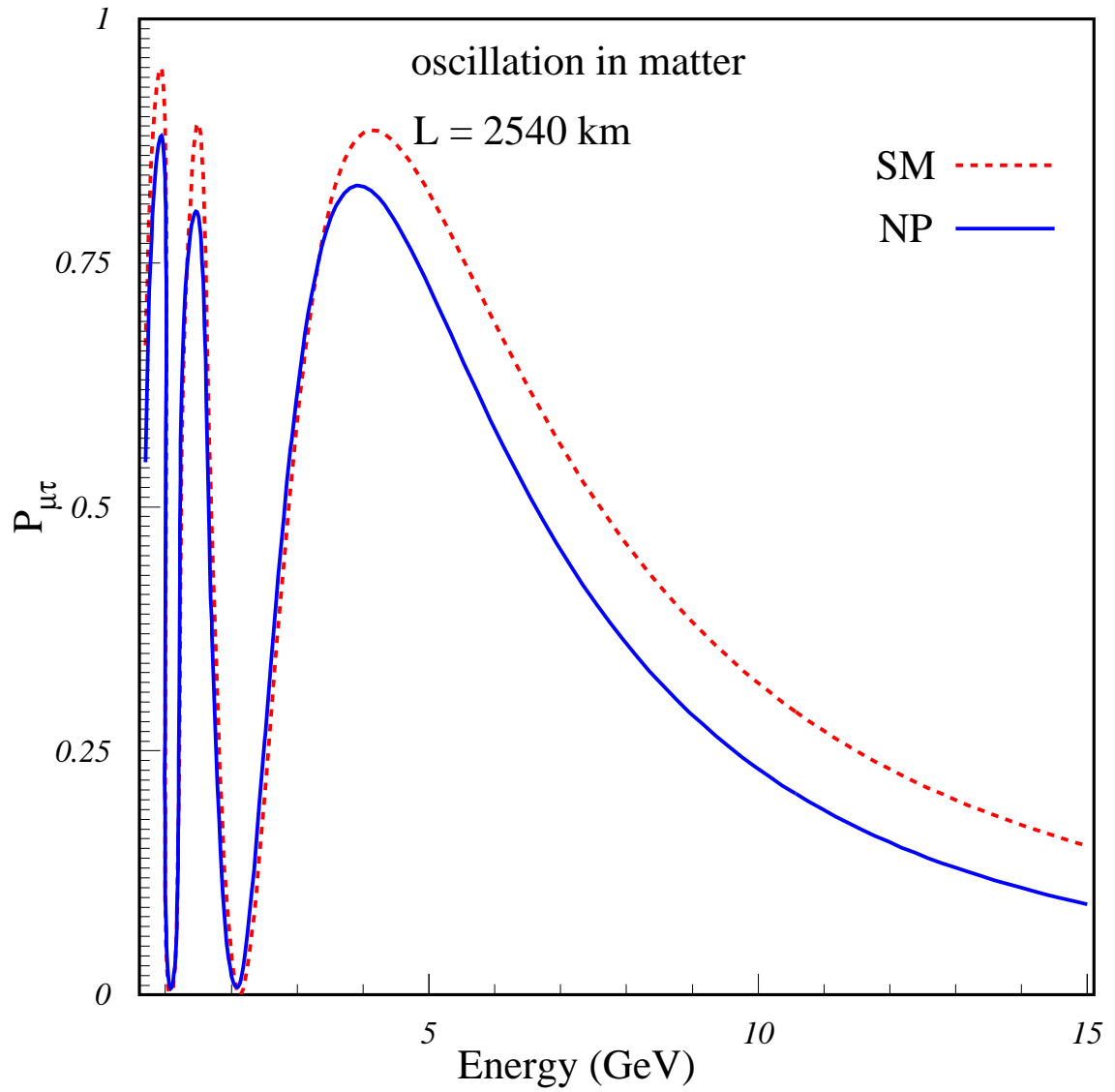


Figure 9.19. Oscillation probability $P_{\mu\tau}$ in matter as a function of energy for the same choice of input parameters as in Fig. 9.18.

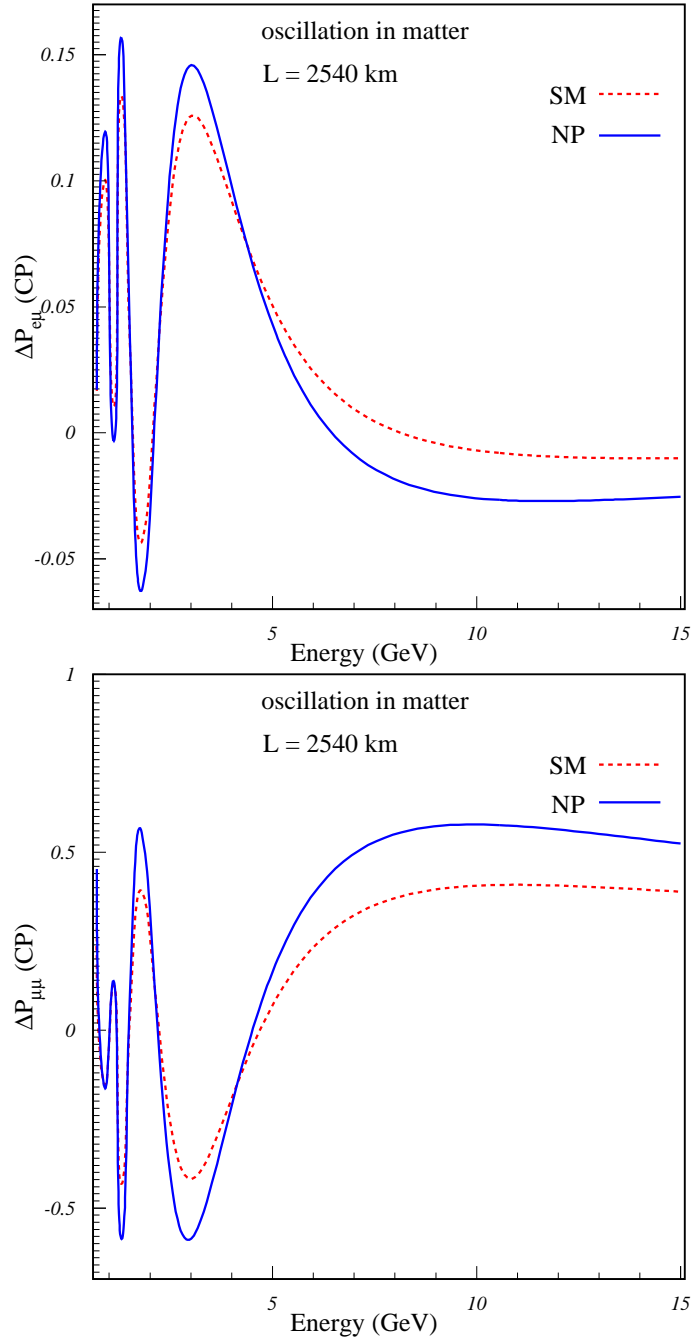


Figure 9.20. Change in oscillation probabilities $\Delta P_{e\mu}(CP) = P_{e\mu} - P_{\bar{e}\bar{\mu}}$ and $\Delta P_{\mu\mu}(CP) = P_{\mu\mu} - P_{\bar{\mu}\bar{\mu}}$ in matter as a function of energy for the same choice of input parameters as in Fig. 9.18.

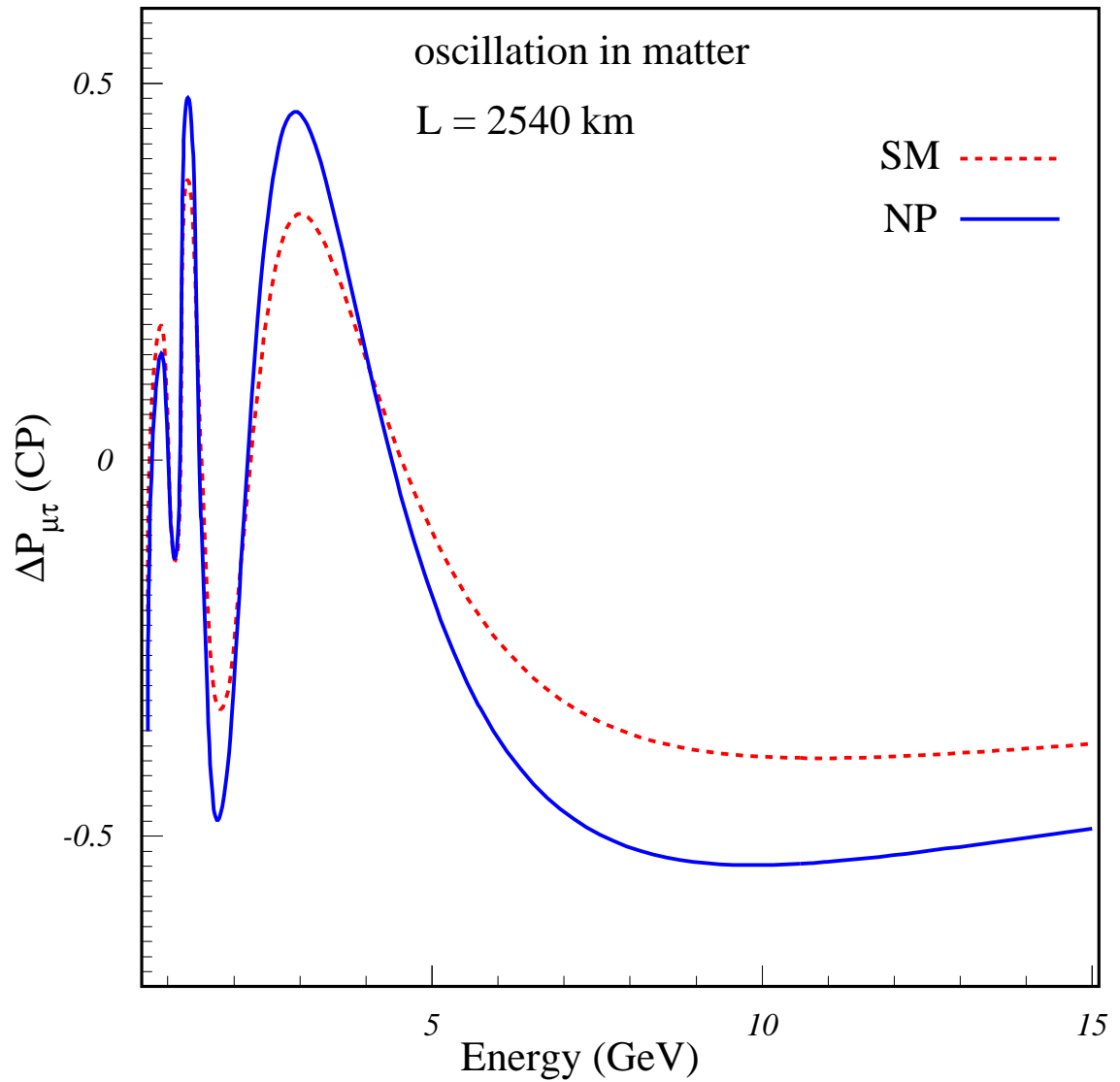


Figure 9.21. Change in oscillation probability $\Delta P_{\mu\tau} (CP) = P_{\mu\tau} - P_{\bar{\mu}\bar{\tau}}$ in matter as a function of energy for the same choice of input parameters as in Fig. 9.18.

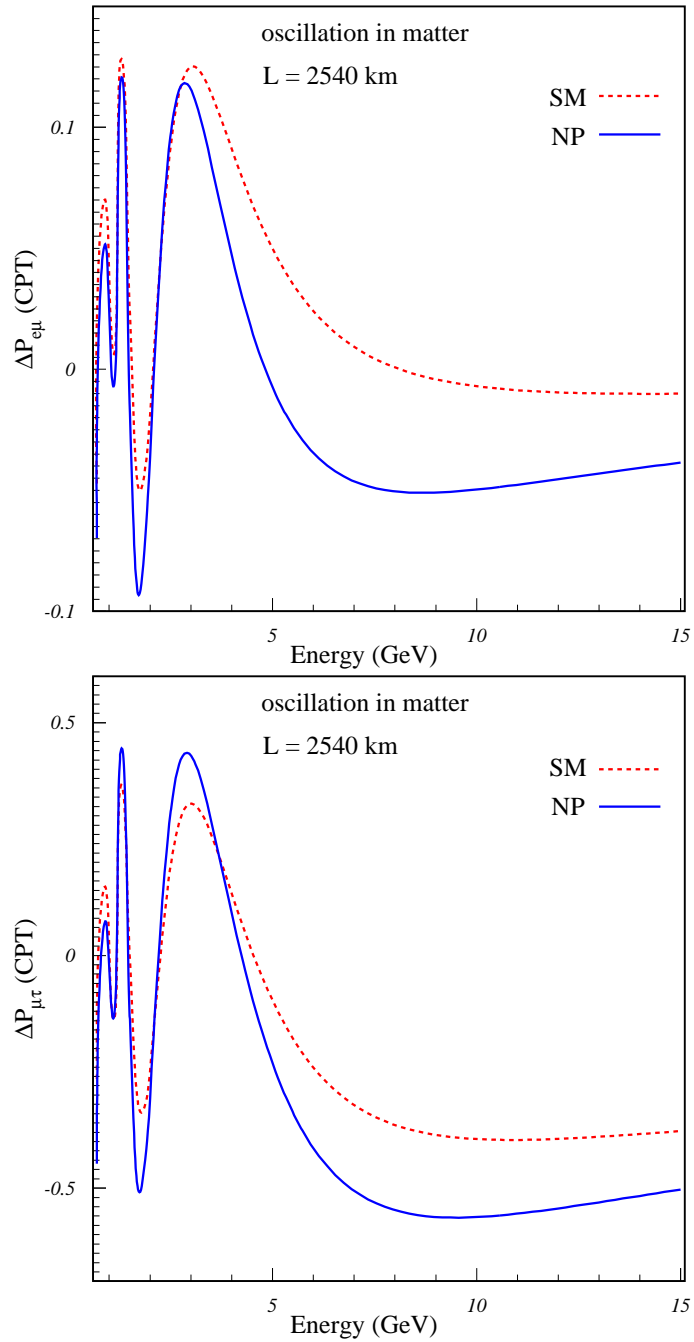


Figure 9.22. Apparent CPT violation parameters $\Delta P_{e\mu} (CPT) = P_{e\mu} - P_{\bar{\mu}e}$ and $\Delta P_{\mu\tau} (CPT) = P_{\mu\tau} - P_{\bar{\tau}\mu}$ in matter as a function of energy for the same choice of input parameters as in Fig. 9.18.

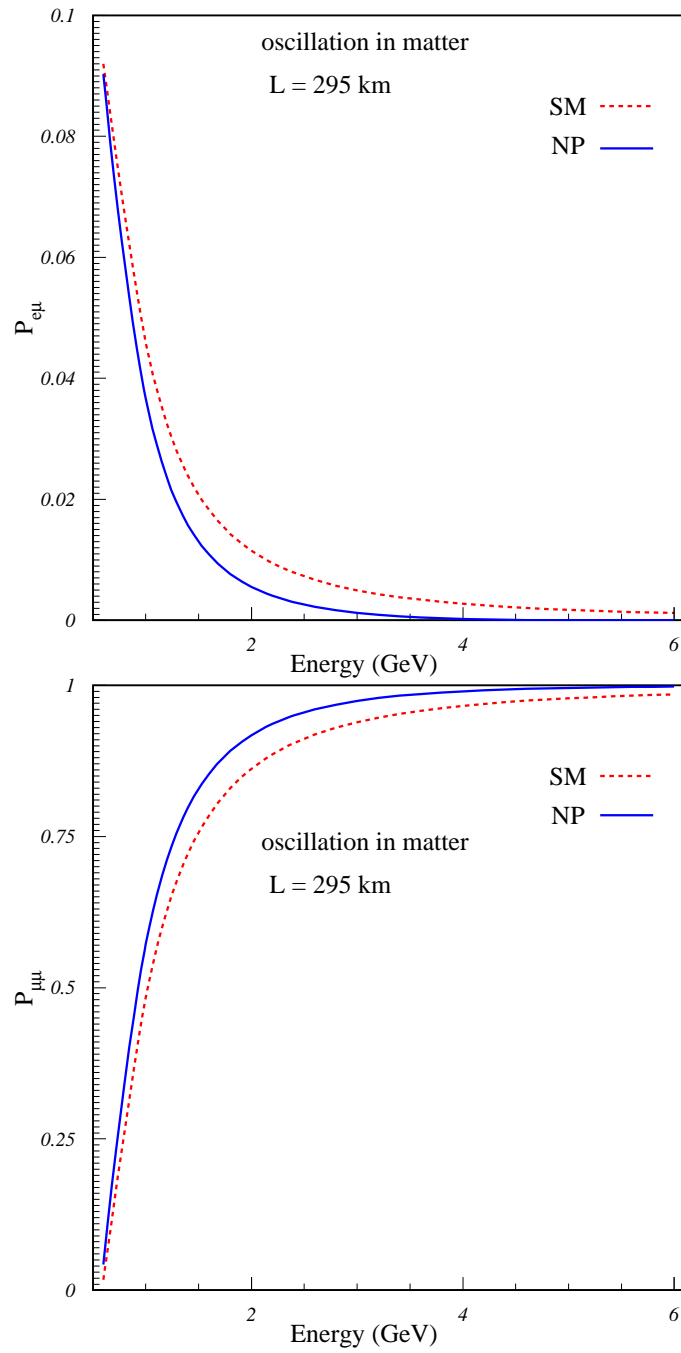


Figure 9.23. Oscillation probabilities $P_{e\mu}$ and $P_{\mu\mu}$ in matter as a function of energy for fixed length $L = 295$ km. All other parameters are the same as in Fig. 9.18.

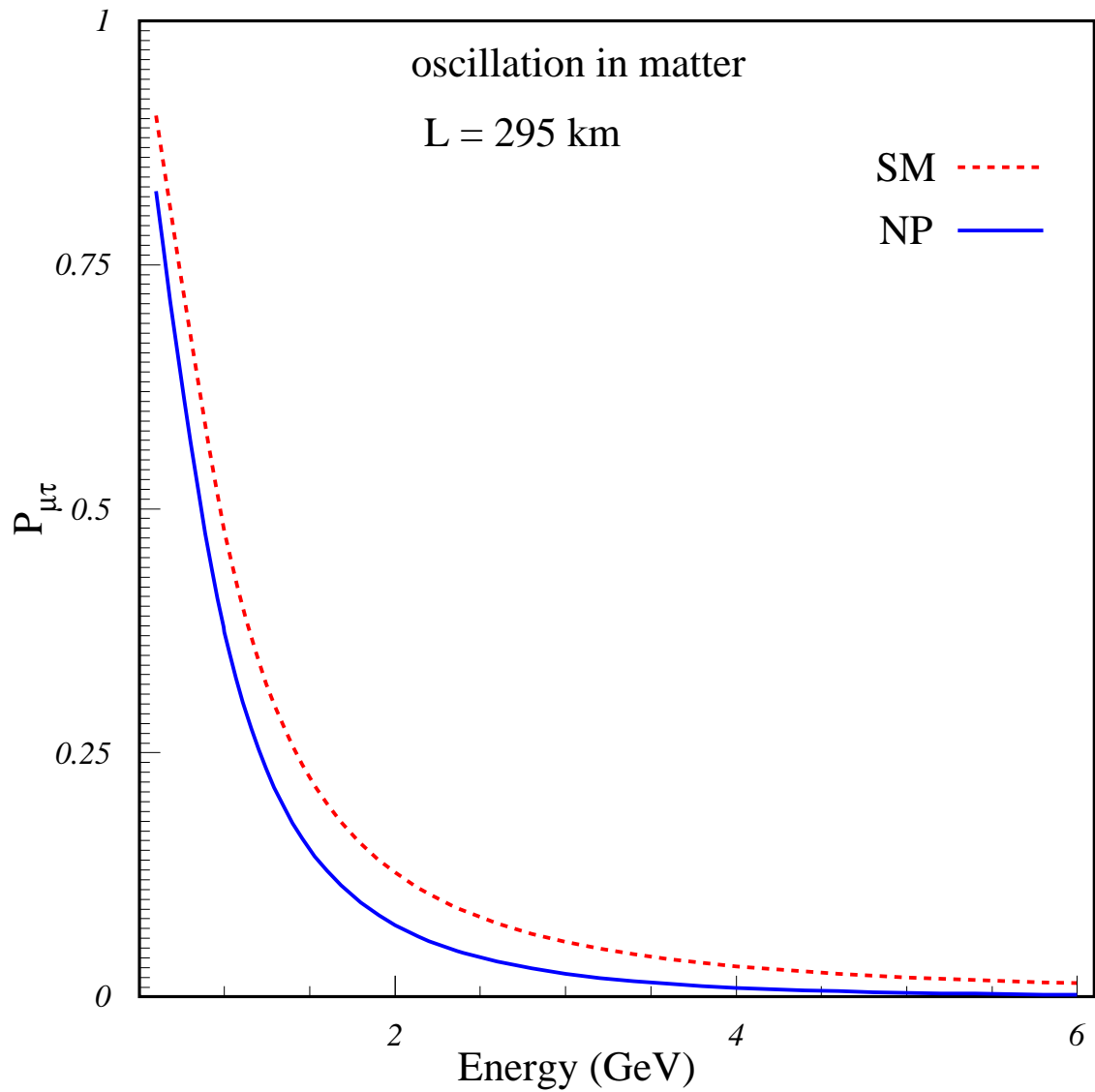


Figure 9.24. Oscillation probability $P_{\mu\tau}$ in matter as a function of energy for the same choice of input parameters as in Fig. 9.18.

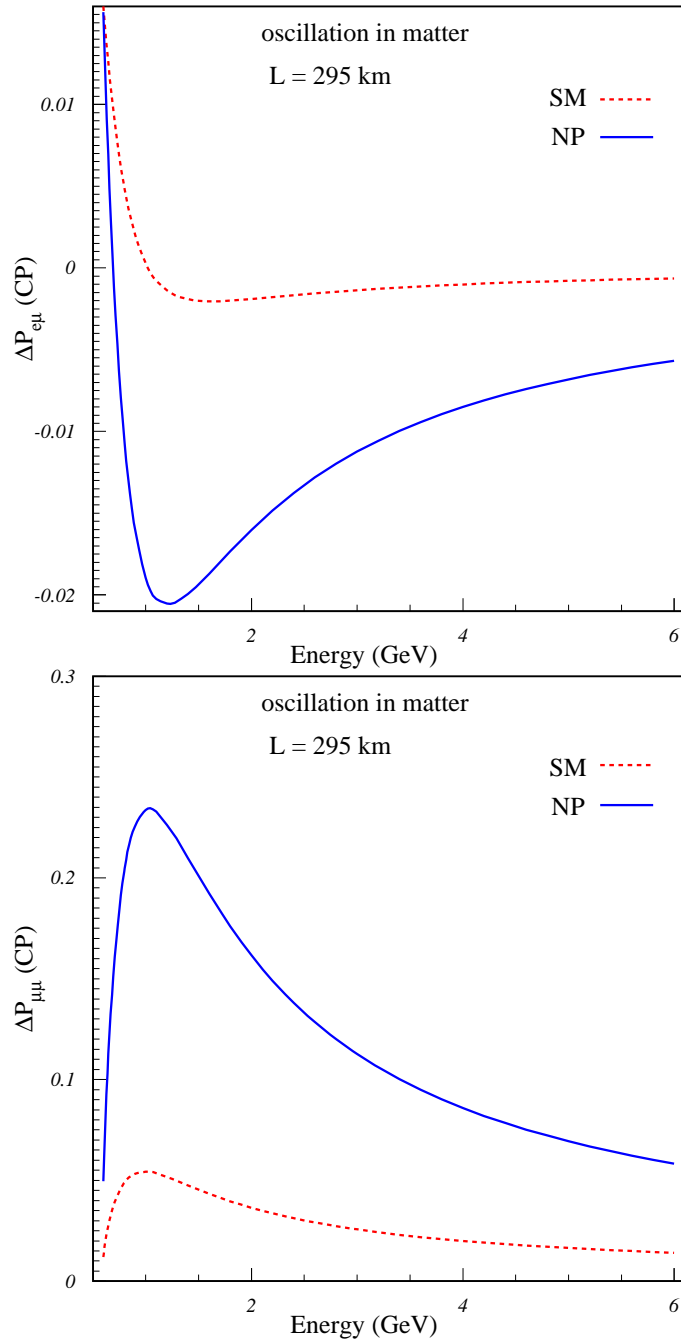


Figure 9.25. Change in oscillation probabilities $\Delta P_{e\mu} (CP) = P_{e\mu} - P_{\bar{e}\bar{\mu}}$ and $\Delta P_{\mu\mu} (CP) = P_{\mu\mu} - P_{\bar{\mu}\bar{\mu}}$ in matter as a function of energy for the same choice of input parameters as in Fig. 9.18.

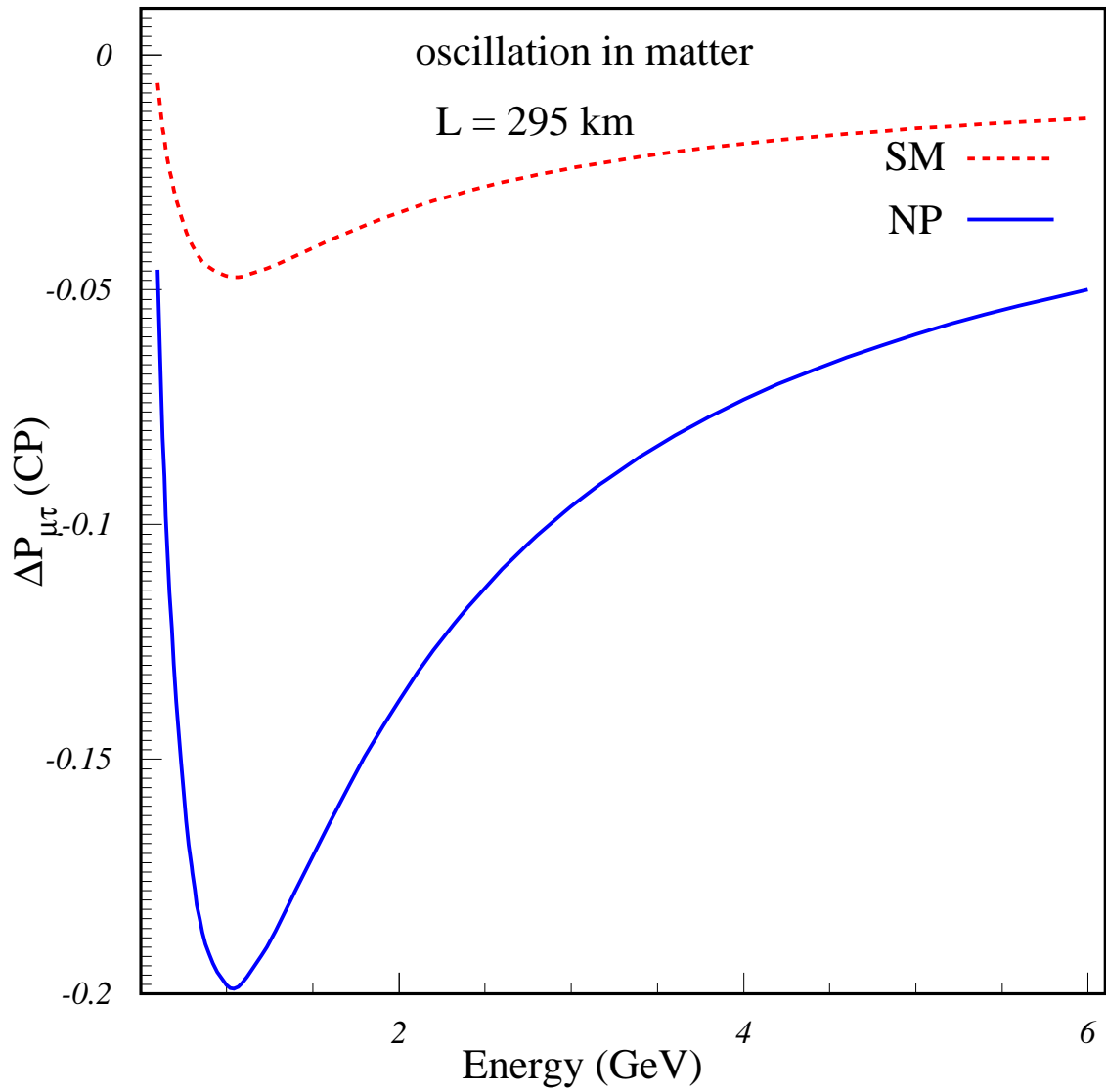


Figure 9.26. Change in oscillation probability $\Delta P_{\mu\tau} (CP) = P_{\mu\tau} - P_{\bar{\mu}\bar{\tau}}$ in matter as a function of energy for the same choice of input parameters as in Fig. 9.18.

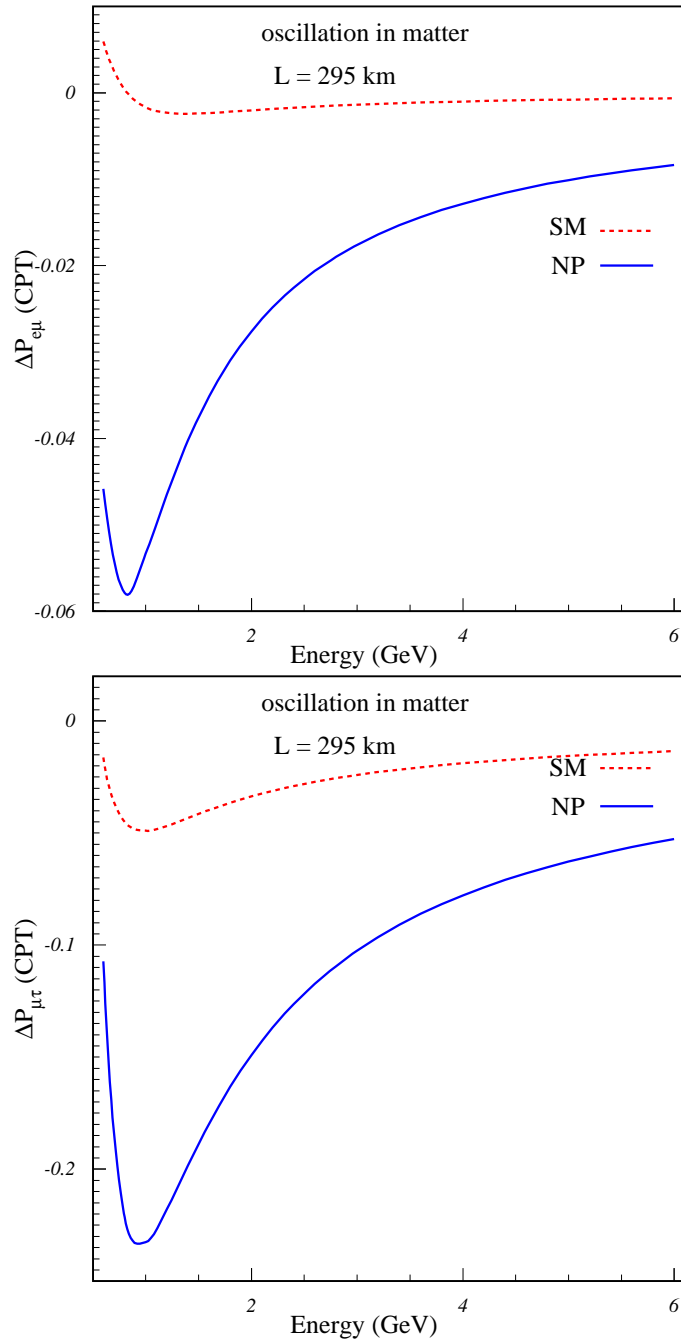


Figure 9.27. Apparent CPT violation parameters $\Delta P_{e\mu} (CPT) = P_{e\mu} - P_{\bar{\mu}e}$ and $\Delta P_{\mu\tau} (CPT) = P_{\mu\tau} - P_{\bar{\tau}\mu}$ in matter as a function of energy for the same choice of input parameters as in Fig. 9.18.

9.5 Summary

In this chapter we have presented a simple analysis on how new physics can affect neutrino oscillation data. These new physics effects can contribute to neutrino oscillations roughly up to 10 %. In the usual two generation vacuum oscillation, there is no CP violation but with new physics, one can have a CP asymmetry which is evident by merely taking the source eigenstate different from the detector eigenstate. There is no reason a priori to assume that the source eigenstate should be equal to the detection eigenstate.

In the three generation vacuum oscillations, we give explicit formulas for the probabilities and CP asymmetries. From the plots, we see that new the physics effects may be quite large. It is possible to be able to separate out these new physics effects from the usual standard CP effect. In the matter effect case, we only give the numerical plots of the oscillation probabilities, CP and CPT asymmetries. One will be able to separate out these new physics contributions from the matter effect contributions since we know the new physics effect from the vacuum oscillation case. We hope that in the near future experiments will be able to see these effects.

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APPENDICES

APPENDIX A

TeV scale Horizontal Symmetry

In this Appendix we give the one-loop anomalous dimension, beta-function and the soft masses for the TeV scale horizontal symmetry model.

A.1 Anomalous dimensions

The one loop anomalous dimensions for the fields in our model are:

$$16\pi^2\gamma_{L_\alpha} = Y_{E_\alpha}^2 - \frac{3}{10}g_1^2 - \frac{3}{2}g_2^2 - \frac{8}{3}g_4^2, \quad (\text{A.1})$$

$$16\pi^2\gamma_{e_\alpha^c} = 2Y_{E_\alpha}^2 - \frac{6}{5}g_1^2 - \frac{8}{3}g_4^2, \quad (\text{A.2})$$

$$16\pi^2\gamma_{Q_{ij}} = (Y_d Y_d^\dagger)_{ji} + (Y_u Y_u^\dagger)_{ji} - \delta_i^j \left(\frac{1}{30}g_1^2 + \frac{3}{2}g_2^2 + \frac{8}{3}g_3^2 \right), \quad (\text{A.3})$$

$$16\pi^2\gamma_{U_{ij}} = 2(Y_u^\dagger Y_u)_{ij} - \delta_i^j \left(\frac{8}{15}g_1^2 + \frac{8}{3}g_3^2 \right), \quad (\text{A.4})$$

$$16\pi^2\gamma_{D_{ij}} = 2(Y_d^\dagger Y_d)_{ij} - \delta_i^j \left(\frac{2}{15}g_1^2 + \frac{8}{3}g_3^2 \right), \quad (\text{A.5})$$

$$16\pi^2\gamma_{H_d} = 3Y_{d_3}^2 - \frac{3}{10}g_1^2 - \frac{3}{2}g_2^2, \quad (\text{A.6})$$

$$16\pi^2\gamma_{H_u} = 3Y_{u_3}^2 - \frac{3}{10}g_1^2 - \frac{3}{2}g_2^2, \quad (\text{A.7})$$

$$16\pi^2\gamma_{\phi_i} = 2\kappa^2 + 8\lambda^2 - \frac{8}{3}g_4^2, \quad (\text{A.8})$$

$$16\pi^2\gamma_\eta = 10\lambda^2 - \frac{8}{3}g_4^2, \quad (\text{A.9})$$

$$16\pi^2\gamma_{\bar{\eta}} = -\frac{8}{3}g_4^2. \quad (\text{A.10})$$

A.2 Beta functions

The beta functions for the Yukawa couplings appearing in the superpotential, Eq. (5.1), are:

$$\beta(Y_{d_3}) = \frac{Y_{d_3}}{16\pi^2} \left(6Y_{d_3}^2 + Y_{u_3}^2 - \frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 \right), \quad (\text{A.11})$$

$$\beta(Y_{u_3}) = \frac{Y_{u_3}}{16\pi^2} \left(6Y_{u_3}^2 + Y_{d_3}^2 - \frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 \right), \quad (\text{A.12})$$

$$\beta(Y_{E_\alpha}) = \frac{Y_{E_\alpha}}{16\pi^2} \left(4Y_{E_\alpha}^2 + 3Y_{d_3}^2 - \frac{9}{5}g_1^2 - 3g_2^2 \right), \quad (\text{A.13})$$

$$\beta(\lambda) = \frac{\lambda}{16\pi^2} (28\lambda^2 + 2\kappa^2 - 8g_4^2), \quad (\text{A.14})$$

$$\beta(\kappa) = \frac{3\kappa}{16\pi^2} \left(2\kappa^2 + 8\lambda^2 - \frac{8}{3}g_4^2 \right). \quad (\text{A.15})$$

The gauge beta function of our model are

$$\beta(g_i) = b_i \frac{g_i^3}{16\pi^2}, \quad (\text{A.16})$$

where $b_i = (\frac{33}{5}, 1, -3, -3)$ for $i = 1 - 4$.

A.3 A terms

The trilinear soft SUSY breaking terms are given by

$$A_Y = -\frac{\beta(Y)}{Y} M_{aux}, \quad (\text{A.17})$$

where $Y = (Y_{u_i}, Y_{d_i}, Y_{E_\alpha}, k, \lambda)$.

A.4 Gaugino masses

The soft masses of the gauginos are given by:

$$M_i = \frac{\beta(g_i)}{g_i} M_{aux}, \quad (\text{A.18})$$

where $i = 1, 2, 3, 4$, corresponding to the gauge groups $U(1)_Y$, $SU(2)_W$, $SU(3)_C$ and $SU(3)_H$, with $\beta(g_i)$ given as in Eq. (55).

A.5 Soft SUSY masses

The soft masses of the squarks and the sleptons are given in the text. For the H_u , H_d Φ_i , η_i , $\bar{\eta}$ fields they are:

$$(\tilde{m}_{soft}^2)_{H_u}^{H_u} = \frac{M_{aux}^2}{16\pi^2} \left(3Y_{u_3}\beta(Y_{u_3}) - \frac{3}{10}g_1\beta(g_1) - \frac{3}{2}g_2\beta(g_2) \right), \quad (\text{A.19})$$

$$(\tilde{m}_{soft}^2)_{H_d}^{H_d} = \frac{M_{aux}^2}{16\pi^2} \left(3Y_{d_3}\beta(Y_{d_3}) - \frac{3}{10}g_1\beta(g_1) - \frac{3}{2}g_2\beta(g_2) \right), \quad (\text{A.20})$$

$$(\tilde{m}_{soft}^2)_{\Phi_i}^{\Phi_i} = \frac{M_{aux}^2}{16\pi^2} \left(2\kappa\beta(\kappa) + 8\lambda\beta(\lambda) - \frac{8}{3}g_4\beta(g_4) \right), \quad (\text{A.21})$$

$$(\tilde{m}_{soft}^2)_{\eta}^{\eta} = \frac{M_{aux}^2}{16\pi^2} \left(10\lambda\beta(\lambda) - \frac{8}{3}g_4\beta(g_4) \right), \quad (\text{A.22})$$

$$(\tilde{m}_{soft}^2)_{\bar{\eta}}^{\bar{\eta}} = \frac{M_{aux}^2}{16\pi^2} \left(-\frac{8}{3}g_4\beta(g_4) \right). \quad (\text{A.23})$$

APPENDIX B

$SU(2)_H$ Symmetry

In this Appendix we give the one-loop anomalous dimension, beta-function and the soft SUSY breaking masses for the various fields in the $SU(2)_H$ symmetry model.

B.1 Anomalous dimensions

The one-loop anomalous dimensions for the fields in our model are:

$$16\pi^2\gamma_\psi = f_{e\mu}^2 - \left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2 + \frac{3}{2}g_4^2 \right), \quad (\text{B.1})$$

$$16\pi^2\gamma_{\psi^c} = 2f_{e\mu}^2 + f_{eE}^2 - \left(\frac{6}{5}g_1^2 + \frac{3}{2}g_4^2 \right), \quad (\text{B.2})$$

$$16\pi^2\gamma_{\psi_\tau} = f_\tau^2 + f_{\tau E}^2 - \left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2 \right), \quad (\text{B.3})$$

$$16\pi^2\gamma_{\tau^c} = 2f_\tau^2 - \frac{6}{5}g_1^2, \quad (\text{B.4})$$

$$16\pi^2\gamma_{Q_{ij}} = (Y_d Y_d^\dagger)_{ji} + (Y_u Y_u^\dagger)_{ji} - \delta_i^j \left(\frac{1}{30}g_1^2 + \frac{3}{2}g_2^2 + \frac{8}{3}g_3^2 \right), \quad (\text{B.5})$$

$$16\pi^2\gamma_{U_{ij}} = 2(Y_u^\dagger Y_u)_{ij} - \delta_i^j \left(\frac{8}{15}g_1^2 + \frac{8}{3}g_3^2 \right), \quad (\text{B.6})$$

$$16\pi^2\gamma_{D_{ij}} = 2(Y_d^\dagger Y_d)_{ij} - \delta_i^j \left(\frac{2}{15}g_1^2 + \frac{8}{3}g_3^2 \right), \quad (\text{B.7})$$

$$16\pi^2\gamma_{H_d} = 3Y_b^2 + 4f_{e\mu}^2 + f_{\tau E}^2 + f_\tau^2 - \frac{3}{10}g_1^2 - \frac{3}{2}g_2^2, \quad (\text{B.8})$$

$$16\pi^2\gamma_{H_u} = 3Y_t^2 - \frac{3}{10}g_1^2 - \frac{3}{2}g_2^2, \quad (\text{B.9})$$

$$16\pi^2\gamma_{\phi_d} = -\frac{3}{2}g_4^2, \quad (\text{B.10})$$

$$16\pi^2\gamma_{\phi_u} = f_{eE}^2 - \frac{3}{2}g_4^2, \quad (\text{B.11})$$

$$16\pi^2\gamma_E = 2f_{eE}^2 - \frac{6}{5}g_1^2, \quad (\text{B.12})$$

$$16\pi^2\gamma_{E^c} = 2f_{\tau E}^2 - \frac{6}{5}g_1^2. \quad (\text{B.13})$$

B.2 Beta functions

The beta functions for the Yukawa couplings appearing in the superpotential, Eq. (4), are:

$$\beta(Y_b) = \frac{Y_b}{16\pi^2} \left(6Y_b^2 + Y_t^2 + f_\tau^2 + f_{\tau E}^2 + 4f_{e\mu}^2 - \frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 \right), \quad (\text{B.14})$$

$$\beta(Y_t) = \frac{Y_t}{16\pi^2} \left(6Y_t^2 + Y_b^2 - \frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 \right), \quad (\text{B.15})$$

$$\beta(Y_\tau) = \frac{Y_\tau}{16\pi^2} \left(4Y_\tau^2 + 3Y_b^2 + 2f_{\tau E}^2 + 2f_{e\mu}^2 - \frac{9}{5}g_1^2 - 3g_2^2 \right), \quad (\text{B.16})$$

$$\beta(f_{eE}) = \frac{f_{eE}}{16\pi^2} \left(4f_{eE}^2 + 2f_{e\mu}^2 - \frac{12}{5}g_1^2 - 3g_2^2 \right), \quad (\text{B.17})$$

$$\beta(f_{\tau E}) = \frac{f_{\tau E}}{16\pi^2} \left(4f_{\tau E}^2 + 2f_\tau^2 + 4f_{e\mu}^2 + 3Y_b^2 - \frac{9}{5}g_1^2 - 3g_2^2 \right), \quad (\text{B.18})$$

$$\beta(f_{e\mu}) = \frac{f_{e\mu}}{16\pi^2} \left(7f_{e\mu}^2 + 2f_{\tau E}^2 + 2f_\tau^2 + 2f_{eE}^2 + 3Y_b^2 - \frac{9}{5}g_1^2 - 3g_2^2 - 3g_4^2 \right) \quad (\text{B.19})$$

The gauge beta function of the model are

$$\beta(g_i) = b_i \frac{g_i^3}{16\pi^2}, \quad (\text{B.20})$$

where $b_i = (\frac{39}{5}, 1, -3, -3)$ for $i = 1 - 4$ with g_4 being the gauge coupling associated with the $SU(2)_H$ gauge group.

B.3 A terms

The trilinear soft SUSY breaking terms are given by

$$A_Y = -\frac{\beta(Y)}{Y} M_{aux}, \quad (\text{B.21})$$

where $Y = (Y_{u_i}, Y_{d_i}, Y_{l_i}, f_{eE}, f_{\tau E}, f_\tau)$.

B.4 Gaugino masses

The soft masses of the gauginos are given by:

$$M_i = \frac{\beta(g_i)}{g_i} M_{aux}, \quad (\text{B.22})$$

where $i = 1, 2, 3, 4$, corresponding to the gauge groups $U(1)_Y$, $SU(2)_W$, $SU(3)_C$, $SU(2)_H$ with $\beta(g_i)$ given as in Eq. (B.20).

B.5 Soft SUSY masses

The soft masses of the squarks and the sleptons are given in the text. For the H_u , H_d , ν^c , S_+ , S_- fields they are:

$$(\tilde{m}_{soft}^2)_{H_u}^{H_u} = \frac{M_{aux}^2}{16\pi^2} \left(3Y_t\beta(Y_t) - \frac{3}{10}g_1\beta(g_1) - \frac{3}{2}g_2\beta(g_2) - 2\left(\frac{x}{2}\right)^2 g_4\beta(g_4) \right), \quad (\text{B.23})$$

$$\begin{aligned} (\tilde{m}_{soft}^2)_{H_d}^{H_d} &= \frac{M_{aux}^2}{16\pi^2} \left(3Y_b\beta(Y_b) + Y_\tau\beta(Y_\tau) + Y_{\tau E}\beta(Y_{\tau E}) - \frac{3}{10}g_1\beta(g_1) - \frac{3}{2}g_2\beta(g_2) \right. \\ &\quad \left. - 2\left(-\frac{x}{2}\right)^2 g_4\beta(g_4) \right), \end{aligned} \quad (\text{B.24})$$

$$(\tilde{m}_{soft}^2)_{\phi_u}^{\phi_u} = \frac{M_{aux}^2}{16\pi^2} \left(f_{eE}\beta(f_{eE}) - \frac{3}{2}g_4\beta(g_4) \right), \quad (\text{B.25})$$

$$(\tilde{m}_{soft}^2)_{\phi_d}^{\phi_d} = \frac{M_{aux}^2}{16\pi^2} \left(-\frac{3}{2}g_4\beta(g_4) \right). \quad (\text{B.26})$$

APPENDIX C

$U(1)_x$ Model

In this Appendix we give the one-loop anomalous dimension, beta-function and the soft SUSY breaking masses for the various fields in Z' model.

C.1 Anomalous dimensions

The one-loop anomalous dimensions for the fields in our model are:

$$16\pi^2\gamma_{L_{ij}} = (Y_l Y_l^\dagger)_{ji} - \delta_i^j \left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2 + 2\left(1 - \frac{x}{2}\right)^2 g_x^2 \right), \quad (\text{C.1})$$

$$16\pi^2\gamma_{e_{ij}^c} = 2(Y_l^\dagger Y_l)_{ij} - \delta_i^j \left(\frac{6}{5}g_1^2 + 2(-1+x)^2 g_x^2 \right), \quad (\text{C.2})$$

$$16\pi^2\gamma_{Q_{ij}} = (Y_d Y_d^\dagger)_{ji} + (Y_u Y_u^\dagger)_{ji} - \delta_i^j \left(\frac{1}{30}g_1^2 + \frac{3}{2}g_2^2 + \frac{8}{3}g_3^2 + 2\left(\frac{x}{6} - \frac{1}{3}\right)^2 g_x^2 \right) \quad (\text{C.3})$$

$$16\pi^2\gamma_{U_{ij}} = 2(Y_u^\dagger Y_u)_{ij} - \delta_i^j \left(\frac{8}{15}g_1^2 + \frac{8}{3}g_3^2 + 2\left(\frac{2}{3}x + \frac{1}{3}\right)^2 g_x^2 \right), \quad (\text{C.4})$$

$$16\pi^2\gamma_{D_{ij}} = 2(Y_d^\dagger Y_d)_{ij} - \delta_i^j \left(\frac{2}{15}g_1^2 + \frac{8}{3}g_3^2 + 2\left(\frac{x}{3} + \frac{1}{3}\right)^2 g_x^2 \right), \quad (\text{C.5})$$

$$16\pi^2\gamma_{H_d} = 3Y_{d_3}^2 + Y_{l_3}^2 - \frac{3}{10}g_1^2 - \frac{3}{2}g_2^2 - 2\left(-\frac{x}{2}\right)^2 g_x^2, \quad (\text{C.6})$$

$$16\pi^2\gamma_{H_u} = 3Y_{u_3}^2 - \frac{3}{10}g_1^2 - \frac{3}{2}g_2^2 - 2\left(-\frac{x}{2}\right)^2 g_x^2, \quad (\text{C.7})$$

$$16\pi^2\gamma_{\nu_i^c} = 4f_{\nu_i^c}^2 - 2g_x^2, \quad (\text{C.8})$$

$$16\pi^2\gamma_{\nu^c} = 4f_{\nu^c}^2 - 2g_x^2, \quad (\text{C.9})$$

$$16\pi^2\gamma_{\bar{\nu}^c} = 4h^2 - 2g_x^2, \quad (\text{C.10})$$

$$16\pi^2\gamma_{S_+} = 2\sum_{i=1}^3 f_{\nu_i^c}^2 + 2f_{\nu^c}^2 - 8g_x^2, \quad (\text{C.11})$$

$$16\pi^2\gamma_{S_-} = 2h^2 - 8g_x^2. \quad (\text{C.12})$$

C.2 Beta functions

The beta functions for the Yukawa couplings appearing in the superpotential, Eq. (4), are:

$$\beta(Y_{d_3}) = \frac{Y_{d_3}}{16\pi^2} \left(6Y_{d_3}^2 + Y_{u_3}^2 + Y_{l_3}^2 - \frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 - \frac{(4+2x+7x^2)}{9}g_x^2 \right), \quad (C.13)$$

$$\beta(Y_{u_3}) = \frac{Y_{u_3}}{16\pi^2} \left(6Y_{u_3}^2 + Y_{d_3}^2 - \frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 - \frac{(4-10x+13x^2)}{9}g_x^2 \right), \quad (C.14)$$

$$\beta(Y_{l_3}) = \frac{Y_{l_3}}{16\pi^2} \left(4Y_{l_3}^2 + 3Y_{d_3}^2 - \frac{9}{5}g_1^2 - 3g_2^2 - (4-6x+3x^2)g_x^2 \right), \quad (C.15)$$

$$\beta(f_{\nu_e}) = \frac{f_{\nu_e}}{16\pi^2} \left(10f_{\nu_e}^2 + 2f_{\nu_\mu}^2 + 2f_{\nu_\tau}^2 + 2f_{\nu^c}^2 - 12g_x^2 \right), \quad (C.16)$$

$$\beta(f_{\nu_\mu}) = \frac{f_{\nu_\mu}}{16\pi^2} \left(10f_{\nu_\mu}^2 + 2f_{\nu_e}^2 + 2f_{\nu_\tau}^2 + 2f_{\nu^c}^2 - 12g_x^2 \right), \quad (C.17)$$

$$\beta(f_{\nu_\tau}) = \frac{f_{\nu_\tau}}{16\pi^2} \left(10f_{\nu_\tau}^2 + 2f_{\nu_\mu}^2 + 2f_{\nu_e}^2 + 2f_{\nu^c}^2 - 12g_x^2 \right), \quad (C.18)$$

$$\beta(f_{\nu^c}) = \frac{f_{\nu^c}}{16\pi^2} \left(10f_{\nu^c}^2 + 2f_{\nu_\mu}^2 + 2f_{\nu_\tau}^2 + 2f_{\nu_e}^2 - 12g_x^2 \right), \quad (C.19)$$

$$\beta(h) = \frac{h}{16\pi^2} (10h - 12g_x^2). \quad (C.20)$$

The gauge beta function of our model are

$$\beta(g_i) = b_i \frac{g_i^3}{16\pi^2}, \quad (C.21)$$

where $b_i = (\frac{33}{5}, 1, -3, (11x^2 - 16x + 26))$ for $i = 1, 2, 2, 3, x$.

C.3 A terms

The trilinear soft SUSY breaking terms are given by

$$A_Y = -\frac{\beta(Y)}{Y} M_{aux}, \quad (C.22)$$

where $Y = (Y_{u_i}, Y_{d_i}, Y_{l_i}, f_{\nu_i^c}, f_{\nu^c}, h)$.

C.4 Gaugino masses

The soft masses of the gauginos are given by:

$$M_i = \frac{\beta(g_i)}{g_i} M_{aux}, \quad (C.23)$$

where $i = 1, 2, 3, x$, corresponding to the gauge groups $U(1)_Y$, $SU(2)_W$, $SU(3)_C$, $U(1)_x$ with $\beta(g_i)$ given as in Eq. (C.21) with $M_x = M'_1$.

C.5 Soft SUSY masses

The soft masses of the squarks and the sleptons are given in the text. For the H_u , H_d , ν^c , S_+ , S_- fields they are:

$$\begin{aligned} (\tilde{m}_{soft}^2)_{H_u} &= \frac{M_{aux}^2}{16\pi^2} \left(3Y_{u_3}\beta(Y_{u_3}) - \frac{3}{10}g_1\beta(g_1) - \frac{3}{2}g_2\beta(g_2) - 2\left(\frac{x}{2}\right)^2 g_x\beta(g_x) \right), \quad (C.24) \\ (\tilde{m}_{soft}^2)_{H_d} &= \frac{M_{aux}^2}{16\pi^2} \left(3Y_{d_3}\beta(Y_{d_3}) + Y_{l_3}\beta(Y_{l_3}) - \frac{3}{10}g_1\beta(g_1) - \frac{3}{2}g_2\beta(g_2) \right. \\ &\quad \left. - 2\left(-\frac{x}{2}\right)^2 g_x\beta(g_x) \right), \quad (C.25) \end{aligned}$$

$$(\tilde{m}_{soft}^2)_{S_+} = \frac{M_{aux}^2}{16\pi^2} \left(2 \sum_{i=1}^3 f_{\nu_i^c} \beta(f_{\nu_i^c}) + 2f_{\nu^c} \beta(f_{\nu^c}) - 8g_x \beta(g_x) \right), \quad (C.26)$$

$$(\tilde{m}_{soft}^2)_{S_-} = \frac{M_{aux}^2}{16\pi^2} (2h\beta(h) - 8g_x\beta(g_x)), \quad (C.27)$$

$$(\tilde{m}_{soft}^2)_{\nu_i^c} = \frac{M_{aux}^2}{16\pi^2} (4f_{\nu_i^c} \beta(f_{\nu_i^c}) - 2g_x \beta(g_x)), \quad (C.28)$$

$$(\tilde{m}_{soft}^2)_{\nu^c} = \frac{M_{aux}^2}{16\pi^2} (4f_{\nu^c} \beta(f_{\nu^c}) - 2g_x \beta(g_x)), \quad (C.29)$$

$$(\tilde{m}_{soft}^2)_{\bar{\nu}^c} = \frac{M_{aux}^2}{16\pi^2} (4h\beta(h) - 2g_x\beta(g_x)). \quad (C.30)$$

APPENDIX D

Quark-Lepton Supersymmetric Model

In this Appendix we give the one-loop anomalous dimension, beta-function and the soft masses for the Quark-Lepton Supersymmetry model.

D.1 Anomalous dimensions

The one loop anomalous dimensions for the fields in our model are:

$$16\pi^2\gamma_Q = Y_d Y_d^\dagger + Y_u Y_u^\dagger + 2Y_Q Y_Q^\dagger - \left(\frac{1}{18}g_x^2 + \frac{3}{2}g_2^2 + \frac{8}{3}g_3^2 \right), \quad (\text{D.1})$$

$$16\pi^2\gamma_{u^c} = 2Y_u^\dagger Y_u + 2Y_{Q'}^\dagger Y_{Q'} - \left(\frac{8}{9}g_x^2 + \frac{8}{3}g_3^2 \right), \quad (\text{D.2})$$

$$16\pi^2\gamma_d = 2Y_d^\dagger Y_d + 2Y_{Q'}^\dagger Y_{Q'} - \left(\frac{2}{9}g_x^2 + \frac{8}{3}g_3^2 \right), \quad (\text{D.3})$$

$$16\pi^2\gamma_F = Y_e Y_e^\dagger + Y_\nu Y_\nu^\dagger + 2Y_F Y_F^\dagger - \left(\frac{1}{18}g_x^2 + \frac{3}{2}g_2^2 + \frac{8}{3}g_\ell^2 \right), \quad (\text{D.4})$$

$$16\pi^2\gamma_E = 2Y_e^\dagger Y_e + 2Y_N^\dagger Y_N - \left(\frac{8}{9}g_x^2 + \frac{8}{3}g_\ell^2 \right), \quad (\text{D.5})$$

$$16\pi^2\gamma_N = 2Y_\nu^\dagger Y_\nu + 2Y_N^\dagger Y_N - \left(\frac{2}{9}g_x^2 + \frac{8}{3}g_\ell^2 \right), \quad (\text{D.6})$$

$$16\pi^2\gamma_{H_d} = 3Tr(Y_d^\dagger Y_d) + 3Tr(Y_e^\dagger Y_e) - \frac{3}{10}g_x^2 - \frac{3}{2}g_2^2, \quad (\text{D.7})$$

$$16\pi^2\gamma_{H_u} = 3Tr(Y_u^\dagger Y_u) + 3Tr(Y_\nu^\dagger Y_\nu) - \frac{3}{10}g_x^2 - \frac{3}{2}g_2^2, \quad (\text{D.8})$$

$$16\pi^2\gamma_{\chi_1} = 4Tr(Y_F Y_F^\dagger) - \frac{2}{9}g_x^2 - \frac{8}{3}g_\ell^2, \quad (\text{D.9})$$

$$16\pi^2\gamma_{\bar{\chi}_1} = 2Tr(Y_N Y_N^\dagger) - \frac{2}{9}g_x^2 - \frac{8}{3}g_\ell^2, \quad (\text{D.10})$$

$$16\pi^2\gamma_{\chi_2} = 4Tr(Y_Q Y_Q^\dagger) - \frac{2}{9}g_x^2 - \frac{8}{3}g_3^2, \quad (\text{D.11})$$

$$16\pi^2\gamma_{\bar{\chi}_2} = 2Tr(Y_{Q'} Y_{Q'}^\dagger) - \frac{2}{9}g_x^2 - \frac{8}{3}g_3^2. \quad (\text{D.12})$$

D.2 Beta functions

The beta functions for the Yukawa couplings appearing in the superpotential, Eq. (8.3), are:

$$\beta(Y_d) = \frac{Y_d}{16\pi^2} \left(3Y_d Y_d^\dagger + Y_u Y_u^\dagger + 2Y_Q Y_Q^\dagger + 2Y_{Q'}^\dagger Y_{Q'} - \frac{7}{9}g_x^2 - 3g_2^2 - \frac{16}{3}g_3^2 \right) \quad (\text{D.13})$$

$$\beta(Y_u) = \frac{Y_u}{16\pi^2} \left(3Y_u Y_u^\dagger + Y_d Y_d^\dagger + 2Y_Q Y_Q^\dagger + 2Y_{Q'}^\dagger Y_{Q'} - \frac{13}{9}g_x^2 - 3g_2^2 - \frac{16}{3}g_3^2 \right) \quad (\text{D.14})$$

$$\beta(Y_e) = \frac{Y_e}{16\pi^2} \left(3Y_e Y_e^\dagger + Y_\nu Y_\nu^\dagger + 2Y_F Y_F^\dagger + 2Y_N^\dagger Y_N - \frac{13}{9}g_x^2 - 3g_2^2 - \frac{16}{3}g_\ell^2 \right) \quad (\text{D.15})$$

$$\beta(Y_\nu) = \frac{Y_\nu}{16\pi^2} \left(3Y_\nu Y_\nu^\dagger + Y_e Y_e^\dagger + 2Y_F Y_F^\dagger + 2Y_N^\dagger Y_N - \frac{7}{9}g_x^2 - 3g_2^2 - \frac{16}{3}g_\ell^2 \right), \quad (\text{D.16})$$

$$\beta(Y_F) = \frac{Y_F}{16\pi^2} \left(2Y_e Y_e^\dagger + 2Y_\nu Y_\nu^\dagger + 4Y_F Y_F^\dagger + 4\text{Tr}(Y_F Y_F^\dagger) - \frac{1}{3}g_x^2 - 2g_2^2 - 8g_\ell^2 \right), \quad (\text{D.17})$$

$$\beta(Y_N) = \frac{Y_N}{16\pi^2} \left(4Y_N Y_N^\dagger + 2\text{Tr}(Y_N Y_N^\dagger) + 2Y_e Y_e^\dagger + 2Y_\nu Y_\nu^\dagger - \frac{4}{3}g_x^2 - 8g_\ell^2 \right), \quad (\text{D.18})$$

$$\beta(Y_Q) = \frac{Y_Q}{16\pi^2} \left(2Y_u Y_u^\dagger + 2Y_d Y_d^\dagger + 4Y_Q Y_Q^\dagger + 4\text{Tr}(Y_Q Y_Q^\dagger) - \frac{1}{3}g_x^2 - 2g_2^2 - 8g_3^2 \right), \quad (\text{D.19})$$

$$\beta(Y_{Q'}) = \frac{Y_{Q'}}{16\pi^2} \left(4Y_{Q'} Y_{Q'}^\dagger + 2\text{Tr}(Y_{Q'} Y_{Q'}^\dagger) + 2Y_u Y_u^\dagger + 2Y_d Y_d^\dagger - \frac{4}{3}g_x^2 - 8g_3^2 \right). \quad (\text{D.20})$$

The gauge beta function of our model are

$$\beta(g_i) = b_i \frac{g_i^3}{16\pi^2}, \quad (\text{D.21})$$

where $b_i = (\frac{40}{3}, 4, -2, -2)$ for $i = x, 2, 3, \ell$.

D.3 A terms

The trilinear soft SUSY breaking terms are given by

$$A_Y = -\frac{\beta(Y)}{Y} M_{aux}, \quad (\text{D.22})$$

where $Y = (Y_u, Y_d, Y_e, Y_N, Y_\nu, Y_F, Y_Q, Y_{Q'})$.

D.4 Gaugino masses

The soft masses of the gauginos are given by:

$$M_i = \frac{\beta(g_i)}{g_i} M_{aux}, \quad (\text{D.23})$$

where $i = x, 2, 3, \ell$, corresponding to the gauge groups $U(1)_x$, $SU(2)_L$, $SU(3)_q$ and $SU(3)_\ell$.

D.5 Soft SUSY masses

The soft masses of the squarks and the sleptons are given in the text. For the H_u , H_d , χ_1 , $\bar{\chi}_1$, χ_2 , $\bar{\chi}_2$, F , E^c , N^c fields are:

$$(\tilde{m}_{soft}^2)_{H_u}^{H_u} = \frac{M_{aux}^2}{16\pi^2} \left[3Tr(Y_u\beta(Y_u) + Y_\nu\beta(Y_\nu)) - \frac{1}{2}g_x\beta(g_x) - \frac{3}{2}g_2\beta(g_2) \right], \quad (\text{D.24})$$

$$(\tilde{m}_{soft}^2)_{H_d}^{H_d} = \frac{M_{aux}^2}{16\pi^2} \left[3Tr(Y_d\beta(Y_d) + Y_e\beta(Y_e)) - \frac{1}{2}g_x\beta(g_x) - \frac{3}{2}g_2\beta(g_2) \right], \quad (\text{D.25})$$

$$(\tilde{m}_{soft}^2)_{\chi_1}^{\chi_1} = \frac{M_{aux}^2}{16\pi^2} \left[4Tr(Y_F\beta(Y_F)) - \frac{2}{9}g_x\beta(g_x) - \frac{8}{3}g_\ell\beta(g_\ell) \right], \quad (\text{D.26})$$

$$(\tilde{m}_{soft}^2)_{\bar{\chi}_1}^{\bar{\chi}_1} = \frac{M_{aux}^2}{16\pi^2} \left[2Tr(Y_N\beta(Y_N)) - \frac{2}{9}g_x\beta(g_x) - \frac{8}{3}g_\ell\beta(g_\ell) \right], \quad (\text{D.27})$$

$$(\tilde{m}_{soft}^2)_{\chi_2}^{\chi_2} = \frac{M_{aux}^2}{16\pi^2} \left[4Tr(Y_Q\beta(Y_Q)) - \frac{2}{9}g_x\beta(g_x) - \frac{8}{3}g_3\beta(g_3) \right], \quad (\text{D.28})$$

$$(\tilde{m}_{soft}^2)_{\bar{\chi}_2}^{\bar{\chi}_2} = \frac{M_{aux}^2}{16\pi^2} \left[2Tr(Y'_Q\beta(Y'_Q)) - \frac{2}{9}g_x\beta(g_x) - \frac{8}{3}g_3\beta(g_3) \right], \quad (\text{D.29})$$

$$(\tilde{m}_{soft}^2)_{E^c}^{E^c} = \frac{M_{aux}^2}{(16\pi^2)} \left[2Y_e\beta(Y_e) + 2Y_N\beta(Y_N) - \left(\frac{8}{9}g_x\beta(g_x) + \frac{8}{3}g_\ell\beta(g_\ell) \right) \right], \quad (\text{D.30})$$

$$(\tilde{m}_{soft}^2)_{N^c}^{N^c} = \frac{M_{aux}^2}{(16\pi^2)} \left[2Y_\nu\beta(Y_\nu) + 2Y_N\beta(Y_N) - \left(\frac{2}{9}g_x\beta(g_x) + \frac{8}{3}g_\ell\beta(g_\ell) \right) \right], \quad (\text{D.31})$$

$$\begin{aligned} (\tilde{m}_{soft}^2)_F^F &= \frac{M_{aux}^2}{(16\pi^2)} [Y_e\beta(Y_e) + Y_\nu\beta(Y_\nu) + 2Y_F\beta(Y_F) \\ &\quad - \left(\frac{3}{2}g_2\beta(g_2) + \frac{1}{18}g_x\beta(g_x) + \frac{8}{3}g_\ell\beta(g_\ell) \right)]. \end{aligned} \quad (\text{D.32})$$

APPENDIX E

Two Generation Neutrino Oscillation Model

Consider the production by β -decay with new physics interaction and detection by leptonic interaction with no new physics. The production Lagrangian can be expressed as

$$L^{prod} = \frac{G_F V_{us}}{\sqrt{2}} \bar{u}_L \gamma_\mu (1 - \gamma_5) d \bar{e} \gamma^\mu (1 - \gamma_5) \begin{pmatrix} 1 + \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} + H.C, \quad (E.1)$$

where ϵ 's are the new physics parameters.

For simplicity we consider the two generation case. We can write the production Lagrangian as

$$L^{prod} = \frac{G_F V_{us}}{\sqrt{2}} \bar{u} \gamma_\mu (1 - \gamma_5) d \bar{e} \gamma^\mu (1 - \gamma_5) e^{i\gamma} \begin{pmatrix} 1 + \epsilon_{11} & \epsilon_{12} \end{pmatrix} \\ \times \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta e^{-2i\alpha} & \cos \theta e^{-2i\alpha} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} + H.C.$$

The charged current part of the detection Lagrangian is given by

$$L_{det}^{cc} = \frac{G_F}{\sqrt{2}} \bar{\mu} \gamma_\mu (1 - \gamma_5) \nu_\mu \bar{\nu}_e \gamma^\mu (1 - \gamma_5) e, \quad (E.2)$$

$$= \frac{G_F}{\sqrt{2}} \bar{\mu} \gamma_\mu (1 - \gamma_5) \underbrace{(-\sin \theta \nu_1 + \cos \theta \nu_2)}_{observed} \underbrace{(\cos \theta \bar{\nu}_1 + \sin \theta \bar{\nu}_2)}_{unobserved} \gamma^\mu (1 - \gamma_5) e \quad (E.3)$$

The probability of $\nu_e \rightarrow \nu_\mu$ is given by

$$P_{\nu_e \rightarrow \nu_\mu} = |A(\nu_e \rightarrow \nu_\mu)|^2 \\ = \left| \frac{-\cos \theta \sin \theta (1 + \epsilon_{11}) + \sin^2 \theta \epsilon_{12} e^{-2i\alpha}}{\sqrt{|1 + \epsilon_{11}|^2 + |\epsilon_{12}|^2}} \right|^2 \\ + \left| \frac{\cos \theta \sin \theta (1 + \epsilon_{11}) + \cos^2 \theta \epsilon_{12} e^{-2i\alpha}}{\sqrt{|1 + \epsilon_{11}|^2 + |\epsilon_{12}|^2}} \right|^2$$

$$\begin{aligned}
& + e^{i(E_1-E_2)t} \left[\frac{-\cos\theta \sin\theta(1+\epsilon_{11}) + \sin^2\theta\epsilon_{12}e^{-2i\alpha}}{\sqrt{|1+\epsilon_{11}|^2 + |\epsilon_{12}|^2}} \right] \\
& \times \left[\frac{\cos\theta \sin\theta(1+\epsilon_{11}^*) + \cos^2\theta\epsilon_{12}^*e^{2i\alpha}}{\sqrt{|1+\epsilon_{11}|^2 + |\epsilon_{12}|^2}} \right] \\
& + e^{i(E_2-E_1)t} \left[\frac{-\cos\theta \sin\theta(1+\epsilon_{11}^*) + \sin^2\theta\epsilon_{12}^*e^{2i\alpha}}{\sqrt{|1+\epsilon_{11}|^2 + |\epsilon_{12}|^2}} \right] \\
& \times \left[\frac{\cos\theta \sin\theta(1+\epsilon_{11}) + \cos^2\theta\epsilon_{12}e^{-2i\alpha}}{\sqrt{|1+\epsilon_{11}|^2 + |\epsilon_{12}|^2}} \right], \\
& \equiv \sin^2\theta_d \cos^2\theta_s + \cos^2\theta_d \sin^2\theta_s \\
& - 2\sin\theta_s \sin\theta_d \cos\theta_s \cos\theta_d \cos \left[\frac{\Delta m_{12}^2 t}{2E} + \beta \right] \tag{E.4}
\end{aligned}$$

where

$$\begin{aligned}
\beta & = \arg([\cos\theta_s(1+\epsilon_{11}) - \sin\theta_s\epsilon_{12}e^{-2i\alpha}][\sin\theta_s(1+\epsilon_{11}^*) + \cos\theta_s\epsilon_{12}^*e^{2i\alpha}]), \\
& \simeq 2\frac{|\epsilon_{12}|\sin\varphi}{\sin 2\theta_d}, \tag{E.5}
\end{aligned}$$

$$\cos\theta_s = \left| \frac{\cos\theta(1+\epsilon_{11}) - \sin\theta\epsilon_{12}e^{-2i\alpha}}{\sqrt{|1+\epsilon_{11}|^2 + |\epsilon_{12}|^2}} \right| \simeq \cos\theta_d - \sin\theta_d|\epsilon_{12}|\cos\varphi, \tag{E.6}$$

$$\sin\theta_s = \left| \frac{\sin\theta(1+\epsilon_{11}) + \cos\theta\epsilon_{12}e^{-2i\alpha}}{\sqrt{|1+\epsilon_{11}|^2 + |\epsilon_{12}|^2}} \right| \simeq \sin\theta_d + \cos\theta_d|\epsilon_{12}|\cos\varphi. \tag{E.7}$$

Where we have use the parametrization

$$\epsilon_{12} = |\epsilon_{12}|e^{i\phi_{12}} \text{ and } 1 + \epsilon_{11} = |1 + \epsilon_{11}|e^{i\phi_{11}}, \quad \varphi = 2\alpha - \phi_{12} + \phi_{11}. \tag{E.8}$$

We find the probabilities to be:

$$\begin{aligned}
P_{\nu_e \rightarrow \nu_\mu} & \simeq \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 t}{4E} \right) \\
& + |\epsilon_{12}| \sin 2\theta \left[2 \cos 2\theta \cos(\varphi) \sin^2 \left(\frac{\Delta m^2 t}{4E} \right) + \sin \varphi \sin \left(\frac{\Delta m^2 t}{2E} \right) \right], \tag{E.9}
\end{aligned}$$

$$\begin{aligned}
P_{\bar{\nu}_e \rightarrow \bar{\nu}_\mu} & \simeq \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 t}{4E} \right) \\
& + |\epsilon_{12}| \sin 2\theta \left[2 \cos 2\theta \cos \varphi \sin^2 \left(\frac{\Delta m^2 t}{4E} \right) - \sin \varphi \sin \left(\frac{\Delta m^2 t}{2E} \right) \right]. \tag{E.10}
\end{aligned}$$

The CP asymmetry (\mathcal{A}) of $\nu_e \rightarrow \nu_\mu$ defined as:

$$\mathcal{A}(\nu_e \rightarrow \nu_\mu) = \frac{P_{\nu_e \rightarrow \nu_\mu} - P_{\bar{\nu}_e \rightarrow \bar{\nu}_\mu}}{P_{\nu_e \rightarrow \nu_\mu} + P_{\bar{\nu}_e \rightarrow \bar{\nu}_\mu}}, \tag{E.11}$$

$$\simeq \frac{2|\epsilon_{12}| \sin 2\theta \sin \varphi \sin \left(\frac{\Delta m^2 t}{2E} \right)}{2 \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 t}{4E} \right)}, \quad (\text{E.12})$$

$$\simeq 2|\epsilon_{12}| \sin \varphi \left[\frac{\sqrt{1 - \frac{\rho}{\sin^2 2\theta}}}{\sqrt{\rho}} \right], \quad (\text{E.13})$$

where

$$\rho = \sin^2 2\theta \sin^2 \left[\frac{\Delta m^2 t}{4E} \right]. \quad (\text{E.14})$$

We observed that because of the introduction of new physics, $P_{\nu_e \rightarrow \nu_\mu} \neq P_{\bar{\nu}_e \rightarrow \bar{\nu}_\mu}$ and hence there will be an observable CP asymmetry as shown above. This Asymmetry is due to the fact that we assumed source \neq detector. This new physics effect can contribute up to 10% deviation from Standard Model and hence can give a large CP violation.

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Scope and Method of Study: In this thesis, we propose four different scenarios that solves the tachyonic slepton mass problem of Anomaly Mediated Supersymmetry Breaking (AMSB). We also address the question of neutrino oscillation using non standard interactions. In the first two chapters we introduce the Standard Model (SM) of particle physics and Supersymmetry (SUSY). We review models of SUSY breaking in the third chapter. Chapters four, five, six and seven have our various models that address the negative slepton mass problem of AMSB. In chapter 8, we propose a simple solution to the neutrino oscillation problem based on nonstandard interactions.

Findings and Conclusions: AMSB is an attractive scenario which can neatly solve the flavor changing neutral current problem of SUSY models. However, the simplest such model has tachyonic sleptons, which is unacceptable. The first model we propose is based on a non-Abelian horizontal gauge symmetry broken at the TeV scale. In this model the sleptons receive positive mass-squared from the asymptotically free $SU(3)_H$ gauge sector. The second model is a class of supersymmetric Z' models based on the gauge symmetry $U(1)_x = xY - (B - L)$, where Y is the Standard Model hypercharge. For $1 < x < 2$, the $U(1)_x$ D -term generates positive contribution to the slepton masses. The third model is the quark-lepton symmetric model based on leptonic $SU(3)_\ell$ gauge symmetry. The negative slepton mass problem is cured by virtue of the positive contribution to the slepton masses from the $SU(3)_\ell$ gauge sector. This model also leads to unification of Standard Model gauge couplings in a non trivial way. The fourth model is based on an asymptotically free $SU(2)_H$ gauge symmetry broken at the TeV scale. This model is viable and also solves the tachyonic slepton mass problem of AMSB. Finally in chapter 8, we show how the Liquid Scintillator Neutrino Detector (LSND) experiment puzzle may be solved by adding new physics terms to the standard interactions.

ADVISOR'S APPROVAL:
